On The Spatial Economics of Knowledge Accumulation

DISSERTATION
ZUR ERLANGUNG DES GRADES EINES DOKTORS DER WIRTSCHAFTSWISSENSCHAFT

EINGEREICHT AN DER FAKULTÄT FÜR WIRTSCHAFTSWISSENSCHAFTEN DER
UNIVERSITÄT REGENSBURG

vorgelegt von:
Johannes Stiller

Berichterstatter:
Prof. Gabriel S. Lee, Ph.D., Universität Regensburg
Prof. Dr. Andreas Roider, Universität Regensburg

Tag der Disputation: 12. Juli 2018

Abstract

On The Spatial Economics of Knowledge Accumulation

by Johannes Stiller

My thesis investigates the spatial economics of knowledge accumulation.

The main contributions of my work are the following: First, I explore the theoretical foundations and the economic relevance of the spatial heterogeneity in knowledge accumulation. The distinction between the creation and transmission of knowledge and their respective local determinants are the main focus of this exploration that links endogenous growth theory and recent research on spatial aspects of human capital and innovation. Second, I present a theoretical analysis on the role of face-to-face interactions in knowledge spillovers. This search-theoretic model considers the creation and transmission of knowledge and determines that knowledge externalities do not reach their optimal extent because agents choose their partners for interaction too narrowly. Third, my empirical analysis for European regions shows that geographical and technological proximity foster innovative spillovers between regions. A spatial-autoregressive estimation of the reduced form of the knowledge production function provides the framework for this investigation.

Acknowledgements

I would like to thank my first supervisor Gabriel Lee for his enduring encouragement and advice as well as his helpful comments on my work. I thank my second supervisor Andreas Roider for his advice and helpful comments on chapter 3. Furthermore, I would like to thank my colleague Dirk Assmann for his contributions to chapter 3. Finally, I want to thank my parents for their unwavering support throughout my entire education.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

This thesis investigates the spatial determinants of knowledge accumulation and economic growth. I summarize previous theoretical and empirical work on the spatial dependence of knowledge and growth and relate this research to its roots in endogenous growth theory. I contribute to this literature with my theoretical investigation of knowledge spillovers in an urban search-theoretic framework and my empirical analysis of innovative spillovers between European regions. My main research contributions are the following: First, I investigate the distinction between the creation and transmission of knowledge in endogenous growth theory and connect it to recent research on spatial aspects of human capital and innovation. Second, I integrate knowledge creation and transmission as simultaneous but distinct mechanisms into an urban search-theoretic model. Third, my empirical analysis for European regions explores the role of different proximity dimensions in innovative spillovers.

The exploration of economic growth's main drivers has long been a major subject of economic research. There is little doubt that the continuous increase of individual and collective knowledge is at the core of the economic prosperity that developed economies enjoy today. Historically, growth in output and economic welfare has been steady but slow for many centuries. Only in the wake of the enlightenment and the development of groundbreaking inventions like the steam engine and electricity has economic growth increased to higher levels. This co-movement of innovation and prosperity is not only observed over time, but also across space. Well educated nations with high innovative capacity are the leaders in economic growth and welfare. The pattern also holds within country borders: cities are rich in human capital and outperform the periphery in output and wages. The spatial distribution of knowledge is therefore closely linked to the spatial distribution of economic welfare. This thesis aims to contribute to the understanding of this connection and its underlying mechanisms.

The remainder of this thesis is organized as follows:

Chapter 2 introduces the contribution of endogenous growth theory as foundation of the economic investigation of knowledge accumulation. I apply these models' implications to a spatial setting and review the theoretical and empirical literature that is based on them. I summarize the literature with particular attention to the distinction of creation and transmission of knowledge and link it to my analyses in chapter 3 and 4.

Chapter 3 examines local knowledge spillovers as a source of agglomeration economies. I use a search-theoretic framework to investigate the creation and transmission of knowledge as an outcome of local face-to-face interactions between agents with heterogeneous cognitive background. The model's results show that agents are too picky in the choice of their interaction partners and consequently knowledge accumulation and economic growth do not reach their optimal extent.

Chapter 4 provides empirical evidence on the role of proximity in innovative spillovers between European regions. Using data from 236 European NUTS2-regions, I estimate a reduced form of the knowledge production function in a spatial-autoregressive framework. The results confirm previous findings on the relevance of innovative spillovers and their dependence on geographical and technological proximity. I further propose alternative specifications of internal inputs and spatial weights and explore the simultaneous impact of geographical and technological distance. Chapter 5 summarizes main findings and concludes my thesis.

Chapter 2

Endogenous Growth and Knowledge Accumulation

2.1 Introduction

This chapter explores the determinants of the spatial heterogeneity in economic growth and knowledge accumulation. I review the fundamental role of human capital and innovation in economic growth as prominently established in endogenous growth theory by Lucas Jr (1988) and Romer (1990). I examine the implications of these models for spatial patterns of innovation and growth. Both models imply that the spatial distribution of human capital and innovation is a main driver of the persistent growth differentials between regions. My review then turns to important contributions to the theoretical understanding of spatial determinants of growth and knowledge accumulation. Empirical evidence is reviewed and includes two important strands of literature: namely the North American new economics of urban and regional growth literature and the European research that applies spatial econometric techniques to examine innovation and growth on the regional level. In conclusion, I briefly discuss the distinctions and shortcomings of Lucas's and Romer's models and explore a possible integration of their respective mechanisms.

2.2 Endogenous Growth

Endogenous growth theory has its roots in neo-classical growth models that have been established by Solow (1956) and Swan (1956). While these models already linked economic growth to technological progress, i.e. the accumulation of knowledge, the process of knowledge accumulation was treated as exogenous and its roots remained unexplored. Neo-classical growth theory defined knowledge as a pure public good and therefore left no room for local disparaties. The integration of knowledge accumulation into these growth models is the main goal of endogenous growth theory. To achieve this goal, these models incorporate explicit mechanisms of human capital growth and innovation into the production technology. In the following, I briefly present two of the most prominent contributions to endogenous growth theory. Lucas Jr (1988) and Romer (1990) established the endogenous growth framework and provided large parts of the foundation to this area of economic research. However, it is not for their prominence that these models will be discussed here. The focus will be on their implications for local growth differentials and the role of human capital and innovation in this phenomenon. While both models are built on the importance of knowledge and stress the relevance of its local dissemination, a detailed interpretation of the models' implications in a spatial setting reveals notable differences between the two approaches.

2.2.1 Lucas's Concept

The defining characteristic of Lucas Jr (1988)'s growth model is the introduction of the stock of human capital as an input to production and the explicit modeling of human capital accumulation. Lucas's production technology is defined as

$$Y(A,K,L,l,h) = AK^{\alpha}(lhL)^{1-\alpha} \tag{2.1}$$

where Y, A and K are output, level of technology and capital. l describes the proportion of total labor time spent working, and h is the stock of human capital. Lucas rewrites the production function in per-capita terms as he turns to analyze the individual's optimization problem. The per-capita production function is

$$y(A,K,L,l,h) = Ak^{\alpha}(lh)^{1-\alpha} \tag{2.2}$$

which is constant returns to scale in k and lh. Capital accumulation follows the differential equation

$$\dot{k} = y - c - (\xi + \delta)k \tag{2.3}$$

where capital growth is equivalent to the output net of consumption and the depreciation of capital. Human capital accumulation is governed by the differential equation

$$\dot{h} = \phi h(1 - l) \tag{2.4}$$

where ϕ denotes the efficiency of human capital accumulation. This equation contains the core of Lucas's approach to endogenize knowledge accumulation. It defines the model's implications in three important ways. First, knowledge is embodied in human capital. Growth is driven by the accumulation of individual knowledge and skills, it does not depend on the creation of new knowledge and resulting growth in the technology level A. Second, human capital growth is proportionate to the existing human capital stock. Individual human capital growth exhibits constant returns to scale. Third, human capital growth relies on an investment of labor time into human capital accumulation, the opportunity cost of learning are foregone wages. Lucas establishes the model's socially optimal solution under perfect foresight which is represented by the steady- state growth rate of consumption.

$$\frac{\dot{c}}{c} = \rho^{-1}(\phi - \theta) \tag{2.5}$$

Consumption growth is determined by the differential of ϕ and θ. The key parameter in this model is ϕ, which determines the efficiency of the human capital accumulation process. A positive growth rate is obtained if the future payoff of human capital accumulation outweighs the preference for current consumption, which is captured by the parameter θ. In its basic form, the model does not have immediate spatial implications. Knowledge accumulation is an individual pursuit and the individual's optimization does not depend on its environment. However, Lucas explores an extension to his model that incorporates a human capital externality. This externality is the theoretical foundation to the relevance of spatial proximity and knowledge spillovers. Specifically, in this version of the model the individual's production function is

$$y_i = Ak_i^{\alpha}(l_i h_i)^{1-\alpha}\bar{h}^{\psi} \tag{2.6}$$

where $\bar{h}$ is the local average level of human capital. The introduction of $\bar{h}$ as an input to individual production reflects the idea that each person is more productive if nearby individuals possess high levels of human capital. For this extension of the model, the steady-state growth rate of individual human capital becomes

$$\gamma_h = \left(\frac{\rho^{-1}(\phi - \theta)}{1 + \psi(1 - 1/\rho)/(1 - \alpha)}\right) \tag{2.7}$$

In this representative agent model, individual human capital accumulation corresponds to aggregate human capital accumulation, which in turn determines the aggregate growth rate. Lucas shows that this decentralized equilibrium is suboptimal, because individual consumers do not obtain the full benefits to society of increasing their own stock of knowledge. The socially optimal solution requires greater investment in human capital accumulation than is obtained in the decentralized equilibrium.

In summary, the main implications of Lucas's model for spatial patterns of growth and knowledge are the following. The local growth rate of human capital determines the local growth rate of consumption. The local growth rate of human capital depends on the local average level of human capital and the time allocated to learning. Thus, the model predicts higher growth rates in regions with high shares of highly educated workers and a strong role of schooling. The mechanism of knowledge accumulation is transmission of knowledge between individuals, the model does not require knowledge creation or innovation for economic growth. Proximity between agents facilitates interaction and transmission which implies that population density and spatial proximity to highly educated regions are conducive to local growth. While the model does not directly predict whether specialization or diversity of local industries benefit growth, a plausible interpretation is that homogeneity in agents' knowledge backgrounds facilitates the transmission of knowledge and therefore enhances economic growth.

2.2.2 Romer's Concept

Romer (1990)'s work builds on the author's own model that he established in Romer (1986). While Lucas and Romer share the insight that knowledge accumulation is the main driver of economic growth, the proposed process of knowledge accumulation in Romer's model differs from Lucas's approach in various ways. In Romer's world, it is not the accumulation of individual human capital that drives growth, but the accumulation of ideas that are embodied in product designs. This focus on knowledge creation introduces the relevance of labor allocation to research and development as opposed to individual learning in Lucas's model. Furthermore, Romer explicitly models the intermediate goods market and firms' investment decision in an environment of imperfect competition.

The characteristic feature of Romer's production technology is that it disaggregates capital into an infinite number of distinct types of producer durables. These durables are indexed by an integer i.

Final output Y is expressed as a function of physical labor L, human capital devoted to final output H_Y, and physical capital K. The functional form for output is expressed as the following extension of the Cobb-Douglas production function:

$$Y(H_Y, L, x) = H_Y^{\alpha} L^{\beta} \sum_i x_i^{(1-\alpha-\beta)} \tag{2.8}$$

Capital accumulation is determined by foregone consumption and follows the differential equation

$$\dot{K}_t = Y_t - C_t \tag{2.9}$$

The research sector formalizes the process of knowledge accumulation in Romer's model. Technological growth evolves according to

$$\dot{A} = \delta H_A A \tag{2.10}$$

where the technology level A is represented by the stock of product designs. A is non-rival and can be accessed by all researchers. The output of researcher j is proportional to his human capital level H_j and the stock of ideas A_j he can access. Summing across all researchers yields the differential equation above where H_A is total human capital employed in research. The intermediate goods sector of producer durables is characterized by the firms' maximization problem with respect to the output of the durable good:

$$\max_x \int_0^{\infty} [H_Y^{\alpha} L^{\beta} x_i^{1-\alpha-\beta} - p_i x_i] di \tag{2.11}$$

which yields the firms' inverse demand function for durables:

$$p_i = (1-\alpha-\beta) H_Y^{\alpha} L^{\beta} x_i^{-\alpha-\beta} \tag{2.12}$$

Faced with given values of H_Y, L, and r, a firm that has already incurred the fixed-cost investment in a design will choose a level of output x to maximize its revenue net of the variable cost:

$$\pi = \max_x (1-\alpha-\beta) H_Y^{\alpha} L^{\beta} x^{1-\alpha-\beta} - r\eta x \tag{2.13}$$

The decision to produce a new specialized input is determined by a comparison of the discounted stream of net revenue and the cost P_A of the initial investment in a design. Because the market for designs is competitive, the price for designs is bid up until it is equal to the present value of the net revenue that a monopolist can extract. At every point in time it must therefore be true that

$$\pi_t = r_t P_A \tag{2.14}$$

For a fixed level of A, the model's symmetry implies that all available durable goods are supplied at the same level, henceforth denoted as $\bar{x}$. If they were not, it would be possible to increase profits in the producer durables sector by reducing the output of high-output firms and diverting the capital released in this way to low-output firms. Since A determines the range of durables that can be produced and since η units of capital are required per unit of durable goods, $\bar{x}$ can be determined from the equation $K = \eta A \bar{x}$.
Then output Y can be written as

$$\begin{aligned} Y(H_Y, L, x) &= H_Y^{\alpha} L^{\beta} \int_0^{\infty} x_i^{(1-\alpha-\beta)} di = \\ &= H_Y^{\alpha} L^{\beta} A \bar{x}^{(1-\alpha-\beta)} \\ &= H_Y^{\alpha} L^{\beta} A \frac{K}{\eta A}^{(1-\alpha-\beta)} \\ &= (H_Y A)^{\alpha} (LA)^{\beta} K^{1-\alpha-\beta} \eta^{\alpha+\beta-1} \end{aligned} \tag{2.15}$$

The strategy to characterize the model's equilibrium that is followed by Romer is to solve for an equilibrium in which the variables A, K, and Y grow at constant rates, i.e. the economy's balanced growth path. Equation 2.15 shows, that output grows at the same rate as A if L, H_Y, and $\bar{x}$ are fixed. If $\bar{x}$ is fixed, K grows at the same rate as A, because total capital use is $A\bar{x}\eta$. Let g denote the growth rate of C, A, Y, and K.

$$g = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \delta H_A \tag{2.16}$$

The constraint $H_Y = H - H_A$ implies a relation between the growth rate g and the interest rate r:

$$g = \delta H_A = \delta H - \frac{\alpha}{(1-\alpha-\beta)(\alpha+\beta)} r \tag{2.17}$$

The core of Romer's model is found in the evolution of ideas as described in equation 2.10, which in turn determines the steady-state growth rate. Creation of new ideas depends on the share of R&D workers and their efficiency as well as their access to existing product designs. Firms' investment decisions are driven by the monopoly profits of product innovation and lead to less than optimal investment in R&D because the external effect of innovation on the evolution of knowledge accumulation is not considered by firms. As in Lucas's model with human capital externalities, steady-state knowledge accumulation and consumption growth are below their socially optimal level.

In conclusion, Romer's model implies somewhat different determinants for spatial patterns of growth and knowledge compared to Lucas Jr (1988). The local growth rate depends on the growth rate of product designs, which is equivalent to the growth rate of ideas or knowledge creation. This growth of ideas is determined by the share of R&D workers and the stock of ideas they can access. Thus, the model predicts higher growth rates in regions with high shares of research workers and easy access to scientific knowledge. The mechanism of knowledge accumulation is knowledge creation. Innovations, which are codified as product designs are necessary for economic growth. The focus on transmission of knowledge in Lucas's model as opposed to creation of knowledge in Romer's model as the driver of growth is the most important distinction between the two approaches. These different mechanisms also have distinct implications on the role of specialization and diversification in local growth rates. Romer's work is interpreted as one of the theoretical foundations for the so called MAR-spillovers, where MAR refers to Marshall, Arrow and Romer as explorers of these spillovers that arise from the local specialization of industries. Based on Romer (1990), however, the argument for specialization is not that clear. Romer himself argues for the importance of vertical innovation, which describes the process of creative disruption that is also at the core of Schumpeterian models of innovation. Vertical innovation refers to new product designs that are based on and replace previous designs within the same industry. As vertical innovation happens within industries, specialization enhances this kind of innovation. On the other hand, diversification provides room for horizontal innovation, i.e. new products are introduced in industries based on product designs and inspiration from different industries. It is the relative importance of vertical and horizontal innovation that determines whether specialization or diversification are conducive to growth in Romer's model.

2.3 Theoretical Literature

2.3.1 New Economics of Urban and Regional Growth

The application of endogenous growth theory to spatial economics motivated a substantial number of studies that investigate extent and drivers of the accumulation of knowledge on the regional level. The vast majority of these studies is of empirical nature. While explicit theoretical explorations on the spatial dimension of knowledge accumulation are relatively rare, there are some notable contributions that deliver spatial extensions of the endogenous growth theory and important foundations for empirical analyses.

I follow Roberts, Setterfield et al. (2010) in the distinction between the North American new economics of urban and regional growth literature and its European counterpart, which mainly focuses on the application of spatial econometrics on the regional level. The new economics of urban and regional growth, which is mainly pursued in North America, is based on endogenous growth theory. Its focus on geographically bounded spillovers from face-to-face interactions explains agglomeration effects, increased growth in cities and urban wage premia. These effects are consistent with Lucas Jr (1988)'s predictions. New economics authors point to this link to endogenous growth theory even though an explicit theoretical integration of these effects into comprehensive growth models is usually not undertaken. The following provides a brief overview over theoretical foundations of the effects of urban face-to-face interactions on local wages and growth.

Edward L. Glaeser (1999) sets up a model of urban learning from face-to-face interactions in which the number of meetings increases in workers' density. This mechanism leads to the agglomeration of skilled workers in cities and urban wage premia. In an extension to his model, the author introduces the assumption that learning only takes place if workers from the same industry meet which reflects the idea of more efficient knowledge transmission between workers with similar knowledge backgrounds adopted in our model.

Peri (2001) uses a similar approach that analyzes the accumulation of young urban workers' skills, assuming that the density of educated workers positively affects the accumulation of skills. The resulting equilibrium features a concentration of young and educated workers in cities that is consistent with empirical observations.

More recently, Davis and Dingel (2013) provide a spatial equilibrium model that examines costly exchange of ideas as agglomeration force. The intensity of knowledge exchange depends on the time devoted to the search for exchange partners, the density of workers and their skill level. Consequently, the exchange of ideas, average abilities of individuals and wage premia are higher in larger cities.

Berliant, Reed III and Wang (2006)'s model of knowledge exchange via face-to-face interactions analyzes the matching process of individuals with heterogeneous knowledge backgrounds. This approach incorporates the competing roles of similarity and diversity of knowledge by assuming that there is an optimal distance of knowledge types that leads to the most efficient transmission of knowledge. With the number of meetings increasing in density, the efficiency of learning is higher in more densely populated cities.

All these models capture the higher rate of knowledge spillovers in cities and focus on the transmission of knowledge and the associated buildup of individual skills. My analysis in chapter 3 extends these contributions as it is the first search-theoretic approach that explicitly includes the creation of knowledge as a simultaneous but distinct process from the transmission of knowledge. This addition enriches the theoretical analysis because the two types of spillovers differ in their dependence on knowledge similarity and also in compensation.

This strand of literature focuses on the role of face-to-face interactions in the accumulation of knowledge. As explored in chapter 2.2, this approach is compatible with Lucas's idea of human capital growth embodied in the individuals' skills. The main driver of agglomeration in these models is therefore the greater opportunity to exchange knowledge in a dense urban area with highly educated workers. These drivers are reflected in my analysis in chapter 3 as well. However, this mechanism does not fully capture Romer's idea of endogenous growth. In Romer's world, knowledge is embodied in patents and the accumulation of it does not directly depend on face-to-face interactions but on access to the local stock of ideas and the share of R&D workers. As Romer considers ideas in terms of blueprints for product varieties, his model gives more relevance to codified knowledge and the transmission of tacit knowledge through face-to-face interactions is not as important as in Lucas's world. However, the process of knowledge creation, which is performed by R&D workers, still requires interaction between those researchers and scientists. My contribution in chapter 3 fits this framework if the creation of knowledge is conceptualized as an outcome of interactions in the R&D sector.

2.3.2 European Research on Regional Innovation and Growth

Recent European research on the determinants of spatial heterogeneity in growth and innovation differs from the new economics approach mainly in the focus on more aggregated spatial units and the application of spatial econometric techniques. Both of these characteristics call for a stronger orientation towards macroeconomic theory as opposed to the analysis of individual and firm-level interactions that is central to the new economics of urban growth approach. Consequently, a large part of this literature relies on the estimation of local production functions that include neighboring regions' output as an input to local production. This framework enables researchers to apply spatial econometric models to analyses of growth and innovation at the level of European NUTS-regions. The theoretical foundations for these investigations are closely tied to endogenous growth theory. As established in chapter 2.2, these models are based on the concept of a production function and treat knowledge as an input to production. While there is a large body of literature that applies this concept to explain local heterogeneity in output and wages, this review focuses on heterogeneity of innovation and its spatial determinants. To investigate spatial patterns of innovation, researchers apply the concept of the production function to the creation of knowledge, shifting the outcome of interest from economic output to innovative output. This knowledge production function (KPF) is the basic framework to explore innovation in space in this strand of the literature. The idea of the KPF predates endogenous growth theory. However, with endogenous growth theory and its focus on knowledge accumulation, the concept gained relevance and researchers' attention. Griliches (1979) provides a foundation for the analysis of innovative spillovers in two ways: he introduces the concept of the knowledge production function on the firm level and the concept of technological distance between firms and industries. The production function is applied to estimate returns to the internal input of R&D within the firm. The results confirm the positive impact of R&D intensity on output. Furthermore, the author explores the definition and measurement of R&D capital in the firm which leads him to consider spillover effects of the R&D activities from other firms and the introduction of the concept of technological distance which mediates these effects. Pakes and Griliches (1984) applies the KPF to explain variations of patent activity on the firm level. Adam B Jaffe (1989) builds on the concept of the knowledge production function and refines the definition of the internal inputs to investigate innovative spillovers from university research to firms. The exploration of state-level time-series data on firms' patents shows a significant spillover effect of university research on firms' innovative output. There is also evidence for an indirect effect as university research induces higher local corporate R&D investment. David B Audretsch et al. (2003) find in their review of the literature that the KPF's capacity to explain a firm's innovative output in isolation is limited, but it provides a strong framework to explain innovative output for larger spatial units of observation. The focus of European economists' investigation subsequently shifted from the firm-level to higher levels of aggregation, primarily to the city, county or state-level. With its focus on innovation, this area of research is directly connected to Romer (1990). The very idea to conceptualize the "production", i.e. the creation, of knowledge in form of the KPF demonstrates its focus on knowledge creation. Patents as the unit of measurement are also directly connected to the blueprints for product designs that embody ideas in Romer's model. Thus, the KPF with innovative spillovers is a close representation of Romer's differential equation of idea accumulation described in equation 2.10. Internal inputs to knowledge production are R&D spendings and human capital, which correspond directly to human capital devoted to research (H_A) in Romer's model. The external input of neighboring regions' innovation corresponds directly to the access to the stock of ideas in Romer's framework. Chapter 4 of this thesis provides an explicit analysis of innovative spillovers between European regions based on the KPF framework.

2.4 Empirical Literature

2.4.1 New Economics of Urban and Regional Growth

Both strands of literature that were introduced in chapter 2.3 - new economics of urban growth and European research on regional innovation patterns - are empirically oriented. Main results of important contributions will be summarized below and put into context to Lucas's and Romer's concepts of endogenous growth. Edward L. Glaeser, D. et al. (1992) use data on employment growth between 1956 and 1987 of large industries in 170 U.S. cities. They find that industry-employment growth is significantly positively related to urban density and diversity of industries. These results are consistent with Lucas Jr (1988) if urban density is considered to foster face-to-face interactions and knowledge transmission. Feldmann and David B. Audretsch (1994) employ a more direct approach to measure the connection between innovative output and the composition of economic activities in a city. Using the United States Small Business Administration's Innovation Data Base (SBIDB) they can directly observe innovative activity across cities by looking at the number of product introductions across U.S. cities. Their results coincide with Edward L. Glaeser, D. et al. (1992), i.e. density and local diversity of industries enhance innovative output. However, their study is more directly connected to Romer (1990). The measurement of innovation by the number of product introductions corresponds closely to Romer's concept of knowledge accumulation. The finding of positive innovation effects from diversity speaks for the relevance of horizontal innovation in Romer's framework. Another study that is in line with Lucas Jr (1988) is provided by Rauch (1993). The author finds that the average level of human capital within a city has a significantly positive impact on wages for data from 1980 in 237 SMSAs. This result fits the core prediction of Lucas's model with human capital externalities, where the average level of human capital drives knowledge accumulation and growth. Rauch (1993) further finds that years of schooling as opposed to experience at the job are conducive to growth, which is consistent with another implication of Lucas's model: time allocated to human capital accumulation enhances growth. These results are supported by Edward L Glaeser, Scheinkman and Shleifer (1995). Data from 203 US cities are used to estimate the impact of the initial level of human capital on subsequent local per capita income growth. The effect of the average level of education is significantly positive as predicted by Lucas's model. Further research by Glaeser falls in line with Lucas Jr (1988) as well. Edward L. Glaeser and Maré (2001) show that urban workers increase the wage differential over non-urban workers during the time they work and live in the city. This urban wage premium is not lost even when they move from the city to a rural area, supporting the story of skill acquisition in an urban environment. Once workers leave the city, they keep their skills and therefore continue to earn the same nominal wage in the rural area. This finding is consistent with Lucas's idea of knowledge that is embodied in human capital and does not support Romer's idea that growth is driven by knowledge that is embodied in patents. In another exploration, Edward L. Glaeser and Resseger (2010) show that per-worker productivity is strongly correlated with urban density and conclude that spatial proximity facilitates the transmission of knowledge. Edward L. Glaeser and Resseger (2010) find that learning opportunities are especially strong in cities with high average levels of skills, indicating that the contact between highly educated individuals accelerates the accumulation of human capital and again confirming Lucas's ideas of knowledge transmission. Carlino, Chatterjee and Hunt (2007) show that doubling urban employment density leads to an increase of patent intensity of about 20 percent. This observation indicates that a dense urban environment causes a higher rate of face-to-face interactions and in turn increased innovative output. This study is particularly interesting, because it provides evidence that density and face-to-face interactions are also important for the creation of knowledge. North American research is not only focused on knowledge transmission, there is also notable research on innovative spillovers as evidenced by the locality of patent citations. Adam B. Jaffe, Trajtenberg and R. Henderson (1993) show that patent-citations are clustered at the MSA-level as well as at the state-level. Interactions between researchers and easier access to nearby patents can explain these spillovers in the context of Romer's model. Adam B Jaffe (1986) focuses on the role of firms' position in technological space. The author examines, whether the R&D activity of neighboring firms in technological space has an effect on firms' R&D effectiveness. Firms' position in technological space is determined by their innovative activity in different patent classes. The author finds that firms with high R&D neighbors produce more patents per dollar of R&D spending. This spillover effect is particularly pronounced for firms that invest highly in R&D themselves. These R&D spillovers also fit into Romer's framework of knowledge creation.

2.4.2 European Research on Regional Innovation and Growth

As established in chapter 2.3, European research in the framework of the KPF is closely based on Romer (1990). Therefore, its empirical results can provide insight into the validity of Romer's predictions. Romer's model is based on the firm level, David B Audretsch et al. (2003)'s findings in support of the KPF's greater explanatory power on the state-level are a main contribution that led to the modification and re-interpretation for higher levels of aggregation. Autant-Bernard (2001) examines data from French regions in the framework of the knowledge production function with attention to geographical and technological spillovers. The author estimates spillover effects along each of the two dimensions. Significant externalities occur with stronger evidence in the technological dimension than in the geographical dimension. The results also indicate that human capital is the primary mediator of these externalities. A closely related study is the investigation of R&D externalities between European regions conducted by Bottazzi and Peri (2003). Their approach also relies on the KPF and focuses on the role of neighboring regions' R&D spendings on local innovative output. A distance decay approach which assigns neighboring regions' R&D investments to five different classes of geographical distance is estimated. They find significant spillovers for regions that are closer than 300 kilometers to each other. The magnitude of these effects is small as a doubling of a neighboring region's R&D activity increases local patent output by 2-3%. Moreno, Paci and Usai (2005) follow a similar approach based on the KPF in order to analyze the mechanics of knowledge interdependence across European regions. They estimate the production function with geographical weights and find evidence for the importance of internal factors as well as spatial spillovers. Their results confirm the findings of significant innovative spillovers driven by R&D spendings and additionally show similar spillovers driven by the number of patent applications in neighboring regions. They also find that technological clustering contributes to the spatial dependence pattern. Paci, Marrocu and Usai (2014) employ a spatially auto-regressive model to measure spillover effects on innovation along the geographical, cognitive, organizational and institutional dimensions. They find significantly positive spillover effects in the geographical and cognitive dimension. All of these studies confirm main predictions from Romer (1990)'s framework in a regional setting. Neighboring regions' R&D intensity and patent activity enhance local R&D efficiency and access to ideas which determine local knowledge accumulation.

2.5 Discussion

A brief discussion of this chapter's insights is complicated by the large body of literature and the heterogeneity of results. There is strong evidence, prominently led by Glaeser's studies, for the important role of human capital and urban density in local growth. These findings are in line with Lucas's approach and confirm the relevance of individual skills and knowledge transmission through face-to-face interactions.

On the other hand, notable evidence provided inter alia by David B. Audretsch and Feldmann (1996) and Adam B. Jaffe, Trajtenberg and R. Henderson (1993) validates the importance of knowledge creation and innovation. Spatial innovative spillovers are found on various levels of aggregation: between firms (David B. Audretsch and Feldmann (1996)), MSAs (Adam B. Jaffe, Trajtenberg and R. Henderson (1993)) and regions (Bottazzi and Peri (2003)). European research based on the KPF provides further validation of Romer's predictions. The interpretation of both models in this chapter closely follows the authors' definitions of the knowledge accumulation process. The distinction between creation and transmission of knowledge is not explicitly made in large parts of the reviewed literature. However, the difference between transmission that leads to human capital growth and creation that leads to new patents is substantial, measurable and has significant economic implications.

The main conceptual vulnerability of Lucas's approach is his assumption of constant returns to scale in the process of human capital accumulation. This assumption cannot be justified without the creation of new knowledge. In the long-run, transmission of existing knowledge becomes less effective and does not sustain perpetual growth as agents have already shared their knowledge and do not create any new knowledge to share. From this perspective, Lucas's concept of human capital accumulation as a process of knowledge transmission is not consistent with his assumption of constant returns to scale in this accumulation process.

On the other hand, the conceptual weakness of Romer's approach is the assumption of a fixed total stock of human capital. In contrast to Lucas's model, this specification ignores the transmission of knowledge between agents and its expansive effect on the stock of human capital. Lucas's approach has its strength in explaining short-term dynamics on the individual level. Knowledge transmission drives individual wage growth and the accumulation of human capital can be allocated to the individual agent as wage premia persist after the worker changes location as shown by Edward L. Glaeser and Resseger (2010). These phenomena can be explained in Lucas's framework, but not in Romer's. Romer's model is better suited to explain long-run aggregate growth that is driven by technological change. Evidence includes studies on the co-movement of growth and innovation on the regional and national level such as Bottazzi and Peri (2003) and Moreno, Paci and Usai (2005). These perspectives show, that Lucas's and Romer's approaches are valid and non-exclusive. Their focus differs in the level of aggregation and the type of knowledge accumulation. Recent economic research has provided a large body of evidence for the relevance of transmission and creation of knowledge. This discussion implies a fruitful avenue of future research. The integration of knowledge transmission and creation in an endogenous growth framework is a challenge that builds on both approaches and mitigates their respective conceptual weaknesses. However, there are significant methodological challenges to the integration of both mechanisms into one model. Most notably, the integration of constant growth rates in the accumulation process of individual human capital through transmission as well as in the aggregate accumulation of ideas produces a model without a stable growth path. More refined models of the processes behind the accumulation of human capital and ideas are required for an interior solution in an integrated framework. The model presented in chapter 3 captures transmission and creation of knowledge simultaneously.

Chapter 3

Knowledge Spillovers in Cities

3.1 Introduction

The objective of this paper is to investigate the effects of two different types of knowledge spillovers (transmission and creation) on urban productivity and city size. I present co-authored work, which applies a search-theoretic spatial equilibrium framework to analyze both types of knowledge spillovers from urban face-to-face interactions. In this framework, cities give individuals the opportunity to increase their productivity through the process of knowledge transmission (learning). The process of knowledge creation (innovation) increases the rate of technological change in the city, which raises the productivity of each worker. The transmission of knowledge can be thought of as the result of workers' observation and imitation of each others' techniques. We assume that this process is facilitated when interacting workers have a similar knowledge background. Knowledge creation results in the form of new ideas from the combination of interacting workers' existing knowledge. We adopt the view of Jacobs (1968) and assume that every interaction among workers, independently of their knowledge background, has the potential to bring about innovations. One major difference between these two types of knowledge spillovers is apparent: Workers benefit individually from the process of imitating other workers. The increased productivity directly leads to higher wages. On the contrary innovations are treated as non-excludable local public good in our model. This assumption can be justified as the contribution to the emergence of innovations is often not directly credited to the inventors and thus not fully compensated. The asymmetry in compensation leads to social inefficiencies in workers' choice of face-to-face meetings and location.

This is the first theoretical model that incorporates both types of knowledge spillovers as distinct mechanisms in an urban context. We use a model economy with two asymmetric locations, the city and the periphery. The city provides people with the opportunity to exchange their knowledge via local face-to-face interactions, whereas the periphery does not. Workers in our model can choose the range of other workers in the city they are willing to interact with in order to exchange information. Since individuals do not consider the impact of these interactions on the rate of technological change in the economy, they only accept a range of matches that is smaller than socially optimal.[1] We also show that the resulting suboptimal extent of knowledge spillovers generally leads to socially inefficient city sizes.

The chapter is organized as follows: Section 2 reviews the empirical and theoretical literature on the topic of local knowledge spillovers. In section 3 the assumptions of our model environment are introduced. Section 4 presents the Steady-State Equilibrium of our model economy. In section 5 the market outcome is compared to the outcome that results from the Social Planner's Problem. We further show the different types of inefficiencies that can emerge in our model. Section 6 summarizes and concludes.

3.2 Literature Review

3.2.1 Empirical Literature

Due to the intractable nature of knowledge spillovers through face-to-face interactions, it is difficult to measure their extent and sources. Consequently, empirical evidence on different types of knowledge spillovers and their impact on economic growth is scarce. However, there are a few notable contributions that find evidence on the impact of the local industry mix on innovation and productivity growth. These results do not directly address the role of knowledge composition that is discussed in our model. However, since industrial diversity and diversity of knowledge in a city are closely related, we can interpret these findings as a reasonable indication for the role of knowledge background.

Independently of the degree of urban specialization or diversification, the literature agrees on the fact that urban density accelerates the emergence of new ideas. Both Edward L. Glaeser, D. et al. (1992) and V. Henderson, Kuncoro and Turner (1995) provide significant results and Carlino, Chatterjee and Hunt (2007) show that doubling urban employment density leads to an increase of patent intensity (i.e. patents per capita) of about 20 percent. This observation indicates that a dense urban environment causes a higher rate of face-to-face interactions and

[1] There is a very restricted parameter space that results in a range of matches that is larger than socially optimal. More on that finding can be found in section 3.5.2.

in turn the creation of more innovative output.

Similarly to the creation of knowledge, there is no doubt that urban density positively affects the transmission of knowledge in cities as well. Since denser cities bring about more face-to-face interactions and since knowledge is best transmitted via those real-life face-to-face interactions, the individual productivity should be highest in those urban areas. Edward L. Glaeser and Resseger (2010) show that per-worker productivity is strongly correlated with urban density and hence explain that proximity facilitates the transmission of knowledge.

While the results on the impact of urban density are unanimous, there is some disagreement in the literature when it comes to the role of specialization and diversity. The existing literature distinguishes between two different views of the world. What Edward L. Glaeser, D. et al. (1992) call the Marshall-Arrow-Romer Model suggests that an increased concentration of a particular industry in a city facilitates the exchange and combination of knowledge between individuals and thus leads to the best innovative outcome. This view relies on the idea that sharing the same knowledge background makes it easier for individuals to communicate specific problems in their field. The Marshall-Arrow-Romer Model implies that those innovative meetings are in particular promoted by cities specialized in one specific industry, because those cities feature more face-to-face interactions between people with a similar knowledge background. Silicon Valley, known for its role as pioneer in computer technology is the most famous example for such a highly specialized and innovative region, as was demonstrated by Saxenian (1994).

In contrast, Jacobs (1968) argues that innovations can arise from every face-to-face interaction, independently of the interacting individuals' knowledge background. According to her view the most innovative city is a place where people from all different fields of the economy interact unrestrictedly. Therefore she favors diversified cities with no particular specialization in one industry. Edward L. Glaeser, D. et al. (1992) quote the story of the emergence of the financial industry in New York, where grain and cotton merchants saw the need for national and international financial transactions. It was only that need that gave rise to the invention of the industry of financial services.

Edward L. Glaeser, D. et al. (1992) and Feldmann and David B. Audretsch (1994) both find empirical evidence for so called Jacobian spillovers, i.e. diversity and not specialization of economic activities enhance growth in cities. Their contributions are discussed in section 2.4.1 of this thesis. But the literature also provides evidence for the existence of so called Marshall-Arrow-Romer spillovers. In contrast to Feldmann and David B. Audretsch (1994) and Edward L. Glaeser, D. et al. (1992), V. Henderson, Kuncoro and Turner (1995) find that Marshall-Arrow-Romer spillovers are prevalent for traditional, whereas Jacobian spillovers are prevalent for young high-tech industries. Thus there exists no definite answer to the question which composition of economic activities is best suited for the creation of new knowledge, but there is support for the hypothesis that knowledge creation is at least not harmed by urban diversity.

Besides the fact that knowledge combined in face-to-face interactions leads to the creation of new ideas and thus to a faster rate of technological change, workers can also use these interactions to learn from each other in order to increase their individual productivity. This process is referred to as the transmission of knowledge. There is a wide range of empirical evidence showing that cities are the places that offer the best learning opportunities for workers. Edward L. Glaeser and Maré (2001) show that urban workers increase the wage differential over non-urban workers during the time they work and live in the city. This urban wage premium is not lost even when they move from the city to a rural area, supporting the story of skill acquisition in an urban environment. Once workers leave the city, they keep their skills and therefore continue to earn the same nominal wage in the rural area. Edward L. Glaeser and Resseger (2010) find that learning opportunities are especially strong in cities with a surpassing level of skills, indicating that the contact between highly educated individuals accelerates the accumulation of human capital. The city promoting the optimal environment for individuals to transmit their knowledge in order to increase their productivity is different from a city promoting the optimal conditions to create new knowledge. Having a different knowledge background might be of no harm (or even an advantage) in creating new ideas but the pure transmission of knowledge in face-to-face interactions is clearly facilitated if interacting individuals have a related body of knowledge. Empirical justification for this statement comes from Edward L. Glaeser, D. et al. (1992). They find that specialization of a city in a particular industry leads to a significantly higher rate of wage growth in that industry. This result can be interpreted as the outcome of better learning opportunities in face-to-face interactions among workers with a similar knowledge background.

All these empirical observations can be summarized by three stylized facts presented in table 3.1. We use these empirical findings to make predictions about the outcome of face-to-face interactions that we expect to predominantly happen in cities with a specific composition of knowledge.

Empirical Observation	**Prediction for f-2-f interactions**
Creation and transmission of knowledge are positively affected by urban density.	Dense urban areas bring about more face-to-face interactions.
The creation of knowledge (or innovative output) is not harmed by the diversity of industries in a city.	The creation of knowledge (or innovative output) is not harmed by the diversity of knowledge types.
The transmission of knowledge decreases in diversity of industries in a city.	The transmission of knowledge decreases in diversity of knowledge types.

Table 3.1: Stylized Facts

3.2.2 Theoretical Literature

The existing literature on the theoretical foundations of knowledge spillovers in cities mostly focuses on the impact of workers' density on productivity without further distinguishing the underlying forms of knowledge.

An overview of important contributions by Edward L. Glaeser (1999), Peri (2001) and Davis and Dingel (2013) is provided in section 2.3.1 of this thesis. Our work is most closely related to Berliant, Reed III and Wang (2006). Their model of knowledge exchange via face-to-face interactions explicitly analyzes the matching of individuals with heterogeneous knowledge backgrounds. This approach incorporates the competing roles of similarity and diversity of knowledge by assuming that there exists an optimal distance of knowledge types that leads to the most efficient transmission of knowledge. With the number of meetings increasing in density, the efficiency of learning is higher in more densely populated cities.

While all these models capture the higher rate of knowledge spillovers in cities, they focus on the transmission of knowledge and associated buildup of individual skills. Our model extends these contributions as it is the first to explicitly include the creation of knowledge as a distinct process from the transmission of knowledge. This addition enriches the theoretical analysis because the two types of spillovers differ in their dependence on knowledge similarity and also in compensation.

3.3 Economic Environment

In this section we present the search-theoretic model of a spatial economy incorporating two different types of knowledge spillovers (creation and transmission of knowledge). The model is related to the work of Berliant, Reed III and Wang (2006) and Edward L. Glaeser (1999), but additionally incorporates the creation of knowledge (also referred to as innovative output or innovation in the following) in the city. The basic idea is the following: Cities provide workers with the opportunity to get into contact via face-to-face meetings. We assume that only a dense urban environment provides the environment to engage in face-to-face interactions, whereas a rural area does not (e.g. because the area is too spacious, meeting points like public squares are not prevalent, etc.). In the city, workers are brought together by a random meeting-technology, where the outcome of knowledge transmission and knowledge creation of each interaction is influenced by the combination of the unobservable knowledge types of the meeting partners. The partners' knowledge type and thus the realization of the intensity is unknown before the meeting, but revealed after a first contact. This framework is adopted from Pissarides (2000), who uses this environment in the context of stochastic job matches. In this type of model it is crucial to distinguish between a meeting and a match. Whether a meeting between two workers becomes a match depends on the realized productivity.[2] Meetings with low realizations are canceled after a very first contact because it is worthwhile to wait for a better partner (with a more adequate knowledge type) to be matched with. We further adopt the neoclassical assumption that innovative output is freely available to everyone in the city and workers are not fully compensated for their created knowledge. This approach makes innovative output a local public good. Its existence gives rise to social inefficiencies since the social benefit of generated innovations exceeds the private benefit. Therefore workers accept only the matches that maximize their expected personal outcome, not taking into account that each accepted match contributes to publicly available innovations in the city.

3.3.1 Basic structure of the economy

Our model economy is populated by infinitely-lived workers. It consists of two heterogeneous regions: The city and the periphery. Time is continuous and in each point of time workers decide in which region to be located. All the action takes place in the city, whereas the periphery is modeled as simple as possible. In the city, individuals have the possibility to interact face-to-face. Living in a crowded urban environment is associated with economic cost. Pollution, road congestion and high house prices are only a few examples for the burden of

[2]For the rest of the paper the label "contact" is tantamount to "meeting" and the label "face-to-face-interaction" is tantamount to "match".

urban living. Each worker in the city generates congestion cost of $t > 0$. N denotes the number of individuals living in the city, so the total congestion cost each worker faces upon entering the city are tN.[3] There is no crowding in the periphery, so peripheral workers do not face any congestion cost.

3.3.2 Economic Agents

Workers are heterogeneous in their horizontally differentiated background of knowledge. The variety of the economy's knowledge base is displayed by a unit circle, represented in figure 3.1. The approach of using a unit circle to illustrate the economy's knowledge base is adopted from Helsley and Strange (1990) and was used by Berliant, Reed III and Wang (2006) and Brueckner, Thisse and Zenou (2002) among others. Each worker is endowed with a specific knowledge type k, which is represented by its position on the circle's circumference K.[4] The circumference K can be interpreted as the economy's knowledge space representing all types of knowledge in the economy (e.g. economics, mathematics, physics, etc.). The location $k \in K$ is drawn from a uniform distribution and exogenously assigned to each worker. In figure 3.1 knowledge type k_A is assigned to worker A, whereas knowledge type k_B is assigned to worker B. The distance of k_A and k_B on the circumference is a measure for the horizontal difference between two types of knowledge. There is no vertical differentiation of knowledge types, i.e. all workers have an equal level of education. Furthermore, position k on the unit circle is only of relevance for workers located in the city and irrelevant for workers located in the periphery since only the city facilitates the exchange of knowledge via face-to-face interactions. Workers are heterogeneous in knowledge background, but homogeneous in preferences.

Flow output (equivalent to flow income) y is spent on a homogeneous consumption good. We discuss the determination of flow output y in section 3.3.4. Flow utility is linear in y, yielding

$$U = U(y) = y. \tag{3.1}$$

This implies that maximizing the level of lifetime utility is equivalent to maximizing the level of lifetime income.

[3]The results of the model analysis are robust to well-established transformations of the congestion cost function. E.g. the results stay unaltered when we use quadratic congestion costs in N, i.e. tN^2. Thus we focus on the easiest case of linear congestion costs.

[4]In the following, we label an individual with that characteristic as worker k.

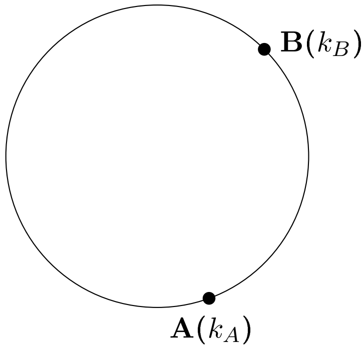


Figure 3.1: Knowledge Space of the Model Economy

3.3.3 Meeting Technology

The incentive for workers to enter the city is the opportunity to increase their productivity by the exchange of knowledge. Before introducing the exact modeling strategy of knowledge spillovers, the emergence of meetings (contacts) in the city is clarified. We apply the framework of stochastic job matching used in Pissarides (2000) and assume that there exists a well-behaved meeting function, which gives the number of contacts as a function of the number of workers searching for face-to-face interactions in the city. By using this framework, we are able to generate a connection between the density of a city and the number of face-to-face meetings taking place. Suppose the city is populated by N individuals. A fraction $m \in (0,1)$ of those N individuals is matched (i.e. currently has a face-to-face interaction). We denote the number of matched individuals as M. Thus the fraction of individuals unmatched is $u = 1 - m$ and we denote the number of unmatched individuals as U, which implies that $N = M + U$. It is important to distinguish a meeting (contact) from a match (face-to-face interaction). Whether a meeting turns into a match is the decision of the individuals who meet and depends on the potential productivity gains. We do not allow for matched workers to search for new partners, so that only unmatched individuals are engaged in the search process. A meeting always requires two parties, one in the first and one in the second position. In Pissarides (2000) the number of job contacts per unit of time depends on the number of firms in the first position and the number of unemployed in the second position. In our modeling framework there are no firms and no unemployed. The number of job contacts is replaced by the number of meetings as well as firms and unemployed are replaced by the number of unmatched workers in the city. Since individuals meet symmetrically, all unmatched workers can either be in the first or second position, thus the meeting function can be described by

$$C = q(U, U). \tag{3.2}$$

The number of meetings per unit of time is denoted by C. Following Pissarides (2000) it is assumed that the meeting function q is increasing and concave in both arguments and homogeneous of degree $\gamma > 1$. The last assumption ensures that the probability of a meeting increases with the density of unmatched workers in the city. Furthermore q is assumed to fulfill the Inada conditions. Figure 3.2 illustrates the behavior of the meeting function.

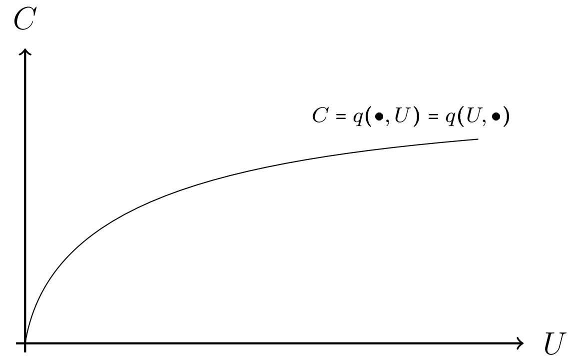


Figure 3.2: Meeting Technology

The meeting technology randomly selects unmatched workers from the set of possible meeting partners U. The meeting rate (the rate at which an unmatched worker has contacts per unit of time) is given by

$$\mu(U) = \frac{C}{U} = \frac{q(U,U)}{U}. \tag{3.3}$$

Using the assumption of homogeneity of degree $\gamma > 1$ we can rewrite the meeting rate $\mu(U)$ as

$$\mu(U) = \frac{q(U,U)}{U} = \frac{U^{\gamma}q(1,1)}{U} = U^{\gamma-1}q(1,1). \tag{3.4}$$

In order to derive a meeting rate which is linear in the number of unmatched workers, we set $\gamma = 2$.[5] The expression $q(1,1)$ determines how many contacts an individual is able to have per unit of time and will thus be denoted as meeting intensity α in the following.[6] Therefore the meeting rate of an unmatched worker in the city can be written as

$$\mu(U) = q(1,1)U = \alpha U. \tag{3.5}$$

[5] This simplifying assumption is also used by Berliant, Reed III and Wang (2006).

[6] A value of $\alpha = 0.1$ indicates that during one period of time an unmatched agent can meet 10 percent of all unmatched individuals in the city. A value of $\alpha = 2$ means that during the same time an unmatched individual can talk to each unmatched individual twice.

3.3.4 Production Technology

We keep production in the periphery as simple as possible in order to focus on the production process in the city. Therefore we assume that the flow output of each individual living in the periphery is equal to the constant $\bar{y}$. For further simplification we set $\bar{y} = 0$.
All structure is put on the production technology in the city. Suppose that an individual of knowledge type k is currently matched with an individual of knowledge type k'. The flow output of worker k crucially depends on the partner's type k' and is represented by

$$y(k, k') = A + e(k, k'). \tag{3.6}$$

The first expression A denotes the urban total factor productivity (TFP).[7] The TFP is common to all individuals in the city. The second expression $e(k, k')$ denotes the personal effectiveness of individual k currently matched with individual k'. If individual k is currently unmatched, it has a personal effectiveness of zero and flow production is solely determined by the TFP A. Both, the TFP A and the personal effectiveness $e(k, k')$, are influenced by knowledge spillovers resulting from face-to-face interactions in the city.

3.3.5 Knowledge Spillovers

This section discusses the impact of the heterogeneity in knowledge types on the extent of knowledge spillovers and clarifies the difference between knowledge transmission and knowledge creation.

Knowledge Transmission

The transmission of knowledge is equivalent to the intellectual exchange described in Berliant, Reed III and Wang (2006). The heterogeneity of workers in terms of their position on the unit circle plays a crucial role for the personal effectiveness. Consider two workers: one endowed with knowledge type $k \in K$, the other endowed with knowledge type $k' \in K$. They are brought together by the random meeting technology and suppose, the two workers both accept to be matched. The matching partners' personal effectiveness depends on the distance of their knowledge types k and k' in the knowledge space K, measured by the

[7]For tractability we assume that total factor productivity is a flow value.

Euclidean metric $d(k,k')$[8] We assume that the highest degree of knowledge transmission and thus the highest personal effectiveness is attained when the two meeting partners are endowed with exactly the same knowledge type ($k = k'$). In this case it is straightforward to communicate and exchange information. They already use the same vocabulary and techniques, so they can start exchanging knowledge and applying the gained knowledge to attain a higher personal effectiveness right away. The assumption of decreasing knowledge transmission with increasing diversity of knowledge types is justified by the stylized empirical facts from section 3.2.1 about the outcome of face-to-face interactions in cities. For illustrative purposes, one can think of two economists working in the field of urban economics. Both use the same terminology and the same techniques and once they are matched (i.e. have a research collaboration), they can immediately start to combine their information and become more productive. As the knowledge distance $d(k,k')$ increases, the transmission of knowledge becomes more cumbersome. Since workers do not have a lot in common, they will have problems understanding each others' technical terminology and will not be able to just imitate each others' techniques. Thus, individuals with very heterogeneous knowledge backgrounds have to put in a lot of effort before the transmission of knowledge can begin and once able to communicate they will find it difficult to apply the gained knowledge in their respective fields. Here one can think of a match between an economist and a dentist. Both will have major problems in understanding the vocabulary and imitating each others' techniques. Once they have managed to communicate, it remains questionable whether they can apply the gained knowledge in their respective occupation. The personal effectiveness $e(k,k')$ can be described through the relationship:

$$e(k,k') = e_0 - e_1 d(k,k') \tag{3.7}$$

The parameter $e_0 > 0$ describes the highest possible level of personal effectiveness that can be attained. This value is achieved when the two meeting partner have exactly the same knowledge background ($k = k'$). The parameter $e_1 > 0$ describes the sensitivity of knowledge transmission to heterogeneity of knowledge types.[9] [10]

[8]Distance in the knowledge space is just used as a measure for diversity of knowledge background and has nothing to do with physical distance.

[9]$e_1 = 0$ indicates that heterogeneity of knowledge types among individuals is irrelevant for the transmission of knowledge and thus for the personal effectiveness. Each match between two workers, independently of the knowledge types they are endowed with, generates the same amount of knowledge transmission. $e_1 \to \infty$ in turn indicates that only workers with the same knowledge background have a chance to increase productivity through the transmission of knowledge. As soon as the knowledge types differ to a minimal extent they are not able to communicate.

[10]This assumption is in contrast to Berliant, Reed III and Wang (2006). They assume that there exists an optimal distance $d^* > 0$ between knowledge types that creates the highest possible productivity. Their justification for this assumption is that "when individuals are too alike, they cannot accomplish much and

Knowledge Creation

Matches in the city not only facilitate the transmission of existing knowledge but also lead to the creation of new knowledge that in turn raises the urban level of technology. How does the distance between knowledge types $d(k,k')$ affect the creation of new knowledge? In the context of knowledge creation, it is not as clear that the outcome should decrease in the distance $d(k,k')$. We can revisit the two examples from above. Two economists working in the same field can form a research collaboration that makes both of them more productive. Additionally they can use the research collaboration to write papers that contribute to the creation of new knowledge. An economist and a dentist will find it very hard to apply the gained information to increase their individual productivity. The used techniques are too different to apply them in their respective occupation and thus we expect the extent of knowledge transmission to be rather low. But it is possible that the combination of their knowledge leads to the creation of new knowledge. For example they could find a way to create a more cost-efficient treatment plan or accounting system. The impact of the knowledge distance $d(k,k')$ on the creation of knowledge is thus less clear than the impact on the transmission of knowledge. Therefore, we are content with the weak assumption that each match, independently of the diversity of knowledge types k and k', creates new knowledge that contributes to the level of technology in the city. This assumption also goes in line with the stylized empirical facts on the extent of knowledge creation in cities. Accordingly, the creation of new knowledge $a(k,k')$ by a currently matched agent with knowledge type k is independent of his partner's knowledge type k' and always equal to $a_0 > 0$, which can be described by

$$a(k,k') = a_0. \tag{3.8}$$

We adopt the neo-classical view and assume that created knowledge is a local public good for all individuals living in the city. In the following we make the simplifying assumption that created knowledge is directly translated into the total factor productivity A in the city. In each point of time the TFP is equal to the created knowledge of a matched worker (a_0) times

little knowledge will be obtained". We do not adopt this assumption because we distinguish between two different types of knowledge spillovers, the transmission and creation of knowledge, whereas Berliant, Reed III and Wang (2006) combine these spillovers into one effect. Once the transmission and creation of knowledge are analyzed separately, it makes sense to assume a maximum effectiveness when agents are alike ($k = k'$), because the pure transmission of knowledge (in the absence of knowledge creation) is easiest when agents do not have to overcome any knowledge barrier.

the number of matched individuals in the city (M). Since the created knowledge is assumed to be a local public good, it is equally distributed across all individuals living in the city (N), regardless of whether they are currently matched or not. Therefore the relationship can be denoted by

$$A = \frac{Ma_0}{N}. \tag{3.9}$$

The existence of a second type of knowledge spillover (i.e. the creation of new knowledge) is a potential source for social inefficiencies. Workers choose the range of acceptable matches in order to maximize their personal effectiveness. However, they do not consider their impact on the creation of knowledge. Section 3.4 provides an extensive discussion of these inefficiencies. Figure 3.3 summarizes the impact of the knowledge distance $d(k, k')$ on the extent of knowledge transmission and knowledge creation in urban face-to-face interactions. In figure 3.3, a_0 is displayed to be larger than e_0. However, since the relation between a_0 and e_0 is an empirical question, our model allows for other relative configurations of a_0 and e_0.

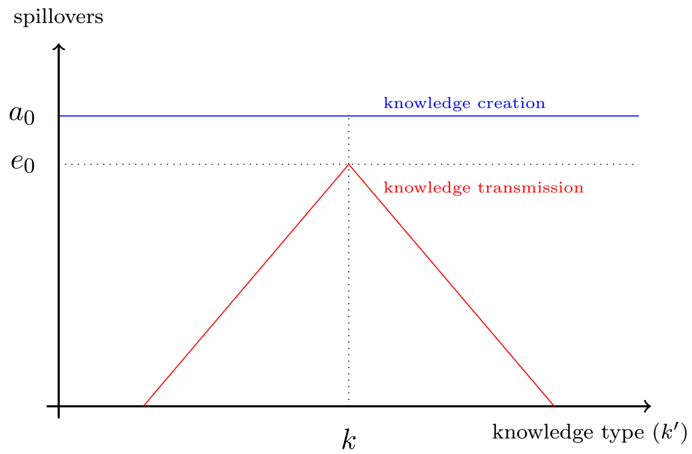


Figure 3.3: Knowledge Transmission and Knowledge Creation in f-2-f Interactions

3.3.6 Choice of the Knowledge Spread

In section 3.3.3 we introduced the meeting technology that determines the rate of contacts an unmatched worker has during one unit of time. We derived the meeting rate $\mu(U) = \alpha U$, which shows that the number of contacts increases linearly at rate α in the density of unmatched individuals U in the city. What determines which meeting turns into a match and which meeting is canceled after a first contact? Suppose we have a meeting between two individuals endowed with knowledge types k and k'. Given the position k on the unit circle, each individual chooses a knowledge spread $\delta_k > 0$, which determines the range of workers individual k is able to have a face-to-face interaction with. The knowledge spread δ_k is geometrically represented by the arc around knowledge type k leading to a knowledge horizon $[k - \delta_k/2, k + \delta_k/2]$. The knowledge horizon can be interpreted as the set of disciplines, individual k has at least elementary knowledge about. This knowledge is indispensable for individual k to exchange knowledge in face-to-face interactions with an individual of knowledge type k'. Only if the knowledge type k' is located within the knowledge horizon of k, i.e. $k' \in [k - \delta_k/2, k + \delta_k/2]$, a face-to-face interaction is possible. Thus extending the knowledge horizon by increasing the knowledge spread δ_k means that individual k interacts with a wider range of workers in the city.

The unit circle represented in figure 4 displays the knowledge space K. Since worker A with knowledge type k_A is located within the knowledge horizon of individual k, a face-to-face interaction is possible. Individual B with knowledge type k_B is situated outside the knowledge horizon of individual k. Consequently no transmission and creation of knowledge can occur, since worker k has no elementary understanding of B's field of knowledge.

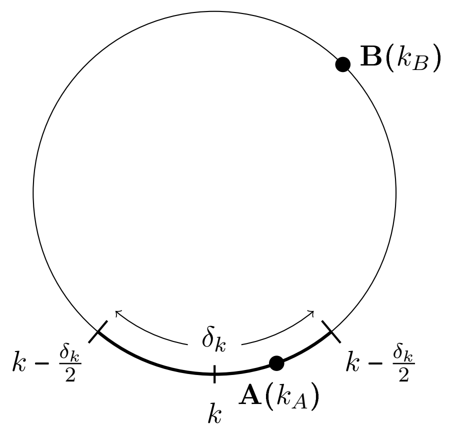


Figure 3.4: Knowledge Horizon

As workers are ex-ante symmetric[11], we only need to consider symmetric equilibria which implies that all workers choose the same knowledge spread ($\delta_k = \delta \;\; \forall k$). In this case either both individuals accept or both reject to be matched.[12] In the next section we clarify how

[11]Knowledge types are revealed only after two workers have met, i.e. workers are heterogeneous ex-post, but indistinguishable ex-ante.

[12]If worker k is not located within the knowledge horizon of k', then worker k' is automatically not located within the knowledge horizon of k.

the choice of δ is determined in the market solution.

3.3.7 Expected Lifetime Utility

We determine the expected lifetime utility of a worker being currently matched (V_m) and the expected lifetime utility of a currently unmatched worker (V_u). The value of being matched depends on the knowledge types of individuals currently having a face-to-face interaction and on the number of unmatched workers in the city U. Thus we have $V_m = V_m(k, k', U)$. Since an unmatched individual of knowledge type k has currently no face-to-face interaction, its expected lifetime utility only depends on its own knowledge type k and and the number of unmatched individuals U, yielding $V_u = V_u(k, U)$.

There exists an exogenous separation rate $\lambda > 0$, at which ongoing face-to-face interactions are split up. This separation rate results from idiosyncratic shocks that arrive to interactions at rate λ. Furthermore, we assume a perfect capital market, in which assets can be traded over time at the exogenous interest rate $r > 0$.

The value of a match between workers k and k', given a number of unmatched individuals U, satisfies the following Bellman Equation:

$$rV_m(k, k', U) = \underbrace{A + e(k, k')}_{y(k,k')} + \lambda\Big[V_u(k, U) - V_m(k, k', U)\Big] \tag{3.10}$$

The value of being matched with worker k' can be interpreted as an asset held by worker k. Given perfect capital markets, the left hand side (capital cost of the asset) has to equal the right hand side (rate of return on the asset). A currently matched worker of knowledge type k has flow income $y(k, k') = A + e(k, k')$ as long as he stays matched with worker k'. The state changes from matched to unmatched at the exogenous separation rate λ with an associated net return of $V_u(k, U) - V_m(k, k', U)$.

The value of worker k being unmatched at a given number of unmatched individuals U in the city, satisfies the following Bellman Equation:

$$rV_u(k, U) = A + \alpha U \int_{k-\frac{\delta}{2}}^{k+\frac{\delta}{2}} \Big[V_m(k, k', U) - V_u(k, U)\Big] dk' \tag{3.11}$$

As above, the value of being unmatched can be interpreted as an asset held by worker k and with perfect capital markets the left hand side equals the right hand side. The first term on the right hand side is the flow of total factor productivity in the city. The meeting rate $\mu(U) = \alpha U$ gives the number of contacts an individual has during one unit of time. Only individuals with a knowledge type $k' \in [k - \delta/2, k + \delta/2]$ are accepted for a face-to-face interaction.[13] The term inside the integral can be interpreted as the net return from changing the state from being unmatched to being matched with a worker of knowledge type k'.

There are two opposing effects to take into account when worker k chooses the knowledge spread δ. First, increasing the knowledge spread δ extends the knowledge horizon. Thus worker k has face-to-face interactions with a wider range of individuals in the city which increases the probability of turning a meeting into a match. On the other hand, increasing the knowledge spread δ also decreases the expected quality of the partners, k accepts for face-to-face interactions, i.e. individual k also enters matches with individuals that have not much in common with him. This diminishes the transmission of knowledge as individual k may be stuck with a bad match.[14] The opportunity cost of a bad match is the possibility of interacting with more adequate partners in the meantime.

We now compute the rate of face-to-face interactions per unit of time. As a fraction δ of all meeting partners are accepted for a face-to-face interaction, the number of matches $p(U)$ per unit of time is given by

$$p(U) = \mu(U)\delta = \alpha\delta U, \tag{3.12}$$

where $\alpha\delta$ can be interpreted as the matching intensity. The matching rate linearly increases in the density of unmatched individuals U. When workers decide where to locate they use the value of being unmatched (V_u) as their benchmark.[15] Using the fact that the distribution of knowledge types on the unit circle is uniform yields that the expected distance from a matched partner's knowledge type is $\delta/4$. Therefore the value of being unmatched V_u can be written as

$$V_u(\delta, U) = \frac{A}{r} + \frac{\alpha\delta U}{r + \lambda + \alpha\delta U}\frac{e_0 - e_1\frac{\delta}{4}}{r}. \tag{3.13}$$

The first term on the right hand side (A/r) is the lifetime income generated from the total factor productivity (TFP). Since A is a local public good, worker k can make use of it independently of being currently matched or not. The second part of the second term on the right-hand side can be interpreted as the expected income premium an individual gets if it is consistently matched compared to being consistently unmatched. Since individual k is not

[13] Other partners are not accepted, because worker k is not able to exchange knowledge with them.

[14] It is a bad match from the perspective of worker k. It might be a good match for the economy (or city) as a whole, since new knowledge is created.

[15] This is the state in which they enter the city.

matched all the time this expected income premium is discounted. The discount rate (first part of the second term) can be interpreted as discounted matching rate.[16]

3.4 Equilibrium Analysis

In this section, we establish the symmetric Steady State Nash Equilibrium. The equilibrium is defined by the workers' choice of knowledge spread δ, the number of unmatched individuals in the city U and the resulting city size N.

3.4.1 Steady State Population

The Steady State Equilibrium requires the number of matched and unmatched workers in the city to be constant over time. In the symmetric case, this relationship implies that the flows into and out of the pool of unmatched workers have to equal.

$$\underbrace{\alpha\delta U}_{\substack{\text{flow out of the pool of}\\ \text{unmatched individuals}}} = \underbrace{\lambda M = \lambda(N - U)}_{\substack{\text{flow into the pool of}\\ \text{unmatched individuals}}} \tag{3.14}$$

Using this identity, we can derive the number of unmatched individuals in the city as an implicit function of the total city population N in steady state:[17]

$$U = \frac{\lambda}{\lambda + \alpha\delta U} N \tag{3.15}$$

3.4.2 Steady State Equilibrium

Workers choose the optimal knowledge spread δ^* by maximizing expected lifetime utility in the city. The optimal knowledge spread δ^* is determined by the trade-off between increasing the probability of having face-to-face interactions and increasing the expected extent of knowledge transmission during a match. Once the optimal knowledge spread δ^* is chosen, individuals move to the city until the levels of lifetime utility in the city and the periphery are equalized and no incentive for moving between the two locations exists (Spatial Equilibrium). Together with equation 3.15 the equilibrium values N^* and U^* are determined. The Steady State Equilibrium, which we will also refer to as Market Solution in the remainder of the

[16] It is the matching rate $\frac{M}{N}$ with the interest rate r in the denominator.
[17] This expression is analogous to the Beveridge Curve in Labor Market Theory.(Pissarides (2000))

text, is defined as follows:

Definition 1: Steady State Equilibrium

The Steady State Equilibrium is an allocation $\{\delta^*, U^*, N^*\}$ that satisfies the following conditions:

(1) Workers maximize expected lifetime utility by choosing their knowledge spread δ: $\delta^* = \text{argmax}_\delta V_u(\delta, U^*)$.

(2) The level of lifetime utility in the city equals the level of lifetime utility in the periphery: $V_u(\delta^*, U^*) - tN^* = 0$.

(3) The condition for Steady State population is satisfied: $U^* = \frac{\lambda}{\lambda + \alpha\delta^* U^*} N^*$.[18]

Workers do not consider the impact of their choice of knowledge spread δ on the steady state population of unmatched individuals, which gives rise to an inefficiency, that we will refer to as matching externality in the following.

Furthermore, workers do not consider their impact on the local flow of innovation, instead they choose their individual knowledge spread δ to maximize their expected lifetime utility from personal effectiveness. We will refer to this inefficiency as innovation externality.[19]

Consequently, their choice of δ satisfies the following first order condition:

$$\frac{\partial V_u}{\partial \delta} = \delta^2 + \frac{2(r+\lambda)}{\alpha U}\left(\delta - \frac{2e_0}{e_1}\right) = 0 \tag{3.16}$$

This condition implicitly defines the equilibrium knowledge spread as a function $\delta(U)$. In the following, we will refer to this condition as the knowledge spread condition KS. We see from KS, that the larger the number of unmatched individuals in the city U, the lower the choice of the knowledge spread δ. Intuitively, this follows from the positive impact of population density on the matching rate, which allows workers to be more picky regarding their interaction partners. This relationship leads to a downward sloping KS-locus in figure 5.

In spatial equilibrium, the levels of lifetime utility have to equalize across locations, such that incentives for relocating disappear. In the context of our model, this definition implies that workers move to the city until the attainable expected lifetime utility in the city $(V_u - tN)$

[18] Conditions (1) and (2) determine the allocation $\{\delta^*, U^*\}$, condition (3) automatically determines N^*.

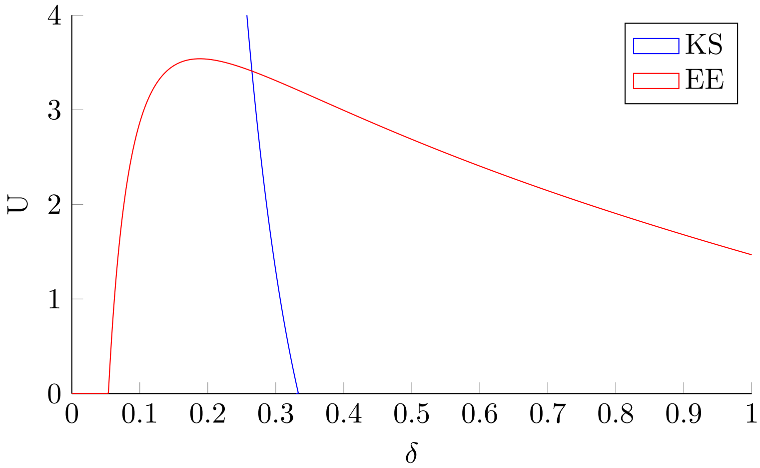
[19] Both, the matching and innovation externality are discussed extensively in section 3.5.

equals lifetime utility in the periphery (0). Therefore the number of unmatched workers living in the city is determined by the following condition:

$$\underbrace{\underbrace{\frac{\alpha\delta U}{\lambda+\alpha\delta U}\frac{a_0}{r}}_{A/r} + \frac{\alpha\delta U}{r+\lambda+\alpha\delta U}\frac{e_0-e_1\frac{\delta}{4}}{r}}_{V_u(\delta,U)} - tN = 0^{20} \tag{3.17}$$

By making use of the steady state population condition, the equilibrium number of unmatched individuals in the city as a function $U(\delta)$ is defined. In the following, we will refer to this relation as the equilibrium entry condition EE.

The influence of the knowledge spread δ on equilibrium population N is twofold. An increase in δ raises the matching rate, but also diminishes the average extent of knowledge transmission. This interaction between knowledge spread δ and equilibrium population N leads to a hump shaped form of the EE-locus in figure 3.5.[21]



Chosen parameter values: $\alpha = 0.2, r = 0.2, \lambda = 0.15, e_0 = 1, e_1 = 6, a_0 = 1, t = 0.5$.
$\{\delta^*, U^*, N^*\} = \{0.265, 3.409, 7.516\}$

Figure 3.5: Steady State Equilibrium

A Steady State Equilibrium emerges for the values of knowledge spread and unmatched workers $\{\delta^*; U^*\}$ in the intersection point of the KS- and the EE-locus as depicted in figure 5. Given the values of δ^* and U^*, equation (15) determines the value of N^*. Using the arbitrarily chosen parameter values in the example above, we derive $\delta^* = 0.265$, which means

[21]Figure 3.5 shows a hump shaped relationship between δ and U. This coherence automatically implies a hump shaped connection between δ and N.

that in Steady State individuals living in the city accept to have face-to-face interactions with 26.5 percent of meeting partners. The size of the city is $N^* = 7.516$ with $U^* = 3.409$ individuals being unmatched.

3.5 Social Inefficiencies

3.5.1 Social Planner's Solution

The previously discussed Steady State Equilibrium gives rise to various inefficiencies. In the following, we will explore the extent and the interactions of those externalities in more detail to assert the social inefficiency of our model economy. In order to do so, we will compare the equilibrium conditions with the optimal choice of a social planner who chooses the knowledge spread δ and the population allocation N simultaneously.

Our model contains three sources of externalities: Congestion externalities arise because individuals do not consider the impact of their location decision on congestion costs. Entering the city bears costs $t > 0$ for every worker living in the city. Matching externalities arise because individuals do not consider the impact of their choice of the knowledge spread δ on the mass of unmatched workers. Most prominently in our model, innovation externalities arise, because individuals do not consider the impact of their choice of the knowledge spread on innovations and thus on the TFP A in the city. The Definition of the Social Planner's solution to our model is as follows:

Definition 2: Social Planner's Solution

The Social Planner's Solution is an allocation $\{\hat{\delta}, \hat{U}, \hat{N}\}$ that satisfies the following conditions:

(1) The Social Planner chooses the knowledge spread and the number of unmatched workers in the city simultaneously in order to maximize the expected lifetime utility of a worker living in the city: $\{\hat{\delta}, \hat{U}\} \in \text{argmax}_{\delta,U}\, V_u(\delta, U)$.

(2) The condition for Steady State population is satisfied: $\hat{U} = \dfrac{\lambda}{\lambda + \alpha\hat{\delta}\hat{U}}\hat{N}$.

The exact optimality conditions are stated in the appendix. We focus on the effects the Social Planner takes into account when choosing the knowledge spread and the population allocation. First, the optimality condition for the knowledge spread:

$$\frac{\partial V_u}{\partial \delta} = \underbrace{\frac{\partial A}{\partial \delta}}_{\substack{\text{innovation extern.} \\ (>0)}} + \underbrace{\frac{\partial A}{\partial U}\frac{\partial U}{\partial \delta}}_{\substack{\text{matching extern.} \\ (<0)}} + \frac{\partial e}{\partial \delta} + \underbrace{\frac{\partial e}{\partial U}\frac{\partial U}{\partial \delta}}_{\substack{\text{matching extern.} \\ (>0)}} = 0 \tag{3.18}$$

The matching externalities lead to an equilibrium knowledge spread that is larger than socially optimal. Individuals are too broad in their acceptance of partners, because they do not consider that they prevent potentially more productive matches by lowering the mass of unmatched workers.

The innovation externality leads to an equilibrium knowledge spread that is smaller than socially optimal. The knowledge spread is too narrow as individuals do not consider the innovative output of interactions with diverse knowledge types. Depending on the relative importance of these externalities, it is possible that workers are either too picky or too generous in their choice of partners. In the following analysis, we will focus on the case of a sufficiently important role of innovations such that the innovation externality outweighs the matching externalities and the equilibrium knowledge spread is smaller than socially optimal.

The impact of the inefficiencies on location decisions is explored in reference to the planner's optimality condition for population allocation: The social planner chooses U such that net lifetime utility for the representative worker in the city is maximized, implying the first order condition:

$$\frac{\partial V_u}{\partial U} = \frac{\partial A}{\partial U} + \frac{\partial e}{\partial U} - t\frac{\partial N}{\partial U} = 0 \tag{3.19}$$

For the case of a smaller than socially optimal equilibrium knowledge spread, the inefficiencies in equilibrium location decisions are again twofold: First, the congestion externality leads to a larger than socially optimal city size as workers do not consider their impact on city-wide congestion costs. Second, the inefficient choice of δ leads to diminished agglomeration forces as knowledge spillovers do not reach their optimal extent. This inefficiency leads to smaller than socially optimal cities.

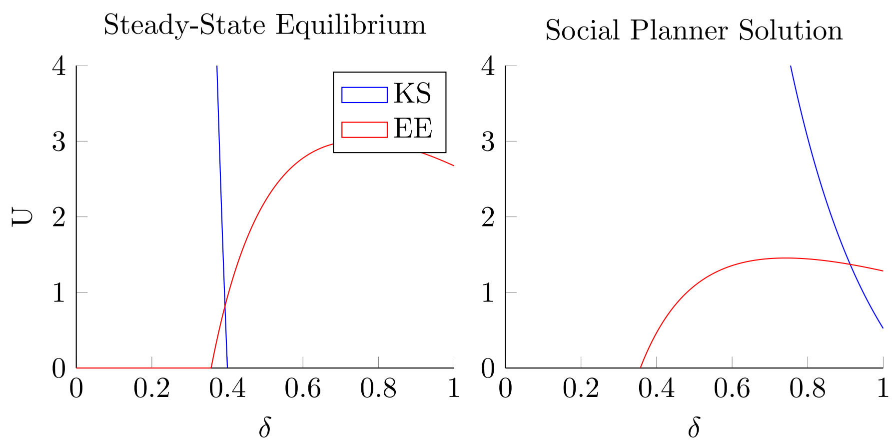
In summary, equilibrium choices of δ and N are generally inefficient. The direction and extent of the inefficiencies depend on the relative importance of innovation for the choice of δ and the importance of overall knowledge spillovers for the choice of N.

3.5.2 Existence of inefficiency patterns

Depending on the parameter configuration, we find three distinct patterns of inefficiency in workers' choices of knowledge spread and location in the market equilibrium relative to the Social Planner's solution: Overselectivity and Underpopulation, Overselectivity and Overpopulation as well as Underselectivity and Overpopulation. In the following, we verify the existence of these inefficiency patterns by construction.

Case 1: Overselectivity and Underpopulation

The first of these cases is marked by excessively narrow knowledge spreads and too small cities in the market equilibrium. Overselectivity can be explained by the relatively large importance of innovation, which workers do not take into account. Underpopulation directly follows from the significant overselectivity in this case, which does not allow the agglomeration force of innovation to develop its full extent. An example for the resulting market equilibrium and social planner allocations are depicted in figure 3.6 below.

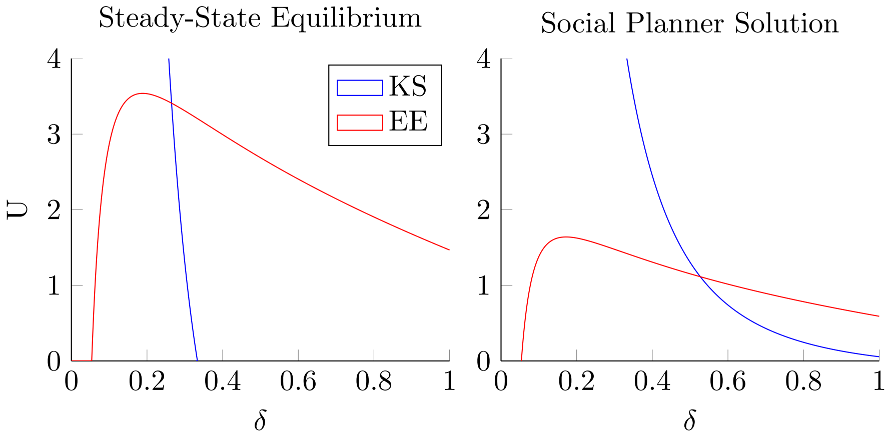


Chosen parameter values: $\alpha = 0.1, r = 0.2, \lambda = 0.8, e_0 = 2, e_1 = 10, a_0 = 2.7, t = 0.8$.
$\{\delta^*, U^*, N^*\} = \{0.3937, 0.8192, 0.8522\}, \{\hat{\delta}, \hat{U}, \hat{N}\} = \{0.9125, 1.3709, 1.5852\}$

Figure 3.6: Case 1: Overselectivity and Underpopulation

Case 2: Overselectivity and Overpopulation

The case of overselectivity and overpopulation in the market equilibrium is similarly driven by an innovation externality. However, in this case, overselectivity is not as pronounced and therefore the interplay of matching and congestion externality leads to overpopulation as illustrated below.

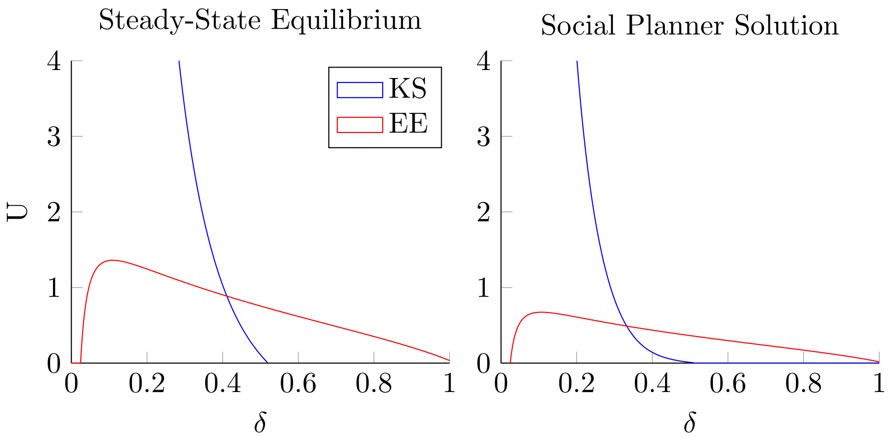


Chosen parameter values: $\alpha = 0.2, r = 0.2, \lambda = 0.15, e_0 = 1, e_1 = 6, a_0 = 1, t = 0.5$.
$\{\delta^*, U^*, N^*\} = \{0.265, 3.409, 7.516\}, \{\hat{\delta}, \hat{U}, \hat{N}\} = \{0.527, 1.114, 1.985\}$

Figure 3.7: Case 2: Overselectivity and Overpopulation

Case 3: Underselectivity and Overpopulation

In the third case of our equilibrium analysis, underselectivity and overpopulation are prevalent in the absence of innovation ($a_0 = 0$). As the innovation externality is zero in this case, the matching externality leads to chosen knowledge spreads that are larger than socially optimal. The interplay with congestion externalities again leads to overpopulation.



Chosen parameter values:$\alpha = 0.16, r = 0.1, \lambda = 0.01, e_0 = 2.6, e_1 = 10, a_0 = 0, t = 0.9$.
$\{\delta^*, U^*, N^*\} = \{0.411, 0.886, 6.051\}, \{\hat{\delta}, \hat{U}, \hat{N}\} = \{0.332, 0.490, 1.767\}$

Figure 3.8: Case 3: Underselectivity and Overpopulation

In summary, these results highlight the relevance of the agglomeration force of innovation in our model. If the role of innovation is sufficiently important, we find that workers are too picky in the choice of their interaction partners. Depending on the magnitude of this externality, cities can be smaller or larger than socially optimal.

If innovation is irrelevant however, we find excessively large knowledge spreads due to the matching externality and cities that are larger than socially optimal.

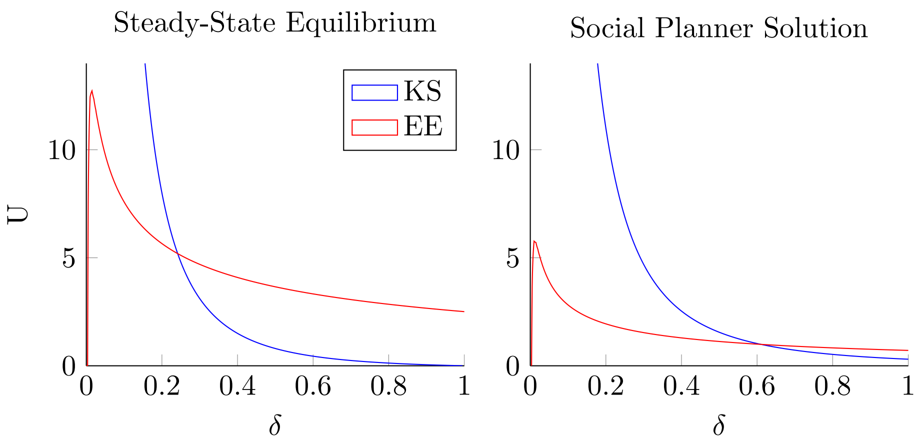
3.5.3 Predicted Inefficiency Pattern

While the preceding section established the existence of three distinct patterns of inefficiency, we now determine which of these patterns is the most empirically relevant. In order to do so, we calibrate our model's key parameters from existing empirical work. Our goal is not to exactly quantify the effects, but rather to establish the model's qualitative predictions.

The key parameters of our model are the ones that govern the relative importance of knowledge creation and transmission, i.e. e_0, e_1 and a_0. It is impossible to measure the parameters directly as the effect of knowledge transmission on productivity is not directly observable. However, observed urban wage premia can be used as an approximation to their respective impact. We interpret measurements of static urban wage premia as an approximation for the role of the technology level and measurements of dynamic wage premia as an approximation for the role of learning. This interpretation follows the logic, that workers immediately benefit from the higher technology level upon moving to the city, while the buildup of know-how happens over time. For the qualitative predictions of our model, it is the relative importance of innovation and learning that is crucial. Recent studies from Carlsen, Rattso and Stokke (2013), D'Costa and Overman (2014), De la Roca and Puga (2014) all find that the static wage premium is more pronounced than the dynamic wage premium. Carlsen, Rattso and Stokke (2013) quantify the share of the static premium in urban lifetime earnings premia as 2/3 while the remaining 1/3 stems from dynamic premia. Controlling for observable and unobservable individual characteristics, they find the static premium for Norwegian urban workers to be 3.3%. Accordingly, we set our parameter a_0 to 0.033. The value of the dynamic premium from learning is consequently set to 0.0165. This value corresponds to $e_0 - e_1\delta/2$ in our model. We normalize $e_0 - e_1 1/2 = 0$. As δ is an endogenous variable, we cannot uniquely determine e_0 and e_1, a reasonable parameter configuration in line with the previous findings is to set $e_0 = 0.02$ and $e_1 = 0.04$.

The qualitative predictions of the model are less sensitive to the remaining parameters as long as their magnitude is broadly in line with reality. The parameters of the arrival and separation rate of matches in our model are chosen in line with the literature on matching in the labor market. Following Hobijn and Sahin (2009), we set the arrival rate $\alpha = 0.3$ and the separation rate $\lambda = 0.05$. Interest rate r is set to 0.01 and congestion costs $t = 0.1$.

For this parameter configuration, the resulting equilibrium pattern as depicted in figure 9 is marked by overpopulation and overselectivity. Thus our model predicts that workers choose a range of interaction partners that is too narrow and that city size is larger than socially optimal. This qualitative pattern is robust to parameter configurations that are in line with the existing literature on urban wage premia and matching rates.



Chosen parameter values: $\alpha = 0.3, r = 0.01, \lambda = 0.05, e_0 = 0.02, e_1 = 0.04, a_0 = 0.033, t = 0.1$.
$\{\delta^*, U^*, N^*\} = \{0.242, 5.193, 44.301\}, \{\hat{\delta}, \hat{U}, \hat{N}\} = \{0.610, 0.991, 4.587\}$

Figure 3.9: Predicted Inefficiency Pattern

3.6 Discussion

The aim of this chapter is to develop a spatial model that explicitly incorporates two different types of knowledge spillovers (the creation and transmission of knowledge) in cities and to show how they affect the migration decision of individuals. We use the framework of stochastic job matching from Pissarides (2000) and apply it in the context of urban face-to-face interactions. Our model economy incorporates two asymmetric locations: The city and the periphery, where only the city provides individuals with the opportunity to exchange knowledge via face-to-face interactions. In each point of time, workers decide where to be located. Furthermore, workers in the city decide over the range of individuals they are willing to interact with. The intensities of knowledge creation and knowledge transmission in those interactions depend on the similarity of knowledge background of the interacting individuals: First, the individual buildup of skills through knowledge transmission increases in the similarity of knowledge backgrounds. And second, the creation of knowledge is independent of knowledge backgrounds.

The market solution exhibits three sources of inefficiencies. Since created knowledge is a local public good in the city, workers only focus on the buildup of their personal skills when deciding about the range of individuals they accept to be matched with (innovation externality). Congestion externalities arise because individuals do not consider the impact of their location decision on city-wide congestion costs. Matching externalities arise because individuals do not consider the impact of their choice of the knowledge spread on the mass of unmatched individuals.

Depending on the parameter values, we find that workers choose a range of matching partners that can be smaller or larger than socially optimal. The more important the role of knowledge creation in face-to-face interactions, the more likely it is, that the chosen range of interaction partners is smaller than socially optimal. This means that people overestimate the importance of interacting with other individuals having a relatively similar knowledge background.

The interplay of agglomeration and dispersion forces determines the allocation of people in spatial equilibrium. Moving to the city provides the chance to benefit from knowledge spillovers in face-to-face interactions. However, these face-to-face interactions come at the price of urban congestion costs. The model analysis shows that the inefficient decision on the range of individuals to interact with also leads to socially inefficient city sizes. Depending on the chosen parameter values the model's equilibrium city size can be smaller or larger than socially optimal.

For parameter values based on the existing empirical literature on urban wage premia and matching, we find that workers are too picky in their choice of partners and the resulting city size is larger than socially optimal.

Chapter 4

Proximity Dimensions and Innovation

4.1 Introduction

This chapter focuses on the spatial interdependence of innovation and uses 236 European regions at the NUTS2-level as units of analysis to examine the mechanisms that drive local innovation. I use the number of patent applications attributed to local inventors as the measure of local innovative output.

The framework builds on a knowledge production function which defines the local number of ideas as a function of local R&D spending and the level of the human capital stock. In addition to these internal inputs, the spillover of ideas from neighboring regions is captured by including the distance-weighted innovative output of other regions as external input to knowledge production.

Building on Bottazzi and Peri (2003), I examine the effect of geographical distance on the exchange of knowledge between regions. I also analyze the effect of technological similarity of the regions' industries. The model predicts that inventors from nearby regions with similar technological background are more likely to exchange knowledge with local inventors as more frequent face-to-face interactions and easier transmission of non-codified knowledge drive knowledge spillovers. A theoretical analysis investigates the connection of the knowledge production function to growth theory. I explore spatial patterns in the data and define the regions' position in technological space. My exploration of the data shows strong evidence for spatial clustering of innovative activity which confirms the findings in previous literature. This chapter further sheds light on the spatial clustering of innovations as I empirically ad-

dress the question how the two dimensions of distance impact local innovative output when simultaneous effects are considered. By including geographical distance and technological distance in a simultaneous regression, I explore whether effective exchange of ideas requires the involved regions to be geographically and technologically close or inventors overcome greater distance in one dimension if they are sufficiently close in the other dimension.
I use the reduced form of the knowledge production function to estimate a spatial-autoregressive regression with autoregressive disturbances (SARAR). A feasible generalized spatial two-stage least squares (FGS2SLS) estimation strategy is used in order to identify the key parameters of interest which measure the spatial dependence of innovation along the two dimensions of distance. I consider alternative internal inputs as well as alternative specifications of the spatial weight matrices. A simultaneous estimation of both proximity dimensions provides insights into the interplay of geographical and technological proximity.
The main results show that internal inputs such as R&D spending and human capital as well as both geographical and technological proximity have a significantly positive impact on local innovative output. The evidence for these spillovers is robust to the introduction of alternative inputs and modifications to the spatial weight matrices. Furthermore, the simultaneous estimation of both proximity dimensions indicates that the role of technological proximity is more important and provides preliminary evidence that geographical and technological proximity complement each other in the facilitation of inter-regional spillovers.
The remainder of this chapter is organized as follows: Section 2 summarizes the related literature, section 3 describes the estimation strategy. The dataset is explored in section 4 and section 5 discusses the main results of the empirical analysis. Section 6 discusses the chapter's main insights.

4.2 Literature Review

A large body of literature investigates the existence and extent of spatial knowledge spillovers. These studies provide insights into the role that proximity plays in innovative spillovers and which forms of proximity are relevant to facilitate these spillovers. In the following, I briefly outline relevant literature and describe my own contributions in this context. The foundations of the knowledge production function and its early applications by Griliches (1979), Pakes and Griliches (1984), Adam B Jaffe (1989) are reviewed in section 2.3.2 of this thesis.

In a more recent contribution, Autant-Bernard (2001) examines data from French regions in the framework of the knowledge production function with attention to geographical and technological spillovers. He conceptualizes spillovers as an external stock of knowledge and estimates spillover effects along each of the two dimensions. Significant externalities occur with stronger evidence in the technological dimension than in the geographical dimension. The results also indicate that human capital is the primary mediator of these externalities. This chapter follows the research line of Bottazzi and Peri (2003), Moreno, Paci and Usai (2005) and Paci, Marrocu and Usai (2014), who apply the concept of the knowledge production function to investigate innovative spillovers between European regions. Their contributions are summarized in section 2.4.2 of this thesis. Another active strand of literature focuses on the differentiation of the economic concept of proximity. In addition to the traditional approach of analyzing the role of geographical distance, researchers explore other forms of proximity such as cognitive, organizational, social or institutional ties between agents. The French School of Proximity includes numerous studies that emphasize the relevance of these non-spatial proximities. An exemplary contribution that summarizes this school of thought is Gilly and Torre (2000). This theoretical exploration focuses on the intersection of industrial and spatial economics. Main areas of research in the theoretical differentiation of proximity dimensions are defined. The research group investigates various forms of interactions between agents on the individual and institutional level and in particular the role of geographical proximity and organizational proximity. Boschma (2005) provides a theoretical foundation to investigate the interplay of these different proximity dimensions in the innovative process. The author establishes that the analysis of geographical proximity's impact leads to biased results when the other dimensions are ignored. Geographical proximity is neither necessary nor sufficient for knowledge transmission. Boschma develops a clear definition for each of the proximity dimensions in order to be able to isolate their effects analytically. Furthermore, he finds that too much proximity may be harmful to innovation due to a lock-in effect that reduces openness to non-local ideas. The author also notes the importance of the dynamic aspects of the proximity dimensions' interplay as geographical proximity may strengthen the other proximity dimensions over time. Carrincazeaux and Coris (2011) provide an excellent review of the most important findings that support the crucial role of social and institutional ties in knowledge exchange. Mattes (2012) finds in a theoretical exploration, that there are important interactions between the different dimensions of proximity in the innovative process and predicts a complementary role of geographical and technological proximity. As established by

Boschma (2005), the isolated effect of geographical proximity is small, but it is of importance in strengthening the other forms of proximity. The theoretical differentiation of the distinct proximity dimensions has inspired a growing number of empirical studies with the goal to capture and disentangle the spillover effects along each proximity dimension. The main empirical challenges in the investigation of spillovers across regions are the measurement of the respective proximity dimensions, the correct specification of the associated spatial weight matrix and the isolation of the spillover effects from other spatial correlations. Various spatial econometric tools have been applied in order to deal with these challenges. Lacombe (2004) first introduced a modification to the econometric approach in a labor market context. The introduction of two weight matrices allows for the simultaneous estimation of the impact of two distinct treatment effects. This approach is the foundation for the estimation of local innovative output and its simultaneous spatial dependence in different proximity dimensions. The application of spatial auto-regressive modeling techniques to the various proximity dimensions and their respective properties is investigated by Corrado and Fingleton (2011). They argue for the modeling of spatial dependence in the error term through a spatial auto regressive model with auto-regressive disturbances (SARAR). The feasible generalized spatial two-stage least squares (FGS2SLS) is established as the most effective estimation method. Carboni (2013) applies the SARAR to firm-level data to investigate inter-industrial spillover effects on R&D spending. Proximity is here measured as a firms' industrial distance which is defined by the trade intensity between sectors. The author finds evidence for the existence of R&D spillovers between neighboring industries. Paci and Usai (2009) examine innovative spillovers measured by patent citation flows between European regions. Their approach refines the spatial weight matrix as it weighs geographical distance with a set of other spatial control variables such as country affiliation and economic conditions. The results show that knowledge spillovers are more pronounced between geographically close regions with particularly high knowledge flows between neighboring regions and regions within the same country. Parent and LeSage (2008) provide another study that exploits variations in patent activity between European regions in order to measure innovative spillovers. This approach exploits the fact that inter-regional relationships may exhibit industry-specific technological linkages or transportation network linkages, which is in contrast to traditional studies relying exclusively on geographical proximity. A series of formal Bayesian model comparisons provides support for a model based on technological proximity combined with spatial proximity, asymmetric knowledge spillovers, and heterogeneity in the disturbances. Estimates of region-specific latent effects parameters structured in this fashion are produced by the model and used to draw inferences regarding the character of knowledge spillovers across the regions. The method is illustrated using sample data on patent activity covering 323 regions in nine European countries. Harris, Moffat and Kravtsova (2011) provides a comprehensive investigation of the prevalent specifications of the spatial weight matrix in spatial analysis. He argues that traditional approaches carry the risk of misspecification as they ignore endogeneity concerns and the impossibility to define the spatial dependence correctly a priori. More sophisticated strategies are proposed, including Bayesian, non-parametric and iteration techniques. The author illustrates the sensitivity of spatial spillover estimations to different specifications of the weight matrix. My analysis follows the approach of this strand of literature and applies the knowledge production function to describe innovative output on a regional level. I use spatial autoregressive models in order to find a measure of innovative spillovers between European regions along the geographical and technological dimension. My analysis confirms previous findings on the relevance of each proximity dimension for detailed current data from 236 European regions. I find that these results are robust to alternative measures of the internal inputs to knowledge production and alternative specifications of spatial weights. A further contribution of my work is the simultaneous estimation of the impact of geographical and technological distance and their complementarity in facilitating innovative spillovers.

4.3 Methodology

The theoretical foundation of this analysis stems from the knowledge production function which is a generalization of Romer (1990)'s production function in endogenous growth theory. Romer establishes a Cobb-Douglas function with inputs of human capital, physical labor and physical capital. Endogenous growth is determined by the technological level A, which describes the stock of designs of durable goods. The evolution of A depends on a productivity parameter, the share of human capital allocated to research and the access to existing ideas. [1]

Bottazzi and Peri (2003) extend Romer's approach by refining the process of technological

[1] For expositional purposes, the theory behind the KPF framework is laid out in the appendix.

growth. The growth of ideas is defined as innovative output, which depends on local human capital and R&D spending. Most importantly, their framework also considers the impact of spillovers from neighboring regions' innovative activity. This extension enables the analysis of the spatial component of technological growth in regions.

In order to do so, the Cobb-Douglas Knowledge Production Function (KPF) is specified with R&D spending and the stock of human capital as internal inputs and distance-weighted innovative activity from neighboring regions as external input.

$$INN_i = RD_i^{\beta_1} HC_i^{\beta_2} INN_j^{\lambda(distance)} controls_i^{\gamma} e^{u_i} \tag{4.1}$$

log-linearizing yields

$$inn_i = \beta_1 rd_i + \beta_2 hc_i + \sum_{k \neq i}^{K} \lambda(distance) inn_j + \gamma\, controls_i + \epsilon_i \tag{4.2}$$

where inn_i is innovative output, rd_i is expenditure on R&D and hc_i is a measure of the human capital stock in region i. $\lambda(distance)$ is a function of the distance between two regions and $controls_i$ denotes further economic and demographic variables that affect local innovative output. The parameters β_1 and β_2 capture innovation's elasticities to the internal inputs R&D spending and human capital stock. The parameter value of λ is central to the spatial component of the model and measures the impact of proximity in innovative spillovers between regions. A positive value for λ indicates that closer regions contribute more spillovers to local innovation.

4.3.1 Spatial-autoregressive Model with Spatial Spillovers

The log-linearized form of the KPF is the foundation for the following empirical analysis. The innovative impact of internal factors can be captured by a linear regression. In order to incorporate the distance-weighted innovative output from neighboring regions as an explanatory variable, the regression model includes a spatially autoregressive component. Additionally, spatially autoregressive errors capture residual spatial patterns that cannot be explained by innovative spillovers. The Spatial-Autoregressive Model with spatial-autoregressive disturbances (SARAR) is characterized by the following system of equations:

$$\mathbf{y} = \lambda\mathbf{W}\mathbf{y} + \mathbf{X}\beta + \mathbf{u} \tag{4.3}$$

$$\mathbf{u} = \rho\mathbf{W}\mathbf{u} + \epsilon \tag{4.4}$$

where $\mathbf{y}$ is the vector of innovative output, $\mathbf{X}$ is the matrix of internal inputs and $\mathbf{W}$ is the spatial weight matrix.

The Spatial-autoregressive model captures knowledge spillovers between regions as spatial dependence in the number of patent applications $\mathbf{y}$ measured by the parameter λ. In the baseline model, the spatial weight matrix $\mathbf{W}$ contains the inverse of the geographical distance between the centroids of each pair of regions. This specification relies on the assumption that geographical distance alone determines the intensity of knowledge flows between regions.

$\mathbf{W}$ is defined as

$$\mathbf{W} = \begin{bmatrix} 0 & \cdots & \cdots & \frac{1}{dist_{J,1}} \\ \frac{1}{dist_{1,2}} & 0 & \cdots & \vdots \\ \vdots & & \ddots & \vdots \\ \frac{1}{dist_{1,J}} & \cdots & \frac{1}{dist_{J-1,J}} & 0 \end{bmatrix}$$

where $dist_i, j$ refers to the geographical distance between the centroids of regions i and j. Internal inputs to innovation are represented by the matrix $\mathbf{X}$ which includes overall R&D expenditure per capita and the share of tertiary educated as a measure of the human capital stock in each region.

The specification further allows for spatially dependent errors $\mathbf{u}$, where ρ captures the spatial dependence and ϵ is an i.i.d. error term. Introducing spatially dependent disturbances avoids that the estimate of λ is inflated by spatial concentration of economic activity that leads to clustering of innovations and is not captured by the internal inputs.

4.3.2 Spatial-autoregressive Model with Spatial and Technological Spillovers

Introducing a second weight matrix yields the modified system of equations:

$$\mathbf{y} = \lambda \mathbf{W} \mathbf{y} + \lambda_T \mathbf{W_T} \mathbf{y} + \mathbf{X}\beta + \mathbf{u} \tag{4.5}$$

$$\mathbf{u} = \rho \mathbf{W} \mathbf{u} + \rho_T \mathbf{W_T} \mathbf{u} + \epsilon \tag{4.6}$$

where $\mathbf{W}$ refers to the first weight matrix, which contains the inverse of the geographical distances and $\mathbf{W_T}$ refers to the second weight matrix containing the technological distances. λ and $\lambda_{\mathbf{T}}$ measure the spatial dependence of patent applications along the geographical and technological dimension respectively.

This specification allows for the simultaneous estimation of spillover effects and therefore gives us an indication of the interplay and relative importance of geographical distance versus cognitive distance in knowledge transfers.

While the baseline model explains the intensity of knowledge flows by geographical distance exclusively, another driving force behind increased transfer of knowledge between neighboring regions is their proximity in cognitive background. Disentangling these two effects is the purpose of this modified specification. Similarly to the baseline model, spatial dependence along both dimensions is captured in order not to inflate the estimates for knowledge flows by the effects of other aspects of spatial concentration.

The position of a region in technological space is based on the distribution of its' patent applications into the eight sections of the International Patent Classification (IPC). The vector T_i contains the share of innovative output in region i for each of the technological sections. Technological proximity between two regions is determined by the angular separation of their technology vectors.

Accordingly, technological proximity $P_{i,j}$ between regions i and j can be written

$$P_{i,j} = \frac{T_i T_j'}{[(T_i T_i')(T_j T_j')]^{0.5}}$$

where $P_{i,j}$ is bounded between 0 and 1. A measure of 0 indicates orthogonal technology vectors and thus minimal technological proximity and a measure of 1 indicates identical technology vectors for both regions.

These pairwise measures of technological proximity are captured in the technological weight matrix $\mathbf{W_T}$.

$$\mathbf{W_T} = \begin{bmatrix} 0 & \dots & \dots & P_{J,1} \\ P_{1,2} & 0 & \dots & \vdots \\ \vdots & 0 & \ddots & \vdots \\ P_{1,J} & \dots & P_{J-1,J} & 0 \end{bmatrix}$$

4.4 Data

The following empirical analysis is based on data from the European Patent Office (EPO) and EUROSTAT for 236 European regions at the NUTS2-level. I include all EU28-regions that report patent activity and have at least one neighboring European region.

Innovative output is measured as the number of patent applications normalized by local GDP in each region, where each patent is allocated to the first inventor's region of residence. I use the average number of patents by priority year for the years 2010-2012. The variable is smoothed in order to decrease the results' sensitivity to short-term fluctuations in the number of patent applications because these fluctuations are quite pronounced at the regional level. The explanatory variables are based on measures from EUROSTAT. R&D spendings per capita are used as the first input into the regional KPF. The human capital stock for each region is measured as the share of tertiary educated. The vector of control variables includes the population density in order to account for agglomeration economies. Furthermore, I consider the local share of per capita output in the manufacturing sector as an indicator of the local industry structure. The share of employment in the science and technology sectors is considered as an alternative measure of the human capital stock.

All explanatory variables are also smoothed and the lagged average measures for the period 2007-2009 are used in order to mitigate endogeneity issues.

Geographical distance is measured as the plane distance between the centroids of each pair of regions.

Descriptive statistics for the dependent and explanatory variables are provided in table 4.1. The empirical analysis exploits the inter-regional variation in the number of patent applications. There is substantial variation between the observed regions, ranging from around

Table 4.1: Summary Statistics

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**	**N**
pat_avg	155.203	136.902	5.044	627.649	236
rd	724.041	635.255	19.3	4342.4	236
hc	29.176	8.228	12.2	51.2	236
manu	0.165	0.059	0.047	0.346	236
pop_dens	425.322	818.193	3.3	7131.1	236

5 patents per year in Slaskie, Poland to over 627 patents per year in Stuttgart, Germany. Figure 4.1 maps patent intensity for the observed NUTS2-regions in 2012.

The map clearly shows a pattern of regional clusters in patent applications. Particularly innovative clusters are located in the southern part of Germany, Scandinavia and the United Kingdom. On the other side of the spectrum, less innovative regions are clustered in the eastern and south-western parts of Europe. However, this regional clustering of patent activity alone is not evidence for knowledge spillovers between regions. The spatial pattern could also be explained by clustering in underlying factors such as human capital and R&D spendings.

The regional pattern of R&D intensity as depicted in Figure 4.2 displays notable similarities to the distribution of patent activity. Scandinavia, England, Germany and the southern part of France stand out as high-investing regions, while Spain, Italy and eastern Europe invest substantially less. There is a visible correlation between high investing regions and high innovations regions. However, innovative activity exhibits a stronger pattern of regional clustering. Compared to R&D investments, neighboring regions are more closely connected in the level of innovative output. While there is a regional pattern in R&D as well, there are also various neighboring regions with vastly different levels of investment. A preliminary conclusion is that R&D spending can explain part of the innovation clustering, however it does not explain the full extent of this spatial pattern. This relationship is reflected in the correlation coefficient between patent activity and R&D intensity of 0.74.

Human capital is measured as the share of tertiary educated in the age group from 30-34. There is no strong clustering in this measure of the human capital stock. The share is highest in Scandinavia, the United Kingdom and the baltic states. Lower levels are observed in Bulgaria, Romania and Turkey. National borders seem to be more relevant than regional borders, which reflects the impact of national government policies on education. Considerable effort has been made in recent years to harmonize education systems within the European Union, but a direct comparison of educational attainment between different countries may still be misleading.

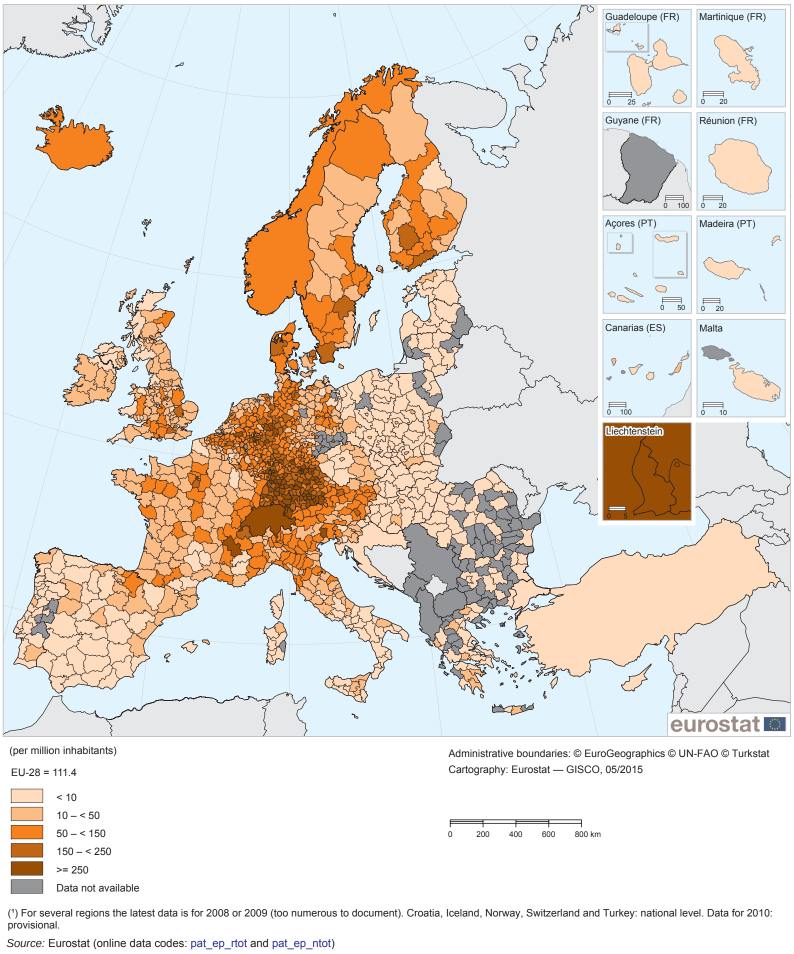


Figure 4.1: Patent Activity by European NUTS3 Regions

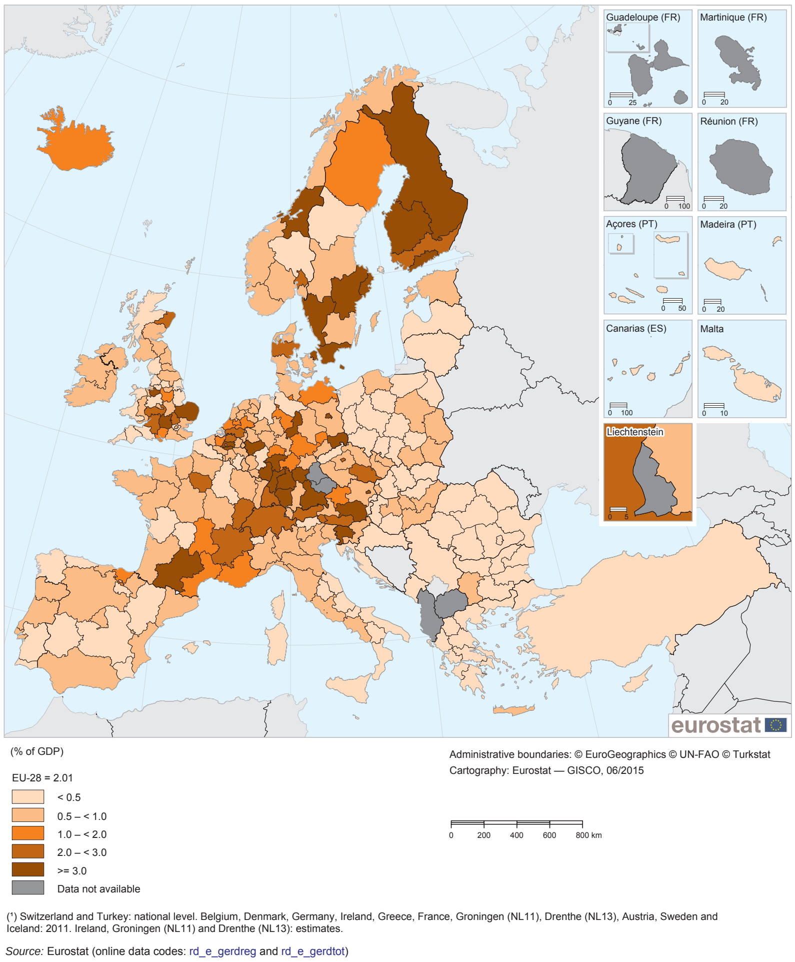


Figure 4.2: R&D Intensity by European NUTS2 Regions

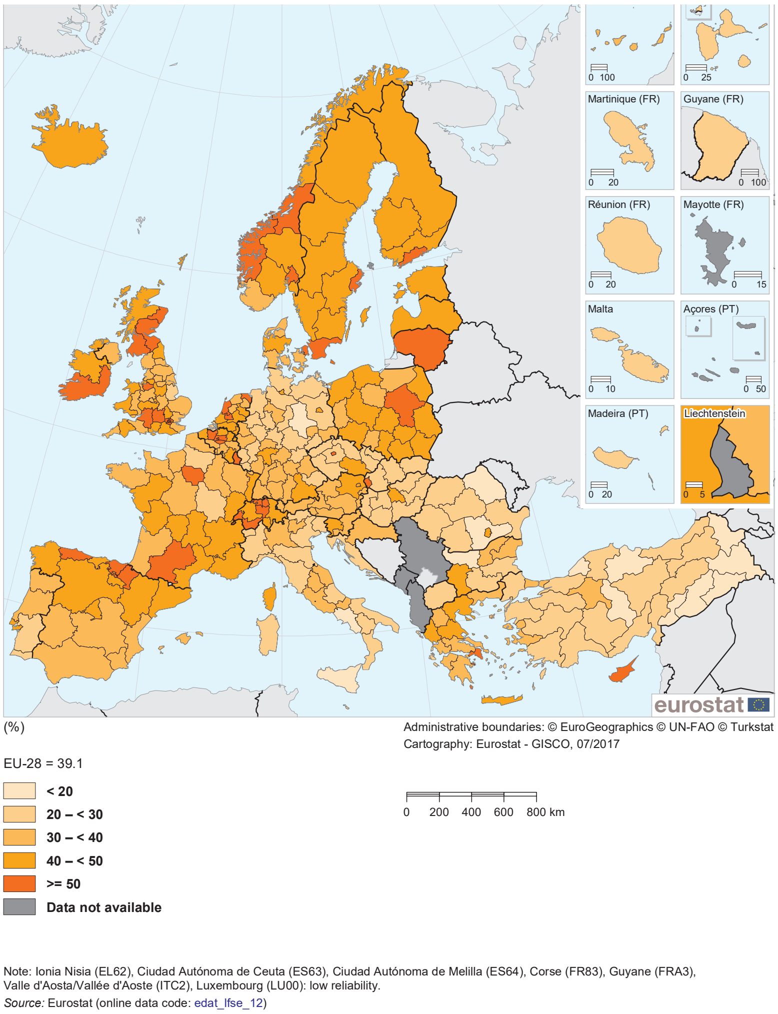


Figure 4.3: Share of Tertiary Educated by European NUTS2 Regions

While there is a positive correlation of 0.35 between innovations and the share of tertiary educated, the explanatory power of this human capital measure is limited. I will therefore control for country effects and consider alternative measures of human capital such as the share of employment in science and technology in the following empirical analysis.

As described before, the computation of technological distance is based on the regions' distribution of innovative output over the eight sections of the International Patent Classification. I use the average number of patent applications of the different sectors for the years 2010-2012 in order to establish the position of each region in technological space. Including only regions that report patents by classes for the observed time frame, the number of observations drops to 148.

Table 4.2 lists the distribution of local patent activity over the eight main classes of the International Patent Classification (IPC). There is substantial variation in the regions' technological profile and their respective positions in technological space. This variation will be exploited in the following analysis of the role of technological distance between regions in innovative spillovers.

Table 4.2: Patent Classification

Patent Class	Mean	Std. Dev.	Min.	Max.	N
A - Human necessities	25.045	25.705	0.132	181.854	148
B - Performing operations	31.961	30.563	0.652	175.013	148
C - Chemistry	17.921	19.776	0.146	124.431	148
D - Textiles	3.109	7.745	0.01	87.578	148
E - Fixed constructions	7.748	8.920	0.049	71.026	148
F - Mechanical engineering	21.609	25.728	0.034	153.616	148
G - Physics	21.259	25.972	0.3	181.981	148
H - Electricity	26.595	36.031	0.506	216.867	148

4.5 Results

All specifications of the SARAR are estimated with a feasible generalized spatial two-stage least squares (FGS2SLS) estimation strategy. As discussed by Corrado and Fingleton (2011), FGS2SLS is more efficient than a maximum likelihood estimation strategy in a SARAR framework without prior information on the transmission mechanism between regions.

4.5.1 SARAR with Spatial Spillovers

The baseline specification of the SARAR for geographical distance includes the average number of patent applications normalized by local GDP in the years 2011-2013 for European NUTS2-region as dependent variable. Normalized R&D spending and the share of tertiary educated as a measure of the human capital stock are the internal inputs to innovation. The estimation strategy captures innovative spillovers in the geographical dimension by the coefficient λ. The FGS2SLS estimates are reported in column 1 of table 4.3 and show a highly significant positive impact of local R&D spending on innovative output while the impact of the human capital stock is not significant at the 5%-level. Most importantly, there is evidence for geographical externalities in patent applications as the estimate for λ is significantly positive at the 1%-level. λ is the key coefficient of this analysis as it measures the spatial dependence in patent activity that cannot be explained by variation in internal inputs and control variables. The significantly positive effect of spatial proximity is therefore an indicator for innovative spillovers between neighboring regions. There is also spatial auto-correlation in the the error-term as captured by the coefficient ρ. This finding indicates that there are is further spatial dependence between regions that affects innovative capacity but cannot be attributed directly to the role of patents in neighboring regions. One possible explanation for this result is that internal inputs in neighboring regions could also positively affect local innovation in the observed region. The second specification of the geographical SARAR

Table 4.3: SARAR with Spatial Spillovers

	(1)	(2)
	Patents	Patents
R&D	0.112***	0.121***
	(10.46)	(11.27)
Human Cap.	1.787	1.843*
	(1.78)	(2.01)
Pop. Density		-0.00235
		(-0.34)
Share of Manuf.		203.1*
		(2.35)
Constant	-53.49	-102.7***
	(-1.60)	(-3.33)
λ	0.0129**	0.0299***
	(3.28)	(3.92)
ρ	0.0659**	0.0274***
	(2.92)	(3.53)
N	236	236

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

includes a vector of control variables in order to account for variation in patent applications that can be explained by regional characteristics beyond the internal inputs of the baseline specification. Population density is included to control for agglomeration economies in innovative output and the share of manufacturing is introduced to capture the role of the structure of the local industry. Results are reported in the second column of table 4.3. The impact of these control variables is not significant at the 5%-level and their inclusion does not qualitatively change the results we obtained in the baseline specification. In particular, the significant role of local R&D spending and geographical clustering of patents persists.

Table 4.4: SARAR with Technological Spillovers

	(1)	(2)
	Patents	Patents
R&D	0.0693***	0.0694***
	(7.78)	(7.72)
Human Cap.	-0.618	0.0642
	(-1.16)	(0.11)
Pop. Density		0.00131
		(0.27)
Share of Manuf.		198.8**
		(2.59)
Constant	15.59	-34.33
	(0.43)	(-0.81)
λ_T	0.0000538***	0.0000521***
	(4.37)	(4.22)
ρ_T	-0.000191***	-0.000192***
	(-4.53)	(-4.79)
Observations	148	148

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

4.5.2 SARAR with Technological Spillovers

The baseline specification of the SARAR for technological distance includes the average number of patent applications normalized by GDP in the years 2011-2013 for European NUTS2-region as dependent variable. Normalized R&D spending and the share of tertiary educated as a measure of the human capital stock are the internal inputs to innovation. The coefficient λ_T captures innovative spillovers in the technological dimension. The FGS2SLS estimates are reported in column 1 of table 4.4 and show a highly significant positive impact of local R&D spending on innovative output while the impact of the human capital stock is not significant at the 5%-level. There is evidence for technological clustering in patent applications as the estimate for λ_T is significantly positive at the 0.1%-level. This finding indicates that technological proximity between regions fosters innovative spillovers. Evidence for spillovers in the technological dimension is slightly stronger than the evidence for the geographical dimension. Spatial auto-correlation in the error-term persists and may indicate further driving forces to the regional clustering of innovations that are not captured by this model. The second specification of the technological SARAR includes a vector of control variables in order to account for variation in patent applications that can be explained by regional characteristics beyond the internal inputs of the baseline specification. Population density is included to control for agglomeration economies in innovative output and the share of manufacturing is introduced to capture the role of the structure of the local industry. The share of manufacturing has a significantly positive impact on patent activity, while population density is insignificant at the 5%-level. Their inclusion does not qualitatively change the results we obtained in the baseline specification. In particular, the significant impact of local R&D spending and technological proximity on local innovation persists.

4.5.3 Alternative Specifications of Local Inputs

The standard measure of human capital is the share of tertiary educated. As education systems are still quite heterogeneous across European countries, the standards for the attainment of tertiary education are not comparable across all countries. Figure 4.3 shows an observable clustering of this measure of human capital on the national level. Therefore I use the share of employment in science and technology (HRST) as an alternative measure of human capital. It is a measure that is directly related to innovative capacity and is comparable across regions

Table 4.5: SARAR with Spatial Spillovers - Alternative Inputs

	(1) Patents	(2) Patents
R&D	0.126***	0.228***
	(11.91)	(6.98)
Human Cap. (HRST)	1.018	
	(1.34)	
Pop. Density	-0.00315	-0.00967
	(-0.45)	(-1.39)
Share of Manuf.	184.7*	167.4*
	(2.13)	(1.96)
Human Cap. (Tert. Ed.)		1.504
		(1.81)
(R&D * Human Cap.)		-0.00254**
		(-3.15)
Constant	-87.31**	-98.56***
	(-2.81)	(-3.29)
λ	0.0309***	0.0211**
	(4.01)	(3.22)
ρ	0.0274**	0.0253*
	(3.27)	(2.51)
N	236	236

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

and countries. Furthermore, it is a direct representation of the share of labor allocated to research, which plays a central role in 's model discussed in chapter 2. Results for the estimation of the SARAR model for geographical distance with this alternative measurement are reported in column 1 of table 4.5. Similarly to the specification with the standard measure of human capital, the impact of human capital is positive but not significant. Compared to the baseline specification, I find no notable changes in the significance of geographical proximity. Column 2 of table 4.5 reports the results for an alternative specification of the geographic SAR that includes an interaction term for R&D spendings and human capital. The interaction term's coefficient is significantly negative. The result is counter-intuitive, theory predicts that R&D investments are more effective when the workforce is highly educated. This finding could be an artifact of country effects in the measurement of the human capital stock.

Table 4.6: SARAR with Technological Spillovers - Alternative Inputs

	(1) Patents	(2) Patents
R&D	0.0730***	0.179***
	(8.02)	(6.96)
Human Cap. (HRST)	-0.512	
	(-0.86)	
Pop. Density	0.00169	-0.00158
	(0.35)	(-0.32)
Share of Manuf.	165.7*	191.1*
	(2.13)	(2.46)
Human Cap. (Tert. Ed.)		2.031**
		(2.81)
(R&D * Human Cap.)		-0.00289***
		(-4.77)
Constant	-14.34	-125.5*
	(-0.34)	(-2.54)
λ_T	0.0000524***	0.0000618***
	(4.23)	(4.74)
ρ_T	-0.000192***	-0.000192***
	(-4.86)	(-5.57)
N	148	148

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

4.5.4 Alternative Specifications of the Weight Matrices

As explored by Harris, Moffat and Kravtsova (2011), the specification of the spatial weight matrix is crucial for the results on spillover effects in spatial settings. In order to get unbiased results, the specification of the weight matrix needs to precisely capture the transmission mechanism. A priori, it is not possible to specify the correct weight matrix as the transmission mechanism is unknown. I therefore estimate the SARAR for each proximity dimension with alternative specifications of the spatial weight matrix.

Table 4.7: SARAR with Spatial Spillovers and Country Effects

	(1) Patents
R&D	0.101***
	(8.83)
Human Cap.	0.255
	(0.24)
Constant	55.47
	(0.55)
λ	0.0144*
	(2.52)
ρ	0.0427*
	(2.02)
N	236

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

I introduce country effects in order to control for the impact of country affiliation on innovative output. There are strong indications for the relevance of country borders from theoretical considerations as well as from the spatial patterns in the data. As education and innovation policies within Europe are still primarily determined on a national level and language barriers play a role in the transmission of knowledge, theory predicts clustering of innovative activity within country borders. Figure 4.1 shows a clear pattern of national clustering of high innovation in regions, in particular for the case of Germany. Estimates for the geographical SARAR with country effects are reported in table 4.7. The results are not qualitatively different from the baseline specification. The key parameter λ is still positive and significantly different from zero at the 5% level. This result indicates, that knowledge externalities are not exclusively driven by nationality but also by geographical proximity across country borders.

Table 4.8: SARAR with Technological Spillovers and Country Effects

	(1) Patents
Patents	
R&D	0.0389
	(1.18)
Human Cap.	1.806
	(0.16)
λ_T	0.0000900
	(0.38)
ρ_T	-0.000195**
	(-3.09)
N	148

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The direction of the effects for technological distance as reported in table 8 is also unchanged. However, the estimates are not significantly different from zero because the introduction of 24 country effects reduces the degrees of freedom and therefore the precision of the estimates substantially.

An alternative specification of the spatial weight matrix is the contiguity matrix. Its elements indicate whether a pair of regions shares a common border or not. The indicator takes on a value of 1 if the two regions are neighboring and 0 otherwise. This structure allows a direct measure of the spillover effects between neighboring regions when compared to the standard specification of the geographical spatial weight matrix which uses the distance between each regions' centroids. The contiguity matrix W_C can be written

$$\mathbf{W_C} = \begin{bmatrix} 0 & \dots & \dots & I_{J,1} \\ I_{1,2} & 0 & \dots & \vdots \\ \vdots & & \ddots & \vdots \\ I_{1,J} & \dots & I_{J-1,J} & 0 \end{bmatrix}$$

where

$$I_i, j = \begin{cases} 1, & \text{if} \quad bnd(i) \cap bnd(j) \neq \varnothing \\ 0, & \text{if} \quad bnd(i) \cap bnd(j) = \varnothing \end{cases}$$

and $bnd(i)$ is the boundary of region i. For the sample of all regions with at least one neighbor, the average number of neighbors is 4.25. There is no analogue to the contiguity matrix in technological space, as regions' position in technological space is defined by a single point and therefore no borders exist. The results for the geographical SARAR with contiguity-based spatial weights are qualitatively similar to the results with distance-based spatial weights as shown in table 9. Evidence for spatial spillovers as measured by λ is significant at the .1%-level.

Table 4.9: SARAR with Contiguity Weights

	(1)
	Patents
R&D	0.114***
	(11.29)
Human Cap.	0.948
	(1.33)
Constant	-28.04
	(-1.51)
λ	0.0991***
	(11.21)
ρ	-0.00220
	(-0.10)
N	236

t statistics in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

4.5.5 Simultaneous Estimation of Spatial and Technological Spillovers

Boschma (2005) argued theoretically and Parent and LeSage (2008) showed empirically that the isolated analysis of one proximity dimension leads to biased results if the true transmission process is based on the interplay of multiple proximity dimensions. Applied to the current analysis, these findings imply, that the isolated estimation of the SARAR for geographical and technological distance respectively leads to biased results if both proximity dimensions affect knowledge transmission. An interaction between geographical and technological distance in the transmission process is plausible. However, it is not a priori clear whether the proximity dimensions act as complements or substitutes. It can be argued, that agents only exchange knowledge primarily with agents that are close to them both geographically and cognitively. This argument implies complementary effects of the proximity dimensions. The opposing view manifests that proximity in one dimension is sufficient for exchange, i.e. agents are willing to overcome large geographical distance to exchange knowledge with a cognitively close agent. This view implies that the proximity dimensions act as substitutes.

Table 4.10: SARAR with Spatial and Technological Spillovers

	(1)
	Patents
R&D	0.0517*
	(2.12)
Human Cap.	1.106
	(0.16)
λ	0.0128
	(1.43)
λ_T	0.000920*
	(2.83)
ρ	0.0156
	(0.21)
ρ_T	-0.000083
	(-0.91)
N	148

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

The results for the SARAR with simultaneous effects of geographical and technological distance are reported in table 4.10. The inclusion of both proximity dimensions allows for the measurement of their interplay. The estimate for λ, which captures the impact of geographical proximity remains positive but becomes insignificant, while the positive impact of technological proximity as measured by λ_T is still significant. The interpretation of these parameters calls for caution. An important concern is that geographical distance and technological distance are not defined independently of each other. As Boschma (2005) describes, a major role of the geographical distance is its reinforcing effect on other proximity dimensions. Geographically close regions are more likely to develop similar industry structures and therefore increasing technological proximity. However, for the observed period there is no significant convergence of the regions' technological distance towards their geographical distance.

4.6 Discussion

This chapter provides empirical evidence on the role of geographical and technological distance in innovative spillovers between European regions. I estimate the reduced form of a regional knowledge production function in a spatially auto-regressive framework using a dataset of 236 European NUTS2-regions. A separate analysis of each proximity dimension confirms previous findings that geographical and technological proximity facilitate innovative spillovers between regions. The consideration of alternative inputs to knowledge production and alternative specifications of the spatial weights shows, that the evidence for these spillovers is robust and slightly stronger for contiguity based spatial weights when compared to spatial weights based on centroid-distance. Furthermore, the simultaneous estimation of both proximity dimensions finds significant impact of technological proximity and indicates that geographical and technological proximity complement each other in the facilitation of inter-regional spillovers. This analysis lays the foundation for further investigation in various directions. The specification of the spatial weight matrix and the estimation of the dynamic aspects of innovative spillovers are two particularly relevant areas of interest. While the use of spatial panel econometrics is limited for the current dataset as it only covers few years for a large sample of European regions, these strategies have great potential in uncovering the temporal scope of spillovers and further disentangling spillover effects from effects due to clustering in inputs to knowledge production. In my analysis, results are robust to different specifications of the geographical weights, however, more refined measurements of the technological distance between regions could provide further insight into the role of cognitive proximity.

Chapter 5

Conclusion

The main goal of this thesis is the exploration of spatial patterns in economic growth and knowledge accumulation. It provides theoretical and empirical insight into extent and determinants of the spatial clustering of knowledge. Chapter 2 summarizes the role of innovation in endogenous growth theory and surveys theoretical and empirical literature on its spatial aspects. Creation and transmission of knowledge are considered as distinct drivers of economic growth. Chapter 3 examines local knowledge spillovers as a source of agglomeration economies. I use a search-theoretic framework to investigate the creation and transmission of knowledge as an outcome of local face-to-face interactions between agents with heterogeneous cognitive background. The model's results show that agents are too picky in the choice of their interaction partners and consequently agglomeration economies do not reach their optimal extent. Chapter 4 provides empirical evidence on the role of proximity in innovative spillovers between European regions. Using data from 236 European NUTS2-regions, I estimate a reduced form of the knowledge production function in a spatial-autoregressive framework. The results confirm previous findings of the relevance of innovative spillovers and their dependence on geographical and technological proximity. I further propose alternative specifications of internal inputs and spatial weights and explore the simultaneous impact of geographical and technological distance. In conclusion, my thesis provides three main contributions to current economic research. Chapter 2 contributes to the distinction of knowledge creation and transmission and reviews evidence for the economic impact of both mechanisms. This distinction is integrated into a search-theoretic framework in chapter 3. Empirical results on the role of geographical and technological proximity dimensions in regional innovation are provided in chapter 4. This thesis provides foundations for future research in various directions. Further theoretical work is required in an effort to integrate the different mechanisms of knowledge accumulation. The search-theoretic framework from chapter 3 can be extended in various dimensions. A formalization of knowledge networks and the dynamics of knowledge dissemination could provide further insights into the role of face-to-face interactions. An important empirical challenge lies in the simultaneous analysis of multiple proximity dimensions and the specification of spatial weights that reflect the underlying mechanisms of knowledge dissemination. Spatial panel analysis has the potential to explore the dynamics of regional innovative spillovers over time and can therefore offer insights into causality and persistence of these effects.

Appendices

Appendix A

Optimality Conditions for the Social Planner

The optimality conditions for the knowledge spread $\frac{\partial V_u}{\partial \delta} = 0$ and for the number of unmatched agents in the city $\frac{\partial V_u}{\partial U} = 0$ can be expressed as follows:

Optimality condition for the knowledge spread:

$$\begin{aligned}
\frac{\partial V_u}{\partial \delta} = & \underbrace{\frac{\lambda \alpha U}{(\lambda + \alpha \delta U)^2} \frac{a_0}{r}}_{\frac{\partial A}{\partial \delta}} + \\
& + \underbrace{\frac{\lambda \alpha \delta}{(\lambda + \alpha \delta U)^2} \frac{a_0}{r}}_{\frac{\partial A}{\partial U}} \underbrace{\left(- \frac{\alpha U^2}{\lambda + 2\alpha \delta U} \right)}_{\frac{\partial U}{\partial \delta}} + \\
& + \underbrace{\frac{(r+\lambda)\alpha U (e_0 - e_1 \frac{\delta}{2}) - (\alpha \delta U)^2 \frac{e_1}{4}}{r(r + \lambda + \alpha \delta U)^2}}_{\frac{\partial e}{\partial \delta}} + \\
& + \underbrace{\frac{(r+\lambda)\alpha \delta (e_0 - e_1 \frac{\delta}{4})}{r(r + \lambda + \alpha \delta U)^2}}_{\frac{\partial e}{\partial U}} \underbrace{\left(- \frac{\alpha U^2}{\lambda + 2\alpha \delta U} \right)}_{\frac{\partial U}{\partial \delta}} = 0
\end{aligned}$$

Optimality condition for the number of unmatched in the city:

$$\begin{aligned}
\frac{\partial V_u}{\partial U} &= \underbrace{\frac{\lambda\alpha\delta}{(\lambda+\alpha\delta U)^2}\frac{a_0}{r}}_{\frac{\partial A}{\partial U}} + \\
&+ \underbrace{\frac{(r+\lambda)\alpha\delta(e_0 - e_1\frac{\delta}{4})}{r(r+\lambda+\alpha\delta U)^2}}_{\frac{\partial e}{\partial U}} - \\
&- t\underbrace{\frac{\lambda+2\alpha\delta U}{\lambda}}_{\frac{\partial N}{\partial U}} = 0
\end{aligned}$$

Appendix B

Theoretical Foundation of the KPF

This exposition of the theoretical foundations of the knowledge production function is adapted from Romer (1990). Romer develops a model of endogenous growth which is driven by technological change that arises from intentional investment decisions made by profit-maximizing agents. The distinguishing feature of the technology as an input is that it is neither a conventional good nor a public good; it is a non-rival, partially excludable good. Because of the non-convexity introduced by a non-rival good, price-taking competition cannot be supported. Instead, the equilibrium is one with monopolistic competition. The main conclusions are that the stock of human capital determines the rate of growth, that too little human capital is devoted to research in equilibrium, that integration into world markets will increase growth rates, and that having a large population is not sufficient to generate growth.
The four basic inputs in this model are capital, labor, human capital, and an index of the level of the technology. Capital is measured in units of consumption goods. Labor services L are skills such as eye-hand coordination that are available from a healthy physical body. They are measured by counts of people. Human capital H is a distinct measure of the cumulative effect of activities such as formal education and on-the-job training. The model separates the rival component of knowledge, H, from the non-rival, technological component, A. Each new unit of knowledge corresponds to a design for a new good, so A is a count of the number of designs.
The formal model of the economy has three sectors. The research sector uses human capital and the existing stock of knowledge to produce new knowledge. Specifically, it produces designs for new producer durables. An intermediate-goods sector uses the designs from the research sector together with forgone output to produce the large number of producer durables that are available for use in final goods production at any time. The final goods sector

uses labor, human capital, and the set of producer durables that are available to produce final output. Output can be either consumed or saved as new capital.

Final output Y in this model is expressed as a function of physical labor L, human capital devoted to final output H_Y, and physical capital K. The unusual feature of the production technology assumed here is that it disaggregates capital into an infinite number of distinct types of producer durables. For now, let these durables be indexed by an integer i.

The functional form for output is the following extension of the Cobb-Douglas production function:

$$Y(H_Y, L, x) = H_Y^{\alpha} L^{\beta} \sum_i x_i^{(1-\alpha-\beta)} \tag{B.1}$$

Thus K_t is equivalent to cumulative forgone output and evolves according to the rule

$$\dot{K}_t = Y_t - C_t \tag{B.2}$$

where C_t denotes aggregate consumption at time t.

All researchers can take advantage of A at the same time. The output of researcher j is proportional to his human capital level H_j and the stock of ideas A_j he can access. With productivity parameter δ, the researcher's output can be written $\delta H_j A_j$. If we sum across all people engaged in research, the aggregate stock of designs evolves according to

$$\dot{A} = \delta H_A A \tag{B.3}$$

where H_A is total human capital employed in research.

Aggregate demand for the durables is determined by solving the maximization problem

$$\max_x \int_0^{\infty} [H_Y^{\alpha} L^{\beta} x_i^{1-\alpha-\beta} - p_i x_i] di \tag{B.4}$$

which yields the inverse demand function:

$$p_i = (1 - \alpha - \beta) H_Y^{\alpha} L^{\beta} x_i^{-\alpha-\beta} \tag{B.5}$$

Faced with given values of H_Y, L, and r, a firm that has already incurred the fixed-cost investment in a design will choose a level of output x to maximize its revenue minus variable cost at every date:

$$\pi = \max_x (1 - \alpha - \beta) H_Y^{\alpha} L^{\beta} x^{1-\alpha-\beta} - r\eta x \tag{B.6}$$

The decision to produce a new specialized input depends on a comparison of the discounted stream of net revenue and the cost P_A of the initial investment in a design. Because the market for designs is competitive, the price for designs will be bid up until it is equal to the present value of the net revenue that a monopolist can extract. At every date t, it must therefore be true that

$$\pi_t = r_t P_A \tag{B.7}$$

For a fixed amount of A, the model's symmetry implies that all available durable goods are supplied at the same level, henceforth denoted as $\bar{x}$. If they were not, it would be possible to increase profits in the producer durable sector by reducing the output of high-output firms and diverting the capital released in this way to low-output goods. Since A determines the range of durables that can be produced and since q units of capital are required per unit of durable goods, it is possible to solve for x from the equation $K = \eta A\bar{x}$. Then output Y can be written as

$$\begin{aligned} Y(H_Y, L, x) &= H_Y^\alpha L^\beta \int_0^\infty x_i^{(1-\alpha-\beta)} di = \\ &= H_Y^\alpha L^\beta A \bar{x}^{(1-\alpha-\beta)} \\ &= H_Y^\alpha L^\beta A \frac{K}{\eta A}^{(1-\alpha-\beta)} \\ &= (H_Y A)^\alpha (LA)^\beta K^{1-\alpha-\beta} \eta^{\alpha+\beta-1} \end{aligned} \tag{B.8}$$

The strategy for characterizing the model that is followed here is to solve for an equilibrium in which the variables A, K, and Y grow at constant exponential rates. This is generally referred to as a balanced growth equilibrium. Equation (7) shows, that output grows at the same rate as A if L, H_Y, and $\bar{x}$ are fixed. If $\bar{x}$ is fixed, then K must grow at the same rate as A, because total usage of capital is $A\bar{x}\eta$. Let g denote the growth rate of A, Y, and K.

$$g = \frac{\dot{C}}{C} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{A}}{A} = \delta H_A \tag{B.9}$$

The constraint $H_Y = H - H_A$ implies a relation between the growth rate g and the interest rate r:

$$g = \delta H_A = \delta H - \frac{\alpha}{(1-\alpha-\beta)(\alpha+\beta)} r \tag{B.10}$$

Most of the content of the model is contained in equation (9), which summarizes the effects of the technological side of the model, including the effects of imperfect competition in the market for producer durables. The intuition behind this equation is related to the investment decision in human capital. The opportunity cost of human capital is the wage income that can be earned instantaneously in the manufacturing sector. The return to investing human capital in research is a stream of net revenue that a design generates in the future. If the interest rate is larger, the present discounted value of the stream of net revenue will be lower. Less human capital will be allocated to research, and the rate of growth will be lower.

The model presented here is essentially the one-sector neoclassical model with technological change, augmented to give an endogenous explanation of the source of the technological change. The central implication of the model is that an economy with a larger total stock of human capital will experience faster growth.

Bibliography

Audretsch, David B. and Maryann P. Feldmann (1996). "R&D Spillovers and the Geography of Innovation and Production". In: *American Economic Review* 86(3), pp. 630–640.

Audretsch, David B et al. (2003). "Handbook of Urban and Regional Economics". In: *Handbook of Urban and Regional Economics* 4.

Autant-Bernard, Corinne (2001). "The geography of knowledge spillovers and technological proximity". In: *Economics of Innovation and New Technology* 10.4, pp. 237–254.

Berliant, Marcus, Robert R. Reed III and Ping Wang (2006). "Knowledge Exchange, matching and agglomeration". In: *Journal of Urban Economics* 60(1), pp. 69–95.

Boschma, Ron (2005). "Proximity and innovation: a critical assessment". In: *Regional studies* 39.1, pp. 61–74.

Bottazzi, Laura and Giovanni Peri (2003). "Innovation and spillovers in regions: Evidence from European patent data". In: *European economic review* 47.4, pp. 687–710.

Brueckner, Jan K., Jaques-Francois Thisse and Yves Zenou (2002). "Local Labor Markets, Job Matching, and Urban Location". In: *International Economic Review* 43(1), pp. 155–171.

Carboni, Oliviero A (2013). "A spatial analysis of R&D: the role of industry proximity". In: *Economics of Innovation and New Technology* 22.8, pp. 820–839.

Carlino, Gerald A., Satyajit Chatterjee and Robert M. Hunt (2007). "Urban density and the rate of invention". In: *Journal of Urban Economics* 61(3), pp. 389–419.

Carlsen, Fredrik, Jorn Rattso and Hildegunn E. Stokke (2013). "Education, experience and dynamic urban wage premium". In: *Working Paper Series Norwegian University of Science and Technology.*

Carrincazeaux, Christophe and Marie Coris (2011). "20 Proximity and innovation". In: *Handbook of regional innovation and growth*, p. 269.

Corrado, Luisa and Bernard Fingleton (2011). "Multilevel modelling with spatial effects". In:

D'Costa, Sabine and Henry Overman (2014). "The urban wage growth premium: Sorting or learning?" In: *Regional Science and Urban Economics* 48, pp. 168–179.

Davis, Don and Jonathan I. Dingel (2013). "A Spatial Knowledge Economy". In: *NBER Working Paper No. 18188*.

De la Roca, Jorge and Diego Puga (2014). "Learning By Working In Big Cities". In:

Feldmann, Maryann P. and David B. Audretsch (1994). "Innovation in cities: science-based diversity, specialization, and localized competition". In: *European Economic Review* 43(2), pp. 409–429.

Gilly, Jean-Pierre and André Torre (2000). *Proximity relations. Elements for an analytical framework*.

Glaeser, Edward L. (1999). "Learning in Cities". In: *Journal of Urban Economics* 46(2), pp. 254–277.

Glaeser, Edward L., Kallal Hedi D. et al. (1992). "Growth in Cities". In: *Journal of Political Economy* 100(6), pp. 1126–1152.

Glaeser, Edward L. and David C. Maré (2001). "Cities and Skills". In: *Journal of Labor Economics* 19(2), pp. 316–342.

Glaeser, Edward L. and Matthew G. Resseger (2010). "The Complementarity between Cities and Skills". In: *Journal of Regional Science* 50(1), pp. 221–244.

Glaeser, Edward L, JoséA Scheinkman and Andrei Shleifer (1995). "Economic growth in a cross-section of cities". In: *Journal of monetary economics* 36.1, pp. 117–143.

Griliches, Zvi (1979). "Issues in assessing the contribution of research and development to productivity growth". In: *The bell journal of economics*, pp. 92–116.

Harris, Richard, John Moffat and Victoria Kravtsova (2011). "In search of W". In: *Spatial Economic Analysis* 6.3, pp. 249–270.

Helsley, Robert W. and William C. Strange (1990). "Matching and agglomeration economies in a system of cities". In: *Regional Sience and Urban Economics* 20(2), pp. 189–212.

Henderson, Vernon, Ari Kuncoro and Matt Turner (1995). "Industrial Development in Cities". In: *Journal of Political Economy* 103(6), pp. 1067–1090.

Hobijn, Bart and Aysegül Sahin (2009). "Job-finding and separation rates in the OECD". In: *Economic Letters* 104(3), pp. 107–111.

Jacobs, Jane (1968). *The Economy of Cities*. New York: Vintage Books.

Jaffe, Adam B (1986). *Technological opportunity and spillovers of R&D: evidence from firms' patents, profits and market value*.

— (1989). “Real effects of academic research”. In: *The American economic review*, pp. 957–970.

Jaffe, Adam B., Manuel Trajtenberg and Rebecca Henderson (1993). “Geographic Localization of Knowledge Spillovers as Evidence by Patent Citations”. In: *Quarterly Journal of Economics* 108(3), pp. 577–598.

Lacombe, Donald J (2004). “Does econometric methodology matter? An analysis of public policy using spatial econometric techniques”. In: *Geographical Analysis* 36.2, pp. 105–118.

Lucas Jr, Robert E (1988). “On the mechanics of economic development”. In: *Journal of monetary economics* 22.1, pp. 3–42.

Mattes, Jannika (2012). “Dimensions of proximity and knowledge bases: innovation between spatial and non-spatial factors”. In: *Regional Studies* 46.8, pp. 1085–1099.

Moreno, Rosina, Raffaele Paci and Stefano Usai (2005). “Spatial spillovers and innovation activity in European regions”. In: *Environment and Planning A* 37.10, pp. 1793–1812.

Paci, Raffaele, Emanuela Marrocu and Stefano Usai (2014). “The complementary effects of proximity dimensions on knowledge spillovers”. In: *Spatial Economic Analysis* 9.1, pp. 9–30.

Paci, Raffaele and Stefano Usai (2009). “Knowledge flows across European regions”. In: *The Annals of Regional Science* 43.3, pp. 669–690.

Pakes, Ariel and Zvi Griliches (1984). “Patents and R&D at the firm level: a first look”. In: *R&D, patents, and productivity.* University of Chicago Press, pp. 55–72.

Parent, Olivier and James P LeSage (2008). “Using the variance structure of the conditional autoregressive spatial specification to model knowledge spillovers”. In: *Journal of Applied Econometrics* 23.2, pp. 235–256.

Peri, Giovanni (2001). “Young workers, learning, and agglomerations”. In: *Journal of Urban Economics* 52(3), pp. 582–607.

Pissarides, Christopher A. (2000). *Equilibrium Unemployment Theory.* Cambridge: MIT Press Books.

Rauch, James E (1993). “Productivity gains from geographic concentration of human capital: evidence from the cities”. In: *Journal of urban economics* 34.3, pp. 380–400.

Roberts, Mark, Mark Setterfield et al. (2010). “Endogenous regional growth: a critical survey”. In: *Handbook of alternative theories of economic growth*, pp. 431–50.

Romer, Paul M (1986). “Increasing returns and long-run growth”. In: *Journal of political economy* 94.5, pp. 1002–1037.

— (1990). “Endogenous technological change”. In: *Journal of political Economy* 98.5, Part 2, S71–S102.

Saxenian, AnnaLee (1994). *Regional Advantage: Culture and Competition in Silicon Valley and Route 128*. Cambridge, MA: Havard University Press.

Solow, Robert M (1956). “A contribution to the theory of economic growth”. In: *The quarterly journal of economics* 70.1, pp. 65–94.

Swan, Trevor W (1956). “Economic growth and capital accumulation”. In: *Economic record* 32.2, pp. 334–361.