Essays on Persistence in Growth Rates and the Success of the British Premium Bond

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Dedicated to Julia

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1. Introduction

1.1. Summary

This dissertation contributes new evidence to two areas of research. The first part of the work aims at analyzing persistence in growth rates of operating performance as an important factor for firm valuations. The second part investigates the tremendous success of a lottery bond, the British Premium Bond.

The first two essays, presented in chapters 2 and 3, perform in-depth analyses on both the predictive power as well as the predictability of persistence in growth rates. In this context, persistence derives from the length and the relative frequency of so called runs. A positive run is registered if a firm produces above-median growth rates for a number of consecutive years. A negative run consists of a series of consecutive below-median growth rates, respectively.

Runs and therefore persistence in growth rates are strongly linked with the valuation of a firm. Many investors, analysts and valuation professionals extrapolate past growth rates to make their forecasts. The reason for this is the wide-spread sentiment among market participants that there is a considerable degree of consistency in a firm's growth rates. This relation between persistence in growth rates and firm valuations leads to the two research questions addressed in the first part of the dissertation: (1) Do investors overestimate the predictive power of a high persistence in sales growth rates? (2) Is it possible to predict a high future persistence in growth rates based on a set of firm-specific financial indicators? These research questions are related to the literature on earnings behavior and investor expectations. De Bondt and Thaler (1987, 1990) analyze returns of stocks that have experienced either extreme capital gains or extreme losses. They argue that investors overreact to past firm performance and conclude that this is the main reason why simple value strategies based on valuation ratios beat growth strategies. In this context, Lakonishok et al. (1994) show that glamour

stocks with consistently high past growth rates in operating performance are rewarded with rich valuations. In the same way, value stocks are punished for previous disappointments after several years of consistently poor growth rates. They conclude that market participants had too high expectations about the future performance of glamour stocks. Confirming this finding, La Porta (1996) and La Porta et al. (1997) show that investors tend to extrapolate past growth too far into the future. As a result, stock market returns of value stocks tend to outperform glamour stocks over the long-term (e.g., Fama and French (1988)). Although investors are often tempted to believe in a high consistency of firm growth rates, research shows that empirically this is not the case. For instance, Little (1962) and Little and Rayner (1966) find that in the UK corporate annual earnings numbers essentially follow random processes. A short time later, Lintner and Glauber (1967) analyze US data and confirm that changes in earnings over time appear to be randomly distributed. The closest related study to the essays presented in this dissertation is the US-based cross-sectional study by Chan et al. (2003). They caution against extrapolating past income growth rates into the future, because there is no persistence beyond chance. However, they do report that there is some persistence in sales growth rates.

Chapter 2 takes up this finding. It follows the question: what are the implications of an increased persistence in sales growth on future income growth rates? In particular, it investigates the hypothesis that investors overestimate the translation of an increased persistence in sales growth into consistently high income growth rates. The initial sample comprises data of more than 54,000 firms from 77 countries and over a sample period of 28 years. For the analysis, a new single measure of persistence in growth, called weighted frequencies-score (wf-score), is developed. It is based on a nonparametric test for serial correlation called “run-test” used by Chan et al. (2003). The new measure allows meaningful comparisons across heterogeneous sets of firms. It also enables to compare persistence in

growth of the three examined performance indicators sales, operating income and net income. The results are as follows. Investors apparently strongly reward runs of above-median growth rates and thus a high persistence in sales growth. It is also shown that an increased persistence in sales growth is a global phenomenon. This supports the finding of Chan et al. (2003). The results furthermore reveal that the higher the persistence in sales growth, the more persistence is lost in the translation into income growth rates. This leads to the hypothesis that firms may trade persistence in income growth for a high persistence in sales growth. In a final test, it is shown that the loss of persistence in sales growth is correlated with consistently high growth rates in operating expenses. In total, the study cautions not to overestimate a high persistence in sales growth as a strong predictor of future profit growth rates.

Chapter 3 analyzes persistence in growth rates from a different perspective. In the previous chapter, the definition of persistence derives on an aggregate firm level. This means, persistence is detected if a group of firms has more runs of a certain length than would occur randomly. The goal is to analyze what past persistence tells about future persistence. In chapter 3, the focus is on individual firms and the predictability of specific runs which consist of combinations of above-median and below-median growth rates. Since firm valuations strongly respond to multiannual runs, it is worth to analyze their predictability and thus investigate the factors indicating or causing future runs in growth. The analysis aims to identify variables that indicate whether a firm is more likely to be particularly successful or unsuccessful within the next couple of years. The research methodology is based on binary response models. Both logit regressions and a multiple discriminate analysis are employed to distinguish between two distinct groups of firms. The first group has a positive run, consisting of a series of above-median growth rates after a given point in time. The second group of firms has a negative run, consisting of below-median growth rates, respectively. The prediction period covers six years. To endogenously identify the parsimonious indicator-

specific set of economically and empirically meaningful variables, stepwise regression is used. In-sample and out-of-sample classification tests are conducted to evaluate the predictive power of the forecast models. The results show that based on a set of widely-used financial variables, predicting positive and negative runs is possible. The accuracy of the prediction depends on the length of the investment period. The most salient prediction variable turns out to be the dividend to price ratio.

In chapter 4, representing the second part of the dissertation, a very successful British lottery bond is in the focus of interest, the Premium Bond. After being launched by the British Exchequer in November 1956, customers had almost 27 million holdings in Premium Bonds totalling about £43 billion by the end of 2011. Although monthly return is solely based on a lottery and therefore uncertain, this financial product is very popular. The study aims to explain what makes the Premium Bond and generally lottery-linked deposit accounts successful. The sample consists of a unique hand-collected set of data provided by the issuer. In total, it covers a period of fifty-four years. The first part of the study considers the expected utility theory (Arrow, 1965; Pratt, 1964). To evaluate the relevance of an investor's individual risk tolerance, the constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) coefficients are calculated at which a saver is indifferent between the Premium Bond and a risk-free investment. The second part of the study searches for factors influencing net sales. To detect relationships, Granger causality tests (Granger, 1969) are employed. Potential explanations based on cumulative prospect theory (Tversky and Kahneman, 1992; Pfiffelmann, 2008) and prize skewness are analysed in detail (Guillén and Tschoegl, 2002; Golec and Tamarkin, 1998; Garrett and Sobel, 1999; Bhattacharya and Garrett, 2008). Finally, autoregressive models are constructed in order to establish a formal relationship between Premium Bond net sales and a variety of potential influential factors. The results show that CARA and CRRA risk coefficients as well as cumulative prospect theory have no

or only limited statistical influence on net sales. However, prize skewness, the number of jackpots and the maximum holding amount are important factors driving net sales.

1.2. Overview of essays

Papers included in the present dissertation:

- Does the persistence in sales growth rates have predictive power?

  (with Sebastian Lobe)

  ACATIS Value Prize 2013, Submitted to European Financial Management

- Predicting above-median and below-median growth rates

  (with Sebastian Lobe)

  Submitted to Review of Managerial Science

- Why are British Premium Bonds so successful? The effect of saving with a thrill

  (with Sebastian Lobe)

  Submitted to Journal of Empirical Finance

Papers not included in the present dissertation:

- The level and persistence of growth rates: International evidence

  (with Sebastian Lobe)

  Working paper, presented at Campus for Finance - Research Conference 2013, WHU Vallendar, January 16/17, 2013

- Perpetuity, bankruptcy, and corporate valuation: The global evidence

  [Ewigkeit, Insolvenz und Unternehmensbewertung: Globale Evidenz]

  (with Sebastian Lobe)

  CORPORATE FINANCE biz 2 (4) (2011), 252–257.

- Happy savers, happy issuer: the UK lottery bond,

  (with Sebastian Lobe)

  Revue Bancaire et Financière/Bank- en Financiewezen, (6-7) (2008), 408–414.

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2. Does the persistence in sales growth rates have predictive power?

(with Sebastian Lobe)

ACATIS Value Prize 2013

Presented at Campus for Finance - Research Conference 2013, WHU Vallendar, January 16-17, 2013

Abstract

Chan, Karceski, and Lakonishok (2003) report that there is some persistence in sales growth rates in the United States. First, we establish that this also holds around the world. Second, we corroborate that investors strongly reward high persistence in sales growth. This suggests that investors tend to overestimate this indicator as a predictor of future profit growth rates. Third, we find evidence that the higher the persistence in sales growth, the more the persistence gets lost in the translation into income growth. Our study issues a warning not to overestimate the predictive power of a high persistence in sales growth.

Keywords: sales growth rates, persistence, prediction

2.1. Introduction

Stocks that have had a long record of superior past growth rates tend to receive rich valuations. However, most of them are not able to live up to these high expectations, and their valuations return to the mean. A prominent interpretation of this effect is offered by both De Bondt and Thaler (1985, 1987) and Lakonishok et al. (1994). They argue that investor overreaction to past firm performance is the main reason why simple value strategies based on valuation ratios (such as book-to-market) surpass growth strategies. Lakonishok et al. (1994) argue that when forecasting future earnings, investors extrapolate past growth too far into the future.[1] Contradicting this strong belief among investors research shows that there is no persistence in long-term earnings growth beyond chance. In their seminal United States (US) based study, Chan et al. (2003) (CKL) reaffirm this notion and forcefully caution against extrapolating past success in income growth into the future. However, they do find some persistence in sales growth. In the following, we call this phenomenon an "increased" or "high" persistence. In other words, more firms than expected under the hypothesis of independence are able to maintain above-median sales growth rates for many consecutive years. This finding prompts the question, what are the implications on growth rates of operating and net income? In the present study, we expand the work of CKL and perform a profound analysis on this topic. We investigate the hypothesis that investors overestimate the predictive power of an increased persistence in sales growth. More specifically, these investors overestimate its translation into consistently high income growth rates. To our knowledge, this article is the first to present empirical evidence on the persistence in sales growth around the world and on its relationship to persistence in income growth. Our results

[1] In sports, a similar phenomenon is known as the belief in "hot hands" (Camerer, 1989). Hendricks et al. (1993) analyze the hot hands effect in mutual fund performance.

should be important to investors as well as analysts in order to avoid being deceived by an alleged useful predictor.

Our sample comprises data from more than 54,000 firms from 77 countries and over a sample period of 28 years. To allow sound comparisons across heterogeneous sets of firms and performance indicators, we develop an innovative measure called the weighted frequencies-score. The indicator is based on the run-test, a nonparametric test for serial correlation, applied by CKL. The weighted frequencies-score further expands upon the original run-test by generating a single measure of persistence in growth. Using this method, we analyze consistency in growth of sales, operating income, and net income. In doing so, we split our sample according to country and industry affiliation, firm size, market valuation, and leverage. In a final analysis, we investigate the hypothesis that firms try to "buy" a high persistence in sales growth at the cost of increasing operating expenses.

Our main findings are as follows. Indicating potential overestimation, we observe that investors strongly reward runs and thus a high persistence in sales growth. In line with the existing US evidence, we find that an increased persistence in sales growth is a global phenomenon. Our results reveal that the higher the persistence in sales growth, the more persistence is lost in the translation into income growth rates. Supporting our hypothesis that firms may trade persistence in income growth for a high persistence in sales growth, we find that the loss of persistence in sales growth is strongly correlated with a high persistence in operating expense growth rates. In total, our study issues a warning not to overestimate a high persistence in sales growth as a strong predictor of future profit growth rates.

The rest of the paper is organized as follows. Section 2.2 reviews the related literature. Section 2.3 discusses our sample and the methodology. Section 2.4 examines how investors evaluate past sales growth in their company valuations. Section 2.5 studies the translation of

persistence in sales growth across a number of subsets of firms. Section 2.6 investigates operating expenses in this context. Section 2.7 concludes the report.

2.2. Literature review

The literature regarding expectations about future growth rates is related to research on why value stocks outperform growth stocks. One possible explanation for this anomaly is investor overreaction. In their research, De Bondt and Thaler (1987, 1990) analyze returns of stocks that have experienced either extreme capital gains or extreme losses. Referring to Kahneman and Tversky (1973), who report that people have a tendency to overweight salient information (such as recent news), they argue that this trend might be explained by biased expectations of the future. De Bondt and Thaler (1990) and Chopra et al. (1992) conduct further analyses that corroborate these findings. Lakonishok et al. (1994) argue that investors tend to extrapolate past growth rates too far into the future. A paper by Barberis et al. (1998) formalizes the same general idea. La Porta (1996) conducts further analyses and argues that analysts and investors rely too heavily on past growth in their forecasts and valuations. La Porta et al. (1997) examine the hypothesis that the superior return to value stocks is the result of expectational errors made by investors. They find that investors may incorrectly assume that there is a significant degree of consistency in growth, so they extrapolate glamour stocks' past superior growth rates (and value stocks' past disappointing growth rates) too far into the future.

Our study is also related to the rather slim literature on the behavior of earnings growth. Early evidence for the United Kingdom (UK) is provided by Little (1962) and Little and Rayner (1966). They find that corporate annual earnings numbers essentially follow random processes and therefore challenge the assumption that a firm's past growth performance is a good predictor of its future growth. In line with these conclusions, Lintner and Glauber (1967) and Brealey (1983) provide evidence for the US and confirm that changes in earnings over time appear to be randomly distributed. Based on these findings, many further studies

investigate earnings predictability by applying time-series models (e.g., Beaver, 1970; Ball and Watts, 1972; Albrecht et al., 1977). However, these studies only focus on short-term forecasting.

One of the few recent studies on persistence in operating performance growth rates is the seminal paper by CKL. They convincingly show that there is no persistence in net income growth rates. Despite this fact, they do identify some persistence in sales growth rates. They suppose that a shrinking profit margin is the reason why growth in sales shows more persistence than growth in profits, but they do not investigate this relationship in detail.

2.3. Data and methodology

2.3.1. Data

Our study is based on a large international sample. The data used are obtained from Thomson Datastream and Worldscope. The sample period runs from 1980 to 2008, as no firm accounting data are available before 1980. The start dates vary across countries and firms because of data availability. First, we select all active and inactive equities recorded in the database. Following CKL, we do not exclude any kind of firms.[2] We then control for multiple collections of the same company, data errors, and missing data. Time-series of inactive firms are included in the dataset during their time of existence. Our initial sample comprises a total of 54,176 firms in 77 countries. At the end of each calendar year, we collect net sales or revenues, operating income, and net income before extraordinary items/preferred dividends for each firm in local currencies.[3]

At the end of each calendar year, we calculate growth in operating performance as follows,

[2] We do not include American depositary receipts (ADRs) and closed-end funds. CKL do not describe their procedure in this context.

[3] Worldscope items WC01001, WC01250, and WC01551.

$$g_{i,t-1,t} = \frac{(PI_{i,t} - PI_{i,t-1})(1 + DY_{i,t})}{PI_{i,t-1}} \tag{2.1}$$

where *g* is the growth rate of firm *i* over the period of time *t-1* to the sample selection year *t*. *PI* denotes the operating performance indicator. Following CKL, we assume that the dividends are reinvested, taking into account different dividend payout policies. We measure growth on a per-share basis and assume that an investor would typically buy and hold shares over a specific period. The number of shares outstanding is adjusted to reflect stock splits and dividends.

In cases where earnings in the base year are negative, growth rates cannot be calculated, so the number of eligible growth rates would be reduced. We therefore also apply the substitution method described in CKL (see page 653). To ensure a robust data basis for comparisons, we drop all countries with an insufficient number of eligible sales growth rates over the entire sample period. Our final sample encompasses 53,435 firms in 48 countries, of which 32,300 exist at the end of our sample period in 2008. In total, the sample includes 531,091 firm-years, with 31.4% of these attributed to US firms. Firms in Japan and the UK account for 12.9% and 7.3% of all firm-year observations, respectively. The remaining 45 countries typically account for less than five percent of the total observations.

2.3.2. Methodology

Our approach is based on the run-test design applied by CKL. First, the median of all eligible growth rates is calculated at every calendar year's end. We then determine how many consecutive years a company is able to beat the median. This row is called the run. Finally, we calculate the percentage of firms with runs in relation to all the firms that survive for the same period of time. Extending the analysis of CKL, our goal is to measure the degree of persistence in growth. We therefore refer to percentages that are higher than we expect under

the hypothesis of independence as an “increased” persistence in growth. To ensure best comparability across sets of firms, we also need to consider some further issues.

2.3.2.1. Nonsurviving firms

When comparing sets of firms, nonsurviving firms may bias our conclusions. The fewer firms survive, the higher the percentage of firms with runs. For instance, consider two groups A and B with 100 eligible firms (e.g. two countries) in the sample selection year. In Group A, three firms have a run for five consecutive years, and all firms survive for the same period of time. We report that 3% of all valid firms have a five-year run. In Group B, three firms have a run for five consecutive years, but now only 90 firms survive for the same period of time. Therefore, we report that 3.33% of all valid firms have a five-year run. It would appear that persistence in growth is higher in Group B than in Group A. In fact, some firms with a particularly poor performance lead to this erroneous conclusion.

2.3.2.2. Comparing run lengths

The run-test produces a combination of percentages, which is difficult to compare with others. Simply adding up the obtained numbers would neglect the fact that a very long run is much more difficult to achieve than a short run.

2.3.2.3. Discrepancy between the groups

Our approach requires two groups of firms. The first group is tested for runs, and the second one provides the basis for median calculation. Typically, the second group would comprise all firms within a country. At sample selection, these groups usually are identical. Without filtering, over time, Group 1 shrinks due to nonsurviving firms. In contrast, Group 2 gains size as each year new firms are added because of new foundations or simply due to improvements in data availability. The longer the test period, the larger the discrepancy. This

finding leads to the problem that it becomes impossible to state precise expected probabilities of beating the median for a number of years.

2.3.2.4. The weighted frequencies-score

To control for these issues, we develop a modified run-test design that we call the "weighted frequencies-score" (wf-score). Limiting our analysis to a rolling five-year horizon reduces the problem of low data availability over long periods of time. At the end of every calendar year, we select all firms that survive for the next five years. We then calculate the median growth rate of this set of firms for each of the next five years after the sample selection. The medians are determined separately for each country in order to avoid biased comparisons due to generally different levels of growth rates. This approach also eliminates the issue of varying inflation rates and accounting conventions across countries. Based on these medians, we determine the percentage of firms with above-median growth rates for a number of consecutive years with respect to the total number of firms in the group and can now accurately determine the percentages expected under the hypothesis of independence. By definition, 50% of all firms have an above-median growth rate in the sample selection year, 25% are expected to have a run for two years, and so on. To factor in the length of the run, we multiply the actual frequency of firms with the inverse of the expected frequency. For instance, if the expected frequency of a four-year run is 6.25%, the weighting factor would be 16. In the final step, we sum up the weighted frequencies to obtain a single, comparable measure of persistence in growth rates. The resulting formula is as follows,

$$wf_{PI,c,t} = \sum_{l=1}^{5} \left(\frac{n_{PI,l,c,t}}{N_{PI,c,t}} \times \frac{1}{0.5^{l}} \right) \tag{2.2}$$

where *wf* is the weighted frequencies-score, *t* specifies the sample selection year, *c* is the group of firms that survive for five years after sample selection, and *PI* denotes the

performance indicator. The run length in years is denoted by *l*; *n* is the number of firms with a run length *l*, and *N* is the total number of firms in group c. If the distribution of above-median growth rates is totally random, the wf-score will be 0.5×2+0.25×4+0.125×8+ 0.0625×16+0.0313×32=5.00. Values above 5.00 suggest persistence beyond pure chance ("increased persistence") and quantify the scale. Values below 5.00 suggest the opposite. The theoretically highest possible wf-score is 31, which would suggest that all firms with above-median growth rates in the first year had a run for five consecutive years. The lowest possible value is 1, indicating that no firm has a run for more than one year.

The focus of our study is to relate persistence in sales growth to other performance indicators. As a measure we use wf-delta which is the difference between the wf-score of income growth and the wf-score of sales growth. Negative values suggest that there is more persistence in sales growth than persistence in income growth. Positive values indicate the opposite.

2.4. Investor appreciation of persistence in sales growth

We begin our study by examining how market valuations are affected by high persistence in growth, especially sales growth. At every calendar year's end, we determine for each firm the length of the current run in both sales growth and net income growth. If a firm has a run, a figure between one (year) and five (years) is assigned. If a firm does not beat the median growth rate, a zero is assigned. We measure the valuation of a company based on its book-to-market ratio (Datastream item MTBV). Table 2.1 shows the results across all firms and the entire sample period. Panel A analyzes the median book-to-market ratio of firms with runs in sales growth. Panel B performs the same analysis with net income growth.

The results clearly indicate that firm valuations become richer with increasing run length. In Panel C, we assume that a firm enjoys a run in both performance indicators at the same time. The ratios suggest that investors not only reward past growth of the bottom line but also of the top line. We next try to isolate how investors appreciate sole persistence in sales growth.

Panel D reports the median book-to-market ratios assuming that a firm has a run in sales growth but no above-median net income growth rate in the current year. Valuations continue to increase with the run length. Panel E tightens the analysis. Firms now have a run in sales growth but no above-median growth in the current and the past year. As is intuitively expected, the overall valuation level is slightly lower than in Panel D but still increases with the length of the sales run. The results also apply to the final ten-year period from 1998 to 2008.

These findings suggest that investors give weight to persistence in past sales growth. The return of an investment, however, primarily depends on net income. If an increased persistence in sales growth does not translate into an increased persistence in income growth, investors are at risk of overestimating an impressive track record of past sales growth rates.

2.5. Relationship between persistence in sales growth and persistence in income growth

We commence with an analysis across the entire sample and sample period. Table 2.2 reports wf-scores measuring the persistence in growth of sales, operating income, and net income.

Consistent with the US results by CKL, we confirm that there is an increased persistence in sales growth. The wf-score of 7.42 surpasses the expected 5.00 under the hypothesis of independence. However, as CKL argue, this persistence vanishes as we get closer to the bottom line. The wf-scores of operating income and net income are only 4.95 and 4.51, respectively. In fact, the probability to achieve a run is slightly lower than we would expect under the hypothesis of independence. To ensure that our results are significantly different from the expected frequencies, we perform chi-square tests to determine the equality of the distributions. We reject the null hypothesis of independence for growth in sales and net income at the 1% level. The persistence of operating income is not significantly different from pure chance. These first results suggest that in general, an increased persistence in sales growth does not translate into persistent high income growth rates.

To exclude the possibility that sales growth actually has become a more accurate predictor over the past decades, we calculate the wf-score for each sample selection year beginning with 1981. The last full five-year period starts in 2004. Table 2.3 presents the wf-scores for every performance indicator over the time periods from 1981 to 2004, 1981 to 1988, 1989 to 1996, and 1997 to 2004.

The results suggest that within 28 years, persistence in sales growth has further increased. In 2004, the wf-score amounts to 8.06. Persistence in net income growth, however, does not follow this trend. It remains relatively stable with a slightly decreasing tendency. In 2004, the wf-score amounts to 4.80. Panel D reports wf-deltas of operating income and sales (OI-S) as well as net income and sales (NI-S). These findings suggest that the persistence of growth diverges over time. The wf-delta between net income and sales increases from -1.92 (1981 to 1988) to -2.56 (1989 to 1996) and finally to -3.31 (1997 to 2004). The same applies to operating income and sales. The results indicate that although persistence in sales growth constantly increases, it is still a weak predictor for persistence in income growth. One possible explanation for this trend is that firms manage their sales growth rates at the cost of income growth.

2.5.1. Analyzing subsets of firms

We hypothesize that even a high persistence in sales growth would provide little information about the corresponding persistence in income growth. Subsets of firms will help us to test this hypothesis.

2.5.1.1. Subset 1: Divided by country

Given the variety of country-specific factors such as the legal system and the extent of investor protection (e.g., Demirgüç-Kunt and Maksimovic, 1998; La Porta et al., 2002; Brockman and Chung, 2003; Beck et al., 2005), it seems likely that persistence is not exactly

equal anywhere around the world. Table 2.4 reports the wf-scores for each country in our sample over the entire sample period. We sort the countries in descending order by their wf-score in sales. As expected, there is an increased persistence in sales growth in almost every country. Mexico, Poland, and France are ranked highest with wf-scores of 8.62, 7.98, and 7.90. In contrast, Turkey, Denmark, and Venezuela only reach scores of 5.84, 5.70, and 3.77. In line with our hypothesis, there seems to be no clear-cut correlation between persistence in sales growth and persistence in income growth. To quantify the link, in Panel D, we calculate the wf-deltas of operating income and sales (OI-S) as well as net income and sales (NI-S). The results indicate that the wf-deltas tend to rise as persistence in sales growth increases. The countries ranked 1 to 15 have an average wf-delta score of -3.10 compared to persistence in net income. The countries ranked 16 to 33 average -2.59, and those ranked 34 to 48 only average -2.13. We find the same pattern when we compare sales and operating income. Apparently, the translation into net income growth becomes weaker as persistence in sales growth increases.

2.5.1.2. Subset 2: Divided by industry

As industries differ in many aspects, such as their sensitivity to business cycles, intensity of competition, and firm financial structure (MacKay and Phillips, 2005), persistence in growth is worth analyzing. The analysis is similar to the previous one, but now we classify firms by their industry affiliations instead of their country affiliations. The median growth rates are still calculated with respect to the individual countries. For industry classifications, we obtain four-digit standard industrial classification (SIC) codes from Worldscope. The industry classification follows Fama and French (1997) distinguishing between 49 industry categories. Firms that do not fit into one of them are labeled as “unclassified.” Table 2.5 presents these results. Again, the list is sorted in descending order by the wf-score in sales.

We find considerable variation across industries. The “Personal Services” (11.98), “Retail” (11.30), and “Healthcare” (10.53) industries are ranked the highest. Consistent with the results of the previous section, we again find that the higher the persistence in sales growth, the less it translates into persistence of income growth. The industries ranked 1 to 15 exhibit a wf-delta (weighted mean) with respect to net income of -4.22. In contrast, the industries ranked 16 to 34 and 35 to 50 only amount to -2.37 and -1.77, respectively. The correlation becomes particularly obvious when considering the top three industries in Panel A. For instance, the “Retail” industry reaches a wf-delta score (NI-S) of -6.14. As a robustness test, we redo the analysis (results not reported) and calculate the median growth rates using industry categories instead of countries. The conclusions are the same.

2.5.1.3. Subset 3: Divided by firm size

Since industries are strongly distinguished from each other in terms of average firm size, we explore how firms of different sizes translate persistence in growth. It is well known that firm size is related to the firm's profitability, productivity, and survival (e.g., Zarowin, 1989; Zarowin, 1990; Beck et al., 2008). We calculate the wf-scores and wf-deltas for large, mid-capitalization, and small firms over the entire sample period. Large firms are ranked in the top two deciles of market capitalization (in US dollars) as of the end of the sample selection year, while small firms fall into the bottom two deciles. Mid-capitalization firms cover all the remaining companies. Size decile breakpoints are computed separately from the entire universe of firms domiciled in the respective country. Panel A in Table 2.6 summarizes the results.

We find that persistence increases with firm size. Large firms have a wf-score (weighted mean) in sales of 9.71, while mid-capitalization firms have a score of 7.42; small firms exhibit only an average score of 4.36. These findings once more support our hypothesis. The higher the persistence in sales growth, the less it translates into persistence of income growth.

According to Table 2.6, large firms have a wf-delta of -4.32 (OI-S) and -4.84 (NI-S). In contrast, the corresponding scores of small firms are positive and average 0.50 (OI-S) and 0.06 (NI-S). In this case, persistence in operating income growth slightly exceeds that in sales growth. By computing the size classification each year, the group of large firms includes more and more past winners. As a check for robustness, we perform the same analysis (results not reported) with fixed size classifications based on the first available firm year. Our conclusions are still the same.

2.5.1.4. Subset 4: Divided by firm valuation

The widespread overestimation of persistence in growth among investors particularly manifests in the existence of value and glamour stocks. Considering the existing evidence (Lakonishok et al., 1994; La Porta et al., 1997) and our findings so far, we expect that glamour stocks would exhibit a relatively high persistence in sales growth, which potentially attracts investors. The translation into consistently high income growth rates, however, is probably weak as research shows that returns of glamour stocks underperform those of value stocks (e.g., Basu, 1977; Jaffe et al., 1989; Chan et al., 1991). In contrast, value stocks may have a relatively low persistence in sales growth but a rather good translation into income growth.

At the end of every calendar year, we split all firms into three distinct groups. Glamour firms are ranked in the bottom three deciles by their book-to-market ratio. The group of value firms comprises firms that are ranked in the top three deciles. The remaining firms are labeled as moderate valuation firms. The decile breakpoints are computed separately for each country to take into account international differences in market valuations. Panel B of Table 2.6 presents the respective wf-scores and wf-deltas. In line with our expectations, the results confirm that the growth rates of glamour firms are more persistent than those of value firms. However, as is observed in the previous subsets of firms, this persistence has a considerably worse

translation. The wf-score (weighted mean) of 10.01 for sales translates into 5.84 for net income, which equals a difference of -4.17. Value firms have a wf-score of 5.18 for sales and 3.53 for net income, which equals a wf-delta of only -1.65.

2.5.1.5. Subset 5: Divided by leverage

The last subset of firms we analyze focuses on the capital structure. According to the pecking order model of financing decisions (Myers, 1984), firms first fund projects out of retained earnings. Since profitable firms generate cash internally, in theory, more profitable firms are supposed to be less leveraged (e.g., Shyam-Sunder and Myers, 1999; Fama and French, 2002). We therefore expect that less-leveraged firms generally would have an increased persistence in sales growth. As a proxy for the debt level of a firm, we use the "debt-to-total-assets ratio" (Remmers et al., 1974) obtained from Worldscope (item WC08236).[4] Following Fama and French (2002), we exclude financial firms (SIC codes 6000 to 6999) because financial intermediaries seem incomparable with other firms in terms of leverage. We also exclude utilities (SIC codes 4900 to 4999) because their capital structure may be influenced by regulation. At the end of every calendar year, we assign each firm to one of three groups. Low leverage firms include firms in the bottom two deciles by their debt-to-total-assets ratios. Median leverage firms comprise stocks ranked in the third through the seventh deciles, and high leverage firms cover firms ranked in the top two deciles. Leverage strongly varies across industries, so decile breakpoints are based on the universe of all firms in one particular industry. This approach additionally ensures that each set of firms include companies from all industries. The median growth rates are still calculated on a country basis. Results are presented in Panel C of Table 2.6 and reveal that persistence of sales growth indeed increases with decreasing leverage. Low leverage firms have a wf-score (weighted mean) in sales of 9.30 across the entire sample period. The corresponding scores of median and high leverage

[4] To control for outliers, we trim the data at the 99th percentile.

firms amount to 7.78 and 5.43, respectively. Analyzing the wf-deltas once more corroborates our previous conclusions.

2.5.2. Robustness test: Firms with a very high persistence in sales growth

In this section, we test for the robustness of our previous results by using as general of an approach as possible. In Panel A of Table 2.7, we construct two strikingly different sets of firms. The first group (Group A1) encompasses only firms with at least one five-year run in sales growth within their time of survival. The second group (Group A2) contains all the remaining firms. These firms do not have a single run in sales growth for more than four years at any given time.

As expected, due to the rigorous selection criteria, the persistence in sales growth of Group A1 is the highest observed in this study. The wf-score of sales amounts to 18.53. Despite this fact, the wf-scores of operating income and net income are only 7.35 and 6.36, respectively. This means that the conversion from persistence in sales growth into persistence in income growth is also the weakest in this study. The wf-deltas amount to -11.18 (OI-S) and -12.17 (NI-S). As expected, the translation is very different when analyzing the results of Group A2. Here, persistence in income growth even exceeds the very low persistence in sales growth (wf-score: 2.68). The wf-delta of operating income and sales is 1.25. With respect to net income and sales, it is 1.05. Obviously, there is a fair amount of firms with long runs in income growth but with shorter or even no runs in sales growth. In Panel B of Table 2.7, we relax the criteria and compare firms with at least one run for four years (Group B1) to firms without a single run for more than three years at any time (Group B2). The results are consistent with those in Panel A. As expected, the wf-scores and wf-deltas of Group B1 are now generally smaller than those of Group A1.

2.6. Relationship between operating expenses and persistence in sales growth

There are a number of conceivable explanations for why persistence in sales growth vanishes on the way to the bottom line. CKL presume that a shrinking profit margin is the reason why growth in sales shows more persistence than growth in profits. Aghion and Stein (2008) argue that firms have to decide whether to focus their efforts either on increasing sales growth or on improving profit margins. Since managerial time and other resources are limited, firms face a strategic tradeoff between these objectives and therefore are confronted with essentially a multitasking problem (e.g., Holmstrom and Milgrom, 1991). Another reason may be that managers know the investors' preferences and actively cater to them. For example, Hong et al. (2003, 2007) examine analyst reports on Amazon.com over the period from 1997 to 2002 and illustrate that analysts initially almost exclusively focused on its long-run revenue potential, while profit margins were virtually neglected.

Our previous findings give reason to believe that managers may trade income growth for momentum in sales growth because they assume that the stock market focuses on growth in sales rather than profit margins. In a last step, we investigate the hypothesis that a high persistence in sales growth is largely consumed by high operating expense growth rates. This process eventually leads to a slightly increased persistence in income growth at best. We focus on the two major items of operating expenses: “cost of goods sold” (CGS) and “selling, general, and administrative expenses” (SGAE).[5] Due to low data availability for all countries except the US, we do not analyze research and development expenses. Table 2.8 lists all subsets of firms previously studied along with the respective wf-deltas based on operating income and sales. The list is sorted from the largest to the smallest loss of persistence in sales. For each subset of firms, we calculate the wf-scores for CGS and SGAE. This approach is the same as the one used for sales, operating income, and net income.

[5] Worldscope items WC01051 and WC01101.

Our results clearly indicate a strong correlation. The group of firms with at least one five-year run in sales growth has the highest wf-delta amounting to -11.18 (OI-S). Interestingly, this group also has the highest persistence in growth rates of CGS (14.82) and SGAE (13.09). To establish a quantitative relationship for all subsets of firms, we calculate Pearson correlation coefficients. Based on the wf-deltas and the wf-scores of CGS, we obtain a correlation coefficient of -99%. The respective result based on the wf-scores of SGAE amounts to -95%. Both correlations are significant at the 1% level. Added together, the results from Table 2.8 suggest two conclusions. First, the higher the loss of persistence in sales growth, the higher the persistence in operating expenses. Second, firms with a low persistence in sales growth tend to enjoy a better-than-expected persistence in operating income growth, since their growth rates in operating expenses are generally lower.

2.7. Conclusion

In this paper, we shed further light on the persistence of growth rates in operating performance as an overestimated predictor for long-term future growth rates. In a first step, we establish that investors do pay a great deal of attention to past consistency in sales growth rates in their company valuations. We therefore focus on the question of how the frequently observed increased persistence in sales growth translates into persistence in income growth. For this purpose, we require an indicator that allows us to consistently quantify persistence in growth rates and to perform meaningful comparisons. We therefore adopt the run-test approach applied by Chan et al. (2003) and develop a measure called the weighted frequencies-score. It analyzes above-median annual growth rates in the operating performance of firms that survive for at least five years and additionally factors in how long a firm outperforms the market. Using this method, we calculate persistence in growth rates for a variety of subsets of firms.

Our results expand the US evidence reported by Chan et al. (2003) and confirm that around the world, sales growth usually has an increased persistence. We also show that this persistence varies remarkably depending on factors like country or industry affiliation, firm size, and market valuation. Our results, however, also provide evidence that the higher the persistence in sales growth, the more persistence gets lost after the translation into income growth. We hypothesize that many firms place great emphasis on a high persistence in sales growth rates and try to “buy” this success. We also examine how the loss of persistence in sales growth is related to persistence in expense growth and find a strong correlation supporting our assumption. Taken together, our study issues a warning to investors and analysts not to overestimate long-term future profit growth, even if a firm has a remarkably high persistence in sales growth.

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Table 2.1: Market valuation of persistence in sales growth and net income growth.
This table analyzes how investors reward persistence in sales growth in their firm valuations. The table reports median book-to-market ratios (BTMV) and available firm-years (N) dependent on the current run length in sales growth and net income growth. Statistics are provided for all firms and the entire sample period from 1980 to 2008.

	Run length					
	No run	1 year	2 years	3 years	4 years	5 years
	Panel A: Run in sales growth					
BTMV	0.719	0.641	0.602	0.562	0.556	0.526
N	119,129	121,133	70,850	42,912	27,047	15,862
	Panel B: Run in net income growth					
BTMV	0.709	0.637	0.588	0.538	0.515	0.463
N	122,802	123,025	59,934	28,140	13,227	5,655
	Panel C: Run in sales growth and net income growth					
BTMV	0.735	0.613	0.541	0.474	0.433	0.364
N	72,495	73,628	26,777	10,365	4,261	1,691
	Panel D: Run in sales growth (no run in net income growth in t)					
BTMV	0.735	0.690	0.649	0.599	0.581	0.543
N	72,495	46,400	43,019	31,725	22,261	13,887
	Panel E: Run in sales growth (no run in net income growth in t and t-1)					
BTMV	0.741	0.690	0.654	0.606	0.592	0.552
N	31,160	46,333	29,944	27,131	20,481	13,179
1998-2008	0.781	0.719	0.667	0.613	0.617	0.578

Table 2.2: Persistence in growth across the entire sample.
This table analyzes persistence in growth across the entire sample of firms. To factor in the length of the run, the actual frequencies of firms with runs are multiplied with weighting factors (WFA) which are the inverse of the expected frequencies. The wf-score is the sum of the weighted frequencies. A wf-score of 5.00 indicates that persistence in growth is randomly distributed. Values above 5.00 indicate and quantify an increased persistence in growth.

	Run length					
	1 year	2 years	3 years	4 years	5 years	wf-score
Expected frequency	50.0%	25.0%	12.5%	6.3%	3.1%	
Weighting factor (WFA)	2	4	8	16	32	
	Panel A: Sales					
Valid firm-years	265,312	265,312	265,312	265,312	265,312	
Firm-years above median	132,655	75,841	45,240	28,292	18,312	
Percent above median	50.0%	28.6%	17.1%	10.7%	6.9%	
Weighted frequencies (Percent*WFA)	1.00	1.14	1.36	1.71	2.21	7.42
	Panel B: Operating income					
Valid firm-years	258,993	258,993	258,993	258,993	258,993	
Firm-years above median	129,498	65,124	31,896	15,852	7,932	
Percent above median	50.0%	25.1%	12.3%	6.1%	3.1%	
Weighted frequencies (Percent*WFA)	1.00	1.01	0.99	0.98	0.98	4.95
	Panel C: Net income					
Valid firm-years	261,919	261,919	261,919	261,919	261,919	
Firm-years above median	130,960	63,084	29,413	13,769	6,598	
Percent above median	50.0%	24.1%	11.2%	5.3%	2.5%	
Weighted frequencies (Percent*WFA)	1.00	0.96	0.90	0.84	0.81	4.51

Table 2.3: Persistence in growth across the sample period.
This table analyzes persistence in growth across the sample period. A wf-score of 5.00 indicates that persistence in growth is randomly distributed. Values above 5.00 indicate and quantify an increased persistence in growth. N denotes the number of available firm-years. Wf-delta is the difference between the wf-scores of operating income and sales (OI-S) as well as net income and sales (NI-S).

	Panel A		Panel B		Panel C		Panel D	
Sample	Sales (S)		Operating income (OI)		Net income (NI)		wf-delta	
period	wf-score	N	wf-score	N	wf-score	N	OI-S	NI-S
1981 to 2004	7.42	265,312	4.95	258,993	4.51	261,919	-2.47	-2.91
1981 to 1988	6.83	34,339	4.72	32,077	4.91	32,306	-2.11	-1.92
1989 to 1996	7.41	79,767	5.11	75,711	4.85	77,083	-2.30	-2.56
1997 to 2004	7.56	151,206	4.92	151,205	4.25	152,530	-2.65	-3.31

Table 2.4: Subset 1: Divided by country.
This table analyzes persistence in growth for each country in our sample. The countries are sorted by the wf-score in sales in descending order. A wf-score of 5.00 indicates that persistence in growth is randomly distributed. Values above 5.00 indicate and quantify an increased persistence in growth. N denotes the number of available firm-years. At the bottom of the table, weighted means for the wf-scores are reported. Wf-delta is the difference between the wf-scores of operating income and sales (OI-S) as well as net income and sales (NI-S).

	Panel A Sales			Panel B Operating income		Panel C Net income		Panel D wf-delta	
Country	Rank	wf-score	N	wf-score	N	wf-score	N	OI-S	NI-S
Mexico	1	8.62	1,209	5.81	1,195	3.43	1,142	-2.81	-5.18
Poland	2	7.98	671	3.78	653	3.71	653	-4.20	-4.27
France	3	7.90	9,698	4.53	9,059	4.72	9,255	-3.37	-3.18
United States	4	7.85	81,882	5.38	80,307	4.72	79,715	-2.47	-3.13
Chile	5	7.83	1,405	4.70	1,474	4.14	1,476	-3.13	-3.69
Japan	6	7.64	44,229	4.69	43,057	4.32	43,384	-2.95	-3.32
Italy	7	7.64	3,396	4.35	3,239	4.28	3,319	-3.29	-3.35
Hungary	8	7.62	220	3.95	210	4.22	220	-3.67	-3.40
Hong Kong	9	7.59	6,020	4.91	5,749	4.35	6,082	-2.68	-3.23
United Kingdom	10	7.55	18,907	6.14	18,709	5.21	19,087	-1.40	-2.34
Germany	11	7.43	9,828	4.17	8,483	4.25	9,280	-3.26	-3.18
Brazil	12	7.37	2,180	4.53	2,039	3.72	2,008	-2.84	-3.65
Switzerland	13	7.35	3,422	4.64	3,327	5.30	3,409	-2.71	-2.05
India	14	7.32	2,998	4.77	2,912	5.03	2,951	-2.54	-2.29
Colombia	15	7.32	328	5.57	319	4.54	327	-1.74	-2.78
Greece	16	7.21	2,253	4.63	2,242	5.02	2,253	-2.57	-2.18
Philippines	17	7.17	1,416	4.67	1,519	4.34	1,612	-2.50	-2.82
South Africa	18	7.15	2,828	4.85	2,859	4.78	2,965	-2.30	-2.37
China	19	7.15	6,827	5.15	6,670	5.45	6,859	-1.99	-1.70
Spain	20	7.01	2,430	4.71	2,196	5.65	2,248	-2.30	-1.37
Sweden	21	7.00	3,204	5.08	2,932	4.20	2,931	-1.92	-2.79
Singapore	22	6.98	3,642	4.16	3,513	4.09	3,675	-2.82	-2.89
Taiwan	23	6.96	5,478	4.32	5,213	4.05	5,234	-2.63	-2.91
Indonesia	24	6.93	2,401	4.47	2,373	3.56	2,398	-2.46	-3.36
Canada	25	6.91	7,947	4.84	8,626	3.97	8,622	-2.07	-2.94
South Korea	26	6.87	5,906	3.88	5,833	3.67	5,806	-2.99	-3.20
Russian Fed.	27	6.81	233	2.60	179	4.00	188	-4.20	-2.81
Ireland	28	6.80	833	5.66	901	5.03	897	-1.15	-1.77
Norway	29	6.73	1,921	4.15	1,818	3.49	1,763	-2.58	-3.24
Peru	30	6.69	548	4.49	509	4.73	523	-2.19	-1.95
Australia	31	6.65	5,040	4.90	6,200	4.12	6,380	-1.75	-2.53
Finland	32	6.63	2,113	4.43	1,859	3.92	1,796	-2.20	-2.71
Thailand	33	6.59	3,529	4.85	3,509	4.54	3,540	-1.74	-2.05
Luxembourg	34	6.59	266	3.87	253	3.74	259	-2.72	-2.85
Argentina	35	6.56	640	4.08	576	3.51	591	-2.49	-3.06
Netherlands	36	6.53	2,561	5.47	2,510	5.39	2,510	-1.06	-1.14
Belgium	37	6.46	2,241	4.14	1,855	4.19	1,971	-2.32	-2.27
Kuwait	38	6.43	51	2.94	51	3.53	51	-3.49	-2.90
Malaysia	39	6.30	6,019	3.97	5,802	3.81	6,064	-2.33	-2.49
Israel	40	6.28	601	4.62	577	3.97	595	-1.66	-2.31
New Zealand	41	6.17	758	5.09	740	4.43	768	-1.09	-1.74
Austria	42	6.09	1,180	4.13	1,114	3.72	1,180	-1.96	-2.38
Portugal	43	6.06	808	4.12	795	4.67	804	-1.95	-1.39
Pakistan	44	6.00	701	4.62	694	4.41	686	-1.38	-1.59
Czech Republic	45	5.98	191	3.54	178	4.15	192	-2.44	-1.83
Turkey	46	5.84	1,354	4.21	1,356	3.50	1,350	-1.64	-2.34
Denmark	47	5.70	2,781	4.04	2,609	3.56	2,690	-1.66	-2.14
Venezuela	48	3.77	218	2.96	200	3.08	210	-0.81	-0.69
All countries		7.42	265,312	4.95	258,993	4.51	261,919	-2.47	-2.91
Countries ranked 1 to 15		7.72	186,393	5.12	180,732	4.62	182,308	-2.60	-3.10
Countries ranked 16 to 33		6.92	58,549	4.65	58,951	4.33	59,690	-2.27	-2.59
Countries ranked 34 to 48		6.18	20,370	4.29	19,310	4.06	19,921	-1.89	-2.13

Table 2.5: Subset 2: Divided by industry.
This table analyzes persistence in growth for each industry category in our sample. The industry definitions follow the method of Fama and French (1997). The industries are sorted by the wf-score in sales in descending order. Wf-scores above 5.00 indicate and quantify an increased persistence in growth. N denotes the number of available firm-years. At the bottom of the table, weighted means for the wf-scores are reported. Wf-delta is the difference between the wf-scores of operating income and sales (OI-S) as well as net income and sales (NI-S).

	Panel A Sales			Panel B Operating income		Panel C Net income		Panel D wf-delta	
Industry	Rank	wf-score	N	wf-score	N	wf-score	N	OI-S	NI-S
Personal services	1	11.98	1,750	7.36	1,690	6.12	1,701	-4.61	-5.85
Retail	2	11.30	12,480	5.71	12,091	5.16	12,039	-5.59	-6.14
Health care	3	10.53	1,715	6.19	1,711	5.18	1,693	-4.34	-5.35
Medical equipment	4	9.65	3,426	6.63	3,426	5.77	3,398	-3.03	-3.88
Communication	5	9.65	5,018	5.36	4,842	4.39	4,820	-4.28	-5.26
Candy & soda	6	9.56	1,480	4.84	1,439	4.49	1,436	-4.72	-5.07
Computer software	7	9.34	9,301	6.60	8,886	5.44	8,943	-2.74	-3.91
Restaraunts, hotels, motels	8	9.33	4,978	5.42	4,829	4.87	4,810	-3.91	-4.46
Insurance	9	9.08	6,439	4.95	6,139	4.96	6,273	-4.14	-4.12
Automobiles and trucks	10	8.98	6,258	4.55	6,123	4.72	6,058	-4.43	-4.27
Computer hardware	11	8.67	3,020	4.74	2,919	4.39	2,916	-3.93	-4.29
Business services	12	8.46	12,498	5.94	12,224	5.44	12,249	-2.52	-3.02
Transportation	13	8.28	7,818	4.17	7,316	3.98	7,387	-4.11	-4.30
Pharmaceutical products	14	8.23	6,066	5.52	6,364	4.95	6,383	-2.71	-3.28
Almost nothing	15	7.76	795	5.53	812	4.48	813	-2.23	-3.28
Wholesale	16	7.74	12,203	4.67	11,725	4.42	11,797	-3.07	-3.32
Rubber and plastic products	17	7.56	2,107	4.87	2,072	4.76	2,065	-2.69	-2.79
Tobacco products	18	7.42	482	8.06	472	7.17	481	0.63	-0.25
Trading	19	7.25	11,226	4.74	10,584	4.32	11,534	-2.51	-2.92
Consumer goods	20	7.15	5,119	4.69	5,043	4.29	5,002	-2.46	-2.85
Measuring and control equipment	21	6.98	3,100	5.44	3,059	5.02	3,042	-1.54	-1.96
Electronic equipment	22	6.92	10,585	5.04	10,139	4.69	10,123	-1.88	-2.23
Apparel	23	6.91	2,868	4.48	2,834	3.87	2,806	-2.43	-3.04
Banking	24	6.87	20,213	5.40	19,636	4.56	19,969	-1.47	-2.31
Construction	25	6.86	9,069	5.75	8,614	6.00	8,816	-1.11	-0.86
Utilities	26	6.80	8,068	3.35	7,824	3.07	7,881	-3.45	-3.73
Entertainment	27	6.80	2,779	4.64	2,773	4.00	2,797	-2.15	-2.80
Unclassified	28	6.76	7,699	5.44	7,649	4.65	7,790	-1.32	-2.11
Machinery	29	6.65	10,515	5.11	10,230	4.85	10,305	-1.54	-1.79
Recreation	30	6.62	2,305	4.13	2,244	3.96	2,254	-2.50	-2.67
Fabricated products	31	6.61	1,033	4.40	990	3.97	988	-2.21	-2.64
Electrical equipment	32	6.52	4,040	4.77	3,938	4.29	3,967	-1.75	-2.23
Food products	33	6.47	6,918	3.88	6,757	3.89	6,777	-2.58	-2.58
Steel works etc	34	6.46	6,495	4.51	6,159	3.99	6,156	-1.95	-2.47
Shipbuilding, railroad equipment	35	6.44	563	6.59	528	5.09	536	0.15	-1.35
Chemicals	36	6.37	8,520	3.90	8,371	3.84	8,357	-2.46	-2.53
Petroleum and natural gas	37	6.29	6,003	4.66	6,184	3.97	6,157	-1.63	-2.32
Non-metallic and industrial metal mining	38	6.25	1,916	5.19	2,464	4.48	2,529	-1.06	-1.77
Construction materials	39	6.24	9,184	4.32	9,029	4.02	9,018	-1.92	-2.22
Shipping containers	40	6.04	1,168	3.46	1,149	3.34	1,147	-2.58	-2.70
Agriculture	41	5.67	1,921	3.70	1,886	3.40	1,864	-1.97	-2.27
Printing and publishing	42	5.65	2,600	5.13	2,546	4.37	2,535	-0.52	-1.27
Business supplies	43	5.62	4,333	3.74	4,212	3.52	4,121	-1.88	-2.10
Beer & liquor	44	5.52	2,024	4.57	1,953	3.84	1,978	-0.95	-1.68
Aircraft	45	5.40	1,222	5.33	1,188	4.75	1,180	-0.07	-0.66
Coal	46	5.33	570	3.96	620	4.37	614	-1.37	-0.96
Real estate	47	5.02	10,028	4.62	8,870	4.26	9,957	-0.39	-0.75
Defense	48	4.76	335	3.23	291	3.14	296	-1.53	-1.62
Precious metals	49	4.50	1,227	3.65	2,402	3.06	2,422	-0.85	-1.44
Textiles	50	3.71	3,832	3.21	3,747	2.90	3,739	-0.51	-0.81
All industries		7.42	265,312	4.95	258,993	4.51	261,919	-2.47	-2.91
Industries ranked 1 to 16		9.15	95,245	5.43	92,536	4.92	92,716	-3.72	-4.22
Industries ranked 17 to 34		6.84	114,621	4.89	111,017	4.47	112,753	-1.95	-2.37
Industries ranked 35 to 50		5.67	55,446	4.28	55,440	3.90	56,450	-1.39	-1.77

Table 2.6: Subsets 3, 4, and 5: Divided by firm size, firm valuation, and leverage.
This table analyzes persistence in growth with respect to firm size, market valuation, and leverage. Wf-scores above 5.00 indicate and quantify an increased persistence in growth. Wf-delta is the difference between the wf-scores of operating income and sales (OI-S) as well as net income and sales (NI-S).

Panel A: Firm size					Panel B: Firm valuation					Panel C: Leverage				
wf-score			wf-delta		wf-score			wf-delta		wf-score			wf-delta	
Sales	Operating income	Net income	OI-S	NI-S	Sales	Operating income	Net income	OI-S	NI-S	Sales	Operating income	Net income	OI-S	NI-S
Large firms					Glamour firms					Low leverage firms				
9.71	5.39	4.87	-4.32	-4.84	10.01	6.52	5.84	-3.50	-4.17	9.30	5.44	4.79	-3.86	-4.51
Mid-cap firms					Moderate valuation firms					Median leverage firms				
7.42	4.81	4.39	-2.61	-3.03	7.75	4.79	4.34	-2.96	-3.41	7.78	4.87	4.50	-2.91	-3.28
Small firms					Value firms					High leverage firms				
4.36	4.86	4.42	0.50	0.06	5.18	3.85	3.53	-1.33	-1.65	5.43	4.75	4.45	-0.68	-0.98

Table 2.7: Robustness test: Firms with a very high persistence in sales growth.
This table compares firms with a very high persistence in sales growth to firms with a low persistence in sales growth. A wf-score of 5.00 indicates that persistence in growth is randomly distributed. Values above 5.00 indicate and quantify an increased persistence in growth. Wf-delta is the difference between the wf-scores of operating income and sales (OI-S) as well as net income and sales (NI-S).

Panel A:

Group A1: Firms with at least one five-year run in sales					Group A2: Firms with less than five-year runs in sales				
wf-score			wf-delta		wf-score			wf-delta	
Sales	Operating income	Net income	OI-S	NI-S	Sales	Operating income	Net income	OI-S	NI-S
18.53	7.35	6.36	-11.18	-12.17	2.68	3.93	3.73	1.25	1.05

Panel B:

Group B1: Firms with at least one four-year run in sales					Group B2: Firms with less than four-year runs in sales				
wf-score			wf-delta		wf-score			wf-delta	
Sales	Operating income	Net income	OI-S	NI-S	Sales	Operating income	Net income	OI-S	NI-S
14.34	6.51	5.71	-7.83	-8.63	1.78	3.69	3.55	1.91	0.00

Table 2.8: Correlation between wf-delta and persistence in growth of operating expenses.
This table analyzes persistence in expense growth using the wf-score approach. Instead of growth rates in operating performance, growth rates in operating expenses ("cost of goods sold" and "selling, general, and administrative expenses") are used. N is the number of firm-years. The wf-deltas are taken from the previous analyses and based on operating income and sales (OI-S). The table reports Pearson correlation coefficients between the wf-deltas and the wf-scores. Coefficients significant at the 1%, 5%, and 10% levels are indicated by ***, **, and *, respectively.

	OI-S	Cost of goods sold		Selling, general and administrative expenses	
Subset of firms	wf-delta	wf-score	N	wf-score	N
Firms with at least one five-year run in sales	-11.18	14.82	62,244	13.09	48,726
Firms with at least one four-year run in sales	-7.83	12.02	92,773	11.18	72,082
Large firms	-4.32	9.09	44,319	10.24	35,281
Low leverage firms	-3.86	8.67	34,356	8.72	27,850
Industries ranked 1 to 16	-3.72	8.37	78,750	8.40	65,098
Glamour firms	-3.50	8.89	53,259	9.15	44,362
Moderate valuation firms	-2.96	7.25	79,236	7.33	61,965
Median leverage firms	-2.91	7.17	113,285	7.28	84,493
Mid-cap firms	-2.61	6.90	115,038	6.82	90,743
Countries ranked 1 to 15	-2.60	7.19	141,721	7.44	121,900
Countries ranked 16 to 33	-2.27	6.48	42,328	5.81	27,814
Industries ranked 17 to 34	-1.95	6.44	73,263	6.66	58,521
Countries ranked 34 to 48	-1.89	5.93	18,157	5.90	9,802
Industries ranked 35 to 50	-1.39	5.38	50,193	5.28	35,897
Value firms	-1.33	4.87	53,639	4.66	41,584
High leverage firms	-0.68	5.09	35,238	5.07	27,661
Small firms	0.50	4.00	31,180	3.28	23,822
Firms with less than five-year runs in sales	1.25	3.42	139,962	4.41	110,790
Firms with less than four-year runs in sales	1.91	2.62	109,433	3.67	87,434
Correlation coefficient		-0,99***		-0,95***	

3. Predicting above-median and below-median growth rates

(with Sebastian Lobe)

Abstract

Multiannual periods of consecutive above-median or below-median growth rates in operating performance, called "runs", have substantial influence on firm valuations. This paper examines the predictability of runs. To utilize information efficiently, we employ a stepwise regression to endogenously identify the parsimonious indicator-specific set of economically and empirically meaningful variables in estimating the probability of an above-median or below-median run. Our novel approach estimates logit models and performs a multiple discriminant analysis to distinguish between firms that will consistently grow above or below the market over a period of six years. In-sample and out-of-sample classification tests corroborate that there is some predictability.

Keywords: operating performance growth rate, persistence, prediction

3.1. Introduction

Prolonged periods of consecutive high or low growth rates in operating performance growth influence stock market valuations and returns. Prior literature shows that firms with consistently high past growth rates are strongly rewarded by the stock market (Lakonishok et al., 1994). At the same time, firms with a multiannual track record of low growth rates suffer severe devaluation. This is because many investors consider past growth as a meaningful predictor for a firm's future performance. Yet, research also shows (La Porta, 1996; La Porta et al., 1997) that investors tend to extrapolate past growth too far into the future. Buying a stock with an impressive record of recent, for instance, above-median growth bears the risk to invest in an overvalued stock which will probably not satisfy the high growth expectations. Clearly, investors are most interested to know ex ante which firms will consistently over- or underperform the market over the next years.

In this paper, we examine the predictability of above-median and below-median growth rates in operating performance over a period of up to six consecutive years. A large body of the literature already deals with predicting various aspects of a firm's future (e.g., Altman, 1968; Palepu, 1986; Fama and French, 1988). However, a rather neglected topic is “persistence” in operating performance growth rates and especially its prediction. One of the few more recent studies in this area is the seminal paper by Chan et al. (2003) (thereafter “CKL”). They define persistence as the ability of a firm to achieve above-median growth rates for a number of consecutive years. After concluding that its own past is a poor predictor for above-median growth in operating performance, they construct Fama and MacBeth (1973) forecasting regressions to predict the magnitude of future growth rates over a period of one to five years. However, they do not explicitly examine the predictability of prolong periods of consecutive above-median growth rates. We want to close this gap because we think that such an analysis has benefits: (1) CKL establish that predicting the magnitude of future growth rates is hardly

possible. It could be easier to predict a binary variable simply indicating whether a firm will or will not grow above the median for a number of years. (2) Predicting the exact magnitude of a future growth rate may not be necessary. Many investors' (e.g., mutual fund managers) primary target is to beat the market. Hence, as a first step, it may be sufficient to estimate the probability that a firm will grow above or below the median within the next several years. (3) Even obtaining a precise forecast of a firm's future growth rate may not be sufficient. Without an estimate of the future median growth rate, even a presumably high predicted growth rate is at risk not to outperform the market. On the other hand, a seemingly poor growth rate may still be adequate in times of bust.

Accounting for this rationale, our research strategy is to compare two distinct groups of firms. The first group has a “positive run”, consisting of a series of above-median growth rates after a given point in time. The second group of firms has a “negative run”, consisting of below-median growth rates. This setting makes it possible to use binary response models. We compare groups of firms with varying future runs in growth rates over a period of six years. Our goal is to examine whether a set of widely used financial variables helps to differentiate these groups and hence to predict series of above-median or below-median growth rates. In a first step, we use pooled logit regressions. By conducting in-sample and out-of-sample classification tests, we evaluate the predictive power of the estimated models. In a second step, we apply a multiple discriminant analysis as an alternative method to check the robustness of our results. We finally test the power of our logit models on a more general level, by trying to assemble new groups of firms with superior performance in terms of growth rates.

We find that predicting positive and negative runs is possible. Predictability depends on the length of the investment period. Over a relatively short period of time like three years, prediction is quite difficult. The evidence shows, however, that it is possible to differentiate

firms with positive or negative runs over a period of five or six years. We also establish that our forecasting models help to assemble new groups which include more firms with positive runs and fewer firms with negative runs than randomly selected ones.

Our analysis is closely related to the term “persistence”. While the literature discusses persistence in many different contexts like, for instance, firm growth (e.g., Dunne and Hughes, 1994), mutual fund performance (e.g., Carhart, 1997), and profitability (e.g., Carey, 1974), there is only a small literature discussing the behavior (and especially consistency) in operating performance growth. Two early studies are Little (1962) and Little and Rayner (1966). They examine the hypothesis that a firm's past growth is a good predictor of its future growth. They find that in a small sample of UK firms corporate annual earnings numbers are essentially random. Lintner and Glauber (1967) and Brealey (1983) confirm that successive changes in US corporate earnings appear to be randomly distributed. Many further studies starting with Beaver (1970) and Ball and Watts (1972) use time-series models in order to analyze the behavior of earnings. In their seminal paper, CKL test for persistence and predictability in growth rates. They focus on the question how well past growth predicts future growth. To the best of our knowledge, there are only two other studies related to CKL. Anagnostopoulou and Levis (2008) examine the sustainability or persistence of operating growth and market performance as a result of R&D investments. Hall and Tochterman (2008) measure the persistence and predictability of sales and earnings growth for Australian listed companies from 1989 to 2006. Our paper contributes to the literature a novel approach providing new evidence on a specific aspect of persistence. We show that periods of consecutive above-median and below-median growth rates are predictable based on a set of financial indicators. Since firm valuations strongly depend on such time periods, it is important to know the factors indicating future above-median or below-median growth rates.

The rest of the paper is organized as follows. Section 3.2 describes our sample and explains how runs are defined and compared. Section 3.3 introduces the logit model and the explanatory variables. Section 3.4 presents the results, conducts a classification test, and performs a multiple discriminant analysis as robustness check. In section 3.5, we confirm the predictability of runs based on a more general setting. Section 3.6 concludes.

3.2. Data and methodology

3.2.1. Data

Data for this study are obtained from Thomson Datastream and Worldscope. In a first step, we select all active and inactive US equities recorded in the database. We include all available types of equities except ADRs and closed-end funds. After screening the data for a multiple collection of the same company, data errors and missing data, the initial sample comprises 17,038 firms. Following the method of CKL, we measure operating performance based on the year-end values of (1) net sales or revenues, (2) operating income, and (3) net income before extraordinary items and preferred dividends (in US dollars).[6] The sample period starts in 1980 and ends in 2008. Time-series of inactive firms are included in the dataset during their time of existence.

At every calendar year-end we calculate growth in operating performance as follows,

$$g_{i,t-1,t} = \frac{PI_{i,t} - PI_{i,t-1}}{PI_{i,t-1}} \tag{3.1}$$

where *g* is the growth rate of firm *i* over the year *t-1* to *t*. *PI* denotes the operating performance indicator. We calculate growth on a per share basis, taking the perspective of an investor who buys and holds shares over a specific holding period. The number of shares outstanding is adjusted to reflect stock splits and dividends. While CKL initially assume that

[6] Worldscope items WC01001, WC01250, and WC01551.

dividends are reinvested taking into account different dividend payout policies, they drop this assumption for their predictive regressions. We therefore do not assume dividend reinvestment, either. We exclude financial firms from our analysis because some financial statement items do not have the same meaning for every firm. For instance, high leverage of nonfinancial firms more likely indicates distress and dwindling profits, while this is a more normal scenario for financial firms. We define financial institutions according to Fama and French (1997). Our final data set encompasses 13,751 US firms of which 5,569 exist at the end of our sample period in 2008.

3.2.2. Runs of above-median or below-median growth rates

Adopting the method of CKL, we define a run in operating performance growth as follows. Each year, based on all available growth rates (e.g., sales) we calculate the median growth rate. We then determine how many consecutive years a firm achieves to beat the median. We call this a positive run. For instance, a firm that realizes growth rates above the median for four years in a row has a four-year positive run. We extend the method of CKL by considering the opposite event as well, which we label a negative run. In this case, a firm performs below the median for several consecutive years. Based on this information, for each firm and each year we obtain an indicator whether a particular firm currently has a positive or negative run and how long it already lasts. Table 3.1 provides an example. The firm starts a three-year positive run in 1991. In 1994, the run ends due to a below-median growth rate. The losing streak from 1994 till 1996 with below-median growth rates represents a three-year negative run.

Table 3.2 summarizes our sample in terms of firm-years with a current positive or negative run length between one and six years. The number of observations beyond six years is very low. The expected probability of a seven-year run is only about 0.8%. In order to ensure a sufficient number of observations, we limit our analysis to a maximum of six years. Our

sample also comprises firms with very long runs. However, these observations are extremely rare. Only one single firm, Walmart Stores Inc., had a maximum 28-year positive run in sales growth during the sample period. According to Table 3.2, there are generally more firms with extended runs in sales growth than firms with extended runs in operating income growth and net income growth. This has two reasons. First, on a technical note there are generally more sales growth rates available. In case of negative accounting figures it is not possible to calculate valid growth rates. As sales accounting figures are significantly less volatile than income figures and usually positive, we obtain more sales growth rates than income growth rates. Second, in line with CKL, we confirm that there is more consistency in sales growth than in income growth. This could be due to the fact that additional drivers like earnings management, production costs and other expenses influence the income number relative to the sales figure which simply expresses supply and demand.

3.3. The logit model

For our research approach, binary logit regressions are well suited. For example, Ou and Penman (1989) use logit regressions and a large set of financial statement items to predict the direction of one-year-ahead earnings changes.[7] We estimate the following pooled logit regression to specify the relationship between firm characteristics and the likelihood *P* of belonging to the "positive run group":

$$\mathrm{P}\left(Y_{i,t,l} = 1\right) = \frac{1}{1+\left(exp\left(-\alpha-\beta x_{i,t}\right)\right)} \tag{3.2}$$

where $Y_{i,t,l}$ is a binary indicator that equals one if firm *i* starts a positive run in year *t+1* for the next *l* years. The indicator is zero if the firm's growth is not consistently above the

[7] This method is also very often used in the literature on bankruptcy prediction (e.g., Martin, 1977; Ohlson, 1980; Shumway, 2001; Campbell et al., 2008), and in the literature on takeover target prediction (e.g., Palepu, 1986; Espahbodi and Espahbodi, 2003; Cremers et al., 2009).

median, defined simply as "negative run group". $x_{i,t}$ is a vector of explanatory variables of firm *i* measured at the end of year *t*, α is the estimated intercept, and β is a vector of coefficients.

3.3.1. Comparing groups of firms with positive and negative runs

We focus on the long-term. Thus, we look at an investment period *l* between three and six ($3 \leq l \leq 6$) years (y). We additionally assume that each firm at least grows above the median in the first year. This assumption helps us to assess the long-term rather than the short-term predictability of runs. The following five scenarios are helpful in distinguishing firms with positive and negative runs.

(1) 3y vs. 1y: The positive run group contains firms that will grow above the median for (at least) the next three years (*Y=1*). The negative run group contains firms that will grow above the median in the first year and below the median for the following two years (*Y=0*). The investment period *l* is three years. Eligible firms require at least three consecutive growth rates.

(2) 4y vs. 1y: The positive run group contains firms that will grow above the median for (at least) the next four years (*Y=1*). The negative run group contains firms that will grow above the median in the first year and below the median for the following three years (*Y=0*). The investment period *l* is four years. Eligible firms require at least four consecutive growth rates.

(3) 5y vs. 1y: The positive run group contains firms that will grow above the median for (at least) the next five years (*Y=1*). The negative run group contains firms that will grow above the median in the first year and below the median for the following four years (*Y=0*). The investment period *l* is five years. Eligible firms require at least five consecutive growth rates.

(4) 6y vs. 1y: The positive run group contains firms that will grow above the median for (at least) the next six years (*Y=1*). The negative run group contains firms that will grow above

the median in the first year and below the median for the following five years (*Y=0*). The investment period *l* is six years. Eligible firms require at least six consecutive growth rates.

(5) 6y vs. 3y: The positive run group contains firms that will grow above the median for (at least) the next six years (*Y=1*). The negative run group contains firms that will grow above the median in the first three years and below the median for the following three years (*Y=0*). The investment period *l* is six years. Eligible firms require at least six consecutive growth rates.

We expect that distinguishing the positive and the negative run groups ex ante becomes easier the longer we extend the investment period *l*. Growth rates of the firms of the "3y vs. 1y" combination behave differently only for at least two years. The firms of the "6y vs. 1y" combination, however, differ over a period of at least five years. We hypothesize that predicting positive and negative runs over long horizons should lead to greater power. The last scenario tightens our analysis, assuming that both groups grow above the median within the first three years.

3.3.2. Explanatory variables

We use accounting and equity market variables which are publicly available. Variables are measured annually by the end of the calendar year before the run starts. We assume that at this point of time all required accounting data is available to the market.

CKL test some variables to predict annual growth rates over one to five years. We adopt most of these variables for our analysis. *PASTGS5* is the growth in sales of the past five years[8], *EP* is the earnings to price ratio, *G* is the sustainable growth rate given by the product of return on equity (income before extraordinary items available to common equity relative to book equity) and the plowback ratio (one minus the ratio of total dividends to common equity divided by income before extraordinary items available to common equity), *RDSALES* is the

[8] We use annualized growth rates.

ratio of research and development expenditures to sales, *BM* is the book-to-market ratio, *PASTR6* is the stock's prior six-month rate of return, and *DP* is the dividend to price ratio. We exclude the dummy variable *TECH* which indicates if a firm is in the pharmaceutical and technology sector. CKL find that this variable has clearly no predictive power. We try our best not to miss out further obvious candidates which might be able to predict a run.

The prediction of bankruptcies and takeovers is also based on operative performance variables and the respective market evaluation. Lending from this research, we collect a range of well-known variables. The following ratios stem from Altman (1968). *WCTA* is working capital to total assets, *RETA* is retained earnings to total assets, *EBITTA* is earnings before interest and taxes to total assets, *METL* is market value equity to total liabilities and *STA* is sales to total assets. The variables *NITA* net income to total assets, *TLTA* total liabilities to total assets, and *CACL* current assets to current liabilities come from Zmijewski (1984). Additionally, we include a wide range of profitability measures. *CPM* is the cross profit margin (sales minus cost of goods sold divided by sales), *OPM* is the operating profit margin, *NPM* is the net profit margin, *ROE* is return on equity, and *OCR* is the overhead cost ratio.[9]

In total, we include 20 independent variables in our logit analysis. Table 3.3 summarizes statistical properties of the variables and reports the expected sign of correlation with future positive runs. All variables except the book-to-market ratio *BM* are winsorized at the first and 99th percentiles of their pooled distributions across all firm-years. We delete all negative values of *BM* and then winsorize at the 99th percentile. Following CKL, we set *RDSALES* to zero if a firm has no R&D spending.

Table 3.4 displays a matrix with pairwise Pearson correlations of the independent variables. Almost all correlations are significant at the 1% level. Only a few variables like net profit

[9] To calculate these variables we use the Datastream and Worldscope items WC01051, WC03501, WC01101, WC05101, WC03351, P, WC03151, WC02999, WC08001, WC18191, WC02201, WC03101, WC03495, MTVB, WC01201, WC09504, WC01001,WC01250, and WC01551.

margin *NPM* and operating profit margin *OPM* are highly correlated (0.969). However, multicollinearity is no severe issue because we control for highly correlated variables with a stepwise regression approach in the next section.

3.3.3. Variable selection

For an efficient use of the information contained in the explanatory variables, we employ a stepwise regression with forward selection and backward elimination to endogenously identify the parsimonious indicator-specific set of variables to be included in estimating the probability of a positive or negative run. It is this parsimony which is one of the advantages of this procedure, while one of its disadvantages is the collapse of standard statistical inference. This shortcoming is a potential concern, but should only deteriorate the power of the parsimoniously extracted variables to explain the out-of-sample variation in the probability of a positive or negative run. Since we are able to replicate reasonably the out-of-sample probability of a positive or negative run, we feel that the advantages of using a stepwise regression procedure outweigh its confinements. Admitting for each of the three operating performance indicators an individual set of independent variables, this selection technique starts with either an empty or a saturated model and tries out all variables one by one. Based on statistical significance the method either includes (forward selection) or excludes (backward elimination) one variable after another. To keep our indicator-specific models parsimonious and to abstain from a data mining exercise, we select the “3y vs. 1y” scenario as the base line model, because this scenario has probably the most difficulties in differentiating the positive and the negative run group. We specify an alpha-to-enter of 0.05 and an alpha-to-remove of 0.1. Firms need to have non-missing values for all predictor variables to be included. For model parsimony, a variable has to be significant at the 10% level in both procedures in order to enter the logit model. We use Wald tests to determine the statistical significance. Unreported tests show that using likelihood ratio tests does not affect the overall

results. The final set of explanatory variables to predict runs in sales growth consists of total liabilities to total assets *TLTA*, the stock's prior six-month rate of return *PASTR6*, and the dividend to price ratio *DP*. The predictors for runs in operating income growth are operating profit margin *OPM*, dividend to price *DP*, and research and development expenditures to sales *RDSALES*. Finally, dividend to price *DP*, the market value of equity to total liabilities *METL*, earnings before interest and taxes to total assets *EBITTA*, and net profit margin *NPM* predict runs in net income growth.

3.4. Results

3.4.1. Logit model estimates

We randomly split our initial sample into a training sample and a hold-out (validation) sample (e.g., Frank et al., 1965). The two sub-samples are divided in a 6:4 split to have a sufficient number of observations for model training, especially with respect to the “6y vs. 1y” and “6y vs. 3y” combinations.[10] Table 3.5 reports the results of logit regression estimates based on the training sample. In Panel A runs are calculated based on sales growth. Panels B and C analyze operating income growth and net income growth. The first four columns in each panel present models for the “3y vs. 1y”, “4y vs. 1y”, “5y vs. 1y” and “6y vs. 1y” combinations of the two groups. In the fifth column we report results of the “6y vs. 3y” scenario.

According to the likelihood ratio chi-square statistics all models except the “6y vs. 3y” net income model are significant at the 1% level. As expected, over an investment period of three years it is unlikely to correctly forecast if a company will either enjoy a three-year positive run or not. The McFadden's pseudo-R^2 coefficients of the “3y vs. 1y” models are only 0.031 for sales growth, 0.031 for operating income growth, and 0.049 for net income growth. The predictive power of the models increases, however, the longer the investment period is. This

[10] Minor deviations from this ratio are due to the random selection procedure.

is especially evident for the “6y vs. 1y” models. The pseudo-R^2 coefficients are 0.211 for sales growth, 0.274 for operating income growth, and 0.222 for net income growth. The results of the “6y vs. 3y” models suggest that it is very difficult to distinguish firms which have a positive run for the first three years. Pseudo-R^2s range between 0.041 (net income) and 0.120 (sales).

The most salient variable is the dividend to price ratio *DP* which is the only one included in all the regression specifications. The sign is consistently negative as expected. This finding is intuitive. Firms paying high dividends have fewer funds for investments and thus lower future growth. CKL also find that a low dividend yield is associated with high future growth in operating performance. Total liabilities to total assets *TLTA* exhibit also the expected negative sign for all sales models. This means that low leverage firms have a higher chance to enjoy a multi-year positive run. This link between capital structure and future investment opportunities is consistent with prior research (Myers, 1977; Myers and Majluf, 1984). The variable rate of return of the past six months *PASTR6*, which is related to momentum strategies (Jegadeesh and Titman, 1993; Jegadeesh and Titman, 2001), shows the expected positive sign. A possible explanation is that investors preferring to buy past winners are likewise attracted to firms generating a high consistency in sales growth rates (Chan et al., 2003). In combination with the fact that this variable is not selected when predicting income growth, it suggests two more things. First, firms are not very successful in translating runs in sales growth into runs in income growth. Second, in line with the investor overreaction hypothesis (De Bondt and Thaler, 1985; De Bondt and Thaler, 1990) past winners are not necessarily long-term future winners.

The coefficients of operating profit margin *OPM* are interestingly negative in the income models. Contrary to intuition, a high operating profit margin does not forecast positive runs in operating income growth. The data suggest that firms with a high operating profit margin

have little potential for further improvements in operating efficiency. Hence, high operating income growth rates need to be generated solely by growth in sales, which may turn out difficult. Firms with a lot of potential for efficiency improvements may compensate growth restrictions on the sales side. The ratio of R&D expenditures to sales *RDSALES* has a positive sign in all operating income models as hypothesized. The coefficients suggest that high R&D investments foster future growth, in particular long-term growth. CKL and other prior studies find a similar relationship (Sougiannis, 1994; Lev and Sougiannis, 1996; Eberhart et al., 2004).

The coefficients of market equity to total liabilities *METL* have the expected positive sign for the net income models. This is basically in line with the evidence on *TLTA*. Although selected by stepwise regression, the variable EBIT to total assets *EBITTA* has little predictive power. Similar to *OPM*, net profit margin *NPM* has a negative sign.

3.4.2. Classification test

We assess the ability of the previously estimated logit models to correctly classify a firm into the two categories of positive and negative runs. For this purpose, we perform in-sample and out-of-sample prediction tests. The drawback of the first method is that identical data is used for model training and validation. As a result, the reported accuracy may be positively biased. A common way to solve this problem is to predict data not used for model training. This approach is called out-of-sample validation. Since there is also evidence that results of in-sample tests are more credible than results of out-of-sample tests (Inoue and Kilian, 2004), we perform both methods. The hold-out sample is set to comprise approximately 40% of the entire sample. The training sample comprises the remaining 60% of the sample. An important factor when performing classification tests is the choice of the cut-off point. Traditionally, it is set to 0.5. In an unbalanced sample this may be inappropriate (Cramer, 1999). For instance, consider 90 healthy firms and 10 unhealthy firms. A logit model simply classifying every firm

as healthy would have an expected classification accuracy of 90%. In order to take into account relative sample frequencies, we calculate the expected probability of selecting a negative run firm and set the cut-off point to that value. Any firm whose predicted probability of belonging to the positive run group exceeds this value is categorized accordingly. The remaining companies are allocated to the negative run group. Although we employ this more precise procedure, in most cases, the number of positive and negative run firms is almost the same.

Table 3.6 reports the results. Panels A1, B1 and C1 test the training sample while Panels A2, B2 and C2 analyze how well the models classify new firms. For each performance indicator we evaluate the entire set of logit models. We report the percentage of firms correctly classified along with the type I error (firms erroneously classified as positive run firms), the type II error (firms misclassified as negative run firms), and the number of observations.

The training sample and the hold-out sample yield almost similar results and reinforce our conclusion that there is some predictability especially over extended investment periods. The classification accuracy of the models corresponds with the pseudo-R^2 reported in Table 3.5. The “3y vs. 1y” models classify on average about 60% of all firms correctly. This rate improves to an average of approximately 72% across the in- and out-of-sample tests of the “6y vs. 1y” models. The “6y vs. 3y” models perform comparably to the “3y vs. 1y” models.

The percentage of correctly classified firms is not the only factor when evaluating the goodness of a model. The risk to invest in the wrong firm is at least as important as the chance to invest in the right firm. The type I error in our analysis stands for the risk of investing in a firm that will not meet the expectations. The type II error reflects the risk to let an opportunity slip. In other words, assuming someone only invests in firms classified as positive run firms, the type I error is very dangerous; the type II error is not. Thus, the primary target of an investor would be to minimize the type I error. Regarding this risk, the models produce quite

large errors. Based on the in-sample prediction, on average 40.9% of all negative run firms are erroneously classified as positive run firms. The respective value based on the out-of-sample prediction is 39.0%. The type II errors are considerably lower and average 25.2% (in-sample) and 27.9% (out-of-sample). In line with the previous results, both types of errors decrease with an increasing investment period. The average type I error of all "3y vs. 1y" models equals 46.6%. The average of all "6y vs. 1y" models is considerably lower but still amounts to 35.1%. The corresponding type II errors fall from 31.1% to 18.2%. Comparing the performance indicators, we conclude that none of them is significantly better predictable.

3.4.3. Multiple discriminant analysis

To check for robustness and a potentially higher predictive power, we redo the preceding analysis using an alternative statistical methodology. In addition to logit regressions, multiple discriminant analysis (MDA) is a well-known technique to distinguish between two groups of firms based on a set of financial variables. The most prominent finance paper using this methodology is probably Altman (1968). Relative to the logit analysis, MDA has plenty of assumptions.[11] Due to frequent violations of these assumptions, maximum likelihood estimation techniques such as logit were recently more utilized. Although MDA is not as general as a logit analysis, for our purpose, it is well suited as an alternative method. In particular, one of the major advantages of MDA is that it requires less data to achieve stable results. In order to make the interpretation of the classification results as easy as possible, we construct two equally sized groups. As a result, the a priori probability of selecting a firm with a negative run is exactly 50%. We again use the set of variables identified in the stepwise regressions and randomly split into a training sample and a hold-out sample according to a 6:4 proportion. Table 3.7 reports the results. For each of the performance indicators, we test the

[11] MDA assumes that the independent variables are normally distributed, have no strong correlations, and that the variance-covariance matrix of the explanatory variables is the same for both groups.

run combinations as in the logit regressions. Panel A analyzes sales, Panel B operating income, and Panel C net income. Columns one, two, and three report the R^2, Wilks' Lambda, and chi-square of each model. The following eight columns display for each sub-sample the percent of correctly classified firms, the type I and II error, and the number of cases.

The results of the MDA corroborate the previous findings. According to the chi-square statistics, all except the “3y vs. 6y” net income model are significant at the 1% level. Similar to the logit regressions, we find a small degree of predictability over long investment periods. The goodness-of-fit of the “3y vs. 1y” models is only 0.035 for sales, 0.043 for operating income, and 0.060 for net income. As expected, the best fit is produced by the “6y vs. 1y” models. The R^2 of the sales model amounts to 0.248, the respective operating income model reaches a value of 0.261, and the R^2 of the net income model amounts to 0.201. The corresponding Wilks' Lambdas suggest the same pattern. The values of the “3y vs. 1y” and “6y vs. 3y” models are close to one, indicating that the two groups are poorly separated. The classification results reflect the model statistics. The “3y vs. 1y” models on average yield approximately a 60% correct classification rate across all firms in the training sample. This is only slightly above the a priori probability of 50%. The “5y vs. 1y” and “6y vs. 1y” models on average correctly classify about 71% of the firms. The out-of-sample results along with the type I and II errors are consistent with the in-sample results. In total, MDA yields almost the same classification results as the logit regressions.

3.5. General test for predictability

So far, we have only tried to discriminate two precisely defined groups of firms with certain patterns of above-median and below-median growth rates. We now extend our analysis to a more general level. We therefore ask whether the previously introduced logit models also help to assemble new groups with a higher share of firms with positive runs and a lower share of firms with negative runs, compared to a randomly selected group of firms.

The approach works as follows. By the end of year *t*, we select all available firms and hold them for the next five years. Out of this, we then construct two sub-groups of firms which we call "positive run group" and "negative run group". Based on the information before year *t*, we estimate a logit model which predicts the probability of a positive or a negative future run for each firm. All firms whose result is greater than 50% enter the positive run group. The remaining firms are allocated to the second group. If the logit model actually helps to predict runs, the positive run group is supposed to perform better than the negative run group. This means, the first group should exhibit a higher share of firms with positive runs and a lower share with negative runs. It is possible every year that a firm either grows above or below the median, so over five years there are $2^5 = 32$ possible growth paths. We focus our comparison on the following five growth paths: Five-year run, four-year run followed by one-year negative run, three-year run followed by two-year negative run, two-year run followed by three-year negative run, one-year run followed by four-year negative run, and five-year negative run. The sixth path we consider is that a firm does not survive for five years.[12] To have as many as possible eligible growth rates we analyze sales.[13] The logit models use the explanatory variables identified in the stepwise regressions and are trained based on the "5y vs. 1y" combination. The previous analyses have shown that this combination offers more eligible growth rates than the "6y vs. 1y" combination and still produces good forecasting models. We repeat the described selection procedure for each year between 1985 and 2003. The start year is 1985 because 1980 is the first year in our sample, and a full five-year period is required for model training. As time progresses more and more years add to the training sample. Table 3.8 reports means and medians of the shares across the time period 1985 to

[12] Due to the comparison of two groups of surviving firms, a potential survivorship bias is basically no issue in our study. However, we test if the logit models can also reduce the share of non-survivors in a group of firms.

[13] In unreported results, we also test operating income and net income with essentially the same conclusions.

2003. To identify significant differences between the two groups we perform two-sided paired t-tests and Wilcoxon rank-sum tests.[14]

Figure 3.1 displays the share of firms with five-year positive runs and five-year negative runs over the entire time period. The figure additionally reports the number of firms allocated to either of the two groups. The positive run group on average includes 523 firms per year, the negative run group 569 firms. The results show that the positive run group indeed contains more firms with positive runs and consistently less firms with negative runs over time. On average, 9.7% of all firms in the positive run group have a five-year run after group selection. In the negative run group on average only 2.4% achieve the same. The t-test indicates that these means are significantly different at the 1% level. The corresponding medians of 10.1% and 3.0% are likewise significantly different according to the Wilcoxon rank-sum test. We also find significantly higher percentages of firms with four-year and three-year positive runs in the positive run group. With respect to firms with extended negative runs, we find that on average 3.7% of all firms in the positive run group suffer five-year negative runs. The according share in the second group is 9.2%. The t-tests indicate a significant difference at the 1% level. The medians support this conclusion. The results further suggest that the positive run group contains slightly fewer non-surviving firms (17.3% compared to 18.6%); however, these differences are not significant. In total, we infer that our logit models help to predict positive and negative runs to some degree.

3.6. Conclusion

Prolonged periods of consecutive above-median or below-median growth rates in operating performance have strong influence on firm valuations. The objective of this study is to

[14] Note that the set of firms is not static. Each year, a newly trained logit model and new set of financial variables is used to allocate the firms to either of the two groups. Therefore we do not need to calculate t-statistics with autocorrelation-consistent standard errors.

explore the predictability of these so called runs. We distinguish between positive runs and negative runs. A positive run is defined as the ability to generate growth rates that exceed the median growth rate of all firms for a number of consecutive years. The opposite event of below-median growth rates for several successive years is called a negative run. To utilize information efficiently, we employ stepwise regression to endogenously identify the parsimonious indicator-specific set of economically and empirically meaningful variables in estimating the probability of a positive or negative run. Using logit regressions and multiple discriminant analysis, we process the information contained in a set of financial variables in order to calculate the likelihood that a firm will have a positive run over the next years. For this purpose, we compare certain groups of firms over a period of three to six years. The estimated models are evaluated by in-sample and out-of-sample classification tests. We find that a set of widely utilized financial variables indeed helps to predict runs. The accuracy improves with increasing run length. An additional test on a general level confirms that our logit models help to assemble new groups of firms which include more firms with positive runs and fewer firms with negative runs than randomly assembled ones.

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Table 3.1: Example of current run length.
This table gives an example how the current length of a positive or negative run is determined. At every calendar year-end, we calculate the annual growth rate in operating performance on a per share basis. Each year, we calculate the median of all growth rates. The number of consecutive years a firm manages to grow above the median is the length of a positive run. The number of consecutive years a firm grows below the median is the length of a negative run. Based on this, we can determine the current run length of each firm by the end of each year in our sample. The example shows the current run length of one particular firm. Positive numbers mark positive runs, negative numbers represent negative runs.

Year	1990	1991	1992	1993	1994	1995	1996	1997
Growth rate above the median	No	Yes	Yes	Yes	No	No	No	Yes
Current run length	-1	1	2	3	-1	-2	-3	1

Table 3.2: Sample summary of current run length.
This table summarizes the number of firm-years with a current positive and negative run length between one and six years. Our sample comprises all US equities with data available from Thomson Worldscope. The sample period is from 1980 till 2008. At every calendar year-end, we calculate the annual growth rate in operating performance (measured by sales, operating income, and net income before extraordinary items) on a per share basis. The number of shares outstanding is adjusted to reflect stock splits and dividends. Each year, we calculate the median of all growth rates. The number of consecutive years a firm manages to grow above the median is the length of the positive run. The number of consecutive years a firm grows below the median is the length of a negative run. Based on this, we can determine the current run length of each firm by the end of each year in our sample.

	Number of firm-years with current run length					
	1 year	2 years	3 years	4 years	5 years	6 years
			Panel A: Sales			
Positive run	25,353	12,516	6,408	3,465	1,972	1,175
Negative run	25,168	12,629	6,698	3,691	2,054	1,226
			Panel B: Operating income			
Positive run	29,040	11,993	5,124	2,256	964	451
Negative run	29,078	11,866	4,818	2,011	898	441
			Panel C: Net income before extraordinary items			
Positive run	29,035	10,909	4,255	1,778	740	361
Negative run	29,287	10,685	3,860	1,375	541	253

Table 3.3: Summary statistics of explanatory variables.
This table presents descriptive statistics on the initial set of explanatory variables. Our sample comprises 13,751 firms. The sample period is from 1980 till 2008. Financial firms (SIC 6000-6999) are excluded. The variables are selected in line with Chan et al. (2003), Altman (1968), and Zmijewski (1984). Additionally, we include five popular profitability measures. For each variable, the table reports the median, mean, the maximum value, the minimum value, the standard deviation, and the expected sign of correlation with a positive run.

Variable	Definition	Median	Mean	Max	Min	Std. Dev.	Expected sign
Chan et al. (2003) variables							
PASTGS5	Growth rate in sales over the past five years	0.0593	0.0615	0.8743	-0.5468	0.2029	+
EP	Earnings to price ratio	0.0288	-0.2068	0.6092	-4.8843	0.8387	+/-
G	Sustainable growth rate (Product of return on equity and plowback ratio)	0.0998	0.1220	0.7380	0.0001	0.1076	+
RDSALES	R&D expenditures to sales ratio	0.0698	0.4943	8.7684	0.0000	1.5494	+
BM	Book to market ratio	0.5348	1.1596	20.0000	0.0000	2.8132	+
PASTR6	Rate of return of the past six months	0.0000	-0.0229	1.9998	-0.8667	0.4532	+
DP	Dividend to price ratio	0.0241	0.0364	0.9889	0.0001	0.0596	-
Altman (1968) variables							
WCTA	Working capital to total assets ratio	0.2171	0.0961	0.8667	-3.2509	0.7267	+
RETA	Retained earnings to total assets ratio	-0.0040	-2.5772	0.7222	-38.2588	7.7797	+
EBITTA	EBIT to total assets ratio	0.0607	-0.1994	0.4115	-3.5106	0.7834	+
METL	Market value equity to total liabilities ratio	2.0503	8.4328	140.8165	0.0214	20.7955	+
STA	Sales to total assets ratio	1.0327	1.2035	4.3238	0.0135	0.9117	+
Zmijewski (1984) variables							
NITA	Net income to total assets ratio	0.0229	-0.2799	0.3045	-4.0381	0.8809	+
TLTA	Total liabilities to total assets ratio	0.5198	0.6843	4.0909	0.0177	0.7702	-
CACL	Current assets to current liabilities ratio	1.8550	2.8176	23.7531	0.0652	3.4902	+
Additional profitability variables							
CPM	Cross profit margin	0.3787	0.3924	1.0000	-1.1004	0.3134	+/-
OPM	Operating profit margin	0.0524	-0.8620	0.6710	-19.3803	3.4738	+/-
NPM	Net profit margin	0.0225	-1.0385	0.9733	-22.0495	3.9654	+/-
ROE	Return on equity	0.0932	0.0161	3.6750	-4.5129	1.1199	+/-
OCR	Overhead cost ratio	0.2744	1.0810	18.4096	0.0138	3.0937	-

Table 3.4: Correlation matrix.

This table reports pairwise Pearson correlations between each of the explanatory variables. *,**, and *** coefficients are significant at the 10%, 5% and 1% level, respectively.

	PASTGS5	EP	G	RDSALES	BM	PASTR6	DP	WCTA	RETA	EBITTA	METL	STA	NITA	TLTA	CACL	CPM	OPM	NPM	ROE	OCR
PASTGS5	1.000																			
EP	0.155***	1.000																		
G	0.297***	0.053***	1.000																	
RDSALES	-0.136***	-0.069***	0.061***	1.000																
BM	-0.087***	-0.302***	-0.168***	-0.036***	1.000															
PASTR6	0.024***	0.279***	-0.023***	-0.016***	-0.147***	1.000														
DP	-0.156***	-0.105***	-0.162***	0.083***	0.260***	-0.155***	1.000													
WCTA	0.192***	0.301***	0.082***	-0.006**	-0.085***	0.110***	-0.124***	1.000												
RETA	0.262***	0.290***	0.086***	-0.127***	-0.035***	0.090***	-0.247***	0.721***	1.000											
EBITTA	0.249***	0.405***	0.357***	-0.208***	-0.045***	0.143***	-0.168***	0.663***	0.761***	1.000										
METL	0.079***	0.069***	0.155***	0.135***	-0.131***	0.178***	-0.057***	0.097***	-0.089***	-0.173***	1.000									
STA	0.044***	-0.014***	0.146***	-0.203***	0.030***	0.011***	-0.110***	-0.113***	-0.079***	0.025***	-0.151***	1.000								
NITA	0.245***	0.415***	0.346***	-0.193***	-0.041***	0.143***	-0.187***	0.692***	0.781***	0.985***	-0.155***	0.007**	1.000							
TLTA	-0.204***	-0.326***	-0.005	0.031***	0.073***	-0.104***	0.088***	-0.907***	-0.740***	-0.669***	-0.122***	0.177***	-0.704***	1.000						
CACL	0.043***	0.097***	0.036***	0.155***	-0.038***	0.036***	-0.050***	0.417***	0.143***	0.112***	0.464***	-0.223***	0.131***	-0.342***	1.000					
CPM	0.108***	0.082***	0.086***	0.040***	-0.082***	0.049***	-0.005	0.044***	0.029***	0.119***	0.077***	-0.153***	0.114***	-0.078***	0.027***	1.000				
OPM	0.278***	0.190***	0.041***	-0.626***	0.014***	0.073***	-0.115***	0.273***	0.427***	0.574***	-0.222***	0.259***	0.562***	-0.268***	-0.121***	0.215***	1.000			
NPM	0.269***	0.235***	0.086***	-0.597***	0.009***	0.081***	-0.157***	0.295***	0.437***	0.604***	-0.203***	0.256***	0.601***	-0.290***	-0.094***	0.206***	0.969***	1.000		
ROE	0.041***	0.038***	0.226***	-0.071***	-0.013***	0.024***	-0.058***	-0.247***	-0.129***	-0.057***	-0.066***	0.095***	-0.071***	0.264***	-0.082***	0.019***	0.044***	0.041***	1.000	
OCR	-0.262***	-0.170***	0.033***	0.681***	-0.022***	-0.064***	0.110***	-0.274***	-0.427***	-0.561***	0.227***	-0.274***	-0.550***	0.265***	0.117***	-0.090***	-0.969***	-0.940***	-0.034***	1.000

Table 3.5: Logit regressions of run indicator on predictor variables.

This table reports results of pooled logit regressions. The sample comprises 13,751 firms. The sample period is 1980 to 2008. Financial firms (SIC 6000-6999) are excluded. The dependent variable is a binary variable indicating if a firm will have a positive (Y=1) or negative (Y=0) run after sample selection. Runs are measured based on sales (Panel A), operating income (Panel B), and net income (Panel C). For each performance indicator, five different models are estimated. The "3y vs. 1y" models try to distinguish firms that will have a run for (at least) three years and firms that will grow above the median in the first year and below the median for the following two years. "4y vs. 1y" compare firms that will have a run for (at least) four years and firms that that will grow above the median in the first year and below the median for the following three years. The "5y vs. 1y" models compare firms that will have a run for (at least) five years and firms that will grow above the median in the first year and below the median for the following four years. The "6y vs. 1y" models compare firms that will have a run for (at least) six years and firms that that will grow above the median in the first year and below the median for the following five years. "6y vs. 3y" tighten the analysis and compare firms that will have a run for (at least) six years and firms that will grow above the median in the first three years and below the median for the following three years. The independent variables are selected by stepwise regression (forward selection and backward elimination) based on the "3y vs. 1y" models. To be selected, a variable has to be significant at the 10% level in both procedures. The used variables are total liabilities to total assets *TLTA*, the stock's prior six-month rate of return *PASTR6*, the dividend to price ratio *DP*, operating profit margin *OPM*, the ratio of research and development expenditures to sales *RDSALES*, market value equity to total liabilities *METL*, earnings before interest and taxes to total assets *EBITTA*, and net profit margin *NPM*. The absolute value of the z-statistics is reported in parentheses. Coefficients significant at the 1%, 5%, and 10% levels are indicated by ***, **, and *, respectively. The table further reports the number of firm-years, the likelihood ratio chi-square, and McFadden's pseudo-R^2.

	Panel A: Sales					Panel B: Operating income					Panel C: Net income				
	3y vs. 1y	4y vs. 1y	5y vs. 1y	6y vs. 1y	6y vs. 3y	3y vs. 1y	4y vs. 1y	5y vs. 1y	6y vs. 1y	6y vs. 3y	3y vs. 1y	4y vs. 1y	5y vs. 1y	6y vs. 1y	6y vs. 3y
TLTA	-1.353	-2.129	-2.242	-2.173	-2.719										
	(-5.22)***	(-5.71)***	(-4.20)***	(-3.21)***	(-3.29)***										
PASTR6	0.620	0.548	1.156	0.307	0.850										
	(3.17)***	(2.16)**	(3.16)***	(0.68)	(1.51)										
DP	-7.167	-11.822	-27.900	-40.390	-29.033	-10.400	-29.413	-41.556	-47.501	-30.009	-11.282	-22.961	-10.625	-8.368	-23.280
	(-4.38)***	(-4.80)***	(-6.46)***	(-6.72)***	(-4.31)***	(-5.96)***	(-8.08)***	(-6.50)***	(-4.60)***	(-3.58)***	(-6.02)***	(-6.27)***	(-2.03)**	(-1.42)	(-2.11)**
OPM						-2.194	-4.340	-7.790	-3.285	-0.487					
						(-3.94)***	(-4.55)***	(-4.50)***	(-1.41)	(-0.27)					
RDSALES						4.974	6.020	18.577	23.538	1.430					
						(2.96)***	(2.30)**	(3.40)***	(3.08)***	(0.33)					
METL											0.045	0.055	0.122	0.028	-0.010
											(4.59)***	(2.85)***	(2.26)**	(0.35)	(-0.23)
EBITTA											-0.173	-2.332	0.427	8.012	2.606
											(-0.19)	(-1.47)	(0.15)	(1.87)*	(0.66)
NPM											-6.944	-9.483	-21.157	-23.975	-4.972
											(-5.87)***	(-4.27)***	(-5.00)***	(-4.14)***	(-0.75)
Constant	0.932	1.382	1.811	2.305	2.719	0.214	0.882	1.343	0.865	0.647	0.410	1.255	0.855	0.899	0.241
	(6.44)***	(6.81)***	(6.21)***	(5.93)***	(6.01)***	(2.31)**	(5.42)***	(5.06)***	(2.03)**	(1.89)*	(3.31)***	(5.76)***	(2.40)**	(1.75)*	(0.44)
N (firm-years)	1,843	1,036	595	361	288	1,937	917	467	205	171	1,563	702	303	140	133
LR chi^2	79.2	90.9	124.6	105.2	43.8	82.3	149.8	129.0	69.4	19.8	104.0	107.5	64.7	43.0	7.4
Pseudo R^2	0.031	0.063	0.151	0.211	0.120	0.031	0.122	0.211	0.274	0.084	0.049	0.111	0.159	0.222	0.041

Table 3.6: Classification tests.
This table reports classification results based on the logit models estimated in Table 3.5. The classification accuracy is based on the training sample and the hold-out sample. The training sample contains 60% of the entire sample, while the hold-out sample contains the remaining 40%. For each model and each performance indicator we report the percent of firms correctly classified, the type I error (firms misclassified as positive run firms), the type II error (firms misclassified as negative run firms), and the number of observations. Panels A1 and A2 analyze sales, Panels B1 and B2 operating income, and Panels C1 and C2 net income.

	Training sample				Hold-out sample			
Model	Correctly classified	Type I error	Type II error	N	Correctly classified	Type I error	Type II error	N
	Panel A1: Sales				Panel A2: Sales			
3y vs. 1y	60.1%	42.0%	37.9%	1,843	61.5%	44.4%	32.5%	1,243
4y vs. 1y	64.2%	40.7%	30.7%	1,036	63.0%	48.6%	23.3%	665
5y vs. 1y	67.9%	37.6%	26.0%	595	70.9%	34.9%	22.9%	371
6y vs. 1y	70.6%	36.5%	21.5%	361	74.1%	36.4%	15.3%	263
6y vs. 3y	70.8%	29.5%	29.0%	288	64.7%	56.8%	20.9%	184
	Panel B1: Operating income				Panel B2: Operating income			
3y vs. 1y	57.3%	53.2%	28.7%	1,937	62.1%	42.8%	31.6%	1,254
4y vs. 1y	64.5%	44.5%	21.6%	917	64.1%	41.8%	27.7%	615
5y vs. 1y	70.7%	36.9%	16.0%	467	70.6%	35.0%	20.0%	282
6y vs. 1y	70.2%	33.1%	22.2%	205	69.7%	29.9%	31.0%	178
6y vs. 3y	59.7%	48.9%	29.9%	171	61.2%	41.7%	35.1%	129
	Panel C1: Net income				Panel C2: Net income			
3y vs. 1y	61.7%	47.5%	26.7%	1,563	59.5%	49.4%	29.5%	1,068
4y vs. 1y	67.5%	42.1%	20.9%	702	68.9%	42.1%	15.0%	441
5y vs. 1y	69.0%	38.0%	20.2%	303	71.7%	21.2%	35.9%	191
6y vs. 1y	77.1%	31.5%	13.4%	140	70.2%	43.3%	5.9%	94
6y vs. 3y	56.4%	51.3%	32.7%	133	55.6%	15.9%	71.7%	90

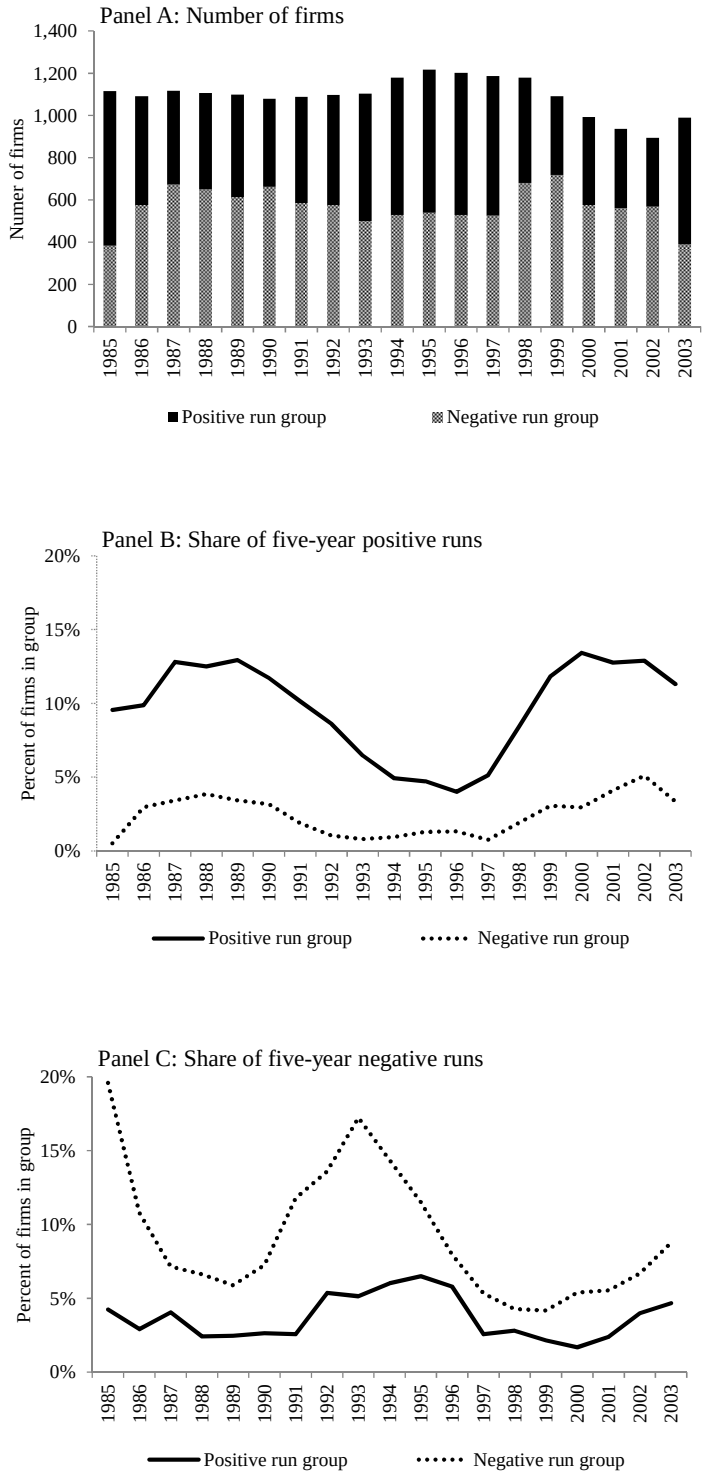
Table 3.7: Multiple discriminant analysis.
This table reports results of the multiple (linear) discriminant analysis. We use the variables from Table 3.5 and estimate five discriminant models for each performance indicator. The first three columns report the R^2 coefficients, the Wilks' Lambda, and the chi-square of each model. The following eight columns present results of the classification test. The classification accuracy is calculated based on the training sample and the hold-out sample. The training sample contains 60% of the entire sample; the hold-out sample contains the remaining 40%. For each model and each performance indicator, we report the percent of firms correctly classified, the type I error (firms misclassified as positive run firms), the type II error (firms misclassified as negative run firms), and the number of observations. Panel A analyzes sales, Panel B operating income, and Panel C net income.

				Training sample				Hold-out sample			
Model	R^2	Wilks' Lambda	Chi^2	Correctly classified	Type I error	Type II error	N	Correctly classified	Type I error	Type II error	N
					Panel (A): Sales						
3y vs. 1y	0.035	0.965	65.5	59.0%	41.1%	40.8%	1,828	61.6%	36.7%	40.0%	1,220
4y vs. 1y	0.093	0.907	94.2	64.6%	38.5%	32.3%	972	64.0%	38.3%	33.6%	648
5y vs. 1y	0.146	0.854	86.8	68.7%	39.9%	22.8%	552	67.7%	36.4%	28.3%	368
6y vs. 1y	0.248	0.752	102.6	73.6%	35.2%	17.6%	364	70.2%	43.8%	15.7%	242
6y vs. 3y	0.189	0.811	41.7	69.3%	32.7%	28.7%	202	70.6%	38.2%	20.6%	136
					Panel (B): Operating income						
3y vs. 1y	0.043	0.957	72.7	60.6%	49.3%	29.5%	1,654	60.5%	50.3%	28.7%	1,102
4y vs. 1y	0.142	0.859	111.8	66.8%	47.0%	19.3%	736	64.1%	51.4%	20.4%	490
5y vs. 1y	0.191	0.809	68.9	70.7%	45.1%	13.4%	328	70.0%	48.2%	11.8%	220
6y vs. 1y	0.261	0.739	47.3	71.9%	38.8%	17.5%	160	67.6%	53.7%	11.1%	108
6y vs. 3y	0.120	0.880	19.9	65.6%	43.8%	25.0%	160	58.3%	61.1%	22.2%	108
					Panel (C): Net income						
3y vs. 1y	0.060	0.940	85.9	62.9%	50.3%	23.9%	1,396	60.6%	53.9%	24.9%	932
4y vs. 1y	0.130	0.870	83.3	70.5%	42.3%	16.7%	600	70.0%	44.0%	16.0%	400
5y vs. 1y	0.195	0.805	54.3	74.0%	39.4%	12.6%	254	68.5%	46.4%	16.7%	168
6y vs. 1y	0.201	0.799	26.4	72.1%	37.7%	18.0%	122	75.0%	37.5%	12.5%	80
6y vs. 3y	0.054	0.946	6.5	63.1%	34.4%	39.3%	122	52.5%	32.5%	62.5%	80

Table 3.8: General test for predictability.
This table performs a more general test for predictability of above-median and below-median growth rates. The test is based on sales growth. The prediction period is five years. In each year *t* between 1985 and 2003, all available firms represent one group. Out of this group, two sub-groups are constructed. For this purpose, each year a logit model is estimated using all available information before year *t*. The models are trained based on the "5y vs. 1y" combination. The first year for model training is 1980. The explanatory variables are total liabilities to total assets *TLTA*, the stock's prior six-month rate of return *PASTR6*, and the dividend to price ratio *DP*. All firms whose estimated probability of enjoying a positive run in the next five years exceeds 50% enter the "positive run group". The remaining firms are allocated to the "negative run group". Each year, the share of firms in the two sub-groups with the following growth paths is calculated: Five-year positive run, four-year positive run followed by one-year negative run, three-year positive run followed by two-year negative run, two-year positive run followed by three-year negative run, one-year positive run followed by four-year negative run, and five-year negative run. Additionally, the share of remaining survivors and non-survivors is calculated. The table reports means and medians across all years between 1985 and 2003. Differences in the means are tested by two-sided paired t-tests, while differences in the medians are tested by Wilcoxon rank-sum tests. Columns three and six report p-values.

Sales growth	Mean (1985 - 2003)			Median (1985 - 2003)		
Survivors	Positive run group	Negative run group	t-test (p-value)	Positive run group	Negative run group	Wilcoxon rank-sum test (p-value)
5-year positive run	9.7%	2.4%	0.0%	10.1%	3.0%	0.0%
4-year positive run, 1-year negative run	4.4%	2.0%	0.0%	3.2%	1.6%	0.2%
3-year positive run, 2-year negative run	3.8%	2.4%	0.2%	3.1%	2.2%	3.0%
2-year positive run, 3-year negative run	3.2%	2.9%	25.9%	3.0%	2.5%	64.0%
1-year positive run, 4-year negative run	3.5%	4.2%	1.8%	3.4%	3.7%	20.9%
5-year negative run	3.7%	9.2%	0.0%	2.9%	7.3%	0.0%
Remaining survivors	54.5%	58.3%	n.a.	54.7%	58.7%	n.a.
Non-survivors	17.3%	18.6%	12.0%	18.0%	18.3%	70.4%
Sum of group	100.0%	100.0%		n.a.	n.a.	

Figure 3.1: Share of firms in the positive run group and the negative run group.
The firms are allocated by the method introduced in Table 3.8. Panel A shows the number of firms in the positive run and negative run group for each year. Panels B and C display the share of firms with five-year positive runs and five-year negative runs over the period 1985 to 2008. The reported year indicates the start of the five-year holding period when the firms are selected.



4. Why are British Premium Bonds so successful? The effect of saving with a thrill

(with Sebastian Lobe)

Presented at:

- Inquire UK's Autumn seminar, Cambridge, UK, September 25, 2007
- The University of Manchester, Manchester Business School, UK, September 25, 2007
- Campus for Finance - Research Conference, WHU Vallendar, January 16-17, 2008
- Midwest Finance Association's 57th Annual Meeting, San Antonio, Texas USA, February 27 - March 1, 2008
- 70. Wissenschaftliche Jahrestagung des Verbands der Hochschullehrer für Betriebswirtschaft e.V., Freie Universität Berlin, May 15-17, 2008
- 27th SUERF Colloquium on New Trends in Asset Management: Exploring the Implications, HVB Forum in Munich, June 12-14, 2008
- Northern Finance Association 2008 Conference, Kananaskis Village, Canada, September 5-7, 2008
- 15th Annual Meeting of the German Finance Association (DGF), Münster, October 10-11, 2008
- 11th Symposium on Finance, Banking, and Insurance, Karlsruhe, December 19, 2008
- 2009 Annual Meeting of the Financial Management Association International, Reno, Nevada USA, October 21-24, 2009

Abstract

The British Premium Bond, which offers a monthly uncertain return solely based on a lottery, is an immense success. Why? Analysing hand-collected data of the past fifty-four years, we find that the bond bears relatively low risk in terms of CARA and CRRA utility. Since prizes are tax-free, the higher an individual's tax bracket, the more it pays to invest in the lottery bond. However, we demonstrate that the CARA and CRRA coefficients (before and after taxes) do not directly influence net sales of the Premium Bond. Rather, our autoregressive models strongly suggest that prize skewness, the maximum holding amount and the number of top prizes are salient influencing factors.

Keywords: Premium Bond, lottery bond, risk tolerance, skewness

4.1. Introduction

Can saving money, without risking the principal, become an adventure? Looking at ordinary savings accounts, one readily answers no. An investor pays an amount of money into a bank account and gets fixed interest payments: a humdrum but safe way of investing. One very popular way of getting a thrill is gambling as people are always happy about winning a prize. Centuries ago, financial products were invented to capitalize on people's fascination for gambling. The idea features saving money with a lottery to make things more exciting. As a result, the issuers usually enjoyed significantly higher sales and profits. Nowadays, lottery bonds or lottery-linked deposit accounts (LLDAs) are available worldwide. One very successful example is the British Premium Bond. Harold MacMillan, Chancellor of the British Exchequer, initially launched the British Premium Bond (PB) in November 1956. After decades of steadily increasing sales, particularly in the last 10 years, the Premium Bonds sky-rocketed. By the end of 2011, around 23 million people in Great Britain had invested about £43 billion in Premium Bonds. What makes these so successful? Because of its longevity, the Premium Bonds are perfect for an empirical analysis on what drives a successful LLDA.

We offer answers to this question by scrutinising a unique, hand-collected set of data provided by the issuer. In total, we have a record over a period of fifty-four years. To understand if the risk attitude attracts savers, we apply the classical Arrow-Pratt constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) approaches to back out the indifference degree of risk tolerance. As the investment alternatives are taxed differentially, individual income taxes play a key role. We first focus on a simple monthly investment period. In doing so, we vary the amount invested and include personal wealth. We then study longer investment periods of five, ten and twenty years. We also discuss further factors potentially influencing the success of Premium Bonds. In this context, we turn our

attention to cumulative prospect theory (Tversky and Kahneman, 1992) and focus on prize skewness. To detect relationships, we conduct Granger causality tests (Granger, 1969). Finally, we present autoregressive models to confirm that skewness, the number of jackpots and the maximum holding amount are indeed factors that encourage net sales.

Much research has already been done on analysing individual risk preferences. Often the central question is what risk preferences do individuals exhibit in certain situations and when do they accept bets with even negative expected returns? While many studies use surveys, e.g. Donkers et al. (2001), others analyse large data samples from TV game shows, e.g. Beetsma and Schotman (2001), or horse races like Jullien and Salanié (2000). Lottery bonds can also be analysed in this context. As these investments are not traded in an artificial environment, it makes them particularly interesting for empirical studies. Guillén and Tschoegl (2002) describe numerous examples of LLDAs with focus on examples located in Latin America. They conclude that these accounts are apparently more a marketing device than a source of funds cheaper than savings deposits. Kearney et al. (2010) survey a broad variety of prize-linked savings (PLS) programs around the world and describe the appeal of PLS programs to US households and issuers. Ukhov (2010) studies the relationship between investor risk preference and asset returns of Russian lottery bonds. He analyses time variations in the risk preferences between 1889 and 1904. Green and Rydqvist (1997, 1999) study Swedish government lottery bonds whose coupon payments are determined by a lottery. They evaluate the rewards of bearing extra lottery risk, finding that prices appear to reflect this risk. They also report that variance reduces lottery prices. In a subsequent paper Rydqvist (2011) investigates risk and effort aversion in the context of tax arbitrage based on Swedish lottery bonds. Florentsen and Rydqvist (2002) analyse the pricing of Danish lottery bonds focusing on tax-based explanations of abnormal ex-day returns. They find that prices fall by more than the lottery mean and also conclude that investors do not enjoy this lottery.

Despite having been continuously operated for more than fifty-four years and their high popularity, there are only very few studies dealing with the Premium Bond. In an early work, Rayner (1969) observes an initial lack of popularity of the Premium Bond program and examines the reasons. He tries to explain how the change in the prize structure affected the demand. He argues that the top prize element should be further increased, while the average yield can be reduced, to cheapen the cost to the Treasury (Rayner, 1969 p. 310). In a second paper Rayner (1970) further studies the prize structure of Premium Bonds. He supposes that the standard deviation is a good approximation to measure the attraction of the risk element in the prize structure. Tufano (2008) analyses the determinants of Premium Bond net sales. He finds that the Premium Bond program has both savings and gambling elements. Pfiffelmann (2007) analyses the optimal design for LLDAs based on the Premium Bonds as an example. In a related paper, Pfiffelmann (2008) continues her research assuming that investors' individual preferences obey cumulative prospect theory. In the work cited above, Guillén and Tschoegl (2002) also state that skewness of returns is a feature to maintain investors' interest in the LLDA. Many studies on gamblers' risk attitudes discuss the importance of the third moment. Golec and Tamarkin (1998) point out that not only mean and variance explain gambling behaviour but also skewness of the returns. Garrett and Sobel (1999) find evidence for the relevance of skewness by examining United States lotteries. Bhattacharya and Garrett (2008) empirically find that the expected return from a lottery game is a decreasing and convex function of the skewness of the lottery game.

The remainder of the paper is organised as follows. Section 4.2 explains the history and the basic design of the bond. In Section 4.3, we introduce our sample and compute the degrees of risk aversion and risk seeking an investor needs to exhibit in order to prefer Premium Bonds. Section 4.4 identifies important factors influencing net sales of the Premium Bond and

focuses on prize skewness as a major factor. In Section 4.5, multivariate autoregressive models combine the previous findings. Section 4.6 concludes.

4.2. History of the Premium Bond and its characteristics

The Premium Bond is issued by National Savings and Investments (NS&I), which has been a government department since 1969. It aims to help reduce the cost to the taxpayer of government borrowing.[15] Launched in 1956, the Premium Bond has been slowly expanding over 35 years. Since 1994, sales have been strongly increasing. The following statistics clearly express this increase. From October 1969 till December 1993, monthly net sales averaged about £25.4 million expressed in April 2006 pounds. In the following twelve years from January 1994 to April 2006, monthly net sales averaged £217.8 million (in April 2006 pounds) which equals an increase by the factor of 8.6. Meanwhile, Premium Bonds definitely enjoy the highest popularity since about 43% of the population own these. The bond is one of the most important investment products in Great Britain for households and it is NS&I's most successful asset. In March 2002, the total amount invested was £17.3 billion which equalled a 27.8% share of the total amount invested in NS&I products. Within ten years, the amount increased to £43.1 billion and the share climbed to 43.6%.

The initial purpose of the Premium Bond was to control inflation and to encourage more people after World War II to save money. For almost thirty years (1950s – 1980s) gambling this way was advertised as a fun way of saving and investing money. The National Lottery was then launched years later in November 1994. Since the 1990s, NS&I changed its marketing strategy and emphasised that Premium Bonds are a serious way of investing money, leading to a huge escalation in sales.

The basic design of the bond is quite simple and has not been altered since its conception: any British citizen aged 16 and over can buy Premium Bonds. It is not possible to hold them

[15] http://www.nsandi.com/about-nsi-what-we-do visited: 20 December 2011.

jointly and they are not transferable to another person. The minimum investment is currently (as at December 2011) £100 or £50 with a monthly standing order. Unlike a common deposit account, the total interest payments per month are subject to a lottery. There are no additional interest payments. The fee for participating in the prize draws is just the forgone interest payment of an alternative investment. For each single pound invested, there is one chance to win. Currently, the maximum amount a person can invest is £30,000. For example, if someone buys Premium Bonds worth £3,000, he or she has 3,000 chances to win. Each bond has exactly the same chance, making time of purchase irrelevant. The prize draws are carried out at the beginning of each month by a sophisticated computer system, which NS&I calls ERNIE (Electronic Random Number Indicator Equipment). The odds of winning a prize are currently 24,000 to 1. This means that an investor holding £24,000 can expect to win once per month on average. Of course, this is not guaranteed. After several changes, the prizes are currently spread from £25 up to £1 million. The total number of prizes per month is calculated by the total number of eligible bond units divided by the odds. The total value of all prizes of a draw is determined by the interest rate that is announced in advance. NS&I can arbitrarily change this rate. On their official web page, NS&I states that 89% of the prize fund is allocated to the lower prize band; 5% to the medium band; and 6% to the higher prize band.[16] Table 4.1 illustrates the distribution of a typical prize draw.

One special feature of the Premium Bond is that all prizes are tax-free, making them even more attractive for potential savers. Unlike a regular lottery, the initial investment is not used up. Moreover, a bond holder can always get the principal refunded at any time. This advantage, plus the maximum holding stipulation, controls the risk of addiction and possible financial ruin.

[16] http://www.nsandi.com/savings-current-interest-rates-premium-bonds-prize-draw-details visited: 20 December 2011.

4.3. Classical risk tolerance analysis

4.3.1. Research method and preliminary considerations

In this section, we analyse the extent to which an investor needs to be risk-averse or risk-seeking in order to consider Premium Bonds a utility maximising investment. A classical approach is the expected utility theory operationalized by Arrow (1965) and Pratt (1964).

$$\text{Constant Absolute Risk Aversion (CARA):} \quad u(x)_{CARA} = -\frac{e^{-\alpha x}}{\alpha} \tag{4.1}$$

$$\text{Constant Relative Risk Aversion (CRRA):} \quad u(x)_{CRRA} = \frac{x^{1-\alpha}}{1-\alpha} \tag{4.2}$$

In the above equations, x stands for the amount of payment, α for the individual risk preference and $u(x)$ for the utility of x. e is the base of the natural logarithm. To obtain the indifference level of risk tolerance, we iteratively calculate the coefficient α which leads to the same utility of a risky Premium Bond and a certain alternative investment. For comparison, we compute both, the constant absolute risk aversion and the constant relative risk aversion. The expected utility of the Premium Bond for a month´s draw is obtained as follows.

$$E\left[u(x)\right] = \sum_{i=1}^{n} p_i \cdot u(x_i), \; p_i = \frac{c_i}{t \cdot o} \tag{4.3}$$

We calculate the utility $u(x_i)$ of each prize x_i of a draw, including the case that nothing is won. Utility components are weighted with the specific probability of occurrence p_i. To calculate these probabilities, we divide the number of prizes in each prize class c_i (e.g., 45 times £10,000) by the total number of prizes of this draw t (e.g., December 2011: 1,788,609). This likelihood is divided by the odds o to obtain the probability p_i that a one-pound bond wins exactly this prize.

Monthly interest payments determine the utility of a certain investment. By iterative calculation, we obtain values for α (CARA, CRRA). An individual investor exhibiting this indifference risk coefficient would be indifferent between the two alternatives. As α is a small number and very sensitive with respect to the accuracy of the interpolation, we perform our calculations with 300 decimal places. Positive (negative) values of α indicate risk aversion (risk seeking) across time. A zero value means risk neutrality. Savers who are less risk-averse or more risk-seeking than the indifference level will choose the Premium Bond since this maximises their utility.

Next, we need to specify reference investments. As we try to employ the longest data record possible, the official Bank of England's (BoE) rate matches this objective nicely. While we are aware that a retail investor cannot invest in a bond delivering the BoE rate, most bonds in the UK should be linked to this rate to a greater or lesser extent. To understand how Premium Bonds perform in comparison to a product an investor can actually purchase, we choose to pick the Income Bond delivering monthly interest payments. This investment, issued as well by NS&I, implies that there will not be a differing issuer's risk premium. Since NS&I is backed by the government, the products are essentially risk-free. Premium Bonds and Income Bonds are similar in terms of the initial investment, the monthly payout structure, the option to withdraw the safe capital at any time and the infinite time to maturity. However, the Income Bond's monthly interest payment is certain, and the interest rate is usually higher but subject to income taxation.

For our analysis, the margin between the interest rate of the Premium Bond and that of other investments is crucial. High expected returns of the lottery bond compared with other investments can encourage even risk-averse investors to buy it. Figure 4.1, illustrating the corresponding time series, shows how the interest rates of the observed investments have changed in the last fifty-four years.

Another key element is taxation. For the fiscal year 2011-12 the UK tax legislation distinguishes between four taxable bands: starting rate (10% rate), basic rate (20% rate), higher rate (40% rate), and additional rate (50% rate). Due to personal allowances, (e.g. 2011-12 £7,475), some savers are not liable for taxation. As previously noted, Premium Bonds enjoy tax exemption which makes them more attractive for savers. For example, the 1.50% interest rate as at December 2011 is equivalent to 3.00% for an additional rate income taxpayer, 2.50% for a higher rate taxpayer, and 1.88% for a basic taxpayer. Therefore, considering after-tax returns, it is possible that Premium Bonds outperform other risk-free investments. Since our analysis covers fifty-four years, we always apply the tax rates valid for that year in consideration. In essence, the tax classes have not changed. The tax rates, however, have been subject to several changes. We were able to obtain UK tax rates from the year 1957 until now.[17] Based on these data, we analyse the four tax bands: no tax, starting rate, basic rate and higher rate. We assume that in the higher rate tax bracket an investor needs to pay the lowest rate within this band. For anyone taxed at higher rates, Premium Bonds would be even more attractive. Also note that the starting tax rate was not raised in all years.

Checking the overall taxpayer distribution for the UK, we find that in 2009-10, 10.4% of all taxpayers were attributed to the higher rate tax, 86.9% to the basic rate tax, and 2.5% to the starting rate tax.[18] This distribution has been relatively similar since 1993. From 1980 to 1993, there was no starting rate and therefore more than 93% of all taxpayers were basic rate taxpayers. Since 27 million Britons own Premium Bonds, which representing about 43% of the recent population, it is reasonable to assume that most bond holders pay the basic rate. On average, each saver possesses about £1,600 in Premium Bonds (calculated from March 2011 figures according to the NS&I Media Centre). In May 2006, NS&I published that more than

[17] We would like to thank Kristian Rydqvist for providing us with data on UK tax rates.

[18] Data on the distribution of UK taxpayers are taken from HM Revenue & Customs (http://www.hmrc.gov.uk/stats/income_tax/table2-1.pdf) downloaded 23 June 2012.

1.5 million people have deposited £5,000 or more, accounting for about 6.5% of all bond holders. The maximum investment of £30,000 was held by 300.000 people, 1.3% of all savers.

4.3.2. Data

The hand-collected data comprise 655 monthly prize draws from the first draw in June 1957 through December 2011. For each month, we have the prize breakdown, the underlying interest rate, the odds of winning, and the maximum individual holding cap. Furthermore, we also gained access to sales records, repayments and net sales from October 1969 to April 2006. To obtain a largely consistent sample period, we supplement the missing data on net sales with approximated values. We therefore estimate monthly net sales as difference between the corresponding total amounts invested in Premium Bonds by the end of each month. Since NS&I publish monthly data on the total prize fund value and the underlying interest rate, it is possible to derive the total number of eligible one-pound bonds (total amount invested). As a check, we compare the original NS&I provided data with the calculated net sales before April 2006. The average accuracy is more than 98%. Using this method, in total we obtain net sales from October 1969 until December 2011. This equals 507 monthly observations.

The Income Bond data contain all the interest rates commencing in July 1982, when the bond was initially launched, until December 2011. To make the savings accounts comparable, we identify the Income Bond interest rate at the beginning of each month, yielding 354 observations. We also collect the official Bank of England base rate at the beginning of each month from June 1957 to December 2011 (655 observations). Additionally, for a long-run analysis, we use 240 Bank of England UK nominal spot curves at the month's beginning (January 1979 till December 1998).

4.3.3. Short-run risk coefficients

Starting off with a myopic approach, we compute the value of α for each month from June 1957 until December 2011. Assuming that an investor deposits £1 and does not intend to get her principal refunded within or right after the time period, then her only concern is the monthly lottery winnings or the interest payments. Furthermore, our investor possesses no additional wealth which influences the CRRA utility function. This simple initial setting will be later extended. By iteration, we can calculate the indifference risk coefficient α. Knowledge of this figure over the whole time frame tells an investor ex post if the decision in favour of the Premium Bond has been utility maximizing or not, with respect to his individual degree of risk tolerance. By tracking the α-values over the full time period, we can assess which individual risk preferences savers need to exhibit in order to consider the Premium Bond an attractive way of saving money and how these change over the past decades.

Since this is the lengthiest data record available, we start with a virtual alternative investment which delivers interest payments equal to the official Bank of England base rate. Our results are based on 655 values in three of the four tax classes. The starting rate tax class only comprises 264 observations because in some years no such tax is raised. Since the higher tax class covers a relatively broad range of tax rates in some years, we consistently use the lowest rate attributed to this class.[19] Panel A in Table 4.2 presents the summary statistics.[20] The results clearly indicate a major change in February 2009. Before this date, the indifference risk coefficients are considerably lower. In years such as 1977, the combination of a Premium Bond interest rate slightly exceeding the BoE rate and the advantage that prizes are tax-free

[19] In unreported results, we also analyse the top tax rates. In some cases rather extreme risk coefficients occur but our conclusions are similar.

[20] In some empirical studies on individual risk preferences, a popular approximation developed by Pratt (1964) is used to calculate the risk coefficients. We take the opportunity to compare our iteratively computed results with this approximation. In total, we conclude that Pratt's approximation and our method produce quite different values for the Premium Bond sample. Detailed results are available on request.

increases the expected utility to such a degree that a risk-averse investor with a CARA $\alpha <$ 0.017 would prefer the risk-carrying Premium Bond. Generally, for higher income taxpayers, an investment in the lottery bond becomes a lot easier attractive in terms of risk tolerance. The lower the individual taxation of an investor, the less risk-averse or more risk-seeking she needs to be. We further observe that between 1993 and 2008 volatility decreases and the trend goes towards risk neutrality due to a better controlled and thus relatively constant margin of interest. As a result, higher rate income taxpayers are still allowed to be risk-averse, however closed to risk neutrality. Although all the other taxpayers require some risk-seeking traits, the values of the CARA α are surprisingly close to risk neutrality during this time. Commencing in February 2009, the BoE base rate rapidly falls below the interest rate of the Premium Bond. Finally from October 2009 till December 2011, the BoE base rate is one third of the 1.50% interest rate paid by the Premium Bond. These circumstances cause that the lottery bond becomes attractive even to quite risk-averse investors. The CARA α of a higher rate taxpayer is, for instance, about 0.199 between January 2010 and December 2011.

Figure 4.2 presents the time series obtained from the CRRA analysis. Note that personal wealth is not included. The CRRA α coefficients are scattered from -0.10862 to 0.13360. The calculation shows that over time, the risk coefficients changes frequently depending on the interest spread between the Premium Bond and the Bank of England rate. While volatility is great until the mid-1990's, it steadily decreases until the sharp increase by the beginning of 2009. In general, the risk coefficients of the separate tax classes follow the same pattern. Before 2009, the Premium Bond interest rate has been adjusted regularly and kept on a fair level compared to the official base rate, which results in risk coefficients relatively closed to zero. As of December 2011, the parameter values of α lay between 0.09483 and 0.13360.

After this first examination, we now compare the results with a product which can be actually purchased – the NS&I Income Bond. Due to the aforementioned shortened data record, there

are no conclusions possible before 1982. The summary statistics are reported in Panel B of Table 4.2. The CARA risk coefficients vary between –0.00002 and 0.06382. In terms of relative risk aversion, we observe values between -0.07845 and 0.07862. Before 2009, the level of risk tolerance for high income taxpayers tends towards risk neutrality. On the other hand, the required degree of risk loving for basic and starting rate taxpayers also decreases in favour of investing in Premium Bonds. In general, both indifference lines converge more and more to the risk neutrality level. Similar to the results based on the BoE base rate, the recent adjustments of the interest rates cause considerable changes of the risk coefficients. Now even a tax-exempt investor may exhibit risk aversion. Comparing our results, we find that based on the Income Bond as an alternative investment one needs to be somewhat less risk-averse or a bit more risk-seeking in order to prefer the Premium Bond than based on the BoE base rate. The mean CRRA coefficient for higher rate taxpayers with the Income Bond as reference is 0.01536, the corresponding value with the BoE rate as reference amounts to 0.02240.

4.3.4. Inclusion of personal wealth and higher investment amounts

We now extend our initial calculations by assuming that an investor possesses additional wealth. As mentioned before, the current average amount invested in Premium Bonds is about £1,600. Thus, we now calculate the CRRA indifference risk coefficients with a £1,600 deposit. Since we lack detailed historic data, we compute equivalent values by adjusting this average deposit with the respective retail price index (RPI) for each month. The basis for the RPI is January 1987.[21] Hence, for example, £1,600 in December 2011 is equivalent to £80 in June 1957. This method makes sure that the assumed money invested is always consistent with the current price level. The situation is similar to the first setting, but we now also take

[21] All RPI data are taken from the Office for National Statistics. (http://www.ons.gov.uk/ons/datasets-and-tables/data-selector.html?cdid=CHAW&dataset=mm23&table-id=2.1) downloaded: 6 April 2013

into account the utility of additional wealth. As a proxy, we use the personal income per year, showing the effects on utility if one had a certain percentage of her yearly income invested. For each tax class, we assume a representative amount of wealth. Inferences on that are drawn by analysing the income tax allowances and the bands for each tax class. The following values are used for our estimation: yearly income of a person who is not liable to tax £3,738, for a starting rate taxpayer £8,755, for a basic rate taxpayer £23,695, and finally £65,975 for a higher rate taxpayer.[22] Again, the income in each tax band is adjusted by the RPI. Panel A in Table 4.3 reports the results. We observe, in line with our previous findings, that measured by the median all investors except the higher income taxpayers need to be risk loving. The means are biased by the time period after 2008 and tend to indicate more risk aversion. The pattern of the indifference lines is equivalent to the simple case. However, now the values are quantitatively larger. The risk coefficients range from -1.77962 to 3.52642. This indicates that, on the one hand, at particular points in time even quite risk-averse savers are indifferent between Premium Bonds and a risk-free investment which yields the BoE base rate. On the other hand, starting rate and non-taxpayers have to be more risk-loving. Comparing the monthly results shows that the higher the tax rates, the higher the risk coefficients are relative to the results without wealth and higher investment amounts. In unreported results, we redo the analysis based on the Income Bond. As expected, the risk coefficients are not significantly different.

Next, we assume that an investor always keeps the highest possible investment. We still use the same time adjusted wealth as before. In the past fifty-four years, the maximum holding was increased in five steps. From June 1957 to March 1964, savers were allowed to hold a maximum of £500. In April 1964, the limit was increased to £1,000. Two further increases followed in April 1980 (to £10,000) and March 1993 (to £20,000). Since May 2003, the

[22] For instance, we estimate the personal wealth of a starting rate taxpayer according to the formula: allowance (person under 65 years) + mean of tax band (here 0-£2,560) = £7,475 + (£2,560/2) = £8,755

maximum holding amount has been £30,000. The results are reported in Panel B of Table 4.3. We find that the distribution of the values is not as broad as in the previous case with a relatively small amount invested. The risk tolerance of higher rate taxpayers declines but still allows being risk-averse. The remaining taxpayers require a slightly lower degree of risk loving. Due to the higher stakes, it is logical that investors have to take on more risks.

4.3.5. Long-term analysis

We next extend the examination to time horizons beyond one month. We look at an individual who intends to invest a lump sum at a particular point of time for several years. The first choice is to buy a risk-free bond with a fixed interest rate depending on the current interest rate level. There are no coupon interest payments during the investment period (zero-coupon bond). Hence, the investor collects all the interest and compounded interest at maturity. The CRRA utility is calculated from this final payment. To simplify matters, we only study the case of a non-taxpayer. As a reference for these calculations, we use the yield curve based on UK government bonds (gilts).[23] Employing this data, we identify the nominal spot rates for investments with investment periods between one month and 25 years. Since the yield curve records start in this year, we begin with January 1979. We assume that an individual invests £1 at the beginning of January 1979. Then we calculate the risk coefficients for three time horizons: twenty years with maturity at the beginning of January 1999, ten years with maturity in January 1989, and five years with maturity in January 1984. At the end of the maturity, the investor gets her principal refunded. For the calculation of the interest payment, we use the monthly discrete interest rates calculated from the compounding interest rates of the spot curves.

[23] Data on UK yield curves are taken from the Bank of England. (http://www.bankofengland.co.uk/statistics/yieldcurve/index.htm) downloaded: 30 August 2006

We construct the following investment strategy for the Premium Bond. The investor buys one bond worth £1 at the end of December 1978. This means that she will participate in the prize draw for the first time at the beginning of February 1979. Now she either wins a prize or not. If she wins, we assume that the prize is invested at the current spot rate exactly for the remaining time period till the beginning of 1999, 1989 or 1984. Then, for each prize in each draw, we calculate the value including interest at maturity.[24] This allows us to compute for each month the expected CRRA utility of the Premium Bond at maturity. For consistency reasons, the principal is refunded together with the last prize draw. To obtain one single indifference risk coefficient, we use the same α in all Premium Bond utility functions and in the utility function of the risk-free investment. The indifference value of α is determined by iteratively finding the value where the sum of all Premium Bond utilities and the utility of the risk-free spot rate investment becomes equal. The results are -0.15719 for the twenty years investment period, -0.12815 for the ten years horizon, and -0.12829 for five years. A further test with £1,000 wealth and £100 invested results -0.17899 for the twenty years maturity.

In total, the previous analyses suggest one potential reason why so many Britons invest in Premium Bonds. While the overall risk, measured by expected utility theory, is relatively small, savers still get a thrill from gambling. Depending on the individual tax rate and the current interest rate, even some risk-averse investors may find the lottery bond attractive.

4.4. Factors influencing net sales

We next try to identify factors that explain the development of net sales over time. The basis for our analysis is monthly data on net sales from October 1969 to December 2011. Before we start our quantitative analysis, we would like to point out an unquantifiable but certainly

[24] We estimate monthly spot rates using the Svensson-Nelson-Siegel approach (Nelson and Siegel, 1987; Svensson, 1994).

important factor: The Premium Bond design is straightforward. As a result, it appeals to virtually every household. In particular, this includes low-income families (Tufano, 2008).

4.4.1. Indifference risk coefficients

We start by examining whether Premium Bond net sales (variable *NETSALES)* were affected by CARA or CRRA coefficients (variables *CARA* and *CRRA*) changing over time. We take the time series of four indifference risk coefficients to clarify whether they have a short-run or long-run influence on net sales. We focus on the basic rate taxpayers representing the largest group. The other groups follow roughly the same pattern, gleaning rather similar results. To identify causal correlations, we employ Granger causality tests (Granger, 1969) allowing us to test whether, after controlling for past values of Y (e.g., *NETSALES*), past values of X (e.g., *CRRA*) help to forecast Y. One of Granger's crucial assumptions for testing causality is that the variables do not follow a distinct trend, implying they must be stationary. Because working with non-stationary variables can lead to spurious regressions and inferences, we first perform an augmented Dickey-Fuller test (ADF test) (Dickey and Fuller, 1979) to discover if the data have unit roots attesting non-stationarity. The variable *NETSALES* is stationary across the time period October 1969 till December 2011. The t-statistic amounts to -3.913 (p-value: 0.002). Conducting this test on all risk coefficient time series shows that these variables are stationary. We thus need not to proceed with first differences, which is a common way of dealing with non-stationary time series. If there is only one unit root in a variable, differencing once generates a stationary time series. However, by doing that, we can only observe the changes in the variables and we lose information included in the levels.

The Granger causality test works like this. First, we test the null hypothesis that the risk coefficient (e.g., *CRRA*) does not Granger-cause *NETSALES*. Therefore, we use an autoregressive model:

Unrestricted regression: $NETSALES_t = \eta_0 + \beta_1 NETSALES_{t-1} + \beta_2 CRRA_{t-1} + \varepsilon_t$ (4.4)

Restricted regression: $NETSALES_t = \eta_0 + \beta_1 NETSALES_{t-1} + \varepsilon_t$ (4.5)

The first regression expresses that *NETSALES* in t depend on *NETSALES* in t-1 and on *CRRA* in t-1. The error term ε_t has an expected value of zero. The second regression is for the significance test. In this equation, the influence of *CRRA* is set to zero. To make sure that there only exists a unidirectional causality, we also test if *NETSALES* Granger-cause *CRRA*. Thus,

Unrestricted regression: $CRRA_t = \eta_0 + \beta_1 CRRA_{t-1} + \beta_2 NETSALES_{t-1} + \varepsilon_t$ (4.6)

Restricted regression: $CRRA_t = \eta_0 + \beta_1 CRRA_{t-1} + \varepsilon_t$ (4.7)

Table 4.4 reports the results with one, three and six lagged months. Past values of the risk coefficients do not help to forecast net sales. Further tests with extended time horizons yield negative results as well. According to our results, in some cases, net sales Granger-cause the risk coefficients. However, there is no meaningful explanation for this.

4.4.2. Analysing the Premium Bond net sales time series

We continue by simply investigating the striking peaks in Premium Bond net sales. First of all, the size of the jackpot seems to be very important. This is in line with theory which suggests that people generally overestimate the very low probability of winning the jackpot (Camerer and Kunreuther, 1989). Interestingly, individuals especially seem to perceive the amount of one million as an important psychological threshold. Although NS&I increased the size of the jackpot six times before 1994, the introduction of the £1 million jackpot in April 1994 marks the first boom in net sales and the cornerstone of the tremendous success in the following decade. The fact that net sales already jumped in February 1994 suggests that the introduction of the £1m jackpot has been pre-announced and many investors made sure to participate in the first draw. The second major jump in net sales in May 2003 can be attributed to the increase of the maximum holding from £20,000 to £30,000. Apparently, many people

grabbed the chance to place additional funds in Premium Bonds. The third peak in net sales occurred in August 2005, when NS&I introduced a second million as a jackpot. In December 2006 and June 2007, six extra £1 million prizes were given away in two special draws. Each draw attracted a massive amount of net sales. Apparently, not only the size of the jackpot is relevant but also the number. Interestingly, individuals obviously do not consider the purchasing power of the prizes. The nominal £1 million of June 2007 was worth £1.43 million expressed in April 1994 pounds. This means that the actual purchasing power of the prize declined substantially since then. One may think that this would make the first prize less attractive, but the facts prove otherwise. So at first glance, it seems that the size and the number of first prizes as well as the maximum individual holding cap play a significant role. We will further analyse these determinants in section 4.5.

4.4.3. Interest rate

We next test an obvious determinant like the Premium Bond interest rate. We use two different time series, the absolute interest rate and the relative interest rate compared to the NS&I Income Bond introduced in section 4.3.1. We again perform Granger tests. The sample periods are October 1969 till December 2011 for the absolute interest rate and July 1982 till December 2011 for the relative interest rate. Both variables are non-stationary, so we use first differences. For consistency reasons, we also use first differences of net sales. The results using multiple lag lengths suggest that the absolute and relative interest rates of the Premium Bond do not Granger-cause net sales. Interestingly, although our tests cannot prove a direct statistical relation, recent developments suggest that Premium Bond holders are, to some degree, interest-sensitive. Between October 1969 and January 2009, the BoE base rate averaged 8.52% while the Premium Bond interest rate averaged only 5.25%. In 463 out of 472 months, the BoE base rate exceeded the Premium Bond interest rate. Apparently, when the reference rate is sufficiently high, holders are willing to forgo a certain part of risk-free

interest in order to participate in the lottery. This attitude obviously changes when the reference rate, and as a consequence the Premium Bond interest rate, becomes too low. Since April 2009, the BoE base rate has been 0.50%. Although the Premium Bond interest rate only fell to 1% and a new £25 prize was introduced, bond holders began to withdraw funds. NS&I eventually reacted in October 2009 and increased the Premium Bond interest rate to 1.5%. As a result, net sales instantly returned to the positive range. The way bond holders obviously do and do not accept certain interest rates supports Tufano (2008) who finds that Premium Bonds have both savings and gambling elements.

4.4.4. Macroeconomic variables

We discussed the potential influence of macroeconomic variables with experts of NS&I. Therefore, we compare, among others, the development of the FTSE100 and the UK unemployment rate with Premium Bond net sales. In no case Granger causality tests can prove a clear statistical link.

4.4.5. Cumulative prospect theory

Premium Bond holders seem to overweight the probability of winning the jackpot. This anomaly from the expected utility theory suggests that cumulative prospect theory (CPT) (Tversky and Kahneman, 1992) may possibly explain the success story of the product. One of the main assumptions is that individuals use an inverse s-shaped weighting function to transform objective probabilities. As a result, extreme outcomes are overvalued. We calculate the CPT valuation for each draw from October 1969 to December 2011 using the original model constructed by Tversky and Kahneman (1992). In the following, we briefly introduce the model.

Consider a gamble with $m + n + 1$ monetary outcomes $x_{-m} < \cdots < x_0 = 0 < \cdots x_n$. The corresponding probabilities of occurrence are $p_{-m}, \ldots, p_n$. Therefore the prospect f is defined

by $f = (x_i, p_i),\ -m \leq i \leq n$. Investors evaluate the contribution of gains and losses to their subjective utility differently, which leads to the following definition:

$$V(f) = V(f^+) + V(f^-), \tag{4.8}$$

where

$$V(f^+) = \sum_{i=0}^{n} \pi_i^+ v(x_i)\,,\ V(f^-) = \sum_{i=-m}^{0} \pi_i^- v(x_i). \tag{4.9}$$

The expression $V(f^+)$ measures the subjective utility of gains. $V(f^-)$ measures the subjective utility of losses, respectively.

The decision weights for gains $\pi_i^+(f^+) = (\pi_0^+, \dots, \pi_n^+)$ and losses $\pi_i^-(f^-) = (\pi_{-m}^-, \dots, \pi_0^-)$ are defined by:

$$\pi_n^+ = w^+(p_n),\ \ \pi_{-m}^- = w^-(p_{-m}), \tag{4.10}$$

$$\pi_i^+ = w^+(p_i + \cdots + p_n) - w^+(p_{i+1} + \cdots + p_n), 0 \leq i \leq n-1, \tag{4.11}$$

$$\pi_i^- = w^-(p_{-m} + \cdots + p_i) - w^-(p_{-m} + \cdots + p_{i-1}), 1-m \leq i \leq 0, \tag{4.12}$$

Tversky and Kahneman (1992) use the following probability weighting functions

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}, \quad w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}, \tag{4.13}$$

satisfying $w^+(0) = w^-(0) = 0$ and $w^+(1) = w^-(1) = 1$.

They further propose the strictly increasing value function

$$v(x) = \begin{cases} x^\propto\ \ if\ x \geq 0, \\ -\lambda(-x)^\beta\ \ if\ x < 0, \end{cases} \tag{4.14}$$

satisfying $v(x_0) = v(0) = 0$. The parameter λ is the loss-aversion coefficient. By conducting experiments, Tversky and Kahneman (1992) estimate the following parameters $\alpha = \beta = 0.88,\ \lambda = 2.25,\ \gamma = 0.61,\ \delta = 0.69$. We use the same parameters for our analysis. To exclusively measure the influence of the prize structure, we assume that the alternative investment offers exactly the same interest rate as the Premium Bond. Therefore, the monetary outcomes are the respective Premium Bond prizes minus the foregone interest payment of the alternative investment. Any valuation $V(f) > 0$ indicates that an investor with

the given set of individual preferences would prefer holding the Premium Bond rather than the alternative investment.

The CPT valuations over the time period October 1969 to December 2011 range from 0.131 to 0.541 and average 0.348, assuming that an investor holds one Premium Bond (£1). Despite the fact that the Premium Bond is obviously considered more attractive than the alternative investment at any time, Figure 4.3 suggests that CPT has difficulties to explain the impressive increase in net sales. The negative correlation (ρ = -0.263, t-statistic = -6.132) contradicts the hypothesis that more and more savers decided in favour of the Premium Bond because they gained attractiveness in terms of valuation based on CPT. Granger causality tests using first differences and lag lengths of 1, 3, and 6 months indicate that past changes in CPT valuation do not generally help to forecast changes in net sales. The f-statistics amount to 0.405 (lag 1), 1.544 (lag 3), and 0.105 (lag 6).

4.4.6. Prize skewness

Previous research on lottery design and gambling argues that the higher moments of the prize distribution are relevant. In unreported tests, we analyse the influence of the prize distribution variance, however we cannot prove the frequently discussed importance (Walker and Young, 2001). The time series show that with the introduction of the £1 million jackpot, the prize variance rose dramatically. In spite of the continuous decline in the following years, net sales expanded rapidly.

Literature also argues that individuals find strongly asymmetric payoffs appealing. Hence, the third moment of the prize distribution is also often considered crucial (e.g., Golec and Tamarkin, 1998; Garrett and Sobel, 1999). Therefore, we test in particular the prize skewness as a factor favouring the decision to purchase and hold Premium Bonds. In the first prize draw in June 1957, NS&I gave away prizes between £25 (19,590 times) and £1,000 (96 times). In the last fifty-four years, the distribution of prizes has been adjusted from time to time

resulting in a change of the prize skewness. For example, NS&I raffled 1,721,067 times £25 and one £1 million in the prize draw in December 2011. This design follows what behavioural theory stipulates: a lottery should offer a large number of small prizes to reduce holder's fatigue from the low likelihood of winning. On the other hand, it should also offer a small number of very large prizes (creating skewness) to keep the thrill (Shapira and Venezia, 1992) and allow individuals to dream (e.g., Forrest et al., 2002). The variable *SKEWNESS* is derived by

$$SKEWNESS_t = \frac{\sum_{i=1}^{n} p_{i,t}(x_i - \bar{x})^3}{\left(\sum_{i=1}^{n} p_{i,t}(x_i - \bar{x})^2\right)^{\frac{3}{2}}} \tag{4.15}$$

where $p_{i,t}$ is the probability of winning a prize of class i in the month t, n is the total number of prize classes, x_i is the value of the prize in class i and $\bar{x}$ is the expected prize. Figure 4.4 shows the time series of *SKEWNESS* and *NETSALES* from October 1969 to December 2011. The pattern suggests that S*KEWNESS* is positively correlated with *NETSALES* in the long-run. The correlation coefficient ρ is 0.494 and significant at the 1% level.

In the short-run, there are obviously exceptions from this correlation. However, if we utilize rolling averages in order to smooth out spikes, the correlation becomes stronger. The correlation coefficient ρ based on six, twelve and twenty-four months is 0.672, 0.748, and 0.847. Each is significant at the 1% level. To test for causality, we apply the Granger test once more. As already mentioned, the variables must not have a unit root. The ADF test on the variable *SKEWNESS* produces a t-statistic of -0.641 (p-value: 0.991), indicating one unit root. We rule out the presence of a second unit root and hence continue with first differences. Table 4.5 reports the results of the Granger test with several lag lengths. Besides the results of the test using a 4 months lag, changes in skewness do not directly Granger-cause net sales. It seems reasonable that small changes in the distribution of prizes, only causing marginal

changes in skewness, have no direct effects on net sales because investors do not recognise them. Big jumps, related with the introduction of a very high prize, for instance, are salient enough to be publicly recognised. Additionally, these events are usually accompanied by considerable marketing effort. It rather seems that the overall skewness level is more important than discrete changes. To further investigate the assumed long-term relationship, we perform a simple univariate regression analysis. Since only the variable *SKEWNESS* is a non-stationary time series, OLS is valid. The dependent variable is *NETSALES*, the independent variable *SKEWNESS*. The regression includes 507 monthly observations from October 1969 till December 2011. The coefficient of *SKEWNESS* is positive and significant at the 1% level. The Newey and West (1987) t-statistic amounts to 12.77 (p-value: 0.00) and the adjusted R-squared of the model is 0.244.

Although our results suggest that net sales increase with prize skewness, this does not hold true each time the number of jackpots was increased. In this case, skewness dropped but it still led to peaks in net sales in the month of the introduction and to increased net sales in the following months. We will further investigate this fact in the next section.

4.5. Regression analysis

In the following section, we construct regression models building on previous results. We analyse the NS&I provided and supplemented data covering the time period October 1969 to December 2011. The dependent variable is net sales (*NETSALES*). Note that this variable measures two investor decisions at the same time. The first one is the decision to buy new or additional bonds. The second one is to sell them. Considering these two different kinds of decisions, net sales is well-suited to analyse especially the influence of prize skewness. According to theory, numerous small prizes are supposed to prevent savers from selling. A very large jackpot has the same effect, but also motivates savers to buy the bond. Therefore, the skewness of the prize distribution should thus have positive effects on net sales.

Our previous analysis supports the assumption that besides the skewness of the prizes, the maximum holding is a factor influencing net sales. We denote the two variables *SKEWNESS* and *MAXINVEST*. As the traditional risk coefficients proved inessential, we exclude them. Due to multicollinearity, we exclude a manifest factor like the value of the first prize. Since skewness is calculated from this figure, the regression would be biased. As discussed before, although investors actually seem to prefer skewed prizes, this general statement proves to be incorrect for changes in the number of the jackpots. To model this phenomenon, we construct a variable denoted *NUMJACKPOTS*, which is equal to the total number of monthly first prizes. The other previously tested factors do not seem to have decisive influence and, furthermore, as tests show, do not improve the quality of the regression models. We therefore restrict our regressions to the three most salient influencing factors. A detailed analysis of the peaks in the time series suggests that Premium Bond investors strongly react to changes in major attributes of the program. One reason may be that changes are broadly published by NS&I and attract substantial media attention. We take account of this behaviour by using first differences of the variables *MAXINVEST* and *NUMJACKPOTS.*

Net sales have been relatively steady until the end of 1993. When NS&I introduced the £1 million top prize, investor demand considerably changed. Consequently, the parameters of the model changed as well. In the following, we split the sample period. The first period ranges from October 1969 to September 1993, shortly before the introduction of the £1 million top prize. The second period covers October 1993 to April 2006, which marks the end of the NS&I provided data. The third period analyses the approximated data on *NETSALES* and runs from May 2006 till December 2011.

4.5.1. Period 1: October 1969 to September 1993

We start with the first period and perform ADF tests. The results indicate that *NETSALES* and *SKEWNESS* are non-stationary (p-values: 0.989 and 0.496). We therefore estimate a

regression with first differences (*D(...)*) of these variables. To control for serial correlations, we use autoregressive processes. An autoregressive model of order p is denoted by AR(p) expressed by the following equation:

$$y_t = \eta + \sum_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t \tag{4.16}$$

According to the Akaike and Schwarz information criteria, an AR(3) model fits best. We again use the Newey and West (1987) method for heteroscedasticity consistent errors and covariance in order to minimize the problem of heteroscedasticity. The results are reported in Table 4.6. The model, denoted "Period 1", includes 284 monthly observations. The adjusted R-squared is 0.249. The coefficient of *D(SKEWNESS)* is not significant (p-value: 0.906). This is in line with the Granger causality tests in the previous section suggesting that there is no short-term relationship between net sales and skewness. As discussed before, we suppose that savers do not perceive small changes in skewness and rather find the total distribution attractive. The variable *D(NUMJACKPOTS)* is not significant, too. This is reasonable because in this period, the number of first prizes was set to one most of the time. Before August 1971, four and five first prizes were alternately given away. Since the value was only £25.000, these changes apparently were too insignificant to affect net sales. With respect to the maximum holding, we find that this variable does influence net sales. *D(MAXINVEST)* has a significant coefficient with a positive sign. The first increase of the maximum holding cap from £1.000 to £10.000, and the second one to £20.000 clearly caused increases in net sales.

4.5.2. Period 2: October 1993 to April 2006

We repeat the previous steps and construct another regression model for the time period October 1993 to April 2006 after demand began to shoot up. We denote the model "Period 2". The ADF tests indicate that *NETSALES* and S*KEWNESS* are now stationary (p-values: 0.019 and 0.036), which eliminates the need to use first differences. Several tests suggest that an

AR(1) model fits best. We again use robust standard errors in order to control for heteroscedasticity. Table 4.6 shows that all variables have a positive sign and are significant at the 1%-level. Although the model looks fairly simple, according to adjusted R-squared, it can still explain 75.7% of the variance. The results suggest that net sales are positively influenced by prize skewness. They are also affected by changes in the maximum holding and changes in the number of the top prizes. We additionally find that obviously most investors did not anticipate these changes. Net sales peaked in the month the change occurred, which means that these newly bought Premium Bonds did not participate in the draw.

4.5.3. Period 3: May 2006 to December 2011

As mentioned at the beginning, we have NS&I provided data available until April 2006. However, in the five years after April 2006, the Premium Bond experienced some very interesting developments. There have been two anniversary specials draws each raffling five times £1 million. Additionally, NS&I introduced a new £25 prize class in April 2009 and reduced the number first prizes from two to one. These events caused considerable changes in the variables *SKEWNESS* and *NUMJACKPOTS*. As described in section 4.3.2, we approximate monthly net sales in order to cover this interesting time period. The dependent variable *NETSALES* is stationary (ADF p-value: 0.00) in this time period. We again include the variable *SKEWNESS*. *D(NUMJACKPOTS)* is supposed to capture the effects of the two special draws. Interestingly, we find that now investors did anticipate these events. As NS&I promoted the 50th anniversary of Premium Bonds with a major TV advertising campaign, investors were well-informed about the forthcoming special draws. We take this fact into account by considering a two-month lead. The variable is denoted *D(NUMJACKPOTS(t+2)).* It is not clear, unfortunately, if savers did or did not anticipate the cut of the second £1 million top prize in April 2009. We therefore prefer a parsimonious model and stick to only one variable. Unlike in the previous two periods there was no change of the maximum holding.

We thus exclude *D(MAXINVEST)*. We choose an AR(3) model and again use robust standard errors. Table 4.6 presents results (denoted "Period 3") estimated based on 68 monthly observations. The adjusted R-squared amounts to 0.449 and is significantly lower than in the second model. The signs of the two explanatory variables are positive and significant at levels of 1%. So the results again indicate that net sales are influenced by the skewness of the prize distribution and the number of first prizes. We note that in unreported results, we test vector autoregressive regressions (VAR) and vector error correction (VEC) models.[25] We obtain no better results than using the autoregressive models presented above.

4.5.4. Forecast tests

In a last step, in-sample forecast tests should help us uncover how well the models work. Since we need all the observations for appropriate parameter estimation, we cannot perform out-of-sample forecast tests. There are two different kinds of in-sample forecasts: static and dynamic forecasts. The static forecast is a sequence of one-step-ahead forecasts. Each month the actual value of the lagged dependent variable is used for the autoregressive term. In the dynamic procedure, the forecasted lagged dependent variables determine the current forecast. The estimations thus become inaccurate the longer the forecasting sample. We evaluate the forecasting accuracy of each model based on the respective time period used for training. Table 4.7 performs in-sample static and dynamic forecasts for each model. The Theil inequality coefficients, especially of the first two models, are relatively close to zero indicating quite accurate forecasts of next month's net sales. Interestingly, the model of the third period performs considerably worse. One reason may be that the first anniversary special draw in December 2006 attracted much more funds than the model predicted. The dynamic

[25] Prize skewness is no entirely exogenous variable. High net sales increase the total number of prizes. Since the number of top prizes is usually fixed, the additional prizes are distributed into the remaining classes. As a result, prizes are slightly more skewed.

forecasts are, as expected, less accurate than the static forecasts. The model of "Period 2" (October 1993 to April 2006) generates the best forecasts. Generally, we conclude that while the static forecasts work quite well especially until 2006, dynamic forecasts provide only a rough estimation. The autoregressive process utilises information contained in net sales and considerably increases model accuracy. This fact suggests that besides the tested variables, Premium Bond net sales depend on a broad variety of further factors. One very important but unquantifiable aspect certainly is the popularity and the mainstream fame the Premium Bonds have gained over the past five decades.

4.6. Conclusion

The objective of this paper is to conduct an empirical analysis of the British Premium Bond. What prompts so many investors to buy and hold a lottery bond with overall risky payouts? In the first step, we calculate the CARA and CRRA risk coefficients at which a saver is indifferent between the Premium Bond and a risk-free investment. A central issue is the discrimination of the different tax classes. Premium Bond prizes are tax-free, making them more or less attractive for certain taxpayers. Basically, we find that the indifference risk coefficients are surprisingly close to risk neutrality and the Premium Bond turns out to be not especially risky using conventional measures. To search for factors that influence net sales, we conduct Granger causality tests. Interestingly, CARA and CRRA risk coefficients have no statistical influence on net sales. We also find that cumulative prospect theory rather explains single peaks in sales than the overall increase. We show that in the short-run, only major changes of prize skewness, such as the introduction of a new first prize, encourage net sales. However, there is evidence of long-run relationships. Using multivariate autoregressive models, we confirm the influence of skewness on net sales. We additionally establish that changes in the maximum holding cap led to jumps in net sales. Our analysis also reveals that not only the size of the jackpot affects net sales, but also the number. This is true although

skewness declines. A strong plus of accuracy originating from the autoregressive processes suggests that Premium Bond net sales additionally depend on factors such as marketing and popularity. Future research could try to confirm our results on the importance of the prize structure based on a quite similar lottery-linked deposit account with a long data record, the Irish Prize Bonds. By the end of 2007, the Prize Bond Company introduced a new prize structure and the monthly jackpot increased to €1 million. In the Annual Report 2007 (p. 3) they state: "*The change was generally welcomed and resulted in greatly increased sales during the last quarter of 2007.*"

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Table 4.1: Number and value of prizes awarded in December 2011.
This table illustrates the details of the December 2011 prize draw as an example.

	Lower value 89%			Medium value 5%		Higher value 6% of prize fund					
Prize £	25	50	100	500	1,000	5,000	10,000	25,000	50,000	100,000	1,000,000
Number	1,721,067	31,544	31,544	3,216	1,072	89	45	17	10	4	1

Total prize fund value	Number of prizes	Interest rate p.a.
£ 53,658,25	1,788,609	1.50%

Table 4.2: Premium Bond compared to alternative investments.
The table reports results of iteratively determined constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA) indifference risk coefficients α. The hand-collected data comprise 655 monthly prize draws from the first draw in June 1957 through December 2011. The invested amount is £1. The reference investments are the Bank of England base rate (Panel A) and the NS&I Income Bond (Panel B). The analysis distinguishes between four income tax bands: no tax, starting rate, basic rate, and higher rate. Positive (negative) values of α indicate risk aversion (risk seeking) across time. A zero value means risk neutrality. Savers who are less risk-averse or more risk-seeking than the indifference level will choose the Premium Bond since this maximises their utility.

α	Mean	Median	StdDev	Maximum	Minimum	N
Panel A: Bank of England base rate as alternative investment						
CARA (no tax)	0.00409	-0.00002	0.01835	0.08979	-0.00070	655
CARA (starting rate)	0.01211	0.00000	0.03191	0.10295	-0.00005	264
CARA (basic rate)	0.00742	0.00000	0.02446	0.11883	-0.00018	655
CARA (higher rate)	0.01296	0.00156	0.03397	0.16401	-0.00011	655
CRRA (no tax)	-0.03337	-0.03729	0.03777	0.09483	-0.10862	655
CRRA (starting rate)	-0.00416	-0.02048	0.04275	0.10308	-0.06895	264
CRRA (basic rate)	0.00279	-0.00160	0.03622	0.11215	-0.07120	655
CRRA (higher rate)	0.02240	0.01789	0.03715	0.13360	-0.05553	655
Panel B: NS&I Income Bond as alternative investment						
CARA (no tax)	0.00013	0.00000	0.00153	0.01766	-0.00002	354
CARA (starting rate)	0.00054	0.00000	0.00277	0.02602	-0.00002	240
CARA (basic rate)	0.00085	0.00000	0.00372	0.03608	-0.00002	354
CARA (higher rate)	0.00446	0.00180	0.00891	0.06382	-0.00001	354
CRRA (no tax)	-0.03412	-0.03237	0.01807	0.03621	-0.07845	354
CRRA (starting rate)	-0.01132	-0.01466	0.01350	0.04525	-0.03947	240
CRRA (basic rate)	-0.00682	-0.00548	0.01519	0.05517	-0.04678	354
CRRA (higher rate)	0.01536	0.01621	0.01605	0.07862	-0.02308	354

Table 4.3: Premium Bond compared to Bank of England base rate with inclusion of personal wealth and higher investment amounts.

The table reports results of iteratively determined constant relative risk aversion (CRRA) indifference risk coefficients. The hand-collected data comprise 655 monthly prize draws from the first draw in June 1957 through December 2011. The invested amount is £1,600 (Panel A) and the maximum holding of £30,000 (Panel B). The analysis distinguishes between four income tax bands: no tax, starting rate, basic rate, and higher rate. For each tax class, a representative amount of wealth is assumed: yearly income of a person who is not liable to tax £3,738, for a starting rate taxpayer £8,755, for a basic rate taxpayer £23,695, and finally £65,975 for a higher rate taxpayer. All values are adjusted by the respective retail price index (RPI) for each month. The reference investment is the Bank of England base. Positive (negative) values indicate risk aversion (risk seeking) across time. A zero value means risk neutrality. Savers who are less risk-averse or more risk-seeking than the indifference level will choose the Premium Bond since this maximises their utility.

α	Mean	Median	StdDev	Maximum	Minimum	N
Panel A: Amount of £1,600 invested						
CRRA (no tax)	-0.07947	-0.10485	0.17587	0.61068	-1.77962	655
CRRA (starting rate)	0.06475	-0.07925	0.36584	1.07497	-0.18555	264
CRRA (basic rate)	0.13386	-0.00851	0.52428	2.39818	-0.29011	655
CRRA (higher rate)	0.48586	0.18884	0.84581	3.52642	-0.33669	655
Panel B: Maximum holding invested						
CRRA (no tax)	-0.05072	-0.06036	0.07744	0.23639	-0.67815	655
CRRA (starting rate)	0.00582	-0.03987	0.11883	0.32179	-0.10785	264
CRRA (basic rate)	0.02198	-0.00382	0.12272	0.48740	-0.14530	655
CRRA (higher rate)	0.11093	0.05843	0.21035	0.94356	-0.15188	655

Table 4.4: Granger causality tests of net sales and risk coefficients.

This table reports results of Granger causality tests between the CARA/CRRA indifference risk coefficients and Premium Bond net sales. The time period is October 1969 till December 2011. CARA denotes the indifference risk coefficients according to the constant absolute risk aversion. CRRA stands for constant relative risk aversion. The analysis distinguishes between two income tax bands (basic rate and higher rate). It considers a £1 investment without any further wealth as well as a £1,600 investment with £23,695 (basic rate tax) / £65,975 (higher rate tax) of wealth. The reference investment is the Bank of England base rate. The table reports results for lag lengths of 1, 3, and 6 months. ***,**,* values are significant at 1%, 5%, and 10%.

		H_0: X does not cause NETSALES		H_0: NETSALES does not cause X	
X	Lag length	F-statistic	p-value	F-statistic	p-value
CARA basic tax	1	0.0007	0.9795	3.1070	0.0786*
(£1 invested)	3	0.5691	0.6356	0.8781	0.4523
	6	1.1981	0.3058	4.5981	0.0001***
CRRA basic tax	1	0.0945	0.7587	0.0275	0.8683
(£1 invested)	3	0.1470	0.9316	0.3506	0.7887
	6	0.3679	0.8993	0.5061	0.8039
CRRA basic tax with wealth	1	0.0039	0.9504	1.5022	0.2209
(£1,600 invested)	3	0.2873	0.8346	0.3305	0.8033
	6	0.9911	0.4305	2.4009	0.0269**
CRRA higher tax with wealth	1	0.0015	0.9690	2.1054	0.1474
(£1,600 invested)	3	0.1736	0.9142	1.0177	0.3844
	6	1.4289	0.2016	1.5594	0.1571

Table 4.5: Granger causality tests of net sales and skewness.
This table reports results of Granger causality tests between Premium Bond net sales and prize skewness. The time period is October 1969 till December 2011. The analysis reports results for lag lengths between 1 and 6 months.
***,**,* values are significant at 1%, 5%, and 10%.

	H_0: D(SKEWNESS) does not cause D(NETSALES)		H_0: D(NETSALES) does not cause D(SKEWNESS)	
Lag length	F-statistic	p-value	F-statistic	p-value
1	1.9157	0.1669	51.4854	0.00***
2	0.7968	0.4514	80.7810	0.00***
3	2.0115	0.1114	53.8222	0.00***
4	8.7712	0.00***	42.6541	0.00***
5	1.5644	0.1685	33.4114	0.00***
6	0.4577	0.8395	25.2407	0.00***

Table 4.6: Multivariate autoregressive models.
This table presents multivariate autoregressive models, divided into three time periods. The dependent variables are *NETSALES* and *D(NETSALES)*. AR(p) is an autoregressive process of order p. The independent variables are prize skewness (*SKEWNESS*), its first difference *D(SKEWNESS)*, the first difference of the maximum holding *D(MAXINVEST)*, and the first difference of the number of jackpots *D(NUMJACKPOTS)*. The variable *D(NUMJACKPOTS(t+2))* considers a two-month lead. Numbers in parentheses are t-statistics computed using Newey-West (heteroscedasticity-adjusted) standard errors. ***,**,* values are significant at 1%, 5% and 10%.

Model	Period 1 Oct-69 to Sep-93	Period 2 Oct-93 to Apr-06	Period 3 May-06 to Dec-11
Dependent variable	D(NETSALES)	NETSALES	NETSALES
SKEWNESS		272,957***	177,996***
		(7.902)	(2.964)
D(SKEWNESS)	1,930.0		
	(0.181)		
D(MAXINVEST)	1,421***	51,135***	
	(3.627)	(10.451)	
D(NUMJACKPOTS)	289,115	233,226,206***	
	(1.124)	(7.163)	
D(NUMJACKPOTS(t+2))			233,158,323***
			(5.377)
AR(1)	-0.295***	0.782***	0.718***
	(-4.806)	(13.488)	(5.605)
AR(2)	-0.360***		-0.402***
	(-2.746)		(-2.717)
AR(3)	-0.293***		0.254*
	(-2.493)		(1.714)
Durbin-Watson stat	1.99	2.17	1.98
Adj. R-squared	0.251	0.757	0.499
N	284	151	68

Table 4.7: Forecast accuracy.
This table analyses the forecast accuracy of the multivariate autoregressive models introduced in Table 4.6. The analysis is divided into three time periods Period 1: October 1969 to September 1993, Period 2: October 1993 to April 2006, and Period 3: May 2006 to December 2011. The table performs two different kinds of in-sample forecasts: static and dynamic forecasts. The static forecast is a sequence of one-step-ahead forecasts. Each month the actual value of the lagged dependent variable is used for the autoregressive term. In the dynamic procedure, the forecasted lagged dependent variables determine the current forecast.

Model	Period 1 Oct-69 to Sep-93	Period 2 Oct-93 to Apr-06	Period 3 May-06 to Dec-11
Static forecast			
Root mean squared error	3,112,928	59,493,050	215,000,000
Mean absolute percent error	53.6	28.3	332.1
Theil inequality coefficient	0.153	0.137	0.349
Dynamic forecast			
Root mean squared error	8,366,202	95,246,589	267,000,000
Mean absolute percent error	235.8	48.1	378.1
Theil inequality coefficient	0.352	0.228	0.442

Figure 4.1: Interest rates of the Premium Bonds compared to the Bank of England base rate and the NS&I Income Bond.
This figure compares the interest rates of the Premium Bonds, the Bank of England (BoE) base rate, and the NS&I Income Bond over the time period from June 1957 to December 2011.

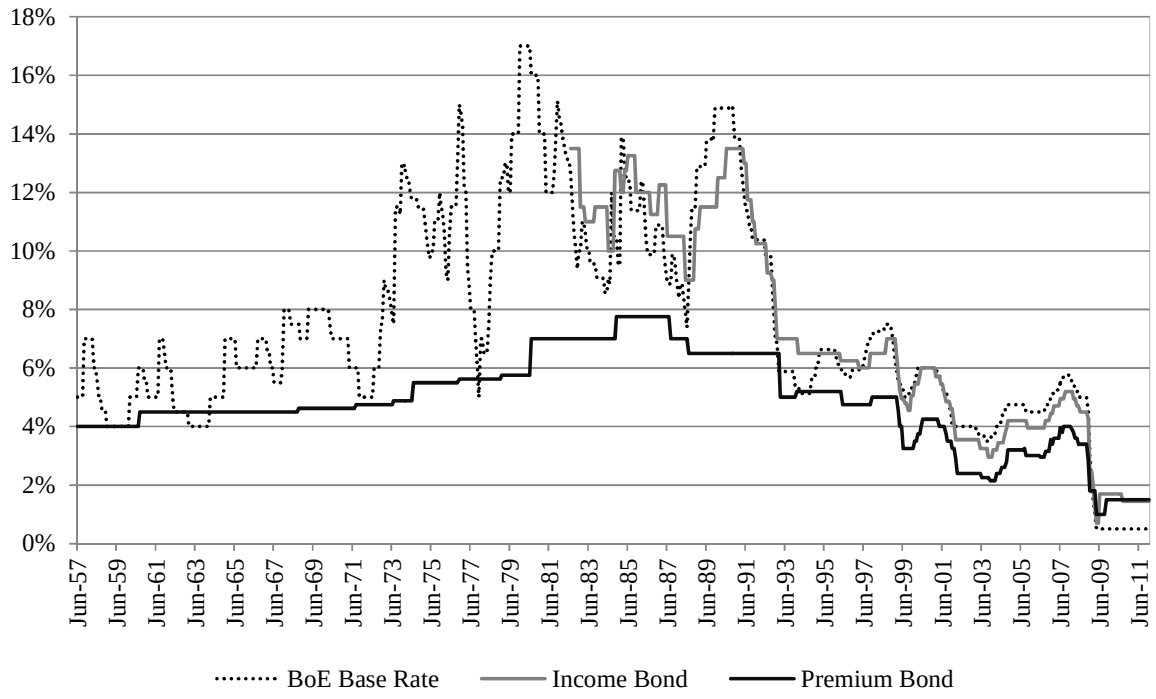

Figure 4.2: Indifference risk coefficients (CRRA) Premium Bond compared to Bank of England base rate.
This figure tracks the constant relative risk aversion *CRRA* indifference risk coefficients over the time period June 1957 to December 2011. The reference investment is the Bank of England base. The analysis distinguishes between four income tax bands: no tax *CRRA(0)*, starting rate *CRRA(S)*, basic rate *CRRA(B)*, and higher rate *CRRA(H)*. Positive (negative) values indicate risk aversion (risk seeking) across time. A zero value means risk neutrality. Savers who are less risk-averse or more risk-seeking than the indifference level will choose the Premium Bond since this maximises their utility.

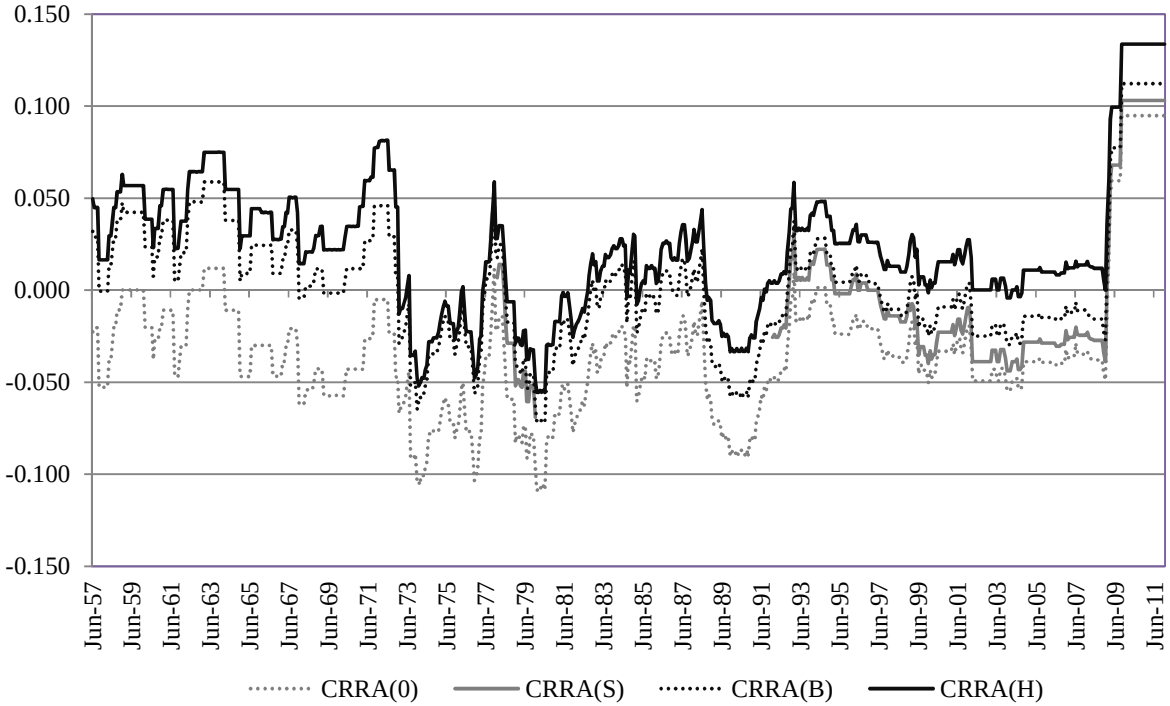

Figure 4.3: Valuation based on cumulative prospect theory compared with Premium Bond net sales.
This figure compares Premium Bond net sales and cumulative prospect theory (CPT) valuation over the time period October 1969 to December 2011. The CPT valuation is based on the theory formalized by Tversky and Kahneman in 1992. The analysis uses the originally estimated parameters $\alpha = \beta = 0.88$, $\lambda = 2.25$, $\gamma^{+} = 0.61$, and $\gamma^{-} = 0.69$.

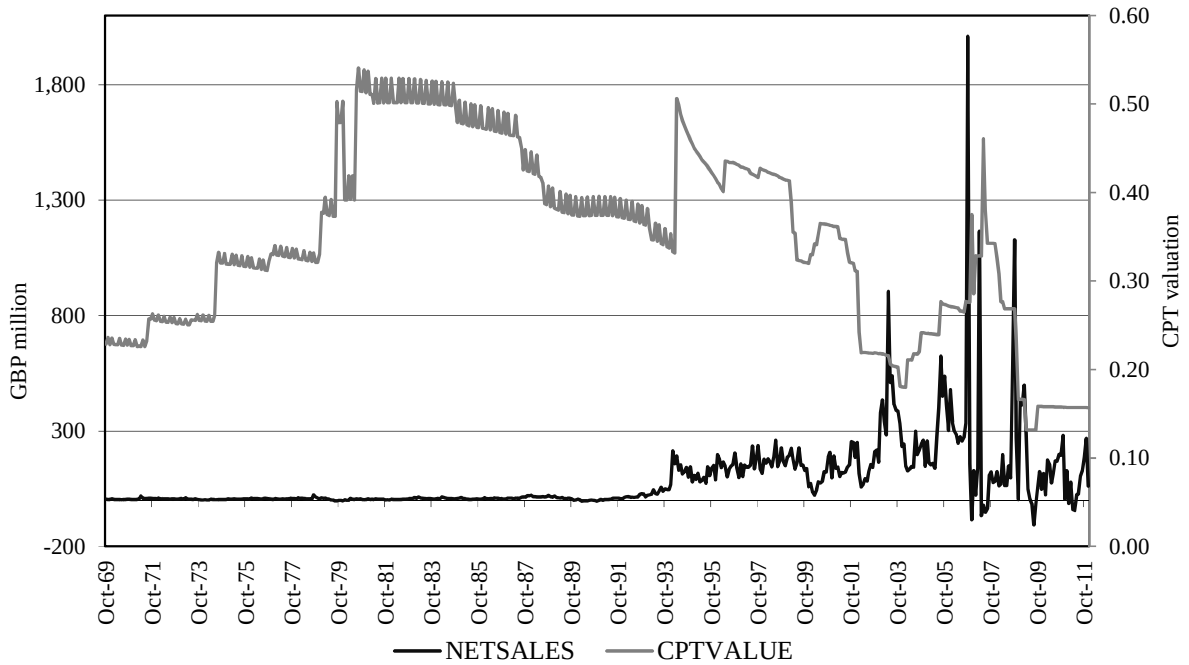
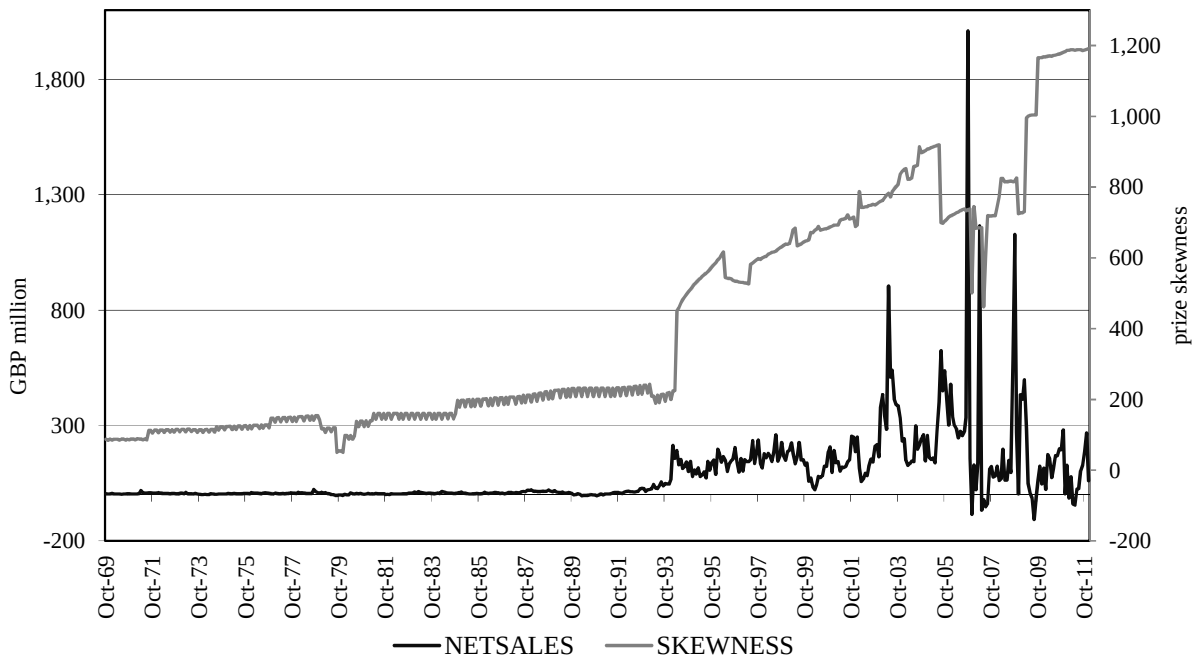
Figure 4.4: Prizes skewness compared with Premium Bond net sales.
This figure compares Premium Bond net sales and prize skewness over the time period October 1969 and December 2011.