Enhancement of many-body quantum interference in chaotic bosonic systems supplementary material

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In this supplementary material we provide some of the details leading to Eq. (6). First, note that the prefactor can be written as

$$A_\gamma(\mathbf{q}_f, \mathbf{q}_i) = \frac{e^{i\pi\alpha/4}}{(2\pi\tilde{\hbar})^{L/2}} \left| \det\left(-\frac{\partial^2 R_\gamma}{\partial q_f^l \partial q_i^{l'}}(\mathbf{q}_f, \mathbf{q}_i) \right) \right|^{1/2} \tag{1}$$

with α the index of inertia of $-\partial \mathbf{q}_i/\partial \mathbf{p}_i$ [1]. Next, starting from Eq. (5), assume that the contributions of non-identical pairs of orbits cancel out because of the time average, and that only short chords, i.e. points such that $\mathbf{q}'_i \simeq \mathbf{q}''_i$ and $\mathbf{q}'_f \simeq \mathbf{q}''_f$ contribute significantly. In such a case, the variables $\mathbf{Q}_{f,i} \equiv (\mathbf{q}''_{f,i} + \mathbf{q}'_{f,i})/2$ and $\delta\mathbf{q}_{f,i} \equiv (\mathbf{q}''_{f,i} - \mathbf{q}'_{f,i})$ can be introduced. The key is to expand in the "small" variables $\delta\mathbf{q}_{f,i}$, which means more specifically keeping only the zero'th order term for the smooth function A_γ but expanding the action R_γ to first order. With $\gamma' = \gamma'' = \gamma$, and using the property $\mathbf{p}_f^{(\gamma)} = \partial R_\gamma/\partial \mathbf{q}_f$; $\mathbf{p}_i^{(\gamma)} = -\partial R_\gamma/\partial \mathbf{q}_i$ (with $\mathbf{p}_i^{(\gamma)}$ and $\mathbf{p}_f^{(\gamma)}$ the initial and final "momenta" of the trajectory γ), gives

$$\begin{aligned}
\langle \hat{f} \rangle(t)_{\rm diag} &= \int d\mathbf{Q}_i d\mathbf{Q}_f d\delta\mathbf{q}_f d\delta\mathbf{q}_i \, \langle \mathbf{q}''_i | \hat{\rho}_0 | \mathbf{q}'_i \rangle \, \langle \mathbf{q}'_f | \hat{f} | \mathbf{q}''_f \rangle . \\
&\times \sum_\gamma |A_\gamma(\mathbf{Q}_f \mathbf{Q}_i)|^2 \exp\left[\frac{i}{\tilde{\hbar}} \left(\mathbf{p}_f^{(\gamma)} \delta\mathbf{q}_f - \mathbf{p}_i^{(\gamma)} \delta\mathbf{q}_i \right) \right] \\
&= \int d\mathbf{Q}_i d\mathbf{Q}_f \sum_\gamma |A_\gamma(\mathbf{Q}_f \mathbf{Q}_i)|^2 [\rho_0]_W(\mathbf{Q}_i, \mathbf{p}_i^{(\gamma)}) \\
&\qquad \times [f]_W(\mathbf{Q}_f, \mathbf{p}_f^{(\gamma)}) \,,
\end{aligned} \tag{2}$$

with $\hat{\rho}_0 \equiv |\Phi_0\rangle\langle\Phi_0|$ the initial density. Note that $-\partial^2 R_\gamma/\partial \mathbf{Q}_f \partial \mathbf{Q}_i = \partial \mathbf{P}_i/\partial \mathbf{Q}_{f\,\gamma}$. Therefore, the determinant in Eq. (1) is just the Jacobian of the transformation from the final "position" $\mathbf{Q}_f$ to the initial "momentum" $\mathbf{P}_i$. Thus, $\sum_\gamma \int d\mathbf{Q}_f |A_\gamma|^2 \mapsto (2\pi\tilde{\hbar})^L \int d\mathbf{P}_i$ leading to Eq. (6).

[1] R. G. Littlejohn, J. Stat. Phys. **68**, 7 (1992).