How Important Are Uncertainty Shocks in the Housing Market?

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Prof. Gabriel Lee
Prof. Kevin Salyer
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Chapter 1

Introduction

This dissertation analyzes the impact of shocks to uncertainty on the macroeconomy and on the housing sector. To this end, I examine different approaches to measuring and modeling uncertainty. My work contributes to this growing literature as follows. Chapter two clarifies one possible source of confusion in the calibration of models using uncertainty shocks, that between ex-ante vs. ex-post uncertainty measures. Chapter three proposes a different approach to modeling uncertainty shocks, that corresponds to the empirical evidence of Jurado, Ludvigson and Ng (2015) and Ludvigson, Ma, and Ng (2016). Chapter four investigates how the factors of production uncertainty, financial intermediation, and credit constrained households can affect housing prices and aggregate economic activity.

 Uncertainty as a factor influencing or governing decisions of economic agents has experienced increasing attention in recent years. Early work, such as Bernanke (1983) and McDonald and Siegel (1986), assumes irreversibility of investments to generate real options effects. More recent work, such as Dorofeenko, Lee, and Salyer (2008, 2014) or Christiano, Motto and Rostagno (2014), focuses on the impact of uncertainty in the context of financial frictions in Dynamic Stochastic General Equilibrium models. In these models, productivity’s time-varying second moment is part of the policy function despite first order approximation and impacts an economy via the optimal contract between borrowers and lenders. The literature on uncertainty and macroeconomics is divided, however, on the (magnitude of the) effects and the propagation mechanism of uncertainty on aggregate fluctuations. Dorofeenko et al. (2008), for instance, examine a 1% unexpected jump in uncertainty and find a large impact on the credit channel but little impact on real variables. Dorofeenko et al. (2014), which combines the model of Dorofeenko et al. (2008) with the multi-sector model of Davis and Heathcote (2005) to examine
the impact of risk on the housing market, show that uncertainty matters for the housing market and especially for housing prices - but not so much for real variables. Christiano et al. (2014), in contrast, add a news component to uncertainty shocks and report that risk shocks are the most important source of business cycle fluctuations. As virtually all the work is done by the exogenous definition of the exogenous risk shocks, as shown by Lee, Salyer and Strobel (2016), it is crucially important to properly model and calibrate uncertainty. To this end, various proxies have been proposed in the literature.

Chapter two, *On Measuring Uncertainty Shocks*, empirically shows systematic differences in those proxies, as realized variables fluctuate more than the measures that are based on forecasts. More precisely, the variation in the realized cross-sectional standard deviation of profit growth and stock returns is larger than the variation in the forecast standard deviation.

Chapter three, *Hump-shape Uncertainty, Agency Costs and Aggregate Fluctuations*, introduces a different approach to modeling uncertainty shocks. The uncertainty measures due to Jurado, Ludvigson and Ng (2015) and Ludvigson, Ma, and Ng (2016) show a hump-shape time path: Uncertainty rises for two years before its decline. Current literature on the effects of uncertainty on macroeconomics, including housing, has not accounted for this observation. This chapter shows that when uncertainty rises and falls over time, then the output displays hump-shape with short expansions that are followed by longer and persistent contractions. And because of these longer and persistent contractions in output, uncertainty is, on average, counter-cyclical. The model builds on the literature combining uncertainty and financial constraints. We model the time path of uncertainty shocks to match empirical evidence in terms of shape, duration and magnitude. In the calibrated models, agents anticipate this hump-shape uncertainty time path once a shock has occurred. Thereby, agents respond immediately by increasing investment (i.e. pre-cautionary savings), but face a substantial drop in investment, consumption and output as more uncertain times lie ahead. With persistent uncertain periods, both risk premia and bankruptcies increase, which cause a further deterioration in investment opportunities. Besides, accounting for hump-shape uncertainty measures can result in a large quantitative effect of uncertainty shock relative to previous literature.

Chapter four, *Housing and Macroeconomy: The Role of Credit Channel, Risk-, Demand- and Monetary Shocks*, uses the standard approach to modeling risk shock to demonstrate that risk (uncertainty) along with the monetary (interest rates) shocks to the housing production

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1. This chapter has been published under the title *On the different approaches of measuring uncertainty shocks* in 2015 in *Economics Letters*, 134, 69-72.
2. This chapter is joint work with Gabriel Lee and Kevin Salyer.
3. This chapter is joint work with Victor Dorofeenko, Gabriel Lee and Kevin Salyer.
sector are a quantitatively important impulse mechanism for the business and housing cycles. Our model framework is that of the housing supply/banking sector model as developed in Drofeenko, Lee, and Salyer (2014) with the model of housing demand presented in Iacoviello and Neri (2010). We examine how the factors of production uncertainty, financial intermediation, and credit constrained households can affect housing prices and aggregate economic activity. Moreover, this analysis is cast within a monetary framework which permits a study of how monetary policy can be used to mitigate the deleterious effects of cyclical phenomenon that originates in the housing sector. We provide empirical evidence that large housing price and residential investment boom and bust cycles in Europe and the U.S. over the last few years are driven largely by economic fundamentals and financial constraints. We also find that, quantitatively, the impact of risk and monetary shocks are almost as great as that from technology shocks on some of the aggregate real variables. This comparison carries over to housing market variables such as the price of housing, the risk premium on loans, and the bankruptcy rate of housing producers.
Chapter 2

On Measuring Uncertainty Shocks

This chapter has been published under the title *On the different approaches of measuring uncertainty shocks* in 2015 in *Economics Letters*, 134, 69-72.

2.1 Introduction

Uncertainty has become increasingly prominent as a source of business cycle fluctuations. Since there is no objective measure of uncertainty, various uncertainty proxies have been proposed in the literature, with “uncertainty” often formalized as time-varying second moment.\(^1\) Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), for instance, use uncertainty proxies derived from both realized and forecast real variables to calibrate their model, while Bloom (2009) uses a measure of forecast stock market volatility. Chugh (2013) and Dorofeenko et al. (2014), in turn, derive uncertainty on a sectoral level based on realized real data.

This paper shows that ex ante, the standard deviation of profit growth and stock returns in the U.S. economy, in the manufacturing sector and in the services sector fluctuates less than ex post by comparing the conditional standard deviation forecast to the realized cross-sectional standard deviation and to the interquartile range (IQR). This finding corroborates the argument of Leahy and Whited (1996, p. 68), that “since uncertainty relates to expectations and not to actual outcomes, it would be incorrect to use the ex post volatility of asset returns as a measure of the variability of the firm’s environment. We therefore need an ex ante measure”. Moreover, my results also show that the forecast standard deviation of profit growth and stock returns are negatively or at times uncorrelated.

\(^1\) A comprehensive survey of the literature can be found in Bloom (2014).
I use a Generalized Autoregressive Conditional Heteroskedasticity-in-mean (GARCH-M) model to forecast the conditional standard deviation of profit growth and stock returns in the manufacturing sector, the services sector and the U.S. economy. The results of the GARCH-M estimation also show that a higher conditional standard deviation increases stock returns due to a higher risk premium and decreases average profit growth.

2.2 Data

For the following analysis, two data sets used in Bloom (2009) are considered. The first data set contains observations on pre-tax profits, sales and industry for a total of 347 firms, 242 of which are in manufacturing and 23 are in the services sector in the United States from 1964Q4 to 2005Q1. The growth rate of quarterly profits\( \Delta \Pi_t \), normalized by sales\( S_t \), is calculated as\( \Delta \tilde{\Pi}_t = \frac{\Pi_t - \Pi_{t-4}}{1/2(S_t + S_{t-4})} \). The second data set contains information on firm-level stock returns for firms in the United States included in the Center for Research in Securities Prices (CRSP) stock-returns file with 500 or more monthly observations. The analysis focuses on the manufacturing sector, the services sector and the whole economy. In the absence of selection bias, mean, and standard deviation can be interpreted as return and risk per month from investing in a representative firm in of the sectors or the economy. As the data are constructed to reflect an average firm’s mean and standard deviation of stock returns and profit growth, the conditional variance reflects uncertainty and innovations to the conditional variance mirror uncertainty shocks in a sector. Using a GARCH-M model, I can predict the conditional standard deviation of stock returns and profit growth of an average firm, test whether uncertainty shocks have an effect on profit growth or stock returns and compare them to the realized cross-sectional standard deviation. Due to its theoretical correspondence, the conditional variance of productivity growth complements the uncertainty proxies.

The mean equation of the GARCH-M model is formulated as\( x_t = \mu + \theta \sigma_t^2 + u_t, u_t | I_{t-1} \sim N(0, \sigma_t^2) \), while the conditional variance\( \sigma_t^2 \) is assumed to follow a GARCH(1,1) process with one-step-ahead predictions given by\( \sigma_{t+1|t} = \omega + \alpha u_t^2 + \beta \sigma_t^2 \) Engle, Lilien, and Robins (1987). \( x_t \) corresponds to stock returns, profit growth or TFP growth, \( \mu \) is the mean, \( \sigma_t^2 \) is the conditional variance.

---

2 A detailed description is included in the Appendix.

3 Profit growth is calculated year-on-year to account for seasonality.

4 More precisely, it contains data on 361 firms, 208 of which are in manufacturing and 10 are in the services sector, ranging from 1962M8 to 2006M12.

5 Selection bias might be an issue, as only firms with 500 or more monthly data are included in the analysis. However, the bias is downward, potentially understating the impact of uncertainty.

variance and $u_t$ is an uncorrelated but serially dependent error. Normality of $u_t$ is a starting point and will be tested for. The one-period forecast of $\sigma_t^2$, based on TFP growth data is this paper’s Benchmark uncertainty estimation. The usefulness of $\sigma_{t+1|t}$ as benchmark is due to four reasons. First, uncertainty shocks are identified as innovations to the conditional one period forecast of the variance. Second, heteroskedasticity is modeled conditional on past information. Third, the GARCH-M approach allows for the conditional variance to affect profit growth, stock returns or TFP and fourth, out of sample forecasts can be done easily.\(^7\)

### 2.3 Results

Table 2.1 reports the distribution of $u_t$ and parameter estimation results. The effect of the conditional variance on profit growth or stock return depends on the sector. A hypothetical increase of

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Profit growth</th>
<th>Stock returns</th>
<th>TFP Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Manufacturing</td>
<td>Services</td>
<td>Economy</td>
</tr>
<tr>
<td>$\mu$</td>
<td>t(8.93)</td>
<td>Normal</td>
<td>Normal</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-16.48***</td>
<td>-0.906</td>
<td>-10.156</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>-0.470***</td>
<td>0.411***</td>
<td>0.004</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.788***</td>
<td>0.807***</td>
<td>0.004</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 2.1: Parameter estimates of the GARCH-M model based on mean profit growth (1964Q4 - 2005Q1), mean stock return (1962M8 - 2006M12) and TFP growth (1950Q1 - 2013Q4) in the manufacturing sector, in the services sector, in the U.S. economy. The distribution for the maximum likelihood estimation is chosen based on the Kolmogorov-Smirnoff test. Test results are reported in table 5 in the Appendix. Asterisks indicate significance at the 1%, 5% and 10% level based on Bollerslev-Wooldridge robust standard errors. Source: Compustat Database, CRSP, Federal Reserve Bank of San Francisco.

50% in the variance across time decreases expected quarterly profit by 29% in the manufacturing sector and by 8% in the services sector, although only the former result is significant.\(^8\)

The risk premia of 1.20% in the services sector, 1.03% in the manufacturing sector and 1.07% in the whole economy seem rather low and might be driven by aggregation and a downward bias, given a p-value of 9.6% in the services sector, 18.6% in the manufacturing sector and 10.8% in the U.S. economy.\(^9\)

Figure 2.1 shows IQR, realized and forecast standard deviation per period, estimated as

---

\(^7\)Test results for the presence of ARCH effects using Engle’s Lagrange multiplier (LM) test are reported in the appendix.

\(^8\)The change in expected quarterly profit growth in the manufacturing sector is calculated as

$$[(.0182247+1.5*.0003164 \times(-20.9759))/(.0182247+.0003164 \times(-20.9759))-1]=-0.2863$$

and analogously in the services sector.

\(^9\)The risk premium is calculated as e.g. $3.026 \times \bar{\sigma}_t^2 = 1.20\%$ in the services sector.
Figure 2.1: IQR, standard deviation and uncertainty proxy for the manufacturing sector, the services sector and the U.S. economy based on normalized profit growth from 1964Q4 to 2005Q1. Source: Federal Reserve Economic Data (FRED), Compustat Database.

explained above using data on profit growth. Forecast fluctuations are lower than the realized ones in the whole economy, as well as in both sectors. In the manufacturing sector, uncertainty increases after recessions, while this is not as clear for the IQR and standard deviation. In the services sector, the fluctuations do not seem to be associated with the occurrence of recessions. A similar pattern is observable for the IQR and standard deviation. Figure 2.2 shows somewhat similar results for stock returns. As can be seen in Figures 2.1 and 2.2, IQR and realized standard deviation fluctuate much more than their predicted counterpart which suggests that realizations of profit growth or stock returns further away from the mean occur more frequently than expected.

To compare these uncertainty proxies to more prominent ones, table 2 shows the pairwise correlation coefficients of Macro Uncertainty of Jurado et al. (2015), the VIX used in Bloom (2009), Policy Uncertainty constructed by Baker, Bloom and Davis (2012), this paper’s forecast-based proxies including the Benchmark, the cyclical component of HP-filtered real GDP and a recession indicator. Interestingly, the correlation of the conditional standard deviation forecast

\footnote{Table 2.4 displays the summary statistics of the time-series, and it can be seen that, on average, the expected conditional standard deviation fluctuates less than the realized standard deviation.}
for profit growth and stock returns are very low or even negative. Moreover, the correlation coefficients of the volatility of stock returns and profit growth are quite different from each other.

Table 2.2: Correlation coefficients of the uncertainty proxies of Jurado et al. (2015), Bloom (2009), this paper’s forecast proxies, the cyclical component of HP-filtered GDP and a recession indicator. PG corresponds to the uncertainty proxy based on profit growth, SM to the uncertainty proxy based on stock market returns; M refers to the manufacturing sector, S to the services sector and E to the whole economy. Source: Jurado et al. (2015), Bloom (2009), FRED.
2.4 Conclusion

This paper presents empirical evidence that ex post, profit growth and stock returns fluctuate more than ex ante. Moreover, fluctuations differ across sectors and depend on whether financial or real variables are used to calculate uncertainty. It is important to calibrate theoretical models accordingly, so as not to overstate the role of uncertainty. Uncertainty shocks decrease profit growth and increase stock returns. Variation in the forecast standard deviation of profit growth is not or negatively correlated with the forecast standard deviation in stock returns.

2.5 Appendix

Compustat and CRSP data were downloaded from, http://www.stanford.edu/~nbloom/quarterly2007a.zip.

Federal Reserve Economic Data

- Real GDP - GDPC1
- NBER Recession Indicator - USREC

Jurado et al.’s Uncertainty Proxy

Downloaded from http://www.econ.nyu.edu/user/ludvigsons/jlndata.zip.

Baker, Bloom and Davis’ Uncertainty Proxy


Tables

<table>
<thead>
<tr>
<th>lags(p)</th>
<th>Benchmark</th>
<th>PG - M</th>
<th>PG - S</th>
<th>SR - M</th>
<th>SR - S</th>
<th>PG - E</th>
<th>SR - E</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.68</td>
<td>0.01</td>
<td>0.00</td>
<td>0.94</td>
<td>0.38</td>
<td>0.00</td>
<td>0.69</td>
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<tr>
<td>2</td>
<td>0.37</td>
<td>0.03</td>
<td>0.02</td>
<td>0.20</td>
<td>0.08</td>
<td>0.00</td>
<td>0.39</td>
</tr>
<tr>
<td>3</td>
<td>0.51</td>
<td>0.03</td>
<td>0.02</td>
<td>0.25</td>
<td>0.16</td>
<td>0.00</td>
<td>0.28</td>
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<tr>
<td>4</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.23</td>
<td>0.14</td>
<td>0.00</td>
<td>0.22</td>
</tr>
<tr>
<td>5</td>
<td>0.01</td>
<td>0.02</td>
<td>0.00</td>
<td>0.32</td>
<td>0.00</td>
<td>0.00</td>
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<td>6</td>
<td>0.04</td>
<td>0.02</td>
<td>0.00</td>
<td>0.41</td>
<td>0.00</td>
<td>0.00</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Table 2.3: LM test results (p-values) ARCH effects. H0: No ARCH effects. PG refers to profit growth in the respective sector, SR to stock market returns.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Period</th>
<th>Obs.</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>1950Q2-2013Q4</td>
<td>255</td>
<td>0.0322011</td>
<td>0.0046514</td>
<td>0.0249326</td>
<td>0.0445141</td>
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<td>PG - Uncertainty - M</td>
<td>1965Q1-2005Q1</td>
<td>161</td>
<td>.0165385</td>
<td>.006576</td>
<td>.0098383</td>
<td>.0492981</td>
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<tr>
<td>PG - SD - M</td>
<td>1964Q4-2005Q1</td>
<td>162</td>
<td>0.1218006</td>
<td>0.081629</td>
<td>0.0338525</td>
<td>0.3934643</td>
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<td>PG - IQR - M</td>
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<td>0.0463093</td>
<td>0.0140203</td>
<td>0.027208</td>
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<td>1965Q1-2005Q1</td>
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<td>.0055934</td>
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<td>PG - SD - E</td>
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<td>SR - SD - M</td>
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<td>.0143971</td>
<td>.0417152</td>
<td>.1368466</td>
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<td>SR - IQR - M</td>
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<td>.0229461</td>
<td>.0293718</td>
<td>.1932621</td>
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<td>SR - Uncertainty - S</td>
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<td>.0083419</td>
<td>.0323597</td>
<td>.0787909</td>
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<td>SR - SD - S</td>
<td>1962M8-2006M12</td>
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<td>SR - IQR - S</td>
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<td>.0219151</td>
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Table 2.4: Summary statistics of the Uncertainty Measures. PG corresponds to profit growth in manufacturing (M), services (S), and the whole economy (E), SR to stock market returns.

<table>
<thead>
<tr>
<th>Uncertainty measure</th>
<th>p-val</th>
<th>$H_0$ : Normality</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0.26</td>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td>Profit Growth M</td>
<td>0.00</td>
<td>t(8.93)</td>
<td>Normal</td>
</tr>
<tr>
<td>Profit Growth S</td>
<td>0.13</td>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td>Profit Growth E</td>
<td>0.44</td>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td>Stock returns M</td>
<td>0.38</td>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td>Stock returns S</td>
<td>0.35</td>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td>Stock returns E</td>
<td>0.53</td>
<td>Normal</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.5: Test results - standardized residuals after GARCH-M estimation. Normality tested using the Kolmogorov-Smirnov test.
Chapter 3

Hump-shape Uncertainty, Agency Costs and Aggregate Fluctuations

3.1 Introduction

This chapter combines uncertainty shocks that rise and fall over time with an agency cost model to provide a further explanation for the observed cyclical fluctuations in output and consumption in the U.S. We model uncertainty (i.e. risk) shocks, changes in the standard deviation around a constant mean, corresponding to the empirical work of Jurado, Ludvigson and Ng (2015) (Macro Uncertainty) and Ludvigson, Ma and Ng (2016) (Financial Uncertainty): We model the time path of uncertainty shocks to match empirical evidence in terms of shape, duration and magnitude. These previously measured uncertainty shocks using the U.S. data show a hump-shape time path: Uncertainty rises for two years before its decline. Current literature on the effects uncertainty on macroeconomics, including housing, has not accounted for this observation. Consequently, the literature on uncertainty and macroeconomics is divided on the effects and the propagation mechanism of uncertainty on aggregate fluctuations. The models examining the effects of uncertainty in the presence of financial constraints, such as Dorofeenko, Lee and Salyer (2008, henceforth DLS), Chugh (2016), Dmitriev and Hoddenbagh (2015) and Bachmann and Bayer (2013) find uncertainty shock plays quantitatively small role in explaining aggregate fluctuations. Whereas Christiano, Motto and Rostagno (2014), however, find the effect of uncertainty shock on aggregate variables is quantitatively large.\footnote{Some other works that find a large uncertainty effect are Bloom (2009), Bloom, Alfaro and Lin (2016) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), and Leduc and Liu (2015). There are other works that find a mixed results such as Gilchrist, Sim, and Zakrajsek (2014), who find a small impact on output and consumption but a large impact on investment.} A common theme on all of
these aforementioned literature on uncertainty, however, is that a risk shock is characterized by an immediate one time peak after the innovation (i.e. non-hump shape).

This paper shows that when uncertainty rises and falls over time, then the output displays hump-shape with short expansions that are followed by longer and persistent contractions. And because of these longer and persistent contractions in output, uncertainty is, on average, counter-cyclical. Our model builds on the literature combining uncertainty and financial constraints as in DLS and Bansal and Yaron (2004). Our first calibration exercise builds on DLS as a benchmark, and incorporates a modified Bansal and Yaron (2004) uncertainty structure while the second calibration exercise includes the preferences due to Greenwood, Hercovwitz and Huffman (1988). Our model’s uncertainty propagation mechanism is, however, different from other models examining the effects of uncertainty in the presence of financial constraints. Unlike other studies that find immediate adverse effects of uncertainty on investment and output following uncertainty shocks that peak immediately after the innovation, we examine the impact of an unexpected shock that does not peak immediately but rises before it falls. In our calibrated models, agents anticipate this hump-shape uncertainty time-path once a shock has occurred. Thereby, agents respond immediately by increasing investment (i.e. precautionary savings), but then substantially reduce investment and consumption (and thus output) as more uncertain times lie ahead. With persistent uncertain periods, both risk premia and bankruptcies increase, which cause a further deterioration in investment opportunities. A hump shape time-varying uncertainty accounts for the majority of the variation in the credit channel variables, although the results are sensitive to the presence and the magnitude of agency costs. In the absence of agency costs, uncertainty shocks cause expansions because there are no adverse effects for households. However, in this case, the shocks do not explain any variation in real (<1% in output and consumption) and financial (<3.5% in the risk premium, the bankruptcy rate and the relative price of capital) variables. Conversely, the more severe the agency friction, i.e. the higher the monitoring costs associated with the friction, the more important uncertainty shocks are. We also show that accounting for hump-shape uncertainty measures can result in a large quantitative effect of uncertainty shock relative to previous literature. We find hump-shaped risk shocks account for 5% of the variation in output and 10% and 16% of the variation in consumption and investment, respectively. Finally, we also analyze the role of the relative risk aversion parameter and uncertainty. We find the relation between explained variation in output and consumption and uncertainty is monotonic - a higher coefficient of relative risk aversion is associated with higher precautionary savings, the associated initial expansion in output is greater and the subsequent contraction is not as severe.
3.2 Motivation

3.2.1 Data

Figure 3.1 shows the Financial Uncertainty and Macro Uncertainty measures proposed by Jurado et al. (2015) and Ludvigson et al. (2016) from the period 1960 to 2015. Uncertainty shocks as defined by Jurado et al. (2015) and Ludvigson et al. (2016) (i) raise between 30% and 73% relative to the median, (ii) exhibit a constant long-run mean and (iii) rise and fall over time with persistence. For example, during the Great Recession period, the Financial Uncertainty measure peaks after rising for 22 months (2006:12 - 2008:10) and peaks after rising for 11 months (2007:11 - 2008:10) after reaching the median during the great recession period. Other uncertainty shocks indicated by Ludvigson et al. (2015) peak after rising for 21 months in the late 1960s (relative to the median, from 1968:7 to the peak in 1970:4); for 26 months in the mid 1970s (1972:11 - 1975:1); for 23 months in the late 1970s (1978:4 - 1980:3); for 10 months

Source: Jurado et al. (2015) and Ludvigson et al. (2016).
in the mid 1980s (1986:3 - 1987:1) and for 8 months in the early 1990s (1989:12 - 1990:8). The \textit{Macro Uncertainty} proxy rose (relative to the median) for 26 months (1972:10 - 1974:12), for 18 months (1978:11 - 1980:5) and for 17 months (2007:5 - 2008:10, with 2007:5, with the trough before the peak slightly above the median). Consequently, the \textit{Financial Uncertainty} and \textit{Macro Uncertainty} measures, depicted in Figure 3.1, strongly suggest that uncertainty is not characterized by jumps as in Bloom (2009) but these measured uncertainty shocks show a hump-shape time path.

### 3.2.2 Empirical Evidence

To show corresponding hump-shapes for output, consumption and investment, we take a simplistic approach to examining the impact of uncertainty on these real variables, while avoiding a contemporaneous jump in uncertainty. We examine the impact of a shock to future uncertainty on today’s output, consumption, investment in a vector autoregression (VAR) model. In doing so, we thus ask, what is the impact on the variables of interest if the anticipated uncertainty is high in the future. We estimate the baseline specification of the VAR using data from 1960Q3 to 2013Q4 with two lags and the cyclical components of output, consumption and investment. Uncertainty is not HP-filtered and expressed as percentage deviation from the median. The results are highly similar if we use the cyclical component of HP-filtered uncertainty or the \textit{Macro Uncertainty} measure. The vector of variables included in the VAR is given by $\begin{bmatrix} \text{Uncertainty}_{t+k} & \text{GDP}_t & \text{Consumption}_t & \text{Investment}_t \end{bmatrix}'$ with $k = 2$ in the baseline specification. Figure 3.2 shows the orthogonalized impulse response functions using this specification.

\textit{Financial Uncertainty} induces hump-shaped responses in output, consumption and investment. However, as opposed to previous analyses, there are no immediate adverse effects if uncertainty is not restricted to jump unexpectedly from one period to another. Instead, a hump-shaped expansion precedes a pronounced contraction. These results are highly robust to different specifications and different orderings - as long as $2 \geq k \geq 8$, i.e. if uncertainty is high in the more distant future. If $k < 2$, the impulse responses show contractions in output, consumption and investment - in line with previous work that analyzes contemporaneous jumps in uncertainty.\footnote{On average, uncertainty peaks for these six shocks after increasing by 48.42\%.}

\footnote{These results are robust to different lag lengths of the VAR. In a second specification, we also include lagged delinquency rates on business loans as a proxy for bankruptcies. The impulse response function of delinquencies is hump-shaped while the responses of output consumption and investment are highly similar for the second specification.}
Figure 3.2: Impulse Response Functions of the VAR \( \left[ \text{Uncertainty}_{t+2}, \text{GDP}_t, \text{Consumption}_t, \text{Investment}_t \right] \)' with two lags.


### 3.3 Model

Carlstrom and Fuerst (1997, henceforth CF) include capital-producing entrepreneurs, who default if they are not productive enough, into a real business cycles (RBC) model. In the CF framework, households and final-goods producing firms are identical and perfectly competitive. Households save by investing in a risk-neutral financial intermediary that extends loans to entrepreneurs. Entrepreneurs are heterogeneous produce capital using an idiosyncratic and stochastic technology with constant volatility. Unlike CF, DLS introduce stochastic shocks to the volatility (uncertainty shocks) of entrepreneurs' technology, such that uncertainty jumps to its peak and converges back to its steady state. While this approach remedies the procyclical bankruptcy rates following TFP shocks it introduces countercyclical bankruptcy rates, DLS is at odds with the measures from Jurado et al. (2015) and Ludvigson et al. (2016). In this paper, we alter the time path and the magnitude of the shocks introduced in DLS, such that they correspond more closely to the *Macro Uncertainty* and *Financial Uncertainty* measures.
Following these changes, the model displays procyclical consumption, precautionary savings and an increase in output following an initial drop. Our model therefore explains the puzzling absence of precautionary savings following uncertainty shocks in the literature, as raised by Bloom (2014).

In the CF framework, the conversion of investment to capital is not one-to-one because heterogeneous entrepreneurs produce capital using idiosyncratic and stochastic technology. If a capital-producing firm realizes a low technology shock, it declares bankruptcy and the financial intermediary takes over production after paying monitoring costs. The timing of events in the model is as follows:

1. The exogenous state vector of technology and uncertainty shocks, denoted \((A_t, \sigma_{\omega,t})\), is realized.

2. Firms hire inputs of labor and capital from households and entrepreneurs and produce the final good output via a Cobb-Douglas production function.

3. Households make their labor, consumption, and investment decisions. For each unit of investment, the household transfers \(q_t\) units of the consumption goods to the banking sector.

4. With the savings resources from households, the banking sector provide loans to entrepreneurs via the optimal financial contract (described below). The contract is defined by the size of the loan, \(i_t\), and a cutoff level of productivity for the entrepreneurs’ technology shock, \(\bar{\omega}_t\).

5. Entrepreneurs use their net worth and loans from the banking sector to purchase the factors for capital production. The quantity of investment is determined and paid for before the idiosyncratic technology shock is known.

6. The idiosyncratic technology shock of each entrepreneur \(\omega_{j,t}\) is realized. If \(\omega_{j,t} \geq \bar{\omega}_t\) the entrepreneur is solvent and the loan from the bank is repaid; otherwise the entrepreneur declares bankruptcy and production is monitored by the bank at a cost proportional to the input, \(\mu_i t\).

7. Solvent entrepreneur’s sell their remaining capital output to the bank sector and use this income to purchase consumption \(c_t\) and (entrepreneurial) capital \(z_t\). The latter will in part determine their net worth \(n_t\) in the following period.
3.3.1 The Impact of Uncertainty Shocks: Partial Equilibrium

The optimal contract is given by the combination of $i_t$ and $\bar{\omega}_t$ that maximizes entrepreneurs’ return subject to participating intermediaries. Financial intermediaries make zero profits due to free entry

$$\max_{i_t, \bar{\omega}_t} q_t i_t f(\bar{\omega}_t; \sigma_{\omega,t})$$

subject to

$$q_t i_t g(\bar{\omega}_t; \sigma_{\omega,t}) \geq i_t - n_t.$$

Net worth is defined as

$$n_t = w_t^e + z_t(r_t + q_t(1 - \delta(u_t))).$$

Entrepreneurs’ share of the expected net capital output is

$$f(\bar{\omega}_t; \sigma_{\omega,t}) = \int_{\bar{\omega}_t}^{\infty} \omega \tilde{\phi}(\bar{\omega}_t; \sigma_{\omega,t}) d\omega - [1 - \tilde{\Phi}(\bar{\omega}_t; \sigma_{\omega,t})] \bar{\omega}_t$$

and the lenders’ share of expected net capital output

$$g(\bar{\omega}_t; \sigma_{\omega,t}) = \int_0^{\bar{\omega}_t} \omega \tilde{\phi}(\bar{\omega}_t; \sigma_{\omega,t}) d\omega + [1 - \tilde{\Phi}(\bar{\omega}_t; \sigma_{\omega,t})] \bar{\omega}_t - \tilde{\Phi}(\bar{\omega}_t; \sigma_{\omega,t}) \mu.$$

To understand the impact of an uncertainty shock, consider the uncertainty shock in partial equilibrium. For this analysis, $q$ and $n$ are assumed to be fixed while $i$ and $\bar{\omega}$ are chosen. In this setting, uncertainty shocks adversely affect the supply of investment as follows. As $\sigma_\omega$ increases, the default threshold $\bar{\omega}$ and lenders’ expected return fall. From the incentive compatibility constraint of entrepreneurs’ problem (3.1), it can be seen that investment has to fall. The effect of an uncertainty shock is summarized graphically, and contrasted with an aggregate technology shock, in Figure 3.3 (taken from DLS).

Whether these results carry over in general equilibrium depends on how the shock is modeled. They are not overturned following a jump in uncertainty, as analyzed in DLS, or if uncertainty reaches its peak quickly. In this case, bankruptcies, the associated agency costs, the risk premium and the price of capital increase. The return to investing falls, saving/investing is less attractive, so investment and output drop while households substitute into consumption. These results are overturned, however, following a shock that is hump-shaped if the peak is sufficiently far in the future.
Figure 3.3: The partial equilibrium impact of an uncertainty shock.

Note: Uncertainty adversely affects capital supply, in contrast to TFP shocks that affect capital demand. Source: DLS.

3.3.2 Modeling Hump-Shaped Uncertainty Shocks

We allow for humps in uncertainty by modifying a subset of equations due to Bansal and Yaron (2004), such that a latent $x_t$ variable affects $\sigma_{\omega,t}$:

\[
\log(\sigma_{\omega,t+1}) = (1 - \rho_{\sigma_{\omega}}) \log(\bar{\sigma}_\omega) + \rho_{\sigma_{\omega}} \log(\sigma_{\omega,t}) + \tilde{\varepsilon}_{t+1} \tag{3.6}
\]

\[
\tilde{\varepsilon}_{t+1} = \varphi_{\sigma} \varepsilon_{\sigma,t+1} + x_{t+1} \tag{3.7}
\]

\[
x_{t+1} = \rho_x x_t + \varphi_x \varepsilon_{x,t+1} \tag{3.8}
\]

\[
\varepsilon_{x,t}, \varepsilon_{\sigma,t} \overset{i.i.d.}{\sim} N(0, 1), \rho_{\sigma}, \rho_x \in [0, 1). \tag{3.9}
\]

$\tilde{\varepsilon}_{t+1}$ is a composite term that enables uncertainty to jump (innovations in the first term $\varphi_{\sigma,t+1} \varepsilon_{\sigma,t+1}$), as in DLS, or increase over time corresponding to the empirical proxies (innovations via the latent variable $x_{t+1}$). Figure 3.4 plots the time series of $\sigma_{\omega,t}$ using different persistence
parameters $\rho_x = [0, 0.5, 0.94, 0.96]$. The horizontal axis measures time in monthly periods, while the vertical axis shows the percentage deviation from the steady state. Setting $\rho_x = 0$ induces a jump in uncertainty, as analyzed in DLS. The larger $\rho_x$, the more pronounced the hump in $\sigma_{\omega,t}$ and the longer uncertainty rises before it peaks.

**Figure 3.4: Modeling Uncertainty Shocks using different persistence parameters.**

Note: The horizontal axis shows monthly periods, while the vertical axis shows the percentage deviation from the steady state. The case with $\rho_x = 0$ corresponds to a jump in uncertainty as analyzed in DLS. The higher $\rho_x$, the more pronounced the hump in uncertainty. In the benchmark case with $\rho_x = .96$, $\sigma_{\omega}$ peaks after rising 25 months, corresponding to the empirical evidence. $\rho_{\sigma_{\omega}}$ is set to $0.9^{1/3}$.

In the benchmark case with $\rho_x = .96$, uncertainty peaks after rising for 25 months, corresponding to the empirical evidence. We match the innovation relative to the steady state using the average increase of an uncertainty shock relative to the long-run mean: We set $\varphi_x = 0.048$ such that $\sigma_{\omega,t}$ increases by 48% relative to the steady state. Our 48% relative increase compares with previous papers as follows. In Bloom (2009) and Bloom, Alfaro and Lin (2016), who use two-state Markov chains to examine the impact of uncertainty, $\sigma_{\omega}$ increases by 100%; in Bloom, Floetotto, Jaimovich, Saporta-Eksten and Terry (2012), $\sigma_{\omega}$ increases between 91% and 330%. Leduc and Liu (2015) introduce an increase of 39.2% relative to the steady state. Christiano, Motto and Rostagno (2014), use a combination of un- and anticipated innovations
over a sequence of eight quarters, and their magnitude of these innovations is between 2.83% and 10% per period. DLS use a 1% innovation, Chugh (2016) and Bachmann and Bayer (2013) examine increases of about 4%, while Dmitriev and Hoddenbagh (2015) use a 3% innovation. These differences are partially driven by differences in measurement; see also Strobel (2015). Not surprisingly, greater innovations in uncertainty are associated with a greater role of uncertainty in terms of variation explained. One special case is the model of Christiano et al. (2014) who introduce a news component to their shocks: Sims (2015) points out potential issues in using news and variance decompositions. Lee, Salyer and Strobel (2016) show that the news component plays a prominent role regarding the importance of uncertainty.

3.3.3 The Impact of Uncertainty Shocks: General Equilibrium

In order to unambiguously identify the change in the impact of a risk shock that is due to its hump-shape, we insert the shock described in the previous section in a framework identical to DLS. For this reason, the model’s exposition is confined to the agents’ optimization problems. The representative household’s objective is to maximize expected utility by choosing consumption $c_t$, labor $h_t$ and savings $k_{t+1}$, i.e.

$$\max_{\{c_t, k_{t+1}, h_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \nu(1 - h_t)]$$ (3.10)

subject to

$$w_t h_t + r_t k_t \geq c_t + q_i i_t$$ (3.11)

$$k_{t+1} = (1 - \delta) k_t + i_t$$ (3.12)

with $w_t$ the wage and $r_t$ the rental rate of capital. These are equal to their marginal products, as the representative final-good’s producing firm faces a standard, static profit maximization problem

$$\max_{K_t, H_t, H_t^e} A_t K_t^{\alpha_K} H_t^{\alpha_H} (H_t^e)^{1 - \alpha_K - \alpha_H} - r_t K_t - w_t H_t - w_t^e H_t^e$$ (3.13)

with $K_t = k_t/\eta$ and $H_t = (1 - \eta) h_t$, where $\eta$ represents the fraction of entrepreneurs in the economy. Total Factor Productivity (TFP) $A_t$ follows an autoregressive process of order one in logs,

$$\log(A_{t+1}) = \rho_A \log(A_t) + \varphi_A \varepsilon_{A,t+1}$$ (3.14)

When solving the model, we follow DLS and assume the share of entrepreneurs labor $(1 - \alpha_K - \alpha_H)$ is approximately zero.
Table 3.1: Benchmark calibration for the monthly frequency.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Function</th>
<th>Value</th>
<th>Rationale / Source (see also discussion in the text)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.9975</td>
<td>Monthly calibration</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital's share of production</td>
<td>0.36</td>
<td>DLS</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Monitoring costs</td>
<td>0.25</td>
<td>DLS</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.2/3</td>
<td>DLS</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>Steady state uncertainty</td>
<td>0.207</td>
<td>Steady State Risk Premium</td>
</tr>
<tr>
<td>$\rho_{\sigma}$</td>
<td>Persistence parameter uncertainty</td>
<td>0.91/3</td>
<td>DLS</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>Persistence parameter hump component</td>
<td>0.96</td>
<td>Jurado et al (2015), Ludvigson et al (2016)</td>
</tr>
<tr>
<td>$\varphi_\sigma$</td>
<td>Innovation in uncertainty (jump)</td>
<td>0.01</td>
<td>DLS</td>
</tr>
<tr>
<td>$\varphi_x$</td>
<td>Innovation in uncertainty (hump)</td>
<td>0.048</td>
<td>Jurado et al (2015), Ludvigson et al (2016)</td>
</tr>
<tr>
<td>$\varpi$</td>
<td>Steady state default threshold</td>
<td>0.557</td>
<td>Steady State Bankruptcy Rate</td>
</tr>
</tbody>
</table>

with $\varepsilon_{A,t} \overset{i.i.d.}{\sim} N(0,1)$. The problem of entrepreneurs is given by

$$\max \{c_t, z_{t+1}\} \Rightarrow E_0 \sum_{t=0}^{\infty} (\gamma \beta)^t c_t$$ (3.15)

subject to

$$n_t = w_t^e + z_t (r_t + q_t (1 - \delta))$$ (3.16)

$$z_{t+1} = n_t \left[ \frac{f(\bar{\omega}_t, \sigma_{\omega,t})}{1 - q_t g(\bar{\omega}_t, \sigma_{\omega,t})} \right] - \frac{c_t^e}{q_t}.$$ (3.17)

The entreprenuers are risk neutral and supply one unit of labor inelastically. Their net worth is defined by sum of labor income $w_t^e$, the income from capital $z_t r_t$ plus the remaining capital $z_t q_t (1 - \delta)$. At the end of a period, entrepreneurial consumption is financed out of the returns from the investment project, which implies the law of motion (3.17). As the equilibrium conditions are described in DLS, we will not list them in this section.

### 3.3.4 Calibration

We calibrate the model for the monthly frequency. Otherwise, the frequency of uncertainty would be too low relative to the empirical counterparts. Table 3.1 shows the benchmark calibration of the key parameters. The household’s monthly discount rate of 0.9975 implies an annual risk free rate of about 3%. Following DLS, we set $\sigma_\omega = 0.207$, which implies an annual risk premium of 1.98%. The slight increase in the risk premium, which is 1.87% in DLS, is due to changes associated with the monthly calibration. The default threshold $\varpi$ targets an annual bankruptcy rate of 3.90%, as in DLS.
3.3.5 Cyclical Behaviour

Because of the assumption on entrepreneurs’ productivity, first order approximation of the equilibrium conditions does not impose certainty equivalence. Instead, uncertainty (time-varying second moment) appears in the policy function as a state variable. Figures 3.5 and 3.6 show the impulse response functions following jumps and humps in uncertainty, i.e. the impulse response function for different values of $\rho_x = [0, 0.5, 0.94, 0.96]$.

Figure 3.5: Impulse responses of output, household consumption and investment following an uncertainty shock for different persistence parameters, $\rho_x = [0, 0.5, 0.96, 0.979]$.

Note: The horizontal axis shows monthly periods, the vertical axis shows the percentage deviation from the steady state.

If $\rho_x = 0$, uncertainty jumps to its peak and an immediate drop in investment and output ensues, which is expected from the partial equilibrium analysis. Household consumption counterfactually increases as households substitute into consumption. The larger $\rho_x$, the longer the shock takes to peak. Interestingly, there is a threshold value of $\rho_x$ that is necessary to induce precautionary savings. For instance, $\rho_x = 0.5$ is insufficient to overcome the partial equilibrium results and to induce precautionary savings. However, values of $\rho_x$ corresponding to the
uncertainty proxies overturn the partial equilibrium results: An uncertainty shock is followed by an increase in investment and a hump-shape response of output following the initial drop. Moreover, in line with the data, consumption is procyclical. The intuition is that immediately after the shock, agency costs are still moderate relative to future periods so investment demand increases, which raises output following the initial drop. While households also substitute into consumption, entrepreneurs greatly reduce consumption after an uncertainty shock because of the increase probability of default and because the higher price of capital results in an increase in investment. The intuition of the model can also be seen in the context of the agency friction. Figure 3.7 shows the impulse responses for different values of $\mu$.

Without agency friction, $\mu = 0$ (actually, for computational reasons, $\mu = 0.0001$), the relative price of capital $q_t$ is unity. An uncertainty shock, then, induces an expansion in output, consumption and investment because there are no adverse effects for households. Although the bankruptcy rate increases, there are no adverse effects. Instead, households benefit from a more productive investment opportunity. If there are agency costs ($\mu > 0$), the relative price of capital $q_t$ increases, as well as risk premia and bankruptcy rates. In this case, households incur
adverse effects of bankruptcies because of the monitoring costs $\tilde{\Phi}(\mu_\omega, \sigma_\omega)\mu$. Unlike shocks that jump, however, hump-shaped shocks increase investment by around 1.7% relative to the steady state, despite the price-increase associated with the agency friction - overturning the partial equilibrium effects. Since expectations are rational, households know that uncertain times of relatively poor investment opportunities are ahead, so they substantially increase saving as soon as they learn about the shock. As shown in Figure 3.7, the magnitude of the initial increase is inversely related to the size of $\mu$ - the higher the monitoring costs, the more capital is destroyed. Without agency friction, consumption does not increase by much in order to invest more. With agency friction and adverse effects of uncertainty for the households, consumption increases the more the greater $\mu$. The initial increase in investment leads to an expansion in output. However, the subsequent deterioration of conditions in the credit channel leads to a drop in investment and to a contraction in output.

Table 3.2 presents the model’s correlation coefficients. The model produces procyclical consumption and investment if uncertainty is not restricted to jump; the degree of procyclicality
Table 3.2: Correlation coefficients of consumption, investment, bankruptcy rate and uncertainty with output.

<table>
<thead>
<tr>
<th>Uncertainty Shock</th>
<th>c</th>
<th>i</th>
<th>BR</th>
<th>σω</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jump</td>
<td>-0.72</td>
<td>0.95</td>
<td>-0.99</td>
<td>-0.69</td>
</tr>
<tr>
<td>Hump</td>
<td>0.19</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: BR refers to the bankruptcy rate. The autocorrelation coefficient of uncertainty is $\rho_{\sigma_\omega} = 0.9^{1/3}$, as in DLS, while uncertainty peaks after rising for 25 months, i.e. $\rho_x = 0.96$.

of consumption depends on the persistence of the latent variable, i.e. on how long the shock takes to peak. The bankruptcy rate and uncertainty are strongly countercyclical for both types of risk shocks.

### 3.4 GHH Preferences and Variable Capital Utilization

The previous section uses the framework of DLS to emphasize the impact of hump-shaped uncertainty shocks: precautionary savings, a hump-shape response of output with a short expansion that is followed by a longer and persistent contraction as well as mildly procyclical consumption. However, the initial drop in output that precedes the short expansion is much stronger compared to the VAR evidence, while the procyclicality of consumption is sensitive to the persistence parameter of the latent variable. To remedy these features, we modify the model by using the preferences due to Greenwood, Hercowitz and Huffman (1988), to eliminate effects of labor supply due to changes in consumption, and allow for variable capital utilization. The representative household thus chooses the capital utilization rate $u_t$, which is impacts the depreciation rate $\delta(u_t)$, with $\delta'(u_t), \delta''(u_t) > 0$. The problem is given by

$$
\max_{\{c_t, h_t+1, u_t, k_t\}} E_0 \sum_{t=0}^{\infty} \beta^t (1-\iota)^{-1} [c_t - \chi h_t^{1+\theta} (1+\theta)]^{1-\iota}
$$

subject to

$$
w_t h_t + r_t(u_t, k_t) \geq c_t + q_t i_t
$$

$$
k_{t+1} = (1 - \delta(u_t))k_t + i_t
$$

$$
\delta(u_t) = \delta_0 + \delta_1 (u_t - 1) + \frac{\delta_2}{2} (u_t - 1)^2.
$$
Table 3.3: Benchmark calibration for the monthly frequency with GHH preferences and variable capital utilization.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Function</th>
<th>Value</th>
<th>Rationale / Source (see also discussion in the text)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ι</td>
<td>Coefficient of relative risk aversion</td>
<td>1</td>
<td>Greenwood et al. (1988)</td>
</tr>
<tr>
<td>1/θ</td>
<td>Intertemporal elasticity of substitution in labor supply</td>
<td>0.8</td>
<td>Greenwood et al. (1988)</td>
</tr>
<tr>
<td>χ</td>
<td>Relative importance of leisure</td>
<td>9.8930</td>
<td>Household works 1/3 of his time</td>
</tr>
<tr>
<td>δ₀</td>
<td>Steady state rate of capital depreciation</td>
<td>0.02/3</td>
<td>DLS</td>
</tr>
<tr>
<td>δ₁</td>
<td>Normalize steady state capital utilization</td>
<td>0.0108</td>
<td>Capital utilization is unity in steady state</td>
</tr>
<tr>
<td>δ₂</td>
<td>Sensitivity of capital utilization</td>
<td>0.2</td>
<td>Schmitt-Grohe and Uribe (2008)*</td>
</tr>
</tbody>
</table>

*Schmitt-Grohe and Uribe (2008) estimate δ₂ = 0.11 but with a relatively large standard error of 0.26. We set δ₂ slightly higher to restrict capital utilization a bit more given the monthly calibration.

The coefficient of relative risk aversion is given by ι, while 1/θ corresponds to the intertemporal elasticity of substitution in labor supply. χ is the relative importance of leisure. The problem of the final-goods’ producing firms is given by

\[
\max_{u_tK_t,H_t,H^e_t} A_t(u_tK_t)^{\alpha K} H^e_t^{\alpha H}(H^e_t)^{1-\alpha K-\alpha H} - r_t(u_tK_t) - w_tH_t - w^e_tH^e_t. \tag{3.22}
\]

The problem of the entrepreneurs and the optimal contract remain unchanged for the most part, except for the depreciation rate, δ(u_t). The calibration of the additional parameters is standard and displayed in Table 3.3. The set of equations determining the equilibrium properties are displayed in the Appendix.

### 3.4.1 Cyclical Behaviour

While the previous section examined an identical economy as DLS, we now consider the impact of the different types of uncertainty shock using the preferences due to Greenwood et al. (1988) to remedy the shortcomings discussed above. We examine the impact on output, investment and household consumption following an innovation that is comparable in magnitude (a 48% innovation). Consider first the impact of the hump-shaped shock displayed in Figure 3.8. The results are quite robust, although the initial adverse impact on output is much smaller while the brief ensuing expansion is (relative to the DLS framework) more pronounced and persistent. However, as shown in Table 3.4 below, the procyclicality of consumption is not sensitive anymore to uncertainty’s time to peak. In contrast, the results change following a jump in uncertainty. Most notably, while output drops as expected, investment increases in the first period, mitigating the drop in output, but then drops persistently below its steady state and output falls again. This is not due to precautionary savings, however. If there is a jump in uncertainty, the amount of capital destroyed immediately after the shock is much larger compared to a hump-shaped shock...
(total costs of default following a jump are more than twice as large after the first two years, which also shows in the bankruptcy rate in Figure 3.9).\textsuperscript{5} Labor, however, is not substituted for capital (labor perfectly comoves with output given the GHH preferences) but the capital stock is simply replaced. This is also reflected by the marginal product of capital, which increases following a jump in uncertainty whereas it falls following a hump in uncertainty.

Figure 3.8: Impulse responses of output, household consumption and investment following a shock (jump and hump) to uncertainty.

<table>
<thead>
<tr>
<th>Output</th>
<th>Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Output Graph" /></td>
<td><img src="image2" alt="Consumption Graph" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Investment</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Investment Graph" /></td>
<td><img src="image4" alt="Uncertainty Graph" /></td>
</tr>
</tbody>
</table>

Note: Uncertainty increases by 48% for both types of uncertainty shock. The horizontal axis shows monthly periods, the vertical axis shows the percentage deviation from the steady state.

Table 3.4 presents a further analysis of the equilibrium characteristics. The model produces procyclical consumption and investment for TFP and risk shocks. As discussed above, this is partially due to the GHH preferences, which are also the reason for perfect comovement of labor and output. As opposed to Carlstrom and Fuerst (1997), the bankruptcy rate is countercyclical for both types of uncertainty shocks. This similarity conceals the fact that following a hump-shaped uncertainty shock, the bankruptcy rate is initially procyclical and becomes countercyclical only later on. The reason is that uncertainty starts to rise while output still expands due to the initial increase in investment. While uncertainty rises, investment and output de-

\textsuperscript{5}Not surprisingly, the marginal product of capital increases following a jump while it initially drops (and is positive only in later periods) following a hump.
Figure 3.9: Lending channel variables following a shock (jump and hump) to uncertainty.

Note: Uncertainty increases by 48% for both types of uncertainty shock. The horizontal axis shows monthly periods, the vertical axis shows the percentage deviation from the steady state, unless indicated otherwise.

crease. In contrast, if uncertainty jumps to its peak, output immediately decreases. Similary, although the correlation between output and uncertainty for the two shocks is not that different, the dynamics are following a shock are.

The correlation of the bankruptcy rate and output implied by the model is considerably higher compared to the data, while the correlation between output and consumption is fairly close to the data for hump-shaped uncertainty shocks. The correlation between output and investment is considerably lower compared to the data because investment is the main driver of the dynamics and leads output. The relative volatilities implied by both types of uncertainty shock are too high for consumption, investment and uncertainty and much lower for hours and bankruptcies, compared to the data. Thus, even though uncertainty, in this model, accounts for the majority of the variation in bankruptcies, uncertainty shocks are not sufficient to explain the observed relative variation. Considering the simplicity of the model and that uncertainty on its own is an unlikely source of bankruptcies, this finding is not too surprising.

---

6Note also that the hump-shaped movements in bankruptcy rates observed in the data are absent if uncertainty moves from steady state to peak from one period to another - simply because of the time path of $\sigma_{\omega,t}$ and the log-normality assumption of entrepreneurs’ productivity. Conversely, they are, to a large extent, present by construction following a hump-shaped shock.
Table 3.4: Business Cycle Characteristics.

<table>
<thead>
<tr>
<th>Shock</th>
<th>$\sigma(y)$</th>
<th>$c$</th>
<th>$i$</th>
<th>BR</th>
<th>$\sigma_w$</th>
<th>$c$</th>
<th>$i$</th>
<th>BR</th>
<th>$\sigma_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>0.20</td>
<td>0.65</td>
<td>4.66</td>
<td>0.003</td>
<td>-</td>
<td>0.96</td>
<td>0.93</td>
<td>0.23</td>
<td>-</td>
</tr>
<tr>
<td>Risk Jump</td>
<td>0.00046</td>
<td>1.48</td>
<td>11.60</td>
<td>0.42</td>
<td>15.12</td>
<td>0.55</td>
<td>0.40</td>
<td>-0.13</td>
<td>-0.33</td>
</tr>
<tr>
<td>Risk Hump</td>
<td>0.038</td>
<td>0.98</td>
<td>10.66</td>
<td>0.14</td>
<td>17.65</td>
<td>0.89</td>
<td>0.46</td>
<td>-0.27</td>
<td>-0.28</td>
</tr>
<tr>
<td>U.S. Data</td>
<td>2.04</td>
<td>0.47</td>
<td>4.03</td>
<td>14.08</td>
<td>7.36</td>
<td>0.78</td>
<td>0.87</td>
<td>-0.81</td>
<td>-0.09*</td>
</tr>
</tbody>
</table>

Note: BR refers to the bankruptcy rate. For this analysis, the innovations to the shocks are such that uncertainty jumps by 1%, as in DLS, and increases over time up to 48% as suggested by the empirical evidence. TFP is highly persistent with an autocorrelation coefficient of $0.9^{1/3}$, and subject to an innovation of 1%. Although the model is calibrated and simulated for 10,000 months, we present quarterly statistics by computing the three month averages. The U.S. Figures for output, consumption, investment and labor are from Dorofeenko et al. (2016). The statistics for bankruptcies and uncertainty are based on own calculations. For the bankruptcy rate, we use quarterly data from 1987Q1 to 2013Q4 and the delinquency rate as a proxy. For uncertainty, we use the Financial Uncertainty measure from 1960 until 2015. *The correlation between output and Macro Uncertainty from 1960Q3 to 2014Q2 is -0.21. Source: FRED, Jurado et al. (2015) and Ludvigson et al. (2016).

Table 3.5: Variance Decomposition: Hump-shaped uncertainty shocks, TFP shocks and the coefficient of relative risk aversion $\iota$.

<table>
<thead>
<tr>
<th>$\iota$</th>
<th>TFP</th>
<th>Hump</th>
<th>TFP</th>
<th>Hump</th>
<th>TFP</th>
<th>Hump</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>94.92</td>
<td>5.08</td>
<td>89.16</td>
<td>10.84</td>
<td>82.40</td>
<td>17.60</td>
</tr>
<tr>
<td>2</td>
<td>96.27</td>
<td>3.73</td>
<td>93.31</td>
<td>6.69</td>
<td>87.03</td>
<td>12.97</td>
</tr>
<tr>
<td>10</td>
<td>96.88</td>
<td>3.12</td>
<td>95.93</td>
<td>4.07</td>
<td>91.96</td>
<td>8.04</td>
</tr>
<tr>
<td>20</td>
<td>96.67</td>
<td>3.33</td>
<td>96.06</td>
<td>3.94</td>
<td>93.27</td>
<td>6.73</td>
</tr>
</tbody>
</table>

Note: Hump refers to uncertainty shocks that are hump-shaped. For emphasis, this analysis omits jumping uncertainty shocks.

Table 3.5 shows the relation between the coefficient of relative risk aversion $\iota$ and the role of uncertainty, which is (inversely) monotonically related. The higher $\iota$, the more agents save as a precaution. The associated initial expansion in output is therefore also greater the higher $\iota$, and the subsequent contraction is not as severe; the volatility of consumption is much smaller the higher $\iota$, and for very high values of $\iota$ consumption negatively deviates from the steady state. For this reason, the (unconditional) variation in output, consumption and investment due to the hump-shape shock is the lower the more risk-averse households are - the initial impact is larger but subsequently the overall variation is diminished.

Finally, to assess the relative importance of TFP and uncertainty in the context of the agency friction, Table 3.6 shows the variance decomposition for different values of $\mu$. Without
Table 3.6: Variance Decomposition: Hump-shaped uncertainty shocks, TFP shocks and the monitoring costs $\mu$.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>TFP</th>
<th>$y$ Hump Jump</th>
<th>TFP</th>
<th>$c$ Hump Jump</th>
<th>TFP</th>
<th>$i$ Hump Jump</th>
<th>TFP</th>
<th>$RP$ Hump Jump</th>
<th>TFP</th>
<th>$BR$ Hump Jump</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>99.82</td>
<td>0.14</td>
<td>0.04</td>
<td>99.89</td>
<td>0.09</td>
<td>0.02</td>
<td>95.97</td>
<td>0.64</td>
<td>3.39</td>
<td>96.59</td>
</tr>
<tr>
<td>0.125</td>
<td>98.27</td>
<td>1.24</td>
<td>0.49</td>
<td>93.06</td>
<td>3.04</td>
<td>3.90</td>
<td>79.61</td>
<td>8.45</td>
<td>11.94</td>
<td>0.52</td>
</tr>
<tr>
<td>0.2</td>
<td>95.59</td>
<td>3.20</td>
<td>1.21</td>
<td>85.45</td>
<td>6.76</td>
<td>7.79</td>
<td>79.14</td>
<td>12.66</td>
<td>8.20</td>
<td>0.47</td>
</tr>
<tr>
<td>0.25</td>
<td>93.19</td>
<td>4.99</td>
<td>1.82</td>
<td>79.91</td>
<td>9.72</td>
<td>10.36</td>
<td>75.79</td>
<td>16.18</td>
<td>8.02</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Note: $BR$ refers to the bankruptcy rate, $RP$ refers to the risk premium. $Hump$ and $Jump$ refer to uncertainty shocks that are hump-shaped or jump, respectively.
agency friction, neither type of uncertainty shock matters. Introducing the friction and setting \( \mu = 0.125 \), uncertainty overall plays a small role for output (1.7%), a non-negligible role for consumption (7%) and it quantitatively matters for investment (20%). Lending-channel variables, in turn, are much more strongly affected, while productivity shocks accounting for less than one percent of the variation in risk premium and bankruptcy rates. Unsurprisingly, the importance of uncertainty for financial variables remains high as monitoring costs (\( \mu = 0.25 \)) double. However, in terms of real variables, uncertainty accounts for 7% of the variation in output, 20% and 25% of the variation in consumption and investment, respectively. In comparison to each other, hump-shaped shaped uncertainty accounts for the lion’s share in real variables, explaining 5%, 10% and 16% of the total variation in output, consumption and investment, respectively. The lending-channel variables are more strongly affected by unexpected changes in uncertainty, with 85% of the variation in bankruptcy rates and 60% of the variation in the risk premium due to shocks that jump.

3.5 Conclusion

We model uncertainty shocks that rise and fall over time. This approach to modeling uncertainty shocks is based on empirical evidence due to Jurado et al. (2015) and Ludvigson et al. (2016). Hump-shaped uncertainty shocks result in a different propagation mechanism compared to previous work combining uncertainty and financial accelerator models. The model’s propagation mechanism resembles business cycles if uncertainty is combined with an agency friction. Changes in the investment supply drive these dynamics. We find that uncertainty shocks, calibrated corresponding to the data, play a non-negligible role for the variation in real and financial variables: they explain 5% of the variation in output and 10% and 16% of the variation in consumption and investment. Comparing the two uncertainty shocks with each other, we find hump-shape uncertainty to matter more strongly for the real variables, while unexpected shocks dominate the lending channel variables. In any case, the contraction and subsequent sluggish recovery due to hump-shaped uncertainty shocks resemble features observed in the recent crisis: there is an expansion, followed by a contraction and a sluggish recovery. We foresee further research in the following line. First, agents in the model anticipate the time path of uncertainty after a shock has occurred. Thus, replacing rational expectations with a learning mechanism could be an interesting extension. Moreover, examining the impact of uncertainty shocks in the context of both equity and debt finance, as in Covas and den Haan (2012), might provide further insights into the choice between different sources of external finance.
3.6 Appendix

3.6.1 Optimality Conditions

The final goods’ production firm’s production function is given by

\[ y_t = A_t (k_t u_t)^\alpha ((1 - \eta) h_t)^{1-\alpha} \]  (3.23)

The aggregate resource constraint is

\[ y_t = (1 - \eta) c_t + \eta c^e_t + \eta i_t \]  (3.24)

The aggregate law of motion is given by

\[ K_{t+1} = (1 - \delta(u_t)) K_t + \eta i_t (1 - \tilde{\Phi} \mu) \]  (3.25)

which is equivalent to

\[ k_{t+1} = (1 - \delta(u_t)) k_t + i_t (1 - \tilde{\Phi} \mu) \]  (3.26)

with

\[ \delta(u_t) = \delta_0 + \delta_1 (u_t - 1) + \frac{\delta_2}{2} (u_t - 1)^2 \]  (3.27)

The household’s problem is described in the text. The intertemporal optimality conditions of the household are as follows:

- **Intratemporal optimality**

  \[ \chi h_t^\theta = A_t (1 - \alpha) (k_t u_t)^\alpha ((1 - \eta) h_t)^{-\alpha} \]  (3.28)

- **Intertemporal optimality**


\begin{align}
q_t (c_t - \chi h_t^{1+\theta} / (1 + \theta))^{-t} &= \\
\beta E \{(c_{t+1} - \chi h_{t+1}^{1+\theta} / (1 + \theta))^{-t} (A_{t+1} \alpha (k_{t+1} u_{t+1})^{\alpha - 1} ((1 - \eta) h_{t+1})^{1-\alpha} u_{t+1} + q_{t+1} (1 - \delta (u_{t+1})) ) \} \\
&= (3.29)
\end{align}

- The stochastic discount factor is given by

\begin{align}
m_{t,t+1} &= E \{ \beta (c_{t+1} - \chi h_{t+1}^{1+\theta} / (1 + \theta))^{-t} / (c_t - \chi h_t^{1+\theta} / (1 + \theta))^{-t} \} \\
&= (3.30)
\end{align}

- The return on investment is

\begin{align}
R^k_{t,t+1} &= (A_{t+1} \alpha (k_{t+1} u_{t+1})^{\alpha - 1} ((1 - \eta) h_{t+1})^{1-\alpha} u_{t+1} + q_{t+1} (1 - \delta (u_{t+1})) ) / q_t \\
&= (3.31)
\end{align}

- The optimal level of capital utilization is

\begin{align}
u_t = 1 + \left( A_t \alpha (k_t u_t)^{\alpha - 1} ((1 - \eta) h_t)^{1-\alpha} / q_t - \delta_1 / \delta_2 \right) \\
&= (3.32)
\end{align}

- The risk premium is

\begin{align}
\text{riskpr} &= q R^k_{t,t+1} - 1 = q (\tilde{\omega}_t / (1 - q g(\tilde{\omega}_t, \sigma_{\omega,t})) - 1 = (\tilde{\omega}_t / g(\tilde{\omega}_t, \sigma_{\omega,t})) - 1 \\
&= (3.33)
\end{align}

The optimal contract, the solution to the problem (3.1) subject to participating lenders, determines investment and the default threshold. Entrepreneurs with \( n_t > 0 \) borrow \( i_t - n_t \) units of consumption and pay back \( (1+r^t)(i_t-n_t) \equiv \tilde{\omega}_t \) which is possible if \( \omega_t \geq \tilde{\omega}_t \). The first order necessary conditions are given by

\begin{align}
q_t = \frac{1}{(1 - \tilde{\Phi}(\omega_t, \sigma_{\omega,t}))(1 + \tilde{\phi}(\omega_t, \sigma_{\omega,t}) \mu + \tilde{\phi}(\omega_t, \sigma_{\omega,t}) \mu f(\omega_t, \sigma_{\omega,t}) / \partial f(\omega_t, \sigma_{\omega,t}) / \partial \omega_t)} \\
&= (3.34)
\end{align}

which governs the default threshold \( \tilde{\omega} \) as a function of \( q \) as well as the leverage ratio

\begin{align}
i_t = \frac{1}{1 - q_t g(\tilde{\omega}_t, \sigma_{\omega,t})} n_t \\
&= (3.35)
\end{align}

which determines investment \( i(\tilde{\omega}(q), q, n) \) and ensures incentive compatibility.
Entrepreneurs’ intertemporal efficiency results from maximizing entrepreneurial consumption, which is linear in $c_t$.

$$q_t = \beta \gamma E_t[q_{t+1}(1 - \delta(u_{t+1})) + A_{t+1}\alpha(u_{t+1}k_{t+1})^{\alpha-1}((1 - \eta)h_{t+1})^{1-\alpha}(\frac{q_{t+1}f(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})}{1 - q_{t+1}g(\bar{\omega}_{t+1}, \sigma_{\omega,t+1})})]$$ (3.36)

The law of motion of entrepreneurs’ capital $z_{t+1}$ is given by the equations

$$\eta \mu_t = z_t[q_t(1 - \delta(u_t)) + A_t\alpha(u_tk_t)^{\alpha-1}((1 - \eta)h_t)^{1-\alpha}] \quad (3.37)$$

$$z_{t+1} = \eta \mu_t[\frac{f(\bar{\omega}_t, \sigma_{\omega,t})}{1 - q_tg(\bar{\omega}_t, \sigma_{\omega,t})}] - \eta c_t^\alpha q_t \quad (3.38)$$

### 3.6.2 Computation of the Steady State

In order to solve the model, I set the steady state default threshold as the inverse of the log-normal distribution for a given target default, which is $0.039/12$

$$\varpi = \Phi^{-1}(0.039/12, \sigma_{\omega}) \quad (3.39)$$

Given $\varpi$, the steady state bankruptcy rate is the log-normal distribution with $\mu_{\omega} = -\sigma_{\omega}^2/2$.

Entrepreneurs’ share of net capital output is the partial expectation of a log-normally distributed variable

$$f(\varpi; \sigma_{\omega}) = \Phi(-\frac{\log(\varpi) - \sigma_{\omega}^2/2 + \varpi^2}{\sigma_{\omega}}) - (1 - \Phi(\varpi; \sigma_{\omega}))\varpi \quad (3.40)$$

where $\Phi$ denotes the normal distribution.

The lenders’ share is

$$g(\varpi; \sigma_{\omega}) = 1 - f(\varpi; \sigma_{\omega}) - \Phi(\varpi; \sigma_{\omega})\mu \quad (3.41)$$

The steady state relative price of capital is

$$q = \frac{1}{1 - \Phi(\varpi; \sigma_{\omega})\mu + \Phi(\varpi; \sigma_{\omega})\mu f(\varpi; \sigma_{\omega})/((\Phi(\varpi; \sigma_{\omega}) - 1))} \quad (3.42)$$

which in turn determines $\gamma$, which prevents self-financing entrepreneurs.
\[ \gamma = \frac{1 - qg(\omega; \sigma_{\omega})}{qf(\omega; \sigma_{\omega})} \] (3.43)

and steady state risk premium

\[ riskpr = \frac{\omega}{g(\omega; \sigma_{\omega})} - 1 \] (3.44)
Chapter 4

Housing and Macroeconomy: The Role of Credit Channel, Risk - , Demand - and Monetary Shocks

4.1 Introduction

Figure 4.1 shows the dramatic rise and fall of real housing prices for the U.S. and some of the selected European countries\(^1\) from the first quarter of 1997 to the second quarter of 2011. The U.S. housing market peaked in the second quarter of 2007 with the price appreciation of 122\%.\(^2\) In comparison to Ireland (300 \%), Greece (185\%), and Spain (163\%) and with each country having slightly different peak periods, the U.S. housing price appreciation is not only in-line with the rest of the economies but also is relatively mild\(^3\). With the subsequent pronounced decline in house prices in these economies from their peaks during the mid 2000’s, there is a growing body of literature that describes these large swings in housing prices as \textit{bubbles} or these sharp increases in housing prices were caused by \textit{irrational exuberance}\(^4\), and subsequently, these

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\(^1\)We have selected Ireland, Spain, Greece, Italy and Portugal for our illustration purpose. Moreover, these countries are purposely chosen as most of them are currently experiencing a great financial difficulties. We also include Germany, who has not experienced any housing or financial crisis, to use as a benchmark comparison.

\(^2\)Unlike the Case-Shiller index that has an appreciation of 122\%, the Office of Federal Housing Enterprise Oversight (OFHEO) experienced 77\% appreciation. The difference between the Office of Federal Housing Enterprise Oversight (OFHEO) and Case-Shiller housing price indices arises largely from the treatment of expensive homes. The OFHEO index includes only transactions involving mortgages backed by the lenders it oversees, Fannie Mae and Freddie Mac, which are capped at $417,000. The Case-Shiller measure has no upper limit and gives more weight to higher-priced homes.

\(^3\)The following countries have experienced less of an appreciation: Portugal (44\%), Italy (70\%) and Germany (-4.2\%) with even depreciation.

\(^4\)The term \textit{Irrational Exuberance} was first coined by Alan Greenspan, the former chairman of the Board of Governors of the Federal Reserve in 1996 at the U.S. Congress testimony.
housing price bubbles then are the causes of the recent global financial instability.

Figure 4.1: Real housing prices for the U.S. and selected European countries from 1997 until 2007.

Given the recent macroeconomic experience of most developed countries, few students of the economy would argue with the following three observations: 1. Financial intermediation plays an important role in the economy, 2. The housing sector is a critical component for aggregate economic behavior and 3. Uncertainty, and, in particular, time-varying uncertainty and monetary shocks are quantitatively important sources of business cycle activity. And while there has no doubt been a concomitant increase in economic research which examines housing markets and financial intermediation, only a few analyses have been conducted in a calibrated, general equilibrium setting; i.e. an economic environment in which the quantitative properties of the model are broadly consistent with observed business cycle characteristics with various shocks. The objective of our paper is to develop a theoretical and computational framework

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5 We define and estimate these uncertainty (risk) shocks as the time variation in the cross sectional distribution of firm level productivities. The detail analysis of the risk shock estimation follows in the later section.

6 Some of the recent works which also examine housing and credit are: Iacoviello and Minetti (2008) and Iacoviello and Neri (2010) in which a new-Keynesian DGSE two sector model is used in their empirical analysis; Iacoviello (2005) analyzes the role that real estate collateral has for monetary policy; and Aoki, Proudman and Vlieghe (2004) analyze house price amplification effects in consumption and housing investment over the business cycle. None of these analyses use risk shocks as an impulse mechanism. Some recent papers that have examined the effects of uncertainty in a DSGE framework include Bloom et al. (2012), Fernandez-Villaverde et al. (2009),
that can help us understand: [1] how does uncertainty in lending channel effect economies (including both financial and housing markets) at different stages of business cycle; [2] what are the effects associated with credit constrained heterogeneous agents on the housing prices and the business cycle; and [3] what types monetary policies might help (or hinder) the process of housing and financial development.

To address the aforementioned questions, we use the framework of the housing supply/banking sector model as developed in Dorofeenko, Lee, and Salyer (2014) with the model of housing demand presented in Iacoviello and Neri (2010). In particular, we examine how the factors of production uncertainty, financial intermediation, and credit constrained households can affect housing prices and aggregate economic activity. Moreover, this analysis is cast within a monetary framework of Carlstrom and Fuerst (2001) which permits a study of how monetary policy can be used to mitigate the deleterious effects of cyclical phenomena that originates in the housing sector.

The Dorofeenko, Lee, and Salyer (2014) model focuses on the effects that housing production uncertainty and bank lending have on housing prices. To do this, their analysis combines the multi-sector housing model of Davis and Heathcote (2005) with the Carlstrom and Fuerst (1997) model of lending under asymmetric information and agency costs since both models had been shown to replicate several key features of the business cycle. In particular, the Davis and Heathcote (2005) model produces the high volatility of residential investment relative to fixed business investment seen in the data. However, the model fails to produce the observed volatility in housing prices. To this basic framework, Dorofeenko, Lee, and Salyer (2014) introduce an additional impulse mechanism, time varying uncertainty (i.e. risk shocks) shocks to the standard deviation of the entrepreneurs’ technology shock affecting only the housing production, and require that housing producers finance the purchase of their inputs via bank loans. Dorofeenko, Lee, and Salyer (2014) model risk shocks as a mean preserving spread in the distribution of the technology shocks affecting only house production and explore quantitatively how changes in uncertainty affect equilibrium characteristics.\footnote{One should note that the time varying risk shocks in this paper are quite different than the pure aggregate or sectoral technology (supply) shocks. First, risk shocks affect only the housing production sector. Second, risk shocks are meant to represent the second moments of the variance. That is, these time varying risk shocks proxy the changes in economic environment uncertainty.}

The importance of understanding how these uncertainty or risk shocks affect the economy is widely discussed in academics and among policymakers. For example, Baker, Bloom, and Davis (2012) demonstrates that a long persistent sluggish economic recovery in the U.S. (e.g. low output growth and unemployment hovering

\footnote{Christiano et al. (2015), and Dorofeenko, Lee and Salyer (2014) with housing markets.}
above 8%) even after the bottoming of the U.S. recession in June 2009 could be attributed to the high levels of uncertainty about economic policy.

In Dorofeenko, Lee, and Salyer (2014), these factors lead to greater house price volatility as housing prices reflect potential losses due to bankruptcy for some housing producers. In fact, the model is roughly consistent with the cyclical behavior of residential investment and housing prices as seen in U.S. data over the sample period 1975-2010. However, Dorofeenko, Lee, and Salyer (2014) model is not consistent with the behavior of housing prices and firm bankruptcy rates as seen in the recent decade. This failure is not surprising since the role of shocks to housing demand combined with changes in household mortgage finance are not present. Consequently, in this paper, we embed key features of the recent model by Iacoviello and Neri (2010) to rectify this omission. As detailed below, the main features of the Iacoviello and Neri (2010) model that we employ are the introduction of heterogenous agents (patient and impatient), a borrowing constraint (which affects impatient households) and a monetary authority that targets inflation via interest rate. We then introduce housing demand shocks (via preferences) and examine how these get transmitted to the economy. Next we examine optimal monetary policy in this setting.

In analyzing the role of the LTV (Loan to Value) ratio on macro and housing variables, we present three different scenarios that are based on LTV ratio: low (80), middle (85) and high (90) borrowing constraints. These different levels of LTV ratio are, for an expositional purpose, to reflect three different European economies: Germany, Italy and Spain. According to IMF (2011) (also shown in Table 4.5 in the Appendix), Germans have one of the lowest LTV ratio, whereas Spanish borrowers have one of the highest LTV ratio with Italy being in between these two levels.8

Unlike some of the recent literature that emphasize the important role of the level of LTV on housing market, our results indicate otherwise: almost no differences between different levels of LTV on the variables that we analyze. On the role of specific shocks, we show that the effects of monetary shocks are huge on most of the macro variables and in particular on the housing investment and the amount of borrowing the households undertake: over 75% and almost 50% of the variation in housing investment and borrowing can be explained by the monetary shocks. On the contrary to monetary shocks, housing demand shocks have a trivial impact on all the variables that we analyze: at most 6% of the variation in housing price can be explained by the preference shocks. Lastly, our endogenous debt financial accelerator model with risk shocks lends a strong support for the important role of risk shocks: over 85% of the variation in housing

8Some of the recent housing market developments for various European countries are discussed in the Appendix.
price is due to risk shocks. Our results, thus, show that there is a clear and an important role for the policy makers to smooth housing price and/or housing investment: to calm markets and to provide and restore market confidence.

4.2 Model: Housing Markets, Financial Intermediation, and Monetary Policy

Our model builds on three separate strands of literature: Davis and Heathcote’s (2005) multi-sector growth model with housing, Dorofeenko, Lee and Salyer’s (2008, 2014) credit channel model with uncertainty, and Iacoviello and Neri’s (2010) model of housing demand. In this paper, however, we do not consider any other New Keynesian economic frictions other than the asymmetric information friction that occurs in the loan contract between the mutual fund and entrepreneurs.

First we specify the households’ optimization problem in a representative agent economy in which the demand for money is motivated by a cash-in-advance constraint. Then, this environment is modified by dividing the households into two groups, patient and impatient (á la Iacoviello and Neri’s (2010)). This will introduce a role for household lending and borrowing. We then introduce financial intermediation (“banking”) sector, where the patient households lend savings to bankers, and the bankers then lend to both impatient households and entrepreneurs (housing producing agents). With the banking sector, the state of the economy (which is measured in this paper by the level of “uncertainty” or “risk”) effects the lending amount and the probability of loan default, and hence effecting the net worth of these financial intermediaries. And consequently, the endogenous net worth of the financial intermediary sector could in fact contribute aggregate movements in various financial and macro variables. Finally, we include government in setting monetary policy rule with a variation of the Taylor Rule.

Here is a brief outline and summary of the environment of this economy: Figure 4.2 shows a schematic of the implied flows for this economy.

- Three types of agents:
  - Risk-averse patient (impatient) households that choose consumption, labor, money holding, and housing service: Lend (borrow) money to (from) the financial intermediaries.
Risk-neutral entrepreneurs (housing developers) that choose consumption, investment, and labor.

* Cost of inputs is financed by borrowing
* Housing production subject to risks shocks.

- **Multisectors: 6 Firms**
  - Three intermediate goods producing: Construction, Manufacturing, and Service
  - Two "final" goods production: Residential Investment and Consumption / Non-residential Investment
  - Housing Production (via entrepreneurs): Residential Investment + Land.

- **A mutual fund: Financial intermediaries** (that guarantees a certain return to households through lending to an infinite number of entrepreneurs).

- **Government**: Lump sum money transfer and taxes.
• **Shocks:** 6 different shocks
  
  – 3 sectoral productivity technology shocks: Construction, Manufacturing and Service
  
  – Idiosyncratic technology shocks ($\omega_t$) affecting housing production.
    
    * Denoting the c.d.f. and p.d.f. of $\omega_t$ as $\Phi(\omega_t; \sigma_{\omega,t})$ and $\phi(\omega_t; \sigma_{\omega,t})$.
    
    * Second moment, $\sigma_{\omega,t}$, (i.e. risk) shocks affecting the distribution of these Idiosyncratic technology shocks.
  
  – Monetary shocks: à la Taylor Rule.
  
  – Housing demand (preference) shocks.

### 4.2.1 Money and cash-in-advance constraint in housing model: Households

This section follows the work of Carlstrom and Fuerst (2001) but adds loans and collateral restrictions on the demand side as in Iacoviello and Neri (2010). Households maximize lifetime utility given by:

$$E_0 \left( \sum_{t=0}^{\infty} \beta^t U(c_t, h_t, 1 - N_t) \right)$$

(4.1)

w.r.t. its consumption $c_t$, labor hours $N_t$, capital $K_t$ and housing $h_t$ stocks, the investment into consumption $i_{k,t}$ and housing $i_{h,t}$ goods sectors, and money holdings $M_t$ subject to the budget constraint (where we let $\lambda$ to denote the Lagrange multiplier):

$$c_t + i_{k,t} + i_{h,t} + \frac{M_{t+1}}{P_{c,t}} \leq K_t (r_t - \delta_k) + N_t w_t (1 - \tau_n) + p_{l,t} x_{l,t} + \frac{M_t + M_{s,t}}{P_{c,t}}.$$

(4.2)

$P_{c,t}$ denotes the nominal consumption price. Note that the new monetary injection, $M_{s,t}$ is distributed by the government at the beginning of the period as a lump-sum transfer (with the aggregate money stock given by $M_{s,t}$). The cash-in-advance constraint (CIA) states that money (post-transfer) must be used to buy investment and consumption goods (the associated Lagrange multiplier is $\kappa$):

$$c_t + i_{k,t} + i_{h,t} \leq \frac{M_t + M_{s,t}}{P_{c,t}}$$

(4.3)

The laws of motion for capital and housing are given by (the respective Lagrange multipliers are $\mu$ and $\nu$):

$$K_{t+1} = F_k(i_{k,t}, i_{k,t-1}) + (1 - \delta_k) K_t$$

(4.4)
$$h_{t+1} = F_h(i_h,t, i_{h,t-1}) + (1 - \delta_h) h_t$$  

(4.5)

Here \(\tau_k\) and \(\tau_n\) are the capital income and labor income taxes correspondingly, \(\delta\) represents the usual capital/housing depreciation rate, \(p_{l,t}x_{l,t}\) is the value of land, and the function \(F_j(i_{j,t}, i_{j,t-1})\) accounts investment adjustment cost. According to Christiano et. al (2005), \(F_j(i_{j,t}, i_{j,t-1}) = (1 - S \left(\frac{i_{j,t}}{i_{j,t-1}}\right)) i_{j,t}\), where \(S(1) = S'(1) = 0\) and \(S''(1) > 0\).

The correspondent Euler equations are:

$$U_{1t} - \kappa_t - \lambda_t = 0$$

$$w_t\lambda_t - U_{3t} = 0$$

$$\mu_t = \beta E_t ((1 - \delta) \mu_{t+1} + ((1 - \tau_k) \tau_t + \tau_k \delta_k) \lambda_{t+1})$$

$$\nu_t = \beta E_t ((1 - \delta_h) \nu_{t+1} + U_{2t+1})$$

$$\kappa_t + \lambda_t = \mu_t F_{c,1} (i_{k,t}, i_{k,t-1}) + \beta E_t (\mu_{t+1} F_{c,2} (i_{k,t+1}, i_{k,t}))$$

$$p_{h,t} (\kappa_t + \lambda_t) = \nu_t F_{h,1} (i_{h,t}, i_{h,t-1}) + \beta E_t (\nu_{t+1} F_{h,2} (i_{h,t+1}, i_{h,t}))$$

$$\frac{\lambda_t}{P_{c,t}} = \beta E_t \left(\frac{\kappa_{t+1} + \lambda_{t+1}}{P_{c,t+1}}\right)$$

Introducing the nominal interest rate \(R_t\) and the shadow prices of the capital and housing good, \(q_{k,t}\) and \(q_{h,t}\) respectively:

$$R_t = 1 + \frac{\kappa_t}{\lambda_t}, \mu_t = q_{k,t} U_{1,t}, \nu_t = q_{h,t} U_{1,t}$$

we obtain:

$$\frac{w_t}{R_t} = \frac{U_{3t}}{U_{1t}}$$  

(4.6)

$$q_{k,t} = \beta E_t \left( (1 - \delta_k) q_{k,t+1} + \frac{(1 - \tau_k) \tau_t + \tau_k \delta_k}{R_{t+1}} \frac{U_{1,t+1}}{U_{1,t}} \right)$$  

(4.7)

$$q_{h,t} = \beta E_t \left( (1 - \delta_h) q_{h,t+1} \frac{U_{1,t+1}}{U_{1,t}} + \frac{U_{2,t+1}}{U_{1,t}} \right)$$  

(4.8)

$$1 = q_{k,t} F_{k,1} (i_{k,t}, i_{k,t-1}) + \beta E_t \left( q_{k,t+1} F_{k,2} (i_{k,t+1}, i_{k,t}) \frac{U_{1,t+1}}{U_{1,t}} \right)$$  

(4.9)

$$p_{h,t} = q_{h,t} F_{h,1} (i_{h,t}, i_{h,t-1}) + \beta E_t \left( q_{h,t+1} F_{h,2} (i_{h,t+1}, i_{h,t}) \frac{U_{1,t+1}}{U_{1,t}} \right)$$  

(4.10)
including the Fisher equation for the nominal interest rate:

\[ 1 = \beta R_t E_t \left( \frac{U_{1,t+1}}{(1 + \Pi_{t+1}) U_{1,t}} \right) \]  \hspace{1cm} (4.11)

where we introduce the inflation rate \( \Pi_t \),

\[ \Pi_{t+1} = \frac{P_{c,t+1}}{P_{c,t}} - 1 \]

The substitution of (4.3) into (4.2) and assuming that both constraints are binding, we have our budget constraint as:

\[ c_{t+1} + i_{k,t+1} + p_{h,t+1}i_{h,t+1} = \frac{K_t ((1 - \tau_k) r_t + \tau_k \delta_k) + N_t w_t (1 - \tau_n) + p_{l,t} x_{l,t} + M_{s,t+1}}{1 + \Pi_{t+1}} \]  \hspace{1cm} (4.12)

**Utility function and demand shocks**

The utility function is assumed to have the form:

\[ U(c, h, l) = \left( \frac{c^{\mu_c} h^{\mu_h} l^{1-\mu_l}}{1-\sigma} \right)^{1-\sigma} \]  \hspace{1cm} (4.13)

where \( \sigma \) denotes the coefficient of relative risk aversion. The housing demand shock can be added to the utility function assuming that the parameter \( \mu_h \) follows the AR(1) processes:

\[ \ln \mu_{h,t+1} = (1 - \rho_h) \ln \mu_{h}^{(0)} + \rho_h \ln \mu_{h,t} + \varepsilon_{h,t+1} \]  \hspace{1cm} (4.14)

We use, in fact, the same utility function as Devis and Heathcote (2005), but slightly deviate from them in notation (\( \mu_l \neq 1 - \mu_c - \mu_{h,t} \)) to avoid the influence of housing demand shocks on the labor supply share \( \mu_l \).

**Heterogeneous agents, borrowing and collateral constraint at the demand side**

As in Iacoviello and Neri (2010), we now introduce two types of agents, patient and impatient. A prime is used to denote impatient household’s parameters and variables (\( \beta', N' \) etc.). The patient households as described in the previous section have a discount factor that is larger than the impatient ones, \( \beta > \beta' \). While both groups may lend or borrow money, the assumption \( \beta > \beta' \) will always lead to the situation where the patient households lend money to the impatient ones.
Adding the borrowing $b_t$ to the patient household’s budget constraint (4.2) produce the inequality (lending corresponds to the negative values of $b_t$):

$$c_t + i_{k,t} + p_{h,t} i_{h,t} + \frac{b_{t-1} R_{b,t-1}}{1 + \Pi_t} + \frac{M_{t+1}}{P_{c,t}} \leq K_t \left(r_t - \tau_k (r_t - \delta_k)\right) + \frac{N_t w_t (1 - \tau_n) + p_{l,t} x_{l,t} + b_t + \frac{M_t + M_{s,t}}{P_{c,t}}}{1 + \Pi_t}$$ (4.15)

In addition to the budget constraint, households face a cash-in-advance constraint:

$$c_t + i_{k,t} + p_{h,t} i_{h,t} \leq \frac{M_t + M_{s,t}}{P_{c,t}}$$ (4.16)

so, the relation (4.12) is changed to the form:

$$c_{t+1} + i_{k,t+1} + p_{h,t+1} i_{h,t+1} = K_t \left(r_t - \tau_k (r_t - \delta_k)\right) + \frac{N_t w_t (1 - \tau_n) + p_{l,t} x_{l,t} + b_t + \frac{R_{b,t-1} b_{t-1}}{1 + \Pi_t} + \frac{M_{t+1}}{P_{c,t}}}{1 + \Pi_{t+1}} + a_M m_{s,t+1}$$ (4.17)

where $m_{s,t} = \left(M_{s,t} + M'_{s,t}\right)/P_{c,t}$. The share $a_M = M_{s,t}/(M_{s,t} + M'_{s,t})$ is chosen equal to its steady-state value and remains constant.

The impatient households have shorter budget constraint, because they don’t own capital and land, so the quantities proportional to $K_t$, $i_{kt}$ and $x_{lt}$ are omitted:

$$c'_t + p_{h,t} i'_{h,t} + \frac{R_{b,t-1} b'_{t-1}}{1 + \Pi_t} + \frac{M'_{t+1}}{P_{c,t}} = N'_t w'_t (1 - \tau_n) + b'_t + \frac{M'_t + M'_{s,t}}{P_{c,t}}$$ (4.18)

and their CIA constraint is:

$$c'_t + p_{h,t} i'_{h,t} \leq \frac{M'_t + M'_{s,t}}{P_{c,t}}$$ (4.19)

which leads to the combined constraint:

$$c'_{t+1} + p_{h,t+1} i'_{h,t+1} = \frac{N'_t w'_t (1 - \tau_n) + b'_t - \frac{R_{b,t-1} b'_{t-1}}{1 + \Pi_t}}{1 + \Pi_{t+1}} + (1 - a_M) m_{s,t+1}$$ (4.20)

The additional (binding) borrowing constraint for the impatient households (collateral con-
straint) restricts the size of the borrowed funds by the value of their housing stock:

\[ b_t' \leq m E_t \left( \frac{p_{h,t+1} (1 + \Pi_{t+1}) h_t'}{R_t} \right) \]  \hspace{1cm} (4.21)

where \( m \) denotes the loan-to-value (LTV) ratio. Iacoviello and Neri (2010) set \( m \) for the impatient fraction of the USA households equal to \( m = 0.85 \). In our calibration exercise, we vary in the range from 0.1 to 0.9 to analyse the effects of \( m \) on our economy; \( m = \{0.1, 0.8, 0.85, 0.9\} \).

The market clearing condition for the borrowing is:

\[ b_t' + b_t = 0 \]  \hspace{1cm} (4.22)

The housing stock growth equations for the patient and impatient householders are:

\[ h_{t+1} + h_{t+1}' = F_h (i_{h,t}, i_{h,t-1}) + F_h (i_{h,t}', i_{h,t-1}') + (1 - \delta_h) (h_t + h_t') \]  \hspace{1cm} (4.23)

The patient households maximize their lifetime utility (4.1) w.r.t. consumption \( c_t \), labor hours \( N_t \), capital \( K_t \) and housing \( h_t \) stocks, the investment into consumption \( i_{k,t} \) and housing \( i_{h,t} \) goods sectors, money holdings \( M_t \) and borrowing \( b_t \) subject to constraints (4.15) and (4.16), stock accumulation equation (4.23) and the capital growth equation (4.4)

Their Euler equations consist of system (4.6) - (4.11) and the additional equation for the borrowing interest rate \( r_{h,t} \):

\[ 1 = \beta R_{h,t} E_t \left( \frac{R_t}{R_{t+1}} \left( \frac{U_{1,t+1}}{U_{1,t}} \right) \right) \]  \hspace{1cm} (4.24)

The impatient households maximize their lifetime utility (4.1) w.r.t. consumption \( c_t' \), labor hours \( N_t' \), housing \( h_t' \), the investment into housing goods sector \( i_{h,t}' \), money holdings \( M_t' \) and borrowing \( b_t' \) subject to constraints (4.18), (4.19), collateral constraint (4.21) and the housing accumulation equation (??). Their Euler equations are:

\[ \frac{u_t'}{R_t'} = \frac{U_{3t}}{U_{1t}} \]  \hspace{1cm} (4.25)

\[ 1 = \beta' R_{t} E_t \left( \frac{U_{1,t+1}}{1 + \Pi_{t+1} U_{1,t}} \right) \]  \hspace{1cm} (4.26)

\[ q'_{h,t} R_t' = 1 - \beta' R_{h,t} E_t \left( \frac{R_t'}{R_{t+1}} \left( \frac{U_{1,t+1}}{U_{1,t}} \right) \right) \]  \hspace{1cm} (4.27)
\[ q'_{h,t} = \beta' E_t \left( \left( 1 - \delta_h \right) q'_{h,t+1} + q'_{b,t+1} h'_{t+1} \frac{U_{1,t+1}}{U_{1,t}} + \frac{U_{2,t+1}}{U_{1,t}} \right) \]  
\[ p_{h,t} = q'_{h,t} F_{h,1} \left( i'_{h,t}, i'_{h,t-1} \right) + \beta' E_t \left( q'_{h,t+1} F_{h,2} \left( i'_{h,t+1}, i'_{h,t} \right) \frac{U_{1,t+1}}{U_{1,t}} \right) \]

(4.28)

(4.29)

Here \( q'_{b,t} \) denotes the shadow price of borrowing. The utility function \( U = U (c'_t, 1 - N'_t, h'_t) \) in equations (4.25) - (4.29) depends on the impatient household’s consumption \( c'_t \), labor hours \( N'_t \) and housing \( h'_t \).

### 4.2.2 Production (Firms): with only one household

We first assume a representative agent framework and then introduce heterogeneous agents as described above. We assume (as in Davis and Heathcote (2005)) that final goods (residential investment and consumption goods) are produced using intermediate goods. The intermediate goods sector consists of three output: building/construction, manufacture, and services, which are produced via Cobb-Douglas production functions:

\[ x_i = k_{it}^{\theta_i} (e^{z_i n_i})^{1-\theta_i} \]  

(4.30)

where \( i = b, m, s \) (building/construction, manufacture, service), \( k_{it}, n_{it} \) and \( z_{it} \) are capital, household labor, entrepreneur labor, and labor augmenting (in log) productivity shock respectively for each sector, with the \( \theta_i \) being the share of capital that differ across sectors. For example, we let in our calibration that \( \theta_b < \theta_m \), reflecting the fact that the manufacturing sector is more capital intensive (or less labor intensive) than the construction sector. Unlike Davis and Heathcote (2005), we include entrepreneurial labor supply in the production function.\(^9\)

The production shocks grows linearly:

\[ z_i = t \ln g_{z,i} + \tilde{z}_i \]  

(4.31)

with the stochastic term \( \tilde{z} = (\tilde{z}_b, \tilde{z}_m, \tilde{z}_s) \) following the vector AR(1) process:

\[ \tilde{z}_{t+1} = B \cdot \tilde{z}_t + \tilde{e}_{t+1} \]  

(4.32)

\(^9\)Although we do not include in the model the entrepreneurial labor income, the assumption of entrepreneurial labor income is necessary as it guarantees a nonzero net worth for each entrepreneur. This nonzero net worth assumption is important as the financial contracting problem is not well defined otherwise. In our calibration section, we let the share of entrepreneur’s labor supply to be quite small but nonzero. Consequently, although the entrepreneurs’ labor supply do not play a role in our equilibrium conditions, small share of entrepreneur’s labor supply does ensure a small but nonzero net worth.
where the the matrix $B$ captures the deterministic part of shocks over time, and the innovation vector $\vec{\varepsilon}$ is distributed normally with a given covariance matrix $\Sigma_\varepsilon$. The shock growth factors $g_{z,i}$ lead to the correspondent growth factors for other variables.

These intermediate firms maximize a conventional static profit function at $t$

$$\max_{\{k_{it}, n_{it}\}} \left\{ \sum_i p_{it} x_{it} - r_t k_t - w_t n_t \right\}$$

subject to equations $k_t \geq \sum_i k_{it}, n_t \geq \sum_i n_{it}$, and non-negativity of inputs, where $r_t, w_t$, and $p_{it}$ are the capital rental, wage, and output prices. A conventional optimization leads to the relations:

$$k_i r = \theta_i p_i x_i$$

$$n_i w = (1 - \theta_i) p_i x_i$$

so that

$$k_i r + n_i w = p_i x_i$$

The intermediate goods are then used as inputs to produce two final goods, $y_j$:

$$y_{jt} = \Pi_{i=b,m,s} x_{1}^{\rho^{i}j}_{ijt},$$

where $j = c, d$ (consumption/capital investment and residential investment respectively), the input matrix is defined by

$$x_{1} = \begin{pmatrix} b_c & b_d \\ m_c & m_d \\ s_c & s_d \end{pmatrix},$$

and the shares of construction, manufactures and services for sector $j$ are defined by the matrix

$$\rho = \begin{pmatrix} B_c & B_d \\ M_c & M_d \\ S_c & S_d \end{pmatrix}.$$
following conditions must also be satisfied:

\[ \sum_i \rho_{ij} = 1 \quad (4.40) \]

and

\[ x_{it} = \sum_j x_{1ijt} \quad (4.41) \]

i.e

\[ x_{bt} = b_{ct} + b_{dt}, \quad x_{mt} = m_{et} + m_{dt}, \quad x_{st} = s_{ct} + s_{dt}. \quad (4.42) \]

With intermediate goods as inputs, the final goods’ firms solve the following static profit maximization problem at \( t \) where the price of consumption good, \( p_{ct} \), is normalized to 1:

\[
\max_{\{b_{jt}, m_{jt}, s_{jt}\}} \left\{ y_{ct} + p_{dt} y_{dt} - \sum_i p_{it} x_{it} \right\}
\]

subject to equation (4.37) and non-negativity of inputs. The optimization of final good firms leads to the relations:

\[ p_{it} x_{1,ijt} = \rho_{ijt} p_{jt} y_{jt} \quad (4.43) \]

where \( i = b, m, s, \; j = c, d \)

Due to CRS property, we obtain:

\[ \sum_{i=b,m,s} p_{it} x_{it} = \sum_{j=c,d} p_{jt} y_{jt} = K_t r_t + w_t N_t \quad (4.44) \]

where

\[ K_t = \sum_{i=b,m,s} k_{it}, N_t = \sum_{i=b,m,s} n_{it} \quad (4.45) \]

Lastly, the housing firms (real estate developers or entrepreneurs) produce the housing good, \( y_{ht} \), given residential investment \( y_{dt} \) and fix amount of land \( x_{lt} \) as inputs, according to

\[ y_{ht} = x_{lt}^{\phi} (1 - \phi) \quad (4.46) \]

where, \( \phi \) denotes the share of land. Output equation (4.46) will be modified later in the section to include idiosyncratic productivity and uncertainty shocks. As mentioned in the introduction, the focus of our paper is on the housing sector in which agency costs with uncertainty and heterogeneity arise: we come back to this modification on the firms’ behavior in the later section.
The optimization defines the price relations:

\[ p_l x_l = \phi p_h y_h, \quad p_d y_d = (1 - \phi)p_h y_h \quad (4.47) \]

**Firms: Production side with two types of households**

With both patient and impatient household, we now need to make small modifications of the production side of the model due to the presence of two types of labor supplying households in the system. Now the production of the intermediate good (4.30) changes to the form\(^1\):

\[ x_i = k_i^{\theta_i} (e^{n_i^\alpha n_i' (1-\alpha)})^{1-\theta_i} \quad (4.48) \]

where \( n_i \) and \( n_i' \) are the hours supplied by the patient and impatient households respectively according to their labor share \( \alpha \). Equations (4.35) and (4.36) now are:

\[ n_i w = \alpha p_i x_i (1 - \theta_i) \quad (4.49) \]

\[ n_i' w' = (1 - \alpha)p_i x_i (1 - \theta_i) \quad (4.50) \]

and

\[ k_i r + n_i' w' + n_i w = p_i x_i. \quad (4.51) \]

Balance equations (4.44) are also changed to:

\[ \sum_{i=b,m,s} p_i x_i = \sum_{j=c,d} p_j y_j = Kr + Nw + N'w'. \quad (4.52) \]

The new variable \( N' \) denotes the total impatient household’s hours:

\[ N' = \sum_{i=b,m,s} n_i'. \quad (4.53) \]

We can introduce now the effective hours \( L \) and the effective wage \( W \):

\[ L = N^\alpha N'^{1-\alpha}, \quad W = \left( \frac{w}{\alpha} \right)^\alpha \left( \frac{w'}{1 - \alpha} \right)^{1-\alpha}. \quad (4.54) \]

\(^{10}\)For the simplicity purpose, we drop the time script in this section.
Then from relation (4.49) and (4.50) follows that

\[ LW = Nw + N'w'. \]  \hspace{1cm} (4.55)

We can also introduce the effective hours

\[ l_i = n_i^\alpha n_i'^{1-\alpha} \]

for building, manufacture and services \((i = b, m, s)\) separately:

\[ l_i = n_i^\alpha n_i'^{1-\alpha}. \]

Then the following equations hold:

\[ l_iW = (1 - \theta_i) p_i x_i = n_i'w' + n_iw, \quad L = \sum_{i=b,m,s} l_i \]

### 4.2.3 Credit Channel with Uncertainty

In this section, we outline how the financial intermediaries decide on the amount of loan that is to be lend out to housing developers (entrepreneurs). One should note that in our lending model, we only focuses on the supply side. That is, we do not address the endogenous lending mechanism for the impatient households (the demand side): The loan for the impatient household is exogenously determined by the collateral constraint in equation (4.21).

**Housing Entrepreneurial Contract**

It is assumed that a continuum of housing producing firms with unit mass are owned by risk-neutral entrepreneurs (developers). The costs of producing housing are financed via loans from risk-neutral intermediaries. Given the realization of the idiosyncratic shock to housing production, some real estate developers will not be able to satisfy their loan payments and will go bankrupt. The banks take over operations of these bankrupt firms but must pay an agency fee. These agency fees, therefore, affect the aggregate production of housing and, as shown below, imply an endogenous markup to housing prices. That is, since some housing output is lost to agency costs, the price of housing must be increased in order to cover factor costs.

The timing of events is critical:

1. The exogenous state vector of technology shocks, uncertainty shocks, housing preference shocks, and monetary shocks, denoted \((z_{i,t}, \sigma_{\omega,t}, \mu_{h,t+1}, R_{t+1})\), is realized.

2. Firms hire inputs of labor and capital from households and entrepreneurs and produce
intermediate output via Cobb-Douglas production functions. These intermediate goods are then used to produce the two final outputs.

3. Households make their labor, consumption, housing, and investment decisions.

4. With the savings resources from households, the banking sector provide loans to entrepreneurs via the optimal financial contract (described below). The contract is defined by the size of the loan \( f_{p,a,t} \) and a cutoff level of productivity for the entrepreneurs' technology shock, \( \bar{\omega}_t \).

5. Entrepreneurs use their net worth and loans from the banking sector in order to purchase the factors for housing production. The quantity of factors (residential investment and land) is determined and paid for before the idiosyncratic technology shock is known.

6. The idiosyncratic technology shock of each entrepreneur is realized. If \( \omega_{a,t} \geq \bar{\omega}_t \) the entrepreneur is solvent and the loan from the bank is repaid; otherwise the entrepreneur declares bankruptcy and production is monitored by the bank at a cost proportional (but time varying) to total factor payments.

7. Solvent entrepreneur’s sell their remaining housing output to the bank sector and use this income to purchase current consumption and capital. The latter will in part determine their net worth in the following period.

8. Note that the total amount of housing output available to the households is due to three sources: (1) The repayment of loans by solvent entrepreneurs, (2) The housing output net of agency costs by insolvent firms, and (3) the sale of housing output by solvent entrepreneurs used to finance the purchase of consumption and capital.

For entrepreneur \( a \), the housing production function is denoted \( G(x_{a,lt}, y_{a,dt}) \) and is assumed to exhibit constant returns to scale. Specifically, we assume:

\[
y_{a,ht} = \omega_{a,t} G(x_{a,lt}, y_{a,dt}) = \omega_{a,t} x_{a,lt}^{\zeta} y_{a,dt}^{1-\zeta}
\]

(4.56)

where, \( \zeta \) denotes the share of land. It is assumed that the aggregate quantity of land is fixed and equal to 1. The technology shock, \( \omega_{a,t} \), is an idiosyncratic shock affecting real estate developers. The technology shock is assumed to have a unitary mean and standard deviation of \( \sigma_{\omega,t} \). The standard deviation, \( \sigma_{\omega,t} \), follows an AR(1) process:

\[
\sigma_{\omega,t+1} = \sigma_0^{1-x} \sigma_{\omega,t}^{x} \exp^{x \sigma_{t+1}}
\]

(4.57)
with the steady-state value \( \sigma_0, \chi \in (0, 1) \) and \( \varepsilon_{\sigma,t+1} \) is a white noise innovation.\(^{11}\)

Each period, entrepreneurs enter the period with net worth given by \( nw_{at} \). Developers use this net worth and loans from the banking sector in order to purchase inputs. Letting \( fp_{at} \) denote the factor payments associated with developer \( a \), we have:

\[
fp_{at} = pdty_{adt} + ptx_{alt}
\]

(4.58)

Hence, the size of the loan is \( (fp_{at} - nw_{at}) \). The realization of \( \omega_{at} \) is privately observed by each entrepreneur; banks can observe the realization at a cost that is proportional to the total input bill.

It is convenient to express these agency costs in terms of the price of housing. Note that agency costs combined with constant returns to scale in housing production (see eq. (4.56)) implies that the aggregate value of housing output must be greater than the value of inputs; i.e. housing must sell at a markup over the input costs, the factor payments. Denote this markup as \( \bar{s}_t \) (which is treated as parametric by both lenders and borrowers) which satisfies:

\[
p_hy_h = \bar{s}_tf_p
\]

(4.59)

Also, since \( E(\omega_t) = 1 \) and all firms face the same factor prices, this implies that, at the individual level, we have\(^{12}\)

\[
p_hG(x_{alt}, y_{adt}) = \bar{s}_tf_p
\]

(4.60)

Given these relationships, we define agency costs for loans to an individual entrepreneur in terms of foregone housing production as \( \mu \bar{s}_t f_{p_{at}} \).

With a positive net worth, the entrepreneur borrows \( (fp_{at} - nw_{at}) \) consumption goods and agrees to pay back \( (1 + r^L_t) (fp_{at} - nw_{at}) \) to the lender, where \( r^L_t \) is the interest rate on loans. The cutoff value of productivity, \( \bar{\omega}_t \), that determines solvency (i.e. \( \omega_{at} \geq \bar{\omega}_t \)) or bankruptcy (i.e. \( \omega_{at} < \bar{\omega}_t \)) is defined by \( (1 + r^L_t) (fp_{at} - nw_{at}) = p_h\bar{\omega}_tF(\cdot) \) (where \( F(\cdot) = F(x_{alt}, y_{adt}) \)). Denoting the c.d.f. and p.d.f. of \( \omega_t \) as \( \Phi(\omega_t; \sigma_{\omega,t}) \) and \( \phi(\omega_t; \sigma_{\omega,t}) \), the expected returns to a

---

\(^{11}\)This autoregressive process is used so that, when the model is log-linearized, \( \hat{\sigma}_{\omega,t} \) (defined as the percentage deviations from \( \sigma_0 \)) follows a standard, mean-zero AR(1) process.

\(^{12}\)The implication is that, at the individual level, the product of the markup \( (\bar{s}_t) \) and factor payments is equal to the expected value of housing production since housing output is unknown at the time of the contract. Since there is no aggregate risk in housing production, we also have \( p_hy_h = \bar{s}_tf_p \).
housing producer is therefore given by:\textsuperscript{13}

\[
\int_{-\infty}^{\infty} \left[ p_{at} \omega F (\cdot) - (1 + r_{L}^t) (f_{p_{at}} - n_{w_{at}}) \right] \phi (\omega; \sigma_{\omega,t}) \, d\omega
\] (4.61)

Using the definition of \( \bar{\omega}_t \) and eq. (4.60), this can be written as:

\[
\bar{s}_t f_{p_{at}} f (\bar{\omega}_t; \sigma_{\omega,t})
\] (4.62)

where \( f (\bar{\omega}_t; \sigma_{\omega,t}) \) is defined as:

\[
f (\bar{\omega}_t; \sigma_{\omega,t}) = \int_{-\infty}^{\bar{\omega}_t} \omega \phi (\omega; \sigma_{\omega,t}) \, d\omega - [1 - \Phi (\bar{\omega}_t; \sigma_{\omega,t})] \bar{\omega}_t
\] (4.63)

Similarly, the expected returns to lenders is given by:

\[
\int_{0}^{\bar{\omega}_t} \quad p_{ht} \omega F (\cdot) \phi (\omega; \sigma_{\omega,t}) \, d\omega + [1 - \Phi (\bar{\omega}_t; \sigma_{\omega,t})] \left(1 + r_{L}^t\right) (f_{p_{at}} - n_{w_{at}}) - \Phi (\bar{\omega}_t; \sigma_{\omega,t}) \mu
\]

\[
\bar{s}_t f_{p_{at}} g (\bar{\omega}_t; \sigma_{\omega,t})
\] (4.64)

Again, using the definition of \( \bar{\omega}_t \) and eq. (4.60), this can be expressed as:

\[
\bar{s}_t f_{p_{at}} g (\bar{\omega}_t; \sigma_{\omega,t})
\] (4.65)

where \( g (\bar{\omega}_t; \sigma_{\omega,t}) \) is defined as:

\[
g (\bar{\omega}_t; \sigma_{\omega,t}) = \int_{0}^{\bar{\omega}_t} \omega \phi (\omega; \sigma_{\omega,t}) \, d\omega + [1 - \Phi (\bar{\omega}_t; \sigma_{\omega,t})] \bar{\omega}_t - \Phi (\bar{\omega}_t; \sigma_{\omega,t}) \mu
\] (4.66)

Note that these two functions sum to:

\[
f (\bar{\omega}_t; \sigma_{\omega,t}) + g (\bar{\omega}_t; \sigma_{\omega,t}) = 1 - \Phi (\bar{\omega}_t; \sigma_{\omega,t}) \mu
\] (4.67)

Hence, the term \( \Phi (\bar{\omega}_t; \sigma_{\omega,t}) \mu \) captures the loss of housing due to the agency costs associated with bankruptcy. With the expected returns to lender and borrower expressed in terms of the size of the loan, \( f_{p_{at}} \), and the cutoff value of productivity, \( \bar{\omega}_t \), it is possible to define the optimal borrowing contract by the pair \( (f_{p_{at}}, \bar{\omega}_t) \) that maximizes the entrepreneur’s return subject to the lender’s willingness to participate (all rents go to the entrepreneur). That is, the optimal contract is determined by the solution to:

\textsuperscript{13}The notation \( \Phi (\omega; \sigma_{\omega,t}) \) is used to denote that the distribution function is time-varying as determined by the realization of the random variable, \( \sigma_{\omega,t} \).
\[
\max_{\bar{\omega}, f p_{at}} \bar{s}_t f p_{at} f (\bar{\omega}_t; \sigma_{\omega,t}) \text{ subject to } \bar{s}_t f p_{at} g (\bar{\omega}_t; \sigma_{\omega,t}) \geq f p_{at} - nw_{at} (4.68)
\]

A necessary condition for the optimal contract problem is given by:
\[
\frac{\partial (.)}{\partial \bar{\omega}_t} : \bar{s}_t f p_{at} \frac{\partial f (\bar{\omega}_t; \sigma_{\omega,t})}{\partial \bar{\omega}_t} = -\lambda_t \bar{s}_t f p_{at} \frac{\partial g (\bar{\omega}_t; \sigma_{\omega,t})}{\partial \bar{\omega}_t} (4.69)
\]

where \(\lambda_t\) is the shadow price of the lender’s resources. Using the definitions of \(f (\bar{\omega}_t; \sigma_{\omega,t})\) and \(g (\bar{\omega}_t; \sigma_{\omega,t})\), this can be rewritten as:\(^{14}\)
\[
1 - \frac{1}{\lambda_t} = \frac{\phi (\bar{\omega}_t; \sigma_{\omega,t})}{1 - \Phi (\bar{\omega}_t; \sigma_{\omega,t})} \mu
\] (4.70)

As shown by eq.(4.70), the shadow price of the resources used in lending is an increasing function of the relevant Inverse Mill’s ratio (interpreted as the conditional probability of bankruptcy) and the agency costs. If the product of these terms equals zero, then the shadow price equals the cost of housing production, i.e. \(\lambda_t = 1\).

The second necessary condition is:
\[
\frac{\partial (.)}{\partial f p_{at}} : \bar{s}_t f (\bar{\omega}_t; \sigma_{\omega,t}) = \lambda_t [1 - \bar{s}_t g (\bar{\omega}_t; \sigma_{\omega,t})] (4.71)
\]

These first-order conditions imply that, in general equilibrium, the markup factor, \(\bar{s}_t\), will be endogenously determined and related to the probability of bankruptcy. Specifically, using the first order conditions, we have that the markup, \(\bar{s}_t\), must satisfy:
\[
\bar{s}_t^{-1} = \left[ f (\bar{\omega}_t; \sigma_{\omega,t}) + g (\bar{\omega}_t; \sigma_{\omega,t}) \right] + \frac{\phi (\bar{\omega}_t; \sigma_{\omega,t}) \mu f (\bar{\omega}_t; \sigma_{\omega,t})}{\frac{\partial f (\bar{\omega}_t; \sigma_{\omega,t})}{\partial \omega_t}}
\] (4.72)
\[
= \left[ 1 - \Phi (\bar{\omega}_t; \sigma_{\omega,t}) \right] A - \frac{\phi (\bar{\omega}_t; \sigma_{\omega,t})}{B} \frac{f (\bar{\omega}_t; \sigma_{\omega,t})}{1 - \Phi (\bar{\omega}_t; \sigma_{\omega,t})}
\]

which then can be written as
\[
\bar{s}_t = \frac{1}{1 - \mu \Phi (\bar{\omega}_t) + \mu f (\bar{\omega}_t) \frac{\varphi (\bar{\omega}_t)}{f (\bar{\omega}_t)}} (4.73)
\]

We make some brief remarks on the markup equation above. First note that the markup factor

\(^{14}\)Note that we have used the fact that \(\frac{\partial f (\bar{\omega}_t; \sigma_{\omega,t})}{\partial \omega_t} = \Phi (\bar{\omega}_t; \sigma_{\omega,t}) - 1 < 0\)
depends only on economy-wide variables so that the aggregate markup factor is well defined. Also, the two terms, $A$ and $B$, demonstrate that the markup factor is affected by both the total agency costs (term $A$) and the marginal effect that bankruptcy has on the entrepreneur’s expected return. That is, term $B$ reflects the loss of housing output, $\mu$, weighted by the expected share that would go to entrepreneur’s, $f(\omega_t; \sigma_{\omega,t})$, and the conditional probability of bankruptcy (the Inverse Mill’s ratio). Finally, note that, in the absence of credit market frictions, there is no markup so that $\bar{s}_t = 1$. In the partial equilibrium setting, it is straightforward to show that equation (4.72) defines an implicit function $\bar{\omega}(\bar{s}_t, \sigma_{\omega,t})$ that is increasing in $\bar{s}_t$.

The incentive compatibility constraint implies

$$f_{p at} = \frac{1}{(1 - \bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t})) n w_{at}}$$

(4.74)

Equation (4.74) implies that the size of the loan is linear in entrepreneur’s net worth so that aggregate lending is well-defined and a function of aggregate net worth.

The effect of an increase in uncertainty on lending can be understood in a partial equilibrium setting where $\bar{s}_t$ and $nw_{at}$ are treated as parameters. As shown by eq. (4.72), the assumption that the markup factor is unchanged implies that the costs of default, represented by the terms $A$ and $B$, must be constant. With a mean-preserving spread in the distribution for $\omega_t$, this means that $\bar{\omega}_t$ will fall (this is driven primarily by the term $A$). Through an approximation analysis, it can be shown that $\bar{\omega}_t \approx g(\bar{\omega}_t; \sigma_{\omega,t})$ (see the Appendix in Dorofeenko, Lee, and Salyer (2008)). That is, the increase in uncertainty will reduce lenders’ expected return ($g(\bar{\omega}_t; \sigma_{\omega,t})$). Rewriting the binding incentive compatibility constraint (eq. (4.74)) yields:

$$\bar{s}_t g(\bar{\omega}_t; \sigma_{\omega,t}) = 1 - \frac{nw_{at}}{f_{p at}}$$

(4.75)

the fall in the left-hand side induces a fall in $f_{p at}$. Hence, greater uncertainty results in a fall in housing production. This partial equilibrium result carries over to the general equilibrium setting.

The existence of the markup factor implies that inputs will be paid less than their marginal products. In particular, profit maximization in the housing development sector implies the following necessary conditions:

$$\frac{p_{lt}}{p_{ht}} = \frac{G_{xl}(x_{lt}, y_{dt})}{\bar{s}_t}$$

(4.76)

$$\frac{p_{dt}}{p_{ht}} = \frac{G_{yd}(x_{lt}, y_{dt})}{\bar{s}_t}$$

(4.77)
These expressions demonstrate that, in equilibrium, the endogenous markup (determined by the agency costs) will be a determinant of housing prices.

The production of new housing is determined by a Cobb-Douglas production with residential investment and land (fixed in equilibrium) as inputs. Denoting housing output, net of agency costs, as \( y_{ht} \), this is given by:

\[
y_{ht} = x_{lt}^{\zeta} y_{dt}^{1-\zeta} \left[ 1 - \Phi (\omega_t; \sigma_{\omega,t}) \right] \mu
\]  
(4.78)

In equilibrium, we require that \( i_{ht} = y_{ht} \); i.e. household’s housing investment is equal to housing output. Recall that the law of motion for housing is given by eq. (4.5)

**Entrepreneurial Consumption and House Prices**

To rule out self-financing by the entrepreneur (i.e. which would eliminate the presence of agency costs), it is assumed that the entrepreneur discounts the future at a faster rate than the patient household. This is represented by following expected utility function:

\[
E_0 \sum_{t=0}^{\infty} (\beta \eta \gamma)^t c_t^e
\]  
(4.79)

where \( c_t^e \) denotes entrepreneur’s per-capita consumption at date \( t \), and \( \gamma \in (0, 1) \). This new parameter, \( \gamma \), will be chosen so that it offsets the steady-state internal rate of return due to housing production.

Each period, entrepreneur’s net worth, \( nw_t \) is determined by the value of capital income and the remaining capital stock.\(^{15}\) That is, entrepreneurs use capital to transfer wealth over time (recall that the housing stock is owned by households). Denoting entrepreneur’s capital as \( k_t^e \), this implies:\(^{16}\)

\[
nw_t = k_t^e \left[ r_t + 1 - \delta_k \right]
\]  
(4.80)

The law of motion for entrepreneurial capital stock is determined in two steps. First, new capital is financed by the entrepreneurs’ value of housing output after subtracting consumption:

\[
\eta k_{t+1}^e = p_{ht} y_{ht} f (\omega_t; \sigma_{\omega,t}) - c_t^e = s_t f p_{at} f (\omega_t; \sigma_{\omega,t}) - c_t^e
\]  
(4.81)

\(^{15}\)As stated in footnote 6, net worth is also a function of current labor income so that net worth is bounded above zero in the case of bankruptcy. However, since entrepreneur’s labor share is set to a very small number, we ignore this component of net worth in the exposition of the model.

\(^{16}\)For expositional purposes, in this section we drop the subscript \( a \) denoting the individual entrepreneur.
Note we have used the equilibrium condition that \( p_h t y_{aht} = \bar{s}_t f p_{at} \) to introduce the markup, \( \bar{s}_t \), into the expression. Then, using the incentive compatibility constraint, eq. (4.74), and the definition of net worth, the law of motion for capital is given by:

\[
\eta k_{t+1}^e = k_t^e (r_t + 1 - \delta_t) - c_t^e
\]  

(4.82)

The term \( \bar{s}_t f (\bar{\omega}_t; \sigma_{\omega,t}) / (1 - \bar{s}_t g (\bar{\omega}_t; \sigma_{\omega,t})) \) represents the entrepreneur’s internal rate of return due to housing production; alternatively, it reflects the leverage enjoyed by the entrepreneur since

\[
\frac{\bar{s}_t f (\bar{\omega}_t; \sigma_{\omega,t})}{1 - \bar{s}_t g (\bar{\omega}_t; \sigma_{\omega,t})} = \frac{\bar{s}_t f p_{at} f (\bar{\omega}_t; \sigma_{\omega,t})}{nw_t}
\]  

(4.83)

That is, entrepreneurs use their net worth to finance factor inputs of value \( f p_{at} \), this produces housing which sells at the markup \( \bar{s}_t \) with entrepreneur’s retaining fraction \( f (\bar{\omega}_t; \sigma_{\omega,t}) \) of the value of housing output.

Given this setting, the optimal path of entrepreneurial consumption implies the following Euler equation:

\[
1 = \beta \eta \gamma E_t \left[ (r_{t+1} + 1 - \delta_t) \frac{\bar{s}_{t+1} f (\bar{\omega}_{t+1}; \sigma_{\omega,t+1})}{1 - \bar{s}_{t+1} g (\bar{\omega}_{t+1}; \sigma_{\omega,t+1})} \right]
\]  

(4.84)

Finally, we can derive an explicit relationship between entrepreneur’s capital and the value of the housing stock using the incentive compatibility constraint and the fact that housing sells at a markup over the value of factor inputs. That is, since \( p_h t F (x_{alt}, y_{adt}) = \bar{s}_t f p_{t} \), the incentive compatibility constraint implies:

\[
p_h t \left( x_{lt}^c y_{dt}^{1-\zeta} \right) = k_t^e \frac{(r_t + 1 - \delta_t)}{1 - \bar{s}_t g (\bar{\omega}_t; \sigma_{\omega,t})} \bar{s}_t
\]  

(4.85)

Again, it is important to note that the markup parameter plays a key role in determining housing prices and output.

Financial Intermediaries

The Capital Mutual Funds (CMFs) act as risk-neutral financial intermediaries who earn no profit and produce neither consumption nor capital goods. There is a clear role for the CMF in this economy since, through pooling, all aggregate uncertainty of capital (house) production can be eliminated. The CMF receives capital from three sources: entrepreneurs sell undepreciated capital in advance of the loan, after the loan, the CMF receives the newly created capital through loan repayment and through monitoring of insolvent firms, and, finally, those entrepreneur’s that
are still solvent, sell some of their capital to the CMF to finance current period consumption. This capital is then sold at the price of \( \bar{s}_t \) units of consumption to households for their investment plans.

### 4.2.4 Government budget constraint

Assuming the absence of government money holdings, its budget constraint equation is:

\[
G_t + \mu \Phi (\varpi_t) p_{h,t} y_{h,t} + m_{s,t} + m_{t}' = K_t (r_t - \delta_k) \tau_k + (N_t w_t + N_t' w_t') \tau_n + m^G_t
\]

where \( G_t \) denotes the real government spending, \( m_{s,t} = M_{st}/P_{c,t}, m_{s,t}' = M_t'/P_{c,t}, \) and \( m^G_t = M^G_t/P_{c,t} \) is a lump-sum money injection into the whole economy. Here we let the money evolves according to

\[
m_{t+1}^G = (1 - \rho_M) m_0^G + \rho_M m_t^G
\]

where, \( \rho_M \in (0, 1) \).

Following Christiano, Eichenbaum and Trabandt (2014), we use the monetary policy rule as:

\[
\log \left( \frac{R_{b,t+1}}{R_b} \right) = \varrho_R \log \left( \frac{R_{b,t}}{R_b} \right) + (1 - \varrho_R) \left( \varrho_\pi \log \left( \frac{1 + \Pi_{t+1}}{1 + \Pi} \right) + \varrho_{\Delta GDP} \log \left( \frac{GDP_{t+1}}{GDP_t} \right) + \varrho_{\Delta GDP} \log \left( \frac{GDP_{t+1}}{GDP_t} \right) \right) + \varepsilon_{R,t+1}
\]

We also use one-period log-difference for GDP instead of averaged four-period difference used by Christiano et al, (2014).

We assume here that the monitoring of defaulting firms is arranged by a government’s institution, but separate the monitoring cost \( \mu \Phi (\varpi_t) p_{h,t} y_{h,t} \) from the other government spendings as each defaulting firms belong to different industries (see eqs (4.89) and (4.90) below). The share of government spendings \( G_t/GDP_t = a_G \) is considered to be a fixed value according to Davis and Heathcote (2005) (see eq (4.91)). The government distributes money injections \( M_{st} \) and \( M_t' \) between the patient and impatient householders proportionally to their steady-state shares \( M_0 \) and \( M_0' \).

---

17Equation (4.87) contains variables with exponential trend removed. The variables without subscript denote steady-state values. We use “borrowing interest rate” of borrowing between patient and impatient households, \( R_b \), in eq (4.87) as one of the targeted values.
The complete system of 62 equilibrium equations of the model is summarized in the Appendix.

4.3 Empirical Results

Our primary empirical objective in this paper is to show the importance of the following key parameters (variables) and shocks on housing variables as well as some of the aggregate macro variables. Consequently, we do not calibrate our model to specific country’s economic parameters, but rather we set the model parameters to our benchmark values of the U.S. economy but with the loan-to-value \((m)\) to vary to reflect different European countries situation. Most of the parameters are based on three sources: Davis and Healthcote (2004), Iacoviello and Neri (2010) and Dorofeenko, Lee and Salyer (2014). Some of the other key parameters that need further explanation are described below. In this section, we do not address some of the housing and business cycles: we do discuss in this paper version, the steady-state, and some of the second moment properties of the model to the data. We, however, focus our results on the dynamics of the model by analyzing impulse response functions and variance decompositions.

4.3.1 Calibration Parameters

We use linear approximation approach to calibrate our model. As mentioned above, we employ our parameters based on the U.S. and various European nations (average values) dataset. We do not claim that the parameters that we employ in this section reflect the true nature of the European economies that we have in mind. For example, the bankruptcy rates across different nations vary as each economy has a different set of bankruptcy laws and rules. Nevertheless, during our calibration exercise, we have checked the robustness of several of the parameters that we thought would lead to an unstable equilibrium case. The parameters that we have finally decided to use do not change much of the empirical results that we are to report in the next section. The crucial parameter that we use to distinguish three different European economies of Germany, Italy, and Spain, we assign the LTV ratio, \(m\), \(\{0.80; 0.85; 0.90\}\)\(^{18}\) respectively.

\(^{18}\)We also include the LTV of 0.1 \((m = 0.1)\) to reflect almost no LTV constraint.
<table>
<thead>
<tr>
<th>Notation</th>
<th>Value</th>
<th>Description</th>
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<td>Bankruptcy rate</td>
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<td>b</td>
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<td>impatient HH’s discount rate</td>
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<td>β</td>
<td>0.951</td>
<td>patient household discount factor</td>
</tr>
<tr>
<td>γ</td>
<td>0.825</td>
<td>extra entrepr discount factor</td>
</tr>
<tr>
<td>η</td>
<td>1.017</td>
<td>population growth rate</td>
</tr>
<tr>
<td>Π₀</td>
<td>0.02</td>
<td>inflation rate</td>
</tr>
<tr>
<td>σ₀</td>
<td>0.231</td>
<td>st.dev.of entrepreneurial ω</td>
</tr>
<tr>
<td>φ</td>
<td>0.106</td>
<td>land share in housing production</td>
</tr>
<tr>
<td>χ</td>
<td>0.001</td>
<td>persistency of σ</td>
</tr>
<tr>
<td>ω₀</td>
<td>0.648</td>
<td>steady-state.of ω</td>
</tr>
<tr>
<td>a_g</td>
<td>0.01</td>
<td>government consump.share of GDP</td>
</tr>
<tr>
<td>δ_k</td>
<td>0.0557</td>
<td>capital depreciation rate</td>
</tr>
<tr>
<td>δ_s</td>
<td>0.0157</td>
<td>res. structure depreciation rate</td>
</tr>
<tr>
<td>δ_h</td>
<td>0.014</td>
<td>housing stock depreciation rate</td>
</tr>
<tr>
<td>θ_b</td>
<td>0.106</td>
<td>construction capital share</td>
</tr>
<tr>
<td>θ_m</td>
<td>0.33</td>
<td>manufacturing capital share</td>
</tr>
<tr>
<td>θ_s</td>
<td>0.248</td>
<td>services capital share</td>
</tr>
<tr>
<td>κ_h</td>
<td>3</td>
<td>housing investment adjustment cost</td>
</tr>
<tr>
<td>κ_k</td>
<td>3</td>
<td>capital investment adjustment cost</td>
</tr>
<tr>
<td>μ_c</td>
<td>0.314</td>
<td>cons.elasticity in utility function</td>
</tr>
<tr>
<td>μ_h</td>
<td>0.0444</td>
<td>housing easitic in utility function</td>
</tr>
<tr>
<td>μ_ℓ</td>
<td>0.642</td>
<td>leisure easitic in utility function</td>
</tr>
<tr>
<td>μ_h</td>
<td>0.25</td>
<td>monitoring cost</td>
</tr>
<tr>
<td>θ_{gdp}</td>
<td>0.012</td>
<td>TR persistency of GDP</td>
</tr>
<tr>
<td>θ_π</td>
<td>1.672</td>
<td>TR persistency of inflation</td>
</tr>
<tr>
<td>θ_R</td>
<td>0.792</td>
<td>TR persistency of interest rate</td>
</tr>
<tr>
<td>θ_{Δgdp}</td>
<td>0.184</td>
<td>TR persistency of GDP change</td>
</tr>
<tr>
<td>θ_h</td>
<td>0.96</td>
<td>persistency of housing demand shock</td>
</tr>
<tr>
<td>θ_m</td>
<td>0.9</td>
<td>persistency of money</td>
</tr>
<tr>
<td>g_{z,b}</td>
<td>0.997</td>
<td>construction productivity growth rate</td>
</tr>
<tr>
<td>g_{z,m}</td>
<td>1.028</td>
<td>manufacturing productivity growth rate</td>
</tr>
<tr>
<td>g_{z,s}</td>
<td>1.016</td>
<td>services productivity growth rate</td>
</tr>
</tbody>
</table>
the data. We use directly the parameter values chosen by the previous authors; readers are directed to their paper for an explanation of their calibration methodology. Parameter values for preferences, depreciation rates, population growth and land’s share are presented in Table 4.1. In addition, the parameters for the intermediate production technologies are presented in Table 4.2.

As in Davis and Heathcote (2005), the exogenous shocks to productivity in the three sectors are assumed to follow an autoregressive process as given in eq. (??). The parameters for the vector autoregression are the same as used in Davis and Heathcote (2005) (see their Table 4, p. 766 for details). In particular, we use the following values (recall that the rows of the $B$ matrix correspond to the building, manufacturing, and services sectors, respectively):

$$B = \begin{pmatrix} 0.707 & 0.010 & -0.093 \\ -0.006 & 0.871 & -0.150 \\ 0.003 & 0.028 & 0.919 \end{pmatrix}$$

Note this implies that productivity shocks have modest dynamic effects across sectors. The contemporaneous correlations of the innovations to the shock are given by the correlation matrix:

$$\Sigma = \begin{pmatrix} \text{Corr}(\varepsilon_b, \varepsilon_b) & \text{Corr}(\varepsilon_b, \varepsilon_m) & \text{Corr}(\varepsilon_b, \varepsilon_s) \\ \text{Corr}(\varepsilon_m, \varepsilon_m) & \text{Corr}(\varepsilon_m, \varepsilon_s) \\ \text{Corr}(\varepsilon_s, \varepsilon_s) \end{pmatrix} = \begin{pmatrix} 1 & 0.089 & 0.306 \\ 0.089 & 1 & 0.578 \\ 0.306 & 0.578 & 1 \end{pmatrix}$$

The standard deviations for the innovations were assumed to be: $(\sigma_{bb}, \sigma_{mm}, \sigma_{ss}) = (0.041, 0.036, 0.018)$.

---

19Davis and Heathcote (2005) determine the input shares into the consumption and residential investment good by analyzing the two sub-tables contained in the “Use” table of the 1992 Benchmark NIPA Input-Output tables. Again, the interested reader is directed to their paper for further clarification.
For the financial sector, we use the same loan and bankruptcy rates as in Carlstrom and Fuerst (1997) in order to calibrate the steady-state value of \( \bar{\omega}_t \), denoted \( \bar{\omega} \), and the steady-state standard deviation of the entrepreneur’s technology shock, \( \sigma_0 \). The average spread between the prime and commercial paper rates is used to define the average risk premium \( rp \) associated with loans to entrepreneurs as defined in Carlstrom and Fuerst (1997); this average spread is 1.87\% (expressed as an annual yield). The steady-state bankruptcy rate \( br \) is given by \( \Phi(\bar{\omega}, \sigma_0) \) and Carlstrom and Fuerst (1997) used the value of 3.9\% (again, expressed as an annual rate). This yields two equations which determine \( (\bar{\omega}, \sigma_0) \):

\[
\begin{align*}
\Phi(\bar{\omega}, \sigma_0) & = 3.90 \\
\frac{\bar{\omega}}{g(\bar{\omega}, \sigma_0)} - 1 & = 1.87
\end{align*}
\]

yielding \( \bar{\omega} \approx 0.65, \sigma_0 \approx 0.23 \).20

The entrepreneurial discount factor \( \gamma \) can be recovered by the condition that the steady-state internal rate of return to the entrepreneur is offset by their additional discount factor:

\[
\gamma \left[ \frac{\bar{\omega} f(\bar{\omega}, \sigma_0)}{1 - \bar{\omega} g(\bar{\omega}, \sigma_0)} \right] = 1
\]

and using the mark-up equation for \( \bar{s} \) in eq. (4.72), the parameter \( \gamma \) then satisfies the relation

\[
\gamma = \frac{g_U}{g_K} \left[ 1 + \frac{\phi(\bar{\omega}, \sigma_0)}{f'(\bar{\omega}, \sigma_0)} \right] \approx 0.832
\]

where, \( g_U \) is the growth rate of marginal utility and \( g_K \) is the growth rate of consumption (identical to the growth rate of capital on a balanced growth path). The autoregressive parameter for the risk shocks, \( \chi \), is set to 0.90 so that the persistence is roughly the same as that of the productivity shocks.

The final two parameters are the adjustment cost parameters \( (\kappa_k, \kappa_h) \). In their analysis of quarterly U.S. business cycle data, Christiano, Eichenbaum and Evans (2005) provide estimates of \( \kappa_k \) for different variants of their model which range over the interval \((0.91, 3.24)\) (their model did not include housing and so there was no estimate for \( \kappa_h \)). Since our empirical analysis involves annual data, we choose a lower value for the adjustment cost parameter and, moreover,

20Note that the risk premium can be derived from the markup share of the realized output and the amount of payment on borrowing: \( \bar{\omega} f_{pt} = (1 + rp)(f_{pt} - nw_t) \). And using the optimal factor payment (project investment), \( f_{pt} \), in equation (4.74), we arrive at the risk premium in equation (4.88).

21It is worth noting that, using financial data, Gilchrist et al. (2008) estimate \( \sigma_0 \) to be equal to 0.36. Moreover, Chugh (2016) using industry level data estimates \( \sigma_0 \) to be exactly 0.23.
we impose the restriction that $\kappa_k = \kappa_h$. We assume that $\kappa_h = \kappa_h = 3$ implying that the (short-run) elasticity of investment and housing with respect to a change in the respective shadow prices is 0.33 (i.e. the inverse of the adjustment cost parameter). Given the estimates in Christiano, Eichenbaum, and Evans (2005), we think that these values are certainly not extreme. We also solve the model with no adjustment costs. As discussed below, the presence of adjustment costs improves the behavior of the model in several dimensions.

**Estimation of Risk Shocks**

In this section, we estimate risk shocks using the U.S. construction firm level data. The main purpose in estimating these shocks is to show that risk shocks defined as the time variation in the cross sectional distribution of firm level productivities are important inputs to a baseline DSGE model. In estimating risk shocks, we use the dataset from the Compustat Industry Specific Quarterly data. For the robustness of our estimation, we estimate for 2 intersecting subsets of firms: i). The firms with S&P GIC sub-industry code 25201030 – Homebuilding (47 firms); ii). The firms with NAICS sub-industry code 23611 (sub-industries 236115-236118) - Residential Building Construction (35 firms).

The procedure we employ in estimating our risk shocks is similar to Chugh (2011), who uses the dataset of Cooper and Haltiwanger’s (2006) U.S. manufacturing dataset. In order to estimate the risk shocks, we first need to estimate the firm-level productivity coefficients via Fama-MacBeth regression as follows: employing the usual Cobb-Douglas production $B_{it} = \alpha c_{it}^{\alpha} l_{it}^{1-\alpha} \exp(\varepsilon_{it})$, where $i$ and $t$ denote firm and time, $B_{it}$ is the Backlogs, i.e., the “dollar value of housing units subject to pending sales contracts” to proxy output, $l_{it}$ is “Land under development”, $m_{it}$ is defined as “Homebuilding inventories Total” - “Land under development” - “Undeveloped inventories owned” and $\varepsilon_{it}$ iid with Normal. Taking the log of the production function, we estimate following regression:

$$\log\left(\frac{B_{it}}{l_{it}}\right) = c + \alpha \log\left(\frac{m_{it}}{l_{it}}\right) + \varepsilon_{it}$$

Given the dataset, the term $\frac{B_{it}}{l_{it}}$ represents the ”profit or productivity” and the term $\frac{m_{it}}{l_{it}}$ denotes the input. The estimates of $\alpha$ for subset (1) and subset (2) are $\alpha_1 = 0.6 \pm 0.05$ and $\alpha_2 = 0.7 \pm 0.04$ respectively with the standard deviations in brackets. With the estimates $\hat{\alpha}$, the

---

22The full description of these NAICS codes are as follows: 23611 Residential Building Construction, 236115 New Single-Family Housing Construction (except Operative Builders), 236116 New Multifamily Housing Construction (except Operative Builders), 236117 New Housing Operative Builders, and 236118 Residential Remodelers.
logarithmic productivity is then defined as the residual:

$$\log(P_t) = \log\left(\frac{B_{it}}{l_{it}}\right) - \hat{\alpha} \log\left(\frac{m_{it}}{l_{it}}\right)$$

where, the aggregate productivity is defined as $P_t \equiv \frac{1}{N} \sum_{i=1}^{N} P_{it}$ and idiosyncratic productivity as $p_{it} \equiv \frac{P_{it}}{P_t}$ (with $\frac{1}{N} \sum_{i=1}^{N} p_{it} = 1$). Consequently, the risk is estimated as cross-sectional standard deviation of $p_{it}$: that is, $\sigma_t = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (p_{it} - 1)^2}$.

Finally, the AR(1) estimates of HP-detrended risk $\sigma_t$, $\log(\sigma_t) = \rho \log(\sigma_{t-1}) + \epsilon_t$ where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$, that we use as our input to our model yield i) for the subset 1, $\rho = 0.28(0.17)$ and $\sigma_\epsilon = 0.23(0.02)$; and ii) for the subset 2, $\rho = 0.26(0.19)$ and $\sigma_\epsilon = 0.24(0.03)$, where the numbers in brackets indicate the standard deviations. Moreover, the corresponding annual value for $\rho^y \approx 0.02$ and $\sigma_\epsilon^y \approx 0.01$.

Figure 4.3: Estimated productivity and risk shocks from 2001 until 2011.

Figure 4.3 shows the estimated productivity and risk shocks from 2001 till 2011. The HP trends for productivity and risk shocks clearly show that the shocks behave opposite: one can think of risk (uncertainty) shocks as "negative" technology shocks in terms of the role the shocks play in our model. These strongly countercyclical construction firm level risk is also a robust finding in micro evidence of Bachmann and Bayer (2010) and Bloom, Floetotto, and Jaimovich (2010).
4.3.2 Dynamics

Loan-to-Value Effect and Various Shocks: Impulse Response Functions

The main questions to be addressed in this section are as follows: i) How do non-standard shocks in relation to the technology shocks effect key housing and macroeconomic variables? ii) How does the collateral constraint on some of the households effect the housing and business cycles?

Figures 4.4 through 4.9 show the impulse response functions (to a 1% innovation in all four shocks) for several macroeconomic and housing variables under one key parameter value\(^{23}\): the LTV ratio, which is set to either 0.1 or 0.85. A few papers have investigated the role of collateral requirements for the transmission of unanticipated shocks and macroeconomic volatility. Campbell and Hercowitz (2004) find that the U.S. mortgage market liberalization of the early 1990s, proxied by an increase in the LTV ratio, played a role in explaining the great moderation. In contrast, Calza, Monacelli and Stracca (2013) show that the transmission of monetary policy shocks to consumption, investment and house prices is dampened by lower LTV ratio. While the results discussed above provide some support for the housing cum credit channel model, the role of the lending channel with collateral constraint is not easily seen because of the presence of the other impulse shocks (i.e., the sectoral productivity shocks).

We first turn to the behavior of three key macroeconomic variables, namely GDP, household consumption (denoted PCE), and total capital when the LTV is set to 10% (\(m=0.1\)) and 85% (\(m=0.85\)) as seen in Figures 4.4 and 4.5. The response to a technology shock to the construction sector has the predicted effect that GDP increases. Consumption also increases, while capital stock responds much bigger. This consumption/savings decision reflects agents response to the expected high productivity (due to the persistence of the shock) in the construction sector. Turning to a risk shock which affects housing production results, we see a modest fall in GDP and in capital stock. Recall, as discussed in the partial equilibrium analysis of the credit channel model, an increase in productivity risk results in a leftward shift in the supply of housing; since residential investment (and hence, the capital stock) is the primary input into housing, it too falls in response to the increased risk. Moreover, changes in uncertainty increases the bankruptcy rate and hence an increase in the agency costs, causing the rate of return on investment to decrease: consequently, resulting in a reduction in investment. Consumption reacts negatively to \(^{23}\)We also have analysed two other parameter dimensions: the monitoring cost (reflecting the agency cost) and capital adjustment cost (reflecting the amplitude of business cycles). The results are not shown as we focus on the effects of the LTV parameter.\(^{23}\)
Figure 4.4: Impulse response of GDP, Total Capital and Consumption (PCE) to a 1% increase in Uncertainty Shocks, Preference Shocks, Sector (Construction) Technology Shocks, and Monetary Shocks: LTV = 0.1 (m=0.1).

Note: The vertical axis is measured as percentage deviation from steady-state values.

A risk shock due to an increase in “pre-cautionary savings” as households face the persistence of the shock. Consumption responds positively to monetary shocks, which is consistent with models that have an investment specific technology shock (e.g. Greenwood, Hercowitz, and Krusell (2000)). An increase in consumption due to monetary shocks also translates into an increase in both GDP and capital stocks. The monetary shocks play a large role in the aforementioned variables: the magnitude of the monetary shock is as big as the technology shocks if not bigger. An increase in preference shocks for housing reflects in a decrease in consumption, and consequently, leading to a decrease in both GDP and capital stocks. Figure 4.5 shows the effects of the aforementioned shocks and three macroeconomic variables for the LTV ratio of 85%. The results are similar both in magnitude and qualitatively to the case where the LTV ratio is set to 10%. The similar results between 10% and 85% of LTV ratios indicate that the collateral constraint equation is binding and the LTV ratio plays almost no role for the key macroeconomic variables mentioned above.
Figure 4.5: Impulse response of GDP, Total Capital and Consumption (PCE) to a 1% increase in Uncertainty Shocks, Preference Shocks, Sector (Construction) Technology Shocks, and Monetary Shocks: LTV = 0.85 (m=0.85).

Note: The vertical axis is measured as percentage deviation from steady-state values.

Figures 4.6 and 4.7 report the impulse response functions of the housing markup, the risk premium on loans to the housing producers and the bankruptcy rate. As mentioned in previous paragraph, as the role of LTV is negligible, we only discuss the scenarios for the 10% LTV. A positive technology shock to the construction sector increases the demand for housing and, ceteris paribus, will result in an increase in the price of housing. This will result in greater lending to the housing producers which will result in a greater bankruptcy rate and risk premium; both of these effects imply that the housing markup will increase. Note the counterfactual implication that both the bankruptcy rate and the risk premium on loans will be procyclical; this was also the case in the original Carlstrom and Fuerst (1997) model and for exactly the same reason. In contrast, a risk shock produces countercyclical behavior in these three variables. Hence, this argues for inclusion of risk shocks as an important impulse mechanism in the economy. With the preference shocks, both the housing markup and risk premium react positively as expected: as
Figure 4.6: Impulse of Housing Markup, Risk Premium on Loans, and Bankruptcy Rate to a 1% increase in Uncertainty Shocks, Preference Shocks, Sector (Construction) Technology Shocks, and Monetary Shocks: LTV = 0.1 (m=0.1).

Note: The vertical axis is measured as percentage deviation from steady-state values.

the demand increase, there is a greater incentive for the housing developers to a higher markup, which then creates an upper pressure on the risk premium. The monetary shocks effect on the housing markup is something that we cannot logically explain.

Finally, we report in Figures 4.8 and 4.9, the impulse response functions of the prices of land, housing and the amount of borrowing to the four shocks. A technology shock to the construction sector results in lower cost of housing inputs due to the increased output in residential investment so that the price of housing falls. However, the price of land, i.e. the fixed factor, increases. For an uncertainty (risk), preference and monetary shocks, the resulting fall in the supply of housing causes the demand for the fixed factor (land) to fall and the price of the final good (housing) to increase. In regards to the borrowing, we clearly see the role of monetary shock: 1% changes in interest rate causes 0.2% decrease in the amount of borrowing. The monetary shock has the biggest effect of all the shocks that are presented.
In ending this section, a word of caution is needed in interpreting the quantitative magnitudes seen in the impulse response functions. In particular, note that the response of housing prices to a preference shock increase is greater than the response due to, say, a risk shock or monetary shock. One might deduce that the housing sector and risks and monetary shocks play a minor role in the movement of housing prices. As the results from the full model (i.e. when the all technology, monetary and risk shocks are present) imply, such a conclusion would be incorrect.

**What drives housing and business cycles? Variance Decompositions**

This section briefly describes the role of various shocks on some of the key macro and housing variables. The main message from Table 4.3 is that the monetary and uncertainty shocks play a major role in accounting for the movements in some of the aggregate as well as housing variables. In other words, there is a large of policy makers in dealing with the volatilities of
these aforementioned variables. On the other hand, the preference shocks play almost no role in any of the macro or housing variables.

Once again, Table 4.3 presents three different scenarios that are based on LTV ratio: low (80), middle (85) and high (90) borrowing constraints. Unlike some of the recent literature that emphasize the important role of the level of LTV on housing market, our results indicate otherwise: almost no differences between different levels of LTV on the variables that we analyze. On the role of specific shocks, Table 4.3 shows that the effects of monetary shocks are huge on most of the macro variables and in particular on the housing investment and the amount of borrowing the households undertake: over 75% and almost 50% of the variation in housing investment and borrowing can be explained by the monetary shocks. On the contrary to monetary shocks, housing demand shocks have a trivial impact on all the variables that we analyze: at most 6% of the variation in housing price can be explained by the preference shocks. Lastly, our endogenous
Figure 4.9: Impulse response of Land Price, Housing Price and Borrowing amount to a 1% increase in Uncertainty Shocks, Preference Shocks, Sector (Construction) Technology Shocks, and Monetary Shocks: LTV = 0.85 (m=0.85).

Note: The vertical axis is measured as percentage deviation from steady-state values.

debt financial accelerator model with risk shocks lends a strong support for the important role of risk shocks: over 85% of the variation in housing price is due to risk shocks.

4.4 Some Final Remarks

Our primary findings fall into two broad categories. First, risk and monetary shocks to the housing producing sector imply a quantitatively large role for uncertainty and monetary policy over the housing and business cycles. Second, there is a great role for the government policies: the effects of both monetary and risk shocks clearly show that having a stable economy can indeed reduce the volatilities of various housing and macroeconomic variables.

For future research, modelling uncertainty due to time variation in the types of entrepreneurs would be fruitful. One possibility would be an economy with a low risk agent whose productivity
Table 4.3: Variance Decomposition of Forecast Error.

<table>
<thead>
<tr>
<th>Shocks</th>
<th>$u_b$</th>
<th>$u_\sigma$</th>
<th>$u_h$</th>
<th>$u_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.87</td>
<td>0.016</td>
<td>0.027</td>
<td>0.086</td>
</tr>
<tr>
<td>PCE</td>
<td>0.46</td>
<td>0.069</td>
<td>0.03</td>
<td>0.44</td>
</tr>
<tr>
<td>Capital Stock</td>
<td>0.54</td>
<td>0.007</td>
<td>0.01</td>
<td>0.44</td>
</tr>
<tr>
<td>House Stock Patient</td>
<td>0.55</td>
<td>0.022</td>
<td>0.044</td>
<td>0.38</td>
</tr>
<tr>
<td>House Stock Impatient</td>
<td>0.50</td>
<td>0.0039</td>
<td>0.029</td>
<td>0.465</td>
</tr>
<tr>
<td>Labor Hour (total)</td>
<td>0.49</td>
<td>0.062</td>
<td>0.049</td>
<td>0.40</td>
</tr>
<tr>
<td>Borrowing</td>
<td>0.495</td>
<td>0.0016</td>
<td>0.023</td>
<td>0.48</td>
</tr>
<tr>
<td>House Price</td>
<td>0.026</td>
<td>0.854</td>
<td>0.065</td>
<td>0.05</td>
</tr>
<tr>
<td>House Investment</td>
<td>0.148</td>
<td>0.085</td>
<td>0.037</td>
<td>0.735</td>
</tr>
</tbody>
</table>

Shocks exhibit low variance and a high risk agent with a high variance of productivity shocks. Because of restrictions on the types of financial contracts that can be offered, the equilibrium is a pooling equilibrium so that the same type of financial contract is offered to both types of agents. Hence the aggregate distribution for technology shocks hitting the entrepreneurial sector is a mixture of the underlying distributions for each type of agent. Our conjecture is that this form of uncertainty has important quantitative predictions and, hence, could be an important impulse mechanism in the credit channel literature that, heretofore, has been overlooked. It also anecdotally corresponds with explanations for the cause of the current credit crisis: a substantial fraction of mortgage borrowers had higher risk characteristics than originally thought.

Moreover, our current model is silent about the optimal loan contract between the impatient households and financial intermediaries. Developing an endogenous household loan model would further shed light on the latest housing and financial boom and bust cycles. A quantitative assessment of the relative importance of the role of monetary policy, as well as the analysis of the optimal conduct of monetary policy, is also left to future research.

Nevertheless, from our analysis, there is a clear and important role for the policy makers to smooth housing price and/or housing investment. The fact that both monetary and uncertainty shocks play a prominent role in explaining the housing and macro business cycles, the monetary policymakers have two instruments on hand to calm markets and provide market confidence. However, one should be cautious in interpreting our empirical results as evidence for policymakers to be directly involved in solving financial and housing problems.
4.5 Appendix

4.5.1 Recent Developments in European Housing Markets: Some Facts

In this section, we briefly discuss some of the recent housing and macroeconomics development for the aforementioned European countries. We start with the supply side by discussing the residential investment, and then focus on the demand side factors: i) household debt for housing loan, ii) borrowing factors; interest rate and loan-to-value.

Residential Investments

Figure 4.10: Annual residential investment from 1997 until 2011.

Figure 4.10 shows that residential investment moves in tandem to house prices to a various degree across countries. Starting with nations that face fairly elastic housing supply, between 1997 and 2007, Spain, Ireland, and Greece’s residential investments approximately increased 120, 80 and 70 % respectively. For Italy and the average EU (15), increases in residential investment have been more modest, despite large house price increases, suggesting that supply is fairly inelastic in these countries. for Germany, residential investment has been stagnating or falling,
but as in the housing price movement, the residential investment has been slightly increasing as of 2009.

**Household Debt**

Figure 4.11: Household Debt: Long Term Loan for House Purchase.

[Graph showing household debt for various European countries from 1997 to 2011. The graph displays clear correlations between house prices, residential investment, and household debt. Germany is the only nation that shows a downturn in household debt among the others. Greece is the most indebted nation, followed by Ireland. An interesting aspect is that Ireland is the only nation that shows a downturn in household debt.

Source: OECD.

Recent product innovations including low and flexible mortgage rate products, which are essentially aimed at restoring housing affordability in the face of rising prices, are well documented for the European countries (e.g. ECB, 2009).
are increasing whereby Spain and Greece are leveling off. High levels of household debt clearly open up the vulnerability of households welfare to changes and shocks to mortgage payments, personal disposable income, and especially to house prices.

Interest rates

Figure 4.12: Lending Rates (> 5 years) for House Purchases.

Figure 4.12 shows European mortgage interest rates (both real and nominal) have come down considerably from early 1990 till mid 2005: the average nominal rate for the European nations decreased from 12 to 4.5%. As can be seen in Figure 4.12, except for Germany, the sample country’s rates have been increasing from 2005 till their peak at late 2008. Subsequently, due to various economic downturns in these countries, the rates have been again falling below the 2005 rates. Table 4.4 shows the loan types for various European and North America nations.

Loan-to-Value

\(^{25}\)If one looks at the average household debt, of which mortgages are the main constituent, represented about one year of household disposable income in 1995. By 2000, debt had risen to about 120% and in 2007 it was close to 170% for the Euro zone countries (OECD, 2010).
Table 4.4: Interest rate type, loan term and lenders across countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>Predominant Interest Rate</th>
<th>Loan Term (years)</th>
<th>Main Lenders</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>variable</td>
<td>25</td>
<td>Banks and nonbank specialist &quot;mortgage originators&quot;</td>
</tr>
<tr>
<td>Austria</td>
<td>fixed</td>
<td>25-30</td>
<td>Banks and Bausparkassen (mainly savings banks)</td>
</tr>
<tr>
<td>Belgium</td>
<td>fixed</td>
<td>20</td>
<td>Banks</td>
</tr>
<tr>
<td>Canada</td>
<td>mixed</td>
<td>25-35</td>
<td>Banks and specialized nondepository mortgage brokers</td>
</tr>
<tr>
<td>Denmark</td>
<td>mixed</td>
<td>30</td>
<td>Mortgage and retails banks</td>
</tr>
<tr>
<td>France</td>
<td>fixed</td>
<td>15-20</td>
<td>Mortgage and retails banks</td>
</tr>
<tr>
<td>Germany</td>
<td>fixed</td>
<td>20-30</td>
<td>Banks and Bausparkassen (mainly savings banks)</td>
</tr>
<tr>
<td>Ireland</td>
<td>variable</td>
<td>21-35</td>
<td>Banks, building societies and mortgage brokers</td>
</tr>
<tr>
<td>Italy</td>
<td>mixed</td>
<td>20</td>
<td>Banks</td>
</tr>
<tr>
<td>Japan</td>
<td>mixed</td>
<td>20-30</td>
<td>Banks and specialized mortgage institutions</td>
</tr>
<tr>
<td>Netherlands</td>
<td>fixed</td>
<td>30</td>
<td>Banks and mortgage banks and brokers</td>
</tr>
<tr>
<td>Portugal</td>
<td>variable</td>
<td>25-35</td>
<td>Banks</td>
</tr>
<tr>
<td>Spain</td>
<td>variable</td>
<td>30</td>
<td>Banks (commercial and savings)</td>
</tr>
<tr>
<td>Sweden</td>
<td>variable</td>
<td>30-45</td>
<td>Bank and mortgage institutions</td>
</tr>
<tr>
<td>U.K.</td>
<td>variable</td>
<td>25</td>
<td>Banks, building societies and mortgage brokers</td>
</tr>
<tr>
<td>U.S.</td>
<td>fixed</td>
<td>30</td>
<td>Banks and mortgage brokers</td>
</tr>
</tbody>
</table>

Source: The "Housing and Finance in the Euro Area, March 2009" report, Table 3.2, European Central Bank.

IMF (2011) reports that there has been a sharp increase in the loan-to-value ratios: during the latest housing upturn, limits on the amount of mortgages have become less stringent than in the past in many markets. Maximum loan-to-value ratios have generally exceeded 80% in OECD countries. According to Table 3.2 in ECB (2009), Table 4.5 shows the maximum Loan to Value ratios on new loans for Germany, Italy, Ireland, Portugal and Spain are 80, 80, 100+, 90, and 100 respectively.

4.5.2 Complete set of equations of the model

The complete system of 62 equilibrium equations of the model is summarized below.

**Household Sectors: 19 Equations (patient and impatient households)**

**Patient households (8 equations)** The household’s modified budget constraint (4.17)

\[
c_{t+1} + i_{k,t+1} + p_{h,t+1} i_{h,t+1} = \frac{K_t (r_t - \tau_k (r_t - \delta_k)) + N_t w_t (1 - \tau_n) + p_{l,t} \lambda_{l,t} + b_t - R_{b,t-1} b_{t-1}}{1 + \Pi_{t+1}} + a_M m_{a,t+1}
\]

Euler equations for the patient household (4.6) - (4.11), (4.24) - (4.29)
Table 4.5: Loan to Value Ratios for selected countries.

<table>
<thead>
<tr>
<th>Country</th>
<th>LTV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>90-100</td>
</tr>
<tr>
<td>Austria</td>
<td>80</td>
</tr>
<tr>
<td>Belgium</td>
<td>100</td>
</tr>
<tr>
<td>Canada</td>
<td>80</td>
</tr>
<tr>
<td>Denmark</td>
<td>80</td>
</tr>
<tr>
<td>France</td>
<td>100</td>
</tr>
<tr>
<td>Germany</td>
<td>80</td>
</tr>
<tr>
<td>Ireland</td>
<td>100+</td>
</tr>
<tr>
<td>Italy</td>
<td>80</td>
</tr>
<tr>
<td>Japan</td>
<td>70-80</td>
</tr>
<tr>
<td>Netherlands</td>
<td>125</td>
</tr>
<tr>
<td>Portugal</td>
<td>90</td>
</tr>
<tr>
<td>Spain</td>
<td>100</td>
</tr>
<tr>
<td>Sweden</td>
<td>80-95</td>
</tr>
<tr>
<td>UK</td>
<td>110</td>
</tr>
<tr>
<td>US</td>
<td>110+</td>
</tr>
</tbody>
</table>

Note: The column LTV refers to the maximum LTV on New Loans. Source: The "Housing and Finance in the Euro Area, March 2009" report, Table 3.2, European Central Bank.

\[
\begin{align*}
\frac{w_t}{R_t} &= \frac{U_{3t}}{U_{1t}} \\
q_{k,t} &= \beta E_t \left( (1 - \delta_k) q_{k,t+1} + \left(1 - \tau_k \right) \frac{r_{t+1} + \tau_k \delta_k}{R_{t+1}} \frac{U_{1,t+1}}{U_{1,t}} \right) \\
q_{h,t} &= \beta E_t \left( (1 - \delta_h) q_{h,t+1} \frac{U_{1,t+1}}{U_{1,t}} + \frac{U_{2,t+1}}{U_{1,t}} \right) \\
1 &= q_{k,t} F_{k,1} (i_{k,t}, i_{k,t-1}) + \beta E_t \left( q_{k,t+1} F_{k,2} (i_{k,t+1}, i_{k,t}) \frac{U_{1,t+1}}{U_{1,t}} \right) \\
p_{h,t} &= q_{h,t} F_{h,1} (i_{h,t}, i_{h,t-1}) + \beta E_t \left( q_{h,t+1} F_{h,2} (i_{h,t+1}, i_{h,t}) \frac{U_{1,t+1}}{U_{1,t}} \right) \\
1 &= \beta R_t E_t \left( \frac{U_{1,t+1}}{(1 + \Pi_{t+1}) U_{1,t}} \right) \\
1 &= \beta R_{h,t} E_t \left( \frac{R_t}{R_{t+1}} \frac{U_{1,t+1}}{U_{1,t}} \right)
\end{align*}
\]

Impatient Households (7 equations) The household’s modified budget constraint (4.20),

\[
c'_{t+1} + p_{h,t+1} i'_{h,t+1} = \frac{N' w' (1 - \tau_n) + b'_t - R_{h,t-1} b'_{t-1} + M'_{s,t+1}}{1 + \Pi_{t+1}} + \frac{M'_{c,t+1}}{R_{c,t+1}}
\]
collateral borrowing constraint (4.21),

\[ b_t' \leq mE_t \left( \frac{p_{h,t+1} h_t'}{R_t} \right) \]

Euler equations:

\[ \frac{w'}{R'} = \frac{U_{3t}}{U_{1t}} \]

\[ 1 = \beta'R_t'E_t \left( \frac{U_{1,t+1}}{U_{1,t}} \right) \]

\[ q_{b,t}'R_t' = 1 - \beta'R_{b,t}E_t \left( \frac{R_{t+1}'}{U_{1,t}} \right) \]

\[ q_{h,t}' = \beta'E_t \left( (1 - \delta_h) q_{h,t+1}' + q_{b,t+1}' h_{t+1}' \right) \frac{U_{1,t+1}}{U_{1,t}} + \frac{U_{2,t+1}}{U_{1,t}} \]

\[ p_{h,t} = q_{h,t}' F_{h,1} \left( i_{h,t}, i_{h,t-1} \right) + \beta'E_t \left( q_{h,t+1}' F_{h,2} \left( i_{h,t+1}, i_{h,t} \right) \frac{U_{1,t+1}}{U_{1,t}} \right) \]

Debt market clearing condition (4.22): 1 equation

\[ b_t' + b_t = 0 \]

Capital growth: 3 equations

\[ h_{t+1} = F_h \left( i_{h,t}, i_{h,t-1} \right) + (1 - \delta_h) h_t \]

\[ h_{t+1}' = F_h \left( i_{h,t}', i_{h,t-1}' \right) + (1 - \delta_h) h_t' \]

\[ K_{t+1} = F_k \left( i_{k,t}, i_{k,t-1} \right) + (1 - \delta_k) K_t \]

Entrepreneur equations: 4 Equations

The entrepreneur equations include

\[ 1 = \beta \eta \gamma E_t \left[ (r_{t+1} + 1 - \delta_h) \frac{s_{t+1} f (\tilde{\omega}_{t+1}; \sigma_{\omega,t+1})}{1 - s_{t+1} g (\tilde{\omega}_{t+1}; \sigma_{\omega,t+1})} \right] \]

\[ \bar{s}_t = \frac{1}{1 - \mu \Phi (\omega_t) + \mu f (\omega_t) \frac{\phi (\omega_t)}{f (\omega_t)}} \]

\[ p_{ht} \left( x_{it} y_{dt} \right) = \kappa_t^e \frac{(r_t + 1 - \delta_h)}{1 - s_{t} g (\tilde{\omega}_t; \sigma_{\omega,t}) s_{t}} \]
\[ \eta k_t^{c+1} = k_t^c (r_t + 1 - \delta) \frac{\tilde{s}_t f(\tilde{\omega}_t; \sigma_{\omega,t})}{1 - \tilde{s}_t g(\tilde{\omega}_t; \sigma_{\omega,t})} - c_t^e \]

Production side equations: 29 Equations for \((i = b, m, s; j = c, d)\)

The production side equations are

\[ x_i = \sum_{j=c,d} x_{1,ij} \]

\[ y_j = \prod_{i=b,m,s} x_{1,i,j}^{p_{1,i,j}} \]

\[ p_i x_{1,i,j} = p_{1,i,j} p_j y_j \]

\[ K = \sum_{i=b,m,s} k_i, N = \sum_{i=b,m,s} n_i \]

\[ y_h = x_i^\phi y_d^{1-\phi} \]

\[ p_l x_l = \phi p_h y_h, p_d y_d = (1-\phi)p_h y_h \]

\[ x_i = k_i^{t,1} \left( e^{\epsilon_1} n_i^a n_i^{1-a} \right)^{1-\theta_i} \]

\[ n_i w = a p_l x_i (1 - \theta_i) \]

\[ n_i' w' = (1 - a) p_l x_i (1 - \theta_i) \]

\[ k_i r + n_i' w' + n_i w = p_l x_i \]

\[ N' = \sum_{i=b,m,s} n_i' \]

Resource constraints: 2 equations

\[ G_t + c_t + c_t' + \epsilon_t + i_{k,t} = y_{c,t} \] (4.89)

\[ i_{h,t} + i_{h,t}' = y_{h,t} (1 - \mu \Phi(\varpi_t)) \] (4.90)
Goverment constraints: 2 equations

The real government spending $G_t$ satisfies budget constraint equation and is assumed to be proportional to the real GDP:

$$G_t + \mu \Phi (\omega_t) p_{h,t} y_{h,t} + \frac{M_{st} + M'_{st}}{P_{c,t}} = K_t (r_t - \delta_k) \tau_k + (N_t w_t + N'_t w'_t) \tau_n + a_G^{G_t}$$

(4.91)

where $q_t = \frac{U_{t+1}}{U_t}$ is the rental rate for housing (see Davis and Heathcote, 2005 for details).

The equations for external Shocks: 6 equations

The housing demand shock

$$\ln \mu_{h,t+1} = (1 - \rho_h) \ln \mu_{h}^{(0)} + \rho_h \ln \mu_{h,t} + \varepsilon_{h,t+1}$$

The money shock: Taylor Rule for interest rate $R_t$:

$$\ln R_{t+1} = \ln R_0 + \rho_\pi \ln \left( \frac{1 + \Pi_t}{1 + \Pi_0} \right) + \rho_{gdp} \ln \left( \frac{GDP_t}{g_k GDP_{t-1}} \right) + \varepsilon_{R,t+1}$$

The intermediate goods production shocks ($i = b, m, s$):

$$z_i = \sum_{j=b,m,s} z_j B_{i,j} + \varepsilon_{i,t+1}$$

The volatility of entrepreneur’s production technology coefficient $\omega_t$:

$$\sigma_{\omega,t+1} = \sigma_0^{1-\chi} \sigma_{\omega,t}^{\chi} e^{\varepsilon_{st+1}}$$

A competitive equilibrium is defined by the decision rules for (aggregate capital, entrepreneurs capital, households (patient and impatient) labor, entrepreneur’s labor, entrepreneur’s net worth, investment, the cutoff productivity level, household (patient and impatient) consumption, and entrepreneur’s consumption) given by the vector: $\{ k_{t+1}, k^e_{t+1}, H_t, H^e_t, X_t, \tilde{\omega}_t, \varepsilon_t, c_t, c^e_t, i'_{h,t}, i''_t, N'_t, w'_t, h'_t, n'_t, m'_{st} \}$, where these decision rules are stationary functions of $\{ K_t, Z_t, h_t, z_i, t \in \{ b, m, s \} \sigma_{\pi,t}, \varepsilon_{M,t+1}, \varepsilon_{h,t+1} \}$, all markets clear and all the firms, households and entrepreneurs solve their respective maxi-
mization problems, along with sets of equations representing the laws of motion for the sector specific shocks \( z_{t,i} \in \{b,m,s\} \), the monetary shock, the preference shock and the uncertainty shock. In total, there are 62 variables:

\[
c, i_k, p_h, i_h, K, r, N, w, p_l, b, R_b, m_s, \Pi, c', i'_h, N', w', h', R, h, q_k, q_h, R', q'_b, q'_h, s, \omega, \sigma, y_h, Z, c^c,
\]

\[
x_b, x_m, x_s, x_{1,i,j}(6), y_c, y_d, p_b, p_m, p_s, p_d, k_b, k_m, k_s, n_b, n_m, n_s, n'_b, n'_m, n'_s, z_b, z_m, z_s, G, \mu_h, m^G
\]

for 62 equations to be solved.
Chapter 5

Conclusion

This dissertation addresses the question, “how important are uncertainty shocks for the housing market?” My findings suggest a quantitatively substantial role of uncertainty for both the housing market and the business cycle, especially in the context of financial frictions. Using a model with asymmetric information, collateral constraints, and a Taylor Rule, uncertainty and monetary shocks are found to play a quantitatively large role for the housing producing sector over the housing- and business cycle. This result suggests a clear role for government policies: reducing the variation in housing and macroeconomic variables by smoothing housing prices and housing investment achieves a more stable economy.

For future research, combining this model with hump-shape uncertainty shocks could be fruitful. My conjecture is that this approach to modeling uncertainty can capture not only the bust but also the preceding run-up in housing- and real variables that was observed in the U.S. and some European countries in the past decade. Therefore, uncertainty’s influence on real business cycle variables might further increase, and remedy the countercyclical consumption implied by jumping risk shocks. This conjecture is based on the results from chapter three, which examines the impact of hump-shaped uncertainty shocks on the economy. Unlike chapter four, there are no collateral constraints but financial frictions which are worsened by hump-shaped uncertainty shocks. These shocks result in a different propagation mechanism compared to previous work combining uncertainty and financial accelerator models, as they induce precautionary savings; output then displays a hump-shaped with short expansions that are followed by longer and persistent contractions. Besides, hump-shaped shocks remedy the counter-factual implication of the DLS framework, that consumption is countercyclical. Moreover, hump-shape shocks that correspond to empirical evidence quantitatively matter for the business cycle, accounting for 5% of the variation in output and 11% and 18% of the variation in consumption and investment,
respectively.

Jurado et al.’s (2015) and Ludvigson et al.’s (2016) empirical evidence, underlying the modeling approach of chapter three, stresses the importance of removing the forecastable component of the uncertainty proxy. Chapter two lends support to this point by presenting empirical evidence regarding the difference between ex post and ex ante fluctuations of two uncertainty proxies - profit growth and stock returns. Moreover, chapter two shows that fluctuations differ across sectors and depend on whether financial or real variables are used to calculate uncertainty. It is important to calibrate theoretical models accordingly, so as not to overstate the role of uncertainty.

Overall, reconciling the disparate findings regarding the importance of uncertainty in both financial accelerator and other types of models remains an important objective in the literature. The results due to Lee, Salyer, and Strobel (2016) suggest a consensus is tightly linked to the modeling approach and calibration, and thus exogenous. Therefore, an interesting extension could be to endogenize uncertainty in financial accelerator models, such that uncertainty is endogenously “accelerated”. This acceleration, in turn, might result from the interaction of heterogeneous entrepreneurs and thus circumvent an exogenous specification.
Chapter 6

Bibliography


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[40] Ludvigson, S., Ma, S., & Ng, S. (2016). “Uncertainty and business cycles: exogenous impulse or endogenous response?”, *mimeo.*


