Teachers’ Content Knowledge and Pedagogical Content Knowledge: The Role of Structural Differences in Teacher Education

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Abstract

Pedagogical content knowledge (PCK) and content knowledge (CK) are key components of teacher competence that affect student progress. However, little is known about how teacher education affects the development of CK and PCK. To address this question, our research group constructed tests to directly assess mathematics teachers’ CK and PCK. Based on these tests, we compared the PCK and CK of four groups of mathematics teachers at different points in their teaching careers in Germany. Confirmatory factor analyses showed that PCK and CK measurement was satisfactorily invariant across the teacher populations considered. As expected, the largest differences in CK and PCK were found between the beginning and the end of initial teacher education. Differences in the structures of teacher education were reasonably well reflected in participants’ CK and PCK.

Keywords

teacher education, content knowledge, pedagogical content knowledge, regression analysis, measurement invariance

In recent decades, educational research has provided compelling evidence that the quality of the learning opportunities created by teachers affects students’ learning and motivation (Hattie, 2009; McCaffrey, Lockwood, Koretz, Louis, & Hamilton, 2004). Particular interest has been directed toward teachers’ knowledge of subject matter: their content knowledge (CK) and pedagogical content knowledge (PCK). Both types of knowledge have been shown to affect teachers’ instructional practice as well as student learning in the domain of mathematics (Baumert et al., 2010; Hill, Rowan, & Ball, 2005).

Given the importance of teacher knowledge for student progress, teacher education can be regarded as a key target and lever of educational reform. However, the understanding of how teacher education programs affect the development of professional knowledge remains limited (Cochran-Smith & Zeichner, 2005). One of the main challenges for research on teacher education lies in the assessment of teacher knowledge. In fact, it is only recently that test instruments have been developed to proximally assess components of teacher knowledge—primarily, in the domain of mathematics (Hill et al., 2005; Krauss, Brunner, et al., 2008; Schmidt et al., 2007; Tato & Senk, 2011).

This article aims at investigating the impact of structural differences in teacher education on teachers’ CK and PCK. Therefore, we conducted a cross-sectional comparison with German pre- and inservice mathematics teachers at different points in their teaching careers.

CK and PCK and Their Impact on Instructional Practice and Student Learning

In the 1980s, Shulman identified research on the content-specific characteristics of teachers and of instruction as the “missing paradigm” of research on learning and instruction (Shulman, 1986, 1987). Stimulated by Shulman’s ideas, a growing body of teacher research has since addressed

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teacher knowledge of subject matter (Ball, Thames, & Phelps, 2008; Woolfolk Hoy, Davis, & Pape, 2006), focusing on two main constructs: CK and PCK (Ball et al., 2008; Borko & Putnam, 1996; Grossman, 1990; Shulman, 1987). Although the definitions of these constructs vary across research groups (Hill et al., 2005; Krauss, Brunner, et al., 2008; Park & Oliver, 2008), there seems to be consensus on some crucial aspects.

CK represents teachers’ understanding of the subject matter taught. According to Shulman (1986), “[t]he teacher need not only understand that something is so, the teacher must further understand why it is so” (p. 9). Thus, the emphasis is on a deep understanding of the subject matter taught at school. Consequently, teachers’ CK differs from the academic research knowledge generated at institutes of higher education as well as from mathematical everyday knowledge that adults retain after leaving school (Krauss, Brunner, et al., 2008).

PCK is the knowledge needed to make subject matter accessible to students (Shulman, 1986). Literature on PCK identified two core facets of that knowledge: knowledge of students’ subject-specific conceptions and misconceptions as well as knowledge of subject-specific teaching strategies and representations (see also Ball et al., 2008; Borko & Putnam, 1996; Park & Oliver, 2008).

Despite the clear theoretical distinction between CK and PCK, findings on their empirical separability are mixed. Hill, Schilling, and Ball (2004) found that elementary teachers’ CK and PCK in mathematics are merged in a single body of knowledge that they termed mathematical knowledge for teaching (MKT). Other studies found that CK and PCK represent two correlated but separable and unique dimensions (Krauss, Brunner, et al., 2008; Phelps & Schilling, 2004). Krauss, Brunner, et al. (2008) concluded that the latent structure of subject-matter knowledge might vary between different teacher populations. There is some consensus and some preliminary evidence for the notion that CK might be a prerequisite for PCK development. We will go into that later.

Especially since Shulman’s publications, research on teachers’ knowledge of subject matter (CK and PCK) has been driven by the assumption that this knowledge is at the heart of their professional competence (Ball, Lubienski, & Mewborn, 2001; Shulman, 1986; Woolfolk Hoy et al., 2006). Indeed, recent studies have provided strong, representative evidence that teachers’ subject-matter knowledge affects their instructional practice and their students’ achievement gains. Hill et al. (2005) found that elementary teachers’ MKT was substantially associated with student gains in mathematical understanding (Hill et al., 2008). Drawing on data from a longitudinal extension to the 2003 cycle of the Organization for Economic Cooperation and Development’s Programme for International Student Assessment (PISA) in Germany, Baumert and colleagues (2010) showed that PCK and CK affect student learning. However, despite the high correlation between CK and PCK, CK had lower predictive power for student progress than did PCK. Furthermore, PCK had the decisive impact on key aspects of instructional quality. Against this background, the question of how teacher education affects the development of teachers’ subject-specific knowledge is crucial to educational reform (Ball et al., 2001).

Research on the Development of Teachers’ CK and PCK

Teacher knowledge develops through pre- and in-service teachers’ engagement with a variety of explicit and implicit learning opportunities (Munby, Russell, & Martin, 2001; Schön, 1987; Sternberg & Grigorenko, 2003). In the following, we systematize the learning environments in which teachers have the opportunity to acquire and develop knowledge of subject matter, and we summarize research on the development of CK and PCK.

Learning Opportunities for the Development of CK and PCK

Teachers gain their knowledge for teaching from various sources (Grossman, 1990); the same can be expected to apply to teacher knowledge of subject matter. Drawing on Grossman’s research, Friedrichsen et al. (2009) distinguished three potential sources of subject-matter knowledge: (a) teachers’ own K-12 learning experiences, (b) teacher education and professional development programs, and (c) teaching experiences. The point that professional knowledge begins to develop even before candidates enter teacher education had already been made by Lortie (1975), who argued that prospective teachers’ professional knowledge and beliefs are significantly shaped by their own school experiences (i.e., the “apprenticeship of observation”; Lortie, 1975). In the context of mathematics, the “pretraining” phase (Feiman-Nemser, 1983) is thought not only to instill (often traditional) approaches to teaching and learning mathematics but also to influence the development of prospective teachers’ understanding of mathematics (Ball et al., 2001).

Clearly, the three types of learning opportunities described by Grossman differ in their levels of formalization and intentional construction (Tynjälä, 2008). Formal learning opportunities are organized and structured by institutions on the basis of learning objectives; they generally lead to qualifications. Formal learning is mainly intentional—That is, the learner has the explicit objective of acquiring knowledge and skills. Informal learning, in contrast, is not intentionally organized and takes place incidentally, as a “side effect” (e.g., of work; Tynjälä, 2008). It has no set objective in terms of learning outcomes and is usually highly contextualized. It is often referred to as learning by experience—or just experience (Tynjälä, 2008; Werquin, 2010). Informal, but deliberative, learning situations (e.g., mentoring, learning in peer groups, and intentional practicing of certain skills or tools) have been described as nonformal learning (Werquin, 2010). In contrast to formal learning, nonformal learning takes place...
outside educational institutions and does not generally lead to qualifications (Werquin, 2010).

Reconsidering Grossman’s three sources of teachers’ professional knowledge in the terms of this classification of learning opportunities, we can conclude not only that the school mathematics curriculum offers formal learning opportunities for acquiring CK in the pretraining phase but also that learning situations prior to teacher education facilitate the informal construction of PCK (e.g., through observation of one’s own teachers). Second, teacher education and professional development programs provide opportunities to acquire CK and PCK by attending workshops and lectures (formal learning opportunities), collaborating with peers, and in teaching practice (nonformal and informal learning opportunities). Third, teaching experience is a prototypical form of informal learning.

How Do These Learning Opportunities Affect the Development of CK and PCK?

Despite the importance attributed to teachers’ knowledge of subject matter, the understanding of how the learning opportunities available during teacher education and professional development affect the development of subject-specific knowledge is still limited (Cochran-Smith & Zeichner, 2005; Darling-Hammond, Wei, Andree, Richardson, & Orphanos, 2009). More and better research on the outcomes of teacher education is urgently needed (Cochran-Smith & Zeichner, 2005).

Qualitative studies. Many of the available studies investigating the development of CK and PCK in teacher education or professional development are small samples or case studies (Cochran-Smith & Zeichner, 2005; Darling-Hammond et al., 2008; De Jong & Van Driel, 2004; Friedrichsen et al., 2009; Richardson & Placier, 2001; Zembal-Saul, Krajcik, & Blumenfeld, 2002). Qualitative studies allow in-depth insights into change in teachers’ knowledge of subject matter and have provided first evidence that teacher education and professional development may affect the development of CK and PCK. Specifically, these studies have addressed two main issues in the development of teachers’ subject-matter knowledge.

CK as a prerequisite for PCK. PCK implies a transformation of subject-matter knowledge, so that it can be used effectively and flexibly in the interaction between teachers and learners in the classroom (Shulman, 1987). In the teacher knowledge literature, there is some consensus that the degree of conceptual understanding of the respective content provides the scope for PCK development (Ball et al., 2001; Baumert et al., 2010; Friedrichsen et al., 2009). It is well documented that pre- and inservice teachers often themselves have misconceptions and fragmented knowledge that limit, for example, their response to student conceptions or their ability to create cognitively challenging learning situations (Haidar, 1997; Halim & Meraah, 2002; Van Driel, Verloop, & De Vos, 1998). CK is therefore regarded as a necessary prerequisite for the development of PCK (Friedrichsen et al., 2009). However, strong CK does not necessarily lead to the development of PCK (Lee, Brown, Luft, & Roehrig, 2007).

The role of reflection and deliberate practice in PCK development. Research on the development of PCK emphasizes the role of teaching experience in the integration of CK with knowledge of student thinking and teaching strategies (Friedrichsen et al., 2009; Lee et al., 2007). However, teaching experience alone does not seem to be sufficient (Friedrichsen et al., 2009). Several studies suggest that teaching experience needs to be coupled with thoughtful reflection on instructional practice, with nonformal learning through interactions with colleagues, and with deliberate formal learning opportunities (Brouwer & Korthagen, 2005; Park & Oliver, 2008; Van Driel, 2010; Zembal-Saul et al., 2002). These results are in line with findings from research on the development of expertise, which emphasize the role of deliberate practice (Ericsson, Krampe, & Tesch-Römer, 1993).

Large-scale studies. Much of the available quantitative research investigating the relationship between teacher knowledge and learning opportunities in teacher education or professional development programs is based on self-report measures (Brouwer & Korthagen, 2005; Darling-Hammond, Chung, & Frelow, 2002; additionally drawing on competence ratings given by cooperating teachers: Garet, Porter, Desimone, Birman, & Yoon, 2001). To date, few studies have assessed teacher knowledge proximally by means of tests.

In a study with experienced inservice teachers, Brunner and colleagues found that secondary mathematics teachers’ job experience (years being a teacher) was not correlated with their scores on a PCK test (Brunner et al., 2006). This result is in line with the findings highlighting the role of reflection and deliberate practice. The Mathematics Teaching in the 21st Century study (MT21; Schmidt et al., 2007), which was a pilot study for the Teacher Education and Development Study in Mathematics (TEDS-M), investigated characteristics of formal teacher education systems in six countries and assessed preservice teachers’ professional knowledge at the end of their formal teacher education. The findings indicate that teacher education affects preservice teachers’ CK and PCK (Schmidt et al., 2007). For example, the authors found that future U.S. teachers whose teacher education programs included rigorous and demanding mathematics courses showed higher mathematical CK than those who attended other programs (Schmidt et al., 2007). Additional cross-sectional analyses with German preservice teachers at the beginning of their teacher education, at the end of the university-based phase, and at the end of the induction phase showed that CK and PCK increased over time (Blömeke et al., 2008; we found similar results in a pilot
study for the present study: Krauss, Baumert, & Blum, 2008. The more recent TEDS-M (Tatto & Senk, 2011) again highlighted the role of deliberative formal learning opportunities for preservice teachers’ subject-matter knowledge. The CK and PCK of mathematics teacher candidates at the end of their initial teacher education varied substantially across the participating countries and across the different types of teacher preparation programs implemented within these countries (Blömeke, Suhl, & Kaiser, 2011; Tatto & Senk, 2011). In sum, the available studies underline the importance of deliberative formal learning opportunities in the development of CK and PCK.

However, this overview of research reveals at least three research desiderata. To date, knowledge of the comparability of CK and PCK test scores across different teacher populations (measurement invariance) is limited. It remains uncertain whether the tests developed allow fair comparisons of the respective groups. Furthermore, it is unclear to what extent the differences in CK and PCK reported in MT21 and TEDS-M were indeed caused by differences in teacher education programs—or by other differences in the teacher populations. For example, the findings showing differences in the CK and PCK of preservice teachers who attended different teacher education programs may be confounded by general cognitive ability. Indeed, Klusmann, Trautwein, Lüdtke, Kunter, and Baumert (2009) showed that preservice teachers enrolled in different teacher education programs differed in several characteristics (e.g., general cognitive ability and high school grade point average [GPA]). Similar problems may affect international comparisons, as teacher candidates’ characteristics may also differ across countries. Finally, it seems promising to consider not only teacher candidates in formal teacher education but also in-service teachers, thus covering a larger span of the teaching career, including a phase characterized by more informal learning.

The Present Study: Investigating the Impact of Structural Differences in Teacher Education on Mathematical CK and PCK—The German Example

The German teacher education system is a prime example for studying the impact of teacher education on teachers’ knowledge of subject matter for two main reasons. First, its two clearly separated phases—a university-based phase and a classroom-based induction phase—offer distinct learning opportunities for CK and PCK. Second, it provides a natural experiment for examining the effects of teacher education because preservice teachers preparing to teach in the academic track are educated separately from those preparing for nonacademic-track schools. In almost all German federal states (Länder), school students approaching the end of Grade 4 are assigned to different secondary tracks—usually in separate schools—on the basis of their aptitude and ability. The number of tracks in the different states ranges from two to four. In all states, however, a clear distinction is made between the academic track (Gymnasium, Grades 5-13) and the nonacademic tracks (all other school types; predominantly, Grades 5-10; Baumert et al., 2010). Furthermore, the content and structure of the first phase of teacher education differs markedly depending on whether candidates intend to teach in academic- or nonacademic-track schools.

Two Phases of Preservice Teacher Education and the Inservice Phase in Germany

First phase of preservice teacher education. The first phase of teacher education in Germany takes place in a university or equivalent institution. It offers primarily formal and also nonformal (e.g., peer learning) learning opportunities for CK and PCK in two subjects as well as general pedagogical studies and internships in schools. Teacher candidates for the academic track study for about nine semesters, with a focus on CK in their two teaching subjects. Candidates for the nonacademic track study for about seven semesters; their classes focus on PCK and pedagogy. Both groups graduate with the First State Examination (Eurydice, 2009/2010; Viebahn, 2003). Relative to the United States, teacher education in Germany is highly standardized by the requirements of this examination.

Second phase of preservice teacher education. The practical induction phase takes 18 to 24 months. During this phase of their education, teacher candidates generally spend 3.5 to 4 days per week at a regular school, during which they observe classroom instruction and gradually take on responsibility for teaching lessons of their own under the supervision of a mentor teacher. The other 1 to 1.5 days per week are spent at teacher education institutes, where they continue their studies of educational theory and subject-related pedagogy. Thus, the induction phase provides broad informal learning opportunities for the development of PCK through lesson observations as well as guided and independent teaching in training schools. In addition, courses at training institutes provide formal learning opportunities for the acquisition of PCK and general pedagogical knowledge (PK). Finally, PCK can develop through nonformal learning—primarily, within peer groups of teacher candidates. In contrast to the first phase of teacher education, there are almost no formal learning opportunities for CK. This second phase of German teacher education is completed with the Second State Examination (Eurydice, 2009/2010; Viebahn, 2003).

Inservice phase. Most learning opportunities for in-service teachers to develop their CK and PCK are informal or
Research Question and Hypotheses

Against this background, we investigated how teachers’ knowledge of subject matter (CK and PCK) differs across the three phases of teacher education in Germany: from the beginning to the end of university studies (first phase), to the end of the induction period (second phase), and finally during inservice teaching (third phase). As mentioned above, professional education programs targeting teacher candidates for the academic track differ from those targeting candidates for the nonacademic tracks in Germany, especially with respect to the quantity of learning opportunities for the development of CK. The following hypotheses therefore guided our investigation of the development of CK and PCK in the three phases of teacher education in Germany:

Hypotheses concerning the development of CK. The broadest formal learning opportunities for CK are provided in the university-based phase of teacher education. However, teacher candidates preparing to teach in academic track schools have up to twice as many learning opportunities for mathematical CK than do those preparing to teach in the nonacademic tracks. We therefore expected the groups’ CK at the beginning of teacher education, as prospective academic-track teachers are known to be more interested in the subject matter and more likely to opt for advanced mathematics courses at school2 (Kleickmann & Anders, in press).

Hypotheses concerning the development of PCK. The first phase of teacher education also provides the broadest formal learning opportunities for the development of PCK; however, the induction phase also seems to offer sound learning opportunities in this respect. Research has identified a combination of teaching experience, guided reflection, and formal learning opportunities as conducive to the development of PCK (De Jong & Van Driel, 2004; Friedrichsen et al., 2009; Lee et al., 2007). As teaching experience alone seems to be insufficient (Brunner et al., 2006) and professional development workshops on PCK during inservice training tend to be fragmented, noncumulative, and voluntary, we expected the inservice phase to have only a weak effect on PCK development. We did not expect the differences between (pre- and inservice) teachers of the academic versus nonacademic tracks to be as pronounced as for CK, as the differences in the available learning opportunities are not as pronounced. However, assuming that CK is an important prerequisite for the development of PCK, differences in CK we hypothesized for the start of teacher education and especially for the end of the university-based phase of teacher education may contribute to differential gains in PCK during pre- and inservice teacher education.

Method

Study Design and Samples

We drew on cross-sectional data from four samples of German pre- and inservice mathematics teachers: (a) Year 1 teacher education students, (b) Year 3 teacher education students, (c) teacher candidates at the end of the induction phase (“student teachers”), and (d) experienced inservice teachers (see Table 1).

All samples derive from the Cognitive Activation in the Classroom (COACTIV) research program conducted at the Max Planck Institute for Human Development, Berlin. COACTIV was initiated to investigate relations between secondary mathematics teachers’ professional knowledge, instructional practice, and student achievement gains as well as the development of teachers’ professional competence (Baumert et al., 2010; Kunter et al., in press).

Preservice mathematics teachers at the beginning versus end of university studies. These two samples comprised 243 mathematics teacher candidates recruited from universities in four cities (Berlin, Kassel, Kiel, and Flensburg). The curricula for mathematics teacher education are relatively similar across German universities and states. Thus, differences in the learning opportunities for CK and PCK across universities can be assumed to be relatively small. Participants were recruited via flyers and announcements made in lectures. The first sample comprised 117 students in the first semester of their university studies (“Year 1 students”) and the second sample, 126 students in at least the fifth semester (“Year 3 students”).

Preservice teachers at the end of the internship phase. This sample stems from the COACTIV–Referendariat study (Loewen, Baumert, Kunter, Krauss, & Brunner, in press). It comprises 539 preservice mathematics teachers in their 2nd (i.e., last) year of the induction phase (student teachers). Participants were recruited in four states (Bavaria, Baden-Württemberg, North Rhine-Westphalia, and Schleswig-Holstein) via announcements in training institutes. Analyses of the selectivity of the sampling process showed that participants did not differ from nonparticipants in terms of school track, GPA at high school, or gender (Kunter & Klusmann,
Data were assessed by a short questionnaire administered to all preservice teachers at the respective training institutes.

**German sample of inservice mathematics teachers.** This sample of 198 German mathematics teachers stems from the main COACTIV study, which was embedded in the longitudinal extension to PISA 2003, which spanned Grade 9 to the end of Grade 10 (Loewen et al., in press). The mathematics teachers of the PISA classes formed the teacher sample for the COACTIV study. However, one type of nonacademic-track schools (Hauptschule) was excluded from the longitudinal study because, in some states, this school type ends with Grade 9. To ensure comparability with the samples of preservice teachers, we therefore also excluded prospective Hauptschule teachers from these samples. Overall, 86.7% of teachers teaching Grade 10 classes sampled in the nationally representative PISA study took part in COACTIV.

As shown in Table 1, the four samples differed in several respects. The German samples of pre- and inservice teachers differed significantly in terms of interest in mathematics, the proportion of females and (prospective) academic track teachers, enrollment in advanced mathematics courses at school, and the distribution of high school GPA. We therefore controlled for these variables when computing differences in teachers’ knowledge of subject matter.

**Tests of CK and PCK**

We used paper-and-pencil tests to assess (prospective) teachers’ mathematical CK and PCK. The CK test comprises 23 items tapping arithmetic, algebra, geometry, and functions. The items are designed to assess conceptual understanding of the contents of the secondary-level mathematics curriculum and require complex mathematical argumentation or proofs (Krauss, Brunner, et al., 2008). A sample item is shown in Figure A1 in the appendix.

The PCK test assesses three facets: students (11 items), instruction (17 items), and tasks (8 items). The student facet assesses the ability to recognize students’ misconceptions, difficulties, and strategies to solve a problem. To this end, teachers are presented with classroom situations and asked to detect, analyze, or predict typical student errors or sources of student misunderstanding. Within the MKT framework, this facet corresponds to the “Knowledge of Content and Students (KCS)” domain (Ball et al., 2008). The instruction facet taps knowledge of different representations and explanations of standard mathematical problems. Having a large repertoire of representations and explanations on one’s disposal should be a key resource for scaffolding student learning. This facet equates to the “Knowledge of Content and Teaching (KCT)” category in the MKT model. Tasks play an important role in

**Table 1.** Descriptive Data for the Four Samples of Pre- and Inservice Mathematics Teachers: Means, Standard Deviations, and Significance Tests

<table>
<thead>
<tr>
<th>Variable</th>
<th>Year 1 students&lt;sup&gt;a&lt;/sup&gt; (n = 117)</th>
<th>Year 3 students&lt;sup&gt;b&lt;/sup&gt; (n = 126)</th>
<th>Student teachers&lt;sup&gt;c&lt;/sup&gt; (n = 539)</th>
<th>Experienced teachers (n = 198)</th>
<th>Test of group differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M (SD)</td>
<td>M (SD)</td>
<td>M (SD)</td>
<td>M (SD)</td>
<td>F / χ²(df) / p</td>
</tr>
<tr>
<td>Age</td>
<td>20.62 (2.12)</td>
<td>24.05 (3.08)</td>
<td>28.43 (3.83)</td>
<td>47.16 (8.50)</td>
<td></td>
</tr>
<tr>
<td>Teaching experience</td>
<td></td>
<td>1.35 (0.48)</td>
<td>21.01 (9.83)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cognitive ability test (figure analogies)</td>
<td>17.34 (3.75)</td>
<td>17.20 (3.83)</td>
<td>17.54 (3.11)</td>
<td></td>
<td>0.60 .55</td>
</tr>
<tr>
<td>Interest in mathematics</td>
<td>2.98 (0.60)</td>
<td>3.13 (0.66)</td>
<td>3.13 (0.57)</td>
<td>3.39 (0.48)</td>
<td>13.96 .00</td>
</tr>
<tr>
<td>Gender (female)</td>
<td>61.54a</td>
<td>66.94a</td>
<td>63.79b</td>
<td>46.96a</td>
<td>6.27 .00</td>
</tr>
<tr>
<td>Academic track</td>
<td>67.83&lt;sup&gt;a&lt;/sup&gt;</td>
<td>33.90&lt;sup&gt;b&lt;/sup&gt;</td>
<td>53.43&lt;sup&gt;a&lt;/sup&gt;</td>
<td>42.93&lt;sup&gt;b&lt;/sup&gt;</td>
<td>11.41 .00</td>
</tr>
<tr>
<td>Advanced math course</td>
<td>72.65&lt;sup&gt;a&lt;/sup&gt;</td>
<td>58.73&lt;sup&gt;b&lt;/sup&gt;</td>
<td>76.99&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td>8.91 .00</td>
</tr>
<tr>
<td>High school GPA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt;2</td>
<td>23.08</td>
<td>26.61</td>
<td>38.21</td>
<td>30.46</td>
<td>24.19 (3) .00</td>
</tr>
<tr>
<td>2.0-2.2</td>
<td>15.38</td>
<td>12.90</td>
<td>16.35</td>
<td>19.54</td>
<td></td>
</tr>
<tr>
<td>2.3-2.5</td>
<td>18.80</td>
<td>15.32</td>
<td>18.06</td>
<td>23.56</td>
<td></td>
</tr>
<tr>
<td>2.6-2.9</td>
<td>23.93</td>
<td>23.39</td>
<td>16.35</td>
<td>18.97</td>
<td></td>
</tr>
<tr>
<td>3.0-3.4</td>
<td>16.24</td>
<td>20.16</td>
<td>10.27</td>
<td>6.90</td>
<td></td>
</tr>
<tr>
<td>&gt;3.4</td>
<td>2.56</td>
<td>1.61</td>
<td>0.76</td>
<td>0.57</td>
<td></td>
</tr>
</tbody>
</table>

Note: GPA = grade point average. Groups with the same subscript letter did not differ significantly on the respective variable. German GPA scores run from 1 to 6 and a 1 indicates the best performance.

<sup>a</sup>First semester students.

<sup>b</sup>Students in at least the fifth semester.

<sup>c</sup>Preservice teachers at the end of the induction phase of teacher education.
the teaching of mathematics. Therefore, the task facet taps the ability to identify multiple ways to solve a mathematical problem. Identifying multiple solutions of a task should be necessary to assess the potential of that task for student learning. Thus, this facet represents aspects of KCT as well as aspects of the “Specialized Content Knowledge (SCK)” category in the MKT framework (Ball et al., 2008). Because tasks are an integral part in the teaching of mathematics, we attributed the task facet to the PCK dimension (for further details, see Baumert et al., 2010; Krauss, Brunner, et al., 2008). A sample item for each facet of the PCK test is presented in Figure A1 in the appendix. For reasons of validity, all questions were open ended. Items with no response or an incorrect response were scored 0; each correct answer was scored 1 (for items requiring several answers, for example, the multiple solution tasks, the sum of the correct answers was calculated). To be able to implement more items and to ensure that we did not exceed 2 hr of test time, the tests were administered in a multimatrix design with three booklets.

The piloting of the CK and the PCK tests involved individual interviews, expert ratings of content validity, and extensive analyses of construct validity (Krauss, Baumert, et al., 2008; Krauss, Brunner, et al., 2008). The main COACTIV study provided evidence for the criterion validity of the CK and the PCK tests: Both were shown to produce measures that were substantially related to teachers’ instructional practice and student achievement gains (Baumert et al., 2010). The tests were conducted by trained test administrators in standardized situations as power tests with no time limits. The mean time to complete both tests was about 2 hr. The items were coded by trained raters following a standardized manual. About 25% of the material was independently coded by two raters. Interrater agreement was satisfactory, with a mean $\rho = .82$ and $SD = 0.16$ (Brennan, 2001; Shavelson & Webb, 1991). Both the CK and the PCK tests showed satisfactory reliability in terms of internal consistency, as shown in Table 2. Because the reliabilities of the PCK facets tasks, instruction, and students were not satisfactory, we conducted our analyses only on the basis of the global PCK score.

Table 2. Reliabilities (Cronbach’s Alpha) of the Tests of CK and PCK by Teacher Subgroup

<table>
<thead>
<tr>
<th>Teacher Subgroup</th>
<th>n</th>
<th>CK Cronbach’s alpha</th>
<th>PCK Cronbach’s alpha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preservice teachers (university phase)</td>
<td>243</td>
<td>.73</td>
<td>.71</td>
</tr>
<tr>
<td>Preservice teachers (induction phase)</td>
<td>539</td>
<td>.76</td>
<td>.73</td>
</tr>
<tr>
<td>German inservice teachers</td>
<td>198</td>
<td>.83</td>
<td>.77</td>
</tr>
</tbody>
</table>

Note: CK = content knowledge; PCK = pedagogical content knowledge.

Investigating Measurement Invariance

The intended group comparisons can be conducted only if the measures of participants’ CK and PCK are comparable—i.e., equivalent—across the four groups of pre- and inservice teachers. To test for measurement invariance between the groups, we therefore conducted a series of CFA following an approach that is well established in the literature on structural equation modeling (Meredith, 1993; Vandenberg & Lance, 2000). In the first step, we tested a configural invariance model, in which the same pattern of fixed and free factor loadings was specified for each group. Building on previous analyses (Krauss, Brunner, et al., 2008), we assumed the CK and PCK items to load on two distinct but correlated factors. In the second step, we tested a metric invariance model, in which factor loadings for like items were set to be invariant across groups. Third, we tested a scalar invariance model, which required the intercepts/thresholds of like items’ regressions on the latent factor to be invariant across groups. Scalar invariance is seen as a necessary condition for comparing the means of different groups (Vandenberg & Lance, 2000). Model fit was evaluated using several goodness-of-fit measures: chi-square, Bentler’s comparative fit index (CFI), Tucker–Lewis index (TLI), and root mean square of approximation (RMSEA; Bollen & Long, 1993). Hu and Bentler (1999) recommended CFI and TLI values of 0.95 or above and RMSEA values of 0.06 or below as indicating good model fit. All analyses of measurement equivalence were conducted using Mplus 5 (Muthén & Muthén, 1998-2007).

Table 3 shows the results of the CFA analyses. The indices indicate that configural and metric invariance can be postulated for the measurement of CK and PCK in the four groups of German teachers. However, the fit indices for the scalar invariance model are slightly below the cutoff scores for good model fit. In other words, the intercepts/thresholds of like items showed some variation between groups, indicating that item difficulties differed to some extent across groups.
To further investigate the comparability of the CK and PCK measures across teacher groups, we conducted analyses of differential item functioning (DIF) using a graded response model (GRM; Samejima, 1997). Analyses computed with the program IRTLRDIFF v2.0b (Thissen, 2001) revealed DIF in the a-parameters (item discriminations) and especially, the b-parameters (item difficulties/thresholds) of some items. These findings are in line with the CFA analyses. However, the analyses also showed that DIF in the item difficulties did not specifically disadvantage any of the teacher groups. In each group, some items were easier and some were more difficult than in other groups.

Building on these analyses, we used item response theory to estimate person parameters using the MULTILOG 7 software (Scientific Software International, Lincolnwood, Illinois). A multigroup GRM (Samejima, 1997) was applied for CK and PCK. The calibration of the a-parameters (item discriminations) and b-parameters (item difficulties/thresholds) was conducted on the basis of all teacher groups (concurrent calibration). Thus, item parameters were constrained to be equal in all groups, ensuring the same metric in the groups. In the second step, the person parameters (maximum likelihood estimates) were computed. Both CK and PCK person parameters were transformed to a scale with a mean of 500 and a standard deviation of 100 within the sample of the inservice teachers (reference group).

**Analysis of Group Differences**

To test for differences between the four German teacher groups, we conducted multiple regression analyses with CK or PCK as the dependent variables and group affiliation (Year 1 students, Year 3 students, student teachers, and inservice teachers) as dummy-coded predictors. In addition, we entered several background variables in the regression to control for selective intake into the different paths of teacher education (academic vs. nonacademic track). As shown in Table 1, there were indeed some substantial between-group differences in participants’ characteristics. As these characteristics may be related to CK or PCK, we controlled for them when computing group differences. Therefore, we controlled for gender, school track, GPA in high school, cognitive abilities—as measured by the KFT (Kognitiver Fähigkeitstest [cognitive ability test]), a measure of nonverbal intelligence (Heller & Perleth, 2000)—enrollment in an advanced mathematics course at upper secondary level, and interest in mathematics. The regression analysis was computed using Mplus 5 (Muthén & Muthén, 1998-2007). The results of the full regression models are shown in Table A1 in the appendix.

In the regression analyses and the analyses of measurement invariance, we used the full information maximum likelihood (FIML) estimation feature in Mplus to deal with missing data. This procedure takes all available information into account (Arbuckle, 1996).

### Results

**German Mathematics Teachers’ CK and PCK at Different Points of the Teaching Career**

As mentioned above, we controlled for selective intake into the different paths of teacher education by entering several background variables in the regression analyses. As shown in Table A1, CK and PCK scores were significantly predicted by gender, GPA, nonverbal cognitive abilities (KFT), interest in mathematics, and enrollment in an advanced mathematics course. Furthermore, school track and the interactions of school track with teacher group were significantly related to CK and/or PCK scores. The interaction effects were included to model differences between school tracks within the four groups of pre- and inservice teachers.

Figures 1 and 2 present the findings showing how CK and PCK differed across the four teacher groups. As hypothesized, differences in the CK of prospective academic- and nonacademic-track teachers were already apparent at their entry to teacher education. First-year students training to teach in the academic track outperformed their counterparts training for the nonacademic tracks by about a half standard deviation (models without control variables). However, when selective intake into the two paths of teacher education was controlled, these differences virtually disappeared (the difference was no longer significant). Thus, the regression analysis with control variables seems to model selective intake into the two programs quite well (for detailed results of the regression models, see Table A1).
Figure 1. Comparison of pre- and inservice mathematics teachers’ CK
Note: CK = content knowledge. Values stem from regression analyses (with and without control variables; see Table A1). Error bars represent 95% confidence intervals. Note that the results are based on a cross-sectional study; the lines should not suggest a longitudinal design.

Figure 2. Comparison of pre- and inservice mathematics teachers’ PCK
Note: PCK = pedagogical content knowledge. Values stem from regression analyses (with and without control variables; see Table A1). Error bars represent 95% confidence intervals. The results are based on a cross-sectional study; the lines should not suggest a longitudinal design.
Our results suggest that the differences in CK from Year 1 students to Year 3 students and from Year 3 students to student teachers—i.e., the span of initial teacher training in Germany—were substantially higher in prospective academic- than nonacademic-track teachers. When covariates were controlled, the difference between Year 1 students and Year 3 students training for the academic track was 0.58 SD and that between Year 3 students and student teachers was 0.44 SD. The corresponding differences for prospective nonacademic-track teachers were much smaller: Year 1 students and Year 3 students differed by 0.13 SD (ns), and Year 3 students and student teachers by 0.23 SD (ns). At the end of teacher education, the difference between prospective academic- and nonacademic-track teachers was about three quarters of a SD (0.77 SD). Experienced inservice teachers showed lower (nonacademic track) or almost the same (academic track) CK scores as student teachers at the end of their teacher education. The difference between experienced inservice teachers in academic- versus nonacademic-track schools was more than 1 SD (1.10 SD). Thus, the data support the idea that CK develops primarily in the first phase of teacher education (university studies), especially with respect to prospective academic-track teachers.

Figure 2 presents the corresponding findings for PCK. Again, in the regression model without control variables, the PCK of future academic- versus nonacademic-track teachers already differed at the beginning of teacher education (0.77 SD). However, when the control variables were considered, the difference (0.37 SD) was no longer significant. Thus, for PCK, too, the regression analysis with control variables seems to model selective intake into the two paths of teacher education quite well.

In contrast to the findings for CK, the differences in PCK between Year 1 students and Year 3 students in the academic track (0.60 SD) resemble those for their peers training for the nonacademic tracks (0.55 SD). The difference between Year 3 students and student teachers in prospective academic-track (0.53 SD) and nonacademic-track teachers (0.52 SD) was also similar. At the end of initial teacher education, the PCK of student teachers training for the academic track differed significantly from that of their peers training for the nonacademic track (0.42 SD). The difference between student teachers and experienced teachers was 0.46 SD for the academic track and 0.13 SD (ns) for the nonacademic track. Experienced academic-track teachers differed significantly from nonacademic-track teachers (0.62 SD). As observed for CK (especially, in the academic track), the first phase of teacher education seems to play an important role in the development of PCK. However, the learning opportunities offered in the induction phase also seem to foster the development of PCK.

### Summary and Conclusion

Drawing on research on teachers’ knowledge of subject matter (i.e., CK and PCK) and its importance for the quality of teaching in schools and for student progress, our study investigated the role of teacher education in the development of this specific knowledge in mathematics teachers. In a cross-sectional comparison study, we investigated the CK and PCK of pre- and inservice teachers at different points of the teaching career. Thereby, we sought to assess teacher knowledge proximally by means of knowledge tests.

### Measurement of CK and PCK Across Different Teacher Populations

Few previous studies have compared test scores across teacher populations. The findings of recent studies on the latent structure of teachers’ knowledge of subject matter have, at first sight, been mixed. In the domain of mathematics, Hill and colleagues found a one-factor structure for elementary teachers (Hill et al., 2004), whereas Krauss and colleagues found a two-dimensional structure for secondary teachers (Krauss, Brunner, et al., 2008). Krauss, Brunner, et al. (2008) explained these contrasting findings as reflecting a different latent structure across teacher populations (see Phelps & Schilling, 2004, for the domain of reading). Thus, a thorough investigation of the measurement of CK and PCK seems necessary, including the latent structure of measures of subject-matter knowledge in different teacher populations.

We therefore analyzed whether the tests used in this study allowed the constructs of CK and PCK to be measured invariantly across teacher groups. The CFA approach we used (Vandenberg & Lance, 2000) provided evidence for configural and metric invariance between the four teacher groups. A latent structure of two correlated but distinct factors for CK and PCK could thus be confirmed in all teacher groups considered. However, the intercepts of the items’ regressions on the latent factors (item difficulties) were not completely invariant. Additional DIF analyses revealed that the difficulties of some items differed between groups. On average, however, this did not advantage or disadvantage any of the groups in terms of their CK or PCK test scores. Thus, despite the very different teacher populations considered, comparisons of group-mean CK and PCK scores seemed to be acceptable.

### The Role of Structural Differences in Teacher Education—The German Example

In a cross-sectional study, we investigated how German teachers’ CK and PCK vary between the different phases of teacher education from the beginning of their university studies to the inservice phase of their career. In so doing, we
analyzed differences between pre- and inservice teachers of the academic and nonacademic tracks, as the learning opportunities available to the two groups during their initial teacher education differ for PCK and especially for CK.

As hypothesized, the first phase of teacher education seems to play a particularly important role in the development of CK. However, considerable differences were already apparent in the mathematical CK of prospective academic- and nonacademic-track teachers at the beginning of their university studies. As hypothesized, learning opportunities in the pretraining phase (Feiman-Nemser, 1983) contribute to this difference. By controlling for background variables and learning opportunities prior to teacher education, we were able to model selective intake to the two paths of teacher education quite well.

Moreover, the results indicate strong differential development of CK during initial teacher education (university and induction phase). Future academic-track teachers showed higher increases in CK from Year 1 to Year 3 of university education as well as from Year 3 to the induction phase than did future nonacademic-track teachers. At the end of teacher education, the differences between teachers of the academic and nonacademic tracks were particularly large. Experienced inservice teachers showed lower (nonacademic track) or almost the same (academic track) CK scores as the respective preservice teachers at their end of teacher education. Thus, the inservice phase does not seem to contribute to substantial further development of CK after initial teacher education.

As hypothesized, the first and the second phases of teacher education seem to play an important role in the development of PCK. In contrast to the findings for CK, academic- and nonacademic-track teachers did not differ greatly in terms of differences in PCK scores from Year 1 to Year 3 of university education or from Year 3 to the induction phase. However, at the end of teacher education, the difference between the two groups was almost half a standard deviation, in favor of future academic-track teachers. In contrast to the findings for CK, inservice academic-track teachers scored higher on PCK than did future academic-track teachers at the end of their initial teacher education. In this group of teachers, the inservice phase seems to contribute to the further, but quite weak, development of PCK after initial teacher education.

As assumed, in addition to the effect of the quantity of learning opportunities, PCK development may be affected by the individually available CK (Ball et al., 2001; Friedrichsen et al., 2009; Halim & Meraah, 2002; Van Driel et al., 1998). Consequently, academic-track teachers might show similar development of PCK during the university phase, although they receive less learning opportunities compared with the prospective nonacademic-track teachers. It may also explain the finding that academic-track teachers seem to benefit from the learning opportunities offered by the inservice phase with regard to PCK, whereas their nonacademic-track colleagues do not. Thus, our findings may again point to the importance of CK in the development of PCK. Higher CK may lead to increased uptake of learning opportunities to acquire PCK, thus moderating the development of PCK.

Drawing on Grossman and Lortie (Grossman, 1990; Lortie, 1975), we expected CK and PCK to begin developing before prospective teachers entered formal teacher education. The differences observed between future academic- and nonacademic-track teachers at the beginning of their teacher education are in line with this expectation. In Germany, for example, future academic-track teachers are more likely to opt for advanced mathematics courses in Grades 11 to 13 than future nonacademic-track teachers. Thus, they have more formal learning opportunities to acquire mathematical knowledge before beginning their professional education.

A further central hypothesis was that formal and nonformal learning opportunities (Werquin, 2010) are especially conducive to the development of CK and PCK, and that teaching experience alone is insufficient (De Jong & Van Driel, 2004; Ericsson et al., 1993; Park & Oliver, 2008). We found support for this hypothesis for CK and PCK. The future academic-track teachers, who have much more formal learning opportunities for CK in the first phase of teacher education, showed considerably higher gains in CK from Year 1 to Year 3 than did future nonacademic-track teachers. Furthermore, the inservice phase, which involves primarily informal learning, does not seem to foster the development of CK and PCK as strongly as the formal and nonformal learning opportunities provided by initial teacher education programs. In line with research on effective professional development, our results suggest that participation in traditional formal professional development during the inservice phase fosters the development of CK and PCK weakly, at best (Brunner et al., 2006; Garet et al., 2001). Research indicates that the success of professional development programs depends on their meeting several criteria: Effective professional development that affected teacher learning, instruction, and student progress consisted of long-term and coherent programs that involved teachers in active learning, and that had a clear focus on content and student learning (Darling-Hammond et al., 2009; Garet et al., 2001). In Germany as well as in the United States, professional development in mathematics and in other domains often fails to meet these criteria. Consequently, effective professional development, as suggested by research on professional development, is not broadly implemented yet (Ball et al., 2001; Darling-Hammond et al., 2009). The fact that a focus on content and related student learning had been identified as one criterion of effective professional development highlights—in line with our result—the need to foster teachers’ CK and PCK during the inservice phase. This could mean a key area for educational reform.

As indicated before, one finding seems to contradict the idea that formal and nonformal learning opportunities play a...
major role in the development of subject-matter knowledge. During the university phase of teacher education, prospective academic- and nonacademic-track teachers showed similar differences in PCK from Year 1 to Year 3, although the latter have more formal learning opportunities for the development of PCK during their university studies. We interpreted this finding as a further point to the importance of CK in the development of PCK. Higher CK may lead to increased uptake of learning opportunities to acquire PCK, thus compensating effects of the quantity of learning opportunities.

Overall, the findings are very much in line with our hypotheses. The findings for PCK are consistent with the particularly low performance of Germany’s nonacademic-track students on the Third International Mathematics and Science Study and PISA assessments—assuming that teachers’ PCK is a crucial prerequisite for student learning (Baumert et al., 2010; Hill et al., 2005). Our study provided further evidence that the deliberative formal and nonformal learning opportunities provided in the context of initial teacher education are crucial for the development of teachers’ subject-matter knowledge. In contrast, informal learning in the form of incidental learning, often referred to as teaching experience, seems to have only a weak effect on the development of teachers’ subject-matter knowledge, especially CK.

The considerable differences in the CK and the PCK of academic-track versus nonacademic-track teachers in Germany highlight a severe social problem (see Baumert et al., 2010). Students attending nonacademic-track schools differ from their peers in the academic track not only in their ability and achievement but also in their social and ethnic backgrounds. Consequently, low-achieving students from families with lower socioeconomic status and immigrant families tend to be taught by teachers who are less competent in terms of CK and PCK. Given the crucial role of subject-matter knowledge for student progress, this means a severe social inequality of learning opportunities (Baumert et al., 2010). Moreover, Baumert et al. (2010) found that school track moderates the relationship between PCK and student learning: The effect of PCK on student achievement gains is larger in nonacademic-track classes—that is, lower achieving students benefit particularly from teachers who are especially competent in terms of PCK (Baumert et al., 2010). However, precisely these students tend to be taught by teachers with low PCK (and CK).

Disparities in the access to high-quality teachers are also a matter of great concern in the United States (Darling-Hammond, 2006). Whereas in our study such disparities resulted from the tracked secondary school system in Germany, in the United States, the unequal distribution of well-trained teachers is primarily a result of differences in the social structure of school districts (Darling-Hammond, 2006; Hill & Lubienski, 2007).

Our findings highlight the urgent need to improve the preparation of future nonacademic-track teachers with respect to subject-matter knowledge (CK and PCK). Moreover, candidates for the academic track already enter teacher education with better subject-matter knowledge, and our findings suggest that these differences persist or even increase across the teaching career. Thus, in addition to improving teacher education, changes in recruitment and selection processes for teacher candidates could help to raise the quality of instruction and student progress in nonacademic-track schools.

**Limitations and Open Questions**

In our study, we addressed structural differences in teacher education and their relations to the CK and PCK of pre- and inservice teachers. Our main emphasis was on differences in the quantity of the learning opportunities for CK available in different teacher education programs. Further research is needed to investigate the impact of differences in the quality of those learning opportunities.

We used a cross-sectional design with different cohorts of (prospective) teachers to examine the development of subject-matter knowledge from Year 1 students to experienced teachers. A longitudinal design would not have been feasible, as it would have had to span decades to reflect development over the different phases of the teaching career. The CK and PCK trajectories displayed in Figures 1 and 2 may therefore be confounded by cohort effects. This may apply especially to the group of experienced teachers, whose teacher education programs may have differed considerably from those attended by the other groups. To improve the comparability of the cohorts, we controlled for several relevant variables. Especially for CK, including these control variables allowed us to model selective intake into the two paths of teacher education quite well. Furthermore, as shown in Table A1, the proportion of variance in CK and PCK explained increased substantially when these control variables were included in the regression model.

Our results showed that the CK of the teacher groups considered here differed significantly. Consequently, in a cross-sectional study investigating the impact of teacher education on the development of PCK, two effects are inseparably intertwined: the effects of the quality and quantity of the learning opportunities made available in teacher education (i.e., the “treatment”) and the effect of individually available CK on the development of PCK. These two effects can only be properly disentangled in longitudinal studies or randomized controlled trails. Beyond teachers’ subject-matter knowledge, moreover, there has to date been very little empirical investigation of how teacher education and inservice training affect the development of teachers’ more generic PK (König, Blömeke, Paine, Schmidt, & Hsieh, 2011; Voss, Kunter, & Baumert, 2011).
## Appendix

<table>
<thead>
<tr>
<th>Knowledge Category (Subscale)</th>
<th>Sample Item</th>
<th>Sample Response (Scoring 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CK</strong></td>
<td>Is it true that $0.999999\ldots = 1$? Please give detailed reasons for your answer.</td>
<td>Let $0.99\ldots = a$&lt;br&gt;Then $10a = 9.99\ldots$, hence,&lt;br&gt;$10a - a = 9.99\ldots - 0.999\ldots$&lt;br&gt;$9a = 9$&lt;br&gt;Therefore $a = 1$; hence, the statement is true.</td>
</tr>
<tr>
<td><strong>PCK: tasks</strong></td>
<td>How does the surface area of a square change when the side length is tripled? Show your reasoning. Please note down as many different ways of solving this problem (and different reasonings) as possible.</td>
<td><em>Algebraic response</em>&lt;br&gt;Area of original square: $a^2$&lt;br&gt;Area of new square is then $(3a)^2 - 9a^2$; i.e., 9 times the area of the original square.&lt;br&gt;<em>Geometric response</em>&lt;br&gt;Nine times the area of the original square</td>
</tr>
</tbody>
</table>
|                              | The area of a parallelogram can be calculated by multiplying the length of its base by its altitude. | ![Parallelogram](image)

Note: The crucial aspect to be covered in this teacher response is that students might run into problems if the foot of the altitude is outside a given parallelogram. |
| **PCK: students**            | A student says: I don’t understand why $(-1) \times (-1) = 1$ Please outline as many different ways as possible of explaining this mathematical fact to your student. | The “permanence principle,” although it does not prove the statement, can be used to illustrate the logic behind the multiplication of two negative numbers and thus foster conceptual understanding: $3 \times (-1) = -3$
$2 \times (-1) = -2$
$1 \times (-1) = -1$
$0 \times (-1) = 0$
$(-1) \times (-1) = 1$
$(-2) \times (-1) = 2$ |
| **PCK: instruction**         | | |

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**Figure A1.** Measures of CK and PCK; sample items and teacher responses

Note: CK = content knowledge; PCK = pedagogical content knowledge.
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Authors’ Note

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Notes

1. A short time after this study was conducted, most states restructured the Gymnasium system to cover Grades 5 to 12.
2. At the time the study was conducted, students had to choose between basic and advanced courses in Grades 11 to 13. As a rule, basic courses involved three lessons per week and advanced courses, five lessons per week.
3. Thus, the groups of pre- and inservice nonacademic-track teachers comprise teachers of the following school types: Realschule, Gesamtschule, and Sekundarschule or the like. All these school types provide lower secondary education, including Grade 10.

4. The fact that the experienced teachers had lower content knowledge test scores than the student teachers can be attributed to the control variables used in the analyses.

References


Table A1. Regression Models Predicting CK and PCK Scores

<table>
<thead>
<tr>
<th></th>
<th>CK</th>
<th></th>
<th>PCK</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M1</td>
<td>M2</td>
<td>M3</td>
<td>M4</td>
</tr>
<tr>
<td>Constant</td>
<td>466.54**</td>
<td>480.55**</td>
<td>371.96**</td>
<td>380.63**</td>
</tr>
<tr>
<td>Year 1 students</td>
<td>−26.84**</td>
<td>−21.27**</td>
<td>−66.94**</td>
<td>−63.96**</td>
</tr>
<tr>
<td>Year 3 students</td>
<td>−17.56**</td>
<td>−4.57</td>
<td>−47.79**</td>
<td>−35.09**</td>
</tr>
<tr>
<td>Student teachers</td>
<td>10.93**</td>
<td>10.14**</td>
<td>−13.32**</td>
<td>−12.82**</td>
</tr>
<tr>
<td>Gender</td>
<td>−21.74**</td>
<td>13.72*</td>
<td>−18.62**</td>
<td>−18.62**</td>
</tr>
<tr>
<td>School track</td>
<td>7.37</td>
<td></td>
<td>13.72*</td>
<td></td>
</tr>
<tr>
<td>School track × Year 1 students</td>
<td>−23.91***</td>
<td></td>
<td>−5.06</td>
<td></td>
</tr>
<tr>
<td>School track × Year 3 students</td>
<td>−13.23***</td>
<td></td>
<td>−5.83</td>
<td></td>
</tr>
<tr>
<td>School track × student teachers</td>
<td>−9.75**</td>
<td></td>
<td>−6.77*</td>
<td></td>
</tr>
<tr>
<td>GPA</td>
<td>−16.16**</td>
<td>7.47**</td>
<td>−13.12**</td>
<td>15.58**</td>
</tr>
<tr>
<td>Cognitive abilities</td>
<td>4.55**</td>
<td></td>
<td>7.47**</td>
<td></td>
</tr>
<tr>
<td>Advanced mathematics course</td>
<td>12.67**</td>
<td>15.35***</td>
<td>13.75**</td>
<td></td>
</tr>
<tr>
<td>Interest in mathematics</td>
<td>15.35***</td>
<td></td>
<td>13.75**</td>
<td></td>
</tr>
</tbody>
</table>

R² .07 .46 .18 .45

Note: CK = content knowledge; PCK = pedagogical content knowledge; GPA = grade point average. M1 and M3 are the initial models, including the group variables only; M2 and M4 are the full models, including all covariates.

*p < .05 (two-tailed). **p < .01 (two-tailed).


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