Constraining gluon poles

I.V. Anikin\textsuperscript{a,b,}\textsuperscript{*}, O.V. Teryaev\textsuperscript{a}

\textsuperscript{a} Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia
\textsuperscript{b} Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

\textbf{A R T I C L E   I N F O}

\textbf{Article history:}
Received 7 August 2015
Received in revised form 21 October 2015
Accepted 29 October 2015
Available online 10 November 2015

Editor: J.-P. Blaizot

Keywords:
Factorization theorem
Gauge invariance
Drell–Yan process

\textbf{A B S T R A C T}

In this letter, we revise the QED gauge invariance for the hadron tensor of Drell–Yan type processes with the transversely polarized hadron. We perform our analysis within the Feynman gauge for gluons and make a comparison with the results obtained within the light-cone gauge. We demonstrate that QED gauge invariance leads, first, to the need of a non-standard diagram and, second, to the absence of gluon poles in the correlators $\langle \bar{\psi} \gamma_\perp A^\mu \psi \rangle$ related traditionally to $dT(x, \tilde{x})/dx$. As a result, these terms disappear from the final QED gauge invariant hadron tensor. We also verify the absence of such poles by analyzing the corresponding light-cone Dirac algebra.

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1. Introduction

In the recent times, we have observed the renaissance in the nucleon structure studies through the Drell–Yan type processes in the existing (Fermilab, Relativistic Heavy Ion Collider, see \cite{1,2}) and future (J-PARC, NICA) experiments. One of the most interesting subjects of such experimental studies in this direction is the so-called single spin asymmetry (SSA) which is expressed with the help of the hadron tensor, see for instance \cite{3} or \cite{4,5}.

Lately, we have reconsidered \cite{6} this process in the contour gauge. We have found that there is a contribution from the non-standard diagram which produces the imaginary phase required to have the SSA. This additional contribution leads to an extra factor of 2 for the asymmetry. This conclusion was supported by analysis of the QED gauge invariance of the hadron tensor.

In comparison, the analysis presented in \cite{7} which uses the axial and Feynman gauge does not support the latter conclusion. For this reason, we perform here the detailed analysis of hadron tensor in the Feynman gauge with the particular emphasis on the QED gauge invariance. We find that the QED gauge invariance can be maintained only by taking into account the non-standard diagram. Moreover, the results in the Feynman and contour gauges coincide if the gluon poles in the correlators $\langle \bar{\psi} \gamma_\perp A^\mu \psi \rangle$ are absent. This is in agreement with the relation between gluon poles and the Sivers function which corresponds to the “leading twist” Dirac matrix $\gamma^\perp$. We confirm this important property by comparing the light-cone dynamics for different correlators.

As a result, we derive the QED gauge invariant hadron tensor which completely coincides with the expression obtained within the light-cone contour gauge for gluons, see \cite{6}.

2. Kinematics

We study the hadron tensor which contributes to the single spin (left-right) asymmetry measured in the Drell–Yan process with the transversely polarized nucleon (see Fig. 1):

\begin{equation}
N^{\perp+}(p_1) + N(p_2) \rightarrow \gamma^\perp(q) + X(P_X) \\
\rightarrow \ell(l_1) + \bar{\ell}(l_2) + X(P_X).
\end{equation}

Here, the virtual photon producing the lepton pair $(l_1 + l_2 = q)$ has a large mass squared $(q^2 = Q^2)$ while the transverse momenta are small and integrated out. The left-right asymmetry means that the transverse momenta of the leptons are correlated with the direction $S \times e_z$ where $S_\mu$ implies the transverse polarization vector of the nucleon while $e_z$ is a beam direction \cite{8}.

Since we perform our calculations within a \textit{collinear} factorization, it is convenient to fix the dominant light-cone directions as

\begin{equation}
p_1 \approx \frac{Q}{x_B \sqrt{2}} n^*, \quad p_2 \approx \frac{Q}{y_B \sqrt{2}} n, \\
n^*_\mu = (1/\sqrt{2}, 0_T, 1/\sqrt{2}), \quad n^\mu = (1/\sqrt{2}, 0_T, -1/\sqrt{2}).
\end{equation}

So, the hadron momenta $p_1$ and $p_2$ have the plus and minus dominant light-cone components, respectively. Accordingly, the quark
and gluon momenta $k_1$ and $\ell$ lie along the plus direction while the antiquark momentum $k_2$—along the minus direction. The photon momentum reads (see Fig. 1)

$$q = l_1 + l_2 = k_1 + k_2$$

which, after factorization, will take the form:

$$q = x_1 p_1 + y p_2 + q_T.$$  

3. The DY hadron tensor

We work within the Feynman gauge for gluons. The standard hadron tensor generated by the diagram depicted in Fig. 1 (the left panel) reads

\[ dW^{\mu\nu}_{(\text{Stand.})} = \int d^4 k_1 d^4 k_2 \delta^{(4)} (k_1 + k_2 - q) \]

\[ \times \int d^4 \ell \Phi^{(A)1\nu}(k_1, \ell) \Phi^{\nu-1}(k_2) \]

\[ \times \text{tr} \left[ \gamma^\mu \gamma^\beta \gamma^\gamma \gamma^\alpha S(\ell - k_2) \right], \]

where

\[ \Phi^{(A)1\nu}(k_1, \ell) = \mathcal{F}_2 \left[ (p_1, S^T | \bar{\psi}(\eta_1) \gamma_\rho A_\rho (z) \psi(0) | S^T, p_1) \right]; \]

\[ \Phi^{\nu-1}(k_2) = \mathcal{F}_1 \left[ (p_2 | \bar{\psi}(\eta_2) \gamma^\nu \gamma^\rho \psi(0) | p_2) \right]. \]

Throughout this paper, $\mathcal{F}_1$ and $\mathcal{F}_2$ denote the Fourier transformation with the measures

\[ d^4 \eta_1 e^{ik_2 \eta_2} \quad \text{and} \quad d^4 \eta_1 e^{-ik_2 \eta_2}, \]

respectively, while $\mathcal{F}_1^{-1}$ and $\mathcal{F}_2^{-1}$ mark the inverse Fourier transformation with the measures

\[ dy e^{iy\lambda} \quad \text{and} \quad dx_1 dx_2 e^{i(x_1 \lambda_1 + i(x_2 - x_1) \lambda_2)} \]

We now implement the factorization procedure (see for instance [9,11]) which contains the following steps: (a) the decomposition of loop integration momenta around the corresponding dominant direction: $k_1 = x_1 p + (k_1 - p) n + k_T$ within the certain light cone basis formed by the vectors $p$ and $n$ (in our case, $n^* = n$); (b) the replacement: $d^4 k_i \rightarrow d^4 k_i dx \delta(x_i - k_i - n)$ that introduces the fractions with the appropriated spectral properties; (c) the decomposition of the corresponding propagator products around the dominant direction. In Eqn. (5), we have (here, $x_{ij} = x_i - x_j$)

\[ S(\ell - k_2) = S(x_{21} p_1 - y p_2) \]

\[ + \frac{\partial S(\ell - k_2)}{\partial k} \bigg|_{k_2 = p_2} + \ldots ; \]

(d) the use of the collinear Ward identity:

\[ \frac{\partial S(k)}{\partial k} = S(k) \gamma_\rho \psi(0) \]

\[ \frac{\partial S(k)}{\partial k} = \gamma_\rho \psi(0) \psi(0); \]

(e) performing of the Fierz decomposition for $\psi_\alpha(z) \bar{\psi}_\beta(0)$ in the corresponding space up to the needed projections.

After factorization, the standard tensor, see Eqn. (5), is split into two terms: the first term includes the correlator without the transverse derivative, while the second term contains the correlator with the transverse derivative, see Eqns. (10) and (16)–(18).

The non-standard contribution comes from the diagram depicted in Fig. 1 (the right panel). The corresponding hadron tensor takes the form [6]:

\[ dW^{\mu\nu}_{(\text{Non-stand.})} = \int d^4 k_1 d^4 k_2 \delta^{(4)} (k_1 + k_2 - q) \text{tr} \left[ \gamma^\mu \mathcal{F}(k_1) \gamma^\nu \Phi(k_2) \right], \]

where the function $\mathcal{F}(k_1)$ reads

\[ \mathcal{F}(k_1) = S(k_1) \gamma^\alpha \int d^4 \eta_1 e^{-i k_1 \eta_1} \]

\[ \times \langle p_1, S^T | \bar{\psi}(\eta_1) A_\rho (0) \psi(0) | S^T, p_1 \rangle. \]

For convenience, we introduce the unintegrated tensor $\bar{W}^{\mu\nu}$ for the factorized hadron tensor $W^{\mu\nu}$ of the process. It reads

\[ \gamma \lambda^{\mu\nu} = \int d^2 \bar{q}_T d\lambda^{\mu\nu} = \frac{2}{q^2} \int d^2 \bar{q}_T \delta^{(2)} (\bar{q}_T) \]

\[ \times i \int dx_1 dx_2 \eta \left[ \delta(x_1 / x_2 - 1) \delta(y / y - 1) \right] \bar{W}^{\mu\nu}. \]

After calculation of all relevant traces in the factorized hadron tensor and after some algebra, we arrive at the following contributions for the unintegrated hadron tensor (which involves all relevant contributions except the mirror one): the standard diagram depicted in Fig. 1, the left panel, gives us

\[ \bar{W}^{\mu\nu}_{(\text{Stand.})} + \bar{W}^{\mu\nu}_{(\text{Stand.}, \partial)} \]

\[ = \bar{q}(y) \left[ -\frac{p_1^\mu}{y} e^{i \delta y} \right] \int dx_2 x_1 - x_2 B^{(1)}(x_1, x_2) \]
1 Generally speaking, in the Feynman gauge the arguments how to derive the

certain complex prescription differ from that we used in [6]. For example, the

prescription can be defined by ordering of operator positions on the light-cone di-

rection.
One can conclude that, in the case of the substantial transverse component of the momentum, there are no sources for the gluon poles at \( x_1 = x_2 \). As a result, the function \( B^{(1)}(x_1, x_2) \) has no gluon poles and, due to T-invariance [11] \( (B^{(2)}(x_1, x_2) = -B^{(2)}(x_2, x_1)) \), obeys \( B^{(2)}(x, x) = 0 \).

On the other hand, if we have \( y^+ \) in the correlator (see Eqn. (16)), the transverse components of gluon momentum are not substantial and can be neglected. That ensures the existence of the gluon poles for the function \( B^{(1)}(x_1, x_2) \). This corresponds to the fact that the Sivers function, being related to gluon poles, contains the “leading twist” projector \( y^+ \). Moreover, we may conclude that the structure \( y^+(\hat{a}A^+) \) does not produce the imaginary part as well as SSA in the Feynman gauge.

### 5. Conclusions and discussions

Working within the Feynman gauge, we derive the QED gauge invariant (unintegrated) hadron tensor for the polarized DY process:

\[
W^{\mu
u}_{\text{Gi}} = W^{\mu
u}_{(\text{Non-stand.})} + W^{\mu
u}_{(\text{Stand.})}
\]

\[
= \tilde{q}(y)
\left[
\frac{p_\mu}{x_B} - p_\mu
\right]
\epsilon^{\nu - \mu - 2T} - p_\mu \int dx_2 B^{(1)}(x_1, x_2).
\]  

(29)

After calculating the imaginary part (or, in other words, after adding the mirror contributions), and then integrating over \( x_1 \) and \( y \) (see Eqn. (13)), we get the QED gauge invariant hadron tensor as

\[
W^{\mu
u}_{\text{Gi}} = \tilde{q}(y)
\left[
\frac{p_\mu}{x_B} - p_\mu
\right]
\epsilon^{\nu - \mu - 2T} - p_\mu \int T(x_B, x_B).
\]  

(30)

This expression fully coincides with the hadron tensor which has been derived within the light-cone gauge for gluons.

Moreover, the factor of 2 in the hadron tensor that we found within the axial-type gauge [6] is still present in the frame of the Feynman gauge. In order to show this factor of 2, let us introduce the mutually orthogonal basis (see [8]) as

\[
Z_\mu = \tilde{p}_1 \mu - \tilde{p}_2 \mu \equiv x_B p_1 \mu - y_B p_2 \mu
\]  

(31)

and

\[
X_\mu = \frac{2}{S} \left[ (Zp_2) (p_1 \mu - \frac{q_\mu}{2x_B}) - (Zp_1) (p_2 \mu - \frac{q_\mu}{2y_B}) \right],
\]

\[
Y_\mu = \frac{2}{S} \bar{\epsilon}_\mu p_1 p_2 q.
\]  

(32)

Here \( \tilde{p}_1 \mu \) and \( \tilde{p}_2 \mu \) are the partonic momenta \( (q^\mu = \tilde{p}_1 \mu + \tilde{p}_2 \mu) \). With the help of (31) and (32), the lepton momenta can be written as (this is the lepton c.m. system)

\[
l_1 \mu = \frac{1}{2} q_\mu + \frac{Q}{2} f_\mu(\theta, \psi; \hat{X}, \hat{Y}, \hat{Z}),
\]

\[
l_2 \mu = \frac{1}{2} q_\mu - \frac{Q}{2} f_\mu(\theta, \psi; \hat{X}, \hat{Y}, \hat{Z}),
\]

(33)

where \( \hat{A} = \hat{A}/\sqrt{-\hat{A}^2} \) and

\[
f_\mu(\theta, \psi; \hat{X}, \hat{Y}, \hat{Z})
\]  

(34)

Within this frame, the contraction of the lepton tensor with the gauge invariant hadron tensor (30) reads

\[
\mathcal{L}_{\mu
u} \cdot W^{\mu
u}_{\text{Gi}} = -2 \cos \theta \epsilon^{\mu
u} \tilde{p}_1 p_2 q \tilde{T}(x_B, x_B).
\]  

(35)

We want to emphasize that this expression in (35) differs by the factor of 2 in comparison with the case where only one diagram (presented in Fig. 1, the left panel) has been included in the (gauge non-invariant) hadron tensor, i.e.

\[
\mathcal{L}_{\mu
u} \cdot W^{\mu
u}_{(\text{Stand.})} = \frac{1}{2} \mathcal{L}_{\mu
u} \cdot W^{\mu
u}_{\text{Gi}}.
\]  

(36)

Therefore, from the practical point of view, if we neglect the diagram in Fig. 1 (right panel) or, in other words, if we use the QED gauge non-invariant hadron tensor, it yields the error of the factor of two.

Further, based on the light-cone dynamics we argue that there are no gluon poles in the correlators \( (\hat{\psi} \gamma_\mu A^+ \hat{\psi}) \). This means that the function \( B^{(2)}(x_1, x_2) \) does not have the representation similar to (19). We also show that the Lorentz and QED gauge invariances of the hadron tensor calculated within the Feynman gauge require that the function \( B^{(2)}(x_1, x_2) \) cannot have gluon poles.

The fact that the function \( B^{(2)}(x_1, x_2) \) cannot be presented in the form of (19) directly leads to the absence of \( dt/dx \) in the final expression of the gauge-invariant hadron tensor. Indeed, from (14), one can see that \( B^{(2)}(x_1, x_2) \) contributes to the standard hadron tensor as

\[
\int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon}.
\]  

(37)

In order to obtain the \( dt/dx \)-contribution, we have to impose the representation (19) on \( B^{(2)}(x_1, x_2) \) and, then perform the integration over \( dx_2 \) by part. However, as shown above, \( B^{(2)}(x_1, x_2) \) does not have the representation (19).
This property seems to be natural from the point of view of gluon poles relation [12] to Sivers functions as the latter is related to the projection $\gamma^+$. As for the function $B^{(1)}(x_1, x_2)$, the transverse derivative of Sivers function resulting from taking its moments may act on both integrand and boundary value. Our result suggests that only the action on the boundary value related to $B^{(1)}(x_1, x_2)$ should produce SSA. It is certainly not unnatural keeping in mind that the integrand differentiation is present even for simple straight-line contours which are not producing SSA.

Acknowledgements

We thank A.V. Efremov and A. Prokudin for useful discussions. The work by I.V.A. was partially supported by the Heisenberg-Landau Program of the German Research Foundation (DFG).

References