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The exclusive limit of the pion-induced Drell–Yan process

S.V. Goloskokov [a], P. Kroll [b,c,*]

[a] Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna 141980, Moscow region, Russia
[b] Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal, Germany
[c] Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

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ABSTRACT

Based on previous studies of hard exclusive leptoproduction of pions in which the essential role of the pion pole and the transversity generalized parton distributions (GPDs) has been pointed out, we present predictions for the four partial cross sections of the exclusive Drell–Yan process, $\pi^- p \to l^- l^+ n$.

1. Introduction

In recent years hard exclusive leptoproduction of mesons and photons has been studied intensively by both experimentalists and theoreticians. It became evident in the course of time that within the handbag approach which is based on QCD factorization in the generalized Bjorken regime of large photon virtuality and large photon–proton center-of-mass energy but fixed x-Bjorken, it is possible to interpret these processes in terms of generalized parton distributions and hard perturbatively calculable subprocesses with, however, occasionally strong power corrections for meson production (for a recent review see [1]). Exploiting the universality property of the GPDs, one may use the set of GPDs extracted from meson leptoproduction, in the calculation of other hard exclusive processes. Of particular interest are processes with time-like virtual photons. Thus in [2] predictions for time-like DVCS ($\gamma p \to l^- l^+ p$) have been given, their experimental examination is still pending. The high-energy pion beam at J-PARC put into operation in the near future, offers the possibility of measuring another exclusive process with time-like virtual photons, namely the exclusive limit of the Drell–Yan process, $\pi^- p \to l^- l^+ n$. The purpose of this letter is to present predictions for the cross sections of this process taking into account what has been learned in the analyses of pion leptoproduction [3,4]. The data on the cross section for π^+ leptoproduction [5,6] demonstrate the prominent role of the contribution from the pion pole at small invariant momentum transfer, t, and it became evident that it is to be calculated as a one-particle-exchange (OPE) term rather than from the GPD $\widetilde{E}$ [7]. In the latter case the pion-pole contribution to the π^+ cross section is underestimated by order of magnitude. A second important observation has been made in [3,4]: The interpretation of the transverse target spin asymmetries in π^+ leptoproduction measured by the HERMES Collaboration [8] necessitates contributions from transversely polarized photons which are to be modeled by transversity GPDs within the handbag approach. This observation is supported by a recent CLAS measurement of π^0 leptoproduction [9].

Since for the process $\pi^- p \to l^- l^+ n$ the same GPDs contribute as for pion leptoproduction and the corresponding subprocesses are just $\hat{s} \leftrightarrow \hat{u}$ crossed ones[1]

$$\mathcal{H}^{\pi^- \to \gamma^*}(\hat{s}, \hat{u}) = -\mathcal{H}^{\gamma^* \to \pi^+}(\hat{u}, \hat{s}) \tag{1}$$

where $\hat{s}$ and $\hat{u}$ denote the subprocess Mandelstam variables, one can exploit the knowledge acquired there. One thus gains predictive power, there is no free parameter or soft hadronic matrix element left for the Drell–Yan process. Our analysis markedly differs from a previous study performed by Berger et al. [11] where only predictions for the longitudinal cross section at leading-twist accuracy have been given. It should be stressed that their and our predictions for that cross section differ by about a factor of 40 due to the different treatment of the pion pole contribution. Our findings may be of help in the preparation of a Drell–Yan experiment [12]. Future data on the exclusive pion-induced Drell–Yan process may reveal whether or not our present understanding of hard exclusive processes in terms of convolutions of GPDs and hard subprocesses also holds for time-like photons. This is a non-trivial

* Corresponding author at: Fachbereich Physik, Universität Wuppertal, D-42097 Wuppertal, Germany.
E-mail addresses: goloskkv@theor.jinr.ru (S.V. Goloskokov), kroll@physik.uni-wuppertal.de (P. Kroll).

[1] A detailed discussion of the space- and time-like connection of the leading-twist amplitudes can be found in [10].

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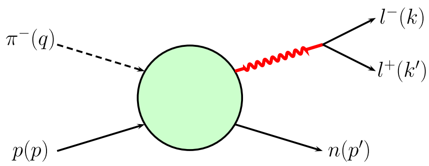

Fig. 1. The exclusive Drell–Yan process. The symbols in brackets denote the momenta of the respective particles.

issue because the physics in the time-like region is complicated and often not understood. Thus, for instance, there is no explanation of the time-like electromagnetic form factors of hadrons [13]. Even the semi-inclusive Drell–Yan process was difficult to understand. It took a long time before the discrepancy between the theoretical predictions and experiment, known as the K-factor, has been explained as threshold logarithms [14,15] representing gluon radiation resumed to next-leading-log (NLL) accuracy.

2. The handbag approach

Here, in this section, we recapitulate the handbag approach. For more details of it we refer to our previous work [3,4]. The process $\pi^- p \to l^- l^+ n$ is depicted in Fig. 1. We work in a center-of-mass frame in which $\mathbf{p}+\mathbf{p}'$ points along the positive 3-axis and we consider the kinematical range of large Mandelstam s $(= (p+q)^2)$ and large photon virtuality,[2] Q'^2, but small

$$\tau = \frac{Q'^2}{s-m^2}\,, \tag{2}$$

the time-like analogue of Bjorken-x (m being the mass of the nucleon). Hence, skewness, defined as

$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+} \approx \frac{\tau}{2-\tau}\,, \tag{3}$$

is also small.

Assuming factorization we can express the helicity amplitudes for $\pi^- p \to \gamma^* n$ in terms of convolutions of GPDs and hard subprocess amplitudes

$$\begin{aligned}
\mathcal{M}_{0+,0+} &= \sqrt{1-\xi^2}\,\frac{e_0}{Q'} \\
&\quad\times\Big[\langle \widetilde{H}^{(3)}\rangle - \frac{\xi^2}{1-\xi^2}\langle \widetilde{E}^{(3)}_{\rm n.p.}\rangle + \frac{2\xi m}{1-\xi^2}\frac{\varrho_\pi}{t-m_\pi^2}\Big],\\
\mathcal{M}_{0-,0+} &= \frac{\sqrt{-t'}}{2m}\,\frac{e_0}{Q'}\Big[\xi\langle \widetilde{E}^{(3)}_{\rm n.p.}\rangle - 2m\frac{\varrho_\pi}{t-m_\pi^2}\Big],\\
\mathcal{M}_{--,0+} &= \sqrt{1-\xi^2}\,\frac{e_0}{Q'^2}\,\mu_\pi\langle H_T^{(3)}\rangle\,,\\
\mathcal{M}_{\pm+,0+} &= \frac{\sqrt{-t'}}{4m}\,\frac{e_0}{Q'^2}\Big[\mu_\pi\langle \bar{E}_T^{(3)}\rangle \mp 8\sqrt{2}m^2\xi\frac{\varrho_\pi}{t-m_\pi^2}\Big],\\
\mathcal{M}_{+-,0+} &\approx 0\,.
\end{aligned} \tag{4}$$

Explicit helicities are labeled by their signs or by zero, e_0 denotes the positron charge and $t' = t - t_0$ where $t_0 = -4m^2\xi^2/(1-\xi^2)$ is the minimal value t corresponding to forward scattering. Terms of order t/Q'^2 are neglected throughout. The amplitudes for negative helicity of the initial state proton are obtained from the set of amplitudes (4) by parity conservation. The residue of the pion pole is given by

$$\varrho_\pi = \sqrt{2}g_{\pi NN}F_{\pi NN}(t)Q'^2F_\pi(Q'^2) \tag{5}$$

[2] The Q'^2-regions of quarkonia states have to be excluded.

where $g_{\pi NN}$ $(= 13.1 \pm 0.3)$ is the familiar pion–nucleon coupling constant and $F_{\pi NN}$ is a form factor that describes the t-dependence of the coupling of the virtual pion to the nucleon. The pion mass, m_π, is neglected except in the pion propagator. As we mentioned in the introduction we treat the pion pole as an OPE term. Therefore the full time-like electromagnetic form factor occurs in (5). Calculating the pion pole contribution from the GPD $\widetilde{E}$ as it is done in [11], one obtains the same expression for it but with the leading-order (LO) perturbative result for the pion form factor. In (4) it is also allowed for a possible non-pole (n.p.) part of $\widetilde{E}$.

For incident π^- mesons the $p \to n$ transition GPDs are required which, as a consequence of isospin invariance, are given by the isovector combination of proton GPDs [7]

$$K^{(3)} = K^u - K^d\,. \tag{6}$$

The convolutions of the GPDs and the amplitudes $\mathcal{H}$ for the subprocess $\pi^- q \to \gamma^* q$ read [3,4]

$$\langle K^{(3)}\rangle = \int dx\mathcal{H}_{\mu\lambda,0+}(x,\xi,Q'^2,t\simeq 0)K^{(3)}(x,\xi,t)\,. \tag{7}$$

The helicity of the final state quark is $\lambda = \mu + 1/2$ with the photon helicity, μ, being either zero or -1. Thus, the asymptotically leading longitudinal amplitude is related to a helicity-non-flip subprocess amplitude while, for transverse photons, a helicity-flip amplitude is convoluted with the transversity GPDs H_T and the combination $\bar{E}_T = 2\widetilde{H}_T + E_T$. As made explicit in (4) the transverse amplitudes are suppressed by μ_π/Q' as compared to the longitudinal ones. The mass parameter μ_π is related to the chiral condensate

$$\mu_\pi = \frac{m_\pi^2}{m_u + m_d} \tag{8}$$

(m_u, m_d are current quark masses). The subprocess amplitudes are calculated to LO of perturbation theory retaining quark transverse momenta, $\mathbf{k}_\perp$, and taking into account Sudakov suppressions while the emission and reabsorption of partons by the nucleon happens collinearly to the nucleon momenta. This so-called modified perturbative approach turns into the leading-twist result [11] for $Q'^2 \to \infty$.

Since the Sudakov factor, $\exp[-S]$, comprises gluonic radiation, resumed to all orders of perturbation theory in NLL approximation [16] which can only be efficiently performed in the impact parameter space, canonically conjugated to the $\mathbf{k}_\perp$-space, one is forced to work in the $\mathbf{b}$-space. Hence,

$$\begin{aligned}
\mathcal{H}_{\mu\lambda,0+} &= \int dz d^2b\,\hat{\Psi}_{-\lambda+}(z,-\mathbf{b})\hat{F}_{\mu\lambda,0+}(x,\xi,z,Q'^2,\mathbf{b})\\
&\qquad\times\alpha_s(\mu_R)\exp[-S(z,\mathbf{b},Q'^2)]\,.
\end{aligned} \tag{9}$$

The Fourier transforms of the hard scattering kernel and the light-cone wave function of the pion are denoted by $\hat{F}$ and $\hat{\Psi}$, respectively. The momentum fraction of the helicity $+1/2$ quark entering the pion is denoted by z; the helicity of the antiquark is $-\lambda$. For the renormalization scale we choose $\mu_R = \max(zQ', (1-z)Q', 1/b)$ and the factorization scale is $1/b$. Following Li and Sterman [16] we only retain the most important quark transverse momenta which appear in the denominators of the parton propagators in the hard scattering kernels. Therefore, we can use the light-cone projector of a $q\bar{q}$ pair on an ingoing pion in collinear approximation [17]

$$\mathcal{P}_\pi = \frac{f_\pi}{2\sqrt{2N_c}}\,\frac{\gamma_5}{\sqrt{2}}\Big\{\not{q}\,\Phi(z)$$

$$-\mu_\pi\Big[\Phi_P(z) - i\frac{\sigma_{\mu\nu}}{2N_c}\Big(\frac{q^\mu n^\nu}{q\cdot n}\Phi_\sigma(z) - q^\mu\frac{d\Phi_\sigma(z)}{dz}\frac{\partial}{\partial k_{\perp\nu}}\Big)\Big]\Big\} \tag{10}$$

and replace the distribution amplitudes by light-cone wave functions. In (10) f_π (= 132 MeV) is the pion decay constant, N_c the number of colors and n is a light-like vector which in a frame where the massless pion moves along the z-direction is $n = [0, 1, 0_\perp]$. Three-particle configurations, $q\bar{q}g$, are neglected. Dirac, flavor and color labels are omitted for convenience. The first term in (10) is the well-known twist-2 part which is employed in the calculation of $\mathcal{H}_{0\pm,0+}$. For the accompanying light-cone wave function we take, as in [3,4],

$$\Psi_{-+} = \frac{\sqrt{2N_c}}{f_\pi}\exp[-a_\pi^2\mathbf{k}_\perp^2/(z(1-z))] \tag{11}$$

with the transverse size parameter $a_\pi = \big[\sqrt{8}\pi f_\pi\big]^{-1}$ fixed from $\pi^0\to\gamma\gamma$ decay [18]. This wave function has frequently been used, see for instance [18,19]. The twist-3 part of (10) is utilized in the calculation of $\mathcal{H}_{--,0+}$. Since for this part quark and antiquark forming the pion have the same helicity orbital angular momentum, represented by factors of $\mathbf{k}_\perp$, is required. As the calculation reveals $\mathcal{H}_{--,0+}$ is dominated by the contribution from Φ_P while the tensor term provides a correction of order t/Q'^2 which is neglected for consistency. For the wave function associated to Φ_P ($\equiv 1$ as follows from the equation of motion [17,20]), we use as in our previous work[3]

$$\Psi_{++} = \frac{16\pi^{3/2}}{\sqrt{2N_c}} f_\pi a_P^3|\mathbf{k}_\perp|\exp[-a_P^2\mathbf{k}_\perp^2]\,. \tag{12}$$

The simple z-independent exponential is forced by the requirements of a constant distribution amplitude and the normalizability of the wave function. For the transverse size parameter, a_P, we take 1.8 GeV^{-1}.

3. Predictions for the partial cross sections

Before we present our predictions for the exclusive Drell–Yan process we specify the various parameters and soft hadronic functions we use in the evaluation. The form factor $F_{\pi NN}$ appearing in (5), is parametrized as

$$F_{\pi NN} = \frac{\Lambda_N^2 - m_\pi^2}{\Lambda_N^2 - t} \tag{13}$$

with $\Lambda_N = 0.44\pm0.04$ GeV. For the time-like pion electromagnetic form factor also occurring in (5), we take the average of the data from CLEO [22] and BaBar [23] as well as a value derived from the $J/\Psi\to\pi^+\pi^-$ decay [24]

$$Q'^2|F_\pi(Q'^2)| = 0.88\pm0.04\ \text{GeV}^2\,. \tag{14}$$

For its phase, $\exp[i\delta(Q'^2)]$, we rely on a recent dispersion analysis [25] which, for 2 GeV$^2 \lesssim Q'^2 \lesssim 5$ GeV^2, provides

$$\delta = 1.014\pi + 0.195(Q'^2/\text{GeV}^2 - 2) - 0.029(Q'^2/\text{GeV}^2 - 2)^2\,. \tag{15}$$

In the absence of any other information on this phase we use this parametrization up to ≈ 8.9 GeV^2 where $\delta = \pi$. For larger values of Q'^2 we take $\delta = \pi$, the asymptotic phase of the time-like pion form factor obtained by analytic continuation of the perturbative result for the space-like form factor [13].

The GPDs are constructed with the help of the familiar double distribution ansatz from the zero-skewness GPDs which are parametrized as[4]

$$K(x, \xi = 0, t) = k(x)\exp[t(b + \alpha'\ln x)] \tag{16}$$

where the forward limit, $k(x)$, is an appropriate parton distribution (PDF) or is parametrized like a PDF with parameters fitted to experiment. The GPD $\widetilde{H}(x, \xi = 0, t)$ (including an error estimate) is taken from the recent analysis of the nucleon form factors [26] which, for this GPD, is based on the DSSV polarized PDFs [27]. A non-pole contribution to $\widetilde{E}$ is neglected, there is no clear signal for it in the data on pion leptoproduction. For the zero-skewness transversity GPDs, H_T and $\bar{E}_T$, the actual values of the parameters are specified in [28]. They are oriented on lattice QCD results [29,30] and lead to fair fits of the pion leptoproduction data [5,8,9] as well as of the spin density matrix elements and transverse target spin asymmetries for vector mesons [31,28]. The errors of the transversity GPDs are estimated from the p-pole fits presented in [29,30]. For the mass parameter (8) that controls the strength of the twist-3 amplitudes, we adopt the value[5] $\mu_\pi = 2$ GeV valid at the scale 2 GeV. For its error we choose +0.55 and −0.15 [24]. The QCD coupling constant, α_s, is evaluated from the one-loop expression for four flavors and $\Lambda_{\text{QCD}} = 182$ MeV. The time-like Sudakov factor is unknown, the continuation from the space-like to the time-like region is not well understood (see [32]). The replacement of Q^2 by $-Q'^2$ (see [32,33]) leads to an oscillating phase but it is unclear whether these oscillations are physical or not. We therefore follow Gousset and Pire [32] and use the space-like Sudakov factor, as utilized in our previous work, also in the time-like region (with $Q^2\to Q'^2$). As shown in [19], for Q'^2 less than 10 GeV^2 the Sudakov factor is always close to unity except near $b = 1/\Lambda_{\text{QCD}}$ where it drops to zero sharply. With the exception of this region the wave function provides the main suppression. Therefore, the detailed behavior of the Sudakov factor is not very important.

The four-fold differential cross section for $\pi^- p\to l^- l^+ n$ reads

$$\frac{d\sigma}{dtdQ'^2d\cos\theta d\phi} = \frac{3}{8\pi}\Big\{\sin^2\theta\frac{d\sigma_L}{dtdQ'^2} + \frac{1+\cos^2\theta}{2}\frac{d\sigma_T}{dtdQ'^2} + \frac{\sin(2\theta)\cos\phi}{\sqrt{2}}\frac{d\sigma_{LT}}{dtdQ'^2} + \sin^2\theta\cos(2\phi)\frac{d\sigma_{TT}}{dtdQ'^2}\Big\} \tag{17}$$

where the angles ϕ and θ, specifying the directions of the leptons, are defined in Fig. 2. The partial cross sections are related to the $\pi^- p\to\gamma^* n$ helicity amplitudes (4) by

$$\frac{d\sigma_L}{dtdQ'^2} = \kappa\sum_{\nu'}|\mathcal{M}_{0\nu',0+}|^2\,,$$

$$\frac{d\sigma_T}{dtdQ'^2} = \kappa\sum_{\mu=\pm1,\nu'}|\mathcal{M}_{\mu\nu',0+}|^2\,,$$

---

[3] It may seem appropriate to use an $l_z = \pm1$ wave function (for a particle moving along the z-direction). Such a wave function has been proposed in [21]. It is proportional to $k_\perp^\pm = k_\perp^x \pm ik_\perp^y$. Its collinear reduction leads to the tensor piece in (10) which goes along with Φ_σ and its derivative; the important term $\sim\Phi_P$ is lacking in this ansatz.

[4] A more complicated profile function is adopted for $\widetilde{H}$, see [26].

[5] According to the recent particle data tables [24] μ_π is rather 2.6 GeV. Using this value the normalizations of H_T and $\bar{E}_T$ have to be altered accordingly since the fit to the pion leptoproduction data fixes the product of μ_π and the transversity GPDs.

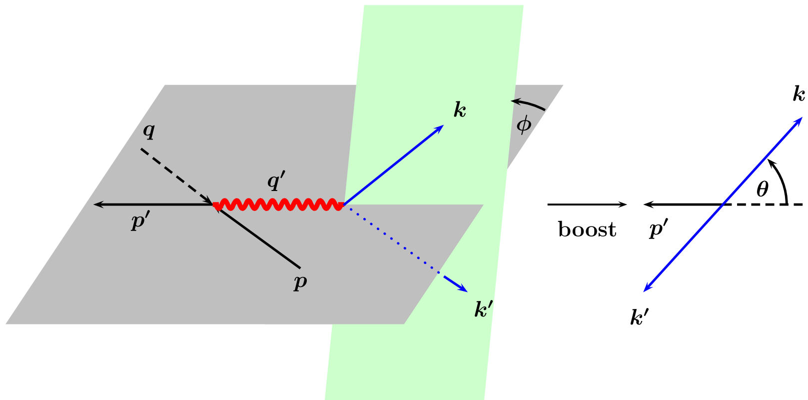

Fig. 2. Definition of the angles ϕ and θ. The latter angle is defined in the rest frame of the virtual photon.

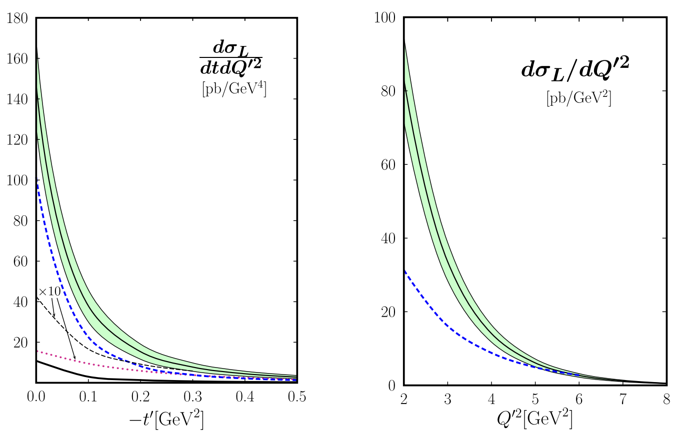

Fig. 3. The longitudinal cross sections $d\sigma_L/dtdQ'^2$ (left) at $Q'^2 = 4$ GeV^2 versus t' and $d\sigma_L/dQ'^2$ (right) versus Q'^2. The thin solid lines with error bands represent our full results at $s = 20$ GeV^2, the thick dashed ones those at 30 GeV^2. The thick solid (dotted, thin dashed) line is the interference term (contribution from $|\langle\widetilde{H}^{(3)}\rangle|^2$, leading twist). The latter two results are multiplied by 10 for the ease of legibility.

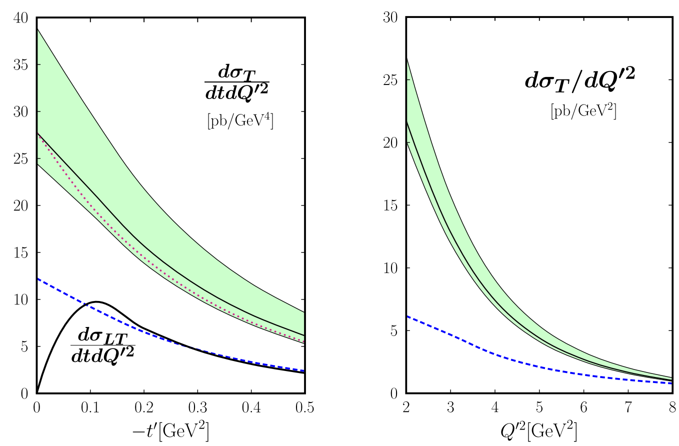

Fig. 4. The transverse cross sections $d\sigma_T/dtdQ'^2$ (left) at $Q'^2 = 4$ GeV^2 versus t' and $d\sigma_T/dQ'^2$ (right) versus Q'^2. The thin solid dashed lines with error bands represent the full result at $s = 20$ GeV^2, the thick dashed ones those at 30 GeV^2 while the dotted line is the contribution from H_T. The thick solid line represents the longitudinal–transverse interference cross section.

$$\frac{d\sigma_{LT}}{dtdQ'^2} = \kappa\ \mathrm{Re}\sum_{\nu'}\left[\mathcal{M}^*_{0\nu',0+}(\mathcal{M}_{+\nu',0+} - \mathcal{M}_{-\nu',0+})\right],$$
$$\frac{d\sigma_{TT}}{dtdQ'^2} = \kappa\ \mathrm{Re}\sum_{\nu'}\left[\mathcal{M}^*_{+\nu',0+}\mathcal{M}_{-\nu',0+}\right]. \qquad (18)$$

The normalization factor reads (lepton masses are neglected)

$$\kappa = \frac{\alpha_{\rm em}}{48\pi^2}\frac{1}{(s-m^2)^2 Q'^2}. \qquad (19)$$

In Fig. 3 we show our predictions for $d\sigma_L/dtdQ'^2$ at $Q'^2 = 4$ GeV^2 and $s = 20$ GeV^2 and $d\sigma_L/dQ'^2$ integrated over t' from 0 to -0.5 GeV^2. The longitudinal cross section is heavily dominated by the contribution from the pion pole, that one from $\widetilde{H}$, including its interference with the pion pole, amounts only to about 10% in the kinematical range of interest. The full result is markedly larger than our leading-twist result which is of the same order as that one quoted in [11]. This amplification is due to the use of the experimental value of the pion form factor (14) instead of its leading-twist result (≈ 0.15 GeV^2). We stress that the OPE contribution from the pion pole does neither rely on QCD factorization nor on a hard scattering. It is therefore not subject to evolution and higher-order perturbative QCD corrections. Because of the dominant contribution from the pion-pole and since we only consider a small range of Q'^2 around 4 GeV^2 the evolution of the GPDs is insignificant and is therefore neglected. As opposed to [11] our interference term is positive. It is generated by the imaginary parts of $\langle\widetilde{H}\rangle$ and the pion-pole contribution while, in an LO leading-twist calculation, it is evidently under control of the corresponding real parts. Constructing $\widetilde{H}$ from the polarized PDFs derived in [34] instead from the DSSV ones [27] alters the predictions for the longitudinal cross section by less than the estimated errors displayed in Fig. 3.

The transverse cross section is shown in Fig. 4. It is substantially smaller than the longitudinal cross section but much larger than the leading-twist result. The uncertainty of our predictions is rather large and asymmetric due to the asymmetric error of μ_π. The transverse cross section can be decomposed as (cf. (18) and (4))[6]

$$\frac{d\sigma_T}{dtdQ'^2} = \kappa\left[|\mathcal{M}_{--,0+}|^2 + 2|\mathcal{M}_{++,0+}(\pi)|^2 + 2|\mathcal{M}_{++,0+}(\bar{E}_T)|^2\right]. \quad (20)$$

The first term in (20), being related to the GPD H_T, is displayed in Fig. 4 separately; it dominates this cross section. The second term, the pion-pole contribution, is rather small; it generates the little difference between the contribution from H_T and the full result for $d\sigma_T$. The contribution from $\bar{E}_T$ is tiny.

[6] The $\bar{E}_T$ (π) term behaves as a natural (unnatural) parity exchange while the H_T has no specific parity behavior [3,4].

The longitudinal–transverse interference cross section is also shown in Fig. 4. The width of its error band is about a half of that of the transverse cross section. $d\sigma_{LT}$ can be written as

$$\frac{d\sigma_{LT}}{dtdQ'^2} = \kappa \, \mathrm{Re}\left[2\mathcal{M}^*_{0+,0+}\mathcal{M}_{++0+}(\pi) - \mathcal{M}^*_{0-,0+}\mathcal{M}_{--,0+}\right]. \tag{21}$$

Both the terms significantly contribute to $d\sigma_{LT}$. The transverse–transverse interference cross section is given by

$$\frac{d\sigma_{TT}}{dtdQ'^2} = \kappa\left[|\mathcal{M}_{++,0+}(\bar{E}_T)|^2 - |\mathcal{M}_{++,0+}(\pi)|^2\right]. \tag{22}$$

This cross section is very small. For instance, at $Q'^2 = 4\ \mathrm{GeV}^2$ and $s = 20\ \mathrm{GeV}^2$ it is less than $\approx 0.3\ \mathrm{pb/GeV}^4$.

The cross sections decrease with growing s. As an example we show results at $s = 30\ \mathrm{GeV}^2$ in the plots. At, say, $s \approx 360\ \mathrm{GeV}^2$ as is available from the pion beam at CERN, the longitudinal cross section is about $30\ \mathrm{fb/GeV}^2$ at $Q'^2 = 4\ \mathrm{GeV}^2$. This is likely too small to be measured.

4. Conclusions

We calculated the partial cross sections for the exclusive Drell–Yan process, $\pi^- p \to l^- l^+ n$, within the handbag approach. In contrast to a previous study of this process [11] we treat the pion pole as an OPE term and take into account transversity GPDs. The parametrizations of the GPDs $\widetilde{H}$, H_T and $\bar{E}_T$ as well as the values of other parameters appearing in the present calculation are taken from previous work [3,4,26]. The generalization of our approach to $K^- p \to l^- l^+ \Lambda$ is straightforward.

Future data on $\pi^- p \to l^- l^+ n$ measured at J-PARC may allow for a test of factorization of the process amplitudes in hard subprocesses and soft GPDs. In contrast to pion leptoproduction where there is a rigorous proof for factorization of the amplitudes for longitudinally polarized photons, factorization of the exclusive Drell–Yan process is an assumption although it seems plausible that the factorization arguments also hold for time-like photons. However, Qiu [35] conjectured that factorization may be broken for the exclusive Drell–Yan process. If however factorization holds to a sufficient degree of accuracy future data on the exclusive Drell–Yan process may improve our knowledge of the GPDs.

The exclusive Drell–Yan process also offers the opportunity to check the dependence of the $\pi\pi\gamma$ vertex on the pion virtuality by comparing data on the time-like form factor measured in $l^+ l^- \to \pi^+\pi^-$ with parametrizations of $\pi^-\pi^{+*} \to l^- l^+$ as part of the Drell–Yan analysis. The extraction of the space-like form factor from $lp \to l\pi^+ n$ data may benefit from that check.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.physletb.2015.07.016.

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