Macroprudential Regulation in Dynamic Stochastic General Equilibrium Models

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Summary

The recent financial crisis in 2008/2009 has highlighted the need for macroprudential policy to ensure the functioning of the financial system. One of the key aspects of macroprudential regulation is reducing procyclicality in the financial intermediation process to dampen fluctuations in the supply of credit. This thesis studies the potential benefits of countercyclical macroprudential policies for financial and macroeconomic stabilization relying on a New Keynesian dynamic stochastic general equilibrium (DSGE) model.

Chapter 2 examines the role of borrowers' indebtedness for the effectiveness of rules on the loan-to-value (LTV) ratio of borrowers. With the rule, the maximum LTV ratio adjusts countercyclically to indicators of financial imbalance. My findings suggest that during housing market booms countercyclical rules that affect borrowers with tight credit constraints induce borrowers to deleverage. Consequently, the regulation curbs macroeconomic activity and produces deflation. In contrast, the rule implemented in mortgage markets with highly indebted borrowers prevents extreme leverage and entails macroeconomic stability. These results imply that countercyclical regulation on highly leveraged borrowers is more efficient.

In chapter 3, I conduct a comparative analysis between end-borrower regulation given by a countercyclical LTV-rule on mortgage borrowers and a lender related instrument in form of a countercyclical rule on the bank capital ratio. Simulating the dynamics of a financial crisis, my findings illustrate that a LTV-rule mitigates the drop of borrowing and enhances the banks' capability to provide loans. Thus, the LTV-rule is more effective than a rule on bank capital. When the transmission from the banking sector to the real economy proceeds over the household mortgage market, I find no evidence for macroeconomic stabilization with either rule.

Chapter 4, joint work with Jürgen Jerger, looks at the interaction between the zero lower bound and rule-based, and therefore flexible macroprudential regulation. The simulations of a demand driven recession show that a countercyclical LTV-rule contributes to financial stability irrespective of the zero lower bound and also to macroeconomic stabilization if the bound binds.

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1 Introduction

1.1 Background

The global financial crisis in 2008/2009 has demonstrated that traditional regulation is insufficient to ensure the soundness of the financial system. This recognition pushed forward a more holistic approach to financial regulation known as macroprudential regulation. Macroprudential policy has the scope to counter growing risks to the stability of the financial system and complements existing microprudential policy. If the financial sector becomes overexposed to the same risk, so that the risk is systemic, the functioning of the system to provide key financial services to the economy is disrupted.[1] Thus, resulting financial imbalance can cause a credit crunch that in turn depresses overall economic activity. Therefore, macroprudential

[1]More precisely, systemic risk arises basically from credit risk, emanating from borrowers' default, or, market risk, that stems from the decline of collateral value, or, liquidity risk that arise from depressed refinancing conditions.

policy aims to mitigate the costs to the economy, in particular, costs to tax payers that accrue from a financial crisis.

The Financial Stability Board, established to safeguard a global financial regulation framework, highly promotes macroprudential policy measures to its G-20 member states. Not only because of the lack of empirical experience with this rather new form of regulation, relatively little is known about the specific policy design and the mechanisms through which these tools affect the real economy. Over the past years, a fast growing body of theoretical literature has developed and proposes a wide range of instruments to address the multitude sources of systemic risk. While the implementation of macroprudential policy tools for banks – the countercyclical capital buffer advocated by Basel III being the most prominent example – has already progressed, tools beyond banking are still at an early stage (ESRB, 2016). In particular, the design of tools to reduce systemic risk from boom and bust episodes in real estate markets are at the center of recent policy debates. Against this background, this dissertation explores the efficiency of macroprudential regulation on mortgage borrowers as stabilization policy using dynamic stochastic general equilibrium (DSGE) models.

Financial frictions have shown to be a key driver of business cycle fluctuations (Kiyotaki and Moore, 1997; Bernanke and Gertler, 1989) making financial stability a relevant objective.

For instance, collateral constraints tied to real estate value can propagate small shocks. When collapsed house prices erode borrowers' net worth, borrowers are forced to fire sale their collateral to repay their loan. This in turn feeds back to lower house prices and even more fire sales. Contrary to this, in booming housing markets the dynamic interaction between credit limits and asset prices leads to a build-up of leverage. Especially, countercyclical varying credit limits are a tool of macroprudential policy that aim to prevent the procyclicality of collateral constraints by operating at business cycle frequencies.[2] This dynamic instrument requires debtors to provide more equity in good times, so that the borrowers' balance sheets are better prepared to absorb losses that build during downturns. Because of borrowers' higher buffers to resist busts, the probability of default and overborrowing are reduced by the regulation.[3] As a result countercyclical credit limits alleviate credit cycles. Since time-varying credit limits pose a rather innovative approach, I confine my research to countercyclical macroprudential instruments.

For the implementation of a sound macroprudential framework, it is also important to fully understand the interactions of macroprudential regulation with monetary policy since monetary pol-

[2]Non-time varying maximum caps on borrowers' leverage are another tool to constraint borrowing and thus excessive leverage.

[3]Moreover, tighter credit constraints restrict potential borrowers from credit and hence from purchasing a property, which reduces property price fluctuations.

icy determines the overall conditions that affect the demand and supply of credit. Nevertheless, the current period of historically low nominal interest rates has led only to a small increase of firms' and households' credit demand to the monetary expansion. One reason for monetary policies' limited efficiency on credit demand is seen in the debt-deflation spiral enforced by missing real interest stimulus. Therefore, this thesis also evaluates the repercussions of macroprudential policy inference on the macroeconomy during a time period of zero interest rates.

1.2 Outline of the thesis

Comprising of three papers, this dissertation provides new insights on the effects of rule-based, and therefore flexible macroprudential regulation (FMR). Each paper is given by one chapter that build upon each other. All papers follow the strand of literature surrounding the seminal work of Iacoviello (2005) relying on a New Keynesian DSGE model with financial frictions and costly external finance. Exploring the financial accelerator mechanism, Iacoviello (2005) provides evidence for the propagation of demand shocks in the presence of housing collateral constraints but not supply shocks. As this amplification effect of credit limits constitutes the rational for macroprudential policy interventions, several papers use a modification of Iacoviello's model for a macroprudential policy analysis, e.g., Christensen (2011), Kolasa (2016) and Lambertini et al. (2013). Among them are also Rubio and Carrasco-Gallego who have published several papers analyzing a macroprudential rule on the loan-to-value (LTV) ratio of borrowers in interaction with monetary policy (Rubio and Carrasco-Gallego, 2014, 2015a,b). In their model economy, the LTV ratio defined as the proportion of the debt burden relative to the residential property value governs the amount of loan that circulates between savers and borrowers. The relatively simple model structure is extremely feasible to scrutinize macroprudential policy instruments attached to the LTV constraint, which is why the model builds the foundation of the following chapters. Nonetheless, this thesis leaves Rubio and Carrasco's debate on the optimal policy mix of macroprudential and monetary policy aside and concentrates on the economic impact of countercyclical macroprudential regulation taking monetary policy as given.

Chapter 2, based on Körner (2016a), studies the usefulness of macroprudential LTV rules in dependence on the level of indebtedness of borrowers.[4] The paper substantiates the introduction of macroprudential regulation for this and the following chapters by demonstrating the collateral effect and the insight that the propagation increases with laxer LTV limits. Describing the decision of the macroprudential authority, various

[4] Chapter 2 is slightly more detailed than the paper version currently under review because the chapter outlines essential mechanisms of the model for the subsequent papers. Additional parts are indicated by a horizontal line.

countercyclical rules on the LTV ratio are parameterized to optimally smooth credit cycles since low credit volatility indicates financial stability. More specifically, the paper analyzes the effects of these rules exercised on highly and low leveraged mortgage borrowers for meeting stabilization objectives. Stochastic simulations of housing demand and technology shocks allow to disentangle the channels of the rules on the macroeconomy.

The model used in chapter 2, however, lacks an authority that conducts the financial intermediation between borrowers and savers. Financial intermediaries contributed significantly to the recent financial crisis as they failed to fulfill their basic function of supplying loans due to frictions in the system. One regulation friction are bank capital requirements that are implemented to insure the institution's solvency, but sharpen a crisis due to their procyclicality. Therefore, chapter 3, built on Körner (2016b), resolves this shortcoming of chapter 2 by introducing a monopolistically competitive banking sector á la Gerali et al. (2010).

The endogenous bank capital accumulation of the banking sector allows to conduct a comparative analysis of a countercyclical non-risk adjusted bank capital (BC) requirement ratio, a countercyclical LTV ratio, and the implementation of both regulations. In contrast to the loss function optimization in chapter 2, this chapter employs a welfare maximization procedure to identify the optimal macroprudential feedback rules. The welfare approach has the benefit that it reflects the welfare effect through the introduction of countercyclical tools for each economic agent, separately. By that the chapter evaluates the regulations' capability for enhancing financial resilience. This paper also sheds light on the significance of particular rules for the development of a comprehensive macroprudential policy framework.

Unlike the previous two chapters, chapter 4, that will be published as Jerger and Körner (2017 forthcoming), focuses on one "rare" economic situation, namely when the zero lower bound on nominal interest rates binds. More precisely, the paper compares the effects of FMR during zero interest rates to the situation when negative interest rates are feasible. Associated therewith, the chapter applies the occasional binding algorithm of Guerrieri and Iacoviello (2015) on the smaller model of chapter 2 to regard the zero lower bound (ZLB) on nominal interest rates, which depicts a non-linear relation. In contrast, the models in chapters 2 and 3 are solved using first and second order perturbation methods with purely linear relations. While several studies on policy coordination consider macroprudential regulation a useful complement to monetary policy during fluctuations stemming from the financial sector, e.g., Kannan et al. (2012), Rubio and Carrasco-Gallego (2014) and Bianchi (2011), chapter 4 explores the role of countercyclical LTV regulation in mitigating the consequences of a constrained monetary policy. Comparing the outcomes obtained with and without a binding ZLB delivers insights in additional stabilization benefits of macroprudential regulation.

Chapter 5 summarizes and concludes the main results of this thesis.

2 Countercyclical collateral constraints and borrowers' leverage ratio

2.1 Introduction

Macroprudential real estate regulation is used to mitigate extensive credit growth and to stabilize the financial system. In times of a housing market boom collateral constraints can amplify business cycles through the self-reinforcing effect of increasing real estate prices and credit growth. As an instrument low caps on loan-to-value (LTV) ratios reduce borrowers' credit acquisition and thus mitigate the propagation mechanism (Liu et al., 2013; Walentin, 2014). Countercyclical varying LTV ratios, however, change the borrowing limit according to the economic situation. This paper analyzes the effects of countercyclial macroprudential rules on different fixed LTV levels of borrowers within a calibrated dynamic stochastic general equilibrium (DSGE) framework and evaluates their impact on welfare. Several papers have demonstrated that countercyclical rules implemented on a pre-set level of debt can reduce macroeconomic fluctuations (Rubio and Carrasco-Gallego, 2015a; Lambertini et al., 2013). Therefore, this paper investigates the relationship between the leverage level of borrowers and the efficiency of countercyclical macroprudential regulation. The results show that the stabilization benefits of a countercyclical macroprudential rule depend on the level of indebtedness of the borrower. Only when borrowers are subject to a high credit limit, a rule on the LTV ratio can reveal its countercyclical leverage effect and mitigates credit cycles. These insights play a role for the design of a macroprudential policy framework.

In several European countries national macroprudential policy authorities have introduced fixed credit limits on LTV ratios to avoid excessive mortgage lending.[1] For instance, in Germany the Finanzstabilitätsrat recently submitted a draft bill to the government for the introduction of caps on the loan-to-real-estate value (Ausschuss für Finanzstabilität, 2015). These LTV limits and also the average LTV ratio of mortgage borrowers are quite distinct across the European countries as table 2.1 documents. Maximum LTV ratios regardless of the limit still allow for more borrowing in booming real estate markets through the appreciated collateral value. Therefore, the European Systemic

[1] In the Netherlands, macroprudential authorities have decided to further reduce the limit on the LTV ratio to 101% in 2017.

Risk Board (ESRB) responsible for the macropudential oversight in the Eurozone recommends countercyclical, so called time-variant LTV ratios. With countercyclical rules, the credit limit tightens in an upswing of a cycle and loosens in a downturn to support credit growth and potentially help to avoid a credit crunch (ESRB, 2014).

Collateral constraints are a relevant factor for macroeconomic volatility: in the mortgage market, the LTV ratio – the fraction of nominal debt and underlying real estate collateral value – can amplify shocks that shift the demand for the collateral asset and inflation in the same direction (Liu et al., 2013). Thus, inflation and rising house prices create a wealth effect for borrowers whereby they can borrow more for consumption and investment. This procyclical effect of LTV ratios in response to demand shocks enhances business cycles.[2] Since housing boom-bust cycles are a common precursor to financial crises (Christensen, 2011), regulating the leverage level of mortgage borrowers is important to prevent spill-overs into the macroeconomy and reduce taxpayers' costs of a crisis. Justiniano et al. (2015) find that rather than credit liberalization house price growth driven by changes in demand accounted for the large debt increase in the recent crises. They propose that real house

[2]However, collateral constraints moderate the effect of shocks generated in the real sector (Mendicino, 2012; Liu et al., 2013). This effect is particularly prevailing in models where the collateral asset is an input factor of the production function.

prices and their evolution are the primary source of credit cycles, while this paper identifies optimal countercyclical rules particularly to mitigate the effects of housing demand shocks.

Walentin (2014), Christensen (2011) and Gruss and Sgherri (2009) provide evidence that macroeconomic volatility increases with relaxation of collateral constraints (higher LTV ratios) due to the larger demand effect of borrowers for a given change in house prices. The studies reinforce a low maximum LTV ratio to be a sound instrument for financial resilience. Estimating a DSGE model for Sweden, Walentin (2014) states that with a constant LTV ratio of 0.95 the effect of a housing demand shock as much as doubles on most macroeconomic variables compared to the outcome with a LTV ratio of 0.85. He finds no amplification to a technology shock. Christensen (2011) confirms that lowering the LTV ratio substantially reduces the magnitude of consumption and debt growth while enhancing financial stability. This paper evaluates if the introduction of a countercyclical rule on low LTV ratios adds not only to financial but also to macroeconomic stabilization.

Several papers highlight the effectiveness of countercyclical rules on LTV ratios. Levine and Lima (2015) show that the deployment of macroprudential regulation together with standard monetary policy improves total welfare even if both authorities react to their own policy targets. Evaluating various policy regimes, Lambertini et al. (2013) document maximal total welfare gains for savers and borrowers when monetary policy follows a standard Taylor rule and the countercyclical LTV-rule responds to credit growth. Further, they find an interest rule reacting to credit growth and a constant LTV ratio improves only savers' welfare. Rubio and Carrasco-Gallego (2015a) introduce a countercyclical LTV-rule reacting to output and house prices in a model with financial frictions. If monetary policy ensures solely price stability and the macroprudential authority's objective is financial stability, they identify the rule that responds relatively more to house price deviation than to output deviation to be socially optimal. Angelini et al. (2012) indicate that the optimal intensity of policy intervention depends on the indicator variable and the origin of the shock. They uncover a countercyclical LTV ratio responding to credit growth to be an effective instrument to achieve stabilization in case of financial shocks or more general demand shocks, as in a number of other papers (Mendicino and Punzi, 2014; Gelain et al., 2013). However, Angelini et al. (2012) and Kannan et al. (2012) find little stabilizing effects of macroprudential regulation in response to technology booms.[3]

A main finding in the literature is the amplification effect of fixed collateral constraints that increases with borrower's lever-

[3] Supporting this finding, Rubio and Carrasco-Gallego (2014) document that monetary and macroprudential policy enter into conflict for supply shocks. Monetary policy reacts to deflation by lowering the interest rate, which facilitates credit growth. Macroprudential policy, however, operates contrary by tightening the credit limit to reduce credit growth.

age. In addition, several studies on macroprudential policy detect welfare gains with countercyclical regulation through smoothed credit cycles. This work combines both research strands by evaluating the effects of countercyclical rules implemented on different borrowers' debt levels on macroeconomic stabilization and welfare. The paper also contributes to the policy debate on the optimal calibration of macroprudential rules.

Following Iacoviello (2005) and Rubio and Carrasco-Gallego (2015a), I use a New Keynesian DSGE model with a private mortgage market calibrated to Eurozone data. As in Kiyotaki and Moore (1997) borrowers are constrained by a constant LTV limit, i.e., a cap on the fraction of debt service to the expected liquidation value of the real estate collateral. In the framework, savers lend directly to borrowers who fully exploit their debt limit. The monetary policy is represented by an inflation targeting interest rule independent from macroprudential policy in line with the current policy situation in Europe.[4]

I assume two benchmark scenarios for the policy exercise. Namely, a high household debt scenario where borrowers face a limit on the LTV ratio of 90% that is always binding due to the linearity of the model. In the low household debt scenario, debtors borrow up to 65%, respectively. In the corresponding policy scenarios, the LTV ratios remaining unchanged for the steady

[4]In the Eurosystem national macroprudential authorities, who are equipped with regulatory power, act independently from the supranational monetary policy regime.

state are linked to a countercyclical rule. The linear LTV rule tightens the borrowing capacity when house prices or when credit growth increases as these indicators are recommended by the ESRB. The policy rules are parameterized so that they reduce credit growth in best way.

The policy exercise uncovers that in response to a housing preference shock a countercyclical rule on borrowers with a high LTV ratio reduces macroeconomic fluctuations. Contrary, output falls below the steady state value in the tight credit scenario with a time-varying optimal rule. That is because after the shock the wealth effect of borrowers with a low credit limit and likewise the amplification effect is smaller. Subsequently, the over-tight constraint with the rule reverses the effect on aggregate demand and produces deflation. My findings suggest that a mortgage market characterized by leveraged borrowers offers regulators ample scope for regulatory inference, while in markets with tight credit constraints countercyclical regulation is redundant.

Moreover, I find welfare benefits through the introduction of the specified rules: borrowers are the winners, while in both scenarios an optimal countercyclical rule on house prices reduces savers' welfare. Beyond, the best policy allocation is a countercyclical rule responding to credit growth on relatively leveraged borrowers because it improves welfare to both households. Thus, this paper contributes to the literature by indi-

Table 2.1: **Average loan-to-value ratios and caps in several European countries.**

Loan-to-value ratios for residential mortgage loans		
	New loans for first-time house buyers	
country	average LTV ratio (2011)[5]	cap on the LTV ratio[6]
Netherlands	101%	max. 103%
Ireland	>100%	max. 90%
Estonia	> 100%[7]	max. 85%
Lithuania	> 100%[8]	max. 85%
Finland	87%	max. 95%
Cyprus	80%	max. 80%
Germany	79%	recommended limit
Belgium	63%	no restriction
Austria	84%	no restriction
France	83%	no restriction
Slovakia	70%	max. 90%
Italy	60%	no restriction

cating the dependence of welfare gains and of macroeconomic stabilization effects on the level of borrowers' indebtedness.

The next section introduces the model. The calibration to Eurozone data is the focus of section three. Section four is about the amplification effects of changing LTV ratios, the specification of the optimal policy rule, and includes the welfare analysis. The discussion of results follows in section five, while section six concludes.

2.2 Model

The model economy is equally populated by two households differing in their intertemporal discount factor. The assumption makes them to savers and borrowers denoted by the index s and b, respectively. The model economy is affected by transitory technology, monetary policy, and for the research focus most essential house preference shocks.

2.2.1 Households

Patient household The representative patient household enters period t holding $h_{s,t-1}$ units of housing and $B_{s,t-1}$ nominal one-period bonds, which yield the gross interest of r_t between the time periods. During period t the household receives $W_{s,t}n_{s,t}$ total nominal factor payments from supplying $n_{s,t}(i)$ hours of labor to each intermediate goods producing firm $i \in [0, 1]$.[9] Further, the household yields D_t nominal dividends from each intermediate goods producing firm $i \in [0, 1]$.[10] The patient household uses its funds to invest in $B_{s,t}$ new bonds at

[5]Banca D'Italia (2013)

[6]Regulatory limits refer to the press releases of national regulation authorities.

[7]EMF Hypostat (2013)

[8]EMF Hypostat (2013)

[9]The household's choices of $n_{s,t}(i)$ must satisfy $n_{s,t} = \int_0^1 n_{s,t}(i)\, di$

[10]Dividends of the intermediate goods producing firm aggregate to $D_t = \int_0^1 D_t(i)\, di$.

the nominal cost, to purchase c_t units of the output good for consumption purposes from the final goods sector at a nominal price P_t, and to buy $h_{s,t}$ housing stock at a nominal price Q_t in accordance to (2.1).[11]

$$c_{s,t} + \frac{B_{s,t}}{P_t} + \frac{Q_t}{P_t} h_{s,t} = \frac{r_{t-1} B_{s,t-1}}{P_t} + \frac{W_{s,t} n_{s,t}}{P_t} + \frac{Q_t}{P_t} h_{s,t-1} + \frac{D_t}{P_t} \quad (2.1)$$

The household chooses $\{c_t, h_{s,t}, n_t, B_{s,t}\}_{t=0}^{\infty}$ to maximize the stream of expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta_s^t \left[\ln(c_{s,t}) + j_t \ln(h_{s,t}) - \frac{n_{s,t}^{\eta}}{\eta} \right], \quad (2.2)$$

where $0 < \beta_s < 1$ denotes the discount factor and $\eta \geq 1$ governs the elasticity of labor supply, while $\frac{1}{\eta-1}$ is the Frisch elasticity.[12] The household derives expected utility from consumption, housing service and leisure according to (2.2). The weight of housing j_t in the utility function represents an intertemporal demand shock for housing that follows an autoregressive process:[13]

$$\ln(j_t) = (1 - \rho_j) \ln(j) + \rho_j \ln(j_{t-1}) + \varepsilon_{j,t}, \quad (2.3)$$

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[11] Lower case letters express variables in terms of consumption goods.

[12] The labor Frisch elasticity describes the change of the wage rate on hours worked while the marginal utility from income remains constant (Hansen, 1985; Rogerson, 1988).

[13] The shock affects an increase of the households' marginal utility of housing.

where $0 < \rho_j < 1$ and the innovation $\varepsilon_{j,t}$ is $i.i.d. \sim N(0, \sigma_j^2)$.

The optimality conditions of the problem are characterized by the budget constraint (2.1), the Euler equation (2.4), the representative saver's labor supply equation (2.5) and the intertemporal housing demand equation (2.6). The Euler equation (2.4) sets the marginal utility of consuming one unit of income today (in t) equal to the discounted future marginal utility of consuming the gross income acquired by saving income in $t+1$.

$$\frac{1}{c_{s,t}} = \beta_s r_t E_t \left[\frac{1}{c_{s,t+1}} \frac{P_t}{P_{t+1}} \right] \tag{2.4}$$

The labor supply equation (2.5) determines how many hours the saving household is willing to work given their real wage rate.

$$n_{s,t}^{\eta-1} c_{s,t} = \frac{W_{s,t}}{P_t} \tag{2.5}$$

The intertemporal equation for housing (2.6) puts in level the costs of housing stock in units of consumption today to the marginal utility of housing services today and the discounted utility gain in consumption units of selling housing stock tomorrow.

$$\frac{1}{c_{s,t}} \frac{Q_t}{P_t} = \frac{j_t}{h_{s,t}} + \beta_s E_t \left[\frac{1}{c_{s,t+1}} \frac{Q_{t+1}}{P_{t+1}} \right] \tag{2.6}$$

Impatient household The representative impatient household enters period t with an existing amount of nominal debt $B_{b,t-1}$ and $h_{b,t-1}$ units of housing. During period t the household receives $W_{b,t}n_{b,t}$ total nominal factor payments from supplying $n_{b,t}(i)$ working hours.[14] Given the budget constraint (2.7), next to labor income the households' funds encompass real estate wealth from last period, $Q_t h_{b,t-1}$, and the amount $B_{b,t}$ of nominal debt issued by the borrower at time t. The household uses its funds to repay outstanding nominal debt $B_{b,t-1}$ plus the interest payment r_{t-1} taking price changes from one period to the next into account. The household purchases $c_{b,t}$ units of the final good for consumption purposes at a nominal price P_t and acquires real estate stock, $h_{b,t}$, at a nominal price Q_t .

$$c_{b,t} + \frac{r_{t-1}B_{b,t-1}}{P_t} + \frac{Q_t}{P_t}h_{b,t} = \frac{Q_t}{P_t}h_{b,t-1} + \frac{B_{b,t}}{P_t} + \frac{W_{b,t}}{P_t}n_{b,t} \quad (2.7)$$

Each period all borrowers have limited access to credit markets, as summarized by the collateral constraint:

$$\frac{r_t B_{b,t}}{P_t} \leq l E_t \left[\frac{Q_{t+1}}{P_t} h_{b,t} \right], \quad (2.8)$$

where l represents the cap on the LTV ratio. Accordingly, the debt service in the next period cannot exceed a proportion of tomorrow's value of today's stock of housing. The exogenous

[14]Symmetric to the saving household, $n_{b,t}(i)$ must satisfy $n_{b,t} = \int_0^1 n_{b,t}(i)di$.

set l impedes the free credit flow of funds and thus acts as a friction. A higher l represents looser collateral requirements, while a lower l states tight credit conditions. $(1 - l)$ stands for the borrowers' equity. In the policy exercise, the macroprudential rule is introduced on l. The representative household chooses $\{c_t, h_{b,t}, n_{b,t}, B_{b,t}\}_{t=0}^{\infty}$ to maximize the stream of expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta_b^t \left[\ln(c_{b,t}) + j_t \ln(h_{b,t}) - \frac{n_{b,t}^{\eta}}{\eta} \right], \tag{2.9}$$

where $0 < \beta_b < 1$ is the borrowers' discount factor that is considerably lower than the savers' discount factor. This assumption assures the flow of funds from savers to borrowers. The collateral constraint binds in steady state because the interest rate is smaller than the inverse of borrowers' discount factor.[15] The intertemporal demand shock j_t is assumed to follow the same autoregressive process as for the patient households (2.3).

The problem's first order conditions comprise the borrowers' budget constraint (2.7), the borrower's Euler equation (2.10), their housing demand equation (2.12), and labor supply equation (2.11), as well as the collateral constraint (2.8). Alike

[15] If the Kuhn-Tucker multiplier μ_t on the collateral constraint is zero, the collateral constraint (2.8) is non-binding. In this model, the multiplier is positive in the steady state and for the given small fluctuations around the steady state.

for patient households, the borrowers' Euler equation (2.10) equates the marginal utility of consuming today to the discounted marginal utility from consuming the gross income acquired tomorrow. For $\mu \neq 0$, the marginal utility of present consumption is larger than the discounted utility of future consumption. Therefore, agents borrow up to the limit to increase current consumption.

$$\frac{1}{c_{b,t}} = \beta_b r_t E_t \left[\frac{1}{c_{b,t+1}} \frac{P_t}{P_{t+1}} \right] + \mu_t r_t \tag{2.10}$$

The intratemporal condition for labor supply (2.11) sets the borrowers' real wage equal to their marginal rate of substitution between working and consuming.

$$n_{b,t}^{\eta-1} c_{b,t} = \frac{W_{b,t}}{P_t} \tag{2.11}$$

Borrowers' housing decision (2.12) sets the costs of an additional unit of housing in terms of consumption equal to the marginal benefits from housing. The benefits encompass the marginal utility of housing services, the discounted resale value of real estate stock in units of consumption in $t+1$, and the utility gain from using housing as collateral.

$$\frac{1}{c_{b,t}} \frac{Q_t}{P_t} = \frac{j_t}{h_{b,t}} + \beta_b E_t \left[\frac{1}{c_{b,t+1}} \frac{Q_{t+1}}{P_{t+1}} \right] + \mu_t l E_t \left[\frac{Q_{t+1}}{P_t} \right] \tag{2.12}$$

2.2.2 Final goods producing firms

The representative final goods producing firm purchases $y_t(i)$ units of each intermediate good $i \in [0, 1]$ at a price $P_t(i)$ to produce y_t units of the final good. Given these prices each firm chooses $\{y_t, y_t(i)\}$ to maximize its profits while manufacturing according to a CES-production function with constant returns to scale technology:

$$y_t \leq \left[\int_0^1 y_t(i)^{\frac{\epsilon_P - 1}{\epsilon_P}} di \right]^{\frac{\epsilon_P}{\epsilon_P - 1}} \tag{2.13}$$

with $\epsilon_P > 1$ denoting the elasticity of substitution between the different intermediate goods $y_t(i)$.

Producing in a perfectly competitive market, profit maximization of the firm leads to the demand function for intermediate goods:

$$y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon_P} y_t. \tag{2.14}$$

The zero profit condition in the final goods market determines the price for the finished good (2.15) as:

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon_P} di \right]^{\frac{1}{1-\epsilon_P}}, \tag{2.15}$$

which applies $\forall t = 1, 2, 3, ...$

2.2.3 Intermediate goods producing firms

Each firm hires patient and impatient workers for $n_{s,t}(i)$ and $n_{b,t}(i)$ working hours to produce $y_t(i)$ units of the intermediate good according to the constant return to scale production function:

$$y_t(i) \leq z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha} \tag{2.16}$$

with $1 > \alpha > 0$ measuring the relative share of patient households in terms of labor income and $1 - \alpha$ of impatient households, respectively. The Cobb-Douglas function implies complementary labor skills of patient and impatient households.[16] The technology parameter z_t follows the autoregressive process:

$$\ln(z_t) = (1 - \rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{z,t}, \tag{2.17}$$

where $1 > \rho_z > 0$ and the technology shock $\varepsilon_{z,t}$ is $i.i.d. \sim N(0, \sigma_z^2)$. Since intermediate goods substitute imperfectly for one another as inputs for the finished goods, intermediate goods producing firms set its product specific price $P_t(i)$ in this monopolistic competitive market. Whereas each firm faces convex

[16] If the labor effort of the heterogeneous households were perfect substitutes, the units worked by one group would influence the total wage income of the other group. In an extreme case, borrowers would not work at all and generate their income by debt (Iacoviello and Neri, 2010). The assumption of complementary is economically justified by the fact that the savers who manage the firms earn higher wages than borrowers (Rubio and Carrasco-Gallego, 2013).

price adjustment costs à la Rotemberg (1982) of the size ϕ_P, measured in units of the final good given by $\frac{\phi_P}{2}\left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right]^2 y_t$ with $\phi_P \geq 0$. π is the gross steady state inflation, $\pi_t = \frac{P_t}{P_{t-1}}$. Real dividends (2.18) of each firm are defined as revenues minus labor and price adjustment costs.

$$\begin{aligned}\frac{D_t(i)}{P_t} &= \left[\frac{P_t(i)}{P_t}\right] y_t(i) - \frac{W_{s,t} n_{s,t}(i) + W_{b,t} n_{b,t}(i)}{P_t} \\ &\quad - \frac{\phi_P}{2}\left[\frac{P_t(i)}{P_{t-1}(i)} - 1\right]^2 y_t. \qquad (2.18)\end{aligned}$$

Due to the (non-linear) adjustment costs, the firm's decision problem becomes dynamic and induces sticky adjustment of prices. Each firm chooses $\{n_{s,t}(i), n_{b,t}(i), y_t(i), P_t(i)\}_{t=0}^{\infty}$ to maximize its total market value:

$$E_0 \sum_{t=0}^{\infty} \beta_s^t \lambda_{s,t} [D_t(i)/P_t]$$

subject to the production function (3.9) and considering that $y_t(i)$ equals the final goods producing firms' demand (2.14).[17] $\frac{\lambda_{s,t}}{P_t}$ measures the period t real marginal utility to the patient households provided by an additional unit of profits in t.

The derived first order conditions encompass the labor demand

[17] The intermediate goods producing firm takes the final price index P_t and final output goods y_t as given.

equation for savers (2.19) and borrowers (2.20), the production function (2.16), and the pricing equation for each intermediate good (2.21), as well as the clearing condition of the production sector (2.22).

$$\frac{W_{s,t}}{P_t} n_{s,t} = \alpha \xi_t z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha} \tag{2.19}$$

$$\frac{W_{b,t}}{P_t} n_{b,t} = (1-\alpha) \xi_t z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha} \tag{2.20}$$

$$\phi_P \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right] \frac{P_t}{\pi P_{t-1}(i)} = \xi_t \epsilon_P \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_P - 1} + (1-\epsilon_P) \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_P} + \beta_s \phi_P E_t \left\{ \frac{c_t}{c_{s,t+1}} \left[\frac{P_{t+1}(i)}{\pi P_t(i)} - 1 \right] \left[\frac{P_{t+1}(i) P_t}{\pi P_t(i)^2} \right] \left(\frac{y_{t+1}}{y_t} \right) \right\} \tag{2.21}$$

$$\left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_P} = z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha} \tag{2.22}$$

The multiplier on the production function $\xi_t(i)$ captures the firm's real marginal costs.[18] In a flexible price specification with $\phi_P = 0$, the price of an intermediate good $P_t(i)$ is the markup $\frac{\epsilon_P}{\epsilon_P - 1}$ over nominal marginal costs $\xi_t(i) P_t$. Log-linear approximation of the pricing function (2.21) around a zero-inflation

[18] The multiplier states by how much costs are reduced if one unit less of $y_t(i)$ is produced.

steady-state yields the Phillip's curve:

$$\hat{\pi}_t = \beta_s E_t \hat{\pi}_{t+1} + \frac{(\epsilon_P - 1)}{\phi_P} \hat{\xi}_t,$$

where hats on a variable denote the variable's deviation from steady state.

2.2.4 Monetary policy

Monetary policy is conducted by a modified Taylor rule (TR):[19]

$$\ln\left(\frac{r_t}{r}\right) = \rho_r \ln\left(\frac{r_{t-1}}{r}\right) + (1 - \rho_r)\left[\omega_\pi \ln\left(\frac{\pi_t}{\pi}\right)\right] + \varepsilon_{v,t}. \qquad (2.23)$$

The monetary authority gradually adjusts the nominal interest rate in response to deviations of current gross inflation π_t from its zero-steady state value, while ρ_r and ω_π are the parameters of the monetary policy rule. The monetary policy shock $\varepsilon_{v,t}$ is $i.i.d. \sim N(0, \sigma_v^2)$.

2.2.5 Model closing equations

In equilibrium the bond market clears according to $B_{s,t} = B_{b,t}$. The goods market is cleared when $y_t = c_{b,t} + c_{s,t} + \left(\frac{\phi^P}{2}\right)$

[19]The inflation-targeting rule coincides with the mandatory objective of the European Central Bank.

$\left(\frac{P_t}{\pi P_{t-1}} - 1\right)^2 y_t$, and the housing market clearing condition satisfies $1 = h_{s,t} + h_{b,t}$, $\forall t$. The constant supply of housing entails price formation by household demand.Assuming symmetric behavior within the intermediate sector implies $P_t(i) = P_t$, $y_t(i) = y_t, n_{s,t}(i) = n_{s,t}, n_{b,t}(i) = n_{b,t}$, and $D_t(i) = D_t$ for $t = 0, 1, 2, ...$ and all $i \in [0, 1]$.

The model is solved using real terms. The equilibrium is defined as the path of allocations $\{c_{s,t}, h_{s,t}, n_{s,t}, b_{s,t}, c_{b,t}, h_{b,t}, n_{b,t}, d_t,$- $y_t, j_t, z_t, \xi_t, \mu_t\}_{t=0}^{\infty}$ and prices $\{r_t, q_t, \pi_t, w_{s,t}, w_{b,t}\}_{t=0}^{\infty}$ that satisfies the households' first order conditions (2.1), (2.4) to (2.8) and (2.7), (2.8) and (2.10) to (2.12) and the firm's optimality conditions (2.16), (2.18) to (2.21) and the Taylor rule (2.23), as well as the shocks' law of motions (2.3) and (2.17).[20] I log-linearize the model around the steady-state and solve it with Dynare using the MATLAB routine. The log-linearized equations are summarized in the appendix 2.A.4 and the applied Klein's solution algorithm is explained in appendix section 2.B.

[20] For the policy scenarios, I add the macroprudential rule to the equilibrium conditions.

2.3 Calibration

Unlike Rubio and Carrasco-Gallego (2015a) who use US date, I calibrate the model to match primarily Eurozone data while assuming a non-inflationary environment. Table 2.2 summarizes the calibration values based on a quarterly frequency. Implying an annual nominal interest rate of four percent, the discount factor β_s is 0.99. Setting the discount factor of impatient households to β_b to 0.975 assures a large enough motive of borrowers to take out a loan, so that the linearization of the binding collateral constraint around the steady state is accurate. With a parameter η equal to two, the Frisch labor elasticity is one. The Eurosystem's household Finance and Consumption Survey (ECB, 2013) ascertains 43.7% of households in the Euro area have some form of debt, while the prevalence of mortgage debt is higher than of non-mortgage debt.[21] In line with these facts, I choose a value of $\alpha = 0.64$ for the income share of non-constrained households, which is also consistent with Rubio and Carrasco-Gallego (2014). The parameter of the price adjustment costs ϕ_P is set to 58, which implies that firms reoptimize their prices on average every 12 months. The elasticity of substitution ϵ_P is fixed to 6. This translates to a markup of 20% of prices over real marginal costs (Keen and Wang, 2007). The following calibration values depend on the LTV

[21] More than half of European indebted households have a mortgage debt, namely 23.1% of all Euro area households.

ratio that is either sixty five or ninety in the two benchmark models. I calibrate the weight on housing in the utility function to 0.1, which induces a steady state housing wealth to annual GDP ratio between 225% (LTV=0.65) and 240% (LTV=0.90). The average ratio from 1994-2009 is around 240% in the Eurozone (Banca D'Italia, 2013). The average mortgage debt to GDP ratio is 40% in the Eurozone (Musso et al., 2011). Correspondingly, the steady-state mortgage loan to annual GDP ratio ranges from 54% when the LTV=0.90, and 27% when steady state LTV is 0.65. For the monetary feedback rule, I assume that the interest rate smoothing parameter is equal to 0.8 and a long-run response to inflation ω_π of 2 to regard the Taylor-principle. The housing demand shock is calibrated so that real house prices increase on average by one percent in both benchmark models creating an impact on output of maximal 0.8 percent. The persistence parameter of this shock is set to 0.95. The productivity shock with $\rho_z = 0.95$ represents an one percent shock to technology, as in Rubio and Carrasco-Gallego (2014). The standard deviation of a monetary policy shock is set to 0.004 in line with the literature.

Table 2.2: **Calibration values.** Parameters are calibrated to match Eurozone data.

Parameter	Description	Value
β_s	discount factor for savers	0.99
β_b	discount factor for borrowers	0.975
η	parameter governing the disutility of labor	2
j	steady state weight of housing	0.1
α	labor share of saver	0.64
ϕ_P	price adjustment costs	58
ϵ_P	price elasticity of demand	6
ρ_r	interest rate smoothing parameter in TR	0.8
ω_π	inflation parameter in TR	2
ρ_j	persistence housing demand shock	0.95
ρ_z	persistence technology shock	0.99
σ_z	standard deviation technology shock	0.01
σ_j	standard deviation housing demand shock	0.06
σ_v	standard deviation monetary policy shock	0.004

2.4 Collateral constraints and optimal countercyclical regulation

2.4.1 Amplification effect of lax credit constraints

The pecuniary externality associated with the collateral constraint *l* plays a central role in the model. Leaving all other parameters unchanged, I alter the fixed LTV ratio from 65% stepwise to 90% in different model specifications to stress the amplification effect of collateral constraints.[22] Table 2.3 lists the standard deviation of output after a housing demand and technology shock.[23] The output variance generated by a housing demand shock increases with laxer credit limits for borrowers, but remains almost unchanged after a technology shock. The amplification with higher credit limits stresses that only housing demand shocks induce a collateral effect: the shift in the households' taste for housing services causes house prices to rise. The tighter the collateral constraint the lower is the wealth effect of the borrower as borrowers profit in two ways by more housing wealth. First, borrowers have a higher propensity to spend than savers because of their discount factor. This is ac-

[22] The externality arises in incomplete credit market where the collateral constraint creates the friction in the market. Fire-sales of borrowers to meet their collateral requirement depress the collateral price, so that economic conditions worsen (Korinek and Simsek, 2016).

[23] I simulate 1000 data series based on the perturbation solution of the model and compute the moments of these simulated series.

Table 2.3: **Simulated output standard deviation** for a selection of models that characterize different LTV ratios after a housing demand and a technology shock.

	Output standard deviation	
model	housing demand shock	technology shock
LTV=0.90	0.0055	0.0200
LTV=0.80	0.0017	0.0195
LTV=0.70	0.0006	0.0194
LTV=0.65	0.0003	0.0194

companied by a higher marginal propensity to consume, which reinforces the effect on aggregate demand conditional on the LTV level. Second, borrowers use housing as collateral, which enables them to increase their borrowing capacity with higher house prices. In turn they spend more income for housing and thereby housing collateral value increases boosting borrowing again. This mechanism is known as the financial multiplier effect (Kiyotaki and Moore, 1997).

Consistent with the findings of Rubio and Carrasco-Gallego (2014), Lambertini et al. (2013), and Walentin (2014), housing demand shocks given collateral constraints are a source for propagating business cycle fluctuations.[24] The fact that a countercyclical macroprudential rule on the LTV ratio induces the

[24] Appendix 2.C.1 provides a detailed explanation of the model's dynamics after a housing demand and a technology shock.

borrower to internalize the externality of the constraint motivates the introduction of countercyclical macroprudential rules on fixed LTV ratios. In this context, the paper defines stabilization effects of macroprudential regulation by lowering variables' variance compared to the situation without regulation.

2.4.2 Macroprudential policy

The macroprudential authority sets the countercyclical LTV ratio according to a linear policy rule as in Lambertini et al. (2013), Angelini et al. (2012), and Brzoza-Brzezina et al. (2013):

$$l_t = (1-\rho_l)\,l + (1-\rho_l)\chi_l\,\ln\left(\frac{x_t}{x}\right) + \rho_l\,l_{t-1} \tag{2.24}$$

where $0 < \rho_l < 1$ is the autoregressive persistence parameter of the rule and $\chi_l < 0$ denotes the response parameter to the chosen indicator variable x_t. The ESRB recommends to use either real estate prices or real estate credit as an indicator (ESRB, 2014). They reason their choice by stating states that strong price growth and high volumes of real estate credit in the real estate sector can induce a banking crisis. With that in mind, deviations of house prices or the credit aggregate, $x_t = \{q_t, b_t\}$, from its steady state value indicate an imbalance in the model. For a countercyclical adjustment of the LTV ratio, χ_l must be negative. Accordingly the LTV ratio tightens (relaxes) in response to positive (negative) deviation of the indicator variable from its steady state value. The following macroprudential policy analysis is conducted for the model specifications with a pre-set LTV ratio of 0.65 and 0.90. In both specifications the rule is introduced making l_t endogenous.

2.4.3 Optimal macroprudential rules

The main objective of macroprudential policy is the prevention of systemic risk and connected with it, excessive borrowing in order to ensure financial stability. To enforce these objectives, I assume the macroprudential authority parametrizes the macroprudential rule (2.24) accordingly. As a proxy for financial stability, the macroprudential authority uses the reduction of credit variance as key argument in its objective function. To parameterize the macroprudential policy rule (2.24), to name it χ_l and ρ_l, the macroprudential authority minimizes its objective function over the two variables.[25] The same procedure is applied by Mendicino and Punzi (2014), Resende et al. (2013) and Angelini et al. (2012). In contrast to others, I do not consider output stabilization as a second argument in the objective function which I justify by Tinbergen's criteria.[26] In line with the Tinbergen principle, this setup with one macroprudential

[25]Several studies analyzed the impact of a countercyclical rule by using an arbitrary parametrization of the rule.

[26]Tinbergen (1952) alludes that every policy objective requires at least one single instrument.

instrument allows for only one objective.[27]

For the parametrization, the countercyclical change of the LTV ratio is, in particular, supposed to abate the financial multiplier triggered by a housing demand shock. Therefore, the optimization is conducted exclusively under a housing demand shock for both rules ($x_t = \{b_t, q_t\}$). In the two benchmark models, the macroprudential regulator minimizes the loss function (2.25) subject to the model equations to find the optimal χ_l and ρ_l for the rule responding to house price or credit growth, separately. I restrict the χ_l on a grid going from -2 to 0.5 in steps of 0.01, which allows a systematic but not to extreme policy reaction. Besides $0 \leq \rho_l < 1$ is assumed.

$$L_{mp} = \min_{\chi_l, \rho_l} Var(b) \quad \text{s. t. the model equations} \tag{2.25}$$

Table 2.4 provides the optimal values of the policy rules in both model specifications. Independent of the indicator variable, a sudden policy reaction effectively reduces credit and output variance compared to a rigid rule; $\rho_l = 0$ is optimal.[28]

[27] By using only one target variable, I circumvent the issue of arbitrarily assigning weights to the arguments in the loss function. As Angelini et al. (2012) and Quint and Rabanal (2014) state, my analysis confirms that weights set to the arguments a priori are sensitive to the result.

[28] Figure 2.6 illustrating the rule on credit growth and figure 2.5 displaying the rule on house price growth emphasize the optimality of $\rho_l = 0$ by plotting the percentage credit change as a function of the intensity parameter χ_l. The rules that react instantaneously curtail credit variance the most.

Table 2.4: **Optimal parameters of the macroprudential policy rule** that reacts to either house price or credit growth after a housing demand shock. The optimal values for ρ_l and χ_l for the two models are results of the loss function minimization.

model	indicator	optimal parameter	
		ρ_l	χ_l
$LTV = 0.90$	$\hat{q}$	0.0	−0.84
$LTV = 0.65$	$\hat{q}$	0.0	−0.70
$LTV = 0.90$	$\hat{b}$	0.0	−2
$LTV = 0.65$	$\hat{b}$	0.0	−2

Regarding the rule responding to house prices, the optimal reaction parameter is relatively strong with −0.84 in the model with a steady-state LTV ratio of 0.9 compared to the model with tight credit conditions with a value of −0.70. The larger collateral effect induced by the higher LTV ratio causes this more intense policy reaction. Turning to the macroprudential policy rule reacting to credit growth, a value of $\chi_l = -2$ is optimal for the model with tight and lax credit conditions. This result underlines that using credit as indicator naturally serves its purpose to stabilize credit volatility.

In a sensitivity analysis, I check the impact of a countercyclical rule on output variance for both scenarios. To do so, figure 2.1 illustrates the rule reacting to house price growth and figure 2.2

displays the rule on credit growth as a function of the intensity parameter χ_l. The results uncover that the rule responding to credit growth is less sensitive to the reaction parameter than a rule on house prices. A rule implemented in the high leverage scenario results in greater output volatility reduction than in the scenario when borrowers provide more equity. For the high leverage scenario, the optimal parameter that mitigates credit variance lies in the same value region as the parameter that reduces output variance. However, this symmetry concerning the optimal parameter is not evident for both rules in the low leverage scenario, where the rule only reduces output variance for small negative parameter values. If the leverage ratio of borrowers is low, the countercyclical rule reduces less likely, simultaneously, the variance of loans and output. Independent of the indicator variable a sudden policy reaction shows to reduce output variance most effectively.

2.4.4 Welfare implications

In order to evaluate the desirability of the specified rules from the above optimization procedure, I compare the welfare implications of macroprudential regulation for the tight and lax credit scenario. Welfare is defined as the present discounted value of lifetime utility (2.26) of borrowers and savers $i = \{b, s\}$, respectively, and is recursively calculated according to (2.27). The separate computation renders possible to identify the winners of the introduction of a macroprudential rule. An accurate approximation to the welfare function requires a second-order expansion to the model equilibrium conditions following Schmitt-Grohé and Uribe (2004b).

$$\Omega_{i,t} \equiv E_t \sum_{m=0}^{\infty} \beta_i^m \left(\log c_{i,t+m} + j \log h_{i,t+m} - \frac{(n_{i,t+m})^{\eta}}{\eta} \right) \tag{2.26}$$

$$\Omega_{i,t} = U(c_{i,t}, h_{i,t}, n_{i,t}) + \beta_i \Omega_{i,t+1} \tag{2.27}$$

Conditional welfare depends on the initial state considered of each model economy, which is the non-stochastic steady state value of welfare captured in the state vector. As the state vector is different for the two scenarios, each benchmark scenario serves as reference. Therefore, I contrast the impact of alternative policies that are consistent with the same non-stochastic steady state to all sources of fluctuations in the model, which refers to the implied volatility of all shocks. In consequence, I compare the welfare change between the considered benchmark model to the welfare outcome with the countercyclical rule for the two indicator variables. I quantify the welfare differences in terms of consumption equivalences: how much is the household willing to pay in consumption units for the implementation of the macroprudential rule because it is welfare improving. A negative value indicates a compensation in percent of the household's consumption stream that the household requires to accept the macroprudential rule.

Table 2.5 summarizes the welfare implications for the two scenarios and the corresponding optimal rules. Compared to a constant LTV ratio, allowing for a countercyclical LTV-rule improves upon total welfare which is the sum of individual welfare gains. Borrowers are the main winners. They obtain a higher welfare level with macropudential policy regardless of the indicator variable because the tightening of the collateral constraint impedes the situation of extreme leverage. Borrowers are unable to smooth their consumption of goods and housing service without a rule, even though they are rational enough to expect a boom or a bust phase. As model technique wise the collateral constraint is always binding, the welfare analysis indicates that the countercyclical rule mitigates the negative externality of the collateral constraint.

If the countercyclical rule reacts to credit growth in the scenario with a LTV ratio of 90%, welfare achievements are the largest. Borrowers receive 0.287% more and savers gain 0.186% more of lifetime consumption they would have without the regulation. In the high leverage scenario, not only borrowers consumption stream is smoothed, but also savers obtain additional welfare units. That is very likely due to the stable stock of housing services from which also savers derive utility. In the low leverage scenario, the rule on credit growth, however, accrues welfare costs for savers.[29]

With a rule on house price growth in the set-up with a LTV ratio of 90%, borrowers' consumption smoothing affects a 0.434% welfare payoff and in the tight credit condition scenario they earn slightly less with 0.135%. These welfare gains with the macroprudential rule on house price growth is at the expense of savers, who lose consumption units. The welfare analysis showed that the welfare achievements through the introduction of the regulation depend on the level of borrowers' indebtedness.

2.5 Discussion of results

Compared to the benchmark, the specified optimal rules alter the transmission of a housing demand shock in the scenarios with an initial LTV ratio of 90% (figure 2.3) and an initial LTV ratio of 65% (figure 2.4).[30] In both scenarios, the shock induces that the rule reacting to either house price growth or credit growth tightens the credit limits different than in the benchmark models with a constant credit limit. In the high leverage

[29] Likewise, Lambertini et al. (2013) find similar welfare gains for both households when the rule responds to credit growth on a LTV ratio of 85% given a standard Taylor rule.

[30] As macroprudential regulation authority is not capable to determine the origin of the shock, the impact of a countercyclical rule under a technology shock is equally essential and is, hence, discussed in the appendix 2.C.2.

Table 2.5: **Conditional welfare gains** with the optimal countercyclical rules. Gains are stated relative to the corresponding benchmark model without regulation, while values are in percent of the household's lifetime consumption stream. Negative values imply costs for the household type.

	rule on house price growth		rule on credit growth	
	LTV=0.65 $\chi_l = -0.70$	LTV=0.90 $\chi_l = -0.84$	LTV=0.65 $\chi_l = -2$	LTV=0.90 $\chi_l = -2$
savers' welfare	−0.065	−0.176	−0.018	0.186
borrowers' welfare	0.135	0.434	0.082	0.287
total effect	0.070	0.258	0.064	0.473

scenario, impatient households may, consequently, borrow less, while the effect is reinforced by a rule on house prices. Comparing the scenario with a LTV ratio of 65% with its benchmark, the rule on credit growth mitigates borrowing and the rule on house prices even leads to deleveraging. Consequently, the effect on output is diverse in the two scenarios. Macroprudential regulation smooths output in the high credit scenario, while output falls in the counter scenario in response to both rules. That is because in the low leverage scenario, the impact on borrowers' wealth, generated by higher house prices, is lower, so the additional amount of credit that the borrower receives for its collateral is lower. By restricting borrowing additionally, borrowers' wealth falls and the impact of the positive housing demand shock reverses in the tight credit scenario. Prices decrease causing interest rates to fall. Because lower interest rates boost borrowers' demand for loans that the countercyclical rule aims to limit, macroprudential and monetary policy are in conflict. Subsequently, dampened credit cycles are at a cost of output destabilization during house price booms when borrowers are subject to strict LTV ratios. In contrast, in the lax credit scenario the countercyclical rule shuts off the amplification effect of the collateral constraint and attenuates output fluctuations. Moreover, in the scenario with a LTV ratio of 65%, both rules cause a drop of debt payments by a lower interest rate, which destabilizes savers' consumption, and supports the finding that savers are socially worse off.

Countercyclical regulation mitigates credit cycles in both scenarios. However, if credit conditions are initially tight, the benefits of a countercyclial rule to a housing demand shock are revoked. Thus, a countercyclical rule can be harmful since it creates unnecessary output variability and deflation. In addition, in this scenario the consequently lowered interest rates oppose tight credit limits, which creates a conflict between policies. If borrowers have initially a high debt level, I find that optimal countercyclical regulation reduces simultaneously output and credit volatility. When borrowers provide less equity, or put differently, exhibit a relatively high leverage ratio, the regulatory scope of the macroprudential authority to safeguard financial stability is larger. These results support findings of Mendicino (2012) who investigates the effects of a low fixed LTV ratio of 50% and a countercyclical rule on a LTV ratio of 85%. In response to a credit shock she also finds that countercyclical caps are successful in dampening credit cycles contrary to tight discretionary credit caps.[31] Though, this paper stands out by adding to the literature that in case of low credit limits the effects of countercyclical regulation are detrimental because the rule induces an output loss and deflation. The regulatory interference with countercyclical regulation shows to be more influential when households are considerably leveraged.

2.6 Conclusion

This paper analyzes the relation between countercyclical macroprudential rules and the debt level of borrowers. In the model framework based on Rubio and Carrasco-Gallego (2015a), the presence of a limit on the LTV ratio of borrowers amplifies the consequences of a housing demand shock for the real economy, while the collateral effect increases with leverage. As technology shocks generate only a weak propagation, the policy anal-

[31] Additionally, she finds no evidence that countercyclical gaps increase the response of output to technology shocks.

ysis focuses on boom and bust cycles in the housing market. In this set-up, I investigate the welfare implications and stabilization benefits of countercyclical LTV-rules for borrowers with a high and a low initial LTV ratio. The rules are parametrized to optimally mitigate credit cycles taking monetary policy as given. Either credit or house price growth indicates countercyclical adjustment of the credit limit.

In response to a housing demand shock, countercyclical rules effectively dampen credit growth independent of the indicator variable and borrowers level of indebtedness. If the rule is implemented on a high cap of the LTV ratio, the rule reduces also output fluctuations contrary to the model with a low cap of the LTV ratio. The rationale behind the different performance of the rules in the scenarios is the following: in an economy with tight credit constraints on borrowers, the time-varying rule additionally tightens the credit limit. The over-tight credit cap prevents a wealth effect for borrowers after the shock, which accounts for a fall of output and leads to deflation. Due to lower nominal interest rate, macroprudential and monetary policy enter into conflict. In addition, the welfare computation supports the introduction of countercyclical rules only in the high leverage scenario.

My findings imply the implementation of a LTV rule in markets with relatively leveraged mortgage borrowers countercyclical macroprudential regulation achieves credit and output stabilization. The rule on credit growth performs thereby better than a rule on house price growth due to higher total welfare gains and smoothed credit cycles. The model results reveal that countercyclical regulation of hardly indebted households delivers credit stabilization at a cost of output variability, which is an important insight for the design of macroprudential real estate regulation.

Figure 2.1: **Output variance change and the rule reacting to house price growth.** Rule as a function of the policy reaction parameter (χ_l) and the percentage output variance change relative to benchmark variance (Y-axis) that results after a housing demand shock. $\rho_l = 0$ refers to an ad hoc adjustment of the LTV ratio and $\rho_l = 0.9$ to a rigid adjustment of the LTV ratio to house price growth. The rule on a LTV ratio of 0.9 and with $\rho_l = 0$ is the dashed blue and with $\rho_l = 0.9$ the solid green line. The rule on a LTV ratio of 0.65 and with $\rho_l = 0$ is the dotted red and with $\rho_l = 0.9$ the turquoise dash-dotted line.

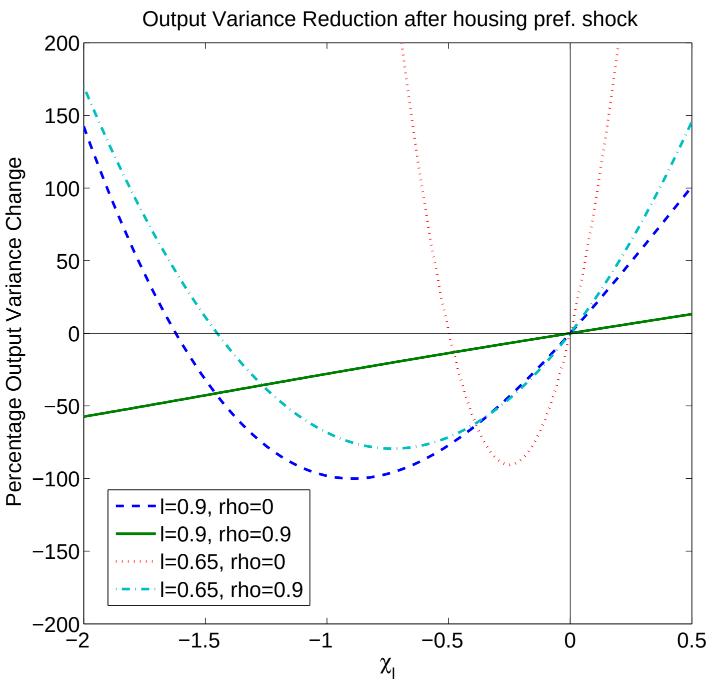


Figure 2.2: **Output variance change and the rule reacting to credit growth.** Rule as a function of the policy reaction parameter (χ_l) and percentage output variance change relative to benchmark variance (vertical axis) that results after a housing demand shock. $\rho_l = 0$ refers to an ad hoc adjustment of the LTV ratio and $\rho_l = 0.9$ to a rigid adjustment of the LTV ratio to credit growth. The rule on a LTV ratio of 0.9 and with $\rho_l = 0$ is the dashed blue and with $\rho_l = 0.9$ the solid green line. The rule on a LTV ratio of 0.65 and with $\rho_l = 0$ is the dotted red and with $\rho_l = 0.9$ the turquoise dash-dotted line.

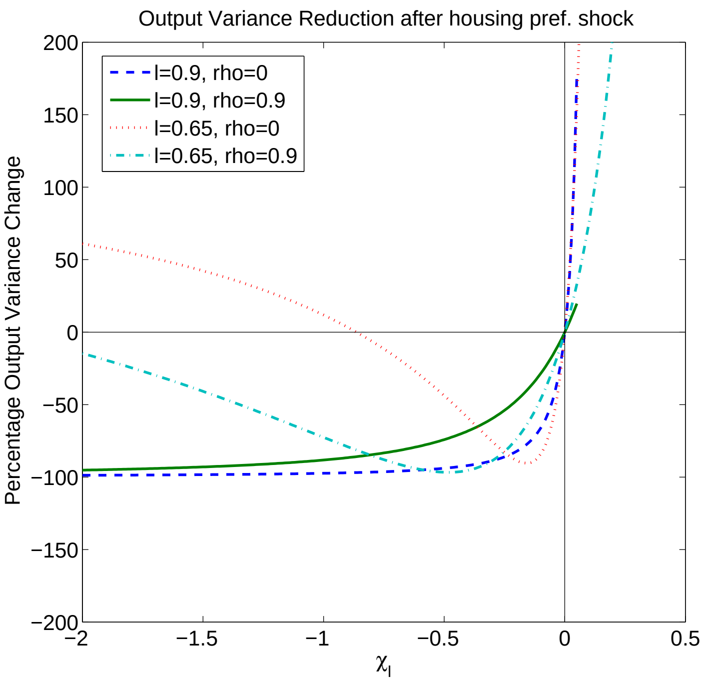


Figure 2.3: **Impulse responses to a housing demand shock in the model with a LTV ratio of 90%.** Variables' reaction with constant LTV ratio (solid red line), with countercyclical LTV-rule on house price (dashed blue line) and credit growth (dotted black line).

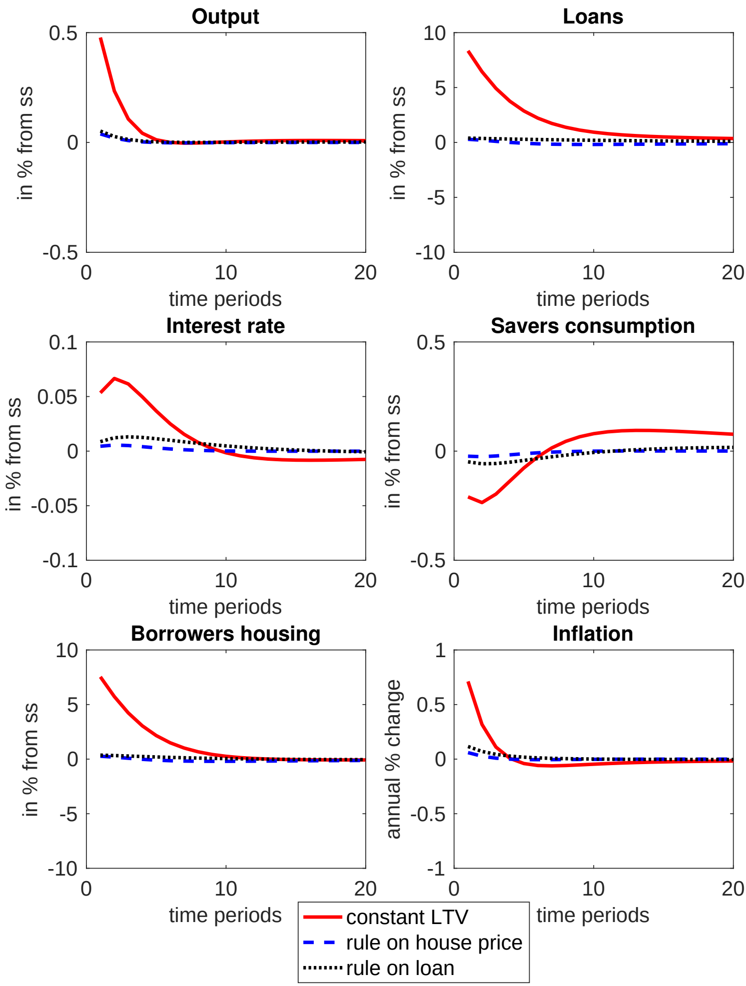


Figure 2.4: **Impulse responses to a housing demand shock in the model with a LTV ratio of 65%.** Variables' reaction with constant LTV ratio (solid red line), with countercyclical LTV-rule on house price (dashed blue line) and credit growth (dotted black line).

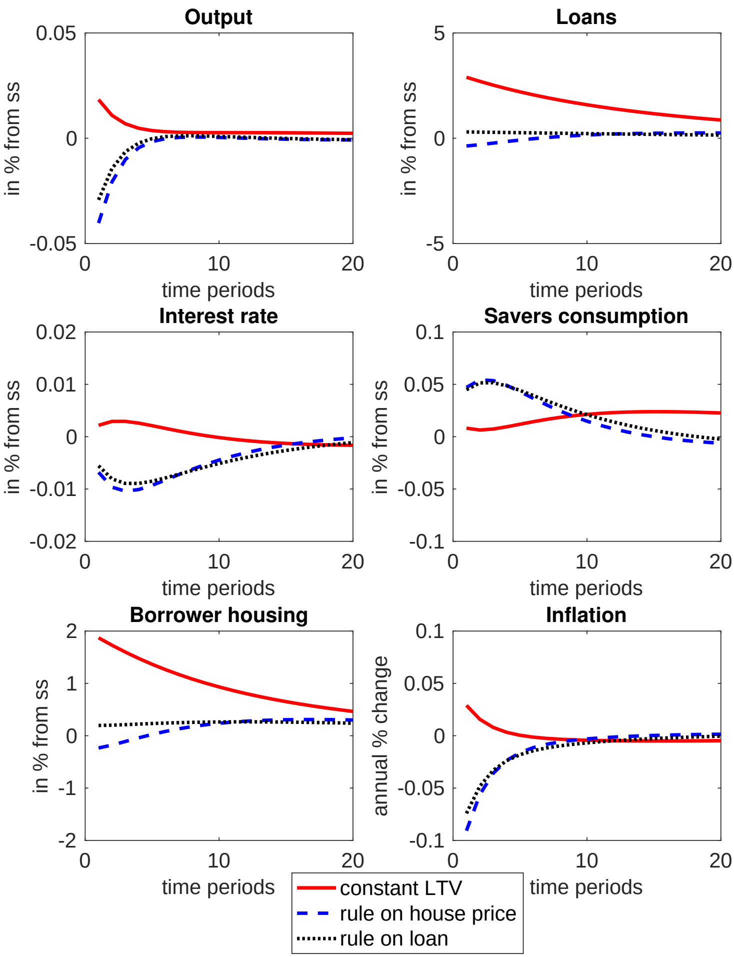


Figure 2.5: **Credit variance change and the rule reacting to house price growth.** Rule as a function of the policy reaction parameter (χ_l) and the percentage credit variance change relative to benchmark variance (vertical axis) that results after a housing demand shock. $\rho_l = 0$ refers to an ad hoc adjustment of the LTV ratio and $\rho_l = 0.9$ to a rigid adjustment of the LTV ratio to house price growth. The rule on a LTV ratio of 0.9 and with $\rho_l = 0$ is the dashed blue and with $\rho_l = 0.9$ the solid green line. The rule on a LTV ratio of 0.65 and with $\rho_l = 0$ is the dotted red and with $\rho_l = 0.9$ the turquoise dash-dotted line.

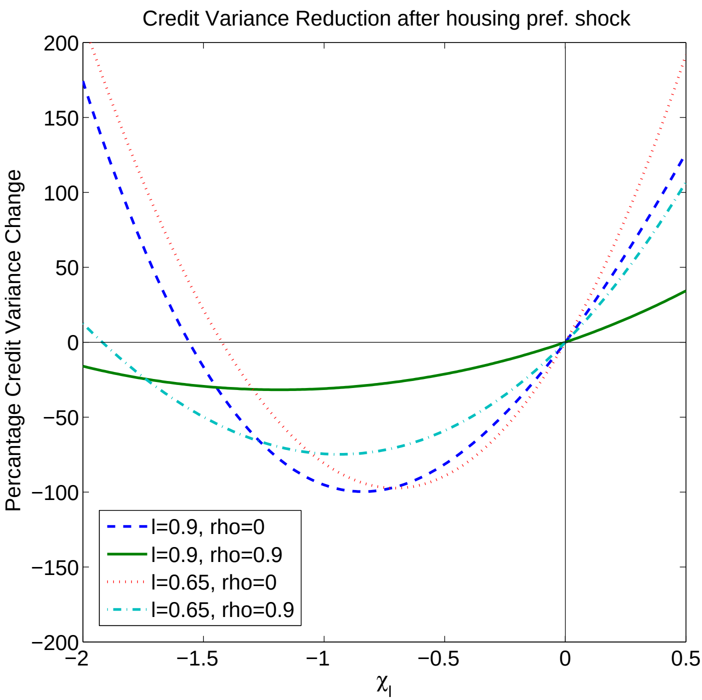
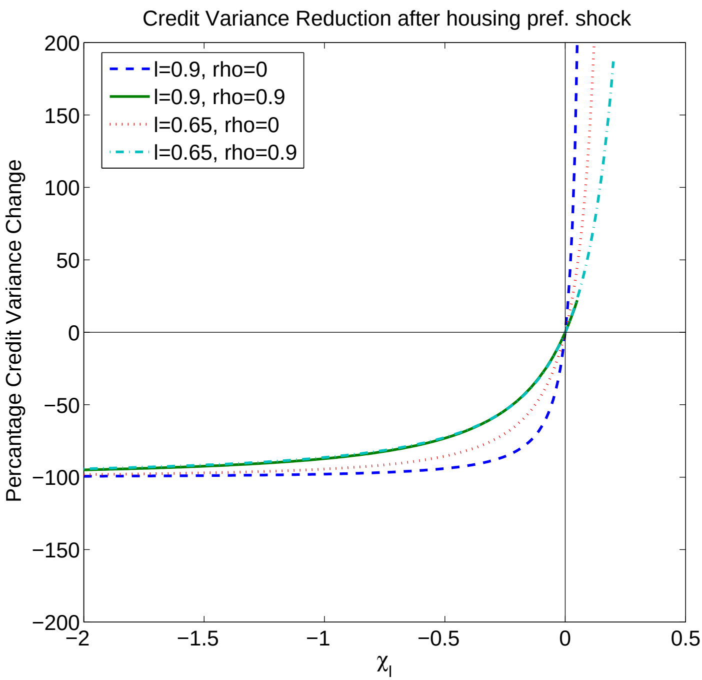


Figure 2.6: **Credit variance change and the rule reacting to credit growth** Rule as a function of the policy reaction parameter (χ_l) and percentage credit variance change relative to benchmark variance (vertical axis) that results after a housing demand shock. $\rho_l = 0$ refers to an ad hoc adjustment of the LTV ratio and with $\rho_l = 0.9$ to a rigid adjustment of the LTV ratio to credit growth. The rule on a LTV ratio of 0.9 and with $\rho_l = 0$ is the dashed blue and with $\rho_l = 0.9$ the solid green line. The rule on a LTV ratio of 0.65 and with $\rho_l = 0$ is the dotted red and with $\rho_l = 0.9$ the turquoise dash-dotted line.



Appendix

2.A Model equations

2.A.1 Optimization problems

- Savers' optimization problem:

$$
\begin{aligned}
&\max_{c_{s,t},h_{s,t},N_{s,t},B_{s,t}} L = E_0 \sum_{t=0}^{\infty} \left\{ \beta_s^t \left[\ln(c_{s,t}) + j_t \ln(h_{s,t}) - \frac{n_{s,t}^{\eta}}{\eta} \right] \right. \\
&-\beta_s^t \lambda_{s,t} \left(c_{s,t} + \frac{B_{s,t}}{P_t} + \frac{Q_t}{P_t} h_{s,t} - \frac{(r_{t-1}) B_{s,t-1}}{P_t} \right. \\
&\left. \left. - \frac{W_{s,t} n_{s,t}}{P_t} - \frac{Q_t}{P_t} h_{s,t-1} - \frac{D_t}{P_t} \right) \right\}
\end{aligned}
$$

First order conditions:

$$\frac{\partial L}{\partial c_{s,t}} = \beta_s^t \frac{1}{c_{s,t}} - \beta_s^t \lambda_{s,t} = 0$$

$$\Longleftrightarrow \frac{1}{c_{s,t}} = \lambda_{s,t}$$

$$\frac{\partial L}{\partial h_{s,t}} = \beta_s^t j_t \frac{1}{h_{s,t}} - \beta_s^t \lambda_{s,t} \frac{Q_t}{P_t} + \beta_s^{t+1} E_t \left[\lambda_{s,t+1} \frac{Q_{t+1}}{P_{t+1}} \right] = 0$$

$$\Longleftrightarrow \lambda_{s,t} \frac{Q_t}{P_t} = \frac{j_t}{h_{s,t}} + \beta_s E_t \left[\lambda_{s,t+1} \frac{Q_{t+1}}{P_{t+1}} \right]$$

$$\frac{\partial L}{\partial n_{s,t}} = \beta_s^t (-\eta) \frac{n_{s,t}^{\eta-1}}{\eta} + \beta_s^t \lambda_{s,t} \frac{W_{s,t}}{P_t} = 0$$

$$\Longleftrightarrow \frac{\eta_{s,t}^{\eta-1}}{\lambda_{s,t}} = \frac{W_{s,t}}{P_t}$$

$$\frac{\partial L}{\partial B_{s,t}} = -\beta_s^t \lambda_{s,t} \frac{1}{P_t} + \beta_s^{t+1} (r_t) E_t \left[\frac{\lambda_{s,t+1}}{P_{t+1}} \right] = 0$$

$$\Longleftrightarrow \lambda_{s,t} = \beta_s (r_t) E_t \left[\lambda_{s,t+1} \frac{P_t}{P_{t+1}} \right]$$

$$\frac{\partial L}{\partial \lambda_{s,t}} = -\beta_s^t \left(c_{s,t} + \frac{B_{s,t}}{P_t} + \frac{Q_t}{P_t} h_{s,t} - (r_{t-1}) \frac{B_{s,t-1}}{P_t} \right.$$

$$\left. - \frac{W_{s,t} n_{s,t}}{P_t} - \frac{Q_t}{P_t} h_{s,t-1} - \frac{D_t}{P_t} \right) = 0$$

$$\Longleftrightarrow c_{s,t} + \frac{B_{s,t}}{P_t} + \frac{Q_t}{P_t} h_{s,t} = (r_{t-1}) \frac{B_{s,t-1}}{P_t} + \frac{W_{s,t} n_{s,t}}{P_t} + \frac{Q_t}{P_t} h_{s,t-1} + \frac{D_t}{P_t}$$

- Borrowers' optimization problem

$$\max_{c_{b,t},h_{b,t},n_{b,t},B_{b,t}} L = E_0 \sum_{t=0}^{\infty} \left\{ \beta_b^t \left[\ln(c_{b,t}) + j \ln(h_{b,t}) - \frac{n_{b,t}^{\eta}}{\eta} \right] - \right.$$

$$\beta_b^t \lambda_{b,t} \left(c_{b,t} + \frac{(r_{t-1}) B_{b,t-1}}{P_t} + \frac{Q_t}{P_t} h_{b,t} - \frac{Q_t}{P_t} h_{b,t-1} - \frac{B_{b,t}}{P_t} \right.$$

$$\left. \left. - \frac{W_{b,t}}{P_t} n_{b,t} \right) - \beta_b^t \mu_t \left(\frac{r_t B_{b,t}}{P_t} - l_t E_t \left[\frac{Q_{t+1}}{P_t} h_{b,t} \right] \right) \right\}$$

First order conditions:

$$\frac{\partial L}{\partial c_{b,t}} = \beta_b^t \frac{1}{c_{b,t}} - \beta_b^t \lambda_{b,t} = 0$$

$$\iff \frac{1}{c_{b,t}} = \lambda_{b,t}$$

$$\frac{\partial L}{\partial h_{b,t}} = \beta_b^t j \frac{1}{h_{b,t}} - \beta_b^t \lambda_{b,t} \frac{Q_t}{P_t} + \beta_b^{t+1} E_t \left[\lambda_{b,t+1} \frac{Q_{t+1}}{P_{t+1}} \right] +$$

$$\beta_b^t \mu_t l_t E_t \left[\frac{Q_{t+1}}{P_t} \right] = 0$$

$$\iff \lambda_{b,t} \frac{Q_t}{P_t} = j \frac{1}{h_{b,t}} + \mu_t l_t E_t \left[\frac{Q_{t+1}}{P_t} \right] + \beta_b E_t \left[\lambda_{b,t+1} \frac{Q_{t+1}}{P_{t+1}} \right]$$

$$\frac{\partial L}{\partial n_{b,t}} = \beta_b^t (-\eta) \frac{n_{b,t}^{\eta-1}}{\eta} + \beta_b^t \lambda_{b,t} \frac{W_{b,t}}{P_t} = 0$$

$$\iff \frac{n_{b,t}^{\eta-1}}{\lambda_{b,t}} = \frac{W_{b,t}}{P_t}$$

$$\frac{\partial L}{\partial B_{b,t}} = -\beta_b^{t+1} E_t \left[\lambda_{b,t+1} \frac{(r_t)}{P_{t+1}} \right] + \beta_b^t \lambda_{b,t} \frac{1}{P_t} - \beta_b^t \mu_t (r_t) \frac{1}{P_t} = 0$$

$$\iff \frac{1}{P_t} \lambda_{b,t} = \mu_t \frac{(r_t)}{P_t} + \beta_b (r_t) E_t \left[\lambda_{b,t+1} \frac{1}{P_{t+1}} \right]$$

$$\iff \lambda_{b,t} = \mu_t (r_t) + \beta_b (r_t) E_t \left[\lambda_{b,t+1} \frac{P_t}{P_{t+1}} \right]$$

$$\frac{\partial L}{\partial \mu_t} = -\beta_b^t \left((r_t) \frac{B_{b,t}}{P_t} - l_t E_t \left[\frac{Q_{t+1}}{P_t} h_{b,t} \right] \right) = 0$$

$$\iff (r_t) \frac{B_{b,t}}{P_t} = l_t E_t \left[\frac{Q_{t+1}}{P_t} h_{b,t+1} \right]$$

$$\frac{\partial L}{\partial \lambda_t} = -\beta_b^t \left(c_{b,t} + (r_{t-1}) \frac{B_{b,t-1}}{P_t} + \frac{Q_t}{P_t} h_{b,t} - \frac{Q_t}{P_t} h_{b,t-1} \right.$$

$$\left. - \frac{B_{b,t}}{P_t} - \frac{W_{b,t}}{P_t} n_{b,t} \right) = 0$$

$$\iff c_{b,t} + (r_{t-1}) \frac{B_{t-1}^b}{P_t} + \frac{Q_t}{P_t} h_{b,t} = \frac{Q_t}{P_t} h_{b,t-1} + \frac{B_{b,t}}{P_t} + \frac{W_{b,t}}{P_t} n_{b,t}$$

- Final good producers' optimization problem

$$\max_{y_t(i)} \Pi^F = P_t y_t - \int_0^1 P_t(i) y_t(i) di$$
$$\Longrightarrow \max_{y_t(i)} \Pi^F = P_t \left[\int_0^1 y_t(i)^{\frac{\epsilon_P - 1}{\epsilon_P}} di \right]^{\frac{\epsilon_P}{(\epsilon_P - 1)}} - \int_0^1 P_t(i) y_t(i) di$$

First order condition:

$$\frac{\partial \Pi^F}{\partial y_t(i)} = P_t \frac{\epsilon_P}{\epsilon_P - 1} \left[\int_0^1 y_t(i)^{\frac{\epsilon_P - 1}{\epsilon_P}} di \right]^{\frac{\epsilon_P}{\epsilon_P - 1} - 1} \frac{\epsilon_P - 1}{\epsilon_P} y_t(i)^{\frac{\epsilon_P - 1}{\epsilon_P} - 1}$$
$$- P_t(i) = 0$$
$$\Longleftrightarrow P_t y_t^{\frac{1}{\epsilon_P}} y_t(i)^{-\frac{1}{\epsilon_P}} = P_t(i)$$
$$\Longleftrightarrow y_t(i)^{\frac{1}{\epsilon_P}} = \frac{P_t}{P_t(i)} y_t^{\frac{1}{\epsilon_P}}$$
$$\Longleftrightarrow y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon_P} y_t$$

The zero-profit condition in the final goods market due to perfect competition determines P_t:

$$y_t = \left[\int_0^1 \left\{\left[\frac{P_t(i)}{P_t}\right]^{-\epsilon_P} y_t\right\}^{\frac{\epsilon_P-1}{\epsilon_P}} di\right]^{\frac{\epsilon_P}{\epsilon_P-1}}$$
$$\iff y_t = P_t^{\epsilon_P}\left[\int_0^1 P_t(i)^{1-\epsilon_P} di\right]^{\frac{\epsilon_P}{\epsilon_P-1}} y_t$$
$$\iff P_t^{-\epsilon_P} = \left[\int_0^1 P_t(i)^{1-\epsilon_P} di\right]^{\frac{\epsilon_P}{\epsilon_P-1}} \iff P_t = \left[\int_0^1 P_t(i)^{1-\epsilon_P} di\right]^{\frac{1}{1-\epsilon_P}}$$

- Intermediate goods producing firms' optimization problem

$$\max_{P_t(i),n_{s,t}(i),n_{b,t}(i)} \Pi^I = E_0 \sum_{t=0}^{\infty}\left\{\beta_s^t\lambda_{s,t}\left[\frac{P_t(i)}{P_t}y_t(i) - \frac{W_{s,t}}{P_t}n_{s,t}(i)\right.\right.$$
$$\left. -\frac{W_{b,t}}{P_t}n_{b,t}(i) - \frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)^2 y_t\right] +$$
$$\left.\xi_t\beta_s\lambda_{s,t}\left(z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha} - y_t\right)\right\}$$
$$\Longrightarrow \max_{P_t(i),n_{s,t}(i),n_{b,t}(i)} \Pi^I = E\sum_{t=0}^{\infty}\left\{\beta_s^t\lambda_{s,t}\left[\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon_P} y_t - \right.\right.$$
$$\left.\frac{W_{s,t}}{P_t}n_{s,t}(i) - \frac{W_{b,t}}{P_t}n_{b,t}(i) - \frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)^2 y_t\right] -$$
$$\left.\beta_s^t\lambda_{s,t}\xi_t\left[\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_P} y_t - z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}\right]\right\}$$

First order conditions:

$$\frac{\partial \Pi^I}{\partial P_t(i)} = \beta_s^t \lambda_{s,t} \left[(1-\epsilon_P) \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon_P} \frac{1}{P_t} y_t - \phi \left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right) \right.$$
$$\left. \frac{1}{\pi P_{t-1}(i)} y_t \right] + \beta_s^{t+1} \lambda_{s,t+1} E_t \left[\phi \left(\frac{P_{t+1}(i)}{\pi P_t(i)} - 1 \right) \right.$$
$$\left. \frac{P_{t+1}(i)}{\pi P_t(i)^2} y_{t+1} \right] + \beta_s^t \lambda_{s,t} \xi_t \left[\epsilon_P \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_P - 1} \frac{1}{P_t} y_t \right] = 0$$

$$\Longleftrightarrow \phi \lambda_{s,t} \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right] \frac{P_t}{\pi P_{t-1}(i)} = \lambda_{s,t} (1-\epsilon_P) \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon_P} +$$
$$\lambda_{s,t} \xi_t \epsilon_P \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_P - 1} + \beta_s \phi E_t$$
$$\left\{ \lambda_{s,t+1} \left[\frac{P_{t+1}(i)}{\pi P_t} - 1 \right] \left[\frac{P_{t+1}(i) P_t}{\pi P_t(i)^2} \right] \frac{y_{t+1}}{y_t} \right\}$$

$$\frac{\partial \Pi^I}{\partial n_{s,t}(i)} = -\beta_s^t \lambda_{s,t} \frac{W_{s,t}}{P_t} + \beta_s^t \lambda_{s,t} \xi_t \alpha z_t n_{s,t}(i)^{\alpha-1} n_{b,t}(i)^{1-\alpha} = 0$$
$$\iff \frac{W_{s,t}}{P_t} n_{s,t}(i) = \alpha \xi_t z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}$$
$$\frac{\partial \Pi^I}{\partial n_{b,t}(i)} = -\beta_s^t \lambda_{s,t} \frac{W_{b,t}}{P_t} + \beta_s^t \lambda_{s,t} \xi_t z_t (1-\alpha) n_{s,t}(i)^{\alpha} n_{b,t}(i)^{-\alpha} = 0$$
$$\iff \frac{W_{b,t}}{P_t} n_{b,t}(i) = (1-\alpha) \xi_t z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}$$
$$\frac{\partial \pi^I}{\partial \xi_t} = y_t(i) - z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha} = 0$$
$$\iff y_t(i) = z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}$$
$$\frac{\partial \Pi^I}{\partial \xi_t} = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_P} y_t - z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha} = 0$$
$$\iff \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_P} y_t = z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}$$

- Since all firms face the same demand given the aggregate output level y_t and the price index P_t, every firm has the same marginal costs ξ_t and consequently sets the same price, so that $P_t(i) = P_t$. The pricing equation in terms of gross inflation $\pi_t = \frac{P_t}{P_{t-1}}$ yields:

$$\phi_P \left[\frac{\pi_t}{\pi} - 1\right] \frac{\pi_t}{\pi} = (1-\epsilon_P) + \xi_t \epsilon_P + \beta_s \phi_P E_t \left\{ \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \left[\frac{\pi_{t+1}}{\pi} - 1\right] \left[\frac{\pi_{t+1}}{\pi}\right] \frac{y_{t+1}}{y_t} \right\}$$

In a zero-inflation environment, the log-linearization of the pricing equation around its steady state, which is $\xi =$

$\frac{\epsilon_P - 1}{\epsilon_P}$, derives the New Keynesian Philipp's curve:

$$\hat{\pi}_t = \beta_s E_t \hat{\pi}_{t+1} + \frac{(\epsilon_P - 1)}{\phi_P} \hat{\xi}_t$$

Inflation is a function of expected inflation and real marginal cost.

- Monetary policy follows a Taylor rule in setting the gross interest rate.

$$\ln\left(\frac{r_t}{r}\right) = \omega_r \ln\left(\frac{r_{t-1}}{r}\right) + (1 - \omega_r)\left(\omega_\pi \ln\left(\frac{\pi_t}{\pi}\right)\right) + v_t$$

- The macroprudential regulation authority sets the LTV ratio of borrowers according to the macroprudential rule.

$$l_t = (1 - \rho_l)l + (1 - \rho_l)\chi_l \ln\left(\frac{x_t}{x}\right) + \rho_l l_{t-1},$$

- Market clearing conditions

  - Goods market

$$y_t = c_{b,t} + c_{s,t} + \left(\frac{\phi^P}{2}\right)\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)^2 y_t$$

– Labor market

$$n_{b,t} = \int_0^1 n_{b,t}(i)di$$
$$n_{s,t} = \int_0^1 n_{s,t}(i)di$$

– Dividend market

$$D_t = \int_0^1 D_t(i)di$$

– Housing market

$$1 = h_{s,t} + h_{b,t}$$

– Bond market

$$B_{s,t} = B_{b,t}$$

- Law of Motion for the housing demand shock j_t, the technology shock z_t, and the monetary policy shock v_t:[32]

$$\ln(j_t) = (1-\rho_j)\ln(j) + \rho_j \ln(j_{t-1}) + \varepsilon_{j,t}$$
$$\ln(z_t) = (1-\rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{z,t}$$
$$\ln(v_t) = \rho_v \ln(v_{t-1}) + \varepsilon_{v,t}$$

2.A.2 Nonlinear system in symmetric equilibrium

The subsequent notation makes use of the bond market clearing condition, $B_{b,t} = B_{s,t}$. In a symmetric equilibrium applies $P_t(i) = P_t, n_{s,t}(i) = n_{s,t}, n_{b,t}(i) = n_{b,t}, D_t(i) = D_t\ \forall t \in [0,1]$. By redefining $\pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t}$, $b_{s,t} = \frac{B_{s,t}}{P_t}$, $q_t = \frac{Q_t}{P_t}$, $d_t = \frac{D_t}{P_t}$, the exposition of the system of the 21 equations for the 21 variables $\{j_t, z_t, v_t, h_{s,t}, n_{s,t}, c_{s,t}, b_{s,t}, h_{b,t}, n_{b,t}, c_{b,t}, \pi_t, w_{s,t}, w_{b,t}, y_t, q_t, l_t, d_t, r_t, \xi_t, \mu_t\}$ is given by:

$$\ln(j_t) = (1-\rho_j)\ln(j) + \rho_j \ln(j_{t-1}) + \varepsilon_{j,t} \tag{2.1}$$

$$\ln(z_t) = (1-\rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \epsilon_{z,t} \tag{2.2}$$

$$\ln(v_t) = \rho_v \ln(v_{t-1}) + \varepsilon_{vt} \tag{2.3}$$

$$\frac{1}{c_{s,t}} q_t = j_t \frac{1}{h_{s,t}} + \beta_s E_t\left[\frac{1}{c_{s,t+1}} q_{t+1}\right] \tag{2.4}$$

[32] Since I assume that a monetary policy shock affects the economy right away, the rigidity parameter ρ_v is zero and redundant.

$$c_{s,t} n_{s,t}^{\eta-1} = w_{s,t} \tag{2.5}$$

$$\frac{1}{c_{s,t}} = \beta_s r_t E_t \left[\frac{1}{c_{s,t+1}} \frac{1}{\pi_{t+1}} \right] \tag{2.6}$$

$$c_{s,t} + b_{s,t} + q_t h_{s,t} = r_{t-1} \frac{b_{s,t-1}}{\pi_t} + w_{s,t} n_{s,t} + q_t h_{s,t-1} + d_t \tag{2.7}$$

$$\frac{1}{c_{b,t}} q_t = j \frac{1}{h_{b,t}} + \mu_t l_t E_t \left[q_{t+1} \pi_{t+1} \right] + \beta_b E_t \left[\frac{1}{c_{b,t+1}} q_{t+1} \right] \tag{2.8}$$

$$c_{b,t} n_{b,t}^{\eta-1} = w_{b,t} \tag{2.9}$$

$$\frac{1}{c_{b,t}} = \mu_t(r_t) + \beta_b(r_t) E \left[\frac{1}{c_{b,t+1}} \frac{1}{\pi_{t+1}} \right] \tag{2.10}$$

$$c_{b,t} + (r_{t-1}) \frac{b_{s,t-1}}{\pi_t} + q_t h_{b,t} = q_t h_{b,t-1} + b_{s,t} + w_{b,t} n_{b,t} \tag{2.11}$$

$$r_t \frac{b_{b,t}}{E_t\left(\pi_{t+1}\right)} = l_t E_t \left[q_{t+1} h_{b,t} \right] \tag{2.12}$$

$$\phi_P \left[\frac{\pi_t}{\pi} - 1 \right] \frac{\pi_t}{\pi} = (1 - \epsilon_P) + \xi_t \epsilon_P + \beta_s \phi_P E_t \left\{ \frac{\lambda_{s,t+1}}{\lambda_{s,t}} \left[\frac{\pi_{t+1}}{\pi} - 1 \right] \left[\frac{\pi_{t+1}}{\pi} \right] \frac{y_{t+1}}{y_t} \right\} \tag{2.13}$$

$$y_t(i) = z_t n_{s,t}^{\alpha} n_{b,t}(i)^{1-\alpha} \tag{2.14}$$

$$w_{s,t} n_{s,t} = \alpha \xi_t z_t n_{s,t}^{\alpha} n_{b,t}^{1-\alpha} \tag{2.15}$$

$$w_{b,t} n_{b,t} = (1 - \alpha) \xi_t z_t n_{s,t}^{\alpha} n_{b,t}^{1-\alpha} \tag{2.16}$$

$$\ln\left(\frac{r_t}{r}\right) = \omega_r \ln\left(\frac{r_{t-1}}{r}\right) + (1 - \omega_r)\left(\omega_\pi \ln\left(\frac{\pi_t}{\pi}\right)\right) + v_t \tag{2.17}$$

$$l_t = (1-\rho_l)l + (1-\rho_l)\chi_l \ln\left(\frac{x_t}{x}\right) + \rho_x l_{t-1} \tag{2.18}$$

$$d_t = y_t - w_{s,t}n_{s,t} - w_{b,t}n_{b,t} - \frac{\phi}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 y_t \tag{2.19}$$

$$y_t = c_{s,t} + c_{b,t} - \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 y_t \tag{2.20}$$

$$1 = h_{b,t} + h_{s,t} \tag{2.21}$$

2.A.3 Steady state calculation

In the absence of three shocks $\varepsilon_{j,t} = \varepsilon_{z,t} = \varepsilon_{v,t} = 0$ for all t the model converges to the local steady state, where all variables are constant. Therefore, in steady state no adjustment costs arise in the intermediate good production sector. By setting the monetary policies inflation target to zero, the gross inflation rate in steady state is one. Variables without time index designate steady state values.

$$j = j \tag{2.1'}$$

$$z = z \tag{2.2'}$$

$$v = v \tag{2.3'}$$

$$\frac{q}{c_s} = j\frac{1}{h_s} + \beta_s \frac{q}{c_s} \tag{2.4'}$$

$$c_s n_s^{\eta-1} = w_s \tag{2.5'}$$

$$\frac{1}{c_s} = \beta_s r \frac{1}{c_s \pi} \tag{2.6'}$$

$$c_s + \frac{b_s}{r} + qh_s = \frac{b_s}{\pi} + w_s n_s + qh_s + d \tag{2.7'}$$

$$\frac{q}{c_b} = \frac{j}{h_b} + \mu l q + \beta_b \frac{q}{c_b} \tag{2.8'}$$

$$c_b n_b^{\eta-1} = w_b \tag{2.9'}$$

$$\frac{1}{c_b} = \frac{\mu r}{\pi} + \beta_b \frac{r}{c_b \pi} \tag{2.10'}$$

$$c_b + \frac{b_s}{\pi} + qh_b = qh_b + \frac{b_b}{r} + w_b n_b \tag{2.11'}$$

$$\frac{b_s}{\pi} = lqh_b \tag{2.12'}$$

$$(\epsilon_P - 1) = \xi \epsilon_P \tag{2.13'}$$

$$y = z n_s^{\alpha} n_b^{1-\alpha} \tag{2.14'}$$

$$w_s n_s = \alpha \xi y \tag{2.15'}$$

$$w_b n_b = (1 - \alpha) \xi y \tag{2.16'}$$

$$\pi = 1 \tag{2.17'}$$

$$l = l \tag{2.18'}$$

$$d = y - w_s n_s - w_b n_b \tag{2.19'}$$

$$y = c_s + c_b \tag{2.20'}$$

$$1 = h_s + h_b \tag{2.21'}$$

Equation (2.6') forms to:

$$r = \frac{\pi}{\beta_s} \tag{2.22}$$

Rewriting (2.10') with (2.22) solves to:

$$\mu = \frac{1}{c_b} (\beta_s - \beta_b) \tag{2.23}$$

Rewriting (2.12') yields:

$$h_b = \frac{r b_b}{\pi q k} \tag{2.24}$$

Plugging (2.24) into (2.10') results in:

$$\frac{q}{c_b} = \frac{j}{h_b} + \frac{1}{c_b} (\beta_s - \beta_b)\, lq + \beta_b \frac{q}{c_b} \tag{2.25}$$

Plugging (2.23) into (2.8') adds up to:

$$\frac{q}{c_b} = \frac{j}{h_b} + lq\frac{1}{c_b} (\beta_s - \beta_b) + \beta_b \frac{q}{c_b} \tag{2.26}$$

Equation (2.13') simplifies to:

$$\xi = \frac{(\epsilon_P - 1)}{\epsilon_P} \tag{2.27}$$

Substituting (2.27) in (2.15') yields:

$$n_s = \alpha y \frac{(\epsilon_P - 1)}{\epsilon_P w_s} \tag{2.28}$$

Substituting (2.27) into (2.16’) results in:

$$n_b = (1-\alpha)y\frac{(\epsilon_P - 1)}{\epsilon_P w_b} \tag{2.29}$$

Plugging (2.28) and (2.22) into (2.7’) delivers:

$$c_s + \frac{b_s\beta_s}{\pi} = \frac{b_s}{\pi} + \alpha y\frac{(\epsilon_P - 1)}{\epsilon_P} + d \tag{2.30}$$

Equation (2.29) combined with (2.22) and plugged into (2.11’) produces:

$$c_b + \frac{b_b}{\pi} = \frac{b_b\beta_s}{\pi} + (1-\alpha)\frac{(\epsilon_P - 1)}{\epsilon_P}y \tag{2.31}$$

Solving (2.31) for b_b results in:

$$b_b = \frac{c_b - (1-\alpha)\frac{(\epsilon_P-1)}{\epsilon_P}y}{\frac{\beta_s}{\pi} - \frac{1}{\pi}} \tag{2.32}$$

Taking (2.24), (2.8’) and (2.32) yields the demand curve:

$$\frac{1}{c_b} = \frac{j\pi k}{\frac{c_b-(1-\alpha)\frac{(\epsilon_P-1)}{\epsilon_P}y}{\frac{\beta_s}{\pi}-\frac{1}{\pi}}} + \frac{1}{c_b}(\beta_s - \beta_b)k + \beta_b\frac{1}{c_b} \tag{2.33}$$

Inserting (2.5’) in (2.28) results in:

$$n_s = \left(\frac{\alpha y\frac{(\epsilon_P-1)}{\epsilon_P}}{c_s}\right)^{\frac{1}{\eta}} \tag{2.34}$$

Likewise, inserting (2.9’) in (2.29) results in:

$$n_b = \left(\frac{(1-\alpha) y^{\frac{(\epsilon_P - 1)}{\epsilon_P}}}{c_b} \right)^{\frac{1}{\eta}} \tag{2.35}$$

Equation (2.34) and (2.35) in (2.14’) yield the supply curve:

$$y = c_b + c_s = z \left(\frac{\alpha (c_s + c_b)^{\frac{(\epsilon_P - 1)}{\epsilon_P}}}{c_s} \right)^{\frac{\alpha}{\eta}} \left(\frac{(1-\alpha)(c_s + c_b)^{\frac{(\epsilon_P - 1)}{\epsilon_P}}}{c_b} \right)^{\frac{1-\alpha}{\eta}} \tag{2.36}$$

Given the demand and the supply curve of the model, one can solve the two equations for the two unknowns $c_{b,t}$ and $c_{s,t}$ and compute the steady states.

2.A.4 Log-linearized equations

For the linearization of the system, I take the natural logarithm of all variables to make use of the approximation that $\ln\left(\frac{x_t}{x}\right) \approx \frac{x_t - x}{x}$. Let $\ln\left(\frac{x_t}{x}\right) = \hat{x}_t$ be the variable’s x_t deviation from its steady state x. Next, I apply a first order Taylor expansion to the system around the steady state.

$$\hat{j}_t = \rho_j \hat{j}_{t-1} + \varepsilon_{j,t} \tag{2.1”}$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t} \tag{2.2”}$$

$$\hat{v}_t = \rho_v \hat{v}_{t-1} + \varepsilon_{v,t} \tag{2.3”}$$

$$\hat{q}_t - \hat{c}_{s,t} = (1-\beta_s)\left(\hat{j}_t - \hat{h}_{s,t}\right) + \beta_s E_t \left[\hat{q}_{t+1} - \hat{c}_{s,t+1}\right] \tag{2.4’}$$

$$\hat{c}_{s,t} + (\eta - 1)\hat{n}_{s,t} = \hat{w}_{s,t} \tag{2.5''}$$

$$E_t\left[\hat{c}_{s,t+1}\right] + E_t[\hat{\pi}_{t+1}] - \hat{c}_{s,t} = \hat{r}_t \tag{2.6''}$$

$$\pi c_s \hat{c}_{s,t} + \pi b_s \hat{b}_{s,t} = \pi q h_s \left(\hat{h}_{s,t-1} - \hat{h}_{s,t}\right) + r b_s \left(\hat{r}_{t-1} + \hat{b}_{s,t-1} - \hat{\pi}_t\right) + \pi w_s n_s \left(\hat{w}_s + \hat{n}_s\right) + \pi d\hat{d}_t \tag{2.7''}$$

$$q h_b \left(\hat{q}_t - \hat{c}_{b,t}\right) = j c_b \left(-\hat{h}_{b,t} + \hat{j}\right) + c_b h_b \mu l q \pi \left(\hat{\mu}_t + \hat{\pi}_{t+1} + \hat{l}_t\right) + h_b q \left(c_b \mu l \pi + \beta_b\right) \hat{q}_{t+1} - \beta_b q h_b E_t \hat{c}_{b,t+1} \tag{2.8''}$$

$$\hat{c}_{b,t} + (\eta - 1)\hat{n}_{b,t} = \hat{w}_{b,t} \tag{2.9''}$$

$$E_t \hat{c}_{b,t+1} + E_t \hat{\pi}_{t+1} - \hat{c}_{b,t} = (1 - \beta_b \beta_s)\hat{\mu}_t + \beta_s \beta_b \hat{r}_t \tag{2.10''}$$

$$c_b \pi \hat{c}_{b,t} + (r) b_s \left(\hat{r}_t + \hat{b}_{s,t-1} - \hat{\pi}_t\right) = q \pi h_b \left(\hat{h}_{b,t-1} - h_{b,t}\right) + b_s \hat{b}_{s,t} + \pi w_b n_b \left(\hat{w}_{b,t} + \hat{n}_{b,t}\right) \tag{2.11''}$$

$$\hat{b}_{s,t} - E_t \hat{\pi}_{t+1} + \hat{r}_t = \hat{l}_t + \hat{q}_{t+1} + \hat{h}_{b,t} \tag{2.12''}$$

$$\hat{\pi}_t = \frac{(\epsilon_P - 1)}{\phi_P} \hat{\xi}_t + \beta_s E_t \hat{\pi}_{t+1} \tag{2.13''}$$

$$\hat{y}_t = \hat{z}_t + \alpha \hat{n}_{s,t} + (1 - \alpha)\hat{n}_{b,t} \tag{2.14''}$$

$$\hat{\xi}_t = \hat{w}_{b,t} + \alpha(\hat{n}_{b,t} - \hat{n}_{s,t}) - \hat{z}_t \tag{2.15''}$$

$$\hat{\xi}_t = \hat{w}_{s,t} + (1 - \alpha)(\hat{n}_{s,t} - \hat{n}_{b,t}) - \hat{z}_t \tag{2.16''}$$

$$\hat{r}_t = \omega_r \hat{r}_{t-1} + (1 - \omega_r)\left(\omega_\pi \hat{\pi}_t\right) + \hat{v}_t \tag{2.17''}$$

$$\hat{l}_t = (1 - \rho_l)\chi_l \hat{q}_t + \rho_l \hat{l}_{t-1} \tag{2.18''}$$

$$d\hat{d}_t = y\hat{y}_t - w_s n_s \hat{w}_{s,t} - n_s w_s \hat{n}_{s,t} - w_b n_b \hat{w}_{b,t} - w_b n_b \hat{n}_{b,t} \tag{2.19''}$$

$$y\hat{y}_t = c_s \hat{c}_{s,t} + c_b \hat{c}_{b,t} \tag{2.20''}$$

$$0 = h_b \hat{h}_{b,t} + h_s \hat{h}_{s,t} \tag{2.21”}$$

2.A.5 Transformed system

The budget constraint of the savers is redundant due to Walras' law, why I leave it out for the following calculation. To solve the model I bring the log-linearized equation in the state-space representation:

$$AE_t(x_{t+1}) = Bx_t + Cv_t, \tag{2.37}$$

whereas predetermined and non-predetermined variables are in
$x_t = [\hat{b}^s_{t-1} \quad \hat{h}_{b,t-1} \quad \hat{r}_{t-1} \quad \hat{h}_{s,t-1} \quad \hat{n}_{b,t} \quad \hat{n}_{s,t} \quad \hat{w}_{b,t} \quad \hat{w}_{s,t} \quad \hat{d}_t \quad \hat{\mu}_t \quad \hat{\xi}_t \quad \hat{l}_t \quad \hat{c}_{s,t} \quad \hat{c}_{b,t} \quad \hat{y}_t \quad \hat{q}_t \quad \hat{\pi}_t]'$
$v_t = [\hat{z}_t \quad \hat{j}_t \quad \hat{\upsilon}_t]'$ and $\varepsilon_t = [\varepsilon_{z,t} \quad \varepsilon_{j,t} \quad \varepsilon_{v,t}]'$
v_t represents the structural shocks which follow:

$$v_t = Pv_{t-1} + \varepsilon_t, \tag{2.38}$$

whereas

$$P = \begin{pmatrix} \rho_z & 0 & 0 \\ 0 & \rho_j & 0 \\ 0 & 0 & \rho_v \end{pmatrix} \tag{2.39}$$

$$
A = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1-\beta_s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_s & 0 & 0 & \beta_s & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & -jc_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_b h_b \mu l q \pi & 0 & 0 & 0 & -\beta_b q h_b & 0 & q h_b (c_b \mu l \pi + \beta_b) & c_b h_b \mu l q \pi \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\beta_s \beta_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
-b_s & q\pi h_b & r b_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -(1-\rho_l)\chi_l & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & h_b & 0 & h_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\tag{2.40}
$$

$$
B = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_s & -c_b & y & 0 & 0 \\
0 & 0 & 0 & (1-\beta_s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & (\eta-1) & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & c_b\mu q l\pi & 0 & c_b h_b \mu l q \pi & 0 & q h_b & 0 & -q h_b & 0 \\
0 & 0 & 0 & 0 & (\eta-1) & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\beta_b\beta_s) & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
-r b_s & \pi q h_b & 0 & 0 & \pi w_b n_b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -c_b\pi & 0 & 0 & r b_s \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{(\epsilon-1)}{\phi} & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & (1-\alpha) & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_l & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -w_b n_b & -n_s w_s & -w_b n_b & -w_s n_s & 0 & 0 & 0 & 0 & 0 & 0 & y & 0 & 0 \\
0 & 0 & 0 & 0 & \alpha & -\alpha & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -(1-\alpha) & (1-\alpha) & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \omega_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-\omega_r)\omega_\pi \\
0 & 0 & 0 & -h_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{2.41}
$$

$$C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -(1-\beta^s) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & jc_b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.42}$$

2.B Klein's model solution technique

This section explains the linear solution method by Klein for solving a system of linear expectational difference equations (Klein, 2000). The algorithm is the basis for the solution method built in Dynare (Villemot, 2011). Linear approximation of the model around its steady state yields the state space representation for the model:[33]

$$AE_t(x_{t+1}) = Bx_t + Cv_t$$

[33] In the appendix section 3.B the applied linear perturbation technique is outlined.

while $x_t = [\hat{b}^s_{t-1} \quad \hat{h}_{b,t-1} \quad \hat{r}_{t-1} \quad \hat{h}_{s,t} \quad \hat{n}_{b,t} \quad \hat{n}_{s,t} \quad \hat{w}_{b,t} \quad \hat{w}_{s,t} \quad \hat{d}_t \quad \hat{\mu}_t \quad \hat{\xi}_t \quad \hat{l}_t \quad \hat{c}_{s,t} \quad \hat{c}_{b,t} \quad \hat{y}_t \quad \hat{q}_t \quad \hat{\pi}_t]'$. So x_t encompasses the variables as their deviation from steady state. According to Klein (2000), the variables are partitioned into n_s predetermined and into n_c non-predetermined variables. Klein defines a predetermined variable as a variable that has an exogenously given initial value and a zero one step prediction error in period zero. In addition, Klein assumes that the prediction error up from period zero follows an exogenous martingale difference process, $\xi_t = x_{t+t} - Ex_{t+1}$. Thus, Klein's definition is a generalization of Blanchard and Kahn (1980)'s interpretation which assumes a predetermined variable has a prediction error of zero in all periods. Given $x_t = [x_{1,t} \quad x_{2,t}]'$, $x_{1,t}$ represents the $(n_s \times 1)$ vector of predetermined variables and $x_{2,t}$ is the $(n_c \times 1)$ vector of non-predetermined variables. The total number of variables is $n = n_s + n_c$. The structural shocks are $v_t = [\hat{z}_t \quad \hat{j}_t \quad \hat{\upsilon}_t]'$. The $(n \times n)$ matrices A, B and the $(n \times n_v)$ matrix C capture parameters, while n_v denotes the number of structural shocks. The shock processes of the model are summarized in:

$$v_t = Pv_{t-1} + \varepsilon_t, \tag{2.43}$$

while the auxiliary random variables of the model are $\varepsilon_t = [\varepsilon_{z,t} \quad \varepsilon_{j,t} \quad \varepsilon_{v,t}]'$ and P has the dimension (3×3) in accordance to the three shocks.

A great advantage of the solution method by Klein is that the matrix A may be singular in contrast to the alternative linear solution method by Blanchard and Kahn (Dejong and Dave, 2011). Thus, the system may exhibit static equilibrium conditions like identities and a system reduction is redundant. Klein's method relies thereby on the complex Schur-decomposition which does not require A to be invertible. The Schur-decomposition is a generalization of a QZ-factorization that allows for complex eigenvalues associated with A and B.[34] Applying the complex Schur-decomposition to the system delivers the unitary complex matrices Q, Z, so that the matrices S and T are upper triangular with diagonal elements that are assumed to be ordered in ascending absolute value moving from left to right (Juillard, 2005).[35]

$$QAZ = S \tag{2.44}$$

$$QBZ = T \tag{2.45}$$

[34] The general eigenvalue problem is defined as $A^{*}v = \lambda B^{*}v$, while v is the generalized eigenvector and λ comprises of the corresponding eigenvalues. Let $A^{*} - \lambda B^{*} = P(\lambda)$ be the linear matrix pencil. Then $\lambda(A^{*}, B^{*})$ captures the set of generalized eigenvalues of P that obey $\det(A^{*} - \lambda_i B^{*}) = 0$ with $\lambda_i \in \mathbb{C}$. If A^{*} and B^{*} are Hermitian matrices, it is important to solve for the generalized eigenvalues using the pencil (A^{*}, B^{*}) instead of $B^{-1}Av = \lambda v$ since the later is generally not Hermitian.

[35] A Hermitian matrix characterizes that the upper triangular portion of the matrix is the negative conjugate of the lower triangular portion of this matrix. So the complex square matrices Q and Z are equal to their conjugate transpose and the product of unitary complex square matrices with its conjugate transpose yields the identity matrix, $Q^{H}Q = Z^{H}Z = I$, while H denotes the Hermitian transpose.

The generalized eigenvalues of A and B are the ratios of the $\{ii\}$'s element of the matrices S and T, i.e.,

$$\lambda(A, B) = \{t_{ii}/s_{ii}\} \quad \forall i = 1, 2, ... \tag{2.46}$$

for $s_{ii} \neq 0$.[36] The arrangement of elements on the diagonal of S and T bring about that the stable eigenvalues smaller than one $|\lambda_i| < 1$ come first and the unstable eigenvalues greater than one and infinite eigenvalues come last (going from left to right).

In general, the number of unstable eigenvalues in a reduced system is critical for determining the dynamic behavior of $\{x_t\}$. For a reduced system, Blanchard and Kahn (1980) distinguishes three possibilities:

1. If the number of unstable eigenvalues is equal to the number of non-predetermined (control) variables, then an appropriate choice for the initial values of the control variables eliminates the explosive behavior. The system is saddle-path stable and yields an unique solution.

2. If more eigenvalues with explosive behavior than non-predetermined variables exist, then the system has no solution and is unstable.

3. If less eigenvalues with explosive behavior than non-pre-

[36] If A is singular, there is a $s_{ii} = 0$ for some i and $\lambda(A, B)$ has fewer than n elements. In line with Klein (2000), I assume that the missing generalized eigenvalue corresponds to an "infinite" eigenvalue for that particular i. The case $|\lambda_i| = 1$ is excluded.

determined variables exist, then for any choice for the controls the system converges to steady state, so there are infinite solutions for this stable system.

The model given in chapter 2 satisfies the Blanchard and Kahn conditions that state that the model has as many eigenvalues larger than one in modulus as it has forward looking variables. More specifically, the saddle-path stable system exhibits five explosive eigenvalues equal to the number of non-predetermined endogenous variables n_c.

Next, I seek the solution of the model which is a feedback rule relating the current endogenous variables to the state variables of the model. Following Klein (2000), the solution approach entails decoupling the system into an explosive and a non-explosive part in a first step. A second step involves solving the individual components, separately. First, I partition Z:

$$Z = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \tag{2.47}$$

The matrix Z_{11} corresponds to the non-explosive eigenvalues with the dimension $(n_s \times n_s)$ that conforms to the number of predetermined variables in $x_{1,t}$.[37] The matrices Z_{12} and Z_{21} have the dimension $(n_s \times n_c)$ and $(n_c \times n_s)$, respectively. The explo-

[37] If there are more stable eigenvalues than predetermined variables, one has more degrees of freedom when pinning down the solution (Klein, 2000). Moreover, it is possible to fix the initial value of forward looking variables to make them predetermined in the sense of Klein.

sive eigenvalues are stored in the $(n_c \times n_c)$ matrix Z_{22}. The aim of the next step is to find a upper triangular system of expectational difference equations in the auxiliary variable z_t defined as:

$$z_t = Z^H x_t \Longleftrightarrow z_t = \begin{pmatrix} Z_{11}^H & Z_{21}^H \\ Z_{12}^H & Z_{22}^H \end{pmatrix} \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix} \tag{2.48}$$

Let Z^H denote the Hermitian transpose of Z. Thus, I obtain

$$z_{1,t} = Z_{11}^H x_{1,t} + Z_{21}^H x_{2,t} \tag{2.49}$$

which contains the stable transformed variables and

$$z_{2,t} = Z_{12}^H x_{1,t} + Z_{22}^H x_{2,t} \tag{2.50}$$

which collects the unstable transformed variables. Since Z is unitary, the relations $Z^H Z = I$ and $Z^H = Z^{-1}$ are feasible. The transformation of the original equation result in:

$$AZE_t(z_{t+1}) = BZz_t + Cv_t. \tag{2.51}$$

Premultiplying the equation (2.51) by Q, I use the relation of the Schur-decomposition (2.44) to replace $A = Q'SZ^H$ and $B =$

$Q'SZ^H$. Next, partitioning the matrices S and T yields:

$$\begin{aligned} \begin{pmatrix} S_{11} & S_{12} \\ 0_{(n_c \times n_s)} & S_{22} \end{pmatrix} E_t \begin{pmatrix} z_{1,t+1} \\ z_{2,t+1} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ 0_{(n_c \times n_s)} & T_{22} \end{pmatrix} \\ \begin{pmatrix} z_{1,t} \\ z_{2,t} \end{pmatrix} + \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} C v_t. \end{aligned} \tag{2.52}$$

Here the matrices S_{11} and T_{11} have the dimension $(n_s \times n_s)$ and are invertible by assumption (modulus greater than one). S_{12} and T_{12} are of size $(n_s \times n_c)$. This upper relation corresponds to the stable part of the system. Thus, the number of predetermined variables with endogenously given initial values is equal to the number of stable eigenvalues. The lower part is unstable because the generalized eigenvalues of $\lambda(A, B)$ given by the diagonal elements of $T_{22}S_{22}^{-1}$ lie all outside the unit cycle. The matrix T_{22} is invertible because the determinant of a triangular matrix is a product of its diagonal entries and the invertibility of a square matrix is equivalent to its determinant being non-zero.

As a second step, I solve for the individual parts of the system. Reformulating the lower, unstable part and solving by forward iteration derives:

$$\begin{aligned} S_{22}E_t(z_{2,t+1}) &= T_{22}z_{2,t} + Q_2 C v_t \\ z_{2,t} &= T_{22}^{-1} S_{22} E_t(z_{2,t+1}) - T_{22}^{-1} Q_2 C v_t. \end{aligned}$$

This implies the expression for $E_t(z_{2,t+1})$:

$$E_t(z_{2,t+1}) = T_{22}^{-1}S_{22}E_t(z_{2,t+2}) - T_{22}^{-1}Q_2Cv_{t+1}.$$

Inserting in each other delivers:

$$z_{2,t} = T_{22}^{-1}S_{22}E_t(T_{22}^{-1}S_{22}E_t(z_{2,t+2}) - T_{22}^{-1}Q_2Cv_{t+1}) - T_{22}^{-1}Q_2Cv_t.$$

The first part of the equation converges to zero: $\lim_{j\to\infty}\left(T_{22}^{-1}S_{22})\right)^j E_t z_{t+j} = 0$. By making use of the shock process relation (2.43), the second part evolves according to:

$$\begin{aligned}\lim_{j\to\infty} z_{2t} &= -\sum_{j=0}^{\infty}(T_{22})^{-(j+1)}S_{22}^{j}Q_2CE_tv_{t+j}\\ \Leftrightarrow z_{2,t} &= -\sum_{j=0}^{\infty}(T_{22})^{-(j+1)}S_{22}^{j}Q_2CP^jv_t.\end{aligned}$$

For the second part to be stable, the following computation must rely on the assumption that the shocks follow a stationary vector autoregressive VAR(1) process. Recalling that $\mathrm{vec}\left((S_{22}T_{22}^{-1})^jQ_2CP^j\right) = ((P^j)'\otimes(S_{22}T_{22}^{-1})^j)\mathrm{vec}(Q_2C)$ and that for the diagonal matrix $P' = P$ applies, the equation (2.53) expresses the non-explosive behavior of $z_{2,t}$ which only depends on v_t:

$$z_{2,t} = -T_{22}^{-1}Rv_t \tag{2.53}$$

with

$$\begin{aligned}
\text{vec}(R) &= \text{vec} \sum_{j=0}^{\infty} (S_{22} T_{22}^{-1})^j Q_2 C P^j \\
&= \sum_{j=0}^{\infty} \text{vec} \left[(S_{22} T_{22}^{-1})^j Q_2 C P^j \right] \\
&= \sum_{j=0}^{\infty} [P^j \otimes (S_{22} T_{22}^{-1})^j]^{-1} \text{vec}(Q_2 C).
\end{aligned}$$

The last step of computing vec(R) entails applying the geometric series, which is applicable because $\frac{1}{t_{ii}/s_{ii}}$ of the unstable generalized eigenvalues captured in the matrix $S_{22} T_{22}^{-1}$ is smaller than one and hence stable.

$$\text{vec}(R) = [I_{(n \cdot n_v) \times (n \cdot n_v)} - P \otimes (S_{22} T_{11}^{-1})]^{-1} \text{vec}(Q_2 C)$$

Next, I recall the unstable part of the system (2.50) and plug in (2.53) to solve the expression for a stable solution:

$$x_{2,t} = -(Z_{22}^H)^{-1} (T_{22}^{-1} R v_t) - (Z_{22}^H)^{-1} Z_{12}^H x_{1,t}. \tag{2.54}$$

The last equation ensures that the system is saddle-path stable and that a unique solution (unique policy function) to the model exists. Since Z is unitary, the following relation holds:

$$\begin{pmatrix} Z_{11}^H & Z_{21}^H \\ Z_{12}^H & Z_{22}^H \end{pmatrix} \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} = \begin{pmatrix} I_{(n_s \times n_s)} & 0 \\ 0 & I_{(n_c \times n_c)} \end{pmatrix}. \tag{2.55}$$

From $Z_{12}^H Z_{11} + Z_{22}^H Z_{21} = 0$ follows that

$$(Z_{22}^H)^{-1} Z_{12}^H Z_{11} Z_{11}^{-1} = -(Z_{22}^H)^{-1} Z_{22}^H Z_{21} Z_{11}^{-1} \tag{2.56}$$

$$\Leftrightarrow (Z_{22}^H)^{-1} Z_{12}^H = -Z_{21}(Z_{11})^{-1}. \tag{2.57}$$

Taking $Z_{12}^H Z_{12} + Z_{22}^H Z_{22} = I$ and multiplying it by $(Z_{22}^H)^{-1}$ results in:

$$(Z_{22}^H)^{-1} Z_{12}^H Z_{12} + (Z_{22}^H)^{-1} Z_{22}^H Z_{22} = (Z_{22}^H)^{-1}. \tag{2.58}$$

Using the result of equation (2.56) transforms equation (2.58) to:

$$-Z_{21} Z_{11}^{-1} Z_{12} + Z_{22} = (Z_{22}^H)^{-1}. \tag{2.59}$$

With the relations of equations (2.58) and (2.59), I reformulate the solution for the unstable part (2.54):

$$x_{2,t} = Z_{21} Z_{11}^{-1} x_{1,t} - M T_{22}^{-1} R v_t \tag{2.60}$$

with $M = Z_{22} - Z_{21} Z_{11}^{-1} Z_{12}$.

In the next step, I seek the solution for the stable part. Therefore, I plug equation (2.60) into the equation for the transformed stable variables (2.49):

$$z_{1,t} = (Z_{11}^H + Z_{21}^H Z_{21} Z_{11}^{-1}) x_{1,t} - Z_{21}^H M T_{22}^{-1} R v_t. \tag{2.61}$$

From $Z_{11}^H Z_{11} + Z_{21}^H Z_{21} = I$, I derive the relation:

$$Z_{11}^H + Z_{21}^H Z_{22} Z_{11}^{-1} = Z_{11}^{-1}. \tag{2.62}$$

I reduce $Z_{21}M = Z_{21}(Z_{22} - Z_{21}^H Z_{11}^{-1} Z_{12})$ to $-Z_{11}^H Z_{12} - (I - Z_{11}^H Z_{11}) Z_{11}^{-1} Z_{12} = -Z_{11}^{-1} Z_{12}$ using $Z_{11}^H + Z_{21}^H Z_{21} = I$ and $Z_{11}^H Z_{12} = -Z_{21}^H Z_{22}$, whereby the equation (2.61) simplifies to:

$$z_{1,t} = Z_{11}^{-1} x_{1,t} + Z_{11}^{-1} Z_{12} T_{22}^{-1} R v_t. \tag{2.63}$$

I take the first row of (2.52) and insert the results for $z_{1,t}$ and $z_{2,t}$ that are the equations (2.63) and (2.53), respectively:

$$\begin{gathered} S_{11}(Z_{11}^{-1} x_{1,t+1} + Z_{11}^{-1} Z_{12} T_{22}^{-1} R v_{t+1}) + S_{12}(-T_{22}^{-1} R v_{t+1}) = \\ T_{11}(Z_{11}^{-1} x_{1,t} + Z_{11}^{-1} Z_{12} T_{22}^{-1} R v_t) + T_{12}(-T_{22}^{-1} R v_t) + Q_1 C v_t. \end{gathered} \tag{2.64}$$

Finally, I define an equation that depends only on predetermined variables:

$$x_{1,t+1} = N x_{1,t} + L v_t \tag{2.65}$$

with $N = Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1}$ and $L = Z_{11} S_{11}^{-1}(T_{11} Z_{11}^{-1} Z_{12} T_{22}^{-1} R + S_{12} T_{22}^{-1} R P + Q_1 C - T_{12} T_{22}^{-1} R) - Z_{12} T_{22}^{-1} R P$. The term in front of the shocks v_t follows the mentioned serially uncorrelated stochastic process of the form $v_t = P v_{t-1}$. The model's solution is the policy equation and the transition equation given by:[38]

$$s_{t+1} = \Gamma_0 s_t + \Gamma_1 \varepsilon_{t+1} \tag{2.66}$$

$$f_t = \Gamma_2 s_t \tag{2.67}$$

where $s_t = [\hat{b}^s_{t-1} \quad \hat{h}_{bt-1} \quad \hat{r}_{t-1} \quad \hat{h}_{s,t} \quad \hat{n}_{b,t} \quad \hat{n}_{s,t} \quad \hat{w}_{b,t} \quad \hat{w}_{s,t}$ $\hat{d}_t \quad \hat{\mu}_t \quad \hat{\xi}_t \quad \hat{l}_t \quad \hat{z}_t \quad \hat{j}_t \quad \hat{v}_t]'$. The vector $s_t = [s_{1,t} \quad s_{2,t}]'$ collects the $s_{1,t}$ endogenous predetermined variables and $s_{2,t}$ are exogenous predetermined variables. The vector f_t captures the controls $f_t = [\hat{c}_{s,t} \quad \hat{c}_{b,t} \quad \hat{y}_t$
$\hat{q}_t \quad \hat{\pi}_t]'$ and $\varepsilon_t = [\varepsilon_{z,t} \quad \varepsilon_{j,t} \quad \varepsilon_{v,t}]'$, where

$$\Gamma_0 = \begin{pmatrix} N & L \\ 0_{(3\times 12)} & P \end{pmatrix}$$

$$\Gamma_1 = \begin{pmatrix} 0_{(12\times 3)} \\ I_{(3\times 3)} \end{pmatrix}$$

$$\Gamma_2 = \left(Z_{21}Z_{11}^{-1} \quad -(Z_{12} - Z_{21}Z_{11}^{-1})T_{22}^{-1}R\right).$$

[38]The equations are sometimes also called state and observation equation, respectively.

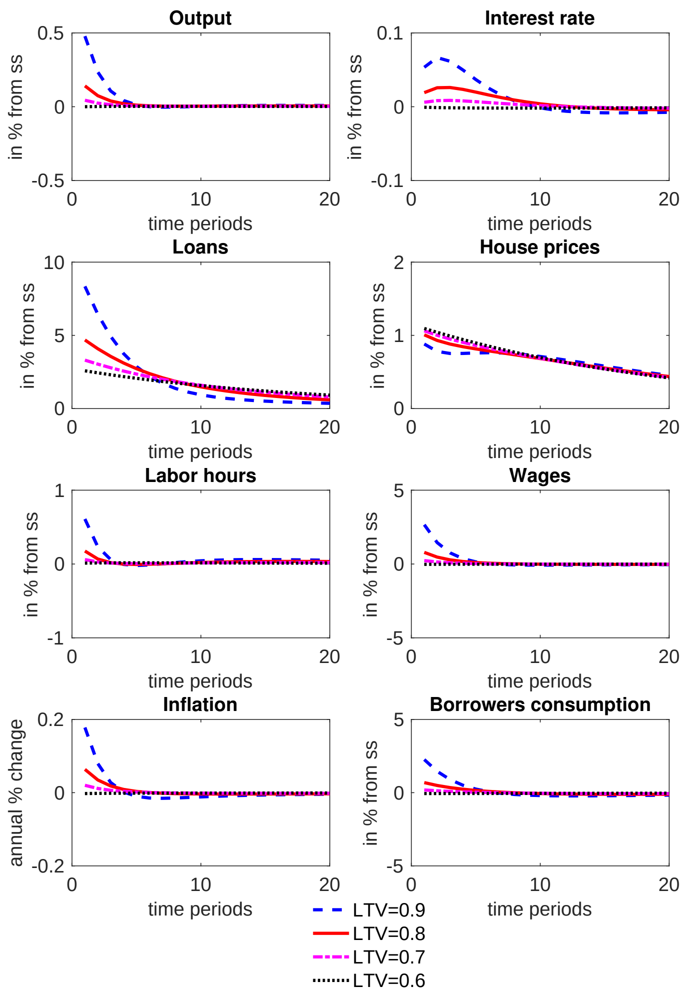
2.C On the amplification effect of collateral constraints

2.C.1 Amplification effect of lax credit constraints in detail

Responses to a housing preference shock A positive housing demand shock shifts the households' taste for housing services up, whereby in all model specifications under consideration the weight of housing in the utility function rises. Figure 2.7 plots the variables' reactions of the models with modified LTV ratio from 60% to 90%. In all model versions, the shock boosts savers' and borrowers' demand for housing. Consequently, due to the higher utility returns from housing, house prices rise almost equivalent. Since the total stock of housing is fixed, the effect on housing demand of borrowers overlaps the demand of savers. As borrowers profit twice from housing wealth, their incentive is greater. First, borrowers have a higher marginal propensity to spend and consume than savers because of their discount factor. Therefore, the higher the LTV ratio, the greater is borrowers' fraction of consumption on aggregate demand. Second, borrowers use housing as collateral. Higher housing wealth enables them to increase their borrowing capacity proportional to their maximum LTV ratio. They invest additional funds from borrowing on housing, which boosts again house prices. As a second round effect the increased housing collateral value promotes borrowing again, creating a financial multiplier effect that is most powerful for the LTV ratio of 90% (Kiyotaki and Moore, 1997). Moreover, higher consumption prices reduce the real value of outstanding debt, affecting borrowers' net worth positively (Fisherian-debt deflation). This results in a positive net effect on aggregate demand that determines output (Iacoviello and Neri, 2010). The net effect of the shock on labor supply is positive, which is mainly driven by savers' decision to work more. Borrowers work less since they gain from the collateral effect in contrast to savers. The assumed complementarity between both types of workers causes the positive net effect on wages. The Taylor rule responds to the shock induced inflation by raising the nominal interest rate. Thus, debt service costs increase, but only marginally affect the borrowing decision of impatient households. Overall, the impact on output is amplified with laxer credit limits.

Responses to a technology shock The figure 2.8 plots the variables' reaction to a transitory technology shock. By raising the technology parameter up, the positive technology shock increases output. Augmented factor payments to both labor types enhance aggregate consumption independent of the LTV ratio in the model economy. Savers work more, while borrowers substitute labor for leisure due to their additional income stream

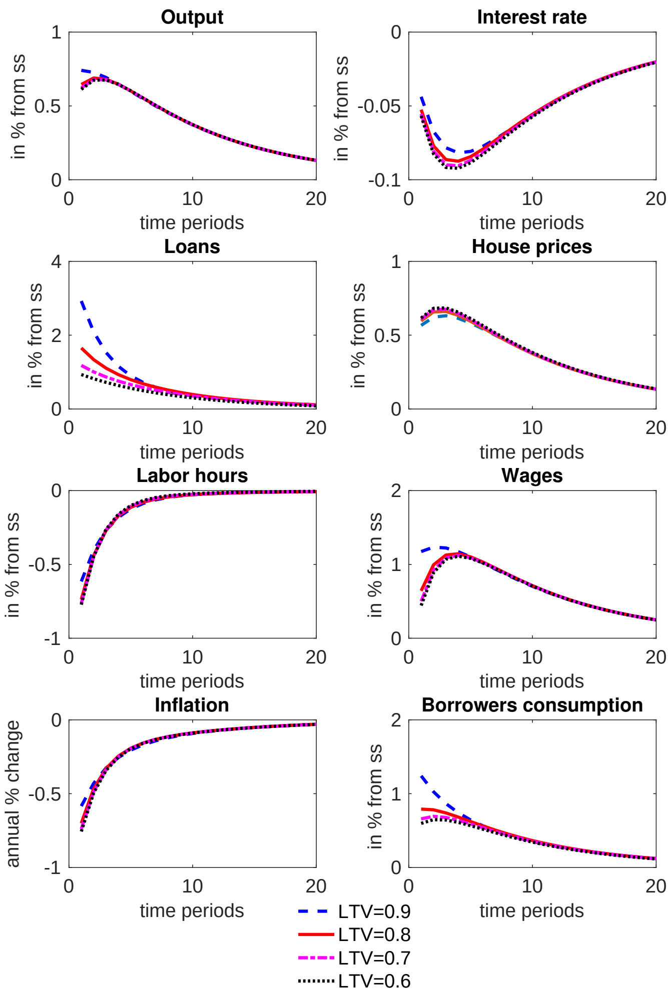
Figure 2.7: **Impulse responses of key variables to a housing demand shock** as in percentage deviation from steady state for different constant LTV ratios. The dashed line represents a LTV ratio of 90%, solid line 80%, dashed-dotted line 70% and the dotted line 60%, respectively.



from a laxer borrowing limit. With higher income, borrowers demand more housing. As consumption prices decrease, the Taylor rule lowers the nominal interest rate reducing borrowing cost. Because house prices rise after a technology shock, borrowers benefit twice from real estate holdings and borrow more, proportionally to their maximum leverage ratio. However, the level of the LTV ratio has only a minor impact on the aggregate output level as well as inflation because the effects on house prices and deflation interfere with each other and cancel each other out. As a result the collateral effect tends to be roughly zero.

Demand shocks moving house prices and inflation in the same direction, create a wealth effect for borrowers whereby output is amplified. However, the impulse response analysis after a technology shock provides no evidence, contingent on the borrowers' leverage ratio, for an amplification, what is in line with the results of Walentin (2014) and Lambertini et al. (2013). The reason seems to lie in the non-credit constrained production sector (Liu et al., 2013). The evolution of house prices determines mortgage market properties but not directly production in the given model. Hence, the evaluation stresses two insights: On the one hand, housing demand shocks shifting the collateral price and inflation in the same direction are a source for business cycle fluctuations. On the other hand, borrower's level of leverage is positively related to macroeconomic volatil-

Figure 2.8: **Impulse responses of key variables to a transitory technology shock** as in percentage deviation from steady state for different constant LTV ratios. The dashed line represents a LTV ratio of 90%, solid line 80%, dashed-dotted line 70% and the dotted line 60%, respectively.



ity. These results motivate the introduction of countercyclical macroprudential rules on a fixed LTV ratio in order to reduce the amplification effect after house price shocks.

2.C.2 Technology shocks and macroprudential regulation

The specified optimal rules in section 2.4 primarily modify the transmission of a housing demand shock as outlined, but the understanding of the transmission of a technology shock is also essential as the macroprudential authority might not be capable to identify the direct source of macroeconomic fluctuation. Figure 2.9 and figure 2.10 illustrate the variables' responses in the scenario with highly leveraged and with low leveraged borrowers, respectively. After a positive impulse to technology in both policy scenarios the hike of loans without regulation is attenuated with the countercyclical rule on credit growth and is even negative with a rule on house price growth. In both scenarios a rule on credit growth smooths borrowers' housing purchases and equivalently provides savers steady housing services because the housing stock is normalized to one. Whereas the effect is more pronounced in the high leverage scenario.[39]

In response to the technology shock, launching the regulation

[39] The induced stabilization in the housing market might be an explanation for savers' welfare gain with a rule on credit growth.

has no impact on the remaining variables regarding the algebraic sign. In the low leverage scenario the reaction of inflation, output, savers consumption, and the interest rate are almost identical to the benchmark model with a constant LTV ratio. This fact underlines the ineffectiveness of macroprudential regulation. In the lax credit limit scenario deflation is slightly more pronounced as well as the interest rate drop than in the corresponding benchmark model, so that Rubio and Carrasco-Gallego (2014) argue that macroprudential policy enters into conflict with monetary policy. That is because macroprudential policy aims to limit the provision of loans during this technological boom time, but monetary policy fuels borrowing by lowering the interest rates.[40] The net effect on output with regulation remains, however, weak in the high leverage scenario and is not detectable in the low leverage scenario. Therefore, I argue that the output costs created by macroprudential regulation during a technological boom are negligible, in particular, when compared to the trade-off of policies that arises in the tight credit scenario as a result of a housing demand shock (see section 2.4).

[40] Indeed, the policies' conflict of goals is also evident in the low leverage scenario, but in the scenario macroprudential policy shows to be in any way inefficient.

Figure 2.9: **Impulse responses to a technology shock in the model with a LTV ratio of 90%.** Variables' reaction with constant LTV ratio (solid red line), with a countercyclical LTV rule on house price (dashed blue line) and credit growth (dotted black line).

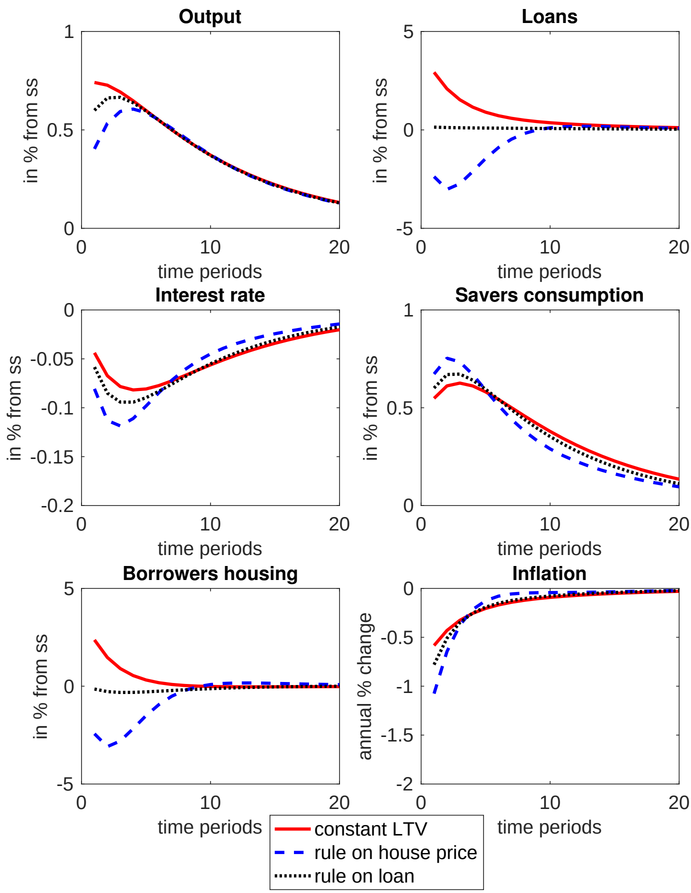
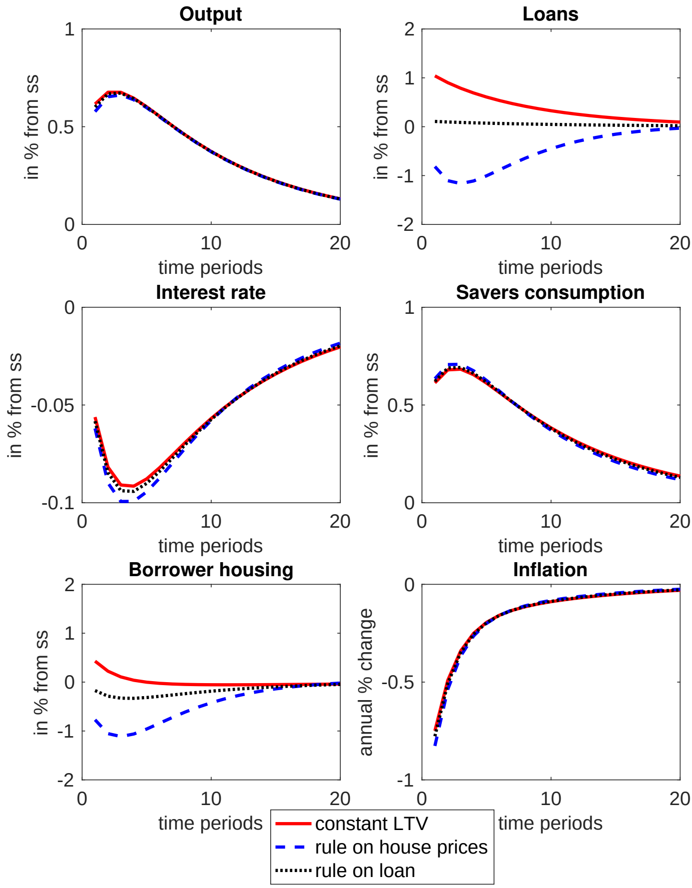


Figure 2.10: **Impulse responses to a technology shock in the model with a LTV ratio of 65%.** Variables' reaction with constant LTV ratio (solid red line), with a countercyclical LTV rule on house price (dashed blue line) and credit growth (dotted black line).



3 Financial intermediation, the mortgage market and macroprudential regulation

3.1 Introduction

During the financial crisis bank capital (BC) regulation and tight credit limits forced borrowers and banks to deleverage. By deleveraging, banks reduce the supply of credit to the economy. This disruption of financial markets amplifies the decline in economic activity (Gruss and Sgherri, 2009; Iacoviello, 2015). The aim of countercyclical macroprudential regulation is a sustainable provision of credit to the economy to ensure financial stability and a fast recovery. Several papers demonstrate that countercyclical risk-weighted BC regulation and lending standards dampen the impact of financial shocks on the macroeconomy, but seem suboptimal in response to other sources of business cycle fluctuations (Unsal, 2013; Angelini et al., 2014;

Angeloni and Faia, 2013). In contrast to these models with financial intermediation, Rubio and Carrasco-Gallego (2014) and Lambertini et al. (2013) analyze the impact of countercyclical loan-to-value (LTV) ratios on borrowers in models with heterogeneous agents without banking. This paper stands out by contrasting the effectiveness of two countercyclical instruments – a rule on the LTV ratio of borrowers and a rule on the leverage ratio of banks, i.e., the BC ratio – in one DSGE model with banking. Moreover, the paper considers the interplay of these macroprudential rules, thereby filling a gap in the literature. While the results predict that both macroprudential policy rules moderate credit volatility and improve welfare, neither rule attenuates the drop of output when the transmission proceeds through the mortgage market. Unlike countercyclical adjusted BC ratios, countercyclical LTV-regulation on borrowers effectively restores lending irrespective of the source of the adverse shock. In addition, this rule has the benefit of dampening the drop of the credit-to-output gap, thus strengthening the banks' resilience against a crisis. The best policy mix is achieved when both rules are active. These insights are essential to derive macroprudential policy implications.

The Basel III Accord obliges banks to maintain a minimum leverage ratio of three percent, defined as Tier 1 capital-to-total-exposure ratio, from 2018 onwards. This complement to the risk-weighted higher capital ratios prevents extreme bank leverage.[1] The recent macroprudential regulation changes also include upper limits for LTV ratios of mortgage borrowers to limit indebtedness. With this regulation borrowers may only borrow up to a fraction of their housing collateral value. In times of financial stress leverage caps on banks and borrowers bear the risk of banks cutting loans more than they would without the regulatory limit whereby the peril of a credit crunch increases. Thus, these frictions can amplify the effects of an adverse shock on aggregate production. The mechanism is known as the financial multiplier effect (Kiyotaki and Moore, 1997).[2] To address the procyclicality of financial frictions, the European Systemic Risk Board (ESRB) – the supervisory institution for financial stability in the EU – recommends countercyclical regulation (ESRB, 2014). Countercyclical non-risk weighted BC regulation represents an innovative extension of present macroprudential policy tools.

The aim of this paper is to assess the capability of dynamic policies for the provision of mortgage loans to households and for banking sector resilience in order to reduce the severity of a financial crisis. More precisely, I compare the impact of i) countercyclical BC ratios, and ii) the countercyclical LTV ratio,

[1] Since the beginning of 2015 banks are obliged to disclose their leverage ratio.

[2] The regulation puts pressure on highly leveraged banks with less risky assets as these also have a potential risk to default. Thus, the measure is a supplement to the risk-based capital ratios, which among others bear the risk of measurement errors and risk model manipulation.

as well as iii) their interplay relative to the situation with constant leverage ratios. In contrast to Angelini et al. (2012), who analyze how these rules reduce the volatility of key variables, conducting a welfare analysis as I do allows the identification of the beneficiaries of the regulation. Moreover, I depart from Angelini et al. (2012)'s focal point on the optimal policy mix between macroprudential and monetary policy by relying on independent policy institutions. The assumed independence of both policies is justified by Tinbergen's criteria.[3] Apart from that, I measure not only the performance of macroprudential regulation with regard to the credit aggregates as earlier studies did, but also with regard to the credit-to-output gap. The credit-to-output ratio prevails as an indicator of the build-up of financial vulnerabilities and serves as an early warning indicator of a banking crises (Drehmann, 2013).

The paper focuses exclusively on the private mortgage market for three reasons. First, there is a common consensus that the financial crisis originated in the mortgage market of households. More specifically, financial innovations of mortgage contracts lead to a build-up of systemic risk. Second, the decline of mortgage loans contributed considerably to the decline of output in the EU at the peak of the crisis (Ciccarelli et al., 2010). Third, Justiniano et al. (2015) stress that decreasing house prices that reduce households' collateral are a major driver of the reces-

[3]The Tinbergen criteria states that every policy objective requires at least one instrument.

sion.[4] In line with this finding, banks identify the lower creditworthiness of borrowers due to the higher risk perception surrounding the economic situation as a reason for tighter credit conditions rather than issues related to costs of funds and banks' balance sheet constraints (ECB, 2010, p.7.). The fact that economic expectations play a role in the appraisal of borrowers' collateral value and credit standards highlights the importance of regulation that counterbalances negative prospects as a remedy to avoid a credit crunch.

To address these issues, I extend the model of Rubio and Carrasco-Gallego (2014) using the banking sector from Gerali et al. (2010). Banks offer mortgage loans to consumers whose consumption decision affects aggregate production.[5] The representative bank's pricing power for loan and deposit rates drives the credit conditions in the economy. Retained earnings from the bank's intermediation activity build up the BC position. The regulatory leverage ratio induces the bank to limit the supply of loans when the BC ratio approaches the regulatory target level in a bust. Saving households provide the banks with financial resources that the banks pass on to borrowers who are constrained

[4] In the bank lending survey the ECB reinforces that the worsening of housing market prospects contributed to the declining loan demand (ECB, 2009a).

[5] The model set-up is similar to Justiniano et al. (2015) who also abstract from debt-constrained firms. They ignore alternative channels through which the credit cycle affects the macroeconomy in order to be able to analyze the important household debt channel in isolation.

by a maximum LTV ratio. Thus, the balance sheet position of banks and borrowers limits the availability of mortgage credit in the economy. A financial shock occurs when the quality of BC deteriorates and lending and saving interest rates decline. Consequently, the increased debt payments of borrowers cause a negative cash-flow effect on consumption and housing demand. The lower borrowing demand puts additional downward pressure on the capital position of banks. The dynamics of the model reflect the financial multiplier effect. Hence, the banks' and borrowers' leverage constraint exhibit a pecuniary externality: both agents fail to internalize that their borrowing decision forces them to deleverage during financial distress, as in Bianchi (2011). This deleveraging process reduces consumption, BC, and credit availability.[6]

The calculation of welfare maximizing policy rules documents that mild countercyclical rules are socially optimal, especially during financial disturbances: the welfare gain of borrowers through consumption smoothing with either rule compensates the welfare loss of savers. The optimal parameterization of the rules is robust to the regime of single or combined exertion,

[6] A pecuniary externality for borrowers arises because they not internalize the spill-over effects on the real economy caused by their accumulation of debt during booms (Jeanne and Korinek, 2010). Banks face a similar credit externality. During busts they sell short on assets to meet bank capital regulation. In turn the deposit interest rate decreases more than in an economy without the regulatory leverage ratio, which deepens the recession. It would be socially optimal for the bank to not exercise such high leverage in good times.

which underlines their complementarity.

Simulation results show that by introducing an optimal countercyclical LTV and BC-rule in the model, borrowers and banks avert the repercussions from a financial and even a negative technology shock. With an LTV-rule, borrowers suffering from a downgraded credit worthiness during the downturn receive alleviated access to mortgage credit due to relaxed credit standards. The countercyclical BC ratio allows the bank to provide the economy with loans in times when constant banking regulation would force them to shorten their balance sheet. The joint implementation of optimal rules successfully mitigates credit cycles and the volatility of BC. The single rule on the LTV ratio performs significantly better in providing credit than the single countercyclical BC-rule. By preventing a further tightening of credit standards, the LTV-rule also dampens the drop of the credit-to-output ratio, thereby reducing the probability of a banking crisis. I find no evidence that either rule stabilizes output due to the non-existent direct transmission from the mortgage market to the production sector. During financial stress a mild countercyclical reaction is socially optimal. Thus, the welfare gain of borrowers through the internalization of the externality compensates the welfare loss of savers. Savers lose in terms of welfare because the regulation only promotes constrained households. These gains do not accrue during technology driven business cycle fluctuations. In this situation strong countercyclical reaction delivers negligible welfare gains.

The analysis shows that in particular countercyclical LTV regulation is a powerful instrument to provide financial sector resilience as it preserves the credit-to-output-ratio during financial turmoil. These findings suggest that LTV-regulation for private mortgage loans represents a valuable complement to the stricter leverage regulation of banks with Basel III. Therefore, the implications are important for the design of a macroprudential policy framework.

The remainder of the paper is organized as follows. In the next section, I discuss the related literature. section 2 introduces the model and the two macroprudential policy instruments. Section 3 describes the calibration and the parametrization of shocks. In section 4, I conduct the welfare analysis to identify the optimal countercyclical rules that are used for a volatility analysis. Finally, I compare the model dynamics for each time-varying rule with the non-macroprudential policy outcome and discuss the results in section 5.

3.2 Related literature

The vast literature on macroprudential regulation in general equilibrium models investigates the interaction between macroprudential and monetary policy. A common finding is that in response to financial shocks, countercyclical regulation achieves financial and macroeconomic stability irrespective of the regime, but mixed results for supply shocks. In a model with a BC requirement rule, Angelini et al. (2014) document that non-coordination between policies achieves only modest stabilization benefits in booms generated by technology increases compared to sizable stabilization gains in response to financial shocks. In a framework with heterogeneous agents and a housing market, Rubio and Carrasco-Gallego (2014) find that macroprudential and monetary policy enter into conflict in response to a technology shock. The conflict arises because monetary policy lowers the key interest rate which in turn stimulates loan demand and macroprudential policy tightens credit limits. Angeloni and Faia (2013) find that mild countercyclical BC regulation and an interest rate rule reacting to financial variables helps to limit the risk taking of banks and improves welfare in risky times. Their results are based on a model in which banks are prone to runs.

Apart from above findings, there are also contradicting contribution that detect no shock-specific desirability of countercyclical regulation. Mendicino (2012) investigates the implications of countercyclical LTV ratios. Her results imply that the regulation successfully dampens credit cycles without increasing the response of output and other macroeconomic variables to real shocks. The recent paper by Lambertini and Uysal

(2015) raises doubt about the general usefulness of countercyclical risk-weighted BC regulation for macroeconomic and financial stability. Their findings indicate that after a negative capital quality shock, countercyclical BC ratios do not alter banks' incentives to reduce lending during a crisis, so that the impact on loans and output is the same as under time-invariant ratios.[7] As a positive side effect they show that the regulation forces banks to issue more stock, so that the volatility of bank variables, such as net income and net worth, and the BC ratio strengthen the banks' resilience.

Another common outcome is the trade-off of beneficiaries from macroprudential regulation. Lambertini et al. (2013) examine welfare implications for savers and borrowers of an extended interest rate rule and an LTV-rule that reacts to a range of variables. According to their results, the use of a countercyclical LTV policy in addition to an interest-rate response to credit growth is welfare improving because of the large gains that accrue to the borrowers. In the model of Rubio and Carrasco-Gallego (2014), savers directly lend to borrowers who are subject to an LTV-rule reacting to credit growth. By conducting a welfare analysis of a countercyclical rule on the LTV ratio, Rubio and Carrasco-Gallego (2014) uncover a trade-off between savers and borrowers. They find that Kaldor-Hicks transfers from borrowers to savers provide total welfare gains.

[7] They also find that steady state output falls under Basel III.

Similar to my analysis Tavman (2015) assumes independent policy institutions. Adopting the model of Gertler and Karadi (2013), he compares the effects of a rule on reserve requirement ratios, risk-weighted BC regulation, and a regulation premium. His findings imply that a countercyclical capital requirement ratio is the most effective macroprudential tool for mitigating the repercussions of the financial multiplier mechanism built into banks' endogenous capital constraints. In his set-up, which allows direct lending to the production sector, this regulation delivers the highest welfare gains and macroeconomic stabilization.

3.3 Model

The model extends the set-up of Rubio and Carrasco-Gallego (2014) with the banking sector of Gerali et al. (2010). The banking sector features endogenous BC accumulation of retained earnings, which is necessary for the analysis of a countercyclical BC measure.

3.3.1 Households

Saving households Following Iacoviello (2005), in the model patient and impatient agents differ in their discount rate. This assumption ensures a flow of funds via the financial intermediary from savers to borrowers. β_s stands for the discount factor of savers. The representative patient household derives utility from consumption $c_{s,t}$, housing services $h_{s,t}$, and leisure $(1 - n_{s,t})$, and thus maximizes the following function:

$$\max_{c_{s,t},h_{s,t},n_{s,t},D_t^s} E_0 \sum_{t=0}^{\infty} \beta_s^t \left[\ln(c_{s,t}) + j^h \ln(h_{s,t}) - \frac{n_{s,t}^{\eta}}{\eta} \right],$$

where j^h denotes the weight of housing in the utility function. The inverse of the Frisch elasticity is given by $\eta - 1$, while $\eta \geq 1$. The budget constraint of savers in real terms reads as:

$$c_{s,t} + \frac{D_t^s}{P_t} + \frac{Q_t}{P_t} h_{s,t} = \frac{D_{t-1}^s R_{t-1}^s}{P_t} + \frac{W_{s,t} n_{s,t}}{P_t} + \frac{Q_t}{P_t} h_{s,t-1} + \frac{X_t}{P_t} + (1-\tau) \frac{G_{t-1}^b}{P_t}, \tag{3.1}$$

where P_t and Q_t are consumption and house prices, respectively. Savers' expenses include consumption $P_t\, c_{s,t}$, expenditure for housing stock $Q_t h_{s,t}$, and savers' investment in deposits D_t^s at the banks, while they have the choice between $[0, 1]$ different deposit products with a maturity of one period. Savers receive funds from nominal wage income $W_{s,t} n_{s,t}$, where $W_{s,t}$ is the nominal wage rate, the stock of housing wealth of last period $Q_t h_{s,t-1}$, profits from the intermediate goods producing firm X_t, as well as dividends from the banks, $(1 - \tau)\, G_{t-1}^b$.[8] Additionally, the financial intermediary pays savers the nominal

[8]The profits of the i intermediate firms aggregate to: $X_t(i) = \int_0^1 X_t(i) di$.

gross deposit interest rate R^s_{t-1} for deposits invested last period, $t-1$.[9]

Banks obtain market power in the deposit and loan market through the introduction of Dixit and Stiglitz (1977) aggregates. Thus, savers allocate their overall amount of D^s_t assets to slightly differentiated deposit contracts $D^s_t(j)$ while the return of contract j is $R^s_t(j)$ (s denotes the specific variable for savers). Accordingly, the deposit products are aggregated by the constant elasticity of substitution technology:

$$D^s_t \geq \left(\int_0^1 (D^s_t(j))^{\frac{\epsilon_{s,t}-1}{\epsilon_{s,t}}} dj \right)^{\frac{\epsilon_{s,t}}{\epsilon_{s,t}-1}}, \tag{3.2}$$

where $\epsilon_{s,t} < -1$ governs the elasticity of substitution between the different contracts that is time varying. The deposit demand elasticity facilitates the steady state interest spread between the policy rate and the deposit rate. Savers choose $D^s_t(j)$ to maximize their financial income given by:

$$\max \int_0^1 R^s_t(j) D^s_t(j) dj$$

subject to (3.2). The resulting demand equation for product j is $D^s_t(j) = D^s_t \left(\frac{R^s_t(j)}{\lambda^{sd}}\right)^{-\epsilon_{s,t}}$. The multiplier on (3.2), λ^{sd} represents the shadow price of the households' choice for $D^s_t(j)$. It can

[9] The first order conditions from the optimization procedure yield the savers' housing demand equation, savers' labor supply equation, and savers' Euler equation, as well as the budget constraint summarized in the appendix.

be replaced by the interest rate index over all j deposit interest rates, R_t^s, so that the demand for the saving product j simplifies to:

$$D_t^s(j) = \left(\frac{R_t^s(j)}{R_t^s}\right)^{-\epsilon_{s,t}} D_t^s \tag{3.3}$$

Combining (3.2) and (3.3) yields the deposit rate index, R_t^s, which is a CES functional of the individual contract prices; $R_t^s = \left[\int_0^1 (R_t^s(j))^{1-\epsilon_{s,t}} dj\right]^{\frac{1}{1-\epsilon_{s,t}}}$. So R_t^s is simply the return the saver receives for its deposits.

Borrowing households The representative impatient household differs from savers along two dimensions. First, they do not earn dividends from either the intermediate goods producing firm or the financial intermediary. Second, their lower discount factor $\beta^b < \beta^s$ makes them into borrowers who are subject to a binding maximum LTV ratio (b denotes the specific variable for borrowers). By pledging their housing wealth $h_{b,t}$ as collateral, they borrow until the amount of debt burden is equal to the fraction of expected tomorrow's housing wealth.[10] The fraction of loan repayment including debt service $R_t^b B_t^b$ to tomorrow's housing wealth $Q_{t+1} h_{b,t}$ may not exceed the regu-

[10] Analogously, savers would face a collateral constraint if they were willing to borrow. Since this is not the case, the collateral constraint is omitted.

latory leverage ratio l:

$$l \geq \frac{\frac{R_t^b B_t^b}{P_t}}{\frac{E_t(Q_{t+1})}{P_t} h_{b,t}}. \tag{3.4}$$

Thus, the level of l represents the credit limit that is constant in the benchmark model but changes according to the counter-cyclical macroprudential rule in the policy scenarios, making it time-dependent l_t.[11] Rising house prices trigger a feed-back loop between higher collateral value and the borrowing capacity known as the financial multiplier (Kiyotaki and Moore, 1997).

Savers and borrowers, alike maximize their stream of utility through consumption $P_t c_{b,t}$, housing wealth $h_{b,t}$, leisure time $1 - n_{b,t}$, and one-period loans B_t^b from the banks:

$$\max_{c_{b,t}, h_{b,t}, n_{b,t}, B_t^s} E_0 \sum_{t=0}^{\infty} \beta_b^t \left[\ln(c_{b,t}) + j^h \ln(h_{b,t}) - \frac{n_{s,t}^{\eta}}{\eta} \right].$$

The impatient households' decision has to match the collateral constraint (3.4) and the budget constraint:[12]

$$c_{b,t} + \frac{R_{t-1}^b B_{t-1}^b}{P_t} + \frac{Q_t}{P_t} h_{b,t} = \frac{Q_t}{P_t} h_{b,t-1} + \frac{B_t^b}{P_t} + \frac{W_{b,t}}{P_t} n_{b,t}. \tag{3.5}$$

Accordingly, the borrower disburses for consumption $P_t \, c_{b,t}$,

[11] $1 - l$ is the borrower's equity. If the representative borrower defaults, this would be the collateral repossession of banks.

[12] As long as the multiplier on collateral constraint satisfies $\mu > 0$, the leverage constraint binds.

housing $Q_t h_{b,t}$, and gross reimbursement for borrowing $R^b_{t-1} B^b_{t-1}$ with R^b_{t-1} being the gross loan interest rate. Borrowers' funds comprise housing wealth from $t-1$, $Q_t h_{b,t-1}$, new loans B^b_t, and wage income $W_{b,t} n_{b,t}$ with $W_{b,t}$ denoting the nominal wage rate for borrowers.[13]

Borrowers face slightly differentiated loan contracts $B^b_t(j)$ for the price of $R^b_t(j)$. They seek a total amount of real loans B^b_t while their individual loan basket is composite of their loan contract with the time varying elasticity of substitution, $\epsilon_{b,t}$, with $\epsilon_{b,t} > 1$:

$$B^b_t \leq \left(\int_0^1 (B^b_t(j))^{\frac{\epsilon_{b,t}-1}{\epsilon_{b,t}}} dj \right)^{\frac{\epsilon_{b,t}}{\epsilon_{b,t}-1}} . \tag{3.6}$$

Thus, impatient households invest in the different loan products $B^b_t(j)$ such that they minimize their financial debt repayments subject to the loan composition technology (3.6):

$$\min \int_0^1 R^b_t(j) B^b_t(j) dj.$$

The demand for loan product j, (3.7) depends negatively on the j's loan interest rates that is put in relation to the aggregate in-

[13]The first order conditions of the borrowers' optimization problem yield borrowers demand for housing, their labor supply, borrowers' Euler equation, as well as the collateral and budget constraint listed in the appendix.

dex of loan interest rate R_t^b and borrowers' overall debt amount.

$$B_t^b(j) = \left(\frac{R_t^b(j)}{R_t^b}\right)^{-\epsilon_{b,t}} B_t^b \tag{3.7}$$

The combination of (3.6) and (3.7) proves that R_t^b is the aggregate index of loan interest rates paid by the representative borrower: $R_t^b = \left[\int_0^1 (R_t^b(j))^{1-\epsilon_{b,t}} dj\right]^{\frac{1}{1-\epsilon_{b,t}}}$.

3.3.2 Firms

Final goods producing firms The final good producer purchases $y_t(i)$ units of each intermediate good $i \in [0,1]$ for the price $P_t(i)$ to compound a homogeneous consumption good. Patient and impatient households consume that final good y_t for the price P_t. The perfectly competitive final goods producer maximizes its profit function:

$$\max_{y_t, y_t(i)} \Pi^F = P_t y_t - \int_0^1 P_t(i) y_t(i) di$$

subject to the final good production technology:

$$y_t \leq \left[\int_0^1 y_t(i)^{\frac{\epsilon_P - 1}{\epsilon_P}} di\right]^{\frac{\epsilon_P}{\epsilon_P - 1}},$$

where $\epsilon_P > 1$. The resulting demand for goods (3.8) shows that the demand for i diminishes with its price.

$$y_t(i) = \left[\frac{P_t(i)}{P_t}\right] y_t \tag{3.8}$$

The price index P_t thereby satisfies: $P_t = \left[\int_0^1 P_t(i)^{1-\epsilon_P} di\right]^{\frac{1}{1-\epsilon_P}}$.

Intermediate goods producing firms Intermediate goods producing firms are monopolistically competitive. By employing both household labor types, they produce differentiated intermediate goods $y_t(i)$ according to the production function:

$$y_t(i) \leq z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha} \tag{3.9}$$

with $1 > \alpha > 0$. The Cobb-Douglas production function with α measuring the labor income share of patient households and $1 - \alpha$ of impatient households assumes that patient and impatient households are not perfect substitutes.[14] The technology z_t follows a log AR (1) process:

$$\ln(z_t) = (1 - \rho_z) \ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{z,t}, \tag{3.10}$$

[14] This assumptions is economically justified by the fact that patient households adopt more demanding management tasks and consequently earn higher income than borrowers. The approach is based on Iacoviello and Neri (2010).

where ρ_z is the autoregressive coefficient, z is the steady state value, and $\varepsilon_{z,t}$ is a technology shock that follows a normal i.i.d. process with zero mean and standard deviation σ_{ε_z}.[15] The firms face quadratic adjustment costs à la Rotemberg if they change the price from one period to the next:

$$\frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)^2 y_t,$$

while ϕ_P is the cost parameter and π denotes steady state inflation. The firms' maximization of its total market value yields the log-linearized Phillip's curve:[16]

$$\hat{\pi}_t = \frac{(\epsilon_P - 1)}{\phi_P}\hat{\xi}_t + \beta_s E_t \hat{\pi}_{t+1}, \tag{3.11}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate, ξ_t refers to the firms' real marginal costs, and hats on the variables denote the values' deviation from the steady state ($\hat{x}_t = \frac{x_t - x}{x_t}$).

3.3.3 Banks

The microfounded banking sector captures the financial intermediation function and profit orientation of banks. At the top of the bank holding, the headquarter manages the flow of funds

[15]The notation also applies to the remaining shocks.

[16]The first order conditions of the firms' optimization problem are listed in the appendix.

from the saving to the borrowing retail branch and the representative bank's capital position (net worth of the bank). BC denoted by K_t^b accumulates out of earnings G_t^b from purchasing the j financial products of the retail branches. While the fraction τ of last period achieved and aggregated bank profits G_{t-1}^b is retained to build up the depreciating BC stock, the fraction $1-\tau$ is disbursed to the patient household as dividends. The depreciation rate δ^b captures the expenses for managing BC that comprises of labor costs and expenses for the technical banking infrastructure. Moreover, the depreciation rate captures the idea that a part of the banks' net worth disappears exogenously, e.g., due to borrowers' default (Angelini et al., 2014), or labor costs. The BC law of motion is:

$$\frac{K_t^b}{P_t} = \frac{1}{s_{kb,t}} \left((1-\delta^b) \frac{K_{t-1}^b}{P_t} + \tau \frac{G_{t-1}^b}{P_t} \right), \tag{3.12}$$

where $s_{kb,t}$ is modeled to represent the BC depreciation shock. The shock follows a log AR (1) process:

$$\ln s_{kb,t} = (1-\rho_{s_{kb}}) \ln s_{kb} + \rho_{s_{kb}} \ln s_{kb,t-1} + \varepsilon_{s_{kb,t}}, \tag{3.13}$$

where $\varepsilon_{s_{kb,t}}$ is the innovation that is $i.i.d. \sim N(0, \sigma^2_{s_{kb}})$. The shock is correlated to the margins of the bank's retail rates. The correlation to the shock on the markup of loan rates is $Cor(\varepsilon_{s_{kb}}, \varepsilon_{mk_b}) = 0.7$. Alike, the correlation between the BC and the markdown interest rate shock is given by $Cor(\varepsilon_{s_{kb}}, \varepsilon_{mk_s}) =$

0.7. This modeling approach of a financial shock generates the severe financial conditions observable during the financial crisis in 2008/09. Moreover, the headquarter ensures the banks fulfils the balance-sheet identity:

$$\frac{B_t}{P_t} = \frac{D_t}{P_t} + \frac{K_t^b}{P_t} \tag{3.14}$$

in all t. Moreover, the headquarter controls that the non-risk weighted BC ratio $\frac{K_t^b}{B_t}$ complies with the regulatory capital requirement ratio υ. This restriction depicts the newly introduced leverage ratio for banks of Basel III. Higher bank leverage is accompanied by a lower ratio. Deviations of $\frac{K_t^b}{B_t}$ from υ imply quadratic adjustment costs given by:[17]

$$\vartheta(K_t^b, B_t) = \frac{\phi_{WS}}{2}\left(\frac{\frac{K_t^b}{P_t}}{\frac{B_t}{P_t}} - \upsilon\right)^2 \frac{K_t^b}{P_t},$$

where ϕ_{WS} is the cost parameter. The macroprudential BC-rule endogenizes the ratio to υ_t, so that it adjusts countercyclically.

In the beginning of t the headquarter receives the stock of deposits D_t for the costs R_t from the deposit branch and allocates the stock of loans B_t to the loan branch for which the headquarter charges the intrabank rate R_t^{WS}. The intrabank deposit rate R_t is equal to the key policy rate in order to close

[17] The banks bears costs for deviations from the regulatory ratio that they transmit on to the prices of financial products. Thus, illegal behavior of the bank is avenged in contrast to the regulatory leverage ratio of borrowers.

the model.[18] Acting in a perfectly competitive market, the representative headquarter maximizes the discounted sum of real cash flows by choosing loans B_t and deposits D_t for t and $t+1$, subject to the banks' balance sheet constraint (3.14) and the capital accumulation relation (3.12):

$$\max_{D_t, B_t} E_0 \sum_{t=0}^{\infty} \beta_s^t \lambda_{s,t} \left[\left(R_t^{WS} \frac{B_t}{P_t} + \frac{D_{t+1}}{P_t} - R_t \right) \frac{D_t}{P_t} - \right.$$
$$\left. \frac{\phi_{WS}}{2} \left(\frac{\frac{K_t^b}{P_t}}{\frac{B_t}{P_t}} - \upsilon \right)^2 \frac{K_t^b}{P_t} - \frac{B_{t+1}}{P_t} + \left(\frac{K_{t+1}^b}{P_t} - \frac{K_t^b}{P_t} \right) \right].$$

$\lambda_{s,t}$ is the Lagrange multiplier from the savers' optimization problem and translates bank profits in terms of units of consumption.

$$R_t^{WS} = R_t - \phi_{WS} \left(\frac{K_t^b}{B_t} - \upsilon \right) \left(\frac{K_t^b}{B_t} \right)^2 \qquad (3.15)$$

The first order condition (3.15) states that deviations of the capital ratio through, e.g., expansion of borrowing, results in a premium on the policy rate, $R_t^{WS} > R_t$. In contrast, overcompliance of the ratio facilitates loan supply by lowering the interbank lending rate below the policy rate. Thus, banks issue a quantity of loans that equalizes the benefits and costs of changing υ. The headquarter's incentive is to keep the BC ratio

[18]Otherwise, if unlimited financing of banks by the central bank's lending facility is assumed, arbitrage opportunities will arise.

as close as possible to the regulatory limit. By doing so, the bank grants as many loans as possible. Because bankers do not consider the fact that by issuing more equity, they would reduce the regulation costs in the case of a negative shock to the economy, banks face a pecuniary externality.[19]

Loan products In the retail department for the bank's loan and deposit products, monopolistic competition persists, so that each developer of a loan or deposit product possesses some price setting power. The loan unit of the bank receives loans B_t from the headquarter at the interbank loan rate R_t^{WS}. The department splits them at no cost into different loan products $j \in [0, 1]$ that are under the hand of each manager. The managers in the loan unit offer the product $B_t(j)$ to impatient households for the corresponding loan rate of $R_t^b(j)$.[20] If the price of the credit product $R_t^b(j)$ changes, the manager faces quadratic adjustment costs ϕ_b, whereby loan rates are sticky. The representative loan product developer sets $R_t^b(j)$ to maximize its

[19]Most studies that evaluate countercyclical policy rules focus on risk-weighted BC ratios, which Basel III promotes, e.g., Angelini et al. (2014), Christensen et al. (2011), and Angeloni and Faia (2013).

[20]The unit's choice must satisfy: $B_t = \int_0^1 B_t(j)dj$.

profits:

$$\max_{R_t^b(j)} E_0 \sum_{t=0}^{\infty} \beta_s^t \lambda_{s,t} \left[R_t^b(j) \frac{B_t^b(j)}{P_t} - R_t^{WS} \frac{B_t(j)}{P_t} - \frac{\phi_b}{2} \left(\frac{R_t^b(j)}{R_{t-1}^b(j)} - 1 \right)^2 \right.$$
$$\left. R_t^b \frac{B_t^b}{P_t} \right]$$

subject to the demand for the loan products (3.7) and $B_t^b(j) = B_t(j)$. The first order condition (3.16) in a symmetric equilibrium implies that changes to the loan rate R_t^b in the past and future yield a surcharge that is transferred to the banks' clients by higher rates.

$$\begin{aligned} \frac{R_t^{WS} \epsilon_{b,t}}{R_t^b} = -(1 + \epsilon_{b,t}) + \phi_b \left(\frac{R_t^b}{R_{t-1}^b} - 1 \right) \frac{R_t^b}{R_{t-1}^b} - \\ \beta_s \left(\frac{c_t^s}{c_{t+1}^s} \phi_b \left(\frac{R_{t+1}^b}{R_t^b} - 1 \right) \left(\frac{R_{t+1}^b}{R_t^b} \right)^2 \frac{B_{t+1}^b P_t}{B_t^b P_{t+1}} \right) \end{aligned} \tag{3.16}$$

When no adjustment costs exist for either the capital-to-asset ratio, $R_t = R_t^{WS}$, or for loan rates, the loan rate is a markup on the policy rate, $R_t^b = \frac{\epsilon_{b,t}}{\epsilon_{b,t}-1} R_t$. The markup is defined by $mk_{b,t} = \frac{\epsilon_{b,t}}{\epsilon_{b,t}-1}$ and is assumed to follow a stochastic log AR(1) process:

$$\ln(mk_{b,t}) = (1-\rho_{mk_b}) \ln(mk_b) + \rho_{mk_b} \ln(mk_{b,t-1}) + \varepsilon_{mk_{b,t}}. \tag{3.17}$$

The markup shock $\varepsilon_{mk_{b,t}}$ thereby is $i.i.d. \sim N(0, \sigma^2_{mk_b})$.

Saving products The developers in the deposit unit of the bank offer different deposit products $D_t^s(j)$ for the deposit rate $R_t^s(j)$ to the patient households. The unit aggregates all deposit products j, as $D_t(j)$, at no cost and passes the funds as a homogenous deposit D_t to the head of the bank holding, thereby meeting $D_t = \int_0^1 D_t(j)dj$. Each product developer receives a reward R_t from the wholesale unit for his passed on product $D_t(j)$ and faces quadratic adjustment costs ϕ_S for changing the deposit rate for saver $R_t^s(j)$. Each deposit product developer maximizes his profits by choosing $R_t^s(j)$:

$$\max_{R_t^s(j)} E_0 \sum_{t=0}^{\infty} \beta_s^t \lambda_{s,t} \left[R_t \frac{D_t(j)}{P_t} - R_t^s(j) \frac{D_t^s(j)}{P_t} - \frac{\phi_s}{2} \left(\frac{R_t^s(j)}{R_{t-1}^s(j)} - 1 \right)^2 R_t^s \frac{D_t^s}{P_t} \right]$$

subject to savers' demand for deposit products (3.3) and the condition that $D_t(j) = D_t^s(j)$. After imposing symmetry, the optimization problem yields:

$$\frac{R_t \epsilon_{s,t}}{R_t^s} = (\epsilon_{s,t} - 1) - \phi_s \left(\frac{R_t^s}{R_{t-1}^s} - 1 \right) \frac{R_t^s}{R_{t-1}^s} + \beta_s \left\{ \frac{c_{t,s}}{c_{t+1,s}} \phi_s \left(\frac{R_{t+1}^s}{R_t^s} - 1 \right) \left(\frac{R_{t+1}^s}{R_t^s} \right)^2 \frac{D_{t+1}^s}{D_t^s} \frac{P_t}{P_{t+1}} \right\}. \tag{3.18}$$

The saving rate R_t^s depends on the policy rate and on the steady state degree of competition in banks' fund raising (the inverse of $\epsilon_{s,t}$) and, inversely, on how important the ϕ_s adjustment costs are. Under flexible rates and without adjustment costs, the deposit rate index is a markdown over the policy rate, $R_t^s = \frac{\epsilon_{s,t}}{\epsilon_{s,t}-1} R_t$. The markdown of the deposit rate is $mk_{s,t} = \frac{\epsilon_{s,t}}{\epsilon_{s,t}-1}$. In addition, the markdown of the deposit rate is assumed to evolve according to a log AR(1) process:

$$\ln(mk_{s,t}) = (1-\rho_{mk_s}) \ln(mk_s) + \rho_{mk_s} \ln(mk_{s,t-1}) + \varepsilon_{mk_{s,t}}. \quad (3.19)$$

The markdown shock $\varepsilon_{mk_{s,t}}$ is $i.i.d. \sim N(0, \sigma^2_{mk_s})$.

Bank profits Overall bank profits are the sum of net earnings from the two retail units minus adjustment costs:

$$\begin{aligned} G_t^b = R_t^b B_t^b - R_t^s D_t^s - \frac{\phi_{WS}}{2} \left(\frac{K_t^b}{B_t} - \upsilon \right)^2 K_t^b - \\ \frac{\phi_s}{2} \left(\frac{R_t^s(j)}{R_{t-1}^s(j)} - 1 \right)^2 R_t^s D_t^s - \frac{\phi_b}{2} \left(\frac{R_t^b(j)}{R_{t-1}^b(j)} - 1 \right) R_t^b B_t^b. \end{aligned} \quad (3.20)$$

3.3.4 Monetary and macroprudential policy

The central bank sets the interest rate according to the Taylor rule that reacts gradually to output and inflation deviations from the steady state. Steady state inflation is zero, $\pi = 1$. ω_π and ω_y measure the interest rate response to inflation and output, respectively. ρ_r is the degree of interest rate smoothing and the monetary policy shock $\varepsilon_{v,t}$ is $i.i.d.N(0, \sigma^2_{\varepsilon_{v,t}})$.

$$R_t = R^{1-\rho_r} R_{t-1}^{\rho_r} \left(\left(\frac{\pi_t}{\pi} \right)^{\omega_\pi} \left(\frac{y_t}{y} \right)^{\omega_y} \right)^{1-\rho_r} \varepsilon_{v,t} \tag{3.21}$$

In line with the recommendations of the Basel Committee on Banking Supervision for the countercyclical risk-based BC ratio, the rule on the BC ratio (3.22) reacts to changes in the credit-to-output ratio, $e_t = \frac{\frac{B_t}{P_t}}{y_t}$, from its steady state, $e = \frac{B}{y}$. The indicator captures the interaction of loan supply and loan demand as the factor is influenced by the co-movement of credit and output. Since positive deviations represent a boom phase and negative deviation a bust phase, the reaction parameter χ_υ is positive for the countercyclical adjustment of the BC ratio, $\chi_\upsilon > 0$. The Basel Committee on Banking Supervision (2010) proposes a prompt release of changes in response to financial stress in the economy, which is why I assume no rigid adjustment of the rule.

$$\upsilon_t = \upsilon \left(\frac{e_t}{e} \right)^{\chi_\upsilon} \tag{3.22}$$

The countercyclical leverage ratio (3.23) changes the credit limit of borrowers in response to deviations of the real credit aggregate, $b_t = \frac{B_t}{P_t}$, from its steady state, $b = \frac{B}{P}$. Several studies identify the credit aggregate as an efficient indicator of credit market imbalance, e.g., Lambertini et al. (2013).[21] Thus, the

[21] Several studies recommend a prompt adjustment of the ratio to the economic imbalance e.g. Körner (2016a).

countercyclical rule with $\chi_l < 0$ implies a tight credit limit in times of credit growth and relaxed credit conditions with declining lending.

$$l_t = l \left(\frac{b_t}{b} \right)^{\chi_l} \tag{3.23}$$

3.3.5 Model closing equations

In a symmetric equilibrium, the goods market is cleared according to:

$$y_t = c_{b,t} + c_{s,t} - \frac{\phi_P}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 y_t - \frac{\phi_{WS}}{2} \left(\frac{K_t^b}{B_t} - \upsilon \right)^2 K_t^b$$
$$- \frac{\phi_s}{2} \left(\frac{R_t^s}{R_{t-1}^s} - 1 \right)^2 R_t^s D_t^s - \frac{\phi_b}{2} \left(\frac{R_t^b}{R_{t-1}^b} - 1 \right)^2 R_t^b B_t^b + \delta^b \frac{K_{t-1}^B}{P_t}. \tag{3.24}$$

The housing market (3.25) clears when housing demand is equal to the fixed supply of one.

$$1 = h_{b,t} + h_{s,t} \tag{3.25}$$

Moreover, labor markets clear. The following analysis of the non-linear system is conducted using real variables denoted by small letters.[22]

[22] All first order conditions, model closing equations and laws of motion are summarized in the appendix 3.A.

3.4 Calibration

The calibrated model primarily matches Eurozone data. Calibration values are summarized in table 3.1. Where it applies, parameters are presented as quarterly numbers. The borrowers' discount factor of 0.975 assures loan demand and a binding borrowing constraint for small deviations from the steady state. The savers' discount factor with $\beta_s = 0.996$ implies a saving rate R_t^s of 1.2 percent per annum (p.a.), while with a steady state interest rate elasticity for deposits of $\epsilon_s = -150$, the policy rate is four percent p.a.. To realize a loan rate of around six percent p.a., the steady state interest rate elasticity of loan demand ϵ_b must be 200. The steady state bank leverage ratio is four percent, which is a little higher than the Basel III regulation for banks' leverage ratio. By assuming a buffer on the regulatory ratio, the approach follows Gerali et al. (2010).[23] To model an imperfect pass-through to loan interest rates, the loan rate adjustment cost parameter ϕ_b is set to 100. I pick a deposit rate adjustment cost parameter ϕ_s of 35 in line with Gerali et al. (2010).[24] Given the retail interest rates, the calibration of the steady state leverage ratio implies a BC depreciation rate δ_B of one. Full depreciation of BC from one period to the next allows

[23] When examining the risk-weights BC ratio, Gerali et al. (2010) assume a ratio of nine percent instead of the regulatory eight percent risk-weighted BC.

[24] Gerali et al. (2010) base their adjustment costs on net rates, which explains why the adjustment costs used here are proportionally higher.

to capture the extreme BC erosion during the last financial crisis in the model.[25] Based on Gerali et al. (2010), the banks' leverage deviation costs are adjusted for gross rates and is set to 100. The fraction τ of total bank profits constructs the bank's capital position, which I set to 77 percent as in Roger and Vlček (2011). Vice versa the fraction $(1 - \tau)$, so 23 percent of profits are disbursed as dividends to savers. As the ECB reports an average housing wealth to gross domestic product ratio of 313 percent (ECB, 2009b, p.20) and Musso et al. (2011) records a ratio of 240 percent for around the same time period from 2000 to 2006, I target a implied steady state housing wealth to annual gross domestic product ratio of 290 percent by setting the weight of housing in the households' utility function to $j^h = 0.06$. Similar to Rubio and Carrasco-Gallego (2015a), the parameter associated with the Frisch elasticity of labor is set to 2. The national average LTV ratios of borrowers within the EU vary from above 100 to 60 percent (Banca D'Italia, 2013). By setting the LTV ratio to 90 percent, the mortgage debt to annual gross domestic product ratio in steady state is 23 percent consistent with the estimate of 22.6 percent for outstanding housing loans to GDP in 2007 (ECB, 2009b, p.21). The price elasticity of demand ϵ_P of six affects a 20 percent markup on prices in steady state. The price adjustment cost parameter ϕ_P is calibrated to 25 which implies that firms change their

[25]This assumption regarding BC depreciation reinforces the financial multiplier by forcing banks to raise new equity in downturns.

prices every 8 months in line with empirical studies. Following the estimate of Rubio and Carrasco-Gallego (2015a) for α, the parameter governing the labor income share for savers is set to 0.64.

The Taylor rule responds to inflation deviation with an intensity of $\omega_\pi = 2$ and to output deviations with $\omega_y = 0.01$ (Gerali et al., 2010). The interest rate smoothing parameter is 0.8 as in Rubio and Carrasco-Gallego (2015a).

The standard deviation of the technology shock is set to 0.02 in order to affect a 1.5 percent change of output. The persistence of the technology shock is 0.9 in line with the estimate of Gerali et al. (2010). The financial shock is a multi-shock that pushes up the costs of loans or, to put it differently, reduces the credit availability to the real sector, as applied in Angelini et al. (2012). Through the implied correlation of 0.7, the BC depreciation shock entails a shock on the markup and markdown of the retail rates. I justify this approach by the fact that the macroprudential authority is confronted with a set of shocks that are hard to disentangle. The BC depreciation shock with a standard deviation of 0.01 is more pronounced than the shock to the bank interest rates with a standard deviation of 0.006. In line with the estimates of Gerali et al. (2010), the persistence is $\rho_{s_{kb}} = 0.81$ for the BC depreciation shock, $\rho_{mk_s} = 0.86$ for the markdown, and $\rho_{mk_b} = 0.78$ for the markup shock, respectively. The shock on BC produces an eight percent reduction of BC, followed by a 12 percent decline in borrowing. The shock provokes a drop in output of two percent.[26] The financial shock mimics quite well the severe developments after the outbreak of the crisis in 2008/09.

3.5 The implications of countercyclical regulation

3.5.1 Welfare implications of optimal policy rules

I use the log-model to quantitatively evaluate the optimal specifications of countercyclical rules on the BC and the LTV ratio measuring the rules' welfare performance. By calculating conditional welfare for the lifetime consumption stream of borrowers and savers separately for the occurrence of different shocks, I shed light on the benefits for the specific type of agent for distinct points of the business cycle. Using the conditional expected discounted lifetime utility of the representative agents, the measure of welfare takes into account the transitional effects from the non-stochastic steady state to the implied stochastic mean of the variables that is attained by each policy rule. I calculate lifetime utility recursively according

[26] For comparison, the BC quality shock in Gertler and Karadi (2013) produces a drop in output of five percent and a drop in loans of 15 percent while BC decreases by 50 percent.

to $\Omega_{i,t} = U(c_{i,t}, h_{i,t}, n_{i,t}) + \beta_i \Omega_{i,t+1}$ for $i = [s, b]$ for the two household types. Alike Mendicino and Pescatori (2008), the total social welfare is simply the weighted sum of the individual welfare gains for the different types of households weighted by their discount factor: $\Omega_{all,t} = (1 - \beta^s)\Omega_{s,t} + (1 - \beta^b)\Omega_{b,t}$.[27] A second-order approximate solution of the model around the non-stochastic steady state is necessary so that the welfare measure captures the effects of time-varying stochastic distortions on the variables' first and second moments (Schmitt-Grohé and Uribe, 2004b).[28]

To assess the implications of each policy rule and their combination, I numerically derive the welfare change with the regulation. Following this approach, the model scenario, in which l and υ are endogenous and respond to the corresponding indicator variable, is compared to the model outcome with constant l and υ. Hence, I conduct a stochastic simulation to receive the welfare level with each single rule and their combination given by $\Omega_{mp,i,t}$, while $mp = \{l_t, \upsilon_t, both\}$. Moreover, I compute the welfare level for the benchmark situation with constant LTV

[27] The weights β^s and β^b ensure that given a constant consumption stream borrowers and savers generate the same individual level of utility, so that both agents contribute equally to total welfare.

[28] Appendix section 3.B.2 describes the second order perturbation method and section 3.C provides a detailed description on computing welfare, respectively.

and BC ratio, which is given by $\Omega_{cr,i,t}$:

$$\Omega_{cr,i,t} = E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\ln\left(c_{cr,i,t}\right) + j \ln h_{cr,i,t} - \frac{n_{cr,i,t}^{\eta}}{\eta} \right) \tag{3.26}$$

The relative welfare gain from benchmark welfare to the welfare level with rule calculated according to (3.27) makes is possible to represent the welfare gain in percent of the agent's life time consumption without regulation which Λ_i captures.

$$\Omega_{mp,i,t} = E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\ln\left((1+\Lambda_i)\, c_{cr,i,t}\right) + j \ln h_{cr,i,t} - \frac{n_{cr,i,t}^{\eta}}{\eta} \right) \tag{3.27}$$

with $i = \{s, b, all\}$. Thus, the value Λ_i indicates how much a household would be willing to pay in consumption units for the regulation to be implemented because it is welfare improving. I conduct a grid search to identify the rules' reaction parameter that maximizes Λ_{all}. To avoid unrealistic high reaction values, I limit the search for the optimal χ_v to the range from 0.1 to 10, with increments of 0.1. The reaction parameter of the LTV-rule χ_l is allowed to lie between -2 to 0, with increments of 0.1. The model is simulated for each parameter value 100000 times which should suffice law of large numbers that the simulation converges to the expected value.

Table 3.1: **Calibration values.** Parameters are calibrated to match Eurozone data.

Parameter	Description	Value
β_s	savers' discount factor	0.996
β_b	borrower' discount factor	0.975
η	parameter associated with labor elasticity	2
j^h	weight of housing in utility function	0.06
l	steady state loan-to-value ratio	0.9
α	labor share of savers	0.64
ϕ_P	price adjustment costs	25
ϵ_P	price elasticity of demand	6
ρ_r	interest rate smoothing parameter in TR	0.8
ω_y	output parameter in TR	0.2
ω_π	inflation parameter in TR	2
$1-\tau$	bank dividend payout ratio	0.23
υ	capital requirement ratio	0.04
ϵ_s	steady state deposit rate elasticity	-150
ϵ_b	steady state loan rate elasticity	200
ϕ_{ws}	banks' leverage deviation costs	100
ϕ_s	saving rate adjustment costs	35
ϕ_b	loan rate adjustment costs	100
δ^b	bank capital management costs	1
ρ_z	persistence technology shock	0.9
$\rho_{s_{kb}}$	persistence capital depreciation shock	0.81
ρ_{mks}	persistence deposit markdown shock	0.86
ρ_{mkb}	persistence loan markup shock	0.78
σ_z	std technology shock	0.02
$\sigma_{s_{kb}}$	std capital shock	0.01
σ_{mkb}	std loan rate markup shock	0.006
σ_{mks}	std deposit rate markdown shock	0.006
$r^2_{bk,mks}$	corr bank capital and markdown shock	0.7
$r^2_{bk,mkb}$	corr bank capital and markup shock	0.7

Table 3.2: **Welfare maximized countercyclical rules for individual implementation** of each policy rule. The optimal parameter values for χ_l and χ_υ maximize total welfare relative to the benchmark without regulation. Gains are computed in percent of consumption equivalence units for savers and borrowers, separately, as well as for both agents.

BC-Rule	optimal χ_υ	total welfare in %	savers' welfare in %	borrowers' welfare in %
financial shock	0.1	0.104	−0.002	0.106
technology shock	7.2	0.00303	−0.00004	0.00307
both shocks	0.2	0.103	−0.001	0.104

LTV-Rule	optimal χ_l	total welfare in %	savers' welfare in %	borrowers' welfare in %
financial shock	−0.1	0.109	-0.002	0.111
technology shock	−0.9	0.00072	0.00003	0.00075
both shocks	−0.2	0.103	−0.002	0.106

The results for the single rules in table 3.3 and for the joint regime in table 3.3 provide three main insights. First, macroprudential rules generate a sizable total welfare gain in response to financial shocks in comparison to only marginal but positive gains in response to technology shocks. The welfare improvement is relatively greater for a countercyclical LTV-rule with 0.109 percent than for a countercyclical BC-rule, for which the households would pay 0.104 percent of their aggregate consumption stream. That is because the LTV-rule influences directly the lending decision of borrowers and thus stimulates credit demand in a bust. Several studies in the literature, e.g., Rubio and Carrasco-Gallego (2014); Kannan et al. (2012) provide similar evidence for the shock-specific social optimality of macroprudential regulation, while adding to the literature, I find countercyclical regulation not to be socially harmful in response to a technology shock. When the household faces the uncertainty of both shocks, the total welfare gain with either rule is 0.103 percent of the households' consumption. Considering the situation when both rules are implemented, the achieved welfare gains are higher than in the single policy rule regimes. Since the welfare analysis uncovers no reduction of the aggregate welfare level for the combined regime, the two rules appear to complement each other or at least do not interfere with each other in a way that diminishes welfare.

Second, the findings show that the welfare loss of savers with the regulation is compensated by a greater welfare gain for borrowers. Without regulation, neither banks nor borrowers anticipate that in response to a negative shock deleveraging is optimal. Since both rules are directly linked to the agents' leverage ratio, either regulation causes borrowers and banks to internalize their pecuniary externality of overborrowing generating a total welfare gain. Borrowers being the winner from macroprudential regulation coincides with the findings of Lambertini et al. (2013) and Rubio and Carrasco-Gallego (2015a). Without regulation, unconstrained savers are also able to accumulate precautionary savings to smooth consumption which explains why they bear marginal welfare losses. The results reveal that regulation leads to a constrained-efficient allocation by reducing the probability of excessive borrowing.

Third, the optimal degree of countercyclicality of the rule depends on the origin of the shock. In response to a financial shock, each rule reacts mildly– with an absolute value close to zero – to changing economic conditions, but in response to a technology shock the optimal macroprudential rule characterizes a high reaction intensity. The result remains valid for the regime with two active policy rules. The dependence of the reaction parameter on the source of the shock poses a challenge for the optimal calibration of macroprudential policy rules.

Table 3.3: **Welfare maximized countercyclical rules for joint implementation.** The optimal parameter values for χ_l and χ_υ maximize total welfare relative to the benchmark without regulation. Gains are computed in percent of consumption equivalence units for savers and borrowers, separately, as well as for both agents.

BC-rule and LTV-rule	optimal χ_l	optimal χ_υ	total welfare in %	savers' welfare in %	borrowers' welfare in %
financial shock	−0.3	0.2	0.110	−0.001	0.111
technology shock	−0.4	4.5	0.0037	−0.00001	0.0037
both shocks	−0.2	0.1	0.105	−0.001	0.106

3.5.2 Volatility analysis

The welfare-maximizing reaction parameter values in response to both shocks (last line of each table) identify the optimal policy rule in the following. Table 3.4 shows the volatility impact of financial and technology shocks on key economic model variables for the benchmark scenario with constant ratios and the three policy regimes. I find that both rules balance the standard deviation of loans in isolation compared to the situation with a constant bank leverage ratio, though only the variance reduction with the LTV-rule is significant.[29] In particular, when both rules are implemented, macroprudential policy attenuates the impact on the credit aggregate. Regarding the variance of BC, the BC-rule performs better than the LTV-rule. Indeed, with the BC-rule the bank uses internal capital for extending loans, which leads to a faster return to the initial level of BC by generating bank profits. The BC ratio is best stabilized by the combination of both rules. The rules do not have a significant effect on inflation, although a few studies as Kolasa (2016) emphasize that macroprudential policy enhances the inflation-output stabilization trade-off. In this model with a distinct mortgage market, macroprudential regulation hardly affects macroeconomic stabilization, unlike the findings of An-

[29] I conduct an F-Test comparing the empirical variances calculated by a 1000 period simulation of each regime.

gelini et al. (2014).[30] I only find a significant reduction in output volatility in the scenario where both macroprudential rules are implemented. The result does not contradict the identified welfare enhancements of borrowers with either rule since the fact that each rule makes borrowers internalize the effects of overborrowing smooths borrowers' stream of utility over time. Besides consumption, borrowers derive utility from housing services that they mainly effort through lending. To sum up, in comparison to the BC-rule the LTV-rule – alone and in combination with the BC-rule – shows to be an effective tool to mitigate credit cycles. In addition the LTV-rule contributes significantly to the health of the financial system by reducing the variance of the credit-to-output ratio.

[30] Angelini et al. (2014) document that macroprudential regulation damages aggregate activity when the economy experiences a technology shock and non-cooperation between macroprudential and monetary policy prevails.

Table 3.4: **Standard deviation of key variables with optimal macroprudential rules after financial and technology shocks.** Asteriks (*) indicate values that are significantly different from the variance in the constant ratio model on a 1 percent level.

	Standard deviation					
	loans	bank capital	capital-to-credit ratio	output	credit-to-output ratio	inflation
constant BC and LTV ratio	0.322313	0.473812	0.763547	0.045397	0.319929	0.017585
LTV-rule ($\chi_l = -0.2$)	0.249279*	0.510366	0.740980	0.045088	0.250730*	0.018581
BC-rule ($\chi_v = 0.2$)	0.311523	0.467938	0.745974	0.045401	0.305242	0.017813
LTV+BC-rule ($\chi_l = -0.2$, $\chi_v = 0.1$)	0.204603*	0.533356	0.724667	0.044990*	0.208015*	0.019026

3.6 Model dynamics and results

This section contrasts the business-cycle implications of a rule on the LTV ratio and the rule on the BC ratio with the benchmark situation with fixed leverage ratios. The parameter in the LTV-rule is optimal at $\chi_l = -0.2$, but I use a slightly higher intensity $\chi_\upsilon = 5$ than optimal for the parameter in the BC-rule in order to illustrate the rule's impact. My approach is reasonable since the volatility analysis indicates that the BC-rule with the optimal mild parametrization affects the key variables only marginal. Figure 3.1 illustrates the impulse responses of the time-invariant regulation economy (blue, solid line) to the economy with countercyclical capital requirements (black, dotted line) and an active countercyclical LTV ratio (red, dashed line).

After a one standard deviation shock on BC accompanied by a shock to the markup and markdown on retail rates, borrowing falls and BC depreciates. The benchmark model predicts that the BC ratio initially increases due to a faster drop in loans than BC. The subsequent drop of the ratio implies higher intrabank costs of lending and thus forces the representative bank to deleverage. The countercyclical BC-rule marginally mitigates that effect by allowing BC to be used for extending loans, so that borrowing drops less. The implementation of a countercyclical LTV-rule also stabilizes the banks' leverage ratio.

The regulation allows borrowers to a higher leverage position, thereby assuring loan demand. Thus, the LTV-rule prevents bank profits that build up bank capital from falling and mitigates the steep decline of loans better than a BC-rule. Moreover, the rule on the LTV ratio dampens the loss of BC, which, however, slightly prolongs the return of net worth to the initial level relative to the benchmark. The LTV-rule stabilizes the plunge of the credit-to-output ratio in contrast to the BC-rule. The spread between the loan retail rate and the policy rate rises without countercyclical regulation as well as when a LTV-rule is implemented. Nonetheless, the BC-rule attenuates the impact on the interest spread as the rule reduces the loan interest rate making banks' internal financing of loans through the bank's equity cheaper. Monetary policy reacts to deflation by lowering the key interest rate, which underlies the increase in the spread. Sticky loan interest rates prevent the lower policy rate being passed on to the retail rates.

In the set-up households' decisions transmit the unanticipated change in credit conditions with the shock to the macroeconomy. In particular, borrowers, but also savers, experience wealth destruction with the shock because of higher loan prices and reduced deposit income. In the benchmark scenario borrowers sell housing stock to savers. Savers smooth their consumption according to their Euler equation by depositing their financial means at the banks. After the shock and induced by the expansionary monetary policy, they substitute their deposits with the bank for investments in housing which pose another saving opportunity in order to smooth utility. Consequently, house prices are hump-shaped (decrease and then increase) because of savers' short-term hike of demand for housing and the exogenous stock of housing. With the countercyclical rule on the LTV ratio and thus greater propensity to borrow, impatient households' demand for housing recovers and leads to a further undesirable rise in house prices.[31] The households reduce labor hours whereby the economy's production falls, while there is no distinct difference between the alternative regimes.

Overall, the fall of the credit aggregate is less pronounced with regulation, in particular with the rule on the LTV ratio. When lending influences only households' consumption demand over the mortgage market, macroprudential regulation fails to mitigate the recession in response to a financial shock.

Figure 3.2 illustrates the impulse responses under the alternative policy regimes when the economy is subject to a negative one standard deviation technology shock. Analyzing the economy in response to this shock allows to check the performance of countercyclical regulation during "normal" business cycle fluctuations. Output and prices of consumer goods decline. The decreased house prices reduce borrowers' collateral posi-

[31]The assumed exogenous fixed supply side of the housing market causes the hump-shaped evolution of house prices. An extension of the model by a more sophisticated supply of housing would moderate this reaction.

tion. Consequently, borrowers are forced to deleverage and sell housing. The poor economic situation induces households to work more. Restrictive monetary policy due to inflation leads to a lower spread between the loan and policy rate. Since this pass-through is sticky, the higher credit costs are not immediately passed on to the loan interest rate by the banks.[32] The effect on the interest rate spread is considerably lower than under a financial shock. Borrowing decreases in the benchmark scenario and only marginally less under a BC-rule. The LTV-rule attenuates the drop of lending and stabilizes BC while the BC-rule amplifies the depreciation of BC below the benchmark reaction in order to finance loans. Again, the BC ratio increases initially and then falls below the steady state because BC depreciates more slowly than loans. The LTV-rule preserves the BC ratio better than the BC-rule since borrowing and BC drop less with the former rule.

In the model macroprudential regulation only affects financial and housing market variables after an exogenous change in technology and has no effect on output. This finding, which contradicts earlier studies, rests on the model framework by virtue of the omitted direct transmission channel to the production sector. The improved financial situation through the relaxed collateral constraint with the LTV-rule stabilizes the credit-to-output ratio in response to both shocks. Thus, the

[32] The development of the interest spread is in line with Gerali et al. (2010) who find that bank inter-meditation mitigates the impact of the shock.

LTV-rule reduces the probability of a sustained banking crisis.

3.7 Conclusion

In a model with a mortgage market, I examine the welfare effects and stabilization properties of three macroprudential policy regimes: a countercylical policy rule on the LTV ratio of borrowers, a countercyclical rule on the BC ratio of banks, and their combined implementation. My analysis is motivated by the recent introduction of a leverage ratio for banks in Basel III and the important role of the mortgage market in recent crises. In the model economy with a banking sector á la Gerali et al. (2010), borrowers are subject to a collateral constraint and banks, respectively, are constrained by regulation in issuing credit. A financial shock deteriorates the balance sheet positions of the agents, thereby triggering the financial multiplier effect. Taking monetary policy as given, I compare the impact of the introduction of the macroprudential tools to the benchmark with constant leverage ratios.

After conducting a welfare analysis for savers and borrowers separately, results show that borrowers attain a welfare gain irrespective of the rule employed. When the rule reacts mildly countercyclically to the changing economic conditions, the welfare increase of borrowers compensates the loss of savers and leads to a pareto improvement. The rationale behind it is that macroprudential regulation induces borrowers to internalize his externality of overborrowing in downturns. In response to a financial shock, both rules, separately and in combination, generate considerably higher welfare gains, but play only a marginal role for uncertainties stemming from technology.

Simulating a financial market collapse shows that the introduction of a LTV-rule effectively stabilizes the credit aggregate, while the effect of a countercyclical BC-rule on credit is relatively small. The mechanism behind it is as follows: the LTV-rule successfully restores borrowers demand for loans and thus dampens the fall of BC after the negative shock. By attenuating credit-to-output fluctuations, the regulation also strengthens the resilience of the banking sector and provides financial stability through a secured lending by banks during downturns. The combined introduction of both rules achieves the best policy outcome. Besides financial stability, I find, however, no evidence for macroeconomic stabilization with either rule when the linkage between the financial and the real sector is exclusively the private mortgage market.

By assessing the macroeconomic performance of countercyclical macroprudential regulation, my findings predict that the LTV-rule performs better than a countercyclical BC-rule as the LTV-rule ensures consistent loan provision during downturns. This insight reveals that countercyclical regulation of credit limits is a useful complement to BC regulation. Moreover, my results show that macroprudential policy alone does not help to moderate the real consequences of a crises. Thus, the study enriches the current policy debate about optimal macroprudential regulation.

Figure 3.1: **Impulse responses to a financial shock** in a economy with constant leverage ratios for banks and borrowers (blue, solid line), with a countercyclical LTV-rule and a constant ratio for the bank (red, dashed line), and a countercyclical BC-rule given a constant LTV ratio (black, dotted line).

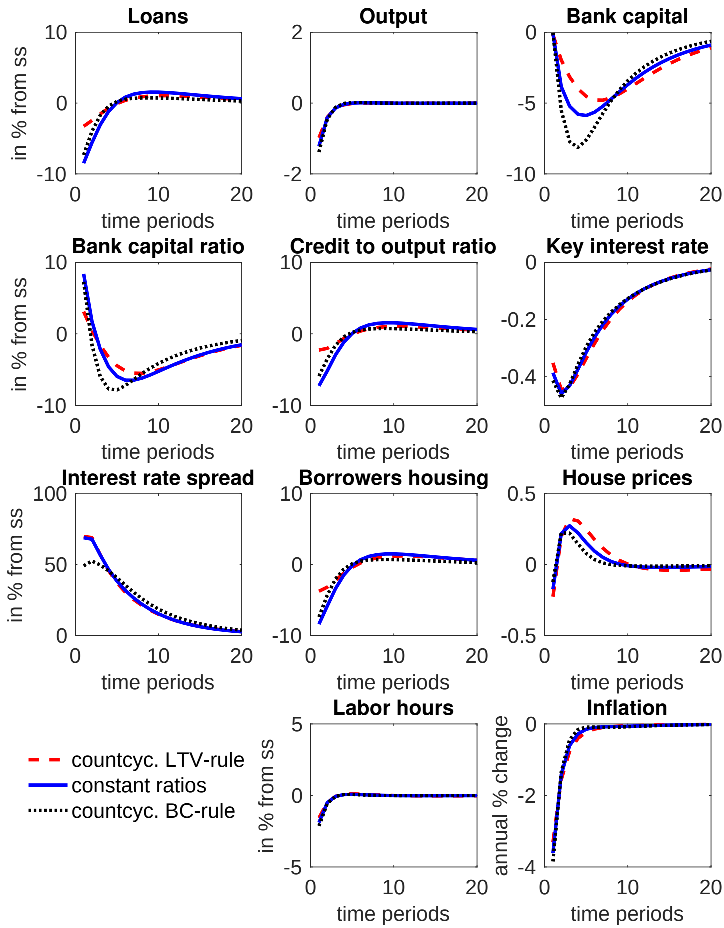
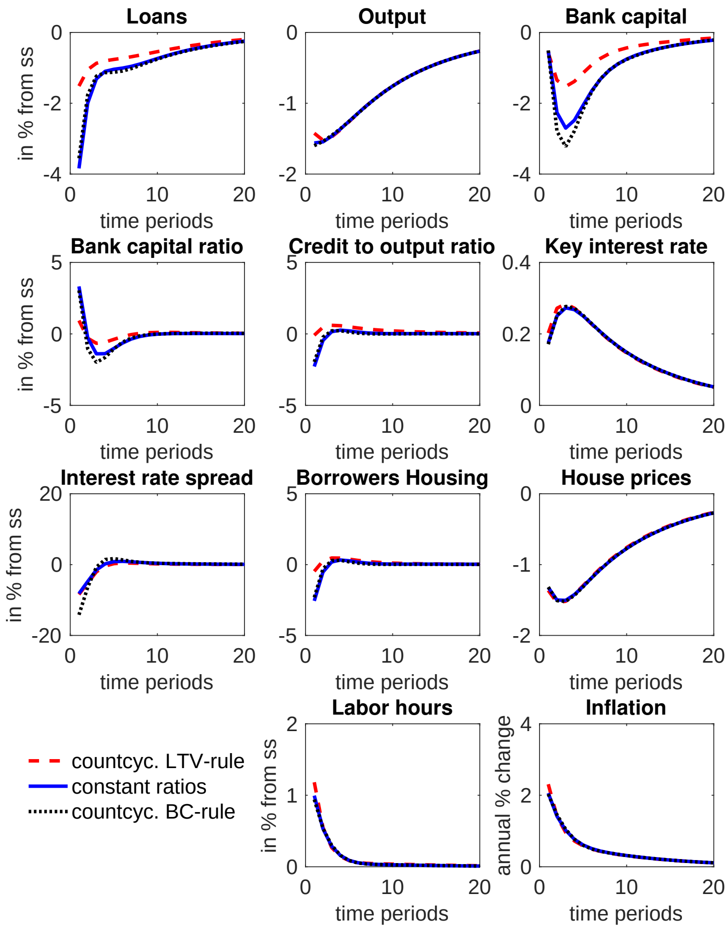


Figure 3.2: **Impulse responses to a technology shock** in an economy with constant leverage ratios for banks and borrowers (blue, solid line), with a countercyclical LTV-rule and a constant ratio for the bank (red, dashed line), and a countercyclical BC-rule given a constant LTV ratio (black, dotted line).



Appendix

3.A Model equations with banking sector

3.A.1 Optimization problems

- Savers' optimization problem:

$$\max_{c_{s,t},h_{s,t},n_{s,t},D_t^s} L = E_0 \sum_{t=0}^{\infty} \left\{ \beta_s^t \left[\ln(c_{s,t}) + j^h \ln(h_{s,t}) - \frac{n_{s,t}^{\eta}}{\eta} \right] - \beta_s^t \lambda_{s,t} \right.$$
$$\left(c_{s,t} + \frac{D_t^s}{P_t} + \frac{Q_t}{P_t} h_{s,t} - \frac{D_{t-1}^s R_{t-1}^s}{P_t} - \frac{W_{s,t} n_{s,t}}{P_t} - \frac{Q_t}{P_t} h_{s,t-1} + \right.$$
$$\left. \left. \frac{X_t}{P_t} + \frac{(1-\omega)}{P_t} G_{t-1}^B \right) \right\}$$

First order conditions:

$$\frac{\partial L}{\partial c_{s,t}} = \beta_s^t \frac{1}{c_{s,t}} - \beta_s^t \lambda_{s,t} = 0$$

$$\iff \frac{1}{c_{s,t}} = \lambda_{s,t}$$

$$\frac{\partial L}{\partial h_{s,t}} = \beta_s^t j^h \frac{1}{h_{s,t}} - \beta_s^t \lambda_{s,t} \frac{Q_t}{P_t} + \beta_s^{t+1} E_t \left[\lambda_{s,t+1} \frac{Q_{t+1}}{P_{t+1}} \right] = 0$$

$$\iff \lambda_{s,t} \frac{Q_t}{P_t} = \frac{j^h}{h_{s,t}} + \beta_s^t E_t \left[\lambda_{s,t+1} \frac{Q_{t+1}}{P_{t+1}} \right]$$

$$\frac{\partial L}{\partial n_{s,t}} = \beta_s^t (-\eta) \frac{\eta_{s,t}^{\eta-1}}{\eta} + \beta_s^t \lambda_{s,t} \frac{W_{s,t}}{P_t} = 0$$

$$\iff \frac{\eta_{s,t}^{\eta-1}}{\lambda_{s,t}} = \frac{W_{s,t}}{P_t}$$

$$\frac{\partial L}{\partial D_t} = -\beta_s^t \lambda_{s,t} \frac{1}{P_t} + \beta_s^{t+1} E_t \left[\frac{\lambda_{s,t+1} R_t^s}{P_{t+1}} \right] = 0$$

$$\iff \lambda_{s,t} = \beta_s E_t \left[\lambda_{s,t+1} R_t^s \frac{P_t}{P_{t+1}} \right]$$

$$\frac{\partial L}{\partial \lambda_{s,t}} = -\beta_t^s \left(c_{s,t} + \frac{D_t^s}{P_t} + \frac{Q_t}{P_t} h_{s,t} - \frac{D_{t-1}^s R_{t-1}^s}{P_t} - \frac{W_{s,t} n_{s,t}}{P_t} - \frac{Q_t}{P_t} h_{s,t-1} - \right.$$
$$\left. \frac{X_t}{P_t} - \frac{(1-\omega)}{P_t} G_t^B \right) = 0$$

$$\iff c_{s,t} + \frac{D_t^s}{P_t} + \frac{Q_t}{P_t} h_{s,t} = \frac{D_{t-1}^s R_{t-1}^s}{P_t} + \frac{W_{s,t} n_{s,t}}{P_t} + \frac{Q_t}{P_t} h_{s,t-1} +$$
$$\frac{X_t}{P_t} + \frac{(1-\omega)}{P_t} G_{t-1}^B$$

- Borrowers' optimization problem

$$\max_{c_{b,t},h_{b,t},n_{b,t},B_t^b} L = E_0 \sum_{t=0}^{\infty} \left\{ \beta_b^t \left[\ln(c_{b,t}) + j^h \ln(h_{b,t}) - \frac{n_{b,t}^{\eta}}{\eta} \right] \right.$$
$$-\beta_b^t \lambda_{b,t} \left(c_{b,t} + \frac{R_{t-1}^b B_{t-1}^b}{P_t} + \frac{Q_t}{P_t} h_{b,t} - \frac{Q_t}{P_t} h_{b,t-1} - \frac{B_t^b}{P_t} - \frac{W_{b,t}}{P_t} n_{b,t} \right) -$$
$$\left. \beta_b^t \mu_t \left(\frac{R_t^b B_t^b}{P_t} - l_t E_t \left[\frac{Q_{t+1}}{P_t} h_{b,t} \right] \right) \right\}$$

First order conditions:

$$\frac{\partial L}{\partial c_{b,t}} = \beta_b^t \frac{1}{c_{b,t}} - \beta_b^t \lambda_{b,t} = 0$$

$$\iff \frac{1}{c_{b,t}} = \lambda_{b,t}$$

$$\frac{\partial L}{\partial h_{b,t}} = \beta_b^t j^h \frac{1}{h_{b,t}} - \beta_b^t \lambda_{b,t} \frac{Q_t}{P_t} + \beta_b^{t+1} E_t \left[\lambda_{b,t+1} \frac{Q_{t+1}}{P_{t+1}} \right] +$$

$$\beta_b^t \mu_t l_t E_t \left[\frac{Q_{t+1}}{P_t} \right] = 0$$

$$\iff \lambda_{b,t} \frac{Q_t}{P_t} = j^h \frac{1}{h_{b,t}} + \mu_t l_t E_t \left[\frac{Q_{t+1}}{P_t} \right] + \beta_b E_t \left[\lambda_{b,t+1} \frac{Q_{t+1}}{P_{t+1}} \right]$$

$$\frac{\partial L}{\partial n_{b,t}} = \beta_b^t (-\eta) \frac{n_{b,t}^{\eta-1}}{\eta} + \beta_b^t \lambda_{b,t} \frac{W_{b,t}}{P_t} = 0$$

$$\iff \frac{n_{b,t}^{\eta-1}}{\lambda_{b,t}} = \frac{W_{b,t}}{P_t}$$

$$\frac{\partial L}{\partial B_{b,t}} = -\beta_b^{t+1} E_t \left[\lambda_{b,t+1} \frac{(R_t^b)}{P_{t+1}} \right] + \beta_b^t \lambda_{b,t} \frac{1}{P_t} - \beta_b^t \mu_t E_t \left[\frac{(R_t^b)}{P_t} \right] = 0$$

$$\iff \frac{1}{P_t} \lambda_{b,t} = \mu_t R_t^b \frac{1}{P_t} + \beta_b E_t \left[\lambda_{b,t+1} \frac{R_t^b}{P_{t+1}} \right]$$

$$\iff \lambda_{b,t} = \mu_t R_t^b + \beta_b E_t \left[\lambda_{b,t+1} \frac{R_t^b P_t}{P_{t+1}} \right]$$

$$\frac{\partial L}{\partial \lambda_{b,t}} = \beta_b^t \Bigg(c_{b,t} + \frac{R_{t-1}^b B_{t-1}^b}{P_t} + \frac{Q_t}{P_t} h_{b,t} - \frac{Q_t}{P_t} h_{b,t-1} - \frac{B_t^b}{P_t} -$$

$$\frac{W_{b,t}}{P_t} n_{b,t} \Bigg) = 0$$

$$\iff c_{b,t} + \frac{R_{t-1}^b B_{t-1}^b}{P_t} + \frac{Q_t}{P_t} h_{b,t} = \frac{Q_t}{P_t} h_{b,t-1} + \frac{B_t^b}{P_t} + \frac{W_{b,t}}{P_t} n_{b,t}$$

$$\frac{\partial L}{\partial \mu_t} = -\beta_b^t \left(\frac{R_t^b B_t^b}{P_t} - l_t E_t \left[\frac{Q_{t+1}}{P_t} h_{b,t} \right] \right) = 0$$

$$\Longleftrightarrow \frac{R_t^b B_t^b}{P_t} = l_t E_t \left[\frac{Q_{t+1}}{P_t} h_{b,t} \right]$$

- Final good producers' optimization problem

$$\max_{y_t(i)} \Pi^F = P_t y_t - \int_0^1 P_t(i) y_t(i) di$$

$$\iff \max_{y_t(i)} \Pi^F = P_t \left[\int_0^1 y_t(i)^{\frac{\epsilon_P - 1}{\epsilon_P}} di \right]^{\frac{\epsilon_P}{\epsilon_P - 1}} - \int_0^1 P_t(i) y_t(i) di$$

First order conditions:

$$\frac{\partial \Pi^F}{\partial y_t(i)} = P_t \frac{\epsilon_P}{\epsilon_P - 1} \left[\int_0^1 y_t(i)^{\frac{\epsilon_P - 1}{\epsilon_P}} di \right]^{\frac{1}{\epsilon_P - 1}} \frac{\epsilon_P - 1}{\epsilon_P} y_t(i)^{\frac{\epsilon_P - 1}{\epsilon_P} - 1} -$$
$$P_t(i) = 0$$

$$\iff P_t y_t^{\frac{1}{\epsilon_P}} y_t(i)^{-\frac{1}{\epsilon_P}} = P_t(i)$$

$$\iff y_t(i)^{\frac{1}{\epsilon_P}} = \frac{P_t}{P_t(i)} y_t^{\frac{1}{\epsilon_P}}$$

$$\iff y_t(i) = \left[\frac{P_t}{P_t(i)} \right]^{\epsilon_p} y_t$$

$$\iff y_t(i) = \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon_p} y_t$$

In order to derive the price index, I include the demand for one intermediate good in the final good producers' production function.

$$
\begin{aligned}
y_t &= \left[\int_0^1 \left\{\left[\frac{P_t(i)}{P_t}\right]^{-\epsilon_P} y_t\right\}^{\frac{\epsilon_P-1}{\epsilon_P}} di\right]^{\frac{\epsilon_P}{\epsilon_P-1}} \\
\Longleftrightarrow y_t &= P_t^{\epsilon_P}\left[\int_0^1 P_t(i)^{1-\epsilon_P} di\right]^{\frac{\epsilon_P}{\epsilon_P-1}} y_t \\
\Longleftrightarrow P_t^{-\epsilon_P} &= \left[\int_0^1 P_t(i)^{1-\epsilon_P} di\right]^{\frac{\epsilon_P}{\epsilon_P-1}} \\
\Longleftrightarrow P_t &= \left[\int_0^1 P_t(i)^{1-\epsilon_P} di\right]^{\frac{1}{1-\epsilon_P}}
\end{aligned}
$$

- Intermediate goods producers' optimization problem

$$
\begin{aligned}
\max_{P_t(i),n_{s,t},n_{b,t}} \Pi^I = E\sum_{t=0}^{\infty}\Bigg\{\beta_s^t\lambda_{s,t}\Bigg[\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon_P} y_t - \frac{W_{s,t}}{P_t}n_{s,t}(i) \\
-\frac{W_{b,t}}{P_t}n_{b,t}(i) - \frac{\phi_p}{2}\left(\frac{P_t(i)}{\pi P_{t-1}(i)}-1\right)^2 y_t\Bigg] - \beta_s^t\lambda_{s,t} \\
\xi_t\left[\left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_P} y_t - z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}\right]\Bigg\}
\end{aligned}
$$

First order conditions:

$$\frac{\partial \Pi^I}{\partial P_t(i)} = \beta_s^t \lambda_{s,t} \left[(1-\epsilon_P) \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon_P} \frac{1}{P_t} y_t - \phi_p \left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right) \right.$$
$$\left. \frac{1}{\pi P_{t-1}(i)} y_t \right] + \beta_s^{t+1} \lambda_{s,t+1} E_t \left[\phi_p \left(\frac{P_{t+1}(i)}{\pi P_t(i)} - 1 \right) \frac{P_{t+1}(i)}{\pi P_t(i)^2} y_{t+1} \right] +$$
$$\beta_s^t \lambda_{s,t} \xi_t \left[\epsilon_P \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_P - 1} \frac{1}{P_t} y_t \right] = 0$$
$$\Longleftrightarrow \phi \lambda_{s,t} \left[\frac{P_t(i)}{\pi P_{t-1}(i)} - 1 \right] \frac{P_t}{\pi P_{t-1}(i)} = \lambda_{s,t} (1-\epsilon_P) \left[\frac{P_t(i)}{P_t} \right]^{-\epsilon_P} +$$
$$\lambda_{s,t} \xi_t \epsilon_P \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_P - 1} + \beta_s \phi_p E_t +$$
$$\left\{ \lambda_{s,t+1} \left[\frac{P_{t+1}(i)}{\pi P_t} - 1 \right] \left[\frac{P_{t+1}(i) P_t}{\pi P_t(i)^2} \right] \frac{y_{t+1}}{y_t} \right\}$$
$$\frac{\partial \Pi^I}{\partial n_{s,t}} = -\beta_s^t \lambda_{s,t} \frac{W_{s,t}}{P_t} + \beta_s^t \lambda_{s,t} \xi_t \alpha z_t n_{s,t}(i)^{\alpha-1} n_{b,t}(i)^{1-\alpha} = 0$$
$$\Longleftrightarrow \frac{W_{s,t}}{P_t} n_{s,t} = \alpha \xi_t z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}$$
$$\frac{\partial \Pi^I}{\partial n_{b,t}} = \beta_s^t \lambda_{s,t} \frac{W_{b,t}}{P_t} + \beta_s^t \lambda_{s,t} \xi_t z_t (1-\alpha) n_{s,t}(i)^{\alpha} n_{b,t}(i)^{-\alpha} = 0$$
$$\Longleftrightarrow \frac{W_{b,t}}{P_t} n_{b,t} = (1-\alpha) \xi_t z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}$$
$$\frac{\partial \Pi^I}{\partial \xi_t} = y_t(i) -_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}$$
$$\Longleftrightarrow y_t(i) = z_t n_{s,t}(i)^{\alpha} n_{b,t}(i)^{1-\alpha}$$

- Banking sector – The headquarters' optimization[33]

[33] $\beta_s \lambda_s$ is the pricing kernel of patient households who are the owners of all banks.

Bank capital is accumulated out of a fraction of retained earnings from $t-1$, τG^b_{t-1}.

$$\frac{K^b_t}{P_t} = \frac{1}{s_{kb,t}} \left((1-\delta^b) \frac{K^b_{t-1}}{P_t} + \tau \frac{G^b_{t-1}}{P_t} \right)$$

The headquarter maximizes its cash flow by taking the intrabank credit rate R^{WS}_t and the intrabank savings rate that is equal to the key interest rate $R^{WS,s}_t = R_t$ as given. Moreover, this department of the representative bank ensures the bank's adherence with the balance sheet identity $B_t = D_t + K^b_t$.

$$\max_{D_t, B_t} \Pi^B = E_0 \sum_{t=0}^{\infty} \left\{ \beta^t_s \lambda_{s,t} \left[\left[R^{WS}_t \frac{B_t}{P_t} - R_t \frac{D_t}{P_t} - \frac{\phi_{WS}}{2} \left(\frac{\frac{K^b_t}{P_t}}{\frac{B_t}{P_t}} - \upsilon \right)^2 \frac{K^b_t}{P_t} + \right. \right. \right.$$
$$\left. \left. \left. \frac{D_{t+1}}{P_t} - \frac{B_{t+1}}{P_t} + \frac{K_{t+1}}{P_t} - \frac{K^b_t}{P_t} \right] - \xi^{WS}_t \left(\frac{B_t}{P_t} - \frac{D_t}{P_t} - \frac{K^b_t}{P_t} \right) \right] \right\}$$

First order conditions

$$\frac{\partial \Pi^B}{\partial D_t} = -\beta_s^t \lambda_{s,t} \frac{R_t}{P_t} + \beta_s^t \lambda_{s,t} \xi_t^{WS} \frac{1}{P_t} + \beta_s^{t-1} \frac{\lambda_{s,t-1}}{P_{t-1}} = 0$$

$$\iff -R_t + \xi_t^{WS} + \beta_s^{t-1} \frac{\lambda_{s,t-1}}{\lambda_{s,t}} \frac{P_t}{P_{t-1}} = 0$$

$$\iff R_t - \beta_s^{t-1} \frac{\lambda_{s,t-1}}{\lambda_{s,t}} \frac{P_t}{P_{t-1}} = \xi_t^{WS}$$

$$\frac{\partial \Pi^B}{\partial B_t} = \beta_s^t \lambda_{s,t} \left[\frac{R_t^{WS}}{P_t} + \phi_{WS} \left(\frac{K_t^b}{B_t} - \upsilon \right) \frac{K_t^b}{P_t} \frac{K_t^b}{B_t^2} \right] - \beta_s^t \frac{\xi_t^{WS}}{P_t} \lambda_{s,t} -$$
$$\beta_s^{t-1} \lambda_{s,t-1} \frac{1}{P_{t-1}} = 0$$

$$\iff R_t^{WS} + \phi_{WS} \left(\frac{K_t^b}{B_t} - \upsilon \right) \left(\frac{K_t^b}{B_t} \right)^2 - \xi_t^{WS} - \beta_s^{t-1} \frac{\lambda_{s,t-1}}{\lambda_{s,t}} \frac{P_t}{P_{t-1}} = 0$$

Inserting both first order conditions in each other links the spread of the intrabank loan rate and the intrabank savings rate and the leverage ratio of banks.

$$0 = R_t^{WS} + \phi_{WS} \left(\frac{K_t^b}{B_t} - \upsilon \right) \left(\frac{K_t^b}{B_t} \right)^2 - R_t +$$
$$\beta_s^{t-1} \frac{\lambda_{s,t-1}}{\lambda_{s,t}} \frac{P_t}{P_{t-1}} - \beta_s^{t-1} \frac{\lambda_{s,t-1}}{\lambda_{s,t}} \frac{P_t}{P_{t-1}}$$

$$\iff R_t^{WS} - R_t + \phi_{WS} \left(\frac{K_t^b}{B_t} - \upsilon \right) \left(\frac{K_t^b}{B_t} \right)^2 = 0$$

$$\iff R_t^{WS} = R_t - \phi_{WS} \left(\frac{K_t^b}{B_t} - \upsilon \right) \left(\frac{K_t^b}{B_t} \right)^2$$

Spread of intrabank rates:

$$s_t^{WS} = R_t^{WS,b} - R_t = -\phi^{WS}\left(\frac{K_t^b}{B_t} - \upsilon\right)\left(\frac{K_t^b}{B_t}\right)^2$$

- Loan product brokers' optimization problem subject to $B_t^b(j) = B_t(j)$ is:

$$\max_{R_t^b(j)} \Pi^L = E_0 \sum_{t=0}^{\infty}\left\{\beta_s^t \lambda_{s,t}\left[R_t^b(j)\left(\frac{R_t^b(j)}{R_t^b}\right)^{-\epsilon_{b,t}}\frac{B_t^b}{P_t} - R_t^{WS}\left(\frac{R_t^b(j)}{R_t^b}\right)^{-\epsilon_{b,t}}\frac{B_t^b}{P_t} - \frac{\phi^b}{2}\left(\frac{R_t^b(j)}{R_{t-1}^b(j)} - 1\right)^2 R_t^b\frac{B_t^b}{P_t}\right]\right\}$$

First order condition:

$$\frac{\partial \Pi^L}{\partial R_t^b(j)} = \beta_s^t \lambda_{s,t}\left[-R_t^b(j)\epsilon_{b,t}\left(\frac{R_t^b(j)}{R_t^b}\right)^{-\epsilon_{b,t}-1}\frac{B_t^b}{P_t}\frac{1}{R_t^b} + \left(\frac{R_t^b(j)}{R_t^b}\right)^{-\epsilon_{b,t}}\frac{B_t^b}{P_t} + R_t^{WS}\epsilon_{b,t}\left(\frac{R_t^b(j)}{R_t^b}\right)^{-\epsilon_{b,t}-1}\frac{B_t^b}{R_t^b P_t} - \phi^b\left(\frac{R_t^b(j)}{R_{t-1}^b(j)} - 1\right)\frac{R_t^b B_t^b}{R_{t-1}^b(j) P_t}\right] + \beta_s^{t+1}\lambda_{s,t+1}\left(\phi^b\left(\frac{R_{t+1}^b(j)}{R_t^b(j)} - 1\right)\left(\frac{R_{t+1}^b}{R_t^b(j)}\right)^2\frac{B_{t+1}P_t}{P_{t+1}}\right) = 0$$

$$\iff R_t^{WS}\epsilon_{b,t}\frac{B_t^b(j)}{R_t^b(j)} = \epsilon_{b,t}B_t^b(j) - B_t^b(j) + \phi^b\left(\frac{R_t^b(j)}{R_{t-1}^b(j)} - 1\right)\frac{R_t^b B_t^b(j)}{R_{t-1}^b(j)} - \beta_s\left(\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\phi^b\left(\frac{R_{t+1}^b(j)}{R_t^b(j)} - 1\right)\left(\frac{R_{t+1}^b}{R_t^b(j)}\right)^2\frac{B_{t+1}^b P_t}{P_{t+1}}\right)$$

In a symmetric equilibrium, $R_t^b(j) = R_t^b$ applies.

$$\begin{aligned}
&\frac{R_t^{WS}\epsilon_{b,t}}{R_t^b} = -(1+\epsilon_{b,t}) + \phi^b\left(\frac{R_t^b}{R_{t-1}^b} - 1\right)\frac{R_t^b}{R_{t-1}^b} - \\
&\beta_s\left(\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\phi^b\left(\frac{R_{t+1}^b}{R_t^b} - 1\right)\left(\frac{R_{t+1}^b}{R_t^b}\right)^2 \frac{B_{t+1}^b P_t}{B_t^b P_{t+1}}\right) \\
&\iff \frac{R_t^{WS}\epsilon_{b,t}}{R_t^b} = \epsilon_{b,t} - 1 + \phi^b\left(\frac{R_t^b}{R_{t-1}^b} - 1\right)\frac{R_t^b}{R_{t-1}^b} - \\
&\beta_s\left(\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\phi^b\left(\frac{R_{t+1}^b}{R_t^b} - 1\right)\left(\frac{R_{t+1}^b}{R_t^b}\right)^2 \frac{B_{t+1}^b P_t}{B_t^b P_{t+1}}\right) \\
&\iff \frac{R_t^{WS}\epsilon_{b,t}}{R_t^b} = \epsilon_{b,t} - 1 + \phi^b\left(\frac{R_t^b}{R_{t-1}^b} - 1\right)\frac{R_t^b}{R_{t-1}^b} - \\
&\beta_s\left(\frac{c_{s,t}}{c_{s,t+1}}\phi^b\left(\frac{R_{t+1}^b}{R_t^b} - 1\right)\left(\frac{R_{t+1}^b}{R_t^b}\right)^2 \frac{B_{t+1}^b}{B_t^b \pi_{t+1}}\right)
\end{aligned}$$

When interest rates are flexible and adjust instantaneously, the relation simplifies to:

$$\begin{aligned}
R_t^b &= \frac{\epsilon_{b,t}}{\epsilon_{b,t} - 1} R_t^{WS} \\
S_t^b \equiv R_t^b - R_t &= \frac{\epsilon_{b,t}}{\epsilon_{b,t} - 1} S_t^{WS} - R_t
\end{aligned}$$

By assuming that the markdown $\frac{\epsilon_{b,t}}{\epsilon_{b,t}-1} = mk_{b,t}$ follows an AR(1) shock process, the first order condition changes to:

$$\frac{mk_{b,t}}{mk_{b,t}-1}\frac{R_t^{WS}}{R_t^b} = \frac{mk_{b,t}}{mk_{b,t}-1} - 1 + \phi^b\left(\frac{R_t^b}{R_{t-1}^b}-1\right)\frac{R_t^b}{R_{t-1}^b} - \beta_s\left(\frac{c_{s,t}}{c_{s,t+1}}\phi^b\left(\frac{R_{t+1}^b}{R_t^b}-1\right)\left(\frac{R_{t+1}^b}{R_t^b}\right)^2\frac{B_{t+1}^b P_t}{B_t^b P_{t+1}}\right)$$

- Saving product brokers' optimization problem subject to $D_t^s(j) = D_t(j)$ is:

$$\max_{r_t^s(j)} \Pi^S = E_0\sum_{t=0}^{\infty}\left\{\beta_s^t\lambda_{s,t}\left[R_t\left(\frac{R_t^s(j)}{R_t^s}\right)^{-\epsilon_{s,t}}\frac{D_t^s}{P_t} - R_t^s(j)\left(\frac{R_t^s(j)}{R_t^s}\right)^{-\epsilon_{s,t}}\frac{D_t^s}{P_t} - \frac{\phi^s}{2}\left(\frac{R_t^s(j)}{R_{t-1}^s(j)}-1\right)^2 R_t^s\frac{D_t^s}{P_t}\right]\right\}$$

First order condition:

$$\frac{\partial\Pi^S}{\partial R_t^s(j)} = \beta_s^t\lambda_{s,t}\left\{-\epsilon_{s,t}R_t\left(\frac{R_t^s(j)}{R_t^s}\right)^{-\epsilon_{s,t}-1}\frac{D_t^s}{P_tR_t^s} + \epsilon_{s,t}R_t^s(j)\left(\frac{R_t^s(j)}{R_t^s}\right)^{-\epsilon_{s,t}-1}\frac{D_t^s}{P_tR_t^s} - \left(\frac{R_t^s(j)}{R_t^s}\right)^{-\epsilon_{s,t}}\frac{D_t^s}{P_t} - \phi^s\left(\frac{R_t^s(j)}{R_{t-1}^s(j)}-1\right)\frac{R_t^sD_t^s}{R_{t-1}^s(j)P_t}\right\} + \beta_s^{t+1}\lambda_{s,t+1}\left\{\phi^s\left(\frac{R_{t+1}^s(j)}{R_t(j)}-1\right)\frac{R_{t+1}^sD_{t+1}^sR_{t+1}^s(j)}{R_t^{s,2}(j)P_{t+1}}\right\} = 0$$

In a symmetric equilibrium $R_t^s(j) = R_t^s$ applies.

$$
\begin{aligned}
\epsilon_{s,t}\frac{R_t D_t^s}{R_t^s P_t} &= (\epsilon_{s,t}-1)\frac{D_t^s}{P_t} - \phi^s\left(\frac{R_t^s}{R_{t-1}^s}-1\right)\frac{R_t^s D_t^s}{R_{t-1}^s P_t} + \beta_s\left\{\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\phi^s\right.\\
&\left.\left(\frac{R_{t+1}^s(j)}{R_t^s}-1\right)\left(\frac{R_{t+1}^s}{R_t^s}\right)^2\frac{D_{t+1}^s}{P_{t+1}}\right\}\\
\Longleftrightarrow \epsilon_{s,t}\frac{R_t}{R_t^s(j)} &= (\epsilon_{s,t}-1) - \phi^s\left(\frac{R_t^s(j)}{R_{t-1}^s(j)}-1\right)\frac{R_t^s}{R_{t-1}^s} + \beta_s\left\{\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\phi^s\right.\\
&\left.\left(\frac{R_{t-1}^s(j)}{R_t^s(j)}-1\right)\left(\frac{R_{t+1}^s}{R_t^s}\right)^2\frac{D_{t+1}^s}{D_t^s}\frac{1}{\pi_{t+1}}\right\}\\
\Longleftrightarrow \epsilon_{s,t}\frac{R_t}{R_t^s(j)} &= (\epsilon_{s,t}-1) - \phi^s\left(\frac{R_t^s(j)}{R_{t-1}^s(j)}-1\right)\frac{R_t^s}{R_{t-1}^s} + \beta_s\left\{\frac{c_{s,t}}{c_{s,t+1}}\phi^s\right.\\
&\left.\left(\frac{R_{t+1}^s(j)}{R_t^s(j)}-1\right)\left(\frac{R_{t+1}^s}{R_t^s}\right)^2\frac{D_{t+1}^s}{D_t^s}\frac{1}{\pi_{t+1}}\right\}
\end{aligned}
$$

The relation simplifies when interest rates are flexible and adjust instantaneously, so that the last two terms equal to zero.

$$
\begin{aligned}
\epsilon_{s,t}\frac{R_t}{R_t^s} &= \epsilon_{s,t}-1\\
R_t^s &= \frac{\epsilon_{s,t}}{\epsilon_{s,t}-1}R_t
\end{aligned}
$$

By assuming that the markdown $\frac{\epsilon_{s,t}}{\epsilon_{s,t}-1} = mk_{s,t}$ follows an AR(1) shock process, the first order condition changes to:

$$\frac{mk_{s,t}}{mk_{s,t}-1}\frac{R_t}{R_t^s(j)} = \frac{mk_{s,t}}{mk_{s,t}-1} - 1 - \phi^s\left(\frac{R_t^s(j)}{R_{t-1}^s(j)} - 1\right)\frac{R_t^s}{R_{t-1}^s} + \beta_s\left\{\frac{c_{s,t}}{c_{s,t+1}}\phi^s\left(\frac{R_{t+1}^s(j)}{R_t^s(j)} - 1\right)\left(\frac{R_{t+1}^s}{R_t^s}\right)^2\frac{D_{t+1}^s}{D_t^s}\frac{1}{\pi_{t+1}}\right\}$$

- Profits of banks

$$G_t^b = R_t^b B_t^b - R_t^d D_t^s - \frac{\phi_{WS}}{2}\left(\frac{K_t^b}{B_t} - \upsilon\right)^2 K_t^b - \frac{\phi^s}{2}\left(\frac{R_t^s(j)}{R_{t-1}^s(j)} - 1\right)^2 R_t^s D_t^s - \frac{\phi^b}{2}\left(\frac{R_t^b(j)}{R_{t-1}^b(j)} - 1\right)^2 R_t^b B_t^b$$

- Borrowers' demand for loans

$$\min_{B_t^b(j)} L^{BD} = \int_0^1 R_t^b(j) B_t^b(j) dj - \lambda^{bd}\left(\left(\int_0^1 (B_t^b(j))^{\frac{\epsilon_{b,t}-1}{\epsilon_{b,t}}} dj\right)^{\frac{\epsilon_{b,t}}{\epsilon_{b,t}-1}} - B_t^b\right)$$

First order condition:

$$\frac{\partial L^{BD}}{\partial B_t^b(j)} = R_t^b(j) - \lambda^{bd}\left(\frac{\epsilon_{b,t}}{\epsilon_{b,t}-1}\left(\int_0^1 (B_t^b(j))^{\frac{\epsilon_{b,t}-1}{\epsilon_{b,t}}} dj\right)^{\frac{\epsilon_{b,t}}{\epsilon_{b,t}-1}-1} \frac{\epsilon_{b,t}-1}{\epsilon_{b,t}}(B_t^b(j))^{\frac{\epsilon_{b,t}-1}{\epsilon_{b,t}}-1}\right) = 0$$

$$\iff R_t^b(j) = \lambda^{bd}(B_t^b)^{\frac{1}{\epsilon_{b,t}}}(B_t^b(j))^{\frac{-1}{\epsilon_{b,t}}}$$

$$\iff B_t^b(j) = B_t^b\left(\frac{R_t^b(j)}{\lambda^{bd}}\right)^{-\epsilon_{b,t}}$$

λ^{bd} is the shadow price for loan products. Since the shadow price is independent of the household i, it can be replaced by the overall price index for loan products R_t^b that is the same for all households. Aggregating the condition over all households i delivers the aggregated nominal demand of patient households for loans:

$$\Longleftrightarrow R_t^b(j) = R_t^b(B_t^b)^{\frac{1}{\epsilon_{b,t}}}(B_t^b(j))^{\frac{-1}{\epsilon_{b,t}}}$$

$$\Longleftrightarrow B_t^b(j) = \left(\frac{R_t^b(j)}{R_t^b}\right)^{-\epsilon_{b,t}} B_t^b$$

The aggregated loan interest rate is:

$$B_t^b(j) = \left[\int_0^1 \left[\left[\frac{R_t^b(j)}{R_t^b}\right]^{-\epsilon_{b,t}} B_t^b\right]^{\frac{\epsilon_{b,t}-1}{\epsilon_{b,t}}} dj\right]^{\frac{\epsilon_{b,t}}{\epsilon_{b,t}-1}}$$

$$R_t^b = \left[\int_0^1 (R_t^b(j))^{1-\epsilon_{b,t}} dj\right]^{\frac{1}{1-\epsilon_{b,t}}}$$

- Savers' demand for deposits:

$$\max_{D_t^s(j)} \Pi^{SD} = \int_0^1 R_t^s(j) D_t^s(j) dj - \lambda^{sd}\left(\left(\int_0^1 (D_t^s(j,i))^{\frac{\epsilon_{s,t}}{\epsilon_{s,t}-1}} dj\right)^{\frac{\epsilon_{s,t}}{\epsilon_{s,t}-1}} - D_t^s(i)\right)$$

First order condition:

$$\frac{\partial \Pi^{SD}}{\partial D_t^s(j)} = R_t^s(j) - \lambda^{sd}\left(\frac{\epsilon_{s,t}}{\epsilon_{s,t}-1}\left(\int_0^1 (D_t^s(j))^{\frac{\epsilon_{s,t}}{\epsilon_{s,t-1}}} dj\right)^{\frac{\epsilon_{s,t}}{\epsilon_{s,t-1}}-1}\right.$$
$$\left.\frac{\epsilon_{s,t}-1}{\epsilon_{s,t}}(D_t^s(j))^{\frac{\epsilon_{s,t-1}}{\epsilon_{s,t}}-1}\right)$$
$$\iff R_t^b(j) = \lambda^{sd}(D_t^s)^{\frac{1}{\epsilon_{s,t}}}(D_t^s(j))^{\frac{-1}{\epsilon_{s,t}}}$$
$$\iff D_t^s(j) = D_t^s\left(\frac{R_t^s(j)}{\lambda^{sd}}\right)^{-\epsilon_{s,t}}$$

λ^{sd} is the shadow price for deposit products. Since the shadow price is independent of the household i, it can be replaced by the overall price index for deposit products R_t^s.

Aggregating the condition over all households i delivers the aggregated nominal demand of patient households for deposits:

$$\iff R_t^s(j) = R_t^s(D_t^s)^{\frac{1}{\epsilon_{s,t}}}(D_t^s(j))^{\frac{-1}{\epsilon_{s,t}}}$$
$$\iff D_t^s(j) = \left(\frac{R_t^s(j)}{R_t^s}\right)^{-\epsilon_{s,t}} D_t^s$$

The aggregated interest rates on deposits to households is:

$$D_t^s(j) = \left[\int_0^1 \left[\left[\frac{R_t^s(j)}{R_t^s} \right]^{-\epsilon_{s,t}} D_t^s \right]^{\frac{\epsilon_{s,t}-1}{\epsilon_{s,t}}} dj \right]^{\frac{\epsilon_{s,t}}{\epsilon_{s,t}-1}}$$

$$R_t^s = \left[\int_0^1 (R_t^s(j))^{1-\epsilon_{s,t}} dj \right]^{\frac{1}{1-\epsilon_{s,t}}}$$

- Monetary policy rule

$$R_t = R^{1-\rho_r} R_{t-1}^{\rho_r} \left(\left(\frac{\pi_t}{\pi} \right)^{\omega_\pi} \left(\frac{y_t}{y} \right)^{\omega_y} \right)^{1-\rho_r} + \varepsilon_{v,t}$$

- The macroprudential policy rules comprise

  – the countercyclical rule on the BC ratio

$$\upsilon_t = \upsilon \left(\frac{e_t}{e} \right)^{\chi_\upsilon}$$

  where $\chi_\upsilon > 0$.

  – and the countercyclical rule on the LTV ratio

$$l_t = l \left(\frac{b_t}{b} \right)^{\chi_l}$$

  where $\chi_l < 0$.

- , Laws of motion for the transitory shocks:

$$\ln(z_t) = (1-\rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{z,t}$$
$$\ln(s_{kb,t}) = (1-\rho_{s_{kb}})\ln(s_{kb}) + \rho_{s_{kb}} \ln(s_{kb,t-1}) + \varepsilon_{s_{kb,t}}$$
$$\ln(mk_{b,t}) = (1-\rho_{mk_b})\ln(mk_b) + \rho_{mk_b} \ln(mk_{b,t-1}) + \varepsilon_{mk_b,t}$$
$$\ln(mk_{s,t}) = (1-\rho_{mk_s})\ln(mk_s) + \rho_{mk_s} \ln(mk_{s,t-1}) + \varepsilon_{mk_s,t}$$

- Model closing equations and market clearing conditions

  – Goods market clearing conditions:

$$y_t = c_{b,t} + c_{s,t} - \frac{\phi_P}{2}\left(\frac{P_t(i)}{\pi P_{t-1}(i)} - 1\right)y_t - \frac{\phi_{WS}}{2}\left(\frac{K_t^b}{B_t} - \upsilon\right)^2 \frac{K_t^b}{P_t} - \frac{\phi^s}{2}\left(\frac{R_t^s(j)}{R_{t-1}^s(j)} - 1\right)^2 R_t^s \frac{D_t^s}{P_t} - \frac{\phi^b}{2}\left(\frac{R_t^b(j)}{R_{t-1}^b(j)} - 1\right) R_t^b \frac{B_t^b}{P_t} + \delta^b \frac{K_{t-1}^B}{P_t}$$

  – Labor market clearing condition

$$n_{b,t} = \int_0^1 n_{b,t}(i)di$$
$$n_{s,t} = \int_0^1 n_{s,t}(i)di$$

- Dividend clearing condition

$$X_t = \int_0^1 X_t(i)di$$

- Housing market clearing condition

$$1 = h_{s,t} + h_{b,t}$$

- Intrabank market clearing condition

$$B_t = \int_0^1 B_t(j)dj$$
$$D_t = \int_0^1 D_t(j)dj$$

3.A.2 Nonlinear system in symmetric equilibrium

To close the model, I consider a symmetric equilibrium, where all intermediate firms and banks make identical decision since they face the same optimization problem. This assumption implies: $P_t = P_t(i), n_{b,t} = n_{b,t}(i), n_{s,t} = n_{s,t}(i), y_t = y_t(i), X_t(i) = X_t, B_t^b = B_t^b(j), D_t^s = D_t^s(j), R_t^b = R_t^b(j), R_t^s = R_t^s(j), G_t^b(j) = G_t^b, \forall i \in [0,1]$ and $\forall j \in [0,1]$. Moreover, I change the variables according to: $\pi_t = \frac{P_t}{P_{t-1}}$, $w_{s,t} = \frac{W_{s,t}}{P_t}$, $w_{b,t} = \frac{W_{b,t}}{P_t}$, $b_t^b = \frac{B_t^b}{P_t}$, $d_t^s = \frac{D_t^s}{P_t}$, $q_t = \frac{Q_t}{P_t}$, $x_t = \frac{X_t}{P_t}$, $\frac{K_t^b}{P_t} = k_t^b$, $g_t^b = \frac{G_t^B}{P_t}$ to represent the nonlinear system in real terms:

$$\ln(z_t) = (1-\rho_z)\ln(z) + \rho_z \ln(z_{t-1}) + \varepsilon_{z,t} \tag{3.1}$$

$$\ln(s_{kb,t}) = (1-\rho_{s_{kb}})\ln(s_{kb}) + \rho_{s_{kb}} \ln(s_{kb,t-1}) + \varepsilon_{s_{kb,t}} \tag{3.2}$$

$$\ln(mk_{b,t}) = (1-\rho_{mk_b})\ln(mk_b) + \rho_{mk_b} \ln(mk_{b,t-1}) + \varepsilon_{mk_{b,t}} \tag{3.3}$$

$$\ln(mk_{s,t}) = (1-\rho_{mk_s})\ln(mk_s) + \rho_{mk_s} \ln(mk_{s,t-1}) + \varepsilon_{mk_{s,t}} \tag{3.4}$$

$$\frac{1}{c_{s,t}} q_t = j^h \frac{1}{h_{s,t}} + \beta_s E_t \left[\frac{1}{c_{s,t+1}} q_{t+1} \right] \tag{3.5}$$

$$c_{s,t} n_{s,t}^{\eta-1} = w_{s,t} \tag{3.6}$$

$$\frac{1}{c_{s,t}} = \beta_s R_t^s E_t \left[\frac{1}{c_{s,t+1}} \frac{1}{\pi_{t+1}} \right] \tag{3.7}$$

$$c_{s,t} + d_t^s + q_t h_{s,t} = \frac{d_{t-1}^s}{\pi_t} R_{t-1}^s + w_{s,t} n_{s,t} + q_t h_{s,t-1} + x_t + (1-\tau)\frac{g_{t-1}^B}{\pi_t} \tag{3.8}$$

$$\frac{1}{c_{b,t}} q_t = j^h \frac{1}{h_{b,t}} + \mu_t l_t E_t \left[q_{t+1} \pi_{t+1} \right] + \beta_b E_t \left[\frac{1}{c_{b,t+1}} q_{t+1} \right] \tag{3.9}$$

$$c_{b,t} n_{b,t}^{\eta-1} = w_{b,t} \tag{3.10}$$

$$\frac{1}{c_{b,t}} = \mu_t R_t^b + \beta_b E_t \left[\frac{R_t^b}{c_{b,t+1}} \frac{1}{\pi_{t+1}} \right] \tag{3.11}$$

$$c_{b,t} + \frac{b_{t-1}^b}{\pi_t} R_{t-1}^b + q_t h_{b,t} = q_t h_{b,t-1} + b_t^b + w_{b,t} n_{b,t} \tag{3.12}$$

$$R_t^b b_t^b = l_t E_t \left[q_{t+1} h_{b,t} \pi_{t+1} \right] \tag{3.13}$$

$$\begin{aligned}\phi_P\left[\frac{\pi_t}{\pi}-1\right]\frac{\pi_t}{\pi} &= (1-\epsilon_P)+\xi_t\epsilon_P\\ +\beta_s\phi_P E_t&\left\{\frac{\lambda_{s,t+1}}{\lambda_{s,t}}\left[\frac{\pi_{t+1}}{\pi}-1\right]\left[\frac{\pi_{t+1}}{\pi}\right]\frac{y_{t+1}}{y_t}\right\}\end{aligned} \tag{3.14}$$

$$y_t = z_t n_{s,t}^{\alpha} n_{b,t}^{1-\alpha} \tag{3.15}$$

$$w_{s,t}n_{s,t} = \alpha\xi_t z_t n_{s,t}^{\alpha} n_{b,t}^{1-\alpha} \tag{3.16}$$

$$w_{b,t}n_{b,t} = (1-\alpha)\xi_t z_t n_{s,t}^{\alpha} n_{b,t}^{1-\alpha} \tag{3.17}$$

$$R_t = R^{1-\rho_r} R_{t-1}^{\rho_r}\left(\left(\frac{\pi_t}{\pi}\right)^{\omega_\pi}\left(\frac{y_t}{y}\right)^{\omega_y}\right)^{1-\rho_r}\varepsilon_{v,t} \tag{3.18}$$

$$\upsilon_t = \upsilon\left(\frac{\frac{b_t}{y_t}}{\frac{b}{y}}\right)^{\chi_\upsilon} \tag{3.19}$$

$$l_t = l\left(\frac{q_t}{q}\right)^{\chi_l} \tag{3.20}$$

$$x_t = y_t - w_{s,t}n_{s,t} - w_{b,t}n_{b,t} - \frac{\phi}{2}\left(\frac{\pi_t}{\pi}-1\right)^2 y_t \tag{3.21}$$

$$k_t^b\pi_t = \frac{1}{s_{kb}}\left((1-\delta^b)k_{t-1}^b + \tau g_{t-1}^b\right) \tag{3.22}$$

$$R_t^{WS,b} = R_t - \phi^{WS}\left(\frac{k_t^b}{b_t}-\upsilon\right)\left(\frac{k_t^b}{b_t}\right)^2 \tag{3.23}$$

$$\begin{aligned}\frac{mk_{b,t}}{mk_{b,t}-1}\frac{R_t^{WS}}{R_t^b} &= \frac{mk_{b,t}}{mk_{b,t}-1} - 1 + \phi^b\left(\frac{R_t^b}{R_{t-1}^b}-1\right)\frac{R_t^b}{R_{t-1}^b} -\\ &\beta_s^t\left(\frac{c_{s,t}}{c_{s,t+1}}\phi^b\left(\frac{R_{t+1}^b}{R_t^b}-1\right)\left(\frac{R_{t+1}^b}{R_t^b}\right)^2\frac{B_{t+1}^b P_t}{B_t^b P_{t+1}}\right)\end{aligned} \tag{3.24}$$

$$\frac{mk_{s,t}}{mk_{s,t}-1}\frac{R_t}{R_t^s} = \frac{mk_{s,t}}{mk_{s,t}-1} - 1 - \phi^s\left(\frac{R_t^s}{R_{t-1}^s}-1\right)\frac{R_t^s}{R_{t-1}^s} + \beta_s\left\{\frac{c_{s,t}}{c_{s,t+1}}\phi^s\left(\frac{R_{t+1}^s}{R_t^s}-1\right)\left(\frac{R_{t+1}^s}{R_t^s}\right)^2\frac{D_{t+1}^s}{D_t^s}\frac{1}{\pi_{t+1}}\right\} \tag{3.25}$$

$$g_t^b = R_t^b b_t^b - R_t^s d_t^s - \frac{\phi_{WS}}{2}\left(\frac{k_t^b}{b_t}-\upsilon\right)^2 k_t^b - \frac{\phi^s}{2}\left(\frac{R_t^s}{R_{t-1}^s}-1\right)^2 R_t^s d_t^s - \frac{\phi^b}{2}\left(\frac{R_t^b}{R_{t-1}^b}-1\right)^2 R_t^b b_t^b \tag{3.26}$$

$$b_t^b = d_t^s + k_t \tag{3.27}$$

$$y_t = c_{b,t} + c_{s,t} + \frac{\phi_P}{2}\left(\frac{\pi_t}{\pi}-1\right)y_t + \frac{\phi_{WS}}{2}\left(\frac{k_t^b}{b_t}-\upsilon\right)^2 k_t^b + \frac{\phi^s}{2}\left(\frac{R_t^s}{R_{t-1}^s}-1\right)^2 R_t^s d_t^s + \frac{\phi^b}{2}\left(\frac{R_t^b}{R_{t-1}^b}-1\right)R_t^b b_t^b + \delta^b\frac{k_{t-1}^b}{\pi_t} \tag{3.28}$$

$$1 = h_{b,t} + h_{s,t} \tag{3.29}$$

3.A.3 Steady state calculation

In the absence of three shocks $\varepsilon_{z,t} = \varepsilon_{v,t} = \varepsilon_{s_{kb,t}} = \varepsilon_{mk_{s,t}} = \varepsilon_{mk_{s,t}} = 0$ for all t, the model converges to the local steady state, where all variables are constant. Therefore, in steady state no adjustment costs arise in the banking sector as well as in the intermediate goods production sector. By setting the monetary policy's inflation target to zero, the gross inflation rate in the steady state is one. Variables without a time index designate steady state values.

$$z = z \tag{3.1'}$$

$$s_{kb} = s_{kb} \tag{3.2'}$$

$$mk_b = mk_b \tag{3.3'}$$

$$mk_s = mk_s \tag{3.4'}$$

$$\frac{1}{c_s}q = j^h \frac{1}{h_s} + \beta_s \left[\frac{1}{c_s}q\right] \tag{3.5'}$$

$$c_s n_s^{\eta-1} = w_s \tag{3.6'}$$

$$\frac{1}{c_s} = \beta_s R^s \left[\frac{1}{c_s}\frac{1}{\pi}\right] \tag{3.7'}$$

$$c_s + d^s + qh_s = \frac{d^s}{\pi}R^s + w_s n_s + qh_s + x + (1-\tau)\frac{g^b}{\pi} \tag{3.8'}$$

$$\frac{1}{c_b}q = j^h \frac{1}{h_b} + \mu l\,[q\pi] + \beta_b \left[\frac{1}{c_b}q\right] \tag{3.9'}$$

$$c_b n_b^{\eta-1} = w_b \tag{3.10'}$$

$$\frac{1}{c_b} = \mu R^b + \beta_b \left[\frac{R^b}{c_b}\frac{1}{\pi}\right] \tag{3.11'}$$

$$c_b + \frac{b^b}{\pi}R^b + qh_b = qh_b + b^b + w_b n_b \tag{3.12'}$$

$$R^b b^b = l\,[qh_b\pi] \tag{3.13'}$$

$$\xi\epsilon_P = (1-\epsilon_P) \tag{3.14'}$$

$$y = z n_s^{\alpha} n_b^{1-\alpha} \tag{3.15'}$$

$$w_s n_s = \alpha \xi z n_s^{\alpha} n_b^{1-\alpha} \tag{3.16'}$$

$$w_b n_b = (1-\alpha) \xi z n_s^{\alpha} n_b^{1-\alpha} \tag{3.17'}$$

$$R = R \tag{3.18'}$$

$$\upsilon = \upsilon \tag{3.19'}$$

$$l = l \tag{3.20'}$$

$$x = y - w_s n_s - w_b n_b \tag{3.21'}$$

$$k^b \pi = (1-\delta^b) k^b + \tau g^b \tag{3.22'}$$

From the steady state relation $\frac{k^b}{b^b} = \upsilon$ follows:

$$R^{WS,b} = R \tag{3.23'}$$

With $\frac{\epsilon_b}{\epsilon_b - 1} = mk_b$ equation (3.24) reduces in steady state to:

$$R^b = \frac{\epsilon_b}{\epsilon_b - 1} R^{WS} \tag{3.24'}$$

With $\frac{\epsilon_s}{\epsilon_s - 1} = mk_s$ equation (3.25) reduces in steady state to:

$$R^s = \frac{\epsilon_s}{\epsilon_s - 1} R \tag{3.25'}$$

$$g^b = R^b b^b - R^s d^s \tag{3.26'}$$

$$b^b = d^s + k^b \tag{3.27'}$$

$$y = c_b + c_s + \delta^b \frac{k^b}{\pi} \tag{3.28'}$$

$$1 = h_b + h_s \tag{3.29'}$$

From the savers' Euler equation (3.7') follows:

$$R^s = \frac{\pi}{\beta_s} \tag{3.30}$$

With equation (3.25') one derives:

$$R = \frac{\epsilon_{s,t} - 1}{\epsilon_{s,t}} \left(\frac{\pi}{\beta^s} \right) \tag{3.31}$$

Rewriting the borrowers' Euler equation (3.11') derives:

$$\mu = \frac{1}{c_b} \left(\frac{\pi}{R_b} - \beta_b \right) \tag{3.32}$$

Plugging (3.32) in the housing demand equation of borrowers (3.9') results in:

$$\frac{1}{c_b} q = \frac{j^h}{h_b} + \frac{1}{c_b} \left(\frac{\pi}{R_b} - \beta_b \right) lq\pi + \beta_b \frac{1}{c_b} q \tag{3.33}$$

From the collateral constraint (3.13') one derives:

$$h_b = R_b \frac{b^b}{\pi} \frac{1}{lq} \tag{3.34}$$

Substituting this in equation (3.33) yields:

$$\frac{1}{c_b} q = \frac{j^h}{R_b \frac{b^b}{\pi} \frac{1}{lq}} + \frac{1}{c_b} \left(\frac{\pi}{R_b} - \beta_b \right) lq\pi + \beta_b \frac{1}{c_b} q \tag{3.35}$$

Equation (3.35) simplifies to:

$$\frac{1}{c_b} = \frac{j^h l\pi}{R_b b^b} + \frac{l}{c_b}\left(\frac{\pi}{R_b} - \beta_b\right)\pi + \beta_b \frac{1}{c_b} \tag{3.36}$$

From equation (3.12'), (3.14') and (3.17') it follows that:

$$b^b = \frac{c_b - \frac{\epsilon_P - 1}{\epsilon_P}(1-\alpha)y}{1 - \frac{R^b}{\pi}} \tag{3.37}$$

Plugging (3.37) in (3.36) results in:

$$\frac{1}{c_b} = \frac{j^h l\pi}{R^b \frac{c_b - \frac{\epsilon_P - 1}{\epsilon_P}(1-\alpha)(c_b + c_s + \delta^b k^b)}{1 - \frac{R^b}{\pi}}} + \frac{l}{c_b}\left(\frac{\pi}{R^b} - \beta_b\right)\pi + \beta_b \frac{1}{c_b} \tag{3.38}$$

Considering the production side of the model economy, equation (3.14') reduces to:

$$\xi = \frac{\epsilon_P - 1}{\epsilon_P} \tag{3.39}$$

Substituting equation (3.39) in savers' labor demand equation (3.16') derives:

$$w_s n_s = \alpha \frac{\epsilon_P - 1}{\epsilon_P} z n_s^{\alpha} n_b^{1-\alpha} \tag{3.40}$$

Likewise, substituting (3.39) in borrowers' labor demand equation (3.17') derives:

$$w_b n_b = (1-\alpha)\frac{\epsilon_P - 1}{\epsilon_P} z n_s^{\alpha} n_b^{1-\alpha} \tag{3.41}$$

Combining the savers' labor supply equation (3.6') with equation (3.40) and the production function (3.15') results in:

$$n_s = \left(\frac{\alpha \frac{1-\epsilon_P}{\epsilon_P} y}{c_s} \right)^{\frac{1}{\eta}} \tag{3.42}$$

The borrowers' labor supply equation (3.10') combined with equation (3.41) yields:

$$n_b = \left(\frac{(1-\alpha) \frac{\epsilon_P - 1}{\epsilon_P} y}{c_b} \right)^{\frac{1}{\eta}} \tag{3.43}$$

Plugging (3.42) and (3.43) in the production function and considering equation (3.28') results in:

$$c_b + c_s + \delta^b k^b = z \left(\frac{\alpha \frac{\epsilon_P - 1}{\epsilon_P} (c_b + c_s + \delta^b k^b)}{c_s} \right)^{\frac{\alpha}{\eta}} \left(\frac{(1-\alpha) \frac{\epsilon_P - 1}{\epsilon_P} (c_b + c_s + \delta^b k^b)}{c_b} \right)^{\frac{1-\alpha}{\eta}} \tag{3.44}$$

From equation (3.22') follows:

$$\delta^b k^b = \tau g_b \tag{3.45}$$

Inserting the saving rate (3.24') and the lending rate (3.25') in equation (3.22') and further partitioning delivers:

$$\delta^b k^b = \tau (R^b b^b - R^s d^s) \tag{3.46}$$

$$\delta^b k^b = \tau R \left(\frac{\epsilon_b}{\epsilon_b - 1} b^b - \frac{\epsilon_s}{\epsilon_s - 1} d^s \right) \tag{3.47}$$

$$\delta^b = \tau R\left(\frac{\epsilon_b}{\epsilon_b - 1}\frac{b^b}{k^b} - \frac{\epsilon_s}{\epsilon_s - 1}\frac{d^s}{k^b}\right) \tag{3.48}$$

Making use of the relation that $\frac{k^b}{b^b} = \upsilon$ leads to:

$$\delta^b = \tau\frac{R}{v}\left(\frac{\epsilon_b(\epsilon_s - 1) + (\epsilon_s v - \epsilon_s)(\epsilon_b - 1)}{(\epsilon_b - 1)(\epsilon_s - 1)}\right) \tag{3.49}$$

$$\delta^b = \tau\frac{R}{v}\left(\frac{\epsilon_b\epsilon_s - \epsilon_b + \epsilon_s v(\epsilon_b - 1) - \epsilon_s\epsilon_b + \epsilon_s}{(\epsilon_b - 1)(\epsilon_s - 1)}\right) \tag{3.50}$$

$$\delta^b = \tau\frac{R}{v}\left(\frac{\epsilon_s - \epsilon_b + v(\epsilon_s(\epsilon_b - 1))}{(\epsilon_b - 1)(\epsilon_s - 1)}\right) \tag{3.51}$$

Inserting the equations (3.26’), (3.21’), (3.51) and (3.45) in (3.8’) solves for

$$d_s = \frac{\left(\frac{(1-\alpha)}{(\epsilon_P-1)}(c_s + c_b + \delta^b k^b) - c_b + \frac{1}{\epsilon_P}(c_s + c_b + \delta^b k^b) + \frac{\delta^b k^b}{\tau}\right)}{\left(1 - \frac{R^s}{\pi}\right)} \tag{3.52}$$

Using (3.52), (3.27’) and (3.37) solves for:

$$k^b = \frac{\left(\frac{(1-\alpha)}{(\epsilon_P-1)}(c_s + c_b + \delta^b k^b) - c_b + \frac{1}{\epsilon_P}(c_s + c_b + \delta^b k^b) + \frac{\delta^b k^b}{\tau}\right)}{\left(1 - \frac{R^s}{\pi}\right)} + \frac{c_b - \frac{\epsilon_P-1}{\epsilon_P}(1-\alpha)(c_s + c_b + \delta^b k^b)}{\left(1 - \frac{R^b}{\pi}\right)} \tag{3.53}$$

Given (3.53) and (3.44) and (3.38) one can solve the system for the three unknowns c_s, c_b, k^b.

3.A.4 Log-linearized equations

For the linearization of the system, I take the natural logarithm of all variables to make use of the approximation that $\ln\left(\frac{x_t}{x}\right) \approx \frac{x_t - x}{x}$. Let $\ln\left(\frac{x_t}{x}\right) = \hat{x}_t$ be the variable's x_t deviation from its steady state x. Next, I apply a first order Taylor expansion to the system around the steady state.

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_{z,t} \tag{3.1”}$$

$$\hat{s}_{kb,t} = \rho_{s_{kb}} \hat{s}_{kb,t-1} + \varepsilon_{s_{kb},t} \tag{3.2”}$$

$$\hat{mk}_{b,t} = \rho_{mk_b} \hat{mk}_{b,t-1} + \varepsilon_{mk_b,t} \tag{3.3”}$$

$$\hat{mk}_{s,t} = \rho_{mk_s} \hat{mk}_{s,t-1} + \varepsilon_{mk_s,t} \tag{3.4”}$$

$$\frac{q}{c_s}(\hat{q}_t - \hat{c}_{s,t}) = -\frac{j^h}{h_s}\hat{h}_{s,t} + \beta_s \frac{q}{c_s} E_t(\hat{q}_{t+1} - \hat{c}_{s,t+1}) \tag{3.5”}$$

$$\hat{c}_{s,t} + (\eta - 1)\hat{n}_{s,t} = \hat{w}_{s,t} \tag{3.6”}$$

$$\hat{c}_{s,t} = \beta_s \frac{R^s}{\pi}(\hat{R}^s_t - E_t\hat{c}_{s,t+1} - E_t\hat{\pi}_{t+1}) \tag{3.7”}$$

$$\begin{aligned}\hat{c}_{s,t} + \hat{d}_{s,t} + qh_s(\hat{h}_{s,t} - E_t\hat{h}_{s,t+1}) = \frac{1}{\pi}R^s d_s(\hat{d}_{s,t-1} + \hat{R}^s_{t-1} - \hat{pi}_t) + \\ w_s n_s(\hat{w}_{s,t} + \hat{n}_{s,t}) + x\hat{x}_t + (1-\tau)\frac{g^b}{\pi}(\hat{g}^B_{t-1} - \hat{\pi}_t)\end{aligned} \tag{3.8”}$$

$$\frac{q}{c_b}(\hat{q}_t - \hat{c}_{b,t}) = -\frac{j^h}{h_s}\hat{h}_{b,t} + \mu lq\pi(\hat{\mu}_t + \hat{l}_t + E_t\hat{q}_{t+1} + E_t\hat{\pi}_{t+1}) + \beta_b\frac{q}{c_b} E_t\left(\hat{q}_{t+1} - \hat{c}_{b,t}\right) \tag{3.9''}$$

$$\hat{c}_{b,t} + (\eta - 1)\hat{n}_{b,t} = \hat{w}_{b,t} \tag{3.10''}$$

$$-c_b\hat{c}_{b,t} = \mu R^b(\hat{\mu}_t - \hat{R}^b_t) + \beta_b\frac{R^b}{c_b\pi}(\hat{R}^b_t - E_t\hat{c}_{b,t+1} - E_t\hat{\pi}_{t+1}) \tag{3.11''}$$

$$c_b\hat{c}_{b,t} + \frac{b_bR^b}{\pi}(\hat{b}_{b,t-1} + \hat{R}_{b,t-1} - \hat{\pi}_t) + qh_b(\hat{h}_{b,t} - \hat{h}_{b,t-1}) = b_b\hat{b}_{b,t} + w_bn_b(\hat{w}_{b,t} - \hat{n}_{b,t}) \tag{3.12''}$$

$$\hat{R}^b_t + \hat{b}^b_t = \hat{l}_t + E_t\hat{q}_{t+1} + \hat{h}_{b,t} + \hat{\pi}_{t+1} \tag{3.13''}$$

$$\hat{\pi}_t = \left(\frac{\epsilon_P - 1)}{\phi_P}\right)\hat{\xi}_t + \beta_sE_t\hat{\pi}_{t+1} \tag{3.14''}$$

$$\hat{y}_t = \hat{z}_t + \alpha\hat{n}_{s,t} + (1-\alpha)\hat{n}_{b,t} \tag{3.15''}$$

$$\hat{\xi}_t = \hat{w}_{b,t} + \alpha(\hat{n}_{b,t} - \hat{n}_{s,t}) - \hat{z}_t \tag{3.16''}$$

$$\hat{\xi}_t = \hat{w}_{s,t} + (1-\alpha)(\hat{n}_{s,t} - \hat{n}_{b,t}) - \hat{z}_t \tag{3.17''}$$

$$\hat{R}_t = \rho_r\hat{R}_{t-1} + (1-\rho_r)\left(\omega_\pi\hat{\pi}_t + \omega_y\hat{y}\right) + \varepsilon_t \tag{3.18''}$$

$$\hat{\upsilon}_t = \chi_\upsilon\left(\frac{\hat{b}}{y}_t\right) \tag{3.19''}$$

$$\hat{l}_t = \chi_l\hat{b}_t \tag{3.20''}$$

$$x\hat{x}_t = y\hat{y}_t - w_sn_s\hat{w}_{s,t} - n_sw_s\hat{n}_{s,t} - w_bn_b\hat{w}_{b,t} - w_bn_b\hat{n}_{b,t} \tag{3.21''}$$

$$k^b\pi(\hat{p}i_t+\hat{k}_t^b) = -k^b\pi\hat{s}_{kb,t}+\frac{1}{s_{kb}}\left((1-\delta^b)k^b\hat{k}_{t-1}^b + \tau\hat{g}_{t-1}^b\right) \quad (3.22")$$

$$R^{WS}\hat{R}^{WS} = R\hat{R}_t + \phi^{WS}(\hat{b}_t - \hat{k}_t^b) \quad (3.23")$$

$$(\epsilon_b - 1 + (1+\beta_s)\phi_b)\hat{R}_t^b = \phi_b\hat{R}_{t-1}^b+\beta_s\phi_b E_t\hat{R}_{t+1}^b+(\epsilon_b-1)\hat{R}_t^{WS}-\hat{\epsilon}_{b,t} \quad (3.24")$$

$$(\epsilon_s - 1 + (1+\beta_s)\phi_s)\hat{R}_t^s = \phi_s\hat{R}_{t-1}^s+\beta_s\phi_s E_t\hat{R}_{t+1}^s+(\epsilon_s-1)\hat{R}_t-\hat{\epsilon}_{s,t} \quad (3.25")$$

$$g^b\hat{g}_t^b = R^b b^b(\hat{R}_t^b + \hat{b}_t^b) - R^s d^s(\hat{R}_t^s\hat{d}_t^s) \quad (3.26")$$

$$b\hat{b}_t = d\hat{d}_t + k^b\hat{k}_t^b \quad (3.27")$$

$$y\hat{y}_t = c_s\hat{c}_{s,t} + c_b\hat{c}_{b,t} \quad (3.28")$$

$$0 = h_b\hat{h}_{b,t} + h_s\hat{h}_{s,t} \quad (3.29")$$

3.A.5 Some remarks

By borrowing the banking sector from Gerali et al. (2010), this model is closely related to their set-up, but the model dynamics differ along two dimensions.

First, after a positive technology shock in the model of Gerali et al., bank profits fall which they reason by the reduced bank interest rate spread. The lower earnings thereby outweigh the increase in intermediated funds. However, in the model at hand the banks' net worth increases when the economy is subject to a positive technology shock. One main model difference is the mentioned structure of the production sector. The broader production sector in the model of Gerali et al. (2010) features next to wage rigidities firms that are reliant on loans from banks to buy physical capital for production. Thus, higher credit availability through the shock directly increases investments in the capital stock, whereby aggregated output increases. As a second round effect the physical capital used as collateral appreciates which rises the firms' borrowing capacity. The mechanism triggers the financial accelerator. Since in my model only households borrow, the aggregated loan demand is less pronounced in my set-up. Therefore, given Gerali's model construction the need for financial intermediation is reinforced. Another reason for the countercyclical properties of bank profits in Gerali's model lies in the formulation of the banks' dividend policy. They assume all bank profits are retained to build up the bank capital stock. Opposing to Gerali et al. (2010), I assume that the representative bank retains only part of bank profits which in turn shows that the banks' market power is lower in my model. Consequently, banks have less influence on the interest rate spread and the intermediation margin which might cause the asymmetric reaction of bank profits to a technology shock across the models.

Second, after an exogenous destruction of bank capital inflation rises in Gerali's model. They justify this effect on prices by higher wage cost. Reacting to inflation,the central bank rises the nominal interest rate, which aggravates loan costs. However, the model in this paper documents benign deflation and nominal interest rate cuts when bank capital depreciates. Also Gertler and Karadi (2013) report deflation to a shock in the banking sector. Besides, deflationary tendencies have been observable in the aftermath of the crises that accompanied expansionary monetary policies.

Despite the fact that financial market conditions have no direct effect on production in my model, the credit conditions affect output indirectly via the households' labor supply and consumption demand. Since the largest share of household liabilities is mortgage debt, my modeling approach is justified. Justiniano et al. (2015) who also abstract from an external financed production sector follows the same reasoning.

3.B Perturbation techniques

3.B.1 First order approximation

The objective of the linearization is to convert the non-linear model equations into a linear system, which can be solved using a linear solution technique.[34] Following the exposition of Juillard (2005) and Faia (2008), in this section I explain how to yield a linear approximation of the policy function, for which

[34]The notation to above relates as follows $x_t = \hat{y}$.

a linear approximation of the model is needed. Moreover, by pointing to the distinction of a first order and a second order approximation, I make clear that for a reliable welfare calculation the variance of future shocks only matters if the model's decision rule and the transition function are second order polynomials (Schmitt-Grohé and Uribe, 2004b). The second order approximation is focus of the subsequent subsection.

The expectational difference equations of the model form a non-linear system. The general representation of a DSGE model is as follows:

$$E_t \{ f(y_{t+1}, y_t, y_{t-1}, v_t; \theta) \} = 0$$

y_t includes all endogenous variables that comprise predetermined and non-predetermined variables.[35] To be consistent with the method used in Dynare, I abstract from deterministic exogenous variables and a further variable partition. $v_t = \sigma \cdot \varepsilon$ are the exogenous stochastic shocks where σ scales the amount of uncertainty in the economy. ε is an auxiliary random variable with $E(\varepsilon_t) = 0$ and $E(\varepsilon\varepsilon') = \sum_\varepsilon$. θ is the vector of parameters.

Let $y_t = g(y_{t-1}, v_t, \sigma)$ be the policy function that solves the system.[36] Plugging in these guesses for $y_t = g(y_{t-1}, v_t, \sigma)$ and for

[35]By assuming stationary variables, the unconditional expectation of the level of y_t is time independent (variables show no long-term growth).

[36]In other words, g is a time recursive (approximated) representation of the model that can generate time series that will approximately satisfy the rational expectation hypothesis contained in the original model.

$y_{t+1} = g(y_t, v_{t+1}, \sigma) = g(g(y_{t-1}, v_t, \sigma), v_{t+1}, \sigma)$ into the function f results in:

$$\begin{aligned} &F(y_{t-1}, v_t, v_{t+1}, \sigma) = \\ &f(g(g(y_{t-1}, v_t, \sigma), v_{t+1}, \sigma), g(y_{t-1}, v_t, \sigma), v_{t-1}, v_t) \end{aligned} \tag{3.54}$$

If the guesses are correct, then $E_t \{F(y_{t-1}, v_t, v_{t+1}, \sigma)\} = 0$ is the solution function and the model is solved (Faia, 2008). Hence, the aim is finding the model's solution function for any given order of approximation that is valid around the neighborhood of the particular steady state of the system. There are several deterministic steady states that satisfy $f(y, y, y, 0)$, whereby y denotes the steady state of y_t. But only one steady state is used for the approximation. Because $F(y, v)$ must be equal to zero for any possible values of y and v, if g is correct, it must be the case that the derivatives of any order of F must be also equal to zero (Schmitt-Grohé and Uribe, 2004b).

$$F_{y^k \sigma^j}(y, \sigma) = 0 \quad \forall y, \sigma, j, k, \tag{3.55}$$

$F_{y^k \sigma^j}(y, \sigma)$ denotes the derivative of F with respect to y taken k times and with respect to σ taken j times.

Next, I take the first order Taylor approximation F^1 of the system (3.54) around the non-stochastic steady state $y = g(y, 0, 0)$ where $y_t = y$, $v_t = 0$ and $\sigma = 0$.

$$E_t\left(F^1(y_{t-1}, v_t, v_{t+1}, \sigma)\right) = E_t\left(f(y, y, y, 0) + f_{y+}\right.$$
$$\left(g_y(g_y\hat{y} + g_v v + g_\sigma\sigma) + g_v \underbrace{v'}_{Ev'=0} + g_\sigma\sigma\right) +$$
$$\left. f_y\left(g_y\hat{y} + g_v v + g_\sigma\sigma\right) + f_{y-}\hat{y} + f_v v\right) = 0$$

with $\hat{y}_t = y_t - y$ expressing deviations of the variable from steady state. The timing notation is as follows: $v = v_t, v' = v_{t+1}$ and the derivative notation is defined by $f_{y+} = \frac{\partial f}{\partial y_{t+1}}$, $f_y = \frac{\partial f}{\partial y_t}$, $f_{y-} = \frac{\partial f}{\partial y_{t-1}}$, $f_v = \frac{\partial f}{\partial v_t}, g_{y-} = \frac{\partial g}{\partial y_{t-1}}, g_v = \frac{\partial g}{\partial v_t}, g_\sigma = \frac{\partial g}{\partial \sigma}$.
Taking expectations yields:

$$E_t\left(F^1(y_{t-1}, v_t, v_{t+1}, \sigma)\right) = \left(f_{y+}g_y g_y + f_y g_y + f_{y-}\right)\hat{y} +$$
$$\left(f_{y+}g_y g_v + f_y g_v + f_v\right)v + \left(f_{y+}g_y \underbrace{g_\sigma}_{=0} + f_y \underbrace{g_\sigma}_{=0}\right)\sigma = 0.$$

The last term is zero since the certainty equivalence holds, $g_\sigma = 0$. The certainty equivalence states that the policy function is independent of the variance-covariance matrix $\sum_\varepsilon$. In general, when approximating the policy function up to first order, second order terms are omitted. As also higher polynomials of the welfare function are discarded, a accurate welfare analysis is not possible (Schmitt-Grohé and Uribe, 2004b). If the focus of research is the effect of uncertainties on the economy, e.g., to evaluate precautionary savings, first order perturbation techniques are not appropriate.

To derive the approximation of the policy function, I conclude from the term in front of v_t that g_v satisfies $-\left(f_{y+}g_y + f_y\right)^{-1} f_v$. g_y can be recovered from $\left(f_{y+}g_yg_y + f_yg_y + f_{y-}\right)\hat{y}$. Taking expectations, the structural state space representation is:

$$\begin{bmatrix} 0 & f_{y+} \\ I & 0 \end{bmatrix} \underbrace{\begin{bmatrix} I \\ g_y \end{bmatrix} g_y\hat{y}}_{x_{t+1}} = \begin{bmatrix} -f_{y-} & -f_y \\ 0 & I \end{bmatrix} \underbrace{\begin{bmatrix} I \\ g_y \end{bmatrix} \hat{y}}_{x_t}, \qquad (3.56)$$

whereas $x_{t+1} = \begin{bmatrix} y_t - y \\ y_{t+1} - y \end{bmatrix}$ and $x_t = \begin{bmatrix} y_{t-1} - y \\ y_t - y \end{bmatrix}$ denoting the variables deviation from its steady state value. Rewriting equation (3.56) results in linear stochastic difference equations written in the state-space representation:

$$AE_t(x_{t+1}) = Bx_t + Cv_t$$

The solution I seek is the first order approximated decision function (3.57), which can be derived by applying the solution technique by Klein (2000) in appendix section 2.B.[37]

$$
\begin{aligned}
y_t &= g(y_{t-1}, v_t, \sigma) \\
y_t &= y + g_y \hat{y} + g_v v
\end{aligned}
\tag{3.57}
$$

The solution represents the time series behavior of y_t as a function of the structural shocks v_t. The expectation of y_t is $E(y_t) = y$ and the variance $\sum_y = g_y \sum_y g'_y + \sigma^2 g_u \sum_\varepsilon g'_u$. Thus, up to first order the unconditional mean of endogenous variables is the same as their non-stochastic steady state value. First order perturbation methods have the advantage that they do not suffer from the ”curse” of dimensionality. As the linear perturbation method is not computationally demanding, the method can be applied in models with a large number of state variables without the need for much computational power.

3.B.2 Second order approximation

This section focuses on a second order approximation to the model equations based on Juillard (2005). Second order perturbation methods account for uncertainties, which affect welfare. Therefore, a welfare analysis requires a second order approximation. The second order Taylor approximation F^2 of (3.54)

[37] The Klein solution algorithm explains how one can exclude explosive trajectories of the system by selecting a stable trajectory (Dejong and Dave, 2011). In particular, it states how to select the stable trajectory given by $g_y = -Z_{22}^{-1} Z_{21}$ (see appendix section 2.B).

is:

$$E_t\left(F^2(y_{t-1},v_t,v_{t+1}\sigma)\right) = E_t\left(F^1(y_{t-1},v_t,v_{t+1},\sigma)+\right.$$

$$0.5\left(F_{y-y-}(\hat{y}\otimes\hat{y}) + F_{vv}(v\otimes v) + F_{v'v'}(\underbrace{v'\otimes v'}_{\sigma^2\vec{\sum}_\varepsilon}) + F_{\sigma\sigma}\sigma^2\right)+$$

$$F_{y-v}(\hat{y}\otimes v) + F_{y-v'}(\hat{y}\otimes\underbrace{v'}_{E=0}) + F_{y-\sigma}\hat{y}\sigma + F_{vv'}(v\otimes v) + F_{v\sigma}v\sigma+$$

$$\left. f_{v'\sigma}\underbrace{v'}_{E=0}\sigma\right).$$

In order to explain the notation used, I assume y_t to be $y = g(s)$ and $f(y) = f(g(s))$. Then the second order derivative is: $\frac{\partial^2 f}{\partial s\partial s} = \frac{f}{\partial y}\frac{\partial^2 g}{\partial s\partial s} + \frac{\partial^2 f}{\partial y\partial y}\left(\frac{\partial g}{\partial s}\otimes\frac{\partial g}{\partial s}\right)$. $\otimes$ refers to the Kronecker product.

As before, all derivatives of F are equal to zero. I recover g_{yv} from: $F_{y-v} = f_{y+}(g_{yy}(g_y\otimes g_v) + g_y g_{yv}) + f_y g_{yv} + B = 0$. The term B contains all terms that are not second derivatives of g. A standard linear problem results: $g_{yv} = -(f_y + g_y + f_y)^{-1}(B + f_y g_{yy}(g_y\otimes g_v))$ (Schmitt-Grohé and Uribe, 2004b). From $F_{y-y-} = f_{y+}(g_{yy}(g_y\otimes g_y) + g_y g_{yy}) + f_y g_{yy} + B = 0$ it stems that $(f_{y+}g_y + f_y)g_{yy} + f_y + g_{yy}(g_y\otimes g_y) = -B$. This Sylvester type equation can be solved with a appropriate algorithm to recover g_{yy}.

Next, finding a solution for g_{vv} from $F_{vv} = f_{y+}(g_{yy}(g_v\otimes g_v) + g_y g_{vv}) + f_y g_{vv} + B = 0$ the following standard linear problem evolves: $g_{vv} = -(f_{y+}g_y + f_y)^{-1}(B + f_{y+}g_{yy}(g_v \otimes g_v))$, where B contains higher order derivatives of $g(\cdot)$.

Since $g_\sigma = 0$ and $F_{y\sigma} = f_y g_y g_{y\sigma} + f_y g_{y\sigma} = 0$ and $F_{v\sigma} = 0$ are likewise zero, one can conclude that also $g_{y\sigma} = g_{v\sigma} = 0$. I derive $g_{\sigma\sigma}$ by rearranging $F_{\sigma\sigma} + F_{v'v'} \sum_\varepsilon = f_{y+}(g_{\sigma\sigma} + f_y g_{\sigma\sigma} + g_y g_{\sigma\sigma}) + (f_{y+y+}(g_v \otimes g_v) + f_{y+}g_{vv}) \sum_\varepsilon = 0$, so that: $g_{\sigma\sigma} = -(f_{y+}(I + g_y) + f_y)^{-1}(f_{y+y+}(g_v \otimes g_v) + f_{y+}g_{vv}) \sum_\varepsilon$.

The second order decision function then simplifies to:

$$y_t = y + 0.5 g_{\sigma\sigma}\sigma^2 + g_y\hat{y} + g_v v + 0.5(g_{yy}(\hat{y}\otimes\hat{y}) + g_{vv}(v\otimes v)) + g_{yv}(\hat{y}\otimes v). \tag{3.58}$$

The second order approximation departs from the certainty equivalence theorem because with $g_{\sigma\sigma}$ the variance of future shocks matters. The result (3.58) shows that the coefficients on the linear and quadratic terms of the state vector are independent of the size of the variance of the underlying shocks but not the constant term, which is significantly altered by the prevailing uncertainty in the model.

Thus, the expectation of y is $E(y_t) = y + (I - g_y)^{-1}(0.5(g_{\sigma\sigma} + g_{yy} \sum_y + g_{vv}\sigma_\varepsilon))$, when $\sigma = 1$ is assumed. The variance is given by $\sum_y = g_y \sum_y g'_y + \sigma^2 g_u \sum_u g'_u$. As the first moments of the system depend on the variance of the shocks given by $0.5 g_{\sigma\sigma}\sigma^2$, the uncertainty in the model affects that the mean of the endogenous variables is different from the non-stochastic steady state. Consequently, the second order approximation to the policy function of a stochastic model differs from its non-stochastic counterpart. Moreover, by applying a second order approximation the non-stochastic and the deterministic steady state differ. The non-stochastic steady state is the point, where the agents decide to stay in the absence of shocks but taking into account the likelihood of future shocks.

3.C Welfare computation

The following section explains the computation of the welfare measure used in this thesis. The welfare measure is defined as the present discounted value of lifetime utility Ω_t. By adding the welfare measure to the endogenous non-predetermined variables, it is possible to calculate the second order approximation to the policy function and simultaneously obtain the second order welfare approximation $\Omega_t = g^{\Omega_t}(y_t, \sigma)$, whereby g^{Ω_t} is just one element of the policy function. So welfare is just one non-linear function of the state vector y_t, which includes the initial states of the economy and σ. The coefficients of the linear and quadratic terms approximated welfare function are independent of the exogenous shocks, but as explained above not the constant terms. Hence, the welfare means with and without policy diverge (Schmitt-Grohé and Uribe, 2004a).[38] To calculate welfare, one can either use the conditional expectation of lifetime utility or the unconditional expectation. In contrast to unconditional welfare, the conditional welfare measure captures the effects of the transition from the non-stochastic steady state without the policy to the steady state induced by the policy of consideration. Thus, one can ensure that the economy begins from the same initial point under all possible policies.

I note that the use of conditional welfare does not imply that results are necessarily tied to some particular initial state (Faia, 2008). Depending on the particular research question one may consider a number of relevant initial values for the state vector and average over those to obtain an average value of conditional welfare (Schmitt-Grohé and Uribe, 2009). Since my research focus is to include the transition path from a non-macroprudential policy situation to a macroprudential policy situation, I tie my results on simply one original state vector.

Comparing alternative macroprudential policies that are implemented in economies with the same non-stochastic steady state, I compute the welfare benefits in percentage of the household's consumption stream that the household receives associated with the particular policy introduction. $\Omega_{cr,i,t}$ captures the lifetime

[38] If welfare were directly approximated to first order, all policies that imply the same non-stochastic steady state will give rise to the same level of welfare. That is because the unconditional expectation and the steady state value of a variable are equal using first order approximation techniques.

utility level of the household i, while i stands either for the lending s or borrowing b household. In fact, calculating lifetime utility recursively, the welfare level with regulation is $\Omega_{mp,i,t}$.

$$\Omega_{cr,i,t} \equiv E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\ln(c_{cr,i,t}) + j \ln(h_{cr,i,t}) - \frac{n_{cr,i,t}^{\eta}}{\eta} \right)$$

$$\Omega_{mp,i,t} \equiv E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\ln(c_{cr,i,t}) + j \ln(h_{cr,i,t}) - \frac{n_{cr,i,t}^{\eta}}{\eta} \right)$$

Following Schmitt-Grohé and Uribe (2004a), the welfare change Λ_i (3.59) is computed by equating the lifetime utility level without regulation $\Omega_{cr,i,t}$ and the lifetime utility level achieved with macroprudential regulation $\Omega_{mp,i,t}$. Thus, Λ_i denotes the welfare gain of adopting macroprudential regulation in consumption units for household i in reference to the benchmark situation without macroprudential regulation. The welfare measure allows to rank the policies according to their desirability for the specific agents.

$$\Omega_{mp,i,t} = E_0 \sum_{t=0}^{\infty} \beta_i^t \left(\ln((1+\Lambda_i)c_{cr,i,t}) + j \ln(h_{cr,i,t}) - \frac{n_{cr,i,t}^{\eta}}{\eta} \right) \tag{3.59}$$

Solving for Λ_i induces:

$$\Omega_{mp,i,t} = E_0 \sum_{t=0}^{\infty} \beta_i^t \ln(1+\Lambda_i) + \Omega_{cr,i,t}$$

$$\Omega_{mp,i,t} = \frac{1}{1-\beta_i} \ln(1+\Lambda_i) + \Omega_{cr,i,t}$$

$$\Lambda_i = \exp((1-\beta_i)(\Omega_{mp,i,t} - \Omega_{cr,i,t})) - 1$$

By performing a second order expansion at the non-stochastic steady state of the policy and non-policy regime, one receives $g^{\Omega_{mp,t}}(y,0)$ and $g^{\Omega_{cp,t}}(y,0)$. Since the initial non-stochastic steady state is the same in both regimes, there is only a change in the system's derivatives with regard to σ. So the resulting second-order-accurate measure of welfare is the percentage gain of consumption for agent i given by:

$$\Lambda_i \approx \exp\left((1-\beta_i) \left[g_{\sigma\sigma}^{\Omega_{mp,i,t}}(y,0) - g_{\sigma\sigma}^{\Omega_{cr,i,t}}(y,0) \right] \frac{\sigma}{2} \times 100 \right) - 1.$$

4 Assessing macroprudential regulation: the role of the zero lower bound

4.1 Introduction

Nominal interest rates near the zero lower bound (ZLB) and slow growth characterize many economies since the economic and financial crisis almost a decade ago. These low rates, however, instead of fostering lending and spending, may and do lead households to deleverage due to low inflation expectations (Guerrieri and Lorenzoni, 2011; Eggertsson and Krugman, 2012). In this paper, we contribute to the literature by looking at the interaction between the ZLB and rule-based, and therefore flexible macroprudential regulation (FMR) that precisely addresses the leverage of household. Macroprudential regulation gained quite some attention both in policymaking and academia; cf. Farhi and Werning (2016), Dogra (2014), and Darbar and Wu (2016).

More specifically, we look at both a fixed and a flexible loan-to-value (LTV) ratio that constrains borrowers. FMR is thus implemented by allowing the maximum LTV ratio to respond to macroeconomic conditions. Monetary policy may or may not be constrained by the ZLB. The unconstrained scenario simply allows for a negative interest rate if an adverse demand shock is sufficiently strong, whereas the constrained scenario does not; see also Fernández-Villaverde et al. (2015).

In section 2, the theoretical model is briefly described, section 3 explains our implementation, section 4 presents and discusses the results. Section 5 concludes.

4.2 The model

4.2.1 Households and production

We use a slightly modified version of Rubio and Carrasco-Gallego (2015a)'s standard two-representative agent model with group specific discount rates β_i, $i \in \{s, b\}$ that determine savers and borrowers. Both groups are of equal size. The assumption $\beta_b < \beta_s$ implies that the more patient savers are lending to borrowers.

Both household types solve the same intertemporal maximization problem which is given by

$$\max_{c_{i,t},h_{i,t},n_{i,t},b_{i,t}} E\sum_{t=0}^{\infty}\beta_i^t\kappa_t\left[\log c_{i,t} + j\log h_{i,t} - \frac{n_{i,t}^{\eta}}{\eta}\right]$$

$c_{i,t}$, $h_{i,t}$, $n_{i,t}$ and j denote consumption, housing services, working hours and the weight of housing, respectively. $\eta - 1 \geq 0$ is the inverse of the Frisch labor supply elasticity. κ_t denotes an intertemporal aggregate demand shock. Following Guerrieri and Iacoviello (2016), it is specified as

$$\ln(\kappa_t) = \rho_\kappa \ln(\kappa_{t-1}) + \varepsilon_{\kappa,t} \tag{4.1}$$

where $0 < \rho_\kappa < 1$ and $\varepsilon_{\kappa,t} \sim N(0, \sigma^2_{\varepsilon_\kappa})$. A decrease of κ_t implies a negative aggregate demand shock since both savers and borrowers are more willing to postpone consumption. Savers face the budget constraint

$$c_{s,t} + b_{s,t} + q_t h_{s,t} = b_{s,t-1}\frac{r_{t-1}}{\pi_t} + w_{s,t}n_{s,t} + q_t h_{s,t-1} + x_t, \tag{4.2}$$

where $b_{s,t}, q_t, r_{t-1}, w_{s,t}$ and π_t denote lending, the price of housing, both in units of consumption, the gross nominal interest rate in $t-1$, the real wage rate earned by savers and overall inflation rate, respectively. x_t are dividend payments from the production sector that are assumed to go to the savers. The budget constraint of borrowers is

$$c_{b,t} + \frac{r_{t-1}}{\pi_t} b_{b,t-1} + q_t h_{b,t} = q_t h_{b,t-1} + b_{b,t} + w_{b,t} n_{b,t} \qquad (4.3)$$

The amount of borrowing is limited by a maximum LTV ratio

$$l \geq E_t \left[\frac{r_t b_{b,t}}{\pi_t q_{t+1} h_{b,t}} \right]. \qquad (4.4)$$

A flexible limit is detailed below.

The production sector is basic. Final output y_t is assembled from a continuum of intermediate goods $y_t(z)$ according to

$$y_t = \int_0^1 \left(y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad (4.5)$$

whereas the $y_t(z)$'s are produced in a continuum of firms $z \in [0, 1]$ that are owned by lenders by means of the CRS technology

$$y_t(z) = n_{s,t}(z)^{\alpha} n_{b,t}(z)^{1-\alpha}, \qquad (4.6)$$

Each firm employs the two types of labor paying $w_{b,t}$ to borrowers and $w_{s,t}$ to savers. (Rationales for this distinction are given e.g. in Iacoviello and Neri (2010).) Prices $P_t(z)$ are set in the usual way observing Rotemberg price adjustment costs ϕ_P and demand $y_t(z) = \left[\frac{P_t(z)}{P_t} \right]^{-\varepsilon} y_t$. Price adjustment costs are introduced in order to allow for different developments of real and nominal interest rates.

Assuming symmetry across firms, profit maximization yields the log-linearized Phillip's curve:

$$\hat{\pi}_t = \frac{(\varepsilon - 1)}{\phi_P}\hat{\xi}_t + \beta_s \kappa_t E_t \hat{\pi}_{t+1} \tag{4.7}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ and ξ_t depict gross inflation and marginal cost, respectively. Hats denote deviations from steady state.

4.2.2 Monetary and macroprudential policy

The central bank sets the interest rate according to the modified Taylor rule

$$r_t = \max\left[1, r^{1-\rho_r} r_{t-1}^{\rho_r}\left(\left(\frac{\pi_t}{\pi}\right)^{\omega_\pi}\left(\frac{y_t}{y}\right)^{\omega_y}\right)^{1-\rho_r}\varepsilon_{v,t}\right] \tag{4.8}$$

when it is constrained by the ZLB. If the ZLB is not binding, interest rates unconditionally follow the second expression in the maximum function. $\epsilon_{v,t}$ captures the monetary policy shock.

Following e.g. Lambertini et al. (2013), FMR is introduced as

$$l_t = l_{t-1}^{\rho_l}\left(l\left(\frac{b_t}{b}\right)^{\chi_l}\right)^{(1-\rho_l)} \tag{4.9}$$

where l denotes the steady state value of the LTV ratio. $0 < \rho_l < 1$ and χ_l denote the persistence and the reaction parameter of the rule, respectively. A countercyclical macroprudential regulation is implied by $\chi_l < 0$. This can be motivated by the fact that individual borrowers do not take into account the negative externality their indebtedness exerts on the macroeconomy in a downturn (Korinek and Simsek, 2016).

In order to close the model, the following market clearing conditions are introduced. Supply of housing is fixed at unity, i.e. $h_{s,t} + h_{b,t} = 1$, goods market clearing is given by $y_t = c_{b,t} + c_{s,t} + \left(\frac{\phi^P}{2}\right)\left(\frac{P_t}{\pi P_{t-1}} - 1\right)^2 y_t$, bond market clearing requires $b_{s,t} = b_{b,t}$, labor market clearing is given by $\int_0^1 n_{i,t}(z)dz = n_{i,t}$, $i \in \{s, b\}$.

4.3 Method and calibration

The model outlined above is simulated in the usual way when the ZLB does not bind or is assumed not to bind. If the ZLB is binding, the policy rule becomes non-linear in the sense of reaction function (4.8). We handle this by using the Occbin toolbox developed by Guerrieri and Iacoviello (2015).[1] In order to calibrate the model, we use parameter values that are commonly used in the relevant literature with the aim of matching data from the Eurozone; see table 4.1.

[1] The occasionally binding solution technique is outlined in the appendix section 4.A.

Table 4.1: Calibration parameters

Parameter	Description	Value
β_s	discount factor saver	0.99
β_b	discount factor borrower	0.975
η	parameter associated with labor elasticity	2
j	weight of housing in utility function	0.1
l	steady state LTV ratio	0.7
α	labor share of saver	0.64
ϕ_P	price adjustment costs	58
ε	price elasticity of demand	6
ρ_r	interest rate smoothing parameter in TR	0.8
ω_y	output parameter in TR	0.1
ω_π	inflation parameter in TR	2
ρ_l	smoothing parameter in LTV-rule	0.2
χ_l	reaction parameter in LTV rule	-2
ρ_κ	persistence preference shock	0.9
σ_κ	standard deviation preference shock	0.02

4.4 Results and discussion

The exogenous shock we are focusing on is a 2 percent reduction of κ_t in $t = 1$. This makes both savers and borrowers willing to postpone consumption and therefore lowers aggregate demand. The shock is strong enough in order to make the ZLB binding. Figures 4.1 and 4.2 show the impulse responses after this shock with and without a binding ZLB, respectively. The solid lines depict the situation when FMR according to (4.9) is in place, where as the dotted lines apply for a fixed value of l.

In all scenarios, house prices increase. This is due to the fact that housing serves as a saving vehicle. Since everybody wants to shift consumption to the future, this vehicle becomes more expensive.

Clearly, the adverse shock triggers an expansionary monetary policy aimed at dampening the effects on output and inflation. If the ZLB binds, however, the real interest nevertheless increases, bringing down indebtedness. This deleveraging leads to a countercyclical increase of the LTV ratio in the presence of FMR. Note that FMR considerably dampens the volatility of output, inflation and, most significantly, debt. Hence, on top of stabilizing financial markets (indebtedness), FMR is a partial substitute for monetary policy in stabilizing the economy after an adverse demand shock.

If the ZLB is assumed not to bind, the real interest rate goes down as a result of the expansionary monetary policy stance. Unlike in the case of a binding ZLB, this leads to an increase in debt, which in turn implies a procyclical downward correction of the LTV ratio in the case of a FMR. Note that this still very successfully dampens indebtedness, whereas there is no additional effect of FMR on the stability of output and inflation if the ZLB does not bind.

On a different note, the results also show the benefits of an unconstrained monetary policy in terms of macroeconomic stabilization. This is a reminder that monetary policy should keep clear of the ZLB in the first place.

4.5 Conclusion

This paper extends the existing literature by documenting that flexible macroprudential regulation helps to attenuate the effects of an adverse aggregate demand shock in the presence of a ZLB. The inability of a constrained monetary policy to bring down real interest rates can thus be partially compensated by flexible macroprudential regulation.

Figure 4.1: Impulse responses after a negative demand shock with ZLB

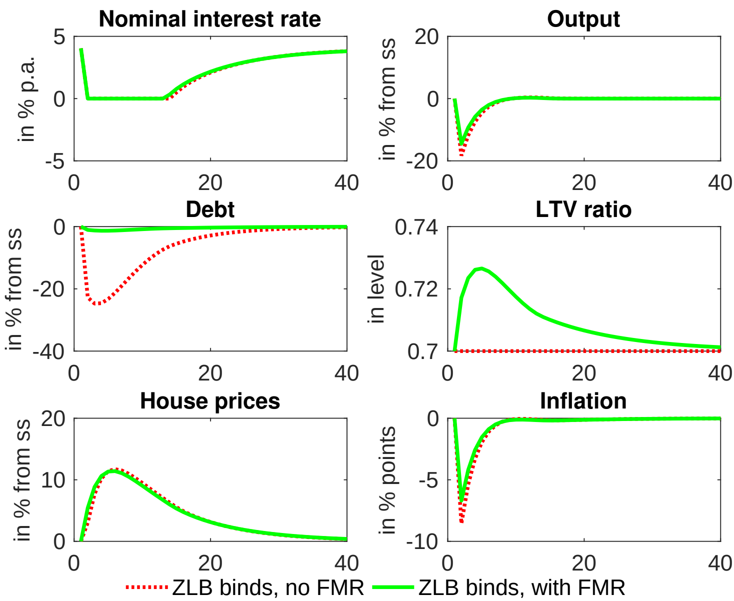
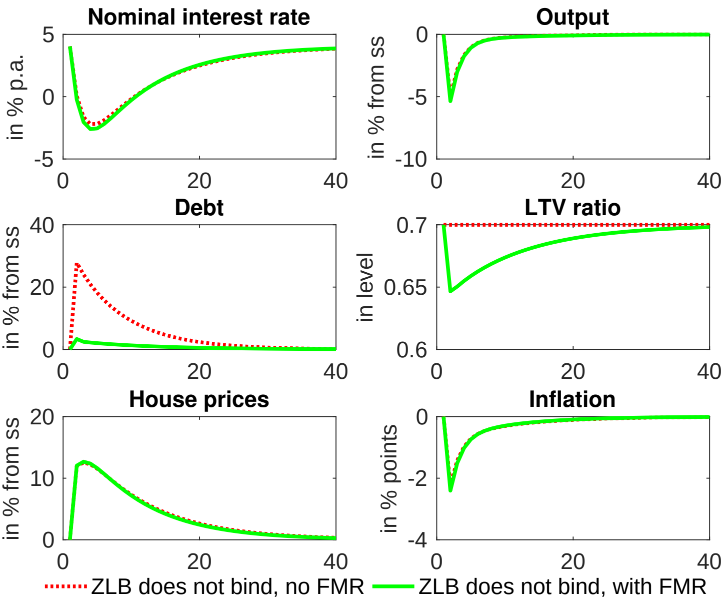


Figure 4.2: Impulse responses after a negative demand shock without ZLB



Appendix

4.A Nonlinear model solution technique

Solving dynamic models with standard linear perturbation methods fails to depict inequality constraints that often arise in economics. Next to several other nonlinear solution methods for medium-size models (e.g., the Smolyak algorithm by Malin et al. (2011), the penalty function approach by Judd (1998)), Guerrieri and Iacoviello (2015) introduce an algorithm to capture occasionally binding constraints in a DSGE model using piecewise linear techniques. The approach assumes that the same model economy comprises of two linear regimes: in one regime dubbed as reference model (R1) the occasionally binding constraint is slack. In the alternative regime (A1) the same constraint is binding. The piecewise linear solution method involves linking the first-order approximation of the model to the same point under each regime. For the existence of a rational expectations solution, the Blanchard and Kahn conditions must hold under the R1 regime. The solution algorithm presumes that a shock hits the model economy in R1 such that the constraint binds. Consequently, the model economy switches to A1. The impact of the shock proceeds regarding the A1 model equations with the binding constraint and abates over a finite time horizon, so that the model returns back to R1. The underlining important assumption is that agents expect no future shocks to occur. Formally, the generic representation of a linearized DSGE model with the occasionally binding constraint being slack is given by:[2]

$$AE_t x_{t+1} + Bx_t + Cx_{t-1} + Ev_t = 0, \qquad (4.10)$$

where the $n \times n$ matrices A, B, C and the $n \times m$ matrix E collect the structural parameters of the linearized system. The vector x_t captures the variables deviation from steady state and v_t comprises of the shock processes. The system linearized around the same non-stochastic steady state with the binding constraint can be expressed as:

$$A^* E_t x_{t+1} + B^* x_t + C^* x_{t-1} + D^* + E^* v_t = 0, \qquad (4.11)$$

where the matrices A^*, B^*, C^* and E^* collect again the structural parameters. The additional vector D^* of size n encompasses constants. D^* includes the terms that arise because the linearization is carried out around the same non-stochastic steady

[2]This representation abstracts from the usual first-order system notation, so that lagged and current state variables are written separately.

state as in R1, even though now the constraint binds. The vector x_t collecting the variables' deviation remains equal across the systems.

Following Guerrieri and Iacoviello (2015), the policy function for the model with an occasionally binding constraint is defined as a function $g(x_{t-1}, v_t) = x_t$ such that the conditions under system (R1) or the system (A1) hold, depending on the evaluation of the occasionally binding constraint. Given the initial steady state vector x_0 and the realization of the shocks v_1, the equation g can be expressed by the set of matrices R_t, the set of matrices P_t, and a matrix Q_1 for the particular point in time such that:

$$x_1 = P_1 x_0 + R_1 + Q_1 v_1. \tag{4.12}$$

Even though $P_t R_t$ are functions of x_{t-1} and the initial shock ϵ_1 only, the matrices are time-varying summarized by:

$$x_t = P_t x_{t-1} + R_t \forall t \in \{2, \infty\}. \tag{4.13}$$

Also Q_t is time-varying, but it fades out in the long-run with the decreasing impact of the initial shock. The above equations (4.12) and (4.13) show that even though R1 and A1 are linear, the solution of the piecewise algorithm is nonlinear. Thus, the applied solution method provides a local approximation as a function of the two models with and without a binding constraint. Next, by a guess-and-verify approach the routine is repeated along the following steps until the right guess is verified. Recall that in $t = 0$ when the shock occurs the model R1 is relevant. The algorithm initially assumes the number of periods in which each regime applies and updates the number if the verification of the guess fails.

1. Given the realized shock v_1, let T be the point in time when the model returns to $R1$ for all consequent periods $t \geq T$. The linear approximation to the decision rule x_t is then: $x_t = Px_{t-1} + Qv_t$ and remains time invariant for any periods, $t \geq T$ since $P_t = P, R_t = 0$ applies.

2. Using the equation $x_T = Px_{T-1}$ and equation 4.11, the solution in period $T - 1$ satisfies: $A^*Px_{T-1} + B^*x_{T-1} + C^*x_{T-2} + D^* = 0$, whereby the underlying assumption that agents expect no shocks beyond the first period is carried out. Solving the equation for x_{T-1} results in the decision rule for x_{T-1} as a function of x_{T-2}: $x_{T-1} = -(A^*P + B^*)^{-1}(C^*X_{T-2} + D^*)$.

3. As a next step, use the last decision rule $x_{T-1} = P_{t-1}x_{T-2}+ R_{T-1}$ with $P_{t-1} = -(A^*P + B^*)^{-1}C^*$ and with $R_{T-1} = -(A^*P + B^*)^{-1}D^*$ to solve for x_{T-2} given x_{T-3}. In doing so consider either model R1 or A1 implied by the current guess.

4. The iteration back proceeds until x_0 is reached considering the prevailing regime by the guess.

5. Depending on whether R1 or A1 is guessed to apply in period one, $Q_1 = -(AP_2+B)^{-1}\epsilon$ or $Q_1 = -(A^*P+B^*)^{-1}\epsilon^*$ is chosen.

6. Using the guess for the solution obtained in steps 1. to 5., compute the paths for x to verify the current guess of regimes. If the guess is verified, stop. Otherwise, update the guess for when regimes (R1) and (A1) apply and return to step 1.

The resulting approximated policy function to the model is highly nonlinear because the shift in regimes is associated with a change of the path of the endogenous variables. More specific, the period of time the model remains in one regime depends on the expectations regarding that regime, which in turn depends on the state vector. This interaction between expectations and the state vector generates the nonlinearity. The approximation based on a first-order perturbation approach has the limitation that it discards all information concerning future shocks.[3] However, the algorithm has is capable to render large scale models and is computational fast.

[3] As agents are unable to foresee the rare possibility of a binding zero lower bound of nominal interest rates, they do not accumulate precautionary savings as lined out in appendix section 3.B.

5 Conclusion

This thesis examines to what extent macroprudential policy contributes to financial and macroeconomic stability using a New Keynesian DSGE model. Adopting countercyclical rules as instruments, the caps on borrowers' LTV ratios are flexible by changing in a countercyclical manner. Thus, the regulation aims to contain the build-up of leverage before a crisis and reduces fire sales of collateral during a crises. Whereas fixed credit limits lead to a procyclical amplification of shocks. To learn more about the explicit policy configuration and its functioning, borrowers face a credit constraint tight to their housing collateral in the underlining model of the following papers.

According to the results of chapter 2, that analyzes borrowers with high and low debt levels, a countercyclical rule attached to the LTV ratio is destabilizing if the leverage level allowed by the rule is relatively low. Simulation results show that during a housing demand boom the tight credit limit induced by the rule impede any economic activity and create a conflict between macroprudential and monetary policy. That is because the amplification effect of collateral constraints coming from less indebted borrowers makes the countercyclical rule irrelevant. However, my insights reveal that a rule on highly indebted borrowers permits a larger policy scope to achieve financial and macroeconomic stability at once and improves total welfare. In addition, credit growth shows to serve as a good indicator for an overheating of the financial sector.

Extending the former model by a banking sector, the paper of chapter 3 tests both, end-borrower regulation in form of a rule on borrowers' LTV ratio and a lender related instrument, namely a rule on the BC ratio of banks. I find that the rule on the LTV ratio mitigates the volatility of mortgage credit more effectively than countercyclical BC regulation during financial and economic downturns. Thus, the LTV rule shows to successfully avert a credit crunch. The rationale behind the dynamics is that the rule on the LTV ratio relaxing the credit limit of borrowers curtails the drop of credit demand, which stabilizes the credit-to-output ratio and the BC position. Further, I uncover that mild countercyclical rules are socially optimal. The welfare gain of borrowers through consumption smoothing with either rule compensates the welfare loss of savers. In conclusion, regulating borrowers credit behavior represents an important complement to macroprudential BC regulation.

My findings in chapter 3 also predict that both rules fail to achieve macroeconomic stabilization if the financial shock occurring in the banking sector affects the macroeconomy exclusively over the private mortgage market. The financial shock is indeed a demand shock moving inflation and output in the same direction that is expected to induce the amplification effect of collateral constraints (Iacoviello, 2005). However, I stress that the amplification of output due to borrowers' collateral constraint arises first and foremost, if the shock occurs on the borrowing agent's utility objective function and changes a variable that is relevant for the collateral constraint, e.g., a housing demand shock.[1]

In chapter 4, I modify the otherwise standard model of chapter 2 to study the consequences of zero interest rates for the macroprudential policy transmission. An exogenous shift of households' preference to consume in the future induces a demand-driven recession in the model, so that output declines sharply. Monetary policy counteracting the abated availability of loans lowers the nominal interest rate until the ZLB binds. The subsequent deflation pushes borrowers to deleverage more. In the light of this debt-deflation cycle, a countercyclical rule on the LTV ratio of borrowers attenuates the acute deflation and supports the recovery of output. Thus, the macroprudential rule corrects the aggregate demand externality that arises if prior to a bust, borrowers do not reflect the economic impact of their

[1] Walentin (2014) provides evidence for the collateral effect of housing demand shocks, but states that the amplification effect is negligible for shocks originating in the housing market.

leveraging during times of a binding ZLB. However, if the ZLB is non-binding, FMR has no effect on the macroeconomy. The findings reveal asymmetries in the macroprudential policy transmission caused by the interaction with either an efficient or a non-efficient monetary policy stance.

In summary, this thesis provides the following research contributions: First, I find that a countercyclical rule on the LTV ratio of borrowers is a powerful instrument to curb the amplification effect of collateral constraints to the real economy that arises in response to shocks emanating from household's behavior. Second, the effectiveness of the rule increases in the presence of highly leveraged borrowers, with the use of credit growth as indicator variable, and when monetary policy is constrained. Third, the countercyclical LTV regulation performs better in moderating the risk of a credit crunch than countercyclical bank capital regulation and is hence capable in strengthening the systems resilience. Overall, the main conclusion from this thesis is that countercyclical tools in the mortgage market successfully enhance financial stability when optimally designed. In addition, they are capable to provide macroeconomic stability during times of a binding ZLB. These insights aid the development of comprehensive macroprudential policy framework in order to reduce the likelihood of a future crises.

My results underline the relevance of macroprudential instrument design for specific mortgage market characteristics and thereby open up new gaps for future research. Research remains to be conducted on the interplay of macroprudential policy and specific mortgage loan properties as, e.g., multi-period loans, variable rate, and fixed loans, and occasionally binding collateral constraints, as well as the interaction with other tools like the borrowers' debt-service to income limit, or the requirements on the amortization period of a mortgage. Research needs also to be done on the specific characteristics of demand shocks which lead in the presence of collateral constraints to the financial multiplier effect on output. Understanding the mechanism behind the collateral effect allows to construct more goal-oriented macroprudential policy measures.

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