Magnetic polarizabilities of light mesons in $SU(3)$ lattice gauge theory.

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We investigate the masses (ground state energies) of neutral pseudoscalar and vector meson in $SU(3)$ lattice gauge theory in strong abelian magnetic field. The energy of ρ^0 meson with zero spin projection $s_z = 0$ on the axis of the external magnetic field decreases, while the energies with non-zero spins $s_z = -1$ and $+1$ increase with the field. The energy of π^0 meson decrease as a function of the magnetic field. We calculated the magnetic polarizabilities of pseudoscalar and vector mesons for lattice volume 18^4. For ρ^0 with spin $|s_z| = 1$ and π^0 meson the extrapolations to zero lattice spacing have been done. We do not observe any evidence in favour of tachyonic mode existence.

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1. INTRODUCTION

Quantum Chromodynamics in abelian magnetic filed of hadronic scale is a reach area for exploration. The investigation of strongly interacting quark-hadronic matter in such field has a deep fundamental meaning. Today it is possible to create a strong magnetic field of $15m_\pi^2 \sim 0.27\,\mathrm{GeV}^2$ [1] in terrestrial laboratories (ALICA, RHIC, NICA, FAIR) during noncentral heavy ion collisions. Our studies have a goal to shed light on the effects that can appear in such experiments. The properties of fundamental particles related to their internal structure are also very important for understanding the effects observed at the experiments.

Let us mention the most famous results concerned QCD physics in strong magnetic fields. The charge asymmetry of emitted charged particles [2–4] in non-central collisions of gold ions at RHIC was explained by chiral magnetic effect [4–6]. Many phenomenological studies have been devoted to understanding of QCD phase structure in strong magnetic fields. Chiral perturbation theory predicts decrease of transition temperature from the confinement to the deconfinement phase with increasing Abelian magnetic field [7]. External magnetic fields also strength the chiral symmetry breaking [8–12]. It was shown in the framework of Nambu-Jona-Lasinio model the QCD vacuum becomes a superconductor in sufficiently strong magnetic field ($B_c = m_\rho^2/e \simeq 10^{16}$ Tl) along the direction of the magnetic field [13–17]. The transition to the superconducting phase is accompanied by a condensation of charged ρ mesons. The strong magnetic fields can also change the order of the phase transition from the confinement to deconfinement phase [18–22].

Lattice studies reveal an interesting effect such as an inverse magnetic catalysis [23]. According to the calculations on the lattice with two types of valence quarks in QCD the critical temperature of the transition from confinement phase to deconfinement phase increases slightly in a strong magnetic field [24]. Calculations in the theory with $N_f = 2 + 1$ [25] with dynamical quarks showed that T_c decreases with increasing magnetic field. Lattice simulations with dynamical overlap fermions in two-flavor lattice QCD also showed the decrease of the critical temperature of confinement - deconfinement transition when the field strength grows [26].

Numerical simulations in QCD with $N_f = 2$ and $N_f = 2 + 1$ indicate the strongly interacting matter in strong magnetic field posses paramagnetic properties in the confinement and deconfinement phases [27–29]. Equation of state of quark-gluon plasma was investigated in [30].

Here we continue our previous work where we studied light mesons in $SU(2)$ lattice gauge theory [31]. We extend this analysis to the $SU(3)$ lattice gauge theory which is more realistic, and calculate the ground state energies of the light mesons as a function of the magnetic field value depending on their spin. Our previous results are in a qualitative agreement with the results of this work. We also calculate several hadronic characteristics such as magnetic polarizabilities of light neutral pseudoscalar and vector mesons. The magnetic polarizability is an important physical quantity which reveals the internal structure of a particle in external magnetic field. We also made the extrapolation to zero lattice spacing where it was possible. Our approach is numerically expensive so we do not take into account dynamical quarks.

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Several articles are devote to the study of meson masses in strong magnetic field. The masses of ρ mesons have been calculated according to the relativistic quark-antiquark model in [32]. Lattice study is given in [33] in the approach with dynamical quarks and agree with our data for $eB < 1$ GeV^2. Phenomenological study was made in [34]. For the case of non-zero spin our results in a qualitative agreement with the results [32–34]. For zero spin our data agree with these results only for small magnetic fields.

2. DETAILS OF CALCULATIONS

For generation of $SU(3)$ gauge configurations the tadpole improved Symanzik action was used

$$S = \beta_{imp} \sum_{\text{pl}} S_{\text{pl}} - \frac{\beta_{imp}}{20u_0^2} \sum_{\text{rt}} S_{\text{rt}}, \tag{1}$$

where $S_{\text{pl,rt}} = (1/3)\text{Tr}\,(1 - U_{\text{pl,rt}})$ is the lattice plaquette (denoted as pl) or $1{\times}2$ rectangular loop (rt), $u_0 = (W_{1\times1})^{1/4} = \langle(1/3)\text{Tr}\,U_{\text{pl}}\rangle^{1/4}$ is the tadpole factor, calculated at zero temperature [35]. This action suppresses ultraviolet dislocations which lead to the appearance of non-physical near zero-modes of Wilson-Dirac operator.

Next, we solve Dirac equation numerically

$$D\psi_k = i\lambda_k\psi_k, \quad D = \gamma^\mu(\partial_\mu - iA_\mu) \tag{2}$$

and find eigenfunctions ψ_k and eigenvalues λ_k for a test quark in the external gauge field A_μ. We find eigenmodes of Dirac operator to calculate the correlators. From the correlators we obtain ground state energies. For the calculation of the fermion spectrum we use the Neuberger overlap operator [36]. This operator allows to investigate the theory in the limit of massless quarks without chiral symmetry breaking and can be written in this form

$$D_{ov} = \frac{\rho}{a}\left(1 + D_W/\sqrt{D_W^\dagger D_W}\right). \tag{3}$$

$D_W = M - \rho/a$ is the Wilson-Dirac operator with a negative ρ/a, a is the lattice spacing in physical units, M is the Wilson term. Fermion fields obey periodical boundary conditions in space and antiperiodical boundary conditions in time. Sign function

$$D_W/\sqrt{(D_W)^\dagger D_W} = \gamma_5 \text{sign}\,(H_W), \tag{4}$$

is calculated using minmax polynomial approximation, where $H_W = \gamma_5 D_W$ is hermitian Wilson-Dirac operator. We investigate behaviour of the meson ground energy state in background gauge field, which is a sum of non-abelian $SU(3)$ gluon field and $U(1)$ abelian uniform magnetic field. Abelian gauge fields interact only with quarks. In our calculations we have neglected the contribution of dynamical quarks. Therefore we add the magnetic field only in overlap Dirac operator. For this reason we use the following ansatz:

$$A_{\mu\,ij} \to A_{\mu\,ij} + A_\mu^B \delta_{ij}, \tag{5}$$

where

$$A_\mu^B(x) = \frac{B}{2}(x_1\delta_{\mu,2} - x_2\delta_{\mu,1}). \tag{6}$$

In order to make this substitution consistent with fermion boundary conditions, one should use the twisted boundary conditions [37]. Magnetic field is directed along z axis and its value is quantized

$$qB = \frac{2\pi k}{(aL)^2}, \quad k \in \mathbb{Z}, \tag{7}$$

where $q = -1/3\,e$.

The quantization condition implies that the magnetic field has a minimal value $\sqrt{eB_{min}} = 380$ MeV for 18^4 lattice volume and $a = 0.125$ fm. We are far from saturation regime, where $k/(L^2)$ is not small because we use k between 0 and 32. For the inversion of overlap Dirac operator we use Gaussian source (with radius $r = 1.0$ in lattice units in space and time direction) and point receiver (the quark position smoothed with Gaussian profile). Our simulations have been carried out on symmetrical lattices with lattice volume 16^4, lattice spacing 0.105 fm and lattice volume 18^4, lattice spacings $a = 0.084$ fm, 0.095 fm, 0.105 fm, 0.115 fm and 0.125 fm. We use statistically independent configurations of gluonic fields for the every value of the quark mass.

3. OBSERVABLES

We calculate the following observables in coordinate space and background gauge field A

$$\langle\psi^\dagger(x)O_1\psi(x)\psi^\dagger(y)O_2\psi(y)\rangle_A, \tag{8}$$

where $O_1, O_2 = \gamma_5, \gamma_\mu$ are Dirac gamma matrices, $\mu, \nu = 1,..,4$ are Lorenz indices, $x = (\mathbf{n}a, n_t a)$ and $y = (\mathbf{n}'a, n'_t a)$ are coordinates on the lattice. The spatial lattice coordinate $\mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3) | n_i = 0, 1, ..., N-1\}$, n_t, n'_t are the numbers of lattice sites in the time direction. In Euclidean space $\psi^\dagger = \bar{\psi}$. In order to calculate the observables (8) we calculate the quark propagators in coordinate space. For the M lowest eigenmodes massive Dirac propagator is represented by the following sum:

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}. \tag{9}$$

In our calculations we use $M = 50$. For the observables (8) the following equation is fulfilled

$$\langle\bar{\psi}O_1\psi\bar{\psi}O_2\psi\rangle_A = -\text{Tr}\,[O_1 D^{-1}(x,y) O_2 D^{-1}(y,x)] \tag{10}$$

$$+\mathrm{Tr}\,[O_1 D^{-1}(x,x)]\mathrm{Tr}\,[O_2 D^{-1}(y,y)]$$

The first term in (10) is the connected part, the second term is the disconnected part. We have checked that in $SU(3)$ theory without dynamical quarks the disconnected part contribution to correlators is zero. We perform Fourier transformation numerically

$$\tilde{\Phi}(\mathbf{p},t) = \frac{1}{N^{3/2}} \sum_{\mathbf{n}\in\Lambda_3} \Phi(\mathbf{n},n_t) e^{-ia\mathbf{n}\mathbf{p}} \qquad (11)$$

The momenta $\mathbf{p}$ has the components $p_i = 2\pi k_i/(aN)$, $k_i = -N/2+1,...,N/2$. For particles with zero momentum their energy is equal to its mass $E_0 = m_0$. As we are interested in the meson ground state energy, we choose $\langle\mathbf{p}\rangle = 0$. To obtain the masses we expand the correlation function to the exponential series

$$\tilde{C}(n_t) = \langle \psi^\dagger(\mathbf{0},n_t) O_1 \psi(\mathbf{0},n_t) \psi^\dagger(\mathbf{0},0) O_2 \psi(\mathbf{0},0)\rangle_A =$$

$$\sum_k \langle 0|O_1|k\rangle\langle k|O_2^\dagger|0\rangle e^{-n_t a E_k}, \qquad (12)$$

$$\tilde{C}(n_t) = A_0 e^{-n_t a E_0} + A_1 e^{-n_t a E_1} + \ldots\,, \qquad (13)$$

A_0, A_1 are constants, E_0 is the ground state energy. E_1 is the energy of first exited state, a is the lattice spacing, n_t is the number of sites in the time direction. From expansion (13) one can see that for large n_t the main contribution origins from the ground state. Because of the periodic boundary conditions the main contribution to the ground state has the following form

$$\tilde{C}_{fit}(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} =$$

$$2A_0 e^{-N_T a E_0/2} \cosh((\frac{N_T}{2} - n_t) a E_0). \qquad (14)$$

The value of the ground state mass can be obtained by fitting the function (14) to the lattice correlator (12). In order to minimize the errors and exclude the contribution of the exited states we take the values of n_t from the interval $6 \le n_t \le N_T - 6$. Masses of the ρ mesons have been obtained from correlator (8), where $O_1, O_2 = \gamma_\mu$. If $O_1, O_2 = \gamma_5$ we get the pseudoscalar π meson. In our calculations u and d quarks are degenerate.

4. THE GROUND STATE ENERGIES OF MESONS IN STRONG MAGNETIC FIELD

Fig. 1 shows the ground state energy of the neutral pion obtained from the correlator $C^{PSPS} = \langle\bar{\psi}(\mathbf{0},n_t)\gamma_5\psi(\mathbf{0},n_t)\bar{\psi}(\mathbf{0},0)\gamma_5\psi(\mathbf{0},0)\rangle$. On this plot we present the data for the smallest bare quark mass $m_q =$ 34.26 MeV, lattice volumes 18^4, 16^4 and lattice spacings $a = 0.105$ fm, 0.115 fm, 0.125 fm. The π^0 energy decreases for the all sets of lattice data and slightly depends on the lattice volume and lattice spacing at moderate magnetic fields. With increase of the field value the lattice effects become more strong, the wave function of light pion becomes of the order or exceeds the lattice size.

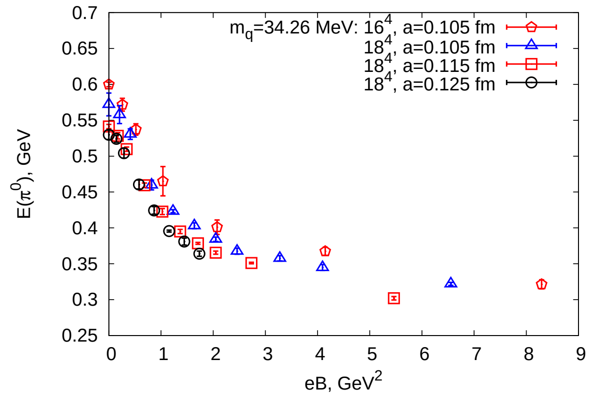

FIG. 1: The energy of the ground state of pseudoscalar neutral π^0 meson obtained from the cosh. fit to the correlator C^{PSPS} as a function of the magnetic field. The data are shown for the bare quark mass $m_q = 34.26$ MeV, lattice volumes 16^4, lattice spacings 0.105 fm and lattice volume 18^4, spacings 0.105 fm, 0.115 fm, 0.125 fm.

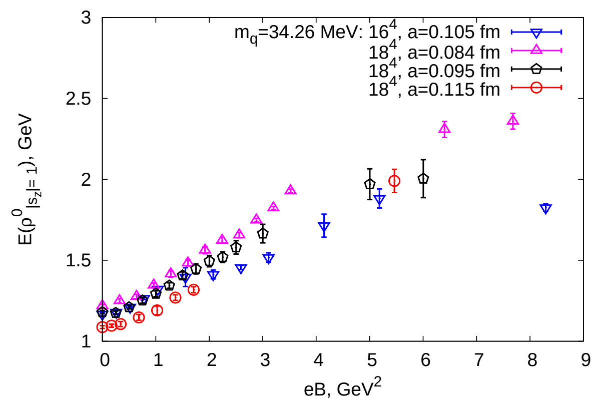

FIG. 2: The energy of the ρ^0 ground state obtained from the cosh. fits to the correlators $C^{VV}_{s_z=\pm1}$ as a function of the magnetic field for various lattice data.

To obtain the energies of neutral vector mesons with various spin projections on the axis of the external magnetic field we use the combinations of the correlators in various spatial dimensions.

$$C^{VV}_{xx} = \langle\bar{\psi}(\mathbf{0},n_t)\gamma_1\psi(\mathbf{0},n_t)\bar{\psi}(\mathbf{0},0)\gamma_1\psi(\mathbf{0},0)\rangle, \qquad (15)$$

$$C^{VV}_{yy} = \langle\bar{\psi}(\mathbf{0},n_t)\gamma_2\psi(\mathbf{0},n_t)\bar{\psi}(\mathbf{0},0)\gamma_2\psi(\mathbf{0},0)\rangle, \qquad (16)$$

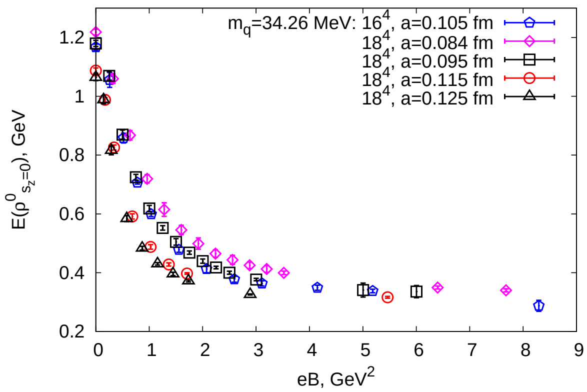

FIG. 3: The energy of the ρ^0 ground state obtained from the cosh. fit to the correlator C^{VV}_{zz} depending on the magnetic field value. The data are shown for the bare quark mass $m_q = 34.26$ MeV, lattice volume 16^4, lattice spacing 0.105 fm and lattice volume 18^4, lattice spacings 0.084 fm, 0.095 fm, 0.115 fm, 0.125 fm.

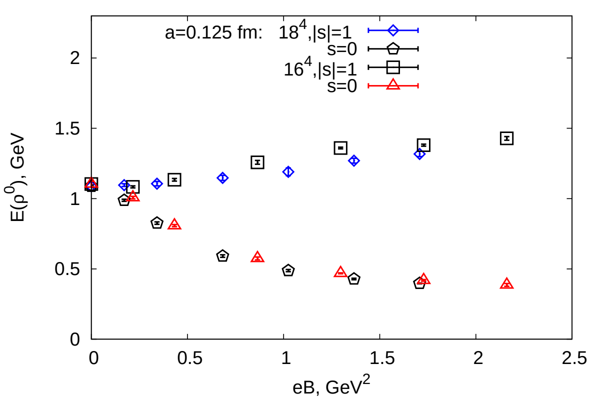

FIG. 4: The energy of the ρ^0 meson for the lattice spacing 0.125 fm and lattice volumes 16^4 and 18^4 for various meson spin projections.

$$C^{VV}_{zz} = \langle\bar\psi(\mathbf{0},n_t)\gamma_3\psi(\mathbf{0},n_t)\bar\psi(\mathbf{0},0)\gamma_3\psi(\mathbf{0},0)\rangle. \qquad (17)$$

The mass of ρ^0 meson with $s_z = 0$ spin projection is obtained from the C^{VV}_{zz} correlator. The combinations of correlators

$$C^{VV}(s_z = \pm 1) = C^{VV}_{xx} + C^{VV}_{yy} \pm i(C^{VV}_{xy} - C^{VV}_{yx}) \qquad (18)$$

give the ground state energies of meson with spin projections $s_z = +1$ and $s_z = -1$.

Fig. 2 represents the energy of the ρ^0 with non-zero spin projection on the axis of the external magnetic field. The energy increases with the field for the all sets of lattice data. At $|eB| \lesssim 2.5$ GeV^2 the energy grows quadratically and at large magnetic fields it looks like a plateau. The terms iC^{VV}_{xy} and iC^{VV}_{yx} in (18) are zero for the case of neutral particles so the ρ^0 masses with $s = -1$ and $s_z = +1$ coincide which should be a consequence of C-parity.

In Fig. 2 we also see that at $eB \gtrsim 2.5$ GeV^2 the data reveals more stronger dependence on the lattice spacing than for the lower values of the field. Simple estimates give the following values of magnetic fields corresponding to the lattice spacing cut-off: 2.5 GeV^2 for $a = 0.125$ fm, 2.9 GeV^2 for lattice spacing $a = 0.115$ fm and $eB \sim$ 3.5 GeV^2 for $a = 0.105$ fm and so on.

In Fig. 3 we depict the ground state energy which we obtained from the correlator C^{VV}_{zz} for various lattice spacings and volumes. For the small magnetic fields we suggest that this energy corresponds to the ρ^0 meson with $s_z = 0$. Also we find the real part of the non-diagonal terms in correlation matrix are equal to zero. At large magnetic field the mixing between $\rho^0(s_z = 0)$ and $\pi^0(s = 0)$ having the same quantum numbers has to be taken into account because the branching $\rho^0 \to \pi^0\gamma$ increases with the magnetic field value. In this work our main goal was to calculate the polarizabilities of ρ^0, but not to distinguish ρ^0 and π^0, this can be done in the following work. In Fig.4 we show for comparison the energy of ρ^0 meson at $a = 0.125$ fm for 16^4 and 18^4 lattices with various spins. Therefore the lattice volume effects are not large.

In Fig.5 we show the ρ^0 energy averaged over three spin components $E(\rho^0) = (E(\rho^0_{s_z=0}) + E(\rho^0_{s_z=-1}) + E(\rho^0_{s_z=+1}))/3$, which corresponds to the energy of unpolarized vector meson, we suppose it has to be a constant value. For the magnetic fields $eB < 2$ GeV^2 the lattice spacing effects can be a possible reason of the mass deviation from a constant value. With the diminishing of the lattice spacing the mass of unpolarized meson becomes closer to a constant value, so it confirms the supposition that a mixing between $\rho^0(s_z = 0)$ and $\pi^0(s = 0)$ states may be weak at $eB < 2$ GeV^2.

From the assumption of constant energy of unpolarized meson and the behaviour of nonzero energy components (Fig.2) we can conclude that there is no tachyonic mode for the explored range of magnetic fields, i.e. the mass of $\rho^0(s = 0)$ doesn't turn to zero. We see the lattice volume effects are small and do not change this conclusion at large magnetic fields. The decrease of mass may be compensated by higher powers of eB for its values larger than 1 GeV or so. These effects can be preventing, in particular, from mass turning to zero and possible emergence of tachyonic mode. The same will be true also for the energies of all spin states of charged [38] ρ mesons. Still, the case of neutral mesons demands further investigation and study of mixing and lattice spacing effects.

Therefore our calculations show that there is a splitting of ground state energy of neutral vector meson in a strong abelian magnetic field that is an interesting physical effect.

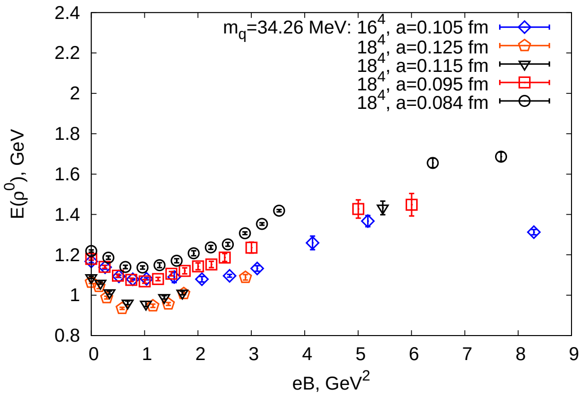

FIG. 5: The energy of the ground state of unpolarized vector neutral ρ^0 meson obtained from the cosh. fits to the correlators as a function of the magnetic field. The data are shown for lattice volume 16^4, lattice spacing 0.105 fm and lattice volume 18^4, lattice spacing 0.084 fm, 0.095 fm, 0.115 fm, 0.125 fm and the bare quark mass $m_q = 34.26$ MeV.

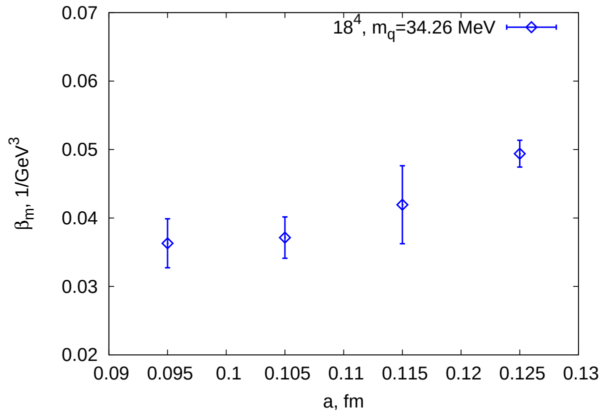

FIG. 7: The magnetic polarizability of π^0 meson for various lattice spacings, the lattice volume 18^4 and bare quark mass $m_q = 34.26$ MeV.

5. MAGNETIC POLARIZABILITIES

The polarizability of meson is an important physical quantity for understanding of its internal structure. The magnetic polarizability of meson shows how current distribution responds to the external magnetic field. In this section we talk about the magnetic polarizabilities of pseudoscalar π^0 and vector ρ^0 mesons.

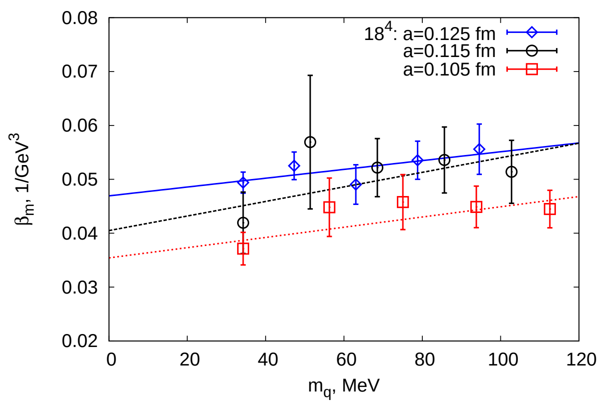

FIG. 8: The magnetic polarizability of pseudoscalar neutral meson π^0 versus the bare lattice quark mass for lattice volume 18^4 and various lattice spacings.

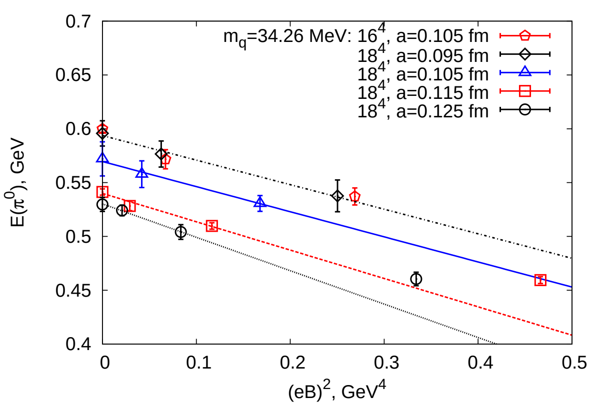

FIG. 6: The energy of the ground state of pseudoscalar neutral π^0 meson as a function of the squared magnetic field. The data are shown for the bare quark mass $m_q = 34.26$ MeV, lattice volume 16^4, lattice spacing 0.105 fm and lattice volume 18^4, lattice spacings 0.095 fm, 0.105 fm, 0.115 fm, 0.125 fm.

In Fig.6 we show the ground state energy of pion as a function of squared magnetic field $(eB)^2$ for fields $(eB)^2 < 0.5$ GeV^4. We fit the data at $(eB)^2 \in [0, 0.3$ GeV$^4]$ by the following function

$$E = E(B=0) - 2\pi\beta_m (eB)^2, \tag{19}$$

where we use "natural" units $\hbar = c = 1$, but $e^2 = 1/137$ in Gaussian units. $E(B=0)$ and β_m are the parameters which we find from the fit. We choose this interval for the fit because we consider the terms $\sim (eB)^4$ give small contribution to the pion energy at such magnetic field values. The dashed-dotted line is for the lattice volume 18^4, lattice spacing $a = 0.095$ fm, the solid line corresponds to the $a = 0.105$ fm, the dashed line is for $a = 0.115$ fm and dotted one corresponds to the case of $a = 0.125$ fm.

The obtained values of polarizabilities for various lattice spacings are summarized in Table 1. We do not observe any functional dependence of the magnetic polarizability on the lattice spacing, the results are presented in Fig.7. We obtain the magnetic polarizability of π^0 meson $\beta_m(\pi^0) = (0.036 \pm 0.004)$ 1/GeV^3 or

$(2.75 \pm 0.31) \cdot 10^{-4}$ fm^3 for the lattice spacing 0.095 fm. This number agrees with sign with the result of of chiral perturbation theory [39], but its value in 2 times larger.

We also explore the dependence of $\beta_m(\pi^0)$ on the bare quark mass, see Fig.8, and find the value of the magnetic polarizability slightly decreases with the diminishing quark mass value.

V_{latt}	a (fm)	$\beta_m^{m_q=34\ MeV}$ (GeV^{-3})	Error (GeV^{-3})	χ^2/d.o.f.
18^4	0.095	0.036	0.004	0.0915051
18^4	0.105	0.037	0.003	0.0501177
18^4	0.115	0.042	0.006	1.03419
18^4	0.125	0.049	0.002	0.0130633

TABLE I: The values of magnetic polarizability of the π^0 meson for the bare quark mass $m_q = 34.26$ MeV, lattice volume 18^4 and various lattice spacings.

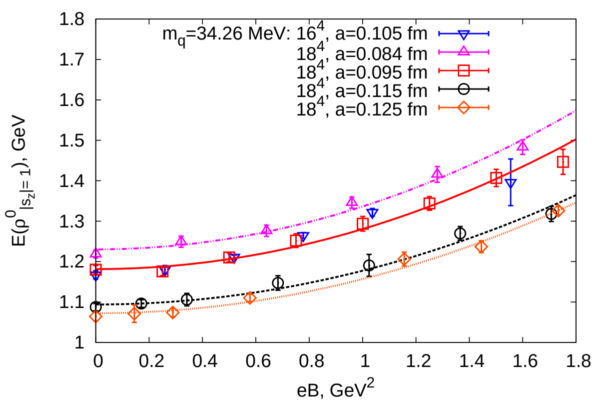

FIG. 9: The ρ^0 meson with the fits $E = E_0(B = 0) - 2\pi\beta_m^{|s|=1}(eB)^2$ to the data on the interval $eB \in [0, 1.8\ \text{GeV}^2]$. All the fits correspond to the 18^4 lattice, the dashed-dotted is for the lattice spacing 0.084 fm, the solid line corresponds to the $a = 0.095$ fm, the dashed one is for the $a = 0.115$ fm and dotted line corresponds to the case of $a = 0.125$ fm.

In Fig.9 in a large scale we depict the energy of the ρ^0 ground state with non-zero spin depending on the magnetic field for the smallest bare quark mass $m_q = 34.26$ MeV, lattice volumes 18^4, lattice spacings 0.105 fm, 0.115 fm, 0.125 fm and lattice volume 16^4, spacing 0.105 fm. To obtain the magnetic polarizability we fit our data to the function $E = E(B = 0) - 2\pi\beta_m^{|s|=1}(eB)^2$ at magnetic fields $0 \le eB < 1.8$ GeV^2, where the data are well described by quadratic law. $E(B = 0)$ and $\beta_m^{|s|=1}$ are the unknown parameters which were found during the fitting procedure. The obtained values $\beta_m^{|s|=1}$ for the ρ^0 meson with spin $|s_z| = 1$ together with errors and lattice parameters are summarized in Table 2 and shown in Fig. 10.

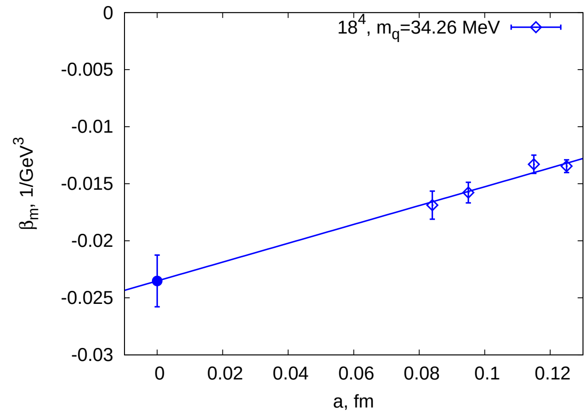

FIG. 10: The magnetic polarizability of ρ^0 meson and its extrapolation by a linear function to zero lattice spacing for the lattice volume 18^4 and bare quark mass $m_q = 34.26$ MeV.

We see the strong dependence of the results on the lattice spacing so we make an extrapolation to the continuum limit. The extrapolation gives the value of magnetic polarizability $\beta_m^{|s|=1}(\rho^0) = (-0.0235 \pm 0.0023)$ 1/GeV^3 for the lattice volume 18^4 and bare quark mass $m_q = 34$ MeV. The magnetic polarizability doesn't depends on the sign of the spin projection of ρ^0 so this quantity has a scalar nature.

V_{latt}	a (fm)	$\beta_m^{m_q=34\ MeV}$ (GeV^{-3})	Error (GeV^{-3})	χ^2/d.o.f.
18^4	0.084	-0.0169	0.0012	1.23546
18^4	0.095	-0.0158	0.0009	0.730133
18^4	0.115	-0.0133	0.0008	0.75398
18^4	0.125	-0.0135	0.0006	0.831648
18^4	$a = 0$ extr.	-0.0235	0.0023	0.560989
		$\beta_m^{ch.\ extr}$ (GeV^{-3})		
18^4	0.115	-0.0138	0.0005	2.64782
18^4	0.125	-0.0161	0.0025	23.8615

TABLE II: The values of magnetic polarizability of the vector ρ^0 meson with non-zero spin for the bare quark mass $m_q = 34.26$ MeV and after chiral extrapolation, lattice volume 18^4 and various lattice spacings.

In Fig.11 the mass of ρ^0 meson with zero spin is depicted for the small magnetic fields. We observe the linear in $(eB)^2$ behaviour only for $(eB)^2 \in [0, 0.1\ \text{GeV}^4]$ and get the value $\beta_m^{s=0}(\rho^0) = (0.47 \pm 0.03)$ 1/GeV^3 for the lattice 18^4 and spacing $a = 0.125$ fm. This value is ~ 35 times larger and opposite in sign compared to the the magnetic polarizability for non-zero spin case $\beta_m^{|s|=1}(\rho^0) = (-0.0135 \pm 0.0006)$ 1/GeV^3. Unfortunately we are limited on the magnetic field value and can't make an extrapolation to zero lattice spacing. Large absolute value of $\beta_m^{s=0}(\rho^0)$ is most probably a lattice spacing effect.

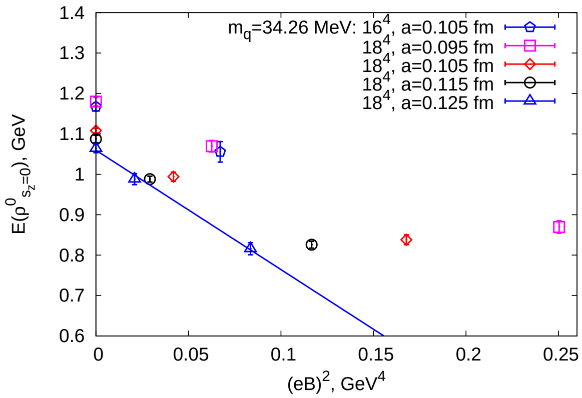

FIG. 11: The energy of the ground state of the vector ρ^0 meson with $s_z = 0$ depending on the squared magnetic field value. The data are shown for the bare quark mass $m_q = 34.26$ MeV, lattice volume 16^4, lattice spacing 0.105 fm and lattice volume 18^4,lattice spacings 0.095 fm, 0.105 fm, 0.115 fm, 0.125 fm.

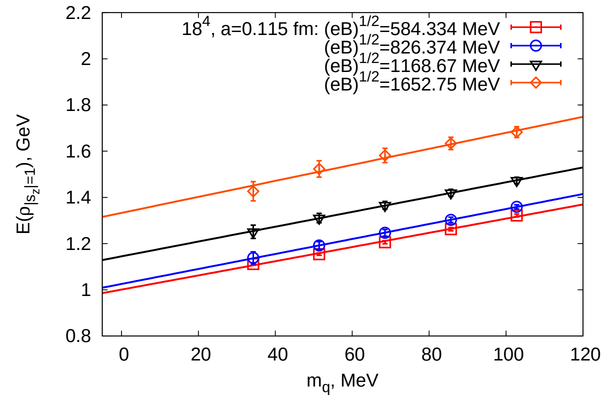

FIG. 12: The ground state energy of the neutral vector ρ meson spin $|s_z| = 1$ for lattice volume 18^4, lattice spacing $a = 0.115$ fm, various quark masses and several values of magnetic fields. The extrapolation was done by the fit (20) to the chiral limit.

Positive value of the magnetic polarizability for zero spin case shows that the external magnetic field increases the size of wavefunction of the $\rho^0(s = 0)$. On the contrary in the case of nonzero spin the magnetic field shrinks the wavefunction of vector neutral meson, the magnetic polarizability has a negative value.

The small value of the magnetic polarizability is not surprising, because we need to apply a very large magnetic field to observe the response of the internal structure of ρ^0 meson composed of charged quarks which have a spin.

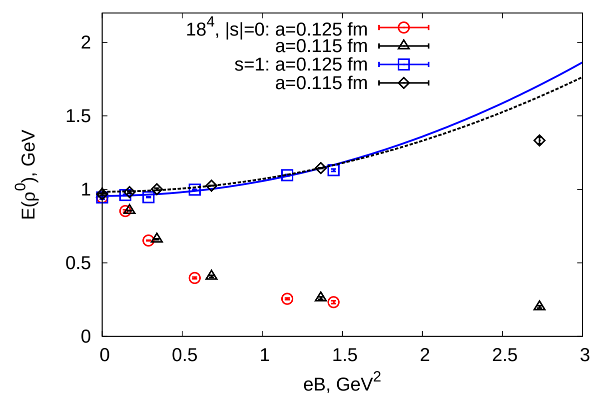

FIG. 13: The mass of the neutral vector ρ^0 meson with various spins on the value of external magnetic field for the lattice volume 18^4 and lattice spacings $a = 0.115$ fm, 0.125 fm after chiral extrapolation. The solid curve is for the fit to the data at 0.125 fm lattice spacing and the dashed curve corresponds to the data at 0.115 fm.

6. QUARK MASS EXTRAPOLATIONS

In Fig.12 we show the quark mass extrapolation of the neutral vector ρ meson spin $|s_z| = 1$ for lattice volume 18^4 and lattice spacing $a = 0.115$ fm. The mass of ρ^0 meson was calculated for several m_q values in the interval $m_q a \in [0.02, 0.06]$. Then we perform a fit by a linear function

$$m_\rho = a_0 + a_1 m_q \tag{20}$$

and find the coefficient a_0 and a_1 from the fit and its errors by χ^2 method. Then we extrapolate $m_\rho(m_q)$ to the chiral limit $m_q = 0$. The result of such extrapolation is presented in Fig.13 for the lattice volume 18^4 and lattice spacings $a = 0.115$ fm, 0.125 fm.

The masses of ρ^0 with zero spin smoothly decrease with magnetic field while the energies with masses spin increase with the field value. In Table 1 in the last three raws we also represent the values of magnetic polarizability obtained from the fits to the data after chiral extrapolation. The values of $\beta_m^{|s|=1}$ after quark mass extrapolations agrees with the values of $\beta_m^{|s|=1}$ obtained for the bare quark mass $m_q = 34$ MeV within the errors for $a = 0.115$ fm. For $a = 0.125$ fm the agreement is not so good, because of the absence of data for the extrapolation.

7. CONCLUSIONS

In this work we are perform lattice QCD simulations to explore the ground state energies of π^0 and ρ^0 mesons. The mass of pseudoscalar meson diminishes with the field value. We observe that the energies of the ground state of neutral vector ρ meson with zero spin projection on the axis of the external magnetic field decrease while the energies with non-zero spin increase as a function of magnetic field. The magnetic polarizability of ρ^0 meson with $s_z = 0$ differs from the magnetic polarizability of ρ^0 meson with $|s_z| = 1$. We consider this phenomena to be the result of the anisotropy created by the strong magnetic field. The energies of ρ^0 with spin $s_z = +1$ and $s_z = -1$ coincides which is the consequence of C-parity.

For vector meson the magnetic polarizability $\beta_m^{|s|=1}(\rho^0) = (-0.0235 \pm 0.0023)$ 1/GeV^3 after extrapolation to zero lattice spacing $a = 0$. For zero spin $\beta_m^{s=0}(\rho^0)$ is opposite in sign to nonzero spin case. The magnetic polarizability of π^0 meson $\beta_m(\pi^0) = (0.036 \pm 0.004)$ 1/GeV^3 for the lattice spacing 0.095 fm. We consider mixing between π^0 and $\rho^0(s = 0)$ states not strong at $eB < 2$ GeV^2, this is the subject for the further detailed investigation. We do not observe any evidence in favour of tachyonic mode existence.

8. ACKNOWLEDGMENTS

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