Spatial String Tension in the Deconfined Phase of (3+1)-Dimensional SU(2) Gauge Theory

G. S. Bali,[1] J. Fingberg,[2] U. M. Heller,[2] F. Karsch,[3,4] and K. Schilling[1]
[1] *Fachbereich Physik, Universität-Gesamthochschule Wuppertal, Gausstrasse 20, D-42097 Wuppertal, Germany*
[2] *Supercomputer Computations Research Institute, The Florida State University, Tallahassee, Florida 32306-44052*
[3] *Hochstleistungsrechenzentrum, c/o Kernforschungsanlage Jülich, D-52425 Jülich, Germany*
[4] *Fakultät für Physik, Universität Bielefeld, P. O. Box 100131, D-33501 Bielefeld, Germany*
(Received 6 July 1993)

We present results of a detailed investigation of the temperature dependence of the spatial string tension in SU(2) gauge theory. We show, for the first time, that the spatial string tension is scaling on the lattice and thus is nonvanishing in the continuum limit. It is temperature independent below T_c and rises rapidly above. For temperatures larger than $2T_c$ we find a scaling behavior consistent with $\sigma_s(T) = (0.136 \pm 0.011)g^4(T)T^2$, where $g(T)$ is the two-loop running coupling constant with a scale parameter determined as $\Lambda_T = (0.076 \pm 0.013)T_c$.

PACS numbers: 11.15.Ha

Non-Abelian SU(N) gauge theories in (3+1) dimensions are known to undergo a deconfining phase transition at high temperature. The physical string tension, characterizing the linear rise of the potential between static quark sources with distance, decreases with increasing temperature and vanishes above T_c. The potential becomes a Debye screened Coulomb potential in the high temperature phase. The leading high temperature behavior as well as the structure of the heavy quark potential for temperatures well above T_c can be understood in terms of a perturbative expansion in a finite volume of size $V^{1/3} < 1/g^2(T)T$ [1,2]. Nonperturbative effects may, however, show up at this length scale, where the generation of a magnetic mass term, $m_m \sim g^2(T)T$ in the gluon propagator may influence the spectrum in the high temperature phase. These nonperturbative effects in the magnetic sector will also manifest themselves in correlation functions for the spatial components of gauge fields.

(3+1)-dimensional renormalizable quantum field theories at high temperature, through dimensional reduction, can be reformulated as effective three-dimensional theories, with the scale of the dimensionful couplings given in terms of the temperature [3]. In the case of an SU(N) gauge theory the effective theory is a three-dimensional gauge theory with adjoint matter (Higgs) fields, emerging from the temporal component of the gauge fields. Basic properties of the gauge invariant correlation functions for spatial components of the gauge fields—the spatial Wilson loops—can be understood in terms of this effective theory. For instance, as this effective theory is confining, it is natural to expect that spatial Wilson loops obey an area law behavior in the high temperature phase

$$W(R,S) = \langle e^{i\oint_{R\times S} dx_\mu A_\mu} \rangle \sim e^{-\sigma_s RS}, \tag{1}$$

where σ_s has been called the spatial string tension, although one should stress that it is not related to properties of a physical potential in the (3+1)-dimensional theory. In the case of QCD the effective theory itself is quite complicated even at high temperatures, as the nonstatic modes do not decouple from the static sector [4]. An analysis of the temperature dependence of the spatial string tension thus yields information on the importance of the nonstatic sector for long-distance properties of high temperature QCD.

The existence of a nonvanishing spatial string tension, σ_s, in the high temperature phase of (3+1)-dimensional SU(N) lattice gauge theory can be proven rigorously at finite lattice spacing [5]. However, despite its basic relevance for a better understanding of the nonperturbative structure of non-Abelian gauge theories at high temperature, little effort has been undertaken to arrive at a quantitative description of the properties of the spatial string tension. In Ref. [6] the question of the temperature dependence of the spatial string tension as well as its scaling behavior has been studied for the first time numerically. The early numerical investigations [6,7] suggested that σ_s stays nonzero but temperature independent in the high temperature phase. Some indications for an increase of σ_s with temperature have been found recently [8]. However, so far no detailed study of the scaling behavior of the spatial string tension with temperature and its behavior in the continuum limit exists.

We present here the results of a detailed, high statistics analysis of the spatial string tension. The finite temperature SU(2) gauge theory has been simulated on lattices of size $N_\tau \times 32^3$, with N_τ ranging from 2 to 32. The simulations have been performed at two values of the gauge coupling, $\beta = 2.5115$ and $\beta = 2.74$, which correspond to the critical couplings for the deconfinement transition on lattices with temporal extent $N_\tau = 8$ and $N_\tau = 16$, respectively [9]. The lattice spacing thus changes by a factor 2.00 ± 0.04, where the error is caused by the uncertainty in both of the critical couplings. We confirm this factor through a calculation of the string tension at

0031-9007/93/71(19)/3059(4)$06.00

low temperatures in the confining phase. On a lattice of size 16×32^3 (32^4) at $\beta = 2.5115$ (2.74) we obtain

$$\sqrt{\sigma} a = \begin{cases} 0.1836 \pm 0.0013, & \beta = 2.5115\,, \\ 0.0911 \pm 0.0008, & \beta = 2.74\,, \end{cases} \tag{2}$$

which corresponds to a change in lattice spacing $a_{\beta=2.5115}/a_{\beta=2.74} = 2.016 \pm 0.023$, and is consistent with the factor 2 obtained from the calculation of the above critical couplings for the deconfinement transition.

We determine the spatial string tension from temperature dependent *pseudopotentials* constructed from Wilson loops of size $R \times S$, where both sides of the loop point into spatial directions,

$$V_T(R) = \lim_{S\to\infty} \ln \frac{W(R,S)}{W(R,S+1)}\,. \tag{3}$$

In the actual calculation we also construct off-axis loops in spatial directions and use standard smearing techniques [10] to improve the convergence of approximants with increasing S.

At fixed gauge coupling the temperature can be varied by varying the temporal extent, N_τ, of the lattice. For $\beta = 2.74$ we have studied the *pseudopotentials* on lattices of size $N_\tau = 16, 12, 8, 6, 4$, and 2, which corresponds to temperatures $T/T_c = 1, 1.33, 2, 2.67, 4$, and 8 in addition to the physical potential at "zero" temperature on a 32^4 lattice. In order to check the scaling behavior of the spatial string tension in the continuum limit we have performed additional calculations at $\beta = 2.5115$ and $N_\tau = 8$, 6, and 4, i.e., $T/T_c = 1, 1.33$, and 2 as well as at zero temperature, approximated by a 16×32^3 lattice. Note that this procedure induces only an overall error into the temperature T/T_c from the uncertainty in the scale T_c, stemming from the error in β_c [9]. Varying β at fixed N_τ to change the temperature, as has been customary so far, would introduce additional errors because the relation between the lattice spacing a and the coupling β is not known sufficiently well.

The pseudopotentials defined through Eq. (3) are shown in Fig. 1 for $\beta = 2.74$. Obviously the effective potentials do not show any significant temperature dependence up to T_c. However, as can also be seen from the figure the slope of the potential rises rapidly above T_c. In order to quantify the temperature dependence of the linearly rising potentials we follow Refs. [11,12] and fit the potentials with the ansatz

$$V_T(R) = V_0 + \kappa R - \frac{e}{R} - f\left(G_L(R) - \frac{1}{R}\right)\,, \tag{4}$$

where G_L denotes the lattice Coulomb potential. This last term takes account of the lattice artifacts present at small distances. We have tried various other fits, including fits where the Coulomb part has been replaced by a logarithmic term, which would be expected in the high temperature limit. Details on these fits as well as a discussion of the short distance part of the pseudopotentials

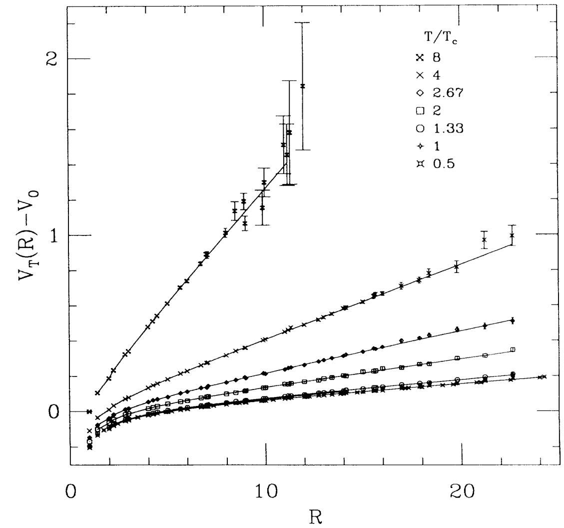

FIG. 1. The pseudopotentials $V_T(R)$ minus the (constant) self-energy contributions V_0 [Eq. (4)] on lattices of size $N_\tau \times 32^3$ for $\beta = 2.74$ as a function of the spatial separation R measured in lattice units.

will be presented elsewhere. In general we found that the fit parameter $\kappa \equiv \sigma_s a^2$ depends only weakly on the actual parametrization of the short distance part of $V_T(R)$. Our results are summarized in Table I. We determine the spatial string tension in units of the critical temperature,

$$\frac{\sqrt{\sigma_s(T)}}{T_c} = \sqrt{\kappa(T)} N_{\tau,c}\,, \tag{5}$$

where $N_{\tau,c} = 8$ (16) for $\beta = 2.5115$ (2.74). These numbers are given in the last column of Table I.

In Fig. 2 we compare the spatial string tension calculated at $\beta = 2.5115$ and 2.74 at different temperatures. We find that our data sets are consistent with each other. Thus, similar to what has been found for the ratio of the physical string tension to the deconfinement transition temperature, scaling violations in the ratio $\sqrt{\sigma_s}/T_c$ are negligible. This demonstrates that the spatial string tension, indeed, is relevant to high temperature QCD as it persists in the continuum limit. Moreover, σ_s coincides with the physical, zero temperature string tension for $T \leq T_c$.

The coupling of the Yang-Mills part of the action of the effective three-dimensional theory, g_3, derived from a (3+1)-dimensional SU(N) gauge theory at high temperature, is given in terms of the temperature and the four-dimensional coupling $g(T)$ as $g_3^2 = g^2(T)T$. Although the temperature will set the scale also for other couplings in the three-dimensional theory, these couplings will in general have a different dependence on the four-dimensional gauge coupling $g^2(T)$ [1]. The functional dependence of $\sigma_s(T)$ on $g^2(T)$ and T is thus not apparent from the general structure of the effective action. Nonetheless, in a

TABLE I. Summary of results from fits to the effective potentials using Eq. (4) on lattices of size $N_\tau \times 32^3$. The values of N_τ, which correspond to the temperatures given in the first column, are described in the text. The third column gives the number of gauge field configurations used in the analysis. They are separated by 100 sweeps of overrelaxed Monte Carlo updates. Remaining autocorrelations have been taken into account in the error analysis.

T/T_c	β	Meas.	V_0	κ	e	f	$\sqrt{\sigma_s}/T_c$
0.5	2.7400	835	0.482(3)	0.0083(1)	0.220(12)	0.13(8)	1.46(1)
1		918	0.475(6)	0.0089(6)	0.210(19)	0.13(12)	1.51(5)
1.33		720	0.474(3)	0.0094(2)	0.207(9)	0.15(6)	1.55(2)
2		279	0.448(5)	0.0152(5)	0.175(11)	0.20(11)	1.97(3)
2.67		477	0.426(6)	0.0231(5)	0.157(11)	0.16(10)	2.43(3)
4		2111	0.390(4)	0.0419(4)	0.135(8)	0.17(7)	3.28(2)
8		8582	0.319(11)	0.1270(18)	0.111(17)	0.28(3)	5.70(4)
0.5	2.5115	550	0.537(4)	0.0337(5)	0.233(8)	0.26(7)	1.46(1)
1		1320	0.543(7)	0.0325(7)	0.250(16)	0.20(10)	1.44(2)
1.33		2580	0.513(4)	0.0381(4)	0.207(7)	0.24(8)	1.56(1)
2		1700	0.443(6)	0.0643(6)	0.142(13)	0.27(6)	2.03(1)

pure three-dimensional SU(N) gauge theory dimensionful quantities are proportional to an appropriate power of the three-dimensional coupling g_3. If the temperature dependence of the pure gauge part of the effective action dominates the spatial string tension we would expect to find

$$\sqrt{\sigma_s(T)} = cg^2(T)T\,, \tag{6}$$

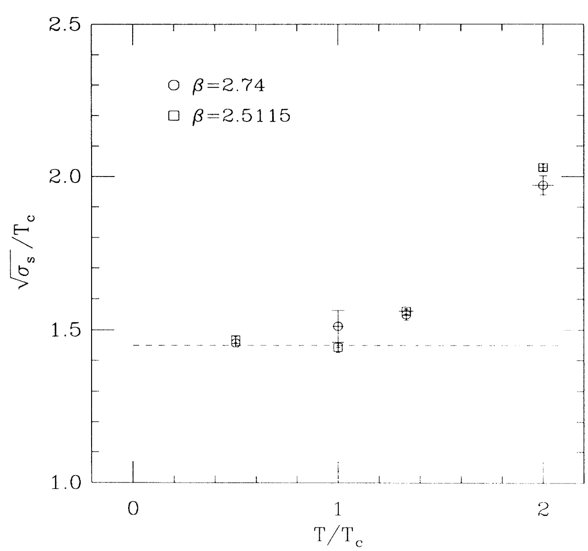

FIG. 2. Square root of the spatial string tension in units of the critical temperature versus temperature calculated at two different values of the gauge coupling. The broken line gives the result for the ratio of the physical string tension to the deconfinement temperature averaged over several values of the critical coupling [9]. The horizontal error bars indicate the uncertainty in the temperature scale due to the statistical errors in the determination of the critical couplings for the deconfinement transition.

where the temperature dependent running coupling constant $g^2(T)$ should, at high temperatures, be determined by the β function of SU(N) in four dimensions.

In Fig. 3 we have plotted $T/\sqrt{\sigma_s(T)}$ against T. From Eq. (6) this ratio is expected to be proportional to $g^{-2}(T)$. We have fitted these data to the two-loop formula for the coupling in SU(2) gauge theory with the scale parameter Λ_T,

$$g^{-2}(T) = \frac{11}{12\pi^2}\ln T/\Lambda_T + \frac{17}{44\pi^2}\ln(2\ln T/\Lambda_T)\,. \tag{7}$$

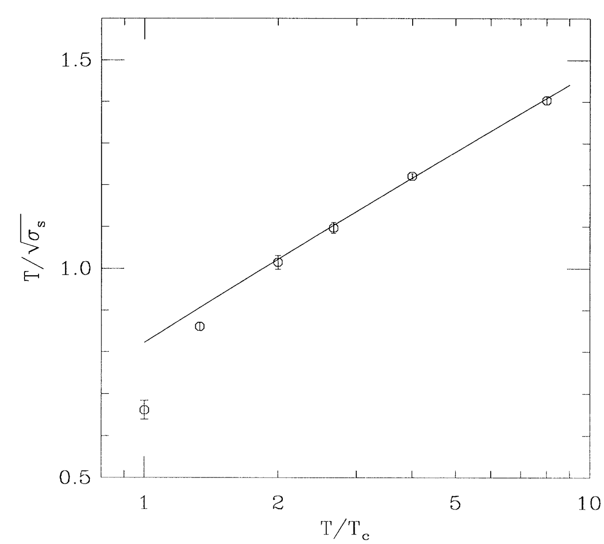

FIG. 3. The ratio of the critical temperature and square root of the spatial string tension versus temperature for $\beta = 2.74$. The line shows a fit to the data in the region $2 \le T/T_c \le 8$ using the two-loop relation for $g(T)$ given in Eq. (7).

We find that the temperature dependence of the spatial string tension is well described by Eqs. (6) and (7) for temperatures above $2T_c$. From the two parameter fit to the data shown in Fig. 3 in the region $T \geq 2T_c$ we obtain

$$\sqrt{\sigma_s(T)} = (0.369 \pm 0.014) g^2(T) T \,, \tag{8}$$

with $\Lambda_T = 0.076(13)T_c$. We note that the second term in Eq. (7) varies only little with temperature. A fit with the one-loop formula thus works almost equally well; it yields $\Lambda_T = 0.050(10)T_c$ and $c = 0.334(14)$ for the coefficient in Eq. (8).

It is rather remarkable that the spatial string tension depends in this simple form on the perturbative SU(2) β function already for $T \geq 2T_c$ and that possible contributions from higher orders in $g^2(T)$ could be absorbed into the scale parameter Λ_T. Moreover, we find that even quantitatively the spatial string tension agrees well with the string tension of the three-dimensional SU(2) gauge theory, $\sqrt{\sigma_3} = (0.3340 \pm 0.0025) g_3^2$ [13]. We take this as an indication that, indeed, the spatial string tension is dominated by the pure gauge part of the effective three-dimensional theory. We note that the value for $g^2(T)$, determined here from long distance properties of the (3+1)-dimensional theory, is about a factor of 2 larger than what has been obtained by comparing the short distance part of the (3+1)-dimensional heavy quark potential with perturbation theory [2].

The computations have been performed on the Connection Machines at HLRZ, SCRI, and Wuppertal. We thank the staff of these institutes for their support. F.K. would like to thank A. Patkos and B. Petersson for discussions on dimensional reduction at high temperature. The work of J.F. and U.M.H. was supported in part by the DOE under Grants No. DE-FG05-85ER250000 and No. DE-FG05-92ER40742. The work of G.S.B. and K.S. was supported by the EC under Grant No. SC1*-CT91-0642. We are grateful to the DFG for supporting the Wuppertal CM-2 project (Grant No. Schi 257/1-4).

[1] T. Reisz, Z. Phys. C **53**, 169 (1992).
[2] P. Lacock, D.E. Miller, and T. Reisz, Nucl. Phys. **B369**, 501 (1992).
[3] T. Appelquist and R.D. Pisarski, Phys. Rev. D **23**, 2305 (1981).
[4] N.P. Landsman, Nucl. Phys. **B322**, 498 (1989).
[5] C. Borgs, Nucl. Phys. **B261**, 455 (1985).
[6] E. Manousakis and J. Polonyi, Phys. Rev. Lett. **58**, 847 (1987).
[7] B.A. Berg and A.H. Billoire, Phys. Rev. D **40**, 550 (1989).
[8] L. Kärkkäinen *et al.*, Bielefeld Report No. BI-TP-93-12 (to be published).
[9] J. Fingberg, U. Heller, and F. Karsch, Nucl. Phys. **B392**, 493 (1993).
[10] APE Collaboration, M. Albanese *et al.*, Phys. Lett. B **192**, 163 (1987); G.S. Bali and K. Schilling, Phys. Rev. D **46**, 2636 (1992).
[11] S.P. Booth *et al.*, Phys. Lett. B **294**, 385 (1992).
[12] G.S. Bali and K. Schilling, Phys. Rev. D **47**, 661 (1993).
[13] M. Teper, Phys. Lett. B **289**, 115 (1992); Report No. OUTP-93-04P (to be published).