Cubic Rashba Spin-Orbit Interaction of a Two-Dimensional Hole Gas in a Strained-Ge/SiGe Quantum Well

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The spin-orbit interaction (SOI) of a two-dimensional hole gas in the inversion symmetric semiconductor Ge is studied in a strained-Ge/SiGe quantum well structure. We observe weak antilocalization (WAL) in the magnetoconductivity measurement, revealing that the WAL feature can be fully described by the $k$-cubic Rashba SOI theory. Furthermore, we demonstrate electric field control of the Rashba SOI. Our findings reveal that the heavy hole (HH) in strained Ge is a purely cubic Rashba system, which is consistent with the spin angular momentum $m_j = ±3/2$ nature of the HH wave function.

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The spin-orbit interaction (SOI) in a two-dimensional system is a subject of considerable interest because the SOI induces spin splitting at a zero magnetic field, which is important in both fundamental research and electronic device applications [1]. Recent developments of SOI-induced phenomena in the solid state demonstrate many possibilities utilizing spin current and the emergence of new physics such as the spin interferometer [2,3], persistent spin helix [4,5], spin Hall effect [6–8], and quantum spin Hall effect [9,10]. Up to now, there have been two well-known SOIs existing in solids: the Dresselhaus SOI [11] due to bulk inversion asymmetry (BIA) in the crystal structure and the Rashba SOI [12,13] due to spatial inversion asymmetry (SIA).

In low-dimensional systems, the Rashba SOI becomes more important because it is stronger at the heterointerface and can be controlled by an external electric field. Many of the pioneering studies on the SOI-induced phenomena mentioned above were performed in two-dimensional electron systems, where the Rashba SOI is described by the $k$-linear Rashba term. In the Hamiltonian, the $k$-linear Rashba term can be written as $H_{R_k} = α_k E_j (k_x σ_+ − k_y σ_−)$, where $σ_{±} = 1/2 (σ_x ± iσ_y)$ denote combinations of Pauli spin matrices, $k_x = k_x + i k_y$, and $k_x$, $k_y$ are the components of the in-plane wave vector $k_{∥}$. The effective magnetic field $Ω_1(k_{∥})$ acting on the transport carrier due to the $k$-linear Rashba term is illustrated in Fig. 1(a).

Recently, a higher-order contribution of the Rashba SOI, the so-called $k^3$ ($k$-cubic) Rashba SOI, has received more attention [14,15]. The Hamiltonian for the $k$-cubic Rashba SOI is expressed as $H_{R_k} = α_k E_j (k_x^3 σ_+ − k_y^3 σ_−)$ [16], and the effective magnetic field $Ω_3(k_{∥})$ in $k$ space is illustrated in Fig. 1(b) [15]. There is a significant difference in the effective field symmetry between the $k$-linear and the $k$-cubic Rashba SOI with one and three rotations in $k$ space, respectively. The $k^3$ symmetry of the SOI is an interesting subject because it influences all of the SOI-induced phenomena as opposed to the $k$-linear Rashba term. For example, in case of the spin Hall effect, the $k$-cubic Rashba term is predicted to give rise to a larger spin Hall conductivity [17–19].

![FIG. 1 (color online). (a),(b) The effective magnetic field direction with respect to the in-plane wave vector $k$ for (a) the linear-Rashba term, $Ω_1$ and (b) the cubic-Rashba term, $Ω_3$. (c) Schematic illustration of the valence band structure forstrained Ge. The HH, LH, and SO bands are indicated.](image-url)
Recently, the cubic-Rashba SOI has been reported in a two-dimensional hole gas (2DHG) in inversion asymmetric semiconductors InGaAs and GaAs [20,21], and a two-dimensional electron gas formed at a surface of the inversion symmetric oxide SrTiO$_3$ [15]. However, in the former case, the InGaAs and GaAs possess both BIA and SIA; thus, they are always influenced by both Dresselhaus and Rashba SOI contributions [22]. Because the Dresselhaus SOI also has $k$-cubic dependence, the cubic-Rashba SOI is difficult to observe independently. In the latter case, the complex electric structure of SrTiO$_3$ makes it difficult to compare with the existing Rashba theory and theoretical development on the spin orbit interaction in the perovskite oxide is still in progress. Consequently, material systems with a simple band structure and pure cubic-Rashba SOI are in high demand. Here, we propose a purely cubic-Rashba SOI system: the single element semiconductor Ge. Because Ge does not have BIA, the Dresselhaus SOI is essentially absent. Thus, the SOI in Ge is described only with the Rashba term.

The valence band structure of strained Ge around the $\Gamma$ point is schematically illustrated in Fig. 1(c). The valence band hole has a $p$-orbital wave function with orbital angular momentum $l = 1$; therefore, it has total angular momentum $j = 3/2$ and $j = 1/2$. The $j = 3/2$ and $j = 1/2$ hole states are separated from each other due to the atomic SOI. The separation between the $HH$ and $LH$ bands can be as large as $\sim 100$ meV with increasing amounts of strain. Due to the lattice mismatch ($\sim 2.1\%$) between the Ge and Si$_3$Ge$_2$ layers, the Ge QW layer is compressively strained. After patterning the wafer in the Hall bar structure, an Al$_2$O$_3$ dielectric layer with a 75-nm thickness is deposited using atomic layer deposition, and then a Pd/Au gate electrode is deposited. The application of the gate voltage enables us to tune the hole density of the 2DHG in the QW. The magnetotransport is measured in a variable temperature cryostat at the temperature range of 1.6–10 K. A magnetic field is applied perpendicular to the sample plane using a superconducting magnet.

First, the change in the sheet resistance $R_{\text{sheet}}$ and hole concentration $n_{\text{hole}}$ at various back gate voltages $V_G$ is shown in Fig. 2(a). When $V_G$ is decreased from $+1.2$ to $-0.4$ V, $R_{\text{sheet}}$ ($n_{\text{hole}}$) monotonically decreases (increases). This change accompanies the variation of hole mobility

![Image](https://example.com/image.png)

FIG. 2 (color online). (a),(b) The $V_G$ dependence of (a) sheet resistance $R_{\text{sheet}}$ and hole concentration $n_{\text{hole}}$ and (b) the hole effective mass $m_{\text{hole}}$. (c) The change of magnetoconductivity $\Delta \sigma(B)$ measured at 1.6 K at various $V_G$. 

These findings are all consistent with the $m_j = \pm 3/2$ characteristics of the $HH$ band in a strained-Ge QW, which obeys the cubic-Rashba Hamiltonian.

A strained-Ge/SiGe single QW is grown by using solid source molecular beam epitaxy. First, a fully relaxed Si$_{0.5}$Ge$_{0.5}$ buffer layer is grown on a Si(001) substrate. On top of this layer, the following QW structure is grown: $10$ p-Si$_{0.5}$Ge$_{0.5}$/10 Si$_{0.5}$Ge$_{0.5}$/20 Ge/30 Si$_{0.5}$Ge$_{0.5}$/3 Si (units in nanometers). Due to the lattice mismatch ($\sim 2.1\%$) between the Ge and Si$_3$Ge$_2$ layers, the Ge QW layer is compressively strained. After patterning the wafer in the Hall bar structure, an Al$_2$O$_3$ dielectric layer with a 75-nm thickness is deposited using atomic layer deposition, and then a Pd/Au gate electrode is deposited. The application of the gate voltage enables us to tune the hole density of the 2DHG in the QW. The magnetotransport is measured in a variable temperature cryostat at the temperature range of 1.6–10 K. A magnetic field is applied perpendicular to the sample plane using a superconducting magnet.

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In this Letter, we experimentally reveal cubic-Rashba SOI in a 2DHG in a strained-Ge/SiGe QW. Comparing the weak localization (WL) and weak antilocalization (WAL) measurements with the analysis of the linear- and cubic-Rashba SOI terms, we found that spin transport in the 2DHG can be fully explained by the cubic-Rashba SOI. The SOI magnitude can be controlled with the gate voltage.
from 2000 to 5200 cm²/Vs. The change in the hole effective mass \( m_{\text{hole}} \) with respect to \( V_G \) is shown in Fig. 2(b). The effective mass of the holes is determined from the temperature dependence of the Shubnikov–de Haas oscillation. The effective mass increases with the hole concentration and this is due to the nonparabolicity of the valence band [25,26].

Next, we show the change of magnetoconductivity \( \Delta \sigma(B) = \sigma(B) - \sigma(0) \) measured at various \( V_G \) in Fig. 2(c). Under a gate sweep of \( V_G = +1.0 \) to \(-0.4 \) V, \( \Delta \sigma(B) \) is significantly modulated. In the high-field region, the conductivity monotonically increases with the external magnetic field, which we attribute to the WL. Together with the WL feature, in the high hole density region \( (V_G < +0.2 \) V) a conductance peak appears when the magnetic field is near zero. This conductance peak is due to the WAL. The cross-over points of \( \Delta \sigma(B) \), which are indicated by arrows in Fig. 2(c), shift position from low magnetic field to high magnetic field with decreasing \( V_G \). The cross-over points are roughly proportional to the magnetic field for the SOI, \( B_{\text{SO}} \); a larger \( B_{\text{SO}} \) produces a larger cross-over point field. Thus, our results suggest that the SOI can be tuned with the gate voltage.

Observed WL and WAL data have been analyzed by using the Iordanskii–Lyanda-Geller–Pikus (ILP) theory for magnetoconductivity, which took into account both the linear- and cubic-Rashba SOI [15,20,27]. The general formula of the ILP theory is provided in the Supplemental Material [28]. We limited our fitting to within the diffusive transport regime where the magnetic field \( B \) is within the range \( B < B_{\text{tr}} = \hbar/2eI_m^0 \), where \( B_{\text{tr}} \) denotes the transport field characterizing the elastic scattering time of the hole, \( \hbar \) is Planck’s constant, \( e \) is the electron charge, and \( I_m \) is the mean free path of the hole. In the ILP theory, we only consider the Rashba SOI since there is no structural inversion asymmetry in Ge. Both \( k \)-linear \( \Omega_1 \) and \( k \)-cubic \( \Omega_3 \) effective magnetic fields in frequency units are incorporated [15]. Under these conditions, the fitting parameters to analyze experimental data are the characteristic magnetic fields for the phase coherence \( B_{\phi} = \hbar/4eD\tau_\phi \), for the \( k \)-linear spin-orbit coupling \( B_{\text{SO}} = (\hbar/4eD)\Omega_1^2 \tau_\text{tr} \), and for the \( k \)-cubic spin-orbit coupling \( B_{\text{SO3}} = (\hbar/4eD)\Omega_3^2 \tau_\text{tr} \), where \( D \) denotes the diffusion constant, \( \tau_\phi \) the phase coherent time of the carrier, and \( \tau_\text{tr} \) the elastic scattering time. By setting either \( B_{\text{SO3}} = 0 \) or \( B_{\text{SO}} = 0 \) in the ILP model, formulas with only the \( k \)-linear term or the \( k \)-cubic term are obtained [28]. In Fig. 3, we present the experimental data (open circles) and fitting results with only the \( k \)-linear term (blue dashed line) or only the \( k \)-cubic term (red solid line). We found that fitting with only the cubic-Rashba term revealed good agreement with the experimental results regardless of the \( V_G \) value, whereas the linear-Rashba term is insufficient for explaining the larger field region of the WAL data. The low magnetic field region can be fitted with both the linear- and cubic- models because this region is determined by the \( B_{\phi} \) term rather than \( B_{\text{SO}} \). However, we observed significantly different behavior between the linear- and cubic-Rashba terms in the larger magnetic field region. The cubic-Rashba model explains the experimental data more precisely; thus, we believe the cubic-Rashba model is relevant for the HH band in Ge.

Figure 4(a) shows \( B_{\text{SO3}} \) and \( B_{\phi} \) obtained by fitting with the cubic-Rashba model. Both \( B_{\text{SO3}} \) and \( B_{\phi} \) systematically change with \( V_G \). When \( B_{\text{SO3}} > B_{\phi} \), WAL is observed, and WL is observed when \( B_{\text{SO3}} < B_{\phi} \). The change of \( B_{\text{SO3}} \) provides direct evidence of the electric field control of the cubic-Rashba SOI. Note that the change of \( B_{\phi} \) with respect to \( V_G \) is due to the change of the hole mobility, and this gives rise to a longer phase coherence time \( \tau_\phi \) with increasing hole concentration. If \( k \)-dependent spin splitting due to the SOI exists, the spin relaxation is described by the Dyakonov-Perel mechanism with \( \tau_{\text{SO}} \propto 1/\Delta^2 \tau_\text{tr} [29] \), where \( \Delta \) is the spin splitting energy and \( \tau_{\text{SO}} \) denotes the spin relaxation time. On the other hand, if spin-flip scattering is the dominant spin relaxation mechanism, the Elliot-Yafet mechanism with \( \tau_{\text{SO}} \propto \tau_\text{tr}/(\Delta g)^2 \) (\( \Delta g \) is the shift of the electron \( g \) factor from that of free electrons) is relevant for describing our results. Therefore, the comparison between \( \tau_{\text{SO}} \) and \( \tau_\text{tr} \) provides additional evidence for the Rashba-type SOI. The \( \tau_{\text{SO}} \) and \( \tau_\text{tr} \) terms can be calculated using the relations \( B_{\text{SO3}} = \hbar/4eD\tau_{\text{SO}} \), \( D = v_F^2 \tau_\text{tr}/2 \), \( v_F = \hbar k_F/m_{\text{hole}} \), and \( k_F = \sqrt{2\pi m_{\text{hole}}} \), where \( v_F \) is the Fermi velocity and \( k_F \) is the Fermi wave vector. We plotted \( \tau_{\text{SO}} \) and \( \tau_\text{tr} \) with respect to the hole concentration as shown in Fig. 4(b). We found that \( \tau_\text{tr} \) increases with increasing carrier concentration, whereas \( \tau_{\text{SO}} \) decreases. This is consistent with the Dyakonov-Perel spin relaxation mechanism. Therefore, it provides further
We now evaluate the spin splitting energy from the $k$ contribution in the 2DHG. Evidence that the Rashba SOI is a dominant spin-orbit contribution in the 2DHG.

The agreement between the experimental data and the $k$-cubic-term-only condition of the ILP theory reveals the existence of the purely cubic-Rashba SOI in the 2DHG. We now evaluate the spin splitting energy from the relationship $\Delta = \hbar \Omega_3$. The change of $\Delta$ with respect to hole concentration is shown in Fig. 4(c). We obtained a $\Delta$ of 0.3–0.4 meV; we find larger spin splitting with increasing hole concentration. This spin splitting provides direct experimental evidence of the cubic-Rashba SOI induced splitting in strained Ge. To analyze the splitting in more detail, we calculated the coefficient $\alpha_3 E_z$ for the cubic-Rashba SOI using the relation $\Delta = \hbar \Omega_3 = \alpha_3 E_z k^3$, as shown in Fig. 4(d). The coefficient $\alpha_3 E_z$ decreases with increasing hole concentration. We believe that this is due to the change of the gate-induced electric field $E_z$ in the QW rather than a change of $\alpha_3$.

Here, the Rashba coefficient $\alpha_3$ for the first HH subband is expressed as follows:

$$\alpha_3 = \frac{-3e\hbar \gamma_3^2}{2m_0^2(E_{HH} - E_{LH})^2},$$

where $\gamma_3$ denotes the Luttinger parameter, and $E_{HH}$ and $E_{LH}$ are the energies of the lowest HH and LH subbands, respectively. Equation (1) is derived from the Luttinger Hamiltonian using the partitioning method presented by Ohkawa and Uemura [13]. Equation (1) suggests that $\alpha_3$ is inversely proportional to the square of the separation between the HH and LH energy. A qualitatively similar expression to Eq. (1) is also reported by other groups [14,20]. In a single heterostructure, the HH-LH separation changes with the external electric field, thus demonstrating gate control of $\alpha_3$ [30]. However, in the QW, the change in the HH and LH separation with external electric field should be small. Furthermore, because the HH-LH separation in the Ge/Si$_{0.5}$Ge$_{0.5}$ QW is initially large due to strain, the gate-induced change of the HH-LH separation is 1 order smaller than its total HH-LH separation energy; therefore, we think that $\alpha_3$ is almost constant over the studied hole concentration range. For these reasons, we believe that the results shown in Fig. 4(d) reflect the change of the effective electric field in the QW under the application of the gate voltage. Because the gate-induced electric field in the QW opposes the initial electric field due to modulation doping on the bottom side of the QW, the increase of the hole concentration in the QW decreases its internal electric field. Thus, $\alpha_3 E_z$ decreases with increasing hole concentration.

Equation (1) suggests another interesting aspect of the cubic-Rashba coefficient. The Rashba coefficient can be strongly tuned with the HH-LH separation. We estimated $\alpha_3$ using Eq. (1) with $\gamma_3 = 5.69$ for Ge [1]. The HH-LH separation for compressively strained-Ge/Si$_{0.5}$Ge$_{0.5}$ is $\sim 110$ meV [31]. We obtained $\alpha_3 = 2.26 \times 10^5 \text{ eA}^2$. The coefficient $\alpha_3$ is about 1 order smaller than for the 2DHG in the GaAs/AlGaAs single heterostructure [21], which is a nearly lattice matched system. The smaller $\alpha_3$ value is due to the large HH-LH separation in Ge/Si$_{0.5}$Ge$_{0.5}$ compared to that of the GaAs/AlGaAs structure; the large HH-LH separation decreases the coefficient $\alpha_3$. In the strained-Ge/SiGe QW, the HH-LH separation can be tuned by the strain due to the SiGe layer; in other words, the composition $x$ of the Si$_{1-x}$Ge$_x$ buffer layer enables a wide range of tuning options for the cubic-Rashba SOI. Note that Ge has a larger $\gamma_3$ value than Si ($\gamma_3 = 1.45$) [1]; thus we expect a larger SOI for the 2DHG in Ge than that of Si. Therefore strained Ge is a promising material for utilizing the cubic-Rashba SOI in a low dimensional system.
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