Financial Trading and the Real Economy: A Competition for Talent

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Financial Trading and the Real Economy: A Competition for Talent

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"The greatest challenge to any thinker is stating the problem in a way that will allow a solution."

Bertrand Russell

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Abstract

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Financial Trading and the Real Economy: A Competition for Talent

by Sebastian ZELZNER

The rise of finance over the last century begs the question of whether financial markets can, and potentially have, become excessive in a way that is detrimental to the real economy. This thesis addresses the brain-drain hypothesis with regards to finance, i.e., the conjecture that the financial sector attracts too much talent, which could produce larger social benefits in other occupations. We set up a new theoretical model, based on the noisy rational expectations equilibrium (REE) model of Grossman and Stiglitz (1980). Agents who specialize in financial trading promote informational efficiency, at the cost that they do not contribute to job creation and output production in the real sector. We find that the equilibrium allocation of talent to financial trading tends to be excessive from a social welfare point of view.

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List of Abbreviations

BEA	**B**ureau of **E**conomic **A**nalysis
CARA	**C**onstant **A**bsolute **R**isk **A**version
CES	**C**onstant **E**lasticity of **S**ubstitution
CE	**C**ertainty **E**quivalent
FE	**F**ull **E**mployment
FOC	**F**irst **O**rder **C**ondition
GI	**G**ains from **I**nformation
GJ	**G**ains from the **J**ob
GMM	**G**eneral **M**ethod of **M**oments
GP	**G**ains from **P**roduction
GS	**G**rossman and **S**tiglitz
GT	**G**ains from **T**rade
HFT	**H**igh **F**requency **T**rading
LIE	**L**aw of **I**terated **E**xpectations
NT	**N**oise **T**rader
OC	**O**ccupational **C**hoice
OLG	**O**ver**l**apping **G**enerations
OTC	**O**ver **T**he **C**ounter
REE	**R**ational **E**xpectations **E**quilibrium
SOC	**S**econd **O**rder **C**ondition
SR	**S**harpe **R**atio
SW	**S**ocial **W**elfare
SWF	**S**ocial **W**elfare **F**unction
UE	**U**n**e**mployment
VA	**V**alue **A**dded

To my grandparents . . .

Chapter 1

Introduction

"I confess to an uneasy Physiocratic suspicion, perhaps unbecoming in an academic, that we are throwing more and more of our resources, including the cream of our youth, into financial activities remote from the production of goods and services, into activities that generate high private rewards disproportionate to their social productivity."

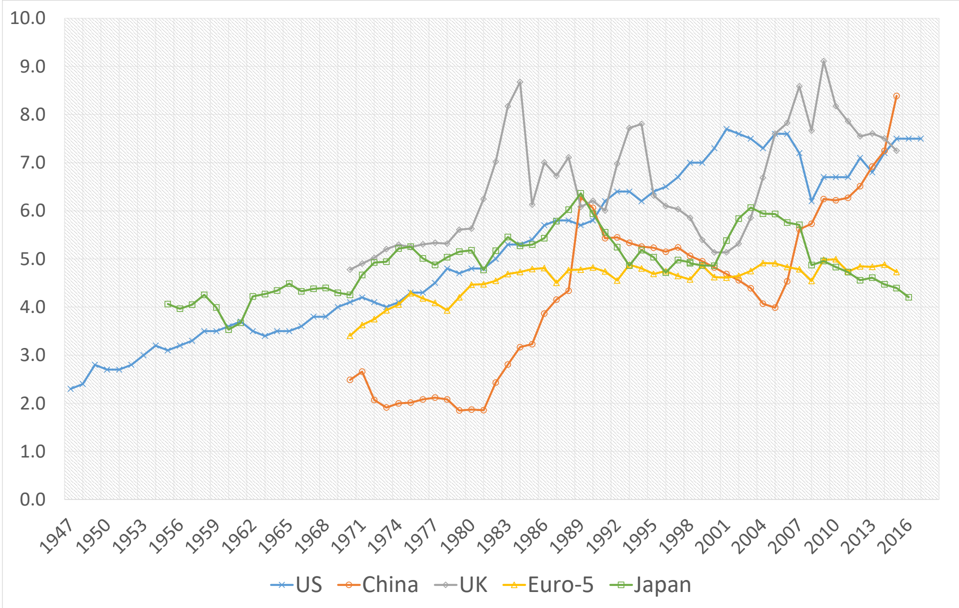
James Tobin, 1984

Over the last century, finance has experienced an era of remarkable growth. Its long-run upward trend in the U.S., as shown by figure 1.1, reached a temporary peak in 2006, just at the dawn of the global financial crisis. By that time, financial sector value added (VA) as a share of GDP was about three times as high as it had been in 1950. In recent years, it recovered from the drop in 2007/08 and is re-approaching pre-crisis highs. Less data is available outside the U.S., but a positive long-run trend also shows up for many other parts of the world. Notably, while finance in the U.S., the U.K. and Japan was seriously affected by the global financial crisis, China's financial sector gathered pace and in 2016 was almost four times as large as in the 1970s. Financial services in the Euro area encountered a steady increase from 1970 to the mid-1980s, but since then expansion has slowed down.

Reoccurring presumptions that the rise of finance might be explained by the rise of the services industry as a whole are rejected by Phillipon and Reshef (2013, p. 75) and Phillipon (2015, p. 1417). Patterns don't change much when looking at finance as a share of services instead of GDP. In response to Stauffer's (2004) criticism on the measurement methods related to finance VA, as well as concerns that VA over GDP could be a misleading indicator when financial services are traded abroad, Bazot (2018) proposes adjusted VA measures. Analyzing data from the U.S. and a number of European countries, he finds that, if anything, "plain" value added has even understated the financial sector's importance in recent decades. Cournède et al. (2015) and Antill et al. (2014) consider other indicators of financial sector size, such as the widely used "Credit-to-GDP" and "Market-Cap-to-GDP". Overall, these measures reveal a similar long-term upward trend as depicted in figure 1.1.[1]

[1] As argued by Philippon (2015, p. 1416), VA to GDP is the conceptually superior measure for the size of the financial sector's share in the economy. Cournède et al. (2015, p. 10) add that while value added has the advantage of providing a single measure that captures all parts of finance, it relies on modeling

FIGURE 1.1: Finance Value Added to GDP (in %)



Source: BEA for U.S. Data, EU KLEMS for Euro-5 and U.K. Data, OECD for CHN Data, Statistics Bureau (SB) for JP Data between 1955-1998, OECD for JP Data between 1998-2016.

Note: From 1970-1974, Euro-5 depicts the weighted average of Germany, Italy, Spain and The Netherlands. From 1975 onwards it also includes France. EU KLEMS derives data on Germany before 1991 from data on West Germany. Different data sources use slightly different definitions of "value added", so level comparisons between countries should be made with caution.

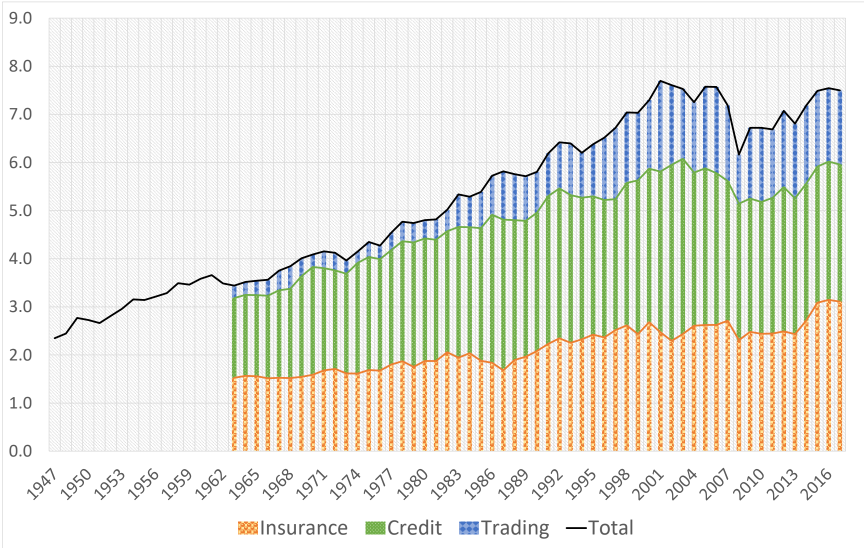
Note: Philippon and Reshef (2013) provide data also for some other countries, such as Canada and Australia. By including historical sources, they get time series dating back until 1850 and find clear long-run upward trends.

The rise of finance. U.S. data from the Bureau of Economic Analysis (BEA) allows for a decomposition of the financial sector, as it is provided also for four different subsectors within finance: (i) "Federal Reserve banks, credit intermediation, and related activities", (ii) "Insurance carriers and related activities", (iii) "Securities, commodity contracts, and investments", and (iv) "Funds, trusts, and other financial vehicles". While the first two subsectors are self-explaining, the remaining two can be subsumed under a single entity called "Other Finance". Greenwood and Scharfstein (2013) further decompose this "Other Finance" and show that more than 80% of it is related to asset management activities and trading (the rest being associated with investment banking activities such as underwriting or M&A). Consequently, Boustanifar et al. (2017) simply call it "trading-related activities". We build on this wording and refer to subsectors (iii)-(iv) as "(Financial) Trading". Figure 1.2 shows U.S. finance value added over GDP for the then three distinct subsectors. It is easy to see that while credit intermediation and insurance roughly doubled since the early 1960s, financial trading increased by a multiple of that. Consequently, financial trading accounted for 21% of finance value added in 2017, compared to only about 7%

assumptions for indirectly remunerated services such as lending or deposit-taking. In contrast, other measures are often restricted to a certain aspect of finance, but have the advantage of being observed directly.

in the 1960-70s.

FIGURE 1.2: U.S. Financial Subsectors' Value Added to GDP (in %)



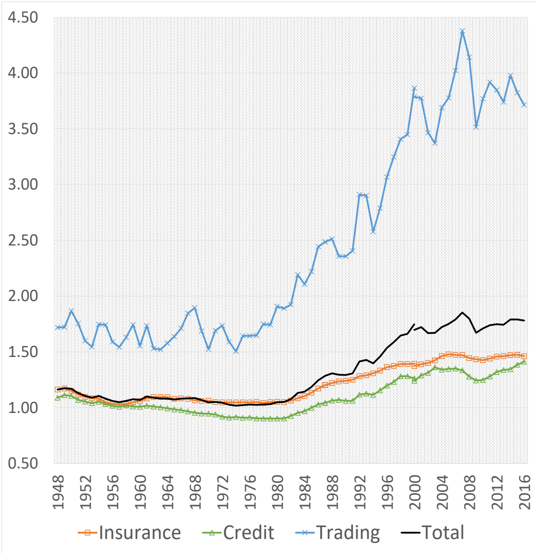
Source: Data from the BEA National Accounts.

Additional insight is gained by breaking up "value added" into its components, namely, returns to the capital factor (i.e., gross operating surplus, or simply "profits") and labor compensation, from which the latter is determined by wages and employment. As data from the BEA shows, profits account for a relatively steady 40-50% of finance VA for most of the time between 1950 and 2016 and with that explain about one-half of the rise of finance VA in the U.S. (see also Cournède et al., 2015). The other half is explained by an increasing relative labor compensation, which means that also finance employees have received their slice of the cake. While in the 1970s the average U.S. financial sector worker earned about the same (full-time equivalent) wage as the average worker in other occupations, figure 1.3a shows that wages and salaries soared since the 1980s. In 2006, the average wage gap between finance and non-finance employees amounted to more than 70%.[2]

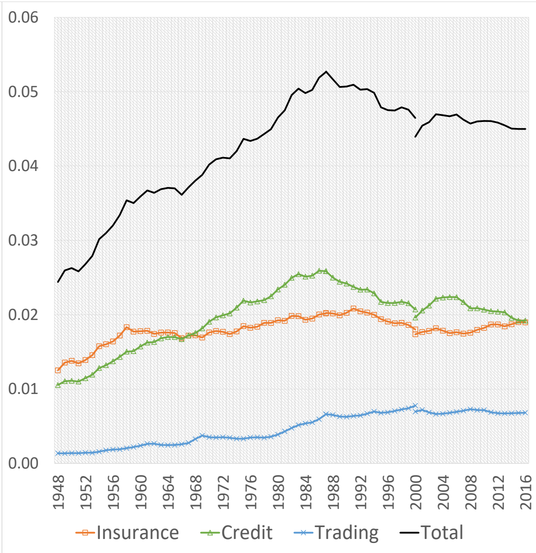
Looking at the three subsectors separately, finance relative wages show a disproportionately strong increase in financial trading. Kaplan and Rauh (2010) point out that the wage gap between financial trading and other industries even widens when looking at the very top positions. In 2004, the combined compensation of the top 25 hedge-fund managers was higher than that of all CEOs from the S&P 500 companies taken together.

[2] Strikingly, Philippon and Reshef (2012) show that this gap remains at a high average of 30-50% when controlling for individual skill background, which implies that even people of similar ability earn a lot more when working in finance rather than in other industries. This "finance wage premium" has been subject to analysis in a growing amount of literature (see, e.g., Oyer, 2008, Bell and Van Reenen, 2013, Axelson and Bond, 2015, Lindley and Mcintosh, 2017, Boustanifar et al., 2017, Böhm et al., 2018, and Célérier and Vallée, 2019). Among other explanations, it is often attributed to the stressful and unstable job environment in finance, high returns to talent, and the participation in industry rents.

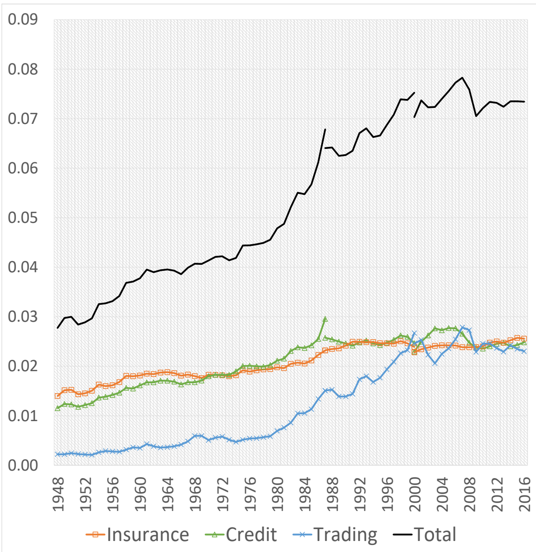
FIGURE 1.3: Decomposition of U.S. Finance Value Added



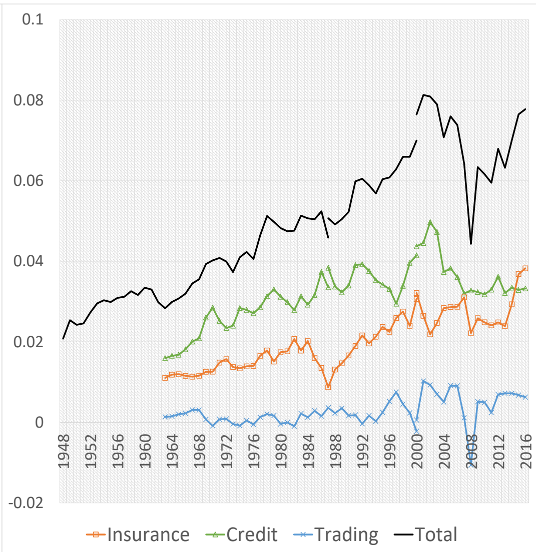
(A) Finance Relative Wages



(B) Finance Share in Employment



(C) Finance Share in Labor Compensation



(D) Finance Share in Profits

Source: Data from the BEA National Accounts and own calculations.

Note: Data frictions in the years 1987 and 2000 are due to changes in the methodology of the BEA (for these years, data is plotted under both the old and the new methodology). Wages include salaries, bonuses and stock options. Compensation additionally includes employer contributions for pension funds, insurance funds and government social insurance.

Note: A somewhat similar analysis for various other countries is provided by Phillippon and Reshef (2013) and Boustanifar et al. (2017).

The financial sector's share of (full-time equivalent) employment, as shown by figure 1.3b, has recently remained relatively constant, after a doubling from about 2.5 to almost 5 percent between 1950 and the 1980s. Looking closer, one can see that also the composition of financial sector employment changed. While relative employment in credit intermediation and insurance has even decreased since the 1980s, financial trading has seen a steady increase. And this increase has actually been quite substantial: financial trading's share in total employment in 2017 was more than two times as large as it was in the late 1970s. Evidence from Philippon and Reshef (2012) suggests that, especially since the 1980s, jobs in finance have become increasingly complex and demanding and, consequently, the financial sector has been claiming an increasing fraction among the well-educated. As they show, the differential in the share of employees with strictly more than high school education in finance vs. in other occupations almost doubled from 12 to 20 percentage points between 1980 and 2005. One can reasonably suggest that the high wages, especially in financial trading, were at least in part geared towards satisfying this increasing demand for talent.

Loosely speaking, combining relative wages and the share in employment gives the financial sector's share in economy-wide labor compensation, as shown by figure 1.3c. For intermediation and insurance, it has been increasing until the late 1980s, but not much has happened after that. This is again totally different for financial trading, where we see a significant increase starting in the early 1980s. With about 2.5% of the economy's labor compensation going to financial trading since the 2000s, it is roughly on par with intermediation and insurance. Insofar as increasing (relative) wages have attracted increasingly skilled employees, changes in the (relative) compensation of employees are a decent indicator for changes in the share of total employed human capital. With this in mind, figure 1.3c illustrates that we have little reason to assume that a disproportionate inflow of talent has recently happened with regards to the intermediation and insurance parts of finance. Strikingly, however, we *do* have every reason to believe that there might have been a brain-drain towards the financial trading industry, which we see to have claimed strongly increasing portions of human resources over the last three to four decades.

To complete the picture, finance's share in economy-wide profits is given by figure 1.3d. Besides being more volatile, with a large negative spike during the financial crisis, they show a pattern similar to that of labor compensation.

What happened? The underlying reasons for the rise of finance are not easily identified. A rather convincing story, which is hinted at by Philippon and Reshef (2013) and Greenwood and Scharfstein (2013), goes as follows. Before the 1980s, the rise of finance was driven mostly by an increasing relative demand for basic financial services, such as credit, investment or insurance. This increase in demand in turn stemmed from increasing incomes (starting from a relatively low level), which created the scope for broad financial participation and allowed people to delegate financial tasks from private provision to the market (see also Buera and Kaboski, 2012, for the services industry as a whole). The composition of finance started to change in the 1980s, when deregulation of the financial sector and the development of IT led to a wave of financial innovation, such as securitization, derivatives trading, financial engineering, hedge funds, private equity and high frequency trading. As a consequence, financial sector growth shifted away from traditional banking and insurance towards financial trading activities[3]. It is hard to argue that this growth of financial trading was driven by demand alone. Professionally managed, high fee investment funds experienced money inflows not only because they pose clear-cut benefits for investors (they actually tend to do not, as shown by the vast amount of literature on the failure of the active funds industry to beat the market; cf., e.g., Fama and French, 2010, and Malkiel, 2019), but rather because they have been heavily advertised (see Malkiel, 2013, and Roussanov et al., 2018).

This story of financial sector growth is consistent with our data as well (cf. figures 1.2 and 1.3). Until 1980, traditional banking and insurance clearly dominated. To keep up with the increasing relative demand for basic financial services, relative employment in these sectors increased. Patterns changed with deregulation and financial innovation in the 1980s. The financial sector as an early, heavy adopter of IT started to delegate routine tasks to machines and computers, which especially hurt low-skilled employment in banking and insurance. In contrast, trading-related activities performed by high-skilled individuals gathered pace. As deregulation and IT are typically considered complementary to skill, this perfectly fits the picture (see Autor et al., 2013, and Boustanifar et al., 2017).

Finance and economic growth I. The enormous growth of the financial sector naturally begs the question of whether this is a good or a bad thing. In pre-crisis times, the common academic view was that a well developed financial sector brings benefits overall. As summarized by Levine (2005), some of the main benefits are (i) the pooling of capital and its efficient allocation to the most promising projects, (ii) the provision of monitoring services, which help reduce problems of asymmetric information, and (iii) the provision of risk sharing and insurance opportunities.

Widely taking the overall advantageousness of well developed financial markets as given, academic discussion during the last century mainly focused on the question of whether an expanding financial system is promoting welfare and economic growth ("the banker ... is the ephor of the exchange economy", Schumpeter, 1911, p. 74), or whether it just reacts to changes in demand from the real sector ("where enterprise leads, finance follows", Robinson, 1952, p. 86). Empirical work on this question was first undertaken by Goldsmith (1969). Using a simple econometric model with cross-country data, he shows that there is a positive correlation between the size of the financial intermediary sector and long-run economic growth, but fails to establish causality. McKinnon (1973) and Shaw (1973) analyze a number of case studies from various countries and find a strong connection between financial and economic development. However, they take more of a descriptive approach than a tangible econometric analysis. So with the absence of any hard evidence on causality, Lucas

[3]Innovation in securitization also led to an increase in household credit, especially mortgage debt. An increasing number of defaults within this sector has been at the center of negative headlines in the U.S. subprime-mortgage crisis in 2008. See Greenwood and Scharfstein (2013) for an evaluation of the rise of household credit.

(1988, p. 6) famously argued that financial development is "very badly over-stressed" in the role it plays for economic growth.

It was five years later that King and Levine (1993), building on Goldsmith (1969) and the work of Barro (1991), found first stressable empirical evidence for causality by showing that the size of the financial intermediary sector is a good indicator for subsequent economic growth. Lots of research followed in the years thereafter. Building on early work by Atje and Jovanovic (1993), Levine and Zervos (1998) found that not only credit, but also equity markets show a strong correlation with economic growth. Evidence for causality further fostered with the work of Rajan and Zingales (1998), Levine et al. (2000), Beck et al. (2000) and many others, who made use of more sophisticated empirical methods, such as panel data econometrics, the instrumental variable approach and GMM estimation.

Even though some skeptics remained and there was still reason to believe that this might not be true for all countries at all times and circumstances (see, e.g., Wachtel, 2003, Rioja and Valev, 2004, and Demetriades and Law, 2006), the prevailing view just before the financial crisis was, as summarized by Levine (2005, p. 921):

> *"A growing body of empirical analyses [...] demonstrate[s] a strong positive link between [...] the financial system and long-run economic growth"*

and

> *"Theory and empirical evidence make it difficult to conclude that the financial system merely – and automatically – responds to economic activity, or that financial development is an inconsequential addendum to the process of economic growth."*

Put simply: Financial development was widely thought to have an unambiguously positive causal effect on long-run economic growth.

Finance and economic growth II. In the aftermath of the financial crisis, this consensus crumbled. Academics increasingly recognized that financial expansion does not only yield benefits, but can also come with serious drawbacks (see, e.g., Zingales, 2015). So, nowadays, the focus of the discussion on the interconnection between financial markets and the real economy mostly lies on the question of whether finance can, and potentially has, become excessive in a way that actually hurts economic welfare. In this sense, Adair Turner (2010, p. 6), who chaired the Financial Services Authority in the U.K. between 2008 and 2013, notably stated:

> *"There is no clear evidence that the growth in the scale and complexity of the financial system in the rich developed world over the last 20 to 30 years has driven increased growth or stability, and it is possible for financial activity to extract rents from the real economy rather than to deliver economic value."*

Generally speaking, the potential risks of a large financial sector include (i) higher economic volatility and an increasing risk of severe financial crises (Reinhart and Rogoff, 2008, Ollivaud and Turner, 2014, Schularick and Taylor, 2012), (ii) excessive risk taking, especially under explicit and implicit state guarantees (Denk et al., 2015, Schich and Aydin, 2014), (iii) exacerbating wealth and income inequality due to the fact that credit and capital services benefit the rich disproportionately (Denk and Cazenave-Lacroutz, 2015, Piketty and Zucman, 2014, de Haan and Sturm, 2017) and due to the "finance wage premium" (cf. footnote 2), and (iv) competition for talent resulting in a brain-drain from other, socially more productive industries (which is the focus of this thesis).

Economists revisiting the data have since then painted a more nuanced picture of the effects of financial development on growth than the one prevailing before. This has become apparent through at least two important facets. First, it was emphasized that it is essential to distinguish between different parts of finance. Beck et al. (2012) show that while corporate credit is positively correlated with economic growth, household credit is not. Beck et al. (2014a) cast a shadow on the role of the sharply increasing financial trading activities, by assessing that financial sector size (measured by VA over GDP) does not have an effect on growth once intermediation is controlled for. Greenwood and Scharfstein (2013) elaborate on this, reasoning that expanding household credit and financial trading may actually hurt welfare. Second, an expansion of the financial sector might benefit the real economy only up to a certain point. Re-estimating King and Levine's (1993) original work, Rousseau and Wachtel (2011) find that the positive relationship between finance and growth that has been found using data from 1960-1989 significantly weakens when including the years up to 2004 and even vanishes when looking at the more recent data from 1990-2004 only. Referring to a kind of Lucas (1976) critique, they argue that the positive early results about the finance-growth relationship may have induced policy to excessively promote the expansion of their financial systems. Haiss et al. (2016) confirm the results by Rousseau and Wachtel (2011) in a set of 26 European countries with data from 1990 to 2009. Other authors stress the fact that the finance-growth relationship seems to be not only non-linear, but also non-monotonic. Simply put, this means that more finance is beneficial when the financial sector is still relatively small, but further expansion turns to be harmful when it is large already. Among the first to emphasize this have been Arcand et al. (2015a) by asking: "Too much Finance?".[4] Lots of other empirical work followed, including Cecchetti and Kharroubi (2012, 2019), Pagano (2013), Gründler and Weitzel (2013), Beck et al. (2014b) Law and Singh (2014), Cournéde et al. (2015), Ductor and Grechyna (2015), Capelle-Blancard and Labonne (2016), Benczúr et al. (2019), Gründler (forthcoming), a meta-analysis by Bijlsma et al. (2018) and reviews of the literature by Panizza (2018) and Popov (2018).[5] As rules of thumb for the point where financial expansion turns from good to bad, this kind of literature provides roughly estimated thresholds such as 100% Credit/GDP, 5% VA/GDP, or 4% of total workforce employed in finance.

[4]The results of the paper have been challenged by William R. Cline from the Peterson Institute for International Economics. This initiated a heated public debate among its authors and Mr. Cline, see Cline (2015a), Arcand et al. (2015b), Cline (2015b), and Panizza (2018, p. 49-50).

[5]For a recent *theoretical* contribution which establishes a non-monotonic relationship between financial development and economic growth in a dynamic model framework á la Lucas (1988), see Bucci and Marsiglio (2019).

As the financial sector did not only grow in size but also changed in composition, it seems natural to assume that the two insights outlined above could be interconnected. Put differently: Insofar as the thresholds just mentioned are established by just looking at total finance or credit, one neglects the fact that the apparent non-linearities in the finance-growth relationship might actually stem from substantial structural changes in the composition of finance. Accounting for Beck et al.'s (2012) results on the effect of household credit on growth, Panizza (2018, p. 48), one of the co-authors of Arcand et al. (2015a), recognizes that "it is thus possible that the 'too much finance' result is really a 'too much household finance' result". Consequently, indiscriminate empirical thresholds are likely to be misleading in the sense that, e.g., a Credit/GDP ratio of more than 100% is actually not a problem per se, but only if it is increasingly related to private debt. Similarly, when looking at recently estimated thresholds for VA/GDP or finance's share in employment, one should keep in mind the driving forces behind the increase in these ratios over recent decades. We have seen that, in this regard, financial trading stands out. The channels through which too much of it can potentially harm the real economy have already been mentioned in points (i)-(iv) above. The literature has related (i) and (ii) mainly to scenarios involving a credit boom and hence to the intermediation part of finance. In contrast, we have seen that both (iii) and (iv) are highly relevant with regards to financial trading, where wages surged and talent followed. The interrelationship between inequality and economic growth has been studied for example by Berg and Ostry (2011) and Cingano (2014), but the connection is not straightforward. In what follows, we will therefore focus on point (iv) and argue why concerns for a brain-drain caused by the extraordinary rise of financial trading have to be taken seriously.

The Wall Street brain-drain. Worries about a brain-drain from other industries into finance often relate to the sharp rise of finance relative wages, which has attracted increasing portions of the well-educated. Some concerns, however, go even further, in that especially the top-paid positions in financial trading attract society's "crème de la crème", its "best and brightest", leading to a scenario in which "finance literally bids rocket scientists away from the satellite industry" and, as a result, "people who might have become scientists, who in another age dreamt of curing cancer or flying to Mars, today dream of becoming hedge fund managers" (Checchetti and Kharroubi, 2012, p. 1-2).

Lots of anecdotal evidence for this conjecture can be found by looking at the flow of U.S. elite university graduates into the job market. Building on university data from Princeton, Yale and Harvard, the *New York Times* reports that between 2000 and 2010, around 20-40% of students who finished university with a Bachelors degree went straight into finance.[6] The *MIT Faculty Newsletter* observes only slightly lower numbers from MIT.[7] Shu (2013, p. 13) finds that even within MIT, finance seems to

[6] Catherine Rampell, "Out of Harvard, and into Finance", *The New York Times*, December 2011, https://economix.blogs.nytimes.com/2011/12/21/out-of-harvard-and-into-finance/.

[7] Daniel Hastings, Steven Lerman, and Melanie Parker, "The Demand for MIT Graduates", *MIT Faculty Newsletter*, January-February 2010, http://web.mit.edu/fnl/volume/223/hastings.html.

attract students with particularly high "raw academic talent". Goldin and Katz (2008) observe that the fraction of male Harvard students who work in finance 15 years after graduation more than tripled between the 1970s and 1990s cohorts. Studying the early career of Stanford MBAs, Oyer (2008) finds that when equity markets boom, entry into finance increases. He adds that people who start on Wall Street are likely to stay there also for their later career. Vivek Wadhwa, technology entrepreneur and director of research at Duke University's Pratt School of Engineering, emphasizes that finance does not only attract graduates from majors related to business and economics. In a testimony to the U.S. House of Representatives in 2006, he remarks that "thirty to forty percent of Duke Masters of Engineering Management students were accepting jobs outside of the engineering profession. They chose to become investment bankers or management consultants rather than engineers."[8] In the same sense, Célérier and Vallée (2019, p. 4029) find that the fraction of engineering graduates from the most selective French universities who work in finance almost tripled from 3% to 8% between 1986 and 2011. Gupta and Hacamo (2019) obtain data from a large U.S. online business networking service (OBNS) and find that "superstar" engineers are significantly more likely to switch job to work in finance in times of high financial sector growth. They add that this can have long-run consequences for startup activity: engineers who worked in finance during their early career are less likely to engage in entrepreneurship later on and, even if they do, their startups tend to be less successful.

Evidence which at least partly alleviates concerns for a finance brain-drain has recently been provided by Böhm et al. (2018). Using detailed scores and performance measures from military aptitude tests in Sweden, they find that the selection of talent into finance has not increased over the 1990-2013 period. Nonetheless, they recognize that finance is still a "high-talent profession" (p. 16), with its employees being significantly more talented than workers in other occupations, on average. One can also question if the situation in Sweden is actually comparable to that in the U.S. Opposed to what we have seen in figure 1.3a, Boustanifar et al. (2017, p. 9) show that Swedish finance relative wages did not increase between 1970 and 2011. Hence, the Swedish financial sector has been lacking an important pull factor for talent. Shu (2016) analyses bachelor graduates from MIT between 1994-2012 and finds that top positions in finance might require different skill sets than those needed for innovating in science and engineering (S&E). He concludes that "finance does *not* systematically attract those who are best prepared at college graduation to innovate in S&E sectors", but adds that "anticipated career incentives influence students' acquisition of S&E human capital during college" (p. 0). Hence, even if high compensation in finance were not to cause a brain-drain induced decline in real sector innovativeness in the short run, by distorting early career aspirations it could still do so in the long run. D'Acunto and Frésard (2018) study the reallocation of skilled workers into finance in a sample of 24 countries from 1970 to 2005. While they find signs for a

[8] Quoted from Philippon (2010, p. 159).

modest brain-drain from other industries, they argue that the magnitude of its effect is probably too small to have significant consequences for overall economic development.

Contrasting evidence, which supports the hypothesis of a significant finance brain-drain, comes from Kneer (2013a, 2013b). Using data on U.S. banking deregulations, she finds that high talent inflows into finance have caused negative effects on productivity in other skill-intensive industries. Boustanifar et al. (2017) show that extraordinarily high wages in finance attract talent even across country borders, thereby imposing negative externalities on the countries of origin. While it is intuitive that affected countries or industries typically suffer from an outflow of talent, it is less clear whether the effect is positive or negative from an overall perspective. A growing amount of literature on the rent-seeking character of modern finance at least suggests that social returns are probably higher in other industries. In this regard, Nobel prize winner Paul Krugman argues that "everything we know suggests that the rapid growth in finance since 1980 has largely been a matter of rent-seeking, rather than true productivity."[9] Paul Woolley (2010, p. 123) from the Paul Woolley Centre for the Study of Capital Market Dysfunctionality at the London School of Economics agrees by stating that "rent extraction has become one of the defining features of finance and goes a long way to explaining the sector's extraordinary growth in recent years". As Luigi Zingales (2015, p. 1328), finance professor at Chicago Booth and winner of the Bernácer Prize emphasizes,

> *"...there is no theoretical reason or empirical evidence to support the notion that all growth in the financial sector over the last 40 years has been beneficial to society. In fact, we have both theoretical reasons and empirical evidence to claim that a component has been pure rent seeking."*

Greenwood and Scharfstein (2013) attribute these rents especially to the financial trading part of finance, where professionally managed mutual funds and hedge funds have attracted increasing amounts of investor money, despite often charging unjustifiably high fees. Recent empirical studies find that the high and increasing relative wages in finance since the 1980s are associated with a participation in increasing industry rents. Lindley and Mcintosh (2017) argue that deregulation together with implicit state guarantees and unintelligible financial instruments helped in creating and extracting rents. They conclude that rent participation seems to be the most convincing explanation for the finance wage premium in the U.K. Similarly, Böhm et al. (2018) find that the wage gap between finance and other industries in Sweden accrues mostly to industry rents.

One of the main arguments that is typically brought forward in favor of financial trading is that the information acquisition process inherent in its activities increases informational efficiency in the market, makes asset prices deviate less from their

[9] Paul Krugman, "Darling, I love you", *The New York Times*, December 2009, https://krugman.blogs.nytimes.com/2009/12/09/darling-i-love-you/.

fundamental value and hence allows society to obtain a more efficient resource allocation.[10] Still, Murphy et al. (1991, p. 506) suspect that these effects are rather small compared to the private returns from trading:

> *"Trading probably raises efficiency since it brings security prices closer to their fundamental values ... But the main gains from trading come from the transfer of wealth to the smart traders ... Even though efficiency improves, transfers are the main source of returns in trading."*

Furthermore, there even is good reason to believe that we have already passed the point where more financial trading makes markets significantly more informative. A good example for this is high-frequency trading (HFT). It is not mainly designed to discover genuinely new information, but rather to create what Hirshleifer (1971) calls "foreknowledge", that is to get a grasp on information just a little earlier than everyone else and then quickly make use of it (for a more nuanced view on HFT, see, e.g., Biais and Woolley, 2012, or Linton and Mahmoodzadeh, 2018). As this makes HFT a game of "the fastest takes it all", the consequence is what the *Bloomberg Markets* magazine calls an "arms race" in finance.[11] Activities like this are obviously prone to the critique of pure rent-seeking, which Stiglitz (1989, p. 5) intuitively illustrates with the following example:

> *"Assume that as a result of some new information, there will be a large revaluation of some security, say from \$10 to \$50. Assume that that information will be announced tomorrow in the newspaper. What is the private versus social return to an individual obtaining the information today? Assume the firm will take no action on the basis of the information – certainly not as a result of knowing the information a day earlier ... The information has only affected who gets to get the return. It does not affect the magnitude of the return. To use the textbook homily, it affects how the pie is divided, but it does not affect the size of the pie."*

Of course the relevant time intervals in HFT are even narrower than "a day earlier". Milliseconds make the difference. Entailing no social value, all resources going to activities like this are wasted from an economy-wide perspective. Bai et al. (2016) give further evidence that recent developments in financial trading did not necessarily come with benefits in terms of informational efficiency. While they do find that the U.S. S&P500 has become significantly more informative between 1960 and 2014, nearly all of this improvement must have happened in the earlier years of the sample. Price efficiency within 2010-2014 shows no significant improvement over that of the 1980s. Farboodi et al. (2019) argue that increases in price informativeness have been even weaker with regards to the stocks of smaller firms outside of the S&P500.

[10] The fact that asset prices can give valuable signals for real decisions, e.g. regarding investment or production, was already highlighted by Hayek (1945). Bond et al. (2012) give a more recent review on the real effects of informational efficiency in financial markets.

[11] Hugh Son and Dakin Campbell, "Wall Street's Big Banks Are Waging an All-Out Technological Arms Race", *Bloomberg Markets*, April 2018, https://www.bloomberg.com/news/features/2018-04-05/wall-street-s-big-banks-are-waging-an-all-out-technological-arms.

After all, Tobin's view, as quoted at the beginning of this Introductory Chapter, may have been unbecoming in academics at his time. It surely is not any more. There is much to suggest that the rise of financial trading, which took off in the 1980s and gathered pace in the 1990s, was fueled mainly by private, not social, returns. And it is likely that the related inflow of talent to Wall Street has caused a significant brain-drain from other industries.

Thesis structure. The goal of this thesis is to contribute to the theoretical literature on the competition for talent between finance and the real sector. For the reasons given in the Introductory Chapter, we focus on the financial trading aspect of finance. Chapter 2 contains an in-depth analysis of the model by Arnold and Zelzner (2020) and considers some variations and extensions. The model includes occupational choice between financial trading and entrepreneurship into the seminal noisy rational expectations equilibrium (REE) framework of Grossman and Stiglitz (1980). Professional traders make the market more informationally efficient, entrepreneurs create output and jobs. The main question is whether the equilibrium amount of talent going into finance is excessive from a social welfare point of view. Before formally setting up the model, we review the related theoretical literature. Details on how we structure the model analysis are given within Chapter 2. Chapter 3 concludes. Proofs and additional material are delegated to the Appendix.

Chapter 2

The Model

"A disease of the economy is the progressive transformation of entrepreneurs into speculators . . . A speculator is a figure similar to what Jesus in the gospels called 'hired-hands' as opposed to good shepherds."

Pope Francis, Genoa 2017

This chapter formalizes the allocation of talent to financial trading versus the real sector using a novel theoretical model built on the noisy rational expectations equilibrium (REE) model of Grossman and Stiglitz (1980, henceforth "GS (1980)"). High potential individuals (hipos) decide whether to engage in speculative financial trading activities (i.e., to become dealers) or in production (to become entrepreneurs). Entrepreneurs create jobs for (lower-skilled) "ordinary" workers and produce output, while dealers contribute to informational efficiency in the asset market. All agents trade in a noisy market environment. We analyze whether the equilibrium allocation of hipos to financial trading is excessive from a social welfare perspective. Our results suggest that this tends to be the case.

The model uses a CARA-Gaussian set-up (i.e., a combination of negative exponential utility and normally distributed random variables), which allows for a tractable analysis and closed-form solutions. Hipos are ex ante identical, so in equilibrium, where occupational choice (OC) is optimal, all dealers and entrepreneurs obtain the same expected utility. Ordinary workers find themselves in a labor market with or without frictions, where they try to find a job in one of the firms set up by entrepreneurs. Firms create a stochastic amount of output, which they partly sell in the asset market. More entrepreneurship is beneficial for workers. Depending on whether we consider a frictionless labor market with full employment or a labor market with wage rigidities and equilibrium unemployment, it either allows them to earn higher wages or decreases their risk of unemployment. In the former case, workers' wage gains from more entrepreneurship come at the expense of firm profitability. In the latter case, however, workers' employment gains do not draw on entrepreneurs' welfare. Hence, the fact that hipos' OC decision does not internalize the positive effect of entrepreneurship on workers gives rise to an externality.

Dealers gather information on asset fundamentals, which gives them a private advantage in trading. Strikingly, this also comes with "informational externalities". The obvious one is asymmetric information. But as private information get partially revealed through the public asset price, there is also a second one: informed trading increases informational efficiency in the market. The last type of agents in the model is a group of noise traders, which exerts an exogenous stochastic asset demand and thereby ensures that the dealers' private information do not leak out to the public perfectly.

To answer the question of whether the allocation of talent to financial trading vs. entrepreneurship is excessive, we conduct a second-best welfare analysis, taking individuals' portfolio and labor market decisions as given. As analytical welfare analysis turns out to be infeasible for stochastic noise trader demand, we take a two-step approach. We first provide a rigorous analysis of the model with non-stochastic noise trader activity, where agents can perfectly infer private information from the public price, information asymmetries vanish and there is either full or "zero" information in the market, depending on whether the mass of dealers is positive or not. In this case, dealers do not earn informational rents. As a second step, we show that the welfare results obtained from this model also hold in the limit for sufficiently small noise volatility and use this as the starting point for a comprehensive numerical analysis of the model with substantial noise trader shocks.

The results obtained from the model without noise are the following. First, the allocation of talent is constrained efficient in the case of a labor market without frictions, i.e., any marginal change in the mass of entrepreneurs, starting from equilibrium, decreases welfare. This is not surprising, as without frictions in the labor market, without information asymmetries and without informational externalities (in that becoming a dealer does not affect informational efficiency at the margin), we essentially obtain a model without market imperfections. Second, the allocation of talent to finance is excessive in the presence of wage rigidity and equilibrium unemployment in the labor market, i.e., a marginal increase in the mass of entrepreneurs, starting from equilibrium, increases welfare in case of labor market frictions. As mentioned before, the reason is entrepreneurship's positive externality on workers' job prospects. Third, we show that under a set of fairly weak conditions, social welfare increases when the possibility to become a dealer is shut down completely. Surprisingly, this is because higher price informativeness as a consequence of informed trading is not generally beneficial. Rather than that, it tends to have a negative welfare effect which is reminiscent of Hirshleifer's (1971) result on the potential harmfulness of information revelation for risk-sharing. Strikingly, impaired risk-sharing especially hurts entrepreneurs, who are the ones setting up enterprises and creating the asset in the first place. Hence, information revelation in the financial market distorts the allocation of talent by discouraging entrepreneurship. In turn, a ban on informed trading enhances risk-sharing, encourages entrepreneurship and real economic activity and increases welfare.

Using Matlab to simulate a wide range of reasonable parameter combinations, we find that our result that the financial sector tends to be too large is not restricted to the analytically tractable case without noise. In fact, it is even reinforced in the presence of noise trader shocks, where dealers earn informational rents and the negative effects of informed trading also apply at the margin: in contrast to the noiseless case, each (additional) dealer makes the price reflect more fundamental information. Therefore, an increase in the mass of entrepreneurs, starting from equilibrium, decreases information revelation in the financial market, enhances mutually beneficial risk-sharing among agents and reduces risk-clustering among entrepreneurs.

In essence, the allocation of talent to GS (1980)-like financial trading in our model is excessive, whenever (i) dealers create negative informational externalities, which relate to deficient risk-sharing and a clustering of risk at entrepreneurs, or (ii) entrepreneurship is associated with positive externalities in the labor market for ordinary workers.

Related theoretical literature. Our model contributes to the growing theoretical literature on the efficiency of the allocation of resources between finance and the real sector (for a glimpse on the empirical literature, see the Introductory Chapter). The two classic papers on the allocation of talent come from Baumol (1990) and Murphy et al. (1991). They broadly distinguish between socially productive industries which create value (e.g., manufacturing and engineering) and rent-seeking industries which just try to acquire portions of the wealth already available (e.g., finance and law). Using rather informal analyses, both argue that economic welfare and growth suffer, if high private returns attract significant amounts of talent to the latter type of industries.

More recently, Philippon (2010) includes OC between entrepreneurs, workers and financiers into an endogenous growth model with overlapping generations. With the help of financiers' monitoring services, entrepreneurs employ workers and create output. Innovation, given by labor productivity growth in the real sector, is modeled on a learning-by-doing basis with knowledge spillovers. It is driven by both physical capital (as in Romer, 1986) in terms of aggregate investment, as well as human capital (as in Lucas, 1988) in terms of the total amount of entrepreneurs. In a second-best scenario with the possibility of direct subsidies on investment and entrepreneurship, there is no need for a discriminatory tax on labor income from the financial versus the real sector. If, however, this kind of intervention is not feasible (which Philippon, 2010, p. 173, argues is the case in most real world scenarios), the third-best solution implies a subsidy to finance when innovation is driven mainly by aggregate investment and a tax on finance when innovation is driven mainly by the mass of entrepreneurs.[1]

Cahuc and Challe (2012) study the welfare effects of rational asset bubbles in a standard OLG setting to which they add occupational choice between finance and

[1]For a model that explores the role of *financial* innovation for economic growth, see Laeven et al. (2015).

production. When provided with financing, the productive sector employs workers and creates output. Production workers earn a fixed wage rate early, which they are willing to reinvest into the productive sector until their consumption period arrives. They can, however, not do so on their own but only with the help of financiers. Frictions in the financial sector ensure that financiers earn an intermediation margin for their services. While Tirole (1985) shows that, by crowding out capital, rational asset bubbles can help to overcome dynamic inefficiency à la Diamond (1965), this is not necessarily the case in the set-up by Cahuc and Challe (2012). As bubbly assets can be traded only by financiers, large rents in the financial sector may arise and crowd out labor from the productive sector. Hence, rational asset bubbles potentially induce an excessively large financial sector and thereby become detrimental to social welfare.

Shakhnov (2017) develops a heterogeneous agents model with matching frictions and an OC decision between banking and entrepreneurship. Entrepreneurs produce output, but need financing from an investor. In the absence of bankers, entrepreneurs and investors do not know each others' type and meet randomly. Bankers, who have superior information on the agents' individual types, enhance this process by efficiently matching each pair of agents. Bankers in this set-up potentially earn large informational rents which result in an excessive financial sector. An appropriate taxation of finance can restore efficiency.

While the papers above emphasize the financial sector's role as an intermediary, Bolton et al. (2016) focus on the financial trading aspect. They propose a model with a dual-structured financial sector, where uninformed investors have the costly option to become "dealers" and participate in an exclusive over-the-counter (OTC) market instead of the organized exchange (for an earlier version of the model with OC between dealers and entrepreneurs, see Bolton et al., 2012). Entrepreneurs (called "originators") are hit by a liquidity shock early and have to sell their business (the "asset") in the market. In contrast to uninformed investors, dealers have precise information on the value of this asset. As a consequence, they are able to "cream-skim" good assets in the OTC market, while the lower quality assets are left to be sold to uninformed investors in the organized exchange (see Fishman and Parker, 2015, for a similar mechanism). Bolton et al. (2016, Section II) show that without any link between dealers' valuation abilities and originators' asset quality, becoming a dealer is driven only by private benefits and entails no social value. Consequently, all resources spent in the process are wasted from a social welfare perspective. The situation changes, if dealers' valuation ability incentivizes originators to make an effort in improving their asset quality (Section III). The size of the dealer market remains generally inefficient, but depending on parameters it can be either too large or too small. Building on the framework by Glosten and Milgrom (1985) and Glosten (1989), Glode and Opp (2020) compare the efficiency of traders' decisions on expertise acquisition in OTC vs. limit-order organized markets. In contrast to Bolton et al.'s (2016) dual-structure of the financial sector, they analyze the two markets separately. Due to differences in market microstructure, private returns to expertise in asset valuation are higher in the OTC market. Whether the resulting acquisition of larger amounts of financial expertise is beneficial or detrimental to welfare depends on whether it is mainly motivated by rent-seeking behavior or includes a significant value-creating component.

Kurlat (2019) proposes a model where banks decide on their amount of expertise in asset valuation. Informed households want to sell their assets to the bank. The acquisition of expertise is costly, but it increases the banks' returns from trading. Besides the private incentive to invest in expertise, there is also a social component to it: more informed banks reduce informational asymmetries between banks and households, mitigate the problem of adverse selection and allow for additional mutually beneficial trades. Kurlat (2019) emphasizes that the mere fact that there are private as well as social returns from the acquisition of expertise does not mean that the two perfectly align. He introduces a measure denoted "r", which gives the ratio of the (marginal) social over the (marginal) private benefits from an additional unit of resources deployed to the acquisition of financial expertise. He estimates a value of $r = 0.16$ for the U.S. junk bond underwriting market, which implies that the private benefits from expertise exceed the social ones by far and, hence, underwriters' investment in expertise is excessive.

Glode et al. (2012) model the behavior of competing financial trading institutions as an "arms race" in finance. Similarly as in Kurlat (2019), each institution has to decide on its optimal investment in financial expertise, which gives access to improved valuation techniques with regards to assets traded with other institutions. In principle, the ability to value assets better than one's trading partners gives an institution an informational advantage in the trading process. In equilibrium, however, trying to gain an edge over the competitors prompts these to act the same way and hence neutralizes any individual advantages. Accumulating costly financial expertise in this set-up is pure rent-seeking (similar to Bolton et al., 2016, Section II) and, hence, deployed resources are wasted from an economy-wide perspective. Simply put, what happens is similar to a prisoners' dilemma, where private incentives push institutions into high investment on financial expertise, while they would be collectively better off without it. Glode and Lowery (2016) build on Glode et al. (2012) and propose a model with competition for talent within the financial sector. Financial institutions compete with each other for a fixed number of potential employees, who can be deployed either to banking or trading (see Bond and Glode, 2014, for a model with OC between banking and bank regulation). While bankers search for profitable investment projects, traders are specialized at valuation tasks which help striking favorable deals with competitor institutions who are required to sell their investments when hit by a liquidity shock. In contrast to banking, trading is pure rent-seeking. Hence, any positive amount of workers in trading is excessive. Still, traders are not only present in equilibrium, but are even paid a higher wage than bankers. The reason is that each trader an institution does not employ itself, does not only imply foregone benefits, but can even bring harm if employed by a competitor instead. Traders hence earn a "defensive premium" over their internal marginal product. The surplus created by investment opportunities identified by bankers, on the other hand, is at risk of not being fully captured by the institution itself, but eventually being appropriated by its competitors. Consequently, bankers face a "wage penalty" on their internal marginal product.

Arping (2013, Section 3) offers another model with a competition for resources between banking and pure rent-seeking trading. In contrast, Arping (2013, Section 4) and Boot and Ratnovski (2016) study models with a genuine social trade-off between banking versus trading. In Arping (2013, Section 4), trading comes with treasury services and the provision of risk management to the bank's borrowers, which makes it complementary to banking. Boot and Ratnovski (2016) study the effects of the allocation of resource between traditional banking vs. trading on bank-level welfare. In their model, banking requires to establish customer relationships, which limits its scalability. In contrast, trading is easily scalable but capital-constrained. For the bank, complementing banking with small-scale trading activity can be beneficial, as the borrower's money can be used to relax trading's capital constraints, while trading creates additional bank profits even when the traditional banking business can't be expanded any further. Large-scale trading, however, gives rise to a time-inconsistency problem and results in an overallocation of resources to trading.

Biais et al. (2015) focus on the trading process in financial markets. Financial institutions have to decide whether to invest in a costly "fast trading" technology or not. If they do, they gain immediate access to a liquid trading venue whenever they desire to execute an asset trade (which creates "search value") and, in addition, receive private information on the asset's value (which creates "speculative value"). If they don't, they find a trading opportunity only with a certain probability and stay uninformed. Adopting the "fast trading" technology in this context entails both social benefits and costs.[2] The gains from a guaranteed trade opportunity do not come at the expense of other agents and hence the private "search value" an institution gains by investing into the technology translates one-to-one into larger social welfare. In contrast, speculative gains come at the expense of trading partners. Even worse, fast institutions' trading on private information creates a negative externality in the form of an increasing bid-ask spread that prevents potentially beneficial trades. As a consequence, equilibrium investment in the "fast trading" technology is generally excessive from a social welfare point of view. Nonetheless, the social optimum investment typically deviates from zero, as at least the aforementioned "search value" positively contributes to social welfare.

[2] See Budish et al. (2015) for a model where high-frequency trading is pure rent-seeking and leads to an "arms race" similar as in Glode et al. (2012).

Axelson and Bond (2015) study optimal contracting and promotion in finance. As the financial sector constitutes a job environment with high level risks of moral-hazard behavior, optimal contracting includes high bonus payments, which in turn help explain why employees in finance earn higher wages than employees in other occupations.[3] In an extension to their basic model, Axelson and Bond (2015, Section VII) also take an eye on the allocation of talent. They find both the possibility that highly skilled agents who would be more productive in other occupations are "lured" into finance by overpay, as well as a "talent-scorned" force, which potentially prevents talented individuals from getting a job in finance, as good outside options make it hard to provide them with proper incentivization.

Our paper also relates to the literature on feedback effects from information revealed by financial markets to the real economy, as recently reviewed by Bond et al. (2012) and Goldstein and Yang (2017). We delegate a brief discussion of this literature to Section 2.5.3.

Building on the GS (1980) model. Our model builds on the noisy REE framework by Grossman and Stiglitz (1980), which by now has become "the workhorse model in the study of financial markets with asymmetric information" (Vives, 2008, p. 112). GS (1980) study information revelation through prices and the limits of price informativeness in financial markets. Only recently, their contribution was acknowledged as one of the "'Top 20' articles published in the *American Economic Review* during its first hundred years" (Arrow et al., 2011, p. 1). Agents in the GS (1980) model can obtain costly private information about the value of a risky asset, which gives them an informational advantage when trading in the market. Trading on their private information, however, partially reveals these information via the public asset price. If, because of positive private information, demand for the asset is high, so is the asset's price. Consequently, also individuals without direct access to the private information can partly infer it from the public price. The reason why they can not do so perfectly, is that the market is exposed to "noise" due to an exogenous asset supply shock. Consequently, a high asset price is not necessarily backed by positive private information on asset fundamentals, but can also stem from "noise". GS (1980) show that informational efficiency, that is the degree to which the public price reveals private information, increases with the amount of informed market activity. Strikingly, however, markets can never be perfectly efficient, as in that case gathering information would no longer entail any private value. All information could already be inferred from the public price and hence no one would have an incentive to invest resources into acquiring these information in the first place.

Crucially, we adopt the GS (1980) notion of financial trading as not only being a zero-sum game where better-informed agents gain at the expense of their counterparties, but as an activity that makes markets more informationally efficient. Obviously, this puts our focus on a very different aspect of finance than the literature

[3]The "cream-skimming" mechanism in Bolton et al. (2016) and the "defense premium" in Glode and Lowery (2016) provide alternative explanations.

that refers to the intermediary role of financial markets. Furthermore, our specification of financial trading also differs substantially from the literature which models trading as either purely rent-seeking or related to social benefits other than informational efficiency (like, e.g., enhanced market liquidity). Further notable parts of the GS (1980) framework that carry over to our model are (i) macroeconomic uncertainty and stochastic asset prices, (ii) simultaneous trading with demand schedules, and (iii) the CARA-normal setting. Unlike idiosyncratic asset risk in the models of, e.g., Bolton et al. (2016) and Glode and Lowery (2016), macroeconomic uncertainty cannot be diversified away and translates into stochastic asset prices. Price volatility is essential for GS (1980)-like trading, as there would be no way for information to be transmitted through the price if it were fixed and known in the first place. The market microstructure is characterized by simultaneously submitted demand schedules, which give each agent's desired amount of assets as a continuous function of the price. A market maker determines the equilibrium price at which demand equals supply. This contrasts with other price determination processes, such as bilateral Nash-bargaining in Bolton et al. (2016) and Cahuc and Challe (2012), or take-it-or-leave-it prices in Glode et al. (2012) and Glode and Lowery (2016). The CARA-normal assumption makes the model analytically tractable.

In order to make proper use of the GS (1980) framework with respect to our research objective, we have to submit it to some modifications and extensions. First of all, GS (1980) do not study welfare. In order to conduct a welfare analysis, we have to specify appropriate welfare measures for all agents in the model. While this is straightforward for the rational agents, it requires us to at least somehow substantiate the origin of market noise. We attribute it to so called "noise traders" (see, e.g., Dow and Gorton, 2008), but keep their aggregate behavior exogenous and do not specify what motivates them to trade. In order to evaluate noise traders' well-being, we assign them a utility function ex-post, which, however, they do not maximize ex-ante.[4] Second, we introduce a productive sector to the GS (1980) pure financial market model and thereby endogenize the asset supply. This allows us to study competition for talent between finance, characterized by GS (1980)-like trading, and the real sector, characterized by the production of goods. Third, we dispose of the direct physical cost of acquiring information and instead take the cost of engaging in the financial sector to be the opportunity cost of not working in the productive sector and vice versa. In contrast to GS (1980), this allows for the existence of a perfectly efficient asset market, which makes a good starting point in our analytical welfare analysis. Fourth, we add a labor market for ordinary workers to account for potential positive side effects of entrepreneurship (innovation would be another one; see

[4]Explicitly modeling noise trader behavior, using (i) boundedly rational investors as in De Long et al. (1990), Mendel and Shleifer (2012) or more recently Vives and Yang (2018) and Eyster et al. (2019), or (ii) "near-rational" investors as in Hassan and Mertens (2017), or (iii) hedging motives as in Rahi (1996) or more recently Bond and Garcia (2019), or (iv) private valuations as in Vives (2014) or Rahi and Zigrand (2018), or (v) discretionary liquidity trading as in Admati and Pfleiderer (1988) or more recently Han et al. (2016), would pose a valuable alternative, but would require us to further leave the GS (1980) set-up.

Phillipon, 2010). It follows that the OC decision of skilled individuals does not only affect their own well-being, but has further consequences for the lower qualified workforce.

Whether the allocation of talent to financial trading within this setting is excessive or deficient is not obvious ex-ante. On the one hand, Murphy et al. (1991, p. 506) suppose that "even though efficiency improves, transfers are the main source of returns in trading" and the evidence presented in the Introductory Chapter lends some support to this view. If this were true and trading a mostly rent-seeking activity, then we would expect the amount of talent in finance to be excessive from a social welfare point of view. On the other hand, Bolton et al. (2016, p. 711) state that in "the standard framework of trading in financial markets first developed by Grossman and Stiglitz (1980)... privately produced information leaks out in the process of trading, and as a result *too little* costly information may be produced", and, hence, "the Grossman-Stiglitz model seems to suggest that the financial sector could be too small" (see Fishman and Parker, 2015, p. 2579, for a similar conjecture). If this were true instead and private incentives to engage in informed trading fall short of its social benefits, the equilibrium amount of talent in finance would be deficient.

Structure of the analysis. The remainder of this chapter builds on the paper by Arnold and Zelzner (2020) and is organized as follows. Chapters 2.1 and 2.2 establish the baseline model and solve for equilibrium, respectively. Chapter 2.3 analyzes the model under the simplifying assumption of non-stochastic noise trader activity. This simplification is necessary in order to derive analytical welfare results, which serve as the starting point for numerical simulation of the model with stochastic noise later. In Chapter 2.4, we extend the model by adding a labor market for ordinary workers within the real sector. We consider both a specification with full employment and a specification with labor market frictions and equilibrium unemployment. Analytical welfare analysis follows in Chapter 2.5. We aim for a second-best solution with regards to the allocation of talent, taking agents' portfolio and labor market decisions as given. In Chapter 2.6 we use Matlab to simulate a wide range of reasonable parameter combinations and check whether our analytical results from the noiseless model carry over to the model with noise trader shocks. The implementation of the second-best solution via appropriate taxation is discussed in Chapter 2.7. Concepts and results from probability theory required for a rigorous analysis of the model are given in Appendix A. Lengthy, technical proofs from the model are delegated to Appendix B. Appendix C contains additional material about the numerical simulation of the model. The Matlab code for the simulation is provided in Appendix D.

2.1 Baseline Model

We consider a CARA-normal economy with a single homogeneous consumption good. Prices are given in terms of this good. There are three "stages". Occupational choice between entrepreneurship and GS (1980)-type financial trading takes place "early". At the "intermediate" stage, all agents trade in the asset market. Uncertainty resolves and consumption is done "late". The baseline model does not include job creation by entrepreneurs. This is added in Chapter 2.4.

Market participants and occupational choice. The model contains three types of agents: a continuum of high professionals (hipos) of mass L; a continuum of passive investors of mass M; and a continuum of noise traders of mass N. Hipos and passive investors have rational expectations and their preferences are given by negative exponential utility:

$$U(\pi) = -\exp(-\rho\pi), \tag{2.1}$$

where π denotes final wealth and ρ indicates the degree of absolute risk aversion. Hipos decide "early" whether to engage in entrepreneurship within the real sector or to become a professional dealer in the financial market. Entrepreneurs set up firms and create output. Dealers acquire information and contribute to market efficiency. Passive investors engage in uninformed market trading. Hipos who neither want to become entrepreneurs nor dealers can also act as passive investors. Noise traders are characterized by exogenous stochastic trading in the asset market. We normalize agents' initial wealth to zero. As decisions under CARA-utility are independent of (non-stochastic) endowments, this is without loss of generality.

The real sector. Denote the mass of hipos who become entrepreneurs by L_E. Each entrepreneur sets up a continuum of firms indexed by the interval $[0, 1/a]$. Consequently, a firm is an element of $[0, L_E] \times [0,\ 1/a]$ and the total mass of firms is given by L_E/a. For each entrepreneur, the subset of firms he owns is of measure zero, so entrepreneurs do not have market power.[5] Each firm produces a stochastic output worth θ, where θ is the combination of two normally distributed random variables s and ε:

$$\begin{aligned} \theta &= s + \varepsilon, \\ s &\sim \mathcal{N}(\bar{s}, {\sigma_s}^2), \\ \varepsilon &\sim \mathcal{N}(0, {\sigma_\varepsilon}^2). \end{aligned} \tag{2.2}$$

The financial market. Firms are tradable in the asset market at the "intermediate" stage. Consequently, they correspond to the risky asset and the payoff of the risky asset is given by θ. Additionally, there is a safe asset in perfectly elastic supply, which allows agents to borrow or lend at zero interest rate. Dealers acquire information on the risky asset and privately observe the fundamental s.[6] Even for them, however,

[5] A more intuitive, albeit less mathematically precise interpretation of this specification would be that setting up a single firm requires a "number" of a entrepreneurs. Hence, the total mass of firms is given by L_E/a and each entrepreneur owns a fraction $1/a$ of the firm he helped to set up.

[6] As s is not firm-specific, we can interpret it as a "macro" fundamental. Hence, the fact that dealers have information on s, while entrepreneurs do not, does not imply that entrepreneurs have inferior insight into their own business. It also entails that, in contrast to Ferreira et al. (2014), entrepreneurs can not hide information by going private.

residual uncertainty ${\sigma_\varepsilon}^2$ regarding the asset payoff remains. Noise traders exogenously exert an aggregate stochastic demand ν for the risky asset, where ν is normally distributed and independent of both s and ε:

$$\nu \sim \mathcal{N}(\bar{\nu}, {\sigma_\nu}^2). \tag{2.3}$$

Rational agents maximize expected utility and submit optimal asset demand schedules to a market maker who determines the price at which demand equals supply.

Free vs. restricted OC. In addition to the setup with free OC, as explained above, we also consider a version of the model where hipos are restricted from becoming dealers. We do this in order to compare the case with professional financial trading to the case where there is no informed trading at all. This helps us isolate the effects of information revelation in financial markets on welfare.

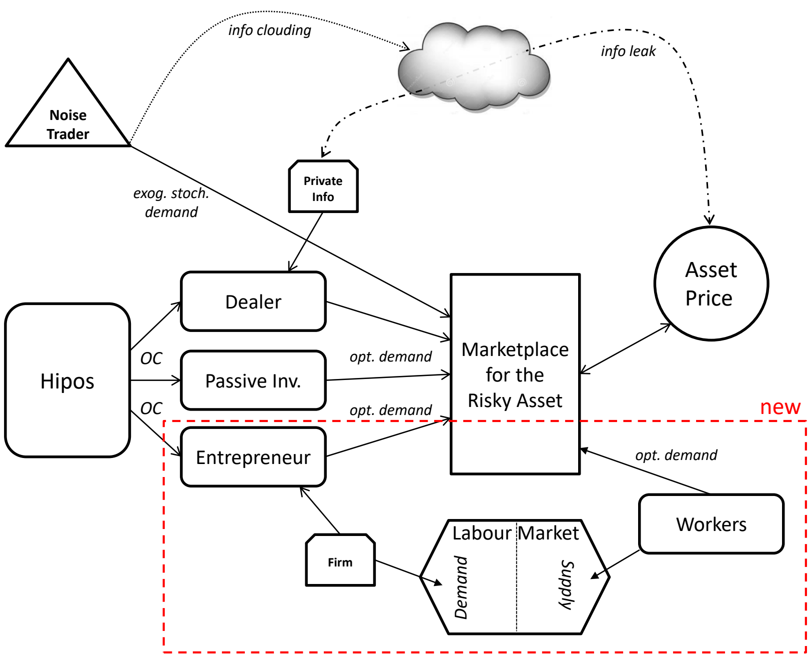
If hipos are allowed to become dealers, they never (strictly) prefer to act as passive investors. Becoming a dealer grants access to a potentially useful signal on the asset payoff and comes without any direct cost. As long as the price is not fully revealing, private access to this signal is of value and, hence, acting as a passive investor is always inferior to becoming a dealer. If the price is fully revealing, hipos are indifferent between becoming a dealer or acting as a passive investor, as they eventually end up with the same information anyway: either by observing it privately, or from the public price. In contrast to the GS (1980) set-up with a direct cost of information, in our model a fully revealing price can be part of an equilibrium with non-stochastic noise trader activity (i.e., with ${\sigma_\nu}^2 = 0$). Without noise, the price is fully revealing whenever there is a non-zero amount of dealers. The precise amount of dealers versus passive investors has no effect on the asset price and agents' welfare. A zero-mass of dealers would immediately imply a non-revealing asset price and, as a consequence, hipos would strictly prefer becoming dealers over staying passive. In any case, without loss of generality, we can confine attention to equilibria where hipos become either dealers or entrepreneurs if there is free occupational choice.

Then, with L_E denoting the amount of hipos engaging in entrepreneurship, $L - L_E$ gives: the mass of dealers, whenever there is free OC; or the mass of hipos acting as passive investors, whenever OC is restricted.

Model variants. As already mentioned, we differentiate between a noisy market environment (${\sigma_\nu}^2 > 0$) and deterministic noise trader activity (${\sigma_\nu}^2 = 0$ and $\nu = \bar{\nu}$). The latter case is important because it allows for analytical welfare results, which are unobtainable in the former. The results from the non-noisy analysis also help us guide the numerical welfare analysis in Chapter 2.6. In order to derive one of our main results (i.e., that information acquisition in financial markets harms risk-sharing and distorts the allocation of talent, cf. Proposition 2.5.2), we have to compare the situation of free vs. restricted OC. Besides the baseline model discussed above, section 2.4 adds a real-sector labor market to the model, either with or without frictions. This allows us to account for potential positive externalities from entrepreneurship and is essential for another main result (i.e., that a marginal increase in the mass of entrepreneurs, starting from equilibrium, can be conducive to welfare, cf. Proposition 2.5.1). Accounting for all of these specifications yields a total of 2x2x3=12 different model variants. To avoid confusion, we always explicitly state the variant which we are currently looking at.

Figure 2.1 illustrates the noisy version of the model with free OC (including a labor market, cf. section 2.4). In a nutshell, it works as follows. Hipos decide on their occupation, then entrepreneurs set up firms and dealers acquire fundamental information. After that, all agents trade in the asset market. Thereby, the dealers' private information partly leak out to the public. Noise prevents the information from being revealed completely: high prices can not only be the result of positive information about fundamentals, but can also stem from high noise trader demand. Everything outside the dashed red box essentially shows the GS (1980) model. Everything inside the box is what we add to it: a real sector and OC between financial trading and entrepreneurship; an endogenous asset supply from firms set up by entrepreneurs; and a real-sector labor market for ordinary workers (cf. section 2.4).

FIGURE 2.1: The Model



2.2 Equilibrium

This and the following sections define and solve for equilibrium. We start with the noisy baseline model with free OC.

Final wealth. The final wealth of a dealer is given by $\pi_D = (\theta - P) I_D$, where I_D denotes asset holdings. A dealer's information set $\mathcal{I}_D$ consists in the public asset price P and the private signal s, that is, $\mathcal{I}_D = \{P, s\}$.[7] For a passive investor, it is $\pi_M = (\theta - P) I_M$ and $\mathcal{I}_M = \{P\}$. As entrepreneurs already own parts of a firm, in order to end up with a portfolio of I_E risky assets, they have to sell a net amount of $\tilde{I_E} = 1/a - I_E$ assets in the market. Hence, $\pi_E = \theta I_E + P(1/a - I_E)$ and $\mathcal{I}_E = \{P\}$.

Definition of equilibrium. Agents' occupational choice L_E, their asset holdings I_D, I_E, I_M and the asset price P jointly determine an equilibrium, iff:

(i) I_D maximizes $\mathbb{E}\left[U(\pi_D)|s, P\right]$,

(ii) I_E maximizes $\mathbb{E}\left[U(\pi_E)|P\right]$ and I_M maximizes $\mathbb{E}\left[U(\pi_M)|P\right]$,

(iii) P clears the asset market, that is $(L - L_E)I_D + MI_M + \nu = L_E(1/a - I_E)$,

(iv) and OC is optimal, i.e.,

- $\mathbb{E}(\pi_E) = \mathbb{E}(\pi_D)$ and $0 < L_E \leq L$, or
- $\mathbb{E}(\pi_E) \geq \mathbb{E}(\pi_D)$ and $L_E = L$.

Equilibrium conditions (i) and (ii) say that, at the "intermediate" stage, agents chose their asset holdings so as to maximize the expected utility of final wealth, conditional on the information they have. The market clearing condition is given by (iii), where the l.h.s. shows asset demand by dealers, passive investors and noise traders and the r.h.s. entrepreneurs' asset supply. The fact that, at the "early" stage, hipos optimally choose the occupation which promises the highest expected utility is entailed in (iv). In an interior equilibrium with both entrepreneurs and dealers, the expected utilities of the two have to be equal to each other. If they were not, then agents would have an incentive to switch occupations towards the one that promises the higher utility. We also allow for a corner equilibrium with entrepreneurs only, which would arise if the expected utility of an entrepreneur exceeds that of a dealer even if all agents become entrepreneurs.[8]

In the timing of the model, occupational choice precedes market trading. Solving for equilibrium works backwards. We first derive agents' optimal portfolio decisions and the equilibrium asset price, given the occupational choice. In a second step, we determine the equilibrium allocation of talent, where individuals optimally decide on the occupation, taking into account their later optimal asset holdings and the resulting asset price.

[7] The price is determined by the auctioneer and therefore is not literally known when the agents make their investment decisions. However, agents do not decide on a fixed amount of assets to buy or sell, but they trade on demand schedules. These are conditioned on the price and, in this sense, P is part of the information set.

[8] We do not consider "equilibria" with a zero mass of entrepreneurs, as this would imply that the risky asset is in zero supply.

2.2.1 Portfolio Holdings and Price Function

Equilibrium conditions (i) and (ii) yield

$$I_D = \frac{s-P}{\rho\sigma_\varepsilon{}^2}, \quad I_E\,(=I_M) = \frac{\mathbb{E}\,(\theta|P)-P}{\rho\mathbb{V}\,(\theta|P)} \tag{2.4}$$

(see Appendix B.1). Agents hold positive amounts of the asset, if its (conditional) expected payoff exceeds its price. Intuitively, they hold more assets, (i) the higher the payoff-to-price differential, (ii) the lower the (conditional) uncertainty regarding the payoff, and (iii) the lower their degree of risk aversion ρ.

Substituting the optimal portfolio choices from (2.4) into the market clearing condition gives the equilibrium price function:

$$P = \frac{w + \frac{L_E+M}{\rho\,\mathbb{V}(\theta|\,w)}\,\mathbb{E}(\theta|\,w) - \frac{L_E}{a}}{\frac{L-L_E}{\rho\sigma_\varepsilon{}^2} + \frac{L_E+M}{\rho\,\mathbb{V}(\theta|\,w)}}, \tag{2.5}$$

where

$$w := \frac{L-L_E}{\rho\sigma_\varepsilon{}^2}s + \nu \tag{2.6}$$

(see Appendix B.2). Both w and P are normally distributed. From Appendix A.3, we know that $\mathbb{E}\,(\theta|w)$ is just a linear function of w and $\mathbb{V}\,(\theta|w)$ is non-random. Hence, the asset price is a linear function of w. As w is the only stochastic variable in the price function, P and w are informationally equivalent in the sense that one can immediately infer P from w and vice versa.

As $\mathbb{E}\,(\theta|w)$ is increasing in w and w is in turn increasing in s, the asset price is the higher, the "better" the asset fundamental. However, as w is also increasing in ν, the price is also the higher, the greater the amount of noise trader asset demand. Consequently, entrepreneurs and passive investors can not perfectly relate a high (low) price P to a "good" ("bad") asset fundamental s. So, while the public price does, to some extent, contain information about s, noise keeps it from being fully revealing.

Restricted occupational choice. Now assume that hipos are no longer allowed to become dealers. Then, given OC, agents' portfolio holdings I_E, I_M and the asset price P jointly determine a (partial) equilibrium, if (i) I_E maximizes $\mathbb{E}\,[U\,(\pi_E)\,|P]$, (ii) I_M maximizes $\mathbb{E}\,[U\,(\pi_M)\,|P]$, and (iii) P clears the asset market, that is $(L - L_E + M)I_M + \nu = L_E\,(1/a - I_E)$.

Without dealers, the price does not contain any information regarding the asset fundamental s. Hence, $\mathbb{E}(\theta|P) = \mathbb{E}(\theta) = \bar{s}$ and $\mathbb{V}(\theta|P) = \mathbb{V}(\theta) = \sigma_s{}^2 + \sigma_\varepsilon{}^2$. From (2.4), it immediately follows that the optimal portfolio decisions are given by (2.7).[9]

[9]It is not completely straightforward to argue that (2.4) remains valid for the case of restricted OC. Some expressions that have been normally distributed with free OC, are non-stochastic with restricted

The market clearing condition simplifies to $(L+M)\, I_M + \nu = L_E/a$ and immediately gives the equilibrium price function according to (2.8):

$$I_E (= I_M) = \frac{\bar{s} - P}{\rho\left(\sigma_s^2 + \sigma_\epsilon^2\right)}, \tag{2.7}$$

$$P = \bar{s} - \frac{\rho\left(\sigma_s^2 + \sigma_\epsilon^2\right)}{L+M}\left(\frac{L_E}{a} - \nu\right). \tag{2.8}$$

As no one observes the asset fundamental s, the best guess on it is just $\mathbb{E}(s) = \bar{s}$. The only price variation left is due to stochastic noise trader demand ν.

2.2.2 Occupational Choice

When deciding on their occupation, hipos compare unconditional expected utilities. Equivalently, they can compare unconditional certainty equivalents (CEs). As its name suggests, the certainty equivalent CE_i of an individual in occupation i, $i \in \{E, D, M\}$, gives the certain amount of wealth that would provide the individual with exactly the same utility as it expects to derive from its uncertain final wealth π_i. Hence, it is implicitly given by $U(\mathsf{CE}_i) = \mathbb{E}\left[U\left(\pi_i\right)\right]$. An agent's CE is equivalent to his expected utility in the sense that the former is just a strictly monotonically increasing function of the latter:

$$\mathsf{CE}_i = U^{-1}\left\{\mathbb{E}\left[U(\pi_i)\right]\right\} = -\frac{1}{\rho}\ln\left\{-\mathbb{E}\left[U\left(\pi_i\right)\right]\right\}. \tag{2.9}$$

Note that we have specified our utility function to take on values only in the negative realm, so $-\mathbb{E}\left[U\left(\pi_i\right)\right]$ is always positive.

Agents' Certainty Equivalents

In what follows, we use the law of iterated expectations to derive the ex-ante (unconditional) certainty equivalents for passive investors, dealers and entrepreneurs in the noisy model with free OC.

Passive investors. With his optimal portfolio holdings I_M according to (2.4), the final wealth of a passive investor is given by

$$\pi_M = \underbrace{I_M(\theta - P)}_{\mathsf{GT}_{M|s,\varepsilon,\nu}} = \mathsf{CE}_{M|s,\varepsilon,\nu}. \tag{2.10}$$

"Late", when all uncertainty resolves, a passive investor's final wealth is not random any more. In this sense, $\mathsf{CE}_{M|s,\varepsilon,\nu}$ denotes his (conditional) certainty equivalent,

OC. Hence, it remains to be checked whether the conditions required to derive (2.4) still hold. The fact that the formulas in Appendix A.3 and A.4 also hold for non-random and degenerate joint normal variables implies that this is the case.

given that he observes s, ε and ν. We denote the part of a passive investor's (conditional) certainty equivalent that relates to gains from trading by $\mathsf{GT}_{\mathsf{M}|s,\varepsilon,\nu}$. As we see, a passive investor gains wealth solely from uninformed trading in the asset market. At the "intermediate" stage, a passive investor only observes the public asset price P. His certainty equivalent, conditional on P, is given by

$$\mathsf{CE}_{M|P} = \underbrace{\frac{[\mathbb{E}(\theta|P) - P]^2}{2\rho\mathbb{V}(\theta|P)}}_{\mathsf{GT}_{M|P}}. \tag{2.11}$$

Finally, there are no information available at all in the "early" stage. A passive investor's unconditional certainty equivalent is given by

$$\mathsf{CE}_M = \underbrace{\frac{\mathbb{E}(z)^2}{1+2\rho\mathbb{V}(z)} + \frac{1}{2\rho}\ln\left[1+2\rho\mathbb{V}(z)\right]}_{\mathsf{GT}_M}, \tag{2.12}$$

where

$$z := \frac{[\mathbb{E}(\theta|P) - P]}{\sqrt{2\rho\mathbb{V}(\theta|P)}} = \sqrt{\mathsf{GT}_{M|P}} \tag{2.13}$$

(see Appendix B.3). Note that z, which is just a monotonic transformation of $\mathsf{GT}_{M|P}$, is proportional to the (conditional) Sharpe Ratio of the risky asset $\mathsf{SR}_{\theta|P}$ (cf. Appendix B.3). Hence, it closely relates to the (conditional) risk-adjusted expected return from uninformed trading in the market.

Dealers. Analogously to the passive investors above, for a dealer we get

$$\pi_D = \underbrace{I_D(\theta - P)}_{\mathsf{GT}_{D|s,\varepsilon,\nu}} = \mathsf{CE}_{D|s,\varepsilon,\nu}. \tag{2.14}$$

Dealers gain wealth solely from informed trading in the asset market. Their certainty equivalent conditional on s and P is given by

$$\mathsf{CE}_{D|s,P} = \underbrace{\frac{(s-P)^2}{2\rho{\sigma_\varepsilon}^2}}_{\mathsf{GT}_{D|s,P}}. \tag{2.15}$$

Similar as before, $\sqrt{\mathsf{GT}_{D|s,P}}$ is proportionate to the (conditional) Sharpe Ratio $\mathsf{SR}_{\theta|s,P}$ and closely relates to the (conditional) risk-adjusted expected return from informed trading in the market (see Appendix B.3).
Dealers' certainty equivalent conditional on P is given by

$$\mathsf{CE}_{D|P} = \underbrace{\frac{1}{2\rho}\ln\frac{\mathbb{V}(\theta|P)}{{\sigma_\varepsilon}^2}}_{\mathsf{GI}_{D|P}} + \underbrace{\frac{[\mathbb{E}(\theta|P) - P]^2}{2\rho\mathbb{V}(\theta|P)}}_{\mathsf{GT}_{M|P}}. \tag{2.16}$$

As $\mathsf{GI}_{D|P}$ is non-random, it carries over one-to-one into the unconditional certainty equivalent and we get

$$\mathsf{CE}_D = \underbrace{\frac{1}{2\rho}\ln\left[1+\frac{\mathbb{V}(s|P)}{\sigma_\varepsilon^2}\right]}_{\mathsf{GI}_D} + \underbrace{\frac{[\mathbb{E}(z)]^2}{1+2\rho\mathbb{V}(z)} + \frac{1}{2\rho}\ln\left[1+2\rho\mathbb{V}(z)\right]}_{\mathsf{GT}_M} \tag{2.17}$$

(see Appendix B.3). As private information come without direct costs and give an informational advantage in trading, a dealer's CE exceeds that of a passive investor. We denote the term which is related to these gains from information by GI_D. It is the higher, (i) the less residual uncertainty ${\sigma_\varepsilon}^2$ remains, and (ii) the larger the dealers' informational advantage (i.e., the less private information leaks to the public and therefore the larger $\mathbb{V}(s|P)$). In case that the price fully reveals the private signal, it is $\mathbb{V}(s|P) = 0$, $\mathsf{GI}_D = 0$ and, hence, the expected utility of a dealer equals that of a passive investor.

Entrepreneurs. An entrepreneur's final wealth π_E can be written as

$$\pi_E = \underbrace{\frac{P}{a}}_{\mathsf{GP}_{E|s,\varepsilon,\nu}} + \underbrace{I_E(\theta - P)}_{\mathsf{GT}_{E|s,\varepsilon,\nu}} = \mathsf{CE}_{E|s,\varepsilon,\nu}. \tag{2.18}$$

The market value created by a single hipo's entrepreneurial activity is denoted by $\mathsf{GP}_{E|s,\varepsilon,\nu}$. "Gross" gains from trade are denoted by $\mathsf{GT}_{E|s,\varepsilon,\nu}$. These gains are "gross" in the sense that they arise from buying an amount of I_E assets in the market, which implies that the entrepreneur has sold his initial $1/a$ assets before. For an alternative illustration of π_E, which instead separates entrepreneurs' "net" gains from trade from the fundamental value of entrepreneurship, see Appendix B.4.
An entrepreneur's certainty equivalent conditional on P is given by

$$\mathsf{CE}_{E|P} = \underbrace{\frac{P}{a}}_{\mathsf{GP}_{E|P}} + \underbrace{\frac{[\mathbb{E}(\theta|P) - P]^2}{2\rho\mathbb{V}(\theta|P)}}_{\mathsf{GT}_{E|P}\ (=\mathsf{GT}_{M|P})}. \tag{2.19}$$

For the unconditional CE, it follows that

$$\mathsf{CE}_E = \underbrace{\mathbb{E}\left(\frac{P}{a}\right) - \frac{\rho}{2}\mathbb{V}\left(\frac{P}{a}\right)}_{\mathsf{GP}_E} + \underbrace{\frac{\left[\mathbb{E}\left(z\right) - \rho\mathrm{Cov}\left(\frac{P}{a}, z\right)\right]^2}{1+2\rho\mathbb{V}(z)} + \frac{1}{2\rho}\ln\left[1+2\rho\mathbb{V}(z)\right]}_{\mathsf{GT}_E} \tag{2.20}$$

(see Appendix B.3). GP_E relates to the expected utility from selling $1/a$ assets for price P in the market. GT_E relates to the expected utility from (re)building the desired asset portfolio I_E after that. Without the covariance term, GT_E would exactly equal a passive investor's gains from trading GT_M. However, an entrepreneur's gains from selling the $1/a$ shares in his firm and the gains from buying back the desired amount of asset holdings I_E are not independent of each other. A higher asset price P increases the former, but hurts the latter. This counter-movement, represented by $\mathrm{Cov}(P, z) < 0$ (see Appendix B.5), mitigates an entrepreneur's risk overall and hence increases GT_E. As a consequence, the "gross" gains from trade for an entrepreneur are always higher than those for passive investors, even though they trade on the same information and eventually hold the same asset portfolio $I_E = I_M$.[10] Again, see Appendix B.4 for an alternative illustration of entrepreneurs' CE, which explicitly states expected utility from "net" trade.

The Equilibrium Allocation of Talent

For each hipo, it is optimal to choose the occupation that yields the highest CE. As we can immediately see by comparing (2.12) to (2.17), becoming a passive investor is never preferred to becoming a dealer. Hence, with free OC, hipos become either dealers or entrepreneurs. In an interior equilibrium $0 < L_E < L$ with both entrepreneurs and dealers, CE_E has to equal CE_D. Otherwise, some agents would voluntarily choose an "inferior" occupation, which is incompatible with optimal behavior. A corner equilibrium with entrepreneurs only may arise if the certainty equivalent of an entrepreneur exceeds that of a dealer even when all agents become entrepreneurs, that is, if $\mathsf{CE}_E \geq \mathsf{CE}_D$ at $L_E = L$.

With the expressions for $\mathbb{V}(s|w)$, $\mathbb{E}(P)$, $\mathbb{V}(P)$, $\mathbb{E}(z)$, $\mathbb{V}(z)$ and $\mathrm{Cov}(P, z)$ given in Appendix B.5, one immediately sees that the agents' certainty equivalents are continuous functions of L_E. Let

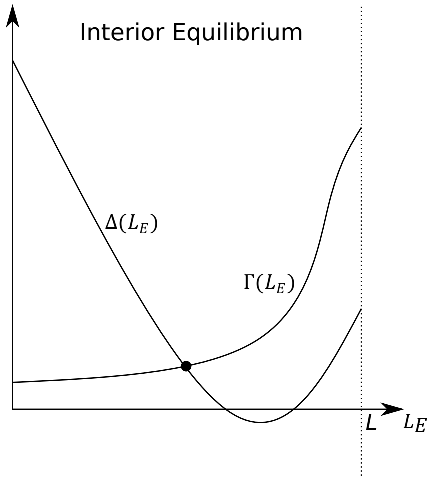
$$\begin{aligned} \Delta(L_E) &:= \mathsf{CE}_E - \mathsf{GT}_M = \mathsf{GP}_E + (\mathsf{GT}_E - \mathsf{GT}_M)\,, \\ \Gamma(L_E) &:= \mathsf{CE}_D - \mathsf{GT}_M = \mathsf{GI}_D. \end{aligned} \tag{2.21}$$

Then, if there is an L_E with $0 < L_E < L$ that solves $\Delta(L_E) = \Gamma(L_E)$, this L_E constitutes an interior equilibrium. If $\Delta(L) \geq \Gamma(L)$, then there is a corner equilibrium with $L_E = L$. Together with the optimal portfolio decisions in (2.4) and the equilibrium asset price (2.5), the equilibrium mass of entrepreneurs L_E constitutes the general equilibrium of the model.

Existence and uniqueness. As figure 2.2 illustrates, continuity of $\Delta(L_E)$ and $\Gamma(L_E)$ implies that a sufficient condition for the existence of an equilibrium with a positive mass of entrepreneurs is given by $\Delta(0) > \Gamma(0)$. As $\Delta(L_E) - \Gamma(L_E)$ is not necessarily monotonic, multiple equilibria are possible. With regards to figure 2.2, this would be the case if $\Delta(L_E)$ crosses $\Gamma(L_E)$ again, from below. The result would be a co-existence of two interior equilibria plus a corner equilibrium. With regards to their properties, these would be quite different. To see this, call an equilibrium L_E "stable", if, after a slight perturbation away from it, the mass of entrepreneurs "returns" to exactly

[10] Strictly speaking, $\mathsf{GT}_E > \mathsf{GT}_M$ also requires $\mathbb{E}(z) > 0$. With (2.13) and (2.11), we can easily show that $z = [(\rho/2)\mathbb{V}(\theta|P)]^{0.5} \cdot I_M$. Hence, $\mathbb{E}(z) > 0$ whenever $\mathbb{E}(I_M) > 0$, that is whenever rational agents are expected not to short the asset.

FIGURE 2.2: Equilibrium L_E in Case of Free OC and ${\sigma_v}^2 > 0$



this equilibrium value.[11] While the interior equilibrium where $\Delta(L_E)$ crosses $\Gamma(L_E)$ from above, as well as the corner equilibrium would obviously be stable, the interior equilibrium where $\Delta(L_E)$ crosses $\Gamma(L_E)$ from below would not: a small perturbation away from it makes the mass of entrepreneurs move to one of the other two equilibria instead.

Strategic substitutability vs. complementarity. Dealers' utility from having an informational advantage over the other agents, given by $\Gamma(L_E)(= \mathsf{GI}_D)$, is strictly increasing in L_E. Intuitively, the informational advantage of a single dealer is the higher, the less other dealers are around.

With regards to entrepreneurs, we see two effects. The first one is, again, one of strategic substitutability: a single entrepreneur's gains from production are the higher, the less other entrepreneurs are around. This is because entrepreneurs are net sellers of the asset and less aggregate entrepreneurship implies a lower total asset supply (and, hence, higher prices). Interestingly, however, we see an additional effect of strategic complementarity: a single entrepreneur benefits from the presence of other entrepreneurs through the fact that more entrepreneurship implies fewer dealers and therefore less information revelation in the market. In fact, as we show in section 2.5.3, the availability of information harms entrepreneurs via an inefficient clustering of risk.

Depending on which of the two effects dominates, $\Delta(L_E)$ is decreasing or increasing. As figure 2.2 shows, we find that for low values of L_E and, hence, a large number of dealers, the first effect tends to dominate. This is not surprising, as $\mathbb{V}(\theta|P)$ as a measure of the degree of information revelation (the higher $\mathbb{V}(\theta|P)$, the less information is revealed) tends to be less sensitive to marginal changes in the mass of dealers when there are relatively many of them (see the pattern of $\Gamma(L_E)(= \mathsf{GI}_D)$ in figure 2.2). In contrast, for high values of L_E and a low presence of dealers, a

[11] Talking about "stability" in a non-dynamic context is controversial. In doing so, we follow the argumentation in Manzano and Vives (2011) and Biais et al. (2015, p. 303).

marginal change in the amount of informed trading has a rather large impact on informational efficiency and, as a result, the second effect gains in strength.

Equilibrium with restricted OC. If hipos are restricted from becoming dealers, the price does not contain any information regarding the risky asset θ. With P according to (2.8), it follows from (2.13) that

$$\begin{aligned} z &= \frac{1}{\sqrt{2\rho}} \frac{\bar{s} - P}{\sqrt{{\sigma_s}^2 + {\sigma_\varepsilon}^2}} \\ &= \sqrt{\frac{\rho}{2} \left(\sigma_s^2 + \sigma_\epsilon^2\right)} \cdot \frac{\frac{L_E}{a} - \nu}{L + M}. \end{aligned} \tag{2.22}$$

The certainty equivalents of passive investors and entrepreneurs are given by (2.12) and (2.20), with P and z according to (2.8) and (2.22). The corresponding closed-form solutions for $\mathbb{E}(P)$, $\mathbb{V}(P)$, $\mathbb{E}(z)$, $\mathbb{V}(z)$ and $\mathrm{Cov}(P, z)$ are given in Appendix B.6.

Without the option to engage in professional trading, hipos decide whether to become entrepreneurs or passive investors. Let again $\Delta(L_E) = \mathsf{CE}_E - \mathsf{GT}_M$. Then, if there is an L_E with $0 < L_E < L$ that solves $\Delta(L_E) = 0$, this L_E constitutes an interior equilibrium with a positive mass of both entrepreneurs and passive investors. If $\Delta(L) \geq 0$, then there is a corner equilibrium with $L_E = L$. Together with the optimal portfolio decisions in (2.7) and the equilibrium asset price (2.8), the equilibrium mass of entrepreneurs L_E constitutes the general equilibrium of the model. The reason why some hipos might prefer staying passive over becoming entrepreneurs is that entrepreneurship entails entrepreneurial risk. Even though the asset price P, according to (2.8), is now independent of the fundamental s, uncertainty remains with regards to noise trader demand ν. Hence, it is $\mathbb{V}(P) > 0$ and with that the gains from entrepreneurship GP_E are not necessarily positive.

Continuity of $\Delta(L_E)$ implies that a sufficient condition for the existence of an equilibrium with a positive mass of entrepreneurs is given by $\Delta(0) > 0$. In contrast to the case of free OC, $\Delta(L_E)$ is strictly monotonically decreasing with restricted OC (see Appendix B.6). Consequently, equilibrium is unique.

2.3 Deterministic Noise Trader Demand

Even though our analysis of the noisy model yields closed-form solutions, complicated expressions for the agents' CEs (cf. Appendix B.3) make it infeasible to derive meaningful analytical welfare results. We therefore take a two-step approach. First, we provide a rigorous analysis of the noiseless model, that is, we assume deterministic noise trader demand (i.e., ${\sigma_\nu}^2 = 0$). In this case, agents can perfectly infer private information from the public price. Hence, information asymmetries vanish and there is either complete or no information in the market, depending on whether the mass of dealers is strictly positive or not. This simplifies expressions enough to allow for analytical welfare analysis. As a second step, we then show that the welfare results we obtain from this model also hold in the limit for small noise volatility and use this as the starting point for our numerical welfare analysis of the model with substantial noise volatility (cf. section 2.6).

Let ${\sigma_\nu}^2 = 0$ and $\nu = \bar{\nu}$. Consider first the case of free OC. If there is a positive mass of dealers, that is $L_E < L$, then with (2.5) and (2.6) we immediately see that the price P fully reveals the private signal s. It follows that $\mathbb{E}(\theta|w) = s$ and $\mathbb{V}(\theta|w) = {\sigma_\varepsilon}^2$. If there are no dealers at all, that is $L_E = L$, the price does not contain information on s and $\mathbb{E}(\theta|w) = \bar{s}$ and $\mathbb{V}(\theta|w) = {\sigma_\varepsilon}^2 + {\sigma_s}^2$. From (2.5), the equilibrium price function is given by:[12]

$$P = \begin{cases} s - \frac{\rho {\sigma_\varepsilon}^2}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right), & \text{for } L_E < L \\ \bar{s} - \frac{\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right)}{L+M}\left(\frac{L}{a} - \bar{\nu}\right), & \text{for } L_E = L \end{cases} \tag{2.23}$$

(see Appendix B.7). From (2.13), it immediately follows that

$$z = \begin{cases} \sqrt{\frac{\rho}{2}{\sigma_\varepsilon}^2} \cdot \frac{\frac{L_E}{a} - \bar{\nu}}{L+M}, & \text{for } L_E < L \\ \sqrt{\frac{\rho}{2}({\sigma_\varepsilon}^2 + {\sigma_s}^2)} \cdot \frac{\frac{L}{a} - \bar{\nu}}{L+M}, & \text{for } L_E = L \end{cases}. \tag{2.24}$$

Hence, z is non-random and $\mathbb{V}(z) = 0$ and $\mathrm{Cov}(P, z) = 0$, from which it immediately follows $\mathsf{GT}_M = \mathsf{GT}_E$. For $L_E = L$, the asset price P is also non-random. The functions $\Delta(L_E)$ and $\Gamma(L_E)$ simplify to

$$\Delta(L_E) = \mathsf{GP}_E = \begin{cases} \frac{1}{a}\left(\bar{s} - \frac{\frac{L_E}{a} - \bar{\nu}}{L+M}\rho{\sigma_\varepsilon}^2 - \frac{\rho{\sigma_s}^2}{2a}\right), & \text{for } L_E < L \\ \frac{1}{a}\left(\bar{s} - \frac{\frac{L}{a} - \bar{\nu}}{L+M}\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right)\right), & \text{for } L_E = L \end{cases} \tag{2.25}$$

and

$$\Gamma(L_E) = \mathsf{GI}_D = \begin{cases} 0, & \text{for } L_E < L \\ \frac{1}{2\rho}\ln\left[1 + \frac{{\sigma_s}^2}{{\sigma_\varepsilon}^2}\right], & \text{for } L_E = L \end{cases}. \tag{2.26}$$

As $\mathsf{GI}_D = 0$ for $L_E < L$, dealers do not earn informational rents in the model without noise. For $L_E < L$, $\Delta(L_E)$ is strictly monotonically decreasing in L_E. Hence, if an interior equilibrium with $L_E < L$ exists, then this is the only interior equilibrium. Obviously, $\Delta(L_E)$ and $\Gamma(L_E)$ are not continuous at $L_E = L$. This implies the possibility for non-existence of equilibrium or two simultaneous equilibria, one with $L_E < L$ and one at $L_E = L$.

For an interior equilibrium with both dealers and entrepreneurs to exist, we require $\Delta(L_E) = 0$ for some L_E with $0 < L_E < L$. This is the case, exactly if

$$\bar{s}_1 < \bar{s} < \bar{s}_2, \tag{2.27}$$

[12] The fact that Appendices A.3 and A.4 also hold for non-random and degenerate joint normal variables implies that the general expressions derived in Chapter 2.2 carry over to the noiseless case.

where

$$\bar{s}_1 := \frac{\rho {\sigma_s}^2}{2a} - \frac{\bar{v}}{L+M}\rho {\sigma_\varepsilon}^2, \quad \bar{s}_2 := \frac{\rho {\sigma_s}^2}{2a} + \frac{\frac{L}{a} - \bar{v}}{L+M}\rho \sigma_\varepsilon^2 \tag{2.28}$$

(see Appendix B.8). If (2.27) holds, then the interior equilibrium L_E is given by

$$L_E = \frac{a(L+M)}{\rho {\sigma_\varepsilon}^2}\left(\bar{s} - \frac{\rho {\sigma_s}^2}{2a}\right) + a\bar{v} \tag{2.29}$$

(see Appendix B.8). Even though dealers do not have an informational advantage over passive investors, the OC decision is non-trivial in the sense that not all agents necessarily want to become entrepreneurs. While the asset price P, according to (2.23), is deterministic with respect to noise traders' asset demand, it still varies with respect to the fundamental s. Hence, it is $\mathbb{V}(P) > 0$ and this entrepreneurial risk implies that the gains from entrepreneurship GP_E are not necessarily positive.

For a corner equilibrium with only entrepreneurs, we require $\Delta(L) \geq \Gamma(L)$, which can be written as

$$\bar{s} \geq \bar{s}_3, \tag{2.30}$$

where

$$\bar{s}_3 := \frac{\frac{L}{a} - \bar{v}}{L+M}\rho\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right) + \frac{a}{2\rho}\ln\left(1 + \frac{{\sigma_s}^2}{{\sigma_\varepsilon}^2}\right) \tag{2.31}$$

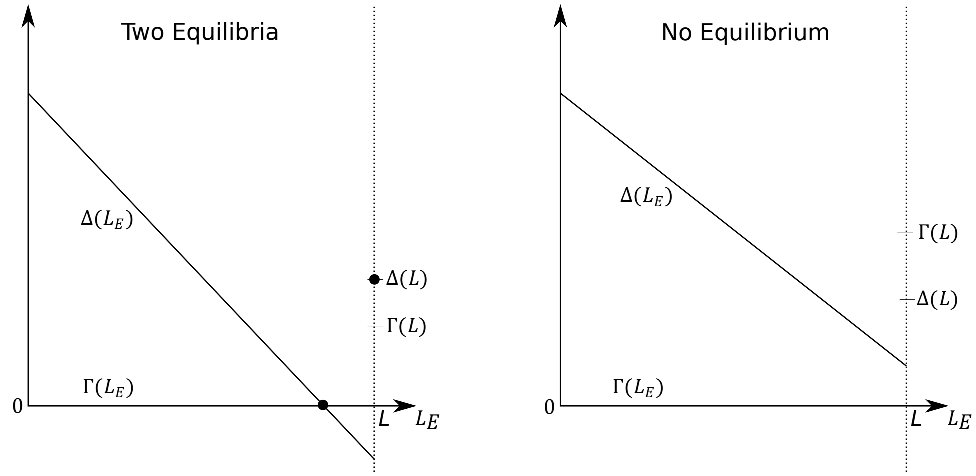
(see Appendix B.8). The fact that $\bar{s}_2 \neq \bar{s}_3$ gives room for the existence of multiple equilibria as well as the possibility of non-existence. Figure 2.3 illustrates the two cases. If $\Delta(L_E) = 0$, for an $L_E < L$, and $\Delta(L) \geq \Gamma(L)$, an interior and a corner equilibrium co-exist. If $\Delta(L_E) > 0$ for $L_E < L$, but $\Gamma(L) > \Delta(L)$, the GS (1980) non-existence result re-arises. As long as $L_E < L$ and the private signal is fully revealed by the public price, all agents prefer to engage in entrepreneurship. Hence, an equilibrium with $L_E < L$ does not exist. For $L_E = L$, however, where the price is uninformative, the (price-taking) agents gain an incentive to acquire private information and, thereby, a (perceived) informational advantage over the other agents, by becoming dealers. Consequently, $L_E = L$ is not an equilibrium either.[13]

If $\Delta(L_E) = 0$ for an $L_E < L$, then an equilibrium where the public price fully reveals the private signal exists. With direct costs of information as in GS (1980), such an equilibrium would not exist: agents would always prefer to become passive investors rather than dealers and free-ride on the information acquired by others. An equilibrium with only entrepreneurs exists if, at $L_E = L$, the opportunity cost of becoming a dealer (i.e., the gain from entrepreneurship) is larger than the (perceived) gains from information. This is similar to GS (1980), where such an equilibrium can only exist if the direct costs of acquiring information are sufficiently large.

Restricted occupational choice. In case of free OC, price informativeness jumps from "perfect", for $L_E < L$, to "zero", at $L_E = L$. Without dealers, the price never

[13]Hellwig (1980) has called these GS(1980)-type agents as schizophrenic, as they do understand how the price conveys information, but do not realize their own informational impact on the price.

FIGURE 2.3: Equilibrium L_E in Case of Free OC and ${\sigma_\nu}^2 = 0$



Note: In contrast to the two cases depicted in the figure, equilibrium is unique, if either (i) $\Delta(L_E) = 0$ for an $L_E < L$ and $\Gamma(L) > \Delta(L)$ (interior equilibrium), or (ii) $\Delta(L_E) > 0$ for $L_E < L$ and $\Delta(L) > \Gamma(L)$ (corner equilibrium).

conveys any information at all. Therefore, there are no discontinuities in case of restricted OC. According to (2.8) and (2.22), with $\bar{\nu}$ instead of ν, we get:

$$P = \bar{s} - \frac{\rho\left(\sigma_s^2 + \sigma_\epsilon^2\right)}{L + M}\left(\frac{L_E}{a} - \bar{\nu}\right), \tag{2.32}$$

$$z = \sqrt{\frac{\rho}{2}\left(\sigma_s^2 + \sigma_\epsilon^2\right)} \cdot \frac{\frac{L_E}{a} - \bar{\nu}}{L + M}. \tag{2.33}$$

P and z are both non-stochastic and hence it is $\mathsf{GT}_M = \mathsf{GT}_E$ as well as $\mathbb{V}(P) = 0$. It follows that

$$\Delta(L_E) = \mathsf{GP}_E = \frac{1}{a}\left(\bar{s} - \frac{\frac{L_E}{a} - \bar{\nu}}{L + M}\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right)\right). \tag{2.34}$$

Obviously, $\Delta(L_E)$ is strictly decreasing in L_E. Together with continuity, this implies that equilibrium is unique. For an interior equilibrium with both entrepreneurs and passive investors to exist, we require $\Delta(L_E) = 0$ at an $0 < L_E < L$, that is:

$$\bar{s}_4 < \bar{s} < \bar{s}_5, \tag{2.35}$$

where

$$\bar{s}_4 := -\frac{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}{L + M}\bar{\nu}, \quad \bar{s}_5 := \frac{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}{L + M}\left(\frac{L}{a} - \bar{\nu}\right) \tag{2.36}$$

(see Appendix B.8). If (2.35) holds, then the interior equilibrium L_E is given by

$$L_E = \frac{a(L + M)}{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}\bar{s} + a\bar{\nu} \tag{2.37}$$

(see Appendix B.8). While this tells us that an equilibrium at which some hipos become entrepreneurs and some act as passive investors is a theoretical possibility, it

is not very intuitive. As the asset price P, according to (2.32), is non-random, there is no entrepreneurial risk. Therefore, the gains from entrepreneurship GP_E are positive, as long as the expected asset price is positive. Put the other way round, an interior equilibrium is possible only if $\mathbb{E}(P)$ becomes negative.
For a corner equilibrium with only entrepreneurs, we require $\Delta(L) \geq 0$, which can be written as

$$\bar{s} \geq \bar{s}_5. \tag{2.38}$$

With (2.35) and (2.38), we again see that an equilibrium with a positive amount of entrepreneurs exists and is unique, whenever $\Delta(0) > 0$.

Comparative Statics

With the (interior) equilibrium L_E in case of free OC given by (2.29), we can easily look at some comparative statics regarding the equilibrium mass of entrepreneurs. First of all, an increase in the expected asset payoff $\bar{s}$ increases the equilibrium mass of entrepreneurs. This is pretty intuitive, as the gains from entrepreneurship stem from the creation and consequent ownership of this asset. An increase in $\bar{v}$ increases aggregate demand and with that the asset's market value. Therefore, the equilibrium mass of entrepreneurs goes up.

In what follows, assume that all agents hold positive amounts of the risky asset (cf. the subsection below). Then, similarly as a variation in $\bar{v}$, an increase in the amount of passive investors M increases aggregate demand and, therefore, the equilibrium level of L_E. While, unsurprisingly, an increase in the total amount of hipos L leads to a higher mass of entrepreneurs L_E, note that the fraction of hipos that decides to engage in entrepreneurship (i.e., L_E/L) is in fact decreasing in L. Interestingly, higher values for a imply a larger amount of entrepreneurs. This might be unexpected, because a higher a effectively means that the amount of assets that is to be gained from entrepreneurship decreases. As it turns out, however, this benefits entrepreneurs on net: less of the asset does not only mean less expected profit, but also less risk; and less of the asset for each entrepreneur implies less aggregate supply for a given mass of entrepreneurs, which in turn benefits the asset's market value. A related question is what happens to the equilibrium aggregate asset supply L_E/a, if a increases. As we can see, a higher a does not only have a positive effect on the mass of entrepreneurs L_E, but that, regarding total equilibrium asset supply L_E/a, this effect even overcompensates the lower asset production per entrepreneur.

Higher payoff risk, i.e., higher ${\sigma_\varepsilon}^2$ or ${\sigma_s}^2$, decreases the equilibrium mass of entrepreneurs. The same holds true for a higher degree of risk aversion ρ. This is not surprising, as entrepreneurs dislike risk related to the asset they create and they dislike this uncertainty the more, the higher their degree of risk aversion.

L_E with free vs. restricted OC. An interesting question concerns the consequences of a ban on dealers for the equilibrium mass of entrepreneurs. To avoid confusion, denote variables that relate to the case of free OC by a superscript "1" and variables that relate to the case of restricted OC by a superscript "0". Hence, the equilibrium mass of entrepreneurs with free or restricted OC is given by L_E^1 or L_E^0, respectively. Starting from an equilibrium with $L_E^1 < L$, we find that for a ban of dealers to strengthen entrepreneurship (i.e., for $L_E^0 > L_E^1$), it is sufficient that

$$\frac{L - a\bar{v}}{L + M} \leq \frac{1}{2} \tag{2.39}$$

(see Appendix B.9). A simple set of sufficient conditions that ensures the validity of (2.39) is given by $\bar{v} \geq 0$ and $M \geq L$. It ensures that noise traders do not short the asset and talent is scarce in the sense that the amount of passive investors exceeds the amount of high potentials.

While these conditions are rather weak, one might still wonder why any conditions are needed here in the first place. Put differently: Why should a ban of dealers ever *decrease* the mass of entrepreneurs? The answer is that, by restricting OC, not only do the "former" dealers have to decide whether to become entrepreneurs or passive investors, but also the "former" entrepreneurs re-evaluate their decision. To understand this, remember that a ban on dealers dramatically changes the informational structure of the market. Only in case that this change benefits entrepreneurs more than passive investors does a ban of dealers increase entrepreneurship.

Trading Volumes

With non-stochastic noise trader demand, the price either perfectly reveals all private information or there are no information at all. In both cases, all agents act on the same information and expectations. Within the original GS (1980) model, this would imply the no-trade result by Milgrom and Stokey (1982).[14] The reason why trade between rational agents is still happening in our model, is because of different initial endowments of the risky asset: while each entrepreneur enters the trading stage with $1/a$ risky assets, each dealer or passive investor enters with zero. Agents' net trading volumes, both with free and restricted OC, are given by

$$\tilde{I}_E = \frac{1}{a} - \frac{\frac{L_E}{a} - \bar{v}}{L + M}, \tag{2.40}$$

$$I_D = I_M (= I_E) = \frac{\frac{L_E}{a} - \bar{v}}{L + M}, \tag{2.41}$$

where I_D drops out in case of restricted OC or $L_E = L$ (see Appendix B.10). Aggregate noise trading is exogenously given by $\bar{v}$. In a (hypothetical) scenario with no entrepreneurs at all, (2.41) shows that trade would happen only because of noise traders. For $0 < L_E \leq L$, trade happens even for $\bar{v} = 0$, because of the aforementioned differences in agents' initial asset endowments.

[14] Strictly speaking, there would be some kind of "noise-trade" left due to noise traders' exogenous asset demand $\bar{v}$, but no further trade between the rational agents.

The fact that equations (2.40) and (2.41) are valid both with free and restricted OC does not imply that individual equilibrium trading volumes are the same in both cases. This is because the (interior) equilibrium values for L_E are not the same, but given by (2.29) and (2.37), respectively. Substituting these into (2.40) and (2.41) yields

$$\begin{aligned} \tilde{I}_E &= \frac{1}{a} - \frac{\frac{L_E^1}{a} - \bar{v}}{L+M} \\ &= \frac{1}{a} - \frac{1}{\rho{\sigma_\varepsilon}^2}\left(\bar{s} - \frac{\rho{\sigma_s}^2}{2a}\right), \end{aligned} \tag{2.42}$$

$$\begin{aligned} I_D = I_M &= \frac{\frac{L_E^1}{a} - \bar{v}}{L+M} \\ &= \frac{1}{\rho{\sigma_\varepsilon}^2}\left(\bar{s} - \frac{\rho{\sigma_s}^2}{2a}\right) \end{aligned} \tag{2.43}$$

in case of free OC (and $L_E^1 < L$), and

$$\begin{aligned} \tilde{I}_E &= \frac{1}{a} - \frac{\frac{L_E^0}{a} - \bar{v}}{L+M} \\ &= \frac{1}{a} - \frac{\bar{s}}{\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right)}, \end{aligned} \tag{2.44}$$

$$\begin{aligned} I_D = I_M(= I_E) &= \frac{\frac{L_E^0}{a} - \bar{v}}{L+M} \\ &= \frac{\bar{s}}{\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right)} \end{aligned} \tag{2.45}$$

in case of restricted OC (and $L_E^0 < L$).

No short-selling. With (2.42)-(2.45), it is easy to identify conditions under which all agents hold positive equilibrium amounts of the risky asset. For the noise traders, this is just given by $\bar{v} \geq 0$. For the rational agents, it comes down to $L_E/a \geq \bar{v}$, with L_E evaluated at equilibrium with free or restricted OC, respectively. The condition simply requires that, in equilibrium, total entrepreneurial output exceeds aggregate noise trader demand. Note that, if all agents hold long positions, entrepreneurs, who are the only agents entering the trading stage with positive amounts of the asset, have to be net sellers. All other agents are net buyers, as their asset holdings equal their respective trading volumes.

If we substitute the equilibrium $L_E^1(< L)$ into $L_E/a \geq \bar{v}$, we immediately see that rational agents do not short the asset in equilibrium with free OC, if $\bar{s} \geq (\rho{\sigma_s}^2)/(2a)$. Analogously, rational agents do not short the asset in equilibrium $L_E^0(< L)$ with restricted OC, if $\bar{s} \geq 0$. It follows that, if agents do not short the asset in equilibrium with free OC, they certainly don't short it with restricted OC. Hence, summing up, no agent ever shorts the asset in equilibrium, if $\bar{v} \geq 0$ and $L_E^1/a \geq \bar{v}$. In order to avoid tedious case distinctions, we will make use of this condition set in the welfare chapter 2.5. Also note that, from a real-life perspective, short-selling is typically both constrained and costly.

Trading volumes with free vs. restricted OC. Regarding the effects of a ban on dealers on trading volumes, we immediately see that, with $L_E^0 > L_E^1$, each rational agent's asset holdings increase. An interesting question is whether this is true also for aggregate net trading. To answer it, note first that, as entrepreneurs are the only group of sellers in the market, aggregate net trading is given by $L_E \cdot \tilde{I}_E$. Comparing (2.42) to (2.44), we immediately see that, for a given amount of entrepreneurs, a ban of dealers has no effect on the trading volume. However, there is an (indirect) effect via the change in the equilibrium mass of entrepreneurs. In Appendix B.10, we show that a ban on dealers typically increases aggregate net trading. Actually, this is not surprising, as the implied increase in entrepreneurship increases aggregate asset supply. The result also emphasizes that a ban on *trading* would have a completely different effect than a ban on professional *traders*. The latter even increases equilibrium trading volumes.

2.4 Adding a Labor Market

Entrepreneurship's only role so far has been the creation of output. We now add a potentially positive additional effect of entrepreneurship: the creation of jobs.[15]

To do so, assume that each of the M passive investors is now endowed with one unit of ("ordinary") labor and hence also called an (ordinary) "worker". The output a firm creates depends on both a macroeconomic shock and the amount of workers employed. Formally, let firm output Y be given by

$$Y = \tilde{\theta} + F(m), \tag{2.46}$$

where

$$\begin{aligned} \tilde{\theta} &= \tilde{s} + \varepsilon, \\ \tilde{s} &\sim N(\hat{s}, {\sigma_s}^2), \\ \varepsilon &\sim N(0, {\sigma_\varepsilon}^2), \end{aligned} \tag{2.47}$$

m is employment at the firm-level and $F(m)$ a standard production function with positive, decreasing marginal returns. To ensure that an interior solution to the entrepreneurs' profit maximization problem exists, let $F(m)$ satisfy the Inada conditions $\lim_{m \to 0} F'(m) = \infty$ and $\lim_{m \to \infty} F'(m) = 0$. Firm profit θ is given by

$$\theta = Y - Wm, \tag{2.48}$$

[15]For empirical evidence on the fact that entrepreneurship creates jobs and drives wage growth, see, e.g., Bednarzik (2000), Acs and Armington (2004), Acs (2006), and Malchow-Møller et al. (2011)

where W is the wage rate per unit of labor. As before, θ corresponds to the payoff of the risky asset. By choosing employment, firms affect the expected value of the asset. A worker's disutility from supplying his unit of labor to a firm is given by $D(\geq 0)$. Hence, he does so only if $W \geq D$.

It seems reasonable to assume that wages and employment are determined at the "intermediate" stage, that is, after the OC decision is made. Note, however, that the additive nature of production Y implies that equilibrium wages and employment are non-stochastic (see below) and, therefore, the timing is basically irrelevant with regard to our results. The fact that only passive investors are endowed with "ordinary" labor implies that becoming a worker is not feasible for the hipos. A possible explanation could be that while only hipos have the mental abilities to engage in professional financial trading or entrepreneurship, hard work in production requires a level of physical fitness met only by workers. As before, only dealers observe $\tilde{s}$. All other assumptions are the same as in the basic version of the model.[16]
In what follows, we will first characterize the scenario of a frictionless labor market with full employment (FE). Afterwards, we introduce labor-market frictions and equilibrium unemployment (UE). Workers benefit from more entrepreneurship in both cases: via increasing wages in the former, via lower risk of unemployment in the latter.

2.4.1 Full Employment

For simplicity, let $D = 0$, so that the aggregate supply of labor equals M for any non-negative wage W. Furthermore, let $\hat{M} = M/(L_E/a)$ denote the number of workers per firm. The labor market is in equilibrium, if (i) firms choose m in order to maximize their entrepreneurs' (conditional) expected utility and (ii) $m = \hat{M}$, so that the labor market clears.

Labor market equilibrium. We can show that maximizing an entrepreneur's (conditional) expected utility w.r.t. m comes down to just maximizing the non-random part of a firm's profits $F(m) - Wm$ (see Appendix B.11). Solving for the optimal m then immediately gives $F^{'}(m) = W$. Using the market clearing condition to substitute m for $\hat{M}$ gives the equilibrium wage $\tilde{W}$ as $\tilde{W} = F^{'}(\hat{M})$.

General equilibrium. Let

$$s := \tilde{s} + F(\hat{M}) - F^{'}(\hat{M})\hat{M}, \quad \text{and} \quad \bar{s} := \mathbb{E}(s). \tag{2.49}$$

Then, $s \sim \mathcal{N}(\bar{s}, {\sigma_s}^2)$, and the equilibrium analysis in the FE model goes through exactly as in the basic version of the model. More formally: Let s and $\bar{s}$ be given by

[16] We neglect the fact that there is job creation also in the financial trading industry (Philippon, 2010, p. 163, makes a similar assumption by ignoring innovation in the financial industry). We justify this by our focus on a job market for "ordinary" workers, not for the high-skilled. As we have seen in the Introductory Chapter, employees in financial trading typically *are* highly skilled. Within our model, these people would be regarded as hipos and not be dependent on an employer: they could decide to engage in informed trading just any time they want.

(2.49). If (I_E, I_D, I_M, P, L_E) is an equilibrium of the basic version of the model, then $(I_E, I_D, I_M, P, L_E, \hat{M}, \tilde{W})$ is an equilibrium of the full employment model (I_D drops out in case of restricted OC).

Existence and uniqueness With $\bar{s}$ given by (2.49), equilibrium analysis from Sections 2.2 and 2.3 carries over to the FE model. At least for a standard Cobb-Douglas production function of the form $F(m) = Am^{1-b}$, where $A > 0$ and $0 < b < 1$, existence of an equilibrium with $L_E > 0$ is ensured by the fact that

$$\begin{aligned}\lim_{L_E \to 0} \bar{s} &= \lim_{L_E \to 0} \left[A\hat{M}^{1-b} - A(1-b)\hat{M}^{-b}\hat{M} \right] \\ &= \lim_{L_E \to 0} bA\hat{M}^{1-b} = \infty \end{aligned} \tag{2.50}$$

and, therefore, $\lim_{L_E \to 0} \Delta(L_E) = \infty$.

As $\bar{s}$ now depends on L_E, it is no longer possible to explicitly solve for the equilibrium L_E, not even in case of non-stochastic NT demand and a standard Cobb-Douglas production function. Still, what we *can* show is that $\bar{s}$ is strictly decreasing in L_E:

$$\begin{aligned}\frac{d\bar{s}}{dL_E} &= F'(\hat{M})(-1)\frac{\hat{M}}{L_E} - \left[F''(\hat{M})(-1)\frac{\hat{M}}{L_E}\hat{M} + (-1)\frac{\hat{M}}{L_E}F'(\hat{M}) \right] \\ &= F''(\hat{M})\frac{\hat{M}^2}{L_E} < 0. \end{aligned} \tag{2.51}$$

It immediately follows that $\Delta(L_E)$ in the noiseless case, given by (2.25) or (2.34), with $\bar{s}$ according to (2.49), is still strictly decreasing in L_E for $L_E < L$. Hence, without noise, the uniqueness properties from the basic model carry over to the FE model (besides the fact that $\lim_{L_E \to 0} \Delta(L_E) = \infty$ and $\Delta(L_E)$ is now *non*-linearly decreasing in L_E, figure 2.3 still applies; see also figure B.2 in Appendix B.12).

In the case of stochastic noise trader demand, the fact that $\bar{s}$ depends on L_E does not change the fact that equilibrium L_E with free OC is not generally unique (qualitatively, figure 2.2 still applies, except that now $\lim_{L_E \to 0} \Delta(L_E) = \infty$). To see that the equilibrium L_E with restricted OC stays unique also in the FE model, note that the fact that $\bar{s}$ decreases in L_E immediately implies that $\Delta(L_E)$ is still strictly decreasing (cf. Appendix B.6; besides the fact that $\lim_{L_E \to 0} \Delta(L_E) = \infty$ and $\Delta(L_E)$ is now *non*-linearly decreasing in L_E, figure B.1 still applies).

Comparative statics. Doing comparative statics with respect to the equilibrium mass of entrepreneurs is less straightforward in the FE model, as we can't solve for L_E explicitly. In the noiseless case, parameter variations that shift $\Delta(L_E)$ up or make it less steep increase the equilibrium level of L_E. From (2.25) and (2.34), with $\bar{s}$ given by (2.49), we immediately see that, analogously as in the basic model, an increase in $\hat{s}$ or $\bar{v}$ increases L_E, as it shifts up $\Delta(L_E)$. An increase in ${\sigma_\varepsilon}^2$ or ${\sigma_s}^2$ or ρ decreases L_E, as it shifts down $\Delta(L_E)$. Regarding a ban of dealers, note that condition (2.39) stays sufficient for $L_E^0 > L_E^1$ (see Appendix B.12).

Workers' job gains To see that workers' labor income benefits from an increasing mass of entrepreneurs via higher equilibrium wages, we take the derivative of $\tilde{W}$ w.r.t. L_E:

$$\frac{d\tilde{W}}{dL_E} = F''(\hat{M})(-1)\frac{aM}{L_E^2} = -F''(\hat{M})\frac{\hat{M}}{L_E} > 0. \tag{2.52}$$

Note, however, that L_E is not exogenous, but endogenously determined in equilibrium. In terms of variation in the exogenous variables, anything that increases the equilibrium mass of entrepreneurs without affecting the other parameters in (2.52) increases workers' equilibrium wage. This is in particular the case if the equilibrium mass of entrepreneurs increases from L_E^1 to L_E^0 as the result of a ban on dealers.

2.4.2 Frictions and Unemployment

In reality, wage rigidities are a common labor market characteristic (see, e.g., Dickens et al., 2007, and Babecký et al., 2010). If wages do not adjust so that demand equals supply, this gives rise to equilibrium unemployment. In what follows, we introduce frictions to the labor market by considering a union wage setting setup as in McDonald and Solow (1981).[17]

Again, the mass of workers per firm is given by $\hat{M}$. Workers organize in firm-level unions. Unions set the wage so as to maximize the (conditional) expected utility of their members. Firms have the right to hire along their optimal labor demand function. If the wage set by unions is greater than the one that clears the market, there is equilibrium unemployment. It amounts to $\hat{M} - m$ workers per firm and, hence, the individual probability of being unemployed is given by $1 - (m/\hat{M})$ for each worker. Unions face a trade-off: on the one hand, setting a high wage benefits employed workers; on the other hand, however, it increases the risk of unemployment. For simplicity, we assume the production function to be Cobb-Douglas with $F(m) = Am^{1-b}$, where $A > 0$ and $0 < b < 1$.

Labor market equilibrium. In Appendix B.13, we show that workers' utility related to gains from the job is separable from their asset trading activities. Hence, unions simply maximize the certainty equivalent of a worker's expected job revenues, which is given by

$$-\frac{1}{\rho}\ln\left(1 - \frac{m}{\hat{M}}\left[1 - \exp\{-\rho(W-D)\}\right]\right) =: \mathsf{GJ}_M, \tag{2.53}$$

with

$$m = \left(\frac{A(1-b)}{W}\right)^{\frac{1}{b}} \tag{2.54}$$

depicting firm-level labor demand. GJ_M is greater than zero, as the argument of the logarithm takes on values between zero and one. Equation (2.54) again points out

[17] In Appendix B.15, we also consider alternative wage-setting regimes: "work or shirk" as in Shapiro and Stiglitz (1984); maximization of the wage bill as in Dunlop (1944); and efficiency wages as in Solow (1979).

to the unions' trade-off: the higher they set the wage W, the lower the firm-level employment m and, hence, the higher the unemployment risk $1 - (m/\hat{M})$.

While we cannot explicitly solve for the wage rate that maximizes (2.53), we can show that such a wage, call it $\tilde{W}$, exists, is unique, and is greater than D (see Appendix B.14). Hence, $\tilde{W}$, together with firms' labor demand m evaluated at $W = \tilde{W}$, call it $\tilde{m}$, constitute a labor market equilibrium in the UE model. In order to have equilibrium unemployment, we require $\tilde{m} < \hat{M}$. A simple sufficient condition for this is $\tilde{m} \leq M/(L/a)$.

General equilibrium. As workers' portfolio decisions are independent of their employment status (cf. Appendix B.13) and we already argued that the analogous holds for entrepreneurs, the equilibrium analysis of the basic version of the model again goes through unchanged by defining

$$s := \tilde{s} + F(\tilde{m}) - \tilde{W}\tilde{m}, \quad \text{and } \bar{s} := \mathbb{E}(s). \tag{2.55}$$

More formally: Let s and $\bar{s}$ be given by (2.55). If (I_E, I_D, I_M, P, L_E) is an equilibrium of the basic version of the model and $\tilde{m} < \hat{M}$, then $(I_E, I_D, I_M, P, L_E, \tilde{m}, \tilde{W})$ is an equilibrium of the UE model (I_D drops out in case of restricted OC). As $\tilde{W}$ and, therefore, also $\tilde{m}$ are independent of L_E (cf. Appendix B.14), $\bar{s}$ given by (2.55) is independent of L_E. Hence, with $\bar{s}$ given by (2.55), the explicit solutions (2.29) and (2.37) for the equilibrium mass of entrepreneurs in the basic model without noise carry over to the UE model. It follows that also the comparative statics from the baseline model go through unchanged.

The fact that $\bar{s}$ is independent of L_E implies that $\Delta(L_E)$ shows qualitatively the same pattern as in the basic version of the model. Hence, for a graphical illustration of how equilibrium is determined in the UE model, we can simply refer to the figures in Sections 2.2 and 2.3. Existence and uniqueness properties then obviously carry over from the basic version of the model as well. Condition (2.39) is sufficient for $L_E^0 > L_E^1$ in the UE model, as with $\bar{s}$ being independent of L_E, the proof in Appendix (B.9) goes through unchanged.

Workers' job gains. As $\tilde{m}/\hat{M}$ is increasing in L_E, workers benefit from an increase in the mass of entrepreneurs through a higher chance of getting employed. To see that the lower risk of unemployment indeed translates into an increase in GJ_M, note that from (2.53):

$$\begin{aligned}\frac{d\mathsf{GJ}_M}{dL_E} &= -\frac{1}{\rho} \cdot \frac{-\tilde{m}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]}{1 - \frac{\tilde{m}}{\hat{M}}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]} \cdot (-1) \cdot \frac{1}{\hat{M}^2} \cdot (-1) \cdot \frac{aM}{{L_E}^2} \\ &= \frac{1}{\rho} \cdot \frac{\frac{\tilde{m}}{\hat{M}}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]}{1 - \frac{\tilde{m}}{\hat{M}}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]} \cdot \frac{1}{L_E} > 0.\end{aligned} \tag{2.56}$$

Hence, everything that increases entrepreneurial activity L_E, without affecting the other parameters in (2.53), increases workers' expected job gains GJ_M. This is in particularly the case if the equilibrium L_E increases as the result of a ban on dealers.

Regarding workers' rate of unemployment, the wage level is of particular interest. This is because a decrease in wages increases job opportunities in two ways. First, (2.54) shows that firm-level employment rises. And second, the number of firms increases as a consequence of an increase in expected firm profit $\bar{s}$:

$$\frac{d\bar{s}}{d\tilde{W}} = \underbrace{\frac{d\tilde{m}}{d\tilde{W}}}_{<0} \underbrace{\left(F'(\tilde{m}) - \tilde{W}\right)}_{=0} - \tilde{m} < 0. \tag{2.57}$$

The wage rate $\tilde{W}$ is endogenously determined in equilibrium. Hence, in terms of variation in the exogenous variables, this implies the following: anything that decreases the equilibrium wage rate without affecting the other parameters in (2.54) increases firm-level employment $\tilde{m}$; and anything that decreases the equilibrium wage rate without affecting the other parameters that influence the equilibrium mass of entrepreneurs increases L_E and, therefore, decreases $\hat{M}$.

2.5 Welfare

This chapter provides analytical welfare results for the case of deterministic noise trader demand, i.e., for ${\sigma_v}^2 = 0$. We define social welfare S as the sum of all agents' certainty equivalents:

$$S = L_E \cdot \mathsf{CE}_E + (L - L_E) \cdot \mathsf{CE}_D + M \cdot \mathsf{CE}_M + N \cdot \mathsf{CE}_N. \tag{2.58}$$

Based on certainty equivalents, social welfare is not affected by how safe income is distributed among agents. Hence, our welfare criterion puts aside any redistributional aspects.[18] Crucially, S includes noise traders' welfare. This requires us to answer the question of how the well-being of agents who exhibit exogenous behavior can be evaluated in the first place. We do this by ex-post assigning them the same CARA-utility function as the rational agents. As noise traders do not maximize this function ex-ante, this either implies that they are hit by some kind of shock that prevents them from maximizing utility and instead makes them randomize their asset demand, or that they are just kind of irrational.[19] The fact that, as we show in Section 2.5.2, equilibrium is constrained efficient in the absence of market imperfections,

[18] A theoretical justification for using a SW-function based on CEs in the presence of uncertainty is given by Chambers and Echenique (2012). For a recent application, see Kawakami (2017, p. 307).

[19] Allen (1984) and Albagli et al. (2018) take a similar approach. Allen (1984) studies welfare in the GS (1980) model and uses a risk-neutral utility function to evaluate NT well-being. Albagli et al. (2018) analyze the effects of limits to arbitrage and noisy information aggregation in financial markets on corporate behavior. They use aggregate expected dividends (accruing to risk-neutral rational agents as well as noise traders) as their welfare criterion (p. 9). Alternative specifications of noise trader behavior have already been discussed in footnote 4. Another possibility would be to just ignore NT well-being in the social welfare analysis. However, as emphasized by Albagli et al. (2018, p. 18), "welfare discussions are incomplete without a proper specification of noise trader welfare."

lends support to our measure of welfare. Furthermore, as we will see, our main analytical results (cf. Propositions 2.5.1-2.5.2) are valid also for $\bar{v} = 0$, that is, in case of no noise trader activity at all.

2.5.1 Social Welfare Function

We proceed to derive agents' certainty equivalents. Aggregating them gives social welfare according to (2.58). We start with the case of free OC.
As z given by (2.24) is non-random, the CE of a passive investor, given by (2.12), simplifies to

$$\mathsf{CE}_M = z^2 = \begin{cases} \frac{\rho\sigma_\varepsilon{}^2}{2}\left(\frac{\frac{L_E}{a}-\bar{v}}{L+M}\right)^2, & \text{for } L_E < L \\ \frac{\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)}{2}\left(\frac{\frac{L}{a}-\bar{v}}{L+M}\right)^2, & \text{for } L_E = L \end{cases}. \tag{2.59}$$

In the FE model, a worker's CE is given by (2.59) plus his wage gains $\tilde{W}(=F'(\hat{M}))$. In the UE model, his CE is given by (2.59) plus his expected gains from the job GJ_M. For $L_E < L$, dealers trade on the same information as the passive investors and, hence, have the same expected utility:

$$\mathsf{CE}_D = \mathsf{CE}_M. \tag{2.60}$$

In contrast to the passive investors, dealers don't gain any additional utility in the labor market models.
The CE of an entrepreneur, given by (2.20), simplifies to

$$\mathsf{CE}_E = \mathsf{GP}_E + z^2 = \mathsf{GP}_E + \begin{cases} \frac{\rho\sigma_\varepsilon{}^2}{2}\left(\frac{\frac{L_E}{a}-\bar{v}}{L+M}\right)^2, & \text{for } L_E < L \\ \frac{\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)}{2}\left(\frac{\frac{L}{a}-\bar{v}}{L+M}\right)^2, & \text{for } L_E = L \end{cases}, \tag{2.61}$$

with GP_E given by (2.25). In the FE model, $\bar{s}$ is given by (2.49). In the UE model, $\bar{s}$ is given by (2.55).
Finally, from Appendix B.16, the CE of a noise trader is given by

$$\mathsf{CE}_N = \begin{cases} \underbrace{\frac{\bar{v}}{N}}_{I_N}\underbrace{\rho\sigma_\varepsilon{}^2\frac{\frac{L_E}{a}-\bar{v}}{L+M}}_{\mathbb{E}(\theta-P)} - \frac{\rho}{2}\underbrace{\left(\frac{\bar{v}}{N}\right)^2}_{I_N{}^2}\underbrace{\sigma_\varepsilon{}^2}_{\mathbb{V}(\theta-P)}, & \text{for } L_E < L \\ \underbrace{\frac{\bar{v}}{N}}_{I_N}\underbrace{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)\frac{\frac{L}{a}-\bar{v}}{L+M}}_{\mathbb{E}(\theta-P)} - \frac{\rho}{2}\underbrace{\left(\frac{\bar{v}}{N}\right)^2}_{I_N{}^2}\underbrace{(\sigma_\varepsilon{}^2+\sigma_s{}^2)}_{\mathbb{V}(\theta-P)}, & \text{for } L_E = L \end{cases}, \tag{2.62}$$

where I_N gives individual noise trader asset demand. As long as all agents hold positive amounts of the asset, NTs' expected gains from trading are positive. Whether their overall utility is positive depends on whether this effect dominates the disutility they face from uncertainty regarding these gains.

Summing up, social welfare S is given by

$$S = \begin{cases} L_E \mathsf{GP}_E + \frac{\rho\sigma_\varepsilon{}^2}{2}\left[\frac{\left(\frac{L_E}{a}\right)^2}{L+M} - \bar{v}^2\left(\frac{1}{L+M}+\frac{1}{N}\right)\right], & \text{for } L_E < L \\ L\mathsf{GP}_E + \frac{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}{2}\left[\frac{\left(\frac{L}{a}\right)^2}{L+M} - \bar{v}^2\left(\frac{1}{L+M}+\frac{1}{N}\right)\right], & \text{for } L_E = L \end{cases} \tag{2.63}$$

(see Appendix B.17). In the FE model, S is given by (2.63) plus the workers' aggregate wage gains $M \cdot \tilde{W}(= M \cdot F'(\hat{M}))$. In the UE model, S is given by (2.63) plus the workers' aggregate expected gains from the job $M \cdot \mathsf{GJ}_M$.

Restricted occupational choice In case of restricted OC, z is given by (2.33) and again non-random. Analogously as above, we get

$$\mathsf{CE}_M = z^2 = \frac{\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)}{2}\left(\frac{\frac{L_E}{a}-\bar{v}}{L+M}\right)^2 \tag{2.64}$$

and workers' CE in the labor market models additionally includes their respective gains from the job. The CE of an entrepreneur is given by

$$\mathsf{CE}_E = \mathsf{GP}_E + z^2 = \mathsf{GP}_E + \frac{\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)}{2}\left(\frac{\frac{L_E}{a}-\bar{v}}{L+M}\right)^2, \tag{2.65}$$

with GP_E given by (2.34). Again, in the labor market models $\bar{s}$ is given by (2.49) or (2.55), depending on whether there is full employment or not. From Appendix B.16, the CE of a noise trader is given by

$$\mathsf{CE}_N = \underbrace{\frac{\bar{v}}{N}}_{I_N} \underbrace{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)\frac{\frac{L_E}{a}-\bar{v}}{L+M}}_{\mathbb{E}(\theta-P)} - \frac{\rho}{2}\underbrace{\left(\frac{\bar{v}}{N}\right)^2}_{I_N{}^2}\underbrace{(\sigma_\varepsilon{}^2+\sigma_s{}^2)}_{\mathbb{V}(\theta-P)}. \tag{2.66}$$

As in case of restricted OC there are no dealers at all, social welfare S, specified as the sum of all agents' CEs, is just given by

$$S = L_E \cdot \mathsf{CE}_E + (L - L_E + M)\cdot \mathsf{CE}_M + N \cdot \mathsf{CE}_N, \tag{2.67}$$

which comes down to

$$S = L_E \mathsf{GP}_E + \frac{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}{2}\left[\frac{\left(\frac{L_E}{a}\right)^2}{L+M} - \bar{v}^2\left(\frac{1}{L+M}+\frac{1}{N}\right)\right] \tag{2.68}$$

(see Appendix B.17). As in the case of free OC, S in the labor market models additionally includes the workers' aggregate gains from the job.

2.5.2 Constrained Efficiency

We first analyze the equilibrium allocation of talent with regards to constrained (in)efficiency, i.e., we check whether a marginal variation in the mass of entrepreneurs, starting from equilibrium, can increase social welfare.
Assume that agents do not short the asset. Then, from the preceding chapter, we immediately see that dealers, passive investors and noise traders benefit from an (exogenous) increase in the mass of entrepreneurs L_E. This is not surprising, as they are all net buyers of the asset and increasing entrepreneurial activity increases total asset supply. Consequently, workers in the labor market models do not only benefit from an (exogenously) increasing L_E via higher gains in the job, but also via higher gains from uninformed market trade. In contrast, entrepreneurs' welfare in all versions of the model decreases in L_E (see Appendix B.18). As they are net sellers of the asset, an increasing total asset supply hurts their market position. It follows that, overall, it is not obvious how an (exogenous) increase in entrepreneurship affects social welfare S.

Second-best analysis. To address this question, we perform a second-best welfare analysis with regards to the socially optimal amount of entrepreneurship L_E, taking the agents' trading and labor market decisions as given. The analysis is further constrained in the sense that we also take as given the state of OC (free vs. restricted) and the labor market environment (none vs. FE vs. UE).[20] The following proposition states the first of our two main analytical welfare results:

PROPOSITION 2.5.1. *Let* ${\sigma_v}^2 = 0$.

(i) ***Free OC.*** *Suppose that an interior equilibrium $L_E < L$ exists, both in the basic version of the model as well as in the labor market economies. Then, this equilibrium L_E maximizes social welfare S on $(0, L)$ in the baseline model and in the FE economy, but falls short of the social welfare maximizing L_E in the UE model.*

(ii) ***Restricted OC.*** *Suppose that an equilibrium $L_E \leq L$ exists, both in the basic version of the model as well as in the labor market economies. Then, this equilibrium L_E maximizes social welfare S on $(0, L]$ in the baseline model and in the FE economy, but falls short of the social welfare maximizing L_E in the UE model.*

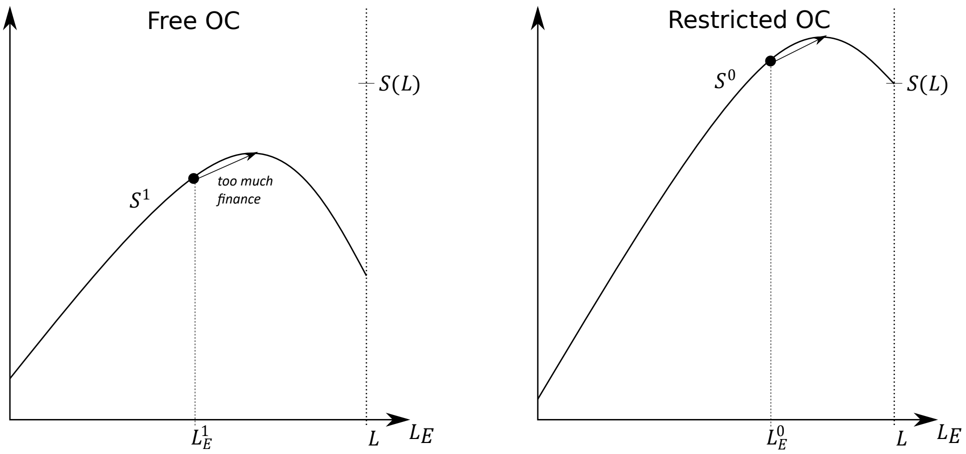
The proof is in Appendix B.19. The proposition states that in the baseline and in the FE model, the equilibrium allocation of talent is (constrained) efficient. This is exactly what we would expect, as what we look at is, essentially, an economy without imperfections: (i) there are no labor market frictions; (ii) without noise, the price is fully revealing and, therefore, information is symmetric; and (iii) without noise, a (marginal) change in the mass of dealers does not affect price informativeness and, hence, does not entail informational externalities. Consequently, the only potential

[20] Another thing we implicitly take as given is agents' "personality traits". This especially concerns the noise traders' non utility-maximizing behavior. If it were possible to nudge them towards behaving more rational, this would increase social welfare (see Appendix B.20).

imperfection remaining is noise traders' non utility-maximizing behavior. And as stated before, the fact that, still, the equilibrium and optimal allocation of talent coincide, lends support to the welfare criterion chosen. Note that the proposition also holds for $\bar{v} = 0$, that is, for the case where there is zero NT activity and hence social welfare consists of only rational agents' well-being. One might wonder why the positive impact entrepreneurship has on workers' wages in the FE model does not translate into a socially deficient equilibrium mass of entrepreneurs. The reason is that the workers' wage gains translate one-to-one into lower firm profitability (see Appendix B.21).

In case of labor market frictions and equilibrium unemployment, the amount of hipos engaging in professional financial trading is too big (see figure 2.4). The reason is that entrepreneurship entails a positive externality: a larger number of firms decreases the workers' risk of unemployment. In contrast to increasing wages in the FE economy, this does not come at the expense of firm profitability. Hence, measures aimed at reallocating talent from finance towards the real sector are beneficial for social welfare.[21] The mass of entrepreneurs that maximizes S on $(0, L)$ in the UE model

FIGURE 2.4: Equilibrium and Optimum SW in the UE Model



with free OC can be approximated by

$$L_E = \frac{a(L+M)}{\rho{\sigma_\varepsilon}^2}\left(\bar{s} - \frac{\rho{\sigma_s}^2}{2a}\right) + a\bar{v} + \frac{a(L+M)}{\rho^2{\sigma_\varepsilon}^2}\tilde{m}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right], \quad (2.69)$$

if this value is smaller than L. The amount of entrepreneurs that maximizes S on $(0, L]$ in case of restricted OC can be approximated by

$$L_E = \frac{a(L+M)}{\rho\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right)}\bar{s} + a\bar{v} + \frac{a(L+M)}{\rho^2\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right)}\tilde{m}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right], \quad (2.70)$$

[21] Remember that we took the labor market environment as given. If we would allow for the possibility to remove labor market frictions, then doing exactly this and thereby moving from the UE economy towards full employment would be clearly superior to any attempt to manipulate OC within the UE economy (see Appendix B.22).

if this value is smaller than L, and by $L_E = L$ otherwise (see Appendix B.23).[22] Comparing (2.69) and (2.70) to (2.29) and (2.37) immediately shows that the optimum L_E exceeds the equilibrium one. Unsurprisingly, it does so by the more, the higher firm level employment $\tilde{m}$ and the higher the "net" wage $\tilde{W} - D$, as this increases the benefits workers get from more entrepreneurship.

Finally, note that, in case of free OC, Proposition 2.5.1 only applies for $L_E \in (0, L)$. It is silent on how equilibrium social welfare $S^1(L_E^1)$, with $L_E^1 < L$, compares to $S^1(L)$. This is because price informativeness jumps from "perfect" to "zero" at $L_E = L$ and, therefore, S^1 is discontinuous at $L_E = L$. Corollary 2.5.2.1 in the next subsection gives further information in this regard.

2.5.3 Informational Efficiency vs. Real Efficiency

While the preceding chapter addressed the effects of a marginal change in the mass of entrepreneurs L_E, starting from equilibrium, we now turn to a comparison of social welfare with free vs. restricted OC. In particular, we show that social welfare with restricted OC exceeds social welfare with free OC, both at equilibrium and the constrained optimum values. To avoid confusion, we indicate expressions that refer to the case of free OC by a superscript "1" and expressions that refer to the case of restricted OC by a superscript "0". Note that for $L_E = L$ we don't have to distinguish the case with free OC from the one with restricted OC, as all hipos engage in entrepreneurship anyway. Comparing, e.g., (2.68) to (2.63) immediately shows that $S^1(L) = S^0(L)$.

Pareto Analysis

We start with individual comparisons. For a passive investor, comparing (2.59) to (2.64) immediately shows that, for any given L_E, $\mathsf{CE}_M^0 > \mathsf{CE}_M^1$. As dealers face the same expected utility as passive investors, this also implies that hipos who decide to become dealers in case of free OC would be better off acting as passive investors with restricted OC. Taking into account that the equilibrium values of L_E differ depending on whether there is free OC or not, a simple set of sufficient conditions for $\mathsf{CE}_M^0(L_E^0) > \mathsf{CE}_M^1(L_E^1)$, which immediately follows from comparing (2.59) to (2.64), is given by $L_E^1/a > \bar{v}$ and $L_E^0 \geq L_E^1$. This holds true also for the labor market models, as workers' (expected) gains on the job are increasing in L_E and, hence, are higher with restricted OC, if $L_E^0 > L_E^1$.

We can show that also for entrepreneurs we have $\mathsf{CE}_E^0 > \mathsf{CE}_E^1$, for any given L_E (see Appendix B.24). With regards to equilibrium values, note that an entrepreneur's equilibrium gains from trading z^2 equal the equilibrium CE of passive investors (and, in case of free OC, also that of dealers). Hence, they are greater without free OC

[22] In fact, the simulation in Chapter 2.6.2 gives evidence that (2.69) and (2.70) are good approximations of the "true" values; cf. table C.16 in Appendix C.7.

under the same conditions as stated above. With free OC, an entrepreneur's equilibrium gains from production are given by $\mathsf{GP}_E^1(L_E^1) = 0$ for $L_E^1 < L$. A ban of dealers either leads to an interior equilibrium $L_E^0 < L$, in which case $\mathsf{GP}_E^0(L_E^0) = 0$ as well, or to a corner equilibrium $L_E^0 = L$ with $\mathsf{GP}_E^0(L) \geq 0$. Either way, it follows that a simple set of sufficient conditions for $\mathsf{CE}_E^0(L_E^0) > \mathsf{CE}_E^1(L_E^1)$ is again given by $L_E^1/a > \bar{v}$ and $L_E^0 \geq L_E^1$. This obviously holds true also for the labor market economies.

Rational agents. In an (interior) equilibrium with free OC, an entrepreneur's welfare equals that of a dealer, which in turn equals that of a passive investor. Under restricted OC, an entrepreneur's equilibrium welfare either equals (if $L_E^0 < L$) or exceeds (if $L_E^0 = L$) that of a passive investor. Hence, from the above it follows that under $L_E^1/a > \bar{v}$ and $L_E^0 \geq L_E^1$, every single hipo benefits from a ban on dealers. Together with the fact that, under this set of conditions, also every single passive investor is better off in equilibrium with restricted OC, this implies that a ban on dealers results in a Pareto improvement for all rational agents in the economy.

Noise traders. Comparing (2.62) to (2.66) immediately shows that, for any given L_E with $L_E/a > \bar{v}$ and $\bar{v} > 0$, banning dealers (i) benefits noise traders through higher expected gains from trading, but, at the same time, (ii) hurts them through a higher volatility of these gains. The same is true also with regards to equilibrium values, if $L_E^1/a > \bar{v} > 0$ and, additionally, $L_E^0 \geq L_E^1$. If effect (i) dominates (ii), then noise traders are better off with restricted OC. Two simple sufficient sets of conditions for that to be the case, i.e., for $\mathsf{CE}_N^0(L_E^0) > \mathsf{CE}_N^1(L_E^1)$, are given by:

(i) $\bar{v} > 0$, $L_E^0 \geq L_E^1$, and
$$\frac{L_E^1}{a} > \bar{v}\left(1 + \frac{L+M}{2N}\right), \tag{2.71}$$
in all versions of the model, or

(ii) $\bar{v} > 0$, $L_E^0 < L$ and
$$\frac{\bar{v}}{N} < \frac{1}{a} \tag{2.72}$$
in the basic version of the model and the UE economy.

The proof is in Appendix B.25. The third condition in (i) is a strengthened version of $L_E^1/a > \bar{v}$. The third condition in (ii) requires noise traders to be sufficiently "small", in the sense that a single noise trader's asset demand is smaller than the amount of assets created by a single entrepreneur. As it stands, condition set (ii) does hold neither for $L_E^0 = L$, nor in the FE model (see Appendix B.25 for adjusted versions of the condition set in these cases). Obviously, if $\bar{v} = 0$, then noise traders are inactive and their final wealth equals zero, independent of the state of OC.

All agents. Condition set (i) implies that besides noise traders, also rational agents benefit from a ban on dealers. It immediately follows that restricting OC leads to a Pareto improvement for *all* agents in the economy, in *all* model variants. Alternatively, combining condition set (ii) with $L_E^1/a > \bar{v}$ and $L_E^0 \geq L_E^1$ gives another set of sufficient conditions for which a ban on dealers results in a Pareto improvement for *all* agents in the economy, in the baseline and the UE model. The result that agents tend to be better off with restricted OC, which implies the absence of information on asset fundamentals, might come as a surprise. We will further elaborate on what drives these results below.

Social Welfare Analysis

In this section, we turn to a comparison of aggregate welfare levels, i.e., to the comparison of social welfare with free vs. restricted OC. The following proposition states the second of our two main analytical welfare results:

PROPOSITION 2.5.2. *Let ${\sigma_v}^2 = 0$. Then, for the baseline model and the FE economy, equilibrium social welfare $S^0(L_E^0)$ in case of restricted OC exceeds equilibrium social welfare $S^1(L_E^1)$ in case of free OC, if*

$$\begin{aligned} \frac{L_E^1}{a} &> \bar{v} \geq 0, \\ \frac{\bar{v}}{N} &\leq \frac{1}{a}. \end{aligned} \tag{2.73}$$

For the UE model, the same holds true by adding $L_E^0 \geq L_E^1$.

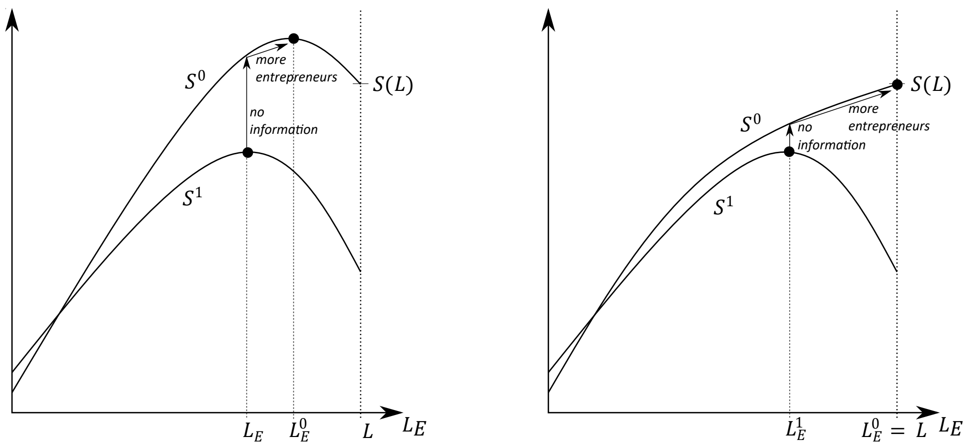
The proof is in Appendix B.26. Proposition 2.5.2 gives a set of simple sufficient conditions under which a ban on dealers raises social welfare. The first condition in (2.73) ensures that all agents hold positive amounts of the asset. The second one states that individual noise trader demand is sufficiently small. In the UE model, where equilibrium welfare differs from the social optimum, we additionally have to assume that $L_E^0 \geq L_E^1$. The set of conditions in Proposition 2.5.2 is weaker than the conditions for a Pareto improvement for all agents. To see this, first note that condition (2.71) is stronger than the set of both $L_E^1/a > \bar{v}$ and $\bar{v}/N \leq 1/a$.[23] And in contrast to the second set of conditions for a Pareto improvement, jointly given by (2.72), $L_E^1/a > \bar{v}$ and $L_E^0 \geq L_E^1$, the condition set in Proposition 2.5.2 does not require $L_E^0 < L$ and also holds for the FE economy. Again, note that the Proposition also holds for $\bar{v} = 0$, i.e., for the case without NT activity and hence a SWF that encompasses only rational agents' welfare. As we can easily see, in this case, the mere existence of an equilibrium L_E^1 already ensures condition set (2.73).

Are information harmful? As mentioned before, the result that social welfare is typically higher with restricted OC might be surprising, especially as we have seen that this is true also for a given mass of entrepreneurs L_E. Holding L_E fixed, the only major difference between the case of free vs. restricted OC is the informational structure in the financial market. While there is full information about the asset fundamental s, available to everyone, in the presence of dealers, in case of restricted OC there are no

[23] It is immediately obvious that (2.71) implies $L_E^1/a > \bar{v}$. Rewriting (2.71) gives $L_E^1/a > \bar{v} + \bar{v}/N \cdot (L + M)/2$. For $M \geq L$, this requires $\bar{v}/N < 1/a$.

information at all. In fact, as information acquisition comes without direct costs, this result implies that the mere presence of information on asset fundamentals tends to be detrimental to social welfare. The reason is that the revelation of fundamental information has a negative impact on ex-ante efficient risk-sharing, as was first highlighted by Hirshleifer (1971).[24] Figure 2.5 shows that we can decompose the transition from equilibrium social welfare with free OC to equilibrium social welfare with restricted OC into two separate effects. The first effect is the one just explained.

FIGURE 2.5: Social Welfare Effects of a Ban on Dealers



Note: Both figures refer to the basic version of the model. They basically look the same in the FE economy. In the UE model, the equilibrium values L_E^1 and L_E^0 are located to the left of the social welfare maximizing L_E's. The left panel depicts the case of an interior equilibrium $L_E^0 < L$, the right panel the case of a corner equilibrium $L_E^0 = L$.

What is even more interesting, however, is the second effect. While impaired risk-sharing hurts all (rational) agents, it does not hurt all of them equally hard. In fact, entrepreneurs suffer disproportionately. As they are the ones who establish firms and create the asset, they, initially, bear all risk. In the trading stage, they engage in mutually beneficial risk-sharing and thereby get rid of some of this risk. Independent of whether there is free or restricted OC, they can always share risk regarding the non-fundamental uncertainty in ε. Strikingly, however, they can share risk regarding the asset fundamental s only if it has not been fully revealed already, which actually is exactly what happens in the presence of dealers (cf. Appendix B.27). This discourages entrepreneurship in case of free OC compared to the case of restricted OC. Simply put, informed trading impedes ex-ante beneficial risk-sharing, leads to a clustering of risk at entrepreneurs and thereby distorts the allocation of talent.

[24]To give an easy and intuitive example, assume that there are only two risk-averse agents and only two possible future states of the world. Agent A gets a payoff $p_A = 1$ in state one of the world and $p_A = 0$ in state two. Agent B gets a payoff $p_B = 0$ in state one of the world and $p_B = 1$ in state two. Without information about the future state of the world, the two agents agree on mutually beneficial insurance and hence get a net-payoff equal to 1/2 each, with certainty. In contrast, if the future state of the world is known from the beginning, one of the agents gets his payoff equal to 1 and the other one his payoff equal to 0. From a social welfare perspective, this is clearly inferior.

Taking these information out of the market via a ban on dealers encourages entrepreneurship, results in increased real economic activity and benefits welfare.

In general, the fact that the revelation of information can harm optimal risk sharing is well known since Hirshleifer (1971). In this sense, also Hu and Qin (2013) find that an increase in informed traders harms welfare in the Grossman (1976) model with diverse information and a fully revealing equilibrium. Similarly, Allen (1984) finds that informational efficiency harms the rational agents in the GS (1980) model and has an ambiguous effect on the noise traders. More recently, Kawakami (2017) shows that information aggregation in an REE framework with heterogeneous information of the type introduced in Diamond and Verrecchia (1981) can harm welfare by decreasing hedging effectiveness. Bond and Garcia (2019) study the equilibrium consequences of indexing vs. active trading in a multi-asset version of Diamond and Verrecchia (1981) and find that increased price informativeness through active trading hurts welfare by constraining agents' risk sharing options. By adding agents with correlated private valuations into a generalized framework of GS (1980) and Hellwig (1980), Rahi and Zigrand (2018) show that more informative prices tend to reduce profitable trading opportunities by bringing asset prices closer to agents' valuations. More generally, the fact that disclosure of information in financial markets can potentially reduce welfare has recently been emphasized also by Morris and Shin (2002), Colombo et al. (2014), Kurlat and Veldkamp (2015), Han et al. (2016), Goldstein and Yang (2017), and Albagli et al. (2018, Chapter 5.3).

Feedback effects to the real economy. Strikingly, these results seem to be at odds with the conventional wisdom that, as argued by Fama (1970, p. 383), an "ideal" market is one in which "prices always 'fully reflect' available information" and, hence, "provide accurate signals for resource allocation". The resolution of this apparent paradox is simple: informational efficiency does not necessarily translate into real efficiency. As made clear by Bond et al. (2012), information revelation is not useful per se, but only if "the price reveals information necessary for decision makers to take value maximizing actions" (p. 6), that is if these information are useful for making better "real" decisions on activities such as production or investment. There are various ways to account for these kind of feedback effects from information in financial markets to the real economy. In Dow and Gorton (1997), managerial pay is linked to the firm's stock market performance and information from asset prices guide real-investment. Goldstein and Yang (2014) consider a model where the funding of financially constrained firms depends on information from secondary financial markets. In Bolton et al. (2016, Section III), traders' asset valuation ability incentivizes entrepreneurs to improve the quality of their firms. Gao and Liang (2013) and Goldstein and Yang (2017, Section 4) allow firms to make more efficient real-investment decisions by observing fundamental information from the financial sector. In Bond and Goldstein (2015), information from asset markets can influence government interventions such as bailouts or state guarantees. Another interesting model recently proposed by Angeletos et al. (2018) considers a two-way feedback effect between the financial and the real sector. Not only can firms guide their decisions by information from asset prices, but also the financial market guides its investment by signals on firm profitability inferred from the degree of startup activity in the real sector. Once a non-fundamental shock to entrepreneurs' sentiment on startup profitability disturbs their decision on market entry, the feedback effects reinforce the shock and give rise to an aggregate behavior called "animal spirits". This results in excessive non-fundamental volatility both in the real and the financial sector and reduces welfare. Benhabib et al. (2019) add a real sector with monopolistic competition as in Dixit and Stiglitz (1977) to the GS (1980) framework. Firms decide on optimal investment under imperfect information and uncertainty. Strikingly, informational interdependence and mutual learning between the financial and the real sector can result in a self-fulfilling surge in uncertainty, accompanied by a drop in investment efficiency and economic output.

Where do we stand? While feedback channels as discussed above might create the scope for further research, their absence does not in any way render our model as-it-stands deficient. The set-up we use allows us to emphasize that informationally efficient markets are not a good thing per se, even if information come without costs. As long as these information are used for portfolio decisions only, impaired risk-sharing tends to reduce welfare. Hence, we re-establish a kind of Hirshleifer (1971) effect in the GS (1980) framework with costless information. The main thing we add to the literature, however, is informed trading's effect on the allocation of talent. Strikingly, the presence of dealers leads to an inefficient clustering of risk at entrepreneurs, discourages real economic activity and distorts the allocation of talent. In fact, this *does* constitute a feedback effect of information in the financial market to the real economy. However, as we have seen, it is actually a negative one. Viewed the other way round, any argumentation that financial trading à la GS (1980) is beneficial to welfare would have to argue that other, positive, feedback effects on allocational efficiency in the real economy outweigh the negative effects we find. In this regard, note that even the literature that accounts for such effects, as cited above, does not establish unambiguously positive welfare effects of information revelation in financial markets.

Model-implied policy recommendations. Proposition 2.5.2 holds true, if we look at maximum instead of equilibrium welfare levels. For the basic version of the model and the FE economy, this follows directly from the fact that the equilibrium L_E equals the social welfare maximizing one (cf. Proposition 2.5.1). When comparing maximum instead of equilibrium levels in the UE model, the conditions of the proposition can even be weakened by disposing of the additional assumption of $L_E^0 \geq L_E^1$ (cf. Appendix B.26). Consequently, combining the results of Propositions 2.5.1 and 2.5.2 immediately gives the following optimal "policy actions":

(i) Implement an outright ban on professional trading.

(ii) In case of labor market frictions, additionally boost entrepreneurship.

Under the conditions of Proposition 2.5.2, the constrained optimum social welfare is always higher in the absence of dealers – therefore, action (i). As Proposition 2.5.1 tells, no second action is required in case of the basic version of the model and the FE economy. In the UE model, one should take additional measures to increase the number of entrepreneurs L_E – therefore, action (ii). As, in reality, policy can typically not directly control the allocation of talent, we discuss implementation via appropriate taxation in Chapter 2.7.

Further remarks. The following corollary summarizes some (rather technical) remarks to Propositions 2.5.1 and 2.5.2.

Corollary 2.5.2.1. *Let* ${\sigma_v}^2 = 0$*.*

(i) Under the conditions of Proposition 2.5.2, it is

$$\lim_{L_E \to L} S^1(L_E) < S^0(L) (= S^1(L)). \tag{2.74}$$

(ii) Assume that the set of conditions stated in Proposition 2.5.2 holds. Additionally, let $M \geq L$ *and* ${\sigma_\epsilon}^2 \leq {\sigma_s}^2$*. Consider the baseline model. Then:*

$$S^1(L_E^1) < S^1(L) (= S^0(L)). \tag{2.75}$$

(iii) An alternative set of conditions which ensures the validity of Proposition 2.5.2 is given by $L_E^0 \geq L_E^1$*,* $\bar{v} \geq 0$*, and*

$$\frac{L_E^0}{a} > \left(1 + \frac{L+M}{N}\right)^{\frac{1}{2}} \bar{v}. \tag{2.76}$$

The proof is in Appendix B.28. If it is $\lim_{L_E \to L} \Delta^1(L_E) \geq 0$ and hence there is no interior equilibrium $L_E^1 < L$, social welfare S^1 is continuously increasing in L_E for $L_E \in (0, L)$. Part (i) of the corollary states that welfare "jumps" up when the price becomes uninformative at $L_E = L$. It follows that if an interior equilibrium L_E^1 does not exist, S^1 is maximum at $L_E = L$. This also implies that keeping a "marginal dealer" in order to reach informational efficiency is not desirable. As explained before, the revelation of information actually tends to hurt social welfare.

Part (ii) of the corollary tells that social welfare S^1 at the local optimum with free OC typically falls short of $S^1(L)$. Starting from the local optimum, a marginal increase in L_E obviously decreases social welfare. At $L_E = L$, however, the price becomes uninformative and, as we have seen in (i), welfare jumps up. Equation (2.75) states that, in fact, it jumps high enough to surpass the local optimum value. Corollary 2.5.2.1(ii) holds under fairly weak conditions. Besides the ones already stated in Proposition 2.5.2 and the condition that talent is scarce, i.e., $M \geq L$, it additionally requires that residual uncertainty is lower than fundamental one, i.e., ${\sigma_\epsilon}^2 \leq {\sigma_s}^2$. Note that forcing $L_E = L$ is not equivalent to a ban on dealers. While the former makes all hipos engage in entrepreneurship, the latter leaves them the choice to become either entrepreneurs or act as passive investors.

Part (iii) of the corollary gives an alternative set of conditions for the validity of Proposition 2.5.2. Essentially, it disposes of the condition that a single noise trader's asset demand falls short of the amount of assets created by a single entrepreneur and instead requires condition (2.76). Compared to the condition that $L_E^1/a > \bar{v}$, condition (2.76) is stronger in the sense that $[1 + (L + M)/N]^{0.5} > 1$, but weaker in the sense that $L_E^0 \geq L_E^1$.

2.6 Stochastic Noise Trader Demand

2.6.1 Small Noise Trader Shocks

The equilibrium analysis in Chapter 2.3 and the welfare results from Chapter 2.5 all hinge on one central assumption: non-stochastic noise trader demand, i.e., ${\sigma_v}^2 = 0$. Now, let ${\sigma_v}^2 > 0$. We will show that these chapters go through unchanged for an infinitesimally small amount of noise trader volatility, i.e., for ${\sigma_v}^2 \to 0$. After that, we take a closer look at what to expect for a ${\sigma_v}^2$ that is greater than but "close" to zero, i.e., for ${\sigma_v}^2 \gtrsim 0$.

Limit analysis. For ${\sigma_v}^2 \to 0$, the noisy versions of $\Delta(L_E)$ and $\Gamma(L_E)$ from Chapter 2.2 converge pointwise to their non-noisy "siblings" given in Chapter 2.3 (see Appendix B.30). Hence, Chapter 2.3 also applies in the limit for ${\sigma_v}^2 \to 0$. Notably, this implies that the equilibrium mass of entrepreneurs L_E in case of NT shocks converges to the one derived for non-stochastic NT demand. Regarding welfare results, Appendices B.30 and B.31 together show that the noisy versions of all agents' CEs converge pointwise to their non-noisy siblings given in Chapter 2.5. This immediately implies that the same is true for the social welfare function S. Together with the fact that the equilibrium L_E converges, it follows that all agents' equilibrium CEs as well as equilibrium social welfare converge. Hence, Chapter 2.5 holds also for ${\sigma_v}^2 \to 0$.

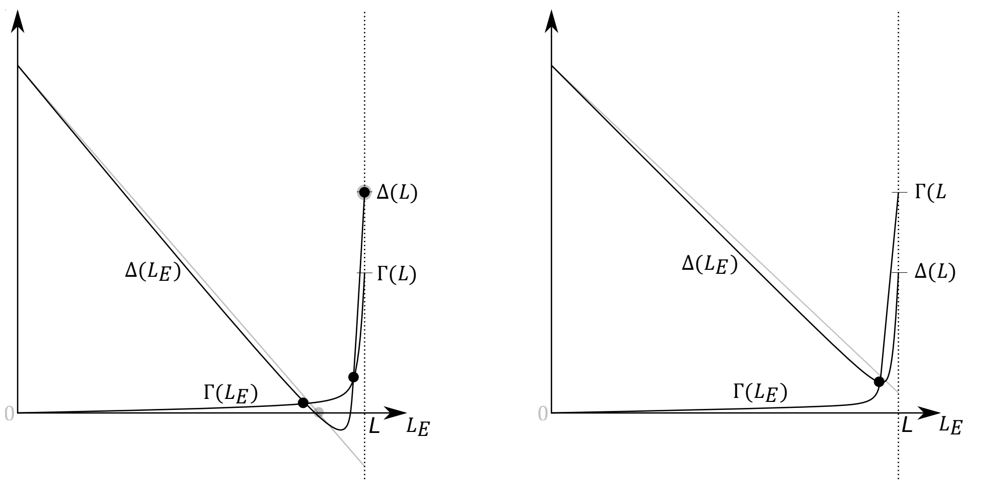
Non-zero NT shocks. The results from the limit analysis let us expect that our main results are likely to (approximately) hold also for positive but small noise trader volatility. We will numerically check on this in the following chapter. Before, however, note that with ${\sigma_v}^2 \gtrsim 0$ at least one important qualitative difference arises compared to the cases of ${\sigma_v}^2 = 0$ and ${\sigma_v}^2 \to 0$: all relevant expressions from Chapter 2.2 as well as social welfare S are continuous in L_E at $L_E = L$.[25] This implies that, with free OC, the existence and uniqueness properties of equilibrium in the former case differ substantially from the two latter ones.

To illustrate this, consider the basic version of the model and the following two examples. First, assume that with ${\sigma_v}^2 = 0$ there is a coexistence of two equilibria, one with $L_E < L$ and one with $L_E = L$. As the left panel of figure 2.6 shows, equilibria close to the ones without noise trader shocks still exist for ${\sigma_v}^2$ greater than zero but

[25]Remember that these expressions lost the continuity property in the limit for ${\sigma_v}^2 \to 0$. In this regard, note that Appendices B.30 and B.31 prove pointwise convergence, but do *not* prove uniform convergence.

small. However, there is an additional (albeit "unstable") equilibrium at L_E close to L, given by the second intersection of $\Delta(L_E)$ and $\Gamma(L_E)$. This additional intersection is due to the fact that instead of the discontinuity in case of ${\sigma_v}^2 = 0$, which makes $\Delta(L_E)$ and $\Gamma(L_E)$ "jump" to $\Delta(L)$ and $\Gamma(L)$ at $L_E = L$, with ${\sigma_v}^2 \gtrsim 0$ these functions are almost kinked and are obviously continuous. As a second example, assume that with ${\sigma_v}^2 = 0$ equilibrium does not exist. In contrast, as is shown by the right panel of figure 2.6, with ${\sigma_v}^2 \gtrsim 0$ the functions $\Delta(L_E)$ and $\Gamma(L_E)$ intersect at L_E close to L and thereby depict an equilibrium. Overall, while we can expect that equilibria close to the ones with ${\sigma_v}^2 = 0$ still exist, additional equilibria with L_E close to L may arise.

FIGURE 2.6: Equilibrium with Small Noise Trader Shocks



Note: Both figures refer to the case of free OC in the basic version of the model. The blurred parts depict $\Delta(L_E)$, $\Gamma(L_E)$, and equilibrium L_E, respectively, for ${\sigma_v}^2 = 0$. Analogous, the bold lines refer to the case of ${\sigma_v}^2$ greater than zero but small.

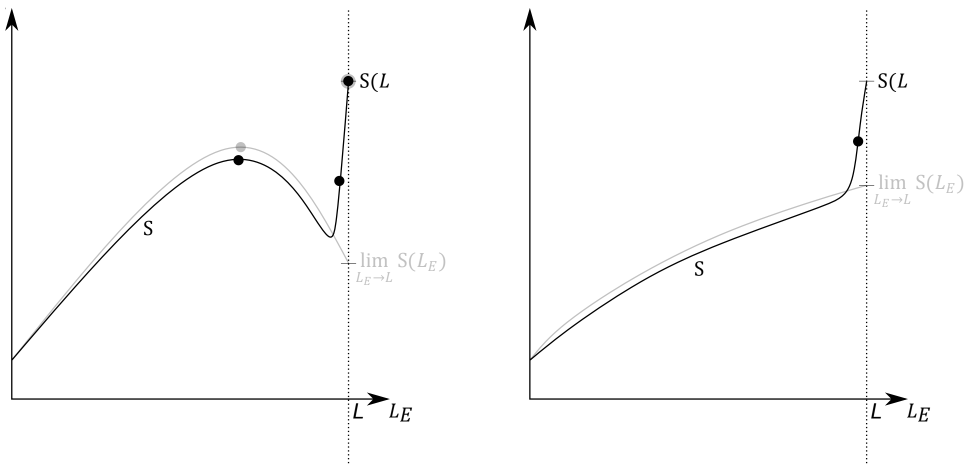
As our welfare results from Chapter 2.5 relate to the equilibrium amount of entrepreneurship, the fact that equilibrium properties change when looking at small but positive noise trader shocks instead of ${\sigma_v}^2 = 0$ implies that Propositions 2.5.1 and 2.5.2 do no longer entail a clear statement. To illustrate this, consider again the first example above. While with ${\sigma_v}^2 = 0$ there is a unique interior equilibrium $L_E < L$, with ${\sigma_v}^2 \gtrsim 0$ there are two. So, if Propositions 2.5.1 and 2.5.2 are still expected to hold approximately, then the question of course is: with regards to which of the two equilibria?

As we can see in the left panel of figure 2.7, we can still expect a local maximum of social welfare close to the equilibrium L_E where $\Delta(L_E)$ crosses $\Gamma(L_E)$ from above (call it $L_E^{1'}$), which in turn is close to the equilibrium $L_E < L$ in case of ${\sigma_v}^2 = 0$. In contrast to the discontinuity in case of ${\sigma_v}^2 = 0$, which makes $S(L_E)$ "jump" to $S(L)$ at $L_E = L$, with ${\sigma_v}^2 \gtrsim 0$ social welfare looks almost kinked in the vicinity of $L_E = L$ and is obviously continuous. Furthermore, if the conditions of Corollary 2.5.2.1(ii) are met, then again $S(L)$ is greater than $S(L_E^{1'})$ and hence S still reaches its global maximum at $L_E = L$. Social welfare at the additional equilibrium with L_E close to L

(call it $L_E^{1''}$) is obviously lower than at $L_E = L$. The left panel of figure 2.7 also shows that besides a local maximum, social welfare now takes on a local minimum in the vicinity of the L_E where $\Delta(L_E)$ crosses zero from below (cf. the left panel of figure 2.6 and Appendix B.19). As $\Gamma(L_E) \geq 0$, its second intersection with $\Delta(L_E)$ occurs to the right of this social welfare minimizing L_E. Hence, S is continuously increasing in L_E for $L_E \in [L_E^{1''}, L]$. Whether $S(L_E^{1'}) \gtreqless S(L_E^{1''})$ is ambiguous, but only of secondary interest anyway: the local social welfare maximum is given at $L_E = L_E^{1'}$, the global maximum at $L_E = L$.

Now consider the second example from above. As equilibrium with ${\sigma_v}^2 = 0$ does not exist (cf. the right panel of figure 2.6), social welfare is strictly increasing for $L_E \in (0, L)$. Furthermore, assume that the conditions of Corollary 2.5.2.1(i) are met and hence $S(L) > \lim_{L_E \to L} S(L_E)$ for ${\sigma_v}^2 = 0$. Then, as the right panel of figure 2.7 shows, with ${\sigma_v}^2 \gtrsim 0$ social welfare is strictly increasing for $L_E \in (0, L]$ and still reaches its global maximum at $L_E = L$. Nothing in particular happens at the newly existing equilibrium $L_E < L$.

FIGURE 2.7: Social Welfare with Small Noise Trader Shocks



Note: Both figures refer to the case of free OC in the basic version of the model. The blurred parts depict social welfare S for ${\sigma_v}^2 = 0$. The bold lines refer to the case of ${\sigma_v}^2$ greater than zero but small.

The reason why, so far, we didn't speak about the case of restricted OC is because there are no qualitative differences with regards to the equilibrium and welfare properties when considering ${\sigma_v}^2 \gtrsim 0$ instead of ${\sigma_v}^2 \to 0$ or ${\sigma_v}^2 = 0$. All relevant expressions are continuous in L_E in any of these cases. Hence, with restricted occupational choice, Chapters 2.3 and 2.5 can be expected to approximately hold for ${\sigma_v}^2 \gtrsim 0$. The labor market models can be illustrated analogous to the basic version of the model. Besides the fact that social welfare takes on its local maximum for an L_E greater than $L_E^{1'}$ in the UE model, figures 2.6 and 2.7 go through more or less unchanged.

In essence, with ${\sigma_v}^2 \gtrsim 0$ we can expect the following. Proposition 2.5.1(ii) holds approximately. Proposition 2.5.1(i) approximately holds with respect to the equilibrium $L_E < L$ close to the interior equilibrium in case of ${\sigma_v}^2 = 0$ (i.e., where $\Delta(L_E)$

first crosses zero from above). With regards to Proposition 2.5.2, note that equilibrium with restricted OC is unique and lies in the vicinity of the L_E that maximizes social welfare S^0 on $[0, L]$. Furthermore, it is $S^0(L) = S^1(L)$. Hence, Proposition 2.5.2 still applies in the sense that equilibrium social welfare without OC is higher than social welfare in any equilibrium with OC that complies with the proposition's conditions.

2.6.2 Large Noise Trader Shocks

So far, we have aimed at analytical results within our model. As these are largely unachievable for the model with noise trader shocks, we turned to a version of the model with deterministic noise trader demand. Our main welfare statements therein, given by Propositions 2.5.1 and 2.5.2, have been the following. First, the equilibrium amount of entrepreneurship is constrained efficient in the basic version of the model and in the FE economy, but falls short of the social welfare maximizing amount in the model with labor market frictions. Second, banning informed trading altogether typically increases social welfare, as the availability of information impairs risk-sharing and distorts the allocation of talent. Subsection 2.6.1 showed that we can expect these results to (approximately) carry over to the case of small noise trader shocks. In this chapter, we perform a comprehensive numerical analysis of the model and check to what extent our results also hold in case of substantially large noise volatility.

Essentially, we ask the following two questions. First, if equilibrium is not constrained efficient for ${\sigma_\nu}^2 > 0$ (as the presence of noise introduces market imperfections), then what is the effect of a marginal increase in the mass of entrepreneurs, starting from equilibrium? Is it positive? Second, is a ban on dealers still conducive to social welfare? Matlab simulations over a wide range of reasonable parameter combinations show that both tends to be the case.

Strategy

The closed-form solutions for $\Delta(L_E)$ and $\Gamma(L_E)$ as well as for social welfare $S(L_E)$, given in Appendices B.5, B.6 and B.29, allow for a numerical analysis of the equilibrium effects of professional trading on welfare for ${\sigma_\nu}^2 > 0$. We use our analysis of the model with deterministic noise trader demand as the starting point for our simulation. To make sure that this starting point is well-behaved, we restrict attention to parameter combinations that imply the existence of a unique interior equilibrium L_E^1 where agents do not short the asset for ${\sigma_\nu}^2 = 0$. Additionally, we consider only parameter combinations that obey the conditions from Proposition 2.5.2. We then introduce noise volatility and gradually increase the standard deviation of aggregate noise trader demand σ_ν from 0.1% up to 50% of the maximum feasible asset supply

L/a and check whether our analytical results prove robust to substantial noise trader volatility.[26]

Consider first the basic version of the model. In table 2.1, we look at a wide range of reasonable parameter combinations within the restrictions just stated. Similar to how we vary σ_v, average aggregate noise trader demand $\bar{v}$ is specified as a fraction of L/a. We assume that talent is scarce by setting M as a multiple (≥ 1) of L. Together, $\bar{v} \geq 0$ and $M \geq L$ ensure $L_E^0 > L_E^1$ in the model with deterministic noise trader demand (cf. equation 2.39). We vary the mass of noise traders N from at least 25% of the mass of rational agents up to a multiple of 25. Together, $\bar{v}$, M and N are set in a way that, for any of the given parameter combinations, it is always $\bar{v}/N \leq 1/a$. To ensure the existence of an equilibrium L_E^1 with $a\bar{v} < L_E^1 < L$ in the model with deterministic noise trader demand (cf. equation 2.29), we set $\bar{s}$ according to row six in table 2.1. We let residual uncertainty ${\sigma_\varepsilon}^2$ be a fraction of ${\sigma_s}^2$, so that knowledge on the asset fundamental s reduces the uncertainty regarding the asset's payoff θ by at least 50% and up to over 90%. As the magnitude of the CARA-parameter ρ cannot be interpreted without context, empirically meaningful and universally valid estimates for it do not exist (cf. Babcock et al., 1993). We set ρ so as to ensure that rational agents are not excessively risk-averse in the model with deterministic noise trader demand. We do so by requiring that rational agent i's certainty equivalent of final wealth π_i does not fall below the 95% confidence interval for π_i. Additionally, we pay attention to not cluster ρ around values that all imply more or less risk-neutral agents (see Appendix C.3).

TABLE 2.1: Parameter Values in the Simulation of the Basic Version of the Model

parameter	values	multiple of
σ_v	0.001, 0.01, 0.05, 0.1, 0.2, 0.5	$\frac{L}{a}$
$\bar{v}$	0.001, 0.01, 0.05, 0.1, 0.2, 0.5	$\frac{L}{a}$
M	1, 2, 3, 5, 10, 100	L
N	1, 2, 3, 5, 10, 100	$0.25(L+M)$
L	100	1
$\bar{s} - \frac{\rho\sigma_s^2}{2a}$	0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99	$\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L}{a} - \bar{v}\right)$
σ_ε^2	0.1, 0.25, 0.5, 0.75, 1	σ_s^2
ρ	0.01, 0.025, 0.05, 0.075, 0.1, 0.25, 0.5, 0.75, 1	$\frac{4}{\sqrt{1.25}}\frac{a}{\sigma_s}$
σ_s^2	1	1
a	10	1

Note: The parameters in the first column are specified as multiples of the magnitudes in the third column.

With parameters set as discussed above, the equilibrium and constrained optimum mass of entrepreneurs both in the model with free and restricted OC are linear homogeneous in L and homogeneous of degree zero in ${\sigma_s}^2$ and a, respectively (see

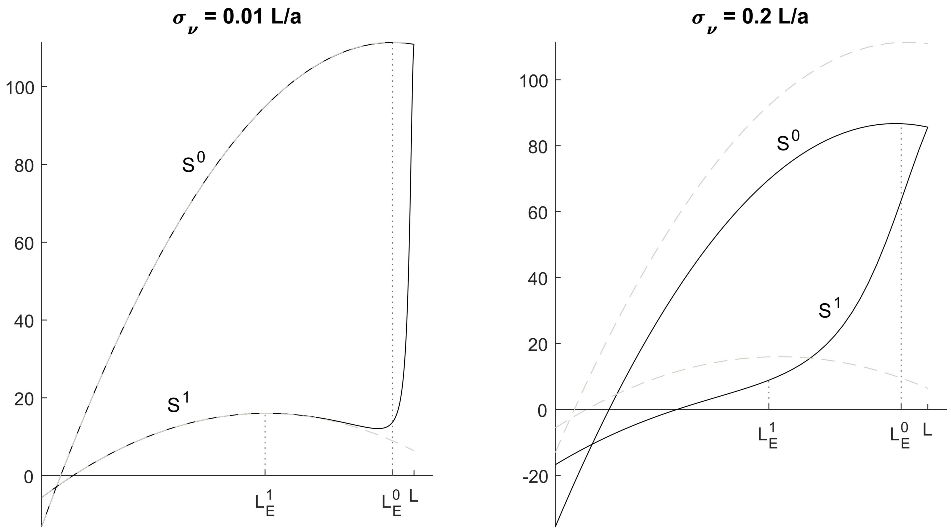
[26] An aggregate asset supply of L/a is realized if all hipos become entrepreneurs. By that, it also states an upper bound for the equilibrium asset supply.

Appendix C.4). In particular, an increase in L by a factor of λ just increases the equilibrium L_E, the social welfare maximizing L_E and the corresponding values of social welfare by factor λ as well. Varying ${\sigma_s}^2$ or a, respectively, does not have any effects on the equilibrium L_E or the social welfare maximizing L_E. The corresponding values of social welfare all simply change by factor λ for a factor λ-increase in σ_s, and by factor $(1/\lambda)$ for a factor λ-increase in a. As a consequence, our welfare analysis is independent of the parameter choice for L, a and ${\sigma_s}^2$ and we can just fix them to some arbitrary values, $L = 100$, $a = 10$, ${\sigma_s}^2 = 1$, say.

Example

Before getting to the simulation, figure 2.8 illustrates a numerical example for the basic version of the model. Social welfare in the case of rather low noise trader volatility, depicted in the left panel of the figure, is very close to the noiseless case (except that S^1 converges to S^0 for $L_E \to L$, cf. Chapter 2.6.1). The equilibrium amounts of entrepreneurship and the corresponding levels of social welfare are in the immediate vicinity of their respective constrained optimum levels. As L_E^1 falls short of the constrained optimum L_E by 0.5%, a marginal increase in the amount of entrepreneurship starting from equilibrium yields a weakly positive effect on social welfare. Equilibrium and maximum social welfare are higher in case of restricted OC compared to free OC by far.

FIGURE 2.8: Social Welfare with Large Noise Trader Shocks



Note: Both panels refer to the basic version of the model. The dashed lines depict social welfare for ${\sigma_v}^2 = 0$. The solid parts refer to the case of ${\sigma_v}^2 > 0$. Parameter values are chosen in accordance with table 2.1, where the values in the second column are 0.01 or 0.2 for σ_v and 0.2, 1, 3, 100, 0.5, 0.75, 0.5, 1, 10, in that order, for the other variables.

For rather large noise trader volatility, depicted in the right panel of figure 2.8, social welfare differs substantially from the noiseless case. While S^0 is still hump-shaped

(Appendix C.2 shows that this is a general property of S^0), S^1 is monotonically increasing in L_E (which, however, is not a general property of S^1). Consequently, a marginal increase in the amount of entrepreneurship, starting from equilibrium L_E^1, increases social welfare. In fact, this should not come as a surprise. Compared to the noiseless case, dealers now earn informational rents. Each additional (marginal) hipo who engages in entrepreneurship instead of professional trading tends to be conducive to welfare, as this implies less informed trading, benefits rational agents' risk sharing, reduces the clustering of risk at entrepreneurs and increases real economic activity (cf. Chapter 2.5.3). Comparing equilibrium social welfare with restricted OC to the case of free OC shows that the former still clearly exceeds the latter.

While figure 2.8 highlights that the presence of information tends to be detrimental to social welfare, especially at equilibrium, it also shows that there is a section with low L_E for which information is actually beneficial. Furthermore, this section seems to expand for higher ${\sigma_v}^2$. In this regard, note that whenever the presence of (or an increase in) dealers actually benefits social welfare, this is usually due to noise traders' part in the social welfare function. For high noise volatility, noise traders tend to dominate the SWF.[27] And, as information bring the asset price closer to fundamentals, noise traders are likely to benefit from the higher informational efficiency that comes with dealers via less risky returns from their (exogenous) trading activities. This weighs especially heavy for high ${\sigma_v}^2$ (and low N, so that individual NT risk is high), in which case the positive effect on NT utility can outweigh the negative effects that dealers typically have on rational agents' welfare (see also simulation tables 2.2 and 2.4).

Simulation

In what follows, we set up the simulation and state the main results. Further details and the Matlab code are delegated to Appendices C and D.

Basic version of the model. The parameters in table 2.1 yield a number of 87,480 combinations for each given value of σ_v and by construction imply that for ${\sigma_v}^2 = 0$ an equilibrium exists and the conditions of Proposition 2.5.2 are satisfied. Additionally, we require equilibrium to be unique, that is $\Delta(L) < \Gamma(L)$ (cf. Chapter 2.3), which rules out 22,368 parameter combinations and leaves us with a remaining total of 65,112 cases.[28] This defines our starting point, for which we know that Propositions 2.5.1 and 2.5.2 hold.

We then use Matlab to simulate the model for positive levels of noise volatility. In doing so, we add two further regulatory conditions. First, we maintain the condition that equilibrium exists, is unique and rational agents do not short the asset on

[27] In fact, for high ${\sigma_v}^2$, the variance $\mathbb{V}(\Psi)$ increases and noise traders certainty equivalent becomes strongly negative (cf. Appendix B.29).

[28] Almost all combinations ruled out entail high values for the risk-aversion parameter ρ. Multiplicity does not occur at all for the first five values of ρ in table 2.1, but for almost 85% of the cases with ρ set to its highest admissible value.

average. Hence, for each ${\sigma_v}^2$, we focus on parametrizations for which $\Delta(a\bar{v}) > \Gamma(a\bar{v})$ and $\Delta(L) < \Gamma(L)$.[29] Second, noise trader welfare for ${\sigma_v}^2 > 0$ is not always well defined, but can converge to minus infinity (cf. Appendix B.29). We rule out such cases. Consequently, the number of admissible parameter combinations further decreases, to a minimum of 16,607 in case of $\sigma_v = 0.5L/a$ (for more details, see table C.1 in Appendix C.5).

Table 2.2 gives the main results from the simulation. The first column states the magnitude of noise volatility, the second column gives the number of admissible parameter combinations and columns three and four show the percentages out of these combinations for which the respective inequalities in the first row hold. Obviously, equilibrium social welfare is almost always higher with restricted OC, that is without any dealers at all. Hence, Proposition 2.5.2 tends to carry over to the case of substantial noise volatility. The marginal effect of a higher mass of entrepreneurs (and, hence, a lower amount of dealers), starting from equilibrium, is positive for the vast majority of cases, at least up to $\sigma_v = 0.2L/a$. The result reverses for very high noise volatility, i.e., at $\sigma_v = 0.5L/a$. The reasons are as explained in the example above. Professional trading tends to impair rational agents' risk-sharing, leads to a clustering of risk at entrepreneurs and implies foregone real economic activity. For high noise volatility, however, the beneficial effect of information on noise traders' return volatility can outweigh these negative effects.

TABLE 2.2: Matlab Simulation of the Basic Version of the Model

$\frac{\sigma_v}{L/a}$	# cases	$\frac{dS^1(L_E^1)}{dL_E} > 0$	$S^0(L_E^0) > S^1(L_E^1)$
0.001	65,112	99.63%	100.00%
0.01	64,938	99.63%	100.00%
0.05	58,614	98.87%	100.00%
0.1	50,114	97.67%	99.90%
0.2	37,658	88.77%	99.55%
0.5	16,607	16.15%	94.61%

Note: Numbers in the last column are similar when comparing constrained optimum levels of SW instead of equilibrium ones.

In Appendix C, we show that while the marginal effect of an increasing L_E, starting from equilibrium, now is typically positive, the equilibrium mass of entrepreneurs still lies in the vicinity of its constrained optimum, if noise volatility is small (cf. table C.2). Social welfare is not only higher with restricted compared to free OC, but the difference is also quite large (cf. table C.3). We also decompose the total effect of a ban on dealers into the two distinct effects illustrated in figure 2.5 and show that they are both positive and of comparable magnitudes for the vast majority of cases (cf. table C.4).

[29] $\Delta(a\bar{v}) > \Gamma(a\bar{v})$ implies $L_E^1 > a\bar{v}$, which in turn implies that $\mathbb{E}(I_D) > 0$ and $\mathbb{E}(I_E) > 0$, cf. equations (2.4) and (B.41).

Full employment model. Consider next the FE model. If not stated otherwise in the upper part of table 2.3, parameters stay the same as in the basic version of the model. In order to simulate the FE model, we specify the firms' production function as Cobb-Douglas: $F(m) = Am^{1-b}$, with $A > 0$ and $0 < b < 1$. Expected firm profit $\bar{s}$ now depends on L_E and is given by $\bar{s}(L_E) = F(\hat{M}) - W\hat{M} + \hat{s} = Ab\hat{M}^{1-b} + \hat{s}$. We set $\hat{s}$ in a way to ensure that an equilibrium L_E^1 with $a\bar{v} < L_E^1 < L$ exists in the model with ${\sigma_v}^2 = 0$ (see the first row in table 2.3 and Appendix C.6). Empirical estimates for the output elasticity of labor suggest a Cobb-Douglas exponent somewhere in between 0.50 and 0.75 (see Douglas, 1976), so we vary $1 - b$ around these values. The efficiency parameter A is set in a way to ensure that the two terms which add up to expected firm profit $\bar{s}$ are of comparable magnitude for ${\sigma_v}^2 = 0$ (see Appendix C.6). The homogeneity properties from the basic model carry over to the FE economy (see Appendix C.4).

TABLE 2.3: Parameter Values in the Simulation of the FE and the UE Model

	parameter	values	multiple of
	$\bar{s}(a\bar{v}) - \frac{\rho\sigma_s^2}{2a}$	0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99	$\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L}{a} - \bar{v}\right) +$ $+\bar{s}(a\bar{v}) - \bar{s}(L)$
FE	b	0.10, 0.25, 0.40, 0.55	1
	A	0.25, 0.5, 0.75, 1	$\frac{\rho\sigma_s^2}{2ab}\left(\frac{\bar{v}}{M}\right)^{1-b}$
	$\bar{s} - \frac{\rho\sigma_s^2}{2a}$	0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 0.9, 0.95, 0.99	$\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L}{a} - \bar{v}\right)$
UE	A	0.25, 0.5, 0.75, 1	$\frac{\rho\sigma_s^2}{2ab}\left(\frac{L}{aM}\right)^{1-b}$
	D	$A(1-b)\left(\frac{L}{aM}\right)^b - \frac{1}{\rho}\ln\left(1 + \rho b(1-b)A\left(\frac{L}{aM}\right)^b\right)$	1

For each given σ_v, the total number of parameter combinations implied from tables 2.1 and 2.3 is 1,399,680. After dropping those for which equilibrium in the noiseless case is not unique, 1,124,190 combinations remain. Again, we also drop all combinations which imply that either equilibrium L_E^1 is not unique and greater than $a\bar{v}$ or that noise trader welfare is not well defined. This leaves us with no less than 943,858 combinations for σ_v up to $0.2L/a$ and with 636,033 combinations for $\sigma_v = 0.5L/a$. The left part of table 2.4 shows the main results from the simulation of the FE model. Similarly to the basic version of the model, a ban on dealers is almost always beneficial to social welfare and a marginal increase in the mass of entrepreneurs starting from equilibrium is most often positive, at least up to $\sigma_v = 0.2L/a$.
Additional material regarding the simulation of the FE model can be found in Appendix C.6.

Unemployment model. Finally, consider the UE model. If not stated otherwise in the lower part of table 2.3, parameter settings stay the same as in the FE model. Expected firm profit $\bar{s}$ is given by (2.55) and independent of L_E. Hence, the first row in the lower part of table 2.3 pins down $\hat{s}$ in a way to ensure that equilibrium L_E^1 in the noiseless case lies within the interval $(a\bar{v}, L)$. The intuition behind setting A is the same as in the FE model and we set the disutility of work parameter D such

TABLE 2.4: Matlab Simulation of the FE and the UE Model

	FE		UE	
$\frac{\sigma_v}{L/a}$	$\frac{dS^1(L_E^1)}{dL_E} > 0$	$S^0(L_E^0) > S^1(L_E^1)$	$\frac{dS^1(L_E^1)}{dL_E} > 0$	$S^0(L_E^0) > S^1(L_E^1)$
0.001	99.60%	100.00%	100.00%	100.00%
0.01	99.60%	100.00%	100.00%	100.00%
0.05	91.63%	99.99%	99.99%	100.00%
0.1	84.53%	99.73%	99.93%	99.94%
0.2	73.58%	98.76%	99.72%	99.67%
0.5	21.66%	92.95%	97.10%	97.48%

that there is equilibrium unemployment for all $L_E < L$ and full employment only for $L_E = L$ (see Appendix C.7). This keeps the positive effect that entrepreneurship has on job creation operative over the whole range of L_E. The homogeneity properties from the basic model carry over to the UE economy (cf. Appendix C.4).

The total number of parameter combinations according to tables 2.1 and 2.3 is given by 1,399,680. After dropping those for which equilibrium in the noiseless case is not unique, 1,041,792 combinations remain. Again, we also drop all combinations which imply that either equilibrium L_E^1 is not unique and greater than $a\bar{v}$ or that noise trader welfare is not well defined. This leaves us with no less than 937,824 combinations for σ_v up to $0.05L/a$, no less than 602,528 combinations for σ_v up to $0.2L/a$ and with 265,712 combinations for $\sigma_v = 0.5L/a$. The right part of table 2.4 shows the main results from the simulation of the UE model. Similar as in the basic and the FE model, equilibrium welfare in case of restricted OC is higher than with free OC for almost all simulated parameter combinations. As entrepreneurship entails a positive externality on workers in the UE model (cf. Proposition 2.5.2), the effect of a marginal increase in entrepreneurship is positive for the vast majority of cases, even for $\sigma_v = 0.5L/a$.

Additional information and results from the simulation of the UE model can be found in Appendix C.7.

2.7 Implementation

As discussed in Chapter 2.5.3, the second-best policy measures to be taken by a social planer would be characterized by, first, a ban on dealers and, only in case of labor market frictions, second, a subsequent increase in the amount of entrepreneurship. In reality, governments can not directly control the allocation of talent by forcing individuals into certain occupations. However, they can influence occupational choice via indirect measures such as preferential tax or regulatory treatments for certain types of jobs. For simplicity, suppose that the government can levy lump-sum taxes τ_i on type-i individuals ($i = E, D, M, N$). Then, a ban on dealers could be (indirectly) implemented via a sufficiently high tax τ_D on dealers and, if desired, a subsequent increase in entrepreneurship could be (indirectly) achieved by a subsidy $-\tau_E$ on entrepreneurs.

Again, it is important to emphasize that a tax on dealers is very different from a tax on trading. While the former targets only professional traders, the latter would target all kinds of trading and thereby also affect the other agents in the model. While this distinction is clear in theory, a tax that hurts professional trading without also affecting uninformed trades seems hard to implement in practice. We can, however, think of it as a proxy for some kind of regulatory requirements aimed at constraining agents and institutions specialized in professional trading activities.

Deterministic Noise Trader Demand

Consider first the case of non-stochastic noise trader demand, i.e., ${\sigma_\nu}^2 = 0$.

Implementation of the second-best allocation. With free OC, the price is fully informative (for $L_E < L$) and, hence, dealers obtain the same utility as passive investors. Consequently, any tax $\tau_D > 0$ on dealers implies that every dealer would be better off as an uninformed investor and, therefore, an equilibrium with a positive amount of dealers cannot exist. To ensure that no one wants to become a dealer even if the price is uninformative (i.e., at $L_E = L$), let $\tau_D > \Gamma(L)$, where $\Gamma(L)$ is given by (2.26). In this case, the tax on dealers is prohibitive in the sense that hipos always prefer uninformed trading over becoming a dealer. This implies that a tax $\tau_D > \Gamma(L)$ is equivalent to a straight ban on dealers and it is all that is needed in order to reach the second-best optimum in the baseline model and the FE economy. In the UE model, additional subsidies to entrepreneurship are required in order to achieve the social welfare maximizing mass of entrepreneurs. Denote this optimal mass of entrepreneurs by $L_E^{'}$ and, for the sake of brevity, assume that $L_E^{'} < L$. Then, the subsidy $-\tau_E$ required to ensure that $L_E^{'}$ is attained as the unique equilibrium outcome is given by $\Delta^0(L_E^{'}) - \tau_E = 0$ and, hence, by $-\tau_E = -\Delta^0(L_E^{'})$, where $\Delta^0(L_E)$ is given by (2.34). Using the approximation for $L_E^{'}$ from (2.70), the optimal subsidy on entrepreneurship is given by

$$-\tau_E^{'} = \frac{\tilde{m}}{a\rho}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right] > 0. \tag{2.77}$$

Total government expenditures equal $L_E^{'}\tau_E^{'}$. The government can raise this money without further affecting OC or social welfare by, e.g., introducing an economy-wide lump sum tax equal to $L_E^{'}\tau_E^{'}/(L + M + N)$ for every single individual (including entrepreneurs).

Implementation of the constrained second-best allocation. In reality, a (prohibitive) tax on dealers might not be practicable. If there are real world forces that make the implementation of a (prohibitive) tax on dealers difficult, then at least the *constrained* optimum allocation can be attained via appropriate subsidies to entrepreneurs.

If social welfare S^1 attains its *(global) optimum at* $L_E = L$ (cf. Corollary 2.5.2.1), then the optimal subsidy on entrepreneurship requires $-\tau_E \geq -\left[\Delta^1(L) - \Gamma^1(L)\right]$. Note, however, that while this subsidy guarantees the existence of an equilibrium at $L_E = L$, it is not necessarily unique. Uniqueness is ensured only if, additionally, $\lim_{L_E \to L} \Delta^1(L_E) - \tau_E \geq 0$ and, hence, $-\tau_E \geq -\lim_{L_E \to L} \Delta^1(L_E)$.[30] To sum up, $L_E = L$ is obtained as the unique equilibrium, if

$$-\tau_E \geq \max\left\{-\left[\Delta^1(L) - \Gamma^1(L)\right], -\lim_{L_E \to L} \Delta^1(L_E)\right\}. \tag{2.78}$$

Just as a (prohibitive) tax on dealers, high subsidies on entrepreneurs or, more generally, a strong manipulation of the allocation of talent, might not be politically practicable.

Hence, we now show how to attain the *(locally) optimal* $L_E < L$ in the presence of dealers. For the sake of brevity, assume that a unique equilibrium $L_E^1 < L$ exists in all versions of the model and that the (locally) optimal $L_E^{'}$ in the UE model, approximated by (2.69), is smaller than L. Then, the optimal subsidy to entrepreneurship is zero in the basic version of the model and the FE economy. In the UE model, the subsidy $-\tau_E$ required to ensure that the (locally) social welfare maximizing $L_E^{'}(< L)$ is attained as an equilibrium outcome is given by $\Delta^1(L_E^{'}) - \tau_E = 0$ and, hence, by $-\tau_E = -\Delta^1(L_E^{'})$, where $\Delta^1(L_E)$ is given by (2.25).[31] Using the approximation for $L_E^{'}$ from (2.69), the optimal subsidy on entrepreneurship $-\tau_E^{'}$ is, again, given by

$$-\tau_E^{'} = \frac{\tilde{m}}{a\rho}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right] > 0. \tag{2.79}$$

In contrast to the implementation of the ("overall") second-best allocation, we do not require a tax on dealers.

Stochastic Noise Trader Demand

Now let ${\sigma_v}^2 > 0$. For ${\sigma_v}^2 \to 0$, implementation works analogously as in case of ${\sigma_v}^2 = 0$. For ${\sigma_v}^2 \gtrsim 0$, we can expect it to work approximately the same as in case of ${\sigma_v}^2 = 0$, at least besides the fact that there is the possibility of additional equilibria in case of free OC (cf. Chapter 2.6.1). In general, a prohibitive tax on dealers is given by $\tau_D > \Gamma(L)$, where $\Gamma(L)$ is given by (2.21) and Appendix B.5. This follows directly from the fact that $\Gamma(L_E)$ is strictly increasing in L_E:

$$\frac{d\mathbb{V}(s|w)}{dL_E} = {\sigma_s}^2{\sigma_v}^2 \underbrace{\frac{d\gamma}{d\alpha}}_{<0} \underbrace{\frac{d\alpha}{dL_E}}_{<0} > 0. \tag{2.80}$$

[30] In general, the possibility of multiple equilibria in the model variants with free OC implies that taxation can not always ensure that a certain OC outcome is attained as the unique equilibrium. In contrast, a social planner does not face this constraint, as he can set L_E directly.

[31] Even though we assume that (i) equilibrium $L_E^1 < L$ is unique, which implies $\Delta^1(L) < \Gamma^1(L)$, and (ii) $L_E^{'} < L$, this subsidy not necessarily establishes $L_E^{'}$ as the unique equilibrium. Uniqueness additionally requires $\Delta^1(L) - \tau_E^{'} < \Gamma^1(L)$.

Again, the amount of entrepreneurship can be controlled by appropriate taxes/subsidies on entrepreneurs. Consider first the case where $\tau_D > \Gamma(L)$, that is, the case of a prohibitive tax on dealers. If the government wants to attain a certain $L_E(< L)$, call it $L_E^{'}$, as the unique equilibrium outcome, it can do so by setting $\Delta^0(L_E^{'}) - \tau_E = 0$ and, hence, $-\tau_E = -\Delta^0(L_E^{'})$, where $\Delta^0(L_E)$ is given by (2.21) and Appendix B.6.

If prohibitive taxes on dealers are not possible, the government can attain a certain $L_E(< L)$, call it $L_E^{'}$, as an equilibrium outcome by setting $\Delta^1(L_E^{'}) - \tau_E = \Gamma^1(L_E^{'}) - \tau_D$ and, hence, $\tau_D - \tau_E = \Gamma^1(L_E^{'}) - \Delta^1(L_E^{'})$, where $\Delta^1(L_E)$ and $\Gamma^1(L_E)$ are given by (2.21) and Appendix B.5. Again, note that while this tax differential guarantees an equilibrium at $L_E = L_E^{'}$, without further assumptions it is not necessarily unique.

Chapter 3

Conclusion

The rise of finance over the last century and the financial crisis of 2007/08 have sparked a discussion about the risks of a large financial sector. An important aspect of this discussion is a potential brain-drain to finance, that is, the question of whether too much talent is lured into high-paid jobs in finance, which could produce larger social benefits in other occupations within the real sector. We identify financial trading as the part of finance where wages and the inflow of talent over the last three to four decades have increased the most by far and which, at the same time, is often accused of providing only little social value.

We address the brain-drain question within a theoretical model that adds a real sector to the seminal noisy rational expectations equilibrium (REE) model of Grossman and Stiglitz (1980). This enables us to study occupational choice between entrepreneurship and financial trading. Individuals who decide to become dealers instead of entrepreneurs increase informational efficiency in the financial market, but at the cost of foregone real economic activity in terms of job creation and the production of output. The model delivers interesting results via a genuinely new mechanism. If the sole benefit of informational efficiency is more informed portfolio decisions, there tends to be too much, rather than too little, professional financial trading. The reason is that more informed trading leads to a clustering of risk at entrepreneurs, which distorts the allocation of talent and discourages real economic activity. Furthermore, in the presence of labor market frictions, entrepreneurship creates additional jobs and hence entails a positive externality on ordinary workers. This fosters the impression that agents' contribution to social welfare is higher in the real sector than in the financial one.

Obviously, our analysis is not "all-encompassing". Like any model, we shed light only on parts of the picture. In particular, we do not consider positive feedback effects of information revelation in the financial market on allocational efficiency in the real economy. Quite to the contrary, the feedback effect we explore is a negative one: informative asset prices can distort occupational choice away from real economic activity. In order to additionally implement positive feedback effects, one could think of either (i) an incentive channel, that is, e.g., the possibility to link managerial pay to firm performance, or (ii) a learning channel, where decision makers can condition their actions on information revealed by secondary financial markets and thereby increase the efficiency of real sector investment, production, etc. In this sense, an interesting idea for future research within the context of our model would be to introduce a positive real effect of information by re-modeling the determination of firm output in a way that makes entrepreneurs' labor demand depend on the stochastic macro variable and, hence, on information about that variable. Professional traders who gain information on that variable and leak it to entrepreneurs via trade in the asset market then create valuable information about the effectiveness of labor, influence entrepreneurs' labor demand and affect the real sphere via wages, employment and production. After all, however, any argumentation that the amount of talent engaged in professional financial trading is in fact deficient, rather than excessive, would have to show that positive feedback effects such as the ones just mentioned, *outweigh* the negative effects we find.

Appendix A

Technical Appendix

The model in the text builds on a CARA-normal framework, which means: negative exponential utility, with normally distributed random variables as input. The underlying probability theory is clearly non-trivial. Hence, this Appendix provides the necessary technical background knowledge.

A.1 Normal Random Variables

Density function. The density of a normally distributed random variable X_1 with expected value μ_1 and variance ${\sigma_1}^2$ is given by

$$f_{X_1}(x_1) = \frac{\exp\left\{-\frac{(x_1-\mu_1)^2}{2{\sigma_1}^2}\right\}}{\sqrt{2\pi{\sigma_1}^2}}. \tag{A.1}$$

The density of two jointly normally distributed random variables (X_1, X_2) with expected values (μ_1, μ_2), variances $({\sigma_1}^2, {\sigma_2}^2)$ and a correlation coefficient equal to ρ, is given by:

$$f_{X_1,X_2}(x_1, x_2) = \frac{\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\frac{(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2\right]\right\}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}. \tag{A.2}$$

Properties. Important properties of normally distributed random variables are the following (see, e.g., Gut, 2009, Section 5):

(i) Two random variables are jointly normal exactly if all of their linear combinations are normal. (A.3)

(ii) Two independent normal random variables are jointly normal. (A.4)

(iii) Two jointly normal random variables are independent exactly if they are uncorrelated. (A.5)

Furthermore, from (i) directly follows that

(iv) If X_1, X_2 are jointly normal, then X_1 is normal and X_2 is normal. (A.6)

From (i) and (ii) jointly follows that

(v) Linear combinations of two independent normal random variables are again normal. (A.7)

A.2 Conditional Expectations

Definition. Let X_1, X_2 be two continuous random variables. The expected value of X_1 is given by:

$$\mathbb{E}\left(X_1\right) = \int_{-\infty}^{\infty} x_1 f_{X_1}(x_1)\mathrm{d}x_1. \tag{A.8}$$

The expected value of X_1 conditional on X_2 is given by

$$\mathbb{E}\left(X_1|X_2\right) = \int_{-\infty}^{\infty} x_1 f_{X_1|X_2}(x_1, x_2)\mathrm{d}x_1. \tag{A.9}$$

Law of the unconscious statistician (LOTUS). In order to calculate the expected value of a function $g(X)$ of a random variable X, no information about the density of this function are needed. It is enough to know the density function of the random variable X:

$$\mathbb{E}(g(X)) = \int_{-\infty}^{\infty} g(x) f_X(x)\mathrm{d}x. \tag{A.10}$$

This also holds for joint distributions, i.e., if $g(X_1, X_2)$ is a function of two random variables X_1, X_2 with joint density function $f_{(X_1,X_2)}(x_1, x_2)$, then:

$$\mathbb{E}(g(X_1, X_2)) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x_1, x_2) f_{(X_1,X_2)}(x_1, x_2)\mathrm{d}x_1\mathrm{d}x_2. \tag{A.11}$$

Conditional expectation of a function of random variables. From (A.9) and (A.11) follows that

$$\mathbb{E}(g(X_1, X_2)|X_3) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x_1, x_2) f_{(X_1,X_2)|X_3}(x_1, x_2, x_3)\mathrm{d}x_1\mathrm{d}x_2. \tag{A.12}$$

Conditioned on itself, a random variable is not random any more. Therefore, a special case of (A.12) is:

$$\mathbb{E}(g(X_1, X_2)|X_2) = \int_{-\infty}^{\infty} g(x_1, x_2) f_{X_1|X_2}(x_1, x_2)\mathrm{d}x_1. \tag{A.13}$$

The law of iterated expectations (LIE). Let X_1 be a random variable and $\mathcal{I}_1$, $\mathcal{I}_2$ two information sets with $\mathcal{I}_2 \subseteq \mathcal{I}_1$. Then:

$$\mathbb{E}(\mathbb{E}(X_1|\mathcal{I}_1)|\mathcal{I}_2) = \mathbb{E}(X_1|\mathcal{I}_2). \tag{A.14}$$

Note that for (A.14) to hold, $\mathcal{I}_2$ necessarily has to be a subset of $\mathcal{I}_1$.

For the simplest version of (A.14), let $\mathcal{I}_1 = \{X_2\}$, where X_2 is just another random variable, and $\mathcal{I}_2$ an empty set, that is $\mathcal{I}_2 = \{\}$. It obviously holds true that $I_2 \subseteq I_1$ and it immediately follows that

$$\mathbb{E}\left(\mathbb{E}\left(X_1|X_2\right)\right) = \mathbb{E}\left(X_1\right). \tag{A.15}$$

For another example of (A.14), consider $\mathcal{I}_1 = \{X_2, X_3\}$ and $\mathcal{I}_2 = \{X_3\}$, where X_3 is a third random variable. It is $\mathcal{I}_2 \subseteq \mathcal{I}_1$ and again it immediately follows that

$$\mathbb{E}(\mathbb{E}(X_1|X_2, X_3)|X_3) = \mathbb{E}(X_1|X_3). \tag{A.16}$$

A.3 Bayesian Updating

Let (X_1, X_2) be jointly normal. Then:

$$X_1|X_2 \sim \mathcal{N}\left(\mathbb{E}(X_1|X_2), \mathbb{V}(X_1|X_2)\right), \tag{A.17}$$

$$\mathbb{E}(X_1|X_2) = \mu_1 + \frac{\mathrm{Cov}(X_1, X_2)}{\sigma_2{}^2}(X_2 - \mu_2), \tag{A.18}$$

$$\mathbb{V}(X_1|X_2) = \sigma_1{}^2 - \frac{\mathrm{Cov}(X_1, X_2)^2}{\sigma_2{}^2}. \tag{A.19}$$

When agents get additional information (here: X_2) on some random variable (here: on X_1), they "update" their initial expectation (i.e., μ_1) by taking these information into account. Analogously for the variance. As these properties are central to our analysis, we offer a short proof below.

Proof. Let $f_{X_2}(x_2) > 0$. Then the conditional density function is by definition given by

$$f_{X_1|X_2}(x_1, x_2) = \frac{f_{(X_1,X_2)}(x_1, x_2)}{f_{X_2}(x_2)}. \tag{A.20}$$

Using (A.1)-(A.2), we get:

$$f_{X_1|X_2}(x_1, x_2) = \frac{\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\frac{(x_1-\mu_1)(x_2-\mu_2)}{\sigma_1\sigma_2} + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2\right] + \frac{(x_2-\mu_2)^2}{2\sigma_2{}^2}\right\}}{\sqrt{2\pi\sigma_1{}^2\left(1-\rho^2\right)}}.$$

Denote

$$\zeta := \frac{1}{\sqrt{2\pi\sigma_1{}^2\left(1-\rho^2\right)}}, \quad \vartheta := \frac{1}{2\sigma_1{}^2\left(1-\rho^2\right)}, \quad \eta(x_2) := \mu_1 + \rho\frac{\sigma_1}{\sigma_2}\left(x_2 - \mu_2\right). \tag{A.21}$$

Rearranging terms gives

$$f_{X_1|X_2}(x_1, x_2) = \zeta \exp\left\{-\vartheta\left[\left(x_1 - \mu_1\right)^2 - 2\rho\frac{\sigma_1}{\sigma_2}\left(x_1 - \mu_1\right)\left(x_2 - \mu_2\right) + \right.\right.$$
$$\left.\left. + \frac{\sigma_1{}^2}{\sigma_2{}^2}\left(x_2 - \mu_2\right)^2 - \frac{\sigma_1{}^2}{\sigma_2{}^2}\left(1 - \rho^2\right)\left(x_2 - \mu_2\right)^2\right]\right\}$$

$$\begin{aligned}
&= \zeta \exp\left\{-\vartheta \left[x_1{}^2 - 2x_1\mu_1 + \mu_1{}^2 - 2\rho\frac{\sigma_1}{\sigma_2}\left(x_1\left(x_2-\mu_2\right) - \right.\right.\right.\\
&\qquad \left.\left.\left. - \; x_2\mu_1 + \mu_1\mu_2\right) + \rho^2\frac{\sigma_1{}^2}{\sigma_2{}^2}\left(x_2-\mu_2\right)^2\right]\right\}\\
&= \zeta \exp\left\{-\vartheta \left[\left(x_1-\eta(x_2)\right)^2 - \eta(x_2)^2 + \mu_1{}^2 - \right.\right.\\
&\qquad \left.\left. - \; 2\rho\frac{\sigma_1}{\sigma_2}\left(-x_2\mu_1 + \mu_1\mu_2\right) + \rho^2\frac{\sigma_1{}^2}{\sigma_2{}^2}\left(x_2-\mu_2\right)^2\right]\right\}\\
&= \zeta \exp\left\{-\vartheta \left[\left(x_1-\eta(x_2)\right)^2 - \mu_1{}^2 - 2\rho\frac{\sigma_1}{\sigma_2}\mu_1\left(x_2-\mu_2\right) - \rho^2\frac{\sigma_1{}^2}{\sigma_2{}^2}\;\cdot\right.\right.\\
&\qquad \left.\left. \cdot\left(x_2-\mu_2\right)^2 + \;\mu_1{}^2 + 2\rho\frac{\sigma_1}{\sigma_2}\mu_1\left(x_2-\mu_2\right) + \rho^2\frac{\sigma_1{}^2}{\sigma_2{}^2}\left(x_2-\mu_2\right)^2\right]\right\}\\
&= \zeta \exp\left\{-\vartheta\left[x_1-\eta(x_2)\right]^2\right\}. \qquad \text{(A.22)}
\end{aligned}$$

Expression (A.22) depicts the density function of a normally distributed random variable with expected value $\eta(x_2)$ and variance $\sigma_1{}^2\left(1-\rho^2\right)$. The proof is complete by noting that

$$\rho = \frac{\operatorname{Cov}\left(X_1, X_2\right)}{\sigma_1\sigma_2}. \qquad \text{(A.23)}$$

Remark: One can easily check that (A.18)-(A.19) also hold if one of the two variables is non-random or if one is just a linear transformation of the other, in which case they are degenerate joint-normal (and informationally equivalent).

A.4 CARA-Utility and Normal Random Variables

A.4.1 Lemma 1

Let X_1, X_2 be jointly normal. Then it holds that

$$\mathbb{E}\left[\exp\left\{X_1 - X_2{}^2\right\}\right] = \frac{\exp\left\{\mu_1 + \frac{1}{2}\sigma_1{}^2 - \frac{\left[\mu_2 + \operatorname{Cov}(X_1,X_2)\right]^2}{1+2\sigma_2{}^2}\right\}}{\sqrt{1+2\sigma_2{}^2}}. \qquad \text{(A.24)}$$

Note that (A.24) also holds if one of the two variables is non-random, or if one is just a linear transformation of the other, in which case they follow a degenerate joint normal distribution (and are informationally equivalent).

Proof. A sketched proof for this result can be found in Demange and Laroque (1995, p. 252-3).[1] We provide an alternative step-by-step proof below. We start by showing that (A.24) holds if X_1, X_2 follow a (non-degenerate) joint normal distribution. We proceed in two steps. First, we derive the expectation conditional on X_2. Second, we use the LIE to get the unconditional expectation.

[1]Demange and Laroque (1995, p. 252–253), however, only assume X_1 and X_2 to be normal, not necessarily *jointly* normal. Strictly speaking, this assumption is not sufficient. In the proof they supply, they implicitly assume joint normality, as they make use of the A.3 properties, which are properties for jointly normal variables (possibly degenerate), but do not generally hold for just X_1 normal and X_2 normal. A simple counterexample can be constructed by letting $X \sim \mathcal{N}(0,1)$ and defining Y by $Y = X$ if $|X| \geq 1$ and $Y = -X$ if $|X| < 1$.

Step 1. With (A.13), we know that

$$\mathbb{E}\left[\exp\left\{X_1 - X_2{}^2\right\}|X_2\right] = \int_{-\infty}^{\infty} \exp\left\{x_1 - x_2{}^2\right\} f_{X_1|X_2}(x_1, x_2)\mathrm{d}x_1.$$

As X_1, X_2 are jointly normal, Appendix A.3 tells us that $X_1|X_2$ is normal, too. Let $\mu_{1|2} := \mathbb{E}(x_1|x_2)$ and $\sigma_{1|2}{}^2 := \mathbb{V}(x_1|x_2)$. Then,

$$\begin{aligned}
\mathbb{E}\left[\exp\left\{X_1 - X_2{}^2\right\}|X_2\right] &= \exp\left\{-x_2{}^2\right\} \int_{-\infty}^{\infty} \exp\{x_1\} \frac{\exp\left\{-\frac{\left[x_1-\mu_{1|2}\right]^2}{2\sigma_{1|2}{}^2}\right\}}{\sqrt{2\pi\sigma_{1|2}{}^2}}\mathrm{d}x_1 \\
&= \exp\left\{-x_2{}^2\right\} \int_{-\infty}^{\infty} \frac{\exp\left\{-\frac{-2x_1\sigma_{1|2}{}^2 + x_1{}^2 - 2x_1\mu_{1|2} + \mu_{1|2}{}^2}{2\sigma_{1|2}{}^2}\right\}}{\sqrt{2\pi\sigma_{1|2}{}^2}}\mathrm{d}x_1 \\
&= \exp\left\{-x_2{}^2\right\} \int_{-\infty}^{\infty} \frac{\exp\left\{-\frac{\left[x_1-\left[\sigma_{1|2}{}^2+\mu_{1|2}\right]\right]^2 - \left[\sigma_{1|2}{}^2+\mu_{1|2}\right]^2 + \mu_{1|2}{}^2}{2\sigma_{1|2}{}^2}\right\}}{\sqrt{2\pi\sigma_{1|2}{}^2}}\mathrm{d}x_1 \\
&= \exp\{-x_2{}^2\} \exp\left\{\frac{2\mu_{1|2}\sigma_{1|2}{}^2 + \sigma_{1|2}{}^4}{2\sigma_{1|2}{}^2}\right\} \int_{-\infty}^{\infty} \frac{\exp\left\{-\frac{\left[x_1-\left[\sigma_{1|2}{}^2+\mu_{1|2}\right]\right]^2}{2\sigma_{1|2}{}^2}\right\}}{\sqrt{2\pi\sigma_{1|2}{}^2}}\mathrm{d}x_1.
\end{aligned}$$

Recall from Appendix A.3, that $\sigma_{1|2}{}^2$ is non-random and $\mu_{1|2}$ depends only on x_2. The expression under the integral sign is the density function of a normal random variable. And, of course, the area under a density function always is unity. Furthermore, applying our knowledge from Appendix A.3, we get:

$$\mathbb{E}\left[\exp\left\{X_1 - X_2{}^2\right\}|X_2\right] = \exp\{-x_2{}^2\} \exp\left\{\mu_{1|2} + \frac{1}{2}\sigma_{1|2}{}^2\right\} \tag{A.25}$$
$$= \exp\left\{-x_2{}^2 + \frac{\mathrm{Cov}(x_1, x_2)}{\sigma_2{}^2}x_2\right\} \exp\left\{\mu_1 + \frac{1}{2}\sigma_1{}^2 - \frac{\mathrm{Cov}(x_1, x_2)}{\sigma_2{}^2}\mu_2 - \frac{\mathrm{Cov}(x_1, x_2)^2}{2\sigma_2{}^2}\right\}.$$

Step 2. Using the LIE yields

$$\begin{aligned}
\mathbb{E}\left[\exp\left\{X_1 - X_2{}^2\right\}\right] &= \mathbb{E}\left[\mathbb{E}\left[\exp\left\{X_1 - X_2{}^2\right\}|X_2\right]\right] \\
&= \exp\left\{\mu_1 + \frac{1}{2}\sigma_1{}^2 - \frac{\mathrm{Cov}(x_1, x_2)}{\sigma_2{}^2}\mu_2 - \frac{\mathrm{Cov}(x_1, x_2)^2}{2\sigma_2{}^2}\right\} \cdot \\
&\quad \cdot \mathbb{E}\left[\exp\left\{-x_2{}^2 + \frac{\mathrm{Cov}(x_1, x_2)}{\sigma_2{}^2}x_2\right\}\right]. \qquad \text{(A.26)}
\end{aligned}$$

For now, let's focus on the remaining expected value term in the expression above. Using (A.10),

$$\mathbb{E}\left[\exp\left\{-x_2{}^2 + \frac{\mathrm{Cov}(x_1, x_2)}{\sigma_2{}^2}x_2\right\}\right] =$$

$$= \int_{-\infty}^{\infty} \exp\left\{-x_2{}^2 + \frac{\mathrm{Cov}(x_1,x_2)}{\sigma_2{}^2}x_2\right\} \frac{\exp\left\{-\frac{(x_2-\mu_2)^2}{2\sigma_2{}^2}\right\}}{\sqrt{2\pi\sigma_2{}^2}} \mathrm{d}x_2$$

$$= \int_{-\infty}^{\infty} \frac{\exp\left\{\frac{-x_2{}^2+2x_2\mu_2-\mu_2{}^2-2\sigma_2{}^2x_2{}^2+2x_2\mathrm{Cov}(x_1,x_2)}{2\sigma_2{}^2}\right\}}{\sqrt{2\pi\sigma_2{}^2}} \mathrm{d}x_2$$

$$= \int_{-\infty}^{\infty} \frac{\exp\left\{\frac{x_2{}^2(-1-2\sigma_2{}^2)+2x_2(\mu_2+\mathrm{Cov}(x_1,x_2))-\mu_2{}^2}{2\sigma_2{}^2}\right\}}{\sqrt{2\pi\sigma_2{}^2}} \mathrm{d}x_2$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_2{}^2}} \exp\left\{\frac{-\left(x_2+\frac{\mu_2+\mathrm{Cov}(x_1,x_2)}{-1-2\sigma_2{}^2}\right)^2+\left(\frac{\mu_2+\mathrm{Cov}(x_1,x_2)}{-1-2\sigma_2{}^2}\right)^2+\frac{\mu_2{}^2}{-1-2\sigma_2{}^2}}{\frac{2\sigma_2{}^2}{1+2\sigma_2{}^2}}\right\} \mathrm{d}x_2$$

$$= \frac{1}{\sqrt{1+2\sigma_2{}^2}} \exp\left\{\frac{\frac{\mu_2{}^2+2\mu_2\mathrm{Cov}(x_1,x_2)+\mathrm{Cov}(x_1,x_2)^2+\mu_2{}^2\left(-1-2\sigma_2{}^2\right)}{(1+2\sigma_2{}^2)^2}}{\frac{2\sigma_2{}^2}{1+2\sigma_2{}^2}}\right\}.$$

$$\cdot \int_{-\infty}^{\infty} \frac{1}{\sqrt{\frac{2\pi\sigma_2{}^2}{1+2\sigma_2{}^2}}} \exp\left\{-\frac{\left(x_2+\frac{\mu_2+\mathrm{Cov}(x_1,x_2)}{-1-2\sigma_2{}^2}\right)^2}{\frac{2\sigma_2{}^2}{1+2\sigma_2{}^2}}\right\} \mathrm{d}x_2.$$

The expression under the integral sign above, again, is the density of a normal random variable. It follows that

$$\mathbb{E}\left[\exp\left\{-x_2{}^2 + \frac{\mathrm{Cov}(x_1,x_2)}{\sigma_2{}^2}x_2\right\}\right] = \frac{\exp\left\{\frac{2\mu_2\mathrm{Cov}(x_1,x_2)+\mathrm{Cov}(x_1,x_2)^2-2\mu_2{}^2\sigma_2{}^2}{(1+2\sigma_2{}^2)2\sigma_2{}^2}\right\}}{\sqrt{1+2\sigma_2{}^2}}. \quad \text{(A.27)}$$

Substituting this expression into (A.26) gives:

$$\mathbb{E}\left[\exp\left\{X_1 - X_2{}^2\right\}\right] = \frac{\exp\left\{\mu_1 + \frac{1}{2}\sigma_1{}^2\right\}}{\sqrt{1+2\sigma_2{}^2}} \cdot$$

$$\cdot \frac{\exp\left\{\frac{-2\mathrm{Cov}(x_1,x_2)\mu_2\left(1+2\sigma_2{}^2\right)-\mathrm{Cov}(x_1,x_2)^2\left(1+2\sigma_2{}^2\right)+2\mu_2\mathrm{Cov}(x_1,x_2)+\mathrm{Cov}(x_1,x_2)^2-2\mu_2{}^2\sigma_2{}^2}{(1+2\sigma_2{}^2)2\sigma_2{}^2}\right\}}{\sqrt{1+2\sigma_2{}^2}} =$$

$$= \frac{\exp\left\{\mu_1 + \frac{1}{2}\sigma_1{}^2 - \frac{[\mu_2+\mathrm{Cov}(x_1,x_2)]^2}{1+2\sigma_2{}^2}\right\}}{\sqrt{1+2\sigma_2{}^2}}. \quad \square$$

In what follows, we show that (A.24) also holds in case that (i) X_1, X_2, or both are non-random, or (ii) X_1, X_2 are degenerate joint normal.

Non-random variables. Let $\bar{X}_1$, $\bar{X}_2$ be non-stochastic and X_1, X_2 two normally distributed random variables. Using (A.24):

- $\mathbb{E}\left(\exp\left\{\bar{X}_1 - \bar{X}_2{}^2\right\}\right) \underbrace{=}_{\text{(A.24)}} \exp\left\{\bar{X}_1 - \bar{X}_2{}^2\right\}.$
- $\mathbb{E}\left(\exp\left\{X_1 - \bar{X}_2{}^2\right\}\right) \underbrace{=}_{\text{(A.24)}} \exp\left\{\mathbb{E}\left(X_1\right) + \frac{1}{2}\mathbb{V}\left(X_1\right) - \bar{X}_2{}^2\right\}$

$$\underbrace{=}_{\text{(A.25)}} \exp\left\{-\bar{X}_2{}^2\right\}\mathbb{E}\left(\exp\left\{X_1\right\}\right)$$

The second equality sign follows from the derivation of (A.25), by just letting $X_2 = \bar{X}_2$ instead of conditioning on X_2.

- $\mathbb{E}\left(\exp\left\{\bar{X}_1 - X_2{}^2\right\}\right) \underbrace{=}_{(A.24)} \frac{\exp\left\{\bar{X}_1 - \frac{\mathbb{E}(X_2)^2}{1+2\mathbb{V}(X_2)}\right\}}{\sqrt{1+2\mathbb{V}(X_2)}}$
  $\underbrace{=}_{(A.27)} \exp\left\{\bar{X}_1\right\}\mathbb{E}\left(\exp\left\{-X_2{}^2\right\}\right)$

  The second equality sign follows from (A.27) and its derivation, by just letting $\mathrm{Cov}(x_1, x_2) = 0$.

These results correspond to what we get from simply taking the expectations directly. Hence, (A.24) holds for non-random variables. In particular, this allows us to use (A.24) with $X_1 = 0$ or $X_2 = 0$.

Degenerate joint-normal variables. If X_2 is just a linear transformation of X_1, then X_1 and X_2 follow a *degenerate* joint normal distribution. As, in that case, $X_1|X_2$ is no longer normal, the proof as it stands for (non-degenerate) joint-normality does not exactly go through.

Now, assume that Y_1 is normal and $Y_2 = a + bY_1$, with a, b constant. We can write $Y_1 - Y_2{}^2$ as

$$\begin{aligned} Y_1 - Y_2{}^2 &= Y_1 - a^2 - 2abY_1 - b^2 Y_1{}^2 \\ &= -\Bigg(\underbrace{bY_1 - \frac{1-2ab}{2b}}_{=:Y}\Bigg)^2 + \left(\frac{1-2ab}{2b}\right)^2 - a^2. \end{aligned}$$

Y is normal and from above we know that we are allowed to apply (A.24) with $X_1 = 0$ and $X_2 = Y$:

$$\begin{aligned} \mathbb{E}\left(\exp\left\{Y_1 - Y_2{}^2\right\}\right) &= \exp\left\{\left(\frac{1-2ab}{2b}\right)^2 - a^2\right\}\mathbb{E}\left(\exp\left\{-Y^2\right\}\right) \\ &= \exp\left\{\left(\frac{1-2ab}{2b}\right)^2 - a^2\right\}\frac{\exp\left\{-\frac{\left(b\mu_1 - \frac{1-2ab}{2b}\right)^2}{1+2b^2\sigma_1{}^2}\right\}}{\sqrt{1+2b^2\sigma_1{}^2}} \\ &= \frac{\exp\left\{\frac{\left(\frac{1}{4b^2} - \frac{a}{b}\right)\left(1+2b^2\sigma_1{}^2\right) - b^2\mu_1{}^2 + \mu_1(1-2ab) - \frac{1}{4b^2} + \frac{a}{b} - a^2}{1+2b^2\sigma_1{}^2}\right\}}{\sqrt{1+2b^2\sigma_1{}^2}} \\ &= \frac{\exp\left\{\frac{\left(\frac{1}{2} - 2ab\right)\sigma_1{}^2 - a^2 - b^2\mu_1{}^2 + \mu_1(1-2ab)}{1+2b^2\sigma_1{}^2}\right\}}{\sqrt{1+2b^2\sigma_1{}^2}}. \end{aligned} \tag{A.28}$$

Now we just have to check that using formula (A.24) with $X_1 = Y_1$ and $X_2 = Y_2$ gives the same result:

$$\mathbb{E}\left(\exp\left\{Y_1 - Y_2{}^2\right\}\right) = \frac{\exp\left\{\mu_1 + \frac{1}{2}\sigma_1{}^2 - \frac{\left(a + b\mu_1 + b\sigma_1{}^2\right)}{1+2b^2\sigma_1{}^2}\right\}}{\sqrt{1+2b^2\sigma_1{}^2}}$$

$$= \frac{\exp\left\{\frac{\mu_1+2b^2\mu_1\sigma_1{}^2+\frac{1}{2}\sigma_1{}^2+b^2\sigma_1{}^4-a^2-2ab\mu_1-b^2\mu_1{}^2-2ab\sigma_1{}^2-2b^2\mu_1\sigma_1{}^2-b^2\sigma_1{}^4}{1+2b^2\sigma_1{}^2}\right\}}{\sqrt{1+2b^2\sigma_1{}^2}}$$

$$= \frac{\exp\left\{\frac{\left(\frac{1}{2}-2ab\right)\sigma_1{}^2-a^2-b^2\mu_1{}^2+\mu_1(1-2ab)}{1+2b^2\sigma_1{}^2}\right\}}{\sqrt{1+2b^2\sigma_1{}^2}}. \quad \square \tag{A.29}$$

In the model, e.g., P and z are degenerate joint normal (cf. Appendix B.3).

A.4.2 Lemma 2

Let X_1, X_2 be jointly normal. Then, if $\left(1-2\sigma_2{}^2\right)>0$, it holds that

$$\mathbb{E}\left[\exp\left\{X_1+X_2{}^2\right\}\right] = \frac{\exp\left\{\mu_1+\frac{1}{2}\sigma_1{}^2+\frac{[\mu_2+\mathrm{Cov}(X_1,X_2)]^2}{1-2\sigma_2{}^2}\right\}}{\sqrt{1-2\sigma_2{}^2}}. \tag{A.30}$$

Proof. This a slightly modified version of Lemma 1 (cf. Appendix A.4.1). The proof is similar.

Step 1 proceeds completely analogously as before. It yields

$$\mathbb{E}\left[\exp\left\{X_1+X_2{}^2\right\}|X_2\right] = \exp\left\{x_2{}^2+\frac{\mathrm{Cov}(x_1,x_2)}{\sigma_2{}^2}x_2\right\}\exp\left\{\mu_1+\frac{1}{2}\sigma_1{}^2- -\frac{\mathrm{Cov}(x_1,x_2)}{\sigma_2{}^2}\mu_2-\frac{\mathrm{Cov}(x_1,x_2)^2}{2\sigma_2{}^2}\right\}.$$

Step 2. Using the LIE gets us

$$\begin{aligned}\mathbb{E}\left[\exp\left\{X_1+X_2{}^2\right\}\right] &= \mathbb{E}\left[\mathbb{E}\left[\exp\left\{X_1+X_2{}^2\right\}|X_2\right]\right] = \\ &= \exp\left\{\mu_1+\frac{1}{2}\sigma_1{}^2-\frac{\mathrm{Cov}(x_1,x_2)}{\sigma_2{}^2}\mu_2-\frac{\mathrm{Cov}(x_1,x_2)^2}{2\sigma_2{}^2}\right\}\cdot \\ &\cdot \mathbb{E}\left[\exp\left\{x_2{}^2+\frac{\mathrm{Cov}(x_1,x_2)}{\sigma_2{}^2}x_2\right\}\right].\end{aligned} \tag{A.31}$$

Again, just focus on the remaining expected value term. Using (A.10),

$$\mathbb{E}\left[\exp\left\{x_2{}^2+\frac{\mathrm{Cov}(x_1,x_2)}{\sigma_2{}^2}x_2\right\}\right] =$$

$$= \int_{-\infty}^{\infty}\exp\left\{x_2{}^2+\frac{\mathrm{Cov}(x_1,x_2)}{\sigma_2{}^2}x_2\right\}\frac{\exp\left\{-\frac{(x_2-\mu_2)^2}{2\sigma_2{}^2}\right\}}{\sqrt{2\pi\sigma_2{}^2}}\mathrm{d}x_2$$

$$= \int_{-\infty}^{\infty}\frac{\exp\left\{\frac{-x_2{}^2+2x_2\mu_2-\mu_2{}^2+2\sigma_2{}^2x_2{}^2+2x_2\mathrm{Cov}(x_1,x_2)}{2\sigma_2{}^2}\right\}}{\sqrt{2\pi\sigma_2{}^2}}\mathrm{d}x_2$$

$$= \int_{-\infty}^{\infty}\frac{\exp\left\{\frac{x_2{}^2(-1+2\sigma_2{}^2)+2x_2(\mu_2+\mathrm{Cov}(x_1,x_2))-\mu_2{}^2}{2\sigma_2{}^2}\right\}}{\sqrt{2\pi\sigma_2{}^2}}\mathrm{d}x_2$$

$$= \int_{-\infty}^{\infty}\frac{1}{\sqrt{2\pi\sigma_2{}^2}}\exp\left\{\frac{\left(x_2+\frac{\mu_2+\mathrm{Cov}(x_1,x_2)}{-1+2\sigma_2{}^2}\right)^2-\left(\frac{\mu_2+\mathrm{Cov}(x_1,x_2)}{-1+2\sigma_2{}^2}\right)^2-\frac{\mu_2{}^2}{-1+2\sigma_2{}^2}}{\frac{2\sigma_2{}^2}{-1+2\sigma_2{}^2}}\right\}\mathrm{d}x_2$$

$$= \frac{1}{\sqrt{1-2\sigma_2{}^2}} \exp\left\{ \frac{\frac{-\mu_2{}^2-2\mu_2\mathrm{Cov}(x_1,x_2)-\mathrm{Cov}(x_1,x_2)^2-\mu_2{}^2\left(-1+2\sigma_2{}^2\right)}{\left(-1+2\sigma_2{}^2\right)^2}}{\frac{2\sigma_2{}^2}{-1+2\sigma_2{}^2}} \right\} \cdot$$

$$\cdot \int_{-\infty}^{\infty} \frac{1}{\sqrt{\frac{2\pi\sigma_2{}^2}{1-2\sigma_2{}^2}}} \exp\left\{ -\frac{\left(x_2+\frac{\mu_2+\mathrm{Cov}(x_1,x_2)}{-1+2\sigma_2{}^2}\right)^2}{\frac{2\sigma_2{}^2}{1-2\sigma_2{}^2}} \right\} \mathrm{d}x_2.$$

For $\left(1-2\sigma_2{}^2\right) > 0$, the integrand in the expression above is the density of a normal random variable. It follows that

$$\mathbb{E}\left[\exp\left\{x_2{}^2+\frac{\mathrm{Cov}(x_1,x_2)}{\sigma_2{}^2}x_2\right\}\right] = \frac{\exp\left\{\frac{-2\mu_2\mathrm{Cov}(x_1,x_2)-\mathrm{Cov}(x_1,x_2)^2-2\mu_2{}^2\sigma_2{}^2}{(-1+2\sigma_2{}^2)2\sigma_2{}^2}\right\}}{\sqrt{1-2\sigma_2{}^2}}.$$

Substituting this expression into (A.31) gives

$$\mathbb{E}\left[\exp\left\{X_1+X_2{}^2\right\}\right] = \frac{\exp\left\{\mu_1+\frac{1}{2}\sigma_1{}^2\right\}}{\sqrt{1-2\sigma_2{}^2}} \cdot$$

$$\cdot \frac{\exp\left\{\frac{-2\mathrm{Cov}(x_1,x_2)\mu_2\left(-1+2\sigma_2{}^2\right)-\mathrm{Cov}(x_1,x_2)^2\left(-1+2\sigma_2{}^2\right)-2\mu_2\mathrm{Cov}(x_1,x_2)-\mathrm{Cov}(x_1,x_2)^2-2\mu_2{}^2\sigma_2{}^2}{(-1+2\sigma_2{}^2)2\sigma_2{}^2}\right\}}{\sqrt{1-2\sigma_2{}^2}} =$$

$$= \frac{\exp\left\{\mu_1+\frac{1}{2}\sigma_1{}^2+\frac{[\mu_2+\mathrm{Cov}(x_1,x_2)]^2}{1-2\sigma_2{}^2}\right\}}{\sqrt{1-2\sigma_2{}^2}}. \quad \square$$

Remark: Just as we did for Lemma 1 in Appendix A.4.1, one can show that (A.30) also holds if one of the two variables is non-random or if one is just a linear transformation of the other, in which case they are degenerate joint-normal.

Appendix B

Model Proofs

B.1 Optimal Portfolio Holdings

Dealers. The expected utility of a dealer given his information set $I_D = \{s, P\}$ is

$$\mathbb{E}\left[U\left(\pi_D\right)|s, P\right] = \mathbb{E}\left[-\exp\left\{-\rho\pi_D\right\}|s, P\right].$$

If $\pi_D|s, P$ is normal, we can apply (A.24). As $\pi_D = I_D(s + \varepsilon - P)$, this is the case exactly if $\varepsilon|s, P$ is normal. As ε and s are independent, this is again the case exactly if $\varepsilon|P$ is normal. The asset price P can depend on the known moments of ε, but as ε is unobservable and materializes only in the last stage of the model (after all trading took place), it cannot be related to ε itself.[1] Hence, $\varepsilon|P \sim \mathcal{N}\left(0, \sigma_\varepsilon^2\right)$ and applying (A.24) with $X_2 = 0$ yields:

$$\mathbb{E}\left[-\exp\left\{-\rho\pi_D\right\}|s, P\right] = -\exp\left\{-\rho\left[\mathbb{E}\left(\pi_D|s, P\right) - \frac{\rho}{2}\mathbb{V}\left(\pi_D|s, P\right)\right]\right\} \tag{B.1}$$

The certainty equivalent (CE) of π_D conditional on knowing s and P (denoted by $\mathsf{CE}_{D|s,P}$), is implicitly given by $U\left(\mathsf{CE}_{D|s,P}\right) = \mathbb{E}\left[U\left(\pi_D\right)|s, P\right]$. It states the certain amount of wealth the individual would value exactly the same as its uncertain final wealth $\pi_D|s, P$. With the CARA-utility function given by (2.1), solving for the CE gives

$$\begin{aligned}\mathsf{CE}_{D|s,P} &= U^{-1}\left\{\mathbb{E}\left[U\left(\pi_D\right)|s, P\right]\right\} = \\ &= -\frac{1}{\rho}\ln\left\{-\mathbb{E}\left[U\left(\pi_D\right)|s, P\right]\right\}.\end{aligned} \tag{B.2}$$

As we can easily see, this is just a strictly positive monotonic transformation of expected utility which preserves the agent's preference ordering. Hence, maximization of the CE is equivalent to maximization of expected utility. From (B.1) and (B.2),

$$\begin{aligned}\mathsf{CE}_{D|s,P} &= \mathbb{E}\left(\pi_D|s, P\right) - \frac{\rho}{2}\mathbb{V}\left(\pi_D|s, P\right) \\ &= I_D(s - P) - \frac{\rho}{2}{I_D}^2\sigma_\varepsilon^2.\end{aligned} \tag{B.3}$$

[1] Note that ε was assumed to be independent of the other two random variables in the model, s and ν, so it is not possible that P is indirectly affected by ε via s or ν.

The first order condition for a dealer's optimal asset demand is given by

$$\frac{\partial \mathsf{CE}_{D|s,P}}{\partial I_D} = (s - P) - \rho I_D \sigma_\varepsilon^2 = 0 \tag{B.4}$$

and we get

$$I_D = \frac{s - P}{\rho \sigma_\varepsilon^2}. \tag{B.5}$$

As (B.4) immediately tells us that the second derivative is negative, this indeed represents the optimal portfolio decision of a dealer.

Passive investors. The expected utility of a passive investor given his information set $I_M = \{P\}$ is

$$\mathbb{E}\left[U\left(\pi_M\right)|P\right] = \mathbb{E}\left[-\exp\left\{-\rho\pi_M\right\}|P\right]. \tag{B.6}$$

Again, if $\pi_M|P$ is normal, we can apply (A.24). As $\pi_M = I_M(s + \varepsilon - P)$ and from above we know that ε and P must be independent of each other, this is the case exactly if $s|P$ is normal. From Appendix A.3, this is the case if s and P are jointly normal. As s and ν are normal and independent, with (A.4) follows that they are jointly normal and with (A.3) that all of their linear combinations are normal. Now assume that P is linear in s and ν.[2] Then, except for a constant, all linear combinations of s and P can also be obtained from linear combinations of s and ν. Hence, all linear combinations of s and P are normal, which again with (A.3) tells that s and P are jointly normal. Applying (A.24) yields:

$$\mathbb{E}\left[-\exp\left\{-\rho\pi_M\right\}|P\right] = -\exp\left\{-\rho\left[\mathbb{E}\left(\pi_M|P\right) - \frac{\rho}{2}\mathbb{V}\left(\pi_M|P\right)\right]\right\} \tag{B.7}$$

and it follows that

$$\begin{aligned} \mathsf{CE}_{M|P} &= \mathbb{E}\left(\pi_M|P\right) - \frac{\rho}{2}\mathbb{V}\left(\pi_M|P\right) \\ &= I_M(\mathbb{E}(s|P) - P) - \frac{\rho}{2}{I_M}^2\mathbb{V}(s + \varepsilon|P) \\ &= I_M(\mathbb{E}(\theta|P) - P) - \frac{\rho}{2}{I_M}^2\mathbb{V}(\theta|P). \end{aligned} \tag{B.8}$$

The first order condition for a passive investor's optimal asset demand is given by

$$\frac{\partial \mathsf{CE}_{M|P}}{\partial I_M} = (\mathbb{E}(\theta|P) - P) - \rho I_M \mathbb{V}(\theta|P) = 0 \tag{B.9}$$

and we get his optimal I_M by

$$I_M = \frac{\mathbb{E}(\theta|P) - P}{\rho\mathbb{V}(\theta|P)}. \tag{B.10}$$

[2] This is the standard "conjecture" for the price function in a CARA-normal model (see Vives, 2008, p. 116-117). As we will show in Appendix B.2, such an equilibrium price indeed exists.

Entrepreneurs. The expected utility of an entrepreneur given his information set $I_E = \{P\}$ is

$$\mathbb{E}\left[U\left(\pi_E\right)|P\right] = \mathbb{E}\left[-\exp\left\{-\rho\pi_E\right\}|P\right]. \tag{B.11}$$

Again, if $\pi_E|P$ is normal, we can apply (A.24). As $\pi_E = P/a + I_E(\theta - P)$ and $P|P$ is non-random, using the same argument as for the passive investors above, we see that $\pi_E|P$ is indeed normal. Applying A.24 and transforming for the CE gives

$$\begin{aligned}\mathsf{CE}_{\pi_E|P} &= \mathbb{E}\left(\pi_E|P\right) - \frac{\rho}{2}\mathbb{V}\left(\pi_E|P\right)\\ &= \frac{P}{a} + I_E(\mathbb{E}(\theta|P) - P) - \frac{\rho}{2}{I_E}^2\mathbb{V}(\theta|P).\end{aligned} \tag{B.12}$$

The first order condition for an entrepreneur's optimal asset holdings is given by

$$\frac{\partial \mathsf{CE}_{\pi_E|P}}{\partial I_E} = \mathbb{E}(\theta|P) - P - \rho I_E \mathbb{V}(\theta|P) = 0 \tag{B.13}$$

and we get

$$I_E = \frac{\mathbb{E}(\theta|P) - P}{\rho\mathbb{V}(\theta|P)}. \tag{B.14}$$

B.2 Equilibrium Price Function

Substituting the optimal portfolio holdings from (2.4) into the market clearing condition (B.15) below gives:

$$\begin{aligned}(L - L_E)I_D + MI_M + \nu &= L_E\left(1/a - I_E\right); \qquad \text{(B.15)}\\ (L - L_E)\frac{s - P}{\rho{\sigma_\varepsilon}^2} + (L_E + M)\frac{\mathbb{E}(\theta|P) - P}{\rho\mathbb{V}(\theta|P)} &= \frac{L_E}{a} - \nu;\\ -\left[\frac{L - L_E}{\rho{\sigma_\varepsilon}^2} + \frac{L_E + M}{\rho\mathbb{V}(\theta|P)}\right]P &= \frac{L_E}{a} - \nu - \frac{L - L_E}{\rho{\sigma_\varepsilon}^2}s - \frac{L_E + M}{\rho\mathbb{V}(\theta|P)}\mathbb{E}(\theta|P)\end{aligned}$$

and with that

$$\begin{aligned}P &= \frac{-\frac{L_E}{a} + \nu + \frac{L - L_E}{\rho{\sigma_\varepsilon}^2}s + \frac{L_E + M}{\rho\mathbb{V}(\theta|P)}\mathbb{E}(\theta|P)}{\frac{L - L_E}{\rho{\sigma_\varepsilon}^2} + \frac{L_E + M}{\rho\mathbb{V}(\theta|P)}}\\ &= \frac{w + \frac{L_E + M}{\rho\mathbb{V}(\theta|P)}\mathbb{E}(\theta|P) - \frac{L_E}{a}}{\frac{L - L_E}{\rho{\sigma_\varepsilon}^2} + \frac{L_E + M}{\rho\mathbb{V}(\theta|P)}},\end{aligned} \tag{B.16}$$

where

$$w := \frac{L - L_E}{\rho{\sigma_\varepsilon}^2}s + \nu.$$

As we know that $\mathbb{V}(\theta|P)$ is non-random and $\mathbb{E}(\theta|P)$ is linear in P (cf. Appendix A.3), equation (B.16) implicitly gives P as a linear function of w. Hence, P and w are informationally equivalent, which technically means that the information sets

$\{w, P\}$, $\{w\}$, $\{P\}$ are equivalent to each other. It immediately follows that

$$P = \frac{w + \frac{L_E + M}{\rho \mathbb{V}(\theta|w)} \mathbb{E}(\theta|w) - \frac{L_E}{a}}{\frac{L - L_E}{\rho \sigma_\varepsilon{}^2} + \frac{L_E + M}{\rho \mathbb{V}(\theta|w)}}.$$

Note that this price is indeed linear in s and ν and, hence, consistent with the "conjecture" made in order to derive equation (B.7). With (A.7), it follows that both P and w are normal.

B.3 Agents' Certainty Equivalents

Passive investors. Substituting the optimal portfolio decision I_M from (B.10) into equation (B.8) gives

$$\begin{aligned}
\mathsf{CE}_{M|P} &= \frac{\mathbb{E}(\theta|P) - P}{\rho \mathbb{V}(\theta|P)} \left(\mathbb{E}(\theta|P) - P\right) - \frac{\rho}{2} \left[\frac{\mathbb{E}(\theta|P) - P}{\rho \mathbb{V}(\theta|P)}\right]^2 \mathbb{V}(\theta|P) = \\
&= \frac{\left[\mathbb{E}(\theta|P) - P\right]^2}{2\rho \mathbb{V}(\theta|P)}. \qquad \text{(B.17)}
\end{aligned}$$

Note that as the safe asset in our setup yields a rate of return equal to zero, this is just a function of the (conditional) Sharpe Ratio of the risky asset θ, given its price P:

$$\begin{aligned}
\mathsf{CE}_{M|P} = \mathsf{GT}_{M|P} &= \frac{1}{2\rho} \frac{\left[\mathbb{E}(\theta|P) - P\right]^2}{\mathbb{V}(\theta|P)} \\
&= \frac{1}{2\rho} \left[\frac{\mathbb{E}(\frac{\theta - P}{P} - 0|P)}{\sqrt{\mathbb{V}(\frac{\theta - P}{P} - 0|P)}}\right]^2 \\
&= \frac{1}{2\rho} \left(S_{\theta|P}\right)^2, \qquad \text{(B.18)}
\end{aligned}$$

where $S_{\theta|P}$ denotes the (conditional) Sharpe Ratio. Written the other way around:

$$S_{\theta|P} = \overset{+}{\scriptstyle(-)} \sqrt{2\rho} \sqrt{\mathsf{GT}_{M|P}}. \qquad \text{(B.19)}$$

This result is closely related to the concept of a maximum certainty equivalent return in Pézier (2012). It implies that the Sharpe Ratio is equivalent to $\mathsf{GT}_{M|P}$ in the sense that the former is just a strictly monotonically increasing function of the latter.

We now want to derive the (unconditional) certainty equivalent CE_M. This is implicitly given by $U(\mathsf{CE}_M) = \mathbb{E}\left[U(\pi_M)\right]$. Using the LIE, we know that

$$\mathbb{E}\left[U(\pi_M)\right] = \mathbb{E}\left[\mathbb{E}\left[U(\pi_M)|P\right]\right] = \mathbb{E}\left[U\left(\mathsf{CE}_{M|P}\right)\right]. \qquad \text{(B.20)}$$

As P is normal (see Appendix B.2), with the use of (A.24) it follows that

$$U(\mathsf{CE}_M) = \mathbb{E}\left[U\left(\mathsf{CE}_{M|P}\right)\right]$$

$$
\begin{aligned}
&= -\mathbb{E}\left[\exp\left\{-\rho\cdot\mathsf{CE}_{M|P}\right\}\right] \\
&= -\mathbb{E}\left[\exp\left\{-\left(\sqrt{\rho}z\right)^2\right\}\right] \\
&= -\frac{\exp\left\{-\rho\frac{\mathbb{E}(z)^2}{1+2\rho\mathbb{V}(z)}\right\}}{\sqrt{1+2\rho\mathbb{V}(z)}},
\end{aligned}
\tag{B.21}
$$

where

$$
z := \frac{\mathbb{E}(\theta|P)-P}{\sqrt{2\rho\mathbb{V}(\theta|P)}}. \tag{B.22}
$$

Consequently,

$$
\mathsf{CE}_M = \frac{\mathbb{E}(z)^2}{1+2\rho\mathbb{V}(z)} + \frac{1}{2\rho}\ln\left[1+2\rho\mathbb{V}(z)\right]. \tag{B.23}
$$

Dealers. Substituting the optimal portfolio decision I_D from (B.5) into equation (B.3) gives

$$
\begin{aligned}
\mathsf{CE}_{D|s,P} &= \frac{s-P}{\rho{\sigma_\varepsilon}^2}(s-P) - \frac{\rho}{2}\left(\frac{s-P}{\rho{\sigma_\varepsilon}^2}\right)^2{\sigma_\varepsilon}^2 = \\
&= \frac{(s-P)^2}{2\rho{\sigma_\varepsilon}^2}.
\end{aligned}
\tag{B.24}
$$

Again, this can be expressed as a function of the (conditional) Sharpe Ratio of the risky asset θ, given its price P and the fundamental s:

$$
\begin{aligned}
\mathsf{CE}_{D|s,P} = \mathsf{GT}_{D|s,P} &= \frac{(s-P)^2}{2\rho{\sigma_\varepsilon}^2} = \\
&= \frac{1}{2\rho}\left[\frac{\mathbb{E}(\frac{\theta-P}{P}-0|s,P)}{\sqrt{\mathbb{V}(\frac{\theta-P}{P}-0|s,P)}}\right]^2 = \\
&= \frac{1}{2\rho}\left(S_{\theta|s,P}\right)^2.
\end{aligned}
\tag{B.25}
$$

To derive the (unconditional) certainty equivalent CE_D, we proceed in two steps. First, we compute the conditional certainty equivalent $\mathsf{CE}_{D|P}$, which, using the LIE, is implicitly given by $U\left(\mathsf{CE}_{D|P}\right) = \mathbb{E}\left[U\left(\mathsf{CE}_{D|s,P}\right)|P\right]$. As a second step, we again use the LIE to obtain CE_D, which is implicitly given by $U\left(\mathsf{CE}_D\right) = \mathbb{E}\left[U\left(\mathsf{CE}_{D|P}\right)\right]$.
Step 1. Denote

$$
\hat{z} := \sqrt{\mathsf{CE}_{D|s,P}}. \tag{B.26}
$$

As $s|P$ is normal, $\hat{z}|P$ is normal too and we can use (A.24) to get

$$
\begin{aligned}
U\left(\mathsf{CE}_{D|P}\right) &= \mathbb{E}\left[U\left(\mathsf{CE}_{D|s,P}\right)|P\right] \\
&= -\mathbb{E}\left[\exp\left\{-\rho\cdot\mathsf{CE}_{D|s,P}\right\}|P\right] \\
&= -\mathbb{E}\left[\exp\left\{-\left(\sqrt{\rho}\hat{z}\right)^2\right\}|P\right] \\
&= -\frac{1}{\sqrt{1+2\rho\mathbb{V}\left(\hat{z}|P\right)}}\exp\left\{-\frac{\rho\mathbb{E}\left(\hat{z}|P\right)^2}{1+2\rho\mathbb{V}\left(\hat{z}|P\right)}\right\}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{\sqrt{1+\frac{1}{\sigma_\varepsilon{}^2}\mathbb{V}(s|P)}} \exp\left\{ -\frac{\left[-\frac{P}{\sqrt{2\sigma_\varepsilon{}^2}} + \frac{1}{\sqrt{2\sigma_\varepsilon{}^2}}\mathbb{E}(s|P)\right]^2}{1+\frac{1}{\sigma_\varepsilon{}^2}\mathbb{V}(s|P)} \right\} \\
&= -\frac{1}{\sqrt{\frac{\mathbb{V}(\theta|P)}{\sigma_\varepsilon{}^2}}} \exp\left\{ -\frac{\frac{[\mathbb{E}(\theta|P)-P]^2}{2\sigma_\varepsilon{}^2}}{\frac{\mathbb{V}(\theta|P)}{\sigma_\varepsilon{}^2}} \right\} \\
&= -\sqrt{\frac{\sigma_\varepsilon{}^2}{\mathbb{V}(\theta|P)}} \exp\left\{ -\frac{[\mathbb{E}(\theta|P)-P]^2}{2\mathbb{V}(\theta|P)} \right\} \\
&= -\sqrt{\frac{\sigma_\varepsilon{}^2}{\mathbb{V}(\theta|P)}} \exp\left\{-\rho\cdot z^2\right\}. \qquad \text{(B.27)}
\end{aligned}
$$

It follows that

$$
\mathsf{CE}_{D|P} = \frac{1}{2\rho}\ln\frac{\mathbb{V}(\theta|P)}{\sigma_\varepsilon{}^2} + z^2. \qquad \text{(B.28)}
$$

Step 2. As z is normal, again using (A.24) gives

$$
\begin{aligned}
U\left(\mathsf{CE}_D\right) &= \mathbb{E}\left[U\left(\mathsf{CE}_{D|P}\right)\right] \\
&= -\sqrt{\frac{\sigma_\varepsilon{}^2}{\mathbb{V}(\theta|P)}}\mathbb{E}\left[\exp\left\{-\rho\cdot z^2\right\}\right] \\
&= -\sqrt{\frac{\sigma_\varepsilon{}^2}{\mathbb{V}(\theta|P)}} \frac{\exp\left\{-\rho\frac{\mathbb{E}(z)^2}{1+2\rho\mathbb{V}(z)}\right\}}{\sqrt{1+2\rho\mathbb{V}\left(z\right)}} \qquad \text{(B.29)}
\end{aligned}
$$

and it follows that

$$
\mathsf{CE}_D = \frac{1}{2\rho}\ln\frac{\mathbb{V}(\theta|P)}{\sigma_\varepsilon{}^2} + \frac{\mathbb{E}\left(z\right)^2}{1+2\rho\mathbb{V}\left(z\right)} + \frac{1}{2\rho}\ln\left[1+2\rho\mathbb{V}\left(z\right)\right]. \qquad \text{(B.30)}
$$

Entrepreneurs. Substituting the optimal portfolio decision I_E from (B.14) into equation (B.12) gives

$$
\begin{aligned}
\mathsf{CE}_{E|P} &= \frac{P}{a} + \frac{\mathbb{E}(\theta|P)-P}{\rho\mathbb{V}(\theta|P)}\left[\mathbb{E}(\theta|P)-P\right] - \frac{\rho}{2}\left[\frac{\mathbb{E}(\theta|P)-P}{\rho\mathbb{V}(\theta|P)}\right]^2\mathbb{V}(\theta|P) \\
&= \frac{P}{a} + z^2. \qquad \text{(B.31)}
\end{aligned}
$$

As P is normal and z is just a linear function of P, all linear combinations of z and P are linear in P and hence normal. With (A.3), it follows that z and P have a (degenerate) joint normal distribution. Applying (A.24) yields

$$
\begin{aligned}
U\left(\mathsf{CE}_E\right) &= \mathbb{E}\left[U(\mathsf{CE}_{E|P})\right] \\
&= -\mathbb{E}\left[\exp\left\{-\rho\cdot\mathsf{CE}_{E|P}\right\}\right] \\
&= -\mathbb{E}\left[\exp\left\{-\rho\frac{P}{a} - (\sqrt{\rho}z)^2\right\}\right]
\end{aligned}
$$

$$= -\frac{\exp\left\{-\rho\mathbb{E}\left(\frac{P}{a}\right) + \frac{\rho^2}{2}\mathbb{V}\left(\frac{P}{a}\right) - \rho\frac{\left[\mathbb{E}(z)-\rho\text{Cov}\left(\frac{P}{a},z\right)\right]^2}{1+2\rho\mathbb{V}(z)}\right\}}{\sqrt{1+2\rho\mathbb{V}(z)}} \tag{B.32}$$

and consequently,

$$\mathsf{CE}_E = \mathbb{E}\left(\frac{P}{a}\right) + \frac{\rho}{2}\mathbb{V}\left(\frac{P}{a}\right) + \frac{\left[\mathbb{E}(z)-\rho\text{Cov}\left(\frac{P}{a},z\right)\right]^2}{1+2\rho\mathbb{V}(z)} + \frac{1}{2\rho}\ln\left[1+2\rho\mathbb{V}(z)\right]. \tag{B.33}$$

B.4 Alternative Representation of an Entrepreneur's CE

We can also write an entrepreneur's final wealth as

$$\pi_E = \underbrace{\frac{\theta}{a}}_{\mathsf{FGP}_{E|P,\theta}} + \underbrace{\tilde{I}_E(P-\theta)}_{\mathsf{NGT}_{E|P,\theta}} = \mathsf{CE}_{E|P,\theta}, \tag{B.34}$$

where $\tilde{I}_E = 1/a - I_E$ is an entrepreneur's net asset supply. $\mathsf{FGP}_{E|P,\theta}$ gives the fundamental value created by an agent's entrepreneurial activity. The gains from actual "net" trade are denoted by $\mathsf{NGT}_{E|P,\theta}$. These gains are "net" in the sense that they arise from the actual amount of assets $\tilde{I}_E$ an entrepreneur trades in the market. An entrepreneur's certainty equivalent conditional on P is given by

$$\mathsf{CE}_{E|P} = \underbrace{\mathbb{E}\left(\frac{\theta}{a}|P\right) - \frac{\rho}{2}\mathbb{V}\left(\frac{\theta}{a}|P\right)}_{\mathsf{FGP}_{E|P}} + \underbrace{\frac{\left[\frac{\rho}{a}\mathbb{V}(\theta|P) - (\mathbb{E}(\theta|P)-P)\right]^2}{2\rho\mathbb{V}(\theta|P)}}_{\mathsf{NGT}_{E|P}}, \tag{B.35}$$

which one can easily show to be equivalent to (B.31). Now let

$$\begin{aligned}\tilde{z} &:= \frac{\left[\frac{\rho}{a}\mathbb{V}(\theta|P) - (\mathbb{E}(\theta|P)-P)\right]}{\sqrt{2\rho\mathbb{V}(\theta|P)}} \\ &= \sqrt{\mathsf{NGT}_{E|P}} \\ &= \sqrt{\frac{\rho}{2}\mathbb{V}\left(\frac{\theta}{a}|P\right)} - z \\ &= \sqrt{\frac{\rho}{2}\mathbb{V}(\theta|P)}\cdot\tilde{I}_E.\end{aligned} \tag{B.36}$$

Then, analogously as in Appendix B.3,

$$\begin{aligned}U\left(\mathsf{CE}_E\right) &= \mathbb{E}\left[U(\mathsf{CE}_{E|P})\right] \\ &= -\mathbb{E}\left[\exp\left\{-\rho\cdot\mathsf{CE}_{E|P}\right\}\right] \\ &= -\mathbb{E}\left[\exp\left\{-\rho\left(\mathbb{E}\left(\frac{\theta}{a}|P\right) - \frac{\rho}{2}\mathbb{V}\left(\frac{\theta}{a}|P\right)\right) - (\sqrt{\rho}\tilde{z})^2\right\}\right]\end{aligned}$$

$$= -\frac{\exp\left\{-\rho\mathbb{E}\left(\frac{\theta}{a}\right) + \frac{\rho^2}{2}\mathbb{V}\left(\frac{\theta}{a}|P\right) + \frac{\rho^2}{2}\mathbb{V}\left[\mathbb{E}\left(\frac{\theta}{a}|P\right)\right] - \rho\frac{\left[\mathbb{E}(\tilde{z}) - \rho\mathrm{Cov}\left(\mathbb{E}\left(\frac{\theta}{a}|P\right),\tilde{z}\right)\right]^2}{1+2\rho\mathbb{V}(\tilde{z})}\right\}}{\sqrt{1+2\rho\mathbb{V}(\tilde{z})}}. \tag{B.37}$$

With the law of total variance, it follows that

$$\mathsf{CE}_E = \underbrace{\mathbb{E}\left(\frac{\theta}{a}\right) - \frac{\rho}{2}\mathbb{V}\left(\frac{\theta}{a}\right)}_{\mathsf{FGP}_E} + \underbrace{\frac{\left[\mathbb{E}(\tilde{z}) - \rho\mathrm{Cov}\left(\mathbb{E}\left(\frac{\theta}{a}|P\right),\tilde{z}\right)\right]^2}{1+2\rho\mathbb{V}(\tilde{z})} + \frac{1}{2\rho}\ln\left[1+2\rho\mathbb{V}(\tilde{z})\right]}_{\mathsf{NGT}_E}. \tag{B.38}$$

Even though (B.38) and (2.20) are mathematically equivalent, (B.38) allows for a different perspective on the composition of entrepreneurs expected utility. More specifically, it shows which part of an entrepreneur's expected utility stems from the fundamental value of entrepreneurship and which from additionally having an asset exchange that allows for beneficial trades. FGP_E relates to an entrepreneur's expected utility from receiving the asset payoff θ for each of the $1/a$ assets he creates. NGT_E relates to an entrepreneur's additional utility from the possibility to sell an amount of $\tilde{I}_E$ out of his $1/a$ assets in the market, instead of holding all of them "to maturity".

Two terms make NGT_E differ from the trading gains GT_M of a passive investor. The covariance term stems from the fact that the fundamental value of entrepreneurship and the gains from trading are not independent of each other. A higher asset price P is a signal for high θ, which benefits $\mathsf{FGP}_{E|P}$, as well as it increases the gains from asset sales, which benefits $\mathsf{NGT}_{E|P}$. This co-movement, depicted by $\mathrm{Cov}\left(\mathbb{E}\left((\theta/a)|P\right),\tilde{z}\right) > 0$, increases overall risk and hence decreases NGT_E.[3] The term which makes $\tilde{z}$ differ from z essentially stems from the fact that $\tilde{I}_E \neq I_M (= I_E)$.

Actual "net" gains from trade for an entrepreneur can be higher or lower than that for passive investors. This is easy to see. If $I_E = I_M = 0$, then $z = 0$ and $\mathsf{GT}_M = 0$, while $\mathsf{NGT}_E > 0$, as $\tilde{I}_E \neq 0$ and with that $\tilde{z} \neq 0$. On the other hand, if $\tilde{I}_E = 0$, then $\tilde{z} = 0$ and $\mathsf{NGT}_E = 0$, while $\mathsf{GT}_M > 0$, as $I_E = I_M \neq 0$ and with that $z \neq 0$. While (2.20) is more convenient to use within our social welfare analysis, (B.38) can give additional information in case that the actual composition of entrepreneurs' expected utility is of interest.

B.5 Closed-Form Solutions - Free OC

Let

$$\alpha := \frac{L - L_E}{\rho\sigma_\varepsilon{}^2}, \quad \beta := \frac{L_E + M}{\rho\mathbb{V}(\theta|w)}, \quad \gamma := \frac{1}{\alpha^2\sigma_s{}^2 + \sigma_v{}^2}. \tag{B.39}$$

[3] As $\mathbb{E}\left((\theta/a)|P\right)$ is linearly increasing in P, and $\tilde{z}$ is linearly decreasing in z, $\mathrm{Cov}(P,z) < 0$ implies $\mathrm{Cov}\left(\mathbb{E}\left((\theta/a)|P\right),\tilde{z}\right) > 0$. Again, $\mathbb{E}(\tilde{z})$ is positive as long as $\tilde{I}_E > 0$, which is the case if agents don't short the asset (on average).

Then, it is

$$\mathbb{V}(\theta|w) = \sigma_\varepsilon{}^2 + \sigma_s{}^2\sigma_v{}^2\gamma, \tag{B.40}$$

$$\mathbb{E}(P) = \bar{s} - \frac{\frac{L_E}{a} - \bar{v}}{\alpha+\beta}, \tag{B.41}$$

$$\mathbb{V}(P) = \frac{1}{\gamma}\left(\frac{1+\alpha\beta\gamma\sigma_s{}^2}{\alpha+\beta}\right)^2, \tag{B.42}$$

$$\mathbb{E}(z) = \frac{\frac{L_E}{a} - \bar{v}}{(\alpha+\beta)\sqrt{2\rho\left(\sigma_\varepsilon{}^2 + \sigma_s{}^2\sigma_v{}^2\gamma\right)}}, \tag{B.43}$$

$$\mathbb{V}(z) = \frac{\gamma\sigma_v{}^4}{2\rho\left(\sigma_\varepsilon{}^2 + \sigma_s{}^2\sigma_v{}^2\gamma\right)(\alpha+\beta)^2}, \tag{B.44}$$

$$\mathrm{Cov}(P,z) = -\sqrt{\mathbb{V}(P)\mathbb{V}(z)}. \tag{B.45}$$

Proof. With the definition of w in (2.6) and the Bayesian updating rules in (A.3), we get

$$\begin{aligned}\mathbb{V}(\theta|w) &= \sigma_s{}^2 + \sigma_\varepsilon{}^2 - \frac{\alpha^2\sigma_s{}^4}{\alpha^2\sigma_s{}^2 + \sigma_v{}^2}\\ &= \sigma_\varepsilon{}^2 + \frac{\alpha^2\sigma_s{}^4 + \sigma_s{}^2\sigma_v{}^2 - \alpha^2\sigma_s{}^4}{\alpha^2\sigma_s{}^2 + \sigma_v{}^2}\\ &= \sigma_\varepsilon{}^2 + \sigma_s{}^2\sigma_v{}^2\gamma \end{aligned} \tag{B.46}$$

and with that

$$\mathbb{V}(s|w) = \sigma_s{}^2\sigma_v{}^2\gamma. \tag{B.47}$$

With P according to (2.5) and as $\mathbb{V}(\theta|w)$ is non-random, it is

$$\begin{aligned}\mathbb{E}(P) &= \frac{\mathbb{E}(w)}{\alpha+\beta} + \frac{\beta\mathbb{E}(\theta)}{\alpha+\beta} - \frac{\frac{L_E}{a}}{\alpha+\beta}\\ &= \frac{\alpha\bar{s}+\bar{v}}{\alpha+\beta} + \frac{\beta\bar{s}}{\alpha+\beta} - \frac{\frac{L_E}{a}}{\alpha+\beta}\\ &= \bar{s} - \frac{\frac{L_E}{a} - \bar{v}}{\alpha+\beta}\end{aligned} \tag{B.48}$$

and

$$\begin{aligned}\mathbb{V}(P) &= \frac{\mathbb{V}\left(w + \beta\left[\bar{s} + \frac{\mathrm{Cov}(\theta,w)}{\mathbb{V}(w)}\left(w - \mathbb{E}(w)\right)\right] - \frac{L_E}{a}\right)}{(\alpha+\beta)^2}\\ &= \frac{\mathbb{V}\left(\left[1 + \frac{\beta\mathrm{Cov}(\theta,w)}{\mathbb{V}(w)}\right]w\right)}{(\alpha+\beta)^2}\\ &= \frac{\left[1 + \frac{\beta\alpha\sigma_s{}^2}{\alpha^2\sigma_s{}^2+\sigma_v{}^2}\right]^2}{(\alpha+\beta)^2}\left(\alpha^2\sigma_s{}^2 + \sigma_v{}^2\right)\end{aligned}$$

$$= \frac{1}{\gamma}\left(\frac{1+\alpha\beta\gamma{\sigma_s}^2}{\alpha+\beta}\right)^2. \tag{B.49}$$

With z according to (2.13) and with (B.46) and (B.48) from above, we get

$$\begin{aligned}
\mathbb{E}(z) &= \frac{\mathbb{E}(\theta)-\mathbb{E}(P)}{\sqrt{2\rho\mathbb{V}(\theta|w)}} \\
&= \frac{\bar{s}-\left(\bar{s}-\frac{\frac{L_E}{a}-\bar{v}}{\alpha+\beta}\right)}{\sqrt{2\rho\left({\sigma_\varepsilon}^2+{\sigma_s}^2{\sigma_v}^2\gamma\right)}} \\
&= \frac{\frac{L_E}{a}-\bar{v}}{(\alpha+\beta)\sqrt{2\rho\left({\sigma_\varepsilon}^2+{\sigma_s}^2{\sigma_v}^2\gamma\right)}}
\end{aligned} \tag{B.50}$$

and

$$\begin{aligned}
\mathbb{V}(z) &= \frac{\mathbb{V}\left(\bar{s}+\frac{\mathrm{Cov}(\theta,w)}{\mathbb{V}(w)}\left(w-\mathbb{E}(w)\right)-\frac{w+\beta\left[\bar{s}+\frac{\mathrm{Cov}(\theta,w)}{\mathbb{V}(w)}\left(w-\mathbb{E}(w)\right)\right]-\frac{L_E}{a}}{\alpha+\beta}\right)}{2\rho\mathbb{V}(\theta|w)} \\
&= \frac{\mathbb{V}\left(\left[\frac{\mathrm{Cov}(\theta,w)}{\mathbb{V}(w)}-\frac{1+\beta\frac{\mathrm{Cov}(\theta,w)}{\mathbb{V}(w)}}{\alpha+\beta}\right]w\right)}{2\rho\mathbb{V}(\theta|w)} \\
&= \frac{\left[\frac{\mathrm{Cov}(\theta,w)(\alpha+\beta)-\mathbb{V}(w)-\beta\mathrm{Cov}(\theta,w)}{(\alpha+\beta)\mathbb{V}(w)}\right]^2\mathbb{V}(w)}{2\rho\mathbb{V}(\theta|w)} \\
&= \frac{\left[\alpha^2{\sigma_s}^2(\alpha+\beta)-\alpha^2{\sigma_s}^2-{\sigma_v}^2-\beta\alpha{\sigma_s}^2\right]^2}{2\rho\left({\sigma_\varepsilon}^2+{\sigma_s}^2{\sigma_v}^2\gamma\right)(\alpha+\beta)^2\left(\alpha^2{\sigma_s}^2+{\sigma_v}^2\right)} \\
&= \frac{\gamma{\sigma_v}^4}{2\rho\left({\sigma_\varepsilon}^2+{\sigma_s}^2{\sigma_v}^2\gamma\right)(\alpha+\beta)^2}.
\end{aligned} \tag{B.51}$$

To compute $\mathrm{Cov}(P,z)$, note that, with Appendix A.3, we know that P and z are both linear in w and it follows that P can be written as a linear function of z. Consequently, the correlation between P and z is either perfectly positive or perfectly negative, depending on whether P is increasing or decreasing in z. Obviously, P is increasing in z exactly if P and z are either both increasing in w or both decreasing in w. Analogously, P is decreasing in z exactly if one of the two is increasing in w while the other one is decreasing in w. From (2.5) and Appendix A.3, we immediately see that P is linearly increasing in w. Regarding z, note that

$$\begin{aligned}
\mathbb{E}(\theta|w)-P &= \mathbb{E}(\theta|w)\left(1-\frac{\beta}{\alpha+\beta}\right)-\frac{w}{\alpha+\beta}+\frac{\frac{L_E}{a}}{\alpha+\beta} \\
&= \left(\bar{s}+\frac{\mathrm{Cov}(\theta,w)}{\mathbb{V}(w)}\left[w-\mathbb{E}(w)\right]\right)\left(\frac{\alpha}{\alpha+\beta}\right)-\frac{w}{\alpha+\beta}+\frac{\frac{L_E}{a}}{\alpha+\beta} \\
&= \text{"non-random term"}+\left(\frac{\mathrm{Cov}(\theta,w)}{\mathbb{V}(w)}\frac{\alpha}{\alpha+\beta}-\frac{1}{\alpha+\beta}\right)w
\end{aligned}$$

and the expression in parentheses is negative exactly if

$$\frac{\mathrm{Cov}(\theta, w)}{\mathbb{V}(w)}\alpha < 1$$
$$\frac{\alpha^2 {\sigma_s}^2}{\alpha^2 {\sigma_s}^2 + {\sigma_v}^2} < 1,$$

which is obviously the case. So z is linearly decreasing in w and it follows that P must be linearly decreasing in z, that is $\mathrm{Corr}(P, z) = -1$. This immediately yields

$$\begin{aligned}\mathrm{Cov}(P, z) &= \mathrm{Corr}(P, z)\sqrt{\mathbb{V}(P)\mathbb{V}(z)} \\ &= -\sqrt{\mathbb{V}(P)\mathbb{V}(z)}.\end{aligned}$$

B.6 Closed-Form Solutions - Restricted OC

With P and z given by (2.8) and (2.22), it immediately follows that

$$\mathbb{E}(P) = \bar{s} - \frac{\rho\left({\sigma_s}^2 + {\sigma_\epsilon}^2\right)}{L+M}\left(\frac{L_E}{a} - \bar{v}\right), \tag{B.52}$$

$$\mathbb{V}(P) = \left[\frac{\rho\left({\sigma_s}^2 + {\sigma_\epsilon}^2\right)}{L+M}\right]^2 {\sigma_v}^2, \tag{B.53}$$

$$\mathbb{E}(z) = \sqrt{\frac{\rho}{2}\left({\sigma_s}^2 + {\sigma_\epsilon}^2\right)} \cdot \frac{\frac{L_E}{a} - \bar{v}}{L+M}, \tag{B.54}$$

$$\mathbb{V}(z) = \frac{\rho\left({\sigma_s}^2 + {\sigma_\epsilon}^2\right)}{2(L+M)^2}{\sigma_v}^2, \tag{B.55}$$

$$\mathrm{Cov}(P, z) = -\sqrt{\mathbb{V}(P)\mathbb{V}(z)}. \tag{B.56}$$

With (2.20) and (2.12), it holds that

$$\begin{aligned}\Delta(L_E) &= \mathsf{CE}_E - \mathsf{GT}_M \\ &= \mathbb{E}\left(\frac{P}{a}\right) - \frac{\rho}{2}\mathbb{V}\left(\frac{P}{a}\right) + \frac{\rho^2 \mathrm{Cov}\left(\frac{P}{a}, z\right)^2 - 2\rho\mathbb{E}(z)\mathrm{Cov}\left(\frac{P}{a}, z\right)}{1 + 2\rho\mathbb{V}(z)}.\end{aligned} \tag{B.57}$$

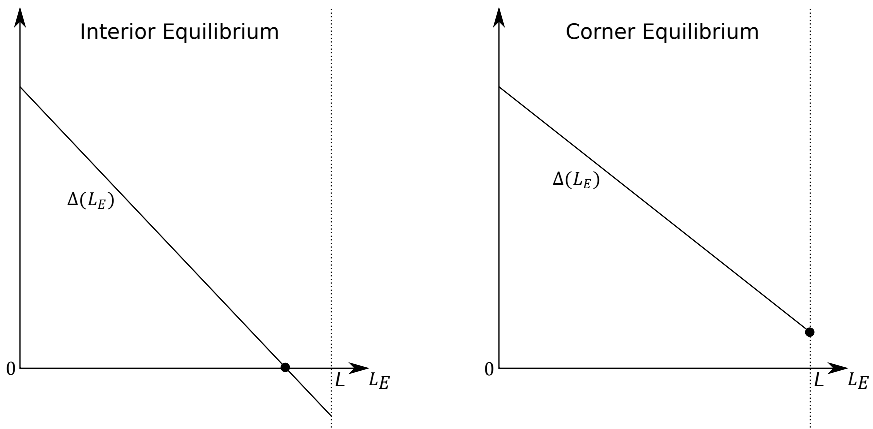
As (B.52)-(B.56) tell, $\mathbb{V}(P)$, $\mathbb{V}(z)$ and with that also $\mathrm{Cov}(P, z)$ are independent of L_E. Hence, differentiating yields

$$\begin{aligned}\frac{\mathrm{d}\Delta(L_E)}{\mathrm{d}L_E} &= \left[-\frac{\rho\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right)}{a^2(L+M)}\right] + \frac{-2\rho Cov(P, z)\sqrt{\rho\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right)}}{a^2(L+M)\sqrt{2}\left(1 + 2\rho V(z)\right)} \\ &= \frac{\rho\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right)}{a^2(L+M)}\left[\frac{-2\left(-\sqrt{\rho V(P)V(z)}\right)}{\sqrt{2}\left(1 + 2\rho V(z)\right)\sqrt{{\sigma_s}^2 + {\sigma_\varepsilon}^2}} - 1\right]\end{aligned}$$

$$
\begin{aligned}
&= \frac{\rho\left(\sigma_s{}^2+\sigma_\varepsilon{}^2\right)}{a^2(L+M)} \left[\frac{2\sqrt{\frac{\rho^4\left(\sigma_s{}^2+\sigma_\varepsilon{}^2\right)^3\sigma_\nu{}^4}{2(L+M)^4}}}{\sqrt{2}\left[1+\frac{\rho^2(\sigma_s{}^2+\sigma_\varepsilon{}^2)\sigma_\nu{}^2}{(L+M)^2}\right]\sqrt{\sigma_s{}^2+\sigma_\varepsilon{}^2}} - 1\right] \\
&= \underbrace{\frac{\rho\left(\sigma_s{}^2+\sigma_\varepsilon{}^2\right)}{a^2(L+M)}}_{>0} \underbrace{\left[\frac{\frac{\rho^2\sigma_\nu{}^2\left(\sigma_s{}^2+\sigma_\varepsilon{}^2\right)}{(L+M)^2}}{1+\frac{\rho^2\sigma_\nu{}^2(\sigma_s{}^2+\sigma_\varepsilon{}^2)}{(L+M)^2}} - 1\right]}_{<0} < 0.
\end{aligned}
\tag{B.58}
$$

Figure B.1 graphically illustrates the result and shows that equilibrium with restricted OC is always unique. For an explicit solution for the equilibrium L_E, see Appendix C.1.

FIGURE B.1: Equilibrium L_E in Case of Restricted OC and $\sigma_\nu{}^2 > 0$



B.7 Equilibrium Price Function for $\sigma_\nu{}^2 = 0$

Let $\sigma_\nu{}^2 = 0$. If $L_E < L$, then from (2.5) follows that:

$$
\begin{aligned}
P &= \frac{\frac{L-L_E}{\rho\sigma_\varepsilon{}^2}s + \bar{\nu} + \frac{L_E+M}{\rho\sigma_\varepsilon{}^2}s - \frac{L_E}{a}}{\frac{L+M}{\rho\sigma_\varepsilon{}^2}} \\
&= \frac{L-L_E}{L+M}s + \frac{\rho\sigma_\varepsilon{}^2}{L+M}\bar{\nu} + \frac{L_E+M}{L+M}s - \frac{\rho\sigma_\varepsilon{}^2}{L+M}\frac{L_E}{a} \\
&= s - \frac{\rho\sigma_\varepsilon{}^2}{L+M}\left(\frac{L_E}{a} - \bar{\nu}\right).
\end{aligned}
\tag{B.59}
$$

If $L_E = L$, then from (2.5) follows that:

$$
\begin{aligned}
P &= \frac{\bar{\nu} + \frac{L+M}{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}\bar{s} - \frac{L}{a}}{\frac{L+M}{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}} \\
&= \bar{s} - \frac{\rho\left(\sigma_\varepsilon{}^2+\sigma_s{}^2\right)}{L+M}\left(\frac{L}{a} - \bar{\nu}\right).
\end{aligned}
\tag{B.60}
$$

Alternatively, (B.60) also follows directly from (2.8) with $L_E = L$ and $\nu = \bar{\nu}$.

B.8 The Equilibrium Mass of Entrepreneurs

Let ${\sigma_v}^2 = 0$. Consider first the case of free OC. An interior equilibrium exists if $\Delta(L_E)$ given by (2.25) equals zero for an L_E between zero and L. Setting $\Delta(L_E) = 0$, we get

$$\begin{aligned}
\bar{s} - \frac{\frac{L_E}{a} - \bar{v}}{L+M}\rho{\sigma_\varepsilon}^2 - \frac{1}{2a}\rho{\sigma_s}^2 &= 0; \\
\bar{s} - \frac{\rho{\sigma_s}^2}{2a} &= \frac{\rho{\sigma_\varepsilon}^2}{L+M}\left(\frac{L_E}{a} - \bar{v}\right); \\
L_E &= \frac{a(L+M)}{\rho{\sigma_\varepsilon}^2}\left(\bar{s} - \frac{\rho{\sigma_s}^2}{2a}\right) + a\bar{v}. \qquad \text{(B.61)}
\end{aligned}$$

This is an interior equilibrium, if

$$\begin{aligned}
\frac{a(L+M)}{\rho{\sigma_\varepsilon}^2}\left(\bar{s} - \frac{\rho{\sigma_s}^2}{2a}\right) + a\bar{v} &> 0; \\
\bar{s} &> \frac{\rho{\sigma_s}^2}{2a} - \frac{\rho{\sigma_\varepsilon}^2}{L+M}\bar{v} \qquad \text{(B.62)}
\end{aligned}$$

and

$$\begin{aligned}
\frac{a(L+M)}{\rho{\sigma_\varepsilon}^2}\left(\bar{s} - \frac{\rho{\sigma_s}^2}{2a}\right) + a\bar{v} &< L; \\
\bar{s} &< \frac{\rho{\sigma_s}^2}{2a} + \frac{\rho{\sigma_\varepsilon}^2}{L+M}\left(\frac{L}{a} - \bar{v}\right). \qquad \text{(B.63)}
\end{aligned}$$

A corner equilibrium with $L_E = L$ exists, if $\Delta(L)$ given by (2.25) is greater than or equal to $\Gamma(L)$given by (2.26). We can write this condition as

$$\begin{aligned}
\frac{1}{a}\left(\bar{s} - \frac{\frac{L}{a} - \bar{v}}{L+M}\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right)\right) &\geq \frac{1}{2\rho}\ln\left[1 + \frac{{\sigma_s}^2}{{\sigma_\varepsilon}^2}\right]; \\
\bar{s} &\geq \frac{a}{2\rho}\ln\left[1 + \frac{{\sigma_s}^2}{{\sigma_\varepsilon}^2}\right] + \frac{\frac{L}{a} - \bar{v}}{L+M}\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right). \qquad \text{(B.64)}
\end{aligned}$$

Restricted occupational choice. Consider now the case of restricted OC. An interior equilibrium exists if $\Delta(L_E)$ given by (2.34) equals zero for some L_E between zero and L. Setting $\Delta(L_E) = 0$, we get

$$\begin{aligned}
\bar{s} - \frac{\frac{L_E}{a} - \bar{v}}{L+M}\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right) &= 0; \\
\frac{L_E}{a} - \bar{v} &= \frac{L+M}{\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right)}\bar{s}; \\
L_E &= \frac{a(L+M)}{\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right)}\bar{s} + a\bar{v}. \qquad \text{(B.65)}
\end{aligned}$$

This is an interior equilibrium, if

$$\begin{aligned}\frac{a(L+M)}{\rho\left({\sigma_\varepsilon}^2+{\sigma_s}^2\right)}\bar{s}+a\bar{v}&>0;\\ \bar{s}&>-\frac{\rho({\sigma_\varepsilon}^2+{\sigma_s}^2)}{L+M}\bar{v}\end{aligned}\tag{B.66}$$

and

$$\begin{aligned}\frac{a(L+M)}{\rho\left({\sigma_\varepsilon}^2+{\sigma_s}^2\right)}\bar{s}+a\bar{v}&<L;\\ \bar{s}&<\frac{\rho({\sigma_\varepsilon}^2+{\sigma_s}^2)}{L+M}\left(\frac{L}{a}-\bar{v}\right).\end{aligned}\tag{B.67}$$

A corner equilibrium with $L_E = L$ exists, if $\Delta(L)$ given by (2.34) is greater or equal to zero. We can write this condition as

$$\begin{aligned}\bar{s}-\frac{\frac{L}{a}-\bar{v}}{L+M}\rho\left({\sigma_\varepsilon}^2+{\sigma_s}^2\right)&\geq 0;\\ \bar{s}&\geq\frac{\rho({\sigma_\varepsilon}^2+{\sigma_s}^2)}{L+M}\left(\frac{L}{a}-\bar{v}\right).\end{aligned}\tag{B.68}$$

B.9 Equilibrium L_E with Free vs. Restricted OC

Let ${\sigma_v}^2 = 0$. For an equilibrium $L_E^1 < L$ to exist, we require $\bar{s} < \bar{s}_2$, which according to (2.28) can be written as

$$\bar{s}-\frac{\rho{\sigma_s}^2}{2a}-\frac{L-a\bar{v}}{a(L+M)}\rho{\sigma_\varepsilon}^2<0.\tag{B.69}$$

Now consider such an equilibrium, given by (2.29), as the "starting point". Furthermore, assume that in case of restricted OC an equilibrium with $L_E^0 < L$ exists, which is then given by (2.37). It is $L_E^0 > L_E^1$, that is, banning dealers increases the equilibrium mass of entrepreneurs, exactly if

$$\begin{aligned}\frac{a(L+M)}{\rho\left({\sigma_\varepsilon}^2+{\sigma_s}^2\right)}\bar{s}+a\bar{v}&>\frac{a(L+M)}{\rho{\sigma_\varepsilon}^2}\left(\bar{s}-\frac{\rho{\sigma_s}^2}{2a}\right)+a\bar{v};\\ \frac{1}{{\sigma_\varepsilon}^2+{\sigma_s}^2}\bar{s}&>\frac{1}{{\sigma_\varepsilon}^2}\left(\bar{s}-\frac{\rho{\sigma_s}^2}{2a}\right);\\ {\sigma_\varepsilon}^2\bar{s}-\left({\sigma_\varepsilon}^2+{\sigma_s}^2\right)\left(\bar{s}-\frac{\rho{\sigma_s}^2}{2a}\right)&>0;\\ -{\sigma_s}^2\bar{s}+\frac{\rho{\sigma_s}^2}{2a}\left({\sigma_\varepsilon}^2+{\sigma_s}^2\right)&>0;\\ \bar{s}-\frac{\rho}{2a}({\sigma_s}^2+{\sigma_\varepsilon}^2)&<0.\end{aligned}\tag{B.70}$$

Finally, note that (B.69) already implies (B.70), if

$$\bar{s} - \frac{\rho {\sigma_s}^2}{2a} - \frac{L - a\bar{v}}{a(L+M)}\rho{\sigma_\varepsilon}^2 \geq \bar{s} - \frac{\rho}{2a}({\sigma_s}^2 + {\sigma_\varepsilon}^2), \tag{B.71}$$

which simplifies to

$$\begin{aligned} -\frac{\rho}{2a}\left({\sigma_s}^2 + 2\frac{L - a\bar{v}}{L+M}{\sigma_\varepsilon}^2\right) &\geq -\frac{\rho}{2a}\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right); \\ 2\frac{L - a\bar{v}}{L+M}{\sigma_\varepsilon}^2 &\leq {\sigma_\varepsilon}^2; \\ \frac{L - a\bar{v}}{L+M} &\leq \frac{1}{2}. \end{aligned} \tag{B.72}$$

In the special case of $L_E^1 < L$ and $L_E^0 = L$, we obviously get $L_E^0 > L_E^1$ as well. And if $L_E^1 < L$ is not satisfied in the first place, then there are no dealers anyway and, consequently, banning them has no effect at all.

B.10 Trading Volumes with Free vs. Restricted OC

Let ${\sigma_v}^2 = 0$. Then, with (2.4) and (2.23) follows that

$$\tilde{I}_E = \begin{cases} \frac{1}{a} - \frac{s-P}{\rho{\sigma_\varepsilon}^2} = \frac{1}{a} - \frac{\frac{L_E}{a} - \bar{v}}{L+M}, & \text{for } L_E < L \\ \frac{1}{a} - \frac{\bar{s}-P}{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)} = \frac{1}{a} - \frac{\frac{L}{a} - \bar{v}}{L+M}, & \text{for } L_E = L \end{cases}, \tag{B.73}$$

$$I_M = I_E = \frac{1}{a} - \tilde{I}_E = \begin{cases} \frac{\frac{L_E}{a} - \bar{v}}{L+M}, & \text{for } L_E < L \\ \frac{\frac{L}{a} - \bar{v}}{L+M}, & \text{for } L_E = L \end{cases}, \tag{B.74}$$

$$I_D = \frac{s-P}{\rho{\sigma_\varepsilon}^2} = \frac{\frac{L_E}{a} - \bar{v}}{L+M}, \quad \text{for } L_E < L, \tag{B.75}$$

in case of free OC. With (2.7) and (2.32) follows that

$$\tilde{I}_E = \frac{1}{a} - \frac{\bar{s} - P}{\rho\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right)} = \frac{1}{a} - \frac{\frac{L_E}{a} - \bar{v}}{L+M}, \quad \text{for } L_E \leq L, \tag{B.76}$$

$$I_M = I_E = \frac{1}{a} - \tilde{I}_E = \frac{\frac{L_E}{a} - \bar{v}}{L+M}, \quad \text{for } L_E \leq L, \tag{B.77}$$

in case of restricted OC. Summarizing equations (B.73)-(B.77) gives equations (2.40)-(2.41) in the text.

Aggregate trading volumes. To see that aggregate net trading $L_E \cdot \tilde{I}_E$ is increasing in L_E, note that

$$\begin{aligned} \frac{d\left(L_E \cdot \tilde{I}_E\right)}{dL_E} &= \frac{1}{a} - \frac{\frac{L_E}{a} - \bar{v}}{L+M} - \frac{\frac{L_E}{a}}{L+M} \\ &= \frac{1}{a}\left(1 - \frac{2L_E}{L+M}\right) + \frac{\bar{v}}{L+M}, \end{aligned} \tag{B.78}$$

for which to be positive, it is sufficient that $M \geq L$ and $\bar{v} \geq 0$. Consequently, a ban on dealers, which increases the equilibrium mass of entrepreneurs (cf. Appendix B.9) without affecting the other parameters in $\tilde{I}_E$, increases aggregate net trading.

B.11 Entrepreneurs' Optimal Labor Demand

If all other firms make profit θ with employment m, then firm j that chooses employment m^j makes profit $\theta^j = \theta + \delta$, with $\delta := \left(F(m^j) - Wm^j\right) - \left(F(m) - Wm\right)$. Since the firm's profit differs from the other firms' profit by the non-random amount δ, arbitrage-freeness implies that also the respective share prices differ by δ, i.e., $P^j = P + \delta$.

The final wealth of an entrepreneur in firm j is given by

$$\pi_E^j = \frac{P^j}{a} + (\theta^j - P^j)I_E^{j^*} + (\theta - P)I_E^{j'}, \tag{B.79}$$

where $I_E^{j^*}$ denotes the amount of shares the entrepreneur holds in his own firm and $I_E^{j'}$ the amount of shares he holds in other firms. As we know that $\theta^j - P^j = \theta - P$, the entrepreneur's final wealth is the same irrespective of whether he trades shares in his own firm or shares in other firms. Then, if we just let $I_E^j := I_E^{j^*} + I_E^{j'}$ denote the total amount of assets the entrepreneur holds, (B.79) simplifies to

$$\begin{aligned} \pi_E^j &= \frac{P^j}{a} + (\theta - P)I_E^j \\ &= \frac{P}{a} + \frac{\delta}{a} + (\theta - P)I_E^j. \end{aligned} \tag{B.80}$$

A firm j entrepreneur maximizes his conditional expected utility with respect to both m^j and I_E^j. Proceeding as in Appendix B.1, we immediately get that the optimal I_E^j corresponds to I_E given by (2.4) or (2.7), depending on whether there is free or restricted OC. This is not surprising, as δ is non-random and optimal portfolio decisions under CARA-utility are independent of (non-stochastic) wealth. The fact that δ is non-random and does not interact with any other random variables also implies that maximizing the conditional expected utility with respect to m^j is equivalent to straight maximization of π_E^j. Hence, the entrepreneur's FOC w.r.t. m^j is simply given by

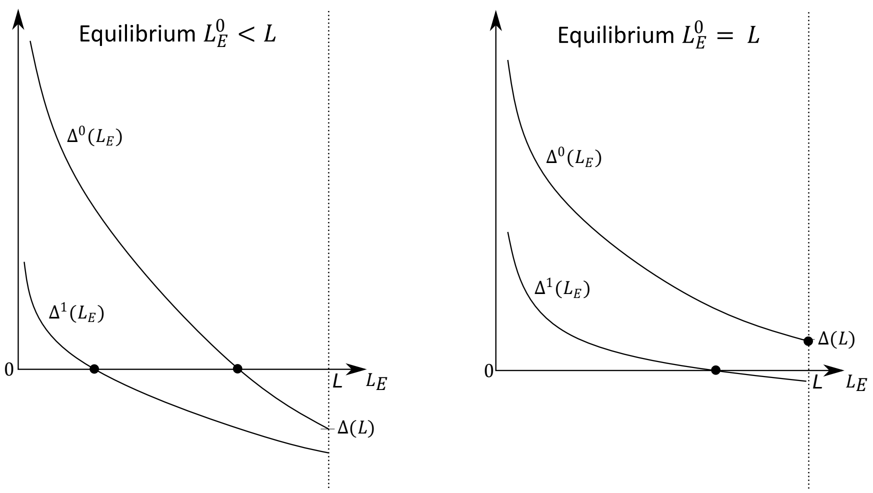
$$\frac{\partial \pi_E^j}{\partial m^j} = \frac{\partial \delta}{\partial m^j} = 0;$$
$$F'(m^j) = W. \tag{B.81}$$

Hence, in (symmetric) equilibrium of the FE model, entrepreneurs' optimal employment and portfolio decisions are given by (B.81) and (2.4) or (2.7), respectively. For a given mass of entrepreneurs L_E, the wage W is the same with free and restricted OC.

B.12 Equilibrium L_E with Free vs. Restricted OC - FE Model

Let ${\sigma_v}^2 = 0$. Let condition (2.39) hold. Then, we can show that it is $L_E^0 > L_E^1$ also in the FE model. In contrast to the basic model, however, we cannot solve for the equilibrium L_E's explicitly. Hence, we have to use a different approach than in Appendix B.9.

FIGURE B.2: Equilibrium L_E with Free vs. Restricted OC in the Noiseless FE Model



To avoid confusion, denote $\Delta(L_E)$ in case of free OC by $\Delta^1(L_E)$ and in case of restricted OC by $\Delta^0(L_E)$. Note that, according to (2.25) and (2.34), $\Delta^1(L_E)$ and $\Delta^0(L_E)$ are both strictly decreasing for $L_E < L$. For any given $L_E < L$, the difference between the two yields

$$\Delta^0(L_E) - \Delta^1(L_E) = \frac{\rho {\sigma_s}^2}{a}\left[\frac{1}{2a} - \frac{\frac{L_E}{a} - \bar{v}}{L+M}\right]. \tag{B.82}$$

If $L_E^1 < L$, then (B.82) evaluated at equilibrium $L_E = L_E^1$ is obviously greater than when evaluated at $L_E = L$, that is

$$\Delta^0(L_E^1) - \Delta^1(L_E^1) > \frac{\rho {\sigma_s}^2}{a}\left[\frac{1}{2a} - \frac{\frac{L}{a} - \bar{v}}{L+M}\right]. \tag{B.83}$$

With condition (2.39), the r.h.s. of (B.83) is obviously positive. It follows that $\Delta^0(L_E^1) > \Delta^1(L_E^1)$. As we know that $\Delta^1(L_E^1) = 0$, this immediately gives $\Delta^0(L_E^1) > 0$. The fact that $\Delta^0(L_E)$ is strictly decreasing with $\lim_{L_E \to \infty} \Delta^0(L_E) = -\infty$ implies that $\Delta^0(L_E)$ equals zero for an $L_E > L_E^1$, so that $L_E^0 > L_E^1$. Figure B.2 illustrates the result.

B.13 Unions' Wage Setting Problem

Unions maximize workers' conditional expected utility. In what follows, we argue that this comes down to the maximization of (2.53).

From Appendix B.11, we already know that firms choose employment m according to $F'(m) = W$. If all other unions choose a wage rate W, which leads to firm employment m and firm profit θ, then if a union j chooses wage W^j, its firm's labor demand is given by m^j and its firm's profits are given by $\theta^j = \theta + \delta$, where $\delta = F(m^j) - W^j m^j - (F(m) - Wm)$. Again, as δ is non-random, arbitrage-freeness implies $P^j = P + \delta$.
The final wealth of a worker in union j is given by

$$\pi_M^j = \begin{cases} W^j - D + (\theta^j - P^j) I_{M|e}^{j^*} + (\theta - P) I_{M|e}^{j'}, & \text{if he is employed} \\ (\theta^j - P^j) I_{M|u}^{j^*} + (\theta - P) I_{M|u}^{j'}, & \text{if he is unemployed} \end{cases}, \tag{B.84}$$

where, if worker j is employed, $I_{M|e}^{j^*}$ denotes his position in the asset of "his" firm and $I_{M|e}^{j'}$ his position in other firms' assets; or if worker j is unemployed, $I_{M|u}^{j^*}$ denotes his position in the asset of "his" firm and $I_{M|u}^{j'}$ his position in other firms' assets. As with CARA-utility a worker's portfolio decision does not depend on whether he gets an additional fixed job income of $W^j - D$ or not, it is $I_{M|e}^{j^*} = I_{M|u}^{j^*}$ and $I_{M|e}^{j'} = I_{M|u}^{j'}$. Moreover, it is $(\theta - P) = (\theta^j - P^j)$, so regarding his final wealth it doesn't make any difference, if the worker trades the asset of "his" firm or the other firms' assets. Then, if we just let $I_M^j := I_{M|e}^{j^*} + I_{M|e}^{j'} = I_{M|u}^{j^*} + I_{M|u}^{j'}$ denotes a worker's total asset holdings, (B.84) simplifies to

$$\pi_M^j = (\theta - P) I_M^j + \begin{cases} W^j - D, & \text{if he is employed} \\ 0, & \text{if he is unemployed} \end{cases}. \tag{B.85}$$

As a firm j worker's probability of being employed is given by $(m^j / \hat{M})$, his conditional expected utility is given by

$$\begin{aligned} \mathbb{E}[U(\pi_M^j)|P] &= \mathbb{E}[\mathbb{E}[U(\pi_M^j)|s, \varepsilon, \nu]|P] \\ &= \mathbb{E}\left[\frac{m^j}{\hat{M}} U\left((\theta - P) I_M^j + W^j - D\right) + \left(1 - \frac{m^j}{\hat{M}}\right) U\left((\theta - P) I_M^j\right) |P\right] \\ &= \frac{m^j}{\hat{M}} \mathbb{E}[U((\theta - P) I_M^j + W^j - D)|P] + \left(1 - \frac{m^j}{\hat{M}}\right) \mathbb{E}[U((\theta - P) I_M^j)|P] \end{aligned}$$

$$
\begin{aligned}
&= \frac{m^j}{\hat{M}} \mathbb{E}\left[-\exp\left\{-\rho((\theta-P)I_M^j)\right\}\exp\left\{-\rho(W^j-D)\right\}|P\right] \\
&\qquad + \left(1-\frac{m^j}{\hat{M}}\right)\mathbb{E}\left[-\exp\left\{-\rho((\theta-P)I_M^j)\right\}|P\right] \\
&= -\frac{m^j}{\hat{M}}\exp\left\{-\rho(W^j-D)\right\}\mathbb{E}\left[\exp\left\{-\rho((\theta-P)I_M^j)\right\}|P\right] \\
&\qquad - \left(1-\frac{m^j}{\hat{M}}\right)\mathbb{E}\left[\exp\left\{-\rho((\theta-P)I_M^j)\right\}|P\right] \\
&= \mathbb{E}\left[\exp\left\{-\rho((\theta-P)I_M^j)\right\}|P\right]\left(-\frac{m^j}{\hat{M}}\exp\left\{-\rho(W^j-D)\right\}-\left(1-\frac{m^j}{\hat{M}}\right)\right) \\
&= \mathbb{E}\left[\exp\left\{-\rho((\theta-P)I_M^j)\right\}|P\right]\cdot\left(\frac{m^j}{\hat{M}}\left[1-\exp\left\{-\rho(W^j-D)\right\}\right]-1\right). \qquad \text{(B.86)}
\end{aligned}
$$

The first equality uses the LIE. Note that from knowing s, ε and ν, one also knows P. Hence, it is $\mathcal{I}_2 := \{P\} \subseteq \mathcal{I}_1 := \{s, \varepsilon, \nu\}$ and we can apply (A.14).
The worker's respective (conditional) certainty equivalent is then given by

$$
\begin{aligned}
\mathsf{CE}^j_{M|P} &= -\frac{1}{\rho}\ln\left(\mathbb{E}\left[\exp\left\{-\rho((\theta-P)I_M^j)\right\}|P\right]\right) \\
&\quad -\frac{1}{\rho}\ln\left(1-\frac{m^j}{\hat{M}}\left[1-\exp\left\{-\rho(W^j-D)\right\}\right]\right). \qquad \text{(B.87)}
\end{aligned}
$$

We have already argued that the worker's optimal I_M^j does not depend on his gains from the job. Hence, I_M^j corresponds to I_M given by (2.4) or (2.7), depending on whether there is free or restricted OC. The union's maximization problem then comes down to just maximizing the second term in (B.87) w.r.t. W^j, taking into account that the firm will optimally respond with the respective labor demand m^j (cf. Appendix B.14).
Hence, in (symmetric) equilibrium of the UE model, unions set the wage so as to maximize (2.53) and workers' optimal portfolio decisions are given by (2.4) or (2.7), respectively. For a given mass of entrepreneurs L_E, workers' gains from job are the same with free and restricted OC.

B.14 Unions' Optimal Wage Setting

Unions anticipate that firms respond to the wage W they set by choosing employment m according to $F'(m) = W$. With a Cobb-Douglas production function of the form $F(m) = Am^{1-b}$, where $A > 0$ and $0 < b < 1$, this gives a firm's labor demand as

$$
m = \left(\frac{A(1-b)}{W}\right)^{\frac{1}{b}}. \qquad \text{(B.88)}
$$

Substituting into the second factor in equation (B.86), which is just a strictly monotonically increasing transformation of (2.53), gives the unions objective function:

$$\frac{\left(\frac{A(1-b)}{W}\right)^{\frac{1}{b}}}{\hat{M}}\left[1-\exp\left\{-\rho(W-D)\right\}\right]-1. \tag{B.89}$$

Performing another strictly monotonically increasing transformation of (B.89) by using the transformation function $g(x)=(x+1)\hat{M}\left(A(1-b)\right)^{-\frac{1}{b}}$ simplifies the firm's objective function to

$$\left(\frac{1}{W}\right)^{\frac{1}{b}}\left[1-\exp\left\{-\rho(W-D)\right\}\right]=:\tilde{\text{GJ}}_M. \tag{B.90}$$

Taking the derivative w.r.t. W yields

$$\frac{d\tilde{\text{GJ}}_M}{dW}=-\frac{1}{b}W^{-\frac{1}{b}-1}\left(1-\exp\left\{-\rho(W-D)\right\}\right)-W^{-\frac{1}{b}}(-\rho)\exp\left\{-\rho(W-D)\right\}. \tag{B.91}$$

A union's FOC is given by

$$\frac{d\tilde{\text{GJ}}_M}{dW}=0; \tag{B.92}$$

$$-\frac{1}{b}W^{-\frac{1+b}{b}}+\frac{1}{b}W^{-\frac{1+b}{b}}\exp\left\{-\rho(W-D)\right\}+\rho W^{-\frac{1}{b}}\exp\left\{-\rho(W-D)\right\}=0;$$

$$\frac{1}{b}W^{-\frac{1+b}{b}}\exp\left\{-\rho(W-D)\right\}\left[-\exp\left\{\rho(W-D)\right\}+1+\rho bW\right]=0;$$

$$\underbrace{1+\rho bW-\exp\left\{\rho(W-D)\right\}}_{=:\psi(W)}=0. \tag{B.93}$$

Obviously, we cannot explicitly solve (B.93) for W. However, we can show that the $W(>0)$ that solves (B.93), call it $\tilde{W}$, is (i) unique, (ii) greater than D and (iii) maximizes (B.90).

Proof. To prove properties (i)-(ii), we show that the l.h.s. of (B.93), denoted by $\psi(W)$, has a single zero for $W>0$ and it takes on this zero at a wage $W>D$. In order to do so, note first that $\psi(W)$ is continuous and differentiable in W and has only one critical point, which is a maximum:

$$\frac{d\psi}{dW}=\rho b-\rho\exp\left\{\rho(W-D)\right\}=0;$$

$$W=\frac{\ln b}{\rho}+D<D, \tag{B.94}$$

with

$$\frac{d^2\psi}{dW^2}=-\rho^2\exp\left\{\rho(W-D)\right\}<0. \tag{B.95}$$

Moreover,

$$\psi(W=0)=1-\exp\left\{-\rho D\right\}>0, \tag{B.96}$$

$$\psi(W = D) = \rho b D > 0. \tag{B.97}$$

Taken together, (B.94)-(B.97) tell us that ψ takes on a positive value at $W = 0$, then increases until it reaches its maximum at a $W < D$. After that, it strictly decreases, but at $W = D$ is still positive. Finally, the fact that $\lim_{W\to\infty} \psi(W) = -\infty$ ensures that ψ must eventually hit zero at a $W > D$, which consequently is the unique positive solution to (B.93). As stated before, we denote this solution by $\tilde{W}$.

That $\tilde{W}$ indeed maximizes (B.90), i.e., that property (iii) holds, follows from the fact that the objective function's first derivative, given by (B.91), changes sign from positive to negative at $W = \tilde{W}$. As the objective function is continuous and differentiable for all $W > 0$ and has a single critical point, to show this, it is sufficient to show that the objective function's derivative is positive for some arbitrary W with $0 < W < \tilde{W}$ and negative for some arbitrary $W > \tilde{W}$. Rewriting (B.91) yields

$$\frac{\tilde{\mathrm{GJ}}_M}{dW} = W^{-\frac{1+b}{b}} \left(-\frac{1}{b} + \frac{1}{b} \exp\{-\rho(W - D)\} + \frac{\rho W}{\exp\{\rho(W - D)\}} \right). \tag{B.98}$$

Evaluated at $W = D (< \tilde{W})$, we get

$$\left.\frac{d\tilde{\mathrm{GJ}}_M}{dW}\right|_{W=D} = D^{-\frac{1+b}{b}} \left(-\frac{1}{b} + \frac{1}{b} + \rho D \right) = \rho D^{-\frac{1}{b}} > 0. \tag{B.99}$$

And for $W \to \infty (> \tilde{W})$:

$$\left.\frac{d\tilde{\mathrm{GJ}}_M}{dW}\right|_{W\to\infty} = 0^+ \cdot \left(-\frac{1}{b} + 0^+ + 0^+ \right) = 0^-, \tag{B.100}$$

where the notation 0^+ (0^-) indicates that the respective term converges to zero "from above" ("from below"). The convergence of the last term in parentheses follows from L'Hôpital's rule.

B.15 Alternative Wage-Setting Regimes

Besides the union wage setting model from the main text, we also consider three other potential sources of real wage rigidities: a "work or shirk" job environment as in Shapiro and Stiglitz (1984); maximization of the wage bill as in Dunlop (1944); and efficiency wages as in Solow (1979). If we denote the equilibrium wage that results from the respective wage-setting regime by $\tilde{W}$ and the corresponding equilibrium firm-level labor demand by $\tilde{m}$, then everything that follows equation (2.55) in chapter 2.4.2 goes through unchanged. In particular, our main propositions with regards to welfare in the UE model from chapter 2.5 hold irrespective of the specific source of real wage rigidity. In what follows, we use our notation from the main text.

Work or shirk. Assume that workers can "work" or "shirk" at their workplace. Firms cannot perfectly monitor their employees and detect a shirker with probability q. A

worker who "works" earns a wage W, but faces a disutility of work D. A worker who "shirks" without being detected also earns wage W, but does not face any disutility from work. A worker who is caught shirking gets nothing. A firm that employs m workers makes profit $\theta = \tilde{\theta} + F(m) - Wm$ if all of its workers work and profit $\theta = \tilde{\theta} + 0 - 0$ if all of its workers shirk. We have already shown that a firm chooses m by simply maximizing $F(m) - Wm$ (cf. Appendix B.11). Hence, given a firm's optimal employment decision m, $F(m) - Wm$ is certainly non-negative. Consequently, firms optimally offer a wage W that is just high enough to prevent workers from shirking and subsequently choose their optimal labor demand m.
The conditional expected utility of a worker who "works" is given by

$$\begin{aligned}\mathbb{E}[U(\pi_M)|P] &= \mathbb{E}[-\exp\{-\rho(I_M(\theta - P) + W - D)\}|P] \\ &= -\mathbb{E}[\exp\{-\rho(I_M(\theta - P))\}|P] \cdot \exp\{-\rho(W - D)\}. \end{aligned} \tag{B.101}$$

The conditional expected utility of a worker who "shirks" is given by

$$\begin{aligned}\mathbb{E}[U(\pi_M)|P] &= \mathbb{E}[\mathbb{E}[U(\pi_M)|s,\varepsilon,\nu]|P] \\ &= \mathbb{E}[-q\exp\{-\rho(I_M(\theta - P))\} - (1-q)\exp\{-\rho(I_M(\theta - P) + W)\}|P] \\ &= q\mathbb{E}[-\exp\{-\rho(I_M(\theta - P))\}|P] \\ &\quad - (1-q)\exp\{-\rho W\}\,\mathbb{E}[\exp\{-\rho(I_M(\theta - P))\}|P] \\ &= -\mathbb{E}[\exp\{-\rho(I_M(\theta - P))\}|P] \cdot [q(1 - \exp\{-\rho W\}) + \exp\{-\rho W\}]. \end{aligned} \tag{B.102}$$

As argued before, the optimal portfolio holdings I_M are independent of whether the worker decides to "work" or to "shirk" (cf. Appendix B.13). A worker refrains from shirking, if (B.101)$\geq$(B.102), that is if

$$\begin{aligned}\exp\{-\rho(W - D)\} &\leq q(1 - \exp\{-\rho W\}) + \exp\{-\rho W\}; \\ \exp\{-\rho W\}\exp\{\rho D\} &\leq q(1 - \exp\{-\rho W\}) + \exp\{-\rho W\}; \\ \exp\{\rho D\} &\leq q(\exp\{\rho W\} - 1) + 1; \\ \exp\{\rho W\} &\geq \frac{\exp\{\rho D\} - 1}{q} + 1; \\ W &\geq \frac{1}{\rho}\ln\left[1 + \frac{\exp\{\rho D\} - 1}{q}\right]. \end{aligned} \tag{B.103}$$

Denote the r.h.s. of (B.103) by $\tilde{W}$. The firm chooses the lowest possible wage that prevents shirking, that is, $W = \tilde{W}$. Subsequently, it chooses employment $\tilde{m} = (F')^{-1}(\tilde{W})$. There is equilibrium unemployment, if $\tilde{m} < \hat{M}$. A simple sufficient condition for this is $\tilde{m} \leq M/(L/a)$.
Maximization of the wage bill. Unions are organized as in the main text. They set the wage, anticipating the entrepreneurs' optimal response regarding labor demand. Instead of maximizing workers' utility, now assume that unions maximize the wage

bill Wm. Let the production function display constant elasticity of substitution (CES):

$$F(m) = A\left[b + (1-b)m^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{B.104}$$

where $A > 0$, $0 < b < 1$ and $0 < \eta < 1$. As before, a firm optimally sets $F'(m) = W$, which with (B.104) comes down to

$$\begin{aligned}
A \cdot \frac{\eta}{\eta-1} \cdot \left[b + (1-b)m^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}-1} \cdot (1-b) \cdot \frac{\eta-1}{\eta} \cdot m^{\frac{\eta-1}{\eta}-1} &= W;\\
A(1-b)m^{-\frac{1}{\eta}}\left[b + (1-b)m^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}} &= W;\\
A(1-b)\left[bm^{-\frac{\eta-1}{\eta}} + (1-b)\right]^{\frac{1}{\eta-1}} &= W;\\
bm^{-\frac{\eta-1}{\eta}} + (1-b) &= \left(\frac{W}{A(1-b)}\right)^{\eta-1};\\
m &= \left[\frac{1}{b}\left(\left(\frac{A(1-b)}{W}\right)^{1-\eta} - (1-b)\right)\right]^{\frac{\eta}{1-\eta}}.
\end{aligned} \tag{B.105}$$

Note that without further restrictions on η, the last step above is valid only if the term in brackets is non-negative, which then in turn implies $m \geq 0$. The CES production function and with that also the firm's profit function is concave, so that this solution meets the firm's SOC for a maximum. Unions anticipate firm behavior and maximize Wm, with m given by (B.105). A union's FOC is given by

$$\begin{aligned}
\frac{d(Wm)}{dW} &= 0;\\
m + W\frac{dm}{dW} &= 0;\\
m + W \cdot m \cdot m^{\frac{\eta-1}{\eta}} \cdot \frac{\eta}{1-\eta} \cdot \frac{1}{b} \cdot (\eta-1) \cdot W^{\eta-2} \cdot (A(1-b))^{1-\eta} &= 0;\\
m^{\frac{1-\eta}{\eta}} - \frac{\eta}{b}\left(A(1-b)\right)^{1-\eta} W^{\eta-1} &= 0;\\
\left(\frac{A(1-b)}{W}\right)^{1-\eta} - (1-b) - \eta\left(A(1-b)\right)^{1-\eta} W^{\eta-1} &= 0;\\
W^{\eta-1}(1-\eta)\left(A(1-b)\right)^{1-\eta} &= 1-b;\\
W &= \left[\frac{1-b}{(1-\eta)\left(A(1-b)\right)^{1-\eta}}\right]^{\frac{1}{\eta-1}};\\
W &= A\left[\frac{1-\eta}{(1-b)^{\eta}}\right]^{\frac{1}{1-\eta}} > 0.
\end{aligned} \tag{B.106}$$

Denote this solution by $\tilde{W}$. As it is the unique solution to the union's FOC and the objective function is continuous and differentiable on the relevant part of its domain, to prove that this solution indeed maximizes the wage bill, it is sufficient to show

that Wm evaluated at both, an arbitrary W with $0 < W < \tilde{W}$ and an arbitrary W with $W > \tilde{W}$ (but below the value above which the term in brackets in (B.105) gets negative), is lower than it is for $W = \tilde{W}$. Evaluated at $\tilde{W}$, m is given by $\tilde{m}$ according to (B.107) and with that the wage bill is $\tilde{W}\tilde{m} = A(1-\eta)\,(\eta/b)^{\eta/(1-\eta)}\,(> 0)$. For $W \to 0 (< \tilde{W})$, the wage bill Wm converges to zero, as W converges to zero "faster" than m to infinity. For $W = A(1-b)^{-\eta/(1-\eta)} (> \tilde{W})$, the wage bill equals zero, as m equals zero.
Substituting $\tilde{W}$ given by (B.106) into (B.105) gives equilibrium employment $\tilde{m}$:

$$\begin{aligned}
\tilde{m} &= \left[\frac{1}{b}\left(\left(\frac{(1-b)}{\left[\frac{1-\eta}{(1-b)^{\eta}}\right]^{\frac{1}{1-\eta}}}\right)^{1-\eta} - (1-b)\right)\right]^{\frac{\eta}{1-\eta}} \\
&= \left[\frac{1}{b}\left((1-\eta)^{-1}(1-b) - (1-b)\right)\right]^{\frac{\eta}{1-\eta}} \\
&= \left[\frac{1-b}{b}\left((1-\eta)^{-1} - 1\right)\right]^{\frac{\eta}{1-\eta}} \\
&= \left[\frac{1-b}{b}\frac{\eta}{1-\eta}\right]^{\frac{\eta}{1-\eta}} > 0. \qquad \text{(B.107)}
\end{aligned}$$

There is equilibrium unemployment, if $\tilde{m} < \hat{M}$.
Efficiency wages. Workers choose the level of effort E they deploy on their job depending on how much wage they are paid: $E = E(W)$, with $dE/dW > 0$. Firm output is given by $F(E(W)m)$. The higher the wage, the higher the workers' motivation and effort level and, hence, the higher the output. Firms choose the wage-employment pair (W, m) that maximizes profit $\Pi = F(E(W)m) - Wm$. The firm's FOCs are given by

$$\frac{\partial \Pi}{\partial W} = F'(E(W)m)m\frac{\partial E}{\partial W} - m = 0, \qquad \text{(B.108)}$$

$$\frac{\partial \Pi}{\partial m} = F'(E(W)m)E(W) - W = 0. \qquad \text{(B.109)}$$

Dividing equation (B.108) by m and by equation (B.109) gives

$$\frac{\partial E}{\partial W} = \frac{E(W)}{W}, \qquad \text{(B.110)}$$

which, if a unique solution W exists, pins down the wage. Substituting into (B.109) yields employment.

As an example, let the production function be given by $F(E(W)m) = [E(W)m]^{1-b}$, where $0 < b < 1$, and assume an effort function $E(W) = \ln W$. The firm's objective function is then given by $\Pi = [(\ln W)m]^{1-b} - Wm$. From (B.110), we get the optimal wage as $\tilde{W} = \exp\{1\}$. Substituting into (B.109) gives optimal employment $\tilde{m} = [(1-b)/\exp\{1\}]^{1/b}$. To make sure that this pair indeed maximizes profit, we check that the objective function's Hessian is negative definite at $(\tilde{W}, \tilde{m})$:

$$\left.\frac{\partial^2 \Pi}{\partial W^2}\right|_{W=\tilde{W}, m=\tilde{m}} = -(1-b)(1+b)\exp\{-2\}\left(\frac{1-b}{\exp\{1\}}\right)^{\frac{1-b}{b}} < 0, \tag{B.111}$$

$$\left.\frac{\partial^2 \Pi}{\partial m^2}\right|_{W=\tilde{W}, m=\tilde{m}} = -b(1-b)\left(\frac{1-b}{\exp\{1\}}\right)^{-\frac{1+b}{b}} < 0 \tag{B.112}$$

and

$$\begin{aligned}\left.\frac{\partial^2 \Pi}{\partial W^2}\right|_{W=\tilde{W}, m=\tilde{m}} \cdot \left.\frac{\partial^2 \Pi}{\partial m^2}\right|_{W=\tilde{W}, m=\tilde{m}} - \left(\left.\frac{\partial^2 \Pi}{\partial m \partial W}\right|_{W=\tilde{W}, m=\tilde{m}}\right)^2 &= b(1+b) - (-b)^2 \\ &= b > 0. \end{aligned} \tag{B.113}$$

Equilibrium output equals $F(E(\tilde{W})\tilde{m}) = F(\tilde{m})$. There is equilibrium unemployment, if $\tilde{m} < \hat{M}$.

B.16 Expected NT Utility with $\sigma_v{}^2 = 0$

Let $\sigma_v{}^2 = 0$. Assume noise traders to be symmetric, that is, an aggregate noise trader demand $\bar{v}$ implies an individual noise trader demand $I_N = \bar{v}/N$. Exogenous behavior combined with CARA-utility for an ex-post evaluation of well-being gives a noise trader's expected utility as

$$\mathbb{E}\left[U(\pi_N)\right] = -\mathbb{E}\left[\exp\left\{-\rho \cdot \frac{\bar{v}}{N}(\theta - P)\right\}\right]. \tag{B.114}$$

As $(\theta - P)$ is normal, we can use (A.4.1) to get

$$\mathbb{E}\left[U(\pi_N)\right] = -\exp\left\{-\rho\frac{\bar{v}}{N}\mathbb{E}(\theta - P) + \frac{1}{2}\rho^2\left(\frac{\bar{v}}{N}\right)^2 \mathbb{V}(\theta - P)\right\}. \tag{B.115}$$

In case of free OC, P is given by (2.23), from which follows that, for $L_E < L$,

$$\begin{aligned}\mathbb{E}\left[U(\pi_N)\right] &= -\exp\left\{-\rho\frac{\bar{v}}{N}\left(\bar{s} - \bar{s} + \rho\sigma_\varepsilon{}^2\frac{\frac{L_E}{a} - \bar{v}}{L+M}\right) + \frac{1}{2}\rho^2\left(\frac{\bar{v}}{N}\right)^2\sigma_\varepsilon{}^2\right\} \\ &= -\exp\left\{-\rho^2\frac{\bar{v}}{N}\sigma_\varepsilon{}^2\frac{\frac{L_E}{a} - \bar{v}}{L+M} + \frac{1}{2}\rho^2\left(\frac{\bar{v}}{N}\right)^2\sigma_\varepsilon{}^2\right\};\end{aligned} \tag{B.116}$$

$$\mathsf{CE}_N = \rho\sigma_\varepsilon{}^2\frac{\bar{v}}{N}\left[\frac{\frac{L_E}{a} - \bar{v}}{L+M} - \frac{1}{2}\frac{\bar{v}}{N}\right] \tag{B.117}$$

and, for $L_E = L$,

$$\mathbb{E}\left[U(\pi_N)\right] = -\exp\left\{-\rho\frac{\bar{v}}{N}\left(\bar{s} - \bar{s} + \rho(\sigma_\varepsilon{}^2 + \sigma_s{}^2)\frac{\frac{L}{a} - \bar{v}}{L+M}\right) + \frac{1}{2}\rho^2\left(\frac{\bar{v}}{N}\right)^2(\sigma_\varepsilon{}^2 + \sigma_s{}^2)\right\}$$

$$= -\exp\left\{-\rho^2\frac{\bar{v}}{N}({\sigma_\varepsilon}^2+{\sigma_s}^2)\frac{\frac{L}{a}-\bar{v}}{L+M}+\frac{1}{2}\rho^2\left(\frac{\bar{v}}{N}\right)^2({\sigma_\varepsilon}^2+{\sigma_s}^2)\right\}; \quad (B.118)$$

$$\mathsf{CE}_N = \rho({\sigma_\varepsilon}^2+{\sigma_s}^2)\frac{\bar{v}}{N}\left[\frac{\frac{L}{a}-\bar{v}}{L+M}-\frac{1}{2}\frac{\bar{v}}{N}\right]. \quad (B.119)$$

Restricted occupational choice. In case of restricted OC, P is given by (2.32), from which follows that

$$\mathbb{E}\left[U(\pi_N)\right] = -\exp\left\{-\rho\frac{\bar{v}}{N}\rho({\sigma_\varepsilon}^2+{\sigma_s}^2)\frac{\frac{L_E}{a}-\bar{v}}{L+M}+\frac{1}{2}\rho^2\left(\frac{\bar{v}}{N}\right)^2({\sigma_\varepsilon}^2+{\sigma_s}^2)\right\}; \quad (B.120)$$

$$\mathrm{CE}_N = \rho({\sigma_\varepsilon}^2+{\sigma_s}^2)\frac{\bar{v}}{N}\left[\frac{\frac{L_E}{a}-\bar{v}}{L+M}-\frac{1}{2}\frac{\bar{v}}{N}\right]. \quad (B.121)$$

B.17 Social Welfare

Let ${\sigma_v}^2 = 0$ in the basic version of the model. Consider first the case of free OC. For $L_E < L$, substituting equations (2.59)-(2.62) into (2.58) yields

$$\begin{aligned} S &= L_E(\mathsf{GP}_E+z^2)+(L-L_E+M)z^2+N\cdot\mathsf{CE}_N \\ &= L_E\mathsf{GP}_E+(L+M)\frac{\rho{\sigma_\varepsilon}^2}{2}\frac{\left(\frac{L_E}{a}-\bar{v}\right)^2}{(L+M)^2}+N\rho{\sigma_\varepsilon}^2\frac{\bar{v}}{N}\left(\frac{\frac{L_E}{a}-\bar{v}}{L+M}-\frac{1}{2}\frac{\bar{v}}{N}\right) \\ &= L_E\mathsf{GP}_E+\rho{\sigma_\varepsilon}^2\left[\frac{1}{2(L+M)}\left(\left(\frac{L_E}{a}\right)^2-2\frac{L_E}{a}\bar{v}+\bar{v}^2\right)+\bar{v}\frac{\frac{L_E}{a}}{L+M}-\frac{\bar{v}^2}{L+M}-\frac{1}{2}\frac{\bar{v}^2}{N}\right] \\ &= L_E\mathsf{GP}_E+\frac{\rho{\sigma_\varepsilon}^2}{2}\left[\frac{\left(\frac{L_E}{a}\right)^2}{L+M}-\bar{v}^2\left(\frac{1}{L+M}+\frac{1}{N}\right)\right], \end{aligned} \quad (B.122)$$

with GP_E given by (2.25). For $L_E = L$, we get

$$\begin{aligned} S &= L(\mathsf{GP}_E+z^2)+M\cdot z^2+N\cdot\mathsf{CE}_N \\ &= L\mathsf{GP}_E+(L+M)\frac{\rho({\sigma_s}^2+{\sigma_\varepsilon}^2)}{2}\left(\frac{\frac{L}{a}-\bar{v}}{L+M}\right)^2+N\rho({\sigma_\varepsilon}^2+{\sigma_s}^2)\frac{\bar{v}}{N}\left[\frac{\frac{L}{a}-\bar{v}}{L+M}-\frac{1}{2}\frac{\bar{v}}{N}\right] \\ &= L\mathsf{GP}_E+\frac{\rho({\sigma_\varepsilon}^2+{\sigma_s}^2)}{2}\left[\frac{\left(\frac{L}{a}\right)^2}{L+M}-\bar{v}^2\left(\frac{1}{L+M}+\frac{1}{N}\right)\right]. \end{aligned} \quad (B.123)$$

Restricted occupational choice. In case of restricted OC, substituting equations (2.64)-(2.66) into (2.67) yields

$$\begin{aligned} S &= L_E(\mathsf{GP}_E+z^2)+(L-L_E+M)z^2+N\cdot\mathsf{CE}_N \\ &= L_E\mathsf{GP}_E+(L+M)\frac{\rho({\sigma_s}^2+{\sigma_\varepsilon}^2)}{2}\left(\frac{\frac{L_E}{a}-\bar{v}}{L+M}\right)^2+N\rho({\sigma_\varepsilon}^2+{\sigma_s}^2)\frac{\bar{v}}{N}\left[\frac{\frac{L_E}{a}-\bar{v}}{L+M}-\frac{1}{2}\frac{\bar{v}}{N}\right] \end{aligned}$$

$$= L_E \mathsf{GP}_E + \frac{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}{2} \left[\frac{\left(\frac{L_E}{a}\right)^2}{L+M} - \bar{v}^2 \left(\frac{1}{L+M} + \frac{1}{N} \right) \right], \tag{B.124}$$

with GP_E given by (2.34).

B.18 Proof that an Entrepreneur's CE is Decreasing in L_E

Let ${\sigma_v}^2 = 0$. Consider first the basic version of the model with free OC. Further, assume that entrepreneurs are net sellers, i.e., $I_E < 1/a$. Calculating the derivative of CE_E with respect to L_E gives

$$\begin{aligned} \frac{d\mathsf{CE}_E}{dL_E} &= -\frac{\rho{\sigma_\varepsilon}^2}{a^2(L+M)} + \rho{\sigma_\varepsilon}^2 \left(\frac{\frac{L_E}{a} - \bar{v}}{L+M} \right) \frac{1}{a(L+M)} \\ &= -\frac{\rho{\sigma_\varepsilon}^2}{a(L+M)} \left(\frac{1}{a} - \frac{\frac{L_E}{a} - \bar{v}}{L+M} \right) \\ &= -\frac{\rho{\sigma_\varepsilon}^2}{a(L+M)} \left(\frac{1}{a} - I_E \right) < 0. \end{aligned} \tag{B.125}$$

The same result applies in the UE model. In the FE economy, $\bar{s}$ negatively depends on L_E and the derivative of CE_E with respect to L_E is given by (B.125) plus the additional term $(1/a) \cdot (d\bar{s}/dL_E) < 0$. The fact that entrepreneurs' CEs are decreasing also in case of restricted OC can be shown analogously.

B.19 Proof of Proposition 2.5.1

Let ${\sigma_v}^2 = 0$. Consider first the case of free OC with $L_E < L$.

Basic model. In the basic version of the model, taking the derivative of social welfare S given by (2.63) with respect to L_E yields

$$\begin{aligned} \frac{dS}{dL_E} &= L_E \frac{d\mathsf{GP}_E}{dL_E} + \mathsf{GP}_E + \frac{\rho{\sigma_\varepsilon}^2}{L+M} \frac{L_E}{a^2} \\ &= L_E \left[-\frac{\rho{\sigma_\varepsilon}^2}{a^2(L+M)} \right] + \mathsf{GP}_E + \frac{\rho{\sigma_\varepsilon}^2}{L+M} \frac{L_E}{a^2} \\ &= \mathsf{GP}_E, \end{aligned} \tag{B.126}$$

where GP_E is given by (2.25). The FOC for a social welfare optimum requires $\mathsf{GP}_E = 0$, which corresponds to the equation that determines an interior equilibrium (cf. Chapter 2.3). As the second derivative gives

$$\frac{d^2S}{d{L_E}^2} = \frac{d\mathsf{GP}_E}{dL_E} = -\frac{\rho{\sigma_\varepsilon}^2}{a^2(L+M)} < 0, \tag{B.127}$$

the equilibrium $L_E < L$ given by (2.29) maximizes social welfare S on $(0, L)$.

Full employment model. In the FE model, social welfare S includes the workers' gains from the job, which is just their wage $\tilde{W} = F'(\hat{M})$. Hence, S is given by

$$S = L_E \mathsf{GP}_E + \frac{\rho \sigma_\varepsilon{}^2}{2} \left[\frac{\left(\frac{L_E}{a}\right)^2}{L+M} - \bar{v}^2 \left(\frac{1}{L+M} + \frac{1}{N} \right) \right] + M\tilde{W}, \tag{B.128}$$

with GP_E given by (2.25) and $\bar{s}$ given by (2.49). Taking the derivative with respect to L_E yields

$$\begin{aligned}
\frac{dS}{dL_E} &= L_E \frac{d\mathsf{GP}_E}{dL_E} + \mathsf{GP}_E + \frac{\rho \sigma_\varepsilon{}^2}{L+M} \frac{L_E}{a^2} + M \cdot F''(\hat{M}) \frac{d\hat{M}}{dL_E} \\
&= \frac{L_E}{a} \left[\frac{d\bar{s}}{dL_E} - \frac{\rho \sigma_\varepsilon{}^2}{a(L+M)} \right] + \mathsf{GP}_E + \frac{\rho \sigma_\varepsilon{}^2}{L+M} \frac{L_E}{a^2} + M \cdot F''(\hat{M})(-1) \frac{\hat{M}}{L_E} \\
&= \frac{L_E}{a} F''(\hat{M}) \frac{\hat{M}^2}{L_E} + \mathsf{GP}_E - F''(\hat{M}) \frac{1}{a} \frac{aM}{L_E} \hat{M} \\
&= F''(\hat{M}) \frac{\hat{M}^2}{a} + \mathsf{GP}_E - F''(\hat{M}) \frac{\hat{M}^2}{a} \\
&= \mathsf{GP}_E.
\end{aligned} \tag{B.129}$$

By the same reasoning as above, this implies that the equilibrium $L_E < L$ maximizes social welfare S on $(0, L)$.

Unemployment model. In the UE model, social welfare S includes the workers' expected gains from the job GJ_M, given by (2.53). Hence:

$$S = L_E \mathsf{GP}_E + \frac{\rho \sigma_\varepsilon{}^2}{2} \left[\frac{\left(\frac{L_E}{a}\right)^2}{L+M} - \bar{v}^2 \left(\frac{1}{L+M} + \frac{1}{N} \right) \right] + M \cdot \mathsf{GJ}_M, \tag{B.130}$$

with GP_E given by (2.25) and $\bar{s}$ given by (2.55). Taking the derivative with respect to L_E yields

$$\begin{aligned}
\frac{dS}{dL_E} &= L_E \frac{d\mathsf{GP}_E}{dL_E} + \mathsf{GP}_E + \frac{\rho \sigma_\varepsilon{}^2}{L+M} \frac{L_E}{a^2} + M \frac{d\mathsf{GJ}_M}{dL_E} \\
&= \mathsf{GP}_E + M \frac{d\mathsf{GJ}_M}{dL_E}.
\end{aligned} \tag{B.131}$$

The FOC requires $dS/dL_E = 0$. From (2.56) we know that $d\mathsf{GJ}_M/dL_E > 0$, so any L_E that solves the FOC has to be greater than the L_E that solves the equilibrium condition $\mathsf{GP}_E = 0$. The second derivative of S with respect to L_E gives

$$\frac{d^2 S}{dL_E{}^2} = \frac{d\mathsf{GP}_E}{dL_E} + M \frac{d^2 \mathsf{GJ}}{dL_E^2}, \tag{B.132}$$

where we know that the first part is negative. If we use the approximation for GJ_M given in Appendix B.23, then $d\mathsf{GJ}_M/dL_E$ is independent of L_E, the FOC has a unique solution and the second order condition for a maximum is met. Hence, if the L_E that solves the FOC is smaller than L, then it maximizes S on $(0, L)$. If the L_E that solves the FOC is greater than L, then $L_E \to L$ maximizes S on $(0, L)$.

If we don't use an approximation for GJ_M but instead continue with GJ_M given by (2.53), the analysis is a little more complicated. However, we can still show the following: if there is no $L_E < L$ that solves the FOC, then $L_E \to L$ maximizes S on $(0, L)$; if there is a unique $L_E < L$ that solves the FOC, then it maximizes S on $(0, L)$; and if there are multiple $L_E < L$ that solve the FOC, either one of these solutions or $L_E \to L$ maximizes S on $(0, L)$. In any case, the equilibrium L_E falls short of the L_E that maximizes S on $(0, L)$.

Proof: Let $\tilde{m} < M/(L/a)$, so that job creation by entrepreneurs is operative over the whole range of L_E. With regards to the second term on the r.h.s. of (B.132), from (2.56), we get

$$M \cdot \frac{d\mathsf{GJ}_M}{dL_E} = \frac{M}{\rho} \frac{\frac{\tilde{m}}{aM}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]}{1 - L_E \frac{\tilde{m}}{aM}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]} \tag{B.133}$$

and hence

$$\begin{aligned} M\frac{d^2\mathsf{GJ}_M}{dL_E{}^2} &= \frac{M}{\rho}(-1)\frac{\frac{\tilde{m}}{aM}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]}{\left(1 - L_E\frac{\tilde{m}}{aM}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]\right)^2} \cdot \\ &\quad \cdot (-1)\frac{\tilde{m}}{aM}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right] \\ &= \frac{M}{\rho}\left(\frac{\frac{\tilde{m}}{aM}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]}{1 - L_E\frac{\tilde{m}}{aM}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]}\right)^2 > 0. \end{aligned} \tag{B.134}$$

It follows that the sign of $d^2S/dL_E{}^2$ is not unambiguous and may depend on L_E. Hence, dS/dL_E is not necessarily monotonic, which implies the possibility of multiple solutions to the FOC. While the sign of $d^2S/dL_E{}^2$ may change with L_E, at least we know that $d^2S/dL_E{}^2$ is strictly increasing in L_E. This follows directly from the fact that $d\mathsf{GP}_E/dL_E$ is independent of L_E and that $d^2\mathsf{GJ}_M/dL_E{}^2$, given by (B.134), is strictly increasing in L_E. As from the existence of an equilibrium with $0 < L_E < L$ follows that $\mathsf{GP}_E|_{L_E=0} > 0$, and we know that $d\mathsf{GJ}_M/dL_E > 0$, it is $(dS/dL_E)|_{L_E=0} > 0$. If $(d^2S/dL_E{}^2)|_{L_E=0} > 0$, then an interior solution to the FOC does not exist (as $d^2S/dL_E{}^2$ is strictly increasing in L_E), and $L_E \to L$ maximizes S on $(0, L)$. If, instead, $(d^2S/dL_E{}^2)|_{L_E=0} < 0$, there are three possibilities: (i) an interior solution to the FOC still does not exist, as dS/dL_E stays above zero for all $L_E < L$. Then, again, $L_E \to L$ maximizes S on $(0, L)$; (ii) a unique interior solution to the FOC exists, as $dS/dL_E = 0$ for a single $L_E < L$. As dS/dL_E has to change sign from positive to negative in this case, this solution maximizes S on $(0, L)$; (iii) two interior solutions to the FOC exist, as dS/dL_E first crosses zero from above and then again from below (both for $L_E < L$). In this case, the "first" solution constitutes a local maximum, the second one a local minimum. Whether the value that maximizes S on $(0, L)$ is then given by the "first" solution to the FOC or by $L_E \to L$ is not obvious. Note, however, that both values exceed the equilibrium L_E. As $d^2S/dL_E{}^2$ is strictly increasing in L_E, no other cases besides the ones explained above are possible.[4]

Restricted Occupational Choice. Consider now the case of restricted OC and let $L_E \leq L$. Social welfare S in the **baseline model** is given by (2.68), with GP_E given by (2.34). Proceeding analogously as in the unrestricted case above yields $dS/dL_E = \mathsf{GP}_E$ and $d^2S/dL_E{}^2 < 0$. As S is continuous in $L_E \leq L$, it follows that the equilibrium $L_E(\leq L)$ maximizes S on $(0, L]$.

Social welfare S in the **FE economy** is given by (2.68), with GP_E given by (2.34) and $\bar{s}$ given by (2.49). Additionally, it includes workers aggregate wage gains $M \cdot \tilde{W}$. Proceeding analogously as in the unrestricted case shows that the equilibrium $L_E(\leq L)$ maximizes S on $(0, L]$.

Social welfare S in the **UE model** is given by (2.68), with GP_E given by (2.34) and $\bar{s}$ given by (2.55). Additionally, it includes workers aggregate expected gains from the job $M \cdot \mathsf{GJ}_M$. Proceeding analogously as in the unrestricted case shows that if there is an interior equilibrium $L_E < L$, then it falls short of the L_E that maximizes S on $(0, L]$. If there is a corner equilibrium with $L_E = L$, then $L_E = L$ also maximizes S on $(0, L]$.

B.20 Welfare Effects of Rational "Noise Traders"

Let $\sigma_v{}^2 = 0$. Rewriting (2.62) gives a noise trader's CE in the presence of dealers as

$$\mathsf{CE}_N = \frac{\rho\sigma_\varepsilon{}^2}{2}\left[\left(\frac{\frac{L_E}{a} - \bar{v}}{L+M}\right)^2 - \left(\frac{\bar{v}}{N} - \frac{\frac{L_E}{a} - \bar{v}}{L+M}\right)^2\right]. \tag{B.135}$$

Comparing this to a passive investor's CE immediately shows $\mathsf{CE}_M > \mathsf{CE}_N$. Hence, each noise trader would be better off as a rational passive investor. Rewriting (2.66) and proceeding analogous, the same can be shown for the case of restricted OC.

Now, assume that all noise traders act as passive investors, i.e.,

$$(I_N =)\frac{\bar{v}}{N} = \frac{\frac{L_E}{a} - \bar{v}}{L+M}(= I_M), \tag{B.136}$$

from which follows that

$$\bar{v} = \frac{N}{L+M+N}\frac{L_E}{a}. \tag{B.137}$$

Consider first the basic version of the model in the presence of dealers. Denote social welfare in an economy with rational "noise traders", i.e., with $\bar{v}$ set according to (B.137), as S'. From (2.63), for given L_E, the social welfare difference in the economy

[4] At least not besides one last (special) case, where S has a single saddle point at an $L_E < L$ and no other extrema. Then, again, $L_E \to L$ maximizes S on $(0, L)$.

without vs. with noise traders is given by

$$\begin{aligned}
S^{'} - S &= \rho\sigma_\varepsilon{}^2 \frac{\frac{L_E}{a}}{L+M}\left(\frac{L_E}{a}\frac{N}{L+M+N} - \bar{v}\right) + \\
&+ \frac{\rho\sigma_\varepsilon{}^2}{2}\left(\frac{1}{L+M} + \frac{1}{N}\right)\left[-\left(\frac{L_E}{a}\right)^2\left(\frac{N}{L+M+N}\right)^2 + \bar{v}^2\right] \\
&= \frac{\rho\sigma_\varepsilon{}^2}{2}\frac{1}{L+M}\left[2\left(\frac{L_E}{a}\right)^2\frac{N}{L+M+N} - 2\frac{L_E}{a}\bar{v} + \right. \\
&+ \left.\left(1 + \frac{L+M}{N}\right)\left[\bar{v}^2 - \left(\frac{L_E}{a}\right)^2\left(\frac{N}{L+M+N}\right)^2\right]\right] \\
&= \frac{\rho\sigma_\varepsilon{}^2}{2}\frac{N}{(L+M)(L+M+N)}\left[2\left(\frac{L_E}{a}\right)^2 - 2\frac{L_E}{a}\bar{v}\frac{L+M+N}{N} + \right. \\
&+ \left.\left(\frac{L+M+N}{N}\right)^2\bar{v}^2 - \left(\frac{L_E}{a}\right)^2\right] \\
&= \frac{\rho\sigma_\varepsilon{}^2}{2}\frac{N}{(L+M)(L+M+N)}\left(\frac{L_E}{a} - \frac{L+M+N}{N}\bar{v}\right)^2 > 0. \qquad \text{(B.138)}
\end{aligned}$$

According to Proposition 2.5.1, the equilibrium mass of entrepreneurs L_E^1 maximizes social welfare S. As Proposition 2.5.1 holds true for all values of $\bar{v}$ and, hence, also for $\bar{v}$ according to (B.137), equilibrium $L_E^{1'}$ in the economy with rational "noise traders" maximizes $S^{'}$. It follows that $S^{'}(L_E^{1'}) \geq S^{'}(L_E^1) > S(L_E^1)$, that is, equilibrium social welfare is higher when noise traders act rational. This also applies in case of restricted OC and in the FE economy (proofs analogous). In the UE model, the same holds true when comparing the respective social welfare optima (it also applies with regards to the respective equilibrium values under the additional assumption of $L_E^{1'} \geq L_E^1$).

B.21 Workers' Wages and Firm Profitability - FE Model

Let $\sigma_v{}^2 = 0$. A single worker's wage gains from an increase in L_E are given by (2.52). A single firm's loss in profitability from an increase in L_E is given by (2.51). As there is a total of M workers and L_E/a firms, the net effect is given by

$$\begin{aligned}
\frac{L_E}{a}\frac{d\bar{s}}{dL_E} + M\frac{d\tilde{W}}{dL_E} &= \frac{L_E}{a}F^{''}(\hat{M})\frac{\hat{M}^2}{L_E} + M(-1)F^{''}(\hat{M})\frac{\hat{M}}{L_E} \\
&= \frac{L_E}{a}F^{''}(\hat{M})\frac{\hat{M}^2}{L_E} - \hat{M}\frac{L_E}{a}F^{''}(\hat{M})\frac{\hat{M}}{L_E} \\
&= 0. \qquad \text{(B.139)}
\end{aligned}$$

B.22 Social Welfare in the UE vs. the FE Model

Let ${\sigma_v}^2 \geq 0$. Compare social welfare in the FE model vs. the UE model. To ensure comparability, let $D = 0$ in the UE model, as this is what we also assumed in the FE model. For any given mass of entrepreneurs L_E, social welfare in the two model variants differs only in two aspects. First, $\bar{s}$ is given by (2.49) in case of FE and by (2.55) in case of UE. Second, workers equilibrium gains on the job are given by $M \cdot \tilde{W}$ in the FE model, while they are given by $M \cdot \mathsf{GJ}_M$ in the UE model. As CE_N, GI_D, GT_E, GT_M and $\mathbb{V}(P)$ are independent of $\bar{s}$, they cancel out with regards to the difference in social welfare for given L_E.

The following proof that social welfare is higher in the FE economy holds true for both free and restricted OC. Indicating variables by a superscript "*FE*" or "*UE*", depending on whether they relate to the FE model or the UE model, the difference in social welfare for given L_E comes down to

$$\begin{aligned}
S^{FE} - S^{UE} &= L_E \cdot \mathsf{GP}_E^{FE} + M \cdot \tilde{W}^{FE} - L_E \cdot \mathsf{GP}_E^{UE} - M \cdot \mathsf{GJ}_M \\
&= \frac{L_E}{a}\left(\mathbb{E}(P)^{FE} - \mathbb{E}(P)^{UE}\right) + M\left(\tilde{W}^{FE} - \mathsf{GJ}_M\right) \\
&= \frac{L_E}{a}\left(\bar{s}^{FE} - \bar{s}^{UE}\right) + M\left(\tilde{W}^{FE} - \mathsf{GJ}_M\right) \\
&= \frac{L_E}{a}\left(\hat{s} + F(\hat{M}) - \tilde{W}^{FE}\hat{M} - \hat{s} - F(\tilde{m}) + \tilde{W}^{UE}\tilde{m}\right) + \hat{M}\frac{L_E}{a}\left(\tilde{W}^{FE} - \mathsf{GJ}_M\right) \\
&= \frac{L_E}{a}\left(F(\hat{M}) - F(\tilde{m}) + \tilde{W}^{UE}\tilde{m} - \hat{M} \cdot \mathsf{GJ}_M\right) \\
&= \frac{L_E}{a}\left(F(\hat{M}) - F(\tilde{m}) + \tilde{W}^{UE}\tilde{m} + \frac{\hat{M}}{\rho}\ln\left(1 - \frac{\tilde{m}}{\hat{M}}\left[1 - \exp\left\{-\rho\tilde{W}^{UE}\right\}\right]\right)\right).
\end{aligned} \tag{B.140}$$

As equilibrium unemployment in the UE model requires $\tilde{m} < \hat{M}$, it follows that $F(\hat{M}) - F(\tilde{m}) > 0$. Hence, for (B.140) to be positive, it is sufficient to show that

$$\begin{aligned}
\tilde{W}^{UE}\tilde{m} + \frac{\hat{M}}{\rho}\ln\left(1 - \frac{\tilde{m}}{\hat{M}}\left[1 - \exp\left\{-\rho\tilde{W}^{UE}\right\}\right]\right) &> 0; \\
\exp\left\{-\rho\tilde{W}^{UE}\frac{\tilde{m}}{\hat{M}}\right\} &< 1 - \frac{\tilde{m}}{\hat{M}}\left[1 - \exp\left\{-\rho\tilde{W}^{UE}\right\}\right]; \\
\underbrace{\frac{\tilde{m}}{\hat{M}}\left[1 - \exp\left\{-\rho\tilde{W}^{UE}\right\}\right]}_{=:\phi_l(\tilde{m})} &< \underbrace{1 - \exp\left\{-\rho\tilde{W}^{UE}\frac{\tilde{m}}{\hat{M}}\right\}}_{=:\phi_r(\tilde{m})}.
\end{aligned} \tag{B.141}$$

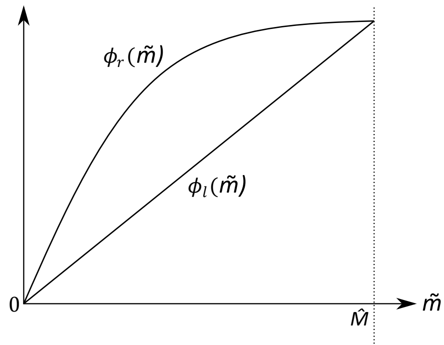
Consider the l.h.s. and the r.h.s. as functions of $\tilde{m}$, denoted by $\phi_l(\tilde{m})$ and $\phi_r(\tilde{m})$, respectively. We know that $\tilde{m} \in (0, \hat{M})$. For $\tilde{m} \to 0$ and $\tilde{m} \to \hat{M}$, equation (B.141) gives $0 = 0$ and $1 - \exp\left\{-\rho\tilde{W}^{UE}\right\} = 1 - \exp\left\{-\rho\tilde{W}^{UE}\right\}$, respectively, so the left hand side equals the right hand side. For all $\tilde{m}$ in between, we immediately see that

$\phi_l(\tilde{m})$ is linearly increasing in $\tilde{m}$. For $\phi_r(\tilde{m})$, we see that

$$\begin{aligned}\frac{\partial\phi_r(\tilde{m})}{\partial\tilde{m}} &= -\exp\left\{-\rho\tilde{W}^{UE}\frac{\tilde{m}}{\hat{M}}\right\}(-1)\rho\tilde{W}^{UE}\frac{1}{\hat{M}}\\ &= \rho\frac{\tilde{W}^{UE}}{\hat{M}}\exp\left\{-\rho\tilde{W}^{UE}\frac{\tilde{m}}{\hat{M}}\right\} > 0,\\ \frac{\partial^2\phi_r(\tilde{m})}{\partial\tilde{m}^2} &= -\rho^2\left(\frac{\tilde{W}^{UE}}{\hat{M}}\right)^2\exp\left\{-\rho\tilde{W}^{UE}\frac{\tilde{m}}{\hat{M}}\right\} < 0. \end{aligned} \tag{B.142}$$

Hence, $\phi_r(\tilde{m})$ is increasing and concave in $\tilde{m}$. Figure B.3 sums up. It shows that (B.141) holds for all $\tilde{m} \in (0, \hat{M})$ and, hence, $S^{FE} > S^{UE}$ for any given L_E.

FIGURE B.3: Welfare in the UE vs. the FE Model



As the equilibrium L_E^{FE} maximizes S^{FE} (cf. Proposition 2.5.1), it follows that equilibrium social welfare in the FE model is higher than both equilibrium and maximum social welfare in the UE model.

B.23 The (Approximately) Optimal L_E in the UE Model

Let $\sigma_v{}^2 = 0$. As is well known, $\ln(1+x) \approx x$ for small x. Applying this to workers' gains on the job GJ_M, given by (2.53), yields:[5]

$$\mathsf{GJ}_M \approx \frac{1}{\rho}\frac{m}{\hat{M}}\left[1-\exp\left\{-\rho(W-D)\right\}\right]. \tag{B.143}$$

Let again indicate superscripts "1" and "0" the case of free OC or the case of restricted OC, respectively. Consider first the case of free OC and let $L_E < L$. From (B.131), the FOC for a local optimum is given by

$$\mathsf{GP}_E^1 + M\frac{d\mathsf{GJ}_M}{dL_E} = 0. \tag{B.144}$$

[5]As both $m/\hat{M}$ and $1-\exp\left\{-\rho(W-D)\right\}$ are positive and smaller than one, their product should be rather small. Hence, this approximation seems reasonable.

Substituting GP_E^1 from (2.25) and using (B.143), evaluated at the labor market equilibrium $W = \tilde{W}$ and $m = \tilde{m}$, yields

$$\frac{1}{a}\left(\bar{s} - \frac{\frac{L_E}{a} - \bar{v}}{L+M}\rho{\sigma_\varepsilon}^2 - \frac{\rho{\sigma_s}^2}{2a}\right) + \frac{\tilde{m}}{a\rho}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right] = 0;$$

$$\bar{s} - \frac{\rho{\sigma_s}^2}{2a} + \frac{\tilde{m}}{\rho}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right] = \frac{\frac{L_E}{a} - \bar{v}}{L+M}\rho{\sigma_\varepsilon}^2;$$

$$L_E = \frac{a(L+M)}{\rho{\sigma_\varepsilon}^2}\left(\bar{s} - \frac{\rho{\sigma_s}^2}{2a}\right) + a\bar{v} + \frac{a(L+M)}{\rho^2{\sigma_\varepsilon}^2}\tilde{m}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]. \quad \text{(B.145)}$$

As $-\frac{\rho{\sigma_\varepsilon}^2}{a^2(L+M)} < 0$, the second order condition for a maximum is met. Hence, if the L_E given by (B.145) is smaller than L, then it gives the amount of entrepreneurship that (approximately) maximizes S^1 on $(0, L)$. If the L_E given by (B.145) is greater than L, then $L_E \to L$ maximizes S^1 on $(0, L)$.

Restricted occupational choice. Consider now the case of restricted OC. From (B.131), the FOC for a local optimum is given by

$$\mathsf{GP}_E^0 + M\frac{d\mathsf{GJ}_M}{dL_E} = 0. \quad \text{(B.146)}$$

Substituting GP_E^0 from (2.34) and again using (B.143), evaluated at the labor market equilibrium $W = \tilde{W}$ and $m = \tilde{m}$, yields

$$\frac{1}{a}\left(\bar{s} - \frac{\frac{L_E}{a} - \bar{v}}{L+M}\rho\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right)\right) + \frac{\tilde{m}}{a\rho}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right] = 0;$$

$$\bar{s} + \frac{\tilde{m}}{\rho}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right] = \frac{\frac{L_E}{a} - \bar{v}}{L+M}\rho\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right);$$

$$L_E = \frac{a(L+M)}{\rho\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right)}\bar{s} + a\bar{v} + \frac{a(L+M)}{\rho^2\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right)}\tilde{m}\left[1 - \exp\left\{-\rho(\tilde{W} - D)\right\}\right]. \quad \text{(B.147)}$$

As $-\frac{\rho\left({\sigma_s}^2 + {\sigma_\varepsilon}^2\right)}{a^2(L+M)} < 0$, the second order condition for a maximum is met. Hence, if the L_E given by (B.147) is smaller than L, then it gives the amount of entrepreneurship that (approximately) maximizes S^0 on $(0, L]$. If the L_E given by (B.145) is greater than L, then $L_E = L$ maximizes S^0 on $(0, L]$.

B.24 Entrepreneurs' CE with Free vs. Restricted OC

Let ${\sigma_v}^2 = 0$. In all versions of the model, the difference between an entrepreneur's CE without and with free OC, for a given value of $L_E < L$, is

$$\mathsf{CE}_E^0 - \mathsf{CE}_E^1 = \mathsf{GP}_E^0 - \mathsf{GP}_E^1 + \frac{\rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)}{2}\left(\frac{\frac{L_E}{a} - \bar{v}}{L+M}\right)^2 - \frac{\rho{\sigma_\varepsilon}^2}{2}\left(\frac{\frac{L_E}{a} - \bar{v}}{L+M}\right)^2$$

$$= -\frac{\frac{L_E}{a} - \bar{v}}{a(L+M)}\rho{\sigma_s}^2 + \frac{\rho}{2a^2}{\sigma_s}^2 + \frac{\rho{\sigma_s}^2}{2}\left(\frac{\frac{L_E}{a} - \bar{v}}{L+M}\right)^2$$

$$= \frac{\rho{\sigma_s}^2}{2a^2(L+M)^2}\left(L+M-L_E+a\bar{v}\right)^2 > 0. \tag{B.148}$$

B.25 NT Equilibrium Utility with Free vs. Restricted OC

Let ${\sigma_v}^2 = 0$. Let $L_E^1 < L$, as otherwise a ban of dealers has no effect anyway.

Conditions (i)

Comparing (B.117) to (B.121), it is obvious that NT welfare is higher in equilibrium without free OC, if $\bar{v} > 0$, $L_E^0 \geq L_E^1$ and the term in brackets is positive, which is the case if

$$\begin{aligned}
\frac{\frac{L_E^1}{a} - \bar{v}}{L+M} &> \frac{1}{2}\frac{\bar{v}}{N} \\
\frac{L_E^1}{a(L+M)} &> \frac{1}{2}\frac{\bar{v}}{N} + \frac{\bar{v}}{L+M} \\
\frac{L_E^1}{a} &> \left(1 + \frac{1}{2}\frac{L+M}{N}\right)\bar{v}.
\end{aligned} \tag{B.149}$$

Note that according to (2.39) and Appendix B.12, $M \geq L$ and $\bar{v} \geq 0$ jointly imply $L_E^0 > L_E^1$ in all versions of the model.

Conditions (ii)

The difference between a noise trader's CE in equilibrium with free vs. restricted OC can be written as

$$\mathsf{CE}_N^0(L_E^0) - \mathsf{CE}_N^1(L_E^1) = \frac{\rho{\sigma_\varepsilon}^2}{L+M}\frac{\bar{v}}{N}\left(\frac{L_E^0}{a} - \frac{L_E^1}{a}\right) + \rho{\sigma_s}^2\frac{\bar{v}}{N}\left(\frac{\frac{L_E^0}{a} - \bar{v}}{L+M} - \frac{1}{2}\frac{\bar{v}}{N}\right). \tag{B.150}$$

Basic model and UE model. In the basic version of the model and the UE model, $\bar{s}$ is independent of L_E. Substituting the equilibrium L_E^1 from (2.29) into (B.150) yields

$$\begin{aligned}
&\frac{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}{L+M}\frac{\bar{v}}{N}\frac{L_E^0}{a} - \frac{\rho{\sigma_\varepsilon}^2}{L+M}\frac{\bar{v}}{N}\left(\frac{L+M}{\rho{\sigma_\varepsilon}^2}\left(\bar{s} - \frac{\rho{\sigma_s}^2}{2a}\right) + \bar{v}\right) - \frac{\rho{\sigma_s}^2}{L+M}\frac{\bar{v}}{N}\bar{v} - \frac{\rho{\sigma_s}^2}{2}\left(\frac{\bar{v}}{N}\right)^2 = \\
&= \frac{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}{L+M}\frac{\bar{v}}{N}\frac{L_E^0}{a} - \frac{\bar{v}}{N}\bar{s} + \frac{\bar{v}}{N}\frac{\rho{\sigma_s}^2}{2a} - \frac{\rho{\sigma_\varepsilon}^2}{L+M}\frac{\bar{v}}{N}\bar{v} - \frac{\rho{\sigma_s}^2}{L+M}\frac{\bar{v}}{N}\bar{v} - \frac{\rho{\sigma_s}^2}{2}\left(\frac{\bar{v}}{N}\right)^2 = \\
&= -\frac{\bar{v}}{N}\underbrace{\left[\bar{s} - \frac{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}{L+M}\left(\frac{L_E^0}{a} - \bar{v}\right)\right]}_{=\Delta^0(L_E^0)} + \frac{\bar{v}}{N}\frac{\rho{\sigma_s}^2}{2}\left(\frac{1}{a} - \frac{\bar{v}}{N}\right).
\end{aligned} \tag{B.151}$$

The term in brackets equals $\Delta^0(L_E^0)$ according to (2.34). If $L_E^0 < L$, then $\Delta^0(L_E^0) = 0$ and a sufficient set of conditions for $\mathsf{CE}_N^0 > \mathsf{CE}_N^1$ is given by $\bar{v} > 0$ and $\bar{v}/N < 1/a$. If, by contrast, $L_E^0 = L$, then $\Delta^0(L) \geq 0$ and the set of conditions just stated is not sufficient any more. A sufficient set of conditions for $L_E^0 = L$ would be given by

$\bar{v} > 0$ and

$$\frac{\rho {\sigma_s}^2}{2}\left(\frac{1}{a} - \frac{\bar{v}}{N}\right) - \Delta^0(L) > 0;$$
$$\frac{\bar{v}}{N} < \frac{1}{a} - \frac{2}{\rho {\sigma_s}^2}\Delta^0(L). \tag{B.152}$$

FE model. In the FE model, $\bar{s}$ depends on L_E. Denoting

$$\bar{s}^1 := \bar{s}|_{L_E = L_E^1}, \quad \bar{s}^0 := \bar{s}|_{L_E = L_E^0}, \tag{B.153}$$

equation (B.151) becomes

$$-\frac{\bar{v}}{N}\left[\bar{s}^1 - \frac{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}{L+M}\left(\frac{L_E^0}{a} - \bar{v}\right)\right] + \frac{\bar{v}}{N}\frac{\rho {\sigma_s}^2}{2}\left(\frac{1}{a} - \frac{\bar{v}}{N}\right) =$$
$$= -\frac{\bar{v}}{N}\underbrace{\left[\bar{s}^0 - \frac{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}{L+M}\left(\frac{L_E^0}{a} - \bar{v}\right)\right]}_{=\Delta^0(L_E^0)} + \frac{\bar{v}}{N}(\bar{s}^0 - \bar{s}^1) + \frac{\bar{v}}{N}\frac{\rho {\sigma_s}^2}{2}\left(\frac{1}{a} - \frac{\bar{v}}{N}\right). \tag{B.154}$$

Again, the term in brackets equals $\Delta^0(L_E^0)$ according to (2.34). If $L_E^0 < L$, then $\Delta^0(L_E^0) = 0$. However, as $d\bar{s}/dL_E < 0$, for $L_E^0 \geq L_E^1$ it is $\bar{s}^0 \leq \bar{s}^1$ and hence the second term in (B.154) is negative. It follows that the set of conditions given for the basic version of the model and the UE model above is not sufficient in the FE model. For $L_E^0 < L$, a sufficient set of conditions in the FE model would be given by $\bar{v} > 0$ and

$$\frac{\rho {\sigma_s}^2}{2}\left(\frac{1}{a} - \frac{\bar{v}}{N}\right) + (\bar{s}^0 - \bar{s}^1) > 0;$$
$$\frac{\bar{v}}{N} < \frac{1}{a} - \frac{2}{\rho {\sigma_s}^2}(\bar{s}^1 - \bar{s}^0). \tag{B.155}$$

In contrast, for $L_E^0 = L$, even this set of conditions is not sufficient, as $\Delta^0(L) \geq 0$. For $L_E^0 = L$, a set of sufficient conditions in the FE model is given by $\bar{v} > 0$ and

$$\frac{\rho {\sigma_s}^2}{2}\left(\frac{1}{a} - \frac{\bar{v}}{N}\right) + (\bar{s}^0 - \bar{s}^1) - \Delta^0(L) > 0;$$
$$\frac{\bar{v}}{N} < \frac{1}{a} - \frac{2}{\rho {\sigma_s}^2}\left[\Delta^0(L) + (\bar{s}^1 - \bar{s}^0)\right]. \tag{B.156}$$

B.26 Proof of Proposition 2.5.2

Let ${\sigma_v}^2 = 0$. We conduct the proof in two steps. First, we show that, under the conditions in (2.73), the difference in social welfare without vs. with free OC is greater than zero for any given L_E with $L_E > a\bar{v}$. Second, we argue that this implies that equilibrium social welfare is higher without than with free OC.

Step 1. For the difference in social welfare for a given $L_E(< L)$, from (2.63) and (2.68) we get:[6]

$$S^0(L_E) - S^1(L_E) =$$
$$= L_E\left(\Delta^0(L_E) - \Delta^1(L_E)\right) + \frac{\rho{\sigma_s}^2}{2}\left[\frac{\left(\frac{L_E}{a}\right)^2}{L+M} - \bar{v}^2\left(\frac{1}{L+M} + \frac{1}{N}\right)\right] =$$
$$= L_E\frac{\rho{\sigma_s}^2}{2a^2} - \frac{\rho{\sigma_s}^2}{L+M}\left(\frac{L_E}{a}\right)^2 + L_E\frac{\rho{\sigma_s}^2}{a(L+M)}\bar{v}+$$
$$+\frac{\rho{\sigma_s}^2}{2(L+M)}\left(\frac{L_E}{a}\right)^2 - \frac{\rho{\sigma_s}^2}{2}\left(\frac{1}{L+M} + \frac{1}{N}\right)\bar{v}^2 =$$
$$= \frac{\rho{\sigma_s}^2}{2(L+M)}\left[-\left(\frac{L_E}{a}\right)^2 + 2\frac{L_E}{a}\left(\frac{L+M}{2a} + \bar{v}\right) - \left(1 + \frac{L+M}{N}\right)\bar{v}^2\right]. \quad \text{(B.157)}$$

Evaluated at $L_E = a\bar{v}$,

$$S^0(a\bar{v}) - S^1(a\bar{v}) =$$
$$= \frac{\rho{\sigma_s}^2}{2(L+M)}\left[-\bar{v}^2 + 2\bar{v}\left(\frac{L+M}{2a} + \bar{v}\right) - \left(1 + \frac{L+M}{N}\right)\bar{v}^2\right], \quad \text{(B.158)}$$

which, for $\bar{v} \geq 0$, is greater than (or equal to) zero, if

$$\frac{L+M}{a}\bar{v} - \frac{L+M}{N}\bar{v}^2 \geq 0;$$
$$\frac{\bar{v}}{N} \leq \frac{1}{a}. \quad \text{(B.159)}$$

Evaluated at $L_E \to L$,

$$\lim_{L_E \to L}\left[S^0(L_E) - S^1(L_E)\right] =$$
$$= \frac{\rho{\sigma_s}^2}{2(L+M)}\left[-\left(\frac{L}{a}\right)^2 + 2\frac{L}{a}\left(\frac{L+M}{2a} + \bar{v}\right) - \left(1 + \frac{L+M}{N}\right)\bar{v}^2\right], \quad \text{(B.160)}$$

which is greater than zero, if

$$\frac{LM}{a^2} + \bar{v}\left(\frac{L}{a} - \bar{v}\right) + \frac{L}{a}\bar{v} - \frac{L+M}{N}\bar{v}^2 > 0, \quad \text{(B.161)}$$

for which, with $\bar{v} \geq 0$ and $L_E^1/a > \bar{v}$, it is sufficient that

$$\frac{LM}{a^2} + \frac{L}{a}\bar{v} - \frac{L+M}{N}\bar{v}^2 > 0;$$
$$\bar{v}L\left(\frac{1}{a} - \frac{\bar{v}}{N}\right) + M\left(\frac{L}{a}\frac{1}{a} - \bar{v}\frac{\bar{v}}{N}\right) > 0, \quad \text{(B.162)}$$

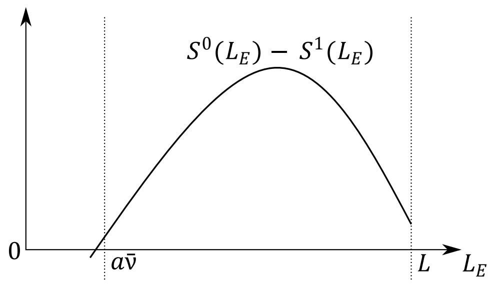
[6] Note that, for given L_E, workers' job gains in the labor market economies are the same with free and restricted OC. Hence, they cancel out.

for which to hold, it is in turn sufficient that $\bar{v}/N \leq 1/a$.
Having shown that $S^0(L_E) - S^1(L_E)$ is positive both at $L_E = a\bar{v}$ and for $L_E \to L$, we now check how it behaves in the interval $(a\bar{v}, L)$. The first derivative is given by

$$\begin{aligned} \frac{d[S^0(L_E) - S^1(L_E)]}{dL_E} &= \frac{\rho {\sigma_s}^2}{2(L+M)} \left[-2\frac{L_E}{a^2} + \frac{2}{a}\left(\frac{L+M}{2a} + \bar{v} \right) \right] \\ &= \frac{\rho {\sigma_s}^2}{a(L+M)} \left[\left(\frac{L+M}{2a} + \bar{v} \right) - \frac{L_E}{a} \right]. \end{aligned} \tag{B.163}$$

Obviously, the social welfare difference is concave, with a unique maximum at $L_E = a\bar{v} + (L+M)/2$, i.e., at an $L_E > a\bar{v}$. As illustrated by figure B.4, it immediately follows that $S^0(L_E) - S^1(L_E) > 0$ for any given L_E within $(a\bar{v}, L)$.

FIGURE B.4: Social Welfare Difference with Free vs. Restricted OC



Step 2. In the **baseline model** and the **FE economy**, we know from Proposition 2.5.1 that $L_E = L_E^1$ maximizes $S^1(L_E)$ and $L_E = L_E^0$ maximizes $S^0(L_E)$. Hence, $S^0(L_E^0) \geq S^0(L_E^1)$. From step 1, it is $S^0(L_E^1) > S^1(L_E^1)$, for $L_E^1 > a\bar{v}$. Taken together, it follows that $S^0(L_E^0) > S^1(L_E^1)$. In the **UE model**, we know from Proposition 2.5.1 that $L_E = L_E^0$ falls short of the L_E that maximizes $S^0(L_E)$. This implies that, for $L_E^1 \leq L_E^0$, $S^0(L_E)$ is increasing in L_E within $[L_E^1, L_E^0]$ and, hence, $S^0(L_E^0) \geq S^0(L_E^1)$. As, for $L_E^1 > a\bar{v}$, from step 1 we know $S^0(L_E^1) > S^1(L_E^1)$, it follows that $S^0(L_E^0) > S^1(L_E^1)$.

B.27 Agents' Risk with Free vs. Restricted OC

Let ${\sigma_v}^2 = 0$. Assume that agents do not short the asset, that is, $\bar{v} \geq 0$ and $L_E/a > \bar{v}$. This also implies that entrepreneurs sell parts of their initial assets, i.e., $I_E < 1/a$. Let $L_E < L$, as otherwise there is no difference between the case with free and restricted OC.
Entrepreneurs. An entrepreneur's final wealth is given by $\pi_E = I_E\theta + (1/a - I_E)P$. For given L_E, the difference in the variance of final wealth with and without free OC is given by

$$\mathbb{V}(\pi_E^1) - \mathbb{V}(\pi_E^0) = I_E^2 {\sigma_\varepsilon}^2 + \left(\frac{1}{a}\right)^2 {\sigma_s}^2 - \left(I_E^2 {\sigma_\varepsilon}^2 + I_E^2 {\sigma_s}^2\right)$$

$$= \left(\frac{1}{a^2} - I_E^2\right) {\sigma_s}^2 > 0. \tag{B.164}$$

With free OC, entrepreneurs carry the full risk of their initial asset holdings $1/a$ with regards to the asset fundamental s. With restricted OC, they can share this risk with the other agents in the economy (see below).

Dealers and passive investors. A dealer's or passive investor's final wealth is given by $\pi_D = \pi_M = I_D(\theta - P)$. It is $I_D = I_M = I_E$. For given L_E, the difference in the variance of final wealth with and without free OC is given by

$$\begin{aligned} \mathbb{V}(\pi_D^1) - \mathbb{V}(\pi_D^0) &= I_D^2 {\sigma_\varepsilon}^2 - \left(I_D^2 {\sigma_\varepsilon}^2 + I_D^2 {\sigma_s}^2\right) \\ &= -I_D^2 {\sigma_s}^2 < 0. \end{aligned} \tag{B.165}$$

With free OC, dealers and passive investors carry no risk with regards to s. With restricted OC, they take some of this risk from the entrepreneurs.

Noise traders. A noise trader's final wealth is given by $\pi_N = I_N(\theta - P)$. For given L_E, the difference in the variance of final wealth with and without free OC is given by

$$\begin{aligned} \mathbb{V}(\pi_N^1) - \mathbb{V}(\pi_N^0) &= I_N^2 {\sigma_\varepsilon}^2 - \left(I_N^2 {\sigma_\varepsilon}^2 + I_N^2 {\sigma_s}^2\right) \\ &= -I_N^2 {\sigma_s}^2 \leq 0. \end{aligned} \tag{B.166}$$

With free OC, noise traders carry no risk with regards to s. With restricted OC, they take some of this risk from the entrepreneurs.

B.28 Proof of Corollary 2.5.2.1

Let ${\sigma_v}^2 = 0$.

(i)

This follows directly from the fact that (B.160) is greater than zero under the conditions of Proposition 2.5.2 (cf. Appendix B.26) and that S^0 is continuous at $L_E = L$ and hence $\lim_{L_E \to L} S^0(L_E) = S^0(L)$.

(ii)

Note that here we do *not* require $L_E = L$ to be an equilibrium in case of restricted OC. If it were, Proposition 2.5.2 would apply. Consider the basic version of the model. With (2.63), (2.68) and (2.29), for $L_E^1 < L$, the difference between $S^1(L)$ and $S^1(L_E^1)$ can be written as

$$S^1(L) - S^1(L_E^1) =$$
$$= L\Delta^0(L) + \frac{\rho {\sigma_\varepsilon}^2}{2(L+M)} \left[\left(\frac{L}{a}\right)^2 - \left(\frac{L_E^1}{a}\right)^2\right] +$$

$$
\begin{aligned}
&+\frac{\rho\sigma_s{}^2}{2(L+M)}\left[\left(\frac{L}{a}\right)^2-\bar{v}^2\left(1+\frac{L+M}{N}\right)\right]= \\
&=\frac{L}{a}\left[\bar{s}-\frac{\frac{L}{a}-\bar{v}}{L+M}\rho\left(\sigma_\varepsilon{}^2+\sigma_s{}^2\right)\right]+\frac{\rho\sigma_\varepsilon{}^2}{2(L+M)}\left[\left(\frac{L}{a}\right)^2-\right. \\
&\left.-\left(\frac{(L+M)}{\rho\sigma_\varepsilon{}^2}\left(\bar{s}-\frac{\rho\sigma_s{}^2}{2a}\right)+\bar{v}\right)^2\right]+\frac{\rho\sigma_s{}^2}{2(L+M)}\left[\left(\frac{L}{a}\right)^2-\bar{v}^2\left(1+\frac{L+M}{N}\right)\right]= \\
&=\frac{L}{a}\left[\bar{s}-\frac{\frac{L}{a}-\bar{v}}{L+M}\rho\left(\sigma_\varepsilon{}^2+\sigma_s{}^2\right)\right]+\frac{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}{2(L+M)}\left(\frac{L}{a}\right)^2-\frac{L+M}{2\rho\sigma_\varepsilon{}^2}\left(\bar{s}-\frac{\rho\sigma_s{}^2}{2a}\right)^2- \\
&-\left(\bar{s}-\frac{\rho\sigma_s{}^2}{2a}\right)\bar{v}-\frac{\rho\sigma_\varepsilon{}^2}{2(L+M)}\bar{v}^2-\frac{\rho\sigma_s{}^2}{2(L+M)}v^2\left(1+\frac{L+M}{N}\right)= \\
&=\frac{L}{a}\left[\bar{s}-\frac{\frac{L}{a}-\bar{v}}{L+M}\rho\left(\sigma_\varepsilon{}^2+\sigma_s{}^2\right)\right]+\frac{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}{2(L+M)}\left[\left(\frac{L}{a}\right)^2-\bar{v}^2\right]- \\
&-\frac{L+M}{2\rho\sigma_\varepsilon{}^2}\left(\bar{s}-\frac{\rho\sigma_s{}^2}{2a}\right)^2-\bar{s}\bar{v}+\frac{\rho\sigma_s{}^2}{2}\bar{v}\left(\frac{1}{a}-\frac{\bar{v}}{N}\right)= \\
&=\bar{s}\left(\frac{L}{a}-\bar{v}\right)+\frac{L}{a}\bar{v}\frac{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}{L+M}-\frac{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}{2(L+M)}\left[\left(\frac{L}{a}\right)^2+\bar{v}^2\right]- \\
&-\frac{L+M}{2\rho\sigma_\varepsilon{}^2}\left(\bar{s}-\frac{\rho\sigma_s{}^2}{2a}\right)^2+\frac{\rho\sigma_s{}^2}{2}\bar{v}\left(\frac{1}{a}-\frac{\bar{v}}{N}\right)= \\
&=\bar{s}\left(\frac{L}{a}-\bar{v}\right)-\frac{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}{2(L+M)}\left(\frac{L}{a}-\bar{v}\right)^2-\frac{L+M}{2\rho\sigma_\varepsilon{}^2}\left(\bar{s}-\frac{\rho\sigma_s{}^2}{2a}\right)^2+\frac{\rho\sigma_s{}^2}{2}\bar{v}\left(\frac{1}{a}-\frac{\bar{v}}{N}\right).
\end{aligned}
\tag{B.167}
$$

We proceed to show that this expression is positive under the conditions of Corollary 2.5.2.1(ii). Consider (B.167) as a function of $\bar{s}$. It takes on a unique maximum at

$$
\bar{s}=\frac{\rho\sigma_\varepsilon^2}{L+M}\left(\frac{L}{a}-\bar{v}\right)+\frac{\rho\sigma_s^2}{2a}. \tag{B.168}
$$

An interior equilibrium $L_E^1 < L$ implies that $\bar{s}$ is less than this maximizing value. Hence, $S^1(L)-S^1(L_E^1)$ is an increasing function for the admissible values of $\bar{s}$. The condition that rational agents do not short the asset in equilibrium, i.e., $L_E^1/a \geq \bar{v}$, puts a lower bound on the set of admissible values of $\bar{s}$, that is, $\bar{s}=\rho\sigma_s^2/(2a)$. Evaluating $S^1(L)-S^1(L_E^1)$ at this value gives

$$
\frac{\rho\sigma_s^2}{2a}\left(\frac{L}{a}-\bar{v}\right)-\frac{\rho(\sigma_\varepsilon{}^2+\sigma_s{}^2)}{2(L+M)}\left(\frac{L}{a}-\bar{v}\right)^2+\frac{\rho\sigma_s{}^2}{2}\bar{v}\left(\frac{1}{a}-\frac{\bar{v}}{N}\right). \tag{B.169}
$$

If $1/a \geq \bar{v}/N, \bar{v} \geq 0, L_E^1 \geq a\bar{v}$ and, additionally, $M \geq L$ and $\sigma_s^2 \geq \sigma_\varepsilon^2$, then a sufficient condition for (B.169) to be greater than (or equal to) zero is given by

$$
\frac{\rho\sigma_s{}^2}{2a}-\frac{\rho 2\sigma_s{}^2}{2(2L)}\left(\frac{L}{a}-\bar{v}\right)\geq 0;
$$

$$\frac{1}{a} - \frac{\frac{L}{a} - \bar{\nu}}{L} \geq 0;$$
$$\frac{\bar{\nu}}{L} \geq 0, \tag{B.170}$$

which, for $\bar{\nu} \geq 0$, obviously holds. It follows that, for $L_E^1/a > \bar{\nu}$, (B.169) is strictly positive, from which in turn follows that (B.167) is strictly positive as well.

(iii)
Consider (2.63), evaluated at equilibrium $L_E^1 (< L)$. Additionally, consider (2.68), evaluated at equilibrium $L_E^0 (\leq L)$. In the **baseline model**, subtracting the former expression from the latter gives

$$\begin{aligned}
&S^0(L_E^0) - S^1(L_E^1) = \\
&= L_E^0 \cdot \mathsf{GP}_E^0(L_E^0) + \frac{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}{2}\left[\frac{\left(\frac{L_E^0}{a}\right)^2}{L+M} - \bar{\nu}^2\left(\frac{1}{L+M} + \frac{1}{N}\right)\right] - \\
&\quad - L_E^1 \cdot 0 - \frac{\rho{\sigma_\varepsilon}^2}{2}\left[\frac{\left(\frac{L_E^1}{a}\right)^2}{L+M} - \bar{\nu}^2\left(\frac{1}{L+M} + \frac{1}{N}\right)\right] = \\
&= L_E^0 \cdot \mathsf{GP}_E^0(L_E^0) + \frac{\rho{\sigma_\varepsilon}^2}{2(L+M)}\left[\left(\frac{L_E^0}{a}\right)^2 - \left(\frac{L_E^1}{a}\right)^2\right] + \\
&\quad + \frac{\rho{\sigma_s}^2}{2(L+M)}\left[\left(\frac{L_E^0}{a}\right)^2 - \left(1 + \frac{L+M}{N}\right)\bar{\nu}^2\right].
\end{aligned} \tag{B.171}$$

$\mathsf{GP}_E^0(L_E^0)$ is either equal to or greater than zero, depending on whether $L_E^0 < L$ or $L_E^0 = L$, so the first term in (B.171) is positive. For $L_E^0 \geq L_E^1$, the second term is positive as well. And for $\bar{\nu} \geq 0$, the third term is positive, if

$$\frac{L_E^0}{a} > \left(1 + \frac{L+M}{N}\right)^{\frac{1}{2}} \bar{\nu}. \tag{B.172}$$

In the **labor market models**, social welfare additionally contains workers' aggregate gains on the job. As these are increasing in L_E, the proof above is sufficient for $S^0(L_E^0) > S^1(L_E^1)$ also in the FE economy and the UE model.

B.29 Expected NT Utility with $\sigma_\nu{}^2 > 0$

Let $\sigma_\nu{}^2 > 0$. Assume noise traders to be symmetric, that is, an aggregate noise trader demand ν implies an individual noise trader demand $I_N = \nu/N$. Exogenous behavior combined with CARA-utility gives a noise trader's expected utility as

$$\mathbb{E}\left[U(\pi_N)\right] = -\mathbb{E}\left[\exp\left\{-\rho \cdot \frac{\nu}{N}(\theta - P)\right\}\right]. \tag{B.173}$$

Proceeding as usual, we first calculate $\mathbb{E}\left[U(\pi_M)|\nu\right]$ and then use the LIE to get the unconditional expected utility.
Step 1. As $(\theta - P)|\nu$ is normal, we can use (A.4.1) and (A.18) to get

$$
\begin{aligned}
\mathbb{E}\left[U(\pi_N)|\nu\right] &= -\mathbb{E}\left[\exp\left\{-\rho\cdot\frac{\nu}{N}(\theta-P)\right\}|\nu\right] = \\
&= -\exp\left\{-\rho\frac{\nu}{N}\mathbb{E}(\theta-P|\nu)+\frac{1}{2}\rho^2\left(\frac{\nu}{N}\right)^2\mathbb{V}(\theta-P|\nu)\right\} = \\
&= -\exp\left\{-\rho\frac{\nu}{N}\left[\mathbb{E}(\theta-P)+\frac{\mathrm{Cov}(\theta-P,\nu)}{\sigma_\nu{}^2}(\nu-\bar{\nu})\right]+\frac{1}{2}\rho^2\left(\frac{\nu}{N}\right)^2\mathbb{V}(\theta-P|\nu)\right\} \\
&= -\exp\left\{\underbrace{-\rho\frac{\nu}{N}\left[\mathbb{E}(\theta-P)+\frac{\mathrm{Cov}(P,\nu)}{\sigma_\nu{}^2}\bar{\nu}\right]}_{=:\Phi}+\right. \\
&\qquad\left.+\underbrace{\rho\left(\frac{\nu}{N}\right)^2\left[\frac{\rho}{2}\mathbb{V}(\theta-P|\nu)+N\frac{\mathrm{Cov}(P,\nu)}{\sigma_\nu{}^2}\right]}_{=:\Psi^2}\right\} \\
&= -\exp\left\{\Phi+\Psi^2\right\}.
\end{aligned} \tag{B.174}
$$

Step 2. As Φ and Ψ are both linear in ν, they are (degenerate) joint normal and we can use (A.4.1) to get

$$
\begin{aligned}
\mathbb{E}\left[U(\pi_N)\right] &= -\mathbb{E}\left(\exp\left\{\Phi+\Psi^2\right\}\right) \\
&= -\frac{\exp\left\{\mathbb{E}(\Phi)+\frac{1}{2}\mathbb{V}(\Phi)+\frac{[\mathbb{E}(\Psi)+\mathrm{Cov}(\Phi,\Psi)]^2}{1-2\mathbb{V}(\Psi)}\right\}}{\sqrt{1-2\mathbb{V}(\Psi)}};
\end{aligned} \tag{B.175}
$$

$$
\mathsf{CE}_N = -\frac{1}{\rho}\mathbb{E}(\Phi)-\frac{1}{2\rho}\mathbb{V}(\Phi)-\frac{[\mathbb{E}(\Psi)+\mathrm{Cov}(\Phi,\Psi)]^2}{\rho(1-2\mathbb{V}(\Psi))}+\frac{1}{2\rho}\ln\left[1-2\mathbb{V}(\Psi)\right], \tag{B.176}
$$

as long as $\mathbb{V}(\Psi) < 0.5$. For $\mathbb{V}(\Psi) \geq 0.5$, the integral in Appendix A.4.2 does not exist (is "infinity"), therefore $\mathbb{E}(\exp\left\{\Phi+\Psi^2\right\})$ does not exist (is "infinity") and, hence, noise trader welfare $\mathbb{E}\left[U(\pi_N)\right] = -\mathbb{E}\left(\exp\left\{\Phi+\Psi^2\right\}\right)$ does not exist (is "minus infinity"). Consequently, let $\mathbb{V}(\Psi) < 0.5$. From (B.174), the moments of Φ and Ψ are given by

$$
\mathbb{E}(\Phi) = -\rho\frac{\bar{\nu}}{N}\left[\bar{s}-\mathbb{E}(P)+\frac{\mathrm{Cov}(P,\nu)}{\sigma_\nu{}^2}\bar{\nu}\right], \tag{B.177}
$$

$$
\mathbb{V}(\Phi) = \left(\frac{\mathbb{E}(\Phi)}{\bar{\nu}}\right)^2\sigma_\nu{}^2, \tag{B.178}
$$

$$
\mathbb{E}(\Psi) = \sqrt{\rho}\frac{\bar{\nu}}{N}\left(\frac{\rho}{2}\mathbb{V}(\theta-P|\nu)+N\frac{\mathrm{Cov}(P,\nu)}{\sigma_\nu{}^2}\right)^{\frac{1}{2}}, \tag{B.179}
$$

$$
\mathbb{V}(\Psi) = \left(\frac{\mathbb{E}(\Psi)}{\bar{\nu}}\right)^2\sigma_\nu{}^2, \tag{B.180}
$$

$$\mathrm{Cov}(\Phi, \Psi) = \frac{\mathbb{E}(\Phi)}{\bar{\nu}} \frac{\mathbb{E}(\Psi)}{\bar{\nu}} \sigma_\nu{}^2. \tag{B.181}$$

Step 3. Equations (B.177)-(B.181) contain the expressions $\mathrm{Cov}(P, \nu)$ and $\mathbb{V}(\theta - P|\nu)$, for which we did not yet offer closed-form solutions. In case of free OC, $\mathbb{E}(P)$ in (B.177) is given by (B.41) and with P according to (2.5) and the definitions in Appendix B.5, we get

$$\begin{aligned}
\mathrm{Cov}(P, \nu) &= \frac{1}{\alpha+\beta}\mathrm{Cov}(w + \beta\mathbb{E}(\theta|w), \nu) \\
&= \frac{1}{\alpha+\beta}\left[\mathrm{Cov}(w, \nu) + \beta\mathrm{Cov}(\mathbb{E}(\theta|w), \nu)\right] \\
&= \frac{1}{\alpha+\beta}\left[\sigma_\nu{}^2 + \beta\frac{\mathrm{Cov}(\theta, w)}{\mathbb{V}(w)}\sigma_\nu{}^2\right] \\
&= \frac{1}{\alpha+\beta}\sigma_\nu{}^2\left[1 + \beta\frac{\mathrm{Cov}(\theta, w)}{\mathbb{V}(w)}\right] \\
&= \sigma_\nu{}^2\frac{1 + \beta\alpha\gamma\sigma_s{}^2}{\alpha+\beta} \\
&= \sigma_\nu{}^2\sqrt{\mathbb{V}(P)\gamma},
\end{aligned} \tag{B.182}$$

where $\mathbb{V}(P)$ is given by (B.42). Furthermore,

$$\begin{aligned}
\mathrm{Cov}(P, s) &= \frac{1}{\alpha+\beta}\mathrm{Cov}(w + \beta\mathbb{E}(\theta|w), s) \\
&= \frac{1}{\alpha+\beta}\left[\mathrm{Cov}(w, s) + \beta\mathrm{Cov}(\mathbb{E}(\theta|w), s)\right] \\
&= \frac{1}{\alpha+\beta}\left[\alpha\sigma_s{}^2 + \beta\frac{\mathrm{Cov}(\theta, w)}{\mathbb{V}(w)}\alpha\sigma_s{}^2\right] \\
&= \frac{1}{\alpha+\beta}\alpha\sigma_s{}^2\left[1 + \beta\frac{\mathrm{Cov}(\theta, w)}{\mathbb{V}(w)}\right] \\
&= \alpha\sigma_s{}^2\sqrt{\mathbb{V}(P)\gamma},
\end{aligned} \tag{B.183}$$

from which follows

$$\begin{aligned}
\mathbb{V}(\theta - P) &= \sigma_\varepsilon{}^2 + \mathbb{V}(s - P) \\
&= \sigma_\varepsilon{}^2 + \mathbb{V}(s) + \mathbb{V}(P) - 2\mathrm{Cov}(s, P) \\
&= \sigma_\varepsilon{}^2 + \sigma_s{}^2 + \mathbb{V}(P) - 2\alpha\sigma_s{}^2\sqrt{\mathbb{V}(P)\gamma}.
\end{aligned} \tag{B.184}$$

With that, we finally get

$$\begin{aligned}
\mathbb{V}(\theta - P|\nu) &= \mathbb{V}(\theta - P) - \frac{\mathrm{Cov}(\theta - P, \nu)^2}{\sigma_\nu{}^2} \\
&= \mathbb{V}(\theta - P) - \frac{\mathrm{Cov}(P, \nu)^2}{\sigma_\nu{}^2} \\
&= \mathbb{V}(\theta - P) - \frac{\sigma_\nu{}^4\mathbb{V}(P)\gamma}{\sigma_\nu{}^2}
\end{aligned}$$

$$
\begin{aligned}
&= \sigma_\varepsilon{}^2 + \sigma_s{}^2 + \mathbb{V}(P) - 2\alpha\sigma_s{}^2\sqrt{\mathbb{V}(P)\gamma} - \sigma_\nu{}^2\mathbb{V}(P)\gamma \\
&= \sigma_\varepsilon{}^2 + \sigma_s{}^2 + \mathbb{V}(P)(1-\gamma\sigma_\nu{}^2) - 2\alpha\sigma_s{}^2\sqrt{\mathbb{V}(P)\gamma}, \qquad \text{(B.185)}
\end{aligned}
$$

with $\mathbb{V}(P)$ given by (B.42).

Restricted occupational choice. In case of restricted OC, $\mathbb{E}(P)$ in (B.177) is given by (B.52) and with P according to (2.8), we get

$$
\operatorname{Cov}(P,\nu) = \frac{\rho(\sigma_s{}^2 + \sigma_\varepsilon{}^2)}{L+M}\sigma_\nu{}^2 \qquad \text{(B.186)}
$$

and

$$
\begin{aligned}
\mathbb{V}(\theta - P|\nu) &= \sigma_\varepsilon{}^2 + \mathbb{V}(s-P|\nu) \\
&= \sigma_\varepsilon{}^2 + \sigma_s{}^2. \qquad \text{(B.187)}
\end{aligned}
$$

B.30 Convergence of $\Delta(L_E)$ and $\Gamma(L_E)$

Let $\sigma_\nu{}^2 \to 0$. Then, as we show below, the $\sigma_\nu{}^2 > 0$ expressions for $\Delta(L_E)$ and $\Gamma(L_E)$, as given in Chapter 2.2.2 and Appendices B.3 and B.6, converge to their $\sigma_\nu{}^2 = 0$ counterparts, as given in Chapter 2.3.

Consider first the case of free OC. Then, for all $L_E < L$, the expressions in Appendix B.5 converge to

$$
\begin{aligned}
\gamma &\to \frac{1}{\alpha^2\sigma_s{}^2}, \qquad \text{(B.188)} \\
\mathbb{V}(\theta|w) &\to \sigma_\varepsilon{}^2, \qquad \text{(B.189)} \\
\beta &\to \frac{L_E + M}{\rho\sigma_\varepsilon{}^2}, \qquad \text{(B.190)} \\
\mathbb{E}(P) &\to \bar{s} - \rho\sigma_\varepsilon{}^2\frac{\frac{L_E}{a} - \bar{\nu}}{L+M}, \qquad \text{(B.191)} \\
\mathbb{V}(P) &\to \alpha^2\sigma_s{}^2\left(\frac{1+\alpha\beta\sigma_s{}^2\frac{1}{\alpha^2}\sigma_s{}^2}{\alpha+\beta}\right)^2 \\
&= \sigma_s{}^2\left(\frac{\alpha+\beta}{\alpha+\beta}\right)^2 \\
&= \sigma_s{}^2, \qquad \text{(B.192)} \\
\mathbb{E}(z) &\to \frac{\rho\sigma_\varepsilon{}^2}{\sqrt{2\rho\sigma_\varepsilon{}^2}}\frac{\frac{L_E}{a} - \bar{\nu}}{L+M} \\
&= \sqrt{\frac{\rho\sigma_\varepsilon{}^2}{2}}\frac{\frac{L_E}{a} - \bar{\nu}}{L+M}, \qquad \text{(B.193)} \\
\mathbb{V}(z) &\to 0, \qquad \text{(B.194)} \\
\operatorname{Cov}(P,z) &\to 0. \qquad \text{(B.195)}
\end{aligned}
$$

With these expressions and equations (2.12), (2.17) and (2.20) follows that

$$\begin{aligned}\Delta(L_E) &\to \frac{1}{a}\left(\bar{s} - \rho{\sigma_\varepsilon}^2\frac{\frac{L_E}{a} - \bar{v}}{L+M}\right) - \frac{\rho}{2a^2}{\sigma_s}^2\\ &= \frac{1}{a}\left(\bar{s} - \frac{\frac{L_E}{a} - \bar{v}}{L+M}\rho{\sigma_\varepsilon}^2 - \frac{1}{2a}\rho{\sigma_s}^2\right), \qquad \text{(B.196)}\\ \Gamma(L_E) &\to \frac{1}{2\rho}\ln\left[1+0\right]\\ &= 0, \qquad \text{(B.197)}\end{aligned}$$

which, for $L_E < L$, equals the ${\sigma_v}^2 = 0$ expressions for $\Delta(L_E)$ and $\Gamma(L_E)$ given by (2.25) and (2.26). For $L_E = L$, we get

$$\begin{aligned}\alpha &= 0, \qquad \text{(B.198)}\\ \gamma &\to \infty, \qquad \text{(B.199)}\\ \gamma{\sigma_v}^2 &= 1, \qquad \text{(B.200)}\\ \mathbb{V}(\theta|w) &= {\sigma_\varepsilon}^2 + {\sigma_s}^2, \qquad \text{(B.201)}\\ \beta &= \frac{L+M}{\rho({\sigma_\varepsilon}^2 + {\sigma_s}^2)}, \qquad \text{(B.202)}\\ \mathbb{E}(P) &= \bar{s} - \rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)\frac{\frac{L}{a} - \bar{v}}{L+M}, \qquad \text{(B.203)}\\ \mathbb{V}(P) &\to 0, \qquad \text{(B.204)}\\ \gamma\mathbb{V}(P) &= \frac{1}{\beta^2}, \qquad \text{(B.205)}\\ \mathbb{E}(z) &= \frac{\rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)}{\sqrt{2\rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)}}\frac{\frac{L}{a} - \bar{v}}{L+M}\\ &= \sqrt{\frac{\rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)}{2}}\frac{\frac{L}{a} - \bar{v}}{L+M}, \qquad \text{(B.206)}\\ \mathbb{V}(z) &\to 0, \qquad \text{(B.207)}\\ \mathrm{Cov}(P,z) &\to 0. \qquad \text{(B.208)}\end{aligned}$$

With these expressions and equations (2.12), (2.17) and (2.20) follows that

$$\begin{aligned}\Delta(L) &\to \frac{1}{a}\left(\bar{s} - \rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)\frac{\frac{L}{a} - \bar{v}}{L+M}\right), \qquad \text{(B.209)}\\ \Gamma(L) &= \frac{1}{2\rho}\ln\left[1 + \frac{{\sigma_s}^2}{{\sigma_\varepsilon}^2}\right], \qquad \text{(B.210)}\end{aligned}$$

which equals the ${\sigma_v}^2 = 0$ expressions for $\Delta(L)$ and $\Gamma(L)$ given by (2.25) and (2.26). Combining (B.196)-(B.197) and (B.209)-(B.210), it follows that, for all $L_E \leq L$, $\Delta(L_E)$ and $\Gamma(L_E)$ are continuous in ${\sigma_v}^2$, even at ${\sigma_v}^2 = 0$.

Restricted occupational choice. In case of restricted OC, the expressions in Appendix B.6 converge to

$$\mathbb{E}(P) = \bar{s} - \frac{\rho\left({\sigma_s}^2 + {\sigma_\epsilon}^2\right)}{L+M}\left(\frac{L_E}{a} - \bar{v}\right), \tag{B.211}$$

$$\mathbb{V}(P) \to 0, \tag{B.212}$$

$$\mathbb{E}(z) = \sqrt{\frac{\rho}{2}\left({\sigma_s}^2 + {\sigma_\epsilon}^2\right)} \cdot \frac{\frac{L_E}{a} - \bar{v}}{L+M}, \tag{B.213}$$

$$\mathbb{V}(z) \to 0, \tag{B.214}$$

$$\mathrm{Cov}(P, z) \to 0. \tag{B.215}$$

With these expressions and equations (2.12) and (2.20) follows that

$$\Delta(L_E) \to \frac{1}{a}\left(\bar{s} - \rho\left({\sigma_s}^2 + {\sigma_\epsilon}^2\right)\frac{\frac{L_E}{a} - \bar{v}}{L+M}\right), \tag{B.216}$$

which, for $L_E \leq L$, equals the ${\sigma_v}^2 = 0$ expression for $\Delta(L_E)$ given by (2.34). Hence, it follows that, for all $L_E \leq L$, $\Delta(L_E)$ is continuous in ${\sigma_v}^2$, even at ${\sigma_v}^2 = 0$.

B.31 Convergence of Noise Trader Utility

Let ${\sigma_v}^2 \to 0$. Then, as we show below, the ${\sigma_v}^2 > 0$ expression for CE_N given in Appendix B.29 converges to its ${\sigma_v}^2 = 0$ counterpart given in Appendix B.16. Consider first the case of free OC. For $L_E < L$, from Appendix B.29 and Appendix B.30 we get

$$\begin{aligned}\mathbb{E}(\Phi) &\to -\rho\frac{\bar{v}}{N}\left[\bar{s} - \bar{s} + \rho{\sigma_\varepsilon}^2\frac{\frac{L_E}{a} - \bar{v}}{L+M} + \sqrt{{\sigma_s}^2\frac{1}{\alpha^2{\sigma_s}^2}} \cdot \bar{v}\right] \\ &= -\rho\frac{\bar{v}}{N}\left[\rho{\sigma_\varepsilon}^2\frac{\frac{L_E}{a} - \bar{v}}{L+M} + \frac{\bar{v}}{\alpha}\right],\end{aligned} \tag{B.217}$$

$$\mathbb{V}(\Phi) \to 0, \tag{B.218}$$

$$\begin{aligned}\mathbb{E}(\Psi) &\to \sqrt{\rho}\frac{\bar{v}}{N}\left[\frac{\rho}{2}\left({\sigma_\varepsilon}^2 + {\sigma_s}^2 + {\sigma_s}^2 - 2\alpha{\sigma_s}^2\frac{1}{\alpha}\right) + N\frac{1}{\alpha}\right]^{\frac{1}{2}} \\ &= \sqrt{\rho}\frac{\bar{v}}{N}\left[\frac{\rho}{2}{\sigma_\varepsilon}^2 + \frac{N}{\alpha}\right]^{\frac{1}{2}},\end{aligned} \tag{B.219}$$

$$\mathbb{V}(\Psi) \to 0, \tag{B.220}$$

$$\mathrm{Cov}(\Phi, \Psi) \to 0. \tag{B.221}$$

With (B.175), it follows that

$$\mathsf{CE}_N \to -\frac{1}{\rho}\left[\mathbb{E}(\Phi) + \mathbb{E}(\Psi)^2\right]$$

$$\begin{aligned}
&\to \frac{\bar{v}}{N}\left[\rho\sigma_\varepsilon{}^2\frac{\frac{L_E}{a}-\bar{v}}{L+M}+\frac{\bar{v}}{\alpha}\right]-\left(\frac{\bar{v}}{N}\right)^2\left[\frac{\rho}{2}\sigma_\varepsilon{}^2+\frac{N}{\alpha}\right] \\
&= \frac{\bar{v}}{N}\left[\rho\sigma_\varepsilon{}^2\frac{\frac{L_E}{a}-\bar{v}}{L+M}\right]-\left(\frac{\bar{v}}{N}\right)^2\frac{\rho}{2}\sigma_\varepsilon{}^2, \qquad \text{(B.222)}
\end{aligned}$$

which, for $L_E < L$, equals the $\sigma_v{}^2 = 0$ expression for CE_N given by (B.117). For $L_E = L$, we get

$$\begin{aligned}
\mathbb{E}(\Phi) &= -\rho\frac{\bar{v}}{N}\left[\bar{s}-\bar{s}+\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)\frac{\frac{L}{a}-\bar{v}}{L+M}+\frac{1}{\beta}\bar{v}\right] \\
&= -\rho\frac{\bar{v}}{N}\left[\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)\frac{\frac{L}{a}-\bar{v}}{L+M}+\frac{\bar{v}}{\beta}\right], \qquad \text{(B.223)} \\
\mathbb{V}(\Phi) &\to 0, \qquad \text{(B.224)} \\
\mathbb{E}(\Psi) &= \sqrt{\rho}\frac{\bar{v}}{N}\left[\frac{\rho}{2}\left(\sigma_\varepsilon{}^2+\sigma_s{}^2-2\alpha\sigma_s{}^2\frac{1}{\beta}\right)+N\frac{1}{\beta}\right]^{\frac{1}{2}} \qquad \text{(B.225)} \\
&= \sqrt{\rho}\frac{\bar{v}}{N}\left[\frac{\rho}{2}\left(\sigma_\varepsilon{}^2+\sigma_s{}^2\right)+N\frac{1}{\beta}\right]^{\frac{1}{2}}, \qquad \text{(B.226)} \\
\mathbb{V}(\Psi) &\to 0, \qquad \text{(B.227)} \\
\mathrm{Cov}(\Phi,\Psi) &\to 0. \qquad \text{(B.228)}
\end{aligned}$$

Again, with (B.175) it follows that, for $L_E = L$,

$$\begin{aligned}
\mathsf{CE}_N &\to -\frac{1}{\rho}\left[\mathbb{E}(\Phi)+\mathbb{E}(\Psi)^2\right] \\
&= \frac{\bar{v}}{N}\left[\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)\frac{\frac{L}{a}-\bar{v}}{L+M}+\frac{\bar{v}}{\beta}\right]-\left(\frac{\bar{v}}{N}\right)^2\left[\frac{\rho}{2}\left(\sigma_\varepsilon{}^2+\sigma_s{}^2\right)+N\frac{1}{\beta}\right] \\
&= \frac{\bar{v}}{N}\left[\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)\frac{\frac{L}{a}-\bar{v}}{L+M}\right]-\left(\frac{\bar{v}}{N}\right)^2\frac{\rho}{2}\left(\sigma_\varepsilon{}^2+\sigma_s{}^2\right), \qquad \text{(B.229)}
\end{aligned}$$

which, for $L_E = L$, equals the $\sigma_v{}^2 = 0$ expression for CE_N given by (B.119). Combining (B.222) and (B.229), it follows that for all $L_E \leq L$, CE_N is continuous in $\sigma_v{}^2$, even at $\sigma_v{}^2 = 0$.

Restricted occupational choice. In case of restricted OC, from Appendix B.29 and Appendix B.30 we get

$$\begin{aligned}
\mathbb{E}(\Phi) &= -\rho\frac{\bar{v}}{N}\left[\bar{s}-\bar{s}+\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)\frac{\frac{L_E}{a}-\bar{v}}{L+M}+\frac{\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)}{L+M}\bar{v}\right] \\
&= -\rho\frac{\bar{v}}{N}\left[\rho(\sigma_s{}^2+\sigma_\varepsilon{}^2)\frac{\frac{L_E}{a}}{L+M}\right], \qquad \text{(B.230)} \\
\mathbb{V}(\Phi) &\to 0, \qquad \text{(B.231)}
\end{aligned}$$

$$\mathbb{E}(\Psi) = \sqrt{\rho}\frac{\bar{v}}{N}\left[\frac{\rho}{2}\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right) + N\frac{\rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)}{L+M}\right]^{\frac{1}{2}}, \tag{B.232}$$

$$\mathbb{V}(\Psi) \to 0, \tag{B.233}$$

$$\mathrm{Cov}(\Phi, \Psi) \to 0. \tag{B.234}$$

With (B.175), it follows that

$$\begin{aligned}
\mathsf{CE}_N &\to -\frac{1}{\rho}\left[\mathbb{E}(\Phi) + \mathbb{E}(\Psi)^2\right] \\
&= \frac{\bar{v}}{N}\left[\rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)\frac{\frac{L_E}{a}}{L+M}\right] - \left(\frac{\bar{v}}{N}\right)^2\left[\frac{\rho}{2}\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right) + N\frac{\rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)}{L+M}\right] \\
&= \frac{\bar{v}}{N}\left[\rho({\sigma_s}^2 + {\sigma_\varepsilon}^2)\frac{\frac{L_E}{a} - \bar{v}}{L+M}\right] - \left(\frac{\bar{v}}{N}\right)^2\frac{\rho}{2}\left({\sigma_\varepsilon}^2 + {\sigma_s}^2\right),
\end{aligned} \tag{B.235}$$

which, for $L_E \leq L$, equals the ${\sigma_v}^2 = 0$ expression for CE_N given by (B.121). Hence, it follows that, for all $L_E \leq L$, CE_N is continuous in ${\sigma_v}^2$, even at ${\sigma_v}^2 = 0$.

Appendix C

Simulation

C.1 Equilibrium L_E^0

Let ${\sigma_\nu}^2 > 0$. Superscript "0" indicates that we refer to the case of restricted OC. The condition that pins down the equilibrium L_E is $\Delta^0(L_E) = 0$, where $\Delta^0(L_E)$ is given by (2.21). Consider first the **baseline model**. Using (B.52)-(B.57) yields

$$
\begin{aligned}
&\Delta^0(L_E) = \\
&= \frac{1}{a}\left[\bar{s} - \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{L+M}\left(\frac{L_E}{a}-\bar{v}\right)\right] - \frac{\rho}{2a^2}\left[\frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{L+M}\right]^2{\sigma_\nu}^2 + \frac{1}{1+2\rho\mathbb{V}(z)}\cdot \\
&\cdot\left[\frac{\rho^2}{a^2}\frac{\rho^3\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)^3}{2(L+M)^4}{\sigma_\nu}^4 + 2\frac{\rho}{a}\left(\frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{2}\right)^{\frac{1}{2}}\frac{\frac{L_E}{a}-\bar{v}}{L+M}\left(\frac{\rho^3\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)^3}{2(L+M)^4}{\sigma_\nu}^4\right)^{\frac{1}{2}}\right] = \\
&= \frac{1}{a}\bar{s} - \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a^2(L+M)}\Bigg[(L_E-a\bar{v}) + \rho(L+M)\mathbb{V}(z) - \frac{2\rho\mathbb{V}(z)}{1+2\rho\mathbb{V}(z)}\cdot \\
&\qquad\cdot[\rho(L+M)\mathbb{V}(z) + (L_E-a\bar{v})]\Bigg] = \\
&= \frac{1}{a}\bar{s} - \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a^2(L+M)}[(L_E-a\bar{v}) + \rho(L+M)\mathbb{V}(z)]\left(1-\frac{2\rho\mathbb{V}(z)}{1+2\rho\mathbb{V}(z)}\right) = \\
&= \frac{1}{a}\bar{s} - \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a^2(L+M)}[(L_E-a\bar{v}) + \rho(L+M)\mathbb{V}(z)]\frac{1}{1+2\rho\mathbb{V}(z)} \overset{!}{=} 0. \qquad \text{(C.1)}
\end{aligned}
$$

Solving for L_E gives the unique equilibrium L_E^0:

$$
\begin{aligned}
L_E^0 &= \frac{a(L+M)}{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}\bar{s}\left[1+2\rho\mathbb{V}(z)\right] + a\bar{v} - \rho(L+M)\mathbb{V}(z) \\
&= \frac{a(L+M)}{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}\bar{s} + a\bar{v} + \frac{\rho a {\sigma_\nu}^2}{L+M}\bar{s} - \frac{\rho^2\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right){\sigma_\nu}^2}{2(L+M)} \\
&= \frac{a(L+M)}{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}\bar{s} + a\bar{v} + \frac{a^2\rho{\sigma_\nu}^2}{L+M}\underbrace{\left(\frac{\bar{s}}{a} - \frac{\rho}{2}\frac{{\sigma_s}^2+{\sigma_\varepsilon}^2}{a^2}\right)}_{\mathbb{E}(\frac{\theta}{a})-\frac{\rho}{2}\mathbb{V}(\frac{\theta}{a})}, \qquad \text{(C.2)}
\end{aligned}
$$

if this expression is smaller than L, and $L_E^0 = L$ otherwise. For ${\sigma_\nu}^2 = 0$, this corresponds to the equilibrium L_E^0 in equation (2.37). The underbraced term gives the certainty equivalent of an entrepreneur's production output θ/a. If it is greater than zero, the equilibrium mass of entrepreneurs with ${\sigma_\nu}^2 > 0$ exceeds the one with ${\sigma_\nu}^2 = 0$.

In the **UE model**, $\bar{s}$ is given by (2.55), everything else stays the same as above. In the **FE economy**, $\bar{s}$ is given by (2.49) and depends non-linearly on L_E. Hence, there is no explicit representation of equilibrium L_E^0.

C.2 Social Welfare S^0 and the Optimum L_E

Let ${\sigma_\nu}^2 > 0$. Consider first the **baseline model**. Social welfare S^0 in case of restricted OC is given by

$$S^0 = L_E \Delta^0 + (L+M)\mathsf{GT}_M^0 + N\mathsf{CE}_N^0, \tag{C.3}$$

with Δ^0 and GT_M^0 given by (2.21) and (2.12), and the moments of P and z given by (B.52)-(B.56). CE_N^0 is given by (B.176) and the corresponding moments of Φ and Ψ are given by (B.177)-(B.181) and (B.186)-(B.187).

Taking the derivative of S^0 with respect to L_E, we get

$$\frac{dS^0}{dL_E} = \Delta^0 + L_E \frac{d\Delta^0}{dL_E} + (L+M)\frac{d\mathsf{GT}_M^0}{dL_E} + N\frac{d\mathsf{CE}_N^0}{dL_E}. \tag{C.4}$$

Δ^0 is given by (C.1) and $L_E(d\Delta^0/dL_E)$ can be easily derived from that expression. Furthermore,

$$\begin{aligned}
(L+M)\frac{d\mathsf{GT}_M^0}{dL_E} &= (L+M)\frac{2\mathbb{E}(z)}{1+2\rho\mathbb{V}(z)}\frac{d\mathbb{E}(z)}{dL_E} \\
&= \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)\frac{\frac{L_E}{a}-\bar{\nu}}{a(L+M)}}{1+\rho^2{\sigma_\nu}^2\frac{{\sigma_s}^2+{\sigma_\varepsilon}^2}{(L+M)^2}} \\
&= \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a^2(L+M)}\frac{1}{1+2\rho\mathbb{V}(z)}(L_E - a\bar{\nu}).
\end{aligned} \tag{C.5}$$

As $\mathbb{E}(\Psi)$, $\mathbb{V}(\Psi)$, $\mathrm{Cov}(P,\nu)$ and $\mathbb{V}(\theta - P|\nu)$ are independent of L_E, the last term in (C.4) comes down to

$$\begin{aligned}
N\frac{d\mathsf{CE}_N^0}{dL_E} &= N\left[-\frac{1}{\rho}\frac{d\mathbb{E}(\Phi)}{dL_E} - \frac{1}{2\rho}\frac{d\mathbb{V}(\Phi)}{dL_E} - \frac{2\mathbb{E}(\Psi)\frac{d\mathrm{Cov}(\Phi,\Psi)}{dL_E} + 2\mathrm{Cov}(\Phi,\Psi)\frac{d\mathrm{Cov}(\Phi,\Psi)}{dL_E}}{\rho\left(1-2\mathbb{V}(\Psi)\right)}\right] \\
&= -\frac{N}{\rho}\left[\frac{d\mathbb{E}(\Phi)}{dL_E} + \frac{{\sigma_\nu}^2}{\bar{\nu}^2}\mathbb{E}(\Phi)\frac{d\mathbb{E}(\Phi)}{dL_E} + 2\frac{{\sigma_\nu}^2}{\bar{\nu}^2}\mathbb{E}(\Psi)\frac{d\mathbb{E}(\Phi)}{dL_E}\frac{\mathbb{E}(\Psi)+\mathrm{Cov}(\Phi,\Psi)}{1-2\mathbb{V}(\Psi)}\right] \\
&= -\frac{N}{\rho}\frac{d\mathbb{E}(\Phi)}{dL_E}\left[1 + \frac{{\sigma_\nu}^2}{\bar{\nu}^2}\mathbb{E}(\Phi) + 2\frac{{\sigma_\nu}^2}{\bar{\nu}^2}\mathbb{E}(\Psi)^2\frac{1+\frac{{\sigma_\nu}^2}{\bar{\nu}^2}\mathbb{E}(\Phi)}{1-2\mathbb{V}(\Psi)}\right] \\
&= -\frac{N}{\rho}\frac{d\mathbb{E}(\Phi)}{dL_E}\left[1 + \frac{{\sigma_\nu}^2}{\bar{\nu}^2}\mathbb{E}(\Phi)\right]\left[1 + 2\frac{{\sigma_\nu}^2}{\bar{\nu}^2}\mathbb{E}(\Psi)^2\frac{1}{1-2\mathbb{V}(\Psi)}\right] \\
&= \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\bar{\nu}\left[1+\frac{2\mathbb{V}(\Psi)}{1-2\mathbb{V}(\Psi)}\right]\left[1+\frac{{\sigma_\nu}^2}{\bar{\nu}^2}\mathbb{E}(\Phi)\right]
\end{aligned}$$

$$= \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\bar{v}\frac{1}{1-2\mathbb{V}(\Psi)}\left[1-\frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\frac{\bar{v}}{N}\frac{{\sigma_v}^2}{\bar{v}^2}L_E\right]. \tag{C.6}$$

With $\bar{v} > 0$ and $\mathbb{V}(\Psi) < 0.5$ (cf. Appendix B.29), the expression outside of the brackets is positive.
Using (C.1), (C.5) and (C.6), equation (C.4) becomes

$$\begin{aligned}
\frac{dS^0}{dL_E} &= \Delta^0 - \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\frac{\bar{v}}{1+2\rho\mathbb{V}(z)} \\
&\quad + \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\bar{v}\frac{1}{1-2\mathbb{V}(\Psi)}\left[1-\frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\frac{\bar{v}}{N}\frac{{\sigma_v}^2}{\bar{v}^2}L_E\right] \\
&= \Delta^0 + \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\left[\bar{v}\left(\frac{1}{1-2\mathbb{V}(\Psi)}-\frac{1}{1+2\rho\mathbb{V}(z)}\right)\right. \\
&\quad \left. - \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\frac{{\sigma_v}^2}{N}\frac{1}{1-2\mathbb{V}(\Psi)}L_E\right] \\
&= \Delta^0 + \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\left[2\bar{v}\frac{\rho\mathbb{V}(z)+\mathbb{V}(\Psi)}{[1-2\mathbb{V}(\Psi)][1+2\rho\mathbb{V}(z)]}\right. \\
&\quad \left. - \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\frac{{\sigma_v}^2}{N}\frac{1}{1-2\mathbb{V}(\Psi)}L_E\right] \\
&= \Delta^0 + \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\frac{1}{1-2\mathbb{V}(\Psi)}\left[\underbrace{\frac{2\mathbb{V}(\Psi)+2\rho\mathbb{V}(z)}{1+2\rho\mathbb{V}(z)}}_{\in(0,1)}\bar{v} - \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\frac{{\sigma_v}^2}{N}L_E\right].
\end{aligned} \tag{C.7}$$

As we know that Δ^0 is linearly decreasing in L_E, it immediately follows that dS^0/dL_E is linearly decreasing in L_E and $dS^0/dL_E = 0$ has a unique solution. This implies that $S^0(L_E)$ is hump-shaped with a unique maximum. Whether the social welfare maximizing L_E exceeds or falls short of the equilibrium L_E^0 depends on whether the term in brackets in (C.7), evaluated at $L_E = L_E^0$, is greater or smaller than zero.[1]

As $S^0(L_E)$ is hump-shaped, it attains it's unique maximum where $dS^0/dL_E = 0$. Using (C.7), we can explicitly state the social welfare maximizing L_E:

$$-\Delta^0 + \frac{\rho^2\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)^2}{a(L+M)^2}\frac{1}{1-2\mathbb{V}(\Psi)}\frac{{\sigma_v}^2}{N}\frac{L_E}{a} = \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{a(L+M)}\frac{2\mathbb{V}(\Psi)+2\rho\mathbb{V}(z)}{[1-2\mathbb{V}(\Psi)][1+2\rho\mathbb{V}(z)]}\bar{v};$$

$$\frac{1}{1+2\rho\mathbb{V}(z)}\frac{L_E}{a} + \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{L+M}\frac{1}{1-2\mathbb{V}(\Psi)}\frac{{\sigma_v}^2}{N}\frac{L_E}{a} = \frac{1}{1-2\mathbb{V}(\Psi)}\cdot$$

$$\cdot\frac{2\mathbb{V}(\Psi)+2\rho\mathbb{V}(z)}{1+2\rho\mathbb{V}(z)}\bar{v} + \frac{a(L+M)}{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}\frac{\bar{s}}{a} + \frac{1}{1+2\rho\mathbb{V}(z)}\left(\bar{v}-\frac{\rho}{a}(L+M)\mathbb{V}(z)\right);$$

and hence

$$L_E = \left(\frac{1}{1+2\rho\mathbb{V}(z)} + \frac{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}{L+M}\frac{1}{1-2\mathbb{V}(\Psi)}\frac{{\sigma_v}^2}{N}\right)^{-1}\left(\frac{a(L+M)}{\rho\left({\sigma_s}^2+{\sigma_\varepsilon}^2\right)}\bar{s}+\right.$$

[1] For ${\sigma_v}^2 \to 0$, the term in brackets converges to zero, dS^0/dL_E converges to Δ^0 and hence the optimum L_E again converges to the equilibrium one.

$$+\frac{a\bar{v}-\rho(L+M)\mathbb{V}(z)}{1+2\rho\mathbb{V}(z)}+\frac{2\mathbb{V}(\Psi)+2\rho\mathbb{V}(z)}{[1-2\mathbb{V}(\Psi)][1+2\rho\mathbb{V}(z)]}a\bar{v}\Bigg), \tag{C.8}$$

if this expression is smaller than L, and $L_E = L$ otherwise.

In the **UE model**, social welfare S^0 contains the additional term $M \cdot \mathsf{GJ}_M$. In labor market equilibrium, GJ_M is given by (2.53), evaluated at $m = \tilde{m}$ and $W = \tilde{W}$. Again, $\bar{s}$ is given by (2.55). If GJ_M is approximated by (B.143), then it is linear in L_E and hence its derivative is independent of L_E. In this case, the optimal L_E is given by

$$\begin{aligned}L_E = &\left(\frac{1}{1+2\rho\mathbb{V}(z)}+\frac{\rho\left(\sigma_s{}^2+\sigma_\varepsilon{}^2\right)}{L+M}\frac{1}{1-2\mathbb{V}(\Psi)}\frac{\sigma_v{}^2}{N}\right)^{-1}\left(\frac{a(L+M)}{\rho\left(\sigma_s{}^2+\sigma_\varepsilon{}^2\right)}\bar{s}+\right.\\ &+\frac{a\bar{v}-\rho(L+M)\mathbb{V}(z)}{1+2\rho\mathbb{V}(z)}+\frac{2\mathbb{V}(\Psi)+2\rho\mathbb{V}(z)}{[1-2\mathbb{V}(\Psi)][1+2\rho\mathbb{V}(z)]}a\bar{v}+\\ &\left.+\frac{\tilde{m}}{a\rho}\left[1-\exp\left\{-\rho(\tilde{W}-D)\right\}\right]\right),\end{aligned} \tag{C.9}$$

which differs from (C.8) only by the last term within the second pair of parentheses and by the definition of $\bar{s}$. Without the approximation in (B.143), GJ_M is non-linear in L_E and the social welfare maximizing L_E cannot be stated explicitly.

In the **FE economy**, social welfare S^0 contains the additional term $M \cdot W$. In labor market equilibrium, W is given by $W = \tilde{W}$, which depends non-linearly on L_E. Furthermore, also $\bar{s}$ given by (2.49) depends non-linearly on L_E. Hence, there is no explicit representation of the social welfare maximizing L_E.

C.3 Specification of Agents' Risk-Aversion Parameter ρ

The final wealth of rational agent i is given by π_i. With CARA-utility and with π_i being normally distributed, the agent's certainty equivalent is given by $\mathbb{E}(\pi_i) - (\rho/2)\mathbb{V}(\pi_i)$. The 95% confidence interval for π_i is (approximately) given by $[\mathbb{E}(\pi_i) - 2\sqrt{\mathbb{V}(\pi_i)},\ \mathbb{E}(\pi_i) + 2\sqrt{\mathbb{V}(\pi_i)}]$. Because of symmetry, this means that the probability of final wealth below the lower boundary of this interval is 2.5%. We call an agent excessively risk-averse, if his CE is lower than $\mathbb{E}(\pi_i) - 2\sqrt{\mathbb{V}(\pi_i)}$. Hence, to ensure that agents are not excessively risk-averse, it needs to hold that

$$\begin{aligned}\mathbb{E}(\pi_i) - (\rho/2)\mathbb{V}(\pi_i) &\geq \mathbb{E}(\pi_i) - 2\sqrt{\mathbb{V}(\pi_i)};\\ \rho &\leq \frac{4}{\sqrt{\mathbb{V}(\pi_i)}}.\end{aligned} \tag{C.10}$$

Now let $\sigma_v{}^2 = 0$. The volatility of final wealth is maximum for agents who become entrepreneurs in the presence of dealers (cf. Appendix B.27). Hence, $\mathbb{V}(\pi_E)$, where $\pi_E = P/a + I_E(\theta - P)$ with P given by (2.23) and I_E given by (2.41), gives an upper bound for any rational agent's uncertainty $\mathbb{V}(\pi_i)$. It follows that if ρ meets condition (C.10) for $\mathbb{V}(\pi_i) = \mathbb{V}(\pi_E)$, then it does so also for all other rational agents' $\mathbb{V}(\pi_i)$. It is

$$\mathbb{V}(\pi_E) = \frac{{\sigma_s}^2}{a^2} + \left(\frac{\frac{L_E}{a} - \bar{v}}{L+M}\right)^2 {\sigma_\varepsilon}^2$$
$$= \frac{1}{a^2}\left[{\sigma_s}^2 + \underbrace{\left(\frac{L_E - a\bar{v}}{L+M}\right)^2}_{\leq 0.25 \text{ (as } M \geq L)} {\sigma_\varepsilon}^2\right] \tag{C.11}$$

and, as we set ${\sigma_\varepsilon}^2 \leq {\sigma_s}^2$ (cf. table 2.1), it follows that

$$\mathbb{V}(\pi_E) \leq 1.25\frac{{\sigma_s}^2}{a^2}. \tag{C.12}$$

Combining (C.10) with (C.12), rational agents are not excessively risk-averse for

$$\rho \leq \frac{4}{\sqrt{1.25\frac{{\sigma_s}^2}{a^2}}};$$
$$\rho \leq \frac{4a}{\sqrt{1.25}\sigma_s}. \tag{C.13}$$

Condition (C.13) ensures that agents are not overly risk-averse. On the other hand, we also want to avoid that our parameter choices for ρ cluster around values that imply more or less risk-neutral agents. In this regard, note that the 4% confidence interval for π_i is (approximately) given by $[\mathbb{E}(\pi_i) - 0.05\sqrt{\mathbb{V}(\pi_i)}, \mathbb{E}(\pi_i) + 0.05\sqrt{\mathbb{V}(\pi_i)}]$.[2] We call an agent almost risk-neutral, if his CE is higher than $\mathbb{E}(\pi_i) - 0.05\sqrt{\mathbb{V}(\pi_i)}$. Hence, agents are *not* almost risk-neutral if

$$\mathbb{E}(\pi_i) - (\rho/2)\mathbb{V}(\pi_i) \leq \mathbb{E}(\pi_i) - 0.05\sqrt{\mathbb{V}(\pi_i)};$$
$$\rho \geq \frac{0.1}{\sqrt{\mathbb{V}(\pi_i)}}, \tag{C.14}$$

or, when using the "estimates" for $\mathbb{V}(\pi_i)$ from above, if

$$\rho \geq \frac{0.1a}{\sqrt{1.25}\sigma_s}. \tag{C.15}$$

C.4 Homogeneity Properties

Let ${\sigma_v}^2 > 0$. The easiest way to get an impression of the homogeneity properties is to run some numerical examples. For the sketch of a comprehensive proof, see below.

Basic model. As can be seen from Appendices B.5, B.6 and B.29, agents' individual CEs, both with free and restricted OC, are homogeneous of degree zero in L, M, N, v, σ_v and L_E jointly. It follows that both $\Delta(L_E)$ and $\Gamma(L_E)$, as well as social welfare

[2] Note that for $X \sim \mathcal{N}(\mu_x, {\sigma_x}^2)$, it is $\mathbb{P}(\mu_x - z\sigma_x \leq X \leq \mu_x + z\sigma_\mu) = \Phi(z) - \Phi(-z) = \Phi(z) - (1 - \Phi(z))$, where Φ denotes the cumulative distribution function of a standard normal distribution and the second equality follows from symmetry. Using a standard normal table (also called a "z-table"), one can easily see that $\mathbb{P}(\mu_x - z\sigma_x \leq X \leq \mu_x + z\sigma_\mu) \approx 0.04$ for $z = 0.05$.

per capita are homogeneous of degree zero in L, M, N, ν, σ_v and L_E jointly. As we set M, N, ν and σ_v as multiples of L in the simulation, it immediately follows that both the equilibrium and the constrained optimum mass of entrepreneurs vary proportionately with L.

Similarly, it can be seen that agents' individual CEs change by a factor $1/\lambda$ for a change in parameters from $(a, \rho, \bar{v}, \sigma_v)$ to $(\lambda a, \lambda\rho, \bar{v}/\lambda, \sigma_v/\lambda)$. It follows that for such a change in parameters, both $\Delta(L_E)$ and $\Gamma(L_E)$ as well as social welfare change by a factor $1/\lambda$. As we set ρ as a multiple of a, and $\bar{v}$ and σ_v as multiples of $1/a$ in the simulation, it immediately follows that both the equilibrium and the constrained optimum mass of entrepreneurs are independent of a.

Finally, it can be seen that agents' individual CEs change by a factor λ for a change in parameters from $(\sigma_s, \rho, \sigma_\epsilon, \bar{s})$ to $(\lambda\sigma_s, \rho/\lambda, \lambda\sigma_\epsilon, \lambda\bar{s})$. It follows that for such a change in parameters, both $\Delta(L_E)$ and $\Gamma(L_E)$ as well as social welfare change by a factor λ. As we set ρ as a multiple of $1/\sigma_s$, and σ_ϵ and $\bar{s}$ as multiples of σ_s in the simulation, it immediately follows that both the equilibrium and the constrained optimum mass of entrepreneurs are independent of σ_s.

Labor market models. Similarly as done above, it can be shown for both the FE and the UE model that when choosing parameters according to tables 2.1 and 2.3, the equilibrium and constrained optimum mass of entrepreneurs are linear homogeneous in L and homogeneous of degree zero in ${\sigma_s}^2$ and a, respectively. In particular, an increase in L by a factor of λ just increases the equilibrium L_E, the social welfare maximizing L_E and the corresponding values of social welfare by factor λ as well. Varying ${\sigma_s}^2$ or a, respectively, does not have any effects on the equilibrium L_E or the social welfare maximizing L_E. The corresponding values of social welfare change by factor λ for a factor λ-increase in σ_s, and by factor $(1/\lambda)$ for a factor λ-increase in a.

C.5 Simulation of the Basic Model

Dropped parameter combinations. The main results regarding the simulation of the basic version of the model can be found in table 2.2 in the running text. We start with 65,112 parameter combinations for $\sigma_v = 0.001L/a$, but this number decreases to 16,607 combinations for $\sigma_v = 0.5L/a$. Further information on the reasons why parameter combinations have been dropped in the simulation process are given by table C.1. Elimination was done in column order. First, we dropped all combinations that satisfy the condition of the second column. From the remaining combinations, we dropped those which satisfy the condition of the third column, and so on. With higher ${\sigma_v}^2$, the number of cases omitted due to non-existence of a unique interior equilibrium without short-selling (columns 2-3) and undefined noise trader welfare (column 4) increases. As the condition in column 2 already ensures $\Delta(L) < \Gamma(L)$ for the remaining cases, multiple L_E^1 in column 3 occur very rarely. The numbers outside parentheses in column 4 represent the number of combinations dropped due to the fact that $\mathbb{V}(\Psi) \geq 0.5$ at equilibrium L_E^1 or L_E^0 (this is the relevant number with respect to table 2.2), the numbers within parentheses give the additional number of parameters that were dropped due to the fact that $\mathbb{V}(\Upsilon) \geq 0.5$ for any other value of L_E (these had to be dropped for calculating the social welfare optimum values in tables C.2-C.3).

TABLE C.1: Omitted Parameter Combinations in the Simulation of the Basic Model

$\frac{\sigma_v}{L/a}$	# $\Delta(a\bar{v}) < \Gamma(a\bar{v})$ or $\Delta(L) > \Gamma(L)$	# mult. $L_E^{\mathbf{1}}$	# $V(\Upsilon) \geq 0.5$
0.001	0	0	0
0.01	174	0	0
0.05	6,492	6	0
0.1	14,868	0	130 (+106)
0.2	26,556	0	898 (+151)
0.5	46,554	0	1,951 (+107)

Equilibrium vs. optimum outcomes. Table 2.2, column 3, shows that, for σ_v up to $0.2L/a$, the equilibrium mass of entrepreneurs tends to fall short of the constrained (local) optimum. We now want to evaluate the magnitude of the average difference between equilibrium and optimum values for the mass of entrepreneurs as well as the respective differences in social welfare. Take for example the difference between equilibrium L_E^1 and the respective constrained (local) optimum, denoted by $\hat{L}_E^1$. Then, what we aim for is a measure of relative difference between those two, averaged over all $(L_E^1, \hat{L}_E^1)$ combinations we got from the simulation. The obvious choice for such a measure would be (standard) percentage change. However, this is problematic in our case mainly for the following reason: its value range is not bound from above but ranges from zero to infinity. As a consequence, its average over many $(L_E^1, \hat{L}_E^1)$ combinations is sensitive to outliers (see below for an example).

We therefore define a "new" measure $\Delta_m(x, y)$ to evaluate the relative difference between two variables x and y:

$$\Delta_m(x, y) = \frac{|x - y|}{\max(|x|, |y|)}. \tag{C.16}$$

As is usually done, we confine the domain of $\Delta_m(x, y)$ to x and y with the same sign (cf. Toernqvist et al., 1980, p. 3).[3] We look at the *absolute* difference between x and y in the numerator of (C.16) as we do not want opposite signs to cancel out when calculating the average $\Delta_m(x, y)$ over many (x, y) combinations. For the scaling into a *relative* difference, we use the maximum operator, so that $\Delta_m(x, y)$ is bound within

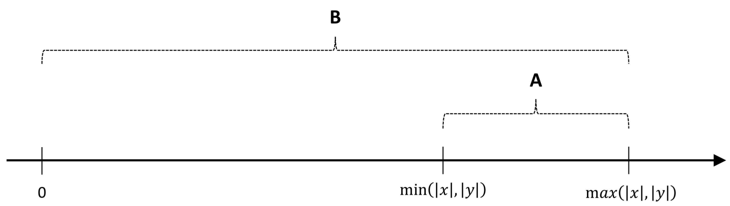
[3]Otherwise, measures of relative differences can give misleading results. This is not only true for our measure Δ_m, but also for (standard) percentage difference. Be, e.g., $y_1 = 5$ and $x_1 = -10$. Then $\frac{|x_1 - y_1|}{|x_1|} = 1.5$. Now let $y_2 = 5$ and $x_2 = -20$. It is $\frac{|x_2 - y_2|}{|x_2|} = 1.25$, which is smaller than 1.5, even though the difference between x and y grew larger.

$[0, 1]$ and has a nice intuition, which can be seen by rearranging (C.16):

$$\begin{aligned}
\Delta_m \cdot \max(|x|, |y|) &= |x - y|; \\
\Delta_m \cdot \max(|x|, |y|) &= \max(|x|, |y|) - \min(|x|, |y|); \\
\Delta_m &= 1 - \frac{\min(|x|, |y|)}{\max(|x|, |y|)}; \\
\min(|x|, |y|) &= (1 - \Delta_m) \cdot \max(|x|, |y|). \qquad \text{(C.17)}
\end{aligned}$$

Now, what Δ_m tells us is that the (absolute) smaller value is $(1 - \Delta_m) \cdot 100\%$ of the (absolute) larger value. This intuition becomes even clearer with figure C.1, where $\Delta_m(x, y) = A/B$ and thereby tells which portion of the total distance $[0, \max(|x|, |y|)]$ is accounted for by the distance between $|x|$ and $|y|$. For $x/y \approx 1$, Δ_m is close to zero,

FIGURE C.1: $\Delta_m(x, y) = \mathrm{A/B}$



just like standard percentage difference. For $x/y \to \infty$, Δ_m converges to one, while standard percentage difference (with y as the reference value) converges to infinity.

The fact that Δ_m is bound within $[0, 1]$ makes its average over many (x, y) combinations less sensitive to outliers. To see that, let, e.g., $x_1 = 0.0001$, $y_1 = 10$, $x_2 = 10$, $y_2 = 20$. Then, what we get is an average standard percentage difference equal to $\frac{1}{2}\left(\frac{10-0.0001}{0.0001} + \frac{20-10}{10}\right) = 5{,}000{,}000\%$, that is, on average, y is $= 5{,}000{,}000\%$ higher than x. Using our measure from (C.16), we get $\Delta_m = \frac{1}{2}\left(\frac{10-0.0001}{10} + \frac{20-10}{20}\right) \approx 75\%$, that is, on average, x is 25% of y. Note also that $\Delta_m(x, y)$ is symmetric both with regards to x and y, i.e., $\Delta_m(x, y) = \Delta_m(y, x)$, as well as around $(0, 0)$, that is $\Delta_m(x, y) = \Delta_m(-x, -y)$. Hence, Δ_m is independent of whether we take x or y as the reference value and we just treat negative values as if they were positive.

Now, as before, let $\hat{L}_E^1$ denote the mass of entrepreneurs that constitutes the constrained (local) social optimum mass of entrepreneurs in the presence of dealers. If $S^1(L_E)$ does not have a local optimum, $\hat{L}_E^{\mathbf{1}}$ is not defined.[4] We denote the value of social welfare corresponding to a mass of entrepreneurs equal to $\hat{L}_E^{\mathbf{1}}$ by $\hat{S}^1$. The entries in columns 2 & 3 of table C.2 give averages (in percent) and standard deviations (in percentage points) over the subset of parameter combinations for which $\hat{L}_E^{\mathbf{1}}$ is defined and $\Delta_m(x, y)$ is well behaved, i.e., where x and y are of the same sign (this is not necessarily the case for $S^1(L_E^1)$ vs. $\hat{S}^1$). The average difference between equilibrium L_E^1 and the local optimum $\hat{L}_E^1$ is quite small, at least up to $\sigma_\nu^2 = 0.2L/a$ (table C.2,

[4] Comparing L_E^1 to the constrained globally optimal L_E would be rather meaningless, as this is typically given by $L_E = L$ even in the noiseless case; cf. Corollary 2.5.2.1(ii).

column 2). The corresponding differences in social welfare are even smaller (column 3).

TABLE C.2: Equilibrium vs. Constrained Optimum Outcomes in the Basic Model

$\frac{\sigma_v}{L/a}$	$\Delta_m\left(L_E^1, \hat{L}_E^1\right)$	$\Delta_m\left(S^1\left(L_E^1\right), \hat{S}^1\right)$	$\Delta_m\left(L_E^0, \hat{L}_E^0\right)$	$\Delta_m\left(S^0\left(L_E^0\right), \hat{S}^0\right)$
0.001	0.03% (0.12%)	≈ 0% (≈ 0%)	≈ 0% (≈ 0%)	≈ 0% (≈ 0%)
0.01	0.21% (0.57%)	0.01% (0.14%)	≈ 0% (0.01%)	≈ 0% (≈ 0%)
0.05	0.97% (2.16%)	0.16% (1.59%)	0.05% (0.37%)	≈ 0% (0.03%)
0.1	1.75% (4.08%)	0.25% (2.06%)	0.23% (1.47%)	0.04% (0.77%)
0.2	2.53% (6.87%)	0.50% (3.58%)	0.75% (4.08%)	0.39% (4.20%)
0.5	9.03% (17.80%)	0.89% (5.23%)	1.92% (8.61%)	1.43% (7.47%)

Note: Δ_m defined according to (C.16).
Note: The number of admissible parameter combinations with regards to $\Delta_m\left(L_E^1, \hat{L}_E^1\right)$ is given by 57,105 for $\sigma_v = 0.001L/a$, decreases to 31,717 for $\sigma_v = 0.2L/a$, and to 14,313 for $\sigma_v = 0.5L/a$. With regards to $\Delta_m\left(S^1\left(L_E^1\right), \hat{S}^1\right)$, the analogous numbers are given by 57,105, 31,668, and 14,313, respectively. With regards to $\Delta_m\left(S^0\left(L_E^0\right), \hat{S}^0\right)$, they are 65,112, 37,431, and 16,484, respectively.

Columns 4 & 5 give the corresponding numbers in the absence of dealers, with only one difference: $\hat{L}_E^0$ denotes the global optimum of $S^0(L_E)$, that is, we also consider parameter combinations which imply $\hat{L}_E^0 = L$. The resulting numbers tend to be even smaller than the ones in case of free OC.

Free vs. restricted OC. While the average differences between equilibrium and constrained optimum outcomes tend to be rather small (cf. table C.2), table C.3 shows that those between social welfare with free vs. restricted OC are large.[5] On average, equilibrium social welfare with free OC is just about 6% of equilibrium social welfare with restricted OC, for ${\sigma_v}^2 = 0.001L/a$, for example. Again, combinations which imply that $\hat{L}_E^1$ is not defined or the two arguments of Δ_m differ in their sign are omitted from the calculations in the respective columns.

What is still missing are more detailed information on the two effects that apply when moving from equilibrium welfare with free OC, i.e., from $S^1(L_E^1)$, to equilibrium welfare with restricted OC, i.e., to $S^0(L_E^0)$. As shown by figure 2.5, these effects are: first, the effect of making the market less informative, call it the "i-effect", given by $S^0(L_E^1) - S^1(L_E^1)$; and second, the related increase in the mass of entrepreneurs, call it the "r-effect", given by $S^0(L_E^0) - S^0(L_E^1)$. Column 2 in table C.4 shows that, for all simulated parameter combinations, the number of entrepreneurs increases when dealers are banned. Columns 2 & 3 tell that both the "i-effect" and the "r-effect" are positive for the vast majority of parameter combinations up to a σ_v of $0.2L/a$. Numbers for the "i-effect" are not so clear any more for $\sigma_v = 0.5L/a$, which is again mainly

[5] These already large numbers from our measure Δ_m also indicate that using (standard) percentage difference instead would have produced exorbitantly large, messy numbers. In this regard, also note that making use of the max-operator in Δ_m is necessary, because it is not clear whether $S^1(L_E^1)$ is greater or smaller than $S^0(L_E^0)$, especially for large ${\sigma_v}^2$ (cf. table 2.2).

TABLE C.3: Free vs. Restricted OC in the Basic Model

$\frac{\sigma_\nu}{L/a}$	$\Delta_m\left(S^1(L_E^1), S^0(L_E^0)\right)$	$\Delta_m\left(\hat{S}^1, \hat{S}^0\right)$
0.001	93.76% (9.57%)	94.83% (8.45%)
0.01	93.24% (10.19%)	95.03% (7.98%)
0.05	91.53% (12.40%)	93.88% (8.49%)
0.1	90.26% (13.70%)	92.87% (8.73%)
0.2	90.20% (14.67%)	93.19% (8.22%)
0.5	48.59% (28.17%)	43.90% (26.78%)

Note: Δ_m defined according to (C.16).
Note: The number of admissible parameter combinations with regards to $\Delta_m\left(S^1(L_E^1), S^0(L_E^0)\right)$ is given by 54,260 for $\sigma_\nu = 0.001L/a$, decreases to 19,285 for $\sigma_\nu = 0.2L/a$, and to 2,000 for $\sigma_\nu = 0.5L/a$. With regards to $\Delta_m\left(\hat{S}^1, \hat{S}^0\right)$, the analogous numbers are given by 46,515, 14,769, and 1,339, respectively.

because with high magnitudes of σ_ν, noise traders' return volatility is likely to benefit from informationally efficient markets and their utility tends to dominate social welfare (see also table 2.2).

TABLE C.4: The Two Effects of a Ban of Dealers in the Basic Model

$\frac{\sigma_\nu}{L/a}$	$L_E^0 > L_E^1$	i-effect>0	r-effect>0	$\frac{\text{i-effect}}{\text{total effect}}$
0.001	100%	100%	100%	60.77%
0.01	100%	99.99%	100%	59.27%
0.05	100%	99.97%	99.89%	55.93%
0.1	100%	99.64%	99.35%	54.07%
0.2	100%	97.40%	98.45%	50.51%
0.5	100%	77.82%	97.22%	28.33%

With regards to their average relative magnitudes, column 4 gives the "i-effect" as a percentage of the total effect. By implication, the relative magnitude of the "r-effect" is given by 100% minus the relative "i-effect". To avoid misleading numbers, the calculations in column 4 are restricted to parameter combinations for which both effects are positive. We see that while the "i-effect" slightly dominates the "r-effect" for low and medium noise volatility, this turns around for large noise trader shocks.
Price variance. Interestingly, the ex-ante price variance tends to be lower rather than higher in case of positive noise volatility. This is shown by table C.5, which gives the average of the percentage difference between the price variance in equilibrium with stochastic noise, denoted $\mathbb{V}_\sigma(P)$, and the price variance with deterministic noise, denoted $\mathbb{V}_0(P)$. The reason is that the direct positive impact of volatility with regards to noise trader demand ν on the asset price P tends to be more than offset by the effect that more noise makes P less sensitive to the macro fundamental s.

TABLE C.5: Price Variance

$\frac{\sigma_v}{L/a}$	0.001	0.01	0.05	0.1	0.2	0.5
$\frac{\mathbb{V}_\sigma(P)-\mathbb{V}_0(P)}{\mathbb{V}_0(P)}$	-0.01%	-0.18%	-0.89%	-1.58%	-2.32%	-1.77%

C.6 Simulation of the FE Model

Setting $\hat{s}$. As $\Delta(L_E)$ is strictly decreasing in L_E for $L_E \in (0, L)$ in the noiseless FE model (cf. Chapter 2.4.1), for an equilibrium L_E^1 with $a\bar{v} < L_E^1 < L$ to exist, we require $\Delta(a\bar{v}) > 0$ and $\lim_{L_E \to L} \Delta(L_E) < 0$. From (2.25) and (2.49), this requires

$$\begin{aligned} \bar{s}(a\bar{v}) - \frac{\rho {\sigma_s}^2}{2a} &> 0; \\ \hat{s} + Ab\left(\frac{M}{\bar{v}}\right)^{1-b} - \frac{\rho {\sigma_s}^2}{2a} &> 0; \\ \hat{s} &> \frac{\rho {\sigma_s}^2}{2a} - Ab\left(\frac{M}{\bar{v}}\right)^{1-b} \end{aligned} \tag{C.18}$$

as well as

$$\begin{aligned} \bar{s}(L) - \frac{\frac{L}{a} - \bar{v}}{L+M}\rho {\sigma_\varepsilon}^2 - \frac{\rho {\sigma_s}^2}{2a} &< 0; \\ \hat{s} + Ab\left(\frac{aM}{L}\right)^{1-b} - \frac{\frac{L}{a} - \bar{v}}{L+M}\rho {\sigma_\varepsilon}^2 - \frac{\rho {\sigma_s}^2}{2a} &> 0; \\ \hat{s} &< \frac{\rho {\sigma_s}^2}{2a} - Ab\left(\frac{aM}{L}\right)^{1-b} + \frac{\frac{L}{a} - \bar{v}}{L+M}\rho {\sigma_\varepsilon}^2. \end{aligned} \tag{C.19}$$

Hence, the first row in the upper part of table 2.3 pins down $\hat{s}$ such that L_E^1 varies in between the interval $(a\bar{v}, L)$ for ${\sigma_v}^2 = 0$.

Setting A. Expected firm profit is given by $\bar{s}(L_E) = Ab\hat{M}^{1-b} + \hat{s}$. As a starting point, consider an equilibrium $L_E^1 = a\bar{v}$, which from (C.18) requires that $\hat{s} = \frac{\rho {\sigma_s}^2}{2a} - Ab\left(\frac{M}{\bar{v}}\right)^{1-b}$ in the noiseless case. If we set

$$A = \xi \cdot \frac{\rho {\sigma_s}^2}{2a} \frac{\left(\frac{\bar{v}}{M}\right)^{1-b}}{b}, \tag{C.20}$$

then at $L_E^1 = a\bar{v}$:

$$Ab\hat{M}^{1-b} = Ab\left(\frac{M}{\bar{v}}\right)^{1-b} = \xi \cdot \frac{\rho {\sigma_s}^2}{2a} \tag{C.21}$$

and

$$\begin{aligned} \hat{s} &= \frac{\rho {\sigma_s}^2}{2a} - \xi \cdot \frac{\rho {\sigma_s}^2}{2a} \\ &= (1-\xi) \cdot \frac{\rho {\sigma_s}^2}{2a}. \end{aligned} \tag{C.22}$$

For $\xi = 0.5$, (C.21) and (C.22) are exactly equal to each other. The third row in the upper part of table 2.3 varies ξ around 0.5, so that A given by (C.20) ensures that the two terms are of comparable magnitude. Note that for $\xi = 0.5$ and $L_E^1 > a\bar{v}$ it is $Ab\hat{M}^{1-b} < \hat{s}$, which is why we focus mainly on $\xi \geq 0.5$ and don't consider values for ξ below 0.25. Table C.6 shows that in the simulation $\hat{s}$ accounts for 60 to 70 percent of $\bar{s}$ at equilibrium with free OC, on average.

TABLE C.6: The Expected Fundamental's Part in Expected Firm Profit in the FE Model

$\frac{\sigma_v}{L/a}$	0.001	0.01	0.05	0.1	0.2	0.5
$\frac{\hat{s}}{\bar{s}}$	62.68%	62.64%	62.59%	62.84%	64.25%	67.46%

Simulation tables. The main results regarding the simulation of the FE model can be found in the left part of table 2.4 in the running text. We start with 1,124,190 parameter combinations for $\sigma_v = 0.001L/a$, but this number decreases to 636,033 combinations for $\sigma_v = 0.5L/a$. Table C.7 gives information on the reasons why parameter combinations are eliminated in the simulation process. It can be read analogous to table C.1 for the basic version of the model, except that we additionally include the remaining admissible combinations for each σ_v in the last column. Tables C.8-C.10 are analogous to the respective tables for the basic model. Results are similar.

TABLE C.7: Omitted Parameter Combinations in the Simulation of the FE Model

$\frac{\sigma_v}{L/a}$	# $\Delta(a\bar{v}) < \Gamma(a\bar{v})$ or $\Delta(L) > \Gamma(L)$	# mult. L_E^1	# $V(\Psi) \geq 0.5$	# rem. cases
0.001	0	0	0	1,124,190
0.01	0	12	0	1,124,178
0.05	11,928	98	0	1,112,164
0.1	55,332	33	2,298 (+1,982)	1,066,527
0.2	161,904	29	18,400 (+4,201)	943,857
0.5	430,266	24	57,867 (+4,373)	636,033

TABLE C.8: Equilibrium vs. Constrained Optimum Outcomes in the FE Model

$\frac{\sigma_v}{L/a}$	$\Delta_m\left(L_E^1, \hat{L}_E^1\right)$	$\Delta_m\left(S^1\left(L_E^1\right), \hat{S}^1\right)$	$\Delta_m\left(L_E^0, \hat{L}_E^0\right)$	$\Delta_m\left(S^0\left(L_E^0\right), \hat{S}^0\right)$
0.001	0.01% (0.06%)	≈ 0% (≈ 0%)	≈ 0% (≈ 0%)	≈ 0% (≈ 0%)
0.01	0.08% (0.31%)	≈ 0% (≈ 0%)	≈ 0% (0.01%)	≈ 0% (≈ 0%)
0.05	0.39% (1.18%)	0.01% (0.30%)	0.06% (0.36%)	≈ 0% (0.16%)
0.1	0.72% (2.06%)	0.04% (0.91%)	0.22% (1.31%)	0.03% (0.79%)
0.2	1.17% (3.38%)	0.12% (1.93%)	0.61% (3.26%)	0.24% (3.05%)
0.5	4.14% (10.94%)	0.55% (4.39%)	1.38% (6.45%)	0.69% (5.15%)

TABLE C.9: Free vs. Restricted OC in the FE Model

$\frac{\sigma_v}{L/a}$	$\Delta_m\left(S^1(L_E^1), S^0(L_E^0)\right)$	$\Delta_m\left(\hat{S}^1, \hat{S}^0\right)$
0.001	73.67% (22.37%)	74.48% (22.35%)
0.01	73.63% (22.45%)	74.81% (22.39%)
0.05	72.71% (22.66%)	74.05% (22.39%)
0.1	71.57% (22.80%)	72.95% (22.34%)
0.2	70.88% (23.17%)	72.10% (22.65%)
0.5	65.50% (26.47%)	65.75% (26.47%)

TABLE C.10: The Two Effects of a Ban of Dealers in the FE Model

$\frac{\sigma_v}{L/a}$	$L_E^0 > L_E^1$	i-effect>0	r-effect>0	$\frac{\text{i-effect}}{\text{total effect}}$
0.001	100%	100%	100%	48.11%
0.01	100%	99.99%	100%	47.73%
0.05	100%	97.87%	99.92%	46.57%
0.1	100%	94.53%	99.61%	45.67%
0.2	100%	90.35%	99.19%	43.27%
0.5	100%	77.69%	98.61%	32.59%

C.7 Simulation of the UE Model

Setting *D*. From (2.54), firms' labor demand is given by

$$m = \left(\frac{A(1-b)}{W}\right)^{\frac{1}{b}}. \tag{C.23}$$

There is unemployment for $L_E < L$ and full employment at $L_E = L$ exactly if

$$W = A(1-b)\left(\frac{L}{aM}\right)^{b}, \tag{C.24}$$

as in this case it is $m < \hat{M}$ for $L_E < L$ and $m = \hat{M}$ at $L_E = L$. The wage W is set by firm level unions according to the following optimality condition (cf. Appendix B.14):

$$1 + \rho b W - \exp\{\rho(W - D)\} = 0. \tag{C.25}$$

Now the question is: Is it possible to set D such that the wage stated in (C.24) follows from (C.25)? The answer is yes, which can easily be seen by rearranging (C.25):

$$\begin{aligned} \ln(1+\rho b W) &= \rho(W-D); \\ D &= W - \frac{1}{\rho}\ln(1+\rho b W), \end{aligned} \tag{C.26}$$

with W given by (C.24). It remains to be checked, whether this D satisfies $D < W$ and $D > 0$. From (C.26), we immediately see that $D < W$. To see that $D > 0$, note that this is the case exactly if

$$\rho W - \ln(1 + \rho b W) > 0, \tag{C.27}$$

for which to hold, it is sufficient that

$$\underbrace{\rho W - \ln(1 + \rho W)}_{=:\vartheta(\rho W)} > 0, \tag{C.28}$$

as $0 < b < 1$. Consider the l.h.s. of (C.28) as a function of ρW, denoted by $\vartheta(\rho W)$. Let $\rho W = 0$ first. Then $\vartheta(0) = 0 - \ln(1 + 0) = 0$. As we have

$$\frac{d\vartheta}{d(\rho W)} = 1 - \frac{1}{1 + \rho W} > 0, \tag{C.29}$$

$\vartheta(\rho W)$ is increasing in ρW. Hence, (C.28) is always satisfied and in turn the D we set is always greater than zero.

Setting A. With D set as discussed above, the equilibrium wage is given by (C.24). Together with the optimal labor demand (2.54), expected firm profit is

$$\begin{aligned}
\bar{s} &= Am^{1-b} - Wm + \hat{s} \\
&= A\left(\frac{A(1-b)}{A(1-b)\left(\frac{L}{aM}\right)^b}\right)^{\frac{1-b}{b}} - A(1-b)\left(\frac{L}{aM}\right)^b\left(\frac{A(1-b)}{A(1-b)\left(\frac{L}{aM}\right)^b}\right)^{\frac{1}{b}} + \hat{s} \\
&= A\left(\frac{aM}{L}\right)^{1-b} - A(1-b)\left(\frac{aM}{L}\right)^{1-b} + \hat{s} \\
&= bA\left(\frac{aM}{L}\right)^{1-b} + \hat{s}.
\end{aligned} \tag{C.30}$$

Again, consider an equilibrium $L_E^1 = a\bar{v}$, which requires $\bar{s} = \frac{\rho {\sigma_s}^2}{2a}$ and hence $\hat{s} = \frac{\rho {\sigma_s}^2}{2a} - bA\left(\frac{aM}{L}\right)^{1-b}$ in the noiseless case. If we set

$$A = \xi \cdot \frac{\rho {\sigma_s}^2}{2a} \frac{\left(\frac{L}{aM}\right)^{1-b}}{b}, \tag{C.31}$$

then at $L_E^1 = a\bar{v}$:

$$bA\left(\frac{aM}{L}\right)^{1-b} = \xi \cdot \frac{\rho {\sigma_s}^2}{2a} \tag{C.32}$$

and

$$\hat{s} = (1 - \xi) \cdot \frac{\rho {\sigma_s}^2}{2a}. \tag{C.33}$$

For $\xi = 0.5$, (C.32) and (C.33) are exactly equal to each other. The second row in the lower part of table 2.3 varies ξ around 0.5, so that A given by (C.31) ensures that the two terms are of comparable magnitude. Note that for $\xi = 0.5$ and $L_E^1 > a\bar{v}$ it is $bA\left(\frac{aM}{L}\right)^{1-b} < \hat{s}$ (as a higher L_E^1 is related to a higher $\hat{s}$), which is why we focus mainly on $\xi \geq 0.5$ and don't consider values for ξ below 0.25. Table C.11 shows that, in the simulation, $\hat{s}$ accounts for 40 to 50 percent of $\bar{s}$ at equilibrium with free OC, on average.

TABLE C.11: The Expected Fundamental's Part in Expected Firm Profit in the UE Model

$\frac{\sigma_v}{L/a}$	0.001	0.01	0.05	0.1	0.2	0.5
$\frac{\hat{s}}{\bar{s}}$	42.52%	42.53%	43.06%	43.92%	45.48%	49.44%

Simulation tables. The main results regarding the simulation of the UE model can be found in the right part of table 2.4 in the running text. We start with 1,041,792 parameter combinations for $\sigma_v = 0.001L/a$, but this number decreases to 265,712 combinations for $\sigma_v = 0.5L/a$. Analogous to table C.7 in the FE model, table C.12 gives information on the reasons why parameter combinations are eliminated in the simulation process. Tables C.13-C.14 are analogous to the respective tables for the basic and the FE model. Unsurprisingly, the difference between equilibrium and constrained optimum outcomes is a lot higher in the UE model (cf. Proposition 2.5.1).

TABLE C.12: Omitted Parameter Combinations in the Simulation of the UE Model

$\frac{\sigma_v}{L/a}$	# $\Delta(a\bar{v}) < \Gamma(a\bar{v})$ or $\Delta(L) > \Gamma(L)$	# mult. $L_E^{\mathbf{1}}$	# $V(\Psi) \geq 0.5$	# rem. cases
0.001	0	0	0	1,041,792
0.01	2,784	12	0	1,039,008
0.05	103,872	96	0	937,824
0.1	237,888	0	2,080 (+1,696)	801,824
0.2	424,896	0	14,368 (+2,416)	602,528
0.5	744,864	0	31,216 (+1,712)	265,712

TABLE C.13: Equilibrium vs. Constrained Optimum Outcomes in the UE Model

$\frac{\sigma_v}{L/a}$	$\Delta_m\left(L_E^1, \hat{L}_E^1\right)$	$\Delta_m\left(S^1\left(L_E^1\right), \hat{S}^1\right)$	$\Delta_m\left(L_E^0, \hat{L}_E^0\right)$	$\Delta_m\left(S^0\left(L_E^0\right), \hat{S}^0\right)$
0.001	67.88% (22.15%)	54.88% (26.64%)	1.90% (5.77%)	0.61% (2.06%)
0.01	55.00% (30.79%)	47.12% (30.97%)	1.90% (5.77%)	0.61% (2.07%)
0.05	49.48% (29.75%)	43.22% (30.60%)	2.10% (6.02%)	0.67% (2.17%)
0.1	47.62% (27.24%)	41.26% (29.60%)	1.94% (5.67%)	0.63% (2.09%)
0.2	47.37% (23.22%)	38.50% (24.89%)	2.27% (6.36%)	0.87% (3.54%)
0.5	56.71% (21.35%)	43.76% (30.66%)	2.22% (7.94%)	1.45% (6.99%)

Table C.15 is similar to table C.10 in the FE model. It gives detailed information on the three effects that apply when moving from equilibrium welfare with free OC $S^1(L_E^1)$ to maximum welfare with restricted OC $\hat{S}^0$. The first and second effect are again given by the already explained "i-effect" and "r-effect". The "e-effect" emerges,

TABLE C.14: Free OC vs. Restricted OC in the UE Model

$\frac{\sigma_v}{L/a}$	$\Delta_m\left(S^1(L_E^1), S^0(L_E^0)\right)$	$\Delta_m\left(\hat{S}^1, \hat{S}^0\right)$
0.001	79.46% (15.93%)	85.57% (9.40%)
0.01	79.90% (15.69%)	77.19% (14.12%)
0.05	80.59% (15.34%)	72.49% (13.78%)
0.1	80.96% (14.99%)	72.42% (12.93%)
0.2	82.08% (13.62%)	75.18% (12.32%)
0.5	85.47% (19.51%)	79.10% (18.54%)

as in the UE model the equilibrium outcome with restricted OC does not coincide with the constrained optimum. It is given by $\hat{S}^0 - S^0(L_E^0)$. By definition, this effect can never be negative. Concerning its relative magnitude, columns 6 & 7 in table C.15 show that it accounts for only about 0.7% of the total effect, on average. This is in part because both equilibrium and optimum L_E with restricted OC are in many cases just given by $L_E = L$. However, the effect remains at a low average of about 4.5% of the total effect even if cases with $L_E^0 = \hat{L}_E^0 = L$ are excluded from the calculations. The other numbers in table C.15 are of comparable magnitudes as in the basic and the FE model.

TABLE C.15: From $S^1(L_E^1)$ to $\hat{S}^0$ in the UE Model - Three Effects

$\frac{\sigma_v}{L/a}$	$L_E^0 > L_E^1$	i-effect> 0	r-effect> 0	e-effect> 0	$\frac{\text{i-effect}}{\text{total effect}}$	$\frac{\text{r-effect}}{\text{total effect}}$
0.001	100%	100%	100%	100%	54.26%	45.04%
0.01	100%	99.99%	100%	100%	52.17%	47.12%
0.05	100%	99.97%	99.99%	100%	47.37%	51.85%
0.1	100%	99.75%	99.93%	100%	44.62%	54.64%
0.2	100%	97.64%	99.52%	100%	40.00%	59.24%
0.5	100%	78.33%	98.63%	100%	20.50%	78.91%

Evidence that the approximation used for calculating the optimal mass of entrepreneurs in the noiseless UE model (cf. Appendix B.23) is a rather good one comes with table C.16. It shows that for low levels of noise volatility, the difference between the approximate $\hat{L}_E$ and the "true" $\hat{L}_E$ is rather small, on average.

TABLE C.16: Approximated vs. "True" Constrained Optimum in the Noiseless UE Model

$\frac{\sigma_v}{L/a}$	Δ_m("true" $\hat{L}_E^1$, approx. $\hat{L}_E^1$)	Δ_m("true" $\hat{L}_E^0$, approx. $\hat{L}_E^0$)
0.001	3.84%	0.21%
0.01	2.49%	0.21%

Note: Δ_m defined according to (C.16).

Appendix D

Matlab Code

D.1 Structure and Strategy

The simulation is written in Matlab version R2017a. As the number of simulated parameter combinations is quite large (more than 15 million in total) and computations include the solving of complicated highly non-linear equations (cf. Appendix B.5), it is important for the code to bring together precision and reliability with speed and efficient use of computer memory. We achieve this mainly by a combination of Matlab's symbolic math toolbox on the one hand and numeric arithmetic on the other hand. The remainder of this section explains the structure of the code and the applied strategy. The code itself is delegated to the subsequent sections.

The structure of the code is the same for all versions of the model. Take, e.g., the basic version of the model. The main code file is named "Basic_Model". It calls an auxiliary file named "symaux_Basic", where we use the symbolic toolbox to hand over our closed-form solutions for the agents' certainty equivalents (cf. Appendices B.5, B.6 and B.29) to Matlab and derive some other expressions for later use, e.g., the first and second derivative of the social welfare function. After that, "Basic_Model" sets up the matrix "paramCombs", which contains all parameter combinations that are to be simulated (cf. tables 2.1 and 2.3). Whenever we have to refer to values within this matrix, we take direct access to it. If, e.g., we want to access a value in row b, column c, we just call "paramCombs(b,c)". This may at times slightly reduce the readability of the code, but saves computer memory compared to the alternative options. After the parameter matrix is set up, we call function "Basic_Model_f1" for every value of σ_v. "Basic_Model_f1" is in fact an array function and has the advantage of being a lot faster than the combination of a "normal" function and a "for-loop". It takes as inputs the "paramCombs" matrix and the expressions derived within "symaux_Basic". As output, it returns the equilibrium and optimum values for the mass of entrepreneurs, the respective levels of social welfare and some other variables of interest. The remainder of "Basic_Model" simply processes this output and gives all the information illustrated in tables 2.2 and C.1-C.5 (and more).

As "Basic_Model_f1" is at the heart of the simulation and where essentially all computations take place, it makes sense to look at this function in some more detail. Prior to any calculations, it transforms the symbolic expressions from "symaux_Basic" into numeric functions of L_E only. This is essential for the speed of the simulation, as Matlab's numeric arithmetic is a lot faster than its symbolic one. The first computation within "Basic_Model_f1" concerns the equilibrium L_E^1 with free OC. We use function "fzero" to determine L_E^1, as it is more precise and reliable than possible alternatives (like, e.g., "fsolve"). However, solving for equilibrium is not completely straightforward, as L_E^1 is determined by $\Delta(L_E) = \Gamma(L_E)$, which is highly non-linear in L_E and entails the possibility of multiple solutions. Furthermore, both $\Delta(L_E)$ and $\Gamma(L_E)$ are almost kinked at $L_E = L$ for low σ_ν (cf. Chapter 2.6.1), which can cause numerical algorithms to fail in finding solutions to $\Delta(L_E) = \Gamma(L_E)$ that lie in the vicinity of $L_E = L$. We address these problems by using the function "rmsearch", developed by John D'Errico (2020) and adjusted for our purpose.[1] It identifies a multitude of good starting values for the numerical algorithm of "fzero" and by that makes sure that all solutions are found. Solving for equilibrium L_E^0 with restricted OC is more straightforward, as we have shown that it is unique (cf. Appendix B.6) and the respective expressions that determine equilibrium are continuous also for $\sigma_\nu = 0$.

Within "Basic_Model_f1", we call two auxiliary functions, namely "S1opt" and "S0opt", which help find the constrained optimum values of L_E. Function "S1opt" finds the constrained optimum L_E with regards to S^1 in case of free OC. As multiple solutions to the first order condition as well as non-existence are a possibility, we use the above mentioned combination of "rmsearch" and "fzero". Function "S0opt" does the analogous in case of restricted OC, which again is more straightforward, as we have shown that S^0 is inverse U-shaped (cf. Appendix C.2).

D.2 Basic Model

Code files are given in the following order:

1. The main file "Basic_Model".
2. The auxiliary file "symaux_Basic".
3. The function "Basic_Model_f1", which also contains the functions "S1opt" and "S0opt".
4. Function "rmsearch" can be found online (see the footnote in Chapter D.1). We only state the implemented adjustments.

All of the code is also submitted in digital form.

File "Basic_Model"

```
% Start timer:
tic
```

[1]John D'Errico (2020). RMSEARCH (https://www.mathworks.com/matlabcentral/fileexchange/13733-rmsearch), MATLAB Central File Exchange.

```

% Call auxiliary file "symaux_Basic", within which we state and derive closed
% form expressions required later for calculations within function
% "Basic_Model_f1":
symaux_Basic;

% Parameter values set as described in the text:
L=100;                                                    %(1)
a=10;                                                     %(2)
Vs=1;                                                     %(3)
rho=[0.01 0.025 0.05 0.075 0.1 0.25 0.5 0.75 1];          %(4)
Ve=[0.1 0.25 0.5 0.75 1];                                 %(5)
M=[1 2 3 5 10 100];                                       %(6)
nuq=[0.001 0.01 0.05 0.1 0.2 0.5];                        %(7)
sq=[0.01 0.05 0.1 0.25 0.5 0.75 0.9 0.95 0.99];           %(8)
N=[1 2 3 5 10 100];                                       %(9)
Vnu=[0.001 0.01 0.05 0.1 0.2 0.5];

% Initiate and fill in the matrix "paramCombs", which contains all
% parameter combinations; Each row of the final "paramCombs" matrix
% constitutes one parameter combination:
paramCombs=L';

avec=NaN(size(paramCombs,1), length(a));
for k=1:size(paramCombs,1)
    avec(k,:)=a;
end
avec=reshape(avec,[],1);

paramCombs=repmat(paramCombs,length(a),1);
paramCombs=[paramCombs avec];
clear avec

Vsvec=NaN(size(paramCombs,1), length(Vs));
for k=1:size(paramCombs,1)
    Vsvec(k,:)=Vs;
end
Vsvec=reshape(Vsvec,[],1);

paramCombs=repmat(paramCombs,length(Vs),1);
paramCombs=[paramCombs Vsvec];
clear Vsvec

rhovec=NaN(size(paramCombs,1), length(rho));
for k=1:size(paramCombs,1)
    rhovec(k,:)=rho*(4/(1.25^.5))*vpa(paramCombs(k,2)/(paramCombs(k,3)^.5));
end
rhovec=reshape(rhovec,[],1);

paramCombs=repmat(paramCombs,length(rho),1);
```

```
paramCombs=[paramCombs rhovec];
clear rhovec

Vevec=NaN(size(paramCombs,1), length(Ve));
for k=1:size(paramCombs,1)
    Vevec(k,:)=Ve*paramCombs(k,3);
end
Vevec=reshape(Vevec,[],1);

paramCombs=repmat(paramCombs,length(Ve),1);
paramCombs=[paramCombs Vevec];
clear Vevec

Mvec=NaN(size(paramCombs,1), length(M));
for k=1:size(paramCombs,1)
    Mvec(k,:)=M*paramCombs(k,1);
end
Mvec=reshape(Mvec,[],1);

paramCombs=repmat(paramCombs,length(M),1);
paramCombs=[paramCombs Mvec];
clear Mvec

nuqvec=NaN(size(paramCombs,1), length(nuq));
for k=1:size(paramCombs,1)
    nuqvec(k,:)=nuq*paramCombs(k,1)/paramCombs(k,2);
end
nuqvec=reshape(nuqvec,[],1);

paramCombs=repmat(paramCombs,length(nuq),1);
paramCombs=[paramCombs nuqvec];
clear nuqvec

sqvec=NaN(size(paramCombs,1), length(sq));
for k=1:size(paramCombs,1)
    sqvec(k,:)=vpa(paramCombs(k,4)*paramCombs(k,3)/(2*paramCombs(k,2)))+sq
        *vpa((paramCombs(k,1)-paramCombs(k,2)*paramCombs(k,7))*paramCombs(k
        ,4)*paramCombs(k,5)/(paramCombs(k,2)*(paramCombs(k,1)+paramCombs(k
        ,6))));
end
sqvec=reshape(sqvec,[],1);

paramCombs=repmat(paramCombs,length(sq),1);
paramCombs=[paramCombs sqvec];
clear sqvec

% Rule out multiple equilibria in case of deterministic noise:
for k=1:size(paramCombs,1)
    if 1/paramCombs(k,2)*(paramCombs(k,8)-paramCombs(k,4)*(paramCombs(k,3)
        +paramCombs(k,5))*(paramCombs(k,1)/paramCombs(k,2)-paramCombs(k,7))
        /(paramCombs(k,1)+paramCombs(k,6)))-(0.5/paramCombs(k,4))*log(1+
        paramCombs(k,3)/paramCombs(k,5))>0
```

```
            paramCombs(k,8)=nan;
        end
end

Nvec=NaN(size(paramCombs,1), length(N));
for k=1:size(paramCombs,1)
        Nvec(k,:)=0.25*N*(paramCombs(k,1)+paramCombs(k,6));
end
Nvec=reshape(Nvec,[],1);

paramCombs=repmat(paramCombs,length(N),1);
paramCombs=[paramCombs Nvec];
clear Nvec

disp(['# Parameter combinations for each \sigma_\nu at start of the simulation (incl. multipl. equ. in the noisless case):  ' num2str(size(paramCombs,1))]);

paramCombs=paramCombs(~isnan(paramCombs(:,8)),:);

disp(['# Parameter combinations for each \sigma_\nu at start of the simulation (excl. multipl. equ. in the noisless case):  ' num2str(size(paramCombs,1))]); %// number of parameter combinations which imply a unique equilbrium in the case of deterministic noise for each value of Vnu

timer=(1:size(paramCombs,1))';
paramCombs=[paramCombs timer];

Vnuvec=NaN(size(paramCombs,1),length(Vnu));
for k=1:size(paramCombs,1)
        Vnuvec(k,:)=Vnu.^2*(paramCombs(k,1)/paramCombs(k,2))^2;
end

toc

% Simulate the model for each \sigma_\nu:
for i=1:size(Vnuvec,2)
    disp(['\sigma_\nu= ' num2str(double(Vnu(i)*100)) '% of L/a']);
    paramCombsci=num2cell([paramCombs Vnuvec(:,i)]);

% Call function "Basic_Model_f1" for each parameter combination; the
% "r.h.s." gives the function inputs, the "l.h.s." gives the function
% outputs:
[LE_T, LE_U, S_T, S_U, S_T_deriv, max_LE_U, SU_max_LE_U, Vbl, RVP, multequ,renT,renE,VrenE,max_LE_S,ST_max_LE_T,max_LE_S_global,ST_max_LE_T_global,SU_LET,UM,VUM,SL,wb,utiE,utiM,Vbl_one] = arrayfun(@(n) Basic_Model_f1(paramCombsci{n,:},Vbn,Fn,VFn,SWFn,dSWFn,d2SWFn,SWF2n,rentEn,rentTn,VPn,GTin,VGTin,dVbn), 1:size(paramCombsci,1), 'uni', 1);

toc

```

```
% Get rid of the parameter combinations that led to nan-values
% within "Basic_Model_f1":
paramCombsci=paramCombsci(~isnan(LE_T),:);
LE_T2=LE_T(~isnan(max_LE_S));
LE_U2=LE_U(~isnan(max_LE_U));
S_T2=S_T(~isnan(ST_max_LE_T));
S_U2=S_U(~isnan(SU_max_LE_U));
SL2=SL(~isnan(ST_max_LE_T_global));
LE_T=LE_T(~isnan(LE_T));
LE_U=LE_U(~isnan(LE_U));
S_T=S_T(~isnan(S_T));
S_U=S_U(~isnan(S_U));
S_T_deriv=S_T_deriv(~isnan(S_T_deriv));
max_LE_U=max_LE_U(~isnan(max_LE_U));
SU_max_LE_U=SU_max_LE_U(~isnan(SU_max_LE_U));
Vbl=Vbl(~isnan(Vbl));
RVP=RVP(~isnan(RVP));
multequ=multequ(~isnan(multequ));
renT=renT(~isnan(renT));
renE=renE(~isnan(renE));
VrenE=VrenE(~isnan(VrenE));
max_LE_S=max_LE_S(~isnan(max_LE_S));
ST_max_LE_T=ST_max_LE_T(~isnan(ST_max_LE_T));
max_LE_S_global=max_LE_S_global(~isnan(max_LE_S_global));
ST_max_LE_T_global=ST_max_LE_T_global(~isnan(ST_max_LE_T_global));
SU_LET=SU_LET(~isnan(SU_LET));
UM=UM(~isnan(UM));
VUM=VUM(~isnan(VUM));
SL=SL(~isnan(SL));
wb=wb(~isnan(wb));
utiE=utiE(~isnan(utiE));
utiM=utiM(~isnan(utiM));
Vbl_one=Vbl_one(~isnan(Vbl_one));

% Restrict to combinations for which a local maximum for S^1 exists:
LE_Ta2=LE_T2(max_LE_S~=-1);
max_LE_Sa2=max_LE_S(max_LE_S~=-1);
ST_max_LE_Ta2=ST_max_LE_T(max_LE_S~=-1);
S_Ta2=S_T2(max_LE_S~=-1);

% Auxiliary variables:
aux0=abs(ST_max_LE_Ta2-S_Ta2)./max(abs(ST_max_LE_Ta2),abs(S_Ta2));
aux=aux0(sign(ST_max_LE_Ta2)==sign(S_Ta2));
aux00=(abs(SU_max_LE_U-S_U2))./max(abs(SU_max_LE_U),abs(S_U2));
aux1=aux00(sign(SU_max_LE_U)==sign(S_U2));

% Combinations dropped in the simulation process:
display(['# \Delta(a*\bar \nu)<\Gamma(a*\bar \nu) or \Delta(L)>\Gamma(L) - omitted: ' num2str(length(wb))])
display(['# Multiple equilibria L_E^1 - omitted: ' num2str(length(multequ))]);
```

```
display(['# V(\Psi)>0.5 in L_E^1 or L_E^0 - omitted: ' num2str(length(Vbl))]);
display(['# V(\Psi)<0.5 in L_E^1 and L_E^0, but V(\Psi)>0.5 for some other L_E - omitted for maximizations: ' num2str(length(Vbl_one))]);

% Admissible combinations for checking the effect of a marginal increase in
% L_E (starting from equilibrium) and whether social welfare is higher in
% equilibrium with free or restricted OC:
display(['# Combinations left (without maximizations): ' num2str(length(LE_T))]);

% Admissible combinations  for comparing equilibrium values to optimum values:
display(['# Combinations left (with maximizations): ' num2str(length(max_LE_S_global))]);

disp('--------------------------------------------------------------------------------');

display(['mean, std and max of (VP-VP0)/VP0 in %:  ' num2str(mean(RVP)*100) '   ' num2str(std(RVP)*100) '   ' num2str(max(RVP)*100)]);

disp('--------------------------------------------------------------------------------');

disp(['L_E^0 > L_E^1 in % :  ' num2str((length(LE_T)-length(find(LE_T>LE_U)))/length(LE_T)*100) '%']);
disp(['S^0 > S^1 at equilibrium in % :  ' num2str((length(S_T)-length(find(S_T>S_U)))/length(S_T)*100) '%']);
disp(['S^0 > S^1 at constrained (global) optimum in % :  ' num2str((length(ST_max_LE_T_global)-length(find(ST_max_LE_T_global-SU_max_LE_U>1e-4)))/length(ST_max_LE_T_global)*100) '%']);

disp('--------------------------------------------------------------------------------');

disp(['S^1'' > -1e-3 in % :  ' num2str((length(S_T_deriv)-length(find(S_T_deriv<-1e-3)))/length(S_T_deriv)*100) '%']);
disp(['S^1'' > -1e-6 in % :  ' num2str((length(S_T_deriv)-length(find(S_T_deriv<-1e-6)))/length(S_T_deriv)*100) '%']);
disp(['S^1'' > 0 in % :  ' num2str((length(S_T_deriv)-length(find(S_T_deriv<0)))/length(S_T_deriv)*100) '%']);

ela=abs(S_T_deriv.*LE_T./S_T);
disp(['mean, std of S^1'' * L_E^1/S^1 (elasticity at equilibrium L_E^1): ' num2str(mean(ela)) '   ' num2str(std(ela))]);

disp('--------------------------------------------------------------------------------');

```

```
display(['dealers/entrepreneurs equilibrium rents (equal to GI) with free OC as a fraction of their utility: ' num2str(mean(renE./(renE+UM))) ' ' num2str(std(renE./(renE+UM))) ]);
display(['dealers/entrepreneurs equilibrium rents with restricted OC as a fraction of their utility: ' num2str(mean(VrenE./(VrenE+VUM))) ' ' num2str(std(VrenE./(VrenE+VUM))) ]);

display(['A hipo''s CE over a uninformed investor''s CE in equilibrium with free OC: ' num2str(mean(utiE./(utiM))) ' ' num2str(std(utiE./(utiM))) ]);

disp('------------------------------------------------------------------------');

% Auxiliary variables:
S_Ua=S_U(sign(S_U)==sign(S_T));
S_Ta=S_T(sign(S_U)==sign(S_T));

display(['mean, std of |S^0-S^1|/max(|S^0|,|S^1|) at equilibrium in % (different sign cases omitted): ' num2str(mean(abs(S_Ta-S_Ua)./max(abs(S_Ta),abs(S_Ua)))*100) ' ' num2str(std(abs(S_Ta-S_Ua)./max(abs(S_Ta),abs(S_Ua)))*100) ]);
display(['# combinations used for the above: ' num2str(length(S_Ta))])

% Auxiliary variables:
SU_max_LE_Ua2=SU_max_LE_U(max_LE_S~=-1);
aux0a=abs(SU_max_LE_Ua2-ST_max_LE_Ta2)./max(abs(SU_max_LE_Ua2),abs(ST_max_LE_Ta2));
aux2=aux0a(sign(SU_max_LE_Ua2)==sign(ST_max_LE_Ta2));

% \hat indicates the respective constrained optimum value (see text)
display(['|\hat S^0 - \hatS^1|/max(|\hat S^0|,|\hat S^1|) at constrained (local) optimum in % (different sign cases omitted, local opt. L_E^1 exists): ' num2str(mean(aux2)*100) ' ' num2str(std(aux2)*100) ]);
display(['# combinations used for the above: ' num2str(length(aux2))])

% Auxiliary variables:
S_T2a=S_T2(sign(S_T2)==sign(SU_max_LE_U));
SU_max_LE_Ua=SU_max_LE_U(sign(S_T2)==sign(SU_max_LE_U));

% \hat indicates the respective constrained optimum value (see text)
display(['mean, std of |\hat S^0-S^1|/max(|\hat S^0|,|S^1|) with S^1 at equilibrium and S^0 at its constrained optimum in % (different sign cases omitted): ' num2str(mean(abs(S_T2a-SU_max_LE_Ua)./max(abs(S_T2a),abs(SU_max_LE_Ua)))*100) ' ' num2str(std(abs(S_T2a-SU_max_LE_Ua)./max(abs(S_T2a),abs(SU_max_LE_Ua)))*100) ]);
display(['# combinations used for the above: ' num2str(length(S_T2a))])

disp('------------------------------------------------------------------------');
```

```
disp(['i-effect >0 in %:  ' num2str(length(find(SU_LET>S_T))/length(LE_T)*100) '%']);
disp(['r-effect >0 in %:  ' num2str(length(find(S_U>SU_LET))/length(LE_T)*100) '%']);

display(['i-effect as share of the total effect (restriction: both effects positive): ' num2str(mean((SU_LET((SU_LET-S_T)>0 & (S_U-SU_LET)>0)-S_T((SU_LET-S_T)>0 & (S_U-SU_LET)>0))./(S_U((SU_LET-S_T)>0 & (S_U-SU_LET)>0)-S_T((SU_LET-S_T)>0 & (S_U-SU_LET)>0))))  ])
display(['# combinations used for the above:  '  num2str(length((SU_LET((SU_LET-S_T)>0 & (S_U-SU_LET)>0)-S_T((SU_LET-S_T)>0 & (S_U-SU_LET)>0))))])

disp('--------------------------------------------------------------------------------');

% \hat indicates the respective constrained optimum value (see text)
display(['mean, std of |\hat L_E^1 - L_E^1|/max(|.|,|.|) - in % (only local optima):  ' num2str(mean(abs(max_LE_Sa2-LE_Ta2)./max(LE_Ta2,max_LE_Sa2))*100) '   ' num2str(std(abs(max_LE_Sa2-LE_Ta2)./max(LE_Ta2,max_LE_Sa2))*100)])
display(['# combinations used for the above:  ' num2str(length(LE_Ta2))]);

display(['mean, std of |\hat S^1 - S^1|/max(|.|,|.|) - in % (different sign cases omitted, only local optima):  ' num2str(mean(aux)*100) '   ' num2str(std(aux)*100)]);
display(['# combinations used for the above:  ' num2str(length(aux))]);

display(['mean, std of |\hat L_E^0 - L_E^0|/max(|.|,|.|) - in %:  ' num2str(mean((abs(max_LE_U-LE_U2)./max(LE_U2,max_LE_U)))*100) '   ' num2str(std((abs(max_LE_U-LE_U2)./max(LE_U2,max_LE_U)))*100)]);

display(['mean, std of |\hat S^0 - S^0|/max(|.|,|.|) - in % (different sign cases omitted):  ' num2str(mean(aux1)*100) '   ' num2str(std(aux1)*100)]);
display(['# combinations used for the above:  ' num2str(length(aux1))]);

end

clear aaa hh diff sh Vs L a Ve M N rho nuq i jj k paramCombsc dSWF SWFn dSWFn d2SWFn dSWF2n SWF2n Vbn Fn VFn rentEn rentTn dVbn rentMn s_T_d sqn VPn wagen timer errorflag;

toc
```

File "symaux_Basic"

```
tic

% Create symbolic variables in Matlab:
syms L M N sq Vs Ve rho nq a LE Vn

```

```
% Expressions from the text:
alpha=(L-LE)/(rho*Ve);
gam=1/(alpha^2*Vs+Vn);
VphiGw=Ve+Vs*Vn*gam;
beta=(LE+M)/(rho*VphiGw);
VsGw=VphiGw-Ve;

% Free OC:
EP=sq-(LE/a-nq)/(alpha+beta);
VP = ((Vs*alpha*beta*gam+1)/(alpha+beta))^2/gam;
Ez = (LE/a-nq)/((alpha+beta)*(2*rho*(Vn*Vs*gam+Ve))^.5);
Vz = gam*Vn^2/((2*rho*(Vn*Vs*gam+Ve))*(alpha+beta)^2);
GP = (EP-.5*rho*VP/a)/a;
CovPz = -(VP*Vz)^.5;
GTu = (Ez-rho*CovPz/a)^2/(1+2*rho*Vz)+(.5/rho)*log(1+2*rho*Vz);
GI = (.5/rho)*log(1+VsGw/Ve);
GTi = Ez^2/(1+2*rho*Vz)+(.5/rho)*log(1+2*rho*Vz);

% Restricted OC:
VEP = sq-rho*(Vs+Ve)*(LE/a-nq)/(L+M);
VVP = Vn*rho^2*(Vs+Ve)^2/(L+M)^2;
VEz = rho^.5*(Vs+Ve)^.5*(LE/a-nq)/((L+M)*2^.5);
VVz = Vn*rho*(Vs+Ve)/(2*(L+M)^2);
VCovPz = -(VVP*VVz)^.5;
VGTi = VEz^2/(1+2*rho*VVz)+(.5/rho)*log(1+2*rho*VVz);
VGTu = (VEz-rho*VCovPz/a)^2/(1+2*rho*VVz)+(.5/rho)*log(1+2*rho*VVz);
VGP = (VEP-.5*rho*VVP/a)/a;

% Free OC;
% b corresponds to \Psi in the text,
% c corresponds to \Phi in the text:
CovPn = Vn*(VP*gam)^.5;
VtmP = Vs+Ve+VP*(-Vn*gam+1)-2*alpha*Vs*(VP*gam)^.5;
Eb = rho^.5*nq*((1/2)*rho*VtmP+N*CovPn/Vn)^.5/N;
Vb = (Eb/nq)^2*Vn;
Ec = -rho*nq*(sq-EP+CovPn*nq/Vn)/N;
Vc = (Ec/nq)^2*Vn;
Covbc = (Eb/nq*(Ec/nq))*Vn;
UN = .5*log(1-2*Vb)/rho-Ec/rho-.5*Vc/rho-(Eb+Covbc)^2/(rho*(1-2*Vb));
UM = GTi;

% Restricted OC:
VCovPn = rho*(Vs+Ve)*Vn/(L+M);
VVtmP = Vs+Ve;
VEb = rho^.5*nq*((1/2)*rho*VVtmP+N*VCovPn/Vn)^.5/N;
VVb = (VEb/nq)^2*Vn;
VEc = -rho*nq*(sq-VEP+VCovPn*nq/Vn)/N;
VVc = (VEc/nq)^2*Vn;
VCovbc = (VEb/nq*(VEc/nq))*Vn;
VUN = .5*log(1-2*VVb)/rho-VEc/rho-.5*VVc/rho-(VEb+VCovbc)^2/(rho*(1-2*VVb)
    );
VUM = VGTi;
```

```
% Social welfare:
SWF = LE*(GP+GTu)+(L-LE)*(GTi+GI)+N*UN+M*UM;
SWF2 = LE*(VGP+VGTu)+(L-LE)*VUM+N*VUN+M*VUM;

% First and second derivatives:
dVb=diff(Vb,LE);
dSWF=diff(SWF,LE);
d2SWF=diff(dSWF,LE);

% \Delta(L_E)-\Gamma(L_E):
F=GP+GTu-GI-GTi;
VF=VGP+VGTu-VGTi;

% "Rents":
rentE=GP+GTu-GTi;
rentT=GI;

% Transform expressions into 'character' form:
SWFn=char(SWF);
SWF2n=char(SWF2);
dVbn=char(dVb);
Vbn=char(Vb);
dSWFn=char(dSWF);
d2SWFn=char(d2SWF);
Fn=char(F);
VFn=char(VF);
rentTn=char(rentT);
rentEn=char(rentE);
VPn=char(VP);
GTin=char(GTi);
VGTin=char(VGTi);

clear L M N sq Vs Ve rho nq a LE Vn alpha beta gam VphiGw VsGw EP Ez VP Vz GP GTu GTi GI CovPz VEP VEz VVP VVz VGP VGTu VGTi VGI VCovPz Vb Vc Covbc Eb Ec Vb Vc Covbc VEb VEc VVb VVc VCovbc SWF SWF2 dSWF dSWF2 F VF VCovPn VUM VUN UM UN dVb rentT rentE SWFnN SWF2nN d2SWF dSWFnN

toc
```

Function "Basic_Model_f1"

```
% This is function "Basic_Model_f1". The r.h.s. shows function inputs, the
% l.h.s. shows function outputs:
function [equ_T, equ_U, s_T, s_U, s_T_d, maxLEU, SU_maxLEU, Vbl, rVP, multequ,rT,rE,VrE,maxLES,ST_maxLET,maxLESglobal,ST_maxLETglobal,SU_LET,UM,VUM,SL,wb,utiE,utiM,Vbl_one] = Basic_Model_f1(L,a,Vs,rho,Ve,M,nq,sq,N,t,Vn,Vbn,Fn,VFn,SWFn,dSWFn,d2SWFn,SWF2n,rentEn,rentTn,VPn,GTin,VGTin,dVbn)

% Show simulation progress in the command window:
if mod(t,10000)==0
```

```
display(num2str(t));
toc
else
end

x0=double(L/2);

% Transform 'F' into a function of L_E:
F=eval(['@(LE)' Fn]);

% Set up the function's output variables:
equ_T=NaN;
s_T=NaN;
s_T_d=NaN;
equ_U=NaN;
s_U=NaN;
maxLEU=NaN;
SU_maxLEU=NaN;
Vbl=NaN;
rVP=NaN;
multequ=NaN;
rT=NaN;
rE=NaN;
VrE=NaN;
maxLES=NaN;
ST_maxLET=NaN;
maxLESglobal=NaN;
ST_maxLETglobal=NaN;
SU_LET=NaN;
UM=NaN;
VUM=NaN;
SL=NaN;
wb=NaN;
utiE=NaN;
utiM=NaN;
Vbl_one=NaN;


if F(a*nq)<0 || F(L)>0
    wb=1;
else
% Solve for equilibrium L_E^1; the function "rmsearch" finds good starting points and search intervals:
    [u, ~, errorflag]=rmsearch(F,'fzero',x0,0,L,'InitialSample',100);

        for k1=1:length(errorflag)
        if errorflag(k1)<0
           u(k1)=[];
        end
        end
% Drop the combination, if it implies multiple equilibria:
if length(u)>1
```

```
        multequ=1;
else

Vb=eval(['@(LE)' Vbn]);
Vbu=Vb(u);
VbL=Vb(L); % Vb in case of restriced OC is indep. of L_E

% Drop the combination, if it does not imply well defined noise trader utiltiy:
if Vbu>=0.5 || VbL>=0.5
    Vbl=1;

else

            equ_T=u; % the unique L_E^1

            VF = eval(['@(LE)' VFn]);
            VF0=VF(0);
            VFL=VF(L);
            if sign(VF0)==sign(VFL) && sign(VFL)>0 % no interior solution exists (VF is strictly decreasing, see text)
                equ_U=L; % corner equilibrium L_E^0=L
            elseif sign(VF0)==sign(VFL) && sign(VF0)<0
                equ_U=0;
                disp('Warning: L_E^0=0');
            else
            % Solve for interior equilibrium L_E^0:
            [equ_U, ~, errorflag]=fzero(VF,[0 L]);  % Sufficient, as we know that L_E^0 is unique (see text)

                  if errorflag<0
                    equ_U=[];
                  end
                  if isempty(equ_U)
                     disp('Error: No equ_U')
                  end
            end

            % Transform expressions into functions of L_E:
            VP=eval(['@(LE)' VPn]);
            rentT=eval(['@(LE)' rentTn]);
            rentE=eval(['@(LE)' rentEn]);
            GTi=eval(['@(LE)' GTin]);
            VGTi=eval(['@(LE)' VGTin]);

            VP0=Vs; % Ex-ante price variance in the noiseless case with free OC

            rVP=(VP(equ_T)-VP0)/VP0;

            rT=rentT(equ_T);
            rE=rentE(equ_T);
```

```

                utiE=rE+GTi(equ_T); % CE_E
                utiM=GTi(equ_T);    % CE_M

                UM=GTi(equ_T);

                % Transform expressions into functions of L_E:
                SWF=eval(['@(LE)' SWFn]);
                SWF2=eval(['@(LE)' SWF2n]);
                dSWF=eval(['@(LE)' dSWFn]);
                d2SWF=eval(['@(LE)' d2SWFn]);

                s_T=SWF(equ_T); % equilibrium social welfare with free OC
                s_T_d=dSWF(equ_T); % slope of the social welfare function with free OC at equilibrium L_E^1
                SL=SWF2(L); % social welfare at L_E=L

                VrE=VF(equ_U);
                VUM=VGTi(equ_U);

                s_U=SWF2(equ_U); % equilibrium social welfare with restricted OC
                SU_LET=SWF2(equ_T); % social welfare with restricted OC, evaluated at L_E=L_E^1

        % For maximization of social welfare, noise trader utility has to
        % be defined for all L_E; Hence, we drop all combinations which
        % imply that Vb is greater than 0.5 for any L_E
        dVb=eval(['@(LE)' dVbn]);
        [Vb1, ~, errorflag0]=rmsearch(dVb,'fzero',x0,0,L,'InitialSample',100);
                for k0=1:length(errorflag0)
                if errorflag0(k0)<0
                    Vb1(k0)=[];
                else
                end
                end

                if isempty(Vb1)
                   Vbmaxvalgl=max([Vb(0) Vb(L)]);
                elseif length(Vb1)>1
                    disp('Vb more than one extremum')
                    Vbmaxvalgl=max([Vb(0) Vb(L) Vb(Vb1)]);
                elseif dVb(Vb1+1e-12)>0
                    disp('Vb has a minimum')
                    Vbmaxvalgl=max([Vb(0) Vb(L) Vb(Vb1)]);
                else
                    Vbmaxvalgl=max([Vb(0) Vb(L) Vb(Vb1)]);
                end
        if Vbmaxvalgl>=0.5 % there is an L_E for which Vb>0.5
            Vbl_one=1;
        else
```

```
                % Call function "S1opt" (see below), which finds the local
                % as well as the global maximum of S^1:
                [maxLES, maxLESglobal]=S1opt(dSWF,d2SWF);

                ST_maxLET=SWF(maxLES); % local maximum social welfare S^1
                ST_maxLETglobal=SWF(maxLESglobal); % global maximum social welfare S^1

                % Call function "S0opt" (see below), which finds the
                % maximum of S^0:
                maxLEU=S0opt(SWF2);

                SU_maxLEU=SWF2(maxLEU); % maximum social welfare S^0
        end
end
end
end

% Auxiliary functions "S1opt" and "S0opt" below

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% function "S1opt" finds the L_E that (locally/globally) maximizes S^1:
function [maxLES,maxLESglobal] = S1opt(dSWF, d2SWF)

            [j, ~, errorflag]=rmsearch(dSWF,'fzero',x0,0,L,'InitialSample',100);
                for k2=1:length(errorflag)
                if errorflag(k2)<0
                    j(k2)=[];
                else
                end
                end

                if isempty(j) % no local extremum
                    maxLES=-1;
                    jvalaux=[SWF(0) SWF(L)];
                    jaux=[0 L];
                    maxLESglobal=jaux(jvalaux==max(jvalaux));
                else
                   % check if local extremum is a maximum:
                   jval=NaN(length(j),1);
                   d2aux=NaN(length(j),1);
                   for i1=1:length(j)
                       d2aux(i1)=d2SWF(j(i1));
                       jval(i1)=SWF(j(i1));
                   end
                   j=j(d2aux<0); % keep only maxima
                   jval=jval(d2aux<0);
```

```
                    jvalaux=[jval' SWF(0) SWF(L)];
                    jaux=[j' 0 L];
                    maxLESglobal=jaux(jvalaux==max(jvalaux));

                    if isempty(j) % no local maximum
                         maxLES=-1;
                    elseif length(j)==1
                         maxLES=j;
                    else
                         % in case there are multiple local maxima, choose
                              the one closest to equilibrium L_E^1:
                         lowdis=zeros(length(j),1);
                         for k3=1:length(j)
                              lowdis(k3)=abs(equ_T-j(k3));
                         end
                         maxLES=j(lowdis==min(lowdis));
                    end
               end

end



% function "S0opt" finds the L_E that maximizes S^0:
function maxLEU = S0opt(SWF2)

          opts2 = optimset('TolX',1e-6); % default TolX is 1e-4
          mSWF2=@(LE)-SWF2(LE);
          % fminbnd finds the min of -SWF2 (local if it exists, global
          % otherwise):
          [e, ~, errorflag]=fminbnd(mSWF2,0,L, opts2);

               for k4=1:length(errorflag)
               if errorflag(k4)<0
                    e(k4)=[];
               else
               end
               end
       % S^0 is inverse U-shaped (see text):
       if e>L-1e-3 % no local minimum has been found
                    maxLEU=L;
       elseif e<0+1e-3 % no local minimum has been found
                    maxLEU=0;
                    disp('optimal L_E equals zero')
       else
                    maxLEU=e;
       end

end


end
```

Adjustemts to function "rmsearch"

...

```
    % any crossings found?
    if isempty(k)
        xfinal=NaN;
        ffinal=NaN;
        exitflag=-10;
    else
    xfinal = zeros(length(k),1);
    ffinal = xfinal;
    exitflag = xfinal;
    xstart = xfinal;
    fstart = xfinal;
    % loop over the candidate intervals
    for i = 1:length(k)
      [xfinal(i),ffinal(i),exitflag(i)] = fzero(fun, ...
        [xinitial(k(i)),xinitial(k(i)+1)],par.Options);

      % store the better of the two points in each bracket
      if abs(finitial(k(i))) <= abs(finitial(k(i)+1))
        xstart(i) = xinitial(k(i));
        fstart(i) = finitial(k(i));
      else
        xstart(i) = xinitial(k(i)+1);
        fstart(i) = finitial(k(i)+1);
      end
    end
    end
```

...

D.3 Full Employment Model

Code files are given in the same order as for the basic version of the model. All of the code is also submitted in digital form.

File "FE_Model"

```
% Start timer:
tic

% Call auxiliary file "symaux_FE", within which we state and derive closed
% form expressions required later for calculations within function
% "FE_Model_f1":
symaux_FE;

% Parameter values set as described in the text:
L=100;                                                        %(1)
a=10;                                                         %(2)
```

```
Vs=1;                                                    %(3)
rho=[0.01 0.025 0.05 0.075 0.1 0.25 0.5 0.75 1];         %(4)
Ve=[0.1 0.25 0.5 0.75 1];                                %(5)
M=[1 2 3 5 10 100];                                      %(6)
nuq=[0.001 0.01 0.05 0.1 0.2 0.5];                       %(7)
b=[0.1 0.25 0.4 0.55];                                   %(8)
A=[0.25 0.5 0.75 1];                                     %(9)
sh=[0.01 0.05 0.1 0.25 0.5 0.75 0.9 0.95 0.99];          %(10)
N=[1 2 3 5 10 100];                                      %(11)
Vnu=[0.001 0.01 0.05 0.1 0.2 0.5];

% Initiate and fill in the matrix "paramCombs", which contains all
% parameter combinations; Each row of the final "paramCombs" matrix
% constitutes one parameter combination:
paramCombs=L';

avec=NaN(size(paramCombs,1), length(a));
for k=1:size(paramCombs,1)
    avec(k,:)=a;
end
avec=reshape(avec,[],1);

paramCombs=repmat(paramCombs,length(a),1);
paramCombs=[paramCombs avec];
clear avec

Vsvec=NaN(size(paramCombs,1), length(Vs));
for k=1:size(paramCombs,1)
    Vsvec(k,:)=Vs;
end
Vsvec=reshape(Vsvec,[],1);

paramCombs=repmat(paramCombs,length(Vs),1);
paramCombs=[paramCombs Vsvec];
clear Vsvec

rhovec=NaN(size(paramCombs,1), length(rho));
for k=1:size(paramCombs,1)
    rhovec(k,:)=rho*(4/(1.25^.5))*vpa(paramCombs(k,2)/(paramCombs(k,3)^.5));
end
rhovec=reshape(rhovec,[],1);

paramCombs=repmat(paramCombs,length(rho),1);
paramCombs=[paramCombs rhovec];
clear rhovec

Vevec=NaN(size(paramCombs,1), length(Ve));
for k=1:size(paramCombs,1)
    Vevec(k,:)=Ve*paramCombs(k,3);
end
Vevec=reshape(Vevec,[],1);
```

```
paramCombs=repmat(paramCombs,length(Ve),1);
paramCombs=[paramCombs Vevec];
clear Vevec

Mvec=NaN(size(paramCombs,1), length(M));
for k=1:size(paramCombs,1)
    Mvec(k,:)=M*paramCombs(k,1);
end
Mvec=reshape(Mvec,[],1);

paramCombs=repmat(paramCombs,length(M),1);
paramCombs=[paramCombs Mvec];
clear Mvec

nuqvec=NaN(size(paramCombs,1), length(nuq));
for k=1:size(paramCombs,1)
    nuqvec(k,:)=nuq*paramCombs(k,1)/paramCombs(k,2);
end
nuqvec=reshape(nuqvec,[],1);

paramCombs=repmat(paramCombs,length(nuq),1);
paramCombs=[paramCombs nuqvec];
clear nuqvec

bvec=NaN(size(paramCombs,1), length(b));
for k=1:size(paramCombs,1)
    bvec(k,:)=b;
end
bvec=reshape(bvec,[],1);

paramCombs=repmat(paramCombs,length(b),1);
paramCombs=[paramCombs bvec];
clear bvec

Avec=NaN(size(paramCombs,1), length(A));
for k=1:size(paramCombs,1)
    Avec(k,:)=A*0.5*(paramCombs(k,4)*paramCombs(k,3))/(paramCombs(k,2)*(
        paramCombs(k,8)))*(paramCombs(k,6)/paramCombs(k,7))^(paramCombs(k
        ,8)-1);
end
Avec=reshape(Avec,[],1);

paramCombs=repmat(paramCombs,length(A),1);
paramCombs=[paramCombs Avec];
clear Avec

shvec=NaN(size(paramCombs,1), length(sh));
for k=1:size(paramCombs,1)
```

```
    shvec(k,:)=vpa(paramCombs(k,4)*paramCombs(k,3)/(2*paramCombs(k,2))-((paramCombs(k,8))*paramCombs(k,9)*(paramCombs(k,6)/paramCombs(k,7))^(1-paramCombs(k,8))))+sh*vpa((paramCombs(k,1)-paramCombs(k,2)*paramCombs(k,7))*paramCombs(k,4)*paramCombs(k,5)/(paramCombs(k,2)*(paramCombs(k,1)+paramCombs(k,6)))+((paramCombs(k,8))*paramCombs(k,9)*(paramCombs(k,6)/paramCombs(k,7))^(1-paramCombs(k,8)))-((paramCombs(k,8))*paramCombs(k,9)*(paramCombs(k,2)*paramCombs(k,6)/paramCombs(k,1))^(1-paramCombs(k,8))));
end
shvec=reshape(shvec,[],1);

paramCombs=repmat(paramCombs,length(sh),1);
paramCombs=[paramCombs shvec];
clear shvec

% Rule out multiple equilibria in case of deterministic noise:
for k=1:size(paramCombs,1)
    if paramCombs(k,4)*(paramCombs(k,10)+paramCombs(k,9)*(paramCombs(k,8))*(paramCombs(k,2)*paramCombs(k,6)/paramCombs(k,1))^(1-paramCombs(k,8))-paramCombs(k,4)*(paramCombs(k,3)+paramCombs(k,5))*(paramCombs(k,1)/paramCombs(k,2)-paramCombs(k,7))/(paramCombs(k,1)+paramCombs(k,6)))/paramCombs(k,2)-.5*log(1+paramCombs(k,3)/paramCombs(k,5))>0
        paramCombs(k,10)=nan;
    end
end

Nvec=NaN(size(paramCombs,1), length(N));
for k=1:size(paramCombs,1)
    Nvec(k,:)=0.25*N*(paramCombs(k,1)+paramCombs(k,6));
end
Nvec=reshape(Nvec,[],1);

paramCombs=repmat(paramCombs,length(N),1);
paramCombs=[paramCombs Nvec];
clear Nvec

disp(['# Parameter combinations for each \sigma_\nu at start of the simulation (incl. multipl. equ. in the noisless case): ' num2str(size(paramCombs,1))]);

paramCombs=paramCombs(~isnan(paramCombs(:,10)),:);

disp(['# Parameter combinations for each \sigma_\nu at start of the simulation (excl. multipl. equ. in the noisless case): ' num2str(size(paramCombs,1))]);

timer=(1:size(paramCombs,1))';
paramCombs=[paramCombs timer];

Vnuvec=NaN(size(paramCombs,1),length(Vnu));
for k=1:size(paramCombs,1)
    Vnuvec(k,:)=Vnu.^2*(paramCombs(k,1)/paramCombs(k,2))^2;
```

```
end

toc

% Simulate the model for each \sigma_\nu:
for i=1:size(Vnuvec,2)
    disp(['\sigma_\nu= ' num2str(double(Vnu(i)*100)) '% of L/a']);
    paramCombsci=num2cell([paramCombs Vnuvec(:,i)]);

% Call function "FE_Model_f1" for each parameter combination; the
% "r.h.s." gives the function inputs, the "l.h.s." gives the function
% outputs:
[LE_T, LE_U, S_T, S_U, S_T_deriv, max_LE_U,SU_max_LE_U,Vbl,RVP,multequ, renT,renE,renM,VrenE,VrenM,max_LE_S,ST_max_LE_T,max_LE_S_global, ST_max_LE_T_global,GTi,VGTi,SU_LET,SL,wb,sqq,utiE,utiM,Vbl_one] = arrayfun(@(n) FE_Model_f1(paramCombsci{n,:},Vbn,Fn,VFn,SWFn,dSWFn, d2SWFn,SWF2n,rentEn,rentTn,rentMn,VPn,sqn,GTin,VGTin,dVbn), 1:size( paramCombsci,1), 'uni', 1);

toc

% Get rid of the parameter combinations that led to nan-values
% within "FE_Model_f1":
paramCombsci=paramCombsci(~isnan(LE_T),:);
LE_T2=LE_T(~isnan(max_LE_S));
LE_U2=LE_U(~isnan(max_LE_U));
S_T2=S_T(~isnan(ST_max_LE_T));
S_U2=S_U(~isnan(SU_max_LE_U));
SL2=SL(~isnan(ST_max_LE_T_global));
LE_T=LE_T(~isnan(LE_T));
LE_U=LE_U(~isnan(LE_U));
S_T=S_T(~isnan(S_T));
S_U=S_U(~isnan(S_U));
S_T_deriv=S_T_deriv(~isnan(S_T_deriv));
max_LE_U=max_LE_U(~isnan(max_LE_U));
SU_max_LE_U=SU_max_LE_U(~isnan(SU_max_LE_U));
Vbl=Vbl(~isnan(Vbl));
RVP=RVP(~isnan(RVP));
multequ=multequ(~isnan(multequ));
renT=renT(~isnan(renT));
renE=renE(~isnan(renE));
renM=renM(~isnan(renM));
VrenE=VrenE(~isnan(VrenE));
VrenM=VrenM(~isnan(VrenM));
max_LE_S=max_LE_S(~isnan(max_LE_S));
ST_max_LE_T=ST_max_LE_T(~isnan(ST_max_LE_T));
max_LE_S_global=max_LE_S_global(~isnan(max_LE_S_global));
ST_max_LE_T_global=ST_max_LE_T_global(~isnan(ST_max_LE_T_global));
GTi=GTi(~isnan(GTi));
VGTi=VGTi(~isnan(VGTi));
SU_LET=SU_LET(~isnan(SU_LET));
SL=SL(~isnan(SL));
```

```
wb=wb(~isnan(wb));
sqq=sqq(~isnan(sqq));
utiE=utiE(~isnan(utiE));
utiM=utiM(~isnan(utiM));
Vbl_one=Vbl_one(~isnan(Vbl_one));

% Restrict to combinations for which a local maximum for S^1 exists:
LE_Ta2=LE_T2(max_LE_S~=-1);
max_LE_Sa2=max_LE_S(max_LE_S~=-1);
ST_max_LE_Ta2=ST_max_LE_T(max_LE_S~=-1);
S_Ta2=S_T2(max_LE_S~=-1);

% Auxiliary variables:
aux0=abs(ST_max_LE_Ta2-S_Ta2)./max(abs(ST_max_LE_Ta2),abs(S_Ta2));
aux=aux0(sign(ST_max_LE_Ta2)==sign(S_Ta2));
aux00=(abs(SU_max_LE_U-S_U2))./max(abs(SU_max_LE_U),abs(S_U2));
aux1=aux00(sign(SU_max_LE_U)==sign(S_U2));

display(['# \Delta(a*\bar \nu)<\Gamma(a*\bar \nu) or \Delta(L)>\Gamma(L) - omitted:  ' num2str(length(wb))])
display(['# Multiple equilibria L_E^1 - omitted: ' num2str(length(multequ))]);
display(['# V(\Psi)>0.5 in L_E^1 or L_E^0 - omitted: ' num2str(length(Vbl))]);
display(['# V(\Psi)<0.5 in L_E^1 and L_E^0, but V(\Psi)>0.5 for some other L_E - omitted for maximizations: ' num2str(length(Vbl_one))]);

% Admissible combinations for checking the effect of a marginal increase in
% L_E (starting from equilibrium) and whether social welfare is higher in
% equilibrium with free or restricted OC:
display(['# Combinations left (without maximizations): ' num2str(length(LE_T))]);

% Admissible combinations  for comparing equilibrium values to optimum values:
display(['# Combinations left (with maximizations): ' num2str(length(max_LE_S_global))]);

disp('--------------------------------------------------------------------------------');

display(['mean, std and max of (VP-VP0)/VP0 in %:  ' num2str(mean(RVP)*100) '   ' num2str(std(RVP)*100) '   ' num2str(max(RVP)*100)]);

disp('--------------------------------------------------------------------------------');

disp(['L_E^0 > L_E^1 in % :  ' num2str((length(LE_T)-length(find(LE_T>LE_U)))/length(LE_T)*100) '%']);
disp(['S^0 > S^1 at equilibrium in % :  ' num2str((length(S_T)-length(find(S_T>S_U)))/length(S_T)*100) '%']);
```

```
disp(['S^0 > S^1 at constrained (global) optimum in % :  ' num2str((length(ST_max_LE_T_global)-length(find(ST_max_LE_T_global-SU_max_LE_U>1e-4)))/length(ST_max_LE_T_global)*100) '%']);

disp('------------------------------------------------------------------------');

disp(['S^1'' > -1e-3 in % :  ' num2str((length(S_T_deriv)-length(find(S_T_deriv<-1e-3)))/length(S_T_deriv)*100) '%']);
disp(['S^1'' > -1e-6 in % :  ' num2str((length(S_T_deriv)-length(find(S_T_deriv<-1e-10)))/length(S_T_deriv)*100) '%']);
disp(['S^1'' > 0 in % :  ' num2str((length(S_T_deriv)-length(find(S_T_deriv<0)))/length(S_T_deriv)*100) '%']);

ela=abs(S_T_deriv.*LE_T./S_T);
disp(['mean, std of S^1'' * L_E^1/S^1 (elasticity at equilibrium L_E^1): ' num2str(mean(ela)) '   ' num2str(std(ela))]);

disp('------------------------------------------------------------------------');

display(['dealers/entrepreneurs equilibrium rents (equal to GI) with free OC as a fraction of their utility: ' num2str(mean(renE./(renE+GTi))) '  ' num2str(std(renE./(renE+GTi))) ]);
display(['dealers/entrepreneurs equilibrium rents with restricted OC as a fraction of their utility: ' num2str(mean(VrenE./(VrenE+VGTi))) '  ' num2str(std(VrenE./(VrenE+VGTi))) ]);

display(['equilibrium wage with free OC as a fraction of worker utility: ' num2str(mean(renM./(renM+GTi))) '  ' num2str(std(renM./(renM+GTi))) ]);
display(['equilibrium wage with restricted OC as a fraction of worker utility: ' num2str(mean(VrenM./(VrenM+VGTi))) '  ' num2str(std(VrenM./(VrenM+VGTi))) ]);

display(['ratio of entrepreneurs/dealers equilibrium rents over workers rents (i.e., GI/GJ) with free OC: ' num2str(mean(renE./(renM))) '   ' num2str(std(renE./(renM))) ]);

display(['A hipo`s CE over a worker`s CE in equilibrium with free OC:  ' num2str(mean(utiE./(utiM))) '   ' num2str(std(utiE./(utiM))) ]);

disp('------------------------------------------------------------------------');

% Auxiliary variables:
S_Ua=S_U(sign(S_U)==sign(S_T));
S_Ta=S_T(sign(S_U)==sign(S_T));

```

```
display(['mean, std of |S^0-S^1|/max(|S^0|,|S^1|) at equilibrium in % (different sign cases omitted): ' num2str(mean(abs(S_Ta-S_Ua)./max(abs(S_Ta),abs(S_Ua)))*100) ' ' num2str(std(abs(S_Ta-S_Ua)./max(abs(S_Ta),abs(S_Ua)))*100) ]);
display(['# combinations used for the above: ' num2str(length(S_Ta))])

% Auxiliary variables:
SU_max_LE_Ua2=SU_max_LE_U(max_LE_S~=-1);
aux0a=abs(SU_max_LE_Ua2-ST_max_LE_Ta2)./max(abs(SU_max_LE_Ua2),abs(ST_max_LE_Ta2));
aux2=aux0a(sign(SU_max_LE_Ua2)==sign(ST_max_LE_Ta2));

% \hat indicates the respective constrained optimum value (see text)
display(['|\hat S^0 - \hat S^1|/max(|\hat S^0|,|\hat S^1|) in % (different sign cases omitted, local opt. L_E^1 exists): ' num2str(mean(aux2)*100) ' ' num2str(std(aux2)*100) ]);
display(['# combinations used for the above: ' num2str(length(aux2))])

% Auxiliary variables:
S_T2a=S_T2(sign(S_T2)==sign(SU_max_LE_U));
SU_max_LE_Ua=SU_max_LE_U(sign(S_T2)==sign(SU_max_LE_U));

% \hat indicates the respective constrained optimum value (see text)
display(['|\hat S^0 - S^1)/max(|\hat S^0|,|S^1|) in % (different sign cases omitted): ' num2str(mean(abs(S_T2a-SU_max_LE_Ua)./max(abs(S_T2a),abs(SU_max_LE_Ua)))*100) ' ' num2str(std(abs(S_T2a-SU_max_LE_Ua)./max(abs(S_T2a),abs(SU_max_LE_Ua)))*100) ]);
display(['# combinations used for the above: ' num2str(length(S_T2a))])

disp('------------------------------------------------------------------------');

disp(['i-effect >0 in %: ' num2str(length(find(SU_LET>S_T))/length(LE_T)*100) '%']);
disp(['r-effect >0 in %: ' num2str(length(find(S_U>SU_LET))/length(LE_T)*100) '%']);

display(['i-effect as share of the total effect (restriction: both effects positive): ' num2str(mean((SU_LET((SU_LET-S_T)>0 & (S_U-SU_LET)>0)-S_T((SU_LET-S_T)>0 & (S_U-SU_LET)>0))./(S_U((SU_LET-S_T)>0 & (S_U-SU_LET)>0)-S_T((SU_LET-S_T)>0 & (S_U-SU_LET)>0)))) ])
display(['# combinations used for the above: ' num2str(length((SU_LET((SU_LET-S_T)>0 & (S_U-SU_LET)>0)-S_T((SU_LET-S_T)>0 & (S_U-SU_LET)>0))))])

disp('------------------------------------------------------------------------');

% \hat indicates the respective constrained optimum value (see text)
```

```
display(['mean, std of |\hat L_E^1 - L_E^1|/max(|.|,|.|) - in % (only local optima):  ' num2str(mean(abs(max_LE_Sa2-LE_Ta2)./max(LE_Ta2,max_LE_Sa2))*100) '   ' num2str(std(abs(max_LE_Sa2-LE_Ta2)./max(LE_Ta2,max_LE_Sa2))*100)])
display(['# combinations used for the above:  ' num2str(length(LE_Ta2))]);

display(['mean, std of |\hat S^1 - S^1|/max(|.|,|.|) - in % (different sign cases omitted, only local optima):  ' num2str(mean(aux)*100) '   ' num2str(std(aux)*100)]);
display(['# combinations used for the above:  ' num2str(length(aux))]);

display(['mean, std of |\hat L_E^0 - L_E^0|/max(|.|,|.|) - in %:  ' num2str(mean((abs(max_LE_U-LE_U2)./max(LE_U2,max_LE_U)))*100) '   ' num2str(std((abs(max_LE_U-LE_U2)./max(LE_U2,max_LE_U)))*100)]);

display(['mean, std of |\hat S^0 - S^0|/max(|.|,|.|) - in % (different sign cases omitted):  ' num2str(mean(aux1)*100) '   ' num2str(std(aux1)*100)]);
display(['# combinations used for the above:  ' num2str(length(aux1))]);

disp('-----------------------------------------------------------------------------------');

sd=cell2mat(paramCombsci(:,10)); % \hat s
sqq=sqq'; % \bar s at equilibrium L_E^1
ss=[sd sqq];
sss=[ss(:,1) ss(:,2)-ss(:,1) ss(:,2)];
disp(['avg, std of (\bar s - \hat s) / \bar s at equilibrium L_E^1:  ' num2str(mean(sss(:,2)./sss(:,3))) '   ' num2str(std(sss(:,2)./sss(:,3)))])
end

clear aaa hh diff sh Vs L a A b Ve M N rho nuq i jj k paramCombsc dSWF SWFn dSWFn dSWF2n SWF2n Vbn Fn VFn rentEn rentTn dVbn utilT rentMn GTin multequ s_T_d sqn Vbl VPn wagen timer errorflag jjax VGTin;

toc
```

File "symaux_FE"

```
tic

% Create symbolic variables in Matlab:
syms L M N sh Vs Ve rho nq a LE Vn b A

% Expressions from the text:
alpha=(L-LE)/(rho*Ve);
gam=1/(alpha^2*Vs+Vn);
VphiGw=Ve+Vs*Vn*gam;
beta=(LE+M)/(rho*VphiGw);
VsGw=VphiGw-Ve;
w=A*(1-b)*(M/(LE/a))^(-b); % wage
```

```
sq=sh+A*(M/(LE/a))^(1-b)-w*(M/(LE/a)); % \bar s

% Free OC:
EP=sq-(LE/a-nq)/(alpha+beta);
VP = ((Vs*alpha*beta*gam+1)/(alpha+beta))^2/gam;
Ez = (LE/a-nq)/((alpha+beta)*(2*rho*(Vn*Vs*gam+Ve))^.5);
Vz = gam*Vn^2/((2*rho*(Vn*Vs*gam+Ve))*(alpha+beta)^2);
GP = (EP-.5*rho*VP/a)/a;
CovPz = -(VP*Vz)^.5;
GTu = (Ez-rho*CovPz/a)^2/(1+2*rho*Vz)+(.5/rho)*log(1+2*rho*Vz);
GI = (.5/rho)*log(1+VsGw/Ve);
GTi = Ez^2/(1+2*rho*Vz)+(.5/rho)*log(1+2*rho*Vz);

% Restricted OC:
VEP = sq-rho*(Vs+Ve)*(LE/a-nq)/(L+M);
VVP = Vn*rho^2*(Vs+Ve)^2/(L+M)^2;
VEz = rho^.5*(Vs+Ve)^.5*(LE/a-nq)/((L+M)*2^.5);
VVz = Vn*rho*(Vs+Ve)/(2*(L+M)^2);
VCovPz = -(VVP*VVz)^.5;
VGTi = VEz^2/(1+2*rho*VVz)+(.5/rho)*log(1+2*rho*VVz);
VGTu = (VEz-rho*VCovPz/a)^2/(1+2*rho*VVz)+(.5/rho)*log(1+2*rho*VVz);
VGP = (VEP-.5*rho*VVP/a)/a;

% Free OC;
% b corresponds to \Psi in the text,
% c corresponds to \Phi in the text:
CovPn = Vn*(VP*gam)^.5;
VtmP = Vs+Ve+VP*(-Vn*gam+1)-2*alpha*Vs*(VP*gam)^.5;
Eb = rho^.5*nq*((1/2)*rho*VtmP+N*CovPn/Vn)^.5/N;
Vb = (Eb/nq)^2*Vn;
Ec = -rho*nq*(sq-EP+CovPn*nq/Vn)/N;
Vc = (Ec/nq)^2*Vn;
Covbc = (Eb/nq*(Ec/nq))*Vn;
UN = .5*log(1-2*Vb)/rho-Ec/rho-.5*Vc/rho-(Eb+Covbc)^2/(rho*(1-2*Vb));
UM = GTi+w;

% Restricted OC:
VCovPn = rho*(Vs+Ve)*Vn/(L+M);
VVtmP = Vs+Ve;
VEb = rho^.5*nq*((1/2)*rho*VVtmP+N*VCovPn/Vn)^.5/N;
VVb = (VEb/nq)^2*Vn;
VEc = -rho*nq*(sq-VEP+VCovPn*nq/Vn)/N;
VVc = (VEc/nq)^2*Vn;
VCovbc = (VEb/nq*(VEc/nq))*Vn;
VUN = .5*log(1-2*VVb)/rho-VEc/rho-.5*VVc/rho-(VEb+VCovbc)^2/(rho*(1-2*VVb)
    );
VUM = VGTi+w;

% Social welfare:
SWF = LE*(GP+GTu)+(L-LE)*(GTi+GI)+N*UN+M*UM;
SWF2 = LE*(VGP+VGTu)+(L-LE)*VGTi+N*VUN+M*VUM;
```

```
% First and second derivatives:
dVb=diff(Vb,LE);
dSWF=diff(SWF,LE);
d2SWF=diff(dSWF,LE);

% \Delta(L_E)-\Gamma(L_E):
F=GP+GTu-GI-GTi;
VF=VGP+VGTu-VGTi;

% "Rents":
rentE=GP+GTu-GTi;
rentT=GI;
rentM=w;

% Transform expressions into 'character' form:
SWFn=char(SWF);
SWF2n=char(SWF2);
dVbn=char(dVb);
Vbn=char(Vb);
dSWFn=char(dSWF);
d2SWFn=char(d2SWF);
Fn=char(F);
VFn=char(VF);
rentTn=char(rentT);
rentEn=char(rentE);
rentMn=char(rentM);
VPn=char(VP);
sqn=char(sq);
GTin=char(GTi);
VGTin=char(VGTi);

clear L M N sq Vs Ve rho nq a LE Vn b w alpha beta gam VphiGw VsGw EP Ez
    VP Vz GP GTu GTi GI CovPz VEP VEz VVP VVz VGP VGTu VGTi VGI VCovPz Vb
    Vc Covbc Eb Ec Vb Vc Covbc VEb VEc VVb VVc VCovbc SWF SWF2 dSWF dSWF2 F
    VF VCovPn VUM VUN UM UN dVb rentT rentE rentM wage drSWF rSWF rSWF2
    d2SWF CovPn VtmP VVtmP

toc
```

Function "FE_Model_f1"

```
% This is function "FE_Model_f1". The r.h.s. shows function inputs, the
% l.h.s. shows function outputs:
function [equ_T, equ_U, s_T, s_U, s_T_d, maxLEU, SU_maxLEU, Vbl, rVP,
    multequ,rT,rE,rM,VrE,VrM,maxLES,ST_maxLET,maxLESglobal,ST_maxLETglobal,
    GT_i,VGT_i,SU_LET,SL,wb,sqq,utiE,utiM,Vbl_one] = FE_Model_f1(L,a,Vs,rho
    ,Ve,M,nq,b,A,sh,N,t,Vn,Vbn,Fn,VFn,SWFn,dSWFn,d2SWFn,SWF2n,rentEn,rentTn
    ,rentMn,VPn,sqn,GTin,VGTin,dVbn)

% Show simulation progress in the command window:
if mod(t,10000)==0
display(num2str(t));
```

```
 toc
 else
end

x0=double(L/2);

% Transform 'F' into a function of L_E:
F=eval(['@(LE)' Fn]);

% Set up the function's output variables:
equ_T=NaN;
s_T=NaN;
s_T_d=NaN;
equ_U=NaN;
s_U=NaN;
maxLEU=NaN;
SU_maxLEU=NaN;
Vbl=NaN;
rVP=NaN;
multequ=NaN;
rT=NaN;
rE=NaN;
rM=NaN;
VrE=NaN;
VrM=NaN;
maxLES=NaN;
ST_maxLET=NaN;
maxLESglobal=NaN;
ST_maxLETglobal=NaN;
GT_i=NaN;
VGT_i=NaN;
SU_LET=NaN;
SL=NaN;
wb=NaN;
sqq=NaN;
utiE=NaN;
utiM=NaN;
Vbl_one=NaN;

if F(a*nq)<0 || F(L)>0
    wb=1;

else
    % Solve for equilibrium L_E^1; the function "rmsearch" finds good starting points and search intervals;
    % Note that \bar s and with that E(P), \Delta(L_E) and F(L_E) go to infinity for L_E -> 0:
    [u, ~, errorflag]=rmsearch(F,'fzero',x0,1e-6,L,'InitialSample',100);

        for k1=1:length(errorflag)
        if errorflag(k1)<0
           u(k1)=[];
```

```
            end
            end

% Drop the combination, if it implies multiple equilibria:
 if length(u)>1
                multequ=1;
 else


Vb=eval(['@(LE)' Vbn]);
Vbu=Vb(u);
VbL=Vb(L); % Vb in case of restriced OC is indep. of L_E

% Drop the combination, if it does not imply well defined noise trader utiltiy:
 if Vbu>=0.5 || VbL>=0.5
     Vbl=1;

 else
                    equ_T=u; % the unique L_E^1

                    VF = eval(['@(LE)' VFn]);
                    VF0=VF(1e-6);
                    VFL=VF(L);
                    if sign(VF0)==sign(VFL) && sign(VFL)>0 % no interior solution exists (VF is strictly decreasing, see text)
                        equ_U=L; % corner equilibrium L_E^0=L
                    elseif sign(VF0)==sign(VFL) && sign(VF0)<0
                        equ_U=1e-6; % functions not defined for L_E=0
                        disp('Warning: L_E^0=0');
                    else
                    % Solve for interior equilibrium L_E^0:
                    [equ_U, ~, errorflag]=fzero(VF,[1e-6 L]); % Sufficient, as we know that L_E^0 is unique (see text)

                           if errorflag<0
                             equ_U=[];
                           end
                           if isempty(equ_U)
                               disp('Error: No equilibrium L_E^0')
                           end
                    end

                    % Transform expressions into functions of L_E:
                    VP=eval(['@(LE)' VPn]);
                    rentT=eval(['@(LE)' rentTn]);
                    rentE=eval(['@(LE)' rentEn]);
                    rentM=eval(['@(LE)' rentMn]);
                    sq=eval(['@(LE)' sqn]);
                    GTi=eval(['@(LE)' GTin]);
                    VGTi=eval(['@(LE)' VGTin]);

```

```
            VP0=Vs; % Ex-ante price variance in the noiseless case with free OC

            rVP=(VP(equ_T)-VP0)/VP0;

            rT=rentT(equ_T);
            rE=rentE(equ_T);
            rM=rentM(equ_T);

            utiE=rT+GTi(equ_T); % CE_E
            utiM=rM+GTi(equ_T); % CE_M

            GT_i=GTi(equ_T);

            % Transform expressions into functions of L_E:
            SWF=eval(['@(LE)' SWFn]);
            SWF2=eval(['@(LE)' SWF2n]);
            dSWF=eval(['@(LE)' dSWFn]);
            d2SWF=eval(['@(LE)' d2SWFn]);

            s_T=SWF(equ_T); % equilibrium social welfare with free OC
            s_T_d=dSWF(equ_T); % slope of the social welfare function with free OC at equilibrium L_E^1
            SL=SWF2(L); % social welfare at L_E=L

            sqq=sq(equ_T); % \bar s at equilibrium

            VrE=VF(equ_U);
            VrM=rentM(equ_U);
            VGT_i=VGTi(equ_U);

            s_U=SWF2(equ_U); % equilibrium social welfare with restricted OC
            SU_LET=SWF2(equ_T); % social welfare with restricted OC, evaluated at L_E=L_E^1

        % For maximization of social welfare, noise trader utility has to
        % be defined for all L_E; Hence, we drop all combinations which
        % imply that Vb is greater than 0.5 for any L_E:
        dVb=eval(['@(LE)' dVbn]);
        [Vb1, ~, errorflag0]=rmsearch(dVb,'fzero',x0,0,L,'InitialSample',100);
            for k0=1:length(errorflag0)
            if errorflag0(k0)<0
                Vb1(k0)=[];
            else
            end
            end

            if isempty(Vb1)
               Vbmaxvalgl=max([Vb(0) Vb(L)]);
            elseif length(Vb1)>1
```

```
                disp('Vb more than one extremum')
               Vbmaxvalgl=max([Vb(0) Vb(L) Vb(Vb1)]);
            elseif dVb(Vb1+1e-12)>0
                disp('Vb has a minimum')
                Vbmaxvalgl=max([Vb(0) Vb(L) Vb(Vb1)]);
            else
                Vbmaxvalgl=max([Vb(0) Vb(L) Vb(Vb1)]);
            end
        if Vbmaxvalgl>=0.5 % there is an L_E for which Vb>0.5
            Vbl_one=1;
        else
            % Call function "S1opt" (see below), which finds the local
            % as well as the global maximum of S^1:
            [maxLES, maxLESglobal]=S1opt(dSWF,d2SWF);

            ST_maxLET=SWF(maxLES); % local maximum social welfare S^1
            ST_maxLETglobal=SWF(maxLESglobal); % global maximum social welfare S^1

            % Call function "S0opt" (see below), which finds the
            % maximum of S^0:
            maxLEU=S0opt(SWF2);

            SU_maxLEU=SWF2(maxLEU); % maximum social welfare S^0
        end

end
end
end

% Auxiliary functions "S1opt" and "S0opt" below

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% function "S1opt" finds the L_E that (locally/globally) maximizes S^1:
function [maxLES,maxLESglobal] = S1opt(dSWF,d2SWF)

          [j, ~, errorflag]=rmsearch(dSWF,'fzero',x0,1e-6,L,'InitialSample',100);

            for k2=1:length(errorflag)
            if errorflag(k2)<0
                j(k2)=[];
            else
            end
            end

            if isempty(j) % no local extremum
                maxLES=-1;
                jvalaux=[SWF(1e-6) SWF(L)];
```

```
                    jaux=[1e-6 L];
                    maxLESglobal=jaux(jvalaux==max(jvalaux));
                else
                    % check if local extremum is a maximum:
                    jval=NaN(length(j),1);
                    d2aux=NaN(length(j),1);
                    for i1=1:length(j)
                        d2aux(i1)=d2SWF(j(i1));
                        jval(i1)=SWF(j(i1));
                    end
                    j=j(d2aux<0); % keep only maxima
                    jval=jval(d2aux<0);

                    jvalaux=[jval' SWF(1e-6) SWF(L)];
                    jaux=[j' 1e-6 L];
                    maxLESglobal=jaux(jvalaux==max(jvalaux));

                    if isempty(j) % no local maximum
                         maxLES=-1;
                    elseif length(j)==1
                         maxLES=j;
                    else
                         % in case there are multiple local maxima, choose the one closest to equilibrium L_E^1:
                         lowdis=zeros(length(j),1);
                         for k3=1:length(j)
                             lowdis(k3)=abs(equ_T-j(k3));
                         end
                         maxLES=j(lowdis==min(lowdis));
                    end
                end

end


% function "S0opt" finds the L_E that maximizes S^0:
function maxLEU = S0opt(SWF2)

            opts2 = optimset('TolX',1e-6); % default TolX is 1e-4
            mSWF2=@(LE)-SWF2(LE);
            % fminbnd finds the min of -SWF2 (local if it exists, global
            % otherwise):
            [e, ~, errorflag]=fminbnd(mSWF2,1e-6,L, opts2);

                for k4=1:length(errorflag)
                if errorflag(k4)<0
                    e(k4)=[];
                else
                end
                end
       % S^0 is inverse U-shaped (see text):
       if e>L-1e-3 % no local minimum has been found
```

```
                        maxLEU=L;
        elseif e<0+1e-3 % no local minimum has been found
                        maxLEU=1e-6;
                        disp('optimal L_E equals "zero"')
        else
                        maxLEU=e;
        end

end

end
```

Adjustemts to function "rmsearch"

Adjustments made to "rmsearch" are the same as in the basic version of the model.

D.4 Unemployment Model

Code files are given in the same order as for the basic version of the model and the FE model. All of the code is also submitted in digital form.

File "UE_Model"

```
% Start timer:
tic

% Call auxiliary file "symaux_UE", within which we state and derive closed
% form expressions required later for calculations within function
% "UE_Model_f1":
symaux_UE;

% Parameter values set as described in the text:
L=100;                                                          %(1)
a=10;                                                           %(2)
Vs=1;                                                           %(3)
rho=[0.01 0.025 0.05 0.075 0.1 0.25 0.5 0.75 1];                %(4)
Ve=[0.1 0.25 0.5 0.75 1];                                       %(5)
M=[1 2 3 5 10 100];                                             %(6)
nuq=[0.001 0.01 0.05 0.1 0.2 0.5];                              %(7)
b=[0.1 0.25 0.4 0.55];                                          %(8)
A=[0.25 0.5 0.75 1];                                            %(9)
D=1;                                                            %(10)
sh=[0.01 0.05 0.1 0.25 0.5 0.75 0.9 0.95 0.99];                 %(11)
N=[1 2 3 5 10 100];                                             %(12)
Vnu=[0.001 0.01 0.05 0.1 0.2 0.5];

% Initiate and fill in the matrix "paramCombs", which contains all
% parameter combinations; Each row of the final "paramCombs" matrix
% constitutes one parameter combination:
paramCombs=L';

```

```
avec=NaN(size(paramCombs,1), length(a));
for k=1:size(paramCombs,1)
    avec(k,:)=a;
end
avec=reshape(avec,[],1);

paramCombs=repmat(paramCombs,length(a),1);
paramCombs=[paramCombs avec];
clear avec

Vsvec=NaN(size(paramCombs,1), length(Vs));
for k=1:size(paramCombs,1)
    Vsvec(k,:)=Vs;
end
Vsvec=reshape(Vsvec,[],1);

paramCombs=repmat(paramCombs,length(Vs),1);
paramCombs=[paramCombs Vsvec];
clear Vsvec

rhovec=NaN(size(paramCombs,1), length(rho));
for k=1:size(paramCombs,1)
    rhovec(k,:)=rho*(4/(1.25^.5))*vpa(paramCombs(k,2)/(paramCombs(k,3)^.5));
end
rhovec=reshape(rhovec,[],1);

paramCombs=repmat(paramCombs,length(rho),1);
paramCombs=[paramCombs rhovec];
clear rhovec

Vevec=NaN(size(paramCombs,1), length(Ve));
for k=1:size(paramCombs,1)
    Vevec(k,:)=Ve*paramCombs(k,3);
end
Vevec=reshape(Vevec,[],1);

paramCombs=repmat(paramCombs,length(Ve),1);
paramCombs=[paramCombs Vevec];
clear Vevec

Mvec=NaN(size(paramCombs,1), length(M));
for k=1:size(paramCombs,1)
    Mvec(k,:)=M*paramCombs(k,1);
end
Mvec=reshape(Mvec,[],1);

paramCombs=repmat(paramCombs,length(M),1);
paramCombs=[paramCombs Mvec];
clear Mvec

nuqvec=NaN(size(paramCombs,1), length(nuq));
```

```
for k=1:size(paramCombs,1)
    nuqvec(k,:)=nuq*paramCombs(k,1)/paramCombs(k,2);
end
nuqvec=reshape(nuqvec,[],1);

paramCombs=repmat(paramCombs,length(nuq),1);
paramCombs=[paramCombs nuqvec];
clear nuqvec

bvec=NaN(size(paramCombs,1), length(b));
for k=1:size(paramCombs,1)
    bvec(k,:)=b;
end
bvec=reshape(bvec,[],1);

paramCombs=repmat(paramCombs,length(b),1);
paramCombs=[paramCombs bvec];
clear bvec

Avec=NaN(size(paramCombs,1), length(A));
for k=1:size(paramCombs,1)
    Avec(k,:)=A*0.5*(paramCombs(k,4)*paramCombs(k,3))/(paramCombs(k,2)*(
        paramCombs(k,8)))*(paramCombs(k,2)*paramCombs(k,6)/paramCombs(k,1))
        ^(paramCombs(k,8)-1);
end
Avec=reshape(Avec,[],1);

paramCombs=repmat(paramCombs,length(A),1);
paramCombs=[paramCombs Avec];
clear Avec

Dvec=NaN(size(paramCombs,1),1);
wvec=NaN(size(paramCombs,1),1);
mvec=NaN(size(paramCombs,1),1);
for k=1:size(paramCombs,1)
   wvec(k)=paramCombs(k,9)*(1-paramCombs(k,8))*(paramCombs(k,1)/(
       paramCombs(k,2)*paramCombs(k,6)))^(paramCombs(k,8));
   Dvec(k)=D*(wvec(k)-(1/paramCombs(k,4))*log(1+paramCombs(k,4)*(
       paramCombs(k,8))*wvec(k)));
   mvec(k)=(paramCombs(k,9)*(1-paramCombs(k,8))/wvec(k))^(1/(paramCombs(k
       ,8)));
end
Dvec=reshape(Dvec,[],1);

paramCombs=[paramCombs Dvec];
clear Dvec

shvec=NaN(size(paramCombs,1), length(sh));
for k=1:size(paramCombs,1)
```

```
    shvec(k,:)=vpa(paramCombs(k,4)*paramCombs(k,3)/(2*paramCombs(k,2))-(
        paramCombs(k,9)*mvec(k)^(1-paramCombs(k,8))-wvec(k)*mvec(k)))+sh*
        vpa((paramCombs(k,1)-paramCombs(k,2)*paramCombs(k,7))*paramCombs(k
        ,4)*paramCombs(k,5)/(paramCombs(k,2)*(paramCombs(k,1)+paramCombs(k
        ,6))));
end
shvec=reshape(shvec,[],1);

wvec=repmat(wvec,length(sh),1);
mvec=repmat(mvec,length(sh),1);
paramCombs=repmat(paramCombs,length(sh),1);
paramCombs=[paramCombs shvec];
clear shvec

% Rule out multiple equilibria in case of deterministic noise:
for k=1:size(paramCombs,1)
    if paramCombs(k,4)*(paramCombs(k,11)+paramCombs(k,9)*mvec(k)^(1-
        paramCombs(k,8))-wvec(k)*mvec(k)-paramCombs(k,4)*(paramCombs(k,3)+
        paramCombs(k,5))*(paramCombs(k,1)/paramCombs(k,2)-paramCombs(k,7))
        /(paramCombs(k,1)+paramCombs(k,6)))/paramCombs(k,2) -.5*log(1+
        paramCombs(k,3)/paramCombs(k,5))>0
       paramCombs(k,11)=nan;
    end
end
clear wvec mvec

Nvec=NaN(size(paramCombs,1), length(N));
for k=1:size(paramCombs,1)
    Nvec(k,:)=0.25*N*(paramCombs(k,1)+paramCombs(k,6));
end
Nvec=reshape(Nvec,[],1);

paramCombs=repmat(paramCombs,length(N),1);
paramCombs=[paramCombs Nvec];
clear Nvec

disp(['# Parameter combinations for each \sigma_\nu at start of the
    simulation (incl. multipl. equ. in the noisless case):  ' num2str(size(
    paramCombs,1))]);

paramCombs=paramCombs(~isnan(paramCombs(:,11)),:);

disp(['# Parameter combinations for each \sigma_\nu at start of the
    simulation (excl. multipl. equ. in the noisless case):  ' num2str(size(
    paramCombs,1))]); %// number of parameter combinations for each value
    of Vnu

timer=(1:size(paramCombs,1))';
paramCombs=[paramCombs timer];

Vnuvec=NaN(size(paramCombs,1),length(Vnu));
for k=1:size(paramCombs,1)
```

```
    Vnuvec(k,:)=Vnu.^2*(paramCombs(k,1)/paramCombs(k,2))^2;
end

toc

% Simulate the model for each \sigma_\nu:
for i=1:size(Vnuvec,2)
    disp(['\sigma_\nu= ' num2str(double(Vnu(i)*100)) '% of L/a']);
    paramCombsci=num2cell([paramCombs Vnuvec(:,i)]);

% Call function "UE_Model_f1" for each parameter combination; the
% "r.h.s." gives the function inputs, the "l.h.s." gives the function
% outputs:
[LE_T, LE_U, S_T, S_U, S_T_deriv, max_LE_U, SU_max_LE_U, Vbl, RVP, multequ,renT,renE,renM,VrenE,VrenM,max_LE_S,ST_max_LE_T,max_LE_S_global,ST_max_LE_T_global,Empl,LEopt0,VLEopt0,ST_LE0opt,SU_VLE0opt,GTi,VGTi,SU_LET,SL,wb,sqq,utiE,utiM,Vbl_one] = arrayfun(@(n) UE_Model_f1(paramCombsci{n,:},Vbn,Fn,VFn,SWFn,dSWFn,d2SWFn,SWF2n,rentEn,rentTn,rentMn,VPn,sqn,empln,LEopt0n,VLEopt0n,GTin,VGTin,dVbn), 1:size(paramCombsci,1), 'uni', 1);

toc

% Get rid of the parameter combinations that led to nan-values
% within "UE_Model_f1":
paramCombsci=paramCombsci(~isnan(LE_T),:);
LE_T2=LE_T(~isnan(max_LE_S));
LE_U2=LE_U(~isnan(max_LE_U));
S_T2=S_T(~isnan(ST_max_LE_T));
S_U2=S_U(~isnan(SU_max_LE_U));
SL2=SL(~isnan(ST_max_LE_T_global));
SU_LET2=SU_LET(~isnan(ST_max_LE_T_global));
LE_T=LE_T(~isnan(LE_T));
LE_U=LE_U(~isnan(LE_U));
S_T=S_T(~isnan(S_T));
S_U=S_U(~isnan(S_U));
S_T_deriv=S_T_deriv(~isnan(S_T_deriv));
max_LE_U=max_LE_U(~isnan(max_LE_U));
SU_max_LE_U=SU_max_LE_U(~isnan(SU_max_LE_U));
Vbl=Vbl(~isnan(Vbl));
RVP=RVP(~isnan(RVP));
multequ=multequ(~isnan(multequ));
renT=renT(~isnan(renT));
renE=renE(~isnan(renE));
renM=renM(~isnan(renM));
VrenE=VrenE(~isnan(VrenE));
VrenM=VrenM(~isnan(VrenM));
max_LE_S=max_LE_S(~isnan(max_LE_S));
ST_max_LE_T=ST_max_LE_T(~isnan(ST_max_LE_T));
max_LE_S_global=max_LE_S_global(~isnan(max_LE_S_global));
ST_max_LE_T_global=ST_max_LE_T_global(~isnan(ST_max_LE_T_global));
Empl=Empl(~isnan(Empl));
```

```
LEopt0=LEopt0(~isnan(LEopt0));
VLEopt0=VLEopt0(~isnan(VLEopt0));
ST_LE0opt=ST_LE0opt(~isnan(ST_LE0opt));
SU_VLE0opt=SU_VLE0opt(~isnan(SU_VLE0opt));
GTi=GTi(~isnan(GTi));
VGTi=VGTi(~isnan(VGTi));
SU_LET=SU_LET(~isnan(SU_LET));
SL=SL(~isnan(SL));
wb=wb(~isnan(wb));
sqq=sqq(~isnan(sqq));
utiE=utiE(~isnan(utiE));
utiM=utiM(~isnan(utiM));
Vbl_one=Vbl_one(~isnan(Vbl_one));

% Restrict to combinations for which a local maximum for S^1 exists:
LE_Ta2=LE_T2(max_LE_S~=-1);
max_LE_Sa2=max_LE_S(max_LE_S~=-1);
ST_max_LE_Ta2=ST_max_LE_T(max_LE_S~=-1);
S_Ta2=S_T2(max_LE_S~=-1);

% Auxiliary variables:
aux0=abs(ST_max_LE_Ta2-S_Ta2)./max(abs(ST_max_LE_Ta2),abs(S_Ta2));
aux=aux0(sign(ST_max_LE_Ta2)==sign(S_Ta2));
aux00=(abs(SU_max_LE_U-S_U2))./max(abs(SU_max_LE_U),abs(S_U2));
aux1=aux00(sign(SU_max_LE_U)==sign(S_U2));

display(['# \Delta(a*\bar \nu)<\Gamma(a*\bar \nu) or \Delta(L)>\Gamma(L) - omitted: ' num2str(length(wb))])
display(['# Multiple equilibria L_E^1 - omitted: ' num2str(length(multequ))]);
display(['# V(\Psi)>0.5 in L_E^1 or L_E^0 - omitted: ' num2str(length(Vbl))]);
display(['# V(\Psi)<0.5 in L_E^1 and L_E^0, but V(\Psi)>0.5 for some other L_E - omitted for maximizations: ' num2str(length(Vbl_one))]);

% Admissible combinations for checking the effect of a marginal increase in
% L_E (starting from equilibrium) and whether social welfare is higher in
% equilibrium with free or restricted OC:
display(['# Combinations left (without maximizations): ' num2str(length(LE_T))]);

% Admissible combinations  for comparing equilibrium values to optimum values:
display(['# Combinations left (with maximizations): ' num2str(length(max_LE_S_global))]);

disp('--------------------------------------------------------------------------------');

display(['mean, std and max of (VP-VP0)/VP0 in %:  ' num2str(mean(RVP)*100) '    ' num2str(std(RVP)*100) '    ' num2str(max(RVP)*100)]);
```

```
disp('----------------------------------------------------------------------------');

disp(['L_E^0 > L_E^1 in % :   ' num2str((length(LE_T)-length(find(LE_T>LE_U)))/length(LE_T)*100) '%']);
disp(['S^0 > S^1 at equilibrium in % :   ' num2str((length(S_T)-length(find(S_T>S_U)))/length(S_T)*100) '%']);
disp(['S^0 > S^1 at constrained (global) optimum in % :   ' num2str((length(ST_max_LE_T_global)-length(find(ST_max_LE_T_global-SU_max_LE_U>1e-4)))/length(ST_max_LE_T_global)*100) '%']);

disp('----------------------------------------------------------------------------');

disp(['S^1'' > -1e-3 in % :   ' num2str((length(S_T_deriv)-length(find(S_T_deriv<-1e-3)))/length(S_T_deriv)*100) '%']);
disp(['S^1'' > -1e-6 in % :   ' num2str((length(S_T_deriv)-length(find(S_T_deriv<-1e-6)))/length(S_T_deriv)*100) '%']);
disp(['S^1'' > 0 in % :   ' num2str((length(S_T_deriv)-length(find(S_T_deriv<0)))/length(S_T_deriv)*100) '%']);

ela=abs(S_T_deriv.*LE_T./S_T);
disp(['mean, std of S^1'' * L_E^1/S^1 (elasticity at equilibrium L_E^1): ' num2str(mean(ela)) '   ' num2str(std(ela))]);

disp('----------------------------------------------------------------------------');

display(['Employment ratio:   ' num2str(mean(Empl))]);

disp('----------------------------------------------------------------------------');

display(['dealers/entrepreneurs equilibrium rents (equal to GI) with free OC as a fraction of their utility:  ' num2str(mean(renE./(renE+GTi))) '   ' num2str(std(renE./(renE+GTi))) ]);
display(['dealers/entrepreneurs equilibrium rents with restricted OC as a fraction of their  utility:  ' num2str(mean(VrenE./(VrenE+VGTi))) '   ' num2str(std(VrenE./(VrenE+VGTi))) ]);

display(['equilibrium wage with free OC as a fraction of worker utility:  ' num2str(mean(renM./(renM+GTi))) '   ' num2str(std(renM./(renM+GTi))) ]);
display(['equilibrium wage with restricted OC as a fraction of worker utility:  ' num2str(mean(VrenM./(VrenM+VGTi))) '   ' num2str(std(VrenM./(VrenM+VGTi))) ]);

display(['ratio of entrepreneurs/dealers equilibrium rents over workers rents (i.e., GI/GJ) with free OC:  ' num2str(mean(renE./(renM))) '   ' num2str(std(renE./(renM))) ]);

```

```
display(['A hipo''s CE over a worker''s CE in equilibrium with free OC:  ' num2str(mean(utiE./(utiM))) '   ' num2str(std(utiE./(utiM))) ]);

disp('------------------------------------------------------------------------------------------------');

% Auxiliary variables:
S_Ua=S_U(sign(S_U)==sign(S_T));
S_Ta=S_T(sign(S_U)==sign(S_T));

display(['mean, std of |S^0-S^1|/max(|S^0|,|S^1|) at equilibrium in % (different sign cases omitted):  ' num2str(mean(abs(S_Ta-S_Ua)./max(abs(S_Ta),abs(S_Ua)))*100) '   ' num2str(std(abs(S_Ta-S_Ua)./max(abs(S_Ta),abs(S_Ua)))*100) ]);
display(['# combinations used for the above:  ' num2str(length(S_Ta))])

% Auxiliary variables:
SU_max_LE_Ua2=SU_max_LE_U(max_LE_S~=-1);
aux0a=abs(SU_max_LE_Ua2-ST_max_LE_Ta2)./max(abs(SU_max_LE_Ua2),abs(ST_max_LE_Ta2));
aux2=aux0a(sign(SU_max_LE_Ua2)==sign(ST_max_LE_Ta2));

% \hat indicates the respective constrained optimum value (see text)
display(['|\hat S^0-\hat S^1|/max(|\hat S^0|,|\hat S^1|) at constrained (local) optimum in % (different sign cases omitted, local opt. L_E^1 exists):  ' num2str(mean(aux2)*100) '   ' num2str(std(aux2)*100) ]);
display(['# combinations used for the above:  ' num2str(length(aux2))])

% Auxiliary variables:
S_T2a=S_T2(sign(S_T2)==sign(SU_max_LE_U));
SU_max_LE_Ua=SU_max_LE_U(sign(S_T2)==sign(SU_max_LE_U));

% \hat indicates the respective constrained optimum value (see text)
display(['|\hat S^0-S^1|/max(|\hat S^1|,|S^1|) in % (different sign cases omitted):  ' num2str(mean(abs(S_T2a-SU_max_LE_Ua)./max(abs(S_T2a),abs(SU_max_LE_Ua)))*100) '   ' num2str(std(abs(S_T2a-SU_max_LE_Ua)./max(abs(S_T2a),abs(SU_max_LE_Ua)))*100) ]);
display(['# combinations used for the above:  ' num2str(length(S_T2a))])

disp('------------------------------------------------------------------------------------------------');

disp(['i-effect >0:  ' num2str(length(find(SU_LET2>S_T2))/length(LE_T2)*100) '%']);
disp(['r-effect >0:  ' num2str(length(find(S_U2>SU_LET2))/length(LE_T2)*100) '%']);
disp(['e-effect >0:  ' num2str(length(find((SU_max_LE_U-S_U2)>-1e-4))/length(LE_T2)*100) '%']);
```

```
display(['i-effect as share of the total effect (restriction: all effects positive): ' num2str(mean((SU_LET2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0)-S_T2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0))./(SU_max_LE_U((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0)-S_T2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0))))])
display(['r-effect as share of the total effect (restriction: all effects positive): ' num2str(mean((S_U2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0)-SU_LET2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0))./(SU_max_LE_U((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0)-S_T2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0))))  ])
display(['e-effect as share of the total effect (restriction: all effects positive): ' num2str(mean((SU_max_LE_U((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0)-S_U2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0))./(SU_max_LE_U((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0)-S_T2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0))))  ])
display(['# combinations used for the above:  ' num2str(length((SU_LET2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0)-S_T2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0)))))])

display(['i-effect as share of the total effect (restriction: all effects positive, \hat L_E^0~=L_E^0): ' num2str(mean((SU_LET2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0 & (max_LE_U~=LE_U2))-S_T2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2)))./(SU_max_LE_U((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2))-S_T2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2)))))  ])
display(['r-effect as share of the total effect (restriction: all effects positive, \hat L_E^0~=L_E^0): ' num2str(mean((S_U2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0 & (max_LE_U~=LE_U2))-SU_LET2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2)))./(SU_max_LE_U((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2))-S_T2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2)))))  ])
display(['e-effect as share of the total effect (restriction: all effects positive, \hat L_E^0~=L_E^0): ' num2str(mean((SU_max_LE_U((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2))-S_U2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2)))./(SU_max_LE_U((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2))-S_T2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2)))))  ])
display(['# combinations used for the above:  ' num2str(length((SU_max_LE_U((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2))-S_U2((SU_LET2-S_T2)>0 & (S_U2-SU_LET2)>0& (max_LE_U~=LE_U2)))))])

disp('--------------------------------------------------------------------------------');

% \hat indicates the respective constrained optimum value (see text)
display(['mean, std of |\hat L_E^1 - L_E^1|/max(|.|,|.|) - in % (only local optima):  ' num2str(mean(abs(max_LE_Sa2-LE_Ta2)./max(LE_Ta2,max_LE_Sa2))*100) '   ' num2str(std(abs(max_LE_Sa2-LE_Ta2)./max(LE_Ta2,max_LE_Sa2))*100)])
display(['# combinations used for the above:  ' num2str(length(LE_Ta2))]);
```

```
display(['mean, std of |\hat S^1 - S^1|/max(|.|,|.|) - in % (different sign cases omitted, only local optima):  ' num2str(mean(aux)*100) '   ' num2str(std(aux)*100)]);
display(['# combinations used for the above:  ' num2str(length(aux))]);

display(['mean, std of |\hat L_E^0 - L_E^0|/max(|.|,|.|) - in %:  ' num2str(mean((abs(max_LE_U-LE_U2)./max(LE_U2,max_LE_U)))*100) '   ' num2str(std((abs(max_LE_U-LE_U2)./max(LE_U2,max_LE_U)))*100)]);

display(['mean, std of |\hat S^0 - S^0|/max(|.|,|.|) - in % (different sign cases omitted):  ' num2str(mean(aux1)*100) '   ' num2str(std(aux1)*100)]);
display(['# combinations used for the above:  ' num2str(length(aux1))]);

disp('------------------------------------------------------------------------');

sd=cell2mat(paramCombsci(:,11)); % \hat s
sqq=sqq'; % \bar s at equilibrium L_E^1
ss=[sd sqq];
sss=[ss(:,1) ss(:,2)-ss(:,1) ss(:,2)];
disp(['avg, std of (\bar s - \hat s) / \bar s at equilibrium L_E^1:   ' num2str(mean(sss(:,2)./sss(:,3))) '   ' num2str(std(sss(:,2)./sss(:,3)))])

disp('------------------------------------------------------------------------');

% Auxiliary Variables:
ST_max_LE_Ta3=ST_max_LE_T(max_LE_S~=-1 & LEopt0~=L);
ST_LE0opta2=ST_LE0opt(max_LE_S~=-1 & LEopt0~=L);
aux0i=abs(ST_max_LE_Ta3-ST_LE0opta2)./max(abs(ST_max_LE_Ta3),abs(ST_LE0opta2));
auxi=aux0i(sign(ST_max_LE_Ta3)==sign(ST_LE0opta2));
aux0ii=abs(SU_max_LE_U-SU_VLE0opt)./max(abs(SU_max_LE_U),abs(SU_VLE0opt));
auxii=aux0ii(sign(SU_max_LE_U)==sign(SU_VLE0opt));

% \hat_0 indicates the approximated constrained optimum in the noiseless case
display(['mean, std of |\hat L_E^1-\hat_0 L_E^1|./max(|.|,|.|) in % (local optimum exists):  ' num2str(mean(abs(max_LE_Sa2-LEopt0(max_LE_S~=-1))./max(abs(max_LE_Sa2),abs(LEopt0(max_LE_S~=-1))))*100) '   ' num2str(std(abs(max_LE_Sa2-LEopt0(max_LE_S~=-1))./max(abs(max_LE_Sa2),abs(LEopt0(max_LE_S~=-1))))*100) ]);
display(['# combinations used for the above:  ' num2str(length(abs(max_LE_Sa2-LEopt0(max_LE_S~=-1))))]);
display(['mean, std of |\hat S^1-\hat_0 S^1|./max(|.|,|.|) in % (different sign cases omitted, local optimum exists, \hat_0 L_E^1~=L):  ' num2str(mean(auxi)*100) '   ' num2str(std(auxi)*100) ]);
display(['# combinations used for the above:  ' num2str(length(auxi))]);
```

```
display(['mean, std of |\hat L_E^0-\hat_0 L_E^0|./max(|.|,|.|) in %:  ' num2str(mean(abs(VLEopt0-max_LE_U)./max(abs(VLEopt0),abs(max_LE_U)))*100) '  ' num2str(std(abs(VLEopt0-max_LE_U)./max(abs(VLEopt0),abs(max_LE_U)))*100) ]);
display(['mean, std of |\hat S^0-\hat_0 S^0|./max(|.|,|.|) in % (different sign cases omitted):  ' num2str(mean(auxii)*100) '  ' num2str(std(auxii)*100) ]);
display(['# combinations used for the above:  ' num2str(length(auxii))]);
end

clear w q b aaa hh diff D A sh Vs L a Ve M N rho nuq i iii iiii jj k paramCombsc dSWF SWFn dSWFn dSWF2n SWF2n Vbn Fn VFn rentEn rentTn dVbn rentMn multequ s_T_d sqn Vbl VPn wagen timer errorflag empln LEopt0n VLEopt0n aux0i aux0ii d2SWFn dSWFnNn GTin VGTin;

toc
```

File "symaux_UE"

```
tic

% Create symbolic variables in Matlab:
syms L M N sh Vs Ve rho nq a LE Vn b D q A

% Expressions from the text:
alpha=(L-LE)/(rho*Ve);
gam=1/(alpha^2*Vs+Vn);
VphiGw=Ve+Vs*Vn*gam;
beta=(LE+M)/(rho*VphiGw);
VsGw=VphiGw-Ve;
w=A*(1-b)*(L/(a*M))^b; % wage
m=(A*(1-b)/w)^(1/b); % employment
sq=sh+A*m^(1-b)-w*m; % \bar s
mh=M*a/LE; % \hat M

% Free OC:
EP=sq-(LE/a-nq)/(alpha+beta);
VP = ((Vs*alpha*beta*gam+1)/(alpha+beta))^2/gam;
Ez = (LE/a-nq)/((alpha+beta)*(2*rho*(Vn*Vs*gam+Ve))^.5);
Vz = gam*Vn^2/((2*rho*(Vn*Vs*gam+Ve))*(alpha+beta)^2);
GP = (EP-.5*rho*VP/a)/a;
CovPz = -(VP*Vz)^.5;
GTu = (Ez-rho*CovPz/a)^2/(1+2*rho*Vz)+(.5/rho)*log(1+2*rho*Vz);
GI = (.5/rho)*log(1+VsGw/Ve);
GTi = Ez^2/(1+2*rho*Vz)+(.5/rho)*log(1+2*rho*Vz);

% Restricted OC:
VEP = sq-rho*(Vs+Ve)*(LE/a-nq)/(L+M);
VVP = Vn*rho^2*(Vs+Ve)^2/(L+M)^2;
VEz = rho^.5*(Vs+Ve)^.5*(LE/a-nq)/((L+M)*2^.5);
VVz = Vn*rho*(Vs+Ve)/(2*(L+M)^2);
VCovPz = -(VVP*VVz)^.5;
```

```
VGTi = VEz^2/(1+2*rho*VVz)+(.5/rho)*log(1+2*rho*VVz);
VGTu = (VEz-rho*VCovPz/a)^2/(1+2*rho*VVz)+(.5/rho)*log(1+2*rho*VVz);
VGP = (VEP-.5*rho*VVP/a)/a;

% Free OC;
% b corresponds to \Psi in the text,
% c corresponds to \Phi in the text:
CovPn = Vn*(VP*gam)^.5;
VtmP = Vs+Ve+VP*(-Vn*gam+1)-2*alpha*Vs*(VP*gam)^.5;
Eb = rho^.5*nq*((1/2)*rho*VtmP+N*CovPn/Vn)^.5/N;
Vb = (Eb/nq)^2*Vn;
Ec = -rho*nq*(sq-EP+CovPn*nq/Vn)/N;
Vc = (Ec/nq)^2*Vn;
Covbc = (Eb/nq*(Ec/nq))*Vn;
UN = .5*log(1-2*Vb)/rho-Ec/rho-.5*Vc/rho-(Eb+Covbc)^2/(rho*(1-2*Vb));
UM = GTi-(1/rho)*log(1-m/mh*(1-exp(-rho*(w-D))));

% Restricted OC:
VCovPn = rho*(Vs+Ve)*Vn/(L+M);
VVtmP = Vs+Ve;
VEb = rho^.5*nq*((1/2)*rho*VVtmP+N*VCovPn/Vn)^.5/N;
VVb = (VEb/nq)^2*Vn;
VEc = -rho*nq*(sq-VEP+VCovPn*nq/Vn)/N;
VVc = (VEc/nq)^2*Vn;
VCovbc = (VEb/nq*(VEc/nq))*Vn;
VUN = .5*log(1-2*VVb)/rho-VEc/rho-.5*VVc/rho-(VEb+VCovbc)^2/(rho*(1-2*VVb));
VUM = VGTi-(1/rho)*log(1-m/mh*(1-exp(-rho*(w-D))));

% Social welfare:
SWF = LE*(GP+GTu)+(L-LE)*(GTi+GI)+N*UN+M*UM;
SWF2 = LE*(VGP+VGTu)+(L-LE)*VGTi+N*VUN+M*VUM;

% First and second derivatives:
dVb=diff(Vb,LE);
dSWF=diff(SWF,LE);
d2SWF=diff(dSWF,LE);

% \Delta(L_E)-\Gamma(L_E):
F=GP+GTu-GI-GTi;
VF=VGP+VGTu-VGTi;

% "Rents":
rentE=GP+GTu-GTi;
rentT=GI;
rentM=-(1/rho)*log(1-m/mh*(1-exp(-rho*(w-D))));
empl=m/mh;

% Approximated constrained optimal L_E's in the noiseless case:
LEopt0=a*(L+M)*sq/(rho*Ve)+a*nq-.5*Vs*(L+M)/Ve+a*m*(L+M)*(1-exp(-rho*(w-D)))/(rho^2*Ve);
```

```
VLEopt0=a*(L+M)*sq/(rho*(Vs+Ve))+a*nq+a*m*(L+M)*(1-exp(-rho*(w-D)))/(rho^2*(Vs+Ve));

% Transform expressions into 'character' form:
SWFn=char(SWF);
SWF2n=char(SWF2);
dVbn=char(dVb);
Vbn=char(Vb);
dSWFn=char(dSWF);
d2SWFn=char(d2SWF);
Fn=char(F);
VFn=char(VF);
rentTn=char(rentT);
rentEn=char(rentE);
rentMn=char(rentM);
VPn=char(VP);
sqn=char(sq);
empln=char(empl);
LEopt0n=char(LEopt0);
VLEopt0n=char(VLEopt0);
GTin=char(GTi);
VGTin=char(VGTi);

clear LEopt0 VLEopt0 empl L M N sq Vs Ve rho nq a LE Vn b w alpha beta gam VphiGw VsGw EP Ez VP Vz GP GTu GTi GI CovPz VEP VEz VVP VVz VGP VGTu VGTi VGI VCovPz Vb Vc Covbc Eb Ec Vb Vc Covbc VEb VEc VVb VVc VCovbc SWF SWF2 dSWF dSWF2 F VF VCovPn VUM VUN UM UN dVb rentT rentE rentM wage mh m drSWF rSWF rSWF2 VtmP VVtmP SWFnN dSWFnN CovPn d2SWF

toc
```

Function "UE_Model_f1"

```
% This is function "UE_Model_f1". The r.h.s. shows function inputs, the
% l.h.s. shows function outputs:
function [equ_T, equ_U, s_T, s_U, s_T_d, maxLEU, SU_maxLEU, Vbl, rVP, multequ,rT,rE,rM,VrE,VrM,maxLES,ST_maxLET,maxLESglobal,ST_maxLETglobal,Empl,LEopt0,VLEopt0,ST_LE0opt,SU_VLE0opt,GT_i,VGT_i,SU_LET,SL,wb,sqq,utiE,utiM,Vbl_one] = UE_Model_f1(L,a,Vs,rho,Ve,M,nq,b,A,D,sh,N,t,Vn,Vbn,Fn,VFn,SWFn,dSWFn,d2SWFn,SWF2n,rentEn,rentTn,rentMn,VPn,sqn,empln,LEopt0n,VLEopt0n,GTin,VGTin,dVbn)

% Show simulation progress in the command window:
if mod(t,10000)==0
display(num2str(t));
toc
else
end

x0=double(L/2);

% Transform 'F' into a function of L_E:
```

```
F=eval(['@(LE)' Fn]);

% Set up the function's output variables:
equ_T=NaN;
s_T=NaN;
s_T_d=NaN;
equ_U=NaN;
s_U=NaN;
maxLEU=NaN;
SU_maxLEU=NaN;
Vbl=NaN;
rVP=NaN;
multequ=NaN;
rT=NaN;
rE=NaN;
rM=NaN;
VrE=NaN;
VrM=NaN;
maxLES=NaN;
ST_maxLET=NaN;
maxLESglobal=NaN;
ST_maxLETglobal=NaN;
Empl=NaN;
LEopt0=NaN;
VLEopt0=NaN;
ST_LE0opt=NaN;
SU_VLE0opt=NaN;
GT_i=NaN;
VGT_i=NaN;
SU_LET=NaN;
SL=NaN;
wb=NaN;
sqq=NaN;
utiE=NaN;
utiM=NaN;
Vbl_one=NaN;

if F(a*nq)<0 || F(L)>0
    wb=1;

else
    % Solve for equilibrium L_E^1; the function "rmsearch" finds good
        starting points and search intervals;
    %\hat M not defined for L_E=0:
    [u, ~, errorflag]=rmsearch(F,'fzero',x0,1e-6,L,'InitialSample',100);

        for k1=1:length(errorflag)
        if errorflag(k1)<0
           u(k1)=[];
        end
        end

```

```
% Drop the combination, if it implies multiple equilibria:
if length(u)>1
            multequ=1;
else

Vb=eval(['@(LE)' Vbn]);
Vbu=Vb(u);
VbL=Vb(L);  % Vb in case of restriced OC is indep. of L_E

% Drop the combination, if it does not imply well defined noise trader utiltiy:
if Vbu>=0.5 || VbL>=0.5
   Vbl=1;

else
                equ_T=u; % the unique L_E^1

                VF = eval(['@(LE)' VFn]);
                VF0=VF(1e-6);
                VFL=VF(L);
                if sign(VF0)==sign(VFL) && sign(VFL)>0 % no interior solution exists (VF is strictly decreasing, see text)
                    equ_U=L;  % corner equilibrium L_E^0=L
                elseif sign(VF0)==sign(VFL) && sign(VF0)<0 % functions not defined for L_E=0
                    equ_U=1e-6;
                    disp('Warning: L_E^0=0');
                else
                % Solve for interior equilibrium L_E^0:
                [equ_U, ~, errorflag]=fzero(VF,[1e-6 L]); % Sufficient, as we know that L_E^0 is unique (see text)

                      if errorflag<0
                        equ_U=[];
                      end
                      if isempty(equ_U)
                          disp('Error: No equilibrium L_E^0')
                      end
                end

                % Transform expressions into functions of L_E:
                VP=eval(['@(LE)' VPn]);
                rentT=eval(['@(LE)' rentTn]);
                rentE=eval(['@(LE)' rentEn]);
                rentM=eval(['@(LE)' rentMn]);
                sq=eval(['@(LE)' sqn]);
                empl=eval(['@(LE)' empln]);
                GTi=eval(['@(LE)' GTin]);
                VGTi=eval(['@(LE)' VGTin]);

                VP0=Vs; % Ex-ante price variance in the noiseless case with free OC
```

```
                rVP=(VP(equ_T)-VP0)/VP0;

                rT=rentT(equ_T);
                rE=rentE(equ_T);
                rM=rentM(equ_T);

                utiE=rT+GTi(equ_T); % CE_E
                utiM=rM+GTi(equ_T); % CE_M

                Empl=empl(equ_T);

                GT_i=GTi(equ_T);

                % Transform expressions into functions of L_E:
                SWF=eval(['@(LE)' SWFn]);
                SWF2=eval(['@(LE)' SWF2n]);
                dSWF=eval(['@(LE)' dSWFn]);
                d2SWF=eval(['@(LE)' d2SWFn]);

                s_T=SWF(equ_T); % equilibrium social welfare with free OC
                s_T_d=dSWF(equ_T); % slope of the social welfare function
                    with free OC at equilibrium L_E^1
                SL=SWF2(L); % social welfare at L_E=L

                sqq=sq(equ_T); % \bar s at equilibrium

                VrE=VF(equ_U);
                VrM=rentM(equ_U);
                VGT_i=VGTi(equ_U);

                s_U=SWF2(equ_U); % equilibrium social welfare with
                    restricted OC
                SU_LET=SWF2(equ_T); % social welfare with restricted OC,
                    evaluated at L_E=L_E^1

        % For maximization of social welfare, noise trader utility has to
        % be defined for all L_E; Hence, we drop all combinations which
        % imply that Vb is greater than 0.5 for any L_E:
        dVb=eval(['@(LE)' dVbn]);
        [Vb1, ~, errorflag0]=rmsearch(dVb,'fzero',x0,0,L,'InitialSample'
            ,100);  %rmsearch finds "good" starting points / intervals;
            fminbnd finds the min of -SWF (local if it exists, global
            otherwise)
                for k0=1:length(errorflag0)
                if errorflag0(k0)<0
                    Vb1(k0)=[];
                else
                end
                end

                if isempty(Vb1)
```

```
                    Vbmaxvalgl=max([Vb(0) Vb(L)]);
                elseif length(Vb1)>1
                    disp('Vb more than one extremum')
                    Vbmaxvalgl=max([Vb(0) Vb(L) Vb(Vb1)]);
                elseif dVb(Vb1+1e-12)>0
                    disp('Vb has a minimum')
                    Vbmaxvalgl=max([Vb(0) Vb(L) Vb(Vb1)]);
                else
                    Vbmaxvalgl=max([Vb(0) Vb(L) Vb(Vb1)]);
                end
        if Vbmaxvalgl>=0.5 % there is an L_E for which Vb>0.5
            Vbl_one=1;
        else
                % Approximated constrained optimal L_E's in the noiseless case:
                LEopt0=eval(LEopt0n);
                LEopt0=max(1e-6,min(LEopt0,L));
                ST_LE0opt=SWF(LEopt0);
                VLEopt0=eval(VLEopt0n);
                VLEopt0=max(1e-6,min(VLEopt0,L));
                SU_VLE0opt=SWF2(VLEopt0);

                % Call function "S1opt" (see below), which finds the local
                % as well as the global maximum of S^1:
                [maxLES, maxLESglobal]=S1opt(dSWF,d2SWF);

                ST_maxLET=SWF(maxLES); % local maximum social welfare S^1
                ST_maxLETglobal=SWF(maxLESglobal); % global maximum social welfare S^1

                % Call function "S0opt" (see below), which finds the
                % maximum of S^0:
                maxLEU=S0opt(SWF2);

                SU_maxLEU=SWF2(maxLEU); % maximum social welfare S^0
        end

end
end
end

% Auxiliary functions "S1opt" and "S0opt" below

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% function "S1opt" finds the L_E that (locally/globally) maximizes S^1:
function [maxLES,maxLESglobal] = S1opt(dSWF,d2SWF)

            [j, ~, errorflag]=rmsearch(dSWF,'fzero',x0,1e-6,L,'InitialSample',100);
```

```
                for k2=1:length(errorflag)
                if errorflag(k2)<0
                    j(k2)=[];
                else
                end
                end

                if isempty(j) % no local extremum
                    maxLES=-1;
                    jvalaux=[SWF(1e-6) SWF(L)];
                    jaux=[1e-6 L];
                    maxLESglobal=jaux(jvalaux==max(jvalaux));
                else
                   % check if local extremum is a maximum:
                   jval=NaN(length(j),1);
                   d2aux=NaN(length(j),1);
                   for i1=1:length(j)
                       d2aux(i1)=d2SWF(j(i1));
                       jval(i1)=SWF(j(i1));
                   end
                   j=j(d2aux<0); % keep only maxima
                   jval=jval(d2aux<0);

                   jvalaux=[jval' SWF(1e-6) SWF(L)];
                   jaux=[j' 1e-6 L];
                   maxLESglobal=jaux(jvalaux==max(jvalaux));

                   if isempty(j) % no local maximum
                        maxLES=-1;
                   elseif length(j)==1
                        maxLES=j;
                   else
                        % in case there are multiple local maxima, choose
                            the one closest to the
                        % approx. \hat L_E^1 in the noiseless case:
                        lowdis=zeros(length(j),1);
                        for k3=1:length(j)
                            lowdis(k3)=abs(LEopt0-j(k3));
                        end
                        maxLES=j(lowdis==min(lowdis));
                   end
                end

end


% function "S0opt" finds the L_E that maximizes S^0:
function maxLEU = S0opt(SWF2)

            opts2 = optimset('TolX',1e-6); % default TolX is 1e-4
            mSWF2=@(LE)-SWF2(LE);
```

```
                % fminbnd finds the min of -SWF2 (local if it exists, global
                % otherwise):
                [e, ~, errorflag]=fminbnd(mSWF2,1e-6,L, opts2);

                    for k4=1:length(errorflag)
                    if errorflag(k4)<0
                        e(k4)=[];
                    else
                    end
                    end
          % S^0 is inverse U-shaped (see text):
          if e>L-1e-3 % no local minimum has been found
                        maxLEU=L;
          elseif e<0+1e-3 % no local minimum has been found
                        maxLEU=1e-6;
                        disp('optimal L_E equals "zero"')
          else
                        maxLEU=e;
          end

end

end
```

Adjustemts to function "rmsearch"

Adjustments made to "rmsearch" are the same as in the basic version of the model and the FE model.

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