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Constraining gluon poles

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ABSTRACT

In this letter, we revise the QED gauge invariance for the hadron tensor of Drell–Yan type processes with the transversely polarized hadron. We perform our analysis within the Feynman gauge for gluons and make a comparison with the results obtained within the light-cone gauge. We demonstrate that QED gauge invariance leads, first, to the need of a non-standard diagram and, second, to the absence of gluon poles in the correlators $\langle \bar{\psi}\gamma_\perp A^+\psi\rangle$ related traditionally to $dT(x,x)/dx$. As a result, these terms disappear from the final QED gauge invariant hadron tensor. We also verify the absence of such poles by analyzing the corresponding light-cone Dirac algebra.

1. Introduction

In the recent times, we have observed the renaissance in the nucleon structure studies through the Drell–Yan type processes in the existing (FermiLab, Relativistic Heavy Ion Collider, see [1,2]) and future (J-Parc, NICA) experiments. One of the most interesting subjects of such experimental studies in this direction is the so-called single spin asymmetry (SSA) which is expressed with the help of the hadron tensor, see for instance [3] or [4,5].

Lately, we have reconsidered [6] this process in the contour gauge. We have found that there is a contribution from the *non-standard* diagram which produces the imaginary phase required to have the SSA. This additional contribution leads to an extra factor of 2 for the asymmetry. This conclusion was supported by analysis of the QED gauge invariance of the hadron tensor.

In comparison, the analysis presented in [7] which uses the axial and Feynman gauges does not support the latter conclusion. For this reason, we perform here the detailed analysis of hadron tensor in the Feynman gauge with the particular emphasis on the QED gauge invariance. We find that the QED gauge invariance can be maintained only by taking into account the non-standard diagram. Moreover, the results in the Feynman and contour gauges coincide if the gluon poles in the correlators $\langle \bar{\psi}\gamma_\perp A^+\psi\rangle$ are absent. This is in agreement with the relation between gluon poles and the Sivers function which corresponds to the "leading twist" Dirac matrix γ^+. We confirm this important property by comparing the light-cone dynamics for different correlators.

As a result, we derive the QED gauge invariant hadron tensor which completely coincides with the expression obtained within the light-cone contour gauge for gluons, see [6].

2. Kinematics

We study the hadron tensor which contributes to the single spin (left-right) asymmetry measured in the Drell–Yan process with the transversely polarized nucleon (see Fig. 1):

$$N^{(\uparrow\downarrow)}(p_1) + N(p_2) \to \gamma^*(q) + X(P_X) \to \ell(l_1) + \bar{\ell}(l_2) + X(P_X). \qquad (1)$$

Here, the virtual photon producing the lepton pair ($l_1 + l_2 = q$) has a large mass squared ($q^2 = Q^2$) while the transverse momenta are small and integrated out. The left-right asymmetry means that the transverse momenta of the leptons are correlated with the direction $\mathbf{S} \times \mathbf{e}_z$ where S_μ implies the transverse polarization vector of the nucleon while $\mathbf{e}_z$ is a beam direction [8].

Since we perform our calculations within a *collinear* factorization, it is convenient to fix the dominant light-cone directions as

$$p_1 \approx \frac{Q}{x_B\sqrt{2}}n^*, \quad p_2 \approx \frac{Q}{y_B\sqrt{2}}n,$$
$$n^{*\,\mu} = (1/\sqrt{2},\, \mathbf{0}_T,\, 1/\sqrt{2}), \quad n^\mu = (1/\sqrt{2},\, \mathbf{0}_T,\, -1/\sqrt{2}). \qquad (2)$$

So, the hadron momenta p_1 and p_2 have the plus and minus dominant light-cone components, respectively. Accordingly, the quark

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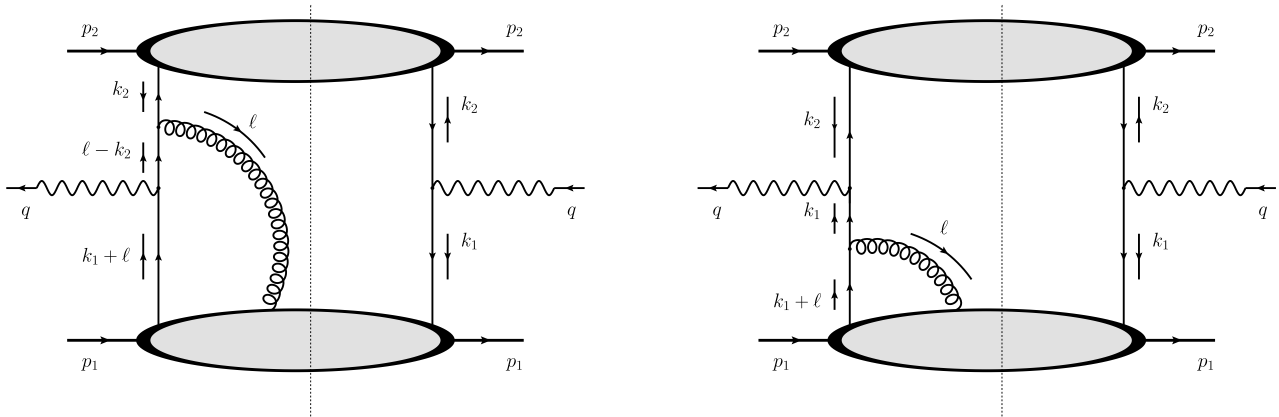

Fig. 1. The Feynman diagrams which contribute to the polarized Drell–Yan hadron tensor.

and gluon momenta k_1 and ℓ lie along the plus direction while the antiquark momentum k_2 – along the minus direction. The photon momentum reads (see Fig. 1)

$$q = l_1 + l_2 = k_1 + k_2 \tag{3}$$

which, after factorization, will take the form:

$$q = x_1 p_1 + y p_2 + q_T. \tag{4}$$

3. The DY hadron tensor

We work within the Feynman gauge for gluons. The standard hadron tensor generated by the diagram depicted in Fig. 1 (the left panel) reads

$$\begin{aligned} d\mathcal{W}^{\mu\nu}_{(\text{Stand.})} &= \int d^4k_1\, d^4k_2\, \delta^{(4)}(k_1+k_2-q) \\ &\times \int d^4\ell\, \Phi^{(A)\,[\gamma_\beta]}_\alpha(k_1,\ell)\, \bar{\Phi}^{[\gamma^-]}(k_2) \\ &\times \mathrm{tr}\big[\gamma^\mu \gamma^\beta \gamma^\nu \gamma^+ \gamma^\alpha S(\ell-k_2)\big], \end{aligned} \tag{5}$$

where

$$\Phi^{(A)\,[\gamma_\beta]}_\alpha(k_1,\ell) = \mathcal{F}_2\Big[\langle p_1, S^T|\bar{\psi}(\eta_1)\gamma_\beta g A_\alpha(z)\psi(0)|S^T, p_1\rangle\Big], \tag{6}$$

$$\bar{\Phi}^{[\gamma^-]}(k_2) = \mathcal{F}_1\Big[\langle p_2|\bar{\psi}(\eta_2)\gamma^-\psi(0)|p_2\rangle\Big]. \tag{7}$$

Throughout this paper, $\mathcal{F}_1$ and $\mathcal{F}_2$ denote the Fourier transformation with the measures

$$d^4\eta_2\, e^{ik_2\cdot\eta_2} \text{ and } d^4\eta_1\, d^4z\, e^{-ik_1\cdot\eta_1 - i\ell\cdot z}, \tag{8}$$

respectively, while $\mathcal{F}_1^{-1}$ and $\mathcal{F}_2^{-1}$ mark the inverse Fourier transformation with the measures

$$dy\, e^{iy\lambda} \text{ and } dx_1 dx_2\, e^{ix_1\lambda_1 + i(x_2-x_1)\lambda_2}. \tag{9}$$

We now implement the *factorization procedure* (see for instance [9,11]) which contains the following steps: (a) the decomposition of loop integration momenta around the corresponding dominant direction: $k_i = x_i p + (k_i \cdot p)n + k_T$ within the certain light cone basis formed by the vectors p and n (in our case, n^* and n); (b) the replacement: $d^4k_i \Longrightarrow d^4k_i\, dx_i \delta(x_i - k_i \cdot n)$ that introduces the fractions with the appropriated spectral properties; (c) the decomposition of the corresponding propagator products around the dominant direction. In Eqn. (5), we have (here, $x_{ij} = x_i - x_j$)

$$\begin{aligned} S(\ell-k_2) &= S(x_{21}p_1 - yp_2) \\ &+ \frac{\partial S(\ell-k_2)}{\partial \ell_\rho}\bigg|^{k_2=yp_2}_{\ell=x_{21}p_1} \ell^T_\rho + \ldots; \end{aligned} \tag{10}$$

(d) the use of the collinear Ward identity:

$$\frac{\partial S(k)}{\partial k_\rho} = S(k)\gamma_\rho S(k), \quad S(k) = \frac{-\not{k}}{k^2 + i\varepsilon};$$

(e) performing of the Fierz decomposition for $\psi_\alpha(z)\,\bar{\psi}_\beta(0)$ in the corresponding space up to the needed projections.

After factorization, the standard tensor, see Eqn. (5), is split into two terms: the first term includes the correlator without the transverse derivative, while the second term contains the correlator with the transverse derivative, see Eqns. (10) and (16)–(18).

The non-standard contribution comes from the diagram depicted in Fig. 1 (the right panel). The corresponding hadron tensor takes the form [6]:

$$\begin{aligned} &d\mathcal{W}^{\mu\nu}_{(\text{Non-stand.})} \\ &= \int d^4k_1\, d^4k_2\, \delta^{(4)}(k_1+k_2-q)\mathrm{tr}\big[\gamma^\mu \mathcal{F}(k_1)\gamma^\nu \bar{\Phi}(k_2)\big], \end{aligned} \tag{11}$$

where the function $\mathcal{F}(k_1)$ reads

$$\begin{aligned} \mathcal{F}(k_1) &= S(k_1)\gamma^\alpha \int d^4\eta_1\, e^{-ik_1\cdot\eta_1} \\ &\times \langle p_1, S^T|\bar{\psi}(\eta_1)\, gA_\alpha(0)\, \psi(0)|S^T, p_1\rangle. \end{aligned} \tag{12}$$

For convenience, we introduce the unintegrated tensor $\overline{\mathcal{W}}_{\mu\nu}$ for the factorized hadron tensor $\mathcal{W}_{\mu\nu}$ of the process. It reads

$$\begin{aligned} \mathcal{W}^{\mu\nu} &= \int d^2\vec{\mathbf{q}}_T d\mathcal{W}^{\mu\nu} = \frac{2}{q^2}\int d^2\vec{\mathbf{q}}_T\, \delta^{(2)}(\vec{\mathbf{q}}_T) \\ &\times i\int dx_1\, dy\,\big[\delta(x_1/x_B - 1)\delta(y/y_B - 1)\big]\overline{\mathcal{W}}^{\mu\nu}. \end{aligned} \tag{13}$$

After calculation of all relevant traces in the factorized hadron tensor and after some algebra, we arrive at the following contributions for the unintegrated hadron tensor (which involves all relevant contributions except the mirror ones): the standard diagram depicted in Fig. 1, the left panel, gives us

$$\begin{aligned} &\overline{\mathcal{W}}^{\mu\nu}_{(\text{Stand.})} + \overline{\mathcal{W}}^{\mu\nu}_{(\text{Stand.},\, \partial_\perp)} \\ &= \bar{q}(y)\Bigg\{ -\frac{p_1^\mu}{y}\,\varepsilon^{\nu S^T - p_2} \int dx_2 \frac{x_1 - x_2}{x_1 - x_2 + i\epsilon} B^{(1)}(x_1, x_2) \end{aligned}$$

$$- \left[\frac{p_2^\nu}{x_1} \varepsilon^{\mu S^T - p_2} + \frac{p_2^\mu}{x_1} \varepsilon^{\nu S^T - p_2} \right] x_1 \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon}$$
$$+ \frac{p_1^\mu}{y} \varepsilon^{\nu S^T - p_2} \int dx_2 \frac{B^{(\perp)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Bigg\}, \quad (14)$$

while the non-standard diagram presented in Fig. 1, the right panel, contributes as

$$\overline{\mathcal{W}}^{\mu\nu}_{\text{(Non-stand.)}} = \bar{q}(y) \frac{p_2^\mu}{x_1} \varepsilon^{\nu S^T - p_2} \times \int dx_2 \left\{ B^{(1)}(x_1, x_2) + B^{(2)}(x_1, x_2) \right\}. \quad (15)$$

Here we introduce the shorthand notation: $\varepsilon^{ABCD} = \varepsilon^{\mu_1\mu_2\mu_3\mu_4} A_{\mu_1} B_{\mu_2} C_{\mu_3} D_{\mu_4}$ with $\varepsilon^{0123} = 1$. Moreover, the parametrizing functions are associated with the following correlators:

$$i\varepsilon^{\alpha + S^T -}(p_1 p_2)\, B^{(1)}(x_1, x_2)$$
$$= \mathcal{F}_2 \left[\langle p_1, S^T | \bar{\psi}(\eta_1)\, \gamma^+\, g A_\perp^\alpha(z)\, \psi(0) | S^T, p_1 \rangle \right], \quad (16)$$

$$i\varepsilon^{+\beta S^T -}(p_1 p_2)\, B^{(2)}(x_1, x_2)$$
$$= \mathcal{F}_2 \left[\langle p_1, S^T | \bar{\psi}(\eta_1)\, \gamma_\perp^\beta\, g A^+(z)\, \psi(0) | S^T, p_1 \rangle \right], \quad (17)$$

$$i p_1^+ \varepsilon^{\rho + S^T -}(p_1 p_2) B^{(\perp)}(x_1, x_2)$$
$$= \mathcal{F}_2 \left[\langle p_1, S^T | \bar{\psi}(\eta_1)\, \gamma^+ \left(\partial_\perp^\rho\, g A^+(z) \right) \psi(0) | S^T, p_1 \rangle \right], \quad (18)$$

where $\eta_1 = \lambda_1 \tilde{n}$, $z = \lambda_2 \tilde{n}$, and the light-cone vector $\tilde{n}$ is a dimensionful analog of n ($\tilde{n}^- = p_2^- / (p_1 p_2)$).

As known from [6], the function $B^{(1)}(x_1, x_2)$ for the DY process can be unambiguously written as

$$B^{(1)}(x_1, x_2) = \frac{T(x_1, x_2)}{x_1 - x_2 + i\varepsilon}, \quad (19)$$

where the function $T(x_1, x_2) \in \Re\mathrm{e}$ parametrizes the corresponding projection of $\langle \bar{\psi}\, G_{\alpha\beta}\, \psi \rangle$, *i.e.*

$$\varepsilon^{\alpha + S^T -}(p_1 p_2)\, T(x_1, x_2)$$
$$= \mathcal{F}_2 \left[\langle p_1, S^T | \bar{\psi}(\eta_1)\, \gamma^+ \tilde{n}_\nu G_T^{\nu\alpha}(z)\, \psi(0) | S^T, p_1 \rangle \right]. \quad (20)$$

Notice that we have derived (see [6]) the certain complex prescription in the *r.h.s.* of (19) within the contour gauge. In this letter, we assume that the same prescription takes place in the Feynman gauge too.[1] With respect to the functions $B^{(2)}(x_1, x_2)$ and $B^{(\perp)}(x_1, x_2)$, we demonstrate below that these functions do not possess the gluon poles and, therefore, cannot be presented in the form of (19).

Summing up all contributions from the standard and non-standard diagrams, we finally obtain the expression for the unintegrated hadron tensor as

$$\overline{\mathcal{W}}^{\mu\nu} = \overline{\mathcal{W}}^{\mu\nu}_{\text{(Stand.)}} + \overline{\mathcal{W}}^{\mu\nu}_{\text{(Stand., }\partial_\perp)} + \overline{\mathcal{W}}^{\mu\nu}_{\text{(Non-stand.)}}$$
$$= \bar{q}(y) \Bigg\{ \left[\frac{p_2^\mu}{x_1} - \frac{p_1^\mu}{y} \right] \varepsilon^{\nu S^T - p_2} \int dx_2\, B^{(1)}(x_1, x_2)$$
$$+ \frac{p_2^\mu}{x_1} \varepsilon^{\nu S^T - p_2} \int dx_2\, B^{(2)}(x_1, x_2)$$

$$- \left[\frac{p_2^\nu}{x_1} \varepsilon^{\mu S^T - p_2} + \frac{p_2^\mu}{x_1} \varepsilon^{\nu S^T - p_2} \right] x_1 \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon}$$
$$+ \frac{p_1^\mu}{y} \varepsilon^{\nu S^T - p_2} \int dx_2 \frac{B^{(\perp)}(x_1, x_2)}{x_1 - x_2 + i\epsilon} \Bigg\}. \quad (21)$$

Notice that the first term in Eqn. (21) coincides with the hadron tensor calculated within the light-cone gauge $A^+ = 0$.

4. QED gauge invariance of hadron tensor

Let us now discuss the QED gauge invariance of the hadron tensor. From Eqn. (21), we can see that the QED gauge invariant combination is

$$\mathcal{T}^{\mu\nu} = \left[\frac{p_2^\mu}{x_1} - \frac{p_1^\mu}{y} \right] \varepsilon^{\nu S^T - p_2},$$
$$\text{with} \quad q_\mu \mathcal{T}^{\mu\nu} = q_\nu \mathcal{T}^{\mu\nu} = 0. \quad (22)$$

We can see that there is a single term with p_2^ν which does not have a counterpart to construct the gauge-invariant combination

$$\frac{p_2^\mu}{x_1} - \frac{p_1^\mu}{y}. \quad (23)$$

Therefore, the second term in Eqn. (14) should be equal to zero. This also leads to nullification of the second term in Eqn. (15).

Hence, the only way to get the QED gauge invariant combination (see (22)) is to combine the first terms in Eqns. (14) and (15). This combination justifies the treatment of gluon pole in $B^{(1)}(x_1, x_2)$ using the complex prescription.

In addition, we conclude that the third term in (14) does not contribute to SSA.

The suggested proof explores only the gauge and Lorentz invariance. Let us consider the other reasoning to justify these properties of correlators, starting with the correlator which generates the function $B^{(2)}(x_1, x_2)$:

$$\int (d\lambda_1\, d\lambda_2) e^{-ix_1\lambda_1 - i(x_2 - x_1)\lambda_2}$$
$$\times \langle p_1, S^T | \bar{\psi}(\lambda_1 \tilde{n})\, \gamma_\perp^\beta\, A^+(\lambda_2 \tilde{n})\, \psi(0) | S^T, p_1 \rangle$$
$$= i\varepsilon^{+\beta S^T -}(p_1 p_2)\, B^{(2)}(x_1, x_2). \quad (24)$$

We are going over to the momentum representation for the correlator from the l.h.s. of Eqn. (24). Schematically, we have

$$\left[\bar{u}(k_1) \gamma_\beta^\perp u(k_2) \right] \times \ldots \times \frac{1}{\ell^2 + i\varepsilon}, \quad (25)$$

where the gluon momentum is $\ell = k_2 - k_1$ and $k_1 = (x_1 p_1^+, k_1^-, \vec{\mathbf{k}}_{1\perp})$, $k_2 = (x_2 p_1^+, k_2^-, \vec{\mathbf{k}}_{2\perp})$. This situation has been illustrated in Fig. 2, see the left panel. Up to the order of g, we are also able to write down that (see Fig. 2, the right panel)

$$\left[\bar{u}(k_1) \gamma_\beta^\perp \mathcal{S}(k_2) u(k_1) \right] \times \ldots \times \frac{1}{\ell^2 + i\varepsilon}, \quad (26)$$

where $\mathcal{S}(k_2) = S(k_2)\gamma^+$. From both these equations, it is clear that to get the non-zero contribution we must have either $\vec{\mathbf{k}}_{1\perp} \neq 0$ or $\vec{\mathbf{k}}_{2\perp} \neq 0$. Indeed,

$$\left[\bar{u}(k_1) \gamma_\beta^\perp \mathcal{S}(k_2) u(k_1) \right] \Rightarrow S_{\beta k_2 + k_1} = k_{2\,\beta}^\perp k_1^+ + k_{1\,\beta}^\perp k_2^+. \quad (27)$$

Therefore, the gluon propagator in Eqns. (25) and (26) takes the following form (cf. [10]):

$$\frac{1}{\ell^2 + i\varepsilon} = \frac{1}{2(x_2 - x_1) p_1^+ \ell^- - \vec{\mathbf{l}}_\perp^2 + i\varepsilon}. \quad (28)$$

[1] Generally speaking, in the Feynman gauge the arguments how to derive the certain complex prescription differ from that we used in [6]. For example, the prescription can be defined by ordering of operator positions on the light-cone direction.

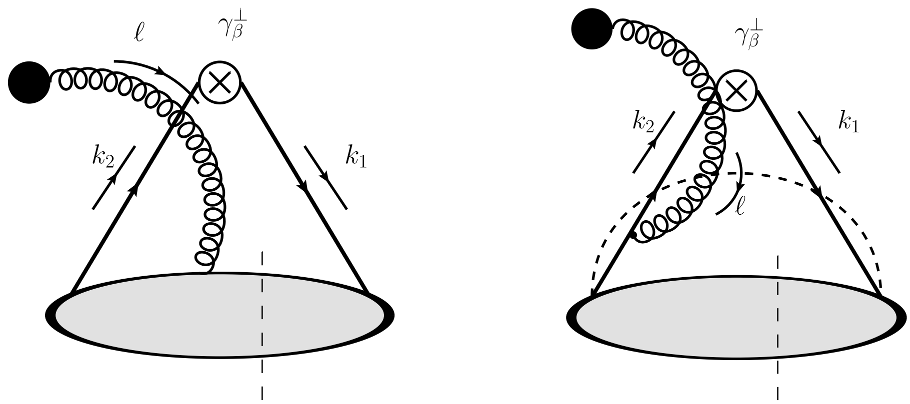

Fig. 2. The matrix element (correlator) of nonlocal twist-3 quark–gluon operator within the momentum representation. Here $\ell = k_2 - k_1$ and $k_1 = (x_1 p_1^+, k_1^-, \vec{\mathbf{k}}_{1\perp})$, $k_2 = (x_2 p_1^+, k_2^-, \vec{\mathbf{k}}_{2\perp})$.

One can conclude that, in the case of the substantial transverse component of the momentum, there are no sources for the gluon poles at $x_1 = x_2$. As a result, the function $B^{(2)}(x_1, x_2)$ has no gluon poles and, due to T-invariance [11] ($B^{(2)}(x_1, x_2) = -B^{(2)}(x_2, x_1)$), obeys $B^{(2)}(x, x) = 0$.

On the other hand, if we have γ^+ in the correlator (see Eqn. (16)), the transverse components of gluon momentum are not substantial and can be neglected. That ensures the existence of the gluon poles for the function $B^{(1)}(x_1, x_2)$. This corresponds to the fact that the Sivers function, being related to gluon poles, contains the "leading twist" projector γ^+. Moreover, we may conclude that the structure $\gamma^+(\partial^\perp A^+)$ does not produce the imaginary part as well as SSA in the Feynman gauge.

5. Conclusions and discussions

Working within the Feynman gauge, we derive the QED gauge invariant (unintegrated) hadron tensor for the polarized DY process:

$$\overline{\mathcal{W}}^{\mu\nu}_{\rm GI} = \overline{\mathcal{W}}^{\mu\nu}_{\rm (Non\text{-}stand.)} + \overline{\mathcal{W}}^{\mu\nu}_{\rm (Stand.)}$$
$$= \bar{q}(y)\Big[\frac{p_2^\mu}{x_1} - \frac{p_1^\mu}{y}\Big]\varepsilon^{\nu S^T - p_2} \int dx_2 B^{(1)}(x_1, x_2). \qquad (29)$$

After calculating the imaginary part (or, in other words, after adding the mirror contributions), and, then integrating over x_1 and y (see Eqn. (13)), we get the QED gauge invariant hadron tensor as

$$W^{\mu\nu}_{\rm GI} = \bar{q}(y_B)\Big[\frac{p_2^\mu}{x_B} - \frac{p_1^\mu}{y_B}\Big]\varepsilon^{\nu S^T - p_2}\, T(x_B, x_B)\,. \qquad (30)$$

This expression fully coincides with the hadron tensor which has been derived within the light-cone gauge for gluons.

Moreover, the factor of 2 in the hadron tensor that we found within the axial-type gauge [6] is still present in the frame of the Feynman gauge. In order to show this factor of 2, let us introduce the mutually orthogonal basis (see [8]) as

$$Z_\mu = \widehat{p}_{1\,\mu} - \widehat{p}_{2\,\mu} \equiv x_B\, p_{1\,\mu} - y_B\, p_{2\,\mu} \qquad (31)$$

and

$$X_\mu = -\frac{2}{s}\Big[(Zp_2)\Big(p_{1\,\mu} - \frac{q_\mu}{2x_B}\Big) - (Zp_1)\Big(p_{2\,\mu} - \frac{q_\mu}{2y_B}\Big)\Big],$$
$$Y_\mu = \frac{2}{s}\,\varepsilon_{\mu p_1 p_2 q}. \qquad (32)$$

Here $\widehat{p}_{i\,\mu}$ are the partonic momenta ($q^\mu = \widehat{p}_{1\,\mu} + \widehat{p}_{2\,\mu}$). With the help of (31) and (32), the lepton momenta can be written as (this is the lepton c.m. system)

$$l_{1\,\mu} = \frac{1}{2}q_\mu + \frac{Q}{2} f_\mu(\theta, \varphi; \hat{X}, \hat{Y}, \hat{Z})\,,$$
$$l_{2\,\mu} = \frac{1}{2}q_\mu - \frac{Q}{2} f_\mu(\theta, \varphi; \hat{X}, \hat{Y}, \hat{Z})\,, \qquad (33)$$

where $\hat{A} = A/\sqrt{-A^2}$ and

$$f_\mu(\theta, \varphi; \hat{X}, \hat{Y}, \hat{Z})$$
$$= \hat{X}_\mu \cos\varphi \sin\theta + \hat{Y}_\mu \sin\varphi \sin\theta + \hat{Z}_\mu \cos\theta\,. \qquad (34)$$

Within this frame, the contraction of the lepton tensor with the gauge invariant hadron tensor (30) reads

$$\mathcal{L}_{\mu\nu} W^{\mu\nu}_{\rm GI} = -2\cos\theta\, \varepsilon^{l_1 S^T p_1 p_2}\, \bar{q}(y_B)\, T(x_B, x_B)\,. \qquad (35)$$

We want to emphasize that this expression in (35) differs by the factor of 2 in comparison with the case where only one diagram (presented in Fig. 1, the left panel) has been included in the (gauge non-invariant) hadron tensor, *i.e.*

$$\mathcal{L}_{\mu\nu} W^{\mu\nu}_{\rm (Stand.)} = \frac{1}{2}\mathcal{L}_{\mu\nu} W^{\mu\nu}_{\rm GI}\,. \qquad (36)$$

Therefore, from the practical point of view, if we neglect the diagram in Fig. 1 (right panel) or, in other words, if we use the QED gauge non-invariant hadron tensor, it yields the error of the factor of two.

Further, based on the light-cone dynamics we argue that there are no gluon poles in the correlators $\langle\bar{\psi}\gamma_\perp A^+\psi\rangle$. This means that the function $B^{(2)}(x_1, x_2)$ does not have the representation similar to (19). We also show that the Lorentz and QED gauge invariances of the hadron tensor calculated within the Feynman gauge require that the function $B^{(2)}(x_1, x_2)$ cannot have gluon poles.

The fact that the function $B^{(2)}(x_1, x_2)$ cannot be presented in the form of (19) directly leads to the absence of dT/dx in the final expression of the gauge-invariant hadron tensor. Indeed, from (14), one can see that $B^{(2)}(x_1, x_2)$ contributes to the standard hadron tensor as

$$\Big[p_2^\nu \varepsilon^{\mu S^T - p_2} + p_2^\mu \varepsilon^{\nu S^T - p_2}\Big] \int dx_2 \frac{B^{(2)}(x_1, x_2)}{x_1 - x_2 + i\epsilon}. \qquad (37)$$

In order to obtain the dT/dx-contribution, we have to impose the representation (19) on $B^{(2)}(x_1, x_2)$ and, then perform the integration over dx_2 by part. However, as shown above, $B^{(2)}(x_1, x_2)$ does not have the representation (19).

This property seems to be natural from the point of view of gluon poles relation [12] to Sivers functions as the latter is related to the projection γ^+. As for the function $B^{(\perp)}(x_1, x_2)$, the transverse derivative of Sivers function resulting from taking its moments may act on both integrand and boundary value. Our result suggests that only the action on the boundary value related to $B^{(1)}(x_1, x_2)$ should produce SSA. It is certainly not unnatural keeping in mind that the integrand differentiation is present even for simple straight-line contours which are not producing SSA.

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