July 29, 2016

Housing and Macroeconomy: The Role of Credit Channel, Risk -, Demand - and Monetary Shocks *

Abstract

This paper demonstrates that risk (uncertainty) along with the monetary (interest rates) shocks to the housing production sector are a quantitatively important impulse mechanism for the business and housing cycles. Our model framework is that of the housing supply/banking sector model as developed in Dorofeenko, Lee, and Salyer (2014) with the model of housing demand presented in Iacoviello and Neri (2010). We examine how the factors of production uncertainty, financial intermediation, and credit constrained households can affect housing prices and aggregate economic activity. Moreover, this analysis is cast within a monetary framework which permits a study of how monetary policy can be used to mitigate the deleterious effects of cyclical phenomenon that originates in the housing sector. We provide empirical evidence that large housing price and residential investment boom and bust cycles in Europe and the U.S. over the last few years are driven largely by economic fundamentals and financial constraints.We also find that, quantitatively, the impact of risk and monetary shocks are almost as great as that from technology shocks on some of the aggregate real variables. This comparison carries over to housing market variables such as the price of housing, the risk premium on loans, and the bankruptcy rate of housing producers.

- *JEL Classification: E4, E5, E2, R2, R3*
- Keywords: hetrogenous households, residential investment, housing prices, credit constraint, loan to value ratio, cash in advance constraint, monetary policy, uncertainty and demand shocks

Victor Dorofeenko
Department of Economics and Finance
Institute for Advanced Studies
Stumpergasse 56
A-1060 Vienna, Austria

Gabriel S. Lee (Corresponding Author)
University of Regensburg
Universtitaetstrasse 31
93053 Regensburg, Germany
And
Institute for Advanced Studies

Kevin D. Salyer
Department of Economics
University of California
Davis, CA 95616, U.S.A.

Johannes Strobel
University of Regensburg
Universtitaetstrasse 31
93053 Regensburg, Germany
Contact Information:
Lee: ++49.941.943.5060; E-mail: gabriel.lee@ur.de
Salyer: ++1.530.752.8359; E-mail: kdsalyer@ucdavis.edu

*We benefitted from comments received during presentations at: the Humboldt University, University of Erlangen-Nuremberg, Computing in Economics and Finance Conference 2014. We are especially indebted to participants in the IHS and the University of Regensburg Macroeconomics Seminars for insightful suggestions that improved the exposition of the paper. We also gratefully acknowledge financial support from Deutsche Forschungsgemeinschaft (DFG Nr. LE 1545/1-1).

1 Introduction

Figure 1 shows the dramatic rise and fall of real housing prices for the U.S. and some of the selected European countries[1] from the first quarter of 1997 to the second quarter of 2011. The U.S. housing market peaked in the second quarter of 2007 with the price appreciation of 122 percent.[2] In comparison to Ireland (300 percent), Greece (185 percent), and Spain (163 percent) and with each country having slightly different peak periods, the U.S. housing price appreciation is not only in-line with the rest of the economies but also is relatively mild[3]. With the subsequent pronounced decline in house prices in these economies from their peaks during the mid 2000's, there is a growing body of literature that describes these large swings in housing prices as *bubbles* or these sharp increases in housing prices were caused by *irrational exuberance*[4], and subsequently, these housing price bubbles then are the causes of the recent global financial instability.

Given the recent macroeconomic experience of most developed countries, few students of the economy would argue with the following three observations: 1. Financial intermediation plays an important role in the economy, 2. The housing sector is a critical component for aggregate economic behavior and 3. Uncertainty, and, in particular, time-varying uncertainty[5] - and monetary shocks are quantitatively important sources of business cycle activity. And while there has no doubt been a concomitant increase in economic research which examines housing markets and financial intermediation, only a few analyses have been conducted in a calibrated, general equilibrium setting; i.e. an economic environment in which the quantitative properties of the model

[1] We have selected Ireland, Spain, Greece, Italy and Portugal for our illustration purpose. Moreover, these countries are purposely chosen as most of them are currently experiencing a great financial difficulties. We also include Germany, who has not experienced any housing or financial crisis, to use as a benchmark comparison.

[2] Unlike the Case-Shiller index that has an appreciation of 122 percent, the Office of Federal Housing Enterprise Oversight (OFHEO) experienced 77 percent appreciation. The difference between the The Office of Federal Housing Enterprise Oversight (OFHEO) and Case-Shiller housing price indices arises largely from the treatment of expensive homes. The OFHEO index includes only transactions involving mortgages backed by the lenders it oversees, Fannie Mae and Freddie Mac, which are capped at $417,000. The Case-Shiller measure has no upper limit and gives more weight to higher-priced homes.

[3] The following countries have experienced less of an appreciation: Portugal (44 percent), Italy (70 percent) and Germany (-4.2 percent) with even depreciation.

[4] The term *Irrational Exuberance* was first coined by Alan Greenspan, the former chairman of the Board of Governors of the Federal Reserve in 1996 at the U.S. Congress testimony.

[5] We define and estimate these uncertainty (risk) shocks as the time variation in the cross sectional distribution of firm level productivities. The detail analysis of the risk shock estimation follows in the later section.

are broadly consistent with observed business cycle characteristics with various shocks.[6] The objective of our paper is to develop a theoretical and computational framework that can help us understand: [1] how does uncertainty in lending channel effect economies (including both financial and housing markets) at different stages of business cycle; [2] what are the effcts associated with credit constrained heterogenous agents on the housing prices and the business cycle; and [3] what types monetary policies might help (or hinder) the process of housing and financial development.

To address the aforementioned questions, we use the framework of the housing supply/banking sector model as developed in Dorofeenko, Lee, and Salyer (2014) with the model of housing demand presented in Iacoviello and Neri (2010). In particular, we examine how the factors of production uncertainty, financial intermediation, and credit constrained households can affect housing prices and aggregate economic activity. Moreover, this analysis is cast within a monetary framework of Carlstrom and Fuerst (2001) which permits a study of how monetary policy can be used to mitigate the deleterious effects of cyclical phenomenon that originates in the housing sector.

The Dorofeenko, Lee, and Salyer (2014) model focuses on the effects that housing production uncertainty and bank lending have on housing prices. To do this, their analysis combines the multi-sector housing model of Davis and Heathcote (2005) with the Carlstrom and Fuerst (1997) model of lending under asymmetric information and agency costs since both models had been shown to replicate several key features of the business cycle. In particular, the Davis and Heathcote (2005) model produces the high volatility of residential investment relative to fixed business investment seen in the data. However, the model fails to produce the observed volatility in housing prices. To this basic framework, Dorofeenko, Lee, and Salyer (2014) introduce an additional impulse mechanism, time varying uncertainty (i.e. risk shocks) shocks to the standard deviation of the entrepreneurs' technology shock affecting only the housing production, and require that housing

[6] Some of the recent works which also examine housing and credit are: Iacoviello and Minetti (2008) and Iacoviello and Neri (2010) in which a new-Keynesian DGSE two sector model is used in their empirical analysis; Iacoviello (2005) analyzes the role that real estate collateral has for monetary policy; and Aoki, Proudman and Vliegh (2004) analyse house price amplification effects in consumption and housing investment over the business cycle. None of these analyses use risk shocks as an impulse mechanism. Some recent papers that have examined the effects of uncertainty in a DSGE framework include Bloom et al. (2012), Fernandez-Villaverde et al. (2009), Christiano et al. (2015), and Dorofeenko, Lee and Salyer (2014) with housing markets.

producers finance the purchase of their inputs via bank loans. Dorofeenko, Lee, and Salyer (2014) model risk shocks as a mean preserving spread in the distribution of the technology shocks affecting only house production and explore quantitatively how changes in uncertainty affect equilibrium characteristics.[7] The importance of understanding how these uncertainty or risk shocks affect the economy is widely discussed in academics and among policymakers. For example, Baker, Bloom, and Davis (2012) demonstrates that a long persistent sluggish economic recovery in the U.S. (e.g. low output growth and unemployment hovering above 8%) even after the bottoming of the U.S. recession in June 2009 could be attributed to the high levels of uncertainty about economic policy.

In Dorofeenko, Lee, and Salyer (2014), these factors lead to greater house price volatility as housing prices reflect potential losses due to bankruptcy for some housing producers. In fact, the model is roughly consistent with the cyclical behavior of residential investment and housing prices as seen in U.S. data over the sample period 1975-2010. However, Dorofeenko, Lee, and Salyer (2014) model is not consistent with the behavior of housing prices and firm bankruptcy rates as seen in the recent decade. This failure is not surprising since the role of shocks to housing demand combined with changes in household mortgage finance are not present. Consequently, in this paper, we embed key features of the recent model by Iacoviello and Neri (2010) to rectify this omisssion. As detailed below, the main features of the Iacoviello and Neri (2010) model that we employ are the introduction of heterogenous agents (patient and impatient), a borrowing constraint (which affects impatient households) and a monetary authority that targets inflation via interest rate. We then introduce housing demand shocks (via preferences) and examine how these get transmitted to the economy. Next we examine optimal monetary policy in this setting.

In analyzing the role of the LTV (Loan to Value) ratio on macro and housing variables, we present three different scenarios that are based on LTV ratio: low (80), middle (85) and high (90) borrowing constraints. These different levels of LTV ratio are, for an expositional purpose, to

[7] One should note that the time varying risk shocks in this paper are quite different than the pure aggregate or sectoral technology (supply) shocks. First, risk shocks affect only the housing production sector. Second, risk shocks are meant to represent the second moments of the variance. That is, these time varying risk shocks proxy the changes in eocnomic environment uncertainty.

reflect three different European economies: Germany, Italy and Spain. According to IMF (2011) (also shown in Table 5 in the Appendix), Germans have one of the lowest LTV ratio, whereas Spanish borrowers have one of the highest LTV ratio with Italy being in between these two levels.[8]

HERE I NEED TO WRITE SOMETHING ELSE OR UPDATE THE RESULTS!!!!

Unlike some of the recent literature that emphasize the important role of the level of LTV on housing market, our results indicate otherwise: almost no differences between different levels of LTV on the variables that we analyze. On the role of specifi shocks, we show that the effects of monetary shocks are huge on most of the macro variables and in particular on the housing investment and the amount of borrowing the households undertake: over 75 percent and almost 50 percent of the variation in housing investment and borrowing can be explained by the monetary shocks. On the contrary to monetary shocks, housing demand shocks have a trivial impact on all the variables that we analyze: at most 6 percent of the variation in housing price can be explained by the preference shocks. Lastly, our endogenous debt financial accelerator model with risk shocks lends a strong support for the important role of risk shocks: over 85 percent of the variation in housing price is due to risk shocks. Our results, thus, show that there is a clear and an important role for the policy makers to smooth housing price and/or housing investment: to calm markets and to provide and restore market confidence.

2 Model: Housing Markets, Financial Intermediation, and Monetary Policy

Our model builds on three separate strands of literature: Davis and Heathcote's (2005) multi-sector growth model with housing, Dorofeenko, Lee and Salyer's (2008, 2014) credit channel model with uncertainty, and Iacoviello and Neri's (2010) model of housing demand. In this paper, however, we do not consider any other New Keynesian economic frictions other than the

[8] Some of the recent housing market developments for various European countries are discussed in the Appendix.

asymmetric information friction that occurs in the loan contract between the mutual fund and entrepreneurs.

First we specify the households' optimization problem in a representative agent economy in which the demand for money is motivated by a cash-in-advance constraint. Then, this environment is modified by dividing the households into two groups, patient and impatient (*á la* Iacoviello and Neri's (2010)) . This will introduce a role for household lending and borrowing. We then introduce financial intermediation ("banking") sector, where the patient households lend savings to bankers, and the bankers then lend to both impatient households and entrepreneurs (housing producing agents). With the banking sector, the state of the economy (which is measured in this paper by the level of "uncertaity" or "risk") effects the lending amount and the probability of loan default, and hence effecting the net worth of these financial intermediaries. And consequently, the endogenous net worth of the finanical intermediary sector could in fact contribute aggregate movements in various financial and macro variables. Finally, we include government in setting monetary policy rule with a variation of the Taylor Rule.

Here is a brief outline and summary of the environment of this economy: Figure 2 shows a schematic of the implied flows for this economy.

- **Three types of agents:**
  - Risk-averse patient (impatient) households that choose consumption, labor, money holding, and housing service: Lend (borrow) money to (from) the financial intermediaries.
  - Risk-neutral entrepreneurs (housing developers) that choose consumption, investment, and labor.
    - Cost of inputs is financed by borrowing
    - Housing production subject to risks shocks.
- **Multisectors: 6 Firms**

- **Three** intermediate goods producing: Construction, Manufacturing, and Service
- **Two** "final" goods production: Residential Investment and Consumption / Non-residential Investment
- **Housing** Production (via entrepreneurs): Residential Investment + Land.

- **A mutual fund: Financial intermediaries** (that guarantees a certain return to households through lending to an infinite number of entrepreneurs).
- **Government**: Lump sum money transfer and taxes.
- **Shocks: 6 different shocks**
  - 3 sectoral productivity technology shocks: Construction, Manufacturing and Service
  - Idiosyncratic technology shocks (ω_t) affecting housing production.
    * Denoting the *c.d.f.* and *p.d.f.* of ω_t as $\Phi(\omega_t; \sigma_{\omega,t})$ and $\phi(\omega_t; \sigma_{\omega,t})$.
    * Second moment, $\sigma_{\omega,t}$, (i.e. risk) shocks affecting the distribution of these Idiosyncratic technology shocks.
  - Monetary shocks: *á la* Taylor Rule.
  - Housing demand (preference) shocks.

2.1 Money and cash-in-advance constraint in housing model: Households

This section follows the work of Carlstrom and Fuerst (2001) but adds loans and collateral restrictions on the demand side as in Iacoviello and Neri (2010). Households maximize lifetime utility given by:

$$E_0 \left(\sum_{t=0}^{\infty} \beta^t U(c_t, h_t, 1 - N_t) \right) \tag{1}$$

w.r.t. its consumption c_t, labor hours N_t, capital K_t and housing h_t stocks, the investment into consumption $i_{k,t}$ and housing $i_{h,t}$ goods sectors, and money holdings M_t subject to the budget constraint (where we let λ to denote the Lagrange multiplier):

$$c_t + i_{k,t} + p_{h,t} i_{h,t} + \frac{M_{t+1}}{P_{c,t}} \leq \tag{2}$$

$$K_t \left(r_t - \tau_k \left(r_t - \delta_k\right)\right) + N_t w_t \left(1 - \tau_n\right) + p_{l,t} x_{l,t} + \frac{M_t + M_{s,t}}{P_{c,t}}.$$

$P_{c,t}$ denotes the nominal consumption price. Note that the new monetary injection, $M_{s,t}$ is distributed by the government at the beginning of the period as a lump-sum transfer (with the aggregate money stock given by $M_{s,t}$). The cash-in-advance constraint (CIA) states that money (post-transfer) must be used to buy investment and consumption goods (the associated Lagrange multiplier is κ):

$$c_t + i_{k,t} + p_{h,t} i_{h,t} \leq \frac{M_t + M_{s,t}}{P_{c,t}} \tag{3}$$

The laws of motion for capital and housing are given by (the respective Lagrange multipliers are μ and ν):

$$K_{t+1} = F_k \left(i_{k,t}, i_{k,t-1}\right) + \left(1 - \delta_k\right) K_t \tag{4}$$

$$h_{t+1} = F_h \left(i_{h,t}, i_{h,t-1}\right) + \left(1 - \delta_h\right) h_t \tag{5}$$

Here τ_k and τ_n are the capital income and labor income taxes correspondingly, δ represents the usual capital/housing depreciation rate, $p_{l,t} x_{l,t}$ is the value of land, and the function $F_j \left(i_{j,t}, i_{j,t-1}\right)$ accounts investment adjustment cost. According to Christiano et. al (2005), $F_j \left(i_{j,t}, i_{j,t-1}\right) = \left(1 - S\left(\frac{i_{j,t}}{i_{j,t-1}}\right)\right) i_{j,t}$, where $S(1) = S'(1) = 0$ and $S''(1) > 0$.

The correspondent Euler equations are:

$$U_{1t} - \kappa_t - \lambda_t = 0$$

$$w_t\lambda_t - U_{3t} = 0$$

$$\mu_t = \beta E_t\left(\left(1-\delta_k\right)\mu_{t+1} + \left(\left(1-\tau_k\right)r_{t+1} + \tau_k\delta_k\right)\lambda_{t+1}\right)$$

$$\nu_t = \beta E_t\left(\left(1-\delta_h\right)\nu_{t+1} + U_{2t+1}\right)$$

$$\kappa_t + \lambda_t = \mu_t F_{c,1}\left(i_{k,t}, i_{k,t-1}\right) + \beta E_t\left(\mu_{t+1}F_{c,2}\left(i_{k,t+1}, i_{k,t}\right)\right)$$

$$p_{h,\,t}\left(\kappa_t + \lambda_t\right) = \nu_t F_{h,1}\left(i_{h,\,t}, i_{h,\,t-1}\right) + \beta E_t\left(\nu_{t+1}F_{h,2}\left(i_{h,\,t+1}, i_{h,\,t}\right)\right)$$

$$\frac{\lambda_t}{P_{c,t}} = \beta E_t\left(\frac{\kappa_{t+1} + \lambda_{t+1}}{P_{c,t+1}}\right)$$

Introducing the nominal interest rate R_t and the shadow prices of the capital and housing good, q_{kt} and $q_{h,t}$ respectively:

$$R_t = 1 + \frac{\kappa_t}{\lambda_t}, \mu_t = q_{k,t}U_{1,t}, \nu_t = q_{h,t}U_{1,t}$$

we obtain:

$$\frac{w_t}{R_t} = \frac{U_{3t}}{U_{1t}} \tag{6}$$

$$q_{k,t} = \beta E_t\left(\left(\left(1-\delta_k\right)q_{k,t+1} + \frac{\left(1-\tau_k\right)r_{t+1} + \tau_k\delta_k}{R_{t+1}}\right)\frac{U_{1,t+1}}{U_{1,t}}\right) \tag{7}$$

$$q_{h,t} = \beta E_t\left(\left(1-\delta_h\right)q_{h,t+1}\frac{U_{1,t+1}}{U_{1,t}} + \frac{U_{2,t+1}}{U_{1,t}}\right) \tag{8}$$

$$1 = q_{k,t}F_{k,1}\left(i_{k,\,t}, i_{k,\,t-1}\right) + \beta E_t\left(q_{k,t+1}F_{k,2}\left(i_{k,\,t+1}, i_{k,\,t}\right)\frac{U_{1,t+1}}{U_{1,t}}\right) \tag{9}$$

$$p_{h,\,t} = q_{h,t}F_{h,1}\left(i_{h,\,t}, i_{h,\,t-1}\right) + \beta E_t\left(q_{h,t+1}F_{h,2}\left(i_{h,\,t+1}, i_{h,\,t}\right)\frac{U_{1,t+1}}{U_{1,t}}\right) \tag{10}$$

including the Fisher equation for the nominal interest rate:

$$1 = \beta R_t E_t\left(\frac{U_{1,t+1}}{\left(1+\Pi_{t+1}\right)U_{1,t}}\right) \tag{11}$$

where we introduce the inflation rate Π_t,

$$\Pi_{t+1} = \frac{P_{c,t+1}}{P_{c,t}} - 1$$

The substitution of (3) into (2) and assuming that both constraints are binding, we have our budget constraint as:

$$c_{t+1} + i_{k,t+1} + p_{h,t+1} i_{h,t+1} = \tag{12}$$
$$\frac{K_t \left((1-\tau_k) r_t + \tau_k \delta_k \right) + N_t w_t (1-\tau_n) + p_{l,t} x_{l,t}}{1+\Pi_{t+1}} + \frac{M_{s,t+1}}{P_{c,t+1}}$$

2.1.1 Utility function and demand shocks

The utility function is assumed to have the form:

$$U(c,h,l) = \frac{\left(c^{\mu_c} h^{\mu_h} l^{-\mu_l} \right)^{1-\sigma}}{1-\sigma} \tag{13}$$

where σ denotes the coefficient of relative risk aversion. The housing demand shock can be added to the utility function assuming that the parameter μ_h follows the AR(1) processes:

$$\ln \mu_{h,t+1} = (1-\rho_h) \ln \mu_h^{(0)} + \rho_h \ln \mu_{h,t} + \varepsilon_{h,t+1} \tag{14}$$

We use, in fact, the same utility function as Devis and Heathcote (2005), but slightly deviate from them in notation ($\mu_l \neq 1 - \mu_c - \mu_{h,t}$) to avoid the influence of housing demand shocks on the labor supply share μ_l.

2.1.2 Heterogeneous agents, borrowing and collateral constraint at the demand side

As in Iacoviello and Neri (2010), we now introduce two types of agents, patient and impatient. A prime is used to denote impatient household's parameters and variables (β', N' etc.). The patient households as described in the previous section have a discount factor that is larger than the impatient ones, $\beta > \beta'$. While both groups may lend or borrow money, the assumption $\beta > \beta'$ will always lead to the situation where the patient households lend money to the impatient ones.

Adding the borrowing b_t to the patient household's budget constraint (2) produce the inequality (lending corresponds to the negative values of b_t):

$$c_t + i_{k,t} + p_{h,t} i_{h,t} + \frac{b_{t-1} R_{b,t-1}}{1 + \Pi_t} + \frac{M_{t+1}}{P_{c,t}} \leq \tag{15}$$
$$K_t \left(r_t - \tau_k \left(r_t - \delta_k\right)\right) + N_t w_t \left(1 - \tau_n\right) + p_{l,t} x_{l,t} + b_t + \frac{M_t + M_{s,t}}{P_{c,t}}$$

In addition to the budget constraint, households face a cash-in-advance constraint:

$$c_t + i_{k,t} + p_{h,t} i_{h,t} \leq \frac{M_t + M_{s,t}}{P_{c,t}} \tag{16}$$

so, the relation (12) is changed to the form:

$$c_{t+1} + i_{k,t+1} + p_{h,t+1} i_{h,t+1} = \tag{17}$$
$$\frac{K_t \left(r_t - \tau_k \left(r_t - \delta_k\right)\right) + N_t w_t \left(1 - \tau_n\right) + p_{l,t} x_{l,t} + b_t - \frac{R_{b,t-1} b_{t-1}}{1+\Pi_t}}{1 + \Pi_{t+1}} + a_M m_{s,t+1}$$

where $m_{s,t} = \left(M_{s,t} + M'_{s,t}\right) / P_{c,t}$. The share $a_M = M_{s,t} / \left(M_{s,t} + M'_{s,t}\right)$ is chosen equal to its steady-state value and remains constant.

The impatient households have shorter budget constraint, because they don't own capital and land, so the quantities proportional to K_t, i_{kt} and x_{lt} are omitted:

$$c'_t + p_{h,t} i'_{h,t} + \frac{R_{b,t-1} b'_{t-1}}{1 + \Pi_t} + \frac{M'_{t+1}}{P_{c,t}} = N'_t w'_t \left(1 - \tau_n\right) + b'_t + \frac{M'_t + M'_{s,t}}{P_{c,t}} \tag{18}$$

and their CIA constraint is:

$$c'_t + p_{h,t} i'_{h,t} \leq \frac{M'_t + M'_{s,t}}{P_{c,t}} \tag{19}$$

which leads to the combined constraint:

$$c'_{t+1} + p_{h,t+1} i'_{h,t+1} = \frac{N'_t w'_t (1-\tau_n) + b'_t - \frac{R_{b,t-1} b'_{t-1}}{1+\Pi_t}}{1+\Pi_{t+1}} + (1-a_M)\, m_{s,t+1} \tag{20}$$

The additional (binding) borrowing constraint for the impatient households (collateral constraint) restricts the size of the borrowed funds by the value of their housing stock:

$$b'_t \leq m E_t \left(\frac{p_{h,t+1} (1+\Pi_{t+1})\, h'_t}{R_t} \right) \tag{21}$$

where m denotes the loan-to-value (LTV) ratio. Iacoviello and Neri (2010) set m for the impatient fraction of the USA households equal to $m = 0.85$. In our calibration exercise, we vary in the range from 0.1 to 0.9 to analyse the effects of m on our economy; $m = \{0.1, 0.8, 0.85, 0.9\}$.

The market clearing condition for the borrowing is:

$$b'_t + b_t = 0 \tag{22}$$

The housing stock growth equations for the patient and impatient householders are:

$$h_{t+1} + h'_{t+1} = F_h (i_{h,t}, i_{h,t-1}) + F_h \left(i'_{h,t}, i'_{h,t-1}\right) + (1-\delta_h)(h_t + h'_t) \tag{23}$$

The patient households maximize their lifetime utility (1) w.r.t. consumption c_t, labor hours N_t, capital K_t and housing h_t stocks, the investment into consumption $i_{k,t}$ and housing $i_{h,t}$ goods sectors, money holdings M_t and borrowing b_t subject to constraints (15) and (16), stock accumulation equation (23) and the capital growth equation (4)

Their Euler equations consist of system (6) - (11) and the additional equation for the borrowing interest rate $r_{b,t}$:

$$1 = \beta R_{b,t} E_t \left(\frac{R_t}{R_{t+1}} \frac{U_{1,t+1}}{(1 + \Pi_{t+1}) U_{1,t}} \right) \tag{24}$$

The impatient households maximize their lifetime utility (1) w.r.t. consumption c_t' , labor hours N_t', housing h_t', the investment into housing goods sector $i_{h,\,t}'$, money holdings M_t' and borrowing b_t' subject to constraints (18), (19), collateral constraint (21) and the housing accumulation equation (**??**). Their Euler equations are:

$$\frac{w_t'}{R_t'} = \frac{U_{3t}}{U_{1t}} \tag{25}$$

$$1 = \beta' R_t' E_t \left(\frac{U_{1,t+1}}{(1 + \Pi_{t+1}) U_{1,t}} \right) \tag{26}$$

$$q_{b,t}' R_t' = 1 - \beta' R_{b,t} E_t \left(\frac{R_t'}{R_{t+1}'} \frac{U_{1,t+1}}{(1 + \Pi_{t+1}) U_{1,t}} \right) \tag{27}$$

$$q_{h,t}' = \beta' E_t \left(\left((1 - \delta_h)\, q_{h,t+1}' + q_{b,t+1}' \frac{b_{t+1}'}{h_{t+1}'} \right) \frac{U_{1,t+1}}{U_{1,t}} + \frac{U_{2,t+1}}{U_{1,t}} \right) \tag{28}$$

$$p_{h,\,t} = q_{h,t}' F_{h,1} \left(i_{h,\,t}', i_{h,t-1}' \right) + \beta' E_t \left(q_{h,t+1}' F_{h,2} \left(i_{h,\,t\,+\,1}', i_{h,t}' \right) \frac{U_{1,t+1}}{U_{1,t}} \right) \tag{29}$$

Here $q_{b,t}'$ denotes the shadow price of borrowing. The utility function $U = U\left(c_t', 1 - N_t', h_t'\right)$ in equations (25) - (29) depends on the impatient household's consumption c_t', labor hours N_t' and housing h_t'.

2.2 Production (Firms): with only one household

We first assume a representative agent framework and then introduce heterogeneous agents as described above. We assume (as in Davis and Heathcote (2005)) that final goods (residential investment and consumption goods) are produced using intermediate goods. The intermediate goods sector consists of three output: building/construction, manufacture, and services, which are produced via Cobb-Douglas production functions:

$$x_i = k_i^{\theta_i} \left(e^{z_i} n_i\right)^{1-\theta_i} \tag{30}$$

where $i = b, m, s$ (building/construction, manufacture, service), k_{it}, n_{it} and z_{it} are capital, household -, entrepreneur labor, and labor augumenting (in log) productivity shock respectively for each sector, with the θ_i being the share of capital that differ across sectors. For example, we let in our calibration that $\theta_b < \theta_m$, reflecting the fact that the manufacturing sector is more capital intensive (or less labor intensive) than the construction sector. Unlike Davis and Heathcote (2005), we include entrepreneurial labor supply in the production function.[9]

The production shocks grows linearly:

$$z_i = t \ln g_{z,i} + \tilde{z}_i \tag{31}$$

with the stochastic term $\vec{z} = (\tilde{z}_b, \tilde{z}_m, \tilde{z}_s)$ following the vector $AR(1)$ process:

$$\vec{z}_{t+1} = B \cdot \vec{z}_t + \vec{\varepsilon}_{t+1} \tag{32}$$

where the the matrix B captures the deterministic part of shocks over time, and the innovation vector $\vec{\varepsilon}$ is distributed normally with a given covariance matrix Σ_ε. The shock growth factors $g_{z,i}$ lead to the correspondent growth factors for other variables.

These intermediate firms maximize a conventional static profit function at t

$$\max_{\{k_{it}, n_{it}\}} \left\{ \sum_i p_{it} x_{it} - r_t k_t - w_t n_t \right\} \tag{33}$$

[9] Although we do not include in the model the entrepreneurial labor income, the assumption of entrepreneurial labor income is necessary as it gurantees a nonzero net worth for each entrepreneur. This nonzero net worth assumption is important as the financial contracting problem is not well defined otherwise. In our calibration section, we let the share of entrepreneur's labor supply to be quite small but nonzero. Consequently, although the entreprenuers' labor supply do not play a role in our equilibrium conditions, small share of entrepreneur's labor supply does ensure a small but nonzero net worth.

subject to equations $k_t \geq \sum_i k_{it}, n_t \geq \sum_i n_{it}$, and non-negativity of inputs, where r_t, w_t, and p_{it} are the capital rental, wage, and output prices. A conventional optimization leads to the relations:

$$k_i r = \theta_i p_i x_i \tag{34}$$

$$n_i w = (1 - \theta_i)\, p_i x_i \tag{35}$$

so that

$$k_i r + n_i w = p_i x_i \tag{36}$$

The intermediate goods are then used as inputs to produce two final goods, y_j:

$$y_{jt} = \underset{i=b,m,s}{\Pi} x1_{ijt}^{\rho_{ij}}, \tag{37}$$

where $j = c, d$ (consumption/capital investment and residential investment respectively), the input matrix is defined by

$$x1 = \begin{pmatrix} b_c & b_d \\ m_c & m_d \\ s_c & s_d \end{pmatrix}, \tag{38}$$

and the shares of construction, manufactures and services for sector j are defined by the matrix

$$\rho = \begin{pmatrix} B_c & B_d \\ M_c & M_d \\ S_c & S_d \end{pmatrix}. \tag{39}$$

The relative shares of the three intermediate inputs differ in producing two final goods. For example, we would set $B_c < B_d$ to represent the fact that the residential investment is more construction input intensive. Moreover, with the CRS property of the production function, the following conditions must also be satisfied:

$$\sum_i \rho_{ij} = 1 \tag{40}$$

and

$$x_{it} = \sum_j x1_{ijt} \tag{41}$$

i.e

$$x_{bt} = b_{ct} + b_{dt}, \quad x_{mt} = m_{ct} + m_{dt}, \quad x_{st} = s_{ct} + s_{dt}. \tag{42}$$

With intermediate goods as inputs, the final goods' firms solve the following static profit maximization problem at t where the price of consumption good, p_{ct}, is normalized to 1:

$$\max_{\{b_{jt}, m_{jt}, s_{jt}\}} \left\{ y_{ct} + p_{dt} y_{dt} - \sum_i p_{it} x_{it} \right\}$$

subject to equation (37) and non-negativity of inputs. The optimization of final good firms leads to the relations:

$$p_{it} x1_{i,jt} = \rho_{i,j} p_{jt} y_{jt} \tag{43}$$

where $i = b, m, s,\ j = c, d$

Due to CRS property, we obtain:

$$\sum_{i=b,m,s} p_{it} x_{it} = \sum_{j=c,d} p_{jt} y_{jt} = K_t r_t + w_t N_t \tag{44}$$

where

$$K_t = \sum_{i=b,m,s} k_{it}, N_t = \sum_{i=b,m,s} n_{it} \tag{45}$$

Lastly, the housing firms (real estate developers or entrepreneurs) produce the housing good,

y_{ht}, given residential investment y_{dt} and fix amount of land x_{lt} as inputs, according to

$$y_{ht} = x_{lt}^{\phi} y_{dt}^{1-\phi} \tag{46}$$

where, ϕ denotes the share of land. Output equation (46) will be modified later in the section to include idiosyncratic productivity and uncertainty shocks. As mentioned in the introduction, the focus of our paper is on the housing sector in which agency costs with uncertainty and hetroegeneity arise: we come back to this modification on the firms' behaviorin the later section. The optimization defines the price relations:

$$p_l x_l = \phi p_h y_h, \qquad p_d y_d = (1-\phi) p_h y_h \tag{47}$$

2.2.1 Firms: Production side with two types of households

With both patient and impatient household, we now need to make small modifications of the production side of the model due to the presence of two types of labor supplying households in the system. Now the production of the intermediate good (30) changes to the form[10]:

$$x_i = k_i^{\theta_i} \left(e^{z_i} n_i^{a} n_i'^{1-\alpha} \right)^{1-\theta_i} \tag{48}$$

where n_i and n_i' are the hours supplied by the patient and impatient households respectively accordingly to their labor share α. Equations (35) and (36) now are:

$$n_i w = \alpha p_i x_i \left(1-\theta_i\right) \tag{49}$$

$$n_i' w' = (1-\alpha) p_i x_i \left(1-\theta_i\right) \tag{50}$$

[10] For the simplicity purpose, we drop the time script in this section.

and

$$k_i r + n_i' w' + n_i w = p_i x_i. \tag{51}$$

Balance equations (44) are also changed to:

$$\sum_{i=b,m,s} p_i x_i = \sum_{j=c,d} p_j y_j = Kr + Nw + N'w'. \tag{52}$$

The new variable N' denotes the total impatient household's hours:

$$N' = \sum_{i=b,m,s} n_i'. \tag{53}$$

We can introduce now the effective hours L and the effective wage W:

$$L = N^{\alpha} N'^{1-\alpha}, \qquad W = \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{w'}{1-\alpha}\right)^{1-\alpha}. \tag{54}$$

Then from relation (49) and (50) follows that

$$LW = Nw + N'w'. \tag{55}$$

We can also introduce the effective hours $l_i = n_i^{\alpha} n_i'^{1-\alpha}$ for building, manufacture and services ($i = b, m, s$) separately:

$$l_i = n_i^{\alpha} n_i'^{1-\alpha}.$$

Then the following equations hold:

$$l_i W = (1 - \theta_i)\, p_i x_i = n_i' w' + n_i w, \qquad\qquad L = \sum_{i=b,m,s} l_i$$

2.3 Credit Channel with Uncertainty

In this section, we outline how the financial intermediaries decide on the amount of loan that is to be lend out to housing developers (entrepreneurs). One should note that in our lending model, we only focuses on the supply side. That is, we do not address the endogenous lending mechanism for the impatient households (the demand side): The loan for the impatient household is exogenously determined by the collateral constraint in equation (21).

2.3.1 Housing Entrepreneurial Contract

It is assumed that a continuum of housing producing firms with unit mass are owned by risk-neutral entrepreneurs (developers). The costs of producing housing are financed via loans from risk-neutral intermediaries. Given the realization of the idiosyncratic shock to housing production, some real estate developers will not be able to satisfy their loan payments and will go bankrupt. The banks take over operations of these bankrupt firms but must pay an agency fee. These agency fees, therefore, affect the aggregate production of housing and, as shown below, imply an endogenous markup to housing prices. That is, since some housing output is lost to agency costs, the price of housing must be increased in order to cover factor costs.

The timing of events is critical:

1. The exogenous state vector of technology shocks, uncertainty shocks, housing preference shocks, and monetary shocks, denoted $(z_{i,t}, \sigma_{\omega,t}, \mu_{h,t+1}, R_{t+1})$, is realized.

2. Firms hire inputs of labor and capital from households and entrepreneurs and produce intermediate output via Cobb-Douglas production functions. These intermediate goods are then used to produce the two final outputs.

3. Households make their labor, consumption, housing, and investment decisions.

4. With the savings resources from households, the banking sector provide loans to entrepreneurs via the optimal financial contract (described below). The contract is defined by the size of

the loan (fp_{at}) and a cutoff level of productivity for the entrepreneurs' technology shock, $\bar{\omega}_t$.

5. Entrepreneurs use their net worth and loans from the banking sector in order to purchase the factors for housing production. The quantity of factors (residential investment and land) is determined and paid for before the idiosyncratic technology shock is known.

6. The idiosyncratic technology shock of each entrepreneur is realized. If $\omega_{at} \geq \bar{\omega}_t$ the entrepreneur is solvent and the loan from the bank is repaid; otherwise the entrepreneur declares bankruptcy and production is monitored by the bank at a cost proportional (but time varying) to total factor payments.

7. Solvent entrepreneur's sell their remaining housing output to the bank sector and use this income to purchase current consumption and capital. The latter will in part determine their net worth in the following period.

8. Note that the total amount of housing output available to the households is due to three sources: (1) The repayment of loans by solvent entrepreneurs, (2) The housing output net of agency costs by insolvent firms, and (3) the sale of housing output by solvent entrepreneurs used to finance the purchase of consumption and capital.

A schematic of the implied flows is presented in Figure 2.

For entrepreneur a, the housing production function is denoted $G\left(x_{alt}, y_{adt}\right)$ and is assumed to exhibit constant returns to scale. Specifically, we assume:

$$y_{aht} = \omega_{at} G\left(x_{alt}, y_{adt}\right) = \omega_{at} x_{alt}^{\zeta} y_{adt}^{1-\zeta} \tag{56}$$

where, ζ denotes the share of land. It is assumed that the aggregate quantity of land is fixed and equal to 1. The technology shock, ω_{at}, is an idiosyncratic shock affecting real estate developers. The technology shock is assumed to have a unitary mean and standard deviation of $\sigma_{\omega,t}$. The standard deviation, $\sigma_{\omega,t}$, follows an $AR(1)$ process:

$$\sigma_{\omega,t+1} = \sigma_0^{1-\chi} \sigma_{\omega,t}^{\chi} \exp^{\varepsilon_{\sigma,t+1}} \tag{57}$$

with the steady-state value $\sigma_0, \chi \in (0,1)$ and $\varepsilon_{\sigma,t+1}$ is a white noise innovation.[11]

Each period, entrepreneurs enter the period with net worth given by nw_{at}. Developers use this net worth and loans from the banking sector in order to purchase inputs. Letting fp_{at} denote the factor payments associated with developer a, we have:

$$fp_{at} = p_{dt} y_{adt} + p_{lt} x_{alt} \tag{58}$$

Hence, the size of the loan is $(fp_{at} - nw_{at})$. The realization of ω_{at} is privately observed by each entrepreneur; banks can observe the realization at a cost that is proportional to the total input bill.

It is convenient to express these agency costs in terms of the price of housing. Note that agency costs combined with constant returns to scale in housing production (see eq. (56)) implies that the aggregate value of housing output must be greater than the value of inputs; i.e. housing must sell at a markup over the input costs, the factor payments. Denote this markup as $\bar{s}_t$ (which is treated as parametric by both lenders and borrowers) which satisfies:

$$p_{ht} y_{ht} = \bar{s}_t fp_t \tag{59}$$

Also, since $E(\omega_t) = 1$ and all firms face the same factor prices, this implies that, at the individual

[11] This autoregressive process is used so that, when the model is log- linearized, $\hat{\sigma}_{\omega,t}$ (defined as the percentage deviations from σ_0) follows a standard, mean-zero AR(1) process.

level, we have[12]

$$p_{ht}G\left(x_{alt}, y_{adt}\right) = \bar{s}_t f p_{at} \tag{60}$$

Given these relationships, we define agency costs for loans to an individual entrepreneur in terms of foregone housing production as $\mu \bar{s}_t f p_{at}$.

With a positive net worth, the entrepreneur borrows $(fp_{at} - nw_{at})$ consumption goods and agrees to pay back $\left(1 + r_t^L\right)(fp_{at} - nw_{at})$ to the lender, where r_t^L is the interest rate on loans. The cutoff value of productivity, $\bar{\omega}_t$, that determines solvency (i.e. $\omega_{at} \geq \bar{\omega}_t$) or bankruptcy (i.e. $\omega_{at} < \bar{\omega}_t$) is defined by $\left(1 + r_t^L\right)(fp_{at} - nw_{at}) = p_{ht}\bar{\omega}_t F(\cdot)$ (where $F(\cdot) = F(x_{alt}, y_{adt})$). Denoting the *c.d.f.* and *p.d.f.* of ω_t as $\Phi(\omega_t; \sigma_{\omega,t})$ and $\phi(\omega_t; \sigma_{\omega,t})$, the expected returns to a housing producer is therefore given by:[13]

$$\int_{\bar{\omega}_t}^{\infty} \left[p_{ht}\omega F(\cdot) - \left(1 + r_t^L\right)(fp_{at} - nw_{at})\right] \phi(\omega; \sigma_{\omega,t})\, d\omega \tag{61}$$

Using the definition of $\bar{\omega}_t$ and eq. (60), this can be written as:

$$\bar{s}_t f p_{at} f(\bar{\omega}_t; \sigma_{\omega,t}) \tag{62}$$

where $f(\bar{\omega}_t; \sigma_{\omega,t})$ is defined as:

$$f(\bar{\omega}_t; \sigma_{\omega,t}) = \int_{\bar{\omega}_t}^{\infty} \omega \phi(\omega; \sigma_{\omega,t})\, d\omega - \left[1 - \Phi(\bar{\omega}_t; \sigma_{\omega,t})\right] \bar{\omega}_t \tag{63}$$

Similarly, the expected returns to lenders is given by:

$$\int_0^{\bar{\omega}_t} p_{ht}\omega F(\cdot)\, \phi(\omega; \sigma_{\omega,t})\, d\omega + \left[1 - \Phi(\bar{\omega}_t; \sigma_{\omega,t})\right]\left(1 + r_t^L\right)(fp_{at} - nw_{at}) - \Phi(\bar{\omega}_t; \sigma_{\omega,t})\, \mu \bar{s}_t f p_{at} \tag{64}$$

[12] The implication is that, at the individual level, the product of the markup ($\bar{s}_t$) and factor payments is equal to the expected value of housing production since housing output is unknown at the time of the contract. Since there is no aggregate risk in housing production, we also have $p_{ht}y_{ht} = \bar{s}_t f p_t$.

[13] The notation $\Phi(\omega; \sigma_{\omega,t})$ is used to denote that the distribution function is time-varying as determined by the realization of the random variable, $\sigma_{\omega,t}$.

Again, using the definition of $\bar{\omega}_t$ and eq. (60), this can be expressed as:

$$\bar{s}_t f p_{at} g\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \tag{65}$$

where $g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)$ is defined as:

$$g\left(\bar{\omega}_t; \sigma_{\omega,t}\right) = \int_0^{\bar{\omega}_t} \omega \phi\left(\omega; \sigma_{\omega,t}\right) d\omega + \left[1 - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\right] \bar{\omega}_t - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \mu \tag{66}$$

Note that these two functions sum to:

$$f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) + g\left(\bar{\omega}_t; \sigma_{\omega,t}\right) = 1 - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \mu \tag{67}$$

Hence, the term $\Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \mu$ captures the loss of housing due to the agency costs associated with bankruptcy. With the expected returns to lender and borrower expressed in terms of the size of the loan, fp_{at}, and the cutoff value of productivity, $\bar{\omega}_t$, it is possible to define the optimal borrowing contract by the pair $\left(fp_{at}, \bar{\omega}_t\right)$ that maximizes the entrepreneur's return subject to the lender's willingness to participate (all rents go to the entrepreneur). That is, the optimal contract is determined by the solution to:

$$\max_{\bar{\omega}_t, fp_{at}} \bar{s}_t f p_{at} f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \text{ subject to } \bar{s}_t f p_{at} g\left(\bar{\omega}_t; \sigma_{\omega,t}\right) \geqslant fp_{at} - nw_{at} \tag{68}$$

A necessary condition for the optimal contract problem is given by:

$$\frac{\partial(.)}{\partial \bar{\omega}_t} : \bar{s}_t f p_{at} \frac{\partial f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{\partial \bar{\omega}_t} = -\lambda_t \bar{s}_t f p_{at} \frac{\partial g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{\partial \bar{\omega}_t} \tag{69}$$

where λ_t is the shadow price of the lender's resources. Using the definitions of $f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)$ and

$g\left(\bar{\omega}_t;\sigma_{\omega,t}\right)$, this can be rewritten as:[14]

$$1-\frac{1}{\lambda_t}=\frac{\phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{1-\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}\mu \tag{70}$$

As shown by eq.(70), the shadow price of the resources used in lending is an increasing function of the relevant Inverse Mill's ratio (interpreted as the conditional probability of bankruptcy) and the agency costs. If the product of these terms equals zero, then the shadow price equals the cost of housing production, i.e. $\lambda_t = 1$.

The second necessary condition is:

$$\frac{\partial\left(.\right)}{\partial fp_{at}}:\bar{s}_t f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)=\lambda_t\left[1-\bar{s}_t g\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\right] \tag{71}$$

These first-order conditions imply that, in general equilibrium, the markup factor, $\bar{s}_t$, will be endogenously determined and related to the probability of bankruptcy. Specifically, using the first order conditions, we have that the markup, $\bar{s}_t$, must satisfy:

$$\begin{aligned}\bar{s}_t^{-1} &= \left[\left(f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)+g\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\right)+\frac{\phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\mu f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\frac{\partial f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{\partial\bar{\omega}_t}}\right] \\ &= \left[1-\underbrace{\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)\mu}_{A}-\underbrace{\frac{\phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}{1-\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}\mu f\left(\bar{\omega}_t;\sigma_{\omega,t}\right)}_{B}\right]\end{aligned} \tag{72}$$

which then can be written as

$$\bar{s}_t=\frac{1}{1-\mu\Phi\left(\varpi_t\right)+\mu f\left(\varpi_t\right)\frac{\varphi(\varpi_t)}{f'(\varpi_t)}} \tag{73}$$

We make some brief remarks on the markup equation above. First note that the markup factor depends only on economy-wide variables so that the aggregate markup factor is well defined. Also,

[14] Note that we have used the fact that $\frac{\partial f(\bar{\omega}_t;\sigma_{\omega,t})}{\partial\bar{\omega}_t}=\Phi\left(\bar{\omega}_t;\sigma_{\omega,t}\right)-1<0$

the two terms, A and B, demonstrate that the markup factor is affected by both the total agency costs (term A) and the marginal effect that bankruptcy has on the entrepreneur's expected return. That is, term B reflects the loss of housing output, μ, weighted by the expected share that would go to entrepreneur's, $f(\bar{\omega}_t; \sigma_{\omega,t})$, and the conditional probability of bankruptcy (the Inverse Mill's ratio). Finally, note that, in the absence of credit market frictions, there is no markup so that $\bar{s}_t = 1$. In the partial equilibrium setting, it is straightforward to show that equation (72) defines an implicit function $\bar{\omega}(\bar{s}_t, \sigma_{\omega,t})$ that is increasing in $\bar{s}_t$.

The incentive compatibility constraint implies

$$fp_{at} = \frac{1}{(1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t}))} nw_{at} \tag{74}$$

Equation (74) implies that the size of the loan is linear in entrepreneur's net worth so that aggregate lending is well-defined and a function of aggregate net worth.

The effect of an increase in uncertainty on lending can be understood in a partial equilibrium setting where $\bar{s}_t$ and nw_{at} are treated as parameters. As shown by eq. (72), the assumption that the markup factor is unchanged implies that the costs of default, represented by the terms A and B, must be constant. With a mean-preserving spread in the distribution for ω_t, this means that $\bar{\omega}_t$ will fall (this is driven primarily by the term A). Through an approximation analysis, it can be shown that $\bar{\omega}_t \approx g(\bar{\omega}_t; \sigma_{\omega,t})$ (see the Appendix in Dorofeenko, Lee, and Salyer (2008)). That is, the increase in uncertainty will reduce lenders' expected return ($g(\bar{\omega}_t; \sigma_{\omega,t})$). Rewriting the binding incentive compatibility constraint (eq. (74)) yields:

$$\bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t}) = 1 - \frac{nw_{at}}{fp_{at}} \tag{75}$$

the fall in the left-hand side induces a fall in fp_{at}. Hence, greater uncertainty results in a fall in housing production. This partial equilibrium result carries over to the general equilibrium setting.

The existence of the markup factor implies that inputs will be paid less than their marginal products. In particular, profit maximization in the housing development sector implies the following necessary conditions:

$$\frac{p_{lt}}{p_{ht}} = \frac{G_{xl}\left(x_{lt}, y_{dt}\right)}{\bar{s}_t} \tag{76}$$

$$\frac{p_{dt}}{p_{ht}} = \frac{G_{yd}\left(x_{lt}, y_{dt}\right)}{\bar{s}_t} \tag{77}$$

These expressions demonstrate that, in equilibrium, the endogenous markup (determined by the agency costs) will be a determinant of housing prices.

The production of new housing is determined by a Cobb-Douglas production with residential investment and land (fixed in equilibrium) as inputs. Denoting housing output, net of agency costs, as y_{ht}, this is given by:

$$y_{ht} = x_{lt}^{\zeta} y_{dt}^{1-\zeta}\left[1 - \Phi\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\mu\right] \tag{78}$$

In equilibrium, we require that $i_{ht} = y_{ht}$; i.e. household's housing investment is equal to housing output. Recall that the law of motion for housing is given by eq. (5)

2.3.2 Entrepreneurial Consumption and House Prices

To rule out self-financing by the entrepreneur (i.e. which would eliminate the presence of agency costs), it is assumed that the entrepreneur discounts the future at a faster rate than the patient household. This is represented by following expected utility function:

$$E_0 \sum_{t=0}^{\infty} \left(\beta\eta\gamma\right)^t c_t^e \tag{79}$$

where c_t^e denotes entrepreneur's per-capita consumption at date t, and $\gamma \in (0,1)$. This new parameter, γ, will be chosen so that it offsets the steady-state internal rate of return due to housing production.

Each period, entrepreneur's net worth, nw_t is determined by the value of capital income and the remaining capital stock.[15] That is, entrepreneurs use capital to transfer wealth over time (recall that the housing stock is owned by households). Denoting entrepreneur's capital as k_t^e, this implies:[16]

$$nw_t = k_t^e \left[r_t + 1 - \delta_\kappa \right] \tag{80}$$

The law of motion for entrepreneurial capital stock is determined in two steps. First, new capital is financed by the entrepreneurs' value of housing output after subtracting consumption:

$$\eta k_{t+1}^e = p_{ht} y_{aht} f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) - c_t^e = \bar{s}_t f p_{at} f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) - c_t^e \tag{81}$$

Note we have used the equilibrium condition that $p_{ht} y_{aht} = \bar{s}_t f p_{at}$ to introduce the markup, $\bar{s}_t$, into the expression. Then, using the incentive compatibility constraint, eq. (74), and the definition of net worth, the law of motion for capital is given by:

$$\eta k_{t+1}^e = k_t^e \left(r_t + 1 - \delta_\kappa \right) \frac{\bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} - c_t^e \tag{82}$$

The term $\bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right) / \left(1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)\right)$ represents the entrepreneur's internal rate of return due to housing production; alternatively, it reflects the leverage enjoyed by the entrepreneur since

$$\frac{\bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} = \frac{\bar{s}_t f p_{at} f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{nw_t} \tag{83}$$

That is, entrepreneurs use their net worth to finance factor inputs of value $f p_{at}$, this produces housing which sells at the markup $\bar{s}_t$ with entrepreneur's retaining fraction $f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)$ of the value of housing output.

[15] As stated in footnote 6, net worth is also a function of current labor income so that net worth is bounded above zero in the case of bankruptcy. However, since entrepreneur's labor share is set to a very small number, we ignore this component of net worth in the exposition of the model.

[16] For expositional purposes, in this section we drop the subscript a denoting the individual entrepreneur.

Given this setting, the optimal path of entrepreneurial consumption implies the following Euler equation:

$$1 = \beta\eta\gamma E_t \left[(r_{t+1} + 1 - \delta_\kappa) \frac{\bar{s}_{t+1}\ f(\bar{\omega}_{t+1}; \sigma_{\omega,t+1})}{1 - \bar{s}_{t+1} g(\bar{\omega}_{t+1}; \sigma_{\omega,t+1})} \right] \tag{84}$$

Finally, we can derive an explicit relationship between entrepreneur's capital and the value of the housing stock using the incentive compatibility constraint and the fact that housing sells at a markup over the value of factor inputs. That is, since $p_{ht} F(x_{alt}, y_{adt}) = \bar{s}_t f p_t$, the incentive compatibility constraint implies:

$$p_{ht} \left(x_{lt}^{\zeta} y_{dt}^{1-\zeta} \right) = k_t^e \frac{(r_t + 1 - \delta_\kappa)}{1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t})} \bar{s}_t \tag{85}$$

Again, it is important to note that the markup parameter plays a key role in determining housing prices and output.

2.3.3 Financial Intermediaries

The Capital Mutual Funds (CMFs) act as risk-neutral financial intermediaries who earn no profit and produce neither consumption nor capital goods. There is a clear role for the CMF in this economy since, through pooling, all aggregate uncertainty of capital (house) production can be eliminated. The CMF receives capital from three sources: entrepreneurs sell undepreciated capital in advance of the loan, after the loan, the CMF receives the newly created capital through loan repayment and through monitoring of insolvent firms, and, finally, those entrepreneur's that are still solvent, sell some of their capital to the CMF to finance current period consumption. This capital is then sold at the price of $\bar{s}_t$ units of consumption to households for their investment plans.

2.4 Government budget constraint

Assuming the absence of government money holdings, its budget constraint equation is:

$$G_t + \mu\Phi\left(\varpi_t\right) p_{h,t} y_{h,t} + m_{s,t} + m'_{s,t} = \tag{86}$$

$$K_t \left(r_t - \delta_k\right) \tau_k + \left(N_t w_t + N'_t w'_t\right) \tau_n + m_t^G$$

where G_t denotes the real government spending, $m_{s,t} = M_{st}/P_{c,t}, m'_{s,t} = M'_t/P_{c,t}$, and $m_t^G = M_t^G/P_{c,t}$ and M_t^G is a lump-sum money injection into the whole economy. Here we let the money evolves according to

$$m_{t+1}^G = \left(1 - \rho_M\right) m_0^G + \rho_M m_t^G$$

where, $\rho_M \in (0, 1)$.

Following Christiano, Eichenbaum and Trabandt (2014), we use the monetary policy rule as:

$$\log\left(\frac{R_{b,t+1}}{R_b}\right) = \varrho_R \log\left(\frac{R_{b,t}}{R_b}\right) + (1 - \varrho_R)\left(\varrho_\pi \log\left(\frac{1+\Pi_{t+1}}{1+\Pi}\right) + \varrho_{\mathrm{gdp}} \log\left(\frac{\mathrm{GDP}_{t+1}}{\mathrm{GDP}}\right) + \varrho_{\Delta\mathrm{gdp}} \log\left(\frac{\mathrm{GDP}_{t+1}}{\mathrm{GDP}_t}\right)\right) + \varepsilon_{R,t+1} \tag{87}$$

We also use one-period log-difference for GDP instead of averaged four-period difference used by Christiano et al, (2014).[17]

We assume here that the monitoring of defaulting firms is arranged by a government's institution, but separate the monitoring cost $\mu\Phi\left(\varpi_t\right) p_{h,t} y_{h,t}$ from the other government spendings as each deafulting firms belong to different industries (see eqs (89) and (90) below). The share of government spendings $G_t/GDP_t = a_G$ is considered to be a fixed value according to Davis and Heathcote (2005) (see eq (91)). The government distributes money injections M_{st} and M'_{st} between the patient and impatient householders proportionally to their steady-state shares M_0

[17] Equation (87) contains variables with exponential trend removed. The variables without subscript denote steady-state values. We use "borrowing interest rate" of borrowing between patient and impatient households, R_b , in eq (87) as one of the targeted values.

and M_0'.

The complete system of 62 equilibrium equations of the model is summarized in the Appendix.

3 Empirical Results

Our primary empirical objective in this paper is to show the importance of the following key parameters (variables) and shocks on housing variables as well as some of the aggregate macro variables. Consequently, we do not calibrate our model to specific country's economic parameters, but rather we set the model parameters to our benchmark values of the U.S. economy but with the loan-to-value (m) to to vary to reflect different european countries situation. Most of the parameters are based on three sources: Davis and Healthcote (2004), Iacoviello and Neri (2010) and Dorofeenko, Lee and Salyer (2014). Some of the other key parameters that need further explanation are described below. In this section, we do not address some of the housing and business cycles: we do discuss in this paper version, the steady-state, and some of the second moment properties of the model to the data. We, however, focus our results on the dynamics of the model by analyzing impulse response functions and variance decompositions.

3.1 Calibration Parameters

We use linear approximation approach to calibrate our model. As mentioned above, we employ our parameters based on the U.S. and various European nations (average values) dataset. We do not claim that the parameters that we employ in this section reflect the true nature of the European economies that we have in mind. For example, the bankrupcy rates across different nations vary as each economy has a different set of bankruptcy laws and rules. Nevertheless, during our calibration excercise, we have checked the robustness of several of the paramameter that we thought would lead to an unstable equilibrium case. The parameters that we have finally decided to use do not change much of the empirical results that we are to report in the next section. The crucial parameter that we use to distinguish three different European economies of Germany, Italy, and Spain, we assign the LTV ratio, $m, \{0.80; 0.85; 0.90\}$ [18] respectively.

A strong motivation for using the Davis and Heathcote (2005) model is that the theoretical constructs have empirical counterparts. Hence, the model parameters can be calibrated to the data. We use directly the parameter values chosen by the previous authors; readers are directed to their paper for an explanation of their calibration methodology. Parameter values for preferences, depreciation rates, population growth and land's share are presented in Table 1. In addition, the parameters for the intermediate production technologies are presented in Table 2.[19]

As in Davis and Heathcote (2005), the exogenous shocks to productivity in the three sectors are assumed to follow an autoregressive process as given in eq. (**??**). The parameters for the vector autoregression are the same as used in Davis and Heathcote (2005) (see their Table 4, p. 766 for details). In particular, we use the following values (recall that the rows of the B matrix correspond to the building, manufacturing, and services sectors, respectively):

$$B = \begin{pmatrix} 0.707 & 0.010 & -0.093 \\ -0.006 & 0.871 & -0.150 \\ 0003 & 0.028 & 0.919 \end{pmatrix}$$

[18] We also include the LTV of 0.1 ($m = 0.1$) to reflect almost no LTV constraint.

[19] Davis and Heathcote (2005) determine the input shares into the consumption and residential investment good by analyzing the two sub-tables contained in the "Use" table of the 1992 Benchmark NIPA Input-Output tables. Again, the interested reader is directed to their paper for further clarification.

Table 1: Key Preference, lending and Production Parameters

Notation	Value	Description
br	0.039	Bankruptcy rate
m_0	0.00868	steady state value of money supply
rp	0.0187	risk premium
a	0.9	patient HH's labor share
b	0.93	impatient HH's discount rate
β	0.951	patient household discount factor
γ	0.825	extra entrepr discount factor
η	1.017	population growth rate
Π_0	0.02	inflation rate
σ_0	0.231	st.dev.of entrepreneurial ω
ϕ	0.106	land share in housing production
χ	0.001	persistency of σ
ω_0	0.648	steady-state.of ω
$a_{\mathcal{G}}$	0.01	gorevnment consump.share of GDP
δ_k	0.0557	capital depreciation rate
δ_s	0.0157	res. structure depreciation rate
δ_h	0.014	housing stock depreciation rate
θ_b	0.106	construction capital share
θ_m	0.33	manufacturing capital share
θ_s	0.248	services capital share
κ_h	3.	housing investment adjustment cost
κ_k	3.	capital investment adjustment cost
μ_c	0.314	cons.easticity in utility function
μ_h	0.0444	housing easticity in utility function
μ_ℓ	0.642	leisure easticity in utility function
$\mu_{\mathbf{h}}$	0.25	monitoring cost
ϱ_{gdp}	0.012	TR persistency of GDP
ϱ_π	1.672	TR persistency of inflation
ϱ_R	0.792	TR persistency of interest rate
$\varrho_{\Delta\mathrm{gdp}}$	0.184	TR persistency of GDP change
ϱ_h	0.96	persistency of housing demand shock
ϱ_m	0.9	persistency of money
$g_{z,b}$	0.997	construction productivity growth rate
$g_{z,m}$	1.028	manufacturing productivity growth rate
$g_{z,s}$	1.016	services productivity growth rate

Table 2: Intermediate Production Technology Parameters

	B	M	S
Input shares for consumption/investment good (B_c, M_c, S_c)	0.031	0.270	0.700
Input shares for residential investment (B_d, M_d, S_d)	0.470	0.238	0.292
Capital's share in each sector $(\theta_b, \theta_m, \theta_s)$	0.132	0.309	0.237
Sectoral trend productivity growth (%) (g_{zb}, g_{zm}, g_{zs})	-0.27	2.85	1.65

Note this implies that productivity shocks have modest dynamic effects across sectors. The contemporaneous correlations of the innovations to the shock are given by the correlation matrix:

$$\Sigma = \begin{pmatrix} Corr(\varepsilon_b, \varepsilon_b) & Corr(\varepsilon_b, \varepsilon_m) & Corr(\varepsilon_b, \varepsilon_s) \\ & Corr(\varepsilon_m, \varepsilon_m) & Corr(\varepsilon_m, \varepsilon_s) \\ & & Corr(\varepsilon_s, \varepsilon_s) \end{pmatrix} = \begin{pmatrix} 1 & 0.089 & 0.306 \\ & 1 & 0.578 \\ & & 1 \end{pmatrix}$$

The standard deviations for the innovations were assumed to be: $(\sigma_{bb}, \sigma_{mm}, \sigma_{ss}) = (0.041, 0.036, 0.018)$.

For the financial sector, we use the same loan and bankruptcy rates as in Carlstrom and Fuerst (1997) in order to calibrate the steady-state value of $\bar{\omega}_t$, denoted ϖ, and the steady-state standard deviation of the entrepreneur's technology shock, σ_0. The average spread between the prime and commercial paper rates is used to define the average risk premium (rp) associated with loans to entrepreneurs as defined in Carlstrom and Fuerst (1997); this average spread is 1.87% (expressed as an annual yield). The steady-state bankruptcy rate (br) is given by $\Phi(\varpi, \sigma_0)$ and Carlstrom and Fuerst (1997) used the value of 3.9% (again, expressed as an annual rate). This yields two equations which determine (ϖ, σ_0):[20]

$$\begin{aligned} \Phi(\varpi, \sigma_0) &= 3.90 \\ \frac{\varpi}{g(\varpi, \sigma_0)} - 1 &= 1.87 \end{aligned} \tag{88}$$

yielding $\varpi \approx 0.65$, $\sigma_0 \approx 0.23$.[21]

[20] Note that the risk premium can be derived from the markup share of the realized output and the amount of payment on borrowing: $\bar{s}_t \bar{\omega}_t fp_t = (1 + rp)(fp_t - nw_t)$. And using the optimal factor payment (project investment), fp_t, in equation (74), we arrive at the risk premium in equation (88).

[21] It is worth noting that, using financial data, Gilchrist et al. (2008) estimate σ_0 to be equal to 0.36. Moreover, Chugh (2016) using industry level data estimates σ_0 to be exactly 0.23.

The entrepreneurial discount factor γ can be recovered by the condition that the steady-state internal rate of return to the entrepreneur is offset by their additional discount factor:

$$\gamma \left[\frac{\bar{s} f(\varpi, \sigma_0)}{1 - \bar{s} g(\varpi, \sigma_0)} \right] = 1$$

and using the mark-up equation for $\bar{s}$ in eq. (72), the parameter γ then satisfies the relation

$$\gamma = \frac{g_U}{g_K} \left[1 + \frac{\phi(\varpi, \sigma_0)}{f'(\varpi, \sigma_0)} \right] \approx 0.832$$

where, g_U is the growth rate of marginal utility and g_K is the growth rate of consumption (identical to the growth rate of capital on a balanced growth path). The autoregressive parameter for the risk shocks, χ, is set to 0.90 so that the persistence is roughly the same as that of the productivity shocks.

The final two parameters are the adjustment cost parameters (κ_k, κ_h). In their analysis of quarterly U.S. business cycle data, Christiano, Eichenbaum and Evans (2005) provide estimates of κ_k for different variants of their model which range over the interval $(0.91, 3.24)$ (their model did not include housing and so there was no estimate for κ_h). Since our empirical analysis involves annual data, we choose a lower value for the adjustment cost parameter and, moreover, we impose the restriction that $\kappa_k = \kappa_h$. We assume that $\kappa_h = \kappa_h = 3$ implying that the (short-run) elasticity of investment and housing with respect to a change in the respective shadow prices is 0.33 (i.e. the inverse of the adjustment cost parameter). Given the estimates in Christiano, Eichenbaum, and Evans (2005), we think that these values are certainly not extreme. We also solve the model with no adjustment costs. As discussed below, the presence of adjustment costs improves the behavior of the model in several dimensions.

3.1.1 Estimation of Risk Shocks

In this section, we estimate risk shocks using the U.S. construction firm level data. The main purpose in estimating these shocks is to show that risk shocks defined as the time variation in the cross sectional distribution of firm level productivities are important inputs to a baseline DSGE model. In estimating risk shocks, we use the dataset from the Compustat Industry Specific Quarterly data. For the robustness of our estimation, we estimate for 2 intersecting subsets of firms: i). The firms with S&P GIC sub-industry code 25201030 – Homebuilding (47 firms); ii). The firms with NAICS sub-industry code 23611 (sub-industries 236115-236118) - Residential Building Construction (35 firms)[22].

The procedure we employ in estimating our risk shocks is similar to Chugh (2011), who uses the dataset of Cooper and Haltiwanger's (2006) U.S. manufacturing dataset. In order to estimate the risk shocks, we first need to estimate the firm-level productivity coefficients via Fama-MacBeth regression as follows: employing the usual Cobb-Douglas production $B_{it} = cm_{it}^{\alpha} l_{it}^{1-\alpha} \exp(\varepsilon_{it})$, where i and t denote firm and time , B_{it} is the Backlogs, i.e., the "dollar value of housing units subject to pending sales contracts" to proxy output, l_{it} is "Land under development", m_{it} is defined as "Homebuilding inventories Total" - "Land under development" - "Undeveloped inventories owned" and ε_{it} *iid* with Normal. Taking the log of the production function, we estimate following regression:

$$\log\left(\frac{B_{it}}{l_{it}}\right) = c + \alpha \log\left(\frac{m_{it}}{l_{it}}\right) + \varepsilon_{it}$$

Given the dataset, the term $\frac{B_{it}}{l_{it}}$ represents the "profit or productivity" and the term $\frac{m_{it}}{l_{it}}$ denotes the input. The estimates of α for subset (1) and subset (2) are $\alpha_1 = 0.6\,(0.05)$ and $\alpha_2 = 0.7\,(0.04)$ respectively with the standard deviations in brackets. With the estimates $\widehat{\alpha}$, the logarithmic productivity is then defined as the residual:

[22] The full description of these NAICS codes are as follows: 23611 Residential Building Construction, 236115 New Single-Family Housing Construction (except Operative Builders), 236116 New Multifamily Housing Construction (except Operative Builders), 236117 New Housing Operative Builders, and 236118 Residential Remodelers.

$$\log\left(P_{it}\right)=\log\left(\frac{B_{it}}{l_{it}}\right)-\widehat{\alpha}\log\left(\frac{m_{it}}{l_{it}}\right)$$

where, the aggregate productivity is defined as $P_t \equiv \frac{1}{N}\sum_{i=1}^{N_t} P_{it}$ and idiosyncratic productivity as $p_{it} \equiv \frac{P_{it}}{P_t}$ (with $\frac{1}{N_t}\sum_{i=1}^{N_t} p_{it} = 1$). Consequently, the risk is estimated as cross-sectional standard deviation of p_{it} : that is,

$$\sigma_t = \sqrt{\frac{1}{N}\sum_{i=1}^{N_t}\left(p_{it}-1\right)^2}.$$

Finally, the AR(1) estimates of HP-detrended risk $\sigma_t, \log\left(\sigma_t\right) = \rho\log\left(\sigma_{t-1}\right) + \varepsilon_t$ where $\varepsilon_t \sim N\left(0, \sigma_\sigma^2\right)$, that we use as our input to our model yield i) for the subset 1, $\rho = 0.28(0.17)$ and $\sigma_\sigma = 0.23(0.02)$; and ii) for the subset 2, $\rho = 0.26(0.19)$ and $\sigma_\sigma = 0.24(0.03)$, where the numbers in brackets indicate the standard deviations. Moreover, the corresponding annual value for $\rho^y \approx 0.02$ and $\sigma_\sigma^y \approx 0.01$.

Figure 3 shows the estimated productivity and risk shocks from 2001 till 2011. The HP trends for productivity and risk shocks clearly show that the shocks behave opposite: one can think of risk (uncertainty) shocks as "negative" technology shocks in terms of the role the shocks play in our model. These strongly countercyclical construction firm level risk is also a robust finding in micro evidence of Bachmann and Bayer (2010) and Bloom, Floetotto, and Jaimovich (2010).

3.2 Dynamics

3.2.1 Loan-to-Value Effect and Various Shocks: Impulse Response Functions

The main questions to be addressed in this sections are as follows: i) How do non-standard shocks in relation to the technology shocks effect key housing and macroeconomic variables? ii) How does the collateral constraint on some of the households effect the housing and business cycles?

Figures 4, 5 and 6 show the impulse response functions (to a 1% innovation in all four shocks) for several macroeconomic and housing variables under one key parameter value[23]: the LTV

[23] We also have analysed two other parameter dimensions: the monitoring cost (reflecting the agency cost) and

ratio, which is set to either 0.1 or 0.85. A few papers have investigated the role of collateral requirements for the transmission of unanticipated shocks and macroeconomic volatility. Campbell and Hercowitz (2004) find that the U.S. mortgage market liberalization of the early 1990s, proxied by an increase in the LTV ratio, played a role in explaining the great moderation. In contrast, Calza, Monacelli and Stracca (2010) show that the transmission of monetary policy shocks to consumption, investment and house prices is dampened by lower LTV ratio. While the results discussed above provide some support for the housing cum credit channel model, the role of the lending channel with collateral constraint is not easily seen because of the presence of the other impulse shocks (i.e., the sectoral productivity shocks).

We first turn to the behavior of three key macroeconomic variables, namely GDP, household consumption (denoted PCE), and total capital when the LTV is set to 10% (m=01) and 85% (m=0.85) as seen in Figures 4a and 4b. The response to a technology shock to the construction sector has the predicted effect that GDP increases. Consumption also increases, while capital stock responds much bigger. This consumption/savings decision reflects agents response to the expected high productivity (due to the persistence of the shock) in the construction sector. Turning to our a risk shock which affects housing production results, we see a modest fall in GDP and in capital stock. Recall, as discussed in the partial equilibrium analysis of the credit channel model, an increase in productivity risk results in a leftward shift in the supply of housing; since residential investment (and hence, the capital stock) is the primary input into housing, it too falls in response to the increased risk. Consumption reacts negatively to a risk shock due to an increas in "pre-cautionary savings" as households face the persistence of the shock.

Consumption responds positively to both technology and monetary shocks, which is consistent with models that have an investment specific technology shock (e.g. Greenwood, Hercowitz, and Krusell (2000)). The monetary shocks play a large role in the aforementioned variables: the

capital adjustment cost (reflecting the amplitude of business cycles). The results are not shown as we focus on the effects of the LTV parameter.

magnitude of the monetary shock is as big as the technology shocks if not bigger.

Figure 5 reports the impulse response functions of the housing markup, the risk premium on loans to the housing producers and the bankruptcy rate. A positive technology shock to the construction sector increases the demand for housing and, ceteris paribus, will result in an increase in the price of housing. This will result in greater lending to the housing producers which will result in a greater bankruptcy rate and risk premium; both of these effects imply that the housing markup will increase. Note the counterfactual implication that both the bankruptcy rate and the risk premium on loans will be procyclical; this was also the case in the original Carlstrom and Fuerst (1997) model and for exactly the same reason. In contrast, a risk shock produces countercyclical behavior in these three variables. Hence, this argues for inclusion of risk shocks as an important impulse mechanism in the economy. With the preference shocks, both the housing markup and risk premium react positively as expected: as the demand increase, there is a greater incentive for the housing developers to a higher markup, which then creates an upper pressure on the risk premium. The monetary shocks effect on the housing markup is something that we cannot logically explain.

Finally, we report in Figure 6, the impulse response functions of the prices of land,housing and the amount of borrowing to the four shocks. A technology shock to the construction sector results in lower cost of housing inputs due to the increased output in residential investment so that the price of housing falls. However, the price of land, i.e. the fixed factor, increases. For an uncertainty (risk), preference and monetary shocks, the resulting fall in the supply of housing causes the demand for the fixed factor (land) to fall and the price of the final good (housing) to increase. In regards to the borrowing, we clearly see the role of monetary shock: 1% changes in interest rate causes 0.2% decrease in the amount of borrowing. The monetary shock has the biggest effect of all the shocks that are presented.

In ending this section, a word of caution is needed in interpreting the quantitative magnitudes seen in the impulse response functions. In particular, note that the response of housing prices to a

Table 3: Variance Decomposition of Forecast Error

	Variance of Decomposition			
	u_b	u_σ	u_h	u_R
Shocks	tech	risk	preference	monetary
GDP	0.87	0.016	0.027	0.086
PCE	0.46	0.069	0.03	0.44
capital stock	0.54	0.007	0.01	0.44
house stock patient	0.55	0.022	0.044	0.38
house stock impatient	0.50	0.0039	0.029	0.465
Labor Hour (total)	0.49	0.062	0.049	0.40
Borrowing	0.495	0.0016	0.023	0.48
House Price	0.026	0.854	0.065	0.05
House Investment	0.148	0.085	0.037	0.735

preference shock increase is greater than the response due to, say, a risk shock or monetary shock. One might deduce that the housing sector and risks and monetary shocks play a minor role in the movement of housing prices. As the results from the full model (i.e. when the all technology, monetary and risk shocks are present) imply, such a conclusion would be incorrect.

3.2.2 What drives housing and business cycles? Variance Decompositions

This section briefly describes the role of various shocks on some of the key macro and housing variables. The main message from Table 3 is that the monetary and uncertainty shocks play a major role in accounting for the movements in some of the aggregate as well as housing variables. In other words, there is a large of policy makers in dealing with the volatilities of these aforementioned variables. On the other hand, the preference shocks play almost no role in any of the macro or housing variables.

Once again, Table 3 presents three different scenarios that are based on LTV ratio: low (80), middle (85) and high (90) borrowing constraints. Unlike some of the recent literature that emphasize the important role of the level of LTV on housing market, our results indicate otherwise: almost no differences between different levels of LTV on the variables that we analyze. On the role of specific shocks, Table 3 shows that the effects of monetary shocks are huge on most of the macro variables and in particular on the housing investment and the amount of borrowing the households undertake: over 75 percent and almost 50 percent of the variation in housing investment and borrowing can be explained by the monetary shocks. On the contrary to monetary shocks, housing demand shocks have a trivial impact on all the variables that we analyze: at most 6 percent of the variation in housing price can be explained by the preference shocks. Lastly, our endogenous debt financial accelerator model with risk shocks lends a strong support for the important role of risk shocks: over 85 percent of the variation in housing price is due to risk shocks.

4 Some Final Remarks

Our primary findings fall into two broad categories. First, risk and monetary shocks to the housing producing sector imply a quantitatively large role for uncertainty and monetary policy over the housing and business cycles. Second, there is a great role for the government policies: the effects of both monetary and risk shocks clearly show that having a stable economy can indeed reduce the volatilities of various housing and macroeconomic variables.

For future research, modelling uncertainty due to time variation in the types of entrepreneurs would be fruitful. One possibility would be an economy with a low risk agent whose productivity shocks exhibit low variance and a high risk agent with a high variance of productivity shocks. Because of restrictions on the types of financial contracts that can be offered, the equilibrium is a pooling equilibrium so that the same type of financial contract is offered to both types of agents. Hence the aggregate distribution for technology shocks hitting the entrepreneurial sector is a mixture of the underlying distributions for each type of agent. Our conjecture is that this form of uncertainty has important quantitative predictions and, hence, could be an important impulse mechanism in the credit channel literature that, heretofore, has been overlooked. It also anecdotally corresponds with explanations for the cause of the current credit crisis: a substantial fraction of mortgage borrowers had higher risk characteristics than originally thought.

Moreover, our current model is silent about the optimal loan contract between the impatient households and financial intermediaries. Developing an endogenous household loan model would further shed light on the latest housing and financial boom and bust cycles. A quantitative assessment of the relative importance of the role of monetary policy, as well as the analysis of the optimal conduct of monetary policy, is also left to future research.

Nevertheless, from our analysis, there is a clear and important role for the policy makers to smooth housing price and/or housing investment. The fact that both monetary and uncertainty shocks play a prominent role in explaining the housing and macro business cycles, the monetary policymakers have two instruments on hand to calm markets and provide market cofidence. However, one should be cautious in interpreting our empirical results as evidence for policymakers to be directly involved in solving financial and housing problems.

References

Aoki, Kosuke, J. Proudman, and J. Vlieghe, (2004), "House Prices, Consumption, and Monetary Policy: A Financial Accelerator Approach." *Journal of Financial Intermediation*, Vol. 13, No. 4, pp. 414–35.

Baker, S., Nicholas Bloom, and Steven J. Davis, (2012), "Measuring Economic Policy Uncertainty", manuscript, Stanford.

Bloom, Nicholas., Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry, (2012), "Really Uncertain Business Cycles," NBER Working Paper No. 18245.

Calza, A., Monacelli, T., and L. Stracca, (2010), "Housing Finance and Monetary Policy", forthcoming Journal of the European Economic Association.

Campbell, J. R. and Z. Hercowitz, (2004), "The Role of Collateralized Household Debt" in Macroeconomic Stabilization", NBER w.p 11330.

Carlstrom, C.T. and T.S. Fuerst, (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, Vol. 87, pp. 893 - 910.

Carlstrom, C.T. and T.S. Fuerst, (1998),"Agency Costs and Business Cycles", *Economic Theory*, Vol. 12, pp. 583-597.

Christiano, L, M. Eichenbaum, and C.L. Evans, (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, Vol. 113, No.1, pp. 1-45.

Christiano, L., R. Motto, and M. Rostagno, (2015), "Risk Shocks", *American Economic Review*, 104, 27-65.

Chugh, Sanja. K. (2016), "Firm Risk and Leverage-Based Business Cycles", *Review of Economic Dynamics*, Vol. 20, p. 111-131.

Davis, Morris and J. Heathcote, (2005), "Housing and the Business Cycle", *International Economic Review*, Vol. 46, No. 3, pp. 751 - 784.

Dorofeenko, Victor, Gabriel Lee and Kevin Salyer, (2008), "Time Varying Uncertainty and the Credit Channel", *Bulletin of Economic Research* , Vol. 60, No. 4, pp. 375-403

- (2014), "Risk Shocks and Housing Markets: Quantitative Analysis," *Journal of Economic and Dynamic Control*, 45, 194-219.

ECB, (2009), "Housing Finance in the Euro Area", *Structural Issues Report*, European Central Bank.

Favilukis, Jack, Sydney Ludvigson and Stijn Van Nieuwerburgh, (2010), "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium", NYU Department of Economics, manuscript.

Greenwood, J., Hercowitz, Z. and Krusell, P.,(2000). "The Role of Investment Specific Technological Change in the Business Cycle", *European Economic Review*, Vol. 44, pp. 91–115.

Iacoviello, Matteo, "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle" (2005), *American Economic Review*, Vol. 95, No. 3, pp. 739-764.

Iacoviello,Matteo and Raoul Minetti,(2008), "The Credit Channel of Monetary Policy: Evidence from the Housing Market", *Journal of Macroeconomics*, Vol. 30, No. 1 (March), pp. 69-96

Iacoviello, Matteo and S.Neri, (2008), "Housing Market Spillovers: Evidence from an Estimated DSGE model", *American Economic Journal: Macroeconomics*, 2 (April): pp. 125–164.

IMF, (2011), "Housing Finance and Financial Stability - Back to Basics?", in Global Financial Stability Report. Durable Financial Stability: Getting there from Here, pp. 111 - 157

Kahn, James, (2009), "What Drives Housing Prices?" Federal Reserve Bank of New York Staff Reports, No. 345.

5 Appendix

5.1 Recent Developments in European Housing Markets: Some Facts

In this section, we briefly discuss some of the recent housing and macroeconomics development for the aforementioned European countries. We start with the supply side by discussing the residential investment, and then focus on the demand side factors: i) household debt for housing loan, ii) borrowing factors; interest rate and loan-to-value.

Residential Investments

Figure 7 shows that residential investment moves in tandem to house prices to a various degree across countries. Starting with nations that face fairly elastic housing supply, between 1997 and 2007, Spain, Ireland, and Greece's residential investments approximately increased 120, 80 and 70 percent respectively. For Italy and the average EU (15), increases in residential investment have been more modest, despite large house price increases, suggesting that supply is fairly inelastic in these countries. for Germany, residential investment has been stagnating or falling, but as in the housing price movement, the residential investment has been slightly increasing as of 2009.

Household Debt

Figure 8 shows the household's long term loan for house purchase. There is a clear correlation between the movements in house prices, residential investment and household debt across the nations. With the rapid rising in house prices for some of the European nations and with an easy access to credit[24], Figure 3 illustrates some of the dramatic increase in the level of household debt (measured by the long term loan for house purchase) over recent years. Except for Germany, the household debt for the rest of the nations have been rapidly increasing over the last few years. Greece leads the pack with being the most indebted amount, followed by Ireland. An interesting aspect from figure 4 is that Ireland is the only nation that shows the downturn on the amount of

[24] Recent product innovations including low and flexible mortgage rate products, which are essentially aimed at restoring housing affordability in the face of rising prices, are well documented for the European countries (e.g. ECB, 2009).

Table 4: Long-run Effects of Uncertainty, Bartik and Interaction term

Country	Predominant Interest Rate Type	Typical Loan Term (years)	Main Lenders
Australia	variable	25	Banks and nonbank specialist ”mortg
Austria	fixed	25-30	Banks and Bausparkassen (mainly sav
Belgium	fixed	20	Banks
Canada	mixed	25-35	Banks and specialized nondepository
Denmark	mixed	30	Mortgage and retails banks
France	fixed	15-20	Mortgage and retails banks
Germany	fixed	20-30	Banks and Bausparkassen (mainly sav
Ireland	variable	21-35	Banks, building societies and mortgag
Italy	mixed	20	Banks
Japan	mixed	20-30	Banks and specialized mortgage instit
Netherlands	fixed	30	Banks and mortgage banks and broke
Portugal	variable	25-35	Banks
Spain	variable	30	Banks (commercial and savings)
Sweden	variable	30-45	Bank and mortgage institutions
UK	variable	25	Banks, building societies and mortgag
US	fixed	30	Banks and mortgage brokers

Source: The report ”Housing and Finance in the Euro Area, March 2009”, Table 3.2, from the European Central Bank.

household debt. Italy and Portugal show that their household debts are increasing whereby Spain and Greece are leveling off.[25] High levels of household debt clearly open up the vulnerability of households welfare to changes and shocks to mortgage payments, personal disposable income, and especially to house prices.

Interest rates

Figure 9 shows European mortgage interest rates (both real and nominal) have come down considerably from early 1990 till mid 2005: the average nominal rate for the European nations decreased from 12 to 4.5 percent. As can be seen in figure 4, except for Germany, the sample country's rates have been increasing from 2005 till their peak at late 2008. Subsequently, due to various economic downturns in these countries, the rates have been again falling below the 2005 rates. Table 4 shows the loan types for various European and North America nations.

Loan-to-Value

[25] If one looks at the average household debt, of which mortgages are the main constituent, represented about one year of household disposable income in 1995. By 2000, debt had risen to about 120% and in 2007 it was close to 170% for the Euro zone countries (OECD, 2010).

Table 5: Loan to Value Ratios

Country	**LTV**	Source
Australia	90-100	ECB
Austria	80	ECB
Belgium	100	ECB
Canada	80	ECB
Denmark	80	ECB
France	100	ECB
Germany	80	ECB
Ireland	100+	ECB
Italy	80	ECB
Japan	70-80	ECB
Netherlands	125	ECB
Portugal	90	ECB
Spain	100	ECB
Sweden	80-95	ECB
UK	110	ECB
US	110+	ECB

Note: The column LTV refers to the maximum LTV on New Loans. Source: The "Housing and Finance in the Euro Area, March 2009"",Table 3.2, from the European Central Bank.

IMF (2011) reports that there has been a sharp increase in the loan-to-value ratios : during the latest housing upturn, limits on the amount of mortgages have become less stringent than in the past in many markets. Maximum loan-to-value ratios have generally exceeded 80% in OECD countries. According to Table 3.2 in ECB (2009), Table 5 shows the maximum Loan to Value ratios on new loans for Germany, Italy, Ireland, Portugal and Spain are 80, 80, 100+, 90, and 100 respectively.

5.2 Complete set of equations of the model

The complete system of 62 equilibrium equations of the model is summarized below.

5.2.1 Household Sectors: 19 Equations (patient and impatient households)

Patient households (8 equations) The household's modified budget constraint (17)

$$c_{t+1}+i_{k,t+1}+p_{h,t+1}i_{h,t+1} = \frac{K_t\left(r_t-\tau_k\left(r_t-\delta_k\right)\right)+N_t w_t\left(1-\tau_n\right)+p_{l,t}x_{l,t}+b_t-R_{b,t-1}b_{t-1}}{1+\Pi_{t+1}}+a_M m_{s,t+1}$$

Euler equations for the patient household (6) - (11), (24) - (29)

$$\frac{w_t}{R_t} = \frac{U_{3t}}{U_{1t}}$$

$$q_{k,t} = \beta E_t \left(\left((1-\delta_k)\, q_{k,t+1} + \frac{(1-\tau_k)\, r_{t+1} + \tau_k \delta_k}{R_{t+1}} \right) \frac{U_{1,t+1}}{U_{1,t}} \right)$$

$$q_{h,t} = \beta E_t \left((1-\delta_h)\, q_{h,t+1} \frac{U_{1,t+1}}{U_{1,t}} + \frac{U_{2,t+1}}{U_{1,t}} \right)$$

$$1 = q_{k,t} F_{k,1} \left(i_{k,\,t}, i_{k,\,t-1} \right) + \beta E_t \left(q_{k,t+1} F_{k,2} \left(i_{k,\,t+1}, i_{k,\,t} \right) \frac{U_{1,t+1}}{U_{1,t}} \right)$$

$$p_{h,\,t} = q_{h,t} F_{h,1} \left(i_{h,\,t}, i_{h,\,t-1} \right) + \beta E_t \left(q_{h,t+1} F_{h,2} \left(i_{h,\,t+1}, i_{h,\,t} \right) \frac{U_{1,t+1}}{U_{1,t}} \right)$$

$$1 = \beta R_t E_t \left(\frac{U_{1,t+1}}{(1+\Pi_{t+1})\, U_{1,t}} \right)$$

$$1 = \beta R_{b,t} E_t \left(\frac{R_t}{R_{t+1}} \frac{U_{1,t+1}}{U_{1,t}} \right)$$

Impatient Households (7 equations) The household's modified budget constraint (20),

$$c'_{t+1} + p_{h,t+1} i'_{h,t+1} = \frac{N'_t w'_t (1-\tau_n) + b'_t - R_{b,t-1} b'_{t-1}}{1+\Pi_{t+1}} + \frac{M'_{s,t+1}}{P_{c,t+1}}$$

collateral borrowing constraint (21),

$$b'_t \leq m E_t \left(\frac{p_{h,t+1} h'_t}{R_t} \right)$$

Euler equations:

$$\frac{w'_t}{R'_t} = \frac{U_{3t}}{U_{1t}}$$

$$1 = \beta' R'_t E_t \left(\frac{U_{1,t+1}}{(1+\Pi_{t+1})\, U_{1,t}} \right)$$

$$q'_{b,t} R'_t = 1 - \beta' R_{b,t} E_t \left(\frac{R'_t}{R'_{t+1}} \frac{U_{1,t+1}}{U_{1,t}} \right)$$

$$q'_{h,t} = \beta' E_t \left(\left((1-\delta_h)\, q'_{h,t+1} + q'_{b,t+1} \frac{b'_{t+1}}{h'_{t+1}} \right) \frac{U_{1,t+1}}{U_{1,t}} + \frac{U_{2,t+1}}{U_{1,t}} \right)$$

$$p_{h,\,t} = q'_{h,t} F_{h,1} \left(i'_{h,\,t}, i'_{h,t-1} \right) + \beta' E_t \left(q'_{h,t+1} F_{h,2} \left(i'_{h,\,t\,+\,1}, i'_{h,t} \right) \frac{U_{1,t+1}}{U_{1,t}} \right)$$

Debt market clearing condition (22)**: 1 equation**

$$b'_t + b_t = 0$$

Capital growth: 3 equations

$$h_{t+1} = F_h \left(i_{h,\,t}, i_{h,t-1} \right) + (1-\delta_h)\, h_t$$

$$h'_{t+1} = F_h \left(i'_{h,\,t}, i'_{h,t-1} \right) + (1-\delta_h)\, h'_t$$

$$K_{t+1} = F_k \left(i_{k,t}, i_{k,t-1} \right) + (1-\delta_k)\, K_t$$

5.2.2 Entrepreneur equations: 4 Equations

The entrepreneur equations include

$$1 = \beta\eta\gamma E_t \left[(r_{t+1} + 1 - \delta_\kappa) \frac{\bar{s}_{t+1}\ f\left(\bar{\omega}_{t+1}; \sigma_{\omega,t+1}\right)}{1 - \bar{s}_{t+1} g\left(\bar{\omega}_{t+1}; \sigma_{\omega,t+1}\right)} \right]$$

$$\bar{s}_t = \frac{1}{1 - \mu\Phi\left(\varpi_t\right) + \mu f\left(\varpi_t\right) \frac{\varphi(\varpi_t)}{f'(\varpi_t)}}$$

$$p_{ht} \left(x_{lt}^{\zeta} y_{dt}^{1-\zeta} \right) = k_t^e \frac{(r_t + 1 - \delta_\kappa)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} \bar{s}_t$$

$$\eta k_{t+1}^e = k_t^e \left(r_t + 1 - \delta_\kappa\right) \frac{\bar{s}_t f\left(\bar{\omega}_t; \sigma_{\omega,t}\right)}{1 - \bar{s}_t g\left(\bar{\omega}_t; \sigma_{\omega,t}\right)} - c_t^e$$

5.2.3 Production side equations: 29 Equations for $(i = b, m, s; j = c, d)$

The production side equations are

$$x_i = \sum_{j=c,d} x1_{i,j}$$

$$y_j = \prod_{i=\{b,m,s\}} x1_{i,j}^{\rho_{i,j}}$$

$$p_i x1_{i,j} = \rho_{i,j} p_j y_j$$

$$K = \sum_{i=b,m,s} k_i, N = \sum_{i=b,m,s} n_i$$

$$y_h = x_l^{\phi} y_d^{1-\phi}$$

$$p_l x_l = \phi p_h y_h, p_d y_d = (1-\phi) p_h y_h$$

$$x_i = k_i^{\theta_i} \left(e^{z_i} n_i^a n_i'^{1-a}\right)^{1-\theta_i}$$

$$n_i w = a p_i x_i \left(1 - \theta_i\right)$$

$$n_i' w' = (1-a) p_i x_i \left(1 - \theta_i\right)$$

$$k_i r + n_i' w' + n_i w = p_i x_i$$

$$N' = \sum_{i=b,m,s} n_i'$$

5.2.4 Resource constraints: 2 equations

$$G_t + c_t + c_t' + c_t^e + i_{k,t} = y_{c,t} \tag{89}$$

$$i_{h,t} + i_{h,t}' = y_{h,t} \left(1 - \mu \Phi \left(\varpi_t\right)\right) \tag{90}$$

5.2.5 Goverment constraints: 2 equations

The real government spending G_t satisfies budget constraint equation and is assumed to be proportional to the real GDP:

$$G_t + \mu\Phi\left(\varpi_t\right) p_{h,t} y_{h,t} + \frac{M_{st} + M'_{st}}{P_{c,t}} =$$

$$K_t\left(r_t - \delta_k\right)\tau_k + \left(N_t w_t + N'_t w'_t\right)\tau_n + m_t^G$$

$$G_t = a_G\left(y_{c,t} + p_{d,t} y_{d,t} + q_t h_t\right) \tag{91}$$

where $q_t = \frac{U_{2,t+1}}{U_{1,t}}$ is the rental rate for housing (see Davis and Heathcote, 2005 for details).

5.2.6 The equations for external Shocks: 6 equations

The housing demand shock

$$\ln \mu_{h,t+1} = (1 - \rho_h)\ln \mu_h^{(0)} + \rho_h \ln \mu_{h,t} + \varepsilon_{h,t+1}$$

The money shock: Taylor Rule for interest rate R_t:

$$\ln R_{t+1} = \ln R_0 + \rho_\pi \ln\left(\frac{1+\Pi_t}{1+\Pi_0}\right) + \rho_{\text{gdp}} \ln\left(\frac{GDP_t}{g_k GDP_{t-1}}\right) + \varepsilon_{R,t+1}$$

The intermediate goods production shocks ($i = b, m, s$):

$$z_i = \sum_{j=b,m,s} z_j B_{i,j} + \varepsilon_{i,t+1}$$

The volatility of entrepreneur's production technology coefficient ϖ_t:

$$\sigma_{\omega,t+1} = \sigma_0^{1-\chi}\sigma_{\omega,t}^{\chi} e^{\varepsilon_{\sigma t+1}}$$

A competitive equilibrium is defined by the decision rules for (aggregate capital, entrepreneurs capital, households (patient and impatien) labor, entrepreneur's labor, entrepreneur's net worth, investment, the cutoff productivity level, household (patient and impatien) consumption, and entrepreneur's consumption) given by the vector:

$\left\{k_{t+1}, k_{t+1}^{e}, H_t, H_t^{e}, X_t, \bar{\omega}_t, c_t, c_t^{e}, c_t', i_{ht}', N_t', w_t', b_t', h_t', n_{bt}', n_{mt}', n_{st}'\right\}$ where these decision rules are stationary functions of $\left\{K_t, Z_t, h_t, z_{i,t\in\{b,m,s\}}\sigma_{\overline{\omega},t}, \varepsilon_{M,t+1}, \varepsilon_{h,t+1}\right\}$, all markets clear and all the firms, households and entrepreneurs solve their respective maximization problems, along with sets of equations representing the laws of motion for the sector specific shocks ($z_{t,i\in\{b,m.s\}}$), the monetary shock, the preference shock and the uncertainty shock. In total, there are 62 variables:

$c, i_k, p_h, i_h, K, r, N, w, p_l, b, R_b, m_s, \Pi, c', i_h', N', w', b', h', R, h, q_k, q_h, R', q_b', q_h', s, \varpi, \sigma, y_h, Z, c^e,$

$x_b, x_m, x_s, x1_{i,j}(6), y_c, y_d, p_b, p_m, p_s, p_d, k_b, k_m, k_s, n_b, n_m, n_s, n_b', n_m', n_s', z_b, z_m, z_s, G, \mu_h, m^G$ for 62 equations to be solved.