Decay form factors for $\Lambda_{b,c}$ and B with QCD sum rules

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Abstract

In this thesis we use the framework of light-cone sum rules to determine on the one hand the $\Lambda_{b,c} \to N^*(1535)$ form factors and semileptonic decay widths, and on the other hand the $B \to f_2(1270)$ form factors.
For the former we use two different methods to determine the form factors. Either we eliminate the negative parity partners of $\Lambda_{b,c}$ or we extract the form factors by choosing the Lorentz-structures with the highest powers of p_+. For the interpolating current we choose an axial-vector one and a pseudoscalar one. For the weak transition current we choose a vector-like and an axial-vector-like current. For the numerical analysis we use two different sets of parameters for the N^* distribution amplitudes obtained from different models. We find that even a rough measurement is sufficient to discriminate these two models.
In order to determine the $B \to f_2(1270)$ form factors we construct the chiral odd quark-antiquark distribution amplitudes for tensor mesons including meson mass corrections. Furthermore we construct quark-antiquark-gluon distribution amplitudes and determine the occurring input parameters with SVZ sum rules. By applying the equations of motion we are able to connect higher-twist distribution amplitudes to leading twist ones. For the calculation of the form factors we use a vector current, an axial-vector one and a tensor one. With the results from the numerical analysis we are able to compare our results to other studies calculating these form factors.

Contents

Chapter 1

Introduction

With the beginning of the 20th century, a golden age for modern physics started. In the mid 1920s E. Schrödinger, W. Heisenberg, M. Born and many others contributed to formulating a theory called quantum mechanics. This was the foundation of the field of quantum physics. Quantum mechanics then became the standard formulation of atomic physics and allowed to answer questions for which classical mechanics failed. In 1928 P. A. M. Dirac combined special relativity with the theory of quantum mechanics and created a theory which describes the behavior of relativistic elementary particles such as electrons and positrons [1].

At the end of the 1920s attempts were made to extend quantum mechanics to quantum field theories. The result of this development was the modern formulation of quantum electrodynamics (QED) by R. P. Feynman, F. Dyson, J. Schwinger, S. Tomonaga[1] and others in the 1940s [2–9]. QED describes the interactions between light and matter. To be more precise, it describes phenomena involving electrically charged particles interacting via the exchange of photons. It is one of the most accurate theories and its predictions (for example the anomalous magnetic moment of the electron or the Lamb shift of the energy levels of the Hydrogen atom) are remarkably precise and in perfect agreement with the experimental values [10, 11].

Simultaneously to these theoretical developments great breakthroughs were achieved on the experimental side as well. With the invention of bubble[2] and spark chambers in the 1950s a large and ever-growing number of particles, called hadrons, were discovered and the question arose if all these particles could be fundamental ones. In the following 20 years a lot of progress was made resulting in a theory describing the strong force called quantum chromodynamics (QCD), a non-Abelian gauge theory describing the interactions of quarks and gluons (see, e.g. [12–15]). The publications mentioned above give only a brief compendium spanning the last 100 years and a more detailed overview of the discoveries in modern physics can be found in [16].

Together with QED and the weak interactions, QCD forms the so called Standard Model of particle physics. The first two have been unified into the theory of electroweak interactions [17–19]. A unification of the electroweak theory with QCD has

[1] In 1965 R. P. Feynman, J. Schwinger and S. Tomonaga won the Nobel Price for their fundamental work in QED.

[2] In 1960 D. A. Glaser won the Nobel Prize for the invention of the bubble chamber.

not been accomplished yet and is still a topic of current research.
Since its development, QCD has been very successful in describing high energy physics phenomena. But although the Lagrange density is known, it is not possible to calculate hadronic properties, such as masses or couplings directly from it. Therefore, as time went by, various approaches and models were developed, for example: lattice QCD [20], chiral perturbation theory [21] and QCD sum rules [22], to name a few. The first one is a non-perturbative theory and a discretized version of QCD on a lattice. The latter two are perturbative approaches valid for certain energy scales.
The method of QCD sum rules was developed in 1979 by M. A. Shifman, A. I. Vainshtein and V. I. Zakharov [23, 24]. SVZ sum rules relate hadronic quantities like form factors or decay constants to the correlation function of quark currents. The three key elements of the theory are:

- Correlation function of quark currents
  This is the basis of every sum rule calculation. The typical correlation function is formed by quark-antiquark current operators sandwiched between the QCD vacuum state.

- Operator product expansion (OPE)
  This expansion provides an analytical expression for the correlation function with a systematical separation of short- and long-distance effects.

- Hadronic dispersion relation
  With the help of the hadronic dispersion relation the OPE expression is linked to the hadronic sum and results in the QCD sum rule.

A combination of SVZ sum rules with light-cone distribution amplitudes was developed at the end of the 1980s [25–28] and is called QCD light-cone sum rules (LCSR). The main difference between SVZ and LCSR is the replacement of the OPE by the light-cone expansion in terms of distribution amplitudes of increasing twist.
The outline of this thesis is as follows: in the next Chapter a brief introduction to the Standard Model and especially QCD is given. In Chapter 3 the theoretical foundation for this thesis is sketched, namely QCD sum rules divided into SVZ sum rules and light-cone sum rules. The two following Chapters 4 and 5 are the main part of this work and are dedicated to the LCSR calculations of the $\Lambda_{b,c} \to N^*(1535)$ and $B \to f_2(1270)$ form factors. For the former also the decay width is calculated. This calculation is supplemented by a numerical analysis including an error analysis. A short summary and conclusion is given in Chapter 6.
In the appendices A-C notations and conventions are given as well as lengthy definitions. In appendix D the two- and three-particle distribution amplitudes (DAs) for the tensor meson $f_2(1270)$ are constructed and equations of motion (EOM) are used to connect leading twist DAs to higher ones. In appendix E the three-particle DA couplings $\xi_3^{\mathcal{T}_{1/2}}$ and $\omega_3^{\mathcal{T}_{1/2}}$ are numerically determined using SVZ sum rules.

Chapter 2

The Standard Model and QCD

In the following sections we will give a short introduction into the Standard Model and especially the foundations of QCD, which provides the framework for our calculations. A broad spectrum of textbooks addressing these topics already exists, see for example [29–34].

2.1 Standard Model

The Standard Model of particle physics is the basis of modern physics. It describes three of the four fundamental forces namely the electromagnetic, the weak and the strong interactions. It is a gauge theory with the underlying symmetry group $SU(3)_c \otimes SU(2)_W \otimes U(1)_Y$ and the interactions happen via the exchange of the corresponding spin-1 gauge fields. To be more precise these gauge fields are eight massless gluons for the strong interactions, one massless photon for the electromagnetic interactions and three massive bosons for the weak interactions. Beside these mediators there are also fundamental spin-$\frac{1}{2}$ particles called fermions which are the building blocks for all the matter surrounding us. Fundamental means that up to the available measuring accuracy these particles are not built-up by other particles. The fermions are organized in three families each consisting of two quarks and two leptons:

	1. generation	2. generation	3. generation
quarks	u	c	t
	d	s	b
leptons	e	μ	τ
	ν_e	ν_μ	ν_τ

plus the corresponding antiparticles. The three generations of fermions have nearly the same properties with respect to their interaction with the gauge fields but differ by their mass and their flavor quantum number. Furthermore, recently confirmed by experiments [35, 36], a scalar spin-0 particle, called the Higgs boson exists, which gives mass to elementary particles via spontaneous symmetry breaking[1] [37, 38].

[1] In 2013 F. Englert and P. Higgs won the Nobel Prize for the prediction of the Higgs boson.

With the help of the Higgs mechanism it was possible to create a gauge invariant theory of electroweak interaction. Nevertheless this interesting topic is not part of this thesis and we would like to refer the reader to the existing literature [39, 40].

2.2 Quantum chromodynamics (QCD)

The focus of this section is the strong interaction of the Standard Model which is described by quantum chromodynamics (QCD). QCD describes the interaction between quarks and gluons and is based on the non-abelian gauge group $SU(3)_c$. Therefore it is formulated in terms of $N_c \times N_f = 3 \times 6$ quarks and $N_c^2 - 1 = 8$ gluon fields. As already mentioned above, the quark fields $q_\alpha^{f,c}$ are massive spin-$\frac{1}{2}$ fermions with flavor ($f = u, d, s, c, b, t$), color ($c = 1, 2, 3$) and Dirac ($\alpha = 1, 2, 3, 4$) indices. The gluon fields are massless spin-1 gauge fields A_μ^a with a color index $a = (1, \ldots, 8)$ and since they are vector fields they have a Lorentz index ($\mu = 0, 1, 2, 3$).
The gauge-invariant QCD Lagrange density is given by

$$\begin{aligned}\mathcal{L}_{QCD} &= \sum_f \sum_{\alpha,\beta} \sum_{i,j} \bar{q}_\alpha^{f,i} (i(\gamma^\mu)_{\alpha\beta} D_\mu^{ij} - (\mathbb{1}_4)_{\alpha\beta} \delta^{ij} m_f) q_\beta^{f,j} - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a} + \cdots \\ &= \sum_f \bar{q}^f (i\not{D} - m_f) q^f - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a} + \cdots , \end{aligned} \tag{2.1}$$

with $\bar{q} = q^\dagger \gamma^0$. The dots denote additional terms like ghost terms and gauge fixing terms which are necessary to guarantee current conservation but are not part of this thesis. In the first line of (2.1) we explicitly show all the indices with the summation going over the above given values. In the second line we show the usual shortened form of the QCD Lagrange density where contracted color indices (i, j) and Dirac indices (α, β) are implicit and the Feynman slash notation is applied.
The covariant derivative in equation (2.1) is given by

$$D_\mu = \partial_\mu - i g_s T^a A_\mu^a, \tag{2.2}$$

with $T^a = \frac{\lambda^a}{2}$ being the generators of $SU(3)$ in the fundamental representation (see also appendix A for more information) and g_s is the coupling constant. The field strength tensor $F_{\mu\nu}^a$ is defined by the commutator of two covariant derivatives

$$F_{\mu\nu}^a T^a = \frac{i}{g_s} [D_\mu, D_\nu], \tag{2.3}$$

and an explicit calculation leads to a representation in terms of the gluon field A_μ^a

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c, \tag{2.4}$$

with the totally antisymmetric structure constants f^{abc} of $SU(3)$ (see appendix A for their definition and values).
There are three fundamental vertices stemming from the QCD Lagrange density (2.1): the quark gluon vertex $\propto g_s$, the three gluon vertex $\propto g_s$ and the four gluon vertex $\propto g_s^2$. The first one is similar to the one occurring in quantum electrodynamics, the latter two are characteristic for a non-abelian theory. Furthermore all these vertices are proportional to only the coupling constant g_s meaning that in the whole theory only a single dimensionless coupling constant appears. The self interactions between gluons lead to various phenomena including the asymptotic freedom of QCD[2], which will be explained in the next section. Other aspects, symmetries and problems of QCD like gauge invariance, CP violation[3], introduction of ghost fields, and many more are not part of this thesis and can be found in many textbooks or publications.

2.2.1 Perturbative QCD

With the help of Feynman rules it is possible to associate analytic expressions with simple diagrams. The latter have several building blocks like quark propagators, gluon propagators and the above mentioned vertices, which can be derived from the QCD Lagrange density (2.1) (the explicit rules can be found for example in [29, 33, 34]). This kind of perturbation theory only makes sense if the perturbative expansion in the coupling constant g_s converges. Since g_s always appears quadratically, it is convenient to define the strong coupling

$$\alpha_s = \frac{g_s^2}{4\pi}, \tag{2.5}$$

as an expansion parameter. The strong coupling α_s is not a constant but rather depends on the renormalization scale μ. With the help of the renormalization group equation (see below) one obtains for the leading order expression

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{\mu^2}{\Lambda_{QCD}^2}\right)}, \tag{2.6}$$

with the leading order coefficient of the β-function

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f. \tag{2.7}$$

Here $N_c = 3$ is the number of colors as above and n_f is the number of active quark-flavors at the scale μ which can take values between one and six. It can easily be seen

[2] In 2004 D. J. Gross, H. D. Politzer and F. Wilczek won the Nobel Prize for the discovery of asymptotic freedom in the theory of the strong interaction.

[3] In 1980 J. W. Cronin and V. L. Fitch won the Nobel Price for the discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons.

that $\beta_0 > 0$ and therefore that the β-functions itself is negative (as long as $n_f \leq 16$, which is true)

$$\beta(\alpha_s) = \mu^2 \frac{d\alpha_s}{d\mu^2} = -\beta_0 \frac{\alpha_s^2}{4\pi} + \mathcal{O}(\alpha_s^3). \tag{2.8}$$

From equation (2.6) and Figure 2.1 it can be seen that for $\mu \to \Lambda_{QCD}$ the coupling

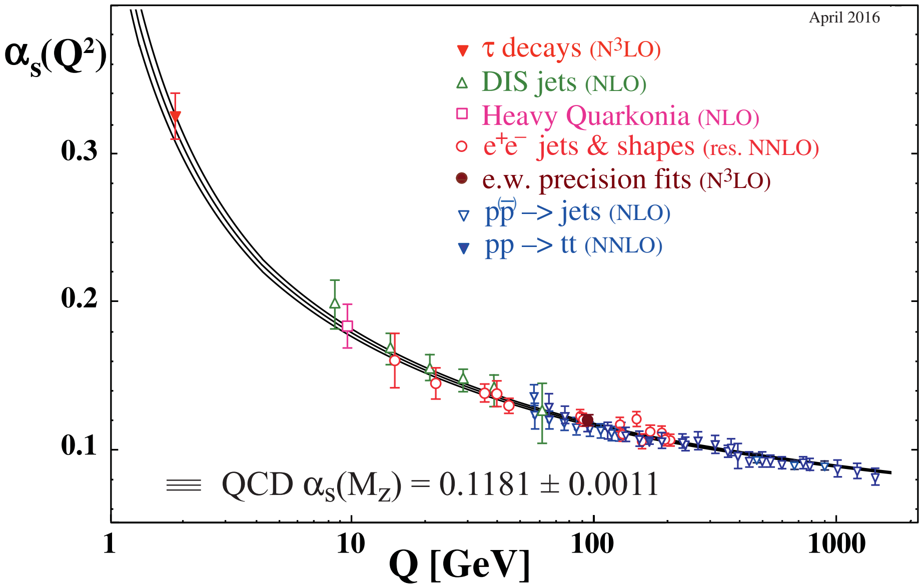

Figure 2.1: Scale dependence of α_s as a function of the energy scale Q^2 from different measurements. (Credit for the figure to the PDG [41])

α_s goes to infinity which results in the quarks forming bound states called hadrons for small energy scales. This effect is called confinement and still needs to be proven with mathematical rigor but experiments and lattice QCD clearly demonstrate this property. Furthermore QCD in this non perturbative region cannot be solved systematically and there is a lot of ongoing research with some promising approaches like lattice QCD (for further reading see for example [42]). On the other hand for large scales μ the coupling α_s decreases which results in the asymptotic freedom of QCD. In this region QCD perturbation theory works well. For our calculation we use a typical hadronic scale $\mu > 1.5$ GeV which justifies perturbative calculations.
Now that we have established a framework in which perturbative calculations are possible we can go back to the Feynman diagrams mentioned earlier. With the help of these we can calculate QCD quantities (for explicit examples see [29]) order by order. The leading order contributions are called tree level contributions where the

momenta of the occurring particles are determined by the initial and final state and therefore are finite. Beside the leading order contributions there are higher order contributions including loops in which integrals over intermediate quark or gluon momenta appear. These integrals can give rise to divergent contributions which have to be treated systematically. This is done by the concept of regularization. There are different regularization schemes, for example Pauli-Villar, momentum cut-off or dimensional regularization. We will use the latter one because compared to Pauli-Villar it does not need auxiliary massive fields and in contrast to momentum cut-off dimensional regularization makes Lorentz invariance explicit (for further reading see for example [39, 43]). The integrals over the intermediate momenta are divergent for $d = 4$ dimension but for a spacetime with different dimension, these integrals are finite. Therefore in dimensional regularization the integrals are calculated by setting the dimension to $d = 4+\epsilon$. For $\epsilon \neq 0$ this results in $\frac{1}{\epsilon}$-poles and thus the integrals can be separated into finite parts and pole parts. By choosing a renormalization scheme these poles are then absorbed into the definition of the physical parameters and the limit $\epsilon \to 0$ can be taken. In order to keep the dimension of the integrals unchanged, a new energy scale μ arises to absorb the change in dimension.

The renormalization in QCD is based on redefining the unrenormalized (bare) quantities in the Lagrange density by the renormalized ones

$$A^a_{\mu,0} = \sqrt{Z_3} A^a_\mu, \qquad q^f_0 = \sqrt{Z_2} q^f, \qquad g_{s,0} = Z_g g_s, \qquad m_{f,0} = Z_m m_f. \tag{2.9}$$

Here the Z-factors contain the divergent parts. After inserting these definitions into the Lagrange density and rearranging it, this multiplicative renormalization can be viewed as introducing counter terms to subtract the divergences. Since the requirement of finite expressions is not unique, one can (besides divergences) also absorb finite terms into the renormalization factors. Therefore there exist various renormalization schemes. We use the modified minimal subtraction ($\overline{MS}$) scheme (introduced in [44]) where not only the $\frac{1}{\epsilon}$-pole is subtracted but also the finite terms $\ln(4\pi)$ and γ_E. The latter is the Euler-Mascheroni constant. For further information and other renormalization schemes see the text books given above or [45].

Due to the renormalization procedure an unphysical parameter, the renormalization scale μ arises. If the calculation would include all orders of perturbation theory, the dependence on the scale μ would vanish and the result would be an independent finite number, as one expects from a physical quantity. But since it is not possible to calculate all orders in perturbation theory, the results depend on the renormalization scale μ. As already mentioned above, for dimensional regularization the scale μ is necessary to restore the initial dimension of the integrals. Furthermore the renormalized quantities and the Z-factors in equations (2.9) also depend on the scale μ. Their μ dependence is given by their respective renormalization group equation (RGE). In general the RGE can be obtained by differentiating the equations (2.9) with respect to μ and using the fact that the bare quantities are μ independent. This leads, for example, to the above given expression for $\alpha_s(\mu^2)$. The scaling behavior of other quantities, for example the quark masses or the Wilson coefficients of the operator product expansion (see below), can be given as a function of the anomalous dimension γ_n and the β function. For this work the leading order γ_0 and β_0 is sufficient and for a given quantity $A(\mu)$ the scaling behavior is

$$A(\mu) = A(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\frac{\gamma_0}{\beta_0}} . \tag{2.10}$$

For the quark masses the anomalous dimension is $\gamma_0 = 4$. In this work the scaling behavior is taken into account. The relevant formulas can be found in the cited references. For more details see in particular [46, 47].

Chapter 3

Theoretical foundations

3.1 QCD Sum Rules

The method of QCD sum rules was developed in 1979 by Shifman, Vainshtein and Zakharov [23, 24]. It is a successful technique to determine non-perturbative hadronic quantities. QCD sum rules are very reliable and agree with a typical accuracy of $10 - 20\%$ with experimental data. They are used for example to determine quark masses, form factors, decay constants and many more hadronic parameters, which makes them a valuable tool for theoretical physicists (which one can tell by the nearly 5000 citations of the original sum rule paper).
The method of QCD light-cone sum rules (LCSR) [25–28] was developed at the end of the 1980s. It uses SVZ techniques combined with light-cone expansion and light-cone distribution amplitudes, which are used in the theory of hard exclusive processes. LCSR are very useful for the calculation of hadronic transition form factors, especially for the decays of heavy to light mesons.
In this section we give a short outline of the two methods, SVZ and light-cone sum rules, which are the foundation of our calculations in Chapters 4 and 5 and also appendix E. A more detailed description of the methods of SVZ sum rules and LCSR can be found for example in [22, 48, 49] or in the original publications [23–28].

3.1.1 SVZ sum rules

The starting point of every sum rule calculation is the correlation function suitable for the underlying physical processes:

$$\Pi(q^2) = i \int d^4x \, e^{iq\cdot x} \langle 0| \, T\{j(x)j(0)\} \, |0\rangle \,, \tag{3.1}$$

where T denotes the time-ordered product, $|0\rangle$ is the QCD vacuum state and j is the interpolating current with the same quantum numbers and quark content as the hadron one is interested in. For simplicity we chose only one current in the correlation function but in practice there can be two different and more complicated currents with different Lorentz- and color-structures. The choice of the interpolating current

depends on the particles involved, e.g. for a nucleon, which consists of three quarks we need a current with three quark operators.
This correlation function is now calculated in two different ways: on the one hand we can insert a complete set of intermediate hadronic states between the two currents. On the other hand we can use operator product expansion (OPE) to write it as a series of local operators. The key feature of sum rules is to equate these two representations which gives a connection between the OPE and the hadronic sum.

The hadronic sum

For the values $q^2 > 0$ it is possible that intermediate hadronic states emerge with the same quantum numbers as the interpolating current. Therefore the formal spectral representation of $\Pi(q^2)$ follows from the basic unitarity relation which is obtained by inserting a complete set of intermediate hadronic states allowed by quantum numbers in the correlation function (3.1),

$$\frac{1}{\pi}\mathrm{Im}\ \Pi(q^2) = \langle 0|\, j\, |h_0\rangle \langle h_0|\, j\, |0\rangle\, \delta(q^2 - m_{h_0}^2) + \rho^h(q^2)\theta(q^2 - s_0^h). \tag{3.2}$$

Here we already isolated the ground-state, denoted by h_0, and we introduced a short-hand notation for the rest of the contributions including excited and continuum states. The threshold for the lowest continuum state is denoted by s_0^h. The appearing matrix elements $\langle 0|\, j\, |h_0\rangle$ and $\langle h_0|\, j\, |0\rangle$ in general give decay constants,or form factors, or other hadronic observables one wants to determine.

The dispersion relation

As we will see, the correlation function (3.1) can be considered for different regions of q^2. For large negative q^2 it can be calculated perturbatively with the use of operator product expansion (see below). For positive q^2 it can be expressed in terms of hadronic observables (see above). Using the Cauchy formula we can derive a dispersion relation linking $\Pi(q^2)$ at an arbitrary point, $q^2 < 0$, to the hadronic sum from above. Therefore we use the integration contour shown in Figure (3.1) for the analytic function $\Pi(q^2)$

$$\Pi(q^2) = \frac{1}{2\pi i}\oint\limits_{\gamma} ds \frac{\Pi(s)}{s-q^2} = \frac{1}{2\pi i}\oint\limits_{|s|=R} ds \frac{\Pi(s)}{s-q^2} + \frac{1}{2\pi i}\int\limits_0^R ds \frac{\Pi(s+i\epsilon) - \Pi(s-i\epsilon)}{s-q^2}. \tag{3.3}$$

We can now let the radius R of the circle go to infinity, which simplifies our calculation. If we require that the correlation function vanishes fast enough for $|q^2| \to \infty$, the integral over the circle goes to zero. If this is not the case, we will show later how to deal with a non vanishing correlation function for $|q^2| \to \infty$; therefore we will not focus on this case right now. For the second integral in equation (3.3) we can use the fact that $\Pi(q^2)$ is real at $q^2 < m_{h_0}^2$. For $q^2 > m_{h_0}^2$ we use the Schwartz reflection principle ,

$$\Pi(q^2 + i\epsilon) - \Pi(q^2 - i\epsilon) = 2i\mathrm{Im}\ \Pi(q^2), \tag{3.4}$$

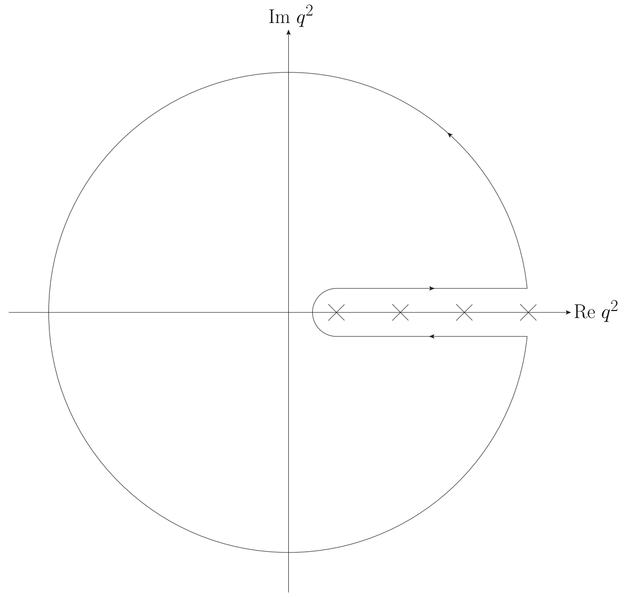

Figure 3.1: The integration contour in the complex q^2-plane. The crosses indicate poles of $\Pi(q^2)$ caused by resonances.

to replace it by an integral over the imaginary part of $\Pi(q^2)$. So we obtain the following dispersion integral

$$\Pi(q^2) = \frac{1}{\pi} \int_{m_{h_0}^2}^{\infty} ds \frac{\text{Im}\,\Pi(s)}{s - q^2 - i\epsilon}. \tag{3.5}$$

From now on we will not show the infinitesimal $-i\epsilon$ explicitly.
If our correlation function is UV divergent, the requirement $\lim_{|q^2| \to \infty} \Pi(q^2) = 0$ is not fulfilled and our dispersion integral is also divergent. In order to get a finite result, we subtract the first few terms of its Taylor expansion from $\Pi(q^2)$

$$\overline{\Pi}(q^2) = \Pi(q^2) - \Pi(0) - q^2 \Pi'(0) - \cdots ,$$

with an arbitrary amount of subtraction terms. But we will not go into detail about these subtraction terms since we will later perform a Borel transformation which will remove such terms.

Operator product expansion

As already mentioned above, for the Euclidean region of q, $q^2 < 0$, we can treat the correlation function (3.1) perturbatively and apply OPE. This can be done because the correlation function is dominated by short-range contributions $x^2 \approx 0$ for the momentum region $-q^2 \gg \Lambda_{QCD}^2$. For a detailed discussion and technical aspects we refer the reader to Ref. [22] or any other publication mentioned above. The main idea of OPE is to expand the product of two currents in a series of local operators which leads to

$$\Pi(q^2) = \sum_d C_d(q^2, \mu^2) \langle 0| O_d |0\rangle (\mu^2), \tag{3.6}$$

with the possible Lorentz- and gauge invariant operators

Dimension	Operator	
0	$O_0 = \mathbb{1}$	perturbative contribution
3	$O_3 = \bar{q}q$	quark condensate
4	$O_4 = G^a_{\mu\nu}G^{a\mu\nu}$	gluon condensate
5	$O_5 = \bar{q}\sigma_{\mu\nu}\frac{\lambda^a}{2}G^{a\mu\nu}q$	quark gluon condensate
6	$(\bar{q}\,\Gamma_1 q)(\bar{q}\,\Gamma_2 q)$	four quark condensate
$\vdots$	$\vdots$	$\vdots$

where Γ_1 and Γ_2 are arbitrary Dirac structures and the dots indicate further condensates which are suppressed by powers of $\Lambda^2_{\text{QCD}}/(-q^2)$. This allows us to truncate the expansion (3.6) after certain terms. As can be seen from equation (3.6) both the Wilson coefficient functions $C_d(q^2, \mu^2)$ and the condensates $\langle 0| O_d |0\rangle (\mu^2)$ are scale dependent. So a key point of OPE is to separate the regions of short and long distances. The short range contributions are included in the Wilson coefficients $C_d(q^2, \mu^2)$, which can be calculated perturbatively. The long range contributions are absorbed into the vacuum condensates, which are non perturbative objects. The dimension 0 coefficient function $C_0(q^2, \mu^2)$ corresponds to the usual perturbative contributions. In Figure E.1 we show diagrams corresponding to both, the perturbative QCD corrections and the contributions of different condensates.

The next step is to determine the Wilson coefficients. For the case of the perturbative contributions one has to contract all quark fields in the correlation function (3.1) using the quark propagator

$$S_0^{ij}(x,y) = -i \langle 0| T\{q^i(x)\bar{q}^j(y)\} |0\rangle = \delta^{ij} \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{\not{p} + m}{p^2 - m^2} + \cdots, \tag{3.7}$$

with the color indices i, j. After the contraction the integrals can be calculated and the final result can be written in the form of a dispersion integral in analogy to the hadronic sum

$$\Pi^{\text{pert}}(q^2) = \frac{1}{\pi} \int_{m^2}^{\infty} ds \frac{\text{Im}\,\Pi^{\text{pert}}(s)}{s - q^2}.$$

Here one must be careful to not misinterpret the imaginary part. Because in QCD quarks are confined and the full quark propagator has no poles there are no intermediate quark states. So there is no physical interpretation as for the hadronic sum and the imaginary part should be treated as a purely mathematical object. Using only

the free propagator in equation (3.7) leads to the free quark loop. The perturbative α_s corrections can be calculated using the Feynman rules of QCD.
In order to determine the Wilson coefficients for $d \neq 0$, it is convenient to use the Fock-Schwinger gauge for the gluon field

$$x_\mu A^\mu = 0,$$

which can be used for the local expansion of the quark fields

$$q(x) = q(0) + x^\mu \partial_\mu q(0) + \cdots = q(0) + x^\mu \overrightarrow{D}_\mu q(0) + \cdots , \tag{3.8}$$

$$\bar{q}(x) = \bar{q}(0) + \bar{q}(0)\overleftarrow{D}_\mu x^\mu + \cdots . \tag{3.9}$$

To determine the Wilson coefficient for the quark condensate we now need to factorize all contributions with one quark and one antiquark field entering the correlation function $\Pi(q^2)$ and contract the remaining quark fields to form free quark propagators. With the expansion for the quark fields we get the following two vacuum matrix elements

$$\begin{aligned} \langle 0| \bar{q}_i^a q_j^b |0\rangle &= \frac{1}{12} \langle \bar{q}q \rangle \delta^{ab} \delta_{ij}, \\ \langle 0| \bar{q}_i^a \overleftarrow{D}_\mu q_j^b |0\rangle &= \frac{im}{48} \langle \bar{q}q \rangle (\gamma_\mu)^{ab} \delta_{ij}. \end{aligned} \tag{3.10}$$

The remaining traces and integrals can be calculated. Here we used the shorthand notation $\langle O_n \rangle \equiv \langle 0| O_n |0\rangle$. For our calculations we also need the gluon condensate which can be derived from the following vacuum matrix elements

$$\begin{aligned} \langle 0| G^a_{\alpha\beta} G^b_{\mu\nu} |0\rangle &= \frac{\langle G^2 \rangle}{96} \delta^{ab} (g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu}), \\ \langle 0| \widetilde{G}^a_{\eta\xi} G^b_{\mu\nu} |0\rangle &= \frac{\langle G^2 \rangle}{96} \delta^{ab} \epsilon_{\eta\xi\mu\nu}, \end{aligned} \tag{3.11}$$

where we used the definition $\widetilde{G}^a_{\eta\xi} = \frac{1}{2} \epsilon_{\eta\xi\mu\nu} G^{a,\mu\nu}$ to derive the second line from the first. For our calculations the quark and the gluon condensates are the only ones contributing, therefore we will not give any further condensates and refer the reader to the literature for more information.

Borel transformation

In order to make the sum rules more stable it is convenient to apply a Borel transformation. This transformation is used in QCD sum rules and light-cone sum rules to remove unknown subtraction terms. It exponentially suppresses the contribution from higher excited or continuum states in the dispersion relation.
The definition is

$$\mathcal{B}_{M^2} F(q^2) = \lim_{\substack{-q^2, n \to \infty \\ -q^2/n = M^2}} \frac{(-q^2)^{(n+1)}}{n!} \left(\frac{d}{dq^2} \right)^n F(q^2), \tag{3.12}$$

with the Borel parameter M^2. For our calculations the two following transformations are important:

$$\mathcal{B}_{M^2}(-q^2)^k = 0,$$
$$\mathcal{B}_{M^2}\left(\frac{1}{(m^2-q^2)^k}\right) = \frac{1}{(k-1)!}\frac{1}{M^{2(k-1)}}e^{-m^2/M^2}, \tag{3.13}$$

with $k > 0$.

Quark-hadron duality

In the previous paragraphs we calculated the correlation function (3.1) in two different ways

$$\Pi^{\text{had}}(q^2) = \frac{\langle 0|\, j\, |h_0\rangle \langle h_0|\, j\, |0\rangle}{s - m_{h_0}^2} + \int_{s_0^h}^{\infty} ds \frac{\rho^h(s)}{s-q^2}, \tag{3.14}$$

and

$$\Pi^{\text{ope}}(q^2) = \frac{1}{\pi}\int_{m^2}^{\infty} ds \frac{\text{Im}\,\Pi^{\text{pert}}(s)}{s-q^2} + \sum_{d>1} C_d(q^2) \langle 0|\, O_d\, |0\rangle\,. \tag{3.15}$$

The spectral density $\rho^h(q^2)$ is in general unknown but can be estimated with the help of the quark-hadron duality. With this duality it is possible, for large negative q^2 values to approximate the spectral density by the imaginary part of $\Pi(q^2)$ calculated in QCD perturbation theory

$$\int_{s_0^h}^{\infty} ds \frac{\rho^h(s)}{s-q^2} \approx \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\text{Im}\,\Pi^{\text{pert}}(s)}{s-q^2}, \tag{3.16}$$

with the effective threshold parameter s_0, which is expected to be close to the mass squared of the first excited state of h_0. After performing the Borel transformation, as mentioned above, to get rid of unknown subtraction terms and to suppress excited states, we can give the final sum rule which contains the two characteristic sum rule parameters s_0 from the quark-hadron duality and M^2 from the Borel transformation. The choice of the Borel parameter window depends on two aspects: on the one hand for too large values the spectral density is no longer suppressed and on the other hand for too small values the higher dimensional condensates, proportional to powers of $1/M^2$, may become too important to neglect. In order to fulfill these contradicting requirements, the highest condensate contribution must be small, while the contribution of the continuum states must be typically less than 30 %. Furthermore in order to have a prediction which is reliable at the $10-20\%$ level, it is required that the sum rule is stable under the variation of M^2 in the according Borel window.
For an explicit example we refer to Ref. [22] where the ρ decay constant f_ρ is determined including α_s corrections and Wilson coefficients up to $d = 6$ are calculated.

3.1.2 Light-cone sum rules

As already mentioned above, light-cone sum rules are based on the SVZ sum rules combined with light-cone distribution amplitudes. Unlike SVZ sum rules the starting point is not a vacuum-to-vacuum correlation function but the time ordered product of two quark currents sandwiched between the vacuum and an on-shell state:

$$F_\mu(p,q) = i\int d^4x\, e^{iq\cdot x}\, \langle h(p)|\, T\{j_\mu(x)\eta(0)\}\, |0\rangle\,. \tag{3.17}$$

For the hadronic sum the procedure is the same as for SVZ sum rules. A complete set of intermediate hadronic states with the corresponding quantum numbers is inserted, leading to a dispersion relation for (3.17).

Light-cone expansion

For large Euclidean momenta $(p+q)^2$ and q^2 the correlation function (3.17) can be systematically calculated in perturbation theory. The x integral in the correlation function is then dominated by small $x^2 \sim 1/q^2 \to 0$ and therefore light-cone expansion is applicable. The leading order diagram is displayed in Figure 5.1 a) and consists of the free quark propagator convoluted with the matrix element of a quark and antiquark sandwiched between the vacuum and an on-shell state:

$$F_\mu(p,q) = i\int d^4x\, e^{iq\cdot x}\, \mathrm{S}(x,0)\, \langle h(p)|\, \bar{q}_1(x)\Gamma_\mu q_2(0)\, |0\rangle + \cdots\,, \tag{3.18}$$

where the dots indicate more contributions, e.g., Figure 5.1 b), and Γ_μ is a generic Dirac matrix structure. Now let us expand the nonlocal quark-antiquark operator in terms of local operators

$$\bar{q}_1(x)\Gamma_\mu q_2(0) = \sum_n \frac{1}{n!}\bar{q}_1(0)(\overleftarrow{D}\cdot x)^n\Gamma_\mu q_2(0). \tag{3.19}$$

This can be decomposed using the fact that the only available scale is the four-momentum p

$$\begin{aligned}\langle h(p)|\, \bar{q}_1(0)\overleftarrow{D}_{\alpha_1}\overleftarrow{D}_{\alpha_2}\ldots\overleftarrow{D}_{\alpha_n}\Gamma_\mu q_2(0)\, |0\rangle =& (-i)^n p_{\alpha_1}p_{\alpha_2}\ldots p_{\alpha_n}p_\mu M_n \\ &+ (-i)^n g_{\mu\alpha_1}p_{\alpha_2}\ldots p_{\alpha_n}M'_n + \cdots\,,\end{aligned} \tag{3.20}$$

where the dots indicate more terms with one or more metric tensors. Now inserting the expansion (3.19) into equation (3.18), performing the integrations and inserting the decomposition one obtains

$$F_\mu(p,q) \sim \frac{1}{q^2}\sum_{n=0}^{\infty}\xi^n M_n + \frac{4}{q^4}\sum_{n=2}^{\infty}\frac{\xi^{n-2}}{n(n-1)}M'_n + \cdots\,, \tag{3.21}$$

with

$$\xi = \frac{2p\cdot q}{m^2-q^2} = \frac{(p+q)^2-q^2}{m^2-q^2}.$$

Because the parameter $\xi \sim 1$, we cannot truncate the above series after a few terms but we see that the second terms M_n' and further terms are suppressed by powers of $1/q^2$ compared to M_n. Furthermore the operators M_n and M_n' have different twist, which is defined as the difference between the dimension and the spin of a local operator. Here every spin corresponds to a definitive symmetrization/anti-symmetrization of the Lorentz indices of the matrix element. So in equation (3.21) the operators are organized by ascending twist and the nonlocal operators in equation (3.18) can be parametrized in terms of distribution amplitudes with ascending twist. We only gave a short outline and recommend the above given literature for further reading.

Distribution Amplitudes

In this paragraph we will give a brief overview on the concept of distribution amplitudes. More than forty years ago distribution amplitudes were introduced to describe exclusive processes with high momentum transfer. [50–54]
In general distribution amplitudes are defined through the vacuum-to-hadron matrix element of nonlocal operators with light-like separation

$$\langle h(p)|\, \bar{q}_1(x)\Gamma_\mu[x,-x]q_2(-x)\,|0\rangle_{x^2=0} = B_\mu \int_0^1 du\, e^{iup\cdot x - i\bar{u}p\cdot x}\phi(u,\mu), \tag{3.22}$$

with the leading order light-cone distribution amplitude $\phi(u,\mu)$ and an arbitrary prefactor B_μ. Here u and $\bar{u} = 1-u$ are the momentum fractions carried by the quark and the antiquark. The definition of the matrix element (3.22) is gauge invariant due to the path-ordered gauge factor connecting the points x and $-x$

$$[x,-x] = \mathcal{P}\mathrm{exp}\left[ig_s \int_0^1 dt(x-(-x))_\mu A^\mu(tx+(1-t)(-x))\right]. \tag{3.23}$$

But as already mentioned above, we use the Fock-Schwinger or light-cone gauge $x_\mu A^\mu = 0$ and therefore the gauge factor is 1 and we will not consider it anymore. The scale μ is the factorization scale separating the short distance contributions at $x^2 < 1/\mu^2$ entering the hard scattering amplitude from the long distance effects at $x^2 > 1/\mu^2$ parametrized by the distribution amplitudes. In analogy to the vacuum condensates, the light-cone distribution amplitudes are universal quantities describing the non perturbative structures of hadrons and the underlying hard scattering amplitude can be described perturbatively by a function $T(q^2,(p+q)^2,u,\mu)$. So after inserting the definition (3.22) into equation (3.18) and performing the x integration, the correlation function can be written in the form of a convolution

$$F_\mu(p,q) = \sum_t \int_0^1 du\, T^{(t)}(q^2,(p+q)^2,u,\mu)\,\phi^{(t)}(u,\mu), \tag{3.24}$$

where the summation goes over the different twists.
The next step would be to determine the light-cone distribution amplitudes $\phi^{(t)}(u,\mu)$

using the conformal symmetry of QCD. With the help of conformal partial wave decomposition it is possible to represent each distribution amplitude as a sum of orthogonal polynomials in the variable u. These polynomials have multiplicatively renormalizable coefficients which have growing anomalous dimensions. So for a sufficiently large renormalization scale μ the expansion can be truncated after a few terms. Schematically

$$a_n(\mu) = a_n(\mu_0) \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_n/\beta_0},$$

with the anomalous dimension γ_n. These coefficients are then determined using experimental data, lattice QCD or other non perturbative approaches. The part of the distribution amplitude with no logarithmic renormalization is called asymptotic. The conformal expansion can be used for distribution amplitudes with various twist. Furthermore using QCD equations of motion it is possible to express higher twist and three-particle distribution amplitudes in terms of the (asymptotic) form of the leading distribution amplitudes (see appendix D). For detailed information about conformal expansion of distribution amplitudes and the scaling behavior of the coefficients we refer to Ref. [55] and for the case of the f_2 DAs to Refs. [56, 57]. Let us add a warning: it is possible that coupling coefficients of DAs mix with each other under renormalization, see for example $\widetilde{\omega}_3$ and ω_3 from the three particle f_2 DAs.

Chapter 4

$\Lambda_{b,c} \to N^* l\nu$ form factors and decay width with LCSR

This Chapter is based on the publication [58] in collaboration with Nils Offen and Andreas Schäfer.

One of the purposes of the Hall B CLAS 12 detector at Jefferson Lab [59], following the 12 GeV upgrade, is measuring the properties of nucleon resonances and explaining them by using the fundamental degrees of freedom of QCD. There exists already a LCSR study calculating the corresponding form factors in terms of the DAs of nucleon resonances [60]. We will analyze a complementary approach to determine nucleon resonance DAs namely the decays of Λ_b and Λ_c baryons, which are produced in large quantities at LHCb (see, e.g. [61]) and the planned PANDA experiment.

In this Chapter we will determine the $\Lambda_{b,c} \to N^*$ form factors and decay widths by using the framework of light-cone sum rules. Our calculation will be similar to the calculation of the $\Lambda_{b,c} \to N$ form factors done in [62–65]. We will use two different methods: on the one hand we will follow the method from Ref. [64] and eliminate the negative parity partner of the $\Lambda_{b,c}$ baryon by taking linear combinations of different Lorentz structures. On the other hand we will extract the form factors by choosing the Lorentz structures leading to the highest possible power of p_+. We will use two different interpolating currents for the $\Lambda_{b,c}$ baryon, namely an axial-vector-like and a pseudoscalar-like current. For the weak transition current we will use a vector one and an axial-vector one. The relevant formulas will partly be given in the appendix. After the theoretical calculation we will calculate the qualitative prediction for two different models for the DAs of the N^*.

With adequate experimental data it will be possible to compare the extracted DAs to the moments of DAs calculated on the lattice [66, 67] or those fitted to electromagnetic N^* form factors [60]. Is is important to have these alternative approaches to get information on the structure of low lying nucleon resonances, because each one of them entails significant systematic uncertainties.

4.1 LCSR calculation

For the rest of this Chapter we give the definitions and derivations for the case of the $\Lambda_c \to N^*$ decay. For the $\Lambda_b \to N^*$ decay one has to replace $c \to b$ everywhere and $d \to u$ in the transition currents.
The starting point for the light-cone sum rule calculation is the correlation function

$$\Pi_a(P,q) = i \int d^4x \;\; e^{iq\cdot x} \left\langle 0 \right| T\{\eta_{\Lambda_c}(0) j_a(x)\} \left| N^*(P) \right\rangle , \tag{4.1}$$

with the weak transition currents

$$j_a(x) = \bar{c}(x)\, \Gamma_a d(x), \quad \Gamma_a = \gamma_\nu,\, \gamma_\nu\gamma_5,$$

and the Λ_c interpolating current

$$\eta_{\Lambda_c} = \epsilon^{ijk}(u_i C \Gamma_1 d_j)\Gamma_2 c_k.$$

We choose two different Lorentz structures for the interpolating current η_{Λ_c}, on the one hand the pseudoscalar one and on the other the axial-vector one with the explicit form:

$$\eta^{(P)}_{\Lambda_c} = (u\, C\gamma_5 d)c \quad \text{and} \quad \eta^{(A)}_{\Lambda_c} = (u\, C\gamma_5\gamma_\lambda d)\gamma^\lambda c.$$

The decay constants for the Λ_c and Λ_c^* baryon are given by

$$\begin{aligned} \left\langle 0 \right| \eta^{(i)}_{\Lambda_c} \left| \Lambda_c(P') \right\rangle &= \lambda^{(i)}_{\Lambda_c} m_{\Lambda_c} u_{\Lambda_c}(P'), \\ \left\langle 0 \right| \eta^{(i)}_{\Lambda_c} \left| \Lambda^*_c(P') \right\rangle &= \lambda^{(i)}_{\Lambda^*_c} m_{\Lambda^*_c} \gamma_5 u_{\Lambda^*_c}(P'). \end{aligned} \tag{4.2}$$

Here i labels the axial-vector A and pseudoscalar P interpolating current. For the vector and axial-vector current the form factors are defined as

$$\begin{aligned} \left\langle \Lambda_c(P') \right| j_\nu \left| N^*(P) \right\rangle &= \bar{u}_{\Lambda_c}(P') \left(f_1(q^2)\gamma_\nu + i\frac{f_2(q^2)}{m_{\Lambda_c}}\sigma_{\nu\mu}q^\mu + \frac{f_3(q^2)}{m_{\Lambda_c}} q_\nu \right) \gamma_5 u_{N^*}(P), \\ \left\langle \Lambda_c(P') \right| j_{\nu 5} \left| N^*(P) \right\rangle &= \bar{u}_{\Lambda_c}(P') \left(g_1(q^2)\gamma_\nu + i\frac{g_2(q^2)}{m_{\Lambda_c}}\sigma_{\nu\mu}q^\mu + \frac{g_3(q^2)}{m_{\Lambda_c}} q_\nu \right) u_{N^*}(P), \end{aligned} \tag{4.3}$$

where $\bar{u}_{\Lambda_c}(P')$ is the Λ_c-bispinor with the four-momentum $P' = P - q$. In the following we drop the form factors f_3 and g_3 since they contribute with coefficients proportional to the lepton mass in semileptonic decays. We can define the Λ_c^* form factors $\tilde{f}_i$ and $\tilde{g}_i$ by replacing Λ_c with Λ_c^* in the above equations and adding a γ_5 after the Λ_c^* spinor. With the help of the equation of motion $(\not{P} - m_{N^*})u_{N^*}(P) = 0$ we can decompose the correlation function (4.1) into six invariant and independent functions:

$$\Pi^{(i)}_\nu(P,q) = \left(\widetilde{\Pi}^{(i)}_1 P_\nu + \widetilde{\Pi}^{(i)}_2 P_\nu \not{q} + \widetilde{\Pi}^{(i)}_3 \gamma_\nu + \widetilde{\Pi}^{(i)}_4 \gamma_\nu \not{q} + \widetilde{\Pi}^{(i)}_5 q_\nu + \widetilde{\Pi}^{(i)}_6 q_\nu \not{q} \right) \gamma_5 u_{N^*}(P), \tag{4.4}$$

and the same for the axial-vector transition current up to the fact that no γ_5 stands in front of the N^* spinor.

Eliminating the Λ_c^* pole

In order to eliminate the Λ_c^* pole we explicitly keep both the Λ_c and the Λ_c^* in the hadronic sum and represent the higher states by a dispersion integral. The fact that the form factors enter in more than one of the structures $\widetilde{\Pi}_j^{(i)}$ in equation (4.4) allows us, after equating the hadronic side and the OPE result, to construct linear combinations in which the contribution of the Λ_c^* is eliminated. After the usual procedure the hadronic sum for the vector transition is

$$
\begin{aligned}
\Pi_\nu^{(i)}(P,q) &= \frac{\lambda_{\Lambda_c}^{(i)} m_{\Lambda_c}}{m_{\Lambda_c}^2-(P-q)^2}\Bigg[2f_1(q^2)P_\nu - 2\frac{f_2(q^2)}{m_{\Lambda_c}}P_\nu\not{q}\\
&+(m_{N^*}+m_{\Lambda_c})\left(f_1(q^2)+\frac{m_{N^*}-m_{\Lambda_c}}{m_{\Lambda_c}}f_2(q^2)\right)\gamma_\nu+\left(f_1(q^2)+\frac{m_{N^*}-m_{\Lambda_c}}{m_{\Lambda_c}}f_2(q^2)\right)\gamma_\nu\not{q}\\
&+\left(-2f_1(q^2)-\frac{m_{N^*}-m_{\Lambda_c}}{m_{\Lambda_c}}(f_2(q^2)+f_3(q^2))\right)q_\nu+\frac{1}{m_{\Lambda_c}}\left(f_2(q^2)-f_3(q^2)\right)q_\nu\not{q}\Bigg]\gamma_5 u_{N^*}(P)\\
&+\frac{\lambda_{\Lambda_c^*}^{(i)} m_{\Lambda_c^*}}{m_{\Lambda_c^*}^2-(P-q)^2}\Bigg[-2\tilde{f}_1(q^2)P_\nu + 2\frac{\tilde{f}_2(q^2)}{m_{\Lambda_c^*}}P_\nu\not{q}\\
&-(m_{N^*}-m_{\Lambda_c^*})\left(\tilde{f}_1(q^2)+\frac{m_{N^*}+m_{\Lambda_c^*}}{m_{\Lambda_c^*}}\tilde{f}_2(q^2)\right)\gamma_\nu-\left(\tilde{f}_1(q^2)+\frac{m_{N^*}+m_{\Lambda_c^*}}{m_{\Lambda_c^*}}\tilde{f}_2(q^2)\right)\gamma_\nu\not{q}\\
&+\left(2\tilde{f}_1(q^2)+\frac{m_{N^*}+m_{\Lambda_c^*}}{m_{\Lambda_c^*}}(\tilde{f}_2(q^2)+\tilde{f}_3(q^2))\right)q_\nu-\frac{1}{m_{\Lambda_c^*}}(\tilde{f}_2(q^2)-\tilde{f}_3(q^2))q_\nu\not{q}\Bigg]\gamma_5 u_{N^*}(P)\\
&+\int_{s_0^h}^{\infty}\frac{ds}{s-(P-q)^2}\Big(\rho_1^{(i)}(s,q^2)P_\nu+\rho_2^{(i)}(s,q^2)P_\nu\not{q}\\
&+\rho_3^{(i)}(s,q^2)\gamma_\nu+\rho_4^{(i)}(s,q^2)\gamma_\nu\not{q}+\rho_5^{(i)}(s,q^2)q_\nu+\rho_6^{(i)}(s,q^2)q_\nu\not{q}\Big)\gamma_5 u_{N^*}(P),
\end{aligned}
\tag{4.5}
$$

where the spectral densities $\rho_j^{(i)}(s,q^2)$ describe excited and continuum states. Here we already see that a linear combination of the first four Lorentz structures yields expressions for the form factors $f_1(q^2)$ and $f_2(q^2)$. We can obtain the hadronic decomposition for the correlation function $\Pi_{\nu 5}^{(i)}(P,q)$ with the axial-vector transition current by replacing $f_i \to g_i$, $\tilde{f}_i \to \tilde{g}_i$, changing the sign of m_{N^*} and dropping the γ_5 in front of the N^* spinor.

Extracting the highest power of p_+

Another possibility to obtain the desired form factors is by projecting out the highest power of p_+. To do so, we introduce two light-like vectors n_μ and p_μ that satisfy

$$q\cdot n=0,\qquad n^2=0,\qquad p_\mu=P_\mu-\frac{1}{2}n_\mu\frac{m_{N^*}^2}{P\cdot n},\qquad p^2=0.$$

We see that $P\to p$ in the infinite momentum frame, $P\cdot n\to\infty$, or for negligible N^* mass, $m_{N^*}\to 0$. Furthermore we can define a projector onto the directions orthogonal to p and n by

$$g_{\mu\nu}^{\perp}=g_{\mu\nu}-\frac{1}{p\cdot n}(p_\mu n_\nu+p_\nu n_\mu).$$

In the hadronic sum we only keep the Λ_c contribution and after contracting the correlation function $\Pi_\nu(P', q)$ with n^ν and multiplying with the projector $\frac{\not p \not n}{2p\cdot n}$ we can write the result as

$$\frac{\not p \not n}{2p \cdot n} n^\nu \Pi_\nu(P, q) \;=\; p \cdot n \left(A(P, q) + \frac{\not q_\perp}{m_{\Lambda_c}} B(P, q) \right) , \tag{4.6}$$

with $\not q_\perp = q^\mu g^\perp_{\mu\nu} \gamma^\nu$. We can then extract the form factors $f_1(q^2)$ and $f_2(q^2)$ from the sum rules for the functions $A(P, q)$ and $B(P, q)$, respectively. In the same manner we can extract the form factors $g_1(q^2)$ and $g_2(q^2)$ with the only difference being that for these there appears one factor γ_5 less.

The OPE part of the light-cone sum rules

For the OPE we contract the two c-quarks in the correlation function (4.1) giving

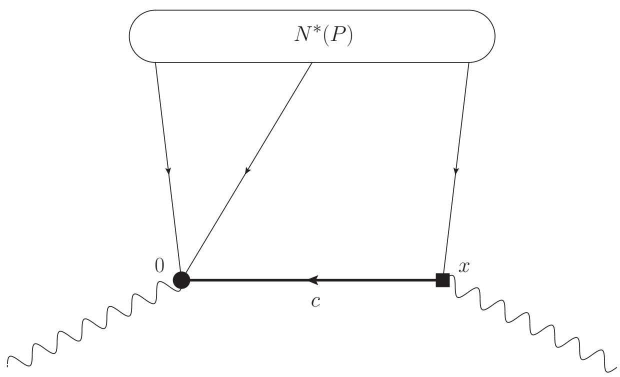

Figure 4.1: Leading order contribution to the correlation function (4.1). The wavy lines represent the external currents, the oval represents the N^* DAs. The straight lines represent the quarks, where the thick one denotes the c-quark propagator.

the free propagator (3.7). The resulting matrix element is decomposed according to (B.1). After this decomposition we are left with a sum over DAs of different twist multiplied by different coefficient functions. With the help of the equations of motion we can identify the contributions to the invariant functions $\widetilde{\Pi}_j$ in (4.4). In general these contributions can be written, neglecting terms which will vanish after Borel transformation,

$$\widetilde{\Pi}^{(i)}_j(P'^2, q^2) = \frac{1}{4} \sum_{n=1}^{3} \int_0^1 dx \frac{w^{(i)}_{jn}(x, q^2)}{D^n}, \tag{4.7}$$

with the denominator

$$D = m_c^2 - (xP - q)^2 = m_c^2 - xP'^2 - \bar{x}q^2 + x\bar{x}m^2_{N^*}, \tag{4.8}$$

where we used $\bar{x} = 1 - x$. We can distinguish the different functions $w_{jn}^{(i)}$ by their indices, as already mentioned above i denotes the interpolating current and can either be A for axial-vector current or P for pseudoscalar current, $j = 1, \cdots, 6$ corresponds to the invariant amplitudes in equation (4.4) to which the functions $w_{jn}^{(i)}$ contribute and $n = 1, 2, 3$ is the power of the denominator. In appendix C we give the functions $w_{jn}^{(i)}$ for the different transition currents and interpolating currents. The next step is to write equation (4.7) as a dispersion integral in P'^2

$$\tilde{\Pi}_j^{(i)}(P'^2, q^2) = \frac{1}{\pi} \int_{m_c^2}^{\infty} \frac{ds}{s - P'^2} \operatorname{Im}_s \tilde{\Pi}_j^{(i)}(s, q^2). \tag{4.9}$$

This we achieve with the following substitution in the denominator (4.8)

$$\begin{aligned} s(x) &= \frac{1}{x}(m_c^2 - \bar{x}q^2 + x\bar{x}m_{N^*}^2), \\ x(s) &= \frac{1}{2m_{N^*}^2}\left[m_{N^*}^2 + q^2 - s + \sqrt{(s - q^2 - m_{N^*}^2)^2 + 4m_{N^*}^2(m_c^2 - q^2)}\right] \end{aligned}$$

and by performing a partial integration if the power of the denominator is larger than one. With the help of quark-hadron duality we can approximate the higher state contributions of the hadronic representation

$$\int_{s_0^h}^{\infty} \frac{ds}{s - P'^2} \rho_j^{(i)}(q^2) \approx \frac{1}{\pi} \int_{s_0}^{\infty} \frac{ds}{s - P'^2} \operatorname{Im}_s \tilde{\Pi}_j^{(i)}(s, q^2), \tag{4.10}$$

with the duality threshold s_0. Next we can subtract the spectral densities $\rho_j^{(i)}$ from the perturbative part. After Borel transformation the final sum rules with subtraction of the Λ_c^*-pole are

$$\begin{aligned} f_1(q^2) &= \frac{e^{m_{\Lambda_c}^2/M^2}}{2m_{\Lambda_c}(m_{\Lambda_c} + m_{\Lambda_c^*})\lambda_{\Lambda_c}^{(i)}} \frac{1}{\pi} \int_{m_c^2}^{s_0} ds\, e^{-s/M^2} \Big[(m_{\Lambda_c} - m_{N^*})\Big(\mathrm{Im}_s\tilde{\Pi}_1^{(i)}(s, q^2) \\ &\quad -(m_{\Lambda_c^*} + m_{N^*})\mathrm{Im}_s\tilde{\Pi}_2^{(i)}(s, q^2)\Big) + 2\mathrm{Im}_s\tilde{\Pi}_3^{(i)}(s, q^2) + 2(m_{\Lambda_c^*} - m_{\Lambda_c})\mathrm{Im}_s\tilde{\Pi}_4^{(i)}(s, q^2)\Big], \\ f_2(q^2) &= \frac{e^{m_{\Lambda_c}^2/M^2}}{2(m_{\Lambda_c} + m_{\Lambda_c^*})\lambda_{\Lambda_c}^{(i)}} \frac{1}{\pi} \int_{m_c^2}^{s_0} ds\, e^{-s/M^2} \Big[\mathrm{Im}_s\tilde{\Pi}_1^{(i)}(s, q^2) \\ &\quad -(m_{\Lambda_c^*} + m_{N^*})\mathrm{Im}_s\tilde{\Pi}_2^{(i)}(s, q^2) - 2\mathrm{Im}_s\tilde{\Pi}_4^{(i)}(s, q^2)\Big]. \end{aligned} \tag{4.11}$$

Without subtraction we obtain

$$\begin{aligned} f_1(q^2) &= \frac{e^{m_{\Lambda_c}^2/M^2}}{2m_{\Lambda_c}\lambda_{\Lambda_c}^{(i)}} \frac{1}{\pi} \int_{m_c^2}^{s_0} ds\, e^{-s/M^2} \mathrm{Im}_s\tilde{\Pi}_1^{(i)}(s, q^2), \\ f_2(q^2) &= \frac{e^{m_{\Lambda_c}^2/M^2}}{2\lambda_{\Lambda_c}^{(i)}} \frac{1}{\pi} \int_{m_c^2}^{s_0} ds\, e^{-s/M^2} \mathrm{Im}_s\tilde{\Pi}_2^{(i)}(s, q^2). \end{aligned} \tag{4.12}$$

For the form factors $g_1(q^2)$ and $g_2(q^2)$ the procedure is the same: the result is obtained from the above sum rules by replacing $\widetilde{\Pi}_j^{(i)}$ with the corresponding ones and changing the sign of m_{N^*}. And, as already mentioned earlier, for the $\Lambda_b \to N^*$ decay one needs to replace the c-quark by a b-quark.
The three procedures: transformation to a dispersion integral, subtraction of the spectral densities and Borel transformation are combined in the following substitution rules for the integrals in equation (4.7), similar to the ones from Ref. [68]:

$$
\begin{aligned}
\int_0^1 dx \frac{f(x)}{D} \to & \int_{x_0}^1 \frac{dx}{x} f(x) e^{-s(x)/M^2}, \\
\int_0^1 dx \frac{f(x)}{D^2} \to & \frac{1}{M^2} \int_{x_0}^1 \frac{dx}{x^2} f(x) e^{-s(x)/M^2} + \frac{f(x_o) e^{-s_0/M^2}}{m_b^2 + x_0^2 m_{f_2}^2 - q^2}, \\
\int_0^1 dx \frac{f(x)}{D^3} \to & \frac{1}{2M^4} \int_{x_0}^1 \frac{dx}{x^3} f(x) e^{-s(x)/M^2} + \frac{1}{2M^2} \frac{f(x_0) e^{-s_0/M^2}}{x_0 (m_b^2 + x_0^2 m_{f_2}^2 - q^2)} \\
& - \frac{1}{2} \frac{x_0^2 e^{-s_0/M^2}}{(m_b^2 + x_0^2 m_{f_2}^2 - q^2)} \frac{d}{dx} \left(\frac{f(x)}{x (m_b^2 + x^2 m_{f_2}^2 - q^2)} \right) \Bigg|_{x=x_0}, \\
\int_0^1 dx \frac{f(x)}{D^4} \to & \frac{1}{6M^6} \int_{x_0}^1 \frac{dx}{x^4} f(x) e^{-s(x)/M^2} + \frac{1}{6M^4} \frac{f(x_0) e^{-s_0/M^2}}{(m_b^2 + x_0^2 m_{f_2}^2 - q^2) x_0^2} \\
& - \frac{1}{6M^2} \frac{x_0^2 e^{-s_0/M^2}}{(m_b^2 + x_0^2 m_{f_2}^2 - q^2)} \frac{d}{dx} \left(\frac{f(x)}{x^2 (m_b^2 + x^2 m_{f_2}^2 - q^2)} \right) \Bigg|_{x=x_0} \\
& + \frac{1}{6} \frac{x_0^4 e^{s_0/M^2}}{(m_b^2 + x_0^2 m_{f_2}^2 - q^2)^2} \left(\frac{d}{dx} \right)^2 \left(\frac{f(x)}{x^2 (m_b^2 + x^2 m_{f_2}^2 - q^2)} \right) \Bigg|_{x=x_0}, \\
\int_0^1 dx \frac{f(x)}{D^5} \to & \frac{1}{24M^8} \int_{x_0}^1 \frac{dx}{x^5} f(x) e^{-s(x)/M^2} + \frac{1}{24M^6} \frac{f(x_0) e^{-s_0/M^2}}{(m_b^2 + x_0^2 m_{f_2}^2 - q^2) x_0^3} \\
& - \frac{1}{24M^4} \frac{x_0^2 e^{-s_0/M^2}}{(m_b^2 + x_0^2 m_{f_2}^2 - q^2)} \frac{d}{dx} \left(\frac{f(x)}{x^3 (m_b^2 + x^2 m_{f_2}^2 - q^2)} \right) \Bigg|_{x=x_0} \\
& + \frac{1}{24M^2} \frac{x_0^4 e^{-s_0/M^2}}{(m_b^2 + x_0^2 m_{f_2}^2 - q^2)^2} \left(\frac{d}{dx} \right)^2 \left(\frac{f(x)}{x^3 (m_b^2 + x^2 m_{f_2}^2 - q^2)} \right) \Bigg|_{x=x_0} \\
& - \frac{1}{24} \frac{x_0^6 e^{-s_0/M^2}}{(m_b^2 + x_0^2 m_{f_2}^2 - q^2)^3} \left(\frac{d}{dx} \right)^3 \left(\frac{f(x)}{x^3 (m_b^2 + x^2 m_{f_2}^2 - q^2)} \right) \Bigg|_{x=x_0}, \quad (4.13)
\end{aligned}
$$

with $x_0 = x(s_0)$.

4.2 Numerical analysis

Before we start with the numerical analysis we have to specify the input parameters. For the baryons we use the masses from Refs. [69, 70]

$$m_{\Lambda_c} = 2.286\,\mathrm{GeV}, \qquad m_{\Lambda_c^*} = 2.595\,\mathrm{GeV},$$
$$m_{\Lambda_b} = 5.620\,\mathrm{GeV}, \qquad m_{\Lambda_b^*} = 5.85\,\mathrm{GeV}.$$

For the shape parameters of the N^* DAs we use two different models. On the one hand we use a set of parameters labeled as LCSR (1) which is obtained by a fit to the form factors $G_1(Q^2)$ and $G_2(Q^2)$ extracted from the measurements of helicity structures in Ref. [71] with the errors added in quadrature. On the other hand we use a second set of parameters labeled as LCSR (2) which is obtained by a fit to helicity amplitudes including all available data at $Q^2 \geq 1.7$ GeV [71–74]. The corresponding values are given in Table 4.1. For the decay constants $\lambda^{(i)}_{\Lambda_{b,c}}$ we use the corresponding leading order two point sum rule instead of inserting fixed values. This has the advantage of reducing the overall uncertainties. For these two point sum rules we use the same two approaches (eliminating the $\Lambda^*_{b,c}$ pole or extracting the highest power of p_+) as for the light-cone sum rules. We take the expressions for the two point sum rules from Ref. [75], see also Ref. [64][1]; we have also checked their result. We follow Ref. [64] and take for the virtual b and c quarks the $\overline{\mathrm{MS}}$ mass

Method	$\lvert\lambda_1^{N^*}/\lambda_1^N\rvert$	$f_{N^*}/\lambda_1^{N^*}$	φ_{10}	φ_{11}	φ_{20}	φ_{21}	φ_{22}	η_{10}	η_{11}	Ref.
LCSR (1)	0.633	0.027	0.36	-0.95	0	0	0	0.00	0.94	[60]
LCSR (2)	0.633	0.027	0.37	-0.96	0	0	0	-0.29	0.23	[60]
LATTICE	0.633(43)	0.027(2)	0.28(12)	-0.86(10)	1.7(14)	-2.0(18)	1.7(26)	-	-	[67]

Table 4.1: Parameters of the $N^*(1535)$ distribution amplitudes at the scale $\mu^2 = 2$ GeV^2 for the two different models. For comparison we also show recent lattice results [67] where only statistical errors are shown. Also in Ref. [67] λ_1^N is given as $10^2\, m_N\, \lambda_1^N = -3.88(2)(19)$ GeV^3 and $\lambda_2^{N^*}$ is given as $10^2\, m_{N^*}\, \lambda_2^{N^*} = 8.97(45)$ GeV^3. Please note that there is a typo in the first column in [60].

given by $\bar{m}_b(\bar{m}_b) = 4.16 \pm 0.03$ GeV and $\bar{m}_c(\bar{m}_c) = 1.28 \pm 0.03$ GeV, calculated with quarkonium sum rules in Ref. [77]. All the scale dependent parameters are evaluated at the factorization scales $\mu_b = 4.0 \pm 1.0$ GeV and $\mu_c = 1.5 \pm 0.5$ GeV, respectively. We see in Figure 4.2 that our sum rules are quite stable with respect to variation of

[1] Please note that there is a typo in the dimension six part of Im $\tilde{F}_1(s)$, equation (D2), in [64]. The correct expression is [76]

$$\mathrm{Im}\tilde{F}_1^{dim\,6}(s) = \pi\frac{\langle\bar{q}q\rangle^2}{72}\delta(s-m_c^2)(11+2b-13b^2)$$

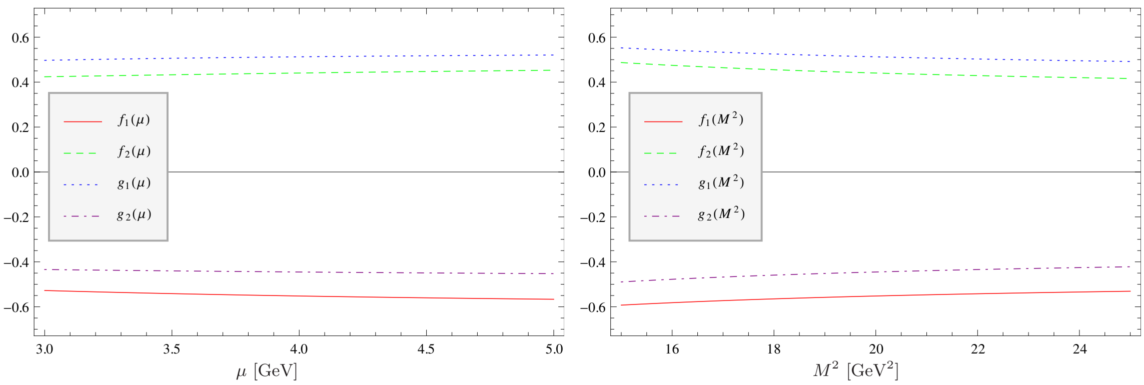

Figure 4.2: μ- and M^2-dependence of the $\Lambda_b \to N^*$ form factors at $q^2 = 0$.

$\mu_{b,c}$. The scaling behavior of the parameters can be found in Ref. [60]. For the Borel parameter and the duality threshold we take

$$\begin{aligned} M_b^2 =& 15 - 25\,\mathrm{GeV}^2, \quad s_0^b = 36 - 40\,\mathrm{GeV}^2, \\ M_c^2 =& 5 - 10\,\mathrm{GeV}^2, \quad s_0^c = 6 - 10\,\mathrm{GeV}^2. \end{aligned}$$

This set of parameters ensures that the usual sum rule quality criteria, namely the suppression of continuum states and of higher twist contributions, are very well fulfilled, see Figure 4.2 for the $\Lambda_b \to N^*$ form factors.
A careful analysis of the two different methods with the input parameters from above reveals that the sum rules derived by eliminating the $\Lambda_{b,c}^*$ pole suffer from various problems:

1. There are large numerical cancellations between different Lorentz structures. The only exceptions are the form factors $f_1(q^2)$ and $g_1(q^2)$ with the pseudoscalar interpolating current.

2. For the pseudoscalar interpolating current very little hierarchy between contributions of different twist, numerical cancellations between different twist contributions and a strong dependence on the variation of the higher twist parameter ξ_{10} occurs.

3. It is not possible to find a set of parameters M^2 and s_0 in such way that the sum rules for all four form factors fulfill our quality criteria.

Therefore as a default we chose the sum rules for the functions $A(P,q)$ and $B(P,q)$ with the axial-vector interpolating current and the corresponding two point sum rule with the highest power of p_+. We chose the axial-vector interpolating current because the pseudoscalar one leads to similar problems as mentioned above. There is no clear hierarchy between different twists and also large numerical cancellations happen for most of the channels. Furthermore the sum rules with the pseudoscalar current strongly depend on the unknown parameter ξ_{10}, and the results changes up

to a factor of 6 when varying this parameter in the range $-0.2 \leq \xi_{10} \leq 0.2$. In contrast the dependence of the axial-vector sum rules on the parameter ξ_{10} is quite moderate. As one can see from Table 4.1 the essential difference between the two models LCSR (1) and LCSR (2) are the values for the twist-four parameters η_{10} and η_{11}, which are related to the p-wave three quark wave functions of N^*, and thus to the distribution of orbital angular momentum. We can use this difference as a measurement for the uncertainties coming from the variation of these two parameters. Furthermore we give the results for the two models separately so that after measuring these decays it is possible to rule out one of the models.
We assume that our sum rule calculations are valid up to $q^2_{max} = 6$ GeV^2 for Λ_b and up to $q^2_{max} = -1$ GeV^2 for Λ_c. For larger q^2 the OPE is not reliable anymore. So in order to calculate the decay widths we need to extrapolate the sum rule results to the whole semileptonic region $q \leq (m_{\Lambda_{b,c}} - m_{N^*})^2$. Therefore we perform a two-parameter fit to our numerical values using the fit function proposed in Ref. [78]

$$\begin{aligned} f_i(q^2) &= \frac{f_i(0)}{1 - \frac{q^2}{m^2_{B^*(1^-)}}} \left\{1 + b_i \left(z(q^2, t_0) - z(0, t_0)\right)\right\}, \\ g_i(q^2) &= \frac{g_i(0)}{1 - \frac{q^2}{m^2_{B^*(1^+)}}} \left\{1 + \tilde{b}_i \left(z(q^2, t_0) - z(0, t_0)\right)\right\}, \end{aligned} \tag{4.14}$$

with the mapping

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \tag{4.15}$$

and the parameters

$$\begin{aligned} t_\pm &= (m_{\Lambda_{b,c}} \pm m_{N^*})^2, \\ t_0 &= t_+ - \sqrt{t_+ - t_-}\sqrt{t_+ - t_{min}}, \end{aligned} \tag{4.16}$$

where $m_{B^*(1^-)} = 5.325$ GeV and $m_{B^*(1^+)} = 5.723$ GeV. Here t_{min} is the lowest value of q^2 mapped to $z(q^2, t_0)$, so that $t_{min} = q^2_{min} \leq q^2 \leq t_-$. For the numerical analysis we extend the fit region by calculating the form factors starting from $q^2_{min} = -6$ GeV^2. Our results for the fit parameters $f_i(0)$, $g_i(0)$, b_i and $\tilde{b}_i$ can be found in Table 4.2 and the fits compared to our sum rule values in Figures 4.3 and 4.4 for the different decays and the two different models. We perform a weighted fit using the uncertainties coming from the variation of the input parameters added in quadrature as weights. For asymmetric errors we take the mean value and shift the central value by the difference of the asymmetric error and the mean value, to get symmetric errors. As already mentioned above, we do not vary the twist-four parameters because we want to use the two models as a measure for the uncertainties from these parameters. As we can see from Figures 4.3 and 4.4, the largest uncertainties come from the twist-four parameters η_{10} and η_{11}. So these decays are very sensitive to the shape of the

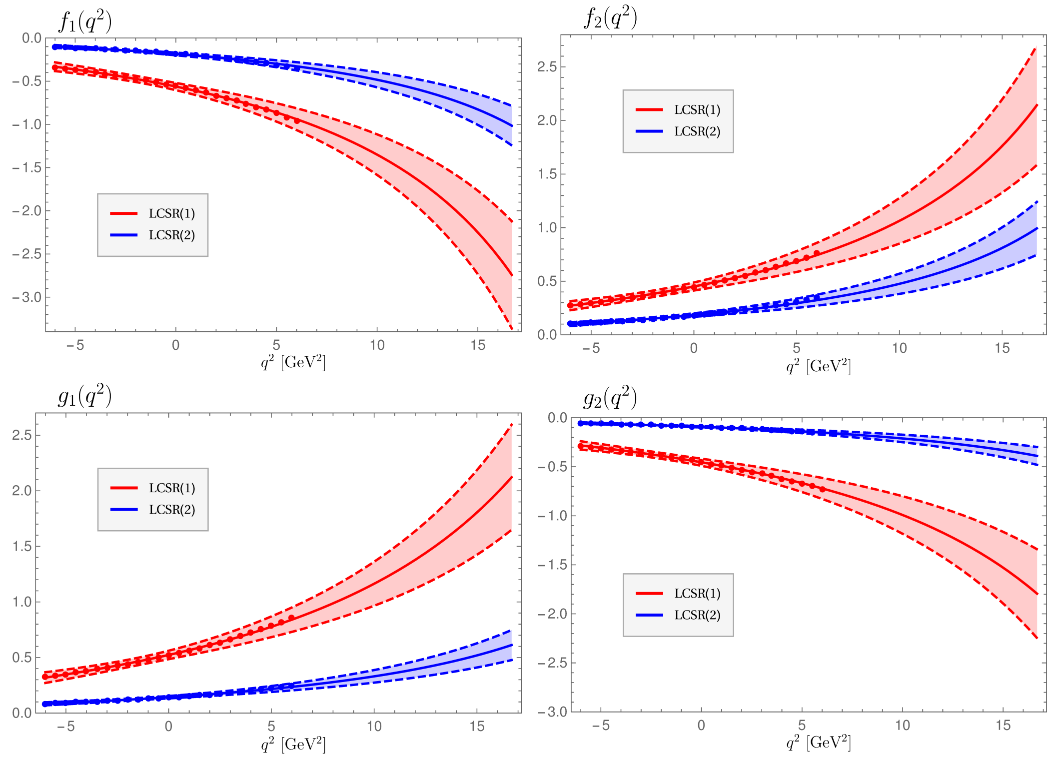

Figure 4.3: The different form factors for the decay $\Lambda_b \to N^*$ with the two different models LCSR (1) and LCSR (2). The solid lines give the central value of the fits to the sum rule results (large dots) and the dashed lines give the uncertainties from the variation of the input parameters except twist-four.

N^* DAs and, after experimental measurements of the form factors, can be used to extract information on the N^* DAs themselves.
We can calculate the decay width by integrating the differential decay rate given by

$$\begin{aligned}\frac{d\Gamma}{dq^2}(\Lambda_{b,c} \to N^* l\nu_l) =& \frac{G_F^2 m_{\Lambda_{b,c}}^3}{192\pi^3}|V_{ub}|^2\lambda^{1/2}(1,r^2,t)\Big\{[(1-r)^2-t][(1+r)^2+2t]|g_1(q^2)|^2\\ &+[(1+r)^2-t][(1-r)^2+2t]|f_1(q^2)|^2\\ &-6t[(1-r)^2-t](1+r)g_1(q^2)g_2(q^2)\\ &-6t[(1+r)^2-t](1-r)f_1(q^2)f_2(q^2)\\ &+t[(1-r)^2-t][2(1+r)^2+t]|g_2(q^2)|^2\\ &+t[(1+r)^2-t][2(1-r)^2+t]|f_2(q^2)|^2\Big\},\end{aligned}$$

over the whole physical region $0 \leq q^2 \leq (m_{\Lambda_{b,c}} - m_{N^*})^2$ using the fit results for the form factors. We used the abbreviations $r = m_{N^*}/m_{\Lambda_{b,c}}$, $t = q^2/m_{\Lambda_{b,c}}^2$ and $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$. As already mentioned above, we neglect

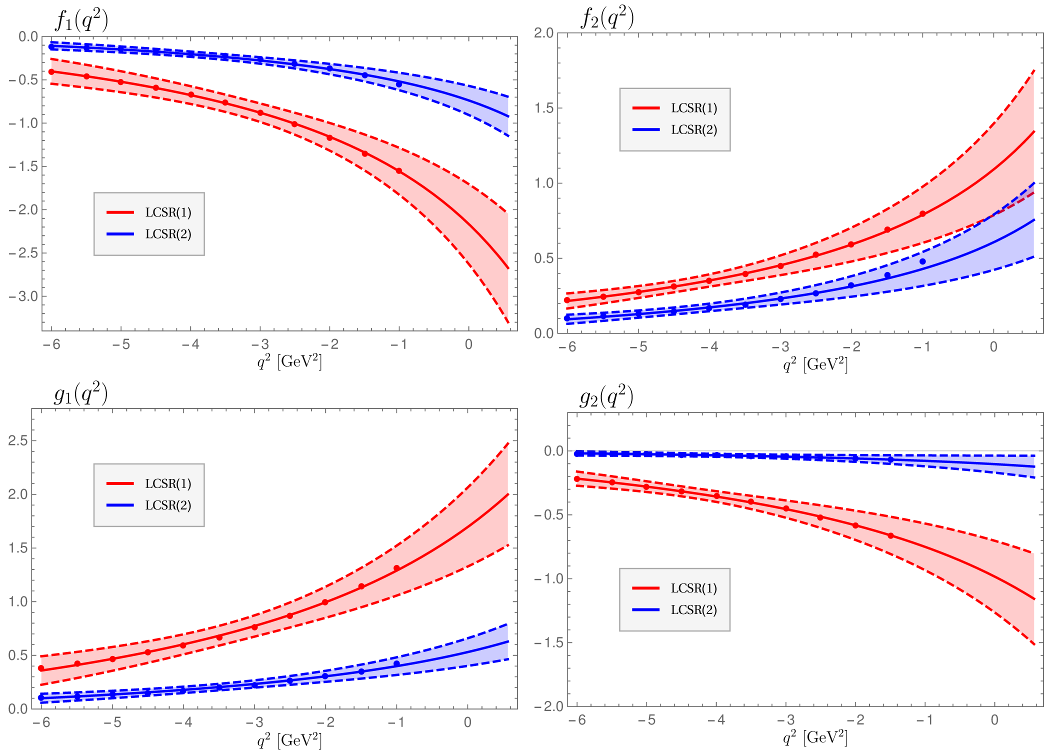

Figure 4.4: The different form factors for the decay $\Lambda_c \to N^*$ with the two different models and the same notation as in Figure 4.3.

the form factors f_3 and g_3 in the decay rate and compared to [64, 65] we exchanged $f_1 \leftrightarrow g_1$ and $f_2 \leftrightarrow g_2$. The normalized differential decay rate can be found in Figure 4.5. Our results for the decay widths are

$$\Gamma(\Lambda_b \to N^*(1535)l\nu) = \left(0.0058^{+0.0010}_{-0.0009}\right) \cdot \left(\frac{V_{ub}}{3.5 \cdot 10^{-3}}\right)^2, \quad \text{LCSR(1)}$$
$$\Gamma(\Lambda_b \to N^*(1535)l\nu) = \left(0.00070^{+0.00012}_{-0.00011}\right) \cdot \left(\frac{V_{ub}}{3.5 \cdot 10^{-3}}\right)^2, \quad \text{LCSR(2)}$$
$$\Gamma(\Lambda_c \to N^*(1535)l\nu) = \left(0.0064^{+0.0012}_{-0.0011}\right) \cdot \left(\frac{V_{cd}}{0.225}\right)^2, \quad \text{LCSR(1)}$$
$$\Gamma(\Lambda_c \to N^*(1535)l\nu) = \left(0.00077^{+0.00016}_{-0.00014}\right) \cdot \left(\frac{V_{cd}}{0.225}\right)^2, \quad \text{LCSR(2)}$$

where we use for the lifetime $\tau_{\Lambda_b} = 1.451 \pm 0.013$ ps and $\tau_{\Lambda_c} = 200 \pm 6$ ps from [41].

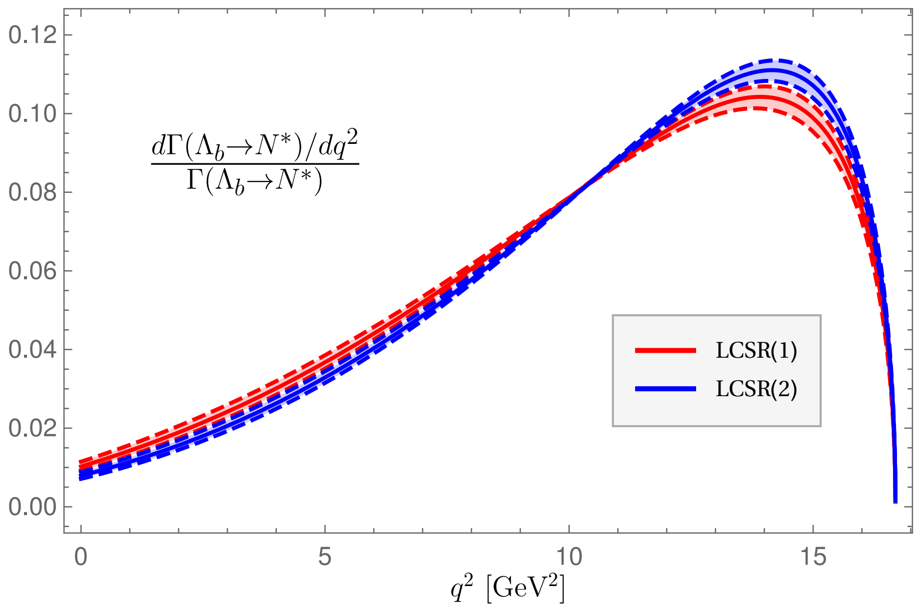

Figure 4.5: Normalized differential decay rate for the $\Lambda_b \to N^*$ decay with the same notation as Figure 4.4.

$\Lambda_b \to N^*$

LCSR(1)			LCSR(2)		
fit parameter	estimate	uncertainty	fit parameter	estimate	uncertainty
$f_1(0)$	-0.562	0.015	$f_1(0)$	-0.185	0.005
b_1	-10.236	1.420	b_1	-12.803	1.505
$f_2(0)$	0.451	0.0133	$f_2(0)$	0.184	0.006
$b_2(0)$	-9.695	1.549	$b_2(0)$	-12.355	1.623
$g_1(0)$	0.523	0.014	$g_1(0)$	0.143	0.004
$\tilde{b}_1$	-10.050	1.401	$\tilde{b}_1$	-11.205	1.460
$g_2(0)$	-0.454	0.013	$g_2(0)$	-0.093	0.003
$\tilde{b}_2$	-9.521	1.500	$\tilde{b}_2$	-10.718	1.483

$\Lambda_c \to N^*$

LCSR(1)			LCSR(2)		
fit parameter	estimate	uncertainty	fit parameter	estimate	uncertainty
$f_1(0)$	-2.17	0.18	$f_1(0)$	-0.74	0.07
b_1	-6.26	1.06	b_1	-7.44	0.89
$f_2(0)$	1.09	0.12	$f_2(0)$	0.61	0.06
$b_2(0)$	-5.91	1.01	$b_2(0)$	-7.14	0.76
$g_1(0)$	1.70	0.14	$g_1(0)$	0.53	0.05
$\tilde{b}_1$	-6.65	1.01	$\tilde{b}_1$	-7.18	1.01
$g_2(0)$	-0.98	0.11	$g_2(0)$	-0.10	0.03
$\tilde{b}_2$	-6.44	0.96	$\tilde{b}_2$	-7.35	2.29

Table 4.2: Our results from fitting to the formula (4.14) for the $\Lambda_{b,c} \to N^*$ form factors with the two models LCSR(1) and LCSR(2).

Chapter 5

$B \to f_2(1270) l\nu$ form factors with LCSR

This Chapter is based on the publication [79] in collaboration with Andreas Schäfer and Matthias Strohmaier.

In recent years the BaBar and Belle experiments measured B decay modes with light tensor mesons in the final state [41, 80, 81]. The possibility of three different polarizations of the final tensor meson can give insights into the helicity structure of the electroweak interaction or can help to find deviations from expectations. The sensitivity reachable with current detectors was recently demonstrated by the measurement of the transition form factor $\gamma^*\gamma \to f_2(1270)$ by Belle [82]. With precise theoretical descriptions of such processes, the physical potential of tensor meson production could be huge.

In earlier work tensor mesons have already been studied. The chiral even and odd DAs were constructed in Ref. [57] where also the $f_2(1270)$ decay constants were calculated. The transition form factor mentioned above, $\gamma^*\gamma \to f_2(1270)$, was recently calculated using LCSR [56]. The definitions of the B to tensor meson form factors are given in Refs. [83–85]. For the decay $B \to f_2(1270)$ a few studies exist, using e.g. a perturbative QCD approach [85] or LCSR approaches [86, 87].

In this Chapter we will use the methods and basic principles discussed in the previous Chapters to calculate the form factors for the $B \to f_2(1270)$ decay. In order to calculate this decay, we will construct, for the first time, chiral odd quark-antiquark DAs, including higher twist contributions and meson mass corrections. We will also construct new three-particle quark-antiquark-gluon DAs; these and the former can be found in appendix D. After the theoretical calculation we will perform a numerical analysis and compare our results to the publications mentioned above.

5.1 LCSR calculation

In this section we give the detailed calculation for the $B \to f_2(1270)$ form factors with the help of LCSR. During this whole calculation we assume that $f_2(1270)$ is a

pure nonstrange isospin singlet state $\frac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$ and that $f_2'(1525)$ is a pure strange quark state $\bar{s}s$. This assumption is equivalent to assuming a vanishing mixing angle [88, 89].
To begin the calculation we consider the correlation function

$$\Pi_a(q, P) = i \int d^4x \, e^{iq\cdot x} \langle f_2^\lambda(P)| \, T\{\bar{q}_1(x)\Gamma_a b(x) j_B(0)\} \, |0\rangle \,, \tag{5.1}$$

with three different Lorentz structures

$$\Gamma_\mu = \gamma_\mu, \quad \Gamma_{\mu 5} = \gamma_\mu \gamma_5, \quad \Gamma_{\mu\nu 5} = \sigma_{\mu\nu}\gamma_5.$$

Here the interpolating current for the B-meson is

$$j_B(0) = \bar{b}(0) i\gamma_5 q_2(0),$$

and the decay constant f_B is defined by the matrix element

$$\langle B(P')| \, \bar{b}(0) i\gamma_5 q_2(0) \, |0\rangle = \frac{f_B m_B^2}{m_b}.$$

The semileptonic $B \to f_2(1270)$ form factors are defined by [83–85]

$$\begin{aligned}
\langle f_2^\lambda(P)| \, \bar{q}_1(0)\gamma_\mu b(0) \, |B(P')\rangle =& \frac{2}{m_B + m_{f_2}} \epsilon_{\mu\nu\alpha\beta} e^{\nu(\lambda)*} P'^\alpha P^\beta \tilde{V}(q^2), && (5.2) \\
\langle f_2^\lambda(P)| \, \bar{q}_1(0)\gamma_\mu\gamma_5 b(0) \, |B(P')\rangle =& i(m_B + m_{f_2}) e_\mu^{(\lambda)*} \tilde{A}_1(q^2) - i\frac{e^{(\lambda)*}\cdot P'}{m_B + m_{f_2}} (P' + P)_\mu \tilde{A}_2(q^2) \\
&- 2i m_{f_2} \frac{e^{(\lambda)*}\cdot P'}{q^2} q_\mu [\tilde{A}_3(q^2) - \tilde{A}_0(q^2)], && (5.3) \\
\langle f_2^\lambda(P)| \, \bar{q}_1(0)\sigma_{\mu\nu}\gamma_5 b(0) \, |B(P')\rangle =& \tilde{A}(q^2) \left[e_\mu^{(\lambda)*}(P + P')_\nu - e_\nu^{(\lambda)*}(P + P')_\mu \right] \\
&- \tilde{B}(q^2) \left[e_\mu^{(\lambda)*} q_\nu - e_\nu^{(\lambda)*} q_\mu \right] \\
&- 2\tilde{C}(q^2) \frac{e^{(\lambda)*}\cdot q}{m_B^2 - m_{f_2}^2} \left[P_\mu q_\nu - P_\nu q_\mu \right], && (5.4)
\end{aligned}$$

with $\tilde{A}_0(0) = \tilde{A}_3(0)$ and $2m_{f_2}\tilde{A}_3(q^2) = (m_B + m_{f_2})\tilde{A}_1(q^2) - (m_B - m_{f_2})\tilde{A}_2(q^2)$.
The tensor form factors can also be defined by the two following matrix elements

$$\begin{aligned}
\langle f_2^\lambda(P)| \, \bar{q}_1(0)\sigma_{\mu\nu} q^\nu b(0) \, |B(P')\rangle =& - 2i\epsilon_{\mu\nu\alpha\beta} P'^\nu P^\alpha e^{\beta(\lambda)*} \tilde{T}_1(q^2), \\
\langle f_2^\lambda(P)| \, \bar{q}_1(0)\sigma_{\mu\nu}\gamma_5 q^\nu b(0) \, |B(P')\rangle =& \tilde{T}_2(q^2) \left[(m_B^2 - m_{f_2}^2) e_\mu^{(\lambda)*} - e^{(\lambda)*}\cdot P'(P' + P)_\mu \right] \\
&+ \tilde{T}_3(q^2) e^{(\lambda)*}\cdot P' \left[q_\mu - \frac{q^2}{m_B^2 - m_{f_2}^2}(P' + P)_\mu \right],
\end{aligned}$$

using

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}.$$

Contracting equation (5.4) with q^ν leads to

$$\begin{aligned}
\tilde{T}_1(q^2) &= \tilde{A}(q^2), \\
\tilde{T}_2(q^2) &= \tilde{A}(q^2) - \frac{q^2}{m_B^2 - m_{f_2}^2}\tilde{B}(q^2), \\
\tilde{T}_3(q^2) &= \tilde{B}(q^2) + \tilde{C}(q^2).
\end{aligned}$$

For the definitions above we used $q_\alpha = P'_\alpha - P_\alpha$ and $e_\alpha^{(\lambda)*} = \frac{e_{\alpha\beta}^{(\lambda)*} q^\beta}{m_B}$. The polarization tensor $e_{\alpha\beta}^{(\lambda)}$ with helicity λ is traceless, symmetric and satisfies the condition $e_{\alpha\beta}^{(\lambda)} P^\alpha = 0$. The normalization is chosen in such a way that $e_{\alpha\beta}^{(\lambda)} e_{\alpha\beta}^{(\lambda')*} = \delta_{\lambda\lambda'}$ and that the completeness relation reads

$$\sum_\lambda e_{\alpha\beta}^{(\lambda)} e_{\mu\nu}^{(\lambda)*} = \frac{1}{2} M_{\alpha\mu} M_{\beta\nu} + \frac{1}{2} M_{\alpha\nu} M_{\beta\mu} - \frac{1}{3} M_{\alpha\beta} M_{\mu\nu},$$

with $M_{\alpha\beta} = g_{\alpha\beta} - P_\alpha P_\beta / m_{f_2}^2$. Furthermore we use the following shorthand notations

$$e_{\alpha q}^{(\lambda)*} = e_{\alpha\beta}^{(\lambda)*} q^\beta, \qquad e_{qq}^{(\lambda)*} = e_{\alpha\beta}^{(\lambda)*} q^\alpha q^\beta.$$

Inserting a complete set of eigenstates, isolating the ground state and applying a Borel transformation we obtain

$$\begin{aligned}
\Pi_\mu(q,P) &= 2 f_B m_B^2 e^{-m_B^2/M^2} \frac{\tilde{V}(q^2)}{(m_B + m_{f_2}) m_b} \epsilon_{\mu\nu\alpha\beta}\, e^{\nu(\lambda)*} q^\alpha P^\beta + \int_{s_0^h}^{\infty} \frac{\rho_\mu(s,q^2)}{s - P'^2}, \\
\Pi_{\mu 5}(q,P) &= \frac{i f_B m_B^2}{m_b} e^{-m_B^2/M^2} \Bigg[(m_B + m_{f_2}) e_\mu^{(\lambda)*} \tilde{A}_1(q^2) - (e^{(\lambda)*} \cdot q)(2P_\mu + q_\mu) \frac{\tilde{A}_2(q^2)}{m_B + m_{f_2}} \\
&\quad - 2 m_{f_2} \frac{e^{(\lambda)*} \cdot q}{q^2} q_\mu \left(\tilde{A}_3(q^2) - \tilde{A}_0(q^2) \right) \Bigg] + \int_{s_0^h}^{\infty} \frac{\rho_{\mu 5}(s,q^2)}{s - P'^2}, \\
\Pi_{\mu\nu 5}(q,P) &= \frac{f_B m_B^2}{m_b} e^{-m_B^2/M^2} \Big[-\tilde{A}(q^2) \left((2P_\mu + q_\mu) e_\nu^{(\lambda)*} - (2P_\nu + q_\nu) e_\mu^{(\lambda)*} \right) \\
&\quad - \tilde{B}(q^2) \left(e_\mu^{(\lambda)*} q_\nu - e_\nu^{(\lambda)*} q_\mu \right) - 2\tilde{C}(q^2) \frac{e^{(\lambda)*} \cdot q}{m_B^2 - m_{f_2}^2} \left(P_\mu q_\nu - P_\nu q_\mu \right) \Big] + \int_{s_0^h}^{\infty} \frac{\rho_{\mu\nu 5}(s,q^2)}{s - P'^2},
\end{aligned}$$

with M^2 being the Borel parameter. Instead of writing the spectral densities for all the Lorentz indices separate, we only give one representative term explicit.
For the OPE we use the quark propagator in a background field [90, 91]

$$\langle 0 | T\{ b^i(x) \bar{b}^j(0) \} | 0 \rangle = -i \int \frac{d^4 k}{(2\pi)^4} e^{-ik\cdot x} \frac{\not{k} + m_b}{m_b^2 - k^2} \delta_{ij}$$

$$- ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ik\cdot x} \int_0^1 dv\, G^{\mu\nu a}(vx) \left(\frac{\lambda^a}{2}\right)^{ij} \left(\frac{\not{k} + m_b}{2(m_b^2 - k^2)^2}\sigma_{\mu\nu} + \frac{1}{m_b^2 - k^2} v x_\mu \gamma_\nu\right). \tag{5.5}$$

With this propagator we obtain the leading two-particle as well as three-particle contributions, see Figure 5.1.

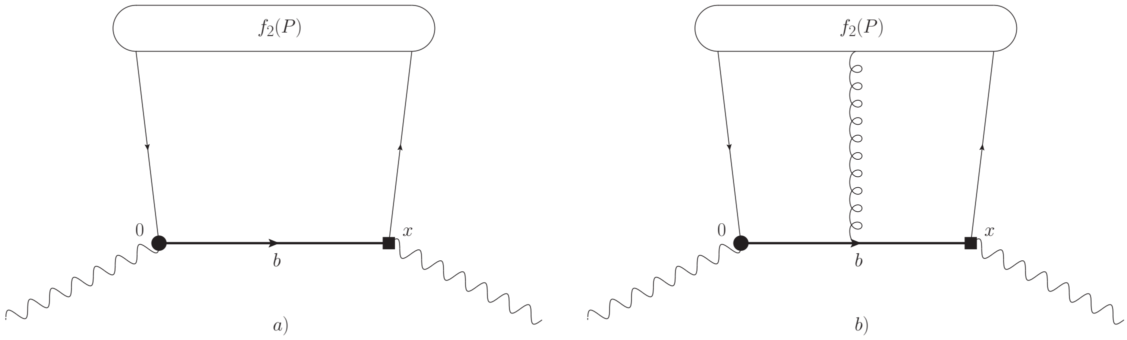

Figure 5.1: Leading two- and three-particle contributions to the correlation function (5.1). The wavy lines represent the external currents, the oval represents the f_2 DAs and the wiggly line depict the gluon. The straight lines represent the quarks, where the thick one denotes the b-quark propagator.

Vector current

We start the perturbative calculation with the vector current, using $\Gamma_\mu = \gamma_\mu$ in the correlation function (5.1). Contracting the b-quarks and inserting the propagator given above leads to

$$\begin{aligned}\Pi_\mu(q,P) = i \int \frac{d^4x\, d^4k}{(2\pi)^4} \frac{e^{ix\cdot(q-k)}}{m_b^2 - k^2} \Big(& m_b \langle f_2^\lambda(P)| \bar{q}_1(x)\gamma_\mu\gamma_5 q_2(0) |0\rangle \\ & + k^\nu \langle f_2^\lambda(P)| \bar{q}_1(x)\gamma_\mu\gamma_\nu\gamma_5 q_2(0) |0\rangle \\ & + \int_0^1 dv \langle f_2^\lambda(P)| \bar{q}_1(x)\gamma_\mu g_s G^{\alpha\beta}(vx) \left(\frac{(\not{k} + m_b)\sigma_{\alpha\beta}}{2(m_b^2 - k^2)} + \frac{v x_\alpha \gamma_\beta}{m_b^2 - k^2}\right)\gamma_5 q_2(0) |0\rangle \Big).\end{aligned}$$

Using the identities

$$\gamma_\mu\gamma_\nu = g_{\mu\nu} - i\sigma_{\mu\nu}, \quad \sigma_{\mu\nu} = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\gamma_5,$$

and

$$\gamma_\mu\gamma_\nu\sigma_{\alpha\beta} = (\sigma_{\mu\beta}g_{\nu\alpha} - \sigma_{\mu\alpha}g_{\nu\beta}) + i(g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}g_{\nu\beta}) - \epsilon_{\mu\nu\alpha\beta}\gamma_5 - i\epsilon_{\nu\alpha\beta\rho}g^{\rho\lambda}\sigma_{\mu\lambda}\gamma_5,$$

the matrix elements can be expressed in terms of the two- and three-particle DAs defined in appendix D. So we obtain

$$\Pi_\mu(q,P) = \int \frac{d^4x d^4k}{(2\pi)^4} \frac{e^{ix\cdot(q-k)}}{m_b^2-k^2} \Bigg(m_b f_{f_2} m_{f_2}^2 (1-\delta_+) \epsilon_{\mu\nu\alpha\beta} \frac{x^\nu P^\alpha}{(P\cdot x)^2} e^{\beta x(\lambda)*} \int_0^1 du\, e^{iu(P\cdot x)} g_a(u)$$

$$+ \frac{k^\nu}{2} \epsilon_{\mu\nu\alpha\beta} f_{f_2}^T \Bigg[m_{f_2} \frac{e^{\alpha x(\lambda)*} P^\beta - e^{\beta x(\lambda)*} P^\alpha}{(P\cdot x)} \int_0^1 du\, e^{iu(P\cdot x)} (A(u) + \frac{m_{f_2}^2 x^2}{4} \mathbb{A}(u))$$

$$+ m_{f_2}^3 (P^\alpha x^\beta - P^\beta x^\alpha) \frac{e_{xx}^{(\lambda)*}}{(P\cdot x)^3} \int_0^1 du\, e^{iu(P\cdot x)} B(u)$$

$$+ \frac{1}{2} \left(e^{\alpha x(\lambda)*} x^\beta - e^{\beta x(\lambda)*} x^\alpha \right) \frac{m_{f_2}^3}{(P\cdot x)^2} \int_0^1 du\, e^{iu(P\cdot x)} C(u) \Bigg] \Bigg) .$$

Here the three-particle DAs do not appear because they give zero, due to the ϵ-tensor structure. In the following calculations, it is possible to replace terms proportional to powers of x_μ in the numerator by the derivative $\left(-i\frac{\partial}{\partial q_\mu} \right)$, based on the following relation:

$$\int d^4x \int \frac{d^4k}{(2\pi)^4} f(k) x^\mu e^{ix\cdot(q-k+uP)} = \int d^4x \int \frac{d^4k}{(2\pi)^4} f(k)(-i) \left(\frac{\partial}{\partial q_\mu} e^{ix\cdot(q-k+uP)} \right)$$

$$= (-i)\frac{\partial}{\partial q_\mu} \left(\int d^4x \int \frac{d^4k}{(2\pi)^4} f(k) e^{ix\cdot(q-k+uP)} \right) = (-i)\frac{\partial}{\partial q_\mu} f(q+uP).$$

Additionally we replace the powers of $(P\cdot x)$ in the denominators with the help of partial integration. In terms like

$$\frac{1}{(P\cdot x)} \int_0^1 du\, e^{iu(P\cdot x)} \mathcal{A}(u),$$

we rewrite $\mathcal{A}(u) = (-1)\frac{d}{du}\hat{\mathcal{A}}(u)$ and obtain

$$\frac{1}{(P\cdot x)} \int_0^1 du\, e^{iu(P\cdot x)} (-1) \left(\frac{d}{du} \right) \hat{\mathcal{A}}(u) \stackrel{PI}{=} i \int_0^1 du\, e^{iu(P\cdot x)} \hat{\mathcal{A}}(u).$$

To have a unique definition for $\mathcal{A}(u)$ and to get rid of the surface terms, $\hat{\mathcal{A}}(u)$ has to fulfill the condition $\hat{\mathcal{A}}(0) = \hat{\mathcal{A}}(1) = 0$. In the same manner we can rewrite $\mathcal{A}(u) = (-1)^2 \left(\frac{d}{du}\right)^2 \hat{\hat{\mathcal{A}}}(u)$ to get rid of $\frac{1}{(P\cdot x)^2}$ terms and $\mathcal{A}(u) = (-1)^3 \left(\frac{d}{du}\right)^3 \hat{\hat{\hat{\mathcal{A}}}}(u)$ to get rid of $\frac{1}{(P\cdot x)^3}$ terms. Here we also require $\hat{\hat{\mathcal{A}}}(0) = \hat{\hat{\mathcal{A}}}(1) = \hat{\hat{\hat{\mathcal{A}}}}(0) = \hat{\hat{\hat{\mathcal{A}}}}(1) = 0$ for uniqueness. After these two steps we can perform the x and k integrations, which lead to the replacement $k \to q + uP$ in the denominator. Performing the derivatives with respect to q_μ we end up with

$$\Pi_\mu(q,P) = \epsilon_{\mu\nu\alpha\beta} e^{\nu q(\lambda)*} q^\alpha P^\beta \int_0^1 du \Bigg[(1-\delta_+) 8 m_b m_{f_2}^2 f_{f_2} \frac{\hat{\hat{g}}_a(u)}{(m_b^2-k^2)^3} - 2 m_{f_2} f_{f_2}^T \frac{\hat{A}(u)}{(m_b^2-k^2)^2} + 4 m_{f_2}^3 f_{f_2}^T \frac{(k^2-4m_b^2)}{(m_b^2-k^2)^4} \hat{\mathbb{A}}(u) \Bigg], \tag{5.6}$$

where $k_\alpha = q_\alpha + uP_\alpha$.

Axial-vector current

For the axial-vector current (using $\Gamma_{\mu 5} = \gamma_\mu \gamma_5$ in the correlation function (5.1)) the line of action is the same as for the axial current. Hence we will only give a short outline. After contracting the b-quarks and inserting the propagator we obtain

$$\begin{aligned}\Pi_{\mu 5}(q,P) = i \int \frac{d^4x\, d^4k}{(2\pi)^4} \frac{e^{ix\cdot(q-k)}}{m_b^2-k^2} \Big[& m_b \langle f_2^\lambda(P)| \bar{q}_1(x)\gamma_\mu q_2(0)|0\rangle \\ & + \langle f_2^\lambda(P)| \bar{q}_1(x)\gamma_\mu \underbrace{\gamma_5\, \not{k}\, \gamma_5}_{-\mathbb{1}} q_2(0)|0\rangle \\ & + \int_0^1 dv\, \langle f_2^\lambda(P)| \bar{q}_1(x)\gamma_\mu\gamma_5 g_s G^{\alpha\beta}(vx) \left(\frac{\not{k}+m_b}{2(m_b^2-k^2)}\sigma_{\alpha\beta} + \frac{v x_\alpha \gamma_\beta}{m_b^2-k^2} \right) \gamma_5 q_2(0)|0\rangle \Big].\end{aligned}$$

In order to rewrite these matrix elements in terms of the two- and three-particle DAs from appendix D, we use the identities given above and an additional one

$$\gamma_\alpha \sigma_{\mu\nu} = i(g_{\mu\alpha}\gamma_\nu - g_{\alpha\nu}\gamma_\mu) + \epsilon_{\alpha\mu\nu\beta}\gamma^\beta\gamma_5.$$

After performing the integrals we end up with

$$\begin{aligned}\Pi_{\mu 5}(q,P) = i \int_0^1 du \Bigg[& P_\mu e_{qq}^{(\lambda)*} \Bigg(\frac{8 m_b m_{f_2}^2 f_{f_2} \hat{\hat{A}}_1(u)}{(m_b^2-k^2)^3} + \frac{96 m_b^3 m_{f_2}^4 f_{f_2} \hat{\hat{A}}_4(u)}{(m_b^2-k^2)^5} + \frac{2 m_{f_2} f_{f_2}^T \hat{A}(u)}{(m_b^2-k^2)^2} \\ & + \frac{4 m_{f_2}^3 f_{f_2}^T (k^2-4m_b^2)\hat{\mathbb{A}}(u)}{(m_b^2-k^2)^4} - \frac{8 u m_{f_2}^3 f_{f_2}^T (1-\delta_+^T) \hat{\hat{h}}_\parallel^{(s)}(u)}{(m_b^2-k^2)^3} + \frac{24 u m_b m_{f_2}^4 f_{f_2} \hat{\hat{\hat{C}}}_1(u)}{(m_b^2-k^2)^4} \\ & - \frac{m_{f_2}^3 f_{f_2}^T \hat{\hat{\hat{B}}}(u)}{(m_b^2-k^2)^3} \left(56 + \frac{48(q^2+uP\cdot q)}{(m_b^2-k^2)} \right) + \frac{4 u m_{f_2}^3 f_{f_2}^T \hat{\hat{C}}(u)}{(m_b^2-k^2)^3} \Bigg) \\ & + q_\mu e_{qq}^{(\lambda)*} \Bigg(\frac{48 m_{f_2}^3 f_{f_2}^T \hat{\hat{\hat{B}}}(u)(uP^2+P\cdot q)}{(m_b^2-k^2)^4} + \frac{24 m_b m_{f_2}^4 f_{f_2} \hat{\hat{\hat{C}}}_1(u)}{(m_b^2-k^2)^4} \\ & - \frac{8 m_{f_2}^3 f_{f_2}^T (1-\delta_+^T) \hat{\hat{h}}_\parallel^{(s)}(u)}{(m_b^2-k^2)^3} + \frac{4 m_{f_2}^3 f_{f_2}^T \hat{\hat{C}}(u)}{(m_b^2-k^2)^3} \Bigg)\end{aligned}$$

$$
\begin{aligned}
&+e_{\mu q}^{(\lambda)*}\Bigg(\frac{2m_b m_{f_2}^2 f_{f_2}\hat{B}_1(u)}{(m_b^2-k^2)^2}-\frac{4m_{f_2}^3 m_b^2 f_{f_2}^T\hat{\hat{C}}(u)}{(m_b^2-k^2)^3}-\frac{4m_{f_2}^3 f_{f_2}^T(1-\delta_+^T)\hat{\hat{h}}_\parallel^{(s)}(u)}{(m_b^2-k^2)^2}\\
&+\frac{8m_b m_{f_2}^4 f_{f_2}\hat{\hat{\hat{C}}}_1(u)}{(m_b^2-k^2)^3}-\frac{2m_{f_2}f_{f_2}^T(uP^2+P\cdot q)\hat{A}(u)}{(m_b^2-k^2)^2}-\frac{4m_{f_2}^3 f_{f_2}^T(k^2-4m_b^2)(uP^2+P\cdot q)\hat{\mathbb{A}}(u)}{(m_b^2-k^2)^4}\\
&+\frac{16m_{f_2}^3 f_{f_2}^T(uP^2+P\cdot q)\hat{\hat{\hat{B}}}(u)}{(m_b^2-k^2)^3}\Bigg)\Bigg]\\
&+i\int_0^1 dv\int\mathcal{D}\underline{\alpha}\frac{f_{f_2}^T v m_{f_2}^5}{(m_b^2-\bar{k}^2)^3}\left(8e_{\mu q}^{(\lambda)*}+\frac{24e_{qq}^{(\lambda)*}\bar{k}_\mu}{m_b^2-\bar{k}^2}\right)\left(\tilde{\tilde{\mathcal{T}}}_1(\underline{\alpha})-\frac{m_{f_2}^2}{2}\tilde{\tilde{\mathcal{T}}}_2(\underline{\alpha})\right)\\
&-i\int_0^1 dv\int\mathcal{D}\underline{\alpha}\frac{f_{f_2}^T v m_{f_2}^3}{(m_b^2-\bar{k}^2)^3}8e_{qq}^{(\lambda)*}P_\mu\left(\tilde{\mathcal{T}}_1(\underline{\alpha})-\frac{m_{f_2}^2}{2}\tilde{\mathcal{T}}_2(\underline{\alpha})\right), \qquad (5.7)
\end{aligned}
$$

where $k_\alpha = q_\alpha + uP_\alpha$ and $\bar{k}_\alpha = q_\alpha + (v\alpha_2+\alpha_3)P_\alpha$. For the three particle DAs we use a notation similar to the one for the two particle DAs: $\mathcal{T}(\underline{\alpha}) = (-1)\frac{d}{d\alpha_3}\tilde{\mathcal{T}}(\underline{\alpha})$, with the requirement $\tilde{\mathcal{T}}(\alpha_3=0)=\tilde{\mathcal{T}}(\alpha_3=1)=0$.

Tensor current

The last Lorentz structure we consider is $\Gamma_{\mu\nu5}=\sigma_{\mu\nu}\gamma_5$, which corresponds to a tensor current in the correlation function (5.1). After contracting the b-quarks and inserting the propagator we get

$$
\begin{aligned}
\Pi_{\mu\nu5}(q,P) =& i\int\frac{d^4x d^4k}{(2\pi)^4}\frac{e^{ix(q-k)}}{m_b^2-k^2}\Bigg(m_b\langle f_2^\lambda(P)|\,\bar{q}_1(x)\sigma_{\mu\nu}\underbrace{\gamma_5\gamma_5}_{\mathbb{1}}q_2(0)\,|0\rangle\\
&+\langle f_2^\lambda(P)|\,\bar{q}_1(x)\sigma_{\mu\nu}\underbrace{\gamma_5\,\not{k}}_{-\mathbb{1}}\gamma_5 q_2(0)\,|0\rangle\\
&+\int_0^1 dv\,\langle f_2^\lambda(P)|\,\bar{q}_1(x)\sigma_{\mu\nu}\gamma_5 g_s G^{\alpha\beta}(vx)\\
&\left(\frac{\not{k}+m_b}{2(m_b^2-k^2)}\sigma_{\alpha\beta}+\frac{vx_\alpha\gamma_\beta}{m_b^2-k^2}\right)i\gamma_5 q_2(0)\,|0\rangle\Bigg).
\end{aligned}
$$

Rewriting the Lorentz structures in terms of the DAs and performing the integrals we obtain

$$
\begin{aligned}
\Pi_{\mu\nu5}(q,P) =& \int_0^1 du\Bigg[\left(e_{q\mu}^{(\lambda)*}P_\nu-e_{q\nu}^{(\lambda)*}P_\mu\right)\Bigg(-\frac{2\,m_b\,m_{f_2}f_q^T\hat{A}(u)}{(m_b^2-k^2)^2}+\frac{12m_b^3 m_{f_2}^3 f_q^T\hat{\mathbb{A}}(u)}{(m_b^2-k^2)^4}\\
&+\frac{16\,m_b\,m_{f_2}^3 f_q^T\hat{\hat{\hat{B}}}(u)}{(m_b^2-k^2)^3}-\frac{4\,u\,m_b\,m_{f_2}^3 f_q^T\hat{\hat{C}}(u)}{(m_b^2-k^2)^3}+\frac{8(1-\delta_+)m_{f_2}^2 f_q(q^2+uP\cdot q)\hat{\hat{g}}_a(u)}{(m_b^2-k^2)^3}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{6(1-\delta_+)m_{f_2}^2 f_q \hat{\hat{g}}_a(u)}{(m_b^2-k^2)^2} - \frac{4m_{f_2}^2 f_q \hat{\hat{A}}_1(u)}{(m_b^2-k^2)^2} + \frac{2\,u\,m_{f_2}^2 f_q \hat{B}_1(u)}{(m_b^2-k^2)^2} + \frac{8(4m_b^2-k^2) f_q m_{f_2}^4 \hat{\hat{A}}_4(u)}{(m_b^2-k^2)^4}\Bigg) \\
&+\left(e_{q\mu}^{(\lambda)*}q_\nu - e_{q\nu}^{(\lambda)*}q_\mu\right)\Bigg(-\frac{4\,m_b\,m_{f_2}^3 f_q^T \hat{\hat{C}}(u)}{(m_b^2-k^2)^3} - \frac{8(1-\delta_+)\,m_{f_2}^2 f_q (uP^2+P\cdot q)\hat{\hat{g}}_a(u)}{(m_b^2-k^2)^3} \\
&+\frac{2\,m_{f_2}^2 f_q \hat{B}_1(u)}{(m_b^2-k^2)^2}\Bigg) \\
&+e_{qq}^{(\lambda)*}\left(P_\mu q_\nu - P_\nu q_\mu\right)\Bigg(-\frac{48\,m_b\,m_{f_2}^3 f_q^T \hat{\hat{\hat{B}}}(u)}{(m_b^2-k^2)^4} + \frac{8(1-\delta_+)\,m_{f_2}^2 f_q \hat{\hat{g}}_a(u)}{(m_b^2-k^2)^3} + \frac{8\,m_{f_2}^2 f_q \hat{\hat{A}}_1(u)}{(m_b^2-k^2)^3} \\
&-\frac{24(5m_b^2-k^2) f_q m_{f_2}^4 \hat{\hat{A}}_4(u)}{(m_b^2-k^2)^5}\Bigg)\Bigg] \\
&+\int_0^1 dv \int \mathcal{D}\underline{\alpha}\,\frac{f_q v\, m_{f_2}^2}{(m_b^2-\bar{k}^2)^2}\Bigg[\frac{8m_{f_2}^2}{m_b^2-\bar{k}^2}(\tilde{\tilde{\mathcal{V}}}(\underline{\alpha}) - \tilde{\tilde{\mathcal{A}}}(\underline{\alpha}))\Big((P_\mu e_{\nu q}^{(\lambda)*} - P_\nu e_{\mu q}^{(\lambda)*}) \\
&+\frac{3e_{qq}^{(\lambda)*}}{m_b^2-\bar{k}^2}(P_\mu q_\nu - P_\nu q_\mu)\Big) + 4\frac{m_{f_2}^2}{m_b^2-\bar{k}^2}(\tilde{\mathcal{V}}(\underline{\alpha}) - \tilde{\mathcal{A}}(\underline{\alpha}))(e_{\mu q}^{(\lambda)*}\bar{k}_\nu - e_{\nu q}^{(\lambda)*}\bar{k}_\mu) \\
&+2(\mathcal{V}(\underline{\alpha}) - \mathcal{A}(\underline{\alpha}))(P_\mu e_{\nu q}^{(\lambda)*} - P_\nu e_{\mu q}^{(\lambda)*})\Bigg],
\end{aligned}
\tag{5.8}
$$

where $k_\alpha = q_\alpha + uP_\alpha$ and $\bar{k}_\alpha = q_\alpha + (v\alpha_2 + \alpha_3)P_\alpha$.

The next step is to write the equations (5.6)-(5.8) as dispersion integrals in P'^2. The general structure, in simplified form, is

$$
\Pi_a(q,P) \sim \sum_{n=1}^{5} \int_0^1 du \frac{\mathcal{A}(u)}{D^n}, \tag{5.9}
$$

where $\mathcal{A}(u)$ is one of the DAs and the denominator is

$$
D = m_b^2 - (q+uP)^2.
$$

For the three-particle DAs the procedure is similar with the replacement $u \to v\alpha_2 + \alpha_3$. With the substitution

$$
\begin{aligned}
s(u) =& \frac{1}{u}(m_b^2 - \bar{u}q^2 + u\bar{u}m_{f_2}^2), \\
u(s) =& \frac{1}{2m_{f_2}^2}\Bigg[m_{f_2}^2 + q^2 - s \\
&+ \sqrt{(s-q^2-m_{f_2}^2)^2 + 4m_{f_2}^2(m_b^2-q^2)}\Bigg],
\end{aligned}
$$

we can write equation (5.9) as a dispersion integral in P'^2

$$
\Pi_a(q,P) \sim \frac{1}{\pi}\int_{m_b^2}^{\infty} \frac{ds}{s-P'^2}\mathrm{Im}_s\mathcal{A}(s),
$$

where we perform a partial integration if the power of the denominator is larger than one. Using quark-hadron duality to approximate the contributions of continuum and excited states

$$\int_{s_0^h}^{\infty} \frac{ds}{s-P'^2}\rho(s) \approx \frac{1}{\pi}\int_{s_0}^{\infty} \frac{ds}{s-P'^2}\mathrm{Im}_s\mathcal{A}(s),$$

where s_0 is the duality threshold and performing a Borel transformation, leads to the final sum rules

$$\begin{aligned}
\tilde{V}(q^2) =& \frac{(m_B+m_{f_2})m_b}{2f_Bm_B}e^{m_B^2/M^2}\frac{1}{\pi}\int_{m_b^2}^{s_0} ds\, e^{-s/M^2}\Big[8(1-\delta_+)m_bm_{f_2}^2f_{f_2}\mathrm{Im}_s\hat{\hat{g}}_a(s) \\
&-2m_{f_2}f_{f_2}^T\mathrm{Im}_s\hat{A}(s)-(3m_b^2+1)m_{f_2}^3f_{f_2}^T\mathrm{Im}_s\hat{\mathbb{A}}(s)\Big] \\
\tilde{A}_1(q^2) =& \frac{m_b\, e^{m_B^2/M^2}}{f_Bm_B(m_B+m_{f_2})}\frac{1}{\pi}\int_{m_b^2}^{s_0} ds\, e^{-s/M^2}\Big[2m_bm_{f_2}^2f_{f_2}\mathrm{Im}_s\hat{B}_1(s)-4m_{f_2}^3m_b^2f_{f_2}^T\mathrm{Im}_s\hat{\hat{C}}(s) \\
&-4(1-\delta_+^T)m_{f_2}^3f_{f_2}^T\mathrm{Im}_s\hat{\hat{h}}_{\parallel}^{(s)}(s)+8m_bm_{f_2}^4f_{f_2}\mathrm{Im}_s\hat{\hat{\hat{C}}}_1(s)-2m_{f_2}f_{f_2}^T(uP^2 \\
&+P\cdot q)\mathrm{Im}_s\hat{A}(s)+(3m_b^2+1)m_{f_2}^3f_{f_2}^T(uP^2+P\cdot q)\mathrm{Im}_s\hat{\mathbb{A}}(s)+16m_{f_2}^3f_{f_2}^T(uP^2 \\
&+P\cdot q)\mathrm{Im}_s\hat{\hat{\hat{B}}}(s)+8\int_0^u d\alpha_3\int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv\, f_{f_2}^Tm_{f_2}^5\mathrm{Im}_s\left(\tilde{\tilde{\mathcal{T}}}_1(\underline{\alpha})-\frac{m_{f_2}^2}{2}\tilde{\tilde{\mathcal{T}}}_2(\underline{\alpha})\right)\Bigg|_{\substack{\alpha_1=1-\alpha_2-\alpha_3\\ \alpha_2=\frac{u-\alpha_3}{v}}}\Bigg] \\
\tilde{A}_2(q^2) =& -\frac{(m_b+m_{f_2})m_b}{2f_Bm_B}e^{m_B^2/M^2}\frac{1}{\pi}\int_{m_b^2}^{s_0} ds\, e^{-s/M^2}\Big[8m_bm_{f_2}^2f_{f_2}\mathrm{Im}_s\hat{\hat{A}}_1(s) \\
&+24m_b^3m_{f_2}^4f_{f_2}\mathrm{Im}_s\hat{\hat{\phi}}_4(s)+2m_{f_2}f_{f_2}^T\mathrm{Im}_s\hat{A}(s)-(3m_b^2+1)m_{f_2}^3f_{f_2}^T\mathrm{Im}_s\hat{\mathbb{A}}(s) \\
&-8(1-\delta_+^T)um_{f_2}^3f_{f_2}^T\mathrm{Im}_s\hat{\hat{h}}_{\parallel}^{(s)}(s)+24um_bm_{f_2}^4f_{f_2}\mathrm{Im}_s\hat{\hat{\hat{C}}}_1(s)+4um_{f_2}^3f_{f_2}^T\mathrm{Im}_s\hat{\hat{C}}(s) \\
&-m_{f_2}^3f_{f_2}^T(56+48(uP\cdot q+q^2))\mathrm{Im}_s\hat{\hat{\hat{B}}}(s)+8\int_0^u d\alpha_3\int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv\, f_{f_2}^Tm_{f_2}^5\Big(3u\mathrm{Im}_s(\tilde{\tilde{\mathcal{T}}}_1(\underline{\alpha}) \\
&-\frac{m_{f_2}^2}{2}\tilde{\tilde{\mathcal{T}}}_2(\underline{\alpha}))-\frac{1}{m_{f_2}^2}\mathrm{Im}_s(\tilde{\tilde{\mathcal{T}}}_1(\underline{\alpha})-\frac{m_{f_2}^2}{2}\tilde{\tilde{\mathcal{T}}}_2(\underline{\alpha}))\Big)\Bigg|_{\substack{\alpha_1=1-\alpha_2-\alpha_3\\ \alpha_2=\frac{u-\alpha_3}{v}}}\Bigg] \\
\tilde{A}_0(q^2) =& \frac{q^2}{2m_{f_2}}\Bigg(\frac{m_b}{f_Bm_b}e^{m_B^2/M^2}\frac{1}{\pi}\int_{m_b^2}^{s_0} ds\, e^{-s/M^2}\Big[48m_{f_2}^3f_{f_2}^T(uP^2+P\cdot q)\mathrm{Im}_s\hat{\hat{\hat{B}}}(s) \\
&+24m_bm_{f_2}^4f_{f_2}\mathrm{Im}_s\hat{\hat{\hat{C}}}(s)-8(1-\delta_+^T)m_{f_2}^3f_{f_2}^T\mathrm{Im}_s\hat{\hat{h}}_{\parallel}^{(s)}(s)+4m_{f_2}^3f_{f_2}^T\mathrm{Im}_s\hat{\hat{C}}(s)
\end{aligned}$$

$$
+24\int_0^u d\alpha_3 \int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv\, m_{f_2}^5 f_{f_2}^T \mathrm{Im}_s\left(\tilde{\tilde{\mathcal{T}}}_1(\underline{\alpha}) - \frac{m_{f_2}^2}{2}\tilde{\tilde{\mathcal{T}}}_2(\underline{\alpha})\right)\Big|_{\substack{\alpha_1=1-\alpha_2-\alpha_3\\ \alpha_2=\frac{u-\alpha_3}{v}}}\Bigg] + \frac{\tilde{A}_2(q^2)}{m_B+m_{f_2}}\Bigg)
$$

$$
+\frac{m_b+m_{f_2}}{2m_{f_2}}\tilde{A}_1(q^2) - \frac{m_b-m_{f_2}}{2m_{f_2}}\tilde{A}_2(q^2)
$$

$$
\tilde{A}(q^2) = -\frac{m_b}{2f_B m_B}e^{m_B^2/M^2}\frac{1}{\pi}\int_{m_b^2}^{s_0} ds\, e^{-s/M^2}\Big[-2m_b m_{f_2} f_{f_2}^T \mathrm{Im}_s\hat{A}(s) + 3m_b^3 m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s\hat{\mathbb{A}}(s)
$$

$$
+16 m_b m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s\hat{\hat{\hat{B}}}(u) + 8(1-\delta_+)m_{f_2}^2 f_{f_2}\mathrm{Im}_s\hat{\hat{g}}_a(s) - 4m_{f_2}^2 f_{f_2}\mathrm{Im}_s\hat{\hat{A}}_1(s)
$$

$$
+2um_{f_2}^2 f_{f_2}\mathrm{Im}_s\hat{B}_1(s) + 2(3m_b^2+1)m_{f_2}^4 f_{f_2}\mathrm{Im}_s\hat{\hat{\phi}}_4(s)
$$

$$
+2\int_0^u d\alpha_3\int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv\, m_{f_2}^2 f_{f_2}\Big[\mathrm{Im}_s(\mathcal{V}(\underline{\alpha}) - \mathcal{A}(\underline{\alpha})) - 4m_{f_2}^2 \mathrm{Im}_s(\tilde{\tilde{\mathcal{V}}}(\underline{\alpha}) - \tilde{\tilde{\mathcal{A}}}(\underline{\alpha}))
$$

$$
+2um_{f_2}^2 \mathrm{Im}_s(\tilde{\mathcal{V}}(\underline{\alpha}) - \tilde{\mathcal{A}}(\underline{\alpha}))\Big]\Big|_{\substack{\alpha_1=1-\alpha_2-\alpha_3\\ \alpha_2=\frac{u-\alpha_3}{v}}}\Bigg]
$$

$$
\tilde{B}(q^2) = \tilde{A}(q^2) + \frac{m_b}{f_B m_B}e^{m_B^2/M^2}\frac{1}{\pi}\int_{m_b^2}^{s_0} ds\, e^{-s/M^2}\Big[-8(1-\delta_+)m_{f_2}^2 f_{f_2}(uP^2 + P\cdot q)\mathrm{Im}_s\hat{\hat{g}}_a(s)
$$

$$
+2m_{f_2}^2 f_{f_2}\mathrm{Im}_s\hat{B}_1(s) + 4\int_0^u d\alpha_3\int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv\, m_{f_2}^4 f_{f_2}\mathrm{Im}_s(\tilde{\mathcal{V}}(\underline{\alpha}) - \tilde{\mathcal{A}}(\underline{\alpha}))\Big|_{\substack{\alpha_1=1-\alpha_2-\alpha_3\\ \alpha_2=\frac{u-\alpha_3}{v}}}\Bigg]
$$

$$
\tilde{C}(q^2) = -\frac{(m_B^2-m_{f_2}^2)m_b}{2f_B m_B}e^{m_B^2/M^2}\frac{1}{\pi}\int_{m_b^2}^{s_0} ds\, e^{-s/M^2}\Big[-48 m_b m_{f_2}^3 f_{f_2}^T \mathrm{Im}_s\hat{\hat{\hat{B}}}(s)
$$

$$
+8(1-\delta_+)m_{f_2}^2 f_{f_2}\mathrm{Im}_s\hat{\hat{g}}_a(s) + 8m_{f_2}^2 f_{f_2}\mathrm{Im}_s\hat{\hat{A}}_1(s) - 6(4m_b^2+1)f_{f_2}\mathrm{Im}_s\hat{\hat{\phi}}_4(s)
$$

$$
+3\int_0^u d\alpha_3\int_{\frac{u-\alpha_3}{1-\alpha_3}}^1 dv\, m_{f_2}^2 f_{f_2}\mathrm{Im}_s(\tilde{\tilde{\mathcal{V}}}(\underline{\alpha}) - \tilde{\tilde{\mathcal{A}}}(\underline{\alpha}))\Big|_{\substack{\alpha_1=1-\alpha_2-\alpha_3\\ \alpha_2=\frac{u-\alpha_3}{v}}}\Bigg].
$$

Here we also used the replacement rules defined in equations (4.13).

5.2 Numerical analysis

In this section we want to analyze the form factors derived in the previous section numerically and compare our results to those of former studies. For this we use the following input values for the masses [41]

$$
m_{f_2} = 1.275 \text{ GeV}, \qquad m_B = 5.279 \text{ GeV},
$$

The decay constants are given, at a scale of $\mu = 1$ GeV, by [56, 57]

$$f_{f_2} = 0.101(10)\ \text{GeV}, \qquad f_{f_2}^T = 0.117(25)\ \text{GeV}.$$

The b-quark pole mass is given by $m_b = 4.8(1)$ GeV, and we use the tree level sum rule from [92] for the B-meson decay constant f_B. The scale dependent parameters are evaluated at the factorization scale $\mu_f = \sqrt{m_B^2 - m_b^2}$. The scaling behavior of the parameters can be found in Refs. [56, 57, 93, 94]. For the Borel parameter and duality threshold we choose values similar to those in a previous study of the B-meson [95]

$$M^2 = 4 - 8\ \text{GeV}^2 \qquad s_0 = 35.5 \pm 2\ \text{GeV}^2.$$

The values for the quark and quark-gluon couplings are given in the appendices E and D. We can parametrize the form factors by a simple three-parameter form

$$F(q^2) = \frac{\tilde{F}(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}, \tag{5.10}$$

with the fit parameters $\tilde{F}(0)$, a and b. We use the uncertainties from varying the input parameters and add them in quadrature as weights to perform a weighted fit. As already described in the previous section, if the errors are asymmetric we shift the central value to get symmetric ones. To determine the parameters $\tilde{F}(0)$, a, b and the corresponding errors, we use a least square fit. Our results from fitting the q^2 dependence within the range $0 \leq q^2 \leq q^2_{\text{max}} = (m_B - m_{f_2})^2$ are given in Table 5.1. In Figure 5.3 we show the q^2 dependence of our form factors. As we can see the three parameter form describes the form factors excellently. This is also confirmed by the values of $\chi^2/$ d.o.f. which are nearly zero as one would expect. The only exceptions

Form factor	$\tilde{F}(0)$	a	b
$\tilde{V}$	0.30 ± 0.03	2.38 ± 0.4	1.50 ± 0.73
$\tilde{A}_1$	0.17 ± 0.01	1.41 ± 0.50	0.35 ± 1.40
$\tilde{A}_2$	0.11 ± 0.02	1.84 ± 1.46	2.30 ± 4.09
$\tilde{A}_0$	0.22 ± 0.02	2.57 ± 0.77	1.89 ± 2.23
$\tilde{T}_1$	0.11 ± 0.02	2.14 ± 1.14	1.34 ± 3.19
$\tilde{T}_2$	0.12 ± 0.01	1.35 ± 1.24	1.11 ± 3.39
$\tilde{T}_3$	-0.02 ± 0.04	1.94 ± 17.51	0.71 ± 49.40

Table 5.1: Our results from fitting the form factors obtained by LCSR to the three-parameter form in (5.10).

are the two form factors $\tilde{A}_2(q^2)$ and $\tilde{T}_2(q^2)$. Their values for $q^2 \geq 10$ GeV^2 should be treated with caution and for these fits we only use values $0 \leq q^2 \leq 10$ GeV^2.

The form factor $\tilde{T}_3(q^2)$ is close to zero for the whole q^2 range because $\tilde{B}(q^2)$ and $\tilde{C}(q^2)$ have nearly the same magnitude but different signs, so we do not show the q^2 dependence. From Figure 5.2 we see that the contributions from the meson mass terms $\mathbb{A}(u)$ and $\phi_4(u)$ to the form factors are not negligible. In detail we find that for $q^2 = 0$ the effect of the meson mass terms is less than 30% for all the form factors. For $q^2 \neq 0$ the effect of the meson mass terms on the form factors $\tilde{A}_2(q^2)$, $\tilde{T}_1(q^2)$, $\tilde{T}_2(q^2)$ and $\tilde{T}_3(q^2)$ increases for higher values of q^2. For the form factors $\tilde{V}(q^2)$ and $\tilde{A}_1(q^2)$ the contributions stay under 30%. For the form factor $\tilde{A}_0(q^2)$ the effect of the meson mass terms is less than 13% for the whole range of q^2 although it depends on $\tilde{A}_1(q^2)$ and $\tilde{A}_2(q^2)$.
Now let us compare our results with other theoretical approaches as illustrated in Figure 5.2 for the $q^2 = 0$ values of the form factors. As one can see, our LCSR approach gives more precise results than the other approaches. In Ref. [85] a "pQCD" approach based on $k_\perp$ factorization is used, and we see that the discrepancies between our and their results are larger than the systematic uncertainties. Refs. [86, 87] are also LCSR calculations, and we can explain the discrepancies by the fact that we calculated higher order contributions. In all the cases the error bars are the result of varying the LCSR input parameters but the effect of neglected higher order terms was not studied. Therefore we should compare our gray bullets in Figure 5.2, which do not contain the meson mass corrections, with the green squares, which are the results from Ref. [86]. We see that even in this case there is a significant difference coming from the higher twist and three-particle DA contributions we take into account. The differences to our black bullets are even larger.

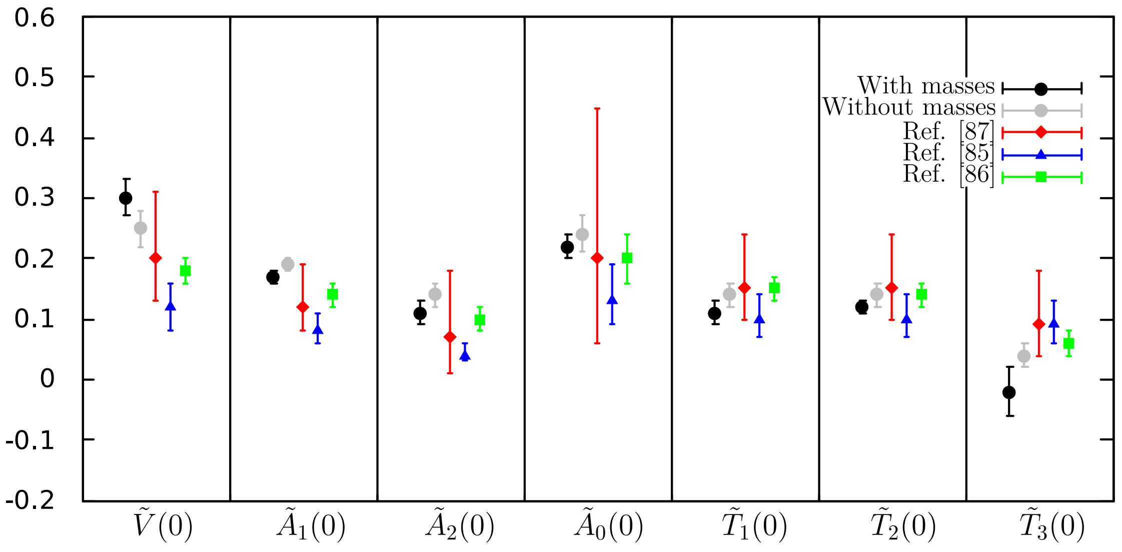

Figure 5.2: The values of the form factors for $q^2 = 0$ from different theoretical approaches.

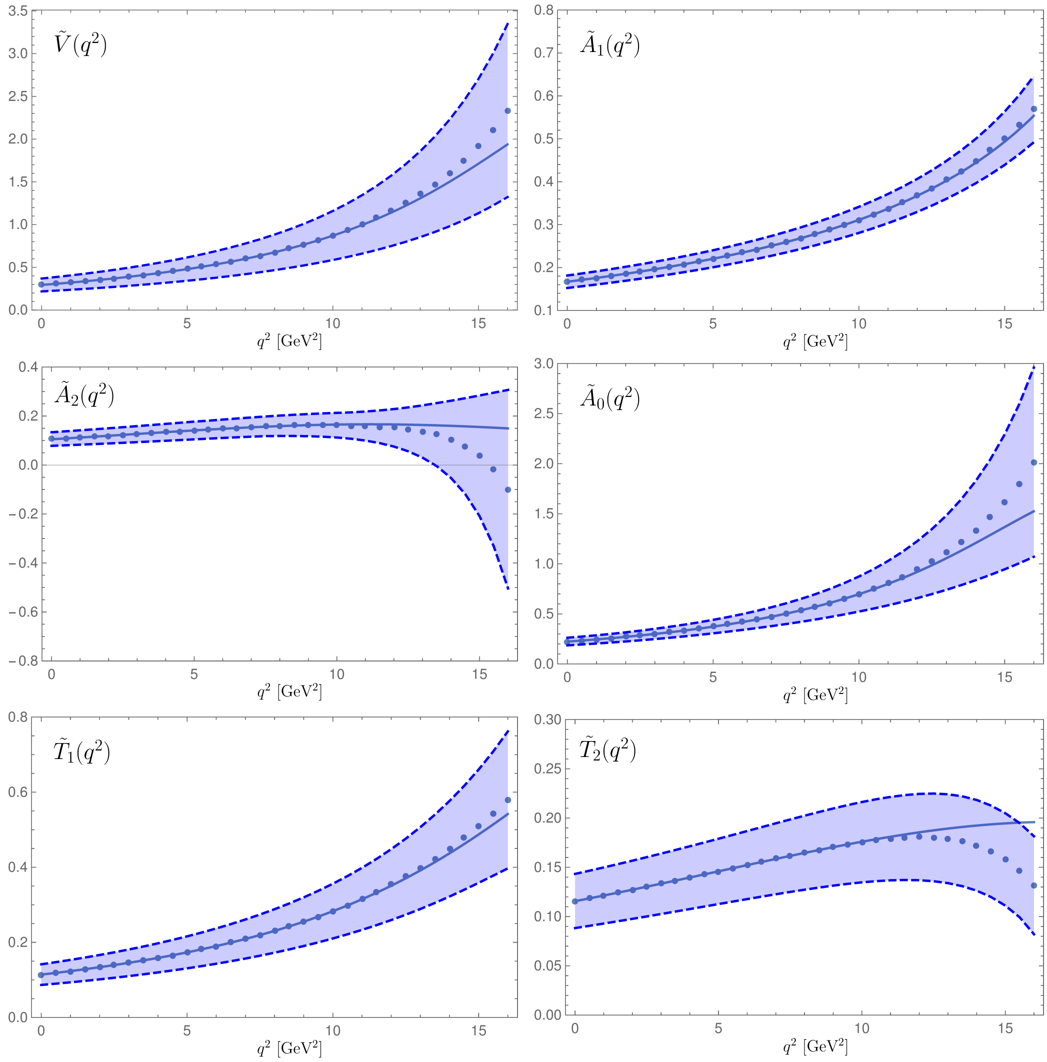

Figure 5.3: q^2 dependence of the form factors. The solid lines give the central value of the fits to the sum rule results (dots) and the dashed lines are the uncertainties from varying the input parameters.

Chapter 6

Summary and conclusion

6.1 Discussion of the results for $\Lambda_{b,c} \to N^* l\nu$ form factors and decay widths

In Chapter 4 we performed a leading order calculation to determine the $\Lambda_{b,c} \to N^*$ form factors and decay widths using the framework of light-cone sum rules and taking into account three-particle Fock-states up to twist-six. We followed the procedure from Ref. [64] by eliminating the $\Lambda^*_{b,c}$ contribution and used two different interpolating currents for the $\Lambda_{b,c}$. With this method we found that our sum rules are plagued by large numerical cancellations between different Lorentz structures. The only exceptions are the two form factors $f_1(q^2)$ and $g_1(q^2)$ if we use the pseudoscalar interpolating current. However these sum rules have the problem that there are large cancellations between contributions of different twist.
Therefore we decided to use another method namely extracting the highest power of p_+. For this method we observed, after comparing the two interpolating currents, that the axial-vector current yields better results for our calculation. With the axial-vector current we have a clear hierarchy of different twist contributions, fewer cancellations and a lower sensitivity to the unknown parameter ξ_{10}.
Our sum rules are dominated by the twist-four contributions, i.e. contributions of angular momentum, compared to the $\Lambda_{b,c} \to N$ case. This is due to the fact that N^* has a higher mass and a smaller normalization factor of the leading order DAs f_{N^*}. In Ref. [60] a similar, but less distinct, behavior was already observed.
Therefore, as already mentioned above, we give the predictions for the decay widths for the two different models separately. Even a rough experimental measurement would allow to distinguish the two models, as one can see from the predictions

$$\Gamma(\Lambda_b \to N^*(1535)l\nu) = \left(0.0058^{+0.0010}_{-0.0009}\right) \cdot \left(\frac{V_{ub}}{3.5 \cdot 10^{-3}}\right)^2, \quad \text{LCSR(1)},$$
$$\Gamma(\Lambda_b \to N^*(1535)l\nu) = \left(0.00070^{+0.00012}_{-0.00011}\right) \cdot \left(\frac{V_{ub}}{3.5 \cdot 10^{-3}}\right)^2, \quad \text{LCSR(2)},$$

$$\Gamma(\Lambda_c \to N^*(1535)l\nu) = \left(0.0064^{+0.0012}_{-0.0011}\right) \cdot \left(\frac{V_{cd}}{0.225}\right)^2, \quad \text{LCSR}(1),$$
$$\Gamma(\Lambda_c \to N^*(1535)l\nu) = \left(0.00077^{+0.00016}_{-0.00014}\right) \cdot \left(\frac{V_{cd}}{0.225}\right)^2, \quad \text{LCSR}(2).$$

The next step would be, with upcoming experimental data and increasing precision for the input parameters, a NLO-analysis since the NLO corrections are expected to be sizable. This will be a very extensive calculation because of the additional mass scale due to the heavy quark and additional structures contributing to the same order in the twist expansion. But with precise experimental data it will be possible to make quantitative statements about the low lying nucleon resonances in terms of the fundamental degrees of freedom of QCD.

6.2 Discussion of the results for $B \to f_2(1270)$ form factors

In Chapter 5 we calculated the $B \to f_2(1270)$ form factors with LCSR using chiral even and chiral odd tensor meson DAs, including, for the first time, twist four meson mass terms. We observed that mass terms have a noticeable impact on the sum rule calculations, and in order to reach high precision, they have to be included. Especially for the region of non-zero q^2 the contributions from these meson mass terms become non negligible. Therefore our results are in fact more precise than any previous study and future studies of tensor mesons should include these meson mass terms as well.
We determined the form factors to be

$$\begin{aligned}
\tilde{V}(0) &= 0.30 \pm 0.03, & \tilde{A}_1(0) &= 0.17 \pm 0.01,\\
\tilde{A}_2(0) &= 0.11 \pm 0.02, & \tilde{A}_0(0) &= 0.22 \pm 0.02,\\
\tilde{T}_1(0) &= 0.11 \pm 0.02, & \tilde{T}_2(0) &= 0.12 \pm 0.01,\\
\tilde{T}_3(0) &= -0.02 \pm 0.04, & &
\end{aligned}$$

for $q^2 = 0$ when fitting our sum rule result to a three-parameter form. The next step would be to calculate the form factors of B decays to other tensor mesons than $f_2(1270)$, e.g. $a_2(1320)$ or $K_2^*(1430)$. For these one should also consider additional $SU(3)$ breaking terms that we have already taken into account in our formulas for the tensor meson DAs (in the numerical analysis we neglected them because for u and d quarks they are small). Since these $SU(3)$ breaking terms depend on the quark mass they can yield important contributions for decays involving for example a strange quark.

Appendix A

Notations and conventions

In this appendix we give a short overview on the notations and conventions we used for the calculations. We used the sign conventions and notations from Bjorken and Drell; for more details see Ref. [30, 96].

Pauli matrices

The Pauli matrices are a set of three 2×2 complex matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{A.1}$$

which are Hermitian and unitary. They obey the following anti- and commutator relations

$$\{\sigma^i, \sigma^j\} = 2\delta^{ij}\mathbb{1}_2, \tag{A.2}$$

$$[\sigma^i, \sigma^j] = 2i\epsilon^{ijk}\sigma^k, \tag{A.3}$$

with the three dimensional Levi-Civita symbol

$$\epsilon^{ijk} = \begin{cases} +1, & \text{for an even permutation of } (1,2,3), \\ -1, & \text{for an odd permutation of } (1,2,3), \\ 0, & \text{otherwise.} \end{cases} \tag{A.4}$$

The Pauli matrices have the following basic properties

$$\det\, \sigma^i = -1, \qquad \mathrm{Tr}\, \sigma^i = 0, \qquad {\sigma^i}^\dagger = \sigma^i. \tag{A.5}$$

The Gell-Mann matrices

The special unitary group $SU(N)$ is the group of unitary $N \times N$ matrices with $UU^\dagger = 1$ and $\det U = 1$. The group $SU(N)$ is a compact, simply connected Lie group with $N^2 - 1$ generators and has rank $N - 1$. For QCD the group $SU(3)$ is relevant, and we give the basic properties of this group. For further reading see [97]. The eight generators of $SU(3)$ are denoted, in the fundamental representation by

$$T^a = \frac{\lambda^a}{2}, \tag{A.6}$$

with the Gell-Mann matrices λ^a.
The Gell-Mann matrices are a set of traceless Hermitian 3×3 matrices fulfilling the commutator relation

$$\left[\lambda^a, \lambda^b\right] = 2if^{abc}\lambda^c, \tag{A.7}$$

with f^{abc} being the totally antisymmetric structure constant of the $SU(3)$ group. An explicit representation of the Gell-Mann matrices is given by

$$\begin{aligned}
\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\
\lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\
\lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{aligned} \tag{A.8}$$

These matrices satisfy

$$\begin{aligned}
\mathrm{Tr}\left(\lambda^a\lambda^b\right) &= 2\delta^{ab}, \qquad \{\lambda^a, \lambda^b\} = \frac{4}{3}\delta^{ab} + 2d^{abc}\lambda^c, \\
\mathrm{Tr}\left(\lambda^a\lambda^b\lambda^c\right) &= 2(d^{abc} + if^{abc}).
\end{aligned} \tag{A.9}$$

The structure constants f^{abc} are the universal structure constants of $SU(3)$ which can be calculated, e.g. , from

$$f^{abc} = \frac{1}{4i}\mathrm{Tr}([\lambda_a, \lambda_b]\lambda_c). \tag{A.10}$$

The nonvanishing elements are

$$\begin{aligned}
f^{123} &= 1, \qquad f^{147} = f^{246} = f^{257} = f^{345} = \frac{1}{2}, \\
f^{156} = f^{367} &= -\frac{1}{2}, \qquad f^{458} = f^{678} = \frac{\sqrt{3}}{2}.
\end{aligned} \tag{A.11}$$

These constants are characteristic for $SU(3)$ and are independent from the representation chosen in equations (A.8). The totally symmetric structure constants d^{abc} can be calculated from

$$d^{abc} = \frac{1}{4}\mathrm{Tr}(\{\lambda_a, \lambda_b\}\lambda_c), \tag{A.12}$$

and the nonzero elements are

$$\begin{aligned}
d^{118} = d^{228} = d^{338} = -d^{888} &= \frac{1}{\sqrt{3}}, \qquad d^{448} = d^{558} = d^{668} = d^{778} = -\frac{1}{2\sqrt{3}}, \\
d^{146} = d^{157} = d^{256} = d^{344} = d^{355} &= \frac{1}{2}, \qquad d^{247} = d^{366} = d^{377} = -\frac{1}{2}.
\end{aligned} \tag{A.13}$$

Dirac matrices

The Dirac matrices are a set of four 4×4 matrices satisfying the Dirac algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}_4, \tag{A.14}$$

with the Minkowski metric $g = \mathrm{diag}(1, -1, -1, -1)$. An explicit representation is given by the so called Dirac representation

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \qquad \gamma^0 = \begin{pmatrix} \mathbb{1}_2 & 0 \\ 0 & -\mathbb{1}_2 \end{pmatrix}, \tag{A.15}$$

with the above defined Pauli matrices σ^i. A fifth gamma matrix can be defined

$$\gamma_5 = \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \frac{i}{4!}\epsilon_{\mu\nu\alpha\beta}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix}, \tag{A.16}$$

with the four dimensional Levi-Civita symbol

$$\epsilon_{\alpha\beta\mu\nu} = \begin{cases} +1, & \text{for an even permutation of } (0,1,2,3) \\ -1, & \text{for an odd permutation of } (0,1,2,3) \\ \ \ 0, & \text{otherwise} \end{cases} . \tag{A.17}$$

From this definition it is clear that $\epsilon^{0123} = -\epsilon_{0123} = -1$. The choice, whether the upper or the lower indices correspond to $+1$ or -1, is conventional and, as already mentioned, we remain with the Bjorken and Drell one[1]. With the definition (A.16) follows

$$\mathrm{Tr}(\gamma_5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta) = 4i\epsilon_{\mu\nu\alpha\beta}. \tag{A.18}$$

The matrix γ^5 has the following properties

$$(\gamma_5)^2 = \mathbb{1}_4, \qquad (\gamma_5)^\dagger = \gamma_5, \qquad \{\gamma_5, \gamma_\mu\} = 0. \tag{A.19}$$

Furthermore we can define the commutator of two gamma matrices by

$$\sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right].$$

In the definition of the N^* distribution amplitudes there appears the charge conjugation matrix C defined such that

$$(\gamma^\mu)^T = -C^{-1}\gamma^\mu C. \tag{A.20}$$

In the Dirac representation the charge conjugation matrix C can be chosen to be real, antisymmetric and unitary

$$C = i\gamma^2\gamma^0 = \begin{pmatrix} 0 & -i\sigma^2 \\ -i\sigma^2 & 0 \end{pmatrix}, \tag{A.21}$$

[1]The other convention would change the sign in equations (A.16, A.18).

and from this explicit form follows

$$C = -C^T = -C^\dagger = -C^{-1}. \tag{A.22}$$

We use the Feynman slash notation defined by

$$\not{A} = A_\mu \gamma^\mu, \tag{A.23}$$

for an arbitrary four vector A_μ.

Appendix B

N^* distribution amplitudes

In this appendix we will give the N^* DAs used to calculate the $\Lambda_{b,c} \to N^*$ form factors in Chapter 4.
The N^* light-cone DAs are defined as matrix elements of nonlocal operators with light-like separation and are easily derived from the nucleon DAs. The standard decomposition for the nucleon light-cone DAs involves 24 invariant functions and was derived in Ref. [98]. After some obvious replacements (see Ref. [60] for more details) the decomposition for N^* is:

$$\begin{aligned}
&4\,\langle 0|\,\epsilon^{ijk} u^i_\alpha(a_1 n) u^j_\beta(a_2 n) d^k_\gamma(a_3 n)\,|N^*(P)\rangle = \\
&= \mathcal{S}_1^{N^*} m_{N^*} C_{\alpha\beta}(u_{N^*})_\gamma - \mathcal{S}_2^{N^*} m_{N^*}^2 C_{\alpha\beta}(\not{n} u_{N^*})_\gamma + \mathcal{P}_1^{N^*} m_{N^*}(\gamma_5 C)_{\alpha\beta}(\gamma_5 u_{N^*})_\gamma \\
&+ \mathcal{P}_2^{N^*} m_{N^*}^2(\gamma_5 C)_{\alpha\beta}(\gamma_5 \not{n} u_{N^*})_\gamma - \left(\mathcal{V}_1^{N^*} + \frac{n^2 m_{N^*}^2}{4}\mathcal{V}_1^{N^*,M}\right)(\not{P} C)_{\alpha\beta}(u_{N^*})_\gamma \\
&+ \mathcal{V}_2^{N^*} m_{N^*}(\not{P} C)_{\alpha\beta}(\not{n} u_{N^*})_\gamma + \mathcal{V}_3^{N^*} m_{N^*}(\gamma_\mu C)_{\alpha\beta}(\gamma^\mu u_{N^*})_\gamma \\
&- \mathcal{V}_4^{N^*} m_{N^*}^2(\not{n} C)_{\alpha\beta}(u_{N^*})_\gamma - \mathcal{V}_5^{N^*} m_{N^*}^2(\gamma_\mu C)_{\alpha\beta}(i\sigma^{\mu\nu} n_\nu u_{N^*})_\gamma \\
&+ \mathcal{V}_6^{N^*} m_{N^*}^3(\not{n} C)_{\alpha\beta}(\not{n} u_{N^*})_\gamma - \left(\mathcal{A}_1^{N^*} + \frac{n^2 m_{N^*}^2}{4}\mathcal{A}_1^{N^*,M}\right)(\not{P}\gamma_5 C)_{\alpha\beta}(\gamma_5 u_{N^*})_\gamma \\
&+ \mathcal{A}_2^{N^*} m_{N^*}(\not{P}\gamma_5 C)_{\alpha\beta}(\not{n}\gamma_5 u_{N^*})_\gamma + \mathcal{A}_3^{N^*} m_{N^*}(\gamma_\mu\gamma_5 C)_{\alpha\beta}(\gamma^\mu\gamma_5 u_{N^*})_\gamma \\
&- \mathcal{A}_4^{N^*} m_{N^*}^2(\not{n}\gamma_5 C)_{\alpha\beta}(\gamma_5 u_{N^*})_\gamma - \mathcal{A}_5^{N^*} m_{N^*}^2(\gamma_\mu\gamma_5 C)_{\alpha\beta}(i\sigma^{\mu\nu} n_\nu\gamma_5 u_{N^*})_\gamma \\
&+ \mathcal{A}_6^{N^*} m_{N^*}^3(\not{n}\gamma_5 C)_{\alpha\beta}(\not{n}\gamma_5 u_{N^*})_\gamma - \left(\mathcal{T}_1^{N^*} + \frac{n^2 m_{N^*}^2}{4}\mathcal{T}_1^{N^*,M}\right)(P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma^\mu u_{N^*})_\gamma \\
&+ \mathcal{T}_2^{N^*} m_{N^*}(n^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(u_{N^*})_\gamma + \mathcal{T}_3^{N^*} m_{N^*}(\sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^{\mu\nu} u_{N^*})_\gamma \\
&+ \mathcal{T}_4^{N^*} m_{N^*}(P^\nu\sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^{\mu\rho} n_\rho u_{N^*})_\gamma - \mathcal{T}_5^{N^*} m_{N^*}^2(n^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma^\mu u_{N^*})_\gamma \\
&- \mathcal{T}_6^{N^*} m_{N^*}^2(n^\mu P^\nu i\sigma_{\mu\nu} C)_{\alpha\beta}(\not{n} u_{N^*})_\gamma - \mathcal{T}_7^{N^*} m_{N^*}^2(\sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^{\mu\nu}\not{n} u_{N^*})_\gamma \\
&+ \mathcal{T}_8^{N^*} m_{N^*}^3(n^\nu\sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^{\mu\rho} n_\rho u_{N^*})_\gamma,
\end{aligned} \tag{B.1}$$

including the $\mathcal{O}(n^2)$ corrections to the lowest twist-three part. For brevity we do not show the gauge links. Here α, β, γ are spinor indices and n_μ is an auxiliary light-like vector $n^2 = 0$. The calligraphic coefficients are related to the integrals containing the N^* DAs, depending on the momentum fractions x_i by the general relation

$$\mathcal{F}(a_1, a_2, a_3, (Pn)) = \int dx_1 dx_2 dx_3\, \delta(1 - x_1 - x_2 - x_3) e^{-i(Pn)\sum_i x_i a_i} F(x_i). \qquad \text{(B.2)}$$

In Table B.1 the corresponding DAs for the different functions $\mathcal{F}$ are given, and the resulting decomposition involves 27 N^* DAs. During the calculation we will use the following notations for the integrals over the DAs

$$\begin{aligned}
\tilde{F}(x_2) &= \int\limits_0^{1-x_2} dx_1 F(x_1, x_2, 1 - x_1 - x_2), \\
\tilde{\tilde{F}}(x_2) &= \int\limits_1^{x_2} dx_2' \int\limits_0^{1-x_2'} dx_1 F(x_1, x_2', 1 - x_1 - x_2'), \\
\tilde{\tilde{\tilde{F}}}(x_2) &= \int\limits_1^{x_2} dx_2' \int_1^{x_2'} dx_2'' \int\limits_0^{1-x_2''} dx_1 F(x_1, x_2'', 1 - x_1 - x_2''),
\end{aligned}$$

where F is one of the DAs.
In Refs. [98–100] these DAs are given up to the next-to-leading order in the conformal spin expansion. All the DAs depend on the scale μ, but for brevity we only write it down explicitly for the twist-three DAs.

- Twist-three DAs:

$$\begin{aligned}
V_1(x_i, \mu) &= 120 x_1 x_2 x_3 \left[\phi_3^0(\mu) + \phi_3^+(\mu)(1 - x_3)\right], \\
A_1(x_i, \mu) &= 120 x_1 x_2 x_3 (x_2 - x_1) \phi_3^-(\mu), \\
T_1(x_i, \mu) &= 120 x_1 x_2 x_3 \left[\phi_3^0(\mu) + \frac{1}{2}(\phi_3^-(\mu) - \phi_3^+(\mu)(1 - 3x_3))\right].
\end{aligned}$$

- Twist-four DAs:

$$\begin{aligned}
V_2(x_i) =& 24 x_1 x_2 \left[\phi_4^0 + \phi_4^+(1 - 5x_3)\right], \quad A_2(x_i, \mu) = 24 x_1 x_2 (x_2 - x_1) \phi_4^-, \\
T_2(x_i) =& 24 x_1 x_2 \left[\xi_4^0 + \xi_4^+(1 - 5x_3)\right], \\
V_3(x_i) =& 12 x_3 \left[\psi_4^0(1 - x_3) + \psi_4^+(1 - x_3 - 10 x_1 x_2) + \psi_4^-(x_1^2 + x_2^2 - x_3(1 - x_3))\right], \\
A_3(x_i) =& 12 x_3 (x_2 - x_1) \left[(\psi_4^0 + \psi_4^+) + \psi_4^-(1 - 2x_3)\right], \\
T_3(x_i) =& 6 x_3 \left[(\phi_4^0 + \psi_4^0 + \xi_4^0)(1 - x_3) + (\phi_4^+ + \psi_4^+ + \xi_4^+)(1 - x_3 - 10 x_1 x_2)\right. \\
& \left. + (\phi_4^- - \psi_4^- + \xi_4^-) x_1^2 + x_2^2 - x_3(1 - x_3))\right], \\
T_7(x_i) =& 6 x_3 \left[(\phi_4^0 + \psi_4^0 - \xi_4^0)(1 - x_3) + (\phi_4^+ + \psi_4^+ - \xi_4^+)(1 - x_3 - 10 x_1 x_2)\right. \\
& \left. + (\phi_4^- - \psi_4^- - \xi_4^-)(x_1^2 + x_2^2 - x_3(1 - x_3))\right], \\
S_1(x_i) =& 6 x_3 (x_2 - x_1) \left[(\phi_4^0 + \psi_4^0 + \xi_4^0 + \phi_4^+ + \psi_4^+ + \xi_4^+)\right.
\end{aligned}$$

$$
\begin{aligned}
&+(\phi_4^- - \psi_4^- + \xi_4^-)(1-2x_3)\Big] \,, \\
P_1(x_i) =& 6x_3(x_1-x_2)\Big[(\phi_4^0 + \psi_4^0 - \xi_4^0 + \phi_4^+ + \psi_4^+ - \xi_4^+) \\
&+(\phi_4^- - \psi_4^- - \xi_4^-)(1-2x_3)\Big] \,.
\end{aligned}
$$

- Twist-five DAs:

$$
\begin{aligned}
V_4(x_i) =& 3\Big[\psi_5^0(1-x_3) + \psi_5^+(1-x_3-2(x_1^2+x_2^2)) + \psi_5^-(2x_1x_2 - x_3(1-x_3))\Big] \,, \\
A_4(x_i) =& 3(x_2-x_1)\Big[-\psi_5^0 + \psi_5^+(1-2x_3) + \psi_5^- x_3\Big] \,, \\
T_4(x_i) =& \frac{3}{2}\Big[(\phi_5^0 + \psi_5^0 + \xi_5^0)(1-x_3) + (\phi_5^+ + \psi_5^+ + \xi_5^+)(1-x_3-2(x_1^2+x_2^2)) \\
&+(\phi_5^- - \psi_5^- + \xi_5^-)(2x_1x_2 - x_3(1-x_3))\Big] \,, \\
V_5(x_i) =& 6x_3\Big[\phi_5^0 + \phi_5^+(1-2x_3)\Big] \,, \quad A_5(x_i) = 6x_3(x_2-x_1)\phi_5^- \,, \\
T_5(x_i) =& 6x_3\Big[\xi_5^0 + \xi_5^+(1-2x_3)\Big] \,, \\
T_8(x_i) =& \frac{3}{2}\Big[(\phi_5^0 + \psi_5^0 - \xi_5^0)(1-x_3) + (\phi_5^+ + \psi_5^+ - \xi_5^+)(1-x_3-2(x_1^2+x_2^2)) \\
&(\phi_5^- - \psi_5^- - \xi_5^-)(2x_1x_2 - x_3(1-x_3))\Big] \\
S_2(x_i) =& \frac{3}{2}(x_2-x_1)\Big[-(\phi_5^0 + \psi_5^0 + \xi_5^0) + (\phi_5^+ + \psi_5^+ + \xi_5^+)(1-2x_3) \\
&+(\phi_5^- - \psi_5^- + \xi_5^-)x_3\Big] \,, \\
P_2(x_i) =& \frac{3}{2}(x_1-x_2)\Big[-(\phi_5^0 + \psi_5^0 - \xi_5^0) + (\phi_5^+ + \psi_5^+ - \xi_5^+)(1-2x_3) \\
&+(\phi_5^- - \psi_5^- - \xi_5^-)x_3\Big] \,.
\end{aligned}
$$

- Twist-six DAs:

$$
\begin{aligned}
V_6(x_i) =& 2\Big[\phi_6^0 + \phi_6^+(1-3x_3)\Big] \,, \quad A_6(x_i) = 2(x_2-x_1)\phi_6^- \,, \\
T_6(x_i) =& 2\Big[\phi_6^0 - \frac{1}{2}(\phi_6^+ - \phi_6^-)(1-3x_3)\Big] \,.
\end{aligned}
$$

Finally the expressions for n^2 corrections to the leading twist DAs V_1, A_1 and T_1 are given in the integrated form by [62, 68, 101]

$$
\begin{aligned}
\tilde{V}_1^M(x_2) &= \int_0^{1-x_2} dx_1 V_1^M(x_1, x_2, 1-x_1-x_2) = \frac{x_2^2}{24}(f_{N^*} C_f^u(x_2) + \lambda_1^{N^*} C_\lambda^u(x_2)), \\
\tilde{A}_1^M(x_2) &= \int_0^{1-x_2} dx_1 A_1^M(x_1, x_2, 1-x_1-x_2) = \frac{x_2^2}{24}(1-x_2)^3(f_{N^*} D_f^u(x_2) + \lambda_1^{N^*} D_\lambda^u(x_2)), \\
\tilde{T}_1^M(x_2) &= \int_0^{1-x_2} dx_1 T_1^M(x_1, x_2, 1-x_1-x_2) = \frac{x_2^2}{48}(f_{N^*} E_f^u(x_2) + \lambda_1^{N^*} E_\lambda^u(x_2)),
\end{aligned}
$$

with

$$\begin{aligned}
C^u_f(x_2) =&(1-x_2)^3\Big[113+495x_2-552x_2^2-10A^u_1(1-3x_2)\\
&+2V^d_1(113-951x_2+828x_2^2)\Big],\\
C^u_\lambda(x_2) =&-(1-x_2)^3\Big[13-20f^d_1+3x_2+10f^u_1(1-3x_2)\Big],\\
D^u_f(x_2) =&11+45x_2-2A^u_1(113-951x-2+828x_2^2)+10V^d_1(1-30x_2),\\
D^u_\lambda(x_2) =&29-45x_2-10f^u_1(7-9x_2)-20f^d_1(5-6x_2),\\
E^u_f(x_2) =&-\Big[(1-x_2)(3(439+71x_2-621x_2^2+587x_2^3-184x_2^4)\\
&+4A^u_1(1-x_2)^2(59-483x_2+414x_2^2)\\
&-4V^d_q(1301-619x_2-769x_2^2+1161x_2^3-414x_2^4))\Big]-12(73-220V^d_1)\ln x_2,\\
E^u_\lambda(x_2) =&-\Big[(1-x_2)(5-211x_2+281x_2^2-111x_2^3\\
&+10(1+61x_2-83x_2^2+33x_2^3)f^d_1-40(1-x_2)^2(2-3x_2)f^u_1)\Big]\\
&-12(3-10f^d_1)\ln x_2.
\end{aligned}$$

For our LCSR calculation the terms with $\ln x_2$ would require a special treatment, but we can omit these terms because they have negligible coefficients. Furthermore

$\mathcal{F}$	r.h.s of (B.2)	$\mathcal{F}$	r.h.s of (B.2)
$\mathcal{S}_1^{N^*}$	$S_1^{N^*}$	$2(P\cdot n)\mathcal{S}_2^{N^*}$	$S_1^{N^*}-S_2^{N^*}$
$\mathcal{P}_1^{N^*}$	$P_1^{N^*}$	$2(P\cdot n)\mathcal{P}_2^{N^*}$	$P_2^{N^*}-P_1^{N^*}$
$\mathcal{V}_1^{N^*}$	$V_1^{N^*}$	$2(P\cdot n)\mathcal{V}_2^{N^*}$	$V_1^{N^*}-V_2^{N^*}-V_3^{N^*}$
$2\mathcal{V}_3^{N^*}$	$V_3^{N^*}$	$4(P\cdot n)\mathcal{V}_4^{N^*}$	$-2V_1^{N^*}+V_3^{N^*}+V_4^{N^*}+2V_5^{N^*}$
$4(P\cdot n)\mathcal{V}_5^{N^*}$	$V_4^{N^*}-V_3^{N^*}$	$4(P\cdot n)^2\mathcal{V}_6^{N^*}$	$-V_1^{N^*}+V_2^{N^*}+V_3^{N^*}+V_4^{N^*}+V_5^{N^*}-V_6^{N^*}$
$\mathcal{A}_1^{N^*}$	$A_1^{N^*}$	$2(P\cdot n)\mathcal{A}_2^{N^*}$	$-A_1^{N^*}+A_2^{N^*}-A_3^{N^*}$
$2\mathcal{A}_3^{N^*}$	$A_3^{N^*}$	$4(P\cdot n)\mathcal{A}_4^{N^*}$	$-2A_1^{N^*}-A_3^{N^*}-A_4^{N^*}+2A_5^{N^*}$
$4(P\cdot n)\mathcal{A}_5^{N^*}$	$A_3^{N^*}-A_4^{N^*}$	$4(P\cdot n)^2\mathcal{A}_6^{N^*}$	$A_1^{N^*}-A_2^{N^*}+A_3^{N^*}+A_4^{N^*}-A_5^{N^*}+A_6^{N^*}$
$\mathcal{T}_1^{N^*}$	$T_1^{N^*}$	$2(P\cdot n)\mathcal{T}_2^{N^*}$	$T_1^{N^*}+T_2^{N^*}-2T_3^{N^*}$
$2\mathcal{T}_3^{N^*}$	$T_7^{N^*}$	$2(P\cdot n)\mathcal{T}_4^{N^*}$	$T_1^{N^*}-T_2^{N^*}-2T_7^{N^*}$
$4(P\cdot n)\mathcal{T}_7^{N^*}$	$T_7^{N^*}-T_8^{N^*}$	$2(P\cdot n)\mathcal{T}_5^{N^*}$	$-T_1^{N^*}+T_5^{N^*}+2T_8^{N^*}$
$\mathcal{A}_1^{N^*,M}$	$A_1^{N^*,M}$	$4(P\cdot n)^2\mathcal{T}_6^{N^*}$	$2T_2^{N^*}-2T_3^{N^*}-2T_4^{N^*}+2T_5^{N^*}+2T_7^{N^*}+2T_8^{N^*}$
$\mathcal{V}_1^{N^*,M}$	$V_1^{N^*,M}$	$4(P\cdot n)^2\mathcal{T}_8^{N^*}$	$-T_1^{N^*}+T_2^{N^*}+T_5^{N^*}-T_6^{N^*}+2T_7^{N^*}+2T_8^{N^*}$
$\mathcal{T}_1^{N^*,M}$	$T_1^{N^*,M}$		

Table B.1: Connection between the terms from the decomposition (B.1) and the N^* DAs via equation (B.2).

it is useful to introduce the following shorthand notation (following Ref. [68]) for the combinations of N^* DAs

$$S_{12}=S_1^{N^*}-S_2^{N^*},\quad P_{21}=P_2^{N^*}-P_1^{N^*},$$

$$V_{1345} = -2V_1^{N^*} + V_3^{N^*} + V_4^{N^*} + 2V_5^{N^*}, \quad V_{43} = V_4^{N^*} - V_3^{N^*},$$
$$V_{123456} = -V_1^{N^*} + V_2^{N^*} + V_3^{N^*} + V_4^{N^*} + V_5^{N^*} - V_6^{N^*}, \quad V_{123} = V_1^{N^*} - V_2^{N^*} - V_3^{N^*},$$

$$A_{1345} = -2A_1^{N^*} - A_3^{N^*} - A_4^{N^*} + 2A_5^{N^*}, \quad A_{34} = A_3^{N^*} - A_4^{N^*},$$
$$A_{123456} = A_1^{N^*} - A_2^{N^*} + A_3^{N^*} + A_4^{N^*} - A_5^{N^*} + A_6^{N^*}, \quad A_{123} = -A_1^{N^*} + A_2^{N^*} - A_3^{N^*},$$

$$T_{78} = T_7^{N^*} - T_8^{N^*}, \quad T_{123} = T_1^{N^*} + T_2^{N^*} - 2T_3^{N^*},$$
$$T_{234578} = 2T_2^{N^*} - 2T_3^{N^*} - 2T_4^{N^*} + 2T_5^{N^*} + 2T_7^{N^*} + 2T_8^{N^*}, \quad T_{127} = T_1^{N^*} - T_2^{N^*} - 2T_7^{N^*},$$
$$T_{125678} = -T_1^{N^*} + T_2^{N^*} + T_5^{N^*} - T_6^{N^*} + 2T_7^{N^*} + 2T_8^{N^*}, \quad T_{158} = -T_1^{N^*} + T_5^{N^*} + 2T_8^{N^*}.$$

The coefficients $\phi_i^{(\pm,0)}, \psi_i^{(\pm,0)}$ and $\xi_i^{(\pm,0)}$ appearing in the N^* DAs can be expressed through eight independent parameters [98].

- For leading conformal spin we have:

$$\phi_3^0 = \phi_6^0 = f_{N^*}, \qquad \phi_4^0 = \phi_5^0 = \frac{1}{2}(f_{N^*} + \lambda_1^{N^*}),$$
$$\xi_4^0 = \xi_5^0 = \frac{1}{6}\lambda_2^{N^*}, \qquad \psi_4^0 = \psi_5^0 = \frac{1}{2}(f_{N^*} - \lambda_1^{N^*}).$$

- For twist-three:

$$\phi_3^- = \frac{21}{2} f_{N^*} A_1^u, \qquad \phi_3^+ = \frac{7}{2} f_{N^*}(1 - 3V_1^d).$$

- For twist-four:

$$\begin{aligned}
\phi_4^+ =& \frac{1}{4}\left[f_{N^*}(3 - 10V_1^d) + \lambda_1^{N^*}(3 - 10f_1^d)\right],\\
\phi_4^- =& -\frac{5}{4}\left[f_{N^*}(1 - 2A_1^u) - \lambda_1^{N^*}(1 - 2f_1^d - 4f_1^u)\right],\\
\psi_4^+ =& -\frac{1}{4}\left[f_{N^*}(2 + 5A_1^u - 5V_1^d) - \lambda_1^{N^*}(2 - 5f_1^d - 5f_1^u)\right],\\
\psi_4^- =& \frac{5}{4}\left[f_{N^*}(2 - A_1^u - 3V_1^d) - \lambda_1^{N^*}(2 - 7f_1^d + f_1^u)\right],\\
\xi_4^+ =& \frac{1}{16}\lambda_2^{N^*}(4 - 15f_2^d), \qquad \xi_4^- = \frac{5}{16}\lambda_2^{N^*}(4 - 15f_2^d).
\end{aligned}$$

- For twist-five:

$$\begin{aligned}
\phi_5^+ =& -\frac{5}{6}\left[f_{N^*}(3 + 4V_1^d) - \lambda_1^{N^*}(1 - 4f_1^d)\right],\\
\phi_5^- =& -\frac{5}{3}\left[f_{N^*}(1 - 2A_1^u) - \lambda_1^{N^*}(f_1^d - f_1^u)\right],
\end{aligned}$$

$$
\begin{aligned}
\psi_5^+ =& -\frac{5}{6}\left[f_{N^*}(5+2A_1^u-2V-1^d)-\lambda_1^{N^*}(1-2f_1^d-2f_1^u)\right],\\
\psi_5^- =& \frac{5}{3}\left[f_{N^*}(2-A_1^u-3V_1^d)+\lambda_1^{N^*}(f_1^d-f_1^u)\right],\\
\xi_5^+ =& \frac{5}{36}=\lambda_2^{N^*}(2-9f_2^d), \qquad \xi_5^- = -\frac{5}{4}\lambda_2^{N^*}f_2^d.
\end{aligned}
$$

- For twist-six:

$$
\begin{aligned}
\phi_6^+ =& \frac{1}{2}\left[f_{N^*}(1-4V_1^d)-\lambda_1^{N^*}(1-2f_1^d)\right],\\
\phi_6^- =& \frac{1}{2}\left[f_{N^*}(1+4A_1^u)+\lambda_1^{N^*}(1-4f_1^d-2f_1^u)\right].
\end{aligned}
$$

The relations between the shape parameters given in Chapter 4.2 and the parameters V_1^d, A_1^d, f_1^d, f_1^u and f_2^d are given by [102]

$$
\begin{aligned}
A_1^u =& \varphi_{10}+\varphi_{11},\\
V_1^d =& \frac{1}{3}-\varphi_{10}+\frac{1}{3}\varphi_{11},\\
f_1^u =& \frac{1}{10}-\frac{1}{6}\frac{f_{N^*}}{\lambda_1^{N^*}}-\frac{3}{5}\eta_{10}-\frac{1}{3}\eta_{11},\\
f_1^d =& \frac{3}{10}-\frac{1}{6}\frac{f_{N^*}}{\lambda_1^{n^*}}+\frac{1}{5}\eta_{10}-\frac{1}{3}\eta_{11},\\
f_2^d =& \frac{4}{15}+\frac{2}{5}\xi_{10}.
\end{aligned}
$$

Appendix C

Correlation functions

In this appendix we will give the coefficient functions $w_{jn}^{(i)}(x, q^2)$ for the different transition and interpolating currents.

C.1 Vector transition current

Pseudoscalar interpolating current

$$
\begin{aligned}
w_{11}^{(\mathcal{P})} &= x_2 m_{N^*} \phi_1^{(\mathcal{P})}, \qquad w_{12}^{(\mathcal{P})} = x_2 m_{N^*}^3 \left[x_2 \phi_2^{(\mathcal{P})} + 2\phi_3^{(\mathcal{P})} \right], \\
w_{13}^{(\mathcal{P})} &= 4 x_2 m_{N^*}^2 m_c^2 \phi_3^{(\mathcal{P})}, \\
\\
w_{21}^{(\mathcal{P})} &= w_{23}^{(\mathcal{P})} = 0, \qquad w_{22}^{(\mathcal{P})} = x_2 m_{N^*}^2 \phi_2^{(\mathcal{P})}, \\
\\
w_{31}^{(\mathcal{P})} &= \frac{m_{N^*}}{2} (m_c + x_2 m_{N^*}) \phi_1^{(\mathcal{P})}, \\
w_{32}^{(\mathcal{P})} &= \frac{m_{N^*}^2}{2} \left[m_c (m_c + x_2 m_{N^*}) \phi_2^{(\mathcal{P})} + 2 x_2 m_{N^*}^2 \phi_3^{(\mathcal{P})} \right], \\
w_{33}^{(\mathcal{P})} &= 2 m_{N^*}^3 m_c^2 (m_c + x_2 m_{N^*}) \phi_3^{(\mathcal{P})}, \\
\\
w_{41}^{(\mathcal{P})} &= \frac{m_{N^*}}{2} \phi_1^{(\mathcal{P})}, \qquad w_{42}^{(\mathcal{P})} = \frac{m_{N^*}^2}{2} \left[m_c \phi_2^{(\mathcal{P})} + 2 m_{N^*} \phi_3^{(\mathcal{P})} \right], \\
w_{43}^{(\mathcal{P})} &= 2 m_{N^*}^3 m_c^2 \phi_3^{(\mathcal{P})}, \\
\\
w_{51}^{(\mathcal{P})} &= -m_{N^*} \phi_1^{(\mathcal{P})}, \qquad w_{52}^{(\mathcal{P})} = -m_{N^*}^3 \left[x_2 \phi_2^{(\mathcal{P})} + 2\phi_3^{(\mathcal{P})} \right], \\
w_{53}^{(\mathcal{P})} &= -4 m_{N^*}^3 m_c^2 \phi_3^{(\mathcal{P})}, \\
\\
w_{61}^{(\mathcal{P})} &= w_{63}^{(\mathcal{P})} = 0, \qquad w_{62}^{(\mathcal{P})} = -m_{N^*}^2 \phi_2^{(\mathcal{P})},
\end{aligned}
$$

where the functions $\phi_i^{\mathcal{P}}$ are

$$
\begin{aligned}
\phi_1^{(\mathcal{P})} =& 2\tilde{A}_1 + 4\tilde{A}_3 + 2\tilde{A}_{123} + 2\tilde{P}_1 + 2\tilde{S}_1 + 6\tilde{T}_1 - 12\tilde{T}_7 - \tilde{T}_{123} - 5\tilde{T}_{127} - 2\tilde{V}_1 + 4\tilde{V}_3 + 2\tilde{V}_{123}, \\
\phi_2^{(\mathcal{P})} =& 3\tilde{\tilde{A}}_{34} + 2\tilde{\tilde{A}}_{123} - \tilde{\tilde{A}}_{1345} - 2\tilde{\tilde{P}}_{21} + 2\tilde{\tilde{S}}_{12} - 12\tilde{\tilde{T}}_{78} - 2\tilde{\tilde{T}}_{123} - 4\tilde{\tilde{T}}_{127} - 6\tilde{\tilde{T}}_{158} + \tilde{\tilde{T}}_{234578} \\
& - 3\tilde{\tilde{V}}_{43} + 2\tilde{\tilde{V}}_{123} + \tilde{\tilde{V}}_{1345}, \\
\phi_3^{(\mathcal{P})} =& - \tilde{A}_1^M - 3\tilde{T}_1^M + \tilde{V}_1^M + \tilde{\tilde{\tilde{A}}}_{123456} - 3\tilde{\tilde{\tilde{T}}}_{125678} + \tilde{\tilde{\tilde{T}}}_{234578} + \tilde{\tilde{\tilde{V}}}_{123456}.
\end{aligned}
$$

Axial-vector interpolating current

$$
\begin{aligned}
w_{11}^{(\mathcal{A})} =& 2\left[-2m_c\phi_1^{(\mathcal{A})} - x_2 m_{N^*}(2\phi_1^{(\mathcal{A})} + \phi_2^{(\mathcal{A})}) + 2m_{N^*}\phi_3^{(\mathcal{A})}\right], \\
w_{12}^{(\mathcal{A})} =& 2m_{N^*}\left[x_2^2 m_{N^*}^2\phi_4^{(\mathcal{A})} - x_2 m_{N^*}m_c\phi_5^{(\mathcal{A})} + 2m_c^2\phi_3^{(\mathcal{A})} + 2x_2 m_{N^*}^2\phi_6^{(\mathcal{A})}\right], \\
w_{13}^{(\mathcal{A})} =& 8m_{N^*}^2 m_c\left[-m_c^2\phi_7^{(\mathcal{A})} + x_2 m_{N^*}m_c\phi_6^{(\mathcal{A})} - x_2^2 m_{N^*}^2\phi_8^{(\mathcal{A})}\right],
\end{aligned}
$$

$$
\begin{aligned}
w_{21}^{(\mathcal{A})} =& -4\phi_1^{(\mathcal{A})}, \qquad w_{23}^{(\mathcal{A})} = 8m_{N^*}^2 m_c\left[-m_c\phi_7^{(\mathcal{A})} - x_2 m_{N^*}\phi_8^{(\mathcal{A})}\right], \\
w_{22}^{(\mathcal{A})} =& 2m_{N^*}\left[2m_c\phi_3^{(\mathcal{A})} + x_2 m_{N^*}\phi_4^{(\mathcal{A})} - 2m_{N^*}\phi_7^{(\mathcal{A})}\right],
\end{aligned}
$$

$$
\begin{aligned}
w_{31}^{(\mathcal{A})} =& -2\frac{m_c^2 - q^2}{x_2}\phi_1^{(\mathcal{A})} + m_{N^*}m_c\phi_2^{(\mathcal{A})} - m_{N^*}^2\phi_9^{(\mathcal{A})} + 2x_2 m_{N^*}^2\phi_{10}^{(\mathcal{A})} + 2\frac{m_{N^*}^2}{x_2}\phi_7^{(\mathcal{A})}, \\
w_{32}^{(\mathcal{A})} =& m_{N^*}^2\Bigg[-2(q^2 - x_2^2 m_{N^*}^2)\phi_3^{(\mathcal{A})} + 2\frac{q^2 + m_c^2}{x_2}\phi_7^{(\mathcal{A})} + x_2 m_{N^*}m_c\phi_{11}^{(\mathcal{A})} \\
& -m_c^2(\phi_5^{(\mathcal{A})} - 2\phi_3^{(\mathcal{A})}) + 2m_{N^*}(m_c - x_2 m_{N^*})\phi_8^{(\mathcal{A})}\Bigg], \\
w_{33}^{(\mathcal{A})} =& 4\frac{m_c^2 m_{N^*}^2}{x_2}\left[-(m_c^2 - q^2)\phi_7^{(\mathcal{A})} + x_2 m_{N^*}m_c\phi_{12}^{(\mathcal{A})} - x_2^2 m_{N^*}^2\phi_8^{(\mathcal{A})}\right],
\end{aligned}
$$

$$
\begin{aligned}
w_{41}^{(\mathcal{A})} =& 2m_{N^*}\left[(\phi_1^{(\mathcal{A})} + \phi_{10}^{(\mathcal{A})}) - \frac{\phi_3^{(\mathcal{A})}}{x_2}\right], \qquad w_{43}^{(\mathcal{A})} = 4m_{N^*}^3 m_c^2\phi_{13}^{(\mathcal{A})}, \\
w_{42}^{(\mathcal{A})} =& \frac{m_{N^*}}{x_2}\left[2(m_c^2 + x_2^2 m_{N^*}^2 - q^2)\phi_3^{(\mathcal{A})} + x_2 m_{N^*}m_c\phi_{11}^{(\mathcal{A})} + 2x_2 m_{N^*}^2\phi_{13}^{(\mathcal{A})}\right],
\end{aligned}
$$

$$
\begin{aligned}
w_{51}^{(\mathcal{A})} =& 2m_{N^*}\phi_2^{(\mathcal{A})}, \qquad w_{53}^{(\mathcal{A})} = 8m_{N^*}^3 m_c\left[m_c\phi_{14}^{(\mathcal{A})} + x_2 m_{N^*}\phi_8^{(\mathcal{A})}\right], \\
w_{52}^{(\mathcal{A})} =& 2m_{N^*}^2\left[-x_2 m_{N^*}\phi_4^{(\mathcal{A})} + m_c(\phi_5^{(\mathcal{A})} + 2\phi_3^{(\mathcal{A})}) + 2m_{N^*}\phi_{14}^{(\mathcal{A})}\right],
\end{aligned}
$$

$$
w_{61}^{(\mathcal{A})} = 0, \qquad w_{62}^{(\mathcal{A})} = -2m_{N^*}^2\phi_4^{(\mathcal{A})}, \qquad w_{63}^{(\mathcal{A})} = 8m_{N^*}^3 m_c\phi_8^{(\mathcal{A})},
$$

where the functions $\phi_i^{\mathcal{A}}$ are

$$
\begin{aligned}
\phi_1^{(\mathcal{A})} &= \tilde{A}_1 + 2\tilde{T}_1 + \tilde{V}_1, \\
\phi_2^{(\mathcal{A})} &= 2\tilde{A}_3 - 2\tilde{P}_1 + 2\tilde{S}_1 - 2\tilde{T}_1 + \tilde{T}_{123} + \tilde{T}_{127} - 2\tilde{V}_3, \\
\phi_3^{(\mathcal{A})} &= \tilde{\tilde{A}}_{123} - \tilde{\tilde{T}}_{123} - \tilde{\tilde{T}}_{127} - \tilde{\tilde{V}}_{123}, \\
\phi_4^{(\mathcal{A})} &= -\tilde{\tilde{A}}_{34} + \tilde{\tilde{A}}_{1345} - 2\tilde{\tilde{P}}_{21} - 2\tilde{\tilde{S}}_{12} - 2\tilde{\tilde{T}}_{127} + 2\tilde{\tilde{T}}_{158} - \tilde{\tilde{T}}_{234578} - \tilde{\tilde{V}}_{43} + \tilde{\tilde{V}}_{1345}, \\
\phi_5^{(\mathcal{A})} &= -\tilde{\tilde{A}}_{34} - 2\tilde{\tilde{A}}_{123} - \tilde{\tilde{A}}_{1345} + 4\tilde{\tilde{T}}_{127} - 4\tilde{\tilde{T}}_{158} + 2\tilde{\tilde{T}}_{234578} - \tilde{\tilde{V}}_{43} + 2\tilde{\tilde{V}}_{123} - \tilde{\tilde{V}}_{1345}, \\
\phi_6^{(\mathcal{A})} &= \tilde{A}_1^M + \tilde{T}_1^M + \tilde{V}_1^M - \tilde{\tilde{\tilde{A}}}_{123456} + \tilde{\tilde{\tilde{T}}}_{125678} - \tilde{\tilde{\tilde{T}}}_{234578} + \tilde{\tilde{\tilde{V}}}_{123456}, \\
\phi_7^{(\mathcal{A})} &= -\tilde{A}_1^M - 2\tilde{T}_1^M - \tilde{V}_1^M + \tilde{\tilde{\tilde{T}}}_{234578}, \\
\phi_8^{(\mathcal{A})} &= \tilde{\tilde{\tilde{A}}}_{123456} - 2\tilde{\tilde{\tilde{T}}}_{125678} + \tilde{\tilde{\tilde{T}}}_{234578} - \tilde{\tilde{\tilde{V}}}_{123456}, \\
\phi_9^{(\mathcal{A})} &= -2\tilde{\tilde{A}}_{34} - 2\tilde{\tilde{A}}_{123} - 2\tilde{\tilde{P}}_{21} - 2\tilde{\tilde{S}}_{12} + 2\tilde{\tilde{T}}_{127} - 2\tilde{\tilde{T}}_{158} + \tilde{\tilde{T}}_{234578} - 2\tilde{\tilde{V}}_{43} + 2\tilde{\tilde{V}}_{123}, \\
\phi_{10}^{(\mathcal{A})} &= \tilde{A}_{123} - \tilde{T}_{123} - \tilde{T}_{127} - \tilde{V}_{123}, \\
\phi_{11}^{(\mathcal{A})} &= 2\tilde{\tilde{A}}_{34} + 2\tilde{\tilde{P}}_{21} + 2\tilde{\tilde{S}}_{12} + 2\tilde{\tilde{T}}_{123} + 2\tilde{\tilde{T}}_{158} - \tilde{\tilde{T}}_{234578} + 2\tilde{\tilde{V}}_{43}, \\
\phi_{12}^{(\mathcal{A})} &= \tilde{T}_1^M + \tilde{\tilde{\tilde{T}}}_{125678} - \tilde{\tilde{\tilde{T}}}_{234578}, \\
\phi_{13}^{(\mathcal{A})} &= -\tilde{A}_1^M - 2\tilde{T}_1^M - \tilde{V}_1^M - \tilde{\tilde{\tilde{A}}}_{123456} + 2\tilde{\tilde{\tilde{T}}}_{125678} + \tilde{\tilde{\tilde{V}}}_{123456}, \\
\phi_{14}^{(\mathcal{A})} &= \tilde{T}_1^M + \tilde{\tilde{\tilde{A}}}_{123456} - \tilde{\tilde{\tilde{T}}}_{125678} - \tilde{\tilde{\tilde{V}}}_{123456}.
\end{aligned}
$$

They agree with those in [64] if one replaces $m_N \to m_{N^*}$ and $m_c \to -m_c$.

C.2 Axial-vector transition current

We can obtain the coefficient functions $w_{jn}^{(i)}$ for the axial-vector transition current from the ones of the previous section by changing the sign at m_c and at $w_{2n}^{(\mathcal{P})}$, $w_{3n}^{(\mathcal{P})}$, $w_{6n}^{(\mathcal{P})}$, $w_{1n}^{(\mathcal{A})}$, $w_{4n}^{(\mathcal{A})}$ and $w_{5n}^{(\mathcal{A})}$.

Appendix D

Distribution amplitudes for the tensor meson f_2

In this appendix we will give the two- and three-particle DAs for the tensor meson f_2 that are needed for the LCSR calculations. We will use equations of motion (EOM) to express certain DAs in terms of leading twist two- and three-particle distributions.

D.1 Two- and three-particle distribution amplitudes

First we construct the two-particle DAs that can be classified into two categories: on the one hand the chiral even ones, and on the other hand the chiral odd ones. As the names suggest the chiral even ones correspond to chirality-conserving Dirac matrix structures while the chiral odd ones correspond to chirality-violating structures. For tensor mesons the chiral even quark-antiquark light-cone DAs have been defined as matrix elements of nonlocal light-ray operators in Refs. [56, 57, 103]

$$\begin{aligned}
\langle f_2(P,\lambda)|\,\bar{q}(z_2n)\gamma_\mu q(z_1n)\,|0\rangle =& f_{f_2}m_{f_2}^2\left[\frac{e_{nn}^{(\lambda)*}}{(p\cdot n)^2}p_\mu\int\limits_0^1 du\,e^{iz_{12}(p\cdot n)}\phi_2(u,\mu)\right.\\
&+\frac{e_{\perp\mu n}^{(\lambda)*}}{(p\cdot n)}\int\limits_0^1 du\,e^{iz_{12}(p\cdot n)}g_v(u,\mu)\\
&\left.-\frac{1}{2}n_\mu m_{f_2}^2\frac{e_{nn}^{(\lambda)*}}{(p\cdot n)^3}\int\limits_0^1 du\,e^{iz_{12}(p\cdot n)}g_4(u,\mu)\right],\\
\langle f_2(P,\lambda)|\,\bar{q}(z_2n)\gamma_\mu\gamma_5 q(z_1n)\,|0\rangle =& -if_{f_2}m_{f_2}^2\,(1-\delta_+)\,\epsilon_{\mu np\beta}\frac{e_{\beta n}^{(\lambda)*}}{(p\cdot n)^2}\int\limits_0^1 du\,e^{iz_{12}(p\cdot n)}g_a(u,\mu).
\end{aligned}\tag{D.1}$$

In the same manner we can construct the chiral odd DAs[1]

$$
\begin{aligned}
\langle f_2(P,\lambda)|\, \bar{q}(z_2 n)\sigma_{\mu\nu}q(z_1 n)\,|0\rangle =& i f_{f_2}^T \Bigg[m_{f_2}\frac{(e^{(\lambda)*}_{\perp n\mu}p_\nu - e^{(\lambda)*}_{\perp n\nu}p_\mu)}{(p\cdot n)}\int_0^1 du\, e^{iz_{12}(p\cdot n)}\phi_\perp(u,\mu) \\
&+ m_{f_2}^3 (p_\mu n_\nu - p_\nu n_\mu)\frac{e^{(\lambda)*}_{nn}}{(p\cdot n)^3}\int_0^1 du\, e^{iz_{12}(p\cdot n)} h_\parallel^{(t)}(u,\mu) \\
&+ \frac{1}{2}(e^{(\lambda)*}_{\perp n\mu}n_\nu - e^{(\lambda)*}_{\perp n\nu}n_\mu)\frac{m_{f_2}^3}{(p\cdot n)^2}\int_0^1 du\, e^{iz_{12}(p\cdot n)} h_4(u,\mu)\Bigg], \\
\langle f_2(P,\lambda)|\, \bar{q}(z_2 n)q(z_1 n)\,|0\rangle =& f_{f_2}^T \frac{e^{(\lambda)*}_{nn}}{(p\cdot n)^2} m_{f_2}^3 \left(1-\delta_+^T\right)\int_0^1 du\, e^{iz_{12}(p\cdot n)} h_\parallel^{(s)}(u,\mu),
\end{aligned}
\tag{D.2}
$$

with

$$
z_{12} = \bar{u}z_1 + u z_2, \quad \bar{u} = 1-u.
$$

Furthermore we have

$$
\begin{aligned}
e^{(\lambda)*}_{\perp\mu n} &\equiv g^\perp_{\mu\nu} e^{(\lambda)*}_{\nu n} = e^{(\lambda)*}_{\mu n} - \frac{e^{(\lambda)*}_{nn}}{(p\cdot n)}p_\mu + \frac{1}{2}n_\mu e^{(\lambda)*}_{nn}\frac{m_{f_2}^2}{(p\cdot n)^2}, \\
g^\perp_{\mu\nu} &= g_{\mu\nu} - \frac{1}{p\cdot n}\left(n_\mu p_\nu + n_\nu p_\mu\right),
\end{aligned}
$$

where the vectors n_μ and $p_\mu = P_\mu - \frac{1}{2}n_\mu \frac{m_{f_2}^2}{p\cdot n}$ are light-like, $n^2 = p^2 = 0$. We parametrize the SU(3) breaking terms by

$$
\delta_\pm = \frac{f_{f_2}^T}{f_{f_2}}\frac{m_{\bar{q}} \pm m_q}{m_{f_2}}, \qquad \delta_\pm^T = \frac{f_{f_2}}{f_q^T}\frac{m_{\bar{q}} \pm m_q}{m_{f_2}}.
$$

For the DAs defined in equations (D.1) and (D.2) we have the following symmetry relations:

$$
\begin{gathered}
\phi_{2/\perp}(u) = -\phi_{2/\perp}(\bar{u}), \quad g_v(u) = -g_v(\bar{u}), \quad g_a(u) = g_a(\bar{u}), \\
h_\parallel^{(s)}(u) = h_\parallel^{(s)}(\bar{u}), \quad h_\parallel^{(t)}(u) = -h_\parallel^{(t)}(\bar{u}),
\end{gathered}
$$

and the normalization is

$$
\int_0^1 du\,(2u-1)\Phi(u) = 1 \text{ for } \Phi \in \{\phi_2(u), -\phi_\perp(u), g_v(u), -h_\parallel^{(t)}(u)\}.
$$

[1] In Ref. [57] the authors already defined the chiral odd DAs but with slightly different prefactors and without the meson mass and SU(3) breaking terms.

The integral of the DAs $g_a(u)$ and $h_\parallel^{(s)}(u)$ vanishes. The DAs $g_4(u)$ and $h_4(u)$ can be expressed in terms of $\phi_2(u)$ and $\phi_\perp(u)$, see below. The nonlocal operators on the left hand side of equations (D.1) and (D.2) are renormalized at a scale μ which results in the DAs also depending on μ. For brevity we will not write down the μ dependence explicitly but we keep in mind that the DAs are scale depended. The scale dependence of all the DAs and coupling constants can be found in the references, as mentioned in the previous Chapters.
Close to the light-cone $x^2 \to 0$ the OPE of the chiral odd DAs takes the form

$$
\begin{aligned}
\langle f_2(P,\lambda)|\,\bar{q}(x)\gamma_\mu q(-x)\,|0\rangle &= f_{f_2} m_{f_2}^2 \left[\frac{e_{xx}^{(\lambda)*}}{(P\cdot x)^2} P_\mu \int_0^1 du\, e^{i\xi(P\cdot x)} \left[A_1(u) + \frac{1}{4} x^2 m_{f_2}^2 A_4(u) \right] \right. \\
&\left. + \frac{e_{\mu x}^{(\lambda)*}}{(P\cdot x)} \int_0^1 du\, e^{i\xi(P\cdot x)} B_1(u) + \frac{1}{2} m_{f_2}^2 x_\mu \frac{e_{xx}^{(\lambda)*}}{(P\cdot x)^3} \int_0^1 du\, e^{i\xi(P\cdot x)} C_1(u) \right], \\
\langle f_2(P,\lambda)|\,\bar{q}(x)\gamma_\mu\gamma_5 q(-x)\,|0\rangle &= -i f_{f_2} m_{f_2}^2 \left(1-\delta_+\right) \epsilon_{\mu\nu\alpha\beta} \frac{x^\nu P^\alpha e_{\beta x}^{(\lambda)*}}{(P\cdot x)^2} \int_0^1 du\, e^{i\xi(P\cdot x)} g_a(u), \\
\langle f_2(P,\lambda)|\,\bar{q}(x)\sigma_{\mu\nu} q(-x)\,|0\rangle &= i f_{f_2}^T \left[m_{f_2} \frac{(e_{x\mu}^{(\lambda)*} P_\nu - e_{x\nu}^{(\lambda)*} P_\mu)}{(P\cdot x)} \int_0^1 du\, e^{i\xi(P\cdot x)} \Big[A(u) \right. \\
&+ \frac{1}{4} x^2 m_{f_2}^2 \mathbb{A}(u) \Big] + m_{f_2}^3 (P_\mu x_\nu - P_\nu x_\mu) \frac{e_{xx}^{(\lambda)*}}{(P\cdot x)^3} \int_0^1 du\, e^{i\xi(P\cdot x)} B(u) \\
&\left. + \frac{1}{2} (e_{x\mu}^{(\lambda)*} x_\nu - e_{x\nu}^{(\lambda)*} x_\mu) \frac{m_{f_2}^3}{(P\cdot x)^2} \int_0^1 du\, e^{i\xi(P\cdot x)} C(u) \right], \\
\langle f_2(P,\lambda)|\,\bar{q}(x) q(-x)\,|0\rangle &= f_{f_2}^T \frac{e_{xx}^{(\lambda)*}}{(P\cdot x)^2} m_{f_2}^3 \left(1-\delta_+^T\right) \int_0^1 du\, e^{i\xi(P\cdot x)} h_\parallel^{(s)}(u),
\end{aligned}
$$

with two new two-particle twist-four DAs $A_4(u)$ and $\mathbb{A}(u)$ which we can express in terms of the other DAs using QCD EOM, see below and $\xi = 2u-1$. By comparing to equations (D.1) and (D.2) we find

$$
\begin{aligned}
A_1(u) &= \phi_2(u) - g_v(u), \\
B_1(u) &= g_v(u), \\
C_1(u) &= 2g_v(u) - \phi_2(u) - g_4(u), \\
A(u) &= \phi_\perp(u), \\
B(u) &= h_\parallel^{(t)}(u) - \frac{\phi_\perp(u)}{2} - \frac{h_4(u)}{2}, \\
C(u) &= h_4(u) - \phi_\perp(u).
\end{aligned}
$$

In equations (D.1) and (D.2), the two particle DAs $\phi_2(u)$, $\phi_\perp(u)$ are leading twist-two, $g_v(u)$, $g_a(u)$, $h_\parallel^{(t)}(u)$, $h_\parallel^{(s)}(u)$ are collinear twist-three and $g_4(u)$, $h_4(u)$ are twist-four.

For the leading twist DAs we will use the asymptotic form

$$\phi_2(u) = -\phi_\perp(u) = 30u\bar{u}(2u-1),$$

where we defined $\phi_\perp(u)$ with a minus sign so that we have the same signs in equation (D.2) as in Ref. [57] from which we take the value for f_{f_2}. Furthermore we also need three-particle quark-antiquark-gluon DAs. In Ref. [56] these three-particle DAs with vector and axial-vector structure are already defined

$$\begin{aligned}
&\langle f_2(P,\lambda)|\,\bar{q}(z_3n)ig_sG_{\mu\nu}(z_2n)\gamma_\alpha q(z_1n)\,|0\rangle = \\
&\qquad -f_{f_2}m_{f_2}^2\frac{p_\alpha}{p\cdot n}\left[p_\mu e^{(\lambda)}_{\perp n\nu} - p_\nu e^{(\lambda)}_{\perp n\mu}\right]\int \mathcal{D}\alpha\, e^{ip\cdot n\sum \alpha_k z_k}\mathcal{V}(\alpha) + \cdots\,, \\
&\langle f_2(P,\lambda)|\,\bar{q}(z_3n)g_s\widetilde{G}_{\mu\nu}(z_2n)\gamma_\alpha\gamma_5 q(z_1n)\,|0\rangle = \\
&\qquad -f_{f_2}m_{f_2}^2\frac{p_\alpha}{p\cdot n}\left[p_\mu e^{(\lambda)}_{\perp n\nu} - p_\nu e^{(\lambda)}_{\perp n\mu}\right]\int \mathcal{D}\alpha\, e^{ip\cdot n\sum \alpha_k z_k}\mathcal{A}(\alpha) + \cdots\,.
\end{aligned}$$

Here, we define a new one for tensor structures

$$\begin{aligned}
&\langle f_2(P,\lambda)|\,\bar{q}(z_3n)\sigma_{\alpha\beta}g_sG_{\mu\nu}(z_2n)q(z_1n)\,|0\rangle = \\
&-f^T_{f_2}\frac{e^{(\lambda)*}_{nn}}{2(p\cdot n)^2}m^3_{f_2}\left[p_\alpha p_\mu g^\perp_{\beta\nu} - p_\beta p_\mu g^\perp_{\alpha\nu} - p_\alpha p_\nu g^\perp_{\beta\mu} + p_\beta p_\nu g^\perp_{\alpha\mu}\right]\int \mathcal{D}\alpha\, e^{ip\cdot n\sum\alpha_k z_k}\mathcal{T}_1(\alpha) \\
&+f^T_{f_2}\frac{m^3_{f_2}}{2}\left[p_\alpha p_\mu e^{\perp(\lambda)*}_{\nu\beta} - p_\beta p_\mu e^{\perp(\lambda)*}_{\nu\alpha} - p_\alpha p_\nu e^{\perp(\lambda)*}_{\mu\beta} + p_\beta p_\nu e^{\perp(\lambda)*}_{\mu\alpha}\right]\int \mathcal{D}\alpha e^{ip\cdot n\sum\alpha_k z_k}\mathcal{T}_2(\alpha) + \cdot\cdot,
\end{aligned}$$

with $e^{\perp(\lambda)*}_{\mu\nu} = e^{(\lambda)*}_{\mu'\nu'}g^\perp_{\mu'\mu}g^\perp_{\nu'\nu}$ (not to be mistaken with $e^{(\lambda)*}_{\perp\mu n} = g^\perp_{\mu\nu}e^{(\lambda)*}_{\nu n}$). The integration measure is defined by

$$\int \mathcal{D}\alpha \equiv \int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \int_0^1 d\alpha_3\, \delta(1-\sum\alpha_k).$$

Here we only give the leading twist DAs . The asymptotic form of the three-particle DAs is taken from Refs. [55, 94]

$$\begin{aligned}
\mathcal{V}(\alpha) &= 360\alpha_1\alpha_2^2\alpha_3\left[\xi_3 + \frac{1}{2}\omega_3(7\alpha_2-3)\right], \\
\mathcal{A}(\alpha) &= 360\alpha_1\alpha_2^2\alpha_3\left[\frac{1}{2}\widetilde{\omega}_3(\alpha_1-\alpha_3)\right], \\
\mathcal{T}_{1/2}(\alpha) &= 360\alpha_1\alpha_2^2\alpha_3\left[\xi_3^{\mathcal{T}_{1/2}} + \frac{1}{2}\omega_3^{\mathcal{T}_{1/2}}(7\alpha_2-3)\right].
\end{aligned}$$

The constants ξ_3, ω_3 and $\widetilde{\omega}_3$ have been determined in Ref. [56] with the help of QCD sum rules and read at a scale of 1 GeV

$$\xi_3 = 0.15(8), \qquad \omega_3 = -0.2(3), \qquad \widetilde{\omega}_3 = 0.06(1).$$

Also by using QCD sum rules (see appendix E for the detailed calculation) we get

$$\left(\frac{m_{f_2}^2\xi_3^{\mathcal{T}_2}}{2}-\xi_3^{\mathcal{T}_1}\right)=0.16(3),$$
$$\left(\frac{m_{f_2}^2\omega_3^{\mathcal{T}_2}}{2}-\omega_3^{\mathcal{T}_1}\right)=-0.33(16).$$

D.2 Equations of motion

In this section we express the twist-three and -four DAs in terms of the leading twist two-particle and three-particle DAs with the help of QCD EOM. We follow the approach from Refs. [90, 94].

Chiral-even DAs

For the chiral-even DAs we start with the following two identities:

$$\begin{aligned}\bar{q}(x)\gamma_\mu q(-x) &= \int_0^1 dt\frac{\partial}{\partial x_\mu}\bar{q}(tx)\not{x}q(-tx) - i\epsilon_{\mu\nu\alpha\beta}\int_0^1 dt\,t\,x^\nu\partial^\alpha\left[\bar{q}(tx)\gamma^\beta\gamma_5 q(-tx)\right]\\ &- \int_0^1 dt\int_{-t}^{t} dv\bar{q}(tx)x^\nu\not{x}\left[tg_s\widetilde{G}_{\mu\nu}(vx)\gamma_5+ivg_sG_{\mu\nu}(vx)\right]q(-tx)\\ &+(m_{\bar{q}}-m_q)\int_0^1 dt\,t\,\bar{q}(tx)\sigma_{x\mu}q(-tx),\end{aligned}\tag{D.3}$$

and

$$\begin{aligned}\bar{q}(x)\gamma_\mu\gamma_5 q(-x) &= \int_0^1 dt\frac{\partial}{\partial x_\mu}\bar{q}(tx)\not{x}\gamma_5 q(-tx) - i\epsilon_{\mu\nu\alpha\beta}\int_0^1 dt\,t\,x^\nu\partial^\alpha\left[\bar{q}(tx)\gamma^\beta q(-tx)\right]\\ &- \int_0^1 dt\int_{-t}^{t} dv\,\bar{q}(tx)x^\nu\not{x}\left[tg_s\widetilde{G}_{\mu\nu}(vx)+ivg_sG_{\mu\nu}(vx)\gamma_5\right]q(-tx)\\ &+(m_{\bar{q}}+m_q)\int_0^1 dt\,t\,\bar{q}(tx)\sigma_{x\mu}\gamma_5 q(-tx).\end{aligned}\tag{D.4}$$

Here we used a short hand notation for the derivative

$$\partial_\alpha\left[\bar{q}(tx)\Gamma q(-tx)\right]\equiv\frac{\partial}{\partial y^\alpha}\left[\bar{q}(tx+y)\Gamma q(-tx+y)\right]\Big|_{y\to 0},$$

with a general Dirac matrix structure Γ. Here we see that the terms proportional to the quark masses break the SU(3) symmetry and relate the twist three DAs in the even (odd) sector to the leading twist DA from the odd (even) sector. After sandwiching them between the f_2 meson state and the vacuum, the operators on both sides of equations (D.3) and (D.4) can be expressed in terms of the DAs defined in the previous section. After dropping all corrections of order x^2 and setting $x_\mu = n_\mu$ we get a system of integral equations:

$$\begin{aligned}\int_0^1 du\, e^{i\xi(p\cdot n)} g_v(u) =& 2\int_0^1 dt \int_0^1 du\, e^{i\xi(p\cdot n)t}\phi_2(u) \\ &+ (1-\delta_+)(ip\cdot n)\int_0^1 dt\, t\int_0^1 du\, e^{i\xi(p\cdot n)t} g_a(u) \\ &+ (ip\cdot n)^2\int_0^1 dt\, t^2\int_{-1}^1 dv\, [\mathcal{A}(v,p\cdot nt) + v\mathcal{V}(v,p\cdot nt)] \\ &- \delta_-(ip\cdot n)\int_0^1 dt\, t\int_0^1 du\, e^{i\xi(p\cdot n)t}\phi_\perp(u),\end{aligned} \tag{D.5}$$

and

$$\begin{aligned}(1-\delta_+)\int_0^1 du\, e^{i\xi(p\cdot n)} g_a(u) =& (ip\cdot n)\int_0^1 dt\, t\int_0^1 du\, e^{i\xi(p\cdot n)t} g_v(u) \\ &- \delta_+(ip\cdot n)\int_0^1 dt\, t\int_0^1 du\, e^{i\xi(p\cdot n)t}\phi_\perp(u) \\ &- (ip\cdot n)^2\int_0^1 dt\, t^2\int_{-1}^1 dv\, [\mathcal{V}(v,p\cdot nt) + v\mathcal{A}(v,p\cdot nt)]\,.\end{aligned} \tag{D.6}$$

Here we use the shorthand notation for the three-particle DAs

$$\mathcal{V}(v,p\cdot nt) = \int \mathcal{D}\alpha\, e^{i(p\cdot n)t(-\alpha_1+v\alpha_2+\alpha_3)}\mathcal{V}(\underline{\alpha}),$$

and the same for $\mathcal{A}(v,p\cdot nt)$.
Now it is convenient to consider moments in an intermediate state to simplify the algebra. Therefore we define

$$M_n^{\phi_{2,\perp}} = \int_0^1 du\, \xi^n \phi_{2,\perp}(u), \qquad M_n^{a,v} = \int_0^1 du\, \xi^n g_{a,v}(u), \tag{D.7}$$

and for the three-particle DAs

$$\mathcal{V}_n(v) = (i)^n \frac{\partial^n}{\partial\tau^n}\mathcal{V}(v,\tau)\bigg|_{\tau\to 0} = \int \mathcal{D}\underline{\alpha}\,(-\alpha_1+v\alpha_2+\alpha_3)^n\mathcal{V}(\alpha_1,\alpha_2,\alpha_3), \tag{D.8}$$

and the same for $\mathcal{A}_n(v)$. Expanding equations (D.5) and (D.6) in powers of $(p \cdot n)$ leads to

$$\begin{aligned}
(n+1)M_n^v =& 2M_n^{\phi_2} + (1-\delta_+)nM_{n-1}^a - \delta_- n M_n^{\phi_\perp} \\
& + n(n-1)\int_{-1}^{1} dv \left[\mathcal{A}_{n-2}(v) + v\mathcal{V}_{n-2}(v)\right], \\
(1-\delta_+)(n+1)M_n^a =& nM_{n-1}^v - \delta_+ n M_{n-1}^{\phi_\perp} - n(n-1)\int_{-1}^{1} dv \left[\mathcal{V}_{n-2}(v) + v\mathcal{A}_{n-2}(v)\right].
\end{aligned}$$

Combining these two equations leads to recurrence equations for $g_a(u)$ and $g_v(u)$

$$\begin{aligned}
&(1-\delta_+)\Big((n+1)M_n^a - (n-1)M_{n-2}^a\Big) = 2M_{n-1}^{\phi_2} - \delta_-(n-1)M_{n-1}^{\phi_\perp} + \delta_+ n M_{n-1}^{\phi_\perp} \\
&- n(n-1)\int_{-1}^{1} dv \left[\mathcal{V}_{n-2}(v) + v\mathcal{A}_{n-2}(v)\right] + (n-1)(n-2)\int_{-1}^{1} dv \left[\mathcal{A}_{n-3}(v) + v\mathcal{V}_{n-3}(v)\right], \\
&(n+1)M_n^v - (n-1)M_{n-2}^v = 2M_n^{\phi_2} - \delta_+(n-1)M_{n-2}^{\phi_\perp} - \delta_- n M_n^{\phi_\perp} \\
&+ n(n-1)\int_{-1}^{1} dv \left[\mathcal{A}_{n-2}(v) + v\mathcal{V}_{n-2}(v)\right] - (n-1)(n-2)\int_{-1}^{1} dv \left[\mathcal{V}_{n-3}(v) + v\mathcal{A}_{n-3}(v)\right].
\end{aligned}$$

We can solve these recurrence equations by transforming them into differential equations. For example for the distribution amplitude $g_a(u)$ we find a first order equation

$$(1-\delta_+)u\bar{u}\, g_a'(u) = \Omega(u),$$

with

$$\begin{aligned}
\Omega(u) =& \phi_2(u) + (\delta_- + \delta_+(2u-1))\phi_\perp'(u) \\
& - \frac{1}{2}\frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3 \frac{1}{\alpha_2}\left(\alpha_1 \frac{d}{d\alpha_1} + \alpha_3 \frac{d}{d\alpha_3}\right)\mathcal{V}(\underline{\alpha}) \\
& - \frac{1}{2}\frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3 \frac{1}{\alpha_2}\left(\alpha_1 \frac{d}{d\alpha_1} - \alpha_3 \frac{d}{d\alpha_3}\right)\mathcal{A}(\underline{\alpha}).
\end{aligned}$$

In the same way we get a differential equation for $g_v(u)$ and as a solution for the two DAs we get

$$\begin{aligned}
(1-\delta_+)g_a(u) &= \int_0^u dv \frac{\Omega(v)}{\bar{v}} - \int_u^1 dv \frac{\Omega(v)}{v}, \\
g_v(u) &= \int_0^u dv \frac{\Omega(v)}{\bar{v}} + \int_u^1 dv \frac{\Omega(v)}{v} + \delta_+ \phi_\perp(u)
\end{aligned}$$

$$
\begin{aligned}
&- \frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3 \frac{1}{\alpha_2}\mathcal{V}(\underline{\alpha}) \\
&- \int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3 \frac{1}{\alpha_2}\left(\frac{d}{d\alpha_1} + \frac{d}{d\alpha_3}\right)\mathcal{A}(\underline{\alpha}),
\end{aligned}
$$

with $\alpha_2 = 1 - \alpha_1 - \alpha_3$.

Chiral-odd DAs

For the chiral-odd DAs the procedure is the same and we only give a short outline. We start with the two operator identities

$$
\begin{aligned}
\frac{\partial}{\partial x_\mu}\left[\bar{q}(x)\sigma_{\mu\nu}x^\nu q(-x)\right] =& i\int_{-1}^{1} dv\, v\bar{q}(x)x^\alpha \sigma_{\alpha\beta}x^\mu g_s G_{\mu\beta}(vx)q(-x) \\
&- ix^\beta \partial_\beta \left[\bar{q}(x)q(-x)\right] - (m_{\bar{q}} - m_q)\bar{q}(x)\not{x}q(-x),
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{q}(x)q(-x) - \bar{q}(0)q(0) =& \int_0^1 dt \int_{-t}^{t} dv\, \bar{q}(tx)x^\alpha \sigma_{\alpha\beta}x^\mu g_s G_{\mu\beta}(vx)q(-tx) \\
&+ i\int_0^1 dt\, \partial^\alpha \left[\bar{q}(tx)\sigma_{\alpha\beta}x^\beta q(-tx)\right] \\
&+ i(m_{\bar{q}} + m_q)\int_0^1 dt\, \bar{q}(tx)\not{x}q(-tx).
\end{aligned}
$$

In analogy to the chiral-even sector we get

$$
\begin{aligned}
(ip\cdot n)(1-\delta_+^T)\int_0^1 du\, e^{i\xi(p\cdot n)}h_\parallel^{(s)} =& -\int_0^1 du\, e^{i\xi(p\cdot n)}\left(3\phi_\perp(u) - (ip\cdot n)\xi h_\parallel^{(t)}(u) - 2h_\parallel^{(t)}(u)\right) \\
&+ (ip\cdot n)^2 \int_{-1}^{1} dv\, v\left(\mathcal{T}_1(v, p\cdot n) - \frac{1}{2}m_{f_2}^2 \mathcal{T}_2(v, p\cdot n)\right) \\
&+ (ip\cdot n)\delta_-^T \int_0^1 due^{i\xi(p\cdot n)}\phi_2(u),
\end{aligned}
$$

and

$$
\begin{aligned}
(1-\delta_+^T)\int_0^1 due^{i\xi(p\cdot n)}h_\parallel^{(s)} =& (ip\cdot n)\int_0^1 dt \int_0^1 due^{i\xi(p\cdot n)t}h_\parallel^{(t)} \\
&+ (ip\cdot n)^2 \int_0^1 dt\, t\int_{-1}^{1} dv\left(\mathcal{T}_1(v, p\cdot nt) - \frac{1}{2}m_{f_2}^2 \mathcal{T}_2(v, p\cdot nt)\right)
\end{aligned}
$$

$$+ (ip\cdot n)\delta_+^T \int_0^1 dt \int_0^1 dv\, e^{i\xi(p\cdot n)t}\phi_2(u).$$

Now we can expand the equations in powers of $(p \cdot n)$ and transform them into relations of moments with the same formulas as already used for the chiral-even ones. After combining these two equations we get recurrence relations for $h_\parallel^{(s)}(u)$ and $h_\parallel^{(t)}(u)$ which we again solve by transforming them into differential equations. Thus we end up with:

$$\begin{aligned}
(1-\delta_+^T)h_\parallel^{(s)}(u) =& \frac{1}{2}\left(\int_0^u dv\,\frac{\Phi(v)}{\bar{v}} - \int_u^1 dv\,\frac{\Phi(v)}{v}\right),\\
h_\parallel^{(t)} =& \frac{1}{2}(u-\bar{u})\left(\int_0^u dv\,\frac{\Phi(v)}{\bar{v}} - \int_u^1 dv\,\frac{\Phi(v)}{v}\right) - \delta_+\phi_2(u)\\
&+ \frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3 \frac{1}{\alpha_2}\left(\mathcal{T}_1(\underline{\alpha}) - \frac{1}{2}m_{f_2}^2\mathcal{T}_2(\underline{\alpha})\right),
\end{aligned}$$

with

$$\begin{aligned}
\Phi(u) =& 3\phi_\perp(u) + \delta_+^T\left(\phi_2(u) - \xi\frac{\phi_2'(u)}{2}\right) + \frac{\delta_-^T}{2}\phi_2'(u)\\
&+ \frac{d}{du}\int_0^u d\alpha_1 \int_0^{\bar{u}} d\alpha_3\,\frac{1}{\alpha_2}\left(\alpha_1\frac{d}{d\alpha_1} + \alpha_3\frac{d}{d\alpha_3} - 1\right)\left(\mathcal{T}_1(\underline{\alpha}) - \frac{1}{2}m_{f_2}^2\mathcal{T}_2(\underline{\alpha})\right).
\end{aligned}$$

Twist-four DAs

In Ref. [56] the twist-four DAs $g_4(u)$ and $A_4(u)$ are given

$$\begin{aligned}
g_4(u) &= 30u\bar{u}(2u-1),\\
A_4(u) &= 100u^2\bar{u}^2(2u-1),
\end{aligned}$$

and we need to determine $\mathbb{A}(u)$ and $h_4(u)$. Therefore we use the following two operator identities [104]

$$\partial_\mu\bar{q}(x)\sigma_{\mu\nu}q(-x) = -i\frac{\partial}{\partial x^\nu}\bar{q}(x)q(-x) + \cdots, \tag{D.9}$$

$$\frac{\partial}{\partial x^\mu}\bar{q}(x)\sigma_{\mu\nu}q(-x) = -i\partial_\nu\bar{q}(x)q(-x) + \cdots, \tag{D.10}$$

where the dots denote three-particle operators which are irrelevant for our purpose. Multiplying equation (D.9= with Q^ν and demanding $P\cdot Q = x\cdot Q = 0$ we get, after the same procedure as above

$$h_4(u) = \phi_\perp(u).$$

From equation (D.10) we find after the same procedure

$$\mathbb{A}(u) = 60u^2\bar{u}^2(2u-1).$$

Appendix E

QCD Sum Rules for the three-particle constants

In this appendix we use QCD sum rules to determine the constants $\xi_3^{\mathcal{T}_{1/2}}$ and $\omega_3^{\mathcal{T}_{1/2}}$ used in this work.
The starting point is the two-point correlation function:

$$\Pi_{\tau\gamma\delta,\alpha\beta\mu\nu} = i\int d^4x\, e^{iq\cdot x} \langle 0|\, T\{j_{\tau\gamma\delta}(x)\eta_{\alpha\beta\mu\nu}(z_1,z_2,z_3;n)\}\, |0\rangle\,. \tag{E.1}$$

The corresponding diagrams are depicted in Figure E.1. In the correlation function the interpolating current is $j_{\tau\gamma\delta}(x) = \bar{q}(x)\sigma_{\tau\gamma} i\overleftrightarrow{D}_\delta q(x)$, which satisfies the relation

$$\langle f_2(P,\lambda)|\, j_{\tau\gamma\delta}(0)\, |0\rangle = f_{f_2}^T m_{f_2}\left(e^{(\lambda)}_{\tau\delta}P_\gamma - e^{(\lambda)}_{\gamma\delta}P_\tau\right),$$

and the three-particle current is $\eta_{\alpha\beta\mu\nu}(z_1,z_2,z_3;n) = \bar{q}(z_1 n)\sigma_{\alpha\beta} i g_s G_{\mu\nu}(z_2 n)q(z_3 n)$. The covariant derivative is defined by $\overleftrightarrow{D}_\mu = \overrightarrow{D}_\mu - \overleftarrow{D}_\mu$ with $\overrightarrow{D}_\mu = \overrightarrow{\partial}_\mu - i g_s A^a_\mu \frac{\lambda^a}{2}$ and $\overleftarrow{D}_\mu = \overleftarrow{\partial}_\mu + i g_s A^a_\mu \frac{\lambda^a}{2}$. Unlike standard sum rule calculations, the three-particle current is put on the light-cone in order to simplify the calculation after contracting the correlation function (E.1) with two light-like vectors n_μ and $\bar{n}_\mu$, $n^2 = \bar{n}^2 = 0$. Furthermore we chose these auxiliary vectors in such way that $(q\cdot\bar{n}) = 0$ and $(q\cdot n) \neq 0$. For the perturbative calculation we need massless propagators in position space (with $\tilde{G}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\mu\nu}G^{\mu\nu}$)

$$\begin{aligned}
\langle 0|\, T\{q(x)\bar{q}(y)\}\, |0\rangle =& i\frac{\Gamma(d/2)\not{\Delta}}{2\pi^{d/2}[-\Delta^2]^{d/2}} - \frac{i}{8\pi^2\Delta^2}\Delta_\alpha g_s \widetilde{G}_{\alpha\beta}(0)\gamma_\beta\gamma_5 \\
&- \frac{1}{4\pi^2}\frac{\not{\Delta}}{\Delta^4} y_\rho x_\sigma g_s G_{\rho\sigma}(0), \\
\langle 0|\, T\{G^a_{\mu\nu}(x)A^b_\lambda(y)\}\, |0\rangle =& -\delta_{ab}\frac{\Gamma[\frac{d}{2}]}{2\pi^{\frac{d}{2}}[-\Delta^2]^{\frac{d}{2}}}(\Delta_\mu g_{\nu\lambda} - \Delta_\nu g_{\mu\lambda}),
\end{aligned}$$

with $\Delta = x - y$. The first one is the quark propagator in a background field [90] [91] (see also equation (5.5) for the propagator in momentum space). The second

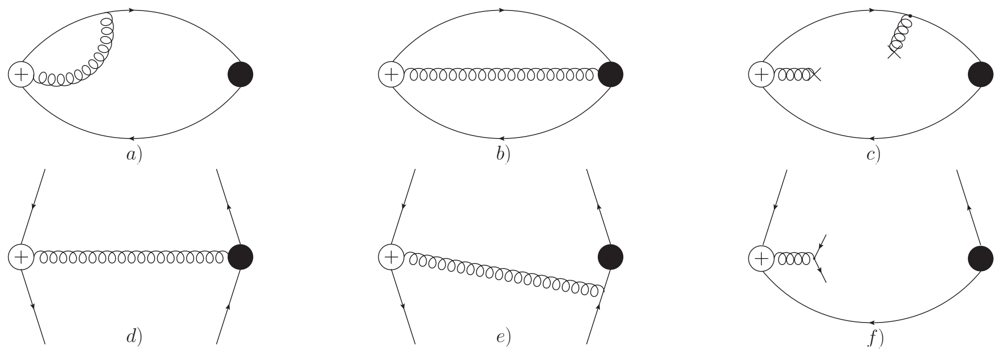

Figure E.1: Diagrams contributing to the correlation functions (E.1): the perturbative QCD corrections (a,b), the gluon (c), quark (d,e) and quark-gluon (f) condensate contributions. Here the filled circle represents the current $j_{\tau\gamma\delta}(x)$ and the other represents the current $\eta_{\alpha\beta\mu\nu}(z_1, z_2, z_3; n)$. The wiggly lines represent gluons and the straight lines quarks.

one can be derived by expanding $G^a_{\mu\nu}(x)$ and then acting with the derivative on the gluon propagator.
The procedure to calculate these diagrams is as follows: first we contract the corresponding fields using the given propagators, second we act with the derivatives, if present, third we contract the correlation function with the light-like vectors n, $\bar{n}$ and the metric $g^{\beta\nu}$, and finally we use the Feynman parametrization to solve the integrals. For the rest of the calculation we will, for a few cases, give the starting expression and then the result after the above procedure.
First we start with the two perturbative $\mathcal{O}(\alpha_s)$ contributions which are diagram (a) and (b) in Figure E.1. For diagram (a) we have

$$\Pi_{\tau\gamma\delta,\alpha\beta\mu\nu} = i\int d^4x\, e^{iq\cdot x} \langle 0|\, T\{j_{\tau\gamma\delta}(x)\eta_{\alpha\beta\mu\nu}(z_1, z_2, z_3; n) i g_s \int d^4y\, \bar{q}(y)\gamma^\lambda A_\lambda(y) q(y)\} \,|0\rangle\,.$$

From the covariant derivative in $j_{\tau\gamma\delta}(x)$ we only use the partial derivatives. After adding the symmetric diagram where the gluon propagator couples to the lower quark, we end up with

$$\Pi_{\bar{n}\gamma\bar{n},n\beta n\beta} = \frac{\alpha_s}{\pi^3}(n\cdot\bar{n})^2 q_\gamma \frac{\Gamma[2-d]}{[-q^2]^{2-d}} \int \mathcal{D}\underline{\alpha}\, e^{iq\cdot n\sum z_k\alpha_k}\alpha_1\alpha_2\alpha_3 \left[\frac{1-2\alpha_1}{1-\alpha_1} + \frac{1-2\alpha_3}{1-\alpha_3}\right].$$

For diagram (b) we have

$$\Pi_{\tau\gamma\delta,\alpha\beta\mu\nu} = i\int d^4x e^{iqx} \langle 0|\, T\{\bar{q}(x)\sigma_{\tau\gamma}(2g_s A_\delta(x))q(x)\bar{q}(z_1 n)\sigma_{\alpha\beta} i g_s G_{\mu\nu}(z_2 n) q(z_3 n)\} \,|0\rangle\,,$$

where we take the gluon gauge field parts from the covariant derivative, which leads to

$$\Pi_{\bar{n}\gamma\bar{n},n\beta n\beta} = \frac{2\alpha_s}{\pi^3}(n\cdot\bar{n})^2 q_\gamma \frac{\Gamma[2-d]}{[-q^2]^{2-d}} \int \mathcal{D}\underline{\alpha}\, \alpha_1\alpha_2\alpha_3 e^{iq\cdot n\sum z_k\alpha_k}.$$

For the gluon condensate diagram (c) we get two contributions: the first one arises from the gluon part of the propagator and the second one from the replacement $A_\delta(x) = \frac{1}{2}x^\eta G_{\eta\delta}(0)$ in the current $j_{\tau\gamma\delta}(x)$. Adding up these two contributions and using the expression (3.11) for the gluon condensate leads to

$$\Pi_{\bar{n}\gamma\bar{n},n\beta n\beta} = \frac{\langle g_s^2 G^2\rangle}{8\pi^2}(n\cdot\bar{n})^2 \int \mathcal{D}\underline{\alpha}\,\alpha_1\alpha_3\delta(\alpha_2)q_\gamma \frac{\Gamma[2-\frac{d}{2}]}{[-q^2]^{2-\frac{d}{2}}} e^{iq\cdot n\sum z_k\alpha_k}.$$

For the first quark condensate diagram (d) we use the same formula as for diagram (b) but here we do not contract the quark. Instead we use the expression (3.10) for the quark condensate, which leads to

$$\Pi_{\bar{n}\gamma\bar{n},n\beta n\beta} = \frac{2}{9}g_s^2\langle\bar{q}q\rangle^2 (n\cdot\bar{n})^2 \frac{q_\gamma}{-q^2}\int \mathcal{D}\underline{\alpha}\, e^{iq\cdot n\sum z_k\alpha_k}\delta(\alpha_1)\delta(\alpha_3).$$

The two remaining diagrams (e) and (f) give no contribution.
The standard procedure for the hadronic representation gives

$$\Pi_{\bar{n}\gamma\bar{n},n\beta n\beta} = \frac{|f_{f_2}^T|^2(n\cdot\bar{n})^2 q_\gamma m_{f_2}^4}{m_{f_2}^2-q^2}\left(\frac{m_{f_2}^2}{2}\mathcal{T}_2(z_k,q\cdot n) - \mathcal{T}_1(z_k,q\cdot n)\right) + (n\cdot\bar{n})^2 q_\gamma \int_{s_0^h}^{\infty} ds\frac{\rho^h(s)}{s-q^2}.$$

Using

$$\frac{\Gamma[2-d]}{(-q^2)^{2-d}} = \frac{1}{2}\int_0^\infty \frac{s^2 ds}{s-q^2},$$

we can rewrite the perturbative contributions as a dispersion integral in q^2 and after applying quark-hadron duality we can subtract the integral over $\rho^h(s)$ from the perturbative part. In order to get sum rules for each coupling separately we need to project them out. Therefore we set $z_1 = z_2 = z_3 = 0$ and integrate over $\mathcal{D}\underline{\alpha}$ for $\xi_3^{\mathcal{T}_{1/2}}$ and over $\mathcal{D}\underline{\alpha}(7\alpha_2-3)$ for $\omega_3^{\mathcal{T}_{1/2}}$. Applying the Borel transformation we obtain the sum rules

$$\begin{aligned}
|f_{f_2}^T|^2 m_{f_2}^4 e^{-\frac{m_{f_2}^2}{M^2}}\left(\frac{m_{f_2}^2\xi_3^{\mathcal{T}_2}}{2} - \xi_3^{\mathcal{T}_1}\right) =& \frac{\alpha_s}{90\pi^3}\int_0^{s_0} ds\, s^2 e^{-\frac{s}{M^2}} + \frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{12}\int_0^{s_0} ds\, e^{-\frac{s}{M^2}} \\
&+ \frac{8}{9}\pi\alpha_s\langle\bar{q}q\rangle^2, \\
|f_{f_2}^T|^2 m_{f_2}^4 e^{-\frac{m_{f_2}^2}{M^2}}\frac{3}{4}\left(\frac{m_{f_2}^2\omega_3^{\mathcal{T}_2}}{2} - \omega_3^{\mathcal{T}_1}\right) =& -\frac{\alpha_s}{240\pi^3}\int_0^{s_0} ds\, s^2 e^{-\frac{s}{M^2}} - \frac{\langle\frac{\alpha_s}{\pi}G^2\rangle}{4}\int_0^{s_0} ds\, e^{-\frac{s}{M^2}} \\
&+ \frac{32}{9}\pi\alpha_s\langle\bar{q}q\rangle^2.
\end{aligned}$$

Using the value $s_0 = 2.53$ GeV^2 and for the Borel parameter the interval $1.0 < M^2 < 1.4$ GeV^2 we obtain from these sum rules:

$$\left(\frac{m_{f_2}^2 \xi_3^{\mathcal{T}_2}}{2} - \xi_3^{\mathcal{T}_1}\right) = 0.16(3),$$
$$\left(\frac{m_{f_2}^2 \omega_3^{\mathcal{T}_2}}{2} - \omega_3^{\mathcal{T}_1}\right) = -0.33(16),$$

with the standard values of the condensates $\langle \bar{q}q \rangle = -(240 \pm 10\text{MeV})^3$ and $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (0.012\text{GeV}^4) \pm 50\%$ from Ref. [22] at a scale of 1 GeV. For $|f_{f_2}^T|^2$ we use the sum rule given in Ref. [57]. The given error corresponds to a 50% uncertainty in the gluon condensate and the variation of the Borel parameter in the given window. The errors from the other input parameters are negligible. Because of cancellations between gluon and quartic condensate, the error in $\omega_3^{\mathcal{T}_{1/2}}$ is very large. The same effect also occurs for the other three-particle constant ω_3 as can be seen in Ref. [56].

List of Figures

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