Long-Term Asset Allocation

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Chapter 1

Introduction

Individual as well as institutional investors face the decision of how to allocate assets in their portfolios. In general, it is distinguished between strategic and tactical asset allocation. While strategic asset allocation concentrates on the allocation and diversification of a portfolio among major asset classes such as cash, bonds, stocks and real estate, tactical asset allocation is a dynamic adjustment of the strategic asset allocation weights with the intent to add value due to a changing fundamental market environment. This thesis focuses on strategic asset allocation and determines optimal portfolios for investors with specific objectives (inflation-hedging) and/or with a focus on long-term investments.

The era of modern portfolio theory starts with the seminal work of Markowitz (1952), which introduces the famous mean-variance optimization framework, an approach that determines optimal portfolios which exhibit minimum risk measured by the variance for a given required return. Although Markowitz optimization is still widely used in academia and in practice, it has the limitation that it is static, i.e. only a one-period investment horizon is considered, which is unrealistic for long-term investors. Due to the fact that the assets' term structure of risk and asset correlations vary substantially over the investment horizon, optimal horizon-dependent asset allocations are also time-varying (Campbell and Viceira, 2005). To take into account this time variation in the term structure of risk, the asset allocation literature is currently focusing on more sophisticated portfolio approaches. The complex dynamics of expected returns and risk are usually captured by stationary vector autoregressive (VAR) models, which typically contain the returns of the assets analyzed and some so-called state variables, which are used to predict these returns. Such VAR models are also used in the following three self-contained chapters/essays of this thesis, but several contributions to the asset allocation literature are made as well.

In the essay *'Inflation-Protecting Asset Allocation: A Downside Risk Analysis'* (Chapter 2), written in cooperation with Steffen Sebastian, we are the first to apply the VAR approach to bootstrap multi-period asset returns, which maintains the asymmetric asset return distributions. Additionally, we determine the inflation-protecting abilities of cash, bonds, stocks and real estate and optimal horizon-dependent inflation-protecting asset allocations within a downside risk framework. This downside risk analysis was inspired by two facts. First, previous studies only analyze correlation statistics between asset returns and the inflation rate to investigate the inflation-hedging qualities of the assets mentioned. However, correlation only measures the linear relation between the return and inflation. Thus, a positive correlation does not necessarily imply that the asset is a good inflation hedge, because the asset return could always be lower than the inflation rate despite the positive correlation. This issue has not been addressed in earlier studies and hence, we compare these correlations to lower partial moments (LPM) which measure the risk of the asset to fall below the inflation rate. Second, we argue that the most widely used risk measure in the asset allocation context, the variance, does not adequately represent the risk perception of investors and downside risk measures are more suitable to investors' risk understanding and in the presence of asymmetric return distributions. Markowitz (1959) already pointed out the advantages of downside risk measures compared to the variance, but it was not possible to handle those risk measures at that time due to their computational complexity.

In the essay *'Modeling Asset Price Dynamics under a Multivariate Cointegration Framework'* (Chapter 3), written in cooperation with Benedikt Fleischmann, we argue that the traditional VAR approach to modeling long-run asset price dynamics ignores common long-run relations between the assets and the state variables. In the presence of so-called cointegration relations, deviations in the long-term comovement of the variables cause forecastable backward movements. To take into account such cointegration relations among the assets and state variables, we capture the asset price dynamics using a vector error correction (VEC) model, which is an extension of the stationary VAR model. We, then, make several comparisons between the VEC and the stationary VAR approach, where both models include T-bills, stocks and bonds and the same set of state variables that have been shown to predict returns (dividend-price ratio, term spread and inflation). These comparisons include the modeled short and long-run behavior, the term structure of risk and the resulting optimal portfolio choice.

In the essay *'Do Stock Prices and Cash Flows Drift Apart? The Influence of Macroeconomic Proxies'* (Chapter 4), written in cooperation with Benedikt Fleischmann, we examine the validity of cointegration between stock prices and cash flows and analyze the additional influence of macroeconomic proxies within a cointegrated VAR framework. This paper is motivated by the fact that the often used state variables in the traditional VAR approach such as the dividend-price or the earnings-price ratio are assumed to be stationary. Since these one-for-one cointegration relations are empirically doubtful and non-stationarity would lead to invalid conclusions about the return forecastability, we investigate possible macroeconomic influences (inflation, short-term interest rates, government and corporate bond yields) which can cause the breakdown of these relations. Thus, although we do not determine optimal asset allocations, this paper is still relevant for the asset allocation literature as return predictions are of significant importance for portfolio choices.

Research Questions

This section gives a short overview of the research questions examined in each essay of this thesis:

Chapter 2: Inflation-Protecting Asset Allocation: A Downside Risk Analysis

- Do correlations and LPMs provide the same results with respect to the inflation-protecting abilities of cash, bonds, stocks and real estate?
- What are optimal horizon-dependent inflation-protecting asset allocations within the downside risk framework?
- How do optimal allocations differ for investors that require a positive real return?

Chapter 3: Modeling Asset Price Dynamics under a Multivariate Cointegration Framework

- Are there differences of the VEC compared to the VAR model with respect to the predictability of cash, stocks and bonds?
- What are the differences in the term structure of risk modeled by the two approaches and what are the sources of these differences?
- How do the optimal portfolio choices suggested by the two models differ?

Chapter 4: Do Stock Prices and Cash Flows Drift Apart? The Influence of Macroeconomic Proxies

- Is the assumption of stationarity of the dividend-price ratio and the earnings-price ratio valid in the multivariate cointegration framework?
- Is the assumption of stationarity of other financial ratios, which are additionally used in the predictability literature, valid in the multivariate cointegration framework?
- How do the macroeconomic variables influence stock prices and cash flows and what are the resulting consequences on total stock returns?

Chapter 2

Inflation-Protecting Asset Allocation: A Downside Risk Analysis

This paper is the result of a joint project with *Steffen Sebastian*.

Abstract

This paper studies the ability of cash, bonds, stocks and direct real estate to hedge inflation and optimal inflation-protecting asset allocations within a downside risk framework. Using a VAR model to capture predictable price dynamics, we find that the inflation-hedging properties of assets substantially change over the investment horizon. Cash is clearly the best hedge against inflation in the short run. However, as the investment horizon increases, bonds, stocks, and real estate become the more attractive options, with real estate exhibiting the best inflation protection qualities on a medium and long-term basis. While cash plays the most important role in short-term portfolios, the weights of the inflation-protecting portfolios shift to real estate, stocks and bonds as the investment horizon increases.

2.1 Introduction

In the United States as well as Europe interest rates are at an all-time low level, resulting in a continuous rise of the money volume. Furthermore, many developed countries are choosing to inject liquidity into their markets in order to stimulate the economy. However, there is a real danger that these monetary policies pave the way to rising inflation bringing the concerns of inflation back to investors' minds. Such fears are not restricted to institutional investors, whose liabilities are often linked to consumer prices or wage levels. They can also affect private investors, who seek to preserve their real capital, as well. Thus, the debate over inflation-protecting properties of assets and how investors can obtain optimal asset allocations to hedge against inflation has been revived.

Since many investors make long-term investments and it is well documented that the term structure of asset risk, asset correlations and the inflation rate can vary over time, the inflation-protecting properties of assets may also change essentially with the investment horizon.

In this paper, we therefore focus on inflation protection not only on short, but on various investment horizons up to 30 years. We consider an investor able to invest in cash, bonds, stocks and direct commercial real estate who seeks inflation protection. In contrast to studies that cover primarily asset and liability management (ALM) (see e.g. Amenc, Martellini, Milhau, and Ziemann, 2009), we assume that the investor's sole objective is to protect her assets against inflation, and that she has no liabilities subject to inflation risk. Using a vector autoregressive (VAR) model to capture predictable asset price and inflation dynamics, we first investigate the time-dependent correlations between the assets and inflation. This is the standard procedure for examining the horizon-dependent inflation-hedging properties of assets (Hoevenaars, Molenaar, Schotman, and Steenkamp, 2008; Amenc, Martellini, Milhau, and Ziemann, 2009; Briere and Signori, 2012). Note, however, that analyzing only correlations can be misleading, because a high positive correlation between an asset class and inflation does not necessarily imply a good inflation-protection ability. The asset return could still be lower than the inflation rate despite the positive correlation. To gain further insight into the inflation-hedging abilities of the assets analyzed, we examine the downside risk measures over various investment horizons based on asset returns generated by the VAR model. In particular, our analysis focuses on lower partial moments (LPM) to measure the risk of assets to fall below the inflation rate. LPMs are also useful for studying optimal inflation-protecting asset allocations over various investment horizons, because they can account for downside return deviations as well as asymmetric return distributions, which are observed empirically. We also argue that the variance, the most commonly used risk measure, does not adequately represent investors' risk perception, because it captures both negative and positive deviations from the mean. Since we consider investors with inflation-hedging motives but no liabilities, it is more suitable to focus only on the negative deviations of a specified target return. Due to concerns about using the variance as risk measure and to account for any the asymmetric asset returns, we apply a downside risk approach instead of the traditional Markowitz (1952) mean-variance approach to determine optimal portfolios.

Comparing the correlation analysis with the LPM analysis, we obtain different results for the inflation-protecting properties of assets and conclude that high correlations do not necessarily imply low LPMs. While the correlation analysis detects cash to be the best inflation hedge over all horizons (which is in line with Hoevenaars, Molenaar, Schotman, and Steenkamp (2008) and Amenc, Martellini, Milhau, and Ziemann (2009)), by analyzing LPMs, however, we find that the inflation-protecting potential of cash, stocks, bonds, and real estate changes substantially over the investment horizon. In the short term, cash is clearly a superior hedge. As the investment horizon increases, bonds, stocks, and real estate become the more attractive options. In fact, in contrast to the correlation results, they are better inflation protectors than cash in the long run, with real estate ultimately exhibiting the best inflation protection qualities for medium and long-term horizons. Bonds outperform equities with respect to inflation protection for medium horizons, but we obtain contrary results in the long run. These findings, consequently, also affect horizon-dependent optimal asset allocations. While cash is the only relevant asset for short-term optimal inflation-hedging portfolios, real estate plays the most important role in medium and long-term portfolios. In our asset allocation analysis, we consider not only an investor desiring to preserve capital, but also an investor with a more performance-oriented goal. Increasing the target return, i.e. considering an investor who aims to achieve a premium in excess of the inflation rate, we find that larger weight is assigned to assets that are riskier than cash. Over a medium and long-term basis, the largest proportion of investor capital should be invested in real estate. Equities also become highly attractive for investors who require a positive real return in the long run.

This paper contributes to the existing literature in two areas: inflation-hedging and strategic asset allocation. A number of studies have investigated the inflation-protecting abilities of various asset classes by employing correlation statistics between assets and inflation or regression models such as Fama and Schwert (1977). A detailed review of these studies is given by Attié and Roache (2009). Most inflation-hedging research has focused only on the relationship between an asset class and inflation for short-term investment horizons. Using a VAR framework, Campbell and Viceira (2005) demonstrate that the risk of T-bills, stocks and bonds, as well as the correlations between these assets, can vary considerably over time, which implies changes in the optimal asset allocation. These effects also have an essential impact on horizon-dependent inflation-protecting abilities of assets and the optimal inflation-hedging asset allocations. This VAR approach to analyzing the term structure of risk and horizon-dependent portfolios was first used in a classical asset allocation context with no reference to inflation-hedging. Fugazza, Guidolin, and Nicodano (2007) and Fugazza, Guidolin, and Nicodano (2009) extend the model of Campbell and Viceira (2005) by including European and U.S. property shares, respectively. MacKinnon and Zaman (2009) include U.S. direct real estate and REIT investments in addition to cash, bonds and stocks, whereas Rehring (2012) analyzes the role of direct real estate in a horizon-dependent mixed-asset portfolio for the U.K. market. Hoevenaars, Molenaar, Schotman, and Steenkamp (2008) and Amenc, Martellini, and Ziemann (2009) are the first to investigate optimal horizon-dependent portfolios with respect to inflation-hedging. Using a VAR methodology[1], they extend the model with alternative asset classes and investigate optimal portfolios for investors with liabilities subject to inflation risk. More recently, Briere and Signori (2012)

[1] Amenc, Martellini, and Ziemann (2009) use a cointegrated VAR model or vector error correction model, which allows for cointegration relations among the included variables.

find that inflation-hedging portfolios can be influenced by macroeconomic regimes. In contrast to previous studies, they do not apply the traditional mean-variance framework to derive optimal allocations of several listed assets, but they instead optimize portfolios with respect to inflation shortfall probabilities. Note, however, that their portfolio optimization focuses only on shortfall probabilities while ignoring the expected shortfall, which is an issue of great interest to investors. Their portfolio model also does not consider co-movements of individual assets. This appears restrictive in the presence of high correlations between asset returns. In this paper, we use the semivariance as downside risk measure for portfolio optimization and we account not only for the shortfall probability, but also for the amount of shortfall with an inflation target as well as co-movements among assets. Several studies have used semivariance approaches in a portfolio context (see e.g. Nawrocki (1999) for an overview; more recent articles are Kroencke and Schindler (2010) and Cumova and Nawrocki (2011)). But to the best of our knowledge, none has examined horizon-dependent asset allocations or inflation-hedging properties of assets. To investigate various investment horizons in a downside risk framework, it is necessary to simulate multi-period returns of the assets analyzed. In contrast to previous studies (such as Amenc, Martellini, and Ziemann (2009) or Briere and Signori (2012)), we do not simulate returns by means of a multivariate normal distribution. This procedure assumes that the asset returns exhibit no asymmetric behavior. However, it is widely accepted and we also find that asset returns are asymmetric and not normally distributed. Taking this into account, we apply a bootstrap resampling method for multi-period return generation. Thus, we believe this paper is the first to study the inflation-protecting abilities of cash, bonds, stocks and direct real estate as well as the optimal inflation-protecting portfolios over various investment horizons within an LPM framework.

The remainder of the paper is organized as follows. After introducing the applied methodology in the next section, we present our dataset and the results of our empirical analysis in Section 2.3. Section 2.4 summarizes the main findings and concludes.

2.2 Methodology

In this section we describe the VAR model used to capture return and inflation dynamics and to generate multi-period returns. We also present the risk measures that are applied to analyze the inflation-protecting properties of cash, bonds, stocks and real estate. Finally, the downside risk optimization problem to calculate optimal inflation-protecting asset allocations is given.

2.2.1 Modeling Asset Return Dynamics

The VAR model is a popular framework for modeling long-run asset price dynamics.[2] Hence, we follow this approach and capture the return dynamics of cash, bonds, stocks and real estate as well as the inflation rate by a first-order VAR model.

Let

$$\mathbf{z}_t = (r_{ca,t}, r_{bo,t}, r_{st,t}, r_{re,t}, infl_t, \mathbf{s}_t)' \tag{2.1}$$

contain the log returns of cash ($r_{ca,t}$), bonds ($r_{bo,t}$), stocks ($r_{st,t}$) and real estate ($r_{re,t}$), the log inflation rate[3] ($infl_t$) and three other state variables (the log of the dividend price ratio, the term spread and the cap rate), which are stacked in $\mathbf{s}_t$ and help to forecast asset returns. Thus, $\mathbf{z}_t$ is a (8×1) vector. We assume the return process is truly generated by the VAR(1) model:

$$\mathbf{z}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{z}_t + \mathbf{u}_{t+1}, \tag{2.2}$$

where $\boldsymbol{\mu}$ is the intercept vector of dimension (8×1) and $\boldsymbol{\Phi}$ is the (8×8) coefficient matrix. The vector $\mathbf{u}_{t+1}$ of dimension (8×1) contains the disturbances of the VAR model which we assume to be IID with zero means and a variance-covariance matrix $\boldsymbol{\Sigma}$.

[2] Authors using the VAR methodology to account for predictability and the horizon effects of asset returns are, e.g.: Campbell and Shiller (1988a,b); Campbell (1991); Campbell and Ammer (1993); Kandel and Stambaugh (1996); Barberies (2000); Campbell, Chan, and Viceira (2003); Campbell and Viceira (2005); Hoevenaars, Molenaar, Schotman, and Steenkamp (2008); Jurek and Viceira (2011).

[3] The log inflation rate is the difference between the logs of the price levels.

We first examine the inflation-protecting abilities of the assets analyzed by considering horizon-dependent correlations between the assets and inflation. These correlations are based on the conditional k-period variance-covariance matrix implied by the VAR model (see e.g. Campbell and Viceira, 2004):

$$\begin{aligned} Var_t\left(\mathbf{z}_{t+1} + \cdots + \mathbf{z}_{t+k}\right) = &\ \mathbf{\Sigma} + \left(\mathbf{I} + \mathbf{\Phi}\right)\mathbf{\Sigma}\left(\mathbf{I} + \mathbf{\Phi}\right)' \\ &+ \left(\mathbf{I} + \mathbf{\Phi} + \mathbf{\Phi}^2\right)\mathbf{\Sigma}\left(\mathbf{I} + \mathbf{\Phi} + \mathbf{\Phi}^2\right)' + \ldots \\ &+ \left(\mathbf{I} + \mathbf{\Phi} + \cdots + \mathbf{\Phi}^{k-1}\right)\mathbf{\Sigma}\left(\mathbf{I} + \mathbf{\Phi} + \cdots + \mathbf{\Phi}^{k-1}\right)', \end{aligned} \quad (2.3)$$

We then use the VAR model in equation (2.2) to generate k-period returns for cash, bonds, stocks and real estate and the inflation rate depending on the investment horizon. In contrast to previous studies (see e.g. Amenc, Martellini, and Ziemann, 2009; Briere and Signori, 2012), we do not simulate returns by drawing variables from a multivariate standard normal distribution. This procedure assumes that the residuals of the VAR model, and also the asset returns, are normally distributed and, therefore, exhibit no asymmetric behavior. However, it is widely accepted that asset returns are usually asymmetric. Taking into account this fact, we apply a bootstrap resampling method to generate multi-period returns following Benkwitz, Lütkepohl, and Wolters (2001) and Lütkepohl (2005), which consists of the following steps:

1. Calculate centered residuals $\hat{\mathbf{u}}_1 - \bar{\mathbf{u}}, ..., \hat{\mathbf{u}}_T - \bar{\mathbf{u}}$, where $\hat{\mathbf{u}}_1, ..., \hat{\mathbf{u}}_T$ are the estimated residuals and $\bar{\mathbf{u}}$ contains the eight usual means of the eight residual series.

2. Draw randomly with replacement from the centered residuals to obtain bootstrap residuals $\boldsymbol{\epsilon}_1^*, ..., \boldsymbol{\epsilon}_T^*$.

3. Recursively calculate the bootstrap time series for the VAR as

$$\mathbf{z}_{t+1}^* = \boldsymbol{\mu} + \mathbf{\Phi}\mathbf{z}_t^* + \boldsymbol{\epsilon}_t^*,\ t = 1, ..., T, \quad (2.4)$$

where $\mathbf{z}_1^* = \mathbf{z}_1$ holds for each generated series.

4. Calculate one-period log returns $\mathbf{z}_1^*, ..., \mathbf{z}_k^*$ by the VAR equation in (2.2) and obtain the k-period log returns by $\mathbf{z}_1^* + \cdots + \mathbf{z}_k^*$.

5. Repeat these steps $10,000$ times.

After bootstrapping these returns, we adjust the k-period returns by transaction costs. This is important because we include direct real estate into the analysis and in comparison to the other assets analyzed, the transaction costs of real estate are much higher and make real estate attractive only for longer investment horizons (details of the transaction cost adjustments are given in the next section). These generated multi-period returns are not only used to analyze the inflation-protecting properties of the asset classes with respect to LPMs over various investment horizons, but also to construct minimum downside risk portfolios for different investment horizons.[4]

2.2.2 The Concept of Lower Partial Moments

In addition to the correlations implied by the VAR model, we also want to measure the risk of the assets to fall below the inflation rate. LPMs, a concept first introduced by Bawa (1975) and Fishburn (1977), are quite useful in this regard because they focus on the downside (left-side) of the return distribution. Moreover, LPMs are also an adequate risk measure for our portfolio analysis. In contrast to the variance, LPMs only capture negative return deviations of a specified target, which is more intuitive since returns above the target are considered desirable and non-risky. Furthermore, asset returns are empirically observed to be non-normally distributed, a feature that LPMs can also handle very efficiently.

Generally, the LPM of order n is defined as:

$$\mathrm{LPM}_n(\tau) = \int_{-\infty}^{\tau} (\tau - r_i)^n f(r_i) dr_i, \tag{2.5}$$

where τ is the target rate, r_i is the return of asset i and $f(r_i)$ is the density function of the ith asset return. Observing T returns of asset i, the discrete version of the LPM of order n is given by:

$$\mathrm{LPM}_n(\tau) = \frac{1}{T} \sum_{t=1}^{T} \left[\max(0, (\tau - r_{it}))\right]^n . \tag{2.6}$$

[4]The bootstrap is theoretically justified as the statistics of interest have a normal limiting distribution (Horowitz, 2001; Barrett and Donald, 2003; Lütkepohl, 2005).

The order n of the LPM can be interpreted as a risk aversion parameter. Higher values of n penalize deviations below the target more strongly. This analysis focuses on three classes of LPMs: the shortfall probability ($n = 0$), the expected shortfall ($n = 1$) and the semivariance ($n = 2$).

In the portfolio context, similarly to the traditional mean-variance framework, co-movements of the LPMs of individual asset returns need to be taken into account. These co-movements are captured by co-lower partial moments (CLPM). There are several definitions for CLPMs between two assets. Nawrocki (1991) presents an asymmetric as well as a symmetric CLPM algorithm. Comparing both concepts empirically, Nawrocki (1991) recommends the usage of the symmetric measure. The most recent approach for measuring the CLPM between two assets was developed by Estrada (2008). In his heuristic framework the semivariance between asset i and asset j is calculated as:

$$\text{CLPM}_{ij}(\tau) = \frac{1}{T}\sum_{t=1}^{T}\left[\max(0,(\tau - r_{it}))\cdot\max(0,(\tau - r_{jt}))\right]. \tag{2.7}$$

In contrast to the symmetric CLPM of Nawrocki (1991), Estrada's CLPM definition considers only downside co-movements, which induces us to apply his approach in this paper.

2.2.3 The Portfolio Choice Problem

There is a great deal of extant literature on long-horizon asset allocation and inflation-hedging in the portfolio context. However, most of the papers use a mean-variance framework with variance as the risk measure to be minimized. Due to concerns about the efficacy of this method and in order to consider non-normally distributed returns, we apply a downside risk framework in our analysis. Markowitz (1959) discussed the advantages using the downside risk measure, but he did not apply this approach due to its computational complexity. The downside risk portfolio choice problem is quite similar to the traditional Markowitz approach, as we can replace the variance matrix by the semivariance of the portfolio and minimize the downside risk measure, which is suggested by Harlow and Rao (1989) and Harlow (1991). In their approach, however, co-movements between assets are disregarded, which is restrictive in the presence of high correlations between the LPMs of the assets. Thus, taking into account co-movements, we approximate the semivariance according to Estrada (2008) and consider the following minimum semivariance portfolio choice problem:

$$\min_{\mathbf{w}} \mathrm{LPM}_{2,p} = \sum_{i=1}^{4}\sum_{j=1}^{4} w_i w_j \mathrm{CLPM}_{ij}(\tau) \tag{2.8}$$

subject to

$$\sum_{i=1}^{4} w_i = 1$$

$$w_i \geq 0, \ \ i = 1, ..., 4,$$

where the vector $\mathbf{w} = (w_1, w_2, w_3, w_4)$ contains the weights of the four assets analyzed in the minimum downside risk portfolio.

In the empirical part of this study, we calculate the semivariance using k-period real asset returns. At first, we assume an investor who seeks to hedge inflation and we calculate minimum semivariance portfolios with the target rate set to 0 (i.e., we minimize the risk of achieving a negative real return). Afterward, we consider a more ambitious investor with respect to the required real return and we determine portfolios with annualized target rates ranging from 1% to 3%.[5]

2.3 Empirical Analysis

This section discusses the results of our empirical analysis. After introducing the dataset, we present the estimated VAR model capturing the asset return and inflation dynamics. Then, the inflation-protecting properties of the assets are analyzed and inflation-hedging portfolios described.

2.3.1 Data

Our empirical application is based on the four most widely used asset classes in the United States: cash, bonds, stocks and direct real estate. We use quarterly data

[5] Throughout the remainder of this paper, we use the term "target rate/return" rather than "annualized target rates/return".

from 1978:Q1 to 2010:Q4.[6] Cash is represented by the 90-day Treasury bill rate. Bonds are the long-term U.S. government bond returns and stock returns (including dividends) are calculated using the S&P 500 index. The data concerning these assets stem from Goyal and Welch (2008).[7] Direct real estate data is obtained from the NCREIF Property Index (NPI).[8]

Real estate returns are determined by appraisal-based capital and income indices. Appraisal-based indices/returns cannot be directly compared to those of liquid assets such as stocks, because they are smoothed and lagged with respect to market movements. Moreover, the systematic risk of appraisal-based returns is lower and less volatile than true real estate market returns (see e.g. Geltner, N. G. Miller, and Eichholtz, 2007, Chap. 25). This underestimation of risk can make appraisal-based returns more attractive than they actually are and incomparable to returns of liquid and more volatile assets. Therefore, we unsmooth the appraisal-based returns following the approach of Geltner (1993). The appraisal-based log real capital returns, c_t^*, are unsmoothed by:

$$c_t = \frac{c_t^* - (1-\omega)c_{t-1}^*}{\omega}, \tag{2.9}$$

where c_t is the log real capital return and ω is the smoothing parameter. This parameter is determined by $\omega = 1/(\bar{L}+1)$, where $\bar{L}$ is the average number of lags. Since we use quarterly data, we have $\bar{L} = 4$ and unsmooth the log real capital returns with $\omega = 0.2$ (as suggested by Geltner, N. G. Miller, and Eichholtz, 2007, Chap. 25). Afterwards, the unsmoothed log real capital returns are retransformed to nominal capital returns (UCR_t) and we use UCR_t to calculate an unsmoothed capital value index (UCV_t). Multiplying the income return (IR_t) with the original capital value index (CV_t), we obtain an income series (In_t) and the unsmoothed income returns ($UIR_t = In_t/UCV_t$), out of which we finally determine the unsmoothed total returns by summing up the unsmoothed income and capital returns ($UIR_t + UCR_t$).

[6] The beginning of the sample was chosen according to the availability of direct real estate data.

[7] We would like to thank Amit Goyal for providing an updated version of this data which is available on his website: http://www.hec.unil.ch/agoyal/.

[8] We would also like to thank NCREIF for providing the data.

We further use some common state variables that have been shown to predict returns: the dividend-price ratio, the term spread and the cap rate, which reflects the real estate market yield and is determined by In_t/UCV_t. The dividend-price ratio is often used as a predictor of future aggregate stock returns (Campbell and Shiller, 1988a,b; Fama and French, 1988; Hodrick, 1992; Goetzmann and Jorion, 1993). The term spread is a business cycle indicator and positively forecasts bond returns (Campbell and Vuolteenaho, 2004; Campbell and Viceira, 2005; Jurek and Viceira, 2011). Fu and Ng (2001) and Plazzi, Torous, and Valkanov (2010) find the cap rate to predict future direct real estate returns. Moreover, we include the inflation rate as a state variable to analyze the inflation-protecting properties of the asset classes and portfolios. With the exception of the cap rate, which is calculated by the NCREIF data, the data of the state variables is also obtained from Goyal and Welch (2008).

Table 2.1 provides an overview of the sample statistics of the variables used. Cash has the lowest nominal return compared to the other asset classes.[9] The most attractive asset class with respect to the return is stocks followed by real estate and bonds. The hierarchy of asset volatilities is the same as for returns. Cash has the lowest variability followed by bonds, real estate and stocks. Considering the skewness of the assets, we detect an asymmetric behavior of asset returns. While cash and bonds are positively skewed, stocks and real estate returns have a negative skewness.[10] Moreover, the (excess) kurtosis of all assets is different from zero[11] and the asset returns do not follow a normal distribution as indicated by the Jarque-Bera test for normality. The test rejects the null of normality for cash and bonds on a 5% and for stocks and real estate on a 1% significance level.

[9] Hereinafter we write "returns" and "rates" instead of "log returns" and "log rates".

[10] According to the D'Agostino test for skewness, the null of no skewness is rejected for stocks on a 5% significance level and for cash and real estate on a 10% significance level. The null is supported for bonds.

[11] According to the Anscombe-Glynn test for kurtosis, the null of no kurtosis is rejected for real estate on a 1% significance level and for bonds and stocks on a 5% significance level. The null is supported for cash.

Table 2.1: Descriptive Statistics

	Mean	Sd	Min	Max	Skew	Kurt	JB-Test
Returns							
Cash (r_{ca})	1.32%	0.80%	0.01%	3.60%	0.57	0.26	7.83*
Bonds (r_{bo})	2.36%	6.05%	-15.68%	21.81%	0.39	0.96	9.14*
Stocks (r_{st})	3.08%	8.17%	-25.57%	19.33%	-0.86	1.23	25.53**
Real estate (r_{re})	2.59%	8.15%	-26.79%	21.66%	-0.56	1.55	21.25**
State Variables							
Log inflation ($infl$)	0.94%	0.99%	-3.99%	4.19%	-0.22	4.55	119.83**
Log term spread (tms)	0.49%	0.37%	-0.77%	1.11%	-0.64	0.22	9.61*
Log dividend price ratio (dp)	-3.64	0.46	-4.49	-2.78	0.02	-1.13	6.58*
Log cap rate (cr)	-3.99	0.16	-4.46	-3.73	-0.85	0.20	16.52**

Notes: This table reports summary statistics of the sample from 1978:Q1 to 2010:Q4. Mean log returns are adjusted by one half of the variance to reflect log mean (gross) returns. "Sd" denotes standard deviation, "Min" denotes minimum, "Max" denotes maximum, "Skew" denotes skewness, "Kurt" denotes kurtosis and "JB-Test" denotes the test statistic of the Jarque-Bera test for normality. * and ** indicate the rejection of the null hypothesis at 5% and 1% significance levels, respectively.

2.3.2 VAR Estimation Results

We capture the predictable return and inflation dynamics by the VAR(1) model given in equation (2.2).[12] Table 2.2 presents the estimation results of the VAR model. Panel A shows OLS estimates with bootstrapped standard errors in parentheses. The bootstrap estimates are calculated from 10,000 paths under the assumption that the initial estimated VAR model truly generates the data process. The R^2 and F-statistics are given in the rightmost column. Panel B reports the covariance structure of the VAR residuals showing the standard deviations of the innovations on the main diagonal and the cross-correlations above the main diagonal.

The first row in Panel A represents the prediction equation for cash, which is highly predictable according to the high R^2 of 89.39%. It shows that the own lag has a high positive influence on T-bills. The second and third rows correspond to bonds and stocks. These variables seem to be more difficult to predict, as they exhibit the lowest R^2's. However, bond returns are explained by T-bills, inflation and the term spread with positive slopes. In the stock equation the dividend-price ratio has a positive influence on equities. Obviously, real estate returns are easier to predict compared to stocks and bonds, as indicated by an R^2 of 35.72%. The cap rate helps to forecast real estate returns with a positive coefficient. The last four rows represent the state variable equations. The equations of the term spread, dividend-price ratio and cap rate reveal the persistent autoregressive behavior of these variables with high coefficients of their own lags. These results are consistent with previous studies such as Campbell and Viceira (2005), Fugazza, Guidolin, and Nicodano (2007) and Briere and Signori (2012).

Turning to the covariance structure of the innovations in Panel B, we see that unexpected inflation is positively correlated with shocks to cash and negatively correlated with shocks to bonds, stocks and real estate. The correlations of the residuals seem to imply that only cash is a good inflation hedge. However, although bonds, stocks and real estate are negatively correlated with inflation innovations, these asset classes may protect investors against inflation as the investment horizon increases.

[12] The lag length one is confirmed by the Schwarz information criterion.

Table 2.2: VAR Estimation Results

Panel A	Coefficients of the Lagged Variables								R^2
	$r_{ca,t}$	$r_{bo,t}$	$r_{st,t}$	$r_{re,t}$	$infl_t$	tms_t	dp_t	cr_t	(F-stat.)
$r_{ca,t+1}$	0.87	0.00	0.01	0.01	0.00	-0.03	0.00	0.00	89.39%
	(0.09)	(0.00)	(0.00)	(0.00)	(0.03)	(0.14)	(0.00)	(0.00)	(127.45)
$r_{bo,t+1}$	5.34	-0.04	-0.07	0.01	0.83	9.46	-0.05	-0.06	14.23%
	(2.02)	(0.09)	(0.07)	(0.07)	(0.73)	(2.90)	(0.03)	(0.05)	(2.51)
$r_{st,t+1}$	-2.61	0.07	0.09	0.01	0.25	-2.90	0.05	0.08	7.31%
	(2.83)	(0.13)	(0.09)	(0.10)	(1.03)	(4.19)	(0.04)	(0.06)	(1.19)
$r_{re,t+1}$	1.45	0.09	0.14	-0.17	4.95	6.60	-0.06	0.06	35.72%
	(2.39)	(0.11)	(0.08)	(0.08)	(0.87)	(3.50)	(0.03)	(0.05)	(8.41)
$infl_{t+1}$	-0.25	-0.03	0.01	0.00	-0.05	-1.45	0.01	0.01	37.29%
	(0.26)	(0.01)	(0.01)	(0.01)	(0.01)	(0.10)	(0.40)	(0.00)	(0.01)
tms_{t+1}	0.02	0.00	0.00	-0.01	-0.01	0.81	0.00	0.00	69.38%
	(0.08)	(0.00)	(0.00)	(0.00)	(0.03)	(0.11)	(0.00)	(0.00)	(34.27)
dp_{t+1}	3.65	-0.03	-0.07	0.00	0.33	3.47	0.94	-0.12	97.06%
	(0.52)	(0.03)	(0.02)	(0.02)	(0.21)	(0.78)	(0.01)	(0.01)	(499.96)
cr_{t+1}	-0.12	-0.13	-0.15	0.13	-4.88	-6.21	0.05	0.93	78.98%
	(2.80)	(0.12)	(0.09)	(0.09)	(0.98)	(4.06)	(0.04)	(0.06)	(56.82)

Continued

Continued

Panel B

	r_{ca}	r_{bo}	r_{st}	r_{re}	$infl$	tms	dp	cr
r_{ca}	(0.26%)	-56.85%	-6.26%	17.51%	29.25%	-82.83%	10.24%	-14.23%
r_{bo}	-	(5.61%)	4.91%	-4.71%	-40.97%	2.65%	-7.20%	8.83%
r_{st}	-	-	(7.88%)	19.29%	-7.69%	-2.59%	-98.00%	-13.16%
r_{re}	-	-	-	(6.56%)	-21.15%	-20.27%	-16.93%	-90.98%
$infl$	-	-	-	-	(0.77%)	-9.02%	12.29%	16.23%
tms	-	-	-	-	-	(0.21%)	-0.79%	12.87%
dp	-	-	-	-	-	-	(7.90%)	10.62%
cr	-	-	-	-	-	-	-	(7.41%)

Notes: This table reports the results of the estimated VAR model. Panel A reports coefficient estimates of the VAR $\mathbf{z}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{z}_t + \mathbf{u}_{t+1}$ with variables: cash, stocks, bonds, real estate, inflation, term spread, dividend-price ratio and cap rate. Bootstrap standard errors are calculated from 10,000 paths under the assumption that the initial estimated VAR model truly generates the data process and are reported in parentheses. The last column reports the R^2 and F-statistic of joint significance. Panel B reports the covariance structure of the VAR residuals showing the standard deviations of the innovations on the main diagonal in parentheses and the cross-correlations above the main diagonal.

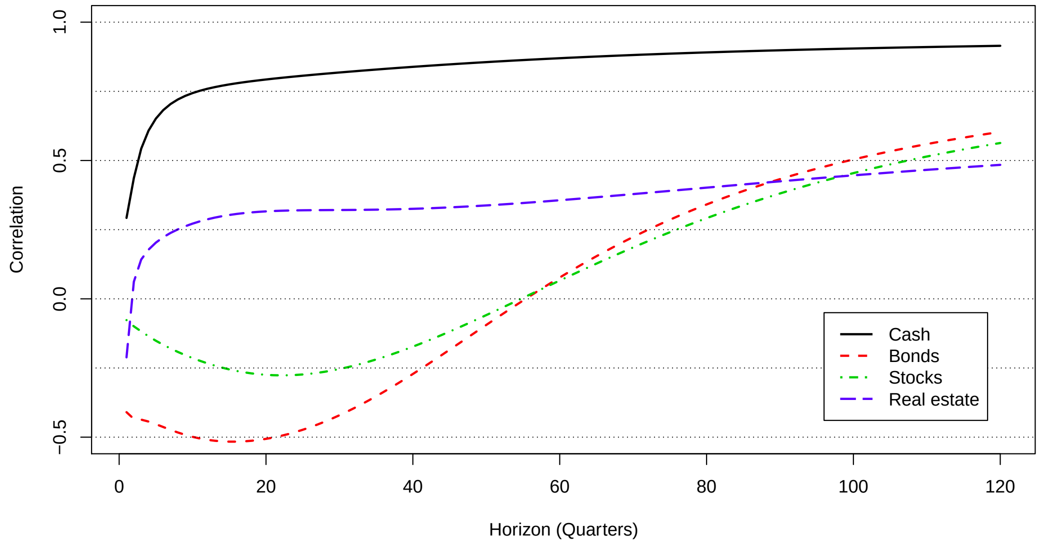
We also test whether the residuals follow a multivariate normal distribution according to Lütkepohl (2005). The test rejects the null of multivariate normality at a 1% significance level. Combined with the asymmetric pattern of the asset returns (illustrated in the previous section), this motivates a residual-based bootstrap method for multi-period return generation in the upcoming downside risk analysis.

2.3.3 Inflation Protection of Individual Assets

To investigate the inflation-hedging properties of the asset classes, we first analyze the correlations between the assets and inflation implied by the VAR model, depending on the investment horizon as in Hoevenaars, Molenaar, Schotman, and Steenkamp (2008), Amenc, Martellini, Milhau, and Ziemann (2009) and Briere and Signori (2012). Figure 2.1 shows the correlations between nominal asset returns and inflation, depending on the investment horizon. The correlations between cash and inflation are always positive and increase over the investment horizon, reaching a coefficient of around 90% at a 30-year horizon. Cash exhibits by far the highest correlations, with real estate exhibiting the second highest (and most positive) correlations up to horizons of 22 years, except for very short horizons. According to the correlations, bonds and stocks exhibit poor inflation-protecting qualities for short and medium horizons, but the hedging abilities of these two assets improve with the investment horizon and are better than those of real estate in the long run.

It is important to note that the correlation statistics only measure the linear relationship between the asset returns and the inflation rate. Thus, any conclusions about inflation-hedging abilities can be misleading. An asset class could move in conjunction with the inflation rate and would have a high positive correlation with inflation. However, if the inflation rate is always higher than the asset return, that asset would be a bad inflation hedge despite its positive correlation with inflation. To further explore the potential of cash, bonds, stocks and real estate to protect against inflation, we measure the risk of the assets to fall below the inflation rate. In particular, we consider LPMs of orders 0 (shortfall probability), 1 (expected shortfall) and 2 (semivariance) for various investment horizons. To calculate these horizon-dependent risk measures, we bootstrap nominal returns of length k, ranging

Figure 2.1: Correlations between Asset Returns and Inflation



Notes: This figure shows correlations between the nominal asset returns and inflation depending on the investment horizon.

from four quarters (one year) to 120 quarters (30 years). Note that real estate, contrary to cash, bonds and stocks, is characterized by high transaction costs, which reduces the returns. To account for this fact, we adjust the generated returns by the transaction costs, except for cash.[13] Following Rehring (2012), we assume round-trip transaction costs for bonds and stocks of 0.1% and 1%, respectively. These costs include bid-ask spreads and brokerage commissions. The round-trip transaction costs for direct real estate are generally assumed to be around 7% and they consist of transfer taxes and professional fees. Table 2.3 gives the LPMs with the inflation rate as the target for the assets analyzed.

We find that the inflation-hedging characteristics of the assets are different than those obtained from the correlation analysis. Although cash is the best inflation hedge for investment horizons up to five years, as indicated by the lowest expected shortfall and semivariance, the LPMs of orders 1 and 2 of this asset class increase with the investment horizon. Moreover, if we follow Briere and Signori (2012), and

[13]Transaction costs for cash are typically very low. Luttmer (1996) reports bid-ask spreads of 3 basis points for T-bills, which we disregard here.

Table 2.3: Lower Partial Moments of the Asset Returns

	Horizon 1			Horizon 2		
	LPM_0	LPM_1	LPM_2	LPM_0	LPM_1	LPM_2
Cash	51.00%	0.94%	0.03%	53.28%	1.78%	0.09%
Bonds	43.12%	3.63%	0.51%	39.00%	4.33%	0.78%
Stocks	43.38%	5.53%	1.17%	38.88%	6.66%	1.90%
Real estate	61.28%	6.91%	1.31%	51.24%	7.08%	1.61%
	Horizon 5			Horizon 10		
	LPM_0	LPM_1	LPM_2	LPM_0	LPM_1	LPM_2
Cash	51.46%	3.43%	0.36%	50.82%	6.33%	1.24%
Bonds	31.12%	4.76%	1.21%	24.40%	4.92%	1.57%
Stocks	30.18%	6.69%	2.44%	20.68%	5.24%	2.26%
Real estate	35.10%	5.74%	1.51%	22.00%	4.02%	1.24%
	Horizon 20			Horizon 30		
	LPM_0	LPM_1	LPM_2	LPM_0	LPM_1	LPM_2
Cash	50.90%	10.67%	3.49%	50.66%	14.39%	6.43%
Bonds	13.70%	3.23%	1.30%	8.36%	2.11%	0.96%
Stocks	10.06%	2.59%	1.15%	5.04%	1.49%	0.78%
Real estate	9.72%	1.89%	0.64%	3.86%	0.68%	0.22%

Notes: This table reports lower partial moments of order zero, one and two with the inflation rate as target for the assets cash, bonds, stocks and real estate and for various investment horizons (1, 2, 5, 10, 20 and 30 years).

consider only shortfall probabilities while disregarding the amount of shortfall to assess inflation-protecting properties, we also obtain different results. The shortfall probabilities of cash are continuously over 50% and are higher than those of all other asset classes for investment horizons longer than one year. According to first and second-order LPMs, bonds, stocks and real estate exhibit weaker inflation-protecting properties in the short run. However, increasing the investment horizon results in a different picture. For investment horizons of 10 years or longer, real estate provides the best downside inflation protection compared to the other assets analyzed, as indicated by the LPMs of all orders for 10 to 30-year horizons (except for the shortfall probability at horizon of 10 years). We find that bonds perform better than stocks with respect to the expected shortfall and the semivariance up to a 10-year investment horizon. For longer horizons, equities are a better inflation hedge than bonds. Cash exhibits the poorest inflation-protecting properties in the long run. While the expected shortfall and semivariance of the other asset classes do not exceed 2.11% and 0.96%, respectively, the expected amount of cash to fall below the inflation rate is 14.39% and the semivariance is 6.43%. In addition, the shortfall probabilities confirm the weak inflation-hedging performance of cash compared to bonds, stocks and real estate in the long run.

If we compare the results of the correlation analysis with those of the LPM analysis, we obtain different results for the inflation-hedging performance of assets. We thus conclude that high correlations do not necessarily imply low LPMs. The tendency for the inflation-hedging abilities of bonds, stocks and real estate to improve along with the investment horizon is indicated by both the correlations and the LPMs. But, the results for cash differ significantly. Furthermore, according to the correlations, real estate exhibits the poorest inflation-hedging qualities in the long run, but the best qualities with respect to the downside risk measures.

2.3.4 Inflation-Protecting Asset Allocation

We now explore the optimal asset allocations for investors who aim to protect their portfolios against inflation. We investigate the optimal portfolios for investors with a real return target of 0% as well as for more ambitious investors with real return targets ranging from 1% to 3%.[14] Our investigations also consider various investment horizons.

As we did for the LPM analysis in the previous section, we again use bootstrapped and transaction cost-adjusted (real) returns of length k (with k varying from four quarters to 120 quarters) for 10,000 paths to construct minimum semivariance portfolios. We begin by examining optimal portfolios for an investor who simply desires to protect her portfolio against inflation, which implies a target real return of 0%. Table 2.4 shows minimum semivariance portfolios with a real return target of 0% at different investment horizons.

Table 2.4: Minimum Semivariance Portfolios with Real Return Target of 0%

Horizon (years)	1	2	5	10	20	30
Cash	1.00	1.00	0.95	0.28	0.00	0.00
Bonds	0.00	0.00	0.02	0.10	0.09	0.04
Stocks	0.00	0.00	0.00	0.15	0.30	0.29
Real estate	0.00	0.00	0.03	0.47	0.61	0.67
Ann. real return	0.00%	0.01%	0.05%	1.86%	2.95%	3.05%
Cumulated real return	0.00%	0.01%	0.26%	18.63%	59.06%	91.40%
Ann. real ret. volatility	2.33%	2.80%	3.75%	6.85%	7.89%	7.83%
LPM_0	51.00%	53.28%	48.98%	19.02%	4.88%	1.54%
LPM_1	0.94%	1.78%	3.20%	2.32%	0.69%	0.23%
LPM_2	0.03%	0.09%	0.36%	0.78%	0.44%	0.18%

Notes: This table reports minimum semivariance portfolios with a real return target of 0% for various investment horizons (1, 2, 5, 10, 20 and 30 years) and corresponding descriptive statistics below. "Ann./ret." are the abbreviations for "Annualized/return".

Note that the shortfall probability and the expected shortfall are much lower for long-horizon investments over 20 or 30 years than for one or two-year investments. Moreover, the annualized real return and the annualized real return volatility increase with the investment horizon (except for the 30-year horizon where the volatility slightly decreases).

[14] Note that our results are not affected by considering real returns with a real return target of 0. Thus, instead of nominal returns with the inflation rate as the target, we switch to real terms from hereon in order to simplify the presentation of the results.

At the one and two-year horizons, the minimum semivariance portfolio is entirely invested in cash. This result is not surprising, however, since we have found that cash would be the best inflation-hedging asset in the short run. Both portfolios perceive real capital, as indicated by slightly positive annualized real returns. The probability of falling below the inflation rate is relatively high with values of 51% and 53% for the one and two-year horizon minimum semivariance portfolios, respectively. These portfolios have an expected shortfall of 0.94% and 1.78% and semivariances close to 0%. Because cash is the only asset included in the portfolios, there are no diversification benefits and the risk measures are equal to the results in Table 2.3. For a five-year investment horizon, the weight of cash remains quite high at 95%, with only 2% and 3% invested in bonds and real estate. If we consider longer investment periods, we see that the weights change substantially. For a 10-year horizon, the optimal inflation-protecting asset allocation is 28% cash, 10% bonds, 15% stocks and 47% real estate. This portfolio has an annualized real return of 1.86% and an annualized real return volatility of 6.85%. We also obtain diversification benefits from allocating various assets. The LPMs of all orders are lower than those of the single assets at a 10-year investment horizon. Note that, in the previous section, stocks were found to exhibit good inflation-hedging qualities as the investment horizon increased, which were taken into account for the long horizon portfolios. While cash plays no role in long horizon inflation-hedging portfolios, the proportion assigned to stocks doubles, with weights of 30% and 29%, respectively, in the optimal portfolio with a horizon of 20 and 30 years. Real estate was found to be the best inflation-hedging asset for investment horizons of 10 years or longer, so more than 50% of investor's capital should be invested in this asset class in the long run. Moreover, the allocations to bonds over the long term are small compared to stocks and real estate. For the optimal portfolio with a 30-year investment horizon, the weight of bonds is only 4%.

Assuming next an investor with a more performance-oriented goal, we also examine optimal semivariance portfolios with higher real return targets. Table 2.5 shows minimum semivariance portfolios with real return targets of 1%, 2% and 3% at various investment horizons.

Intuitively, we would expect that imposing a higher target return would lead to portfolios with more risk. Although the composition of the short-run portfolios and, consequently, the LPMs do not change with the different target rates, the risk measures of the medium and long-term portfolios increase substantially. For example, at a five-year investment horizon, the minimum semivariance portfolio with a target real return of 0% falls below the inflation rate with a probability of 48.98%. Increasing the target real return to 1%, 2% and 3% yields shortfall probabilities of 63.84%, 69.30% and 71.14%, respectively. The other risk measures, the expected shortfall and the semivariance, also rise with an increasing target return. The higher portfolio risk is caused by shifting more weight to assets that are more volatile than cash. Considering a five-year horizon, investors who require an additional premium decrease their allocations to cash (down to 38% for the 3% target) and invest more in bonds, stocks and real estate compared to investors who seek to preserve their capital and invest more than 90 % in cash. Apart from cash, substantial amounts of bonds and real estate are allocated at five-year investment horizons. As horizon and target rates increase, the weights of bonds (and to some extent, real estate) shift to stocks, making equities an interesting asset class for the long run. However, real

Table 2.5: Minimum Semivariance Portfolios with Positive Real Return Targets

Horizon (years)	1	2	5	10	20	30
Target: Real return 1%						
Cash	1.00	1.00	0.82	0.08	0.00	0.00
Bonds	0.00	0.00	0.10	0.15	0.07	0.00
Stocks	0.00	0.00	0.00	0.23	0.36	0.38
Real estate	0.00	0.00	0.08	0.52	0.57	0.62
Ann. real return	0.00%	0.01%	0.36%	2.45%	3.02%	3.16%
Cumulated real return	0.00%	0.01%	1.78%	24.52%	60.49%	94.68%
Ann. real ret. volatility	2.33%	2.80%	4.03%	7.95%	7.96%	7.59%
LPM_0	67.32%	72.68%	63.84%	27.14%	12.66%	6.00%
LPM_1	1.54%	3.05%	5.42%	4.31%	2.27%	1.03%
LPM_2	0.05%	0.19%	0.83%	1.62%	1.30%	0.85%

Continued

Continued

Target: Real return 2%						
Cash	1.00	1.00	0.60	0.00	0.00	0.00
Bonds	0.00	0.00	0.20	0.17	0.00	0.00
Stocks	0.00	0.00	0.06	0.30	0.45	0.41
Real estate	0.00	0.00	0.14	0.53	0.55	0.59
Ann. real return	0.00%	0.01%	0.89%	2.72%	3.12%	3.21%
Cumulated real return	0.00%	0.01%	4.43%	27.23%	62.44%	96.36%
Ann. real ret. volatility	2.33%	2.80%	4.84%	8.49%	8.17%	7.61%
LPM_0	80.24%	87.04%	69.30%	39.40%	27.36%	19.08%
LPM_1	2.28%	4.65%	7.66%	7.35%	6.10%	4.41%
LPM_2	0.09%	0.34%	1.57%	2.94%	3.32%	2.98%
Target: Real return 3%						
Cash	1.00	1.00	0.38	0.00	0.00	0.00
Bonds	0.00	0.00	0.31	0.14	0.00	0.00
Stocks	0.00	0.00	0.11	0.35	0.49	0.48
Real estate	0.00	0.00	0.20	0.51	0.51	0.52
Ann. real return	0.00%	0.01%	1.45%	2.78%	3.16%	3.25%
Cumulated real return	0.00%	0.01%	7.24%	27.75%	63.16%	97.55%
Ann. real ret. volatility	2.33%	2.80%	6.02%	8.60%	8.30%	7.74%
LPM_0	90.10%	94.70%	71.14%	53.72%	45.94%	41.84%
LPM_1	3.14%	6.47%	10.11%	11.91%	13.36%	13.34%
LPM_2	0.14%	0.56%	2.55%	4.99%	7.44%	8.75%

Notes: This table reports minimum semivariance portfolios with real return targets of 1%, 2% and 3% for various investment horizons (1, 2, 5, 10, 20 and 30 years) and corresponding descriptive statistics below. "Ann./ret." are the abbreviations for "Annualized/return".

estate appears to be the most attractive asset for investors willing to hedge inflation and also for performance-oriented investors with a 10-year or longer investment horizon.

2.4 Conclusion

In this paper we investigated the inflation-protecting abilities of cash, bonds, stocks and direct real estate and optimal asset allocations between these assets for investors seeking to preserve real capital or to achieve a return exceeding the inflation rate. We captured the dynamics of asset returns and the inflation rate by a VAR model to analyze horizon-dependent correlations between assets and inflation. Moreover, using the VAR framework, we generated multi-period asset returns to investigate lower partial moment risk measures of the assets over various investment horizons. These bootstrap-based returns were also used to determine horizon-dependent optimal inflation-hedging asset allocations within a downside risk framework.

We find that the standard approach of considering only correlations to analyze the inflation-hedging properties of assets can be misleading. Comparing correlation results with those of an LPM analysis shows very different findings. According to the correlations, cash is the best inflation-hedging asset over all horizons and real estate exhibits the poorest inflation-protecting ability in the long run. In the downside risk analysis, however, we find that cash hedges again inflation best only in the short run, while real estate actually has the best inflation protection qualities for medium and long horizons. We also find that bonds and stocks improve their inflation-hedging abilities along with the investment horizon. These changes in the inflation-hedging properties of cash, stocks, bonds and real estate over the investment horizon also affect the optimal inflation-protecting asset allocations. While cash is the only relevant asset for short-term optimal inflation-hedging portfolios, real estate plays the most important role in medium and long-term portfolios. In our asset allocation analysis, we consider not only an investor who desires to preserve capital, but also an investor with a more performance-oriented target. Increasing the target return, i.e. examining an investor who requires a premium in excess of the inflation rate, we find that larger weights are assigned to more risky assets compared to cash. On a medium and long-term basis, the highest amount of the investor's capital should be invested in real estate. Equities also become highly attractive for investors who require a positive real return in the long run.

Our study has only investigated the role of the most widely used assets: cash, bonds, stocks and real estate, with respect to inflation-hedging but can be extended to other asset classes. It would seem sensible to include inflation-linked bonds in the analysis of inflation protection. However, the length of available time series is too short for an econometric analysis without any concerns. The same argument also applies to the inclusion of alternative investment classes such as infrastructure, which is currently of great interest to many investors.

Chapter 3

Modeling Asset Price Dynamics under a Multivariate Cointegration Framework

This paper is the result of a joint project with *Benedikt Fleischmann*.

Abstract

We show that allowing for cointegration within a vector autoregressive (VAR) framework yields important implications for modeling the asset price dynamics of T-bills, stocks and bonds over all investment horizons. While the stationary VAR approach ignores common stochastic trends of the included variables, the vector error correction (VEC) model captures these common long-run relations and their predictable restorations. We find interesting differences in the term structure of risk of the VEC compared to the traditional VAR. There is a strong positive link between risk premia and real interest rates in the short term and a much more negative and longer-lasting impact of inflation on excess stock and bond returns. Incorporating cointegration significantly shifts downward nominal stock and bond volatilities and incorporates inflation as the driving component of nominal interest rates, which results in a flat risk structure of real interest rates. For an extreme risk-averse investor, the optimal real (nominal) return portfolio is tilted much more towards T-bills (bonds).

3.1 Introduction

In recent empirical finance research, the stationary vector autoregressive (VAR) model is a popular framework for modeling long-run asset price dynamics.[1] In the multi-horizon context, the VAR has some convenient advantages compared to the simple regression model. First, this approach makes it possible to study the interactions between asset prices and economic state variables as well as the pulling and pushing forces going through certain economic channels. Second, long-term effects can easily be explored by iteratively calculating multi-period forecasts. Hence, the model estimated by short-run dynamics is able to capture long-horizon behavior. Third, while simple long-horizon regressions are often criticized due to their statistical properties (biased t-statistics), the VAR shows no econometric issues with respect to long-run forecasts. Therefore, the VAR setup is often used to account for predictability and to capture time-varying investment opportunities of several assets such as cash, stocks and bonds simultaneously.

However, we argue that the stationary VAR approach ignores important additional information as it does not consider the presence of common long-run relations between the assets and the state variables. Deviations in the long-term comovement of the variables cause predictable backward movements. These cointegration effects can be incorporated into an extension of the VAR model, the vector error correction (VEC) model. Allowing for cointegration yields important implications for the interdependencies among the variables at all horizons. We observe a significant change in the horizon-dependent risk structure of the asset returns and ultimately that the optimal portfolio rules are substantially different from the stationary VAR model.

The framework of cointegration and error correction has been used in several other studies and goes back to Granger (1981) and Engle and Granger (1987). Campbell and Shiller (1987) test cointegration between dividends and stock prices as well as long-term bond yields and short-term interest rates. They detect the dividend-

[1] Some examples of authors using the VAR methodology to account for predictability and horizon effects of asset returns include: Campbell and Shiller (1988a,b); Campbell (1991); Campbell and Ammer (1993); Kandel and Stambaugh (1996); Barberies (2000); Campbell, Chan, and Viceira (2003); Campbell and Viceira (2005); Hoevenaars, Molenaar, Schotman, and Steenkamp (2008); Jurek and Viceira (2011).

price ratio and term spread to be stationary and restoring to their means, while the deviations can be quite persistent. Nasseh and Strauss (2000) find significant cointegration relations between stock prices and macroeconomic variables in six European countries. Other academic researchers emphasize a long-run relationship between consumption and dividends containing important information about the variances and means of cash flows and, by implication, their returns (Bansal, Gallant, and Tauchen, 2007; Hansen, Heaton, and Li, 2008; Bansal, Dittmar, and Kiku, 2009; Bansal and Kiku, 2011). Bansal, Dittmar, and Kiku (2009) and Bansal and Kiku (2011) incorporate this error correction information into the VAR framework (EC-VAR) and conclude that the dynamics are better captured compared to the traditional VAR. As a consequence, the risk premium and the term structure of risk can be distorted by neglecting cointegration. Furthermore, Lettau and Ludvigson (2001) and Lettau and Ludvigson (2005) motivate cointegration between consumption, labor income and financial wealth (*cay*). They find that *cay* outperforms popular stock return predictors such as the dividend-price ratio for short as well as long horizons. Hence, cointegration can handle the deviation of asset prices from the fundamental equilibrium in boom and bust cycles and predict their restorations. Additionally, more recent studies use a VEC appproach to model price and return dynamics of financial securities (e.g. Zhong, Darrat, and Anderson, 2003; Blanco, Brennan, and Marsh, 2005; Durre and Giot, 2007; Barnhart and Giannetti, 2009).

These findings and studies motivate an extension of the stationary VAR model by cointegration and the integration of common long-run relationships into a long-run asset pricing analysis. If the time series used in the traditional VAR are differenced in order to obtain stationarity, their stochastic trends are eliminated. Although this procedure is quite common, it is disadvantageous for cointegrated variables and always leads to a distortion of the relationships between the variables analyzed. However, the magnitude and the direction of this bias remain unclear and depend on the investment horizon.

Therefore, we contribute to the literature by comparing the stationary VAR and the VEC model with respect to their modeled short- and long-run behavior, where both models include the same set of investable assets (T-bills, stocks and bonds) and common state variables that have been shown to predict returns (dividend-price ratio, term spread and inflation).[2] Starting from a VAR representation, we find strong evidence for common stochastic trends between the levels of the six variables. The cointegration rank test indicates four cointegration relations among the level variables on a 5% significance level. While the standard VAR model captures only the long-run dynamics of stationary data, the VEC model takes into account information about the four cointegration relations and is able to distinguish between short and long-run effects. The estimation results show an adjusted R^2 that is more than two times higher for the risk premia of stocks and bonds for the VEC. These increases already show the importance of incorporating common long-run effects in the analysis of asset price dynamics. This motivates a further comparison of the long-run dynamics implied by the VAR and VEC, depending on the time horizon, by investigating the variance decompositions of real and nominal asset returns. Therefore, we examine the various risk components of the returns, their interactions and their sources. We find substantial differences between the two models with respect to the term structure of risk. The VEC shows a much higher correlation between the risk premia and real interest rate in short and medium horizons, as well as a much more negative correlation between the risk premia and inflation in the long run. As a further finding, the volatilities of nominal returns are significantly lower over all horizons under cointegration. Turning to real terms, we find the same evidence for stock returns and, moreover, the term structure of T-bills appears roughly flat compared to the mean-averting structure of the stationary VAR. The latter result indicates a strong common stochastic trend between nominal T-bills and inflation. Finally, these differences in the risk structure influence the optimal portfolio choice. Under cointegration and extreme risk aversion, the optimal real (nominal) return portfolio is tilted much more towards T-bills (bonds). In the VEC, a less risk-averse investor has a much higher equity exposure as the investment horizon lengthens

[2] A large amount of literature documents predictability, see for example: Campbell and Shiller (1988a,b); Fama and French (1988, 1989); Hodrick (1992); Campbell and Vuolteenaho (2004); Campbell and Viceira (2005); Jurek and Viceira (2011). However, the evidence of predictability is not unambiguous, especially in the short run. Several authors mention the poor out-of-sample predictability of stock returns. A critical discussion of out-of-sample predictability is given by Goyal and Welch (2008).

and even leverages the position in the very long run. This behavior is borne by a decreasing bond position compared to the VAR model.

The remainder of the paper is organized as follows: In the next section, we describe the methodology of a VAR model, extend it to a VEC model, derive the horizon-dependent variance-covariance matrices for both models and outline the portfolio problem. The third section introduces the data, examines the time series properties for further investigations and presents the results of our empirical short and long-run analysis. Finally, Section 4 summarizes the main findings.

3.2 Methodology

In this section, we introduce the VAR and VEC models capturing the return dynamics of the assets analyzed first. Starting with the commonly used VAR methodology, we extend this framework to a VEC model for analyzing integrated and cointegrated time series. Additionally, the VEC allows us to explicitly distinguish between short- and long-run effects in the dynamic system. Second, the predictable return components have important implications for the investment horizon-dependent risk structure of the assets, and ultimately for the portfolio choice. Finally, the risk structure modeled by multi-period conditional variances and the portfolio choice problem are derived.

3.2.1 VAR Specification

Let $\Delta \mathbf{z_t}$ be a vector that includes $n_a + 1$ log asset returns and n_s additional log state variables that have been identified to predict returns. In the specification of this study, $r_{0,t}$ denotes our benchmark asset, while $\mathbf{x}_t = \mathbf{r}_t - \boldsymbol{\iota} r_{0,t}$ are the n_a excess returns of the other asset returns, $\mathbf{r}_t$, relative to the benchmark (where $\boldsymbol{\iota}$ is a vector of n_a ones) and $\boldsymbol{s}_t$ contains the n_s predictors. Thus:

$$\Delta \mathbf{z}_t = \begin{pmatrix} r_{0,t} \\ \mathbf{x}_t \\ \mathbf{s}_t \end{pmatrix} \tag{3.1}$$

is of order $((1 + n_a + n_s) \times 1)$. Assume that a VAR model of order p captures the dynamic relationships between the $n = 1 + n_a + n_s$ asset returns and the state variables:

$$\Delta \mathbf{z}_t = \boldsymbol{\mu} + \mathbf{B}_1 \Delta \mathbf{z}_{t-1} + \cdots + \mathbf{B}_p \Delta \mathbf{z}_{t-p} + \mathbf{u}_t, \tag{3.2}$$

where the $\mathbf{B}_j$s are the $(n \times n)$ coefficient matrices, $\boldsymbol{\mu}$ is a $(n \times 1)$ vector of constants and $\mathbf{u}_t = (u_{1t}, ..., u_{nt})'$ is an error term. The shocks $\mathbf{u}_t$ are assumed to be IID normal with time-invariant zero means and variance-covariance matrix $\boldsymbol{\Sigma}_u$.[3]

This standard VAR approach is an established framework for modeling asset return distributions at various investment horizons (see Kandel and Stambaugh, 1996; Barberies, 2000; Campbell, Chan, and Viceira, 2003; Campbell and Viceira, 2005; Jurek and Viceira, 2011). The variables (returns, rates and first differences of indices) in $\Delta \mathbf{z}_t$ are assumed to be stationary, i.e. integrated of order zero ($I(0)$) or, alternatively, to have no stochastic trends. But $\Delta \mathbf{z}_t$ can be transformed to a level vector $\mathbf{z}_t$ to obtain important additional information for its joint long-run behavior. In this case the variables have a strong link among each other and the VAR of the first differences is not the most beneficial framework, since it can distort these relationships (Lütkepohl, 2005, pp. 243–244).

3.2.2 VEC Specification

Although a VAR(p) model is generally able to handle $\mathbf{z}_t$ with stochastic trends, it is not the most adequate form in our context because the variables of interest are $\Delta \mathbf{z}_t$. However, we can rewrite the level VAR(p):

$$\mathbf{z}_t = \boldsymbol{\mu} + \mathbf{A}_1 \mathbf{z}_{t-1} + \cdots + \mathbf{A}_p \mathbf{z}_{t-p} + \mathbf{u}_t \tag{3.3}$$

[3]This restrictive assumption is necessary for the portfolio choice section as in Campbell, Chan, and Viceira (2003) or Jurek and Viceira (2011). Although this assumption is perhaps not observed empirically, it is relatively uncritical as we concentrate on long-term asset allocations. Chacko and Viceira (2005) show that the volatility is not persistent and variable enough to have a considerable impact on portfolio choices of long-term investors compared to the portfolio choices of short-term investors.

to an unrestricted VEC model of order $p-1$ by subtracting both sides of Equation (3.3) with $\mathbf{z}_{t-1}$:

$$\Delta\mathbf{z}_t = \boldsymbol{\mu} + \mathbf{\Pi}\mathbf{z}_{t-1} + \mathbf{\Gamma}_1\Delta\mathbf{z}_{t-1} + \cdots + \mathbf{\Gamma}_{p-1}\Delta\mathbf{z}_{t-p+1} + \mathbf{u}_t, \tag{3.4}$$

where $\mathbf{\Pi} = -(\mathbf{I} - \mathbf{A}_1 - \cdots - \mathbf{A}_p)$ and $\mathbf{\Gamma}_j = -(\mathbf{A}_{j+1} + \cdots + \mathbf{A}_p)$ for $j = 1, ..., p-1$. As can be seen in Equation (3.4), matrix $\mathbf{\Pi}$ summarizes the long-run effects, and the short-run effects remain in the $\mathbf{\Gamma}_j$s.

Since we want to account for not only stochastic trends, but also for cointegration relationships, we apply the cointegration-rank test (Johansen, 1988, 1991) to determine the number $r \leq n$ of cointegration relations. This procedure, based on likelihood ratios, tests whether there is a significant difference between the likelihood of the unrestricted model in Equation (3.4) and the likelihood of a model with $\mathbf{\Pi}_r$ restricted to rank r. Hence, stepwise testing indicates the number of cointegration relations r, which is the most restrictive model without obtaining a significantly different likelihood. After obtaining r, we calculate the decomposition of $\mathbf{\Pi}_r = \boldsymbol{\alpha}\boldsymbol{\beta}'$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $(n \times r)$ matrices, which leads to the reduced rank system:

$$\Delta\mathbf{z}_t = \boldsymbol{\mu} + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{z}_{t-1} + \mathbf{\Gamma}_1\Delta\mathbf{z}_{t-1} + \cdots + \mathbf{\Gamma}_{p-1}\Delta\mathbf{z}_{t-p+1} + \boldsymbol{\nu}_t, \tag{3.5}$$

as shown in Johansen (1996). The matrix $\boldsymbol{\beta}$ contains the cointegration relations, $\boldsymbol{\alpha}$ is a matrix of loadings and $\boldsymbol{\nu}_t$ is an error term, which is assumed to be IID normal with zero means and variance-covariance matrix $\mathbf{\Sigma}_\nu$. Note that in case of $r = 0$, i.e. n independent stochastic trends and no cointegration, the matrix $\mathbf{\Pi}_r = \mathbf{0}$ and the VEC($p-1$) in Equation (3.5) equals a VAR($p-1$) as in Equation (3.2). Otherwise the VEC outperforms the VAR in differences, because taking into account cointegration relations leads to an increase in the likelihood.

3.2.3 Horizon-Dependent Variance-Covariance

We investigate the long-run dynamics implied by the stationary VAR and the VEC by examining the term structure of risk and the horizon-dependent variance decompositions of the asset returns analyzed. The risk statistics are based on the covariance matrix of the residuals, i.e. we take into account return predictability. In the following section, we derive the conditional k-period variance-covariance matrices.

For the VAR model, the conditional k-period variance-covariance matrix, scaled by the investment horizon, is calculated as follows (see e.g. Campbell and Viceira, 2004):

$$\begin{aligned}\frac{1}{k}Var_t\left(\Delta\mathbf{z}_{t+1}+\cdots+\Delta\mathbf{z}_{t+k}\right)&=\frac{1}{k}Var_t\left(\mathbf{z}_{t+k}-\mathbf{z}_t\right)=\frac{1}{k}Var_t\left(\mathbf{z}_{t+k}\right)\\&=\frac{1}{k}\mathbf{M}'\Big[\boldsymbol{\Sigma}+\left(\mathbf{I}+\mathbf{B}\right)\boldsymbol{\Sigma}\left(\mathbf{I}+\mathbf{B}\right)'\\&\quad+\left(\mathbf{I}+\mathbf{B}+\mathbf{B}^2\right)\boldsymbol{\Sigma}\left(\mathbf{I}+\mathbf{B}+\mathbf{B}^2\right)'+\cdots\\&\quad+\left(\mathbf{I}+\mathbf{B}+\cdots+\mathbf{B}^{(k-1)}\right)\boldsymbol{\Sigma}\left(\mathbf{I}+\mathbf{B}+\cdots+\mathbf{B}^{(k-1)}\right)'\Big]\mathbf{M},\end{aligned}\tag{3.6}$$

where:

$$\mathbf{B}=\begin{pmatrix}\mathbf{B}_1&\mathbf{B}_2&\mathbf{B}_3&\cdots&\mathbf{B}_p\\\mathbf{I}&\mathbf{0}&\mathbf{0}&\cdots&\mathbf{0}\\\mathbf{0}&\mathbf{I}&\ddots&&\vdots\\\vdots&\ddots&\ddots&\mathbf{0}&\vdots\\\mathbf{0}&\cdots&\mathbf{0}&\mathbf{I}&\mathbf{0}\end{pmatrix},\ \boldsymbol{\Sigma}=\begin{pmatrix}\boldsymbol{\Sigma}_u&\mathbf{0}&\cdots&\mathbf{0}\\\mathbf{0}&\mathbf{0}&\cdots&\mathbf{0}\\\vdots&\ddots&\ddots&\vdots\\\mathbf{0}&\cdots&\cdots&\mathbf{0}\end{pmatrix},\ \mathbf{M}=\begin{pmatrix}\mathbf{I}\\\mathbf{0}\\\vdots\\\mathbf{0}\end{pmatrix}\tag{3.7}$$

and $\mathbf{I}$ is the $(n\times n)$ identity matrix and $\mathbf{0}$ is a $(n\times n)$ matrix filled with zeros.

For the VEC model, we start by retransforming the VEC($p-1$) in Equation (3.5) to a VAR(p) in Equation (3.3) by setting $\boldsymbol{\Pi}_r=\boldsymbol{\alpha}\boldsymbol{\beta}'=-(\mathbf{I}-\mathbf{A}_1-\cdots-\mathbf{A}_p)$ and $\boldsymbol{\Gamma}_j=-(\mathbf{A}_{j+1}+\cdots+\mathbf{A}_p)$ for $j=1,...,p-1$. Afterwards, we write the VAR(p) in a VAR(1) representation, where $\mathbf{A}$ is of the same structure as $\mathbf{B}$ in Equation (3.7), replacing the $\mathbf{B}_j$s by the $\mathbf{A}_j$s and derive $\mathbf{z}_{t+k}$ for this VAR(1):

$$\mathbf{z}_{t+k}=\mathbf{M}'\left[\mathbf{c}+\mathbf{A}^{k-1}\boldsymbol{\nu}^*_{t+1}+\mathbf{A}^{k-2}\boldsymbol{\nu}^*_{t+2}+\cdots+\mathbf{A}\boldsymbol{\nu}^*_{t+k-1}+\boldsymbol{\nu}^*_{t+k}\right],$$

where $\mathbf{c}$ includes all deterministic components and $\boldsymbol{\nu}^*_{t+j}=(\boldsymbol{\nu}_{t+j},\mathbf{0},...,\mathbf{0})'$. Finally, we obtain the conditional k-period variance-covariance matrix of the VEC($p-1$), scaled by the investment horizon:[4]

$$\frac{1}{k}Var_t\left(\mathbf{z}_{t+k}\right) = \frac{1}{k}\mathbf{M}'\left[\mathbf{A}^{k-1}\boldsymbol{\Sigma}^*\left(\mathbf{A}^{k-1}\right)' + \mathbf{A}^{k-2}\boldsymbol{\Sigma}^*\left(\mathbf{A}^{k-2}\right)' + \cdots + \mathbf{A}\boldsymbol{\Sigma}^*\mathbf{A}' + \boldsymbol{\Sigma}^*\right]\mathbf{M}, \tag{3.8}$$

where $\boldsymbol{\Sigma}^*$ is of the same structure as $\boldsymbol{\Sigma}$ in Equation (3.7) replacing the $\boldsymbol{\Sigma}_u$ by the $\boldsymbol{\Sigma}_\nu$.

The investigation and comparison of the long-run dynamics implied by the VAR and VEC models are then based on decompositions of the conditional k-period variance of the assets analyzed.[5] The elements of these decompositions are extracted by appropriate selector vectors and matrices applied to $\frac{1}{k}Var_t\left(\mathbf{z}_{t+k}\right)$, as reported in Campbell and Viceira (2004) for the VAR.

3.2.4 Portfolio Choice Problem

Besides analyzing the term structure of risk, we additionally determine the optimal k-period mean-variance portfolios of a buy-and-hold investor. For this purpose we use the loglinear approximation of the k-period portfolio return introduced by Campbell and Viceira (2002) and used in Campbell and Viceira (2004, 2005), which is given by:

$$r_{p,t+k}^{(k)} = r_{0,t+k}^{(k)} + \boldsymbol{\omega}'(k)\mathbf{x}_{t+k}^{(k)} + \frac{1}{2}\boldsymbol{\omega}'(k)\left(\boldsymbol{\sigma}_x^2(k) - \boldsymbol{\Sigma}_{xx}(k)\boldsymbol{\omega}(k)\right), \tag{3.9}$$

where $\boldsymbol{\omega}(k)$ is the $(n_a \times 1)$ vector containing the asset weights, except the weight on the benchmark, with regard to a k-period investment. In Equation (3.9), $\boldsymbol{\sigma}_x^2(k)$ and $\boldsymbol{\Sigma}_{xx}(k)$ are obtained by decomposing $\frac{1}{k}Var_t\left(\mathbf{z}_{t+k}\right)$ into the following block structure:

$$\frac{1}{k}Var_t\left(\mathbf{z}_{t+k}\right) = \begin{pmatrix} \sigma_0^2(k) & \boldsymbol{\sigma}_{0x}'(k) & \boldsymbol{\sigma}_{0s}'(k) \\ \boldsymbol{\sigma}_{0x}(k) & \boldsymbol{\Sigma}_{xx}(k) & \boldsymbol{\Sigma}_{xs}'(k) \\ \boldsymbol{\sigma}_{0s}(k) & \boldsymbol{\Sigma}_{xs}(k) & \boldsymbol{\Sigma}_{ss}(k) \end{pmatrix} \tag{3.10}$$

[4] Some elements of the variance-covariance matrix of a non-stationary VAR in levels diverge as the horizon $k \to \infty$. However, in case of cointegrated variables, the elements of the VEC variance-covariance matrix can be bounded (Lütkepohl, 2005, pp. 258–262).

[5] For example, as the 90-day nominal T-bill, (nTb), is equal to the real T-bill, (rTb), plus the inflation $(infl)$, the k-period variance of the 90-day nominal T-bill can be decomposed as: $Var_t(nTb_{t+k}) = Var_t(rTb_{t+k}) + 2Cov_t(rTb_{t+k}, infl_{t+k}) + Var_t(infl_{t+k})$.

and defining $\boldsymbol{\sigma}_x^2(k) = \text{diag}(\boldsymbol{\Sigma}_{xx}(k))$. The $(n_a \times n_a)$ matrix $\boldsymbol{\Sigma}_{xx}(k)$ denotes the k-period covariance matrix of excess returns, the $(n_a \times 1)$ vector $\boldsymbol{\sigma}_{0x}(k)$ contains the k-period covariances between benchmark asset and excess returns, and $\sigma_0^2(k)$ is the k-period variance of the benchmark asset. The remaining components, $\boldsymbol{\sigma}_{0s}(k)$, $\boldsymbol{\Sigma}_{xs}(k)$ and $\boldsymbol{\Sigma}_{ss}(k)$, are covariances involving the state variables. From Equation (3.9) one can calculate the conditional k-period variance of the log portfolio return as:

$$Var_t\left(r_{p,t+k}^{(k)}\right) = \boldsymbol{\omega}'(k)\boldsymbol{\Sigma}_{xx}(k)\boldsymbol{\omega}(k) + \sigma_0^2(k) + 2\boldsymbol{\omega}'(k)\boldsymbol{\sigma}_{0x}(k), \tag{3.11}$$

and the k-period log expected portfolio return as:

$$\begin{aligned} E_t\left(r_{p,t+k}^{(k)}\right) + \frac{1}{2}Var_t\left(r_{p,t+k}^{(k)}\right) = {} & E_t\left(r_{0,t+k}^{(k)}\right) + \frac{1}{2}\sigma_0^2(k) \\ & + \boldsymbol{\omega}'(k)\left(E_t(\mathbf{x}_{t+k}^{(k)}) + \frac{1}{2}\boldsymbol{\sigma}_x^2(k) + \boldsymbol{\sigma}_{0x}(k)\right). \end{aligned} \tag{3.12}$$

Equation (3.12) shows how to calculate the approximation of the cumulative log expected portfolio return. Note that the expected log return has to be adjusted by one half of the return variance to obtain the log expected return relevant for portfolio optimization (a Jensen's inequality adjustment; see Campbell and Viceira, 2004). This adjustment depends on the horizon. There are no horizon effects in expected log returns because we assume that they take the values of their sample counterparts. Thus, for the k-period expected log benchmark return we assume that $k\bar{r}_0$ estimates $E_t\left(r_{0,t+k}^{(k)}\right)$, where $\bar{r}_0$ denotes the sample average of the log benchmark return. Similarly, for the vector of log excess returns we estimate $E_t\left(\mathbf{x}_{t+k}^{(k)}\right)$ by $k\bar{\mathbf{x}}$. Even if there were no horizon effects in expected log returns, there would be horizon effects in log expected returns because conditional variances and covariances will not increase in proportion to the investment horizon unless returns are unpredictable.

Campbell and Viceira (2002, 2004) provide the formula for the solution to the mean-variance problem. They assume an investor with power utility function, i.e. CRRA preferences. Thus, the optimization problem is defined as:

$$\max_{\boldsymbol{\omega}(k)} E_t\left(r_{p,t+k}^{(k)}\right) + \frac{1}{2}(1-\gamma)\,Var_t\left(r_{p,t+k}^{(k)}\right), \tag{3.13}$$

and the closed-form solution without any restrictions follows as:

$$\boldsymbol{\omega}(k) = \frac{1}{\gamma}\boldsymbol{\Sigma}_{xx}^{-1}(k)\left(E_t\left(\mathbf{x}_{t+k}^{(k)}\right) + \frac{1}{2}\boldsymbol{\sigma}_x^2(k)\right) + \left(1-\frac{1}{\gamma}\right)\left(-\boldsymbol{\Sigma}_{xx}^{-1}(k)\boldsymbol{\sigma}_{0x}(k)\right), \tag{3.14}$$

where γ is the coefficient of relative risk aversion. $\boldsymbol{\omega}(k)$ is a combination of two portfolios and the mixture depends on the investor's risk aversion. The first portfolio:

$$\boldsymbol{\Sigma}_{xx}^{-1}(k)\left(E_t\left(\mathbf{x}_{t+k}^{(k)}\right)+\frac{1}{2}\boldsymbol{\sigma}_x^2(k)\right) \tag{3.15}$$

is the *growth optimal* portfolio, the second portfolio is the *global minimum variance* portfolio and is the solution for an extreme risk-averse investor ($\gamma \to \infty$):

$$-\boldsymbol{\Sigma}_{xx}^{-1}(k)\boldsymbol{\sigma}_{0x}(k). \tag{3.16}$$

3.3 Empirical Analysis

3.3.1 Data and Time Series Properties

Our empirical application is based on quarterly post-war data spanning the period 1952:Q1–2010:Q4. Thus, we start shortly after the 1951 Fed-Treasury Accord to avoid problems caused by the essentially fixed short-term nominal rates before 1952.

We use the data set from Goyal and Welch (2008)[6] and extract six time series: Stock prices of the S&P 500 Index and the 12-month moving sums of dividends paid on the S&P 500 Index, 90-day T-bill rates, long-term government bond returns as well as their yields and the inflation rates. The original source of stock prices is the Center for Research in Security Prices (CRSP) and the dividends are from Robert Shiller's website. The T-bills (secondary market) are originally obtained from the Federal Reserve Bank of St. Louis (FRED). The source of the long-term government yields and return data is Ibbotson's *Stocks, Bonds, Bills and Inflation Yearbook*, and the source of the inflation rate is the Consumer Price Index (All Urban Consumers) from the Bureau of Labor Statistics.

These series are used to construct logs of the ex-post real T-bill rates, the excess stock returns (including dividends), the excess bond returns, the dividend-price ratio, the inflation rate and the term (yield) spread.[7] Table 3.1 provides the abbreviations of the variables used.

[6] We would like to thank Amit Goyal for providing an updated version of this data, which is available on his website: http://www.hec.unil.ch/agoyal/.

[7] Hereinafter we write returns and rates instead of log returns and log rates.

Table 3.1: Abbreviations

Variable Definition	Abbreviation
Log ex-post real returns on 90-day T-bill	$d(rTb)$
Log excess return on the S&P 500 index	$d(exSt)$
Log excess return on the 10-year constant marturity Treasury Bond index	$d(exBo)$
Log (1+S&P 500 dividend-price ratio)	$d(dp)$
Log yield on a 10-year Treasury Bond minus the log yield of 90-day T-bill	$d(tms)$
Log inflation rate	$d(infl)$

Notes: The table shows the abbreviations of the (differenced) variables used. The levels of the variables are the accumulated differences of the log variables and abbreviated without $d(\cdot)$.

We compare a stationary VAR and a VEC model with respect to their modeled term structure of risk and asset allocation, where both models include data of the same investable assets (T-bills, stocks and bonds) and common state variables that have been shown to predict returns (dividend-price ratio, term spread and inflation). Previous research has shown that the dividend-price ratio positively forecasts future aggregate stock returns (Campbell and Shiller, 1988a,b; Fama and French, 1988, 1989; Hodrick, 1992; Goetzmann and Jorion, 1993). The term spread is a business cycle indicator and positively predicts excess bond returns (Fama and Bliss, 1987; Fama and French, 1989; Campbell and Shiller, 1991; Campbell, Chan, and Viceira, 2003; Campbell and Vuolteenaho, 2004; Campbell and Viceira, 2005; Jurek and Viceira, 2011). The ex-post real interest rate positively forecasts future excess stock and bond returns (Campbell, Chan, and Viceira, 2003; Campbell and Viceira, 2005; Jurek and Viceira, 2011). Moreover, we use the inflation rate instead of the commonly used ex-ante nominal interest rate (Fama and Schwert, 1977; Campbell, 1987; Glosten, Jagannathan, and Runkle, 1993; Campbell, Chan, and Viceira, 2003; Campbell and Viceira, 2005; Jurek and Viceira, 2011) because we can capture nearly the same dynamics with inflation and real interest rates as with real and nominal interest rates, but we can directly extract inflation influence. The VAR model includes differenced variables shown in Table 3.1, whereas the VEC model includes

Table 3.2: Descriptive Statistics

Variable	Mean	Sd	Sharpe	Min	Max	Skew	Kurt
$d(rTb)$	0.28%	0.74%	-	-1.99%	4.00%	0.31	5.82
$d(exSt)$	1.65%	8.02%	0.21	-30.80%	19.31%	-0.88	4.65
$d(exBo)$	0.48%	5.08%	0.09	-19.22%	20.10%	0.43	5.36
$d(dp)$	0.81%	0.30%		0.28%	1.54%	0.31	2.55
$d(tms)$	0.38%	0.34%		-0.77%	1.11%	-0.03	2.87
$d(infl)$	0.90%	0.90%		-3.99%	4.19%	0.09	7.03

Notes: The table reports summary statistics of the sample from 1952:Q1 to 2010:Q4 (236 data points). Mean log returns are adjusted by one half of the variance to reflect log mean (gross) returns. “Sd” denotes standard deviation, “Sharpe” denotes Sharpe ratio, “Min” denotes minimum, “Max” denotes maximum, “Skew” denotes skewness and “Kurt” denotes kurtosis of the time series.

the accumulated differences of these variables, which we denote as level variables. The differenced variables are commonly used in the traditional VAR framework and can be interpreted easily (e.g. returns, rates). While the level representation of the real rate, returns and inflation rate can be interpreted as prices, the interpretation of the accumulated log dividend-price ratio and term spread seems less intuitive. However, the accumulations of the dividend-price ratio and the term spread can be seen as income indices of stocks and bonds, respectively. In detail, the dividend-price ratio is also considered as dividend yield (Fama and French, 1990), the income return of stocks. Accumulating the logs of income returns leads to an income return index which is comparable to the approach of accumulating the logs of total returns to obtain a total return index. Similarly, long-term yields can be understood as income an investor earns if she holds the bond until maturity. Hence, we interpret the accumulation of the logs of term spreads (long-term yields in excess of T-bill rates) as income index of bonds (in excess of T-bill prices).

Tables 3.2 and 3.3 provide an overview of the sample statistics and correlation of the variables used in the VAR and VEC models. We see that real T-bills have a low return and low variability. The risk premium of bonds is only about a third of the equity premium, although the standard deviation is 5.1% compared to 8% and results in a very low Sharpe ratio for bond investments. As we transform the

Table 3.3: Simultaneous and Lagged Correlations

Panel A						
Variable	$d(rTb_t)$	$d(exSt_t)$	$d(exBo_t)$	$d(dp_t)$	$d(tms_t)$	$d(infl_t)$
$d(rTb_t)$	1	0.090	0.257	0.017	-0.072	-0.661
$d(exSt_t)$		1	0.083	-0.086	0.121	-0.172
$d(exBo_t)$			1	-0.065	0.205	-0.337
$d(dp_t)$				1	-0.230	0.303
$d(tms_t)$					1	-0.283
$d(infl_t)$						1

Panel B						
Variable	$d(rTb_t)$	$d(exSt_t)$	$d(exBo_t)$	$d(dp_t)$	$d(tms_t)$	$d(infl_t)$
$d(rTb_{t+1})$	0.278	0.109	0.104	0.030	0.031	-0.066
$d(exSt_{t+1})$	-0.010	0.114	0.095	0.131	0.112	-0.067
$d(exBo_{t+1})$	-0.021	-0.108	-0.058	-0.021	0.185	0.035
$d(dp_{t+1})$	0.018	-0.108	-0.097	0.971	-0.259	0.315
$d(tms_{t+1})$	-0.044	0.052	0.237	-0.190	0.841	-0.224
$d(infl_{t+1})$	-0.076	-0.143	-0.215	0.276	-0.323	0.484

Notes: Panel A reports simultaneous correlations between the variables: 90-day T-bill, stocks, bonds, dividend-price ratio, term spread and inflation. Panel B reports the lagged correlations of these variables with the first-order autocorrelations on the main diagonal.

variables to levels, we calculate the dividend-price ratio as $dp_t = ln(1 + D_t/P_t)$ instead of the normally used $dp_t = ln(D_t/P_t)$ to avoid wrong scaling effects and hence obtain a positive mean for this dividend-price rate. At first sight, the low correlation between the risk premia of stocks and bonds indicates a good diversification potential. Furthermore, the dividend-price ratio and term spread are very persistent variables, as indicated by their high AR(1) coefficients (0.971 and 0.841).

Since we want to account for common long-run behavior, we analyze the time-series properties of our data with respect to unit roots and cointegration. Panel A in Table 3.4 presents the results of the univariate test statistics of the augmented Dickey-Fuller test (ADF), with the null of a unit root. The tests are performed by allowing for an intercept and by setting the number of lagged differences as

Table 3.4: Unit Root and Cointegration Rank Test

Panel A	Level			Difference	
Variable	ADF-Stat.	Lags		ADF-Stat.	Lags
rTb	-0.503	2	$d(rTb)$	-8.553***	1
$exSt$	-1.504	2	$d(exSt)$	-10.598***	1
$exBo$	0.315	2	$d(exBo)$	-11.347***	1
dp	-1.487	2	$d(dp)$	-2.594*	1
tms	2.158	2	$d(tms)$	-4.103***	1
$infl$	-0.231	2	$d(infl)$	-6.281***	1

Panel B				
H_0	LR	10% CV	5% CV	1% CV
$r \leq 5$	0.09	6.5	8.18	11.65
$r \leq 4$	11.8	15.66	17.95	23.52
$r \leq 3$	32.77**	28.71	31.52	37.22
$r \leq 2$	67.38***	45.23	48.28	55.43
$r \leq 1$	106.25***	66.49	70.6	78.87
$r = 0$	162.38***	90.39	85.18	104.2

Notes: Panel A reports the results of the augmented Dickey-Fuller test and Panel B reports the results of the Johansen (1988) trace test. *, **, *** denote significance at the 10%, 5%, 1% levels, respectively. The augmented Dickey-Fuller test including a constant in the model is performed to test for unit root. The Johansen (1988) trace test including a constant in the model is perfomed to determine the cointegration rank. The number of lagged terms used in both tests is chosen as suggested by the SC (see Appendix 3.A.2).

suggested by the Schwarz Criterion (SC) for the multivariate models (see Appendix 3.A.2). The non-stationarity hypothesis can be accepted for all levels, but is strongly rejected for the first differences except for the dividend-price ratio, which is only rejected on a 10% level. There is an ongoing discussion about the non-stationarity of the dividend-price ratio (see e.g. Goyal and Welch, 2003; Cochrane, 2008; Lettau and Van Nieuwerburgh, 2008). However, taking an economical position, we treat the dividend-price ratio as $I(0)$.[8]

[8] Using the present value identity $d_t - p_t = E_t \sum \rho^{j-1}(-\Delta d_{t+j} + r_{t+j})$ derived by Campbell and Shiller (1988a), we infer the stationarity of the dividend-price ratio from the stationarity of Δd_t and r_t (as shown in Goyal and Welch, 2003).

After identifying stochastic trends in each of the level variables, we apply the Johansen trace test to determine the number of cointegration relations. The test is applied by allowing for an intercept and by setting the number of lags in the unrestricted level VAR equal to two, as suggested by the SC (see Appendix 3.A.2). The test indicates four cointegration relations at the 5% level, as shown in Panel B, meaning there are four stationary linear combinations among the level variables that have an influence on the differenced variables.[9]

3.3.2 Estimation Results

Since the VEC model is an extension of the stationary VAR model, we show the VAR estimation results first. Afterwards, we discuss the VEC estimates and investigate the differences of the two models with respect to the term structure of risk and optimal asset allocation.

Table 3.5 presents the OLS estimates of the VAR model of order one. The number of lags is chosen according to the SC statistics (see Appendix 3.A.2).[10] Panel A reports the slope coefficients and adjusted R^2 with bootstrapped 95% confidence intervals and F-statistics given in brackets and parentheses below. Since there is suspicion of autocorrelation and heteroskedasticity in the residuals, we perform a bootstrap to analyze the significance of the point estimates, as the common t-statistics might be biased. This bootstrap is performed under the null hypothesis that the initial estimated model truly generates the data process.[11] The coefficients indicated to be significant by the bootstrap intervals are boldfaced. Panel B shows the standard deviations of the innovations on the main diagonal and the cross-correlations above the main diagonal.

[9] The results of the cointegration rank test can vary with the number of lags included in the VAR. However, for up to four lags in the VAR, our results remain stable around three to four relations depending on different significance levels.

[10] To assure the comparability to other studies (see e.g. Campbell, Chan, and Viceira, 2003; Campbell and Viceira, 2005; Hoevenaars, Molenaar, Schotman, and Steenkamp, 2008; Jurek and Viceira, 2011), we follow the suggestion of the SC and HQ criterion and do not use four lags as recommended by the AIC. The results, however, are robust with respect to the number of lags.

[11] Appendix 3.A.1 provides a more detailed description of the bootstrapping method .

Table 3.5: VAR Parameter Estimates

Panel A	$\mathbf{A}_1$ – Coefficients of Lagged Variables						*adj.* R^2
	$d(rTb_t)$	$d(exSt_t)$	$d(exBo_t)$	$d(dp_t)$	$d(tms_t)$	$d(infl_t)$	(F-stat.)
$d(rTb_{t+1})$	**0.491**	0.009	0.009	-0.096	0.280	**0.284**	11.06%
	[0.254,0.748]	[-0.002,0.02]	[-0.01,0.028]	[-0.495,0.302]	[-0.029,0.703]	[0.091,0.558]	(5.85)
$d(exSt_{t+1})$	-1.554	0.102	0.101	**5.673**	1.859	-1.473	4.22%
	[-4.557,1.488]	[-0.032,0.224]	[-0.11,0.319]	[3.475,12.663]	[-2.121,6.577]	[-4.304,1.377]	(2.717)
$d(exBo_{t+1})$	**0.908**	-0.075	-0.089	-0.433	**3.992**	**0.871**	4.51%
	[0.053,4.084]	[-0.157,0.009]	[-0.221,0.044]	[-3.221,2.969]	[2.348,7.873]	[0.156,3.93]	(2.84)
$d(dp_{t+1})$	0.012	-0.001	-0.001	**0.946**	-0.027	0.011	94.53%
	[-0.018,0.037]	[-0.002,0]	[-0.003,0.001]	[0.881,0.964]	[-0.071,0.008]	[-0.017,0.034]	(674.4)
$d(tms_{t+1})$	0.018	-0.002	0.004	-0.018	**0.852**	0.022	70.53%
	[-0.035,0.109]	[-0.005,0.001]	[-0.001,0.009]	[-0.115,0.115]	[0.731,0.934]	[-0.025,0.108]	(94.4)
$d(infl_{t+1})$	**0.469**	-0.005	-0.010	0.120	-0.211	**0.677**	33.81%
	[0.143,0.667]	[-0.017,0.007]	[-0.03,0.009]	[-0.325,0.512]	[-0.665,0.105]	[0.339,0.828]	(20.9)

Continued

Continued

Panel B

	$\Sigma_\mathbf{u}$ – Residual Correlations and Standard Deviations					
	$d(rTb)$	$d(exSt)$	$d(exBo)$	$d(dp)$	$d(tms)$	$d(infl)$
$d(rTb)$	(0.007)	0.102	0.273	-0.103	-0.210	-0.949
$d(exSt)$	-	(0.078)	0.097	-0.917	0.063	-0.139
$d(exBo)$	-	-	(0.049)	-0.205	0.089	-0.447
$d(dp)$	-	-	-	(0.001)	-0.095	0.166
$d(tms)$	-	-	-	-	(0.002)	-0.064
$d(infl)$	-	-	-	-	-	(0.007)

Notes: Panel A reports coefficient estimates of the VAR $\Delta\mathbf{z}_t = \boldsymbol{\mu} + \mathbf{A}_1\Delta\mathbf{z}_{t-1} + \mathbf{u}_t$ with variables: 90-day T-bill, stocks, bonds, dividend-price ratio, term spread and inflation. Bootstrap intervals are calculated from 10,000 paths under the assumption that the initial estimated VAR model truly generates the data process and are reported in brackets. Bold coefficients imply significance on a 5% level. The last column displays the adjusted R^2 and the F-statistic. Panel B reports the covariance structure of the residuals showing the standard deviations of the innovations on the main diagonal in parentheses and the cross-correlations above the main diagonal.

The first row in Panel A of Table 3.5 represents the prediction equation for the real T-bill rates. It shows that only the own lag and the lagged inflation have a significant positive influence on the real interest rate. The second and third rows correspond to the risk premia of stocks and bonds and these variables seem to be difficult to predict, as they have the lowest R^2s. However, the lagged dividend-price ratio has a positive significant coefficient in the excess stock equation. Excess bond returns are significantly positively explained by the real interest rate, the term spread and inflation. The last three rows represent the state variable equations and show their very persistent autoregressive behavior. Additionally, the real interest rate has a significant positive influence on the inflation rate. Turning to the covariance structure of the innovations in Panel B, we see that the innovations in risk premia are slightly positively correlated to each other and both are positively correlated with shocks to the real T-bill rates. Unexpected excess stock returns are almost perfectly negatively correlated with shocks to the dividend-price ratio. Unexpected excess bond returns are slightly positively correlated to innovations in the term spread and are negatively correlated with shocks to inflation. Moreover, unexpected inflation is almost perfectly negatively correlated with unexpected real interest rates. These findings are in line with Campbell, Chan, and Viceira (2003); Campbell and Viceira (2005); Hoevenaars, Molenaar, Schotman, and Steenkamp (2008) and Jurek and Viceira (2011).

While the standard VAR model captures only the long-run dynamics of stationary data, the VEC model takes into account information of the four cointegration relations and is able to distinguish between short- and long-run effects. The results of the reduced rank first-order VEC presented in Equation (3.5), as detected by the lag length selection criterion (see Appendix 3.A.2), are displayed in Table 3.6. Panel A of Table 3.6 shows the loadings $\boldsymbol{\alpha}$ for the four normalized cointegration relations $\boldsymbol{\beta}'$ displayed in Panel B. $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}'$ describe the long-run behavior of the data. The short-run effects $\boldsymbol{\Gamma}_1$, the adjusted R^2 and the F-statistics for the full system are illustrated in Panel C. In these three panels bootstrapped 95% confidence intervals are given in brackets below the coefficient estimates. The coefficients indicated to be significant by the bootstrap intervals are boldfaced. Panel D at the bottom summarizes the covariance structure of the innovations in the VEC system, where we show the standard deviation of the innovations on the main diagonal and the cross-correlations off the diagonal.

Before we investigate the captured long-run effects on $\Delta\mathbf{z}$ where we must interpret $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}'$ simultaneously, we analyze the cointegration relations between the level variables presented in Panel B. The first row of $\boldsymbol{\beta}'$ represents the long-run cointegration relation among real T-bill prices, the income index of bonds and the Consumer Price Index (CPI), in which only the real T-bill prices are significant. According to this equation, real T-bill prices are independent of the income index of bonds and the CPI in the long run. While the independence between real T-bill prices and the income index of bonds is consequence of the construction of the income index as it is measured in excess of real T-bills, the independence of real T-bill prices and the CPI can be explained by the Fisher hypothesis (Fisher, 1930).

Table 3.6: VEC Parameter Estimates

Panel A

	$\boldsymbol{\alpha}$ – Long-Run Loadings of Level Equations			
	$ect1$	$ect2$	$ect3$	$ect4$
$d(rTb_{t+1})$	0.003	0.005	0.008	**-0.046**
	[-0.065,0.03]	[-0.013,0.018]	[-0.045,0.056]	[-0.088,-0.002]
$d(exSt_{t+1})$	0.221	**-0.113**	**0.641**	**0.598**
	[-0.446,0.606]	[-0.355,-0.003]	[0.077,1.272]	[0.331,1.326]
$d(exBo_{t+1})$	**0.448**	-0.028	0.013	**-0.355**
	[0.203,0.866]	[-0.163,0.062]	[-0.413,0.344]	[-0.768,-0.112]
$d(dp_{t+1})$	-0.004	0.000	-0.005	**-0.003**
	[-0.007,0.002]	[-0.001,0.002]	[-0.01,0]	[-0.009,-0.001]
$d(tms_{t+1})$	**0.013**	-0.003	**0.019**	0.009
	[0.002,0.027]	[-0.007,0.001]	[0.006,0.034]	[-0.002,0.021]
$d(infl_{t+1})$	-0.028	-0.001	-0.032	**0.044**
	[-0.061,0.037]	[-0.014,0.018]	[-0.083,0.023]	[0.004,0.09]

Panel B

	$\boldsymbol{\beta}'$ – Long-Run Level Equations					
	rTb_t	$exSt_t$	$exBo_t$	dp_t	tms_t	$Infl_t$
$ect1$	**1**	0	0	0	0.310	-0.305
					[-0.516,1.577]	[-0.87,0.11]
$ect2$	0	**1**	0	0	**-2.724**	0.216
					[-5.136,-0.686]	[-0.693,1.395]
$ect3$	0	0	**1**	0	**-1.503**	0.086
					[-2.56,-0.78]	[-0.298,0.557]
$ect4$	0	0	0	**1**	-0.504	**-0.459**
					[-1.324,0.401]	[-0.858,-0.024]

Continued

Continued

Panel C	$\mathbf{\Gamma_1}$ – Short-Run Loading of Lagged Differences						*adj.* R^2
	$d(rTb_t)$	$d(exSt_t)$	$d(exBo_t)$	$d(dp_t)$	$d(tms_t)$	$d(infl_t)$	(F-stat.)
$d(rTb_{t+1})$	1.393	0.003	**0.021**	-1.022	1.477	1.258	18.07%
	[-0.759,3.4]	[-0.01,0.014]	[0.002,0.043]	[-3.219,0.58]	[-0.625,3.513]	[-0.902,3.347]	(9.01)
$d(exSt_{t+1})$	19.244	**0.176**	-0.021	7.729	18.284	19.524	10.30%
	[-4.809,43.658]	[0.039,0.31]	[-0.273,0.19]	[-14.721,27.682]	[-5.835,42.567]	[-5.064,44.401]	(3.95)
$d(exBo_{t+1})$	10.342	-0.025	0.007	-1.354	13.070	10.792	13.75%
	[-4.335,26.622]	[-0.101,0.072]	[-0.119,0.168]	[-16.539,10.667]	[-1.804,29.553]	[-4.1,27.307]	(4.38)
$d(dp_{t+1})$	-0.149	**-0.002**	0.000	**0.787**	-0.168	-0.152	94.89%
	[-0.361,0.069]	[-0.003,-0.001]	[-0.002,0.002]	[0.574,0.957]	[-0.38,0.046]	[-0.367,0.069]	(3399.6)
$d(tms_{t+1})$	**0.813**	0.000	0.003	-0.013	**1.541**	**0.835**	73.01%
	[0.277,1.388]	[-0.002,0.004]	[-0.003,0.008]	[-0.443,0.561]	[0.939,2.064]	[0.288,1.428]	(159.1)
$d(infl_{t+1})$	-1.615	-0.003	**-0.023**	1.055	**-2.469**	-1.514	39.04%
	[-3.812,0.516]	[-0.015,0.01]	[-0.045,-0.004]	[-0.647,3.311]	[-4.629,-0.295]	[-3.77,0.653]	(49.8)

Continued

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Panel D						
	$\mathbf{\Sigma}_\nu$ – Residual Correlations and Standard Deviations					
	$d(rTb)$	$d(exSt)$	$d(exBo)$	$d(dp)$	$d(tms)$	$d(infl)$
$d(rTb)$	(0.007)	0.176	0.269	-0.171	-0.193	-0.952
$d(exSt)$	-	(0.075)	0.121	-0.923	-0.016	-0.192
$d(exBo)$	-	-	(0.046)	-0.219	0.059	-0.433
$d(dp)$	-	-	-	(0.001)	-0.022	0.213
$d(tms)$	-	-	-	-	(0.002)	-0.071
$d(infl)$	-	-	-	-	-	(0.007)

Notes: Panel A, B, C report long- and short-run coefficient estimates of the VEC $\Delta \mathbf{z}_t = \boldsymbol{\mu} + \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{z}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{z}_{t-1} + \boldsymbol{\nu}_t$ with the level variables: 90-day T-bill, stocks, bonds, dividend-price ratio, term spread and inflation. Bootstrap intervals are calculated from 10,000 paths under the assumption that the initial estimated VEC model truly generates the data process and are reported in brackets. Bold coefficients imply significance on a 5% level. The last column of Panel C displays the adjusted R^2 and the F-statistic. Panel D reports the covariance structure of the residuals showing the standard deviations of the innovations on the main diagonal in parentheses and the cross-correlations above the main diagonal.

The second and third equations provide the relations between the income index of bonds, the CPI and the stock and bond prices, respectively. Both stock and bond prices are related *positively* to the income index of bonds, but they are not related significantly to the CPI in the long run. The positive long-run connection between stock prices and the income level of bonds might be influenced by the overall market condition. In a good economic condition stock prices as well as interest rates are generally high which also implies a high income level of bonds. For bonds it seems that the income level and the total bond index move together in the long run. The last row of $\boldsymbol{\beta}'$ shows the cointegration relation of income index of stocks and bonds and the CPI. According to this equation, the income level of stocks is related positively to general prices in the long-run which is intuitive as investors require a higher income level in case of rising prices.

Turning to the influence of $\boldsymbol{\beta}'$ on $\Delta\mathbf{z}$, we have to consider the corresponding loadings in $\boldsymbol{\alpha}$ (Panel A) as well. Only the fourth entry of the first row of $\boldsymbol{\alpha}$ is significant, indicating an influence of the fourth cointegration relation between the income index of stocks and CPI on real T-bill rates in the long term. In the presence of a disequilibrium in the income level of stocks and the CPI, it seems that an adjustment on real T-bills takes place. Considering the loadings of the equity premium, we find that a deviation of the long-run comovement between the income level of stocks and the CPI influences the equity premium significantly. The long-run effect of the bond income index is canceled out due to contrary signs in the loadings. Excess bond returns are positively predicted by real T-bill prices. Moreover, as for real T-bills and stocks it seems that also excess bond returns are affected by a long-run divergence between the income level of stocks and the CPI.

Panel C of Table 3.6 summarizes the short-run effects estimated by the VEC model. For the real interest rate equation only the lagged bond premium has a positive and significant coefficient. The row corresponding to excess stock returns shows that their own lagged returns predict stock premia positively and all other predictors are insignificant in this equation. However, none of the differenced variables has significant influence on excess bond returns in the short run. The state variables dividend-price and term spread are strongly influenced by their own lags and show a very persistent autoregressive behavior. Moreover, some short-term cross-forecasting effects can be observed for the state variables.

The covariance structure of the innovations is described in Panel D. We see that the innovations in risk premia are positively correlated to each other and with shocks to the real T-bill rates. Unexpected excess stock returns are almost perfectly negatively correlated with shocks to the dividend-price ratio. Unexpected excess bond returns are weakly positively correlated to innovations in the term spread and are negatively correlated with shocks to inflation. Moreover, unexpected inflation is almost perfectly negatively correlated with unexpected real interest rates.

Comparing the stationary VAR and the VEC model, we obtain a more than two times higher adjusted R^2 for predicting the risk premia of stocks and bonds.[12]

[12] Including two lags in the stationary VAR does not significantly increase the adjusted R^2 compared to the VAR(1) or capture the horizon effects of the VEC(1).

These increases show the importance of incorporating common long-run effects for capturing the realized stock and bond return dynamics. Turning to the estimated coefficients, we can only compare the short-run matrix (influence of the stationary variables) with the VAR coefficients matrix. However, some interesting changes are apparent. Significant predictors for real interest rates (lagged real interest rate and inflation) in the VAR become insignificant in the VEC model, while lagged excess bond returns now have a positive significant influence in the short run. The equity premium shows a momentum effect in the VEC, but in contrast to the stationary VAR, the dividend-price ratio has no significant short-term influence anymore. There are obviously no significant short-run effects for the bond premium in the VEC. Finally, it is interesting to note that the autoregressive component of the inflation disappears in the VEC model. Considering the covariance structure of the innovations, we see that the correlations differ only slightly. These different results motivate a further comparison of the long-run dynamics implied by the VAR and VEC, depending on the time horizon.

3.3.3 Long Horizon Effects

In the following section we analyze the k-period horizon effects by considering the term structure of risk, the horizon-dependent correlations and variance decompositions of real and nominal asset returns. To get a deeper insight into the horizon effects, we appropriately split the k-period return variances into components consisting of risk premium, real interest rate and inflation and integrate the arising covariance terms into our discussion.

Table 3.7 reports the term structure of risk and correlations of the real interest rate and the risk premia implied by the VAR (Panel A) and VEC (Panel B) model depending on the investment horizon (quarters) and the bootstrapped 95% confidence intervals in brackets below. Standard deviations and corresponding bootstrap intervals are reported in percentage. The model comparison shows some important differences in the term structure. While in both models the periodic long-term return volatilities of real T-bills are higher than their short-term volatilities, the mean aversion effect is more pronounced for the VAR model, leading to a nearly 70%

Table 3.7: Term Structure of Risk and Correlations

Panel A	Horizon k (quarters)				
VAR	1	5	10	50	100
$Sd_t(rTb_{t+k})$	**0.697**	**0.840**	**0.852**	**1.081**	**1.419**
	[0.58,0.8]	[0.67,0.98]	[0.68,1.01]	[0.76,1.41]	[0.79,2]
$Sd_t(exSt_{t+k})$	**7.868**	**7.993**	**7.499**	**5.251**	**4.406**
	[6.77,8.71]	[6.48,9.05]	[5.72,8.63]	[3.84,6.68]	[3.44,5.91]
$Sd_t(exBo_{t+k})$	**4.979**	**4.673**	**4.682**	**3.914**	**3.123**
	[4.26,5.55]	[3.68,5.37]	[3.32,5.51]	[2.15,4.99]	[1.86,4.18]
$Cor_t(rTb_{t+k}, exSt_{t+k})$	0.102	0.171	0.149	-0.070	-0.140
	[-0.11,0.31]	[-0.10,0.41]	[-0.13,0.41]	[-0.34,0.38]	[-0.46,0.42]
$Cor_t(rTb_{t+k}, exBo_{t+k})$	**0.273**	0.187	0.077	-0.359	-0.329
	[0.08,0.44]	[-0.07,0.42]	[-0.21,0.37]	[-0.55,0.18]	[-0.56,0.23]
$Cor_t(exSt_{t+k}, exBo_{t+k})$	0.097	0.134	0.235	0.393	0.321
	[-0.07,0.26]	[-0.14,0.37]	[-0.13,0.49]	[-0.21,0.63]	[-0.23,0.57]

Panel B	Horizon k (quarters)				
VEC	1	5	10	50	100
$Sd_t(rTb_{t+k})$	**0.656**	**0.776**	**0.859**	**0.802**	**0.844**
	[0.53,0.75]	[0.58,0.86]	[0.57,0.96]	[0.39,1.01]	[0.33,1.21]
$Sd_t(exSt_{t+k})$	**7.467**	**6.523**	**5.473**	**4.144**	**3.774**
	[6.28,8.16]	[4.91,7.19]	[3.83,6.12]	[2.18,4.82]	[1.74,4.60]
$Sd_t(exBo_{t+k})$	**4.64**	**3.383**	**3.01**	**3.421**	**3.158**
	[3.91,5.14]	[2.61,3.9]	[2.19,3.59]	[1.46,4.13]	[1.16,4.02]
$Cor_t(rTb_{t+k}, exSt_{t+k})$	0.176	**0.463**	**0.565**	0.317	-0.253
	[-0.05,0.38]	[0.18,0.62]	[0.20,0.69]	[-0.44,0.69]	[-0.73,0.62]
$Cor_t(rTb_{t+k}, exBo_{t+k})$	**0.269**	0.230	0.326	0.256	-0.287
	[0.06,0.46]	[-0.04,0.49]	[-0.04,0.60]	[-0.54,0.68]	[-0.74,0.57]
$Cor_t(exSt_{t+k}, exBo_{t+k})$	0.121	0.20	**0.327**	**0.787**	**0.846**
	[-0.06,0.29]	[-0.06,0.43]	[0.01,0.57]	[0.24,0.88]	[0.30,0.92]

Notes: This table reports the term structure of risk and correlations of the assets implied by the VAR (Panel A) and by the VEC model (Panel B) depending on the investment horizon. Bootstrap intervals are calculated from 10,000 paths under the assumption that the initial estimated VAR or VEC model truly generates the data process and are reported in brackets. Bold coefficients imply significance on a 5% level. Standard deviations and corresponding bootstrap intervals are reported in percentage.

higher standard deviation for the 100-period horizon. This mean-aversion arises primarily from the persistent behavior of real T-bills, which is also found by Campbell and Viceira (2005). The weaker mean-averting shape in the VEC is possibly caused by an offsetting influence of inflation. Turning to stocks, we expect a decrease in equity premium variation over the investment horizon due to the positive coefficient of the dividend-price ratio on stock returns and the large negative correlation between their innovations. If prices are decreasing unexpectedly, this is bad news for an investor. On the other hand, the good news is that a low realized return on stocks is usually accompanied by positive shocks to the dividend yield and high dividend yield predicts high returns for the future. The mean-reverting effect for excess stock returns is observed in both models, but risk substantially differs at medium horizons (7.5% in the VAR vs. 5.5% in the VEC at a 10-period horizon). This difference between the models is predominantly caused by a stronger mean-reversion effect of the dividend-price ratio in the VEC. Moreover, under cointegration, this effect is reinforced by the longer lasting influence of inflation.

Turning to excess bond returns, the volatilities are fairly similar under the models for very short and long horizons, but the stationary VAR overestimates the risk by about 50% at intermediate horizons. In the intervening periods the volatilities modeled by the VEC are hump-shaped with a much steeper drop until period 10 and a subsequent backward movement to the VAR term structure in the long run. The general mean-reversion behavior of the bond premium is due to the negative correlation of the shocks between the excess bond returns and inflation and weakened by a mean-averting influence of the term spread. However, in the VEC, the term spread positively affects the bond volatility only in the long run. This missing compensation leads to the steep drop in the first quarters. Turning to the asset correlations, we observe that the correlations estimated by the VAR are lower than the ones of the VEC and, according to the bootstrap intervals, not significantly different from zero for medium and long horizons. However, for the VEC model, we obtain significant positive correlations. Real T-bills and excess stock returns are significantly correlated at medium-term and peaks with 0.60 at a horizon of 20 periods. For the risk premia of stocks and bonds the correlation is over 0.80 at long horizons.

Table 3.8: Variance Decomposition for Treasury Bills

Panel A	Horizon k (quarters)				
VAR	1	5	10	50	100
$Var_t(nTb_{t+k})$	**1.196**	**10.09**	**26.97**	**213.7**	**422.7**
	[0.62,1.90]	[4.75,15.6]	[10.9,41.6]	[32.8,348]	[37,788]
$Var_t(rTb_{t+k})$	**11.16**	**16.22**	**16.70**	**26.87**	**46.28**
	[7.90,14.7]	[10.4,22.0]	[10.6,23.3]	[13.5,46.2]	[15,91]
$2Cov_t(rTb_{t+k}, infl_{t+k})$	**-22.09**	**-31.54**	**-29.32**	39.11	130.1
	[-29.7,-15.3]	[-45.4,-18.4]	[-47.6,-13.3]	[-36.3,93.4]	[-32,271]
$Var_t(infl_{t+k})$	**12.13**	**25.41**	**39.59**	**147.7**	**246.3**
	[8.49,16.2]	[15.9,34.2]	[21.1,55.2]	[29.9,241]	[30,468]

Panel B	Horizon k (quarters)				
VEC	1	5	10	50	100
$Var_t(nTb_{t+k})$	**1**	**5.87**	**12.99**	**117.9**	**308.0**
	[0.54,1.54]	[2.90,8.47]	[5.48,18.8]	[14.9,171]	[17,451]
$Var_t(rTb_{t+k})$	**9.885**	**13.84**	**16.96**	**14.80**	**16.38**
	[6.45,13.1]	[7.85,17.2]	[7.47,21.2]	[3.65,23.8]	[2.59,35]
$2Cov_t(rTb_{t+k}, infl_{t+k})$	**-19.63**	**-27.74**	**-35.52**	-34.37	47.64
	[-26.9,-12.5]	[-36.6,-14.8]	[-47.8,-13.2]	[-64.7,26.1]	[-30,112]
$Var_t(infl_{t+k})$	**10.74**	**19.77**	**31.55**	**137.4**	**244.0**
	[6.90,14.7]	[11.1,25.6]	[14.3,41.4]	[13.4,181]	[12,349]

Notes: This table reports the variance decompositions of nominal Treasury bill returns implied by the VAR (Panel A) and by the VEC model (Panel B) depending on the investment horizon. The first row of each panel shows the variances of nominal T-bills and is normalized to the first entry (one horizon variance) of the VEC panel. The decomposition for the k-period variance of the nominal Treasury bill return is $Var_t(nTb_{t+k}) = Var_t(rTb_{t+k}) + 2Cov_t(rTb_{t+k}, infl_{t+k}) + Var_t(infl_{t+k})$. Bootstrap intervals are calculated from 10,000 paths under the assumption that the initial estimated VAR or VEC model truly generates the data process and are reported in brackets. Bold coefficients imply significance on a 5% level.

In Table 3.8 we report the variance decompositions of nominal T-bill returns implied by the VAR (Panel A) and the VEC model (Panel B), depending on the investment horizon and the bootstrapped 95% confidence intervals in brackets below. This gives us further insight into the interaction of innovations of the short rates and the inflation. The results are interpreted as follows. The first row of each panel shows the variances of nominal T-bills and is normalized to the first entry (one horizon variance) of the VEC panel. The horizon-dependent variance of nominal T-bills is decomposed into the variances and covariance of the real T-bills and inflation.

This normalization of the panels enables us to identify the horizon effects in the variances and covariances (column by column) and the components of the nominal T-bills variance (row by row), as well as to ensure the comparability of the two panels. As we have already found for the real T-bills, the nominal T-bills also show a mean-averting behavior, but with a much steeper increase in volatility. Due to the risk reduction of the covariance at short and medium horizons, the components offset each other, leading to a slightly lower variation of nominal interest rates compared to the real T-bills. In the long-run, this effect turns upside down, makes nominal T-bills much more risky and is less pronounced in the VEC model. By implication, this indicates a strong common relationship between nominal T-bills and inflation in the long term. This relationship is ignored by the VAR model and leads to an overestimation of the T-bills' mean-averting behavior.

Table 3.9 reports the variance decompositions of nominal and real stock returns implied by the two models (Panel A and B), depending on the investment horizon and the bootstrapped 95% confidence intervals in brackets below. In contrast to

Table 3.9: Variance Decomposition for Stock Returns

Panel A	Horizon (quarters)				
VAR	1	5	10	50	100
$Var_t(nSt_{t+k})$	**1.107**	**1.113**	**0.935**	**0.439**	**0.454**
	[0.82,1.35]	[0.73,1.43]	[0.55,1.24]	[0.27,0.69]	[0.23,0.81]
$Var_t(rSt_{t+k})$	**1.143**	**1.204**	**1.06**	**0.503**	**0.354**
	[0.85,1.39]	[0.79,1.53]	[0.62,1.4]	[0.29,0.80]	[0.23,0.60]
$Var_t(exSt_{t+k})$	**1.114**	**1.15**	**1.012**	**0.496**	**0.349**
	[0.83,1.36]	[0.75,1.47]	[0.59,1.34]	[0.26,0.78]	[0.21,0.62]
$2Cov_t(exSt_{t+k}, rTb_{t+k})$	0.02	0.041	0.034	-0.014	-0.032
	[-0.02,0.05]	[-0.02,0.09]	[-0.03,0.09]	[-0.08,0.07]	[-0.14,0.08]
$Var_t(rTb_{t+k})$	**0.009**	**0.013**	**0.013**	**0.021**	**0.036**
	[0.00,0.01]	[0.00,0.01]	[0.00,0.01]	[0.01,0.03]	[0.01,0.07]
$2Cov_t(rTb_{t+k}, infl_{t+k})$	**-0.017**	**-0.025**	**-0.023**	0.031	0.102
	[-0.02,-0.01]	[-0.03,-0.01]	[-0.03,-0.01]	[-0.02,0.07]	[-0.02,0.21]
$Var_t(infl_{t+k})$	**0.009**	**0.02**	**0.031**	**0.116**	**0.193**
	[0.00,0.01]	[0.01,0.02]	[0.01,0.04]	[0.02,0.18]	[0.02,0.36]
$2Cov_t(exSt_{t+k}, infl_{t+k})$	-0.029	**-0.086**	**-0.133**	-0.21	-0.195
	[-0.06,0.02]	[-0.15,-0.00]	[-0.23,-0.01]	[-0.42,0.04]	[-0.53,0.09]

Continued

Continued

Panel B	Horizon (quarters)				
VEC	1	5	10	50	100
$Var_t(nSt_{t+k})$	**1**	**0.765**	**0.523**	**0.156**	**0.156**
	[0.70,1.20]	[0.43,0.92]	[0.24,0.65]	[0.06,0.22]	[0.04,0.24]
$Var_t(rSt_{t+k})$	**1.042**	**0.861**	**0.648**	**0.359**	**0.24**
	[0.74,1.24]	[0.48,1.04]	[0.30,0.80]	[0.09,0.48]	[0.05,0.34]
$Var_t(exSt_{t+k})$	**1.004**	**0.766**	**0.539**	**0.309**	**0.256**
	[0.71,1.20]	[0.43,0.92]	[0.26,0.67]	[0.08,0.42]	[0.05,0.38]
$2Cov_t(exSt_{t+k}, rTb_{t+k})$	0.031	**0.084**	**0.096**	0.038	-0.029
	[-0.01,0.06]	[0.02,0.11]	[0.02,0.12]	[-0.03,0.07]	[-0.10,0.03]
$Var_t(rTb_{t+k})$	**0.008**	**0.011**	**0.013**	**0.012**	**0.013**
	[0.00,0.01]	[0.00,0.01]	[0.00,0.01]	[0.00,0.01]	[0.00,0.02]
$2Cov_t(rTb_{t+k}, infl_{t+k})$	**-0.015**	**-0.022**	**-0.028**	-0.027	0.037
	[-0.02,-0.01]	[-0.02,-0.01]	[-0.03,-0.01]	[-0.05,0.02]	[-0.02,0.08]
$Var_t(infl_{t+k})$	**0.008**	**0.015**	**0.025**	**0.108**	**0.191**
	[0.00,0.01]	[0.00,0.02]	[0.01,0.03]	[0.01,0.14]	[0.00,0.27]
$2Cov_t(exSt_{t+k}, infl_{t+k})$	-0.035	**-0.09**	**-0.122**	**-0.283**	**-0.311**
	[-0.06,0.01]	[-0.12,-0.02]	[-0.17,-0.03]	[-0.39,-0.01]	[-0.50,-0.00]

Notes: This table reports the variance decompositions of nominal and real stock returns implied by the VAR (Panel A) and by the VEC model (Panel B) depending on the investment horizon. The first row of each panel shows the variances of nominal stock returns and is normalized to the first entry (one horizon variance) of the VEC panel. The decomposition for the k-period variance of the nominal stock return is $Var_t(nSt_{t+k}) = Var_t(exSt_{t+k}) + 2Cov_t(exSt_{t+k}, rTb_{t+k}) + Var_t(rTb_{t+k}) + 2Cov_t(rTb_{t+k}, infl_{t+k}) + Var_t(infl_{t+k}) + 2Cov_t(exSt_{t+k}, infl_{t+k})$ and that of the real stock return is $Var_t(rSt_{t+k}) = Var_t(exSt_{t+k}) + 2Cov_t(exSt_{t+k}, rTb_{t+k}) + Var_t(rTb_{t+k})$. Bootstrap intervals are calculated from 10,000 paths under the assumption that the initial estimated VAR or VEC model truly generates the data process and are reported in brackets. Bold coefficients imply significance on a 5% level.

the T-bills in Table 3.8, the variances of nominal stock returns are decomposed in more detail into variances and covariances of the equity premium, real interest rates and inflation. Again, the first row of each panel shows the variances of nominal stocks and is normalized to the first entry of the VEC panel. The components of the last six rows sum up to the nominal return variance and the components of rows three to five sum up to the real return variance, respectively. For both models, nominal and real stock returns are mean-reverting, whereas the amount of risk reduction is much higher for the VEC model, for which the lowest nominal stock volatility is observed in the very long run. In both panels the variation of excess stock return attributes the most variation to real returns. Hence, the horizon effects of the real interest rate variation are small compared to the absolute stock variation. However, the covariation between real T-bills and excess stock returns adds a significant positive amount in the VEC at medium horizons. This effect, already mentioned by Fama and French (1989), is not significant in the stationary model. While the variances of inflation are fairly similar for the VAR and VEC over all horizons, the covariance between the equity premium and inflation is -0.1 compared to -0.16 and hence more than 50% higher under cointegration for a 100-period horizon. These covariance terms decrease the overall variances of nominal stock returns in both models and overcompensate inflation variation. The finding of a negative covariance is in line with the hypothesis of Modigliani and Cohn (1979), who conclude that stock market investors suffer from a specific form of money illusion, disregarding the effect of changing inflation on cash flow growth. When inflation rises unexpectedly, investors increase discount rates but ignore the impact of expected inflation on expected cash flows, leading to an undervalued stock market, and vice versa. This mispricing should eventually diminish, which would indicate the good inflation-hedging properties of stocks in the long run. However, when allowing for cointegration, stocks remain more risky in real than nominal terms.

Table 3.10 reports the variance decompositions of nominal and real bond returns implied by the two models (Panel A and B), depending on the investment horizon and the bootstrapped 95% confidence intervals in brackets below. The decomposition presented and the interpretation of this table are the same as in Table 3.9, replacing excess stocks by excess bonds. For the real bonds the risk structure decreases continuously in the VAR model, whereas the VEC model shows the hump-shaped term structure of the excess returns.

While in both panels the variation of excess bond returns explains the most variation to real bond returns, the variation and covariation of real T-bills contribute only little fractions. For both models nominal bond returns are mean-reverting up to a horizon of 50 quarters for the VAR and up to 70 quarters for the VEC with a value of 1.6% and 2.5%, respectively. Afterwards, they show a mean-averting behavior. The mean-reversion of excess bond returns is initially reinforced by the negative covariances between the real interest rates and inflation and between the bond premium and inflation. The correlations of real interest rates and inflation are roughly equal and significantly negative in both models at short and medium horizons and the correlations between the bond premium and inflation remain negative and significant over all horizons. The latter effect, however, is much more pronounced in the VEC at long horizons. The variances of the inflation are fairly similar and mean-averting for the VAR and VEC over all horizons. In sum, the increase of the nominal T-bill volatility (which is calculated by $Var_t(rTb)+2Cov_t(rTb,infl)+Var_t(infl)$) cannot be offset by the negative covariations between the bond risk premia and nominal T-bills (which is $2Cov_t(exBo,rTb)+2Cov_t(exBo,infl)$) and leads to the mean-averting behavior of nominal bond returns in the very long run. Actually, the cash flows of a (default risk-free) nominal long-term bond are fixed, so the nominal long-term return does not move with inflation. Standard bond indexes, such as the one used in this paper, do, however, represent a security with constant maturity. In terms of inflation hedging, this means that the return on these bond indexes benefits from

Table 3.10: Variance Decomposition for Bond Returns

Panel A	Horizon (quarters)				
VAR	1	5	10	50	100
$Var_t(nBo_{t+k})$	**1.149**	**0.874**	**0.737**	**0.316**	**0.653**
	[0.83,1.42]	[0.55,1.13]	[0.40,0.98]	[0.18,0.47]	[0.17,1.07]
$Var_t(rBo_{t+k})$	**1.328**	**1.173**	**1.136**	**0.657**	**0.433**
	[0.96,1.65]	[0.73,1.55]	[0.58,1.56]	[0.25,1.05]	[0.20,0.70]
$Var_t(exBo_{t+k})$	**1.211**	**1.067**	**1.071**	**0.749**	**0.476**
	[0.87,1.50]	[0.65,1.41]	[0.54,1.49]	[0.23,1.22]	[0.17,0.85]
$2Cov_t(exBo_{t+k},rTb_{t+k})$	**0.093**	0.072	0.03	-0.149	-0.142
	[0.02,0.16]	[-0.02,0.17]	[-0.08,0.14]	[-0.32,0.04]	[-0.37,0.05]
$Var_t(rTb_{t+k})$	**0.024**	**0.034**	**0.035**	**0.057**	**0.098**
	[0.01,0.03]	[0.02,0.04]	[0.02,0.05]	[0.02,0.09]	[0.03,0.19]
$2Cov_t(rTb_{t+k},infl_{t+k})$	**-0.047**	**-0.067**	**-0.062**	0.083	0.276
	[-0.06,-0.03]	[-0.09,-0.03]	[-0.10,-0.02]	[-0.07,0.19]	[-0.06,0.57]
$Var_t(infl_{t+k})$	**0.026**	**0.054**	**0.084**	**0.314**	**0.523**
	[0.01,0.03]	[0.03,0.07]	[0.04,0.11]	[0.06,0.51]	[0.06,0.99]
$2Cov_t(exBo_{t+k},infl_{t+k})$	**-0.158**	**-0.286**	**-0.422**	**-0.738**	**-0.579**
	[-0.23,-0.08]	[-0.42,-0.13]	[-0.63,-0.16]	[-1.28,-0.08]	[-1.29,-0.01]

Continued

Continued

Panel B	Horizon (quarters)				
VEC	1	5	10	50	100
$Var_t(nBo_{t+k})$	**1**	**0.461**	**0.31**	**0.15**	**0.201**
	[0.71,1.21]	[0.28,0.59]	[0.18,0.43]	[0.06,0.21]	[0.04,0.31]
$Var_t(rBo_{t+k})$	**1.153**	**0.648**	**0.561**	**0.672**	**0.447**
	[0.80,1.42]	[0.38,0.85]	[0.29,0.77]	[0.12,0.90]	[0.07,0.67]
$Var_t(exBo_{t+k})$	**1.052**	**0.559**	**0.443**	**0.572**	**0.487**
	[0.74,1.28]	[0.33,0.73]	[0.23,0.63]	[0.10,0.82]	[0.06,0.78]
$2Cov_t(exBo_{t+k}, rTb_{t+k})$	**0.08**	0.059	0.082	0.069	-0.075
	[0.01,0.15]	[-0.01,0.12]	[-0.01,0.15]	[-0.11,0.15]	[-0.24,0.06]
$Var_t(rTb_{t+k})$	**0.021**	**0.029**	**0.036**	**0.031**	**0.035**
	[0.01,0.02]	[0.01,0.03]	[0.01,0.04]	[0.00,0.05]	[0.00,0.07]
$2Cov_t(rTb_{t+k}, infl_{t+k})$	**-0.042**	**-0.059**	**-0.075**	-0.073	0.101
	[-0.05,-0.02]	[-0.07,-0.03]	[-0.10,-0.02]	[-0.13,0.05]	[-0.06,0.23]
$Var_t(infl_{t+k})$	**0.023**	**0.042**	**0.067**	**0.292**	**0.518**
	[0.01,0.03]	[0.02,0.05]	[0.03,0.08]	[0.02,0.38]	[0.02,0.74]
$2Cov_t(exBo_{t+k}, infl_{t+k})$	**-0.134**	**-0.169**	**-0.243**	**-0.741**	**-0.866**
	[-0.21,-0.06]	[-0.25,-0.07]	[-0.36,-0.09]	[-1.02,-0.06]	[-1.33,-0.02]

Notes: This table reports the variance decompositions of nominal and real bond returns implied by the VAR (Panel A) and by the VEC model (Panel B) depending on the investment horizon. The first row of each panel shows the variances of nominal bond returns and is normalized to the first entry (one horizon variance) of the VEC panel. The decomposition for the k-period variance of the nominal bond return is $Var_t(nBo_{t+k}) = Var_t(exBo_{t+k}) + 2Cov_t(exBo_{t+k}, rTb_{t+k}) + Var_t(rTb_{t+k}) + 2Cov_t(rTb_{t+k}, infl_{t+k}) + Var_t(infl_{t+k}) + 2Cov_t(exBo_{t+k}, infl_{t+k})$ and that of the real bond return is $Var_t(rBo_{t+k}) = Var_t(exBo_{t+k}) + 2Cov_t(exBo_{t+k}, rTb_{t+k}) + Var_t(rTb_{t+k})$. Bootstrap intervals are calculated from 10,000 paths under the assumption that the initial estimated VAR or VEC model truly generates the data process and are reported in brackets. Bold coefficients imply significance on a 5% level.

the reassessments of expected inflation that are incorporated into the bond yield, so that the ability of constant maturity bond returns to hedge unexpected inflation should improve with the investment horizon. However, the reassessment seems to be slow for both models and is stronger for the VAR and makes real returns less risky than nominal returns in the very long run. In contrast, allowing for cointegration implies a stronger risk reduction of nominal bond returns over all horizons and also significantly reduces the risk of real returns in the short term.

3.3.4 Asset Allocation Decisions

For a deeper analysis of the horizon effects in the term structure of risk mentioned above, we investigate the optimal mean-variance portfolio allocations of investors with various holding intervals. We analyze two types of portfolios. One portfolio is the global minimum variance (GMV) portfolio and the second portfolio represents a less risk-averse investor with a risk aversion of $\gamma = 20$. For the GMV portfolio, only risk statistics are taken into account for the optimal decision, while for the portfolio of an investor with lower risk-aversion, the term structure of expected returns is also relevant.

Table 3.11 shows four cases of GMV portfolio allocations for investment horizons of up to 25 years and bootstrapped 95% confidence intervals in brackets below. We consider VAR and VEC-based investors allocating their wealth by taking into account real and nominal returns. Panels A and B report the composition of the VAR and VEC models based on real returns. In these cases, very risk-averse investors hold most of their money in cash because it is the least risky investment in real terms over all investment horizons. However, cointegration tilts allocation toward cash in the long run. While a small negative weight is assigned to stock investments in the VEC at medium horizons due to a high positive correlation between T-bills and stocks, the weights are nearly zero under stationarity at all horizons. Starting with a negative weight, the allocations to real bonds increase with the investment horizon in both models. However, cointegration reduces bond allocation in the long run.

Panels C and D report the composition of the VAR and VEC models based on nominal returns. The different nominal term structures change optimal allocations compared to real terms. While in both models all asset weights are roughly equal up to horizon 10, they substantially differ for the longer investment horizons. The VEC-based investor shifts nearly all his wealth to bonds as the horizon increases, whereas he decreases T-bills to zero and assigns stocks a minor role in the nominal portfolio decision. The VAR-based investor diversifies more among the assets by holding 15% T-bills, 17% stocks and 68% bonds in the very long run.

Table 3.11: Global Minimum Variance Portfolios

Panel A	Horizon (quarters)				
VAR real	1	5	10	50	100
T-bills	**1.04**	**1.05**	**1.02**	**0.91**	**0.84**
	[1.02,1.06]	[1.00,1.09]	[0.97,1.08]	[0.84,1.06]	[0.75,1.11]
Stocks	-0.01	-0.02	-0.02	-0.02	0.01
	[-0.02,0.00]	[-0.03,0.00]	[-0.04,0.01]	[-0.07,0.03]	[-0.10,0.08]
Bonds	**-0.04**	-0.03	-0.01	0.11	0.14
	[-0.06,-0.01]	[-0.07,0.01]	[-0.06,0.03]	[-0.03,0.17]	[-0.07,0.23]

Panel B	Horizon (quarters)				
VEC real	1	5	10	50	100
T-bills	**1.05**	**1.08**	**1.12**	**1.06**	**0.92**
	[1.02,1.07]	[1.03,1.13]	[1.04,1.18]	[0.85,1.19]	[0.73,1.15]
Stocks	-0.01	**-0.05**	**-0.08**	-0.06	0.01
	[-0.02,0.00]	[-0.07,-0.02]	[-0.10,-0.02]	[-0.12,0.05]	[-0.15,0.15]
Bonds	**-0.04**	-0.03	-0.04	0.00	0.07
	[-0.06,-0.00]	[-0.09,0.01]	[-0.11,0.02]	[-0.13,0.15]	[-0.12,0.26]

Panel C	Horizon (quarters)				
VAR nominal	1	5	10	50	100
T-bills	**0.97**	**0.89**	**0.80**	**0.38**	**0.15**
	[0.96,0.97]	[0.86,0.92]	[0.77,0.85]	[0.33,0.76]	[0.01,0.94]
Stocks	0.00	**0.01**	**0.02**	0.06	0.17
	[-0.00,0.00]	[0.00,0.02]	[0.00,0.04]	[-0.07,0.15]	[-0.19,0.37]
Bonds	**0.03**	**0.10**	**0.17**	**0.56**	0.68
	[0.01,0.03]	[0.07,0.11]	[0.12,0.20]	[0.20,0.60]	[-0.01,0.85]

Panel D	Horizon (quarters)				
VEC nominal	1	5	10	50	100
T-bills	**0.97**	**0.91**	**0.82**	**0.41**	0.03
	[0.96,0.98]	[0.88,0.93]	[0.78,0.87]	[0.35,0.67]	[-0.01,0.74]
Stocks	0.00	-0.01	-0.01	0.04	-0.07
	[-0.00,0.00]	[-0.01,0.00]	[-0.02,0.02]	[-0.08,0.16]	[-0.34,0.35]
Bonds	**0.03**	**0.10**	**0.19**	**0.55**	**1.03**
	[0.01,0.03]	[0.07,0.12]	[0.12,0.21]	[0.27,0.64]	[0.22,1.14]

Notes: This table reports the four cases of GMV portfolio allocations for investment horizons of up to 25 years. Panels A and B show the portfolio compositions of the VAR and VEC models based on real returns. Panels C and D report the portfolio compositions of the VAR and VEC models based on nominal returns. Bootstrap intervals are calculated from 10,000 paths under the assumption that the initial estimated VAR or VEC model truly generates the data process and are reported in brackets. Bold coefficients imply significance on a 5% level.

Table 3.12 shows four cases of optimal portfolio allocations of investors with a risk aversion $\gamma = 20$ for investment horizons up to 25 years and bootstrapped 95% confidence intervals in brackets below. We consider a VAR and a VEC-based investor allocating his wealth by taking into account real and nominal returns. Panels A and B report the composition of the VAR and VEC model based on real returns. While in both panels all asset weights are roughly equal up to horizon 10, they substantially differ for the longer investment horizons. In the short run, about 80% of the money is assigned to T-bills and the rest to stocks and bonds. As the holding period lengthens, the VAR-based investor shifts his T-bills allocation to stocks and bonds whereas the absolute increase is nearly twice as high for stocks than for bonds. In contrast, the VEC-based investor holds his T-bill exposure roughly constant over all investment horizons. He funds the strong increasing stock allocation with short-selling bonds resulting in a 123% stock and -97% bond position.

Panels C and D report the composition of the VAR and VEC models based on nominal returns. We see once more that the differences in optimal portfolio weights are small at short horizons. Furthermore, in both cases, the T-bills allocations strongly decrease with the investment horizon and are negative for the very long run. An alternative pattern with respect to stock and bond allocations is observed between the models. While the VAR-based investor allocates most of his wealth to bonds as the investment horizon increases, the VEC-based investor shifts his money to stocks, resulting in a leveraged equity position at a 100-period horizon and his preference toward bonds reverses as the holding period lengthens.

The differences in the optimal portfolio allocations between the stationary and the cointegrated model arise due to the different term structures of risk and correlations as well as due to changes in expected returns. While for both types of investors stocks are the riskiest and T-bills are the least risky investment under real returns, the absolute volatilities differ to a large extent. Under cointegration, the risk is almost always lower and especially reduced for bonds in the short run, for T-bills in the long run and for stocks over all horizons. Thus, the real VEC-based investor keeps his exposure to T-bills consistently high and increases the equity exposure with the investment horizon. Turning to nominal returns, the VEC model assigns all assets lower volatilities compared to the VAR model over all horizons, leading to

Table 3.12: Optimal Portfolio Holdings for $\gamma = 20$

Panel A	Horizon (quarters)				
VAR real	1	5	10	50	100
T-bills	**0.83**	**0.84**	**0.83**	**0.64**	0.44
	[0.78,0.86]	[0.73,0.88]	[0.67,0.88]	[0.18,0.76]	[-0.08,0.63]
Stocks	**0.12**	**0.10**	**0.11**	**0.24**	**0.35**
	[0.09,0.14]	[0.07,0.15]	[0.08,0.19]	[0.16,0.41]	[0.22,0.53]
Bonds	**0.05**	**0.06**	**0.06**	**0.12**	**0.20**
	[0.02,0.08]	[0.01,0.13]	[0.00,0.16]	[0.00,0.46]	[0.05,0.62]
Panel B	Horizon (quarters)				
VEC real	1	5	10	50	100
T-bills	**0.83**	**0.80**	**0.81**	0.85	0.74
	[0.76,0.85]	[0.61,0.85]	[0.47,0.89]	[-0.23,0.98]	[-0.80,0.98]
Stocks	**0.13**	**0.12**	**0.15**	**0.72**	**1.23**
	[0.10,0.16]	[0.09,0.23]	[0.12,0.38]	[0.58,1.49]	[0.88,2.53]
Bonds	**0.04**	**0.08**	0.04	-0.58	-0.97
	[0.01,0.08]	[0.00,0.20]	[-0.10,0.23]	[-1.01,0.16]	[-1.81,0.08]
Panel C	Horizon (quarters)				
VAR nominal	1	5	10	50	100
T-bills	**0.76**	**0.68**	**0.61**	0.11	-0.25
	[0.71,0.78]	[0.59,0.72]	[0.46,0.65]	[-0.14,0.30]	[-0.48,0.24]
Stocks	**0.13**	**0.13**	**0.15**	**0.32**	**0.51**
	[0.11,0.15]	[0.11,0.17]	[0.13,0.22]	[0.20,0.47]	[0.20,0.70]
Bonds	**0.11**	**0.18**	**0.24**	**0.57**	**0.74**
	[0.08,0.14]	[0.14,0.24]	[0.18,0.33]	[0.36,0.75]	[0.28,1.01]
Panel D	Horizon (quarters)				
VEC nominal	1	5	10	50	100
T-bills	**0.76**	**0.62**	**0.51**	0.20	-0.15
	[0.69,0.78]	[0.44,0.67]	[0.20,0.59]	[-0.71,0.38]	[-1.35,0.41]
Stocks	**0.14**	**0.16**	**0.23**	**0.82**	**1.15**
	[0.12,0.17]	[0.14,0.27]	[0.19,0.44]	[0.64,1.57]	[0.83,2.58]
Bonds	**0.11**	**0.21**	**0.27**	-0.02	-0.01
	[0.07,0.14]	[0.13,0.32]	[0.11,0.43]	[-0.53,0.57]	[-1.26,0.66]

Notes: This table reports the four cases of optimal portfolio allocations of investors with a risk aversion $\gamma = 20$ for investment horizons up to 25 years. Panel A and B show the portfolio compositions of the VAR and VEC model based on real returns. Panel C and D report the portfolio compositions of the VAR and VEC model based on nominal returns. Bootstrap intervals are calculated from 10,000 paths under the assumption that the initial estimated VAR or VEC model truly generates the data process and are reported in brackets. Bold coefficients imply significance on a 5% level.

over 50% lower risks for stocks and bonds in the long run. This strong risk reduction of stocks and bonds and the mean-aversion effect of short-term interest rates make T-bills the riskiest asset class under cointegration in the long run. Thus, the very risk-averse nominal VEC-based investor shifts his allocation from T-bills to bonds faster than the VAR-based investor. Furthermore, according to the VEC model, the diversification potential between stocks and bonds diminishes with an increasing investment horizon as the correlations are 83% and 73% in real and nominal terms at a 100-period horizon. However, the correlations in the VAR model are considerably lower and increase to 50% for nominal returns but decrease to 31% for real returns at a 100-period horizon. For this reason, under cointegration and depending on the level of risk aversion, the nominal GMV portfolio includes only the less risky bond position and the more aggressive portfolio includes only stocks due to their higher expected return.

3.4 Conclusion

This paper shows that the incorporation of cointegration into the commonly used VAR framework yields important implications for modeling asset price dynamics over all investment horizons. In the presence of common long-run relations, the VEC model captures these effects. Cointegration leads to a significant change in the horizon-dependent risk structure of the asset returns and ultimately in the optimal asset allocations compared to the stationary VAR model.

We find strong evidence that the traditional VAR distorts the term structure of risk because the levels of the variables share common stochastic trends ignored by the stationary VAR. Analyzing the properties of the time series, we detect four cointegration relations between the levels of T-bills, stocks, bonds, dividend-price ratio, term spread and inflation. Since deviations of the long-term comovement of the variables cause predictable backward movements that are captured by the VEC, the cointegration model explains the occurred risk premia of stocks and bonds much better than the stationary VAR.

We find substantial differences between the two models with respect to the term structure of risk. In the VEC the risk structure of real T-bills is much lower than in the VAR model in the long run. While the return variation of excess stock returns is mean-reverting in both models, the effect is much more pronounced for the VEC, especially in the first periods. This difference is predominantly caused by a much stronger mean-reversion effect of the dividend-price ratio under cointegration. Furthermore, the term structure of risk for the bond premium decreases in the stationary model over the horizon, while in the VEC, the volatility is hump-shaped with a much steeper drop in the first periods and a subsequent backward movement to the VAR term structure in the long run. The mean-reversion behavior of the bond premium is the result of negative correlations between excess bond returns and inflation, whereas this effect is weakened by a mean-averting influence of the term spread. However, under cointegration, the term spread affects bond volatility only in the long run.

Furthermore, in a variance decomposition exercise, depending on the time horizon, we examine various risk components of real and nominal returns and their interdependencies. We find inflation to be the driving component of nominal interest rates and detect a strong relationship between nominal T-bills and inflation under cointegration in the long run. Moreover, we observe the excess return variation as the main component of the corresponding real stock and bond return variation in both models. The variation of real T-bills only has a marginal effect on the total variation of real returns. Allowing for cointegration, the volatility of nominal stock and bond returns is significantly decreased by the covariation between both risk premia and inflation at long horizons, whereas the VAR model is not able to capture this effect. Finally, these differences in the risk structure influence the optimal portfolio choice. Under cointegration and extreme risk aversion, the optimal real (nominal) return portfolio is tilted much more towards T-bills (bonds). In the VEC, a less risk-averse investor has a much higher equity exposure as the investment horizon lengthens and even leverages the position in the very long run. This behavior is borne by a decreasing bond position compared to the VAR model.

We have tried to illustrate our findings with the use of variables commonly included in the stationary VAR framework. This enables us to compare and link the results to the related literature. However, our analysis can be extended by incorporating additional or other variables into the cointegration model that can influence the results. Moreover, the traditional VAR analysis often includes parameter uncertainty investigations that also have implications for asset allocation decisions across various investment horizons. Analyzing parameter uncertainty within the cointegration framework is an interesting topic for further research.

3.A Appendix

3.A.1 Bootstrap Method

We apply the residual-based bootstrap method suggested by Benkwitz, Lütkepohl, and Wolters (2001) and Lütkepohl (2005), which consists of the following steps:

1. Estimate the unknown coefficients of the VAR or VEC. Let $\hat{\mathbf{u}}_t$ and $\hat{\boldsymbol{\nu}}_t$ be the estimate of the VAR residuals $\mathbf{u}_t$ and the VEC residuals $\boldsymbol{\nu}_t$, respectively.

2. Calculate centered residuals $\hat{\mathbf{u}}_1 - \bar{\mathbf{u}}, ..., \hat{\mathbf{u}}_T - \bar{\mathbf{u}}$ or $\hat{\boldsymbol{\nu}}_1 - \bar{\boldsymbol{\nu}}, ..., \hat{\boldsymbol{\nu}}_T - \bar{\boldsymbol{\nu}}$, where $\bar{\mathbf{u}}$ and $\bar{\boldsymbol{\nu}}$ are the n usual means for the n residual series.

3. Draw randomly with replacement from the centered residuals to obtain bootstrap residuals $\boldsymbol{\epsilon}_1^*, ..., \boldsymbol{\epsilon}_T^*$.

4. Recursively calculate the bootstrap time series for the VAR as

$$\Delta \mathbf{z}_t^* = \boldsymbol{\mu} + \mathbf{B}_1 \Delta \mathbf{z}_{t-1}^* + \cdots + \mathbf{B}_p \Delta \mathbf{z}_{t-p}^* + \boldsymbol{\epsilon}_t^*, \ t = 1, ..., T, \tag{3.17}$$

   where $(\Delta \mathbf{z}_{-p+1}^*, ..., \Delta \mathbf{z}_0^*) = (\Delta \mathbf{z}_{-p+1}, ..., \Delta \mathbf{z}_0)$ holds for each generated series. For the VEC its level representation is used for data generation and hence the bootstrap time series for the VEC are calculated as in Equation (3.17), replacing $\Delta \mathbf{z}_t^*$ by $\mathbf{z}_t^*$ and using the corresponding coefficient matrices $\mathbf{A}_1, ..., \mathbf{A}_p$.

5. Reestimate the coefficients of the VAR or VEC using the bootstrapped data and calculate the statistic of interest q^*.

6. Repeat these steps N times.

The bootstrap confidence intervals (standard percentile intervals) are then given by

$$CI = \left[s_{\gamma/2}^*, s_{(1-\gamma/2)}^* \right],$$

where $s_{\gamma/2}^*$ and $s_{(1-\gamma/2)}^*$ are the $\gamma/2$- and $1 - (\gamma/2)$-quantiles of the N bootstrap versions of q^*.

3.A.2 Model Selection

The number of lags to be included in the VAR and VEC models is determined by taking into account the suggestions of the Akaike information criterion (AIC), the Schwarz criterion (SC) and the Hannan & Quinn criterion (HQ). Table 3.13 reports the test statistics of these criteria depending on the number of lags.

Panel A reports the results for the VAR model. The SC and HQ criteria suggest one lag for the VAR model as the test statistics are minimized, while the AIC suggests four lags. Panel B reports the results for the VAR in levels, the basis of the VEC model. The SC and the HQ suggest two lags for the VAR model in levels which is equivalent to a VEC(1), while the AIC suggests three lags. For the empirical analysis, we follow the suggestions of the SC and HQ and investigate a VAR and a VEC of order one.

Table 3.13: Lag Length Selection

Panel A	Lags for VAR in Differences			
Criterion	$p = 1$	$p = 2$	$p = 3$	$p = 4$
AIC(p)	-66.228	-66.377	-66.267	-66.409*
HQ(p)	-65.973*	-65.904	-65.575	-65.499
SC(p)	-65.596*	-65.204	-64.553	-64.153

Panel B	Lags for VAR in Levels			
Criterion	$p = 1$	$p = 2$	$p = 3$	$p = 4$
AIC(p)	-62.999	-66.667	-66.692*	-66.614
HQ(p)	-62.744	-66.193*	-66.001	-65.704
SC(p)	-62.367	-65.493*	-64.978	-64.358

Notes: This table reports the test statistics of the Akaike, Schwarz and Hannan & Quinn information criteria to determine the number of lags to be included in the VAR (Panel A) and VEC (Panel B). * denotes the minimum test statistic depending on the number of lags for each information criterion.

Chapter 4

Do Stock Prices and Cash Flows Drift Apart? The Influence of Macroeconomic Proxies

This paper is the result of a joint project with *Benedikt Fleischmann*.

Abstract

The evidence of stationarity of the dividend-price ratio and earnings-price ratio is empirically mixed. Non-stationarity leads to invalid conclusions about return predictability. A breakdown of these relations can be caused by different macroeconomic influences. We investigate the connections of stock prices and cash flows (dividends and earnings) to macroeconomic proxies within a cointegration framework. We find that prices and cash flows are not one-for-one cointegrated and detect a negative inflation link to prices and positive inflation links to cash flows. The risk-free rate significantly decreases dividends and not prices. Government and corporate bond yields have contrary impacts on equity markets.

4.1 Introduction

Several studies in the predictability literature use variables such as the dividend-price ratio and earnings-price ratio to forecast stock returns.[1] According to theory, stock prices are the discounted future cash flows and, therefore, prices should move around their fundamentals (dividends and earnings) in the long run. It is generally assumed that prices and cash flows are cointegrated one-for-one or, alternatively, that the dividend-price ratio and earnings-price ratio are stationary variables, since otherwise the conventional t-statistics lead to wrong conclusions about the evidence of return predictability. However, the stationarity of these valuation ratios is empirically doubtful (Ang and Bekaert, 2007; Lettau and Van Nieuwerburgh, 2008). A natural question arises whether this observation is blurred by the high persistence of the valuation ratio or whether it is based on a change in the payout policy (Fama and French, 2001; Grullon and Michaely, 2002, 2004; Boudoukh, Michaely, Richardson, and Roberts, 2007) or whether it is actually caused by a breakdown of the one-for-one relation due to different macroeconomic influences on prices and dividends (Lettau and Ludvigson, 2001; Lee, 2010).

In this paper, we extend the loglinear Campbell and Shiller (1988a) model to investigate the influences of macroeconomic variables (inflation, short-term interest rates, government and corporate bond yields) on stock prices and cash flows (dividends and earnings) and, consequently, the implied impacts on total stock returns. Our cointegration model shows that (i) prices and cash flows do not form (trend-) stationary relations, and especially do not form stationary one-for-one relations; (ii) dividends and earnings move close together and only minor different macroeconomic influences are observable; (iii) inflation strongly decreases stock prices and increases cash flows; (iv) the risk-free rate negatively influences all equity market variables; and (v) government and corporate bond yields have contrary impacts on the equity market. Since we approximate total stock returns by price changes and dividends, the returns are, therefore, also linked to the macroeconomic effects of prices and dividends.

[1] See, for example, (Fama and Schwert, 1977; Keim and Stambaugh, 1986; Fama and French, 1988, 1989; Kothari and Shanken, 1997).

We start from a vector autoregressive model (VAR) of non-stationary time series and allow for cointegration. The choice of variables and the model setup are well justified. Campbell and Shiller (1987) propose a cointegration VAR framework between prices and dividends and find a weak long-run relation between these variables. Cochrane (1994) and Lee (1995) investigate the permanent and transitory components of this bivariate model, without questioning the validity and implication of the $[1, -1]$ assumption of the dividend-price ratio. Campbell and Shiller (1988b) and Lamont (1998) illustrate the importance of earnings measures (dividend-earnings or earnings-price ratio) to account for dividend predictability, since these ratios mirror actual business success which dividends do not directly reflect. Lee (1996) confirms a link between dividends and earnings within a cointegration framework. While these studies focus only on a relation between equity market variables, Lettau and Ludvigson (2001) see a natural long-run connection between the equity market and the macroeconomy that mirrors business conditions and, consequently, model it as a cointegration relation between dividends, aggregate consumption and labor income. Other studies proxy macroeconomic conditions by variables such as inflation, interest rates, term spread and credit spread (for example: Fama and French, 1988, 1989; Boudoukh and Richardson, 1993; Campbell and Thompson, 2008; Cochrane, 2008; Goyal and Welch, 2008), but they do not examine their long-run influence on the equity market. Following these studies, we also incorporate the inflation rate, short-term interest rates and government and corporate bond yields in the model to analyze their impacts on prices, dividends and earnings.[2]

When detecting the non-stationarity of these time series, we apply a vector error correction (VEC) model to capture the interactions of the seven variables. We show that the system has four cointegration relations or three remaining stochastic trends. The versatile model setup enables us to test the validity of the stationarity of the

[2]It would seem sensible to include a proxy for the real economy such as GDP. However, GDP is subject to large revisions after the primary release and hence, we avoid to include such a proxy as the analysis would be influenced by this reporting bias (Faust, Rogers, and Wright, 2005). The macroeconomic proxies used in this study do not suffer from such a reporting bias. Interest rates and yields are never revised and also the inflation rate is revised only if there are changes in the factors of seasonal adjustments or in the base year (Croushore and Stark, 2001).

dividend-price ratio in a multivariate framework. Moreover, in the same way, we also examine the stationarity of further financial ratios such as dividend-earnings, earnings-price, real short-interest rates, term spread and credit spread, which are additionally used in the predictability literature. Our tests show that the dividend-earnings ratio and the term spread most likely are stationary or, alternatively, that the underlying variables have the same stochastic trends, whereas the null hypothesis of stationarity is rejected for the remaining ratios. Additionally, we find inflation to have a strong impact on the equity market. While other papers often consider returns, prices and dividends in real terms to avoid inflation effects and assume that inflation influences equity market variables identically, we show a negative linkage between nominal prices and inflation and nominal cash flows to be positively associated to inflation in the long run. Thus, nominal total stock returns are reduced by inflation shocks in the short term, but recover for long time horizons. This result connects the contrary findings of Fama and Schwert (1977) and Boudoukh and Richardson (1993). While Fama and Schwert find a negative relation in the short run, Boudoukh and Richardson find a positive relation in the long run. Moreover, we find that interest rates play a similar role for all equity market variables. Although prices, dividends and earnings are reduced by rising interest rates, the magnitude is much more pronounced for cash flows than for stock prices. Finally, we find large positive effects on the equity market for corporate bond yields, but the influence of the government bond yields is negative.

The remainder of this paper is organized as follows: In the next section, we describe the methodology of the econometric model, the tests used for identifying pulling and pushing forces in the system and derive the framework for the long-horizon analysis. Section 3 introduces the data set, examines the time series properties for further investigations and presents the results of our empirical analysis. Finally, Section 4 summarizes the main findings.

4.2 Methodology

In this section, we introduce the VEC model capturing the dynamics of the variables analyzed and, to gain further insights about the pulling and pushing forces acting among the stochastic trends, we then apply four different types of structural hypotheses tests. Finally, we introduce the impulse response analysis to investigate the long-run dynamics implied by the VEC.

4.2.1 The Econometric Model

The unrestricted basic model, a n-dimensional vector autoregressive model VAR(p), is defined as follows:

$$\mathbf{z}_t = \mathbf{A}_1\mathbf{z}_{t-1} + \cdots + \mathbf{A}_p\mathbf{z}_{t-p} + \mathbf{\Psi}\mathbf{d}_t + \mathbf{u}_t, \qquad t = 1, ..., T, \tag{4.1}$$

where $\mathbf{z}_t$ contains the n variables of interest, both of which are assumed to be integrated of order one, ($I(1)$), and the shocks, $\mathbf{u}_t$, are assumed to be IID with time-invariant zero means and variance-covariance matrix $\mathbf{\Sigma}_u$. The matrices $\mathbf{A}_1, ..., \mathbf{A}_p$ are the $(n \times n)$ slope coefficients, while the vector $\mathbf{d}_t$ contains dummy variables, a constant and a time trend and $\mathbf{\Psi}$ is the loading matrix of these deterministic components. Dropping the deterministic components for simplification and without imposing binding restrictions, this VAR(p) can be transformed to a VEC of order $p-1$ by subtracting both sides of Equation (4.1) with $\mathbf{z}_{t-1}$:

$$\Delta\mathbf{z}_t = \mathbf{\Pi}\mathbf{z}_{t-1} + \mathbf{\Gamma}_1\Delta\mathbf{z}_{t-1} + \cdots + \mathbf{\Gamma}_{k-1}\Delta\mathbf{z}_{t-k+1} + \mathbf{u}_t, \tag{4.2}$$

where $\mathbf{\Pi} = -(\mathbf{I} - \mathbf{A}_1 - \cdots - \mathbf{A}_p)$ and $\mathbf{\Gamma}_j = -(\mathbf{A}_{j+1} + \cdots + \mathbf{A}_p)$ for $j = 1, ..., p-1$. As can be seen in Equation (4.2), matrix $\mathbf{\Pi}$ summarizes the long-run effects and the short-run effects remain in the matrices $\mathbf{\Gamma}_1, ..., \mathbf{\Gamma}_{p-1}$. While the $\mathbf{\Gamma}_j$'s are full rank matrices, $\mathbf{\Pi}$ must have reduced rank, otherwise a logical inconsistency would occur.[3] To determine the number $r \leq n$ of cointegration relations, we test the hypothesis

$$H_1(r) : \mathbf{\Pi}_r = \boldsymbol{\alpha}\boldsymbol{\beta}', \tag{4.3}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are both $(n \times r)$ matrices. Hypothesis H_1 is performed stepwise by investigating whether there is a significant difference between the likelihood of the unrestricted model in Equation (4.2) and the likelihood of a model with $\mathbf{\Pi}_r$ restricted to rank r (trace test).[4] The optimal rank r corresponds to the most restrictive model

[3] Assuming $\mathbf{z}_t \sim I(1)$, $\Delta\mathbf{z}_t \sim I(0)$ and considering $\mathbf{\Pi} = \mathbf{I}$, the stationary variable $\Delta\mathbf{z}_t$ on the left-hand side of Equation (4.2) would be equal to the sum of stationary variables $\mathbf{\Gamma}_j\Delta\mathbf{z}_{t-j}$ and a non-stationary term $\mathbf{z}_{t-1}$ (Juselius, 2006, Chap. 5).

[4] A full discussion of this trace test and its distribution is given in Johansen and Juselius (1990).

without obtaining a significantly different likelihood. After specifying the optimal r, we calculate the decomposition of $\mathbf{\Pi}_r = \boldsymbol{\alpha}\boldsymbol{\beta}'$, which leads to the reduced rank system

$$\Delta \mathbf{z}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{z}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{z}_{t-1} + \cdots + \mathbf{\Gamma}_{p-1}\Delta \mathbf{z}_{t-p+1} + \boldsymbol{\nu}_t. \tag{4.4}$$

As shown in Johansen (1996), $\boldsymbol{\beta}'$ transforms the non-stationary $\mathbf{z}_t$ to stationary relations $\boldsymbol{\beta}'\mathbf{z}_t$, which are also known as cointegration relations. The matrix $\boldsymbol{\alpha}$ contains the loadings on the stationary and depended variables $\Delta \mathbf{z}_t$. Note that in case of n independent stochastic trends and no cointegration, the matrix $\mathbf{\Pi}_r$ equals $\mathbf{0}$.

4.2.2 Hypotheses Testing

Since we want to detect not only the number of cointegration relations, but also to gain further insights about the pulling and pushing forces acting among the stochastic trends, we apply four different types of structural hypotheses tests to the matrices capturing the long-run effects. Thus, we can analyze the exclusion and exogeneity of variables and investigate whether the financial ratios exhibit (trend-) stationary behavior. Following Johansen and Juselius (1992), our tests are:

$$H_2 : \boldsymbol{\beta} = \mathbf{H}_2\boldsymbol{\varphi}, \quad \mathbf{H}_2(n \times s), \boldsymbol{\varphi}(s \times r), \quad r \leq s \leq n, \tag{4.5}$$

$$H_3 : \boldsymbol{\beta} = (\mathbf{H}_3, \boldsymbol{\psi}), \quad \mathbf{H}_3(n \times r_1), \boldsymbol{\psi}(n \times r_2), \quad r = r_1 + r_2, \tag{4.6}$$

$$H_4 : \boldsymbol{\beta} = (\mathbf{H}_4\boldsymbol{\varphi}, \boldsymbol{\psi}), \quad \mathbf{H}_4(n \times s), \boldsymbol{\varphi}(s \times r_1), \boldsymbol{\psi}(n \times r_2), \quad r \leq s \leq n, \quad r = r_1 + r_2, \tag{4.7}$$

$$H_5 : \boldsymbol{\alpha} = \mathbf{H}_5\boldsymbol{\xi}, \quad \mathbf{H}_5(n \times m), \boldsymbol{\xi}(m \times r), \quad r \leq m \leq n, \tag{4.8}$$

where $\mathbf{H}_2$, $\mathbf{H}_3$, $\mathbf{H}_4$ and $\mathbf{H}_5$ are appropriately chosen transformation matrices. Hypothesis H_2 sets the same $(n - s)$ restrictions on all r cointegration relations $\boldsymbol{\beta}$, whereas hypothesis H_3 assumes r_1 cointegration relations to be known and the coefficients of the remaining r_2 relations to be estimated. Hypothesis H_4, a combination of H_2 and H_3, sets only a few restrictions on the first r_1 cointegration relations and leaves the remaining coefficients (in the r_1 relations and in $\boldsymbol{\psi}$) to be estimated. There are two special cases: for $r_2 = 0$, the hypothesis H_4 is equal to H_2 and for $r_1 = s$ hypothesis, H_4 reduces to H_3. Hypothesis H_5 tests the exclusion of the influence of certain long-run relations. The corresponding test statistics, which are

χ^2-distributed in each case, and further details are given in Johansen and Juselius (1992).

We use H_2 to test the exclusion of certain variables from all long-run relations, i.e. to test a zero row restriction on $\boldsymbol{\beta}$. If rejected, we cannot omit the variable from the cointegration relations. Moreover, many empirical finance studies use financial ratios, which are assumed to be stationary, to predict asset returns. For example, the dividend-price ratio is assumed to be a one-for-one relation. To test such a hypothesis, we also use H_2 to analyze the validity of this relation in the full system, e.g. dividends (d_t) and prices (p_t) are long-run homogeneous in all cointegration relations. If rejected, we cannot reformulate the long-run relations directly to the dividend-price ratio without a loss of information. If the H_2 constraints are too restrictive, we cannot respecify all of the long-run relations to financial ratios without losing information. In contrast to H_2, we relax the restrictions with H_3 to only r_1 relations. Thus, the test shows that whether e.g. dividends, prices or the dividend-price ratio are (trend-)stationary by themselves in a multivariate framework. In the case where H_3 is rejected, e.g. the one-for-one relation of a financial ratio, we can test a more general relation (linear combination) between the variables of interest where the coefficients have to be estimated. The H_4 test analyzes if there is any stationary linear combination between the variables for each equation, e.g. whether a stationary ratio ($d_t - \beta p_t$) for some estimated value of β exists or not. Moreover, including some additional variables in the r_1 linear combinations, the H_4 test investigates the stationarity of these extended linear combinations.

Last, a natural question is if the variables adjust to, are pushed by or are weakly exogenous to the estimated long-run relations. As a result, we use H_5 to analyze the structural restrictions of the loading effects $\boldsymbol{\alpha}$ of the cointegration relations. For example, in the presence of a disequilibrium between dividends and prices, the issue can be addressed as to whether there is a significant adjustment back to the equilibrium and if it is due to changes in stock prices or dividends.

4.2.3 Long-Run Analyses

To investigate the long-run dynamics implied by the VEC, we examine the horizon-dependent influence of unexpected shocks on the stock return. The corresponding statistics are based on appropriately iterated coefficient matrices of the VEC. Therefore, we start by retransforming the reduced rank VEC($p-1$) in Equation (4.4) back to a VAR(p) by setting $\mathbf{A}_{1,r} = \mathbf{I} + \mathbf{\Pi}_r + \mathbf{\Gamma}_1$, $\mathbf{A}_{j,r} = \mathbf{\Gamma}_j - \mathbf{\Gamma}_{j-1}$ for $j = 2, ..., p-1$ and $\mathbf{A}_{p,r} = -\mathbf{\Gamma}_{p-1}$. Afterwards, we rewrite the VAR(p) as a VAR(1) with the $(pn \times pn)$ coefficient matrix $\mathbf{A}_r$.

The influence of an unexpected shock of one variable on the variables is examined by an impulse-response analysis. The effects of the shocks can be seen in the Wold (moving average) representation theorem:

$$\mathbf{z}_t^* = \mathbf{A}_r^0 \boldsymbol{\nu}_t^* + \mathbf{A}_r^1 \boldsymbol{\nu}_{t-1}^* + \mathbf{A}_r^2 \boldsymbol{\nu}_{t-2}^* + \cdots ,$$

where $\mathbf{z}_t^* = (\mathbf{z}_t, ..., \mathbf{z}_{t-p+1})'$ and $\boldsymbol{\nu}_t^*$ are the residuals of the VEC in Equation (4.4) (stacked with a vector of zeros). Since the variables in $\mathbf{z}_t$ are assumed to be non-stationary, the elements in $\mathbf{A}_r^k$ do not need to converge to zero as $k \to \infty$ and some shocks can consequently have permanent effects.[5]

The horizon-dependent risk statistics are based on the covariance matrix of the residuals and the iterated coefficient matrices. Starting with the future value $\mathbf{z}_{t+k}^*$, which can be described by its current value $\mathbf{z}_t^*$ and a sum of intermediate shocks

$$\mathbf{z}_{t+k}^* = \mathbf{A}_r^k \mathbf{z}_t^* + \mathbf{A}_r^{k-1} \boldsymbol{\nu}_{t+1}^* + \mathbf{A}_r^{k-2} \boldsymbol{\nu}_{t+2}^* + \cdots + \mathbf{A}_r \boldsymbol{\nu}_{t+k-1}^* + \boldsymbol{\nu}_{t+k}^*,$$

we obtain the conditional k-period variance-covariance matrix of the VEC, scaled by the investment horizon:

$$\frac{1}{k} Var_t \left(\mathbf{z}_{t+k}^*\right) = \frac{1}{k} \left[\mathbf{A}_r^{k-1} \mathbf{\Sigma}^* \left(\mathbf{A}_r^{k-1}\right)' + \mathbf{A}_r^{k-2} \mathbf{\Sigma}^* \left(\mathbf{A}_r^{k-2}\right)' + \cdots + \mathbf{A}_r \mathbf{\Sigma}^* \mathbf{A}_r' + \mathbf{\Sigma}^*\right], \tag{4.9}$$

where $\mathbf{\Sigma}^*$ is the $(pn \times pn)$ covariance matrix of the residuals $\boldsymbol{\nu}^*$.

[5]Some elements of the iterated coefficient matrix of a non-stationary VAR in levels diverge as the horizon $k \to \infty$. However, in the case of cointegrated variables, the elements of $\mathbf{A}_r^k$ of the VEC can be bounded (Lütkepohl, 2005, pp. 258–262).

4.2.4 Returns

Using the loglinear framework of Campbell and Shiller (1988a) allows us to derive an approximation of the stock returns, although we only model the movements of stock prices and dividends:

$$\begin{aligned} r_t &= p_t - p_{t-1} + \log(1 + \exp(d_t - p_t)) \\ &= \rho p_t + (1-\rho) d_t - p_{t-1} + c + e_t^*, \end{aligned} \tag{4.10}$$

where $\rho = \left(1 + \exp(\overline{d_t - p_t})\right)^{-1}$, $(\overline{d_t - p_t})$ denotes the average log dividend-price ratio, $c = -\log(\rho) - (1-\rho)\log(1/\rho - 1)$ and e_t^* is an approximation error. Since ρ is close to one considering quarterly data, the weight of the stock price in t on stock returns is large while the impact of dividends in t is small. This approximation holds exactly when the dividend-price ratio $(d_t - p_t)$ is constant over time and accurately when the variation between dividends and prices is small. Nevertheless, Engsted, Pedersen, and Tanggaard (2010) attest the Campbell-Shiller approximation great properties even in the presence of rational explosive bubbles, where d_t and p_t do not move one-for-one and the stationarity of the dividend-price ratio may be questionable.

Modeling prices and dividends in the VEC separately, we do not assume a $[1, -1]$ relationship and handle the cointegration relation more flexibly. To calculate the effects of total returns implied by the VEC model, we apply a selection vector, $\mathbf{m}$, to the corresponding statistics of interest. Setting prices and dividends as the first elements of $\mathbf{z}_t$, the vector $\mathbf{m}$ is defined as

$$\mathbf{m} = (\rho, (1-\rho), 0, ..., 0, -1, 0, ..., 0), \tag{4.11}$$

where ρ and $(1-\rho)$ extracts the first two elements of Equation (4.10) and the -1 subtracts the lagged price impact. The constant c is omitted, since we only want to analyze the return dynamics (not the absolute values of the total returns).

4.3 Empirical Analysis

4.3.1 Data and Time Series Properties

Our empirical application is based on quarterly U.S. data spanning the period 1927:Q1 to 2011:Q4 ($n = 340$ observations) and includes seven variables measured as log: stock price index (p_t), dividends (d_t), earnings (e_t), inflation rate (π_t), 90-day nominal T-bill rates ($tb_t^{\$}$) and long-term government (y_t^g) and corporate bond (y_t^c) yields. These variables are denoted as levels in the sequel.

The stock price index is taken from the Center for Research in Security Prices (CRSP), the quarterly dividends are extracted from the CRSP total and price return data and earnings (the 12-month moving sums) are from Robert Shiller's website.[6] The Treasury bill rates are taken from the National Bureau of Economic Research (NBER) Macrohistory Data-base up to 1934 and then from the Federal Reserve Bank of St. Louis (FRED) subsequently. The source of the inflation rate is the Consumer Price Index (All Urban Consumers) from the Bureau of Labor Statistics and the source of the long-term government and AAA-rated corporate bond yields data is Ibbotson's *Stocks, Bonds, Bills and Inflation Yearbook*.[7]

Table 4.1 presents the univariate unit root and stationarity properties of the time series analyzed. We use the augmented Dickey-Fuller test (ADF) with the null of a unit root and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test with the null of stationarity. Panel A reports the results of the tests including a constant but no deterministic time trend. Panel B reports the results of the tests including a constant and a deterministic time trend. The number of lags used is given in parentheses, where the ADF's lag length is determined by the Akaike Information Criterion (AIC). The KPSS's lag length is determined by the integer value of $(4 \cdot (n/100)^{0.25})$, which depends only on the number of observations. Allowing only for a constant,

[6]Following Cochrane (1991) and Cochrane (2008), we calculate the dividend-price ratio with total return, R_t, and price index, P_t, as $D_t/P_t = (R_t/P_t) - 1$. We, then, obtain dividend growth by the identity $D_t/D_{t-1} = (D_t/P_t)(P_{t-1}/D_{t-1})(P_t/P_{t-1})$. Finally, cumulating the dividend growth leads to the level of dividends.

[7]We would like to thank Amit Goyal for providing the data used in Goyal and Welch (2008), for which an updated version is available on his website: http://www.hec.unil.ch/agoyal/.

Table 4.1: Univariate Stationarity

Panel A: Unit Root Test Regressions with Constant

	Levels			First Differences	
Variable	ADF(q)	KPSS(q)	Variable	ADF(q)	KPSS(q)
p_t	0.08 (4)	5.51*** (5)	Δp_t	-9.23*** (3)	0.10 (5)
d_t	0.28 (15)	5.66*** (5)	Δd_t	-5.26*** (14)	0.08 (5)
e_t	-0.03 (12)	5.58*** (5)	Δe_t	-6.75*** (11)	0.04 (5)
π_t	-2.93**(15)	0.72** (5)	$\Delta \pi_t$	-8.63*** (14)	0.03 (5)
$tb_t^{\$}$	-1.65 (8)	1.97*** (5)	$\Delta tb_t^{\$}$	-7.83*** (7)	0.09 (5)
y_t^g	-1.11 (5)	2.81*** (5)	Δy_t^g	-9.10*** (4)	0.25 (5)
y_t^c	-1.20 (5)	2.82*** (5)	Δy_t^c	-8.17*** (4)	0.21 (5)

Panel B: Unit Root Test Regressions with Constant and Trend

	Levels			First Differences	
Variable	ADF(q)	KPSS(q)	Variable	ADF(q)	KPSS(q)
p_t	-3.60** (4)	0.45*** (5)	Δp_t	-8.17*** (3)	0.05 (5)
d_t	-6.59***(16)	0.26*** (5)	Δd_t	-4.78*** (14)	0.04 (5)
e_t	-5.66***(11)	0.46*** (5)	Δe_t	-6.42*** (11)	0.02 (5)
π_t	-2.77 (15)	0.36*** (5)	$\Delta \pi_t$	-8.53*** (14)	0.01 (5)
$tb_t^{\$}$	-1.74 (8)	0.78*** (5)	$\Delta tb_t^{\$}$	-6.96*** (7)	0.06 (5)
y_t^g	-0.47 (5)	0.79*** (5)	Δy_t^g	-8.16*** (5)	0.13* (5)
y_t^c	-0.93 (5)	0.73*** (5)	Δy_t^c	-7.20*** (5)	0.15** (5)

Notes: The table reports the results of the univariate unit root test regressions. Panel A reports the results of the tests including a constant but no deterministic time trend. Panel B reports the results of the tests including a constant and a deterministic time trend. The number of lags used are given in parentheses; the symbols *, ** and *** denote significance at the 10, 5 and 1% level, respectively. The ADF tests the null of non-stationarity and the lag length is determined by the Akaike Information Criterion (AIC). The KPSS tests the null of stationarity and the lag length is determined by the integer value of $(4 \cdot (n/100)^{0.25})$.

the non-stationarity hypothesis is supported for all levels except for the inflation rate, which is rejected on a 5% level, but non-stationarity is strongly rejected for all first differences. The stationarity hypothesis of the KPSS test is rejected for all levels at 1% significance (except for inflation, which is rejected at 5% significance), but stationarity is supported for all first differences. Including both a constant and a deterministic time trend in the unit root tests, there is no clear evidence whether the levels of prices, dividends and earnings have a unit root. While the ADF test rejects the hypothesis of a unit root for the levels of prices, dividends and earnings, the KPSS test rejects the null of stationarity for those variables. Following previous studies that test these variables with respect to unit roots (Zhong, Darrat, and Anderson, 2003; Koivu, Pennanen, and Ziemba, 2005; Durre and Giot, 2007), we also assume prices, dividends and earnings to exhibit a unit root. The levels of the macroeconomic proxies are non-stationary according to both tests. Since the first differences of all variables seem to be stationary as indicated by the ADF and KPSS tests (except for government and corporate bond yields, where the KPSS test rejects stationarity at 10% and 5% significance, respectively), we assume the levels of all variables to have a stochastic trending behavior and to be $I(1)$.

Descriptive statistics of the stationary first-differences are reported in Table 4.2. Panel A of the table shows the summary statistics of the sample. Panel B reports simultaneous correlations between the variables: price returns, dividends and earnings growth rates, changes in inflation rate, the short-term interest rate and the long-term government and corporate bond yields. Prices, dividends and earnings grow high on average[8] and their changes exhibit high variability. The dividend growth volatility is about twice the price return volatility, which is a result of our calculation methodology since we assume quarterly dividends to be reinvested at stock market rates. Changes measured quarterly in the inflation rate are quite volatile in our sample. The changes in the interest rates and the long-term yields have low variability and nearly no growth over the sample period. All time series have a relatively small skewness but show an extremely high non-normal kurtosis.

[8] Testing the null that the mean is equal to zero, we obtain p-values of 3%, 4% and 7% for prices, dividends and earnings, respectively. The tests are performed with heteroskedasticity and autocorrelation consistent standard errors.

Table 4.2: Descriptive Statistics

Panel A							
Variable	Mean	Sd	Min	Max	Skew	Kurt	Autocor
Δp_t	1.30%	10.84%	-51.59%	63.16%	0.07	10.83	-0.03
Δd_t	1.09%	21.38%	-116.21%	70.90%	-0.48	6.97	-0.64
Δe_t	1.26%	12.92%	-112.75%	140.23%	1.13	63.05	0.52
$\Delta \pi_t$	0.01%	1.29%	-4.32%	6.71%	0.47	6.81	-0.34
$\Delta tb_t^{\$}$	0.00%	0.20%	-1.83%	1.16%	-2.16	29.04	-0.10
Δy_t^g	0.00%	0.11%	-0.52%	0.51%	-0.57	9.05	-0.11
Δy_t^c	0.00%	0.09%	-0.53%	0.50%	-0.22	10.94	-0.06

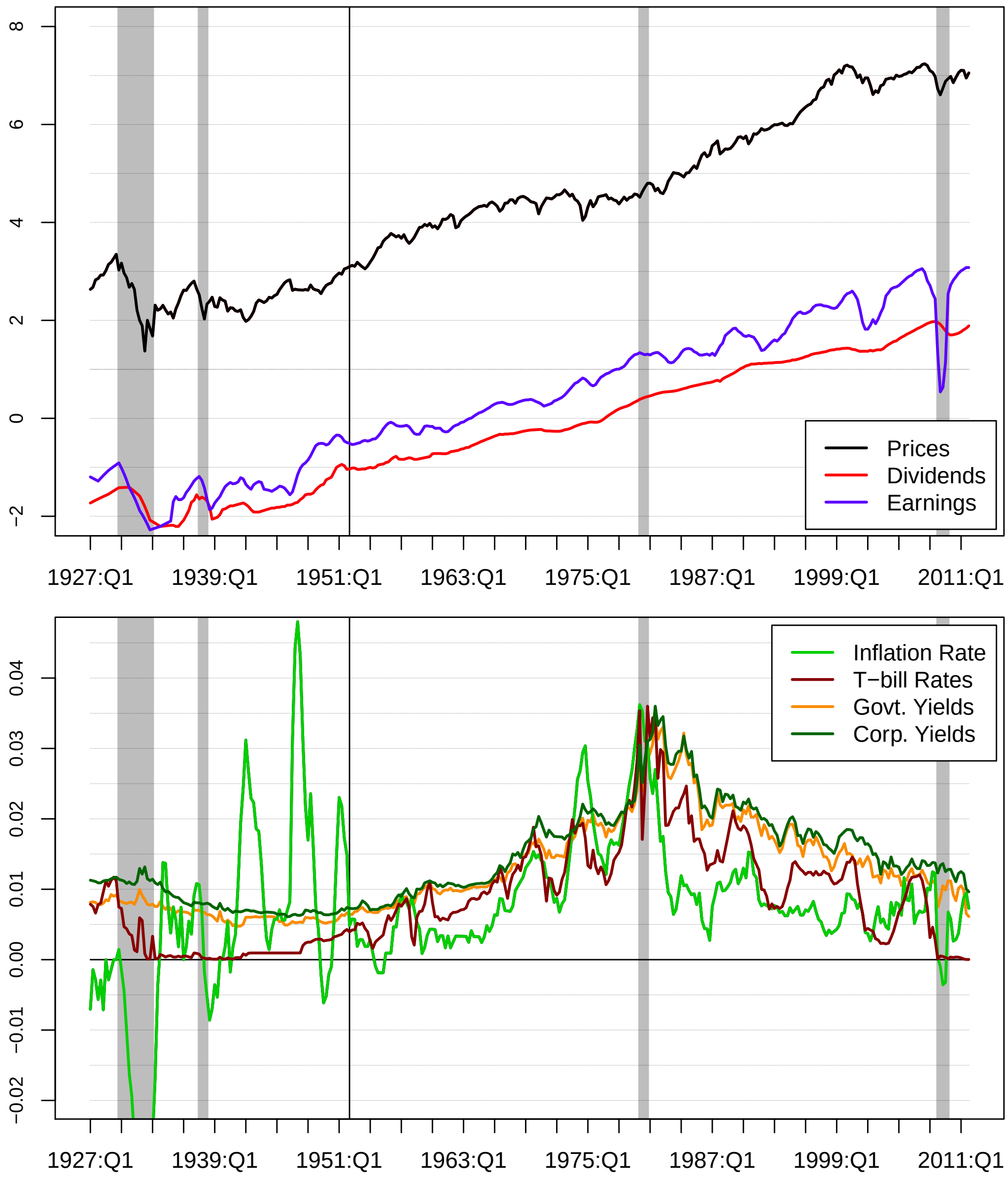
Panel B							
Correlation	Δp_t	Δd_t	Δe_t	$\Delta \pi_t$	$\Delta tb_t^{\$}$	Δy_t^g	Δy_t^c
Δp_t	1.00	0.01	0.14	0.04	-0.02	-0.07	-0.21
Δd_t		1.00	0.11	-0.05	0.07	0.02	0.04
Δe_t			1.00	0.02	0.13	0.13	0.05
$\Delta \pi_t$				1.00	0.10	0.21	0.18
$\Delta tb_t^{\$}$					1.00	0.55	0.63
Δy_t^g						1.00	0.87
Δy_t^c							1.00

Notes: The table reports the descriptive statistics of the variables: price returns, dividends and earnings growth rates, the changes in inflation rate, the short-term interest rate, the long-term yield and the corporate bond yield. Panel A of the table reports summary statistics of the sample from 1927:Q1 to 2011:Q4 (340 data points). "Sd" denotes standard deviation; "Min" denotes minimum; "Max" denotes maximum; "Skew" denotes skewness; "Kurt" denotes kurtosis of the time series; and "Autocor" the first-order autocorrelation. Panel B reports simultaneous correlations between the variables used.

While previous research only focuses on the link between (real) stock prices and (real) cash flows, intuition suggests additional links between the equity market and macroeconomic factors like inflation or short and long-term interest rates. Hence, our set of information for the upcoming analysis is defined as:

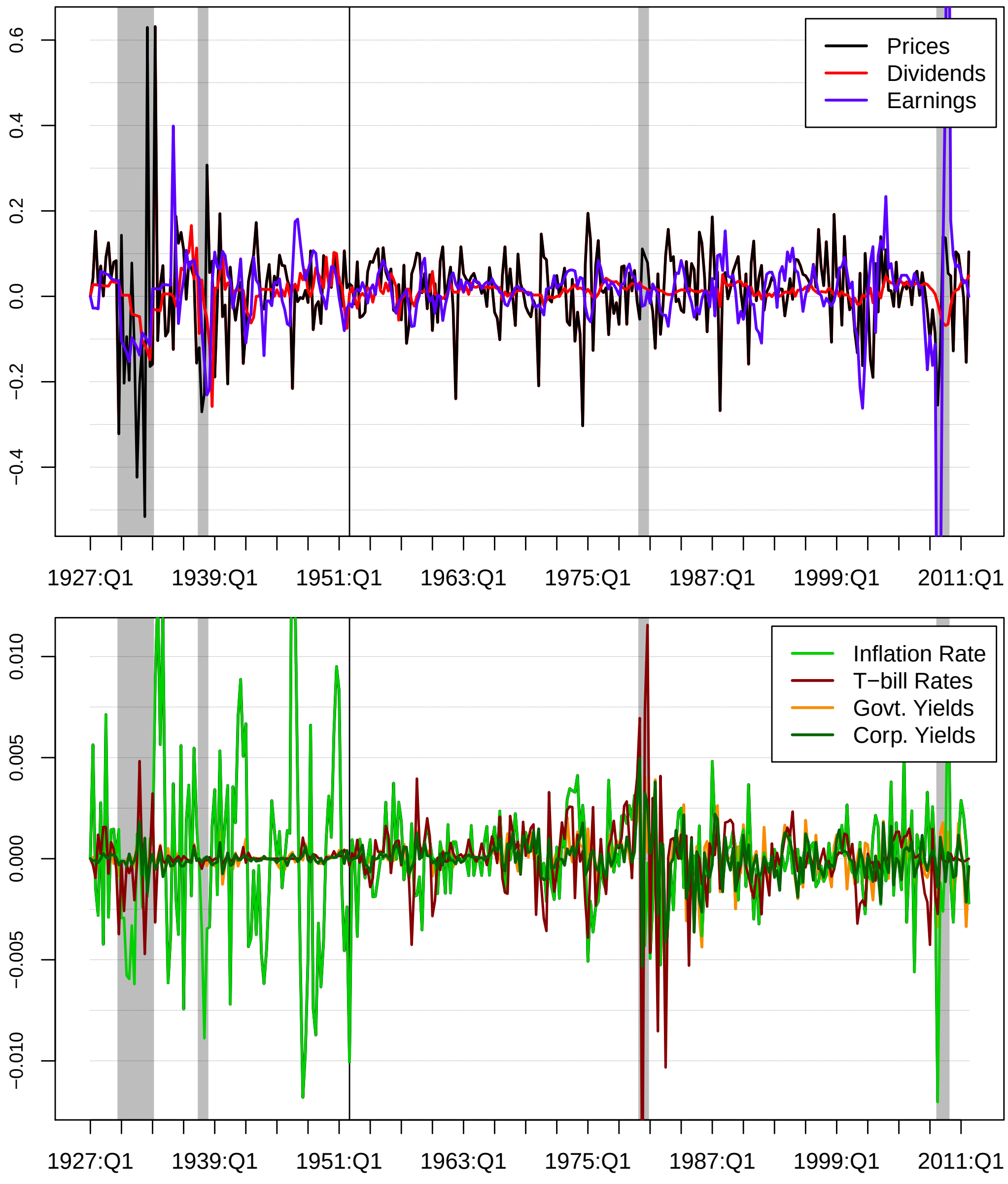
$$\mathbf{z}_t = \left(p_t, d_t, e_t, \pi_t, tb_t^{\$}, y_t^g, y_t^c\right)' . \tag{4.12}$$

Figure 4.1: Level Variables



Notes: This figure plots the logarithms of the level variables. The upper graph contains the time series of p_t, d_t and e_t, and the lower graph π_t, $tb_t^{\$}$, y_t^g and y_t^c. The gray vertical bars denote the extreme events where dummy variables are set. The black vertical line denotes the quarter 1952:Q1.

Figure 4.2: Differenced Variables



Notes: This figure plots the logarithms of the differenced level variables. The upper graph contains the time series of Δp_t, Δd_t and Δe_t, and the lower graph $\Delta \pi_t$, $\Delta tb_t^{\$}$, Δy_t^g and Δy_t^c. The gray vertical bars denote the extreme events where dummy variables are set. The black vertical line denotes the quarter 1952:Q1.

Graphical inspection of the variables $\mathbf{z}_t$ and $\Delta\mathbf{z}_t$ indicates three extreme events in the sample analyzed (see Figures 4.1 and 4.2).[9] First, the Great Depression had dramatic negative effects on the equity market and the inflation rate. Second, we observe extraordinary transitory shocks on inflation, interest rates and yields at the beginning of the Volcker era in around 1980. Third, due to the recent financial crisis the stock market overreacted with a steep drop at the end of 2008 and recovered subsequently. To account for these outliers and to eliminate the greatest sources of non-normal kurtosis, we set a mean-shift dummy, $d_{s,t}$, for the periods $t =$ 1929:Q1,...,1933:Q1 and $t =$ 1937:Q3,...,1938:Q2 and transitory shock dummies, $d_{tr1,t}$ and $d_{tr2,t}$, for the periods $t =$ 1980:Q1,...,1980:Q4 and $t =$ 2008:Q4,...,2009:Q4, respectively.[10] Thus, the deterministic part of the model is defined as $\mathbf{d}_t = (d_{s,t}, d_{tr1,t}, d_{tr2,t}, t, 1)^{'}$. In addition to the extreme events, Figure 4.2 shows visible heteroskedastic behavior in the data. While the volatility of the equity series is quite homogeneous, except during the Great Depression and the recent financial crisis, the fixed-income series is highly volatile at the beginning of Volcker era. The inflation variability is about four times higher before the Treasury-Federal Reserve (FED) Accord of late 1951 compared to the subsequent period, and interest rates are nearly constant during World War II.

4.3.2 Cointegration Rank Analysis

To determine the number of cointegration relations, we apply the Johansen rank test. The trace test has been shown to be more robust than the maximum eigenvalue test in terms of non-normality (Cheung and Lai, 1993) and is not sensitive to heteroskedasticity effects (Lee and Tse, 1996; Rahbek, Hansen, and Dennis, 2002). As a result, we use the trace test to account for the skewness, kurtosis and heteroskedasticity in the data. We find four cointegration relations or, alternatively, three remaining stochastic trends at the 5% level in our model, where the number

[9] For a better visual presentation, we use the 12-month moving average of dividends and inflation, since the two original time series are highly volatile at quarterly frequencies.

[10] With the exclusion of these outliers we capture the common market movement with our model, since the estimation results are not biased by these rare events. Of course, this model forecasts regular market movements and is not able to predict financial crises or other extreme events.

of lags is two, as suggested by the Schwarz (SC) criterion (see Appendix: Table 4.8, Panel A).[11]

To investigate the stochastic trends in our model in more detail, we test the cointegration rank of models by stepwise increasing the dimensionality of the multivariate time series. The results of these nested models $\mathcal{M}_1$ to $\mathcal{M}_6$ are reported in Table 4.3. As for $\mathcal{M}_6$, all submodels are estimated as first-order VECs.[12] Boldfaced values denote the cointegration rank supported at the 5% level. The p-values of the trace test and the modulus of the largest unrestricted characteristic roots, ρ_{max}, are presented for each model and each possible cointegration rank. The latter statistic is taken into consideration when checking the robustness of the trace test. If an additional $(r+1)$th cointegration relation is mistakenly included in the model, the largest characteristic root will take a value close to one, which indicates the non-stationarity of the $(r+1)$th cointegration vector (Juselius, 2006, Chap. 8).

Following Campbell and Shiller (1987), we start with model $\mathcal{M}_1$ and only include stock prices and dividends. The trace test and our model specifications confirm the result of Campbell and Shiller who find some evidence for a cointegration relation between stock prices and dividends. Based on Campbell and Shiller (1988b)'s findings that prices and dividends are connected to a measure of earnings, we extend the first model by earnings. The rank test for the resulting model, $\mathcal{M}_2$, clearly indicates only one cointegration relation. However, the results for models $\mathcal{M}_1$ and $\mathcal{M}_2$ are inconsistent, as theory would suggest two cointegration relations for the second model. Since prices and dividends have the same stochastic trend, earnings should follow this trend because dividends are derived by earnings in the long run. According to the results, however, either earnings would follow another stochastic trend than prices and dividends or the result of the first model is misleading. Previous studies strengthen the latter possibility. Analyzing a trivariate system of prices, dividends and earnings, Lee (1996) finds one cointegration relation and a strong

[11]The results of the cointegration rank test can vary with the number of lags included in the VAR. However, up to five lags in the VAR our results remain stable around three to four relations depending on the significance level considered. The stability of the cointegration rank regarding the lag length is reported in the Appendix Table 4.8, Panel B.

[12]We assume the data generating process of the level variables to have two lags and not to change in the submodels.

Table 4.3: Cointegration Rank

Variables			Cointegration Rank							
		Statistic	0	1	2	3	4	5	6	7
$\mathcal{M}_1$:	p_t, d_t									
		p-value	0.03	**0.82**	-					
		ρ_{max}	0.65	**0.86**	1.00					
$\mathcal{M}_2$:	p_t, d_t, e_t									
		p-value	0.00	**0.10**	0.67	-				
		ρ_{max}	0.66	**0.76**	0.92	1.00				
$\mathcal{M}_3$:	p_t, d_t, e_t,									
	π_t	p-value	0.00	0.00	**0.15**	0.82	-			
		ρ_{max}	0.66	0.67	**0.77**	0.93	1.00			
$\mathcal{M}_4$:	p_t, d_t, e_t,									
	π_t, $tb_t^{\$}$	p-value	0.00	0.00	**0.20**	0.45	0.94	-		
		ρ_{max}	0.66	0.68	**0.76**	0.92	0.96	1.00		
$\mathcal{M}_5$:	p_t, d_t, e_t,									
	π_t, $tb_t^{\$}$, y_t^g	p-value	0.00	0.00	0.00	**0.49**	0.82	0.78	-	
		ρ_{max}	0.66	0.67	0.82	**0.83**	0.90	0.98	0.99	
$\mathcal{M}_6$:	p_t, d_t, e_t,									
	π_t, $tb_t^{\$}$, y_t^g,	p-value	0.00	0.00	0.00	0.02	**0.58**	0.91	0.94	-
	y_t^c	ρ_{max}	0.66	0.67	0.82	0.81	**0.86**	0.90	0.99	1.00

Notes: The table reports the results for the cointegration rank determination. The models $\mathcal{M}_1$ to $\mathcal{M}_6$ contain stepwise increasing dimensionalities of the multivariate time series. The p-values of the trace test and the moduli of the largest unrestricted characteristic roots, ρ_{max}, are presented for each model and each possible cointegration rank. Boldfaced values denote the cointegration rank supported at the 5% level.

comovement between dividends and earnings without detecting a significant link to prices. Lamont (1998) confirms these results with his bivariate cointegration tests between the three variables.

In contrast to Campbell and Shiller (1988b), Lee (1996) and Lamont (1998), who only focus on the link between (real) stock prices and (real) cash flows, we also analyze the links of macroeconomic factors like inflation or short and long-term interest rates to the stock market, since these links can influence the variables of the valuation ratios differently. Therefore, we stepwise extend model $\mathcal{M}_2$ by the macroeconomic variables π_t, $tb_t^{\$}$, y_t^g and y_t^c. In a subsequent analysis, we show the importance of all seven variables in the long run and their significant influence on each other. Adding inflation increases the number of cointegration relations as the tests indicate that the model $\mathcal{M}_3$ has a cointegration rank of two. Thus, π_t forms a new stationary relation with the variables in the system. Adding $tb_t^{\$}$ does not increase the number of cointegration relations, i.e. a variable with a new stochastic trend is added. However, inclusion of y_t^g and y_t^c raises the cointegration rank by one in each case and suggests that the two variables follow stochastic trends already existing in the system, which leads to our full model, $\mathcal{M}_6$, with four cointegration relations.

4.3.3 Restriction Tests

Thus far, the connection between the stochastic trends of the equity market and the macroeconomic variables is no clear-cut. Therefore, we perform various long-run restriction tests on the matrices $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$. Testing the long-run exclusion of a variable (a row of zeros) in $\boldsymbol{\beta}$, we gain insights about whether the tested variable can be excluded or adds new information to the long-run structure. Likewise, the test of weak exogeneity of a variable (a row of zeros) in $\boldsymbol{\alpha}$ can be informative if the tested variable is affected by the long-run equations and a new added variable changes the previous exogeneity and endogeneity characteristics of the remaining variables. Testing the trend-stationarity of a variable (a unit vector) in $\boldsymbol{\beta}$, we analyze whether the tested variables have deterministic growth rates in the multivariate model.

The results of the $\boldsymbol{\beta}$ restriction tests are presented in Table 4.4. Boldfaced values denote the support of the null hypothesis at the 5% level. Panel A shows the results of the exclusion tests of the variables in each model ($\mathcal{M}_1$ to $\mathcal{M}_6$). In our full model, $\mathcal{M}_6$, neither the variables nor the trend can be excluded from the system. Since stock prices are excludable in the model $\mathcal{M}_2$, there is a connection between stock prices and inflation. Dividends and earnings seem to be important long-run pushing components in each model. Moreover, we find that the short-term interest rates can be omitted in the model without long-term yields. Hence, there seems to be no

Table 4.4: $\boldsymbol{\beta}$ Restriction Tests

Panel A: Long-Run Exclusion Test (Zero Row in $\boldsymbol{\beta}$)

Model		p_t	d_t	e_t	π_t	$tb_t^{\$}$	y_t^g	y_t^c	$trend$
$\mathcal{M}_1$:	$\chi^2(1)$	12.33	20.52						6.67
	p-value	0.00	0.00						0.01
$\mathcal{M}_2$:	$\chi^2(1)$	**2.62**	26.87	29.66					**0.90**
	p-value	**0.11**	0.00	0.00					**0.34**
$\mathcal{M}_3$:	$\chi^2(1)$	20.21	24.89	32.07	96.97				**2.43**
	p-value	0.00	0.00	0.00	0.00				**0.30**
$\mathcal{M}_4$:	$\chi^2(2)$	12.26	24.90	27.77	101.14	**5.98**			**4.30**
	p-value	0.00	0.00	0.00	0.00	**0.05**			**0.12**
$\mathcal{M}_5$:	$\chi^2(3)$	17.60	27.02	30.51	106.87	44.50	44.94		12.42
	p-value	0.00	0.00	0.00	0.00	0.00	0.00		0.01
$\mathcal{M}_6$:	$\chi^2(4)$	26.40	26.97	28.19	107.56	50.46	36.74	29.22	14.30
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01

Panel B: Trend-Stationarity Test (Unit Vector in $\boldsymbol{\beta}$)

Model		p_t	d_t	e_t	π_t	$tb_t^{\$}$	y_t^g	y_t^c
$\mathcal{M}_6$:	$\chi^2(3)$	34.18	30.35	29.81	15.98	32.61	32.59	32.07
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: The table reports the results for the restriction tests on the long-run equation matrix $\boldsymbol{\beta}$. Boldfaced values denote the support of the null at the 5% level. Panel A shows the results of the exclusion tests of the variables (zero row in $\boldsymbol{\beta}$) in the models $\mathcal{M}_1$ to $\mathcal{M}_6$. Panel B shows the results of the trend-stationarity tests of each variable (unit vector in $\boldsymbol{\beta}$) in the multivariate model $\mathcal{M}_6$.

direct link between the stochastic trends of T-bills and the equity market. Panel B shows the results of the trend-stationarity tests of each variable in the multivariate

Table 4.5: $\boldsymbol{\alpha}$ Restriction Tests

Weak Exogeneity (Zero Row in $\boldsymbol{\alpha}$)								
		Variable						
Model		p_t	d_t	e_t	π_t	$tb_t^{\$}$	y_t^g	y_t^c
$\mathcal{M}_1$:	$\chi^2(1)$	**2.22**	16.38					
	p-value	**0.14**	0.00					
$\mathcal{M}_2$:	$\chi^2(1)$	**0.00**	24.21	9.51				
	p-value	**0.95**	0.00	0.00				
$\mathcal{M}_3$:	$\chi^2(1)$	20.85	29.55	11.83	88.02			
	p-value	0.00	0.00	0.00	0.00			
$\mathcal{M}_4$:	$\chi^2(2)$	19.87	29.45	13.91	85.08	**0.24**		
	p-value	0.00	0.00	0.00	0.01	**0.89**		
$\mathcal{M}_5$:	$\chi^2(3)$	19.02	36.12	22.58	88.01	9.30	15.15	
	p-value	0.00	0.00	0.00	0.03	0.00	0.00	
$\mathcal{M}_6$:	$\chi^2(4)$	22.89	35.73	25.31	86.11	17.45	18.84	31.05
	p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: The table reports the results of the weak exogeneity tests of the variables (zero row in $\boldsymbol{\alpha}$) in the models $\mathcal{M}_1$ to $\mathcal{M}_6$. Boldfaced values denote the support of the null at the 5% level.

model $\mathcal{M}_6$. The null of trend-stationarity is rejected in each case, strengthening the results of the univariate stationarity tests presented above.[13]

Table 4.5 reports the the results of the weak exogeneity tests of the variables in each model ($\mathcal{M}_1$ to $\mathcal{M}_6$). Boldfaced values denote the support of the null hypothesis at the 5% level. In our full model, $\mathcal{M}_6$, none of the variables are weakly exogenous. However, stock prices can be treated weakly exogenous in the models $\mathcal{M}_1$ and $\mathcal{M}_2$ and are not affected by the long-run equations. Incorporating the macroeconomic variables, stock prices become endogenous in the models and are pushed by them. Dividends and earnings are strongly influenced by the long-run relations in each model. Furthermore, the short-term interest rates are exogenous in the model, $\mathcal{M}_4$,

[13]To save space, the results are only reported for model $\mathcal{M}_6$, but remain stable for all models. Although the trend-stationarity tests are sensitive to the chosen cointegration rank, the rejection of the null is also obtained for $r = 3$ and $r = 5$ in model $\mathcal{M}_6$.

without the long-term yields and, thus, no significant adjustment of T-bills to the equity market components and inflation takes place.

To sum up, all variables analyzed need to be included in the model, since they have significant long-run influences on and adjustments to each other. Moreover, we detect stock prices pushing dividends without adjusting to the dividend-price relation in model $\mathcal{M}_1$. In the second model, stock prices are also obsolete in the long-run equation and, hence, dividends are only related to earnings here. Since stock prices are no longer excludable and weak exogenous in model $\mathcal{M}_3$, there seems to be a strong link between the equity market variables and inflation.

4.3.4 Testing the Financial Ratios

To further analyze common stochastic trends, we test the validity of the stationarity of the dividend-price ratio and additional financial ratios such as dividend-earnings, earnings-price, real short-interest rates, term spread and credit spread, which are often used in the predictability literature. If the financial ratio has a constant mean, then we can infer that the corresponding non-stationary variables follow the same stochastic trend.

Table 4.6 reports the results of the financial ratio stationarity tests in a multivariate framework. We test the following ratios: dividend-price ($d_t - p_t$); dividend-earnings ($d_t - e_t$); earnings-price ($e_t - p_t$); real T-bills ($tb_t^{\$} - \pi_t$); term spread ($y_t^g - tb_t^{\$}$); and credit spread ($y_t^c - y_t^g$). Panel A presents the results for assuming fixed $[1, -1]$ ratios and are performed with hypotheses test H_3.[14] Panel B presents the results for assuming arbitrary $[1, -\beta]$ ratios and are performed with hypotheses test H_4. Boldfaced values denote the support of the null of stationarity at a 5% significance level. If trend-stationarity of the restricted cointegration relation cannot be rejected, more restrictive stationarity tests without a deterministic trend are performed. The null of trend-stationarity of the $[1, -1]$ ratios has to be rejected for dividend-price as well as all other ratios except for $d_t - e_t$ and $y_t^g - tb_t^{\$}$. However, these two ratios have unit roots if the deterministic trend is omitted. The rejection of $\mathcal{H}_{2.2}$ can be

[14] Additionally, we test the stationarity of the $[1, -1]$ financial ratios with the univariate ADF and KPSS test. The results of these tests are reported in Table 4.9 of the Appendix.

Table 4.6: Financial Ratio Stationarity Tests

Panel A: Financial Ratio Stationarity Tests $[1, -1]$				
	Ratio	*trend* $\times 10^4$	$\chi^2(\nu)$	p-value
$\mathcal{H}_1$:	$d_t - p_t$	38.13	31.61 (3)	0.00
$\mathcal{H}_{2.1}$:	$d_t - e_t$	11.68	**3.13 (3)**	**0.37**
$\mathcal{H}_{2.2}$:	$d_t - e_t$	-	11.42 (4)	0.02
$\mathcal{H}_3$:	$e_t - p_t$	24.23	28.83 (3)	0.00
$\mathcal{H}_4$:	$tb_t^{\$} - \pi_t$	-0.26	26.75 (3)	0.00
$\mathcal{H}_{5.1}$:	$y_t^g - tb_t^{\$}$	-0.09	**5.87 (3)**	**0.12**
$\mathcal{H}_{5.2}$:	$y_t^g - tb_t^{\$}$	-	11.50 (4)	0.02
$\mathcal{H}_6$:	$y_t^c - y_t^g$	-42.13	34.23 (3)	0.00

Panel B: Financial Ratio Stationarity Tests $[1, -\beta]$					
	Ratio	β	*trend* $\times 10^4$	$\chi^2(\nu)$	p-value
$\mathcal{H}_7$:	$d_t - \beta \cdot p_t$	0.41	-63.10	22.44 (2)	0.00
$\mathcal{H}_{8.1}$:	$d_t - \beta \cdot e_t$	0.88	-6.11	**2.32 (2)**	**0.31**
$\mathcal{H}_{8.2}$:	$d_t - \beta \cdot e_t$	0.92	-	**2.42 (3)**	**0.49**
$\mathcal{H}_9$:	$e_t - \beta \cdot p_t$	0.46	-66.61	23.63 (2)	0.00
$\mathcal{H}_{10}$:	$tb_t^{\$} - \beta \cdot \pi_t$	3.45	-0.28	13.73 (2)	0.00
$\mathcal{H}_{11.1}$:	$y_t^g - \beta \cdot tb_t^{\$}$	0.96	-0.10	**5.48 (2)**	**0.06**
$\mathcal{H}_{11.2}$:	$y_t^g - \beta \cdot tb_t^{\$}$	1.05	-	10.91 (3)	0.01
$\mathcal{H}_{12}$:	$y_t^c - \beta \cdot y_t^g$	0.97	-0.03	8.65 (2)	0.01

Notes: The table reports the results of the financial ratio stationarity tests. Panel A presents the results for fixed $[1, -1]$ ratios and are performed with hypotheses test H_3. Panel B presents the results for $[1, -\beta]$ ratios and are performed with hypotheses test H_4. Boldfaced values denote the support of the null of stationarity at a 5% significance level. If trend-stationarity cannot be rejected, further stationarity tests without a deterministic trend are performed.

justified by a changed dividend payout policy over the sample period (Fama and French, 2001; Grullon and Michaely, 2002, 2004; Boudoukh, Michaely, Richardson, and Roberts, 2007; Park and Kim, 2012) and makes dividends and earnings move slightly apart with a deterministic trend. One might argue that similar reasons hold for the rejection of the stationarity of the dividend-price ratio, but according to Lettau and Van Nieuwerburgh (2008) "[...] structural changes in payout policies [...] can only explain a small part of the change in the dividend-price ratio." Thus, the gap between stock prices and dividends became far to big to be caused only by a changed payout policy. Furthermore, the short interest rates and the government yields drift apart in a deterministic way, indicating either a steadily increase of the requested bond yield relative to the short rates or a steadily increasing spread because of too low T-bill rates.

Relaxing the $[1, -1]$ conditions, we observe that the adjusted dividend-earnings ratio is stationary and term spread is trend-stationary. As interpreted in Froot and Obstfeld (1991), the coefficient in $(d_t - \beta \cdot e_t)$ is less than one and implies that earnings move more than dividends. The same argumentation holds for $\mathcal{H}_{11.1}$, while in this case the β is much closer to one. For all other ratios, even the relaxation of the $[1, -1]$ assumption does not lead to trend-stationary behavior and, thus, strongly suggests the influence of different stochastic trends. Since the $(d_t - e_t)$ and $(y_t^g - tb_t^\$)$ ratios are trend-stationary in a single relation, we further test whether this result holds for all cointegration vectors. According to the global test results, the use of the dividend-earnings ratio causes no loss of information ($\chi^2_4 = 3.83$ and p-value $= 0.43$), while the hypothesis is strongly rejected for the term spread ($\chi^2_4 = 28.39$ and p-value $= 0.00$).

4.3.5 Level Effects

The previous analysis shows that the null of cointegration between prices and the stock's cash flows is rejected by allowing the influence of other macroeconomic factors. However, we find four cointegration relations among the seven variables analyzed. These four relations can be extracted by the decomposition of long-run effect matrix $\mathbf{\Pi}_r$ in its cointegration matrix $\boldsymbol{\beta}'$ and the adjustments $\boldsymbol{\alpha}$. Since the decomposition in $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}'$ is not unique, we focus on the level matrix $\boldsymbol{\Pi}_r$ to investigate the different long-run impacts on the variables.

Table 4.7 reports the coefficient estimates of the long-run matrix $\boldsymbol{\Pi}_r$ of model $\mathcal{M}_6$. Bootstrap standard errors, which are calculated from 10,000 paths under the assumption that the initial estimated VEC model truly generates the data process, are reported in parentheses.[15] Boldfaced coefficients imply significance on a 5% level according to bootstrapped intervals. The first row of this table represents the error correction equation for stock price returns and shows that only the own lagged level and the lagged inflation rate have a significant negative influence, which illustrates a macroeconomic long-run link. Dividends and earnings, in contrast, are not significant predictors of price changes, even if theory would expect a strong

[15] A more detailed description of the bootstrapping method is given in Appendix 4.A.4.

Table 4.7: $\boldsymbol{\Pi}_r$ Matrix of Model $\mathcal{M}_6$ and $r = 4$

	Variable							
	p_{t-1}	d_{t-1}	e_{t-1}	π_{t-1}	$tb^{\$}_{t-1}$	y^g_{t-1}	y^c_{t-1}	$trend \times 10^3$
Δp_t	**-0.035**	-0.045	0.050	**-2.738**	-0.557	-13.929	15.264	0.396
	(0.022)	(0.047)	(0.036)	(0.734)	(3.090)	(10.407)	(9.850)	(0.506)
Δd_t	**0.079**	**-0.284**	**0.192**	**2.523**	**-8.898**	6.524	3.465	-0.418
	(0.039)	(0.072)	(0.053)	(1.114)	(4.731)	(15.831)	(15.286)	(1.130)
Δe_t	0.013	**0.060**	**-0.044**	0.764	**-4.210**	**10.911**	-6.779	-0.394
	(0.012)	(0.024)	(0.021)	(0.403)	(1.667)	(5.680)	(5.531)	(0.322)
$\Delta \pi_t$	**-0.008**	0.002	0.002	**-0.648**	0.482	-0.843	0.450	0.080
	(0.002)	(0.005)	(0.004)	(0.073)	(0.310)	(1.032)	(1.002)	(0.067)
$\Delta tb^{\$}_t$	**0.001**	0.000	0.000	0.005	**-0.085**	**0.515**	**-0.425**	-0.010
	(0.000)	(0.001)	(0.001)	(0.011)	(0.049)	(0.159)	(0.156)	(0.009)
Δy^g_t	0.000	0.000	0.000	0.001	**0.098**	-0.109	0.003	0.005
	(0.000)	(0.000)	(0.000)	(0.007)	(0.030)	(0.103)	(0.098)	(0.005)
Δy^c_t	0.000	0.000	0.000	0.003	**0.089**	0.067	**-0.165**	0.002
	(0.000)	(0.000)	(0.000)	(0.006)	(0.024)	(0.081)	(0.079)	(0.005)

Notes: The table reports the coefficient estimates of the long-run matrix $\boldsymbol{\Pi}_r$ of model $\mathcal{M}_6$ and $r = 4$. Bootstrap standard errors, which are calculated from 10,000 paths under the assumption that the initial estimated VEC model truly generates the data process, are reported in parentheses. Boldfaced coefficients imply significance on a 5% level according to bootstrapped intervals.

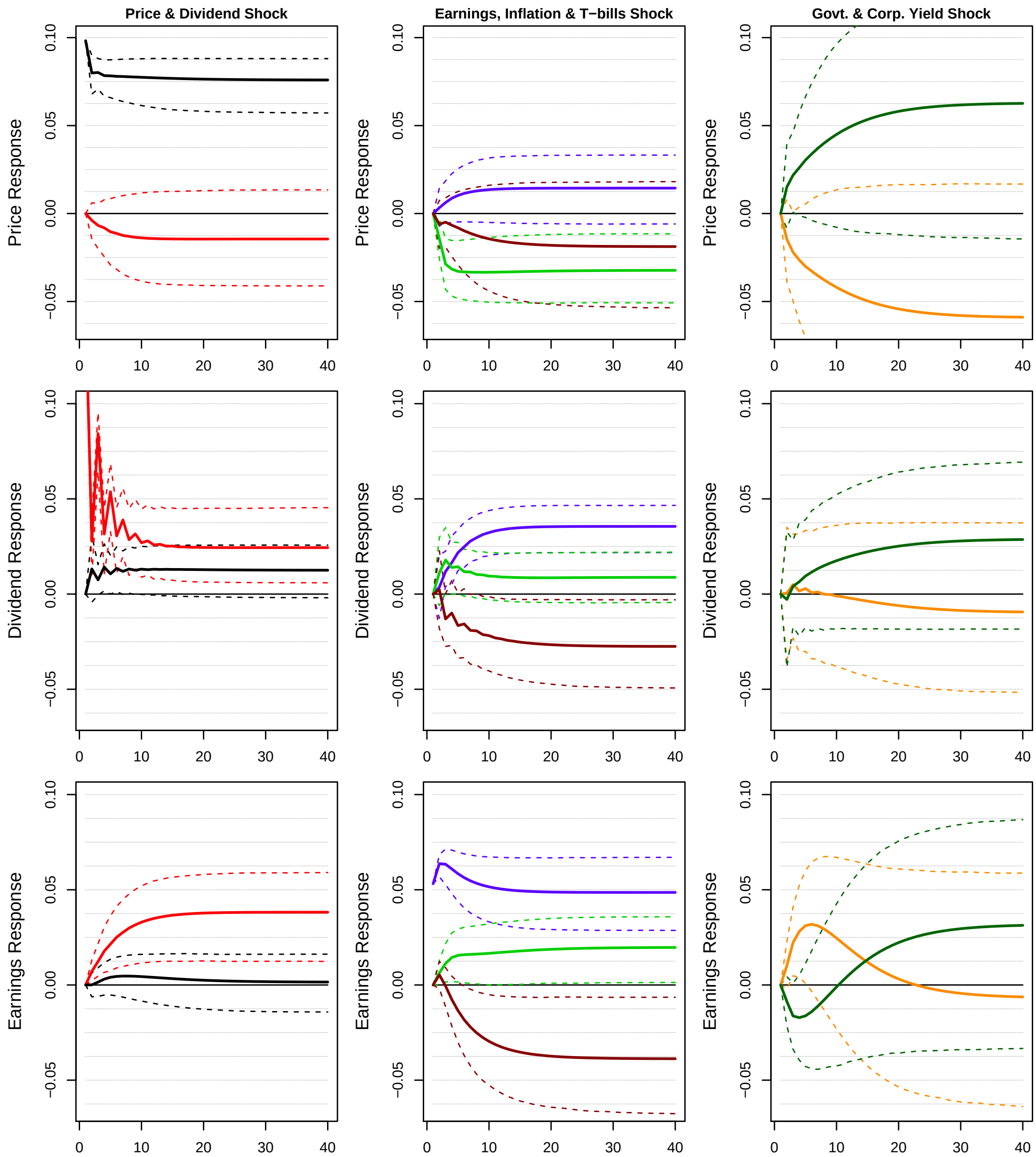
long-run connection. The last three level variables have no significant effects, albeit the coefficients of the yields are large and nearly related $[1, -1]$ almost forming the credit spread. The second row corresponds to the dividend growth, which is significantly explained by lagged prices, dividends, earnings, inflation rate and short-term interest rates. As the third row shows, the influences on earnings growth is less pronounced and only significant for lagged dividends, earnings, T-bills and long-term yields. Comparing the equations of dividends and earnings demonstrates their strong interdependence, although some different effects remain. While the dividends significantly change with prices and inflation, earnings remain nearly unchanged by these two variables. The negative effect of short-term rates is more than twice as high for dividends compared to earnings growth. Government bond yields have only significant level effects on earnings growth. Comparing the influences of the macroeconomic variables on prices and cash flows changes, we observe significant differences: completely contrary inflation effects, T-bills and long-term yields affect only cash flows. As the last four rows show, the equity market has a moderate effect on the macroeconomic variables. Only stock prices predict changes in inflation and T-bills. Furthermore, short-term rates and bond yields are interrelated.

4.3.6 Horizon-Dependent Analysis

Because longer horizon dynamics in the VEC are complicated to assess by considering estimated coefficients matrices, we further investigate the impact of unexpected shocks by an impulse response analysis. This is done by extracting the innovations' influence of the equity market and the macroeconomic variables on prices, dividends and earnings. Furthermore, applying the loglinear approximation for total returns allows us to use the responses of prices and dividends to calculate the stock return response.

Figure 4.3 plots the responses of prices, dividends and earnings (line-by-line) to one standard deviation shocks in prices (black), dividends (red), earnings (blue), inflation (green), T-bills (dark red), long-term yields (orange) and corporate yields (dark green) with the corresponding bootstrapped 5% intervals from left to right. The intervals are calculated from 10,000 paths under the assumption that the ini-

Figure 4.3: Impulse Response Functions



Notes: This figure plots the responses of prices, dividends and earnings (line-by-line) to one standard deviation shocks in prices (black), dividends (red), earnings (blue), inflation (green), T-bills (dark red), long-term government yields (orange) and corporate yields (dark green) with the corresponding bootstrapped 5% intervals from left to right. The intervals are calculated from 10,000 paths under the assumption that the initial estimated VEC model truly generates the data process. The depicted horizon comprises 40 quarters.

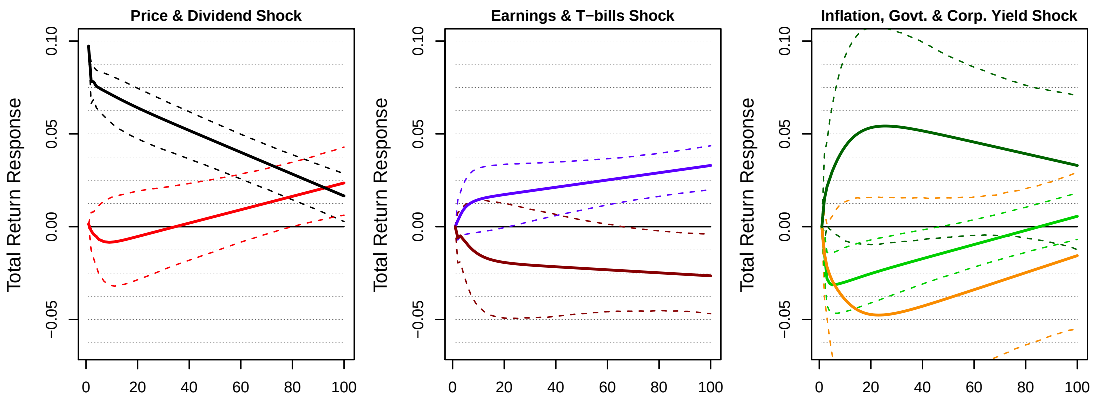
tial estimated VEC model truly generates the data process. The depicted horizon comprises 40 quarters. The figure shows that nearly all shocks on the equity market variables have permanent effects, although not necessarily statistically permanent. Considering stock price responses, we see that a dividend shock cause a negative price reaction and an earnings shock a positive (insignificant) price reaction. An unexpected price impulse never disappears and nearly remains on the initial level. Turning to macroeconomic shocks, all variables (except corporate bond yields) lead to negative effects, but only an inflation innovation reduces prices fast and significantly. Again, the strong link between stock prices and the inflation rate is visible. The dividend response to a price impulse is positive, but only significant in the short term. Hence, dividends adjust due to price shocks. An unexpected change in dividends decays fast (10 quarters) to a sixth of the base level. Opposed shocks of dividends and earnings on each other result in a significant rise of the cash flow variables. While the influence of earnings on dividends is well established, the opposite connection can be interpreted as the managers' ability to adjust today's dividends according to their expected future business success. The responses of the two cash flow variables to macroeconomic impulses are comparable to each other in the long run. Both T-bills and inflation innovations have significant negative and positive influences, respectively. While cash flows respond positively to shocks of corporate bond yields, contrary responses are present for government bond yields in the long term.

Comparing these results to those of Cochrane (1994) and Lamont (1998), we find contrary effects, as they show that prices and dividends are permanently affected by a dividend innovation, and price innovations are transitory for prices and neglectable for dividends. These differences can be caused for three reasons. First, while they allow for only stock prices and cash flows in their models, we extend the system by macroeconomic information. Second, they make the strong assumption that prices and cash flows all share one common trend and, third, that all these variables move one-for-one.

Although we only model stock prices and dividends, we are able to investigate the effects of the innovations of the variables analyzed on total stock returns by applying the loglinear return approximation in Equation (4.10). Figure 4.4 plots the cumulative responses of the total stock returns to one standard deviation shocks in prices (black), dividends (red), earnings (blue), inflation (green), T-bills (dark red), long-term yields (orange) and corporate yields (dark green) with the corresponding bootstrapped 5% intervals from left to right. The intervals are calculated from 10,000 paths under the assumption that the initial estimated VEC model truly generates the data process. The depicted horizon comprises 100 quarters.

In response to a price shock, cumulated returns are expected to decrease subsequently. On the other hand, an innovation in dividends increases returns with a significant positive response in the long run. These two findings are a direct result of the impulse responses between prices and dividends and the loglinear approximation. Due to a price shock, permanent responses of prices and dividends decrease returns because the negative response factor caused by $\rho p_{t+k} - p_{t+k-1}$ dominates the positive factor caused by $(1-\rho)d_{t+k}$. Thus, cumulated returns decline over the investment horizon. In contrast, the typical approach of restricting prices and dividends to move one-for-one results in a vanishing (transitory) cumulative return response as

Figure 4.4: Impulse Response Functions of Returns



Notes: This figure plots the cumulative responses of the total stock returns to one standard deviation shocks in prices (black), dividends (red), earnings (blue), inflation (green), T-bills (dark red), long-term government yields (orange) and corporate yields (dark green) with the corresponding bootstrapped 5% intervals from left to right. The intervals are calculated from 10,000 paths under the assumption that the initial estimated VEC model truly generates the data process. The depicted horizon comprises 100 quarters.

the factors compensate each other and makes discount rate (price) shocks transitory in the long run. Applying the same argumentation for a dividend shock, cumulated returns increase permanently since the factors caused by $\rho p_{t+k} - p_{t+k-1}$ and $(1-\rho)d_{t+k}$ are positive in the long run. In this case, restricting prices and dividends to move one-for-one results in a transitory return response and yields dividend news to be permanent with a positive shift in cumulated returns. Turning to earnings shocks, we see that the positive response of prices and dividends leads to a positive and significant rise of long-horizon returns. On the other side, a T-bill shock causes a negative response of the equity market variables and leads to a negative and significant decrease of long-horizon returns. While an inflation innovation significantly depletes stock prices, cumulative returns (including dividends) recover subsequently after a steep drop in the first periods (which is in line with the findings of Fama and Schwert (1977) for the short run and Boudoukh and Richardson (1993) for the long run). Impulses of the two bond yields lead to large and contrary effects of the cumulated stock returns, which diminish in the long run.

4.4 Conclusion

Valuation ratios such as the dividend-price ratio and earnings-price ratio have been often used in the predictability literature to forecast stock returns. For this matter it is assumed that prices and cash flows are cointegrated one-for-one or, alternatively, that the dividend-price ratio and earnings-price ratio are stationary variables. Otherwise the conventional t-test infers invalid conclusions about the return predictability. However, the empirical evidence on the dividend-price ratio's stationarity is, at best, mixed. This raises the question as to whether this problem is blurred by the high persistence of the valuation ratio, is based on a change in the payout policy or is actually caused by a breakdown of the one-for-one relation due to various macroeconomic influences on prices and dividends.

By extending the bivariate model of prices and dividends with earnings, inflation rate, short-term interest rates and government and corporate bond yields, we investigate the influence of macroeconomic proxies on stock prices and cash flows within a cointegration framework and consequently deduce the impact on total stock returns with the loglinear approximation. We find four cointegration relations among the seven non-stationary time series. Testing the dividend-price and further financial ratios (dividend-earnings, earnings-price, real short-interest rates, term spread and credit spread) for stationarity in a multivariate framework, we reject the null for the dividend-price ratio and only find the dividend-earnings ratio and the term spread to most likely have common stochastic trends. We then empirically analyze the macroeconomic impacts on the equity market. We show that inflation has a very strong impact on stock prices, dividends and earnings. While previous research typically considers the real terms of returns, prices and dividends to eliminate inflation effects and assume that inflation has the same impact on all equity market variables, we find a negative linkage of nominal prices and inflation and nominal cash flows to be positively associated to inflation in the long run. These effects erode nominal returns in the short term and yield returns to recover in the long run. The risk-free rate has the same negative connection to all equity market variables, but the magnitude is much more pronounced for cash flows than for stock prices. Finally, we find large positive effects on the equity market for corporate bond yields, but the influence of government bond yields is negative.

We hope that our results will stimulate the asset pricing and predictability literature. Finding different macroeconomic effects on prices, dividends and earnings, we would suggest considering modified (stationary) valuation ratios to better predict future returns. We leave this topic for further research.

4.A Appendix

4.A.1 Model Selection

The number of lags to be included in the VEC model $\mathcal{M}_6$ is determined by considering the Akaike information criterion (AIC), the Schwarz criterion (SC) and the Hannan & Quinn criterion (HQ). Additionally, the results of the cointegration rank test can vary with the number of lags included in the VAR. Table 4.8, Panel A reports the test statistics of these criteria depending on the number of lags. Panel B shows the stability of the cointegration rank of the Johansen (1988) trace test

Table 4.8: Lag Length Selection and Cointegration Rank Stability

Panel A	Lags p in VAR Model					
Criterion	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$	$p=6$
AIC(p)	-45.64	-46.27	-46.28	**-46.79**	-46.78	-46.76
HQ(p)	-45.29	-45.69	-45.47	**-45.77**	-45.53	-45.29
SC(p)	-44.76	**-44.82**	-44.26	-44.22	-43.64	-43.06

Panel B	Critical Values			Lags p in VAR Model			
Rank	10%	5%	1%	$p=2$	$p=3$	$p=4$	$p=5$
$r \leq 6$	10.49	12.25	16.26	3.36	3.93	3.78	3.18
$r \leq 5$	22.76	25.32	30.45	12.57	16.29	14.29	14.69
$r \leq 4$	39.06	42.44	48.45	**40.45**	**40.35**	30.02	31.81
$r \leq 3$	59.14	62.99	70.05	74.21	68.13	**52.73**	**55.48**
$r \leq 2$	83.2	87.31	96.58	133.15	121.19	91.22	99.54
$r \leq 1$	110.42	114.9	124.75	227.96	177.02	143.07	154.81
$r = 0$	141.01	146.76	158.49	334.41	257.16	228.18	223.00

Notes: The table reports the lag length selection and cointegration rank stability. Panel A shows the test statistics of the Akaike, Schwarz and Hannan & Quinn information criteria to determine the number of lags to be included in the VAR. Boldfaced values denote the minimum test statistic depending on the number of lags for each information criterion. Panel B shows the Johansen (1988) trace test results for various lag lengths. Boldfaced values denote the supported rank at the 5% significance level.

results for various lag lengths. Boldfaced values denote the supported rank at the 5% significance level.

The SC suggests two lags for the VAR model in levels which is equivalent to a VEC(1), while the AIC and the HQ suggest four lags. For the empirical analysis, we follow the suggestions of the SC and investigate a first order VEC. Turning to the cointegration rank stability in Panel B, which has up to five lags in the VAR, our results remain stable at around three to four relations depending on the significance level considered.

4.A.2 Univariate Stationarity of the Financial Ratios

The stationarity of the financial ratios such as dividend-price, dividend-earnings, earnings-price, real short-interest rates, term spread and credit spread is often doubtful. To test these hypotheses, we use the ADF test with the null of a unit root and the KPSS test with the null of stationarity. Both tests are performed by allowing for a constant but not for a deterministic time trend. The number of lags used is given in parentheses, where the ADF's lag length is determined by the AIC. The KPSS's

Table 4.9: Univariate Stationarity of Financial Ratios

	Levels					First Differences			
Variable	$\text{ADF}_\mu(q)$		$\text{KPSS}_\mu(q)$		Variable	$\text{ADF}_\mu(q)$		$\text{KPSS}_\mu(q)$	
$(d_t - p_t)$	-1.99	(4)	3.40***	(5)	$\Delta(d_t - p_t)$	-8.69***	(6)	0.03	(5)
$(e_t - p_t)$	-3.26**	(5)	1.34***	(5)	$\Delta(e_t - p_t)$	-8.71***	(6)	0.02	(5)
$(d_t - e_t)$	-3.00**	(10)	1.96***	(5)	$\Delta(d_t - e_t)$	-8.11***	(9)	0.02	(5)
$(y_t^g - tb_t^\$)$	-5.10***	(5)	0.62**	(5)	$\Delta(y_t^g - tb_t^\$)$	-7.18***	(16)	0.02	(5)
$(y_t^c - y_t^g)$	-3.54***	(6)	0.63**	(5)	$\Delta(y_t^c - y_t^g)$	-8.37***	(7)	0.09	(5)
$(tb_t^\$ - \pi_t)$	-3.46***	(11)	0.26	(5)	$\Delta(tb_t^\$ - \pi_t)$	-8.23***	(14)	0.02	(5)

Notes: The table reports the results of the univariate unit root test regressions of the sample from 1927:Q1 to 2011:Q4. All tests include a constant but no deterministic time trends. The number of lags used is given in parentheses; the symbols *, ** and *** denote significance at the 10, 5 and 1% level, respectively. The ADF tests the null of non-stationarity and the lag length is determined by the *Akaike Information Criterion* (AIC). The KPSS tests the null of stationarity and the lag length is determined by the integer value of $(4 \cdot (n/100)^{0.25})$.

lag length is determined by the integer value of $(4 \cdot (n/100)^{0.25})$, which depends only on the number of observations. Table 4.9 reports the corresponding results.

The non-stationarity hypothesis is rejected for all ratios except for the dividend-price ratio, but non-stationarity is strongly rejected for all the first differences. The stationarity hypothesis of the KPSS test, in contrast, is rejected for all ratios except the real short-term interest rate, but stationarity is supported for all first differences.

4.A.3 Stability of the Long-Run Matrices across the Models

To see the changes in long-run effects, we compare the long-run matrices, $\mathbf{\Pi}_r$, of various models. The results are presented in Table 4.10, which reports the coefficient estimates of the long-run matrices $\mathbf{\Pi}_r$ of models $\mathcal{M}_1$ to $\mathcal{M}_6$ and a model of the macroeconomic variables π_t, $tb_t^{\$}$, y_t^g and y_t^c denoted as $\mathcal{M}_m$. Panels A to G report the row of the corresponding variable in each model. Each model is based on a second-order level VAR. The cointegration rank is set as presented in Table 4.3 and shown in the second column. Boldfaced coefficients imply significance on a 5% level according to bootstrapped intervals and are calculated from 10,000 paths under the assumption that the initial estimated VEC model truly generates the data process. Comparing the results of the submodels to the full model, we obtain stable coefficient estimates with respect to the sign and magnitude in the long-run matrix.

Table 4.10: Stability Analysis of the Long-Run Matrices

Π Matrices of the Models		Variable							
Panel A: Δp_t		p_{t-1}	d_{t-1}	e_{t-1}	π_{t-1}	$tb^{\$}_{t-1}$	y^g_{t-1}	y^c_{t-1}	$trend \times 10^3$
$\mathcal{M}_6$:	$r = 4$	**-0.035**	-0.045	0.050	**-2.738**	-0.557	-13.929	15.264	0.396
$\mathcal{M}_5$:	$r = 3$	-0.019	-0.037	0.038	**-2.723**	-0.610	1.364		0.224
$\mathcal{M}_4$:	$r = 2$	**-0.022**	-0.032	0.032	**-2.810**	1.064			0.293
$\mathcal{M}_3$:	$r = 2$	**-0.031**	-0.023	0.041	**-2.754**				0.212
$\mathcal{M}_2$:	$r = 1$	0.000	-0.002	0.001					0.003
$\mathcal{M}_1$:	$r = 1$	-0.021	0.047						-0.282
Panel B: Δd_t		p_{t-1}	d_{t-1}	e_{t-1}	π_{t-1}	$tb^{\$}_{t-1}$	y^g_{t-1}	y^c_{t-1}	$trend \times 10^3$
$\mathcal{M}_6$:	$r = 4$	**0.079**	**-0.284**	**0.192**	**2.523**	**-8.898**	6.524	3.465	-0.418
$\mathcal{M}_5$:	$r = 3$	**0.081**	**-0.278**	**0.191**	**2.487**	-8.757	9.551		-0.514
$\mathcal{M}_4$:	$r = 2$	**0.059**	**-0.255**	**0.161**	2.127	-0.057			0.098
$\mathcal{M}_3$:	$r = 2$	**0.060**	**-0.261**	**0.160**	**2.080**				0.191
$\mathcal{M}_2$:	$r = 1$	0.040	**-0.280**	**0.187**					0.401
$\mathcal{M}_1$:	$r = 1$	**0.090**	**-0.199**						**1.198**
Panel C: Δe_t		p_{t-1}	d_{t-1}	e_{t-1}	π_{t-1}	$tb^{\$}_{t-1}$	y^g_{t-1}	y^c_{t-1}	$trend \times 10^3$
$\mathcal{M}_6$:	$r = 4$	0.013	**0.060**	**-0.044**	0.764	**-4.210**	**10.911**	-6.779	-0.394
$\mathcal{M}_5$:	$r = 3$	0.004	**0.062**	**-0.037**	**0.761**	**-4.303**	**4.117**		-0.389
$\mathcal{M}_4$:	$r = 2$	-0.005	**0.074**	**-0.052**	0.667	-0.431			-0.145
$\mathcal{M}_3$:	$r = 2$	-0.002	**0.068**	**-0.052**	0.549				-0.122
$\mathcal{M}_2$:	$r = 1$	-0.009	**0.064**	**-0.043**					-0.092
Panel D: $\Delta \pi_t$		p_{t-1}	d_{t-1}	e_{t-1}	π_{t-1}	$tb^{\$}_{t-1}$	y^g_{t-1}	y^c_{t-1}	$trend \times 10^3$
$\mathcal{M}_6$:	$r = 4$	**-0.008**	0.002	0.002	**-0.648**	0.482	-0.843	0.450	0.080
$\mathcal{M}_5$:	$r = 3$	**-0.007**	0.001	0.001	**-0.652**	0.521	-0.405		0.087
$\mathcal{M}_4$:	$r = 2$	**-0.006**	0.001	0.002	**-0.634**	0.219			0.057
$\mathcal{M}_3$:	$r = 2$	**-0.008**	0.003	0.004	**-0.597**				0.036
$\mathcal{M}_m$:	$r = 3$				**-0.624**	0.331	0.124	-0.204	-0.004
Panel E: $\Delta tb^{\$}_t$		p_{t-1}	d_{t-1}	e_{t-1}	π_{t-1}	$tb^{\$}_{t-1}$	y^g_{t-1}	y^c_{t-1}	$trend \times 10^3$
$\mathcal{M}_6$:	$r = 4$	**0.001**	0.000	0.000	0.005	**-0.085**	**0.515**	**-0.425**	-0.009
$\mathcal{M}_5$:	$r = 3$	0.000	0.000	0.000	0.005	**-0.115**	**0.121**		-0.007
$\mathcal{M}_4$:	$r = 2$	0.000	0.000	0.000	0.005	-0.002			0.000
$\mathcal{M}_m$:	$r = 3$				0.003	-0.057	**0.409**	**-0.363**	0.001
Panel F: Δy^g_t		p_{t-1}	d_{t-1}	e_{t-1}	π_{t-1}	$tb^{\$}_{t-1}$	y^g_{t-1}	y^c_{t-1}	$trend \times 10^3$
$\mathcal{M}_6$:	$r = 4$	0.000	0.000	0.000	0.001	**0.098**	-0.109	0.003	0.005
$\mathcal{M}_5$:	$r = 3$	0.000	0.000	0.000	0.001	**0.089**	**-0.095**		0.005
$\mathcal{M}_m$:	$r = 3$				0.001	**0.080**	-0.103	0.020	0.001
Panel G: Δy^c_t		p_{t-1}	d_{t-1}	e_{t-1}	π_{t-1}	$tb^{\$}_{t-1}$	y^g_{t-1}	y^c_{t-1}	$trend \times 10^3$
$\mathcal{M}_6$:	$r = 4$	0.000	0.000	0.000	0.003	**0.089**	0.067	**-0.165**	0.002
$\mathcal{M}_m$:	$r = 3$				0.001	**0.081**	0.044	**-0.133**	0.001

Notes: The table reports the coefficient estimates of the long-run matrices, $\mathbf{\Pi}_r$, of models $\mathcal{M}_1$ to $\mathcal{M}_6$ and a model of the macroeconomic variables π_t, $tb^{\$}_t$, y^g_t and y^c_t denoted as $\mathcal{M}_m$. Each model is based on a second-order level VAR. The cointegration rank is set as presented in Table 4.3 and shown in the second column of the table. Boldfaced coefficients imply significance on a 5% level according to bootstrapped intervals and are calculated from 10,000 paths under the assumption that the initial estimated VEC model truly generates the data process.

4.A.4 Bootstrap Method

We apply the residual-based bootstrap method suggested by Benkwitz, Lütkepohl, and Wolters (2001) and Lütkepohl (2005), which consists of the following steps:

1. Estimate the unknown coefficients of the VEC. Let $\hat{\boldsymbol{\nu}}_t$ be the estimate of the VEC residuals $\boldsymbol{\nu}_t$.

2. Calculate centered residuals $\hat{\boldsymbol{\nu}}_1 - \bar{\boldsymbol{\nu}}, ..., \hat{\boldsymbol{\nu}}_T - \bar{\boldsymbol{\nu}}$, where $\bar{\boldsymbol{\nu}}$ are the n usual means for the n residual series.

3. Draw randomly with replacement from the centered residuals to obtain bootstrap residuals $\boldsymbol{\epsilon}_1^*, ..., \boldsymbol{\epsilon}_T^*$.

4. Recursively calculate the bootstrap time series for the VAR as

$$\mathbf{z}_t^* = \boldsymbol{\mu} + \mathbf{A}_1 \mathbf{z}_{t-1}^* + \cdots + \mathbf{A}_p \mathbf{z}_{t-p}^* + \boldsymbol{\epsilon}_t^*, \; t = 1, ..., T,$$

   where $(\mathbf{z}_{-p+1}^*, ..., \mathbf{z}_0^*) = (\mathbf{z}_{-p+1}, ..., \mathbf{z}_0)$ holds for each generated series.

5. Reestimate the coefficients of the VEC using the bootstrapped data and calculate the statistic of interest q^*.

6. Repeat these steps N times.

The bootstrap confidence intervals (standard percentile intervals) are then given by

$$CI = \left[s_{\gamma/2}^*, s_{(1-\gamma/2)}^* \right],$$

where $s_{\gamma/2}^*$ and $s_{(1-\gamma/2)}^*$ are the $\gamma/2$- and $1 - (\gamma/2)$-quantiles of the N bootstrap versions of q^*.

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