Measuring Counterparty Risk

Development of innovative Methods in Light of Regulatory Reforms

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Introduction

Motivation and area of research

A sound and stable financial sector is a critical and essential building block of modern economies. Financial institutions act as intermediaries and support the alignment of money supply and lending as well as an efficient transfer of risks. The stability of the financial system is considered to be a crucial prerequisite for economic growth (BCBS (2011)). By pursuing their business activities, financial institutions take different types of risks. Some of these risk are taken deliberately, such as credit risk from lending transactions, others are inherited in the business activities themselves, such as operational or business risks. Financial institutions need to manage their risks properly to prevent losses and to ensure they are able to fulfill their contractual obligations at any time. Given the systemic importance of banking institutions and their stability, regulatory requirements are imposed to ensure the appropriate management of risks. For example, banks are required to hold a regulatory defined amount of capital to absorb potential losses from their business activities (BCBS (2011)). The quantification of capital requirements for different types of risk is a central aspect of supervisory oversight and an indispensable element of the regulatory framework. Financial institutions use various instruments to take, transfer and manage financial risks. In the past decades, derivative instruments have played a major role in the financial sector as they offer the possibility to synthetically take or close a risk position. Hence, derivatives can be used to efficiently transfer risks between counterparties. When trading derivatives (bilaterally) over-the-counter (OTC), each counterparty has the risk of the other not meeting its contractual obligations (Gregory (2015)). Within the regulatory framework, this risk is referred to as Counterparty (Credit) Risk (CCR).[1]

[1] The terms "*Counterparty Credit Risk*" and "*Counterparty Risk*" are used synonymously within this thesis.

The most challenging task in counterparty risk management is the modeling of credit exposures. The measurement of credit exposures for derivatives and classical lending transactions differ in various aspects (Picoult (2004)). First, the future exposure of derivatives is uncertain, as it is a function of market parameters, while the exposure of lending risk transactions is usually known. Second, the risk in derivative transactions is bilateral, as a positive exposure can emerge for both counterparties in the future. Lending risk is generally unilateral, as the direction of the credit exposure is clearly defined by the underlying contract. Third, the mitigation of CCR often involves complex and dynamic risk mitigation techniques, such as margining, netting and clearing. The collateralization of lending risk transactions tends to be static and the methodological challenge lies in the valuation of collaterals. Finally, the quantification of CCR exposures is a laborious and complex task, while the exposure of lending risk transactions is deterministic in most cases. Given the bilateral nature of risk, the inherent uncertainty of future market parameters and the economic impact of dynamic risk mitigation techniques, the modeling of credit exposures from derivatives transactions requires advanced statistical concepts and in-depth quantitative knowledge (Picoult (2004)). The regulatory framework provides different approaches for the quantification of counterparty risk exposures. In general, institutions are able to choose between the application of an internal model method (IMM) or a regulatory standardized approach. While an IMM needs to adhere to a set of guidelines and requires supervisory approval, standardized approaches are less complicated, but result in a more conservative risk assessment (see BCBS (2006), BCBS (2014d)).

The Global Financial Crisis (GFC) from 2007-2009 revealed the significance of counterparty risk and the OTC derivatives market for the financial stability of the global economic system (FSB (2010)). During the crisis, financial institutions suffered tremendous losses in their derivatives business activities stemming from actual counterparty defaults as well as increasing Credit Valuation Adjustments (CVA). These losses jeopardized the survival of many financial institutions as well as the stability of the global financial system (Gregory (2010)). The Lehman Brothers bankruptcy in 2008 and the subsequent financial turmoil provided striking evidence for failures and shortcomings in the regulation of financial institutions and the OTC derivatives market. The G20 leader's report of the 2009 Pittsburgh summit stated that *"Major failures of regulation and supervision, plus reckless and irresponsible risk taking by banks and other financial institutions, created dangerous financial fragilities that contributed significantly to the current crisis."* (G20 (2009)). As a response to the financial crisis, governments and supervisory authorities developed additional regulatory requirements. Amongst other aspects, the G20 in 2009 agreed to increase capital standards for banking institutions and to strengthen the regulation of the

OTC derivatives market (G20 (2009)). The corresponding policy measures included regulatory initiatives aiming at the reduction and mitigation of counterparty risk. There were two major areas of regulatory reform in this context. First, additional regulatory capital requirements were imposed to make banks more resilient against losses from counterparty risk. In this context, a variety of changes were made to the CCR capital framework (BCBS (2011)). Second, new rules for the trading of derivatives were imposed, such as mandatory central clearing for certain standard derivatives and the introduction of requirements for the collateralization of non-centrally cleared derivative transactions (FSB (2010)).

When looking retrospectively at supervisory activities in the past decade, it becomes clear that the Global Financial Crisis triggered a reform of the whole regulatory framework for banking institutions. At the time of writing, the process of regulatory conversion is not completed, as various regulatory changes are still to be finalized and implemented (BCBS (2017)). Nevertheless, the reform of the regulatory framework has already led to significant changes in the assessment of regulatory capital for counterparty risk. Under Basel II, the capital requirement for CCR was solely based on a default risk charge which is calculated in line with the capital requirements for credit risk based on a loan equivalent exposure measure (BCBS (2006)). An additional capital charge for CVA risk was introduced under Basel III to safeguard financial institutions against mark-to-market losses caused by the deterioration of the credit quality of counterparties. Furthermore, additional requirements for the application of the IMM, such as the consideration of wrong-way-risk, were added to the framework, leading to higher capital requirements and more extensive qualitative model standards (BCBS (2011)). The existing standardized approaches for counterparty risk are considered outdated and not appropriate given the changed market conditions and standards as well as volatility levels observed during the GFC (BCBS (2013)). Hence, the BCBS published a new standardized approach for measuring counterparty credit risk exposures (SA-CCR) in 2014 (BCBS (2014d)). The SA-CCR will replace the existing standardized approaches going forward. This approach will be a central cornerstone of the future regulatory framework, as its results are used in subsequent regulatory measures, such as leverage ratio (BCBS (2014a)) and the CVA risk capital charge (BCBS (2019a)). Furthermore, under final Basel III rules (BCBS (2017)), the benefit from the application of internal methods will be bounded based on the result of the respective regulatory standardized approach.

Following the G20 declaration (G20 (2009)), additional rules and regulations for trading derivatives were introduced. The aim of these new regulatory initiatives is the reduction of risk in the financial industry and the protection of counterparties from the risk of another counterparty's default. The regulatory efforts are threefold. First, an obligation to clear certain standardized derivatives via central clearing counterparties (CCPs) is introduced. Second, additional reporting requirements with respect to OTC derivatives trading activities are imposed to increase the transparency of the OTC market. Third, rules for the collateralization of non-centrally cleared derivatives are adopted to reduce the effect of counterparty defaults in OTC transactions. These rules include the mandatory exchange of variation margin (VM) and initial margin (IM) for certain bilateral transactions (FSB (2010), BCBS and IOSCO (2019)).

The emerging changes to the regulatory capital framework and the new rules for OTC derivatives trading will have a significant impact on the regulatory required capital for counterparty risk and the valuation of OTC derivatives. This leads to various theoretical and practical challenges for financial institutions when modeling counterparty credit risk exposures. This thesis aims to analyze and tackle three selected issues resulting from the introduction of the new standardized approach (SA-CCR) as well as the mandatory exchange of initial margin for non-centrally cleared OTC derivatives. The selected issues are handled in three independent research papers, which are presented in the chapters 1, 2 and 3 of this thesis. The following paragraphs provide a first introduction on the background, motivation and focus of each research paper.

Research paper I | *Credit Exposure under SA-CCR: Fixing the treatment of equity options*

The SA-CCR will replace the existing supervisory standardized approaches going forward. As discussed above, its introduction will affect a series of regulatory measures and most likely lead to higher capital requirements (ABA et al. (2019)). The approach will be broadly applied and has to be implemented by the majority of financial institutions, as its results will be utilized in the determination of the capital output floor (BCBS (2017)). The banking industry generally welcomes the introduction of the SA-CCR, as the approach offers significant methodological enhancements compared to its predecessors, such as the consideration of risk mitigating effects from margining and over-collateralization. Nevertheless, there is ongoing discussion and criticism regarding certain methodological aspects. Amongst others, the flawed treatment of non-linear products as well as the overly conservative calibration of supervisory parameters, in particular for equity products, have been bones of contention (ABA et al. (2019)). Given the importance of the SA-CCR and its subsequent usage in the regulatory framework, a sound understanding of its methodology, results and weaknesses is a crucial prerequisite for its appropriate application by supervisory authorities and banking institutions. The first research paper (see chapter 1) provides a theoretical and empirical analysis of methodological issues regarding the treatment of equity options under SA-CCR. The research paper aims to increase the understanding of the methodology and its weaknesses as well as to develop measures for improving the SA-CCR.

Research paper II | *Computing valuation adjustments for CCR using a modified supervisory approach*

The calculation of CCR exposures is required for different aspects of risk management and valuation. First, credit exposures are required for the limitation of counterparty risk. Second, exposure results are used as inputs for the calculation of capital requirements. Third, expected exposure profiles are used in the calculation of various valuation adjustments, which are an integral part of derivatives pricing. The calculation of CCR exposures and especially time-dependent exposure profiles is a highly complex and laborious task. Small- and medium-sized financial institutions are, in most cases, not capable of maintaining an advanced CCR exposure model (EBA (2016)). Hence, there is undoubtedly a demand for more simple, but sufficiently accurate semi-analytical methods for exposure quantification. The SA-CCR involves significant enhancements compared to its predecessors. Amongst others, the new approach is able to recognize the risk mitigating effects from margining and provides a more sophisticated approach to netting and diversification (BCBS (2014d)). Furthermore, BCBS (2014b) provides a maximum of transparency on the methodological foundations of the approach. Hence, the SA-CCR offers a series of desirable features for the modeling of exposures while providing an accessible and holistic methodological framework. As discussed above, the SA-CCR has to be implemented by the majority of financial institutions. Hence, the utilization of the SA-CCR for the generation of time-dependent exposure profiles is an option worth considering, in particular for transactions not covered by advanced approaches. The second research paper (see chapter 2) develops a new semi-analytical approach based on the SA-CCR for determining time-dependent exposure profiles in the context of the CVA calculation.

Research paper III | *The KANBAN Approach - A new way to compute forward Initial Margin*

Given the emerging regulatory requirements regarding the collateralization of non-centrally cleared derivatives, the majority of OTC transactions will be supported by the exchange of Initial Margin (IM) in the future. ISDA provides a standard model (ISDA-SIMM™) for the calculation of IM amounts, which essentially equals a sensitivity-based analytical VaR approach (ISDA (2016), ISDA (2019)). This model is expected to become market standard for the calculation of IM amounts for OTC derivatives. The bilateral exchange of IM significantly impacts capital requirements, funding costs and the profitability of derivative transactions. Hence, IM amounts must be considered when calculating CCR exposures. This requires the calculation of future IM requirements. The accurate forecasting of IM requirements is a difficult and challenging task. In particular, the calculation of time- and path-dependent IM amounts in a Monte Carlo framework is complex and anything but straight-forward. In general, an approach for forecasting IM requirements should balance the computational burden and the accuracy of results. Most existing approaches are either inaccurate, hard to implement or not fully developed in order to be applied to complex practical situations. The third research paper (see chapter 3) introduces a new approach for forecasting IM requirements under ISDA-SIMM™.

In summary, this thesis aims to develop innovative solutions for prevailing and emerging issues in the area of CCR exposure modeling. The aforementioned challenges arise from changes in the regulatory framework as well as developments with respect to market standards in the OTC derivatives market. The thesis contributes to a wide field of scientific research on the modeling of counterparty credit risk exposures. The subsequent paragraphs provide an overview on existing literature focussing on methods for exposure quantification and forecasting of IM.

Literature

Over the past decades, the calculation of CCR exposures has been an active field of scientific research leading to a multitude of approaches and methods. According to Gregory (2015), there are three categories of approaches with different levels of sophistication: (1) advanced models, (2) semi-analytical and (3) parametric approaches. The most sophisticated way to model CCR exposures is the application of advanced approaches using Monte Carlo simulation. This involves complex tasks, such as the calibration of stochastic processes for the evolution of risk factors, the consideration of correlations and the modeling of collateralization. There is plenty of academic literature on various issues of the application of advanced exposure models (see, e.g., Picoult (2002), Canabarro and Duffie (2003) Pricso and Rosen (2005), Pykhtin and Zhu (2007)). The literature on advanced methods covers different areas. For example, Picoult (2004) analyses the application of Monte Carlo (MC) simulation in the context of economic capital based on the results of Canabarro et al. (2003). The modeling of collateral and the impact of margining is also an important issue and has been analysed, amongst others, by Gibson (2005) and Pykhtin (2009). The emergence and ongoing improvement of advanced models also led to adoptions in the regulatory framework. Under Basel II (BCBS (2006)), banks are allowed to apply advanced exposure models for the purpose of measuring regulatory capital for counterparty risk for the first time.[2]

[2] Fleck and Schmidt (2005) provide a comprehensive analysis of the Basel II framework for counterparty credit risk.

While advanced methods deliver the most accurate results, their application requires a magnitude of personal and technical resources. Small- and medium-sized banks often lack the capabilities to develop and maintain advanced methods in the area of counterparty risk (Thompson and Dahinden (2013)). This results in a demand for less sophisticated approaches, which avoid the burdensome operation of a Monte Carlo simulation. This aspect has led to the development of various semi-analytical methods, which are based on certain assumptions regarding the evolution of risk factors and market values. Semi-analytical approaches have been designed for various asset classes and products (see, e.g., Brigo and Masetti (2005), Wilde (2005), Leung and Kwok (2005), de Pricso et al. (2007)). One early example is the approach of Sorensen and Bollier (1994). Their approach uses a strip of swaptions to model the exposure profile of an interest rate swap. In addition, the emergence of valuation adjustments for counterparty credit risk has led to various semi-analytical models for the approximation of product-specific CVA results (see, e.g., Kao (2016), Hull and White (2012), Cherubini (2013)). The second research paper (see chapter 2) aims to add an alternative and innovative method for measuring CCR exposures to the library of semi-analytical approaches.

Parametric approaches measure CCR exposures based on a few simple parameters. According to Gregory (2015), most parametric approaches model CCR exposure via a combination of the current exposure and an add-on for the potential future exposure. There is scarce academic literature regarding parametric approaches, but the concept is often utilized by regulators as the basis for regulatory standardized approaches. Under Basel II (BCBS (2006)), banks not applying an IMM are able to choose between two standardized approaches: Current Exposure Method (CEM)[3] and Standard Method (SM). Both approaches are parametric approaches, where the calculation of exposure is based on supervisory prescribed risk-weights.[4] The new standardized approach SA-CCR features elements of semi-analytical and parametric approaches. BCBS (2014b) and BCBS (2014d) provide comprehensive information on the SA-CCR methodology and its model foundations. The introduction of the SA-CCR led to a variety of literature regarding the evaluation of the new approach (see, e.g., Albuquerque et al. (2017), ABA et al. (2019), Berrahoui et al. (2019)). The first research paper (see chapter 1) amends this research by dealing with weaknesses of the SA-CCR regarding the treatment of (non-linear) equity products.

The increasing importance of Initial Margin (IM) in the OTC derivatives market results in the requirement of forecasting IM amounts to model the effect from the exchange of IM on CCR exposures. Various approaches for forecasting IM requirements are discussed in recent

[3] Please note that the CEM has already been introduced in the Basel I framework (BCBS (1988), BCBS (1995)).

[4] A comprehensive discussion of the Basel II standardized appraoches is provided by Fleck and Schmidt (2005).

academic research. A series of research papers propose the calculation of forward IM based on dynamic initial margin models (see Andersen et al. (2017a), Andersen et al. (2017b), Anfuso et al. (2017), McWalter et al. (2018)). These dynamic margin models are based on regression techniques, utilizing existing scenario information from the Monte Carlo simulation. Chan et al. (2017) provide an overview and evaluation of different regression approaches. Under ISDA-SIMM™ the task of forecasting IM reduces to the calculation of forward sensitivities, as these are the only dynamic time-dependent model inputs. There are various concepts for the calculation of forward sensitivities which can be applied to forecast IM amounts. Fries (2019), Fries et al. (2018) and Antonov et al. (2017)) use the concept of Adjoint Algorithmic Differentiation (AAD) based on the work of Giles and Glasserman (2006) and Capriotti (2011). Zeron and Ruiz (2018) utilize the concept of Chebyshev Spectral Decomposition to compute the required forward sensitivities. The third research paper (see chapter 3) broadens this area of research by introducing an innovative approach for forecasting IM requirements in an existing Monte Carlo framework.

Contributions

This thesis contributes to the literature on modeling counterparty credit risk exposures and adds innovative methods to the counterparty risk management toolbox. The main contributions of this thesis can be structured according to the independent research papers. These research papers are presented within the chapters 1, 2 and 3 of this thesis.

Contribution I | *Credit Exposure under SA-CCR: Fixing the treatment of equity options*

The first research paper (see chapter 1) develops and explores potential measures to improve the treatment of equity options under the SA-CCR. Based on a methodological deep-dive on the SA-CCR's model foundations, an empirical analysis is conducted, in which the supervisory parameters are validated and the resulting exposures are benchmarked against an advanced model. The results of the analysis reveal the overly conservative calibration of supervisory parameters for equity transactions as well as the weaknesses of the SA-CCR in coping with non-linear products. Furthermore, it is found that the supervisory calibration for equity transaction clearly lacks granularity, leading to insufficient risk sensitivity of results. Given these findings, measures for improving the SA-CCR methodology are proposed. This includes the alignment of the calibration with the standardized approach for market risk (SA-TB) as well as the utilization of economic delta adjustments for the treatment of non-linear products. The application of economic instead of supervisory delta adjustments offers improvement, but should be accompanied by regulatory guidelines to ensure a consistent implementation across institutions. The alignment of the standardized approaches for CCR and market risk is proven to be beneficial for the risk sensitivity of the SA-CCR and would significantly contribute to the consistency of the regulatory capital framework for derivative products. In summary, the first research paper fosters the sound understanding of the SA-CCR's methodological framework and associated weaknesses regarding the treatment of (non-linear) equity products. It provides measures for improvement of the SA-CCR methodology. In particular, the idea of aligning regulatory standardized approaches for market and counterparty risk is explored by theoretical and empirical analysis based on comprehensive historical data.

Contribution II | *Computing valuation adjustments for CCR using a modified supervisory approach*

The second research paper (see chapter 2) introduces a fast and simple semi-analytical method for the calculation of time-dependent exposure profiles in the context of CVA quantification. This new approach is a modified version of the supervisory SA-CCR. For an appropriate application, various adjustments are conducted on the supervisory framework. Within the research paper, these adjustments are derived and a risk-neutral calibration of the modified SA-CCR is established to ensure consistency with requirements defined by the accounting framework (IFRS 13). The modified SA-CCR offers a holistic framework covering a variety of asset classes and financial instruments. In a benchmark study, the modified SA-CCR is used to calculate exposure profiles and CVA results for a set of hypothetical netting sets involving commonly used derivative products, such as interest rate swaps and FX forwards. The results are compared to the outcome of an advanced benchmark model. The findings clearly indicate that the modified SA-CCR captures time-dependent and product-specific exposure dynamics and produces sufficiently accurate CVA results for accounting purposes. The modified SA-CCR provides an alternative and innovative semi-analytical modeling framework with significant enhancements to current industry practices regarding the calculation of CVA for transactions not covered by advanced models. As the new approach is a modification of the supervisory SA-CCR, institutions are able to leverage on existing or future implementations of the SA-CCR. Hence, the approach is of high practical relevance for all kinds of financial institutions involved in derivative trading activities.

Contribution III | *The KANBAN Approach - A new way to compute forward Initial Margin*

The third research paper (see chapter 3), presents an innovative approach for forecasting IM requirements under ISDA-SIMM™ based on forward sensitivities. This approach offers a framework for the calculation of time- and path-dependent sensitivities in a Monte Carlo based exposure model. The KANBAN approach utilizes existing elements of the exposure model, such as cash flow objects and the pricing of non-linear instruments via American Monte Carlo (AMC). The technical design of the approach is based on principles adopted from industrial just-in-time manufacturing. Cash flows are used as central objects, as they carry the comprehensive information for the production of forward sensitivities which can be interpreted by a central market data service. This enables a lean "on-the-fly" generation of path- and time-dependent sensitivities. The KANBAN approach offers a series of advantages over existing methods for the calculation of forward Initial Margin. First, the approach is much faster compared to classical "bump-and-run" approaches, as each cash flow is processed independently and sensitivities are calculated simultaneously instead of successively. Second, the KANBAN approach uses only information and methodological building blocks which are already implemented in the prevailing model framework. Hence, no additional model risk is added to the counterparty risk and valuation framework. Third, the new approach is applicable to any financial instrument, as the calculation of sensitivities is based on the unified representation of financial instruments as a series of cash flows. The research paper includes the methodological foundation of the KANBAN approach and a case study, in which the methodology is applied to standard financial products and an interest rate swap portfolio.

Structure

The thesis is structured alongside the three independent research papers with varying co-authors.[5] Chapter 1 presents the first research paper on the calculation of credit exposure under the new supervisory standardized approach (SA-CCR) and particularly the associated treatment of equity options. In chapter 2, a modified SA-CCR approach for the calculation of valuation adjustments for counterparty credit risk is introduced and discussed. Chapter 3 is dedicated to the development and application of a new methodology for forecasting initial margin amounts. The Conclusion summarizes the thesis by discussing main results and providing an outlook for future research in the area of counterparty credit risk.

[5] At the beginning of each chapter, information with respect to the current status of the paper and the respective co-author(s) is provided. As the studies have been submitted to different journals with varying formal requirements, there are minor formal differences across the chapters of this thesis.

Chapter 1

Credit Exposure under SA-CCR: Fixing the treatment of equity options

This chapter corresponds to a working paper with the same name (submitted to *Journal of Credit Risk*, currently under review).

Abstract

The new standardized approach for measuring counterparty credit risk exposures (SA-CCR) will replace the existing regulatory standard methods for exposure quantification. There is ongoing discussion with respect to the calibration and appropriate treatment of non-linear products under the SA-CCR. Especially, the calibration of supervisory parameters for equity derivatives has been a bone of contention. Furthermore, the SA-CCR struggles with the adequate reflection of non-standard options. Our paper provides empirical evidence that the SA-CCR parameters are not aligned with historically observed volatilities. We explore a potential alignment of the SA-CCR with the new standardized approach for market risk (SA-TB) as well as the application of economic delta adjustments for path-dependent equity products. Our results demonstrate that an alignment of SA-CCR and SA-TB could lead to a significantly improved risk assessment for equity derivatives.

Keywords: Counterparty Credit Risk; SA-CCR; Regulatory Capital; Credit Exposure

JEL classification: G01, G21, G32

1.1 Introduction

Counterparty Credit Risk (CCR) has been a main source of loss during the Great Financial Crisis (GFC). Furthermore, CCR significantly contributes to banks' overall risk-weighted Assets (RWA). Hence, the assessment of minimum capital requirements for CCR has been a focus of regulators in the past decade. Especially the appropriate measurement of the Exposure at Default (EAD) for derivatives has been subject to ongoing discussions, as the existing approaches are considered to be outdated. The existing standardized approaches are used broadly in the banking industry. In general, even banks with an approved internal model (IMM) for CCR do not have approval to use the IMM for all products and / or subsidiaries (Thompson and Dahinden (2013)). The Basel Committee on Banking Supervision (BCBS) decided to review the standardized methods for measurement of CCR exposures and developed a new approach for exposure quantification (SA-CCR) (BCBS (2014d)).

The EAD calculated under the SA-CCR serves as input for the calculation of other regulatory measures, such as leverage ratio, large exposure framework, CVA risk capital charge and the CCP hypothetical capital calculation (BCBS (2017)). Hence, all banks must implement the SA-CCR irrespectively of the application of an internal model. Furthermore, there is ongoing discussion to use the results produced by regulatory standardized approaches as basis for the limitation of the capital benefits from the application of internal models (BCBS (2017)). Given the broad usage of the SA-CCR and its subsequent impact on various regulatory measures, we believe that the SA-CCR is a major cornerstone of the future regulatory framework. Thereby, systemic misjudgement of risk by the SA-CCR will not only affect capital requirements for CCR, but the regulatory framework as a whole.

The industry generally supports the replacement of the existing standardized approaches by the SA-CCR (OCC et al. (2019)).[1] Nevertheless, there are enduring concerns regarding a significant increase of capital requirements due to the conservative calibration and lack of risk sensitivity of the SA-CCR. There is ongoing discussion with respect to various flaws and shortcomings of the SA-CCR in Europe and the US. One area of concern is the conservatism of the calibration. According to ABA et al. (2019) a recalibration of the supervisory factors and volatilities for equity products would reduce the burden for financial institutions. As stated by OCC et al. (2019), various stakeholders suggested to align the SA-CCR parameters with risk-weights of

[1] This was made clear by leading industry associations throughout the consultation process, which started with the publication of the consultation paper in 2013 (BCBS (2013)).

the new standardized approach for market risk (SA-TB) (BCBS (2019b)). This would lead to a consistent assessment of risk and an increase of the risk sensitivity of the SA-CCR. In addition, potential drawbacks from using a supervisory delta adjustment formula based on Black & Scholes for exotic options were addressed multiple times by various industry bodies. They suggest allowing banks to use their own proprietary delta values to avoid a disconnect between the calculation of capital requirements and actual risk management.

Our paper provides the following contributions. First, we provide a theoretical contemplation of the calibration and treatment of equity options under the SA-CCR. Second, we conduct an empirical analysis to validate the supervisory parameters and to benchmark the results from the SA-CCR against an advanced model. Based on the outcome of this analysis, we identify flaws of the SA-CCR and suggest improvements to the regulatory methodology for measuring CCR exposures. Our paper focuses on the calibration and treatment of equity options under the SA-CCR. Equity derivatives have smaller trading volumes compared to interest rate and foreign exchange derivatives (BCBS (2018b)). Nevertheless, the quantification of their exposure is often subject to standardized approaches, as only 70% of banks with IMM approval use their internal model for plain-vanilla and exotic equity derivatives (Thompson and Dahinden (2013)). Hence, the handling of equity derivatives is considered an important and challenging field of application for standardized approaches.

We find that the calibration of the SA-CCR is overly conservative for most equity underlyings. Furthermore, there is a lack of granularity in calibration, limiting the ability of the approach to properly reflect the risk of most underlyings. These issues lead to a significant over-estimation of risk for equity derivatives. Introducing a more granular and risk sensitive calibration approach would improve the results of the SA-CCR while keeping a reasonable level of complexity. Our results indicate that an alignment of the SA-CCR with the new standardized approach for market risk (SA-TB) would significantly enhance its risk sensitivity. In addition, the usage of economic approaches for the calculation of the delta adjustment parameter would also increase the risk sensitivity of the SA-CCR with respect to barrier options. Nevertheless, the application of economic delta adjustments needs to come along with additional regulatory guidelines to ensure a consistent implementation and results across institutions.

The remainder of this paper is structured as follows. Section 1.2 provides a short overview of existing regulatory approaches for the measurement of CCR exposures and introduces the SA-CCR, including its methodological foundations. A thorough understanding of the SA-CCR methodology is a prerequisite for the subsequent discussion and empirical analysis. Readers acquainted with the regulatory capital framework for CCR and the methodological foundations of the SA-CCR might decide to skip this section. We elaborate on the current regulatory status and ongoing methodological discussions in Section 1.3. The empirical part of this paper is structured as follows. First, we perform a volatility analysis (section 1.4) based on historical data to assess the need for recalibration of the supervisory parameters and the appropriateness of SA-TB risk-weights. Second, we conduct a simulation study (section 1.5) to assess the performance of different SA-CCR calibrations and configurations based on hypothetical European plain-vanilla and barrier options. Section 1.6 summarizes the main conclusions and recommendations.

1.2 Approaches for the determination of CCR exposures

1.2.1 Overview and regulatory developments

In order to determine the default risk capital requirements for derivatives, banks must calculate the *EAD* for those transactions. The calculation of exposure values is performed for a set of positions within a legally enforceable netting agreement. The resulting *EAD* values serve as input for the calculation of regulatory capital requirements for default risk. The calculation of exposure values for CCR is considered a time-consuming and expensive task, as running the necessary simulations requires a high amount of computational power. Hence, there is a multitude of academic research on the optimization of calculation processes and simplification of methodology via the development of semi-analytical approaches. For example, Ghamami and Zhang (2014) provide an efficient Monte Carlo framework to decrease the computational time needed for the estimation of regulatory exposure measures. Orlando and Härtel (2014) develop a parametric approach that estimates the exposure as the sum of current and potential future exposure, while Pykhtin and Rosen (2010) introduce a semi-analytical approach which is capable of reflecting the impact of collateralization on CCR exposures.

Under the current regulatory framework, there are different approaches for calculating the *EAD* for derivative transactions (BCBS (2011)). On the one hand, there are regulatory standardized approaches, while on the other hand institutions may apply internal models. Since the introduction of Basel II (BCBS (2005)), banks are allowed to use their own internal exposure models to estimate the credit exposure of derivative transactions within the regulatory capital framework (**Internal Model Method(IMM)**). The internal model needs to adhere to quantitative and qualitative requirements. The application is subject to supervisory approval. When applying an Internal Model Method (IMM), the EAD for each netting set is defined as the product of a scaling factor (α) and the Effective Expected Positive Exposure ($EEPE$).[2] Under Basel III (BCBS (2011)), the $EEPE$ is defined as the maximum of an $EEPE$ calculated under stressed and current market data.

The regulatory framework currently offers a set of standardized approaches for exposure quantification. The **Current Exposure Method (CEM)** as well as the **Standardized Method (SM)** are simple and parametric approaches providing an approximation of the EAD. While the CEM is used by the majority of financial institutions, the SM is hardly applied within the financial industry (EBA (2016)).[3] CEM provides an approximation of the EAD as the sum of the current exposure (replacement costs) and an add-on for potential future exposure (PFE).[4] CEM has been criticized for various reasons in the past and is considered outdated. The BCBS summarizes the critique towards CEM in the following three main issues (BCBS (2013)). First, there is no differentiation between netting sets with and without margin agreements. Hence, the risk mitigation effects from margining are not rewarded within the regulatory capital framework. Second, the calibration of the supervisory parameters is outdated and does not consider volatility levels observed during the GFC. Third, the CEM involves a very simplistic recognition of diversification, when aggregating trade-level add-ons. In general, there has been ongoing criticism by members of the industry as well as academics.[5] An additional issue is the lack of risk sensitivity. Transactions of the same asset class and the same maturity will always have the same PFE under the CEM, regardless of their further, probably different, features (Gregory (2010)). Especially for options and other non-linear positions the lack of consideration of these features leads to unreasonable results.

Driven by this criticism, the BCBS has developed a new standardized approach (SA-CCR) to overcome the shortcomings of existing approaches and to align the regulatory treatment of derivatives with current market practices (BCBS (2013)). Going forward, the option to use an IMM model will persist, while the CEM and the SM will be replaced by the SA-CCR. The SA-CCR is more complex compared to CEM and aims to provide a reasonable risk sensitive exposure measure considering risk mitigation techniques as well as the moneyness of the netting set and the subsequent positions. Furthermore, the calculation procedure for the PFE takes

[2] The correction-factor (α) is set to 1.4 and is introduced to account for systemic model errors as well as the missing consideration of general wrong-way risk in the regulatory CCR framework. For further information on the interpretation and role of the α-factor, please refer to Lynch (2014).

[3] According to EBA (2016) there are only two banks in Europe applying the SM.

[4] A more detailed outline of the CEM is provided in APPENDIX 1.A.1.

[5] Please refer to Fleck and Schmidt (2005) and Pykhtin (2014) for a comprehensive discussion of critique towards CEM.

into account the non-linearity of products. The new approach can capture risk mitigation through margining, as there is a distinction between margined and unmargined netting sets. For margined netting sets, a shorter risk horizon is applied compared to unmargined netting sets (1 year), when calculating the PFE. Additionally, the SA-CCR allows for hedging and netting within the main asset classes via the introduction of hedging sets and subsets. Another key improvement compared to CEM is the enhanced reflection of over-collateralization.

1.2.2 The new standardized approach (SA-CCR)

General structure and interpretation

According to BCBS (2014b), the SA-CCR shall provide an approximation of the EAD under IMM, which is defined as the product of the α-factor and the Effective Expected Positive Exposure (EEPE). The SA-CCR defines the netting set EEPE as the sum of replacement costs (RC) and potential future exposure (PFE). Therefore, the fundamental structure of the SA-CCR is very close to the CEM but follows the basic idea of the IMM:

$$EAD^{(SA-CCR)} = \alpha \cdot EEPE^{(SA-CCR)} = \alpha \cdot (RC + PFE) \approx EAD^{(IMM)} \tag{1.1}$$

The SA-CCR is calibrated to stressed (historic) market data in order to achieve a conservative approximation. Furthermore, the α-factor is transferred from the IMM formulation. While the structure of the approach is very similar to CEM, the calculation of RC and PFE have been revised. For the calculation of RC the main improvement to CEM is the consideration of margining. In contrast, the calculation of the PFE component has been completely revised and a new methodological framework was introduced. Hence, we focus on the presentation and discussion of the PFE calculation in the subsequent paragraphs.[6] The PFE aims to quantify the risk of an increase in exposure due to a change in the market value of the netting set during a predefined risk horizon. Under the SA-CCR, the PFE term is defined as the product of the aggregated add-on on netting set level and a multiplier. The multiplier is a function of the netting set's market value, volatility-adjusted collateral value and the aggregated add-on. In summary, the multiplier can be interpreted as a scaling factor for the PFE add-on with respect to the moneyness of the netting set.[7]

[6] APPENDIX 1.A.2 provides an overview of the calculation of RC under the SA-CCR.

[7] Detailed information and the derivation of the multiplier formula are available in BCBS (2014b) and BCBS (2014d).

PFE add-on calculation

BCBS (2014b) offers a detailed description of the methodological framework for add-on calculation.[8] The add-on calculation is performed based on the assumption of zero market values ($V_i(t_0) = 0$) and the absence of collateral ($C_A(t_0) = 0$). Furthermore, it is assumed that there are no cash flows within the risk-horizon of 1 year. The market value of transactions is assumed to follow an arithmetic Brownian motion with zero drift and constant volatility. Under these assumptions, we obtain the following analytical solution for the expected exposure (EE) of a netting set (k) at time t:[9]

$$EE_k(t) = \sigma_k(t) \cdot \phi(0) \cdot \sqrt{t} \tag{1.2}$$

where $\sigma_k(t)$ equals the annualized volatility of the netting set's market value at t and $\phi(0)$ is defined as the standard normal probability density: $\phi(0) = 1/\sqrt{2\pi}$. This formulation is the basis for the calculation of add-ons for margined and unmargined netting sets. The add-on for **unmargined** netting sets represents a conservative analytical approximation of the $EEPE$ for a risk horizon of one year. We are able to derive an analytical solution using an EE profile based on equation (1.2) and applying a floor of 1 year to all trade maturities:[10]

$$AddOn_k^{(no-margin)} = EEPE_k = \frac{2}{3} \cdot \phi(0) \cdot \sigma_k(0) \cdot \sqrt{1year} \tag{1.3}$$

This equation can be restated at trade-level.[11] Flooring all trade maturities at 1 year would produce unreasonable results and lead to an over-estimation of hedge effectiveness for short-dated trades. Hence, a maturity factor(MF_i) is introduced at trade-level to account for maturities (M_i) smaller than 1 year:

$$AddOn_i^{(no-margin)} = \frac{2}{3} \cdot \phi(0) \cdot \sigma_i(0) \cdot \sqrt{1year} \cdot MF_i \tag{1.4}$$

where MF_i for transactions in an unmargined netting set is defined as:

$$MF_i^{(no-margin)} = \sqrt{\frac{min(M_i, 10d)}{1year}} \tag{1.5}$$

The add-on for **margined** netting sets is defined as the potential increase in exposure over the Margin Period of Risk (MPOR). Based on the assumptions of zero market value and absence of

[8] The following methodological discussion is based on BCBS (2014b).

[9] For a detailed mathematical representation, please refer to APPENDIX 1.A.2.

[10] For the detailed mathematical derivation, please refer to APPENDIX 1.A.2.

[11] For the respective mathematical proof, please refer to APPENDIX 1.A.2.

collateral, this amount is defined based on equation (1.2):

$$AddOn_k^{(margin)} = EE_k(MPOR) = \phi(0) \cdot \sigma_k(0) \cdot \sqrt{MPOR} \tag{1.6}$$

In accordance with the add-on calculation for unmargined netting sets, we can restate this equation at trade-level. In order to arrive at a consistent formulation for the trade-level add-on, we formulate the add-on for margined netting sets in line with equation (1.4) and introduce a maturity factor for margined netting sets (BCBS (2014b)):

$$MF_i^{(margin)} = \frac{3}{2} \cdot \sqrt{\frac{MPOR}{1year}} \tag{1.7}$$

Hence, the only difference between the add-on calculation for margined and unmargined nettings sets is the definition of the maturity factor. For the calculation of the SA-CCR PFE add-on, each transaction is allocated to one of five risk categories (EQ, IR, CR, CO, FX) based on its primary risk factor.[12] To keep the calculation fast and simple, the SA-CCR does not use trade-level volatilities but a simple set of parameters. According to BCBS (2014d), the add-on at trade-level is defined as the product of a supervisory factor (SF_i), an adjusted notional amount ($d_i^{(a)}$), the supervisory delta adjustment (δ_i) and the maturity factor (MF_i)[13]:

$$AddOn_i = SF_i \cdot d_i^{(a)} \cdot \delta_i \cdot MF_i \tag{1.8}$$

We obtain the following definition of the trade-level market value volatility ($\sigma_i^{(V)}$) at $t = 0$ by inserting equation (1.4) into equation (1.8) and solving for σ_i:

$$\sigma_i^{(V)} = \frac{3 \cdot SF_i}{2 \cdot \phi(0)} \cdot d_i^{(a)} \cdot |\delta_i| \tag{1.9}$$

According to BCBS (2014b), the first factor of equation (1.9) can be interpreted as the one-year volatility of the transaction's primary risk factor ($\sigma_i^{(RF)}$). Hence, we are able to derive the following definition of the **supervisory factor**:

$$SF_i = \frac{2}{3} \cdot \sigma_i^{(RF)} \cdot \phi(0) \tag{1.10}$$

The **supervisory delta adjustments** (δ_i, SDA) is used to account for the direction of the transactions with respect to the primary risk factor (long/short) as well as the moneyness of non-linear

[12] If a transaction has more than one material risk factor, an allocation to multiple risk categories might be required (BCBS (2014d)).

[13] The maturity factor (MF_i) is calculated based on equations (1.5) and (1.7).

positions (e.g. options). For non-linear products, δ_i is calculated based on a simplified Black & Scholes delta formula:

$$\delta_i = \psi \cdot N\left(\omega \cdot \frac{ln(P/K) + 0.5 \cdot \left(\sigma_i^{(reg)}\right)^2 \cdot T}{\left(\sigma_i^{(reg)}\right) \cdot \sqrt{T}}\right) \tag{1.11}$$

where P represents the underlying price, K the strike price and $\sigma_i^{(reg)}$ the (supervisory) volatility. T is defined as the amount of years until the latest exercise date of the option.[14] The parameters ψ and ω are required to cover all combinations of bought/sold and call/put options.[15] According to BCBS (2014d) and BCBS (2018a), banks are not allowed to use their own internal delta results or calculation procedures for the estimation of the supervisory delta adjustment. The SDA for each option has to be calculated based on equation (1.11).[16] The **adjusted notional amount** ($d_i^{(a)}$) represents the size / volume of the transaction. The definition of d_i differs by asset class. For equity derivatives, d_i is defined as the product of the current price and the number of units referenced by the contract.[17] After calculation of trade-level add-ons based on equation (1.8), the results are aggregated to risk category specific add-ons at netting set level. The aggregated add-on at netting set level is defined as the simple sum of risk-category specific add-ons. The aggregation methodology differs by risk categories and considers diversification and hedging benefits.[18] With respect to equity derivatives, trade-level add-ons are aggregated for each entity (index or issuer). The add-on for each entity (j) is defined as the simple sum of the trade-level add-ons. The aggregation of entity level add-ons to an equity add-on at netting set level is performed based on a single-risk-factor model with supervisory correlation parameters (ρ_j).

The required supervisory parameters for equity derivatives are set by BCBS (2014d). Table 1.1 shows the values for these regulatory prescribed parameters. According to BCBS (2013) the supervisory parameters (volatility, supervisory factor) for equities were calibrated using a three-step approach. First, the supervisory parameters were initially calibrated based on historic market data. These results were compared to a supervisory CCR model for different portfolio compositions. Finally, the supervisory authorities conducted a Quantitative Impact Study (QIS) to assess the impact of the calibrated SA-CCR on real-life portfolios.

[14] For options with multiple exercise dates, one might only assume the latest exercise date.

[15] ψ equals (-1) where the transaction is a sold call or a bought put option and $(+1)$ where the transaction is a bought call or sold put option. ω equals (-1) for put and $(+1)$ for call options.

[16] There are specific rules for the estimation of the respective inputs into the formula for exotic options (e.g. Asian, Bermudan, American, Digital). Nevertheless, there is no specification how to deal with Barrier options in the respective documents. Hence, we assume that SDA for Barrier options is calculated based on the simplified Black & Scholes formula in equation (1.11).

[17] Additional background on the calculation of the adjusted notional is available in APPENDIX 1.A.2.

[18] A detailed description of aggregation methodologies is provided in BCBS (2014b).

Table 1.1: Supervisory parameters for equity derivatives

Risk Category	Subclass	$SF_i^{(CEM)}$	$SF_i^{(SA-CCR)}$	$\widehat{=}\sigma_i^{(RF)}$	$\sigma_i^{(reg)}$	ρ_j
Equity	Index	6-10%	20%	75.2%	75%	80%
Equity	Single-Name	6-10%	32%	120.3%	120%	50%

Notes: This table provides values for supervisory SA-CCR parameters as set by BCBS (2014d).

1.3 Regulatory status and discussions

In 2013, the BCBS issued a consultation paper on a new non-internal model method for calculating exposures for the capitalization of CCR exposures (BCBS (2013)). The final SA-CCR paper was published in 2014 (BCBS (2014d)). The transformation of the proposed new standardized approach into the regulatory capital framework differs across jurisdictions in terms of content and timing. In Europe, the new requirements regarding the SA-CCR have been included in the new version of the Capital Requirements Regulation (EC (2019)). The mandatory compliance date is the 28th June 2021. In addition to the proposed approach, the European Commission introduced a simplified SA-CCR approach as well as a revised Original Exposure Method (OEM) to reduce the operational burden for institutions without material derivatives business. In late 2018, the responsible US agencies[19] published a proposed rulemaking for the implementation of the SA-CCR for consultation (OCC et al. (2018)). The final rule to implement the SA-CCR was issued in November 2019 together with details on industry responses and associated comments by the regulators (OCC et al. (2019)). The new rules will become effective as of 1st April 2020. The mandatory compliance date for "advanced approaches banking institutions" was set to 1st January 2022.

Based on the proposed rulemaking, the US agencies collected about 58 responses from various stakeholders on different aspects of the SA-CCR methodology. In general, the agencies received broad endorsement for the implementation of the SA-CCR. Nevertheless, the commenters raised various concerns and suggestions for modifying the SA-CCR. Main areas of concerns are the application of the α-factor, the scheduled time-line for implementation as well as the overall level of conservatism resulting from the conservative design of the PFE multiplier, the overly conservative calibration of certain supervisory parameters and the lack of granularity of the supervisory parameters for some asset classes. In addition, there is ongoing discussion

[19] The agencies involved in the rulemaking for SA-CCR are the Federal Deposit Insurance Corporation (FDIC), Department of Treasury - Office of the Comptroller of the Currency (OCC) and the Federal Reserve System (FED). We will denominate those institutions as "US agencies" in the course of this paper.

with respect to analysis and potential improvements of the SA-CCR. Berrahoui et al. (2019) derive a self-consistent and risk sensitive representation of the SA-CCR applying cash flow decomposition. Roberson (2018) analysis the capabilities of the SA-CCR to properly reflect initial margining, while Albuquerque et al. (2017) provide an assessment of the SA-CCR's performance for cleared transactions.

With respect to calibration, the proposed supervisory parameters for equity derivatives are one of the main concerns raised by various institutions. For instance, ABA et al. (2019) state that the SA-CCR provides an insufficient calibration of supervisory parameters for equity products. First, the calibration is considered to be overly conservative and not in line with observed volatilities in stressed market situations. Second, the risk-weights are significantly higher compared to the current standardized capital framework (CEM) and would thereby produce an increase in *EAD* of 23%. ABA et al. (2019) find that the supervisory factor for equities of 32 % (see table 1.1) is twice as high, as it is required to address the risk of equity contracts in severe market stress. Second, there is a lack of granularity with respect to supervisory parameters for equities. The parameters of the SA-CCR only differentiate between the type of underlying (index, single-name). There is no consideration of other properties which influence the quality of risk. Nevertheless, equity underlyings with different properties show significantly different levels of risk under stressed market conditions.[20] This is not reflected in the SA-CCR. Hence, the risk sensitivity of the approach is diminished. Based on these results, ABA et al. (2019) urge the US agencies to recalibrate the supervisory parameters to more reasonable levels.

Overall, the US agencies received various suggestions to enhance the risk sensitivity of the SA-CCR by increasing the granularity of the supervisory parameters for equities. These suggestions include the differentiation of single-name equity underlyings by size, credit quality, industry sector and region. Some commenters proposed to align the SA-CCR calibration with the risk-weights for equities proposed for the new standardized approach for market risk (BCBS (2019b)). According to OCC et al. (2019), revising the granularity and calibration for equity derivatives could generally contribute to risk sensitivity. Nevertheless, the agencies aim for a simplistic approach generating comparable results among financial institutions. Hence, they refrain from adding additional categories of single-name or index equity underlyings. The alignment of the SA-CCR parameters with the new Basel III rules for market risk might be considered by the agencies after their implementation in the United States.

[20] ABA et al. (2019) show that issuers in emerging markets show a significantly higher volatility compared to issuers in advanced markets.

In general, the treatment of options in the CCR framework has been significantly improved by the introduction of the SA-CCR. The most important improvement is the introduction of a scaling parameter (δ_i) for non-linear products within the calculation of PFE add-ons (ABA et al. (2019)). Hereby, the SA-CCR considers the moneyness of non-linear transactions and their sensitivity to changes in the underlying risk factor. However, there are concerns with respect to the application of the Black & Scholes formula for the calculation of the supervisory delta adjustment (δ_i, SDA) (OCC et al. (2019)). While the simplified Black & Scholes formula proposed by BCBS (2014d) might be suitable for plain-vanilla options, the application to path-dependent financial instruments, such as Barrier, Asian, Bermudan options, is viewed critically. The SDA in its proposed form is not able to capture the behavior of such products. Hence, the SDA might involve a significant over- or underestimation of the real economic delta of the option, which could lead to serious deviations in the calculation of EAD and subsequent capital requirements (ABA et al. (2019)).

Commenters suggest allowing institutions to follow their own, existing internal practices for the modeling of volatilities and the calculation of delta adjustments (OCC et al. (2019)). These internal practices are subject to an internal governance framework and supervisory review. Furthermore, they are already used in the new standardized approach for the trading book (SA-TB) within the fundamental review of the trading book (FRTB) (BCBS (2019b)). According to OCC et al. (2019) the use of internally modeled volatilities and delta adjustments is not compliant with the standardized fashion of the SA-CCR. The SA-CCR shall function as a standardized approach. Hence, the use of internal models for the calculation of delta adjustments would not be in line with the objectives of the SA-CCR and the rules for the calculation of supervisory delta adjustments are implemented as suggested (OCC et al. (2019)).

The main postulations raised by commenters with respect to the treatment of equity options can be summarized as follows. First, multiple stakeholders urge the US agencies to recalibrate the supervisory volatility and supervisory factor (1A). Second, the granularity of the equity parameters should be increased (1B). Third, commenters suggest allowing institutions to use their own internal delta approaches when calculating the SDA (2). As discussed above, these postulations have not yet been taken into account by the US agencies (OCC et al. (2019)). Nevertheless, they leave open the possibility for a closer alignment of the SA-CCR to the new standardized approach for market risk (SA-TB). The SA-TB proposes a more granular approach to calibration for equity derivatives considering the type, region, size and industry sector of the underlying. Furthermore, SA-TB makes use of internally calculated sensitivity data for the calculation of capital requirements for market risk in a standardized framework (BCBS (2019b)). Hence, a closer alignment of the SA-CCR to SA-TB is in line with the postulations above and might have a positive contribution to the risk sensitivity of the SA-CCR.

1.4 Volatility analysis

1.4.1 Methods and data

The volatility analysis aims to provide evidence for the necessity of recalibrating the SA-CCR (postulation 1A). Hence, we conduct a validation of the SA-CCR supervisory volatilities and the corresponding supervisory factors for equities based on historic data. As the volatility and the supervisory factor are directly related, we focus our analysis on the validation of volatilities.[21] According to BCBS (2013), the initial calibration of the SA-CCR was performed based on stressed market data. In the context of SA-CCR, the stress period is set by finding the three-year period with the highest estimation of volatility. BCBS (2013) states that shorter time horizons were used in some cases. There is no specific clarification with respect to the exact time horizons used for the calibration of different asset classes. Furthermore, BCBS (2013) provides no detailed information about the calibration for the equity risk category, other than the use of observed volatilities for main indices and their constituents. The supervisory parameters for equity distinguish between transactions referencing single-name and index underlyings. Hence, our analysis uses market data for both types of underlyings in order to provide a proper validation of parameters.

We use the constituents of the FTSE All-World index (excl. US) and the S&P500 as of 29th March 2019 for the validation of single-name parameters. The validation of equity index parameters is based on historic market data for 42 main indices from different countries and regions.[22] The historic prices are downloaded from Thomson Reuters Eikon for the time period from 31/12/1993 - 29/03/2019.[23]

While the analysis of ABA et al. (2019) works with pre-defined and static stress periods, we follow the approach outlined in BCBS (2013). We apply a rolling window to identify the period with the highest estimation of volatility for each single-name and index underlying

[21] Volatilities can be directly transformed to equivalent supervisory factors, by application of equation (1.10).
[22] The set of indices used to validate the SA-CCR index parameters is provided in Table 1.A.2 of APPENDIX 1.A.5.
[23] Please note that underlyings with insufficient market data for the estimation of volatilities based on the respective time horizon are excluded from the analysis.

independently. Hence, we do not consider a consistent market data period for all underlyings but identify the stress period for each underlying separately. Furthermore, we perform our analysis using a one- and three-year time horizon, as the initial calibration of the SA-CCR could also be based on shorter time horizons. The annualized volatility ($\hat{\sigma}$) for each underlying is estimated based on daily returns and a scaling factor of $\sqrt{T}$ assuming 250 trading days per year ($T = 250$). Based on this approach, we obtain the maximum annualized volatility for each single-name and index underlying, based on a one- and three-year market data period. For an aggregated view on single-name underlyings, we calculate a weighted-average volatility with weights based on the market capitalization in USD as of 29th March 2019.

1.4.2 Validation of SA-CCR parameters

The volatility analysis reproduces step 1 of the calibration approach laid down in BCBS (2013). We follow the respective specification and identify the highest estimation of volatility for each equity underlying based on a three-year and one-year time horizon. Thereby, we obtain a volatility value ($\hat{\sigma_i}^{(hist,xY)}$) as well as the relevant stress period for each equity underlying (i) in the scope of our analysis. By comparing these values to the regulatory volatilities used in the SA-CCR, we are able to draw conclusions with respect to the conservatism involved in the calibration of the SA-CCR as well as the need for recalibration, as suggested by various institutions in the consultation process (OCC et al. (2019)).

Table 1.2: Summary of validation results

Subclass	$SF_i^{(reg)}$	$\widehat{=}\sigma_i^{(reg)}$	$\hat{\sigma_i}^{(hist,3Y)}$	$\widehat{=}SF_i^{(hist,3Y)}$	$\hat{\sigma_i}^{(hist,1Y)}$	$\widehat{=}SF_i^{(hist,1Y)}$
Single-Name	32%	120.3%	53.3%	14.2%	69.9%	18.6%
Index	20%	75.2%	37.8%	10.1%	52.5%	14.0%

Notes: This table provides a summary of the validation results for equity underlyings. $SF_i^{(reg)}$ equals the SA-CCR regulatory supervisory factor and $\sigma_i^{(reg)}$ the equivalent volatility. $\hat{\sigma_i}^{(hist,xY)}$ represents the weighted-mean of the highest volatility estimation for the respective time horizon. $\hat{\sigma_i}^{(hist,xY)}$ is transformed into an equivalent supervisory factor by application of equation (1.10).

Table 1.2 provides an overview of the validation results. Our results show that the SA-CCR parameters are significantly higher compared to historically observed volatilities. Figure 1.1 shows an overview of the validation results for single-name underlyings. The supervisory parameters for these underlyings seem to be overly conservative. Even when applying a shorter time horizon of one year, the weighted-average volatility does not exceed 70%. Based on historical average-weighted stressed volatilities, we obtain an equivalent supervisory factor of 18.6 %. The SA-CCR applies a supervisory factor of 32%. This exceeds the observed volatilities by a factor of 1.72 and equals the 92% quantile of the empirical distribution. When using a three-year time period for the estimation of volatility, this discrepancy between observable market data and supervisory calibration is even more pronounced. The SA-CCR overstates the weighted-average observed volatilities by a factor of 2.25. The SA-CCR volatility equals the 97% quantile of the empirical distribution.

Figure 1.2 shows the validation results for 42 equity indices. When applying a time horizon of three years, the supervisory volatility exceeds the observed volatility by a factor of 1.98. No stressed volatility estimate of our sample exceeds the SA-CCR supervisory volatility. The results for a shorter time horizon of one year are closer to the supervisory results but the SA-CCR still overstates the observed values by a factor of 1.43 (89% quantile).

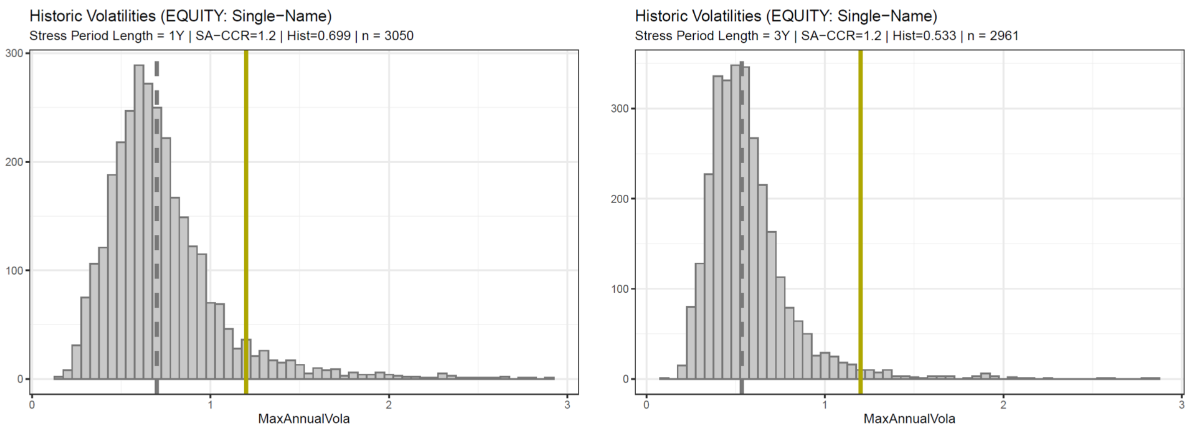

Figure 1.1: Equity single-name validation results. *Note* The figure shows the distribution of the highest volatility estimation for single-name equity underlyings for a stress period length of 1 year (left panel) and 3 years (right panel). The vertical lines indicate the market capitalization weighted-average volatility (dashed line) and the SA-CCR supervisory volatility (solid line).

Our results show that the supervisory volatility and the supervisory factor of the SA-CCR are conservatively calibrated. Even when identifying the stress period for each underlying individually and for a time horizon of one year, we do not observe the volatility levels of the SA-CCR. In summary, we are able to demonstrate that the parameters of the SA-CCR are almost twice as high as indicated by historical observations. Taking these results into consideration, we are certainly justified in saying that the supervisory parameters for equity should be recalibrated to tally with historic observations. According to BCBS (2013), the SA-CCR was not designed to cope with exotic transactions. Hence, an additional level of conservatism has been applied when calibrating the supervisory parameters. Nevertheless, we believe that the very conservative parameters for equity underlyings together with their lack of granularity could lead to a significant deterioration of risk sensitivity.

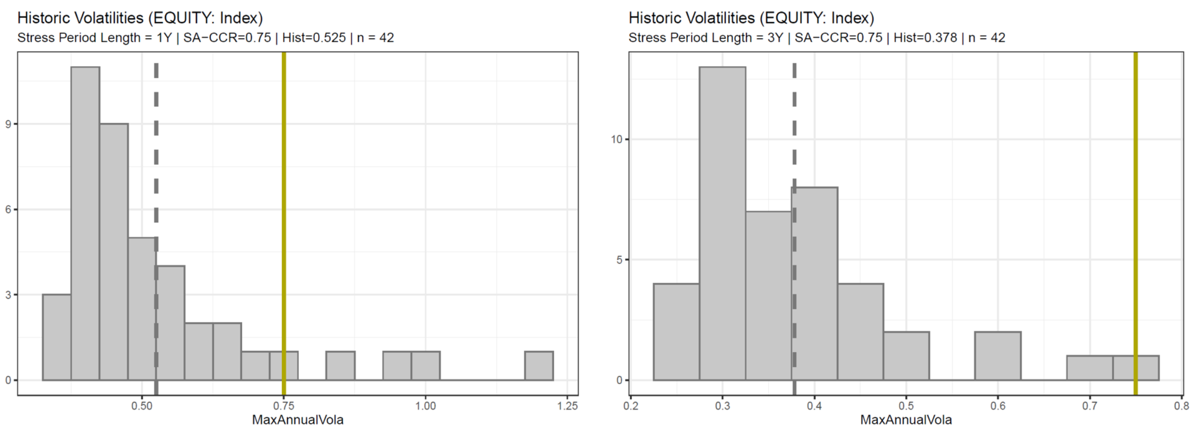

Figure 1.2: Equity index validation results. *Note* The figure shows the distribution of the highest volatility estimation for equity index underlyings for a stress period length of 1 year (left panel) and 3 years (right panel). The vertical lines indicate the average volatility (dashed line) and the SA-CCR supervisory volatility (solid line).

1.4.3 Alignment with SA-TB risk-weights

As discussed above, multiple institutions suggested to increase the granularity of supervisory parameters for equity, as underlyings with different region, credit quality, industry sector and size involve different levels of risk. The analysis of ABA et al. (2019) shows that there are significant and persistent differences between volatility estimates for equity underlyings with different properties. We agree that the granularity of supervisory parameters should be increased with respect to equity underlyings. Nevertheless, the categorization of equity underlyings should be simple, broadly acknowledged and consistent with other regulatory requirements. Hence, we favor recalibrating the SA-CCR parameters in line with the parameters introduced by BCBS (2019b). The linear risk charge for equity risk within the SA-TB offers a granular calibration of supervisory risk-weights, taking into account the type of underlying, the economy, the market capitalization as well as the industry sector. Overall, equity sensitivities are allocated to 13 different buckets. For each bucket a risk-weight is defined based on the annualized volatility of the underlying risk factor. We are able to transform these risk-weights into equivalent SA-CCR supervisory factors based on the following relationship between SA-TB and SA-CCR risk-weights:[24]

$$SF_i = \frac{2}{3} \cdot \frac{RW_i^{(SA-TB)}}{2.3378 \cdot \sqrt{t} \cdot \sqrt{2\pi}} \tag{1.12}$$

where $RW_i^{(SA-TB)}$ represents the risk-weight for a specific underlying under the SA-TB and t equals the standard risk horizon for the linear risk charge under the SA-TB (t=10 days). Given

[24] A detailed derivation of the relationship between SA-TB and SA-CCR risk-weights is provided in APPENDIX 1.A.3.

this relationship, we are able to perform a transformation and validation of SA-TB based SA-CCR parameters. In a first step, the SA-TB risk-weights are transformed to SA-CCR equivalent volatilities ($\sigma_i^{(SA-CCR)}$) based on equations (1.12) and (1.10). Second, the resulting volatilities are compared to the volatility estimates derived in section 1.4.2 for a three- and one-year period. We calculate the volatility estimates of a specific SA-TB bucket as the weighted-average (market capitalization) of all underlyings allocated to the bucket. The allocation of underlyings to the specific bucket is performed based the categorization provided by BCBS (2019b) and the respective instrument data from Thomson Reuters as of March 2019.[25]

Table 1.3: SA-TB risk-weights: Transformation and validation

Bucket	Size	Economy	Industry	$RW_i^{(SA-TB)}$	$\widehat{=}\sigma_i^{(SA-CCR)}$	$\sigma_i^{(reg)}$	$\hat{\sigma_i}^{(hist,1Y)}$	$\hat{\sigma_i}^{(hist,3Y)}$
1	Large	Emerging	Sector 1	0.55	1.18	1.2	0.74	0.58
2			Sector 2	0.60	1.29	1.2	0.74	0.57
3			Sector 3	0.45	0.97	1.2	0.86	0.63
4			Sector 4	0.55	1.18	1.2	0.78	0.59
5	Large	Advanced	Sector 1	0.30	0.64	1.2	0.61	0.48
6			Sector 2	0.35	0.75	1.2	0.65	0.51
7			Sector 3	0.40	0.86	1.2	0.64	0.47
8			Sector 4	0.50	1.07	1.2	0.86	0.63
9	Small	Emerging	Sector 1-4	0.70	1.50	1.2	0.98	0.72
10		Advanced	Sector 1-4	0.50	1.07	1.2	0.72	0.56
11			Other Sector	0.70	1.50	1.2	0.98	0.72
12	Index	Advanced	non-sector specific	0.15	0.32	0.75	0.46	0.34
13		Emerging	non-sector specific	0.25	0.54	0.75	0.69	0.47

Notes: This table provides the summarized results from the validation of volatilities transformed from SA-TB risk-weights based on historically derived volatility estimates for different periods of market data. Definition of industry sectors is provided in APPENDIX 1.A.3.

Table 1.3 provides an overview of the transformation and validation results for each SA-TB bucket.[26] Our results show that volatilities derived from SA-TB are significantly lower compared to the regulatory volatility used in the SA-CCR for advanced market underlyings. Furthermore, the transformed volatilities offer a differentiation between emerging and advanced markets as well as small and large underlyings. The comparison with historic volatility estimates indicates that SA-TB based volatilities involve a sufficient level of conservatism for single-name underlyings. This is different for index underlyings, for which the historic volatility estimates exceed the SA-TB based volatilities. Nevertheless, our results clearly indicate different levels of risk for advanced and emerging market indices. Hence, we believe that a calibration of the SA-CCR to SA-TB sensitivities is generally suitable, but should take into account an additional level of conservatism with respect to index underlyings.

[25] Additional details on the SA-TB bucket definitions are provided in APPENDIX 1.A.3.

[26] Please note that the results for bucket 11 are not explicitly calculated as no underlying within our analysis is mapped to this bucket. Hence, the results for bucket 11 equal the results for bucket 9. We consider this assumption in line with the risk-weights set for SA-TB.

In addition to the highest volatility estimates, we perform a validation of SA-TB volatilities through time to assess the conservatism of SA-CCR and SA-TB risk-weights in different market environments. We use rolling windows (monthly) to calculate the historic volatility estimate per underlying for a one- and three-year period. The volatility estimation per bucket is defined as the weighted-average volatility of all underlyings allocated to the specific bucket. Thereby, we obtain a time series of historic volatility estimates for each SA-TB bucket. Figure 1.3 shows the results for SA-TB bucket 5.[27] In summary, the results for the through-time analysis confirm that the SA-TB based parameters are sufficiently conservative for single-name underlyings.

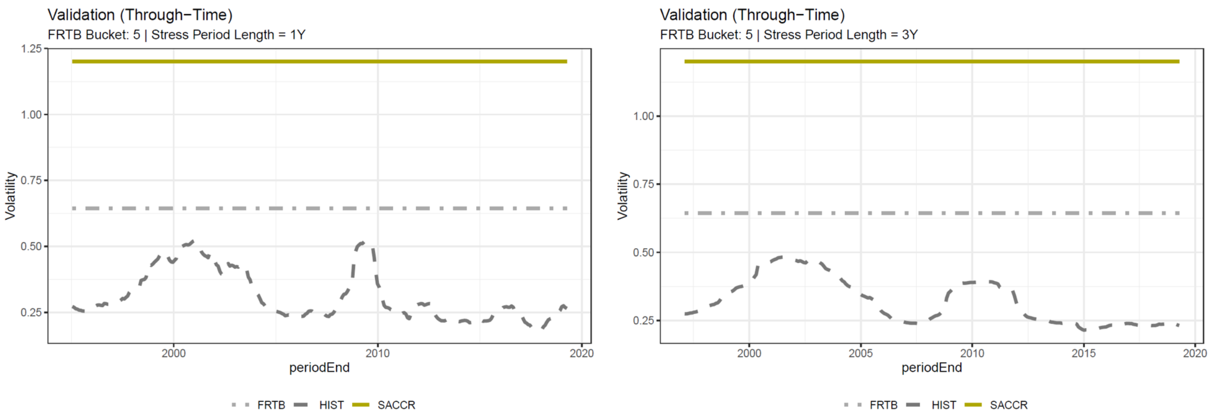

Figure 1.3: Through-time validation (bucket 5). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

With respect to index underlyings, we recognize that the SA-TB based volatilities for advanced economies do not fully capture the volatilities observed during the Great Financial Crisis for the one-year calibration period (see figure 1.4). This observation is in line with the results presented in Table 1.3 and supports our suggestion to include an additional level of conservatism when deriving SA-CCR parameters for indices based on SA-TB risk-weights.

The volatility analysis confirms the fact that SA-CCR parameters are overly conservative and do not align with historically observed volatility levels. Furthermore, the SA-CCR calibration does not reflect the different risk levels between underlyings with different region, sector and market capitalization. Our results support the suggestion to align the SA-CCR calibration with the supervisory risk-weights of SA-TB. Nevertheless, it needs to be ensured that parameters are sufficiently conservative to cover volatility levels observed in stressed market environments. Based on our results, we believe that the alignment of SA-CCR and SA-TB would significantly enhance the risk sensitivity of the SA-CCR. In the subsequent simulation study, we test the

[27] APPENDIX 1.A.6 provides the graphical representation of through-time validation results for further SA-TB buckets.

impact of different parameter calibrations and definitions of the delta adjustment on the risk sensitivity of the SA-CCR based on hypothetical equity options.

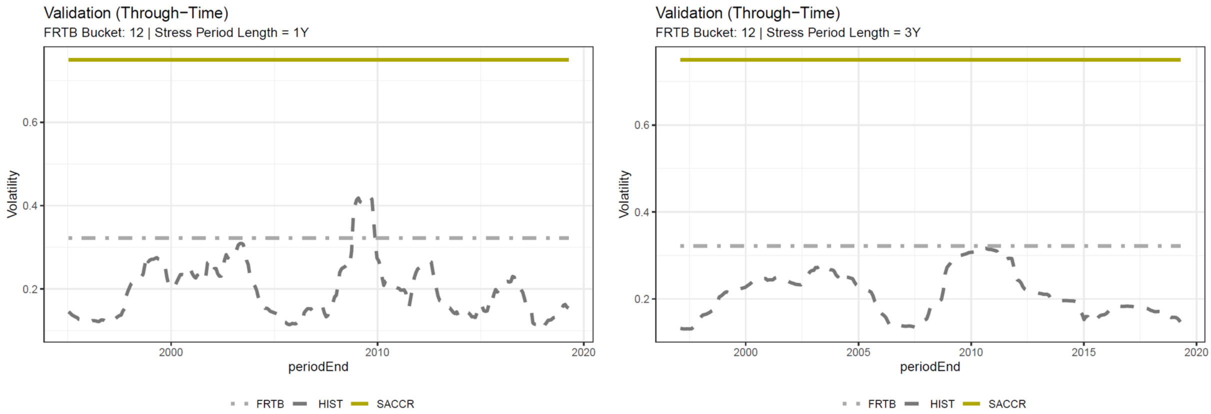

Figure 1.4: Through-time validation (bucket 12). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

1.5 Simulation study

1.5.1 Methods and data

The aim of the simulation study is to assess the adequacy of the SA-CCR supervisory parameters for equity derivatives and to analyze different possibilities for recalibration and improvement. The analysis is based on randomly generated options with a maturity of one year. The generation of those options is performed by utilizing the results of the volatility analysis in section 1.4. An underlying is randomly selected for each generated option from the set of underlyings in scope of the volatility analysis. The price/strike ratio for each option as well as the barrier level (for barrier options) are randomly defined within a predefined range based on a uniform distribution in order to create options with different moneyness and barrier level. The current price of each underlying is defined as the close price as of 29th March 2019.[28] Only bought (long) options are in scope of our analysis, as sold (short) options in individual netting sets have an exposure of zero according to BCBS (2018a). We calculate the market value of each randomly generated option at $t = 0$ based on the respective pricing formulas (Haug (2007)).

The simulation study focuses on the calculation of the PFE add-on as the SA-CCR supervisory parameters are main input factors for PFE calculation. We generate the PFE results based on (1)

[28] Price data is downloaded from Thomson Reuters Eikon.

a Monte Carlo simulation (IMM) and (2) the SA-CCR. By comparing these results, we are able to draw conclusions about the appropriateness of different SA-CCR configurations. In order to normalize the results, we divide the resulting PFE values by the adjusted notional of the respective transaction. Each generated option is treated as its own netting set. Hence, we are able to compare the trade-level PFE result from the Monte Carlo Simulation with the SA-CCR PFE value. The SA-CCR result is defined as:

$$PFE_i^{(SA-CCR)}(\Omega) = \frac{AddOn_i^{(SA-CCR)}(\Omega)}{d_i^{(EQ)}} \tag{1.13}$$

where Ω represents the configuration of the SA-CCR. A specific configuration (Ω) is defined as a combination of the applied calibration and delta adjustment. The delta adjustment can be calculated based on the supervisory formula or using an economic approach (e.g. finite difference method). Table 1.4 provides the definition of different SA-CCR configurations.

Table 1.4: Definition of SA-CCR configurations

Configuration (Ω)	Description	SF_i	δ_i
REG | SDA (regVol)	Use regulatory parameters as defined by BCBS (2014d)	$SF_i^{(reg)}$	$\sigma_i^{(reg)}$
FRTB | SDA (frtbVol)	Calibration based on SA-TB / Supervisory delta	$\sigma_i^{(SA-TB)}$	$\sigma_i^{(SA-TB)}$
FRTB | EDA (frtbVol)	Calibration based on SA-TB / Economic delta	$\sigma_i^{(SA-TB)}$	$\sigma_i^{(SA-TB)}$
HIST | SDA (histVol)	Calibration to historic volatilities / Supervisory delta	$\hat{\sigma}_i^{(hist)}$	$\hat{\sigma}_i^{(hist)}$
HIST | EDA (histVol)	Calibration to historic volatilities / Economic delta	$\hat{\sigma}_i^{(hist)}$	$\hat{\sigma}_i^{(hist)}$

Notes: This table provides the definition of different SA-CCR configurations. The third and fourth column give information about the calibration of the SA-CCR supervisory factor and delta adjustment.

The target measure of the SA-CCR PFE add-on is a conservative approximation of the EEPE of an internal model taking into account the aforementioned assumptions.[29] The applied Monte Carlo simulation is calibrated to the stressed volatility estimates generated in section 1.4. Ideally, the SA-CCR should reproduce the results from the Monte Carlo simulation. In order to assess the quality of the different configurations (Ω), we observe the mean squared error between the IMM and SA-CCR result for each configuration:

$$MSE(\Omega) = \frac{1}{N} \cdot \sum_{i=1}^{N} \left(PFE_i^{(IMM)} - PFE_i^{(SA-CCR)}(\Omega) \right)^2 \tag{1.14}$$

where $PFE_i^{(IMM)}$ represents the PFE result based on the Monte Carlo simulation. N equals the number of randomly generated options. $PFE_i^{(SA-CCR)}(\Omega)$ is the PFE add-on calculated using the SA-CCR with the respective configuration (Ω). $MSE(\Omega)$ can be interpreted as a measure for the risk sensitivity of the SA-CCR under the respective configuration. Based on this measure, we are

[29] A detailed description of the Monte Carlo simulation as well as the according definitions of PFE values are provided in APPENDIX 1.A.4.

able to assess the appropriateness of different configurations. Furthermore, we gain knowledge regarding the application of economic deltas for barrier options. Based on these results, we derive suggestions with respect to potential improvements of the SA-CCR methodology.

1.5.2 Results for plain-vanilla options

The simulation study for plain-vanilla options compares the Monte Carlo (IMM) results with the SA-CCR PFE values for three different configurations. All configurations use the supervisory formula for the calculation of the delta adjustment. As stated by multiple commenters during the consultation process, the SA-CCR formula is considered appropriate for plain-vanilla options. Concerns were only raised with respect to path-dependent options (OCC et al. (2019), ABA et al. (2019)). Hence, we do not consider using economic delta adjustments for plain-vanilla options in the course of this analysis. Figure 1.5 shows the results for unmargined options with single-name underlying and $\hat{\sigma_i}^{(hist)}$ calibrated to a three-year time horizon. The calibration of the SA-CCR parameters to SA-TB risk-weights leads to a significant reduction of the MSE for the set of randomly generated options. The calibration of parameters to historical volatilities (HIST | SDA (histVol)) is displayed for illustrative purposes. This configuration is not a practicable option for the SA-CCR, as this would require a calibration of supervisory parameters to each underlying.

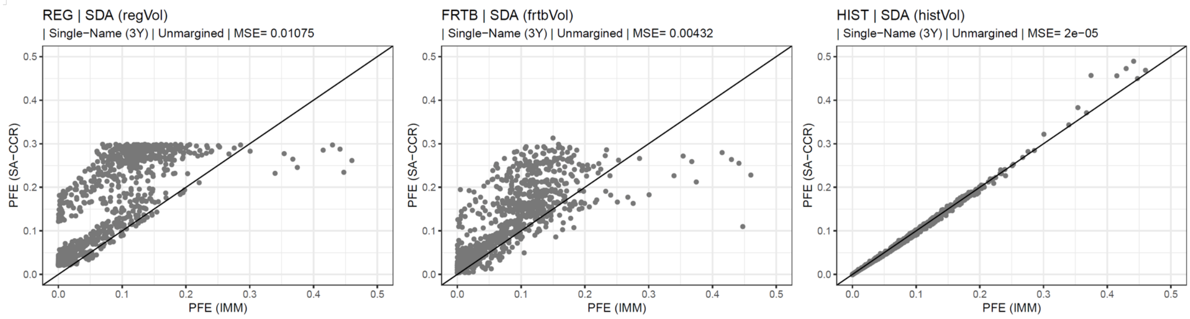

Figure 1.5: MSE results for European options (Single-Name (3Y), unmargined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

We perform the simulation study for index and single-name underlyings calibrated to different market data horizons as well as for margined and unmargined netting sets.[30] Table 1.5 provides the results of the simulation study for those different combinations of inputs and configurations. Our results confirm that the supervisory parameters lead to overly conservative exposure

[30] A graphical representation of all results with respect to this analysis is provided in APPENDIX 1.A.7.

values. Furthermore, we observe that the calibration of SA-CCR parameters to SA-TB risk-weights generally leads to a significant reduction in MSE, while maintaining a sufficient level of conservatism for most equity underlyings.

Table 1.5: MSE results for plain-vanilla European options

Calibration	Margining	REG | SDA (regVol)	FRTB | SDA (frtbVol)	HIST | SDA (histVol)
Single-name (3Y)	Unmargined	0.01075	0.00432	0.00002
	Margined	0.00158	0.00068	0.00000
Single-name (1Y)	Unmargined	0.00851	0.00388	0.00003
	Margined	0.00114	0.00061	0.00000
Index (3Y)	Unmargined	0.00399	0.00045	0.00000
	Margined	0.00055	0.00006	0.00000
Index (1Y)	Unmargined	0.00221	0.00117	0.00001
	Margined	0.00026	0.00013	0.00000

Notes: This table provides the mean-squared error results for European plain-vanilla equity options for different subsets of positions, portfolios and calibrations. The definition of configurations is provided in Table 1.4.

Figure 1.6 shows results for index underlyings calibrated to a one-year time horizon. In line with the volatility analysis (section 1.4), we observe an under-estimation of risk when calibrating the SA-CCR to SA-TB risk-weights. Hence, we emphasize our recommendation to include an additional level of conservatism with respect to index underlyings.

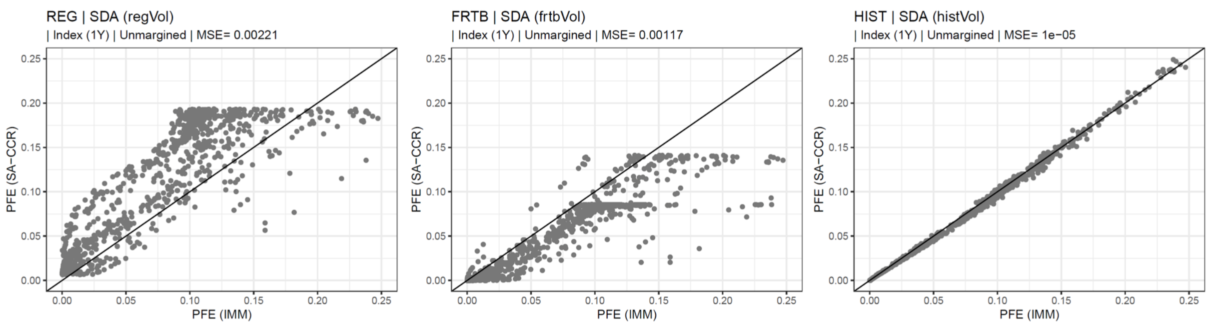

Figure 1.6: MSE results for European options (Index (1Y), unmargined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

1.5.3 Results for barrier options

As discussed in section 1.3, multiple stakeholders suggested introducing a modification to the SA-CCR, allowing banks to use their internal procedures to calculate the SA-CCR delta adjustment (δ_i) for path-dependent options (OCC et al. (2019)). We agree that the simplified Black & Scholes formula is not suitable for path-dependent options and its application could lead to unreasonable exposure results. Nevertheless, the volatility applied to calculate the delta adjustment is used as input for the calibration of the supervisory factor of the SA-CCR. We aim for a configuration that maintains this methodological consistency, while offering an improvement with respect to the treatment of path-dependent options. Hence, we suggest allowing banks to use economic formulas / models to calculate the delta adjustment, but based on (recalibrated) supervisory volatility inputs. In order to assess the implications of this proposal, we conduct a simulation study for European barrier options. We calculate the SA-CCR for different configurations, including the application of an economic delta adjustment (EDA) based on SA-TB volatilities. All randomly generated barrier options have an index underlying and have not touched the barrier at $t = 0$. The EDA for barrier options is calculated numerically based on finite differences (Hull (2011)). The Monte Carlo simulation for each underlying is calibrated to the stressed volatility estimated, based on a three-year period. Figure 1.7 shows

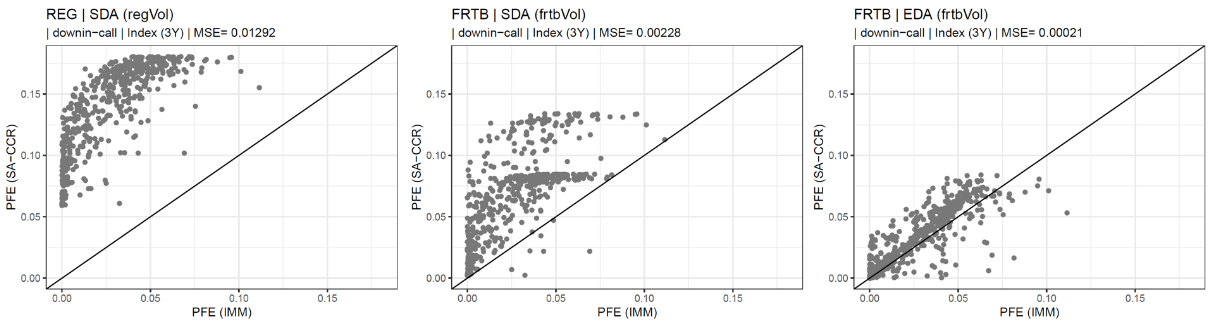

Figure 1.7: MSE results for European barrier options (down-in call). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 500 randomly generated European barrier options. Each panel provides the results for a specific SA-CCR calibration.

the results for down-in call options. The PFE values calculated based on the regulatory SA-CCR (left panel), are over-estimating the IMM results as barrier options are treated as if they were plain-vanilla European options. The recalibration of the parameters to SA-TB volatilities (middle panel) leads to an improvement and a reduction of the MSE. This implies that the overly conservative calibration of the SA-CCR also significantly contributes to the inadequate treatment of path-dependent options. The application of the economic delta adjustment (based on SA-TB volatilities) leads to an additional significant reduction of the MSE. Hence, our results indicate that the application of economic delta sensitivities contributes to the SA-CCR's risk sensitivity. We conduct this analysis for different barrier option types to get a universal picture with respect to the impact of applying an economic delta adjustment. Table 1.6 shows the results of the simulation study for different European barrier options.[31] In general, we observe an improvement of the SA-CCR risk sensitivity when applying an economic delta adjustment together with a recalibration to SA-TB risk-weights. Nevertheless, we recognize that this is

[31] A graphical representation of all results with respect to this analysis is provided in APPENDIX 1.A.8.

Table 1.6: MSE results for European barrier options

Option Type		REG | SDA (regVol)	FRTB | SDA (frtbVol)	FRTB | EDA (frtbVol)	HIST | EDA (histVol)
Down-in	Call	0.01292	0.00228	0.00021	0.00011
	Put	0.00064	0.00033	0.00017	0.00010
Down-out	Call	0.00618	0.00065	0.00092	0.00006
	Put	0.00295	0.00101	0.00009	0.00005
Up-in	Call	0.00503	0.00056	0.00057	0.00011
	Put	0.00201	0.00044	0.00093	0.00058
Up-out	Call	0.01641	0.00371	0.00008	0.00005
	Put	0.00084	0.00023	0.00037	0.00004

Notes: This table provides the mean-squared error results for European barrier options for different subsets of positions, portfolios and calibrations. The definition of configurations is provided in Table 1.4.

accompanied by less conservative SA-CCR results (see for example Figure 1.7, right panel). This might lead to an under-estimation for certain transactions with riskier underlyings. From our point of view, the SA-CCR involves various supervisory elements (e.g. α-factor), which lead to an overall conservative risk assessment. Hence, we believe that the less conservative estimation on transaction-level will not lead to significantly lower exposure values on portfolio-level. In summary, we think that allowing institutions to use economic delta adjustments will significantly improve the risk sensitivity of the SA-CCR. Nevertheless, there need to be regulatory guidelines and specifications with respect to the determination of economic delta adjustments in order to ensure a consistent implementation and comparability of results across institutions.

1.6 Conclusion

This paper picks up open issues and discussions with respect to the treatment of equity options under the new standardized approach for counterparty credit risk (SA-CCR). Our results show that the SA-CCR is calibrated very conservatively with respect to equity derivatives. The supervisory parameters do not align with historically observed volatility levels and are generally more conservative compared to the risk-weights applied in the new standardized approach for market risk (SA-TB). In addition, the SA-CCR calibration shows a lack of granularity, as differences between underlyings with diverging properties (e.g. economy) are not recognized. OCC et al. (2019) acknowledge these issues, but do not recalibrate the parameters and refrain from introducing additional categories for equity derivatives. Nevertheless, they leave open the possibility to align the SA-CCR with the SA-TB, once the new requirements for market risk are implemented in the US. Our paper provides empirical evidence that a recalibration of SA-CCR parameters to SA-TB risk-weights enhances the risk sensitivity of the SA-CCR for single-name underlyings. With respect to parameters for index underlyings, the SA-TB risk-weights should be carefully reviewed before incorporation. We demonstrate that the SA-CCR in its proposed form does not adequately handle path-dependent equity options. The application of a simplified Black & Scholes formula to exotic options leads to a deterioration of risk sensitivity and the quality of SA-CCR results. Our simulation study for barrier options shows that the application of economic delta adjustments together with a recalibration of parameters to more reasonable levels could lead to a significant improvement of the SA-CCR. To render a final judgment on the application of economic delta adjustments, additional analysis with respect to further exotic options referencing underlyings in different asset classes is required. Furthermore, the incorporation of economic delta adjustments should be accompanied by regulatory guidelines and specifications for their determination. In summary, we believe that an alignment of the regulatory standardized approaches for market and counterparty credit risk provides added value with respect to the transparency and quality of the regulatory capital framework.

1.A APPENDIX

1.A.1 Current Exposure Method (CEM)

The Current Exposure Method (CEM) is a simple and parametric approach for the calculation of the Exposure at Default (EAD) for derivative transactions.[32] The EAD under CEM is composed of the replacement costs (RC), representing the current exposure, and the potential future exposure (PFE), which is calculated in form of an add-on ($AddOn$). Hence, the EAD for a given netting set (k) is defined as:

$$EaD_k^{(CEM)} = RC_k + PFE_k = max\left[V_k(t_0) - C_A(t_0), 0\right] + AddOn_k \tag{1.15}$$

where $C_A(t_0)$ represents the volatility-adjusted collateral value at t_0 and $V_k(t_0)$ equals the netting set's market value at t_0, which is defined as the sum of the market value of all transactions (i) in the netting set:

$$V_k(t_0) = \sum_{i \in k} V_i(t_0) \tag{1.16}$$

The PFE add-on at trade-level ($AddOn_i$) is defined as the product of the transaction's notional principle amount (N_i) and a supervisory factor (SF_i):

$$AddOn_i = N_i \cdot SF_i \tag{1.17}$$

For each transaction, the supervisory factor (SF_i) is defined based on its asset class and remaining lifetime. Full offsetting / netting is allowed when calculating the replacement cost component. During the aggregation of trade-level PFE add-ons at netting set level, only partial offsetting is allowed. The aggregated PFE add-on at netting set level is given as:

$$AddOn_k = (0.4 + 0.6 \cdot NGR_k) \cdot \sum_{i \in k} AddOn_i \tag{1.18}$$

NGR represents the Net-Gross-Ratio and is a scaling factor for the recognition of diversification. The Net-to-Gross Ratio is defined as:

$$NGR_k = \frac{max(\sum_{i \in k} V_i(t_0), 0)}{\sum_{i \in k} max(V_i(t_0), 0)} \tag{1.19}$$

The NGR is basically a measure with respect to the netting of current market values of the transactions in a netting set. It is assumed that this level of offsetting also applies for the PFE. This allows netting across different asset classes and underlyings. Formula 1.18 shows, that the add-on at netting set level is floored at 40% of the sum of all trade-level add-ons. Hence, the PFE component of a netting set does not reach zero, even if the transactions within the netting set are perfectly offsetting contracts.

[32] The methodological overview presented in this section is based on BCBS (2006) and Gregory (2015).

1.A.2 SA-CCR Methodology

The methodological representation and analysis of the SA-CCR presented in this section is generally based on BCBS (2014d) and BCBS (2014b).

Calculation of Replacement Costs (RC) under the SA-CCR

The determination of the Replacement Costs component (RC) differentiates between possible states of a netting set with respect to its collateralization. RC for **unmargined**[33] netting sets represents the amount of loss that would occur if the counterparty defaults immediately ($\tau = t_0$). Within a legally enforceable netting agreement, the regulator allows full offsetting for the purpose of RC calculation. Therefore, RC is defined as the maximum of (1) the sum of market values (V) of all positions (i) within a given netting set (k) minus the net volatility-adjusted Collateral Amount received (C_A)[34] and (2) zero:

$$RC_k^{(no-margin)} = max\left(V_k - C_A(1year), 0\right) = max\left(\sum_{i \epsilon k} V_i - C_A(1year), 0\right) \tag{1.20}$$

For an unmargined netting set, all collateral positions are considered to be independent, as the netting set is not supported by a margin agreement and no variation margin is exchanged. Hence, the collateral amount for unmargined netting sets is defined as the Net Independent Collateral Amount ($NICA$). $NICA$ is calculated as the difference between received independent collateral (Initial Margin, IM) and unsegregated, posted independent collateral.

In the case of a **margined** netting set, RC is defined as the amount of loss which would occur if the counterparty defaults at the present time ($\tau = t_0$) or at a future time ($\tau = t$) within the one-year risk horizon. As the time of default as well as the amount of exposure at τ are unknown, the SA-CCR uses a conservative assumption to calculate future replacement costs. It is conservatively assumed that the amount of exposure at τ is high enough to trigger a margin call. This is the case if the exposure amount exceeds the sum of the threshold (TH) and minimum transfer amount (MTA) minus $NICA$. The replacement costs for maringed netting sets are defined as the maximum of (1) current RC, (2) future RC and (3) zero:

$$RC_k^{(margin)} = max(V_k - C_A(MPOR), TH + MTA - NICA, 0) \tag{1.21}$$

[33] According to BCBS (2014b) a netting set is considered unmargined in the absence of a variation margin.

[34] The collateral amounts are calculated via application of respective volatility-adjustments / haircuts for the one-year time horizon (BCBS (2014b)).

Derivation of EE formula

Based on the assumptions introduced by BCBS (2014b), a transaction's market value follows an arithmetic Brownian motion. Hence, the market value (V) of a transcation (i) at a specific future point in time (t) is generally defined as:

$$V_i(t) = V_i(t_0) + \mu_i \cdot dt + \sigma_i(t) \cdot \sqrt{t} \cdot X_i \tag{1.22}$$

where X_i is a standard normal random variable ($X_i \sim N(0,1)$). μ_i equals the drift and $\sigma_i(t)$ the time-dependent volatility of the transaction's market value. The SA-CCR add-on is calculated under the assumption of zero drift ($\mu_i = 0$), fixed volatility (σ_i), an initial market value of 0 ($V_i(t_0) = 0$) and absence of collateral. Furthermore, it is assumed that there are no cash flows between t_0 and t. Based on these assumptions, equation (1.22) reduces to:

$$V_i(t) = 1_{\{M_i \geq t\}} \cdot \sigma_i \cdot \sqrt{t} \cdot X_i \tag{1.23}$$

where $1_{\{\cdot\}}$ represents an indicator variable which recognizes, if the transaction has matured. σ_i represents the (fixed) volatility of the transaction's market value. Hence, for $M_i \geq t$ the market value of the transaction is also normally distributed with $V_i(t) \sim N(0, \sigma_i^2 \cdot t)$.

Based on equation (1.23) and the aforementioned assumptions, the market value of a netting set, representing a group of legally nettable transactions, is defined as the sum of the single market values of all transactions (i) being an element of netting set k:

$$V_k(t) = \sum_{i \in k} V_i(t) = \sum_{i \in k} 1_{\{M_i \geq t\}} \cdot \sigma_i \cdot \sqrt{t} \cdot X_i \tag{1.24}$$

As the sum of joint normally distributed random variables is again normally distributed, the market value of the netting set is normally distributed. This results from the assumption that X_i is a standard normal random variable. Hence, the variance of the $V_k(t)$ can be calculated the following way:

$$\begin{aligned} Var(V_k(t)) &= Var\left(\sum_{i \in k} V_i(t)\right) \\ &= Var\left(\sum_{i \in k} \sigma_i \cdot \sqrt{t} \cdot X_i \cdot 1_{\{M_i \geq t\}}\right) \\ &= t \cdot \left(\sum_{i,j} \sigma_i \cdot \sigma_j \cdot COV(X_i, X_j) \cdot 1_{\{M_i \geq t\}} \cdot 1_{\{M_j \geq t\}}\right) \\ &= t \cdot \left(\sum_{i,j} \sigma_i \cdot \sigma_j \cdot \rho_{ij}\right) \\ &=: t \cdot (\sigma_k(t))^2 \end{aligned} \tag{1.25}$$

Based on this result, $V_k(t)$ is normally distributed with $N \sim (0, t \cdot (\sigma_k(t))^2)$. In the case of a known correlation ρ_{ij} between two normal random variables X_i and X_j, equation 1.24 can be restated as follows:

$$V_k(t) = \sigma_k(t) \cdot \sqrt{t} \cdot Y \tag{1.26}$$

where $Y \sim N(0,1)$ and $\sigma_k(t)$ represents the volatility of the netting set's market value:

$$\sigma_k(t) = \sqrt{\sum_{i,j} \rho_{ij} \cdot \sigma_i \cdot \sigma_j \cdot 1_{\{M_i \geq t\}} \cdot 1_{\{M_j \geq t\}}} \tag{1.27}$$

Please note that the netting set's volatility $\sigma_k(t)$ is a function of t which does not withstand the assumption of fixed volatility. This dependence results from the changing composition of the netting set as transactions mature over time. The target measure ($EE_k(t)$) is formally defined as the expected positive value of the netting set's market value. Hence, we are able to calculate the Expected Exposure of a netting set by:

$$EE_k(t) = \mathbb{E}\left[max(V_k(t), 0)\right] = \mathbb{E}\left[max(\sigma_k(t) \cdot \sqrt{t} \cdot Y, 0)\right] \tag{1.28}$$

As Y is a standard normal variable, we are able to calculate the expected value of Y and hence $EE(t)$ analytically:

$$\begin{aligned}
AddOn_k = EE(t) &= \mathbb{E}\left[max(V_k(t), 0)\right] = \mathbb{E}\left[max(\sigma_k(t) \cdot \sqrt{t} \cdot Y, 0)\right] \\
&= \sigma_k(t) \cdot \sqrt{t} \cdot \mathbb{E}\left[max(Y, 0)\right] \\
&= \sigma_k(t) \cdot \sqrt{t} \cdot \int_0^{+\infty} \frac{1}{\sqrt{2\pi}} \cdot \exp\left[\frac{-y^2}{2}\right] y dy \\
&= \sigma_k(t) \cdot \sqrt{t} \cdot \frac{1}{\sqrt{2\pi}} \cdot \left[-e^{\frac{-y^2}{2}} dy\right]_0^{+\infty} \\
&= \sigma_k(t) \cdot \sqrt{t} \cdot \frac{1}{\sqrt{2\pi}} \cdot \left[-(0-1)\right] \\
&= \sigma_k(t) \cdot \sqrt{t} \cdot \frac{1}{\sqrt{2\pi}} \\
&= \sigma_k(t) \cdot \sqrt{t} \cdot \phi(0)
\end{aligned} \tag{1.29}$$

where $\phi(0)$ is defined as the standard normal probability density: $\phi(0) = 1/\sqrt{2\pi}$.

Derivation of EEPE formula for unmargined netting sets

Within the SACCR's methodological framework (BCBS (2014b)), the $EEPE$ is defined as the time-weighted average of the EE for a one-year risk horizon. By inserting results from equation (1.29), we obtain:

$$EEPE_k = \frac{1}{1year}\int_0^{1year} EE^{no-margin}(t)dt = \frac{1}{1year}\int_0^{1year} \sigma_k(t)\cdot\sqrt{t}\cdot\phi(0) \tag{1.30}$$

For a calculation of the $EEPE$ in this form, it is necessary to determine the volatility of the netting set's market value $\sigma_k(t)$ for several time points (t) within the risk horizon of one year. To avoid complexity in the add-on calculation, an additional assumption is introduced. Hereby, the maturity of all positions (i) in the netting set (k) is floored at the risk horizon (1 year). This implies that the volatility of the netting set's market value remains constant. Hence, we are able to replace $\sigma_k(t)$ with $\sigma_k(0)$. Based on this assumption, we are able to solve equation (1.30) analytically:

$$\begin{aligned}
EEPE_k &= \frac{1}{1year}\cdot\int_0^{1year}\sigma_k(0)\cdot\sqrt{t}\cdot\phi(0)dt \\
&= \frac{1}{1year}\cdot\phi(0)\cdot\sigma_k(0)\cdot\int_0^{1year}\sqrt{t}dt \\
&= \frac{1}{1year}\cdot\phi(0)\cdot\sigma_k(0)\cdot\left[\frac{2}{3}t^{\frac{3}{2}}\right]_0^{1year} \\
&= \frac{1}{1year}\cdot\phi(0)\cdot\sigma_k(0)\cdot\left[\frac{2}{3}(1year)^{\frac{3}{2}}-\frac{2}{3}(0)^{\frac{3}{2}}\right] \\
&= \frac{2}{3}\cdot\phi(0)\cdot\sigma_k(0)\cdot(1year)^{\frac{3}{2}}\cdot(1year)^{-1} \\
&= \frac{2}{3}\cdot\phi(0)\cdot\sigma_k(0)\cdot\sqrt{1year}
\end{aligned} \tag{1.31}$$

Aggregating trade-level add-ons

The SA-CCR aims to calculate aggregated add-ons instead of dealing with trade-level volatilities directly. Hence, equation (1.31) needs to be restated the following way:

$$EEPE_k = AddOn_k = \sqrt{\sum_{i,j}\rho_{ij}\cdot AddOn_i\cdot AddOn_j} \tag{1.32}$$

where $AddOn_i(t)$ represents the EEPE of a netting set with one trade (i):

$$AddOn_i = EEPE_i = \frac{2}{3}\cdot\phi(0)\cdot\sigma_i\cdot\sqrt{1year} \tag{1.33}$$

By inserting equation (1.33) into equation (1.32), we are able to show that trade-level add-ons can be aggregated as if they were standard deviations:

$$
\begin{aligned}
AddOn_k(t) &= \sqrt{\sum_{i,j} \rho_{ij} \cdot AddOn_i \cdot AddOn_j} \\
&= \sqrt{\sum_{i,j} \rho_{ij} \cdot \frac{2}{3} \cdot \phi(0) \cdot \sigma_i(0) \cdot \sqrt{1year} \cdot \frac{2}{3} \cdot \phi(0) \cdot \sigma_j(0) \cdot \sqrt{1year} \cdot \phi(0)} \\
&= \sqrt{\sum_{i,j} \rho_{ij} \cdot \left(\frac{2}{3}\right)^2 \cdot \sigma_i(0) \cdot \sigma_j(0) \cdot 1year \cdot (\phi(0))^2} \\
&= \frac{2}{3} \cdot \phi(0) \cdot \sqrt{1year} \cdot \sqrt{\sum_{i,j} \rho_{ij} \cdot \sigma_i(0) \cdot \sigma_j(0)} \\
&= \frac{2}{3} \cdot \phi(0) \cdot \sqrt{1year} \cdot \sigma_k(0)
\end{aligned}
\tag{1.34}
$$

Equation (1.34) is equal to the definition of the $EEPE$ at netting set level in equation (1.31). Hence, trade-level add-ons can be aggregated as if they were standard deviations. This is the central foundation with respect to the aggregation procedures formulated by BCBS (2014b). This is also true for margined netting sets:

$$
\begin{aligned}
AddOn_k(t) &= \sqrt{\sum_{i,j} \rho_{ij} \cdot AddOn_i \cdot AddOn_j} \\
&= \sqrt{\sum_{i,j} \rho_{ij} \cdot \phi(0) \cdot \sigma_i(0) \cdot \sqrt{MPOR} \cdot \phi(0) \cdot \sigma_j(0) \cdot \sqrt{MPOR}} \\
&= \sqrt{\sum_{i,j} \rho_{ij} \cdot \sigma_i(0) \cdot \sigma_j(0) \cdot MPOR \cdot (\phi(0))^2} \\
&= \phi(0) \cdot \sqrt{MPOR} \cdot \sqrt{\sum_{i,j} \rho_{ij} \cdot \sigma_i(0) \cdot \sigma_j(0)} \\
&= \phi(0) \cdot \sqrt{MPOR} \cdot \sigma_k(0)
\end{aligned}
\tag{1.35}
$$

Adjusted notional for equity positions

Following the argumentation in (BCBS (2014b)), the volatility of a transaction's market value ($\sigma_i^{(V)}$) for equity derivatives is estimated by the product of the stock (index) price volatility, the amount of shares (N_i) referenced by the transaction and the sensitivity of the transaction to changes in the underlying price (δ_i). The volatility of the stock (index) is approximated by the product of the relative (log-normal) volatility of the stock (index) price ($\sigma_i^{(RF)}$) and the current stock (index) price (P_i). Hence, we arrive at the following definition for $\sigma_i^{(V)}$:

$$\sigma_i^{(V)} = \sigma_i^{(RF)} \cdot |\delta_i| \cdot P_i \cdot N_i \tag{1.36}$$

By replacing $\sigma_i^{(RF)}$ with the definition in equation (1.10) and inserting equation (1.36) into equation (1.9), we obtain the following definition of the adjusted notional for equity derivatives ($d_i^{(EQ)}$):

$$d_i^{(EQ)} = P_i \cdot N_i \tag{1.37}$$

1.A.3 Transformation of SA-TB and SA-CCR risk-weights

The new Standardized Approach for Market Risk (SA-TB) was designed to provide an approximation for the Expected Shortfall with a confidence level of 97.5% (BCBS (2019b)). Hence, it provides an analytical approximation of the internal model's target measure under the Fundamental Review of the Trading Book (FRTB). One part of the approach is the linear delta risk charge, which provides a capital amount for the first order risk with respect to the underlying of the position. Hence, one main concept of the linear risk charge is the approximation of the portfolio loss amount at a future point in time ($L(t)$) by a first order taylor term:

$$L(t) = \sum_{i=1}^{N^{(RF)}} s_i \cdot \Delta_i(t) \tag{1.38}$$

where s_i represents the sensitivity of the portfolio to a certain risk factor (i) and Δ_i equals the change of this risk factor over a specific time horizon (t). $N^{(RF)}$ equals the number of risk factors. It is assumed that the movements of risk factors follow a geometric Brownian Motion with zero drift and fixed volatility. Hence, the movements of the risk factors are normally distributed with ($\Delta \sim N(0, C_b)$), where C_b represents the covariance matrix for all risk factors in a specific bucket (b). Based on these assumptions, the loss amount is also normally distributed with $L \sim N(0, s^T C_b s)$. We are able to decompose the covariance matrix (C_b) into:

$$C_b = diag(\sigma) \cdot \rho_b \cdot diag(\sigma) \tag{1.39}$$

where ρ_b is equal to the correlation matrix and σ represents the standard deviation of risk factors. This leads to the following solution for the capital charge at bucket level:

$$\begin{aligned} K_b &= \sqrt{s^T C_b s} \cdot q_\alpha \\ &= \sqrt{s^T diag(\sigma) \rho_b diag(\sigma)} \cdot q_\alpha \\ &= \sqrt{WS^T \rho_b WS} \end{aligned} \tag{1.40}$$

WS represents a one-dimensional vector of weighted sensitivities. The weighted sensitivity (WS) for a specific risk factor (i) is defined as:

$$WS_i = s_i \cdot RW_i = s_i \cdot q_\alpha \cdot \sigma_i \cdot \sqrt{t} \tag{1.41}$$

where σ_i equals the annualized volatility estimate for the risk factor (i). t equals the holding period in years. According to BCBS (2019b), the SA-TB risk-weights have been calibrated to a market data period equal to the SA-TB liquidity horizons. Hence, the liquidity horizon is not incorporated in the calculation and the holding period equals the base time horizon ($t = 10 days$). q_α equals the Expected Shortfall for a given confidence level (ES_α). Under the above mentioned assumptions, q_α solves for:

$$\begin{aligned} q_\alpha &= \mathbb{E}\left[L|L \geq VaR_\alpha\right] \\ &= \mathbb{E}\left[L|L \geq \Phi^{-1}(\alpha)\right] \\ &= \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}. \end{aligned} \tag{1.42}$$

According to BCBS (2019b), the target measure of the new market risk framework is the Expected Shortfall for a confidence level of 97.5 % ($\alpha = 0.975$) for a base time horizon of 10 days. By inserting equation (1.42) into equation (1.41), we obtain the following definition for the SA-TB bucket level risk-weight:

$$RW_i^{(SA-TB)} = \frac{\phi(\Phi^{-1}(0.975))}{1-0.975} \cdot \sigma_i \cdot \sqrt{t} \approx 2.3378 \cdot \sigma_i \cdot \sqrt{t} \tag{1.43}$$

The SA-CCR supervisory factor (SF_i) can be interpreted as the SA-CCR risk-weight. According to equation (1.10), SF_i is defined as:

$$SF_i^{(SA-CCR)} = \frac{2}{3} \cdot \sigma_i \cdot \phi(0) = \frac{2}{3} \cdot \frac{\sigma_i}{\sqrt{2\pi}} \tag{1.44}$$

By solving equations (1.44) and (1.43) for σ_i, we obtain the following relationship between the SA-CCR supervisory factor and the SA-TB bucket level risk-weight:

$$SF_i^{(SA-CCR)} = \frac{2}{3} \cdot \frac{RW_i^{(SA-TB)}}{2.3378 \cdot \sqrt{t} \cdot \sqrt{2\pi}} = \frac{2}{3} \cdot \frac{RW_i^{(SA-TB)} \cdot \phi(0)}{2.3378 \cdot \sqrt{t}} \tag{1.45}$$

Based on equation (1.45), we are able to transform the SA-TB bucket level risk-weights introduced by BCBS (2019b) into SA-CCR supervisory factors and volatilities. Table 1.A.1 provides the results for the transformation as well as the SA-TB bucket definition.

Table 1.A.1: SA-TB risk-weights: Bucket definition and risk-weight transformation

Bucket	Market Cap	Economy	Sector	$RW_i^{(SA-TB)}$	$\widehat{=}\sigma_i^{(SA-CCR)}$	$\widehat{=}SF_i^{(SA-CCR)}$
1	Large	Emerging	Sector 1	0.55	1.18	0.31
2			Sector 2	0.60	1.29	0.34
3			Sector 3	0.45	0.97	0.26
4			Sector 4	0.55	1.18	0.31
5	Large	Advanced	Sector 1	0.30	0.64	0.17
6			Sector 2	0.35	0.75	0.20
7			Sector 3	0.40	0.86	0.23
8			Sector 4	0.50	1.07	0.28
9	Small	Emerging	Sector 1-4	0.70	1.50	0.40
10		Advanced	Sector 1-4	0.50	1.07	0.28
11			Other Sector	0.70	1.50	0.4
12	Index	Advanced	non-sector specific	0.15	0.32	0.09
13		Emerging	non-sector specific	0.25	0.54	0.14

Notes: This table provides the SA-TB bucket definition based on BCBS (2019b) as well as the transformation of the bucket level risk-weights to SA-CCR parameters.

Based on BCBS (2019b) the following definitions apply:

- **Market Capitalization:** *"Large market cap is defined as a market capitalisation equal to or greater than USD 2 billion and small market cap is defined as a market capitalisation of less than USD 2 billion."*
- **Economy:** *"The advanced economies are Canada, the United States, Mexico, the euro area, the non-euro area western European countries (the United Kingdom, Norway, Sweden, Denmark and Switzerland), Japan, Oceania (Australia and New Zealand), Singapore and Hong Kong SAR."*
- **Sector:** Definition of industry sectors 1-4 according to the following grouping:
    1. Consumer goods and services, transportation and storage, administrative and support service activities, healthcare, utilities
    2. Telecommunications, industrials
    3. Basic materials, energy, agriculture, manufacturing, mining and quarrying
    4. Financials including government-backed financials, real-estate activities, technology

1.A.4 Methodological foundations of the simulation study

For the simulation study in section 1.5, we need to produce exposure results based on a Monte Carlo simulation that are consistent with the add-on methodology of the SA-CCR. According to BCBS (2014b), the SA-CCR add-on equals a conservative approximation of the Effective Expected Positive Exposure (EEPE) under the assumption of an initial market value of zero and absence of collateral. Furthermore, it is assumed that the market value of a transaction follows an arithmetic drift-less Brownian motion with fixed volatility. To assess the appropriateness of the SA-CCR calibration, we apply an advanced model based on the assumptions used for the derivation of the SA-CCR add-on. In order to arrive at consistent and comparable results, we apply the following steps:

Scenario generation: We produce $N_{(s)}$ path-dependent market scenarios (s_k) for the evolution of each option's underlying price ($S_i(t)$). In accordance with the SA-CCR assumptions, we use a fixed volatility and assume a drift of zero ($\mu = 0$). Based on the aforementioned assumptions, the evolution process for the underlying price can be written as (Hull (2011)):

$$S_i(t_j) = S_i(t_{j-1}) \cdot \exp\left[\left(\frac{\sigma_i^2}{2}\right) \cdot \left(t_j - t_{j-1}\right) + \sigma_i \cdot X \cdot \sqrt{t_j - t_{j-1}}\right] \tag{1.46}$$

where X is a standard normal variable and $S_i(t_j)$ equals the underlying price for transaction (i) at time t_j. For a proper comparison with SA-CCR results, we calibrate the model to stressed market data based on the result of the volatility analysis[35] ($\sigma_i = \hat{\sigma_i}^{(hist)}$). For our calculations we simulate 10.000 market scenarios ($N_{(s)} = 10.000$) and use 40 time steps per year ($N_{(t)} = 40$).[36]

Instrument valuation: Based on the simulated market scenarios, we calculate the market value for each hypothetical option for each scenario path and point in time ($V_i(t_j, s_k)$) by applying the respective analytical pricing formulas (Haug (2007)). Thereby, we obtain $N_{(s)}$ scenario paths for the evolution of an option's market value.

Exposure calculation: Given the scenarios for the development of the market value for each option, we are able to calculate the **Expected Exposure (EE)** for each point in time ($EE_i(t_j)$):

$$EE_i(t_j) = \mathbb{E}\left[max\left(V_i(t_j, s_k) - V_i(t_0), 0\right)\right] \approx \frac{1}{N_{(s)}} \sum_{k=1}^{N_{(s)}} max\left(V_i(t_j, s_k) - V_i(t_0), 0\right) \tag{1.47}$$

Please note that the initial market value ($V_i(t_0)$) is subtracted in order to recognize the associated assumption in the SA-CCR (initial market value=0). Based on the resulting exposure profile, we

[35] see section 1.4.

[36] For our simulation study on barrier options we reduce the amount of market scenarios to 5.000.

calculate the **Effective Expected Exposure (EEE)** for each point in time ($EEE_i(t_j)$) based on the following formula (BCBS (2006)):

$$EEE_i(t_j) = max\left(EE_i(t_j), EEE_i(t_{j-1})\right) \tag{1.48}$$

The **Effective Expected Positive Exposure (EEPE)** is defined as the average time-weighted EEE over the risk horizon of 1 year (BCBS (2006)):

$$EEPE_i = \frac{1}{1year} \cdot \int_0^{1year} EEE_i(t)dt \approx \sum_{j=1}^{N_{(t)}} EEE_i(t_j) \cdot (t_j - t_{(j-1)}) \tag{1.49}$$

The resulting $EEPE_i$ is based on the same assumptions as the SA-CCR PFE add-on for **un-margined** netting sets. For the purpose of this analysis, we define:

$$PFE_i^{(IMM)} = \frac{EEPE_i^{(IMM)}}{d_i^{(EQ)}} \tag{1.50}$$

where $d_i^{(EQ)}$ is defined according to equation (1.38). For **margined** netting sets, $PFE_i^{(SA-CCR)}$ is defined as the expected exposure at $t = MPOR$, where $MPOR$ equals the margin period of risk for the respective netting set (BCBS (2014b)). Hence, we define $PFE_i^{(IMM)}$ for **margined** netting sets as:

$$PFE_i^{(IMM)} = \frac{EE_i^{(IMM)}(MPOR)}{d_i^{(EQ)}} \tag{1.51}$$

1.A.5 Selected indices for validation of SA-CCR parameters

Table 1.A.2: Selected indices for validation of SA-CCR parameters

	RIC	Index Name	Bucket (SA-TB)
1	.AAX	AEX ALL-SHARE INDEX	12
2	.AEX	AEX-INDEX	12
3	.MAR	S&P MERVAL ARGENTINA INDEX (ARS)	12
4	.ATG	ATHEX COMPOSITE SHARE PRICE INDEX	13
5	.ATX	WIENER BOERSE ATX	12
6	.BFX	BEL 20	12
7	.BUX	BUDAPEST STOCK INDEX	13
8	.SOFIX	Bulgarian Stock Exchange SOFIX Index	13
9	.CRBEX	CROBEX Index	13
10	.CYMAIN	Main Market Index	13
11	.GDAXI	DAX PERFORMANCE-INDEX TR	12
12	.DJI	DOW JONES INDUSTRIAL AVERAGE	12
13	.DJU	DOW JONES UTILITY AVERAGE	12
14	.STOXX	STOXX EUROPE 600	12
15	.STOXX50E	EURO STOXX 50 INDEX	12
16	.N100	EURONEXT 100 INDEX	12
17	.FCHI	CAC40	12
18	.FTSE	FTSE 100	12
19	.FTMC	FTSE 250	12
20	.FTAS	FTSE ALL SHARE	12
21	.KLSE	FTSE BURSA MALAYSIA KLCI	13
22	.FTEU1	FTSE EUROTOP 100 INDEX	12
23	.FTMIB	FTSE MIB	12
24	.HSI	HANG SENG INDEX	12
25	.IBEX	IBEX 35	12
26	.IXIC	NASDAQ COMPOSITE	12
27	.ISEQ	ISEQ ALL SHARE	12
28	.KS11	KOSPI COMPOSITE	12
29	.KS200	KOSPI 200 INDEX	12
30	.LUXX	LUXEMBOURG SE LUXX INDEX	12
31	.MDAXI	MDAX PERFORMANCE INDEX	12
32	.IMOEX	MOEX RUSSIA INDEX	13
33	.SPX	S&P 500	12
34	.OMXH25	OMX HELSINKI 25 INDEX	12
35	.OMXIPI	OMX ICELAND ALL SHARE PRICE INDEX	13
36	.OMXSPI	OMX STOCKHOLM ALL SHARE PRICE INDEX	12
37	.OMXS30	OMX STOCKHOLM 30 INDEX (INDEX) SWEDEN	12
38	.OMXTGI	OMX TALLINN INDEX	13
39	.PX	WB PRAGUE INDEX	13
40	.SAX	SAX Index	13
41	.SBITOP	Ljubljana Stock Exchange SBI TOP Index	13
42	.SSMI	SWX SMI	12

Notes: This table provides the set of indices in scope of the volatility analysis (section 1.4).

1.A.6 Through-time validation of SA-TB based volatilities

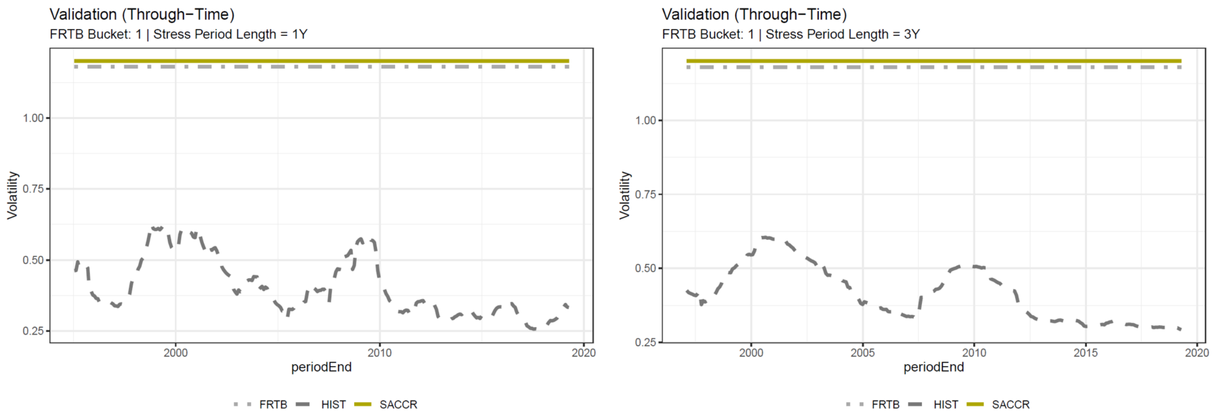

Figure 1.A.1: Through-time validation (bucket 1). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

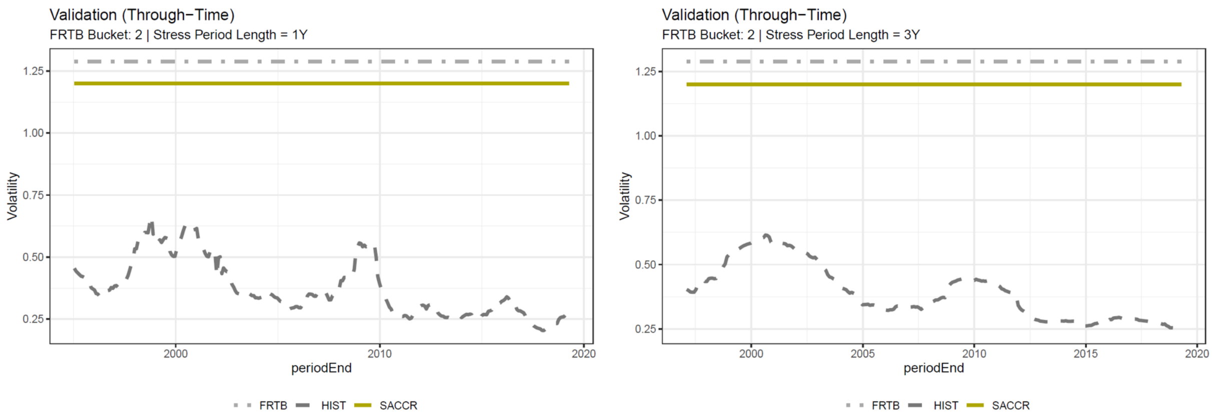

Figure 1.A.2: Through-time validation (bucket 2). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

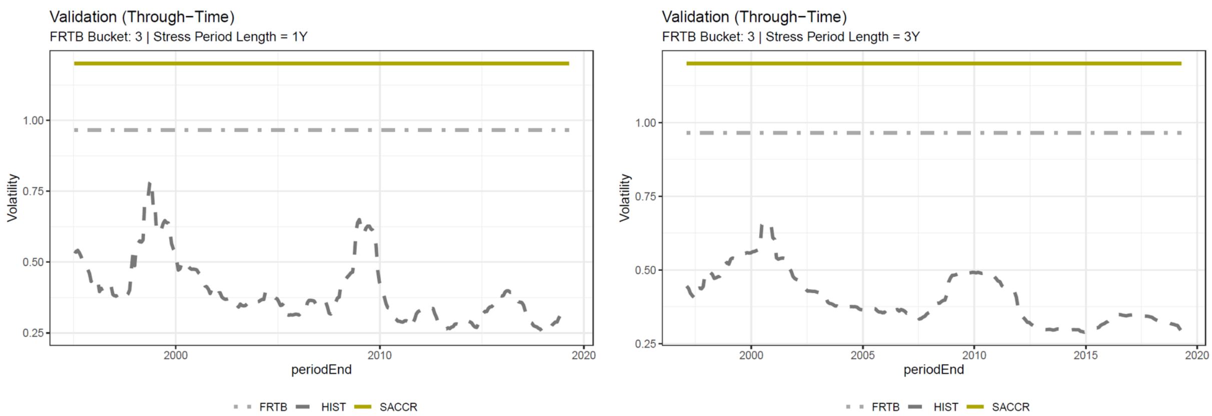

Figure 1.A.3: Through-time validation (bucket 3). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

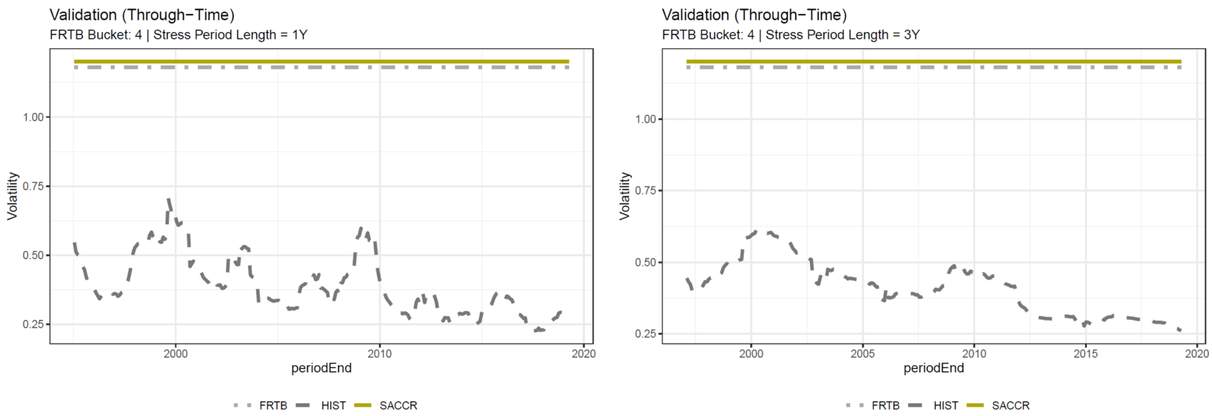

Figure 1.A.4: Through-time validation (bucket 4). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

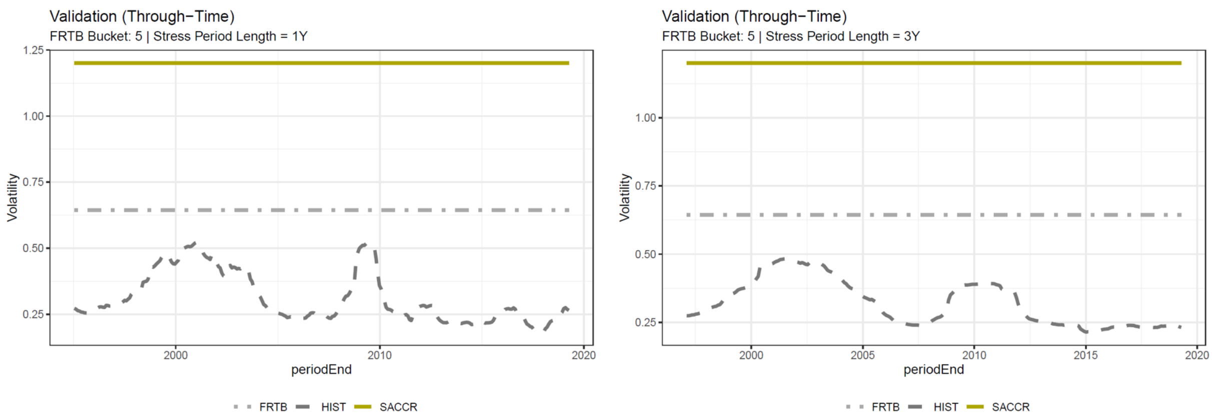

Figure 1.A.5: Through-time validation (bucket 5). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

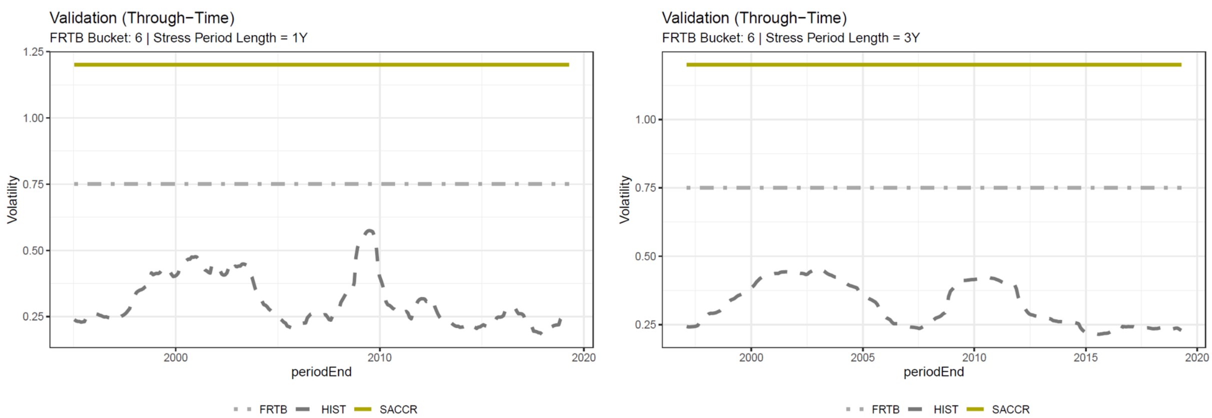

Figure 1.A.6: Through-time validation (bucket 6). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

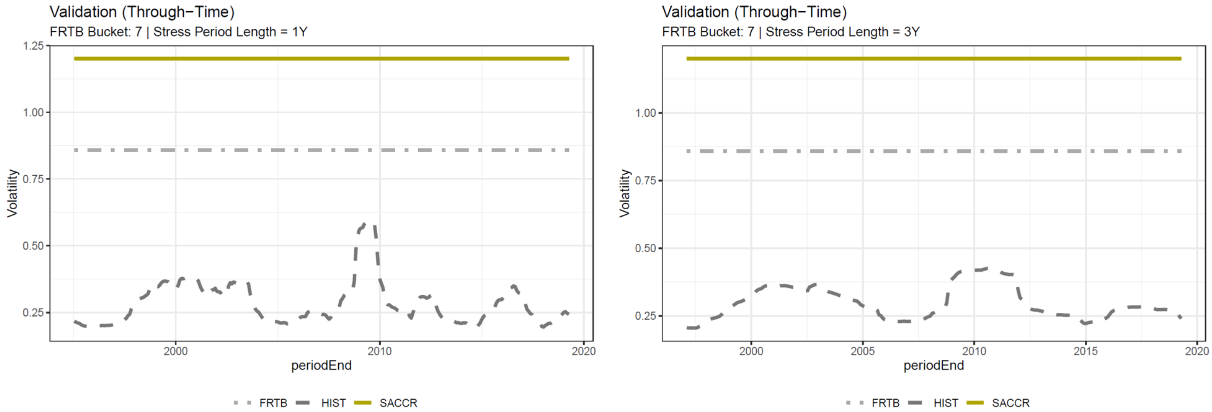

Figure 1.A.7: Through-time validation (bucket 7). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

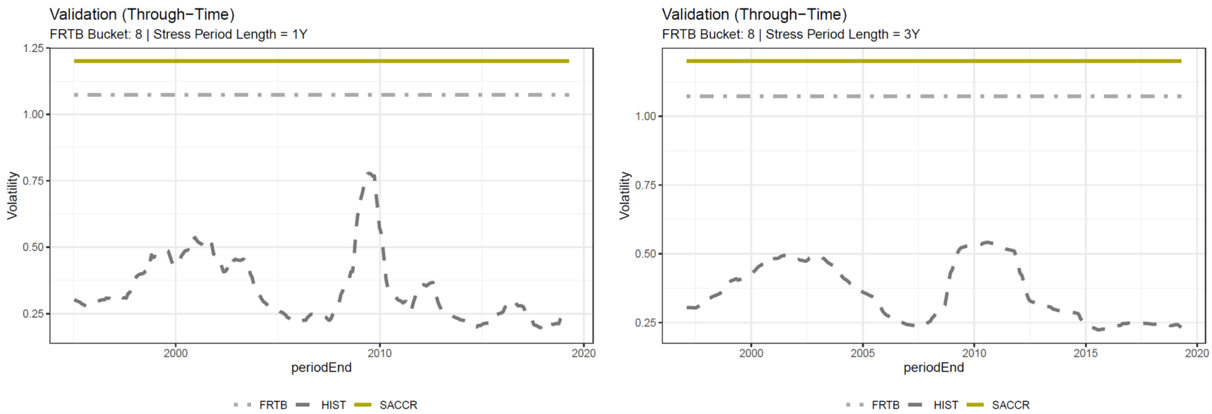

Figure 1.A.8: Through-time validation (bucket 8). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

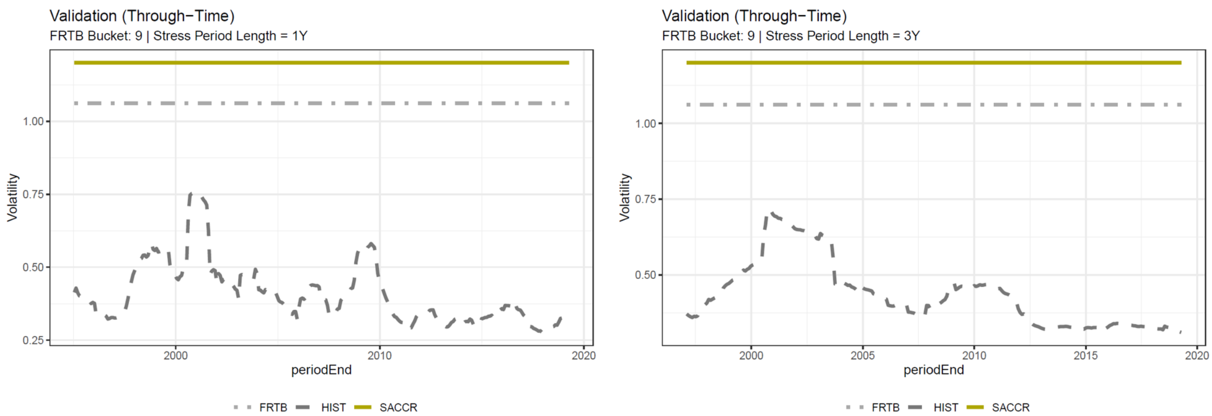

Figure 1.A.9: Through-time validation (bucket 9). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

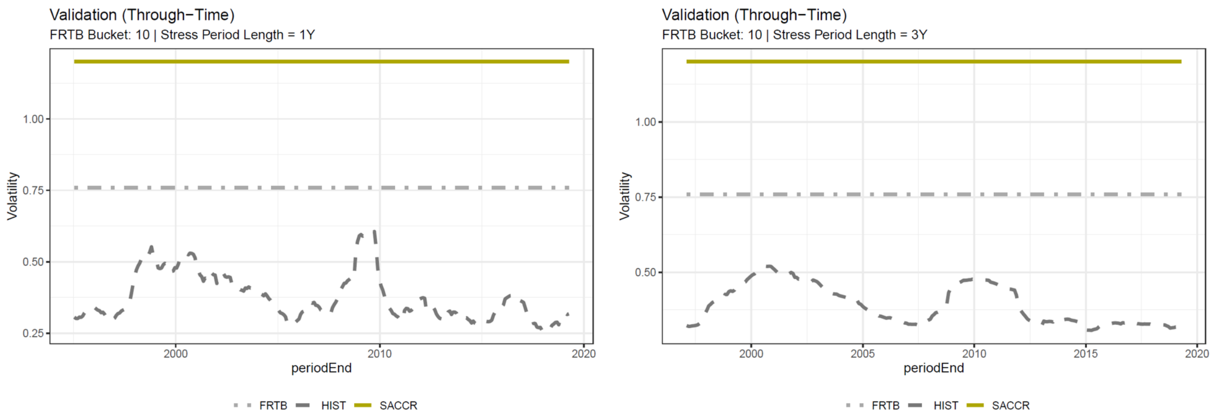

Figure 1.A.10: Through-time validation (bucket 10). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

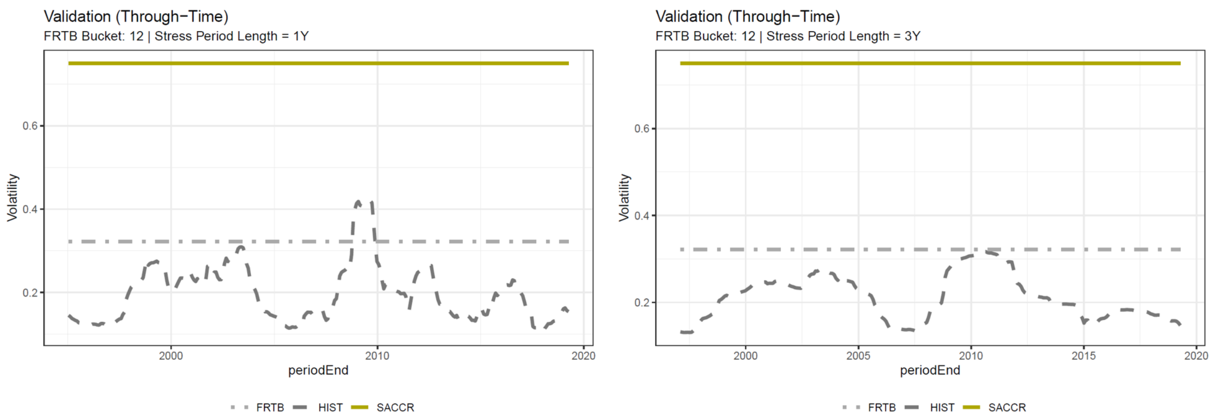

Figure 1.A.11: Through-time validation (bucket 12). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

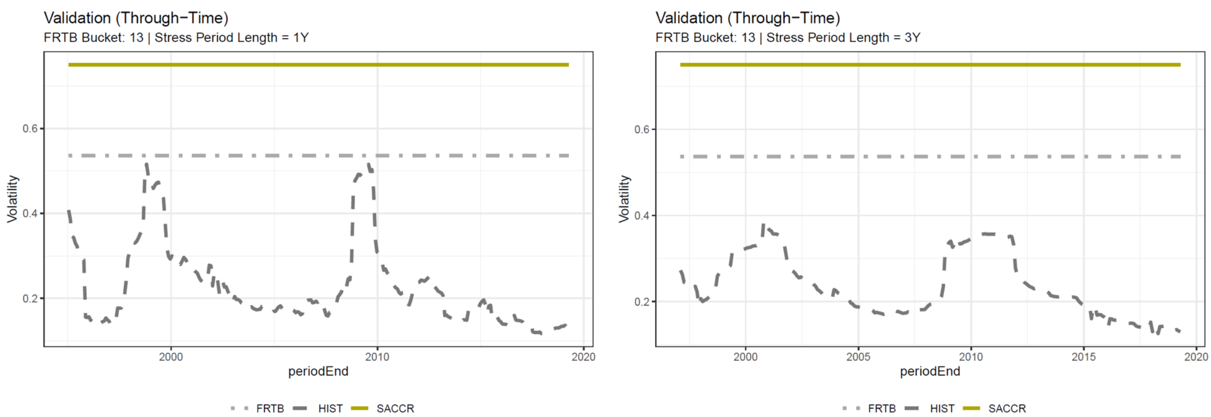

Figure 1.A.12: Through-time validation (bucket 13). *Note* The graph includes the historic volatility estimates (dashed line), the SA-CCR regulatory volatility (solid line) and the SA-CCR equivalent volatility based on SA-TB risk-weights (dash-dotted line).

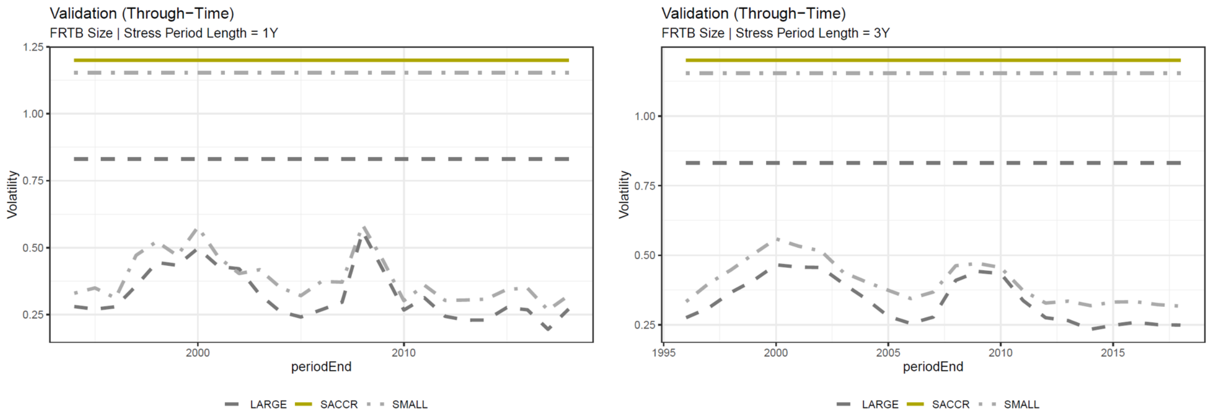

Figure 1.A.13: Through-time validation (SA-TB category: Size). *Note* The graph compares the historic volatility estimates for small (dash-dotted lines) and large (dashed lines) market cap underlyings. The horizontal lines represent the respective SA-TB volatility and the SA-CCR volatility (solid line).

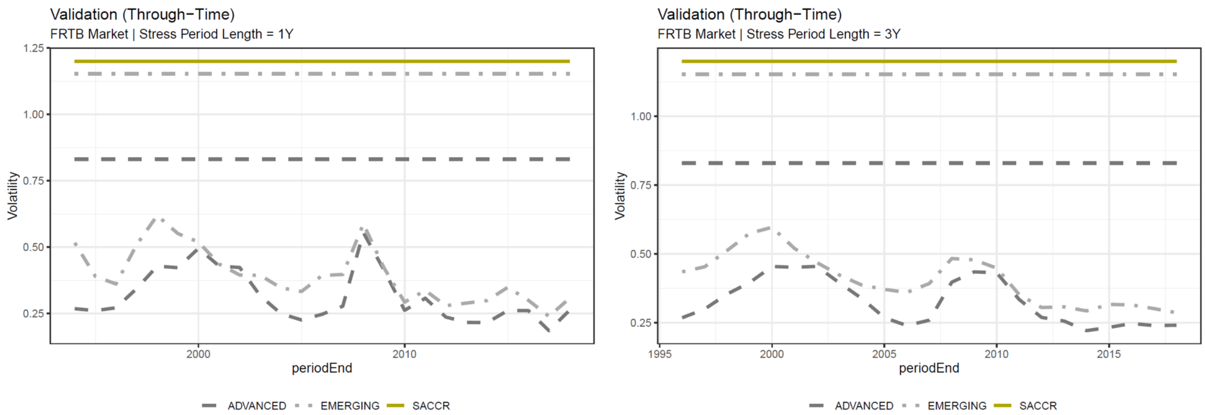

Figure 1.A.14: Through-time validation (SA-TB category: Economy). *Note* The graph compares the historic volatility estimates for emerging (dash-dotted lines) and advanced (dashed lines) underlyings. The horizontal lines represent the respective SA-TB volatility and the SA-CCR volatility (solid line).

1.A.7 Results of the simulation study - Plain-vanilla options

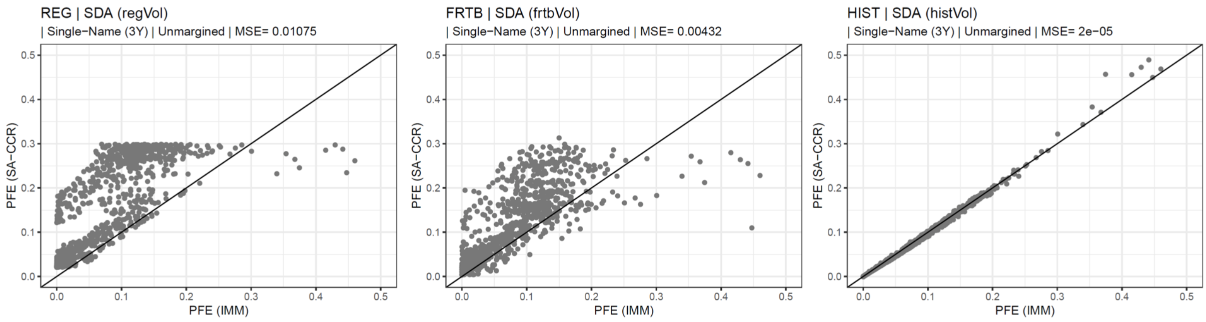

Figure 1.A.15: MSE results for European options (Single-Name (3Y), unmargined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

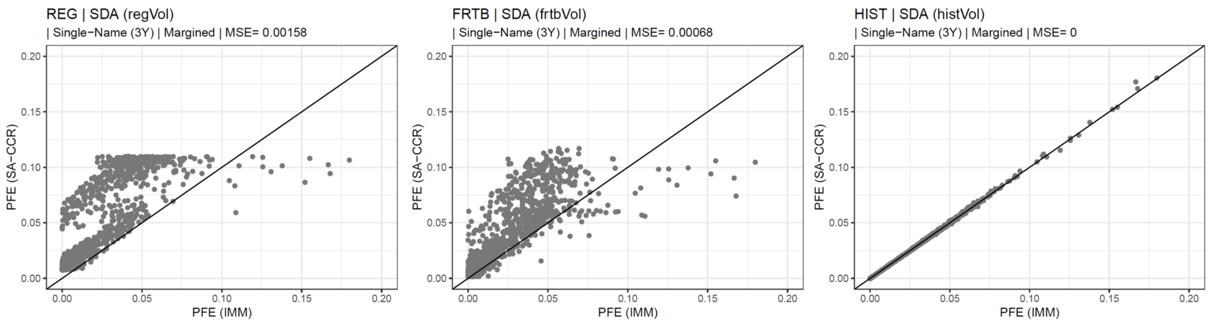

Figure 1.A.16: MSE results for European options (Single-Name (3Y), margined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

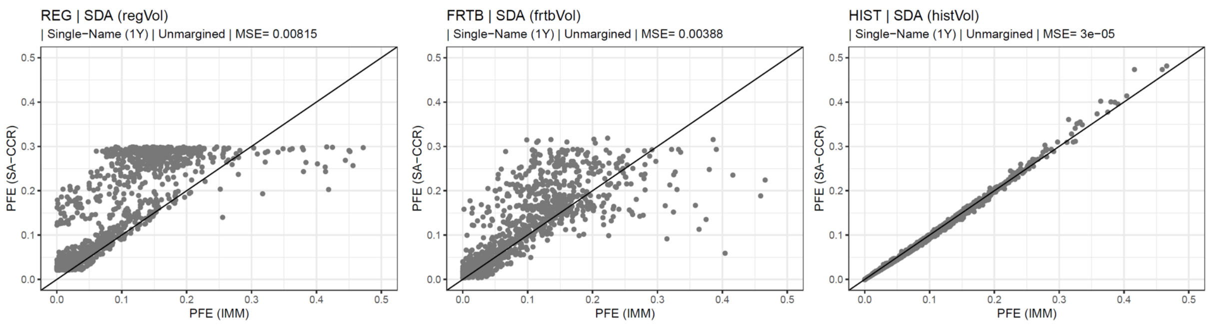

Figure 1.A.17: MSE results for European options (Single-Name (1Y), unmargined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

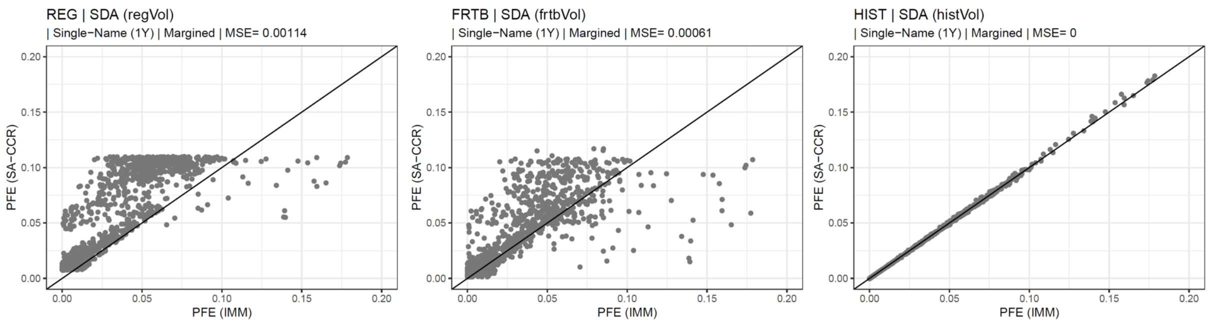

Figure 1.A.18: MSE results for European options (Single-Name (1Y), margined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

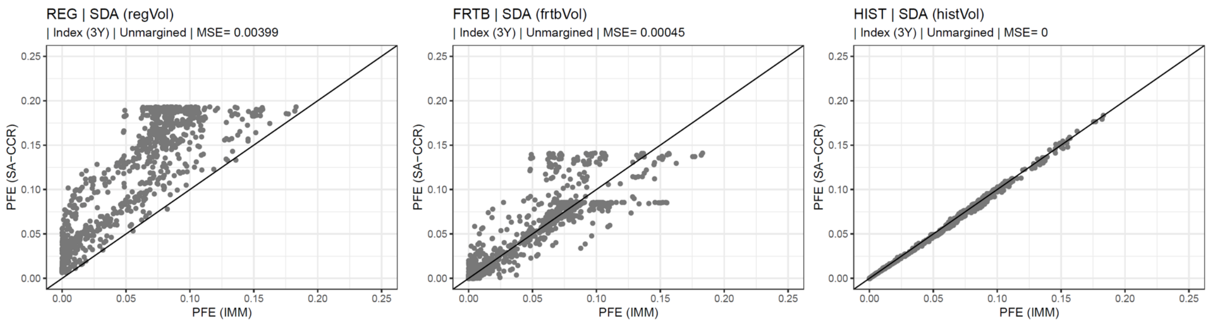

Figure 1.A.19: MSE results for European options (Index (3Y), unmargined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

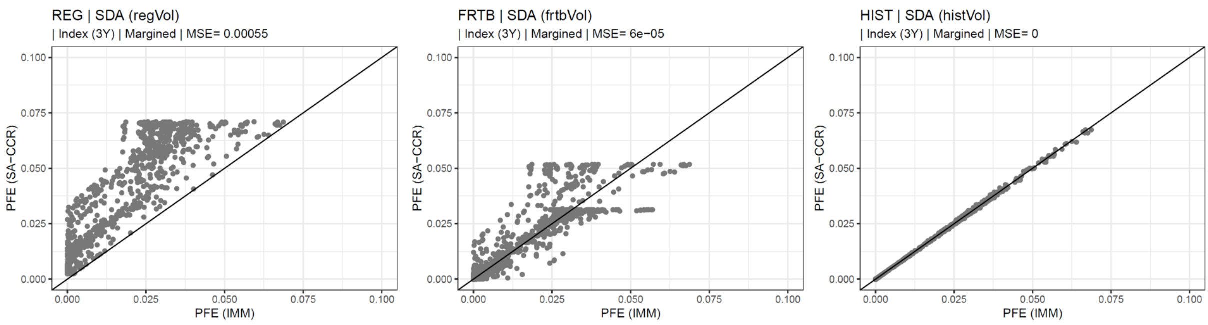

Figure 1.A.20: MSE results for European options (Index (3Y), margined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

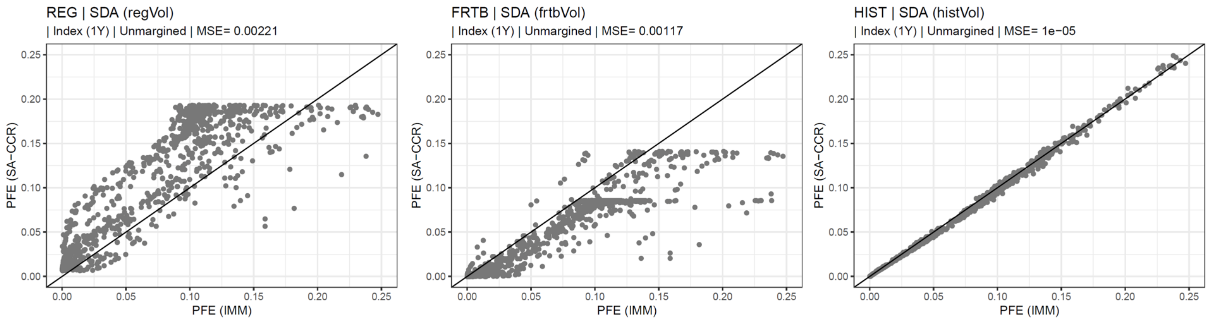

Figure 1.A.21: MSE results for European options (Index (1Y), unmargined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

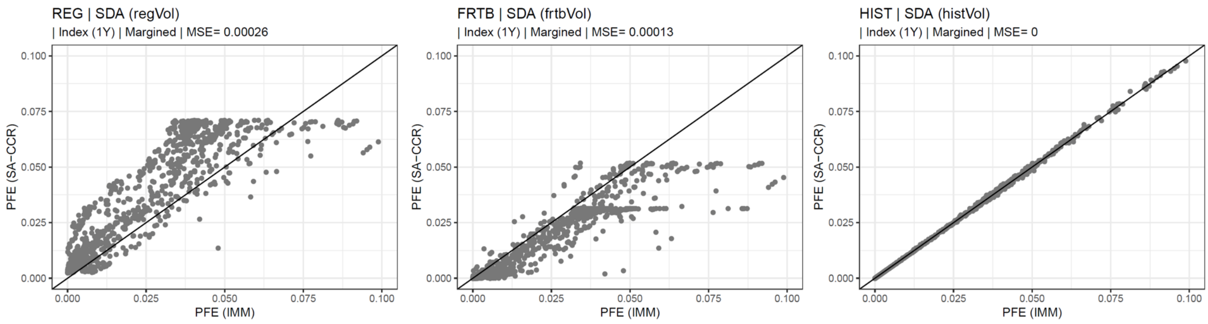

Figure 1.A.22: MSE results for European options (Index (1Y), margined). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 1,000 randomly generated options. Each panel provides the results for a specific SA-CCR configuration.

1.A.8 Results of the simulation study - Barrier options

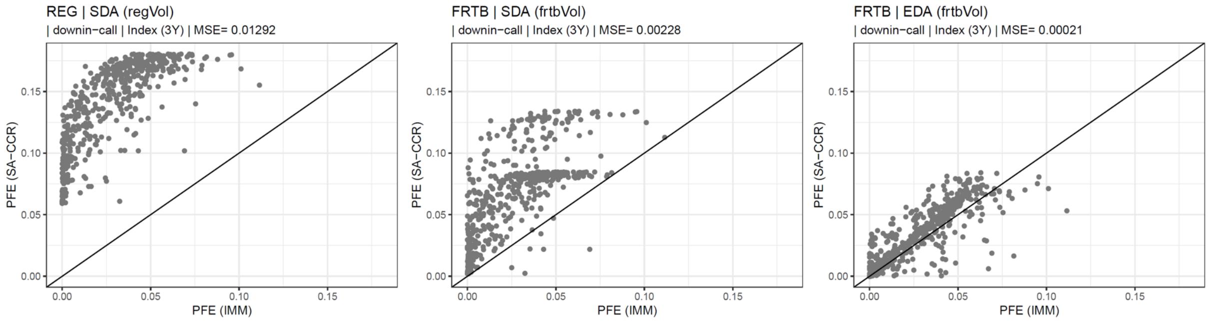

Figure 1.A.23: MSE results for European barrier options (down-in call). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 500 randomly generated European barrier options. Each panel provides the results for a specific SA-CCR configuration.

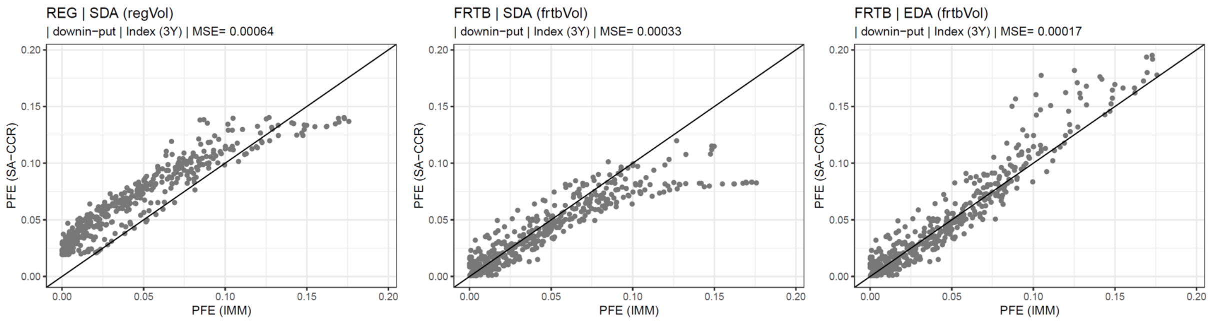

Figure 1.A.24: MSE results for European barrier options (down-in put). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 500 randomly generated European barrier options. Each panel provides the results for a specific SA-CCR configuration.

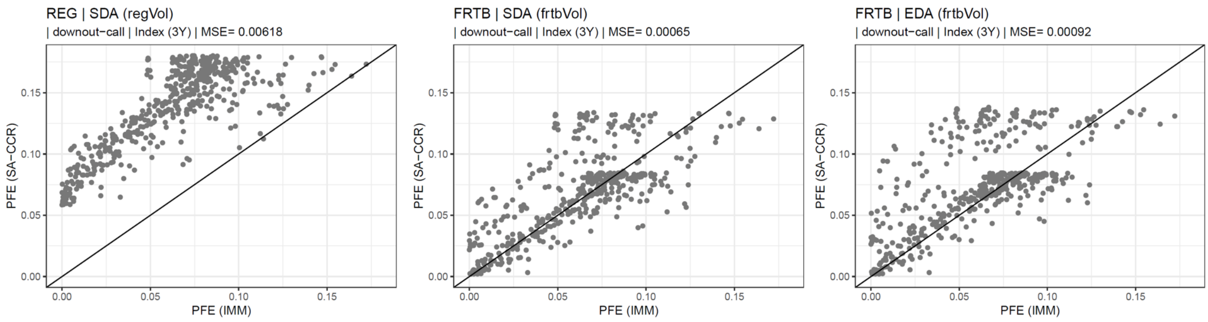

Figure 1.A.25: MSE results for European barrier options (down-out call). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 500 randomly generated European barrier options. Each panel provides the results for a specific SA-CCR configuration.

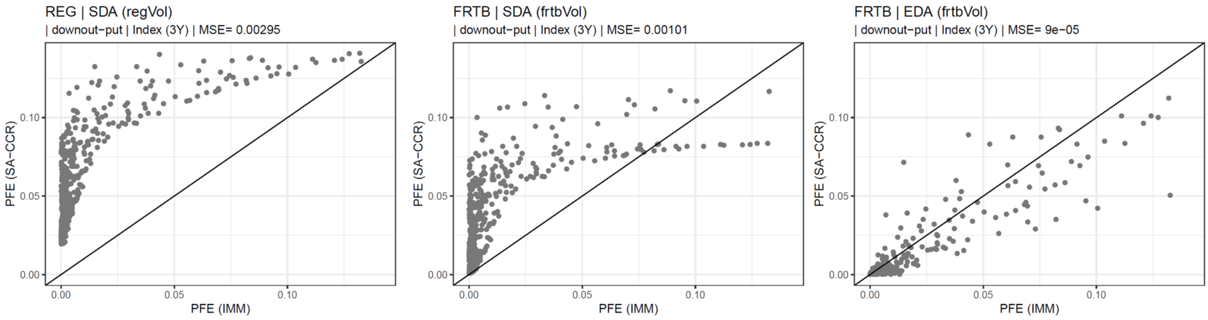

Figure 1.A.26: MSE results for European barrier options (down-out put). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 500 randomly generated European barrier options. Each panel provides the results for a specific SA-CCR configuration.

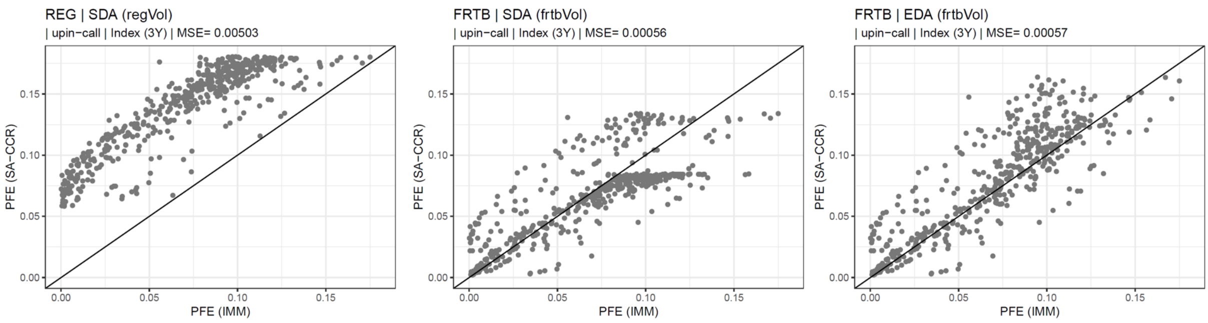

Figure 1.A.27: MSE results for European barrier options (up-in call). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 500 randomly generated European barrier options. Each panel provides the results for a specific SA-CCR configuration.

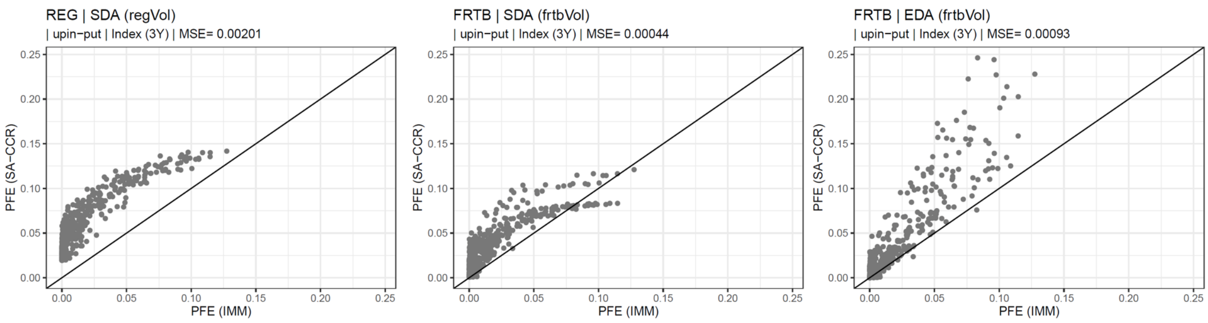

Figure 1.A.28: MSE results for European barrier options (up-in put). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 500 randomly generated European barrier options. Each panel provides the results for a specific SA-CCR configuration.

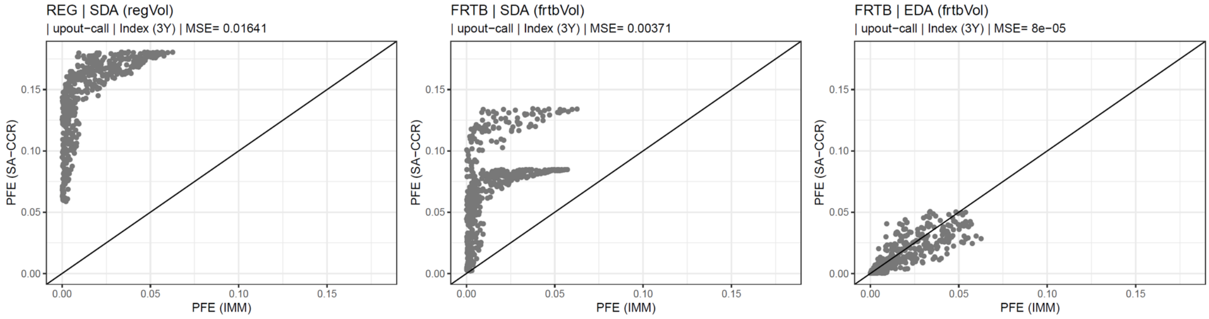

Figure 1.A.29: MSE results for European barrier options (up-out call). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 500 randomly generated European barrier options. Each panel provides the results for a specific SA-CCR configuration.

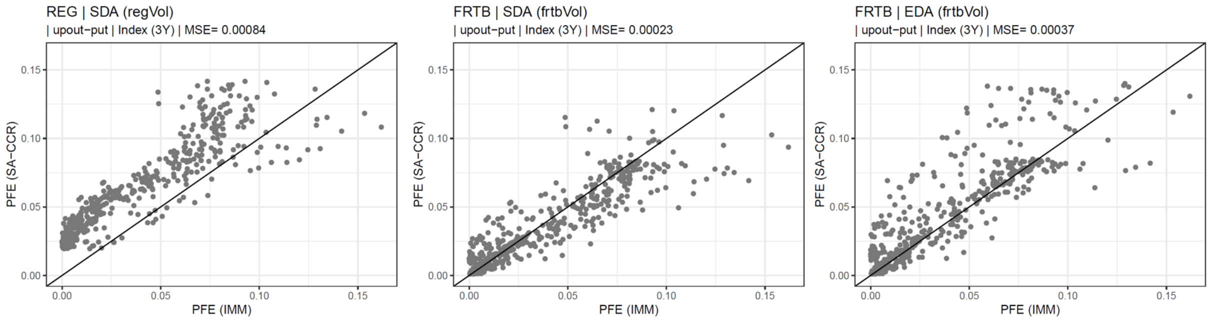

Figure 1.A.30: MSE results for European barrier options (up-out put). *Note* The figure shows the SA-CCR and IMM PFE results in percentage of the adjusted notional amount for 500 randomly generated European barrier options. Each panel provides the results for a specific SA-CCR configuration.

Chapter 2

Computing valuation adjustments for counterparty credit risk using a modified supervisory approach

This chapter is joint work with Patrick Büchel* and Daniel Rösch† published as:

Büchel, P., Kratochwil, M. & Rösch, D. (2020): Computing valuation adjustments for counterparty credit risk using a modified supervisory approach. ***Review of Derivative Research.***
https://doi.org/10.1007/s11147-019-09165-w.

Abstract
Considering counterparty credit risk (CCR) for derivatives using valuation adjustments (CVA) is a fundamental and challenging task for entities involved in derivative trading activities. Particularly calculating the expected exposure is time consuming and complex. This paper suggests a fast and simple semi-analytical approach for exposure calculation, which is a modified version of the new regulatory standardized approach (SA-CCR). Hence, it conforms with supervisory rules and IFRS 13. We show that our approach is applicable to multiple asset classes and derivative products, and to single transactions as well as netting sets.

Keywords: Counterparty Credit Risk; Credit Valuation Adjustments (CVA); Credit Exposure; Standardized Approach for Measuring Counterparty Credit Risk Exposures (SA-CCR)

JEL classification: G21, G32

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2.1 Introduction

The financial crisis and its aftermath have revealed the importance of counterparty credit risk (CCR) in over-the-counter (OTC) derivative transactions. Today, the consideration of CCR is market standard and the calculation of credit valuation adjustments (CVA) has evolved to be a fundamental task for entities involved in derivatives trading due to several reasons. Firstly, market participants need to consider CCR when pricing derivatives. Secondly, international financial reporting standards (IFRS 13) require all entities involved in derivative transactions to consider CCR in the accounting fair value.[1] Thirdly, financial institutions are expected to calculate minimum capital requirements for CVA risk under Basel III, which implies the calculation of CVA as well as CVA sensitivities. The most time-consuming and complex part of xVA calculation is the determination of the expected exposure. Given the lack of clear methodological guidance in IFRS 13, a wide range of methods has been developed by regulators, financial institutions and scientists alike. As many market participants may not be able to apply highly complex and sophisticated methods, there is a need for simpler semi-analytical and parametric approaches. Most existing approaches are either too simplistic to be robust, only applicable on transaction level or suitable for a small range of products. Hence, most of these methods are not applicable to multi-dimensional netting sets.

Our paper provides the following contributions. Firstly, we develop a fast and simple semi-analytical method for exposure calculation, which is a modified version of the new supervisory standardized approach for measuring counterparty credit risk exposures (SA-CCR). We derive the necessary adjustments to the regulatory SA-CCR in order to ensure consistency with IFRS 13. The approach has a flexible structure and is able to capture risk mitigating effects from margining and collateralization. Secondly, we show that our approach is applicable to multiple asset classes and on a single-transaction as well as a netting set level. To ensure the usability of our approach, we compare our results with an advanced model approach for an illustrative set of interest rate and foreign exchange derivatives.

We find that our modified SA-CCR approach is able to produce expected exposure profiles capturing the main exposure dynamics of interest rate and foreign exchange positions. Hence, we are able to mirror exposure profiles generated by advanced methods, which might serve as input for CVA calculation. As we maintain the key building blocks and methodological assumptions

[1] IFRS 13 also requires the inclusion of an entity's own credit risk in the fair value measurement via the calculation of Debit Valuation Adjustments (DVA). In this paper we focus solely on the calculation of CVA based on the expected positive exposure. It is possible to adapt our approach for calculation of further xVAs.

of the supervisory SA-CCR, we offer a flexible and consistent approach to calibration based on market-implied volatilities, yet simple enough to be adopted by smaller institutions with limited personal resources.

The remainder of the paper is structured as follows. Section 2.2 provides an overview and categorization of existing methods for exposure quantification. In section 2.3 we derive the necessary adjustments to the SA-CCR based on central model foundations. The calibration of the modified SA-CCR is lined out in section 2.4. The methodology and results of the empirical analysis are presented in section 2.5. Section 2.6 concludes this paper.

2.2 Methods for exposure quantification

Calculation of xVAs requires the quantification of the expected exposure at time t. The lack of clear guidance from accountants and supervisors as well as the need for complex and simpler methods have led to the development of a wide range of approaches. According to Gregory (2015), these methods can be divided into advanced, parametric and semi-analytical approaches. Using **advanced approaches** is the most sophisticated way to quantify CCR exposures, and there is plenty of academic literature on their application (see Pykhtin and Zhu (2007),Pricso and Rosen (2005), Picoult (2004), Canabarro and Duffie (2003), Picoult (2004)). An advanced approach provides the most realistic risk assessment, but requires in-depth quantitative knowledge, a multitude of input data and a powerful infrastructure. Especially the simulation of potential market scenarios and the valuation of transactions for each scenario and viewpoint are complex and laborious tasks. Developing and maintaining an advanced model is complex and associated with high costs. While advanced approaches are usually applied in larger financial institutions, small- and medium-sized market participants often do not have the capabilities to operate a complex exposure simulation model. Thompson and Dahinden (2013) find that even banks applying advanced models are often unable to cover all asset classes and products within these models. Therefore, we are certainly justified in saying that there is a need for alternative, less sophisticated approaches. To avoid an operational burdensome simulation model, various **semi-analytical methods** have been developed. These approaches are based on assumptions with respect to the development of risk factors driving the market value of a product or netting set. One prominent example for semi-analytical methods is the swaption approach introduced by Sorensen and Bollier (1994). They measure the exposure of an interest rate swap by valuing a series of swaptions, which a party would theoretically enter into in case of the counterparty's default. There are several other semi-analytical methods for interest rate swaps and other derivative products (such as Leung and Kwok (2005) for credit default swaps). In the past years, there has been a lot of work on the development and enhancement of reduced-form and structural models for CVA calculation (e.g. Kao (2016), Hull and White (2012), Cherubini (2013)). While semi-analytical methods are considered the best choice for modeling CCR exposure on transaction level, their application is limited. Semi-analytical methods are generally suited for a limited number of products and designed for a specific asset class. Hence, it is difficult to apply these methods for products with multiple underlying risk factors (e.g. cross-currency-swaps) and multi-dimensional netting sets. In general, CCR exposures are calculated on netting set / counterparty level and require an aggregation of product-specific exposure profiles, which is something most of these semi-analytical methods are not able to provide. Most semi-analytical approaches ignore diversification and effects from collateralization, netting and margining. Even extensions are only able to recognize these effects in a very limited way. For example, Brigo and Masetti (2005) develop an analytical approach for interest rate portfolios in a single currency.

Parametric approaches are considered to be the most simplistic way of quantifying CCR exposures. They provide an approximation based on a limited number of simple parameters. Most parametric approaches calculate the exposure as the sum of current exposure (CE) and an add-on for potential future exposure (PFE). By calibrating the aforementioned simple parameters to more complex methods, the outcome of parametric approaches is aligned with the results from more sophisticated models. Especially regulatory standardized approaches are based on the idea of simplification and calibration. When calculating the exposure at default (EAD) for the assessment of minimum regulatory capital requirements, banks currently have the option to choose between using an advanced Internal Model Method (IMM) or one of two standardized approaches (Standardized Method (SM), Current Exposure Method (CEM)).[2] According to EBA (2016) the Current Exposure Method (CEM) is the most widespread approach for calculating CCR exposures in the European banking sector for regulatory purposes. This method was introduced by the Basel Committee on Banking Supervision (BCBS) in 1996 (BCBS (1996)) and is still valid after it was adjusted in the course of Basel II (BCBS (2005)). A majority of financial institutions uses methods based on the CEM for accounting and pricing purposes. These approaches are often referred to as "mark-to-market plus add-on" methods. In the past, especially the CEM was criticized for several reasons. From the perspective of BCBS (2014d) the

[2] Institutions with very limited trading business have the possibility to use an even simpler method, the Original Exposure Method (OEM), for the purpose of calculating minimum regulatory capital requirements.

main issues are (1) the lack of risk sensitivity, (2) the outdated calibration of risk-weights, (3) the missing ability to recognize credit risk mitigation techniques (in particular margining) as well as (4) a too simplistic attempt to capture netting effects.[3]

Driven by this criticism, the financial crisis and the increasing importance of bilateral margining in OTC derivatives markets[4], a new regulatory standardized approach was developed by the BCBS (BCBS (2014d)). The SA-CCR will replace the existing standardized approaches (SM and CEM). With the development of the SA-CCR, the BCBS was striving for a holistic approach applicable to a variety of derivative products. Furthermore, the SA-CCR was intended to overcome the weaknesses of existing approaches while keeping complexity on a reasonable level.

The SA-CCR can be classified as a semi-analytic method. It uses a rule based calculation scheme and simple parameters. Nevertheless, the derivation of the approach is based on detailed assumptions with respect to the distribution of market values and model based aggregation algorithms. The SA-CCR has several major advantages compared to its predecessors. First, the SA-CCR is able to distinguish between margined and unmargined netting sets. Effects of margining are considered in the current and potential future exposure component. This is an important feature in light of the rising importance of bilateral margining and central clearing. As stated in BCBS (2014d) the SA-CCR is also able to cope with complex situations (e.g. several netting sets are covered by one margin agreement). Second, the SA-CCR applies a more sophisticated approach to netting and diversification. This adds additional complexity, but should lead to higher risk sensitivity in the approximation of exposures (BCBS (2014d)). The structure of the calculation of potential future exposure is flexible and allows to add or delete elements where necessary.[5] Third, the SA-CCR takes over-collateralization, moneyness of transactions and the netting set into account. As excess collateral and transactions with negative values guard against rising exposures, this should lead to more realistic results. Overall, the SA-CCR is more complex compared to the popular CEM, but financial institutions might be able to leverage on the improved risk sensitivity and flexibility. The SA-CCR provides a consistent exposure calculation framework for all asset classes, while accounting for specific

[3] For a comprehensive discussion of critique of CEM, please refer to Fleck and Schmidt (2005) and Pykhtin (2014)).

[4] As a result of the global financial crises, regulators all over the world set regulations for reducing risk in the financial industry, especially in the OTC market, and to protect counterparties from the risk of a potential default of the other counterparty. These are the obligation to clear certain derivative products as well as the obligation to reduce the risk of non-cleared OTC derivative contracts by exchanging collateral in form of initial and variation margin (BCBS and IOSCO (2015), ESAs (2016)).

[5] There are discussions to give national competent authorities the option to adjust the add-on structure for institutions with complex commodity trading activities.

aspects of different financial products (such as equity options, swaptions, etc.). Considering these facts alongside the aforementioned improvements and the transparency with respect to its model foundations, the application of SA-CCR for CVA pricing and accounting purposes is an interesting option for all kinds of entities involved in derivatives trading.

According to Marquart (2016) the application of regulatory approaches (foremost CEM) is considered best practice when calculating exposures for CVA pricing. She analysed the impact on accounting CVA when switching from CEM to SA-CCR, under the assumption of using a simple CVA formula $CVA = PD \cdot LGD \cdot EAD$, where EAD is defined as the Effective Expected Positive Exposure ($EEPE$) resulting from SA-CCR, or CEM respectively. The application of the supervisory SA-CCR for CVA pricing compasses several issues. First, the SA-CCR aims for an approximation of the exposure at default (EAD) under the Internal Model Method (IMM). Under IMM, the EAD is defined as the product of the Effective Expected Positive Exposure ($EEPE$) and a factor ($\alpha = 1.4$), which is used to convert the $EEPE$ into a loan equivalent exposure.[6] For CVA pricing, a time dependent expected exposure profile $EE(t)$ is required. Hence, the target measure of the supervisory SA-CCR is not appropriate. Second, the SA-CCR is calculated for a risk horizon of up to one year for unmargined netting sets. For the purpose of CVA pricing, an exposure profile for the life-time of a netting set is required. Using the $EEPE$ or EAD as a scalar when calculating CVA would ignore the time dependency of exposure. Third, the SA-CCR is calibrated to a period of stress. This means resulting exposures are calculated under the real-world measure. According to IFRS 13, the calculation of CVA needs to be conform to the expectations of market participants. This requires a calibration under the risk-neutral measure. Additionally, the SA-CCR contains a set of conservative elements which should not be applied when calculating exposure for CVA pricing. In conclusion, we find that the SA-CCR in its supervisory form does not conform to IFRS 13. Hence, modifications to the regulatory SA-CCR are required to deploy the approach for CVA pricing and accounting purposes.

2.3 Derivation of the modified SA-CCR

This paper aims to define modifications to the supervisory SA-CCR to derive an approach for the calculation of expected exposure profiles. As stated above, the adjustments are necessary in order to calculate exposure values suitable for CVA calculations. While adjusting the SA-CCR,

[6] For information on the calibration and theoretical background of α, please refer to ISDA et al. (2003), Lynch (2014) and Gregory (2015).

we aim to retain the basic structure and main building blocks. This allows the application of a consistent approach across asset classes and enables financial institutions to leverage on future implementations of the supervisory SA-CCR. The following presentation of the SA-CCR methodology and the derivation of its adjustments is based on the content and structure of BCBS (2014b).

The calculation of CVA requires an expected exposure profile as the main input. In the absence of collateral, the expected exposure of a netting set (k) is defined as the expected positive value of the netting set's market value (V_k) at a future point in time (t):

$$EE_k(t) = \mathbb{E}^Q\left[max\left(V_k(t), 0\right)\right] \tag{2.1}$$

In its supervisory form, the target measure of the SA-CCR is a conservative Effective Expected Positive Exposure ($EEPE$) on netting set level under the real-world measure (calibrated to historic stressed volatilities). Hence, the main adjustment when deriving our approach is the change of target measure to an $EE_k(t)$ under the risk-neutral measure. To retain the general structure of the SA-CCR, we define $EE_k(t)$ as the combination of replacement costs ($RC_k(t)$) and potential future exposure ($PFE_k(t)$):

$$EE_k(t) = RC_k(t) + PFE_k(t) \tag{2.2}$$

Please note that both components of the modified SA-CCR are a function of time (t). Following our approach, $RC_k(t)$ captures the deterministic component, while potential future exposure quantifies the stochastic component of $EE_k(t)$. In the following sections, we derive the modified formulas for calculation of these components on netting set level. Finally, we transfer our results to the SA-CCR specific parameters for exposure calculation.

2.3.1 Replacement Costs

For the derivation of the replacement costs formula, we first introduce the following assumptions. **(A1)** A transaction's market value follows a driftless brownian motion. For the formulation of replacement costs, we set the volatility to zero. **(A2)** We assume no cash flows between (t_0, t). **(A3)** Furthermore, the transaction's netting set is unmargined and therefore not supported by a margin process.[7] These assumptions are used implicitly by BCBS (2014b) for the derivation

[7] A netting set is considered to be unmargined if there is no exchange of variation margin (VM). Nevertheless, other types of collateral (such as initial margin (IM)) might be present.

of RC for unmargined netting sets. Under these assumptions, the future market value of a transaction (V_i) at a specific point in time (t) is defined as:

$$V_i(t) = V_i(t_0) + \sigma_i(t) \cdot \sqrt{t} \cdot X_i \tag{2.3}$$

where $V_i(t_0)$ represents today's market value and X_i is a standard normal random variable ($X_i \sim N(0,1)$). $\sigma_i(t)$ represents the volatility of the transaction's market value at time t. Applying assumption **A1**, the expected future market value is equal to today's market value as the second term of equation (2.3) becomes zero. As stated above, $RC(t)$ should capture the deterministic movements of a transaction's market value. In particular, interest rate and credit default swaps involve regular payments resulting in a change of the transaction's market value over time. In order to cover these deterministic effects, we relax assumption **A2**. Given assumption **A1**, the future market value of a transaction is deterministic and can be calculated based on the transaction's future cash flows. This may generally be written as:

$$\hat{V}_i(t) = \sum_{j=t}^{T} CF_{REC}(t_j) \cdot DF(t,t_j) - \sum_{j=t}^{T} CF_{PAY}(t_j) \cdot DF(t,t_j) \tag{2.4}$$

where $CF_{REC}(t_j)$ is the cash flow received at time t_j, $CF_{PAY}(t_j)$ equals the cash flow paid at time t_j and $DF(t,t_j)$ represents the discount factor from time t_j to time t.

For more complex derivatives or in case no information regarding future cash flows is available, we introduce a time-dependent and product-specific scaling factor (s_i) to provide an approximation of the future market value of a transaction ($\hat{V}_i(t)$).

$$\hat{V}_i(t) = V_i(t_0) \cdot s_i(t) \tag{2.5}$$

For interest rate or credit default swaps, this scaling factor might be based on a simplified duration measure for the respective product:

$$s_i(t) = \frac{D_i(t)}{D_i(t_0)} \cdot 1_{\{M_i \geq t\}} \tag{2.6}$$

where $1_{\{M_i \geq t\}}$ is an indicator variable which has the value of 1 if the transaction has not expired at t (i.e., maturity M_i is greater or equal than t). The duration measure $D_i(t)$ is defined as:[8]

$$D_i(t) = \frac{\exp(-r \cdot max(S_i,t)) - \exp(-r \cdot E_i)}{r} \tag{2.7}$$

[8] For the derivation of $D_i(t)$ please refer to section 2.A.4.

where S_i is the start date of the transaction and E_i its end date. r is defined as the current interest rate level. For simple products in other asset classes, $s_i^{(a)}$ could be represented by the indicator variable. Nevertheless, our approach offers the flexibility to define a transaction specific scaling factor for all kinds of (exotic) products. This allows a recognition of deterministic developments of the transaction's market value in a flexible and consistent setting.

Within a legally enforceable netting set (k), the offsetting between transactions with positive and negative market values ($\hat{V}_i(t)$) is allowed. Hence, a netting set's market value at time t is defined as:

$$\hat{V}_k(t) = \sum_{i \in k} \hat{V}_i(t) \tag{2.8}$$

As stated above, replacement costs do not involve stochastic elements. Thus, the expectation of the future market value is solely driven by deterministic movements and hence represented by $\hat{V}_k(t)$. This leads to the following formulation of replacement costs for **unmargined** and **uncollateralized** netting sets:

$$RC_k(t) = \mathbb{E}^{Q}\left[max\left(V_k(t), 0\right)\right] = max\left(\hat{V}_k(t), 0\right) \tag{2.9}$$

In the presence of **collateral**, the market value of the netting set is reduced by the cash-equivalent value of net collateral received ($C_{CE}(t)$). Under assumption **A3**, all collateral posted or received has the form of independent collateral. Given the lack of a margin process, no adjustment to the notional amount of collateral posted/received is required. The time dependency of the collateral value is limited to the volatility of the collateral value itself. In accordance with the supervisory SA-CCR, we calculate cash-equivalent values of collateral ($C_{CE}(t)$) using collateral haircuts. There are two main adjustments to the supervisory approach. Firstly, we do not use a fixed time horizon, but calculate the cash-equivalent value for specific points in time (t). Secondly, we do not apply regulatory prescribed haircuts, but values based on institutions' own volatility estimates. Given these adjustments, $C_{CE}(t)$ is defined as:

$$C_{CE}(t) = \sum_{c \in k} V_c^{rec}(t_0) \cdot (1 - h_c(t)) - \sum_{c \in k} V_c^{post,unseg}(t_0) \cdot (1 + h_c(t)) \tag{2.10}$$

where V_c equals the market value of a received ($V_c^{rec}(t_0)$) or unsegregated posted ($V_c^{post,unseg}(t_0)$) collateral position at time $t = t_0$. The haircut applicable to a specific collateral position is represented by $h_c(t)$. Please note that segregated posted collateral is not relevant for the calculation of replacement costs, as it is placed in a bankruptcy remote account and will therefore not increase exposure to the relevant counterparty. Including collateral positions in the calculation of replacements costs for unmargined netting sets leads to:

$$RC_k(t) = max\left(\hat{V}_k(t) - C_{CE}(t), 0\right) \tag{2.11}$$

In order to derive a formulation for **margined** netting sets, we need to relax assumption **A3**. Within the modified SA-CCR, we introduce the possibility to model collateral dynamics directly. We calculate the future expected market value of each transaction at each point in time t based on known cash flows using equation (2.4) or by applying a scaling factor (see equation(2.5)). Hence, we know the expected future market value of the netting set ($\hat{V}_k(t)$) at each t. Based on this information, we are able to derive an expected collateral path including the consideration of margin parameters like threshold (TH) and Minimum Transfer Amount (MTA). In case of ($TH \neq 0$) collateral is only exchanged, when the threshold is exceeded. This implies that the incremental amount above the threshold is exchanged in form of collateral. We define the amount above the threshold as the collateral demand ($\hat{C_D}(t)$). If ($MTA \neq 0$), collateral is only exchanged, when the absolute difference between the current collateral position ($\hat{C}(t)$) and the collateral demand exceeds the MTA. Under the assumption of symmetric MTA and TH, absence of rounding, daily margining and instantaneous processing of collateral exchange, the expected collateral position at t_j is defined as:[9]

$$\begin{aligned}\hat{C}(t_j) = \hat{C}(t_{j-1}) + max\Big(max\Big(\hat{C}_D(t_j) - \hat{C}(t_{j-1}), 0\Big) - MTA, 0\Big) + \\ min\Big(min\Big(\hat{C}_D(t_j) - \hat{C}(t_{j-1}), 0\Big) + MTA, 0\Big)\end{aligned} \tag{2.12}$$

where the collateral demand ($\hat{C_D}(t)$) is defined as:

$$\begin{aligned}\hat{C_D}(t_j) = max\Big(max\Big(\hat{V}(t_j), 0\Big) - TH, 0\Big) + \\ min\Big(min\Big(\hat{V}(t_j), 0\Big) + TH, 0\Big)\end{aligned} \tag{2.13}$$

Based on this definition, the replacement costs for a margined netting set at t are defined as:[10]

$$RC_k^{margin}(t) = max\left(\hat{V}_k(t) - \hat{C}(t) + NICA, 0\right) \tag{2.14}$$

where $NICA$ represents the Net Independent Collateral Amount defined as:

$$NICA = IM_{rec} - IM_{post}^{unseg} \tag{2.15}$$

[9] Please note that our approach offers the possibility to integrate additional collateral parameters, such as independent amounts, rounding or other re-margining periods.

[10] Please note that the application of haircuts is also required for margined netting sets. In case of a margined netting set, the risk horizon for the application of haircuts is set to the MPOR.

In addition to modeling collateral dynamics directly, we introduce an optional (alternative), more simplistic approximation for the recognition of collateral in margined netting sets. This conservative approximation follows the methodology described in BCBS (2014b). We assume that the latest exchange of variation margin is not known at time t. Hence, we estimate $RC(t)$ of margined netting sets as the maximum of replacement costs of an equivalent unmargined netting set, the highest exposure amount which would not trigger a margin call and zero. In general, a margin call is triggered if the uncollateralized market value is equal to the sum of TH and MTA. This amount is reduced by the net independent collateral amount ($NICA$).[11]

Under a margin agreement, changes in the netting set's market value will lead to changes in the amount of variation margin posted or received. Therefore, we introduce a time-dependent adjustment for variation margin (VM) based on the change of the market value of the netting set. Based on this adjustment, we arrive at the following approximation of replacement costs for **margined netting** sets:

$$RC_k^{margin}(t) \approx \max\Big(\hat{V}_k(t) - \hat{C}_{CE}(t), TH + MTA - NICA, 0\Big) \tag{2.16}$$

where $\hat{C}_{CE}(t)$ is defined as:

$$\hat{C}_{CE}(t) = \left(\sum_{c \in k} V_c^{VM}(t_0) \cdot (1 \pm h_c(MPOR))\right) \cdot \frac{\hat{V}_k(t)}{V_k(t_0)} + NICA \tag{2.17}$$

where $V_c^{VM}(t_0)$ is defined as today's market value of a variation margin collateral position.[12] In general, we cap the expected exposure of a margined netting set at the expected exposure of an equivalent netting set without any form of margin agreement. This is equal to the assumption that a netting set is treated as unmargined as long as no collateral is exchanged (e.g. the sum of MTA and TH is not exceeded). This procedure is required to avoid overly conservative results due to high thresholds and minimum transfer amounts.

[11] In this case $NICA$ has to include differential of the independent amounts used as parameters within the calculation of variation margin amounts.

[12] Please note that a change in sign of $\hat{V}_k(t)$ will also lead to a change in sign of variation margin. In case of posted VM, segregated collateral needs to be eliminated from the calculation of $\hat{C}_{CE}(t)$. Hence, assumptions on the properties of potentially posted and received variation margin are required.

2.3.2 Potential Future Exposure

In line with BCBS (2014b), we define the potential future exposure (PFE) as the product of a multiplier (m_k) and an aggregated add-on ($AddOn_k$) for each netting set (k):

$$PFE_k(t) = m_k(t) \cdot AddOn_k(t) \tag{2.18}$$

where $m_k(t)$ is a function of $\hat{C}_{CE}(t)$, $\hat{V}_k(t)$ as well as the calculated aggregated add-on ($AddOn_k(t)$) of the respective netting set (k). In our approach, the aggregated add-on represents an analytical approximation of $EE_k(t)$ on netting set level, assuming a market value of zero and the absence of collateral. The multiplier is introduced to account for market value and collateral amounts different from zero. The regulatory SA-CCR approach reflects the benefit of excess collateral and negative market values, as only these are mitigants against potential future exposure. Please note that the multiplier as well as the aggregated add-on are a function of time (t). In the subsequent paragraphs we provide the derivation of add-ons as well as the multiplier formula.

Add-ons for unmargined netting sets

The netting set level add-on for unmargined netting sets represents an estimate of the expected exposure (EE) at time t. The assumptions of the regulatory SA-CCR presented in BCBS (2014b) are maintained in order to build a consistent and integrated framework. Hence, our approach is based on the following main assumptions:

- **AO1:** The market value of all transactions is zero ($V_i(t) = 0$). This assumption implies that the market value of the netting set is zero ($V_k(t) = 0$).
- **AO2:** There is neither received nor posted collateral ($C_{CE}(t) = 0$).
- **AO3:** There are no cash-flows within the time period (t_0, t)
- **AO4:** The evolution of each transaction's market value follows an arithmetic brownian motion with zero drift.

Under these assumptions, the expected exposure of a netting set at time t is defined as:[13]

$$EE_k(t) = \mathbb{E}^{Q}\left[max(V_k(t), 0)\right] = \mathbb{E}^{Q}\left[max(\sigma_k(t) \cdot \sqrt{t} \cdot Y, 0)\right] \tag{2.19}$$

[13] For detailed derivation of equation (2.19), please refer to APPENDIX 2.A.1.

with $\sigma_k(t)$ representing the annualized volatility of the netting set's market value at t. As Y is a standard normal variable, we can calculate $EE_k(t)$ analytically. Hence, the expected exposure solves for:[14]

$$EE_k(t) = \sigma_k(t) \cdot \sqrt{t} \cdot \phi(0) \tag{2.20}$$

where $\phi(0)$ is defined as the standard normal probability density: $\phi(0) = 1/\sqrt{2\pi}$. According to BCBS (2014b) and in line with the above foundations, we are able to restate this equation at trade-level in order to calculate an expected exposure at trade-level $EE_i(t)$.

$$AddOn_i(t) = EE_i(t) = \sigma_i(t) \cdot \sqrt{t} \cdot \phi(0) \tag{2.21}$$

Please note that contrary to BCBS (2014b) the volatility of the market value on trade-level ($\sigma_i(t)$) is a function of t as we estimate the volatility of each transaction's market value as a function of t. Nevertheless, we are generally able to use the same structure and aggregation methodology for the calculation of add-ons as proposed by BCBS (2014b).[15]

Add-ons for margined netting sets

The add-on for margined netting sets aims to estimate the expected increase of exposure between time of default ($\tau = t$) and the final close-out of positions ($t + MPOR$). Given assumptions **AO1**, **AO2** and **AO3** and in accordance with the argumentation of BCBS (2014b), the calculation of this amount on netting set level can be reduced to:

$$AddOn_k^{margin}(t) = EE_k^{margin}(t) = \sigma_k(t) \cdot \phi(0) \cdot \sqrt{MPOR} \tag{2.22}$$

For a netting set with only one trade, we can restate formula (2.22) and arrive at the formulation for the trade-level add-on for transactions in a margined netting set.[16]

$$AddOn_i^{margin}(t) = \sigma_i(t) \cdot \phi(0) \cdot \sqrt{MPOR} \tag{2.23}$$

[14] For the respective derivation of the analytical formulation of $EE_k(t)$, please refer to 2.A.1.
[15] The validity of this assumption under the new target measure $EE(t)$ is proven in APPENDIX 2.A.2.
[16] As shown in APPENDIX 2.A.2, the aggregation of trade-level add-ons also holds true when aggregating margined trade-level add-ons.

Structure of add-on calculations

The regulatory SA-CCR has a specific structure for the calculation of PFE add-ons. Aggregation procedures are used to calculate netting set level add-ons from trade-level add-ons. These aggregation rules are based on the central idea that add-ons can be aggregated like standard deviations. While deriving our modified approach for add-on calculation, we apply similar assumptions as used for developing the regulatory SA-CCR. We have shown that the general principles of the SA-CCR are still valid under the new target measure ($EE(t)$). Hence, we are generally able to apply the same basic structure and methodology for aggregation as provided by the regulatory SA-CCR.

The first step for calculating the aggregated add-on on netting set level is the determination of an add-on at **trade-level**. In line with the regulatory SA-CCR, the calculation of trade-level add-ons is asset class specific, but has common features for all derivative transactions. Hence, each transaction is allocated to at least one of five asset classes based on the primary risk factor.[17] For products with more than one material risk factor, the assignment to multiple asset classes is required.[18]

Following the supervisory SA-CCR, we operate with simple trade-level parameters instead of trade-level volatilities ($\sigma_i(t)$) directly. Hence, we define a transaction's add-on at time t as the product of an exposure factor (EF_i), the adjusted notional amount (d_i), its delta (δ_i) and a scaling factor with respect to time ($\sqrt{t}$ or $\sqrt{MPOR}$).[19]

$$AddOn_i(t) = EF_i \cdot d_i(t) \cdot \delta_i(t) \cdot \sqrt{t} \tag{2.24}$$

$$AddOn_i^{margin}(t) = EF_i \cdot d_i(t) \cdot \delta_i(t) \cdot \sqrt{MPOR} \tag{2.25}$$

By inserting equation (2.21) into equation (2.24) and solving for $\sigma_i(t)$, we arrive at the following approximation for the volatility of the transaction's market value at t:

$$\sigma_i(t) = \frac{EF_i}{\phi(0)} \cdot d_i(t) \cdot |\delta_i(t)| \tag{2.26}$$

[17] Within this paper, we share the number and set-up of asset classes and hedging sets proposed by the Basel Committee. Nevertheless, the general structure of our approach allows for further modification with respect to the amount and definition of asset classes, hedging and subsets.

[18] Details with respect to this requirement are still under discussion. A first discussion paper has been published by EBA (2017).

[19] The maturity factor (MF_i) used in the supervisory SA-CCR is applied as correction for trades maturing within the risk horizon of 1 year for unmargined netting sets. Under the target measure $EE(t)$, this is not necessary, as no averaging over a dedicated risk horizon is applied. Hence, a maturity factor is not required.

In accordance with BCBS (2014b), the ratio of EF_i and $\phi(0)$ can be interpreted as the annualized standard deviation of the transaction's primary risk factor ($\sigma_i^{(RF)}$):

$$\sigma_i^{(RF)} = \frac{EF_i}{\phi(0)} \tag{2.27}$$

Please note that the volatility of the risk factor ($\sigma_i^{(RF)}$) is assumed to be constant over time. Hence, the time dependence of the volatility of the transaction's market value is solely resulting from $d_i(t)$ and $\delta_i(t)$. The **exposure factor** (EF_i) can be interpreted as an approximation of the expected exposure of a netting set with one directional trade, which has the size of one unit adjusted notional at $t = 1year$.

$$EF_i = \sigma_i^{(RF)} \cdot \phi(0) \tag{2.28}$$

This relationship allows a calibration of EF_i based on the (implied) volatility of the transaction's primary risk factor ($\sigma_i^{(RF)}$). The supervisory SA-CCR provides supervisory factors (SF_i) on subclass level.[20] We introduce a more granular approach to the calibration of the exposure factor in section 2.4.

The **delta parameter** (δ_i) is a function of the direction of the trade with respect to the primary risk factor (long / short). For products with a non-linear relationship to the primary risk factor, δ_i serves as a scaling factor with respect to the moneyness of the product.[21] For plain vanilla options we use a delta formula based on the formula provided by the supervisory SA-CCR (BCBS (2014d)):

$$\delta_i(t) = \psi \cdot N\left(\omega \cdot \frac{ln(\hat{P}(t)/K) + 0.5 \cdot \left(\sigma_i^{(impl)}\right)^2 \cdot (T-t)}{\left(\sigma_i^{(impl)}\right) \cdot \sqrt{(T-t)}}\right) \tag{2.29}$$

where K represents the strike price and $\sigma_i^{(impl)}$ the (implied) volatility of the underlying of the option. T is defined as the amount of time (in years) between today and the expiry date of the option.[22] $\hat{P}(t)$ equals an estimation of the spot price of the underlying at time t.[23] If an estimation of $P(t)$ via the forward price is not possible, we assume $P(t) = P(t_0)$. The parameters ψ and ω are required to cover all combinations of bought/sold and call/put options.[24]

[20] such as rating categories within asset class credit

[21] Please note that supervisory approach is offering a specific delta formula for CDOs which uses detachment and attachment points as inputs for the calculation of the delta parameter.

[22] For options with multiple exercise dates, one might only assume the latest exercise date.

[23] Example: For FX options, we are able to estimate the forward price at time t based on the interest rate curves of the involved currencies.

[24] ψ equals (-1) where the transaction is a sold call option or a bought put option and $(+1)$ where the transaction is a bought call option or sold put option. ω equals (-1) for put and $(+1)$ for call options.

For more complex and exotic options, equation (2.29) might not be appropriate as a lot of these products are path-dependent. As $\delta_i(t)$ is defined on trade-level, our approach offers the flexibility to include the actual economic deltas for these products. Nevertheless, this requires an assumption on the development of the products delta over time, especially if those deltas are not calculated analytically. Hence, the methodology for calculation of $\delta_i(t)$ needs to be defined for each product type based on the availability and quality of the respective data.

The **adjusted notional amount** (d_i) captures the size of a transaction. For interest rate and credit derivatives, d_i is also used to recognize the duration of the instrument and thereby its sensitivity to changes in underlying risk factors. In general, the add-on under SA-CCR is proportional to the adjusted notional. The adjustments to the asset class specific formulation of d_i are presented in section 2.A.4.

Aggregation of trade-level add-ons

With respect to the aggregation of trade-level add-ons and structure of subsets and hedging sets, we basically follow the procedures and definitions of the supervisory SA-CCR BCBS (2014b):

In case of **interest rate derivatives**, all transactions (i) are allocated to a hedging subset based on their currency (c) and maturity. For each currency, three maturity buckets ($0-1y$, $1-5y$, $>5y$) are defined. Within these maturity buckets (b), trade-level add-ons of long and short positions are aggregated assuming a correlation of 100%. Hence, we arrive at the following definition for the add-on (X) for each hedging subset (X_{cb}) at time t:

$$X_{cb}(t) = \sum_{i \in \{ccy_c, MB_b\}} X_i(t) = \sum_{i \in \{ccy_c, MB_b\}} EF_i \cdot d_i(t) \cdot \delta_i(t) \cdot \sqrt{t} \tag{2.30}$$

In a next step we calculate an add-on for each hedging set (currency) based on the following equation:

$$X_c(t) = \sqrt{\sum_b (X_{cb}(t))^2 + \sum_b \sum_{b \neq d} \rho_{bd} \cdot X_{cb}(t) \cdot X_{cd}(t)} \tag{2.31}$$

where ρ_{bd} is defined as the correlation between two maturity buckets.[25] Given equation (2.31), we are able to account for offsetting effects between long and short transactions in the same currency and different maturity bucket. Based on the adjustments of trade-level parameters

[25] In line with BCBS (2014b) the correlation between maturity buckets ($0-1y$) and ($>5y$) is set to 30%, while all correlations between other maturity buckets are set to 70%. Please note, that we assume constant correlation over time in line with our methodological framework (see equation (2.42)).

($d_i(t)$, $\delta_i(t)$) over time, we model implicitly the time-variant development of the sensitivity of the transactions and the netting set to changes in the underlying risk factors. Hence, we are able to capture the offsetting effects of long and short positions in the same currency and with different maturities over time.

Within the supervisory SA-CCR framework, the regulatory PFE add-on on for IR derivatives on asset class level is defined as the sum of all hedging set add-ons:

$$X_{reg}^{(IR)}(t) = \sum_{c} X_c(t) \tag{2.32}$$

Given the formulation above, the regulatory PFE add-on for IR derivatives does not recognize diversification effects between exposure in different currencies.

The add-on calculation for **FX derivatives** does not require an allocation of transactions to hedging subsets. All transactions referencing the same currency pair are allocated to a hedging set directly. Within the resulting hedging sets, full offsetting of long and short positions is allowed. Hence, the aggregation of trade-level add-ons (X_i) to the specific hedging set add-on (X_c) for each currency pair (c) can be written as:

$$X_c(t) = \sum_{i \in ccy_c} X_i(t) = \sum_{i \in ccy_c} EF_i \cdot d_i(t) \cdot \delta_i(t) \cdot \sqrt{t} \tag{2.33}$$

For FX derivatives, the regulatory PFE add-on on asset class level ($X^{(FX)}$) is defined as the simple sum of all hedging set add-ons (X_c):

$$X_{reg}^{(FX)}(t) = \sum_{c} X_c(t) \tag{2.34}$$

Hence, the regulatory PFE add-on for FX derivatives does not take diversification effects between different currency pairs into account. The aggregated PFE add-on across asset classes is defined as the simple sum of all add-ons on asset class level. Hence, diversification effects across asset classes are also not considered during the aggregation of the PFE add-on.

The aggregation procedures for **equity, credit and commodity derivatives** involve offsetting of transactions via the application of a single-factor model. Hence, offsetting of transactions with different underlying reference entity (CR), issuer (EQ) or commodity type (COM) is considered, when calculating the PFE add-on.[26]

[26] For further details on the aggregation procedures for those asset classes, we refer to BCBS (2014b) and BCBS (2014d).

In general, the recognition of diversification effects in the add-on calculation of the supervisory SA-CCR is deemed conservative. We perceive the missing consideration of offsetting effects across currencies (IR) and currency pairs (FX) as too conservative for the purpose of CVA calculation. Our approach provides the flexibility for implementing improvements to the methodology for the aggregation of PFE add-ons. As a simple example, we introduce a modified aggregation procedure for the calculation of IR and FX add-ons on asset class level. This method is based on the new standardized approach for market risk (BCBS (2019b)), where this concept is used to aggregate bucket level results (FX=currency pairs, IR=currencies) to asset class results. Following BCBS (2019b), we define the asset class level add-on of the modified SA-CCR for asset classes FX and IR as:

$$X_{mod}^{(IR/FX)}(t) = \sqrt{\sum_{b}(X_b(t))^2 + \sum_{b}\sum_{b \neq c}\gamma_{bc}^{(a)} \cdot X_b(t) \cdot X_c(t)} \tag{2.35}$$

where $X_b(t)$ equals the add-on of the respective hedging set (b). γ_{bc} represents the correlation between two currency pairs (FX) or currencies (IR). In line with BCBS (2019b) we set $\gamma_{bc}^{(IR)} = 0.5$ and $\gamma_{bc}^{(FX)} = 0.6$. In the modified SA-CCR, equation (2.35) is used to replace equations (2.32) and (2.34).

As mentioned above, the modified SA-CCR framework offers the possibility to include more complex aggregation methodologies to model correlations between hedging sets and asset classes. The derivation of more complex modified procedures for the aggregation of add-ons would require a comprehensive discussions and analysis of different aggregation methods for each asset class. Within this paper we focus on providing detailed insights in the methodological foundations of the modified SA-CCR. Hence, we choose a simple aggregation methodology to account for diversification effects between hedging sets in the asset classes IR and FX.

2.3.3 Multiplier

When deriving the PFE multiplier, the assumptions **AO1** and **AO2** are relaxed. Hence, the market value of a netting set can be different from zero and received or posted collateral might be present. The multiplier is defined as a fraction of PFE. Thereby the PFE is corrected for the fact that market value and collateral amounts are different from zero. Based on these assumptions, the expected exposure ($EE(t)$) of an unmargined netting set (k) at a certain point in time t is defined as:

$$EE_k(t) = \mathbb{E}^Q\left[max\left((\hat{V}_k(t) + \sigma_k(t)\cdot\sqrt{t}\cdot Y) - C_{CE}(t), 0\right)\right] \tag{2.36}$$

where:

- $\hat{V}_k(t)$ is the (deterministic) market value of the netting set at time t,
- $C_{CE}(t)$ represents the value of net collateral received at t,
- $\sigma_k(t)$ is the volatility of the netting set at time t and
- Y is defined as a standard normal random variable.

Based on equation (2.36) and the aforementioned assumptions, the multiplier formula is derived analytically for unmargined netting sets. For details on assumptions and analytical calculation of the multiplier formula, please refer to APPENDIX 2.A.3. The multiplier for a netting set (k) at time t is defined as:

$$m(t) = y\cdot\Phi\left[\phi(0)y\right] + \frac{\phi\left[\phi(0)y\right]}{\phi(0)} - \frac{max\left(\hat{V}_k(t) - C_{CE}(t), 0\right)}{AddOn_k(t)} \tag{2.37}$$

where $y = \frac{\hat{V}_k(t) - C_{CE}(t)}{AddOn_k(t)}$, $\Phi(.)$ is the standard normal cumulative distribution function and $\phi(.)$ the standard normal probability density function. This differs from the model-based multiplier formula derived by BCBS (2014b). The supervisory SA-CCR applies a more conservative multiplier function to account for the possibility that future MtM values are not normally distributed. Additionally only cases are considered where $\hat{V}_k(t) - C_{CE}(t)$ is less than zero. Futhermore, a floor is introduced in order to prevent the multiplier from reaching zero. The modified SA-CCR uses the multiplier as defined in equation (2.37) without further modifications. As shown in APPENDIX 2.A.3, the same formulation applies for margined and unmargined netting sets.

2.4 Calibration

2.4.1 Background and general considerations

In its regulatory setup, the SA-CCR is calibrated under the real-world measure based on historical data. According to BCBS (2013), the regulatory SA-CCR parameters were estimated using a three step approach. In a first step, the supervisory parameters were calibrated based on market data from different markets. Volatilities and correlations were evaluated based on a stress period which, in most cases, was defined as the three-year period with the largest historically observed volatility. The BCBS applied different approaches by asset class to perform this initial calibration.[27] The second step was based on a comparison of SA-CCR exposure outcomes with results from simplified IMM models for a set of hypothetical portfolios. This comparison was carried out for small portfolios involving hypothetical trades for each asset class. The third and final step involved a benchmarking exercise based on contributions by a set of IMM banks via Quantitative Impact Studies. The model outcomes were averaged and compared with CEM and SA-CCR exposures to derive final adjustments to the regulatory parameters.

The outcome of this process is regulatory prescribed parameters (option volatilities, supervisory factors and correlations) for each asset class.[28] For some asset classes, such as interest rates or foreign exchange, the supervisory factor is defined on asset class level. Hence, there is no differentiation between more granular risk factors such as currencies or tenors. For some asset classes additional levels (subclasses) were introduced to increase the granularity and risk sensitivity of the approach.[29] The BCBS tried to limit the granularity of the regulatory approach, aiming for a total number of risk factors close to the Current Exposure Method (CEM). As the SA-CCR was not designed to cover exotic products or more complex risk factors, a certain degree of conservatism was included when developing the approach and defining model parameters (BCBS (2013)).

The supervisory parameters given by the BCBS are not appropriate when generating exposure profiles for accounting and pricing purposes. In summary, the main issues are the calibration under the real-world measure based on historical (stressed) volatilities, the lack of granularity with respect to risk factors, as well as the high degree of conservatism applied to the overall

[27] For a detailed overview of the calibration by asset class, please refer to BCBS (2013).
[28] The supervisory parameters are available in BCBS (2014d).
[29] Example: For credit derivatives, the supervisory factor is defined based on the underlying type (index or single-name) and the underlying's credit quality.

calibration approach. Hence, a re-calibration of the parameters is required in order to use the modified SA-CCR for CVA calculation.

2.4.2 Calibration of the modified SA-CCR

The calibration of the modified SA-CCR has two main objectives. First, we aim to increase the granularity of risk factors aiming for more risk sensitive exposure calculation. Second, the calibration is performed under the risk-neutral measure based on market-implied volatilities, in order to meet the expectation of market participants. Overall, the main driver of exposure is the volatility of the primary risk factor of each transaction. Hence, we focus on the calibration of the exposure factor (EF_i). We do not provide a re-calibration of the SA-CCR correlation parameters, but apply a modified methodology for the aggregation of hedging set add-ons involving additional correlation parameters (see section 2.3.2). We perform a calibration of option volatilities for the calculation of the delta parameter (see equation (1.11)). The calibration approach differs by asset class and depends on data availability for the respective risk factors. Within this section, we provide a general overview and present proposals for the calibration of the modified SA-CCR, without discussing all asset class specific details. We focus on the asset classes IR and FX, as these are relevant for the subsequent empirical analysis.

In general, the calibration of **exposure factors** is based on market-implied volatilities obtained from options. Hence, the approach offers the possibility to consider the maturity and the moneyness of a certain position. For **interest rate derivatives**, the exposure factor is calibrated based on market-implied at-the-money (ATM) swaption volatilities for the respective currency, taking the volatility term structure into account. Hence, the risk factor is defined by the combination of currency and tenor. In case of missing data we propose to use the supervisory factor (0.5%).

In line with BCBS (2013), we calibrate the exposure factor for **FX derivatives** directly from implied FX option volatilities using the relationship stated in equation (2.28). While the supervisory SA-CCR only applies one distinct supervisory factor for the whole FX asset class, we consider each currency pair a single risk factor. Hence, the volatility used to calculate the exposure factor for a specific transaction is a function of its base and reference currency. The market-implied ATM volatilities are provided based on the following term structure: 1D, 1W, 1M, 2M, 3M, 6M, 9M, 12M, 2Y, 3Y, 4Y, 5Y, 10Y. In order to calculate the relevant volatility value for a specific transaction, we use linear interpolation. If there is no valid and appropriate data on implied volatilities for a certain currency pair, we use the supervisory factor (4%).

For other asset classes (Equity, Credit, Commodity) the granularity of exposure factors can be chosen based on available data and desired complexity of implementation. For equity and credit derivatives, the calibration for each underlying can be carried out independently or via a beta-approach as described in BCBS (2013), where exposure factors for single-name positions are obtained from index volatilities and the respective beta. For commodity positions, different dimensions, such as underlying, grade and delivery location can be considered when calibrating exposure factors. In general, one could also decide to calibrate the exposure factor on broad commodity types similar to the supervisory approach. By maintaining the key building blocks of the regulatory SA-CCR, we provide a flexible framework for the calibration of the modified SA-CCR. This leads to a general trade-off between the risk sensitivity of the approach and the amount of data required for its calibration.

2.5 Empirical analysis

2.5.1 Methodology

In this section we assess the modified SA-CCR's ability to provide a risk sensitive and accurate approximation of the exposure calculated by an advanced model. As the main criterion, we compare the resulting credit valuation adjustment (CVA) from both approaches. The analysis is performed based on illustrative examples of netting sets. These are composed of hypothetical interest rate (IR) and foreign exchange (FX) transactions. The netting sets involve products with different maturity, underlying, direction and moneyness. Our empirical study comprises the following main steps:

SA-CCR (re)calibration: First, we calibrate the relevant exposure factors (EF_i) of the modified SA-CCR to market-implied volatilities as described in section 2.4.[30] We construct a series of netting sets for the empirical analysis including margined and unmargined netting sets as well as netting sets with non-linear products. An overview of the transactions and netting sets is provided in 2.B.2. The calibration of the relevant parameters is carried out based on a market data set as of 28th September 2018.

[30] Please note that other parameters, such as correlations, are not re-calibrated. Hence, we use the supervisory parameters given by the regulatory SA-CCR.

Exposure calculation: Second, we calculate $EE_k(t)$ based on the modified SA-CCR and an advanced benchmark model (BMM). For interest rates, we apply a model based on the approach of Trolle and Schwartz (2009). The modeling of FX rates is based on a Heston model (Heston (1993)). The parameters of the Trolle-Schwartz model are calibrated to European swaptions. The calibration process is based on swaption prices derived from quoted implied volatilities for a series of European swaptions with different underlying and option tenor as well as different strike price. Parameters of the Heston model are calibrated based on implied log-normal volatilities quoted for FX option strategies. Quotes are available for different maturity and moneyness. These strategies are transformed to European FX options and their corresponding prices, which are used as input for the calibration process.[31]

From our point of view, the applied models provide state-of-the-art stochastic processes for the evolution of all major risk factors. The models also cover the dependency of the implied volatility to moneyness (volatility skew). Accuracy is a critical issue when calculating exposures for CVA pricing and accounting purposes. Hence, we decided to use an advanced, state-of-the-art benchmark model involving a comprehensive set of risk factors to assess the accuracy of the modified SA-CCR, rather than a more simplistic exposure model.

CVA calculation: Based on the exposure profiles obtained from the modified SA-CCR and the benchmark model, we calculate the CVA using the following discretized formula under the assumption of a flat credit spread curve (40 basis points) and a recovery rate of 40% (Gregory (2010)).

$$CVA \approx (1-R) \cdot \sum_{i=1}^{M} DF(t_i) \cdot EE(t_i) \cdot PD(t_{i-1}, t_i) \tag{2.38}$$

Analysis: Finally, we compare the exposure profiles and the CVA results for the benchmark model with the modified SA-CCR. The targets of this comparison are the assessment of the applicability for the purpose of CVA calculation, the validation of results as well as the identification of shortcomings and areas of future work. Hence, we use illustrative examples for different products and situations.

[31] For additional details on the calibration of the applied benchmark model, please refer to APPENDIX 2.B.1.

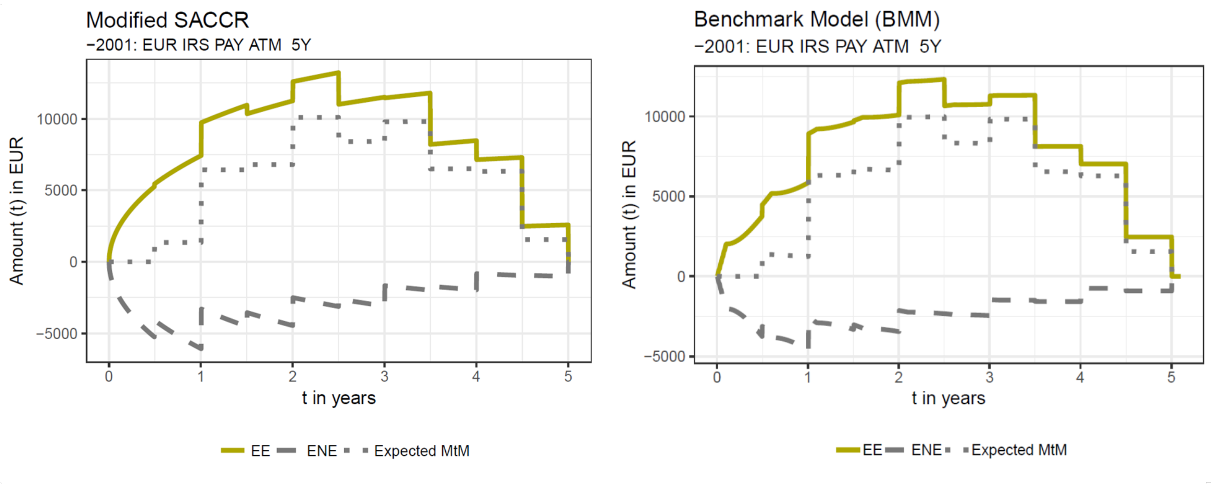


Figure 2.1: Exposure profiles of an EUR 5Y ATM IR (payer) swap. *Note* The figure shows different types of exposure profiles for an EUR 5Y ATM IR (payer) swap, calculated with the modified SA-CCR approach (left panel) and a benchmark model (right panel).

2.5.2 Results

Results for interest rate swaps

The expected exposure profile of an IR swap is driven by two contrary effects. The uncertainty with respect to future payments leads to an increasing exposure (dispersion), whereas the roll-off of swap payments has a decreasing effect on the exposure (amortization) over time. The combination of these effects results in the typical humped shape of the exposure profile (Gregory (2010)). Within the modified SA-CCR, the dispersion effect is reflected by scaling the volatility with the square root of time ($\sqrt{t}$) when calculating the PFE add-on.[32] As shown above, the modified SA-CCR approach is also able to recognize the amortization effect due to the consideration of cash flows in replacement costs and the adjustment of the duration parameter, which is used to calculate the PFE add-on. Our approach allows the generation of expected (positive) exposure (EE), expected negative exposure (ENE) and expected market value profiles ($EMtM$).[33] These different types of exposure profiles are presented in figure 2.1 for an EUR 5Y at-the money (ATM) IR payer swap, with differing payment frequencies (fix = annual, float = semi-annual).

The shape of the exposure profiles modeled with the modified SA-CCR is in line with the results of the advanced benchmark model (BMM). By calibrating the modified SA-CCR approach to market-implied volatilities, we are able to reflect a similar level of risk. Additionally, figure 2.1

[32] see equation (2.21)

[33] Please note that the following presentation of results is focused on the calculation of CVA and thereby the generation of expected (positive) exposure (EE).

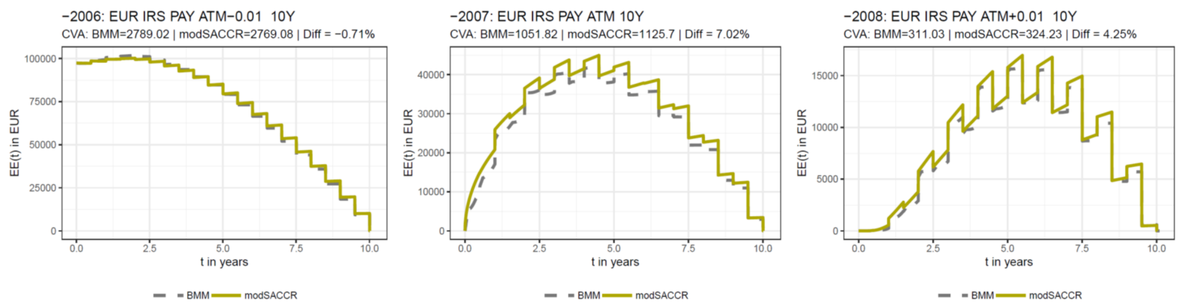


Figure 2.2: Expected exposure of EUR 10Y IR payer swaps. *Note* The figure shows the expected exposure profile ($EE(t)$) of IR (payer) swaps with a maturity of 10 years, which are in-the-money (left), at-the-money (middle) and out-of-the-money (right). The results of the modified SA-CCR (solid line) are close to the results retrieved from the benchmark model (dashed line). This is also indicated by the corresponding CVA results shown in the header of each graph.

shows that the modified SA-CCR is able to recognize the asymmetry between payer and receiver swaps, as well as the differing payment frequencies of the fixed and floating leg. By using an adjusted SA-CCR multiplier formula, we consider the effect of the current market value on future exposure uncertainty. Figure 2.2 provides expected exposure profiles and corresponding CVA results of EUR 10Y ATM IR payer swaps with different moneyness.

The results show that our approach mirrors the exposure dynamics produced by an advanced model. Based on the generated exposure profiles, we are able to calculate Credit Valuation Adjustments (CVA) by using equation (2.38) and the aforementioned assumptions with respect to the credit spread curve and recovery rate (see section 2.5.1).

In order to assess the quality of the approximation by the modified SA-CCR, we apply the approach to an illustrative set of IR swaps with different underlying currencies, tenors and moneyness. We use 14 hypothetical IR swap transactions. Each transaction is put into a separate uncollateralised and unmargined netting set. All hypothetical transactions are fix-to-floating IR swaps with identical payment frequencies (fixed = anual, float = semi-annual). The test data comprises swaps with different underlying currencies (EUR, USD), tenors (5Y, 7Y, 10Y), direction (pay, receive) and moneyness (ATM+0.01, ATM, ATM-0.01).[34] Table 2.1 provides an overview of the CVA results for the different netting sets.[35]

In general, we observe that the expected market value estimated by the modified SA-CCR is very close to the results from the benchmark model for all analysed netting sets. This is a result of the cash flow based calculation of replacement costs. The excellent approximation of the

[34] For additional details on the hypothetical transactions, please refer to 2.B.2.

[35] A graphical representation of the expected exposure profiles for these netting sets are provided in APPENDIX 2.B.3.

Table 2.1: CVA results for hypothetical IR swaps

Id	Description	CVA (SA-CCR)	CVA (BMM)
-2000	EUR IRS PAY ATM-0.01 5Y	703.82	702.95
-2001	EUR IRS PAY ATM 5Y	167.68	155
-2002	EUR IRS PAY ATM+0.01 5Y	11.55	11.21
-2003	EUR IRS REC ATM-0.01 5Y	0.52	0.12
-2004	EUR IRS REC ATM 5Y	55.22	43.57
-2005	EUR IRS REC ATM+0.01 5Y	490.03	491.33
-2006	EUR IRS PAY ATM-0.01 10Y	2,769.08	2,789.02
-2007	EUR IRS PAY ATM 10Y	1,125.70	1,051.82
-2008	EUR IRS PAY ATM+0.01 10Y	324.23	311.03
-2009	EUR IRS PAY ATM-0.01 7Y	1,370.16	1,363.46
-2010	EUR IRS PAY ATM 7Y	410.23	400.67
-2011	EUR IRS PAY ATM+0.01 7Y	57.68	69.2
-2012	USD IRS PAY ATM-0.01 5Y	470.25	457.78
-2013	USD IRS PAY ATM 5Y	120.95	96.75
-2014	USD IRS PAY ATM+0.01 5Y	20.20	13.2

Notes: This table provides an overview of the CVA results for different hypothetical IR swaps calculated with (1) the modified SA-CCR approach and (2) the benchmark model (BMM). We provide a graphical representation of the expected exposure profile for each netting set in APPENDIX 2.B.3.

expected market value leads to very good results for in-the-money (ITM) transactions, as their future exposure is mainly driven by movements in the expected MtM (see table 2.1). Given the granular calibration approach of the modified SA-CCR, we are able to consider different levels of risk with respect to the underlying currency and tenor of the transactions. The calibration to at-the-money (ATM) market-implied volatilities leads to an appropriate approximation for most netting sets, even if the respective transaction is not at-the-money (see figure 2.2 and table 2.1).

The modified SA-CCR is able to provide an adequate and risk sensitive approximation of the expected exposure profile for IR swaps. We are able to use these as input for the calculation of CVA and receive very good results on single-transaction level. The approach is capable of reflecting all major exposure dynamics and estimating a reasonable level of risk. Furthermore, the modified SA-CCR is sensitive to different underlying currencies, tenors and the moneyness of transactions.

Results for FX forwards

The risk of FX forwards is dominated by the notional exchange at maturity (T) of the transaction. FX forwards do not involve any further cash flows between (t) and (T). Hence, the exposure is driven by the uncertainty regarding future payments at maturity. The exposure monotonically increases with time and is also driven by small effects from interest rate risk (Gregory (2010)). We are able to recognize these characteristics in the modified SA-CCR via a cash flow based calculation of replacement costs as well as the scaling of the add-on by $\sqrt{t}$ (see equation 2.24).

Figure 2.3 provides the expected exposure profile of 3Y EUR/USD FX forwards with different moneyness.

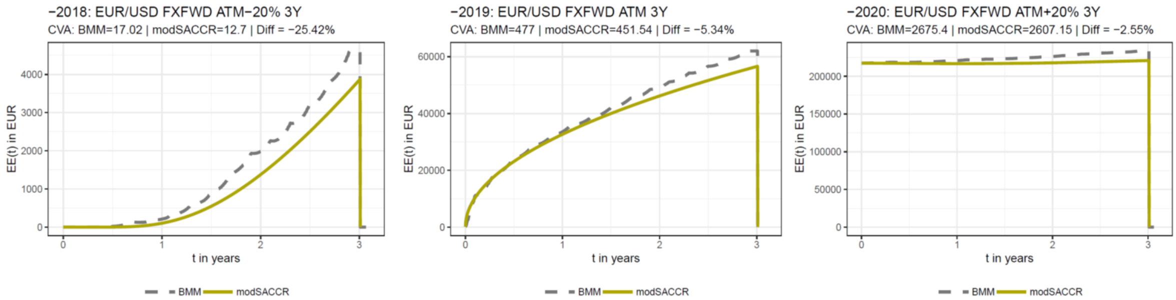


Figure 2.3: Expected exposure profile for FX forwards with different moneyness. *Note* The figure shows the expected exposure profile and CVA results for three EUR/USD FX forwards with maturity of 3 years and different moneyness. The results of the modified SA-CCR (solid line) are very close to the results from the benchmark model (dashed line) for ITM and ATM transactions.

The results presented in figure 2.3 show that the modified SA-CCR reflects the influence of current market values on the uncertainty of future exposures. Furthermore, the shape of the generated exposure profiles are in line with the outcome of the benchmark model. Based on the resulting expected exposure profiles, we are able to calculate the CVA for different products and netting sets. For an illustrative validation of the modified SA-CCR, we have composed 18 single-transaction nettings sets. Furthermore, we assume the absence of collateral and margin agreements. The netting sets are comprised of FX forwards with different maturity, underlying currency pair, as well as moneyness. Table 2.2 provides an overview of the CVA results for these hypothetical netting sets.[36]

In general, the expected exposure of FX forwards is more sensitive to calibration compared to IR swaps. As discussed, the risk of FX forwards is concentrated on payments at maturity. Hence, the uncertainty of future exposure increases over time. There are no exposure-reducing effects on the expected exposure from roll-off of payments during the life-time of the transaction. Hence, ATM and OTM transactions are very sensitive to uncertainty in calibration. In contrast, ITM transactions are less sensitive to calibration, as the expected exposure is mainly driven by the current and expected market value. Hence, the modified SA-CCR generates reasonable expected exposure profiles for ITM transactions, which are very close to the outcome of the benchmark model. The expected exposure of OTM transactions is mainly driven by the PFE add-on. Hence, results are more sensitive to calibration of the add-on. As we calibrate the modified SA-CCR to ATM volatilities, we are not able to correctly reflect the level of risk estimated by the benchmark model for all currencies and tenors.

[36] The visualization of $EE(t)$ profiles is available in APPENDIX (2.B.4).

Table 2.2: CVA results for hypothetical FX forwards

Id	Description	CVA (SA-CCR)	CVA (BMM)
-2015	EUR/USD FXFWD ATM-20% 1Y	0.05	0.08
-2016	EUR/USD FXFWD ATM 1Y	81.41	80.19
-2017	EUR/USD FXFWD ATM+20% 1Y	827.75	827.96
-2018	EUR/USD FXFWD ATM-20% 3Y	12.70	17.02
-2019	EUR/USD FXFWD ATM 3Y	451.54	477.00
-2020	EUR/USD FXFWD ATM+20% 3Y	2,607.15	2,675.40
-2021	EUR/JPY FXFWD ATM-20% 3Y	41.98	27.41
-2022	EUR/JPY FXFWD ATM 3Y	548.06	509.16
-2023	EUR/JPY FXFWD ATM+20% 3Y	2,431.10	2,538.73
-2024	EUR/GBP FXFWD ATM-20% 3Y	22.71	45.89
-2025	EUR/GBP FXFWD ATM 3Y	487.13	462.00
-2026	EUR/GBP FXFWD ATM+20% 3Y	2,489.46	2,495.68
-2027	EUR/CHF FXFWD ATM-20% 1Y	0.01	0.45
-2028	EUR/CHF FXFWD ATM 1Y	67.95	64.50
-2029	EUR/CHF FXFWD ATM+20% 1Y	800.52	799.50
-2030	USD/GBP FXFWD ATM-20% 3Y	41.07	83.08
-2031	USD/GBP FXFWD ATM 3Y	559.32	595.09
-2032	USD/GBP FXFWD ATM+20% 3Y	2,486.28	2,369.26

Notes: This table provides an overview of the CVA results for different hypothetical FX forwards calculated with (1) the modified SA-CCR approach and (2) the benchmark model (BMM). We provide a graphical representation of the expected exposure profile for each netting set in APPENDIX 2.B.4.

Nevertheless, the modified SA-CCR considers the exposure mitigating effect of current negative market values and is able to reproduce the shape of the exposure profile. Hence, an advanced calibration approach considering the dependency of market-implied volatility on moneyness should lead to more appropriate results. For ATM FX forwards, the modified SA-CCR provides reasonable exposure profiles and an appropriate approximation of the CVA estimated by the benchmark model.

Taking the results for the illustrative examples into account, we are justified in saying that our approach is able to provide an expected exposure profile, which could serve as a reasonable basis for the approximation of the CVA. The approach incorporates the specific risks of different currency pairs and tenors. Furthermore, the risk mitigating effect of current negative market values is considered via the PFE multiplier. Nevertheless, we have uncovered calibration issues with respect to OTM transactions. These could be solved by taking the dependency of the market-implied volatility to the moneyness of the transaction into account.

Results for multi-transaction netting sets

In addition to the analysis of single transactions, we apply our approach to illustrative multi-transaction netting sets. The supervisory SA-CCR provides a holistic framework for the aggregation of trade-level add-ons across hedging (sub)sets and asset classes. The supervisory methodology is considered to be conservative, as it proscribes institutions to recognize diversification effects between risk factors which are considered to be correlated in some cases (e.g. interest rate risk in different currencies).[37] As stated in section 2.3.2, we apply an adjusted aggregation procedure for the aggregation of hedging set add-ons to results on asset class level using regulatory correlation parameters introduced in BCBS (2019b). Hence, our approach is still expected to produce conservative results for netting sets with transactions in different hedging sets.

The hypothetical netting sets are designed to cover the aggregation across different elements of the SA-CCR (e.g hedging sets, asset classes, sub-hedging sets). In total, we examine nine hypothetical netting sets.[38] Table 2.3 provides an overview of the CVA results and the composition of these netting sets.

Table 2.3: CVA results for hypothetical combined netting sets

Id	Trade Id	Description	CVA (SA-CCR)	CVA (BMM)
-2033	-2001	EUR IRS PAY ATM 5Y	0.00	0.00
	-2004	EUR IRS REC ATM 5Y		
-2034	-2001	EUR IRS PAY ATM 5Y	1,509.91	1,616.82
	-2007	EUR IRS PAY ATM 10Y		
	-2010	EUR IRS PAY ATM 7Y		
-2035	-2013	USD IRS PAY ATM 5Y	236.43	113.26
	-2001	EUR IRS PAY ATM 5Y		
-2036	-2012	USD IRS PAY ATM-0.01 5Y	362.86	290.24
	-2013	USD IRS PAY ATM 5Y		
	-2014	USD IRS PAY ATM+0.01 5Y		
-2037	-2019	EUR/USD FXFWD ATM 3Y	895.19	730.41
	-2022	EUR/JPY FXFWD ATM 3Y		
-2038	-2019	EUR/USD FXFWD ATM 3Y	462.59	415.14
	-2031	USD/GBP FXFWD ATM 3Y		
-2039	-2028	EUR/CHF FXFWD ATM 1Y	609.61	591.44
	-2031	USD/GBP FXFWD ATM 3Y		
-2040	-2019	EUR/USD FXFWD ATM 3Y	532.95	535.98
	-2016	EUR/USD FXFWD ATM 1Y		
-2041	-2019	EUR/USD FXFWD ATM 3Y	501.31	491.82
	-2004	EUR IRS REC ATM 5Y		

Notes: This table provides an overview of the CVA results for different hypothetical netting sets with multiple transactions, calculated with (1) the modified SA-CCR approach and (2) the benchmark model (BMM). We provide a graphical representation of the expected exposure profile for each netting set in APPENDIX 2.B.5.

Nettings sets 2033, 2034 and 2036 are composed of IR derivatives referencing the same underlying currency, while netting set 2035 comprises two IR swaps with different currencies. For netting set 2033, the expected exposure profile is considered to be zero, as the involved transactions have the same currency and tenor, but the opposite direction. We are allowed to fully offset the expected exposure of these transactions resulting in an expected exposure and CVA of zero. The results for netting set 2034 show that the aggregation procedures of the

[37] For more detailed information on the aggregation mechanisms, please refer to BCBS (2014d) and BCBS (2014b).
[38] Details on the composition of theses netting sets are provided in the Appendix (2.B.2).

supervisory SA-CCR produce an adequate result for multiple transactions in the same currency, but with different maturity. The exposure of netting set 2035 is over-estimated. This results from the obviously insufficient recognition of diversification effects between different currencies via the modified aggregation formula (see equation (2.35)).

When calculating the FX add-on, the modified SA-CCR allows netting between different currency pairs. Nevertheless, the expected exposures for netting sets 2037, 2038 and 2039 are slightly overestimated. The result for netting set 2040 shows, that our approach is suitable for netting sets composed of transactions with the same underlying currency, but different tenors. Netting set 2041 provides an illustrative example for a portfolio with transactions in multiple asset classes. The SA-CCR does not allow the recognition of netting effects across asset classes when calculating the add-on. Nevertheless, the CVA results for netting set 2041 are very close to the outcome of the benchmark model.

The results for multi-transaction netting sets affirm that the aggregation methodology is an important element of the modified SA-CCR. Especially, the aggregation across different currencies without recognition of offsetting effects is a critical issue with respect to the accuracy of the modified SA-CCR. Hence, there needs to be additional work on the incorporation of diversification effects between hedging sets. The aggregation across asset classes does not seem to be a major issue for netting sets composed of IR and FX transactions. Notwithstanding, it should be mentioned that the illustrative examples only cover a limited number of compositions and situations. To fully assess the applicability of the modified SA-CCR, it is necessary to extend the empirical analysis to additional examples based on hypothetical and real-world data.

Results for collateralized portfolios

In order to assess the treatment of collateral in the modified SA-CCR, we calculate CVA for 6 selected netting sets based on the assumption of a perfect CSA.[39] The respective netting sets have already been presented in the previous sections on the results of IR swaps and FX forwards. The assumption of a perfect CSA leads to replacement costs of zero and a PFE multiplier of 1 as the amount of collateral is equal to the market value of the netting set at all future points in time t. Hence, the expected exposure under the modified SA-CCR is solely driven by the PFE add-on, which is calculated for a risk horizon equal to the margin period of risk.

[39] The existence of a perfect CSA implies: $TH = 0$, $MTA = 0$, absence of rounding, no inital margin and instantaneous collateral exchange.

Table 2.4: CVA results for collateralized portfolios (perfect CSA)

Id	Description	CVA (SA-CCR)	CVA (BMM)
-2000	EUR IRS PAY ATM-0.01 5Y	16.33	10.47
-2001	EUR IRS PAY ATM 5Y	16.33	10.64
-2002	EUR IRS PAY ATM+0.01 5Y	16.33	10.83
-2015	EUR/USD FXFWD ATM-20% 1Y	24.17	24.02
-2016	EUR/USD FXFWD ATM 1Y	24.17	23.98
-2017	EUR/USD FXFWD ATM+20% 1Y	24.17	23.94

Notes: This table provides an overview of the CVA results for different hypothetical netting sets under the assumption of a perfect CSA calculated with (1) the modified SA-CCR approach and (2) the benchmark model (BMM). We provide a graphical representation of the expected exposure profile for each netting set in APPENDIX 2.B.6.

Table 2.4 provides an overview of the CVA results for the selected netting sets under the assumption of a perfect CSA. The results show that the modified SA-CCR is capable of capturing the effects from margining on the expected exposure and the resulting CVA. For FX forwards (2015, 2016, 2017) we receive nearly identical results compared to the benchmark model. The results of the modified SA-CCR do not differ by moneyness as the market value is fully collateralized and therefore not impacting the replacement costs or the PFE multiplier. With respect to IR swaps, we recognize a difference in the CVA results. Nevertheless, we are able to mirror the key exposure dynamics produced by the BMM (see section 2.B.6).

In addition to the analysis of netting sets under the assumption of a perfect CSA, we consider the same netting sets given an imperfect CSA ($TH = 5.000, MTA = 1.000$). The following calculations are preformed under the assumption of instantaneous fulfilment of collateral calls. Table 2.5 provides an overview of the CVA results for the selected netting sets under the assumption of an imperfect CSA.

The results for FX forwards are basically in line with the results from the BMM. Nevertheless, there are differences with respect to the level of expected exposure. These differences are mainly driven by the fact that the expected market value of the modified SA-CCR differs slightly from the BMM. The expected future market value of the modified SA-CCR does not consider all risk factors, such as cross-currency basis spreads. Hence, the market value differs from the BMM. The difference in the expected market value leads to a different collateral path. These effects result in a different consideration of collateral and CVA values. The results for IR swaps are also well in line with the CVA values from the BMM. The modified SA-CCR is able to capture the combination of collateral parameters and the decreasing duration of an IR swap over time. In summary, we are certainly justified in saying that the modified SA-CCR is able to capture the main exposure dynamics of margined netting sets. Nevertheless, there is room for additional analysis, validation and potential improvements of the collateral treatment.

Table 2.5: CVA results for collateralized portfolios (CSA: TH=5.000, MTA=1.000)

Id	Description	CVA (SA-CCR)	CVA (BMM)
-2000	EUR IRS PAY ATM-0.01 5Y	99.2	108.52
-2001	EUR IRS PAY ATM 5Y	79.71	63.98
-2002	EUR IRS PAY ATM+0.01 5Y	0.76	0.58
-2015	EUR/USD FXFWD ATM-20% 1Y	15.43	15.97
-2016	EUR/USD FXFWD ATM 1Y	24.51	30.85
-2017	EUR/USD FXFWD ATM+20% 1Y	35.55	43.96

Notes: This table provides an overview of the CVA results for different hypothetical netting sets under the assumption of an imperfect CSA ($TH = 5.000, MTA = 1.000$) calculated with (1) the modified SA-CCR approach and (2) the benchmark model (BMM). We provide a graphical representation of the expected exposure profile for each netting set in APPENDIX 2.B.7.

Results for non-linear products

In addition to the presented examples, we analyse the treatment of non-linear products. For this purpose, we use 6 different FX options in individual netting sets. These FX options are all referencing the EUR/USD exchange rate and have a maturity of 1 year, but differ with respect to option type (call/put) and moneyness (ITM/OTM/ATM).[40] When calculating the exposure under the modified SA-CCR, we assume a constant market value ($\hat{V}_i(t) = V_i(t_0)$) as well as a constant exchange rate ($\hat{P}_i(t) = P_i(t_0)$). This is a simplification in line with the methodology presented in section 2.3.2. The option volatility used for the calculation of the option price at ($t = t_0$) and $\delta_i(t)$ is calibrated based on FX option strategies as described in section 2.4.

Table 2.6 provides an overview of the CVA results for the different options. The results for ATM and ITM options are reasonable and well in line with the results from the BMM (2126, 2128, 2130, 2133). With respect to OTM options, the modified SA-CCR does not meet the results produced by the BMM. This is consistent with the outcome for FX forwards and results form the fact that we are calibrating the modified SA-CCR to ATM options. Hence, we are not able to cover the FX volatility skew in the modified SA-CCR. This leads to an underestimation of exposure for OTM FX options. An improvement of the calibration to capture the volatility surface as a whole should significantly increase the accuracy for OTM options.

2.5.3 Implications for the supervisory SA-CCR

When deriving our modified approach, we are keeping key building blocks and elements of the supervisory SA-CCR. The EaD calculated under the superviory SA-CCR will serve as input

[40] The ITM call is defined setting the price/strike ratio to 1.2, while the OTM call is constructed with a price/strike ratio of 0.8. The put options are defined vice versa.

Table 2.6: CVA results for FX options

Id	Description	CVA (SA-CCR)	CVA (BMM)
-2126	EUR/USD FX Call Option ATM (1Y)	127.92	124.34
-2128	EUR/USD FX Put Option ATM (1Y)	127.26	115.86
-2130	EUR/USD FX Call Option ITM (1Y)	801.10	807.98
-2131	EUR/USD FX Put Option OTM (1Y)	8.97	0.12
-2132	EUR/USD FX Call Option OTM (1Y)	0.12	0.91
-2133	EUR/USD FX Put Option ITM (1Y)	801.10	803.44

Notes: This table provides an overview of the CVA results for different hypothetical FX options calculated with (1) the modified SA-CCR approach and (2) the benchmark model (BMM). We provide a graphical representation of the expected exposure profile for each netting set in APPENDIX 2.B.8.

for the calculation of other regulatory measures, such as leverage ratio (BCBS (2014a)), large exposure framework (BCBS (2014c)) and the CVA risk capital charge (BCBS (2017)). Hence, all banks will have to implement the SA-CCR irrespectively of the application of an internal model. Furthermore, there is ongoing discussion to limit the benefit from the application of internal models by introducing a capital floor based on the outcome of regulatory standardized approaches (BCBS (2017)). Given the broad application of the SA-CCR and its subsequent impact on various regulatory measures, we are certainly justified in saying that the SA-CCR is of significant importance for the regulatory framework as a whole. Hence, systemic misjudgement of risk by the supervisory SA-CCR is not an isolated issue, but will propagate through other regulatory measures.

Our results imply that the risk sensitivity of the supervisory SA-CCR can be significantly improved by adjustments to its methodological framework and its calibration. First, the multiplier formula of the supervisory SA-CCR is very conservative as it involves a floor and uses a more conservative function than analytically implied. This leads to an insufficient recognition of risk-mitigating effects from over-collateralization (especially with respect to initial margin). Abolishing the supervisory floor and adjusting the multiplier formula would significantly improve risk sensitivity for a multitude of portfolios, especially in light of new margin requirements for OTC derivatives. Second, the lack of granularity of the supervisory factor and volatility leads to a lack of risk sensitivity. IR and FX transactions with different currency are all treated with a single risk-weight. Our results show, that the calibration of the approach for each currency has strong benefits with respect to its risk sensitivity. From our point of view the calibration of the supervisory approach should be more granular to account for the characteristics of different risk factors. Third, our results reveal that the aggregation procedures are a critical issue with respect to the accuracy of results for multi-transaction netting sets. Especially netting sets with IR or FX transactions referencing different currencies are sensitive to an insufficient consideration of those effects. Nevertheless, the supervisory SA-CCR framework is flexible enough to include more complex aggregation approaches. Hence, we strongly recommend to review the supervisory aggregation procedures and consider other forms of aggregation for IR and FX add-ons.

2.6 Conclusion

The calculation of CVA based on expected exposure profiles is crucial for financial institutions involved in derivative trading activities. As a lot of market participants are not capable of implementing and maintaining an advanced model for the estimation of the expected exposure, there is a practical need for simple, semi-analytical approaches. Existing semi-analytical approaches are often designed to reflect the properties of a specific asset class and fail to provide a holistic approach across asset classes. Furthermore, they are usually not capable of capturing effects from collateralization and margining.

This paper proposes a modified supervisory approach based on the Standardized Approach for Measuring Counterparty Credit Risk Exposures (SA-CCR). We derive necessary adjustments to the supervisory methodology and calibration, obtaining an approach which is applicable for the calculation of CVA for accounting and pricing purposes. Main adjustments to the supervisory SA-CCR are the change of target measure and risk-neutral calibration to market-implied volatilities. While deriving our approach, we maintain key building blocks of the supervisory approach.

Our results indicate that the modified SA-CCR is able to capture main exposure dynamics on single-transaction level for the most important asset classes (FX and IR). The risk-neutral calibration results in a reasonable level of exposure. Based on the resulting expected exposure profiles, we provide an approximation of CVA, which is close to the results produced by an advanced model. By analysing multi-transaction netting sets, we reveal that in some cases appropriate aggregation mechanisms are essential to provide a reasonable approximation of the expected exposure profile. Nevertheless, the structure of the SA-CCR allows the inclusion of additional adjustments to the aggregation of results for hedging sets and asset classes. Additional analysis regarding the application of the modified SA-CCR to further asset classes and more complex products is subject for future work.

The modified SA-CCR offers a holistic and consistent framework for the calculation of exposure profiles. All asset classes and product types are modeled based on common methodological foundations. The consistent treatment and aggregation of exposure from different product types and asset classes allows the recognition of risk-mitigating effects, such as collateralization and margining, on portfolio level. Our approach is capable of estimating the impact from margin parameters and collateral on the exposure profile. Given the high amount of flexibility with respect to the treatment of products on transaction level and the aggregation of add-ons, it is possible to add additional complexity, where needed. By keeping the key building blocks and structure of the supervisory approach, institutions have the possibility to leverage on the implementation of the supervisory SA-CCR, when applying the modified approach. Based on the empirical results presented in this paper and the high importance of accuracy in CVA calculations, we do not consider the modified SA-CCR as replacement for existing advanced models. Nevertheless, our approach is suitable for the calculation of exposure profiles for products not covered by advanced methods and offers an alternative approach for institutions not capable of maintaining own advanced models. Taking everything into consideration, we believe that our approach is of significant practical relevance and offers improvements to current industry practices with respect to the calculation of CVA outside advanced models.

2.A APPENDIX | Analytical derivation of SA-CCR model foundations

2.A.1 Expected Exposure

Based on the assumptions introduced by BCBS (2014b), a transaction's market value follows an arithmetic Brownian motion. Hence, the market value (V) of a transcation (i) at a specific future point in time (t) is generally defined as:

$$V_i(t) = V_i(t_0) + \mu_i \cdot dt + \sigma_i(t) \cdot \sqrt{t} \cdot X_i \tag{2.39}$$

where X_i is a standard normal random variable ($X_i \sim N(0,1)$). μ_i equals the drift and $\sigma_i(t)$ the time-dependent volatility of the transaction's market value. The SA-CCR add-on is calculated under the assumption of zero drift ($\mu_i = 0$), an initial market value of 0 ($V_i(t_0) = 0$) and absence of collateral. Furthermore, it is assumed that there are no cash flows between t_0 and t. Based on these assumptions, equation (2.39) reduces to:

$$V_i(t) = 1_{\{M_i \geq t\}} \cdot \sigma_i(t) \cdot \sqrt{t} \cdot X_i \tag{2.40}$$

where $1_{\{\cdot\}}$ represents an indicator variable, which recognizes if the transaction has matured. σ_i represents the volatility of the transaction's market value. Hence, for $M_i \geq t$ the market value of the transaction at time t is also normally distributed with $V_i(t) \sim N(0, \sigma_i(t)^2 \cdot t)$. Based on equation (2.40) and the aforementioned assumptions, the market value of a netting set, representing a group of legally nettable transactions, is defined as the sum of the single market values of all transactions (i) being an element of netting set k:

$$V_k(t) = \sum_{i \in k} V_i(t) = \sum_{i \in k} 1_{\{M_i \geq t\}} \cdot \sigma_i(t) \cdot \sqrt{t} \cdot X_i \tag{2.41}$$

As the sum of joint normally distributed random variables is again normally distributed, the market value of the netting set at time t is normally distributed. This results from the assumption that X_i is a standard normal random variable. Hence, under the assumption of fixed correlations between the market value of a netting set's transactions, the variance of the

$V_k(t)$ can be calculated the following way:[41]

$$\begin{aligned}
Var(V_k(t)) &= Var\left(\sum_{i\in k} V_i(t)\right) \\
&= Var\left(\sum_{i\in k} \sigma_i(t)\cdot\sqrt{t}\cdot X_i\cdot 1_{\{M_i\geq t\}}\right) \\
&= t\cdot\left(\sum_{i,j} \sigma_i(t)\cdot\sigma_j(t)\cdot COV(X_i,X_j)\cdot 1_{\{M_i\geq t\}}\cdot 1_{\{M_j\geq t\}}\right) \\
&= t\cdot\left(\sum_{i,j} \sigma_i(t)\cdot\sigma_j(t)\cdot\rho_{ij}\cdot 1_{\{M_i\geq t\}}\cdot 1_{\{M_j\geq t\}}\right) \\
&=: t\cdot(\sigma_k(t))^2
\end{aligned} \tag{2.42}$$

Based on this result, $V_k(t)$ is normally distributed with $N \sim (0, t\cdot(\sigma_k(t))^2)$. In case of a known correlation ρ_{ij} between two normal random variables X_i and X_j, equation 2.41 can be restated as follows:

$$V_k(t) = \sigma_k(t)\cdot\sqrt{t}\cdot Y \tag{2.43}$$

where $Y \sim N(0,1)$ and $\sigma_k(t)$ represents the volatility of the netting set's market value:

$$\sigma_k(t) = \sqrt{\sum_{i,j} \rho_{ij}\cdot\sigma_i(t)\cdot\sigma_j(t)\cdot 1_{\{M_i\geq t\}}\cdot 1_{\{M_j\geq t\}}} \tag{2.44}$$

Please note that the netting set's volatility $\sigma_k(t)$ is itself a function of t. This dependence results from the changing composition of the netting set, as transactions mature over time as well as changes in the volatility of the transactions' market values. The target measure ($EE_k(t)$) is formally defined as the expected positive value of the netting set's market value. Hence, we are able to calculate the Expected Exposure of a netting set by:

$$EE_k(t) = \mathbb{E}^{Q}\left[max(V_k(t),0)\right] = \mathbb{E}^{Q}\left[max(\sigma_k(t)\cdot\sqrt{t}\cdot Y,0)\right] \tag{2.45}$$

[41] The assumption of fixed correlations implies that correlations between the market value of transactions in the netting set are not a function of time.

As Y is a standard normal variable, we are able to calculate the expected value of Y and hence $EE(t)$ analytically:

$$\begin{aligned}
AddOn_k = EE(t) &= \mathbb{E}^Q\left[max(V_k(t),0)\right] = \mathbb{E}^Q\left[max(\sigma_k(t)\cdot\sqrt{t}\cdot Y,0)\right] \\
&= \sigma_k(t)\cdot\sqrt{t}\cdot\mathbb{E}^Q\left[max(Y,0)\right] \\
&= \sigma_k(t)\cdot\sqrt{t}\cdot\int_0^{+\infty}\frac{1}{\sqrt{2\pi}}\cdot\exp\left[\frac{-y^2}{2}\right]ydy \\
&= \sigma_k(t)\cdot\sqrt{t}\cdot\frac{1}{\sqrt{2\pi}}\cdot\left[-e^{\frac{-y^2}{2}}dy\right]_0^{+\infty} \\
&= \sigma_k(t)\cdot\sqrt{t}\cdot\frac{1}{\sqrt{2\pi}}\cdot\left[-(0-1)\right] \\
&= \sigma_k(t)\cdot\sqrt{t}\cdot\frac{1}{\sqrt{2\pi}} \\
&= \sigma_k(t)\cdot\sqrt{t}\cdot\phi(0)
\end{aligned}\tag{2.46}$$

where $\phi(0)$ is defined as the standard normal probability density: $\phi(0) = 1/\sqrt{2\pi}$.

2.A.2 Aggregating trade level add-ons

The regulatory and modified SA-CCR are aiming for calculation of aggregated add-ons instead of dealing with trade-level volatilities directly. Hence, equation (2.46) needs to be restated the following way:

$$EE_k(t) = AddOn_k(t) = \sqrt{\sum_{i,j}\rho_{ij}\cdot AddOn_i(t)\cdot AddOn_j(t)}\tag{2.47}$$

where $AddOn_i(t)$ represents the expected exposure of a netting set with one trade (i) at t:

$$AddOn_i(t) = EE_i(t) = \sigma_i(t)\cdot\sqrt{t}\cdot\phi(0)\tag{2.48}$$

By inserting equation (2.48) into equation (2.47), we are able to show that trade level add-ons can be aggregated as if they were standard deviations:

$$\begin{aligned}
AddOn_k(t) &= \sqrt{\sum_{i,j} \rho_{ij} \cdot AddOn_i(t) \cdot AddOn_j(t)} \\
&= \sqrt{\sum_{i,j} \rho_{ij} \cdot \sigma_i(t) \cdot \sqrt{t} \cdot \phi(0) \cdot \sigma_j(t) \cdot \sqrt{t} \cdot \phi(0)} \\
&= \sqrt{\sum_{i,j} \rho_{ij} \cdot \sigma_i(t) \cdot \sigma_j(t) \cdot t \cdot (\phi(0))^2} \\
&= \phi(0) \cdot \sqrt{t} \cdot \sqrt{\sum_{i,j} \rho_{ij} \cdot \sigma_i(t) \cdot \sigma_j(t)} \\
&= \phi(0) \cdot \sqrt{t} \cdot \sigma_k(t) \qquad (2.49)
\end{aligned}$$

Equation (2.49) is equal to the definition of the expected exposure at netting set level $EE_k(t)$ in equation (2.46). Hence, trade-level add-ons can be aggregated at each point in time (t) as if they were standard deviations. This is the central foundation with respect to the aggregation procedures formulated by BCBS (2014b). As shown above, this foundation is still valid after switching to the new target measure ($EE_k(t)$) for the modified SA-CCR.

2.A.3 PFE multiplier formulation

Multiplier formula for unmargined netting sets

As discussed in section 2.3.3, the expected exposure ($EE(t)$) of an unmargined netting set (k) at time t under the presence of collateral and a market value different from zero is defined as follows:

$$EE_k(t) = \mathbb{E}^{\mathbb{Q}}\left[max\left(\hat{V}_k(t) + \sigma_k(t) \cdot \sqrt{t} \cdot Y - C_{CE}(t), 0\right)\right] \qquad (2.50)$$

where:

- $\hat{V}_k(t)$ is the (deterministic) market value of the netting set at time t,
- $C_{CE}(t)$ equals the cash-equivalent collateral value at time t as defined by equation, (2.10)
- $\sigma_k(t)$ is the volatility of the netting set at time t,
- Y is defined as a standard normal random variable.

For the derivation of the multiplier formula, the following definitions are applied:

$$X := \hat{V}_k(t) + \sigma_k(t) \cdot \sqrt{t} \cdot Y - C_{CE}(t) \tag{2.51}$$

$$\mathbb{E}[X] = \hat{V}_k(t) - C_{CE}(t) =: x_0 \tag{2.52}$$

$$Var[X] = (\sigma_k(t))^2 \cdot t =: \sigma^2 \tag{2.53}$$

These definitions imply that X is normally distributed with $X \sim N\left(x_0, \sigma^2\right)$. Hence, the Expected Exposure ($EE_k(t)$) can be calculated analytically by solving:

$$EE_k(t) = \mathbb{E}^Q\left[max(X,0)\right] \tag{2.54}$$

Applying the probability density function of a normally distributed random variable leads to the following formulation for $EE_k(t)$:

$$EE_k(t) = \int_0^\infty x \cdot \frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}} \cdot \exp\left(-\frac{(x - x_0)^2}{2\sigma^2}\right) dx \tag{2.55}$$

In order to solve equation (2.55), the following substitutions have to be applied: $y = \frac{x - x_0}{\sigma}$ and $\sigma \cdot dy = dx$.

$$EE_k(t) = \frac{1}{\sqrt{2\pi}} \int_{\frac{-x_0}{\sigma}}^\infty (x_0 + \sigma \cdot y) \cdot \exp\left(-\frac{y^2}{2}\right) dy \tag{2.56}$$

Equation (2.56) can be solved analytically:

$$\begin{aligned}
EE_k(t) &= \frac{1}{\sqrt{2\pi}} \int_{\frac{-x_0}{\sigma}}^\infty (x_0 + \sigma \cdot y) \cdot \exp\left(-\frac{y^2}{2}\right) dy \\
&= x_0 \cdot \int_{\frac{-x_0}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) dy + \sigma \cdot \int_{\frac{-x_0}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}} \cdot y \cdot \exp\left(-\frac{y^2}{2}\right) dy \\
&= x_0 \cdot [\Phi(y)]_{\frac{-x_0}{\sigma}}^\infty + \sigma \cdot \left[-\frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{y^2}{2}\right)\right]_{\frac{-x_0}{\sigma}}^\infty \\
&= x_0 \cdot \left[1 - \Phi\left(-\frac{x_0}{\sigma}\right)\right] + \sigma \cdot \left[-0 + \frac{1}{\sqrt{2\pi}} \cdot \exp\left(\frac{-x_0^2}{2\sigma^2}\right)\right] \\
&= x_0 \cdot \Phi\left(\frac{x_0}{\sigma}\right) + \sigma \cdot \phi\left(\frac{x_0}{\sigma}\right)
\end{aligned} \tag{2.57}$$

After replacing x_0 and σ with the values defined in equations (2.52) and(2.53) we arrive at:

$$EE_k(t) = \left[\hat{V}_k(t) - C_{CE}(t)\right] \cdot \Phi\left(\frac{\hat{V}_k(t) - C_{CE}(t)}{\sigma_k(t) \cdot \sqrt{t}}\right) + \sigma_k(t) \cdot \sqrt{t} \cdot \phi\left(\frac{\hat{V}_k(t) - C_{CE}(t)}{\sigma_k(t) \cdot \sqrt{t}}\right) \tag{2.58}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and $\phi(\cdot)$ the standard normal probability density function. This equation is equivalent to the result for calculating the EE(t) for unmargined netting sets according to equation (41) in BCBS (2014b). By assuming $\left[\hat{V}_k(t) - C_{CE}(t)\right] = 0$, we receive $EE(t)$ equal to the formulation of equation (2.20):

$$AddOn_k(t) = EE_k(t) = \sigma_k(t) \cdot \sqrt{t} \cdot \phi(0) \tag{2.59}$$

As we want to express the multiplier in terms of $AddOn_k(t)$ rather than netting set volatilities, we solve equation (2.59) for $\sigma_k(t)$. This results in:

$$\sigma_k(t) = \frac{AddOn_k(t)}{\sqrt{t} \cdot \phi(0)} \tag{2.60}$$

Inserting equation (2.60) into equation (2.58) leads to:

$$\begin{aligned} EE_k(t) = {} & \left[\hat{V}_k(t) - C_{CE}(t)\right] \cdot \Phi\left(\phi(0) \cdot \frac{\hat{V}_k(t) - C_{CE}(t)}{AddOn_k(t)}\right) \\ & + \frac{AddOn_k(t)}{\phi(0)} \cdot \phi\left(\phi(0) \cdot \frac{\hat{V}_k(t) - C_{CE}(t)}{AddOn_k(t)}\right) \end{aligned} \tag{2.61}$$

Based on equation (2.62), we are able to derive the multiplier formula in accordance with BCBS (2014b) by isolating the PFE portion of equation (2.62). Therefore, we need to subtract the replacement costs from $EE_k(t)$:

$$\begin{aligned} PFE_k(t) = {} & \left[\hat{V}_k(t) - C_{CE}(t)\right] \cdot \Phi\left(\phi(0) \cdot \frac{\hat{V}_k(t) - C_{CE}(t)}{AddOn_k(t)}\right) \\ & + \frac{AddOn_k(t)}{\phi(0)} \cdot \phi\left(\phi(0) \cdot \frac{\hat{V}_k(t) - C_{CE}(t)}{AddOn_k(t)}\right) - RC_k(t) \end{aligned} \tag{2.62}$$

Based on equation (2.18), the multiplier of a netting set at time t ($m_k(t)$) is defined as the ratio of potential future exposure and $AddOn_k(t)$:

$$m_k(t) = \frac{PFE_k(t)}{AddOn_k(t)} \tag{2.63}$$

Hence, we arrive at the following formula for the multiplier:

$$\begin{aligned} m_k(t) = {} & \frac{\left[\hat{V}_k(t) - C_{CE}(t)\right]}{AddOn_k(t)} \cdot \Phi\left(\phi(0) \cdot \frac{\hat{V}_k(t) - C_{CE}(t)}{AddOn_k(t)}\right) \\ & + \frac{1}{\phi(0)} \cdot \phi\left(\phi(0) \cdot \frac{\hat{V}_k(t) - C_{CE}(t)}{AddOn_k(t)}\right) - \frac{max\left(\hat{V}_k(t) - C_{CE}(t), 0\right)}{AddOn_k(t)} \end{aligned} \tag{2.64}$$

By substituting $\frac{\hat{V}_k(t)-C_{CE}(t)}{AddOn_k(t)} = y$ we arrive at the following final formulation for the multiplier:

$$m_k(t) = y \cdot \Phi\left[\phi(0)y\right] + \frac{\phi\left[\phi(0)y\right]}{\phi(0)} - \frac{max\left(\hat{V}_k(t) - C_{CE}(t), 0\right)}{AddOn_k(t)} \tag{2.65}$$

Multiplier formula for margined netting sets

Based on assumptions presented in section 2.3.3, the expected exposure of a margined netting set is defined as:

$$EE_k(t) = \mathbb{E}^Q\left[max\left(\hat{V}_k(t) + \sigma_k(t) \cdot \sqrt{MPOR} \cdot Y - \hat{C}_{CE}(t), 0\right)\right] \tag{2.66}$$

where:

- $\hat{V}_k(t)$ is the (deterministic) market value of the netting set at time t,
- $\hat{C}_{CE}(t)$ represents the (deterministic) cash-equivalent collateral value at t as defined by equation (2.17),
- $\sigma_k(t)$ is the volatility of the netting set at time t,
- Y is defined as a standard normal random variable.

Based on equation (2.66) the following definitions are applied:

$$X := \hat{V}_k(t) + \sigma_k(t) \cdot \sqrt{MPOR} \cdot Y - \hat{C}_{CE}(t) \tag{2.67}$$

$$\mathbb{E}[X] = \hat{V}_k(t) - \hat{C}_{CE}(t) =: x_0 \tag{2.68}$$

$$Var[X] = (\sigma_k(t))^2 \cdot \left(\sqrt{MPOR}\right)^2 =: \sigma^2 \tag{2.69}$$

These definitions imply that X is normally distributed with $X \sim N\left(x_0, \sigma^2\right)$. Hence, the Expected Exposure ($EE_k(t)$) can be calculated analytically by solving:

$$EE_k(t) = \mathbb{E}^Q\left[max(X, 0)\right] \tag{2.70}$$

Based on this formulation, we are able to solve the equation analytically by applying the same steps as presented in section 2.A.3. Hence, we arrive at the following formulation for the multiplier:

$$m_k(t) = \frac{\left[\hat{V}_k(t) - \hat{C}_{CE}(t)\right]}{AddOn_k^{margin}(t)} \cdot \Phi\left(\phi(0) \cdot \frac{\hat{V}_k(t) - \hat{C}_{CE}(t)}{AddOn_k^{margin}(t)}\right)$$
$$+ \frac{1}{\phi(0)} \cdot \phi\left(\phi(0) \cdot \frac{\hat{V}_k(t) - \hat{C}_{CE}(t)}{AddOn_k^{margin}(t)}\right) - \frac{max\left(\hat{V}_k(t) - \hat{C}_{CE}(t), 0\right)}{AddOn_k^{margin}(t)} \tag{2.71}$$

By substituting $\frac{\hat{V}_k(t) - \hat{C}_{CE}(t)}{AddOn_k^{margin}(t)} = y$ we arrive at the following formula for the multiplier:

$$m_k(t) = y \cdot \Phi\left[\phi(0)y\right] + \frac{\phi\left[\phi(0)y\right]}{\phi(0)} - \frac{max\left(\hat{V}_k(t) - \hat{C}_{CE}(t), 0\right)}{AddOn_k^{margin}(t)} \tag{2.72}$$

2.A.4 Adjusted notional calculation by asset class

In general, we follow the asset class specific calculation procedures for the determination of the adjusted notional amount ($d_i(t)$) introduced by BCBS (2014b). Hence, the following paragraphs do not provide a detailed discussion of the asset class specific features of the SA-CCR, but concentrate on elements that have been modified when developing our approach.

For the derivation of the adjusted notional for **interest rate (IR) swaps**, we allow cash flows during the life-time of **interest rate derivatives**. The main purpose of the add-on calculation for IR transactions within the modified SA-CCR is to capture the diffusion and the amortization effect. BCBS (2014b) derives the calculation of the adjusted notional for interest rate derivatives based on the following pricing formula for a fixed-to-floating swap:

$$V_i^{swap}(t) = \left[SR_i(t) - FR_i\right] \cdot \int_{max(S_i,t)}^{E_i} N_i(\tau) \cdot DF(t,\tau) d\tau \tag{2.73}$$

where $SR_i(t)$ equals the swap rate at time t and FR_i is the fixed rate of the swap. $DF(t,\tau)$ represents the discount factor from time τ to time t. E_i represents the time until the end date of the transaction at time t while S_i equals the time until the start date of the transaction at time t (in years). The volatility of the swap rate describes the diffusion effect of the exposure and is captured by the exposure factor (EF_i) in equation (2.25). Thus the adjusted notional parameter is meant to provide an approximation for the integral at time t. A deviation from the supervisory approach is introduced. Instead of calculating the value under the integral at $t = t_0$

or the start date of the swap respectively, we estimate the value at time (t^*):

$$d_i^{(IR)}(t) = \bar{N}_i \cdot D_i(t) = \bar{N}_i \int_{t^*}^{E_i} DF(t,\tau)d\tau \tag{2.74}$$

where t^* is defined as the maximum of t and the start date of the interest rate swap (S_i). $\bar{N}_i$ is the average of the swap notional between t and the end date of the transaction (E_i). $D_i(t)$ represents the approximation of the integral under the assumption of a flat interest rate curve. $D_i(t)$ is a measure for the duration of the swap and solves for:

$$D_i(t) = \int_{t^*}^{E_i} \exp(-r\tau)d\tau = \frac{\exp(-r \cdot t^*) - \exp(-r \cdot E_i)}{r} \tag{2.75}$$

With increasing t, $D_i(t)$ decreases. This behaviour captures the amortization effect of interest rates swaps. In case of a cash flow based calculation of the expected future market value for replacement costs (see equation (2.4)) the duration parameter ($D_i(t)$) is replaced by the actual (absolute) duration of the swap position.

In accordance with BCBS (2014b), equation (2.75) is also applied to **credit derivatives**. Hence, the adjusted notional amount for credit and interest rate derivatives is a function of time and defined as:

$$d_i^{(IR/CR)}(t) = \bar{N}_i \cdot D_i(t) = \bar{N}_i \cdot \frac{\exp(-r \cdot t^*) - \exp(-r \cdot E_i)}{r} \tag{2.76}$$

For **FX derivatives** the adjusted notional amount is defined as the notional of the foreign currency leg denominated in the reporting / domestic currency (D):

$$d_i^{(FX)}(t) = \bar{N}_i^{(A_i)} \cdot \hat{P}_{\frac{A_i}{D}}(t) \tag{2.77}$$

where A_i represents the foreign currency of the transaction and D the reporting currency. $P_{\frac{A_i}{D}}$ equals the price of one unit foreign currency in domestic currency. As $P_{\frac{A_i}{D}}$ is not known for future points in time, an estimation of $P_{\frac{A_i}{D}}(t)$ is required. For FX products, we use the respective forward exchange rate for determination of the adjusted notional amount at time t:

$$\hat{P}_{\frac{A_i}{D}}(t) = F_{\frac{A_i}{D}}(t_0, t) = P_{\frac{A_i}{D}}(t_0) \cdot e^{(r_D - r_{A_i}) \cdot t} \tag{2.78}$$

If both legs of the FX product are denominated in foreign currency (A_i and B_i), d_i is calculated as follows:

$$d_i^{(FX)}(t) = \left(\bar{N}_i^{(A_i)} \cdot \hat{P}_{\frac{A_i}{D}}(t) + \bar{N}_i^{(B_i)} \cdot \hat{P}_{\frac{B_i}{D}}(t)\right) \cdot 0.5 \tag{2.79}$$

In line with BCBS (2014d) the adjusted notional amount for **equity and commodity** derivatives is defined as product of the number of units referenced by the transaction (η_i) and the price of one unit (P_i). In accordance with our approach for FX derivatives, we use the respective forward price for future dates as estimation for the price of one unit at time t.

$$d_i^{(EQ/FX)}(t) = \eta_i \cdot \hat{P}_i(t) \tag{2.80}$$

If there are deterministic changes in the notional of a transaction over time (e.g. amortizing interest rate swap), the future notional amount at t is known. Our approach offers the flexibility to include a product specific notional profile ($N_i(t)$) into the calculation of the adjusted notional.

2.B APPENDIX | Empirical analysis

2.B.1 Calibration of the benchmark model (BMM)

In order to benchmark the results of the modified SA-CCR, we calculate exposure profiles based on an advanced exposure model. The model involves stochastic processes for the simulation of the development of risk factors over time. The parameters of these stochastic processes have to be calibrated. In case of generation of exposure profiles for CVA calculation, the calibration is performed under the risk-neutral framework based on observable market prices for specific financial instruments. The aim of the calibration process is to find the set of model parameters (Ω) that minimizes the deviation between the prices of calibration instruments (n) calculated by the model (P_n^{model}) and their market price (P_n^{market}):

$$r^2(\Omega) = \sum_{n=1}^{N} \left(P_n^{market} - P_n^{model}(\Omega) \right)^2 \tag{2.81}$$

Hence, the main inputs of the calibration process are financial instruments with observable market prices as well as their properties (such as maturity, strike, underlying). Within this section, we provide an overview of the instruments and data used for the calibration of the benchmark model. However, we do not provide a detailed information on the models and the corresponding pricing functions.[42]

Trolle-Schwartz Model: For the simulation of interest rates, we apply a model based on Trolle and Schwartz (2009). The model simulates the evolution of instantaneous forward rates with stochastic volatility. We use an extended form of the model considering a multi-curve setting under the assumptions of deterministic tenor-basis spreads. The calibration of the model parameters for each currency is based on European swaptions. Available swaption data has the following main dimensions and values:

- Option Tenor: 1M, 3M, 6M, 9M, 1Y, 2Y, 5Y, 10Y, 15Y, 20Y, 30Y
- Swap Tenor: 1Y, 2Y, 5Y, 10Y, 15Y, 20Y, 30Y
- Strike (ATM ± bp): 12.5, 25, 50, 75, 100, 150, 200, 300

[42] Please refer to Heston (1993) and Trolle and Schwartz (2009) for additional information.

The prices of these swaptions are not directly observable in the market, as swaptions are quoted in terms of implied volatility. Hence, the observable implied volatilities have to be transformed into swaption prices. Depending on the volatility definition (log-normal/normal) we use a Black or Bachelier pricing formula to arrive at the swaption prices. These prices are used to calibrate the parameters of the Trolle-Schwartz model. Based on the available input data, we are able to recognize dependence of the volatility on the strike, underlying and option tenor.

Heston Model: The exchange rate is simulated via a stochastic model based on Heston (1993)). The exchange rate equals the price of one unit foreign currency, expressed in domestic currency. The parameters for modeling the interest rate process of the foreign and domestic currency are given by the calibration of the Trolle-Schwartz model. The other parameters of the model are calibrated based on European options on exchange rates. There are no quotes for FX options directly available in the market. Quotes are only directly observable for FX option strategies in the form of log-normal volatilities as function of delta. Available FX option strategy data for each exchange rate has the following main dimensions and values:

- Strategy type: Risk Reversal, Straddle, Strangle
- Maturity: 1D, 1W, 2W, 3W, 1M, 2M, 3M, 6M, 9M, 1Y, 2Y, 3Y, 4Y, 5Y, 7Y, 10Y
- Delta: 0.5, 0.25, 0.1

For each maturity, each of these strategies is converted into European call and put options. This results in a set of five options for each maturity (3 call and 2 put options). The respective implied volatility of each option is transferred into a market price. This set of options with the corresponding market prices is used to calibrate the parameters of the Heston model. The resulting model is able to capture the dependency of the implied volatility to the moneyness and maturity.

2.B.2 Input: Transactions and netting sets

Table 2.B.1: Hypothetical netting sets for empirical analysis in section 2.5

Id	tradeId	class	description	notional(EUR)	mtm(EUR)
-2000	-2000	IR	EUR IRS PAY ATM-0.01 5Y	1,000,000	49,937.01
-2001	-2001	IR	EUR IRS PAY ATM 5Y	1,000,000	-0.01
-2002	-2002	IR	EUR IRS PAY ATM+0.01 5Y	1,000,000	-49,937.03
-2003	-2003	IR	EUR IRS REC ATM-0.01 5Y	1,000,000	-49,937.01
-2004	-2004	IR	EUR IRS REC ATM 5Y	1,000,000	0.01
-2005	-2005	IR	EUR IRS REC ATM+0.01 5Y	1,000,000	49,937.03
-2006	-2006	IR	EUR IRS PAY ATM-0.01 10Y	1,000,000	97,419.36
-2007	-2007	IR	EUR IRS PAY ATM10Y	1,000,000	-0.02
-2008	-2008	IR	EUR IRS PAY ATM+0.01 10Y	1,000,000	-97,419.39
-2009	-2009	IR	EUR IRS PAY ATM-0.01 7Y	1,000,000	69,387.70
-2010	-2010	IR	EUR IRS PAY ATM 7Y	1,000,000	0
-2011	-2011	IR	EUR IRS PAY ATM+0.01 7Y	1,000,000	-69,387.71
-2012	-2012	IR	USD IRS PAY ATM-0.01 5Y	1,000,000	46,129.85
-2013	-2013	IR	USD IRS PAY ATM 5Y	1,000,000	0.01
-2014	-2014	IR	USD IRS PAY ATM+0.01 5Y	1,000,000	-46,129.83
-2015	-2015	FX	EUR/USD FWD ATM-20% 1Y	1,000,000	-205,592.64
-2016	-2016	FX	EUR/USD FWD ATM 1Y	1,000,000	171.95
-2017	-2017	FX	EUR/USD FWD ATM+20% 1Y	1,000,000	205,936.54
-2018	-2018	FX	EUR/USD FWD ATM-20% 3Y	1,000,000	-217,299.87
-2019	-2019	FX	EUR/USD FWD ATM 3Y	1,000,000	144.92
-2020	-2020	FX	EUR/USD FWD ATM+20% 3Y	1,000,000	217,589.71
-2021	-2021	FX	EUR/JPY FWD ATM-20% 3Y	1,000,000	-200,390.47
-2022	-2022	FX	EUR/JPY FWD ATM 3Y	1,000,000	-16.85
-2023	-2023	FX	EUR/JPY FWD ATM+20% 3Y	1,000,000	200,356.78
-2024	-2024	FX	EUR/GBP FWD ATM-20% 3Y	1,000,000	-206,765.66
-2025	-2025	FX	EUR/GBP FWD ATM 3Y	1,000,000	54.23
-2026	-2026	FX	EUR/GBP FWD ATM+20% 3Y	1,000,000	206,874.12
-2027	-2027	FX	EUR/CHF FWD ATM-20% 1Y	1,000,000	-199,214.46
-2028	-2028	FX	EUR/CHF FWD ATM 1Y	1,000,000	-22.31
-2029	-2029	FX	EUR/CHF FWD ATM+20% 1Y	1,000,000	199,169.84
-2030	-2030	FX	USD/GBP FWD ATM-20% 3Y	1,000,000	-200,050.19
-2031	-2031	FX	USD/GBP FWD ATM 3Y	1,000,000	-80.85
-2032	-2032	FX	USD/GBP FWD ATM+20% 3Y	1,000,000	199,888.48
-2033	-2001	IR	EUR IRS PAY ATM 5Y	1,000,000	-0.01
	-2004	IR	EUR IRS REC ATM 5Y	1,000,000	0.01
-2034	-2001	IR	EUR IRS PAY ATM 5Y	1,000,000	-0.01
	-2007	IR	EUR IRS PAY ATM10Y	1,000,000	-0.02
	-2010	IR	EUR IRS PAY ATM 7Y	1,000,000	0
-2035	-2013	IR	USD IRS PAY ATM 5Y	1,000,000	0.01
	-2001	IR	EUR IRS PAY ATM 5Y	1,000,000	-0.01
-2036	-2012	IR	USD IRS PAY ATM-0.01 5Y	1,000,000	46,129.85
	-2013	IR	USD IRS PAY ATM 5Y	1,000,000	0.01
	-2014	IR	USD IRS PAY ATM+0.01 5Y	1,000,000	-46,129.83
-2037	-2019	FX	EUR/USD FWD ATM 3Y	1,000,000	144.92
	-2022	FX	EUR/JPY FXFWD ATM 3Y	1,000,000	-16.85
-2038	-2019	FX	EUR/USD FWD ATM 3Y	1,000,000	144.92
	-2031	FX	USD/GBP FWD ATM 3Y	1,000,000	-80.85
-2039	-2028	FX	EUR/CHF FWD ATM 1Y	1,000,000	-22.31
	-2031	FX	USD/GBP FWD ATM 3Y	1,000,000	-80.85
-2040	-2019	FX	EUR/USD FWD ATM 3Y	1,000,000	144.92
	-2016	FX	EUR/USD FWD ATM 1Y	1,000,000	171.95
-2041	-2019	FX	EUR/USD FWD ATM 3Y	1,000,000	144.92
	-2004	IR	EUR IRS REC ATM 5Y	1,000,000	0.01
-2126	-2126	FX	EUR/USD FX Call Option ATM 1Y	1.000.000	29,993.39
-2128	-2128	FX	EUR/USD FX Put Option ATM 1Y	1.000.000	29,993.39
-2130	-2130	FX	EUR/USD FX Call Option ITM 1Y	1.000.000	200,028.82
-2131	-2131	FX	EUR/USD FX Put Option OTM 1Y	1.000.000	28.82
-2132	-2132	FX	EUR/USD FX Call Option OTM 1Y	1.000.000	28.82
-2133	-2133	FX	EUR/USD FX Put Option ITM 1Y	1.000.000	200,028.82

Notes: This table provides an overview of the hypothetical netting sets, which are used within the empirical analysis presented in section 2.5 of this paper. All IR trades are fix-to-float IR swaps.

2.B.3 Output: IR profiles

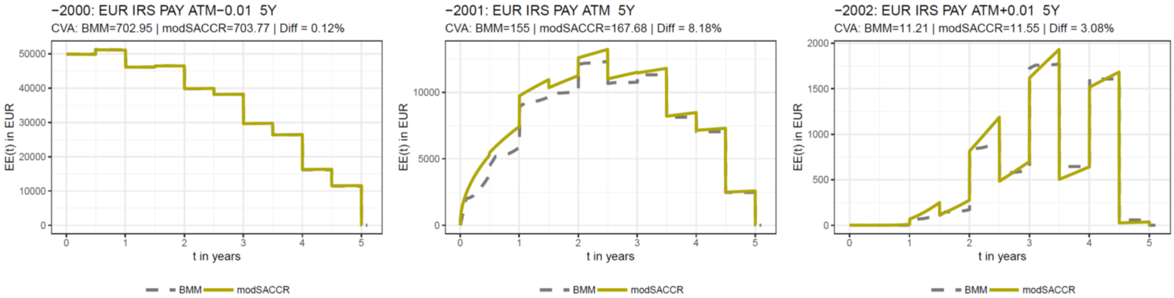


Figure 2.B.1: Expected exposure of EUR 5Y IR payer swaps. *Note* The figure shows the expected exposure profile and CVA results for EUR 5Y IR payer swaps with different moneyness calculated with the modified SA-CCR (solid line) and the benchmark model (dashed line). Please notice that in case of a payer swap, ATM-0.01 (left panel) is equal to an in-the-money position.

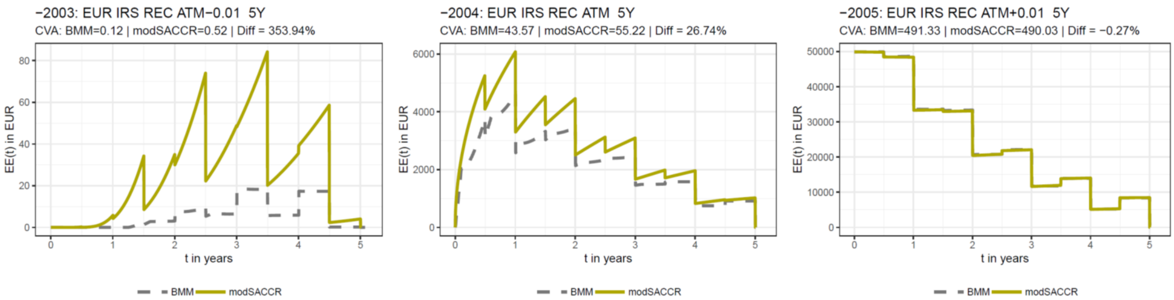


Figure 2.B.2: Expected exposure of EUR 5Y IR receiver swaps. *Note* The figure shows the expected exposure profile and CVA results for EUR 5Y IR receiver swaps with different moneyness calculated with the modified SA-CCR (solid line) and the benchmark model (dashed line). Please notice that in case of a receiver swap, ATM-0.01 (left panel) is equal to an out-the-money position.

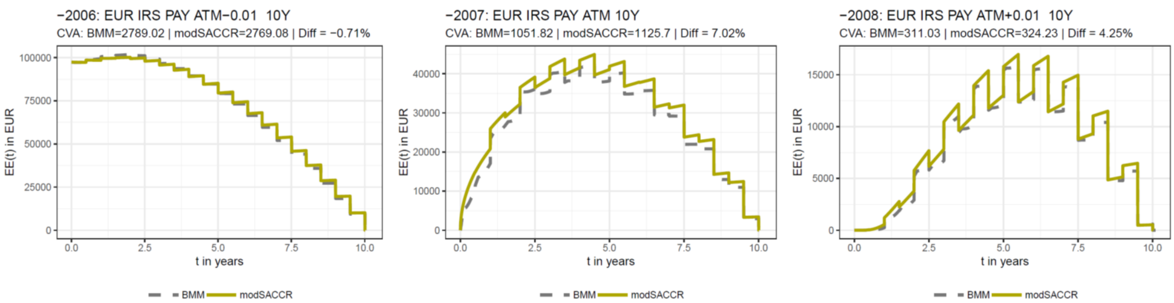


Figure 2.B.3: Expected exposure of EUR 10 IR payer swaps. *Note* The figure shows the expected exposure profile and CVA results for EUR 10Y IR payer swaps with different moneyness calculated with the modified SA-CCR (solid line) and the benchmark model (dashed line).

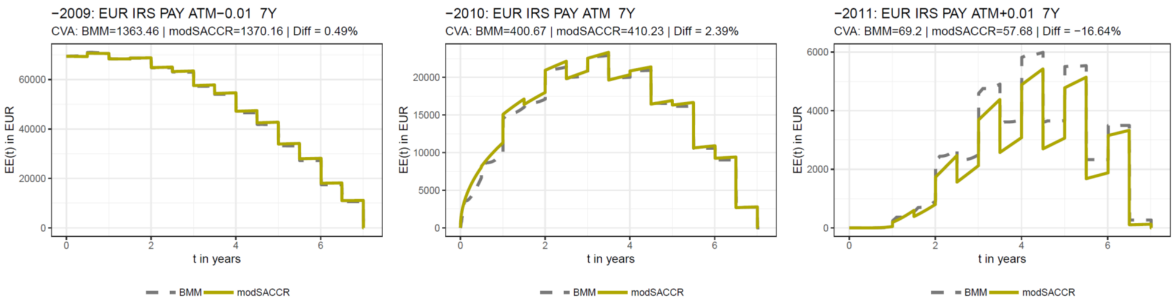


Figure 2.B.4: Expected exposure of EUR 7Y IR payer swaps. *Note* The figure shows the expected exposure profile and CVA results for EUR 7Y IR payer swaps with different moneyness calculated with the modified SA-CCR (solid line) and the benchmark model (dashed line).

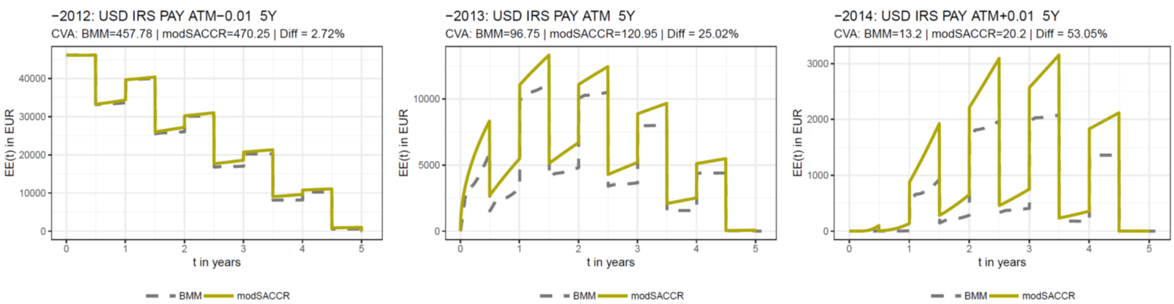


Figure 2.B.5: Expected exposure of USD 5Y IR payer swaps. *Note* The figure shows the expected exposure profile and CVA results for USD 5Y IR payer swaps with different moneyness calculated with the modified SA-CCR (solid line) and the benchmark model (dashed line).

2.B.4 Output: FX profiles

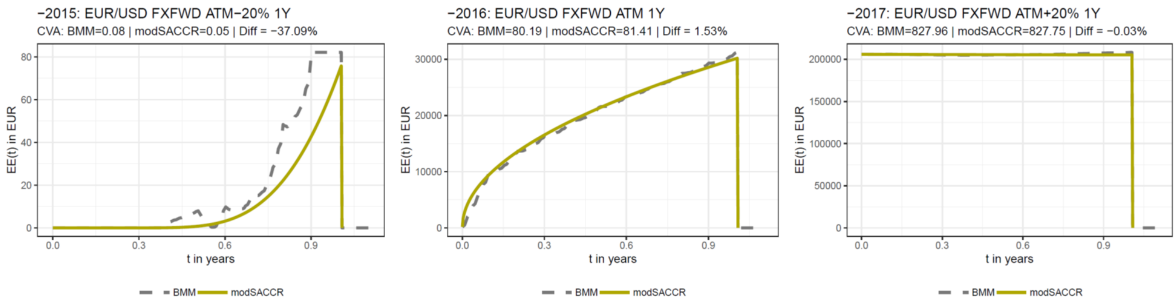


Figure 2.B.6: Expected exposure of EUR/USD 1Y FX forwards. *Note* The figure shows the expected exposure profile and CVA results of EUR/USD 1Y FX forwards with different moneyness. The solid line represents the result from the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

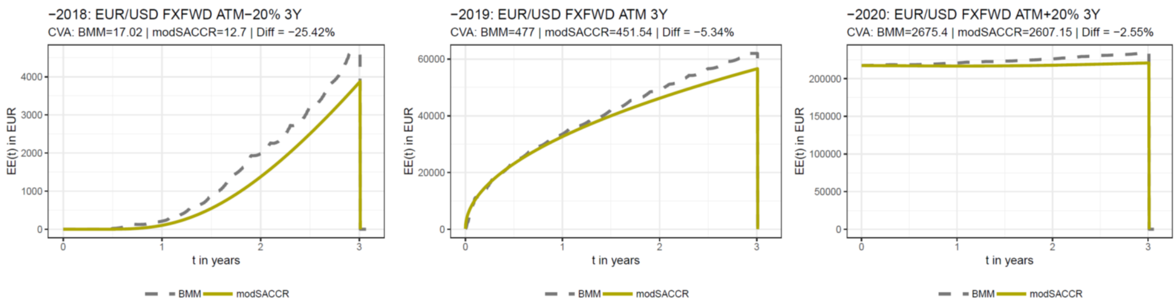


Figure 2.B.7: Expected exposure of EUR/USD 3Y FX forwards. *Note* The figure shows the expected exposure profile and CVA results of EUR/USD 3Y FX forwards with different moneyness.The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

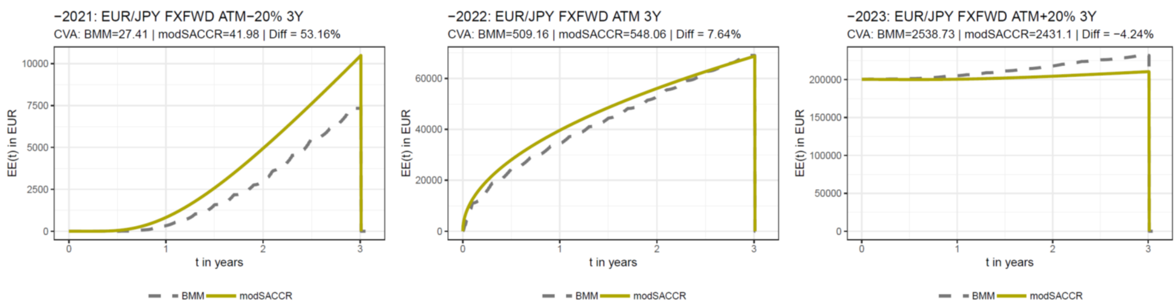


Figure 2.B.8: Expected exposure of EUR/JPY 3Y FX forwards. *Note* The figure shows the expected exposure profile and CVA results of EUR/JPY 3Y FX forwards with different moneyness. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

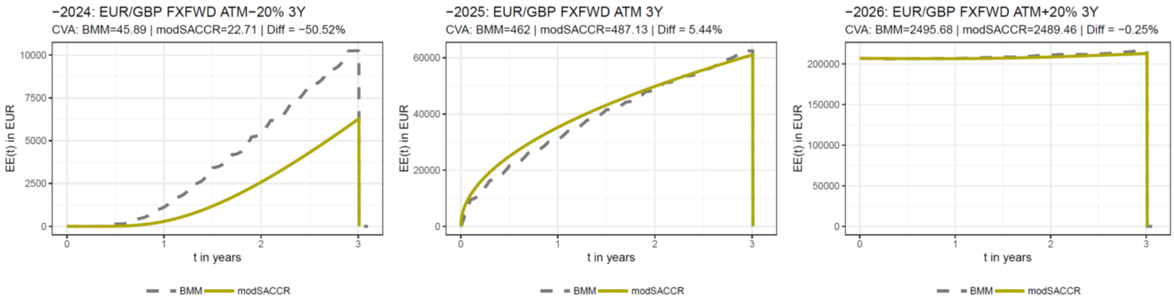


Figure 2.B.9: Expected exposure of EUR/GBP 3Y FX forwards. *Note* The figure shows the expected exposure profile and CVA results of EUR/GBP 3Y FX forwards with different moneyness. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

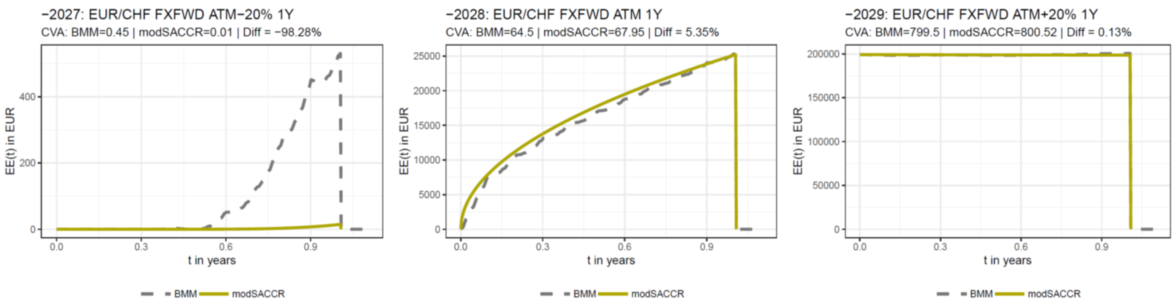


Figure 2.B.10: Expected exposure of EUR/CHF 1Y FX forwards. *Note* The figure shows the expected exposure profile and CVA results of EUR/CHF 1Y FX forwards with different moneyness. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

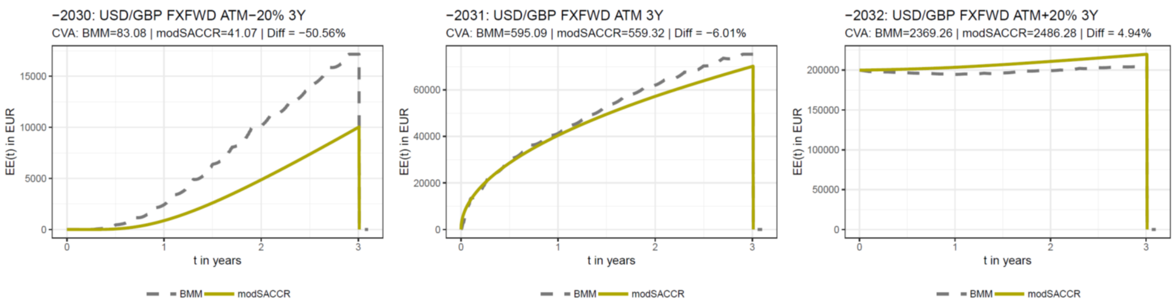


Figure 2.B.11: Expected exposure of USD/GBP 3Y FX forwards. *Note* The figure shows the expected exposure profile and CVA results of USD/GBP 3Y FX forwards with different moneyness. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

2.B.5 Output: Profiles for combined netting sets

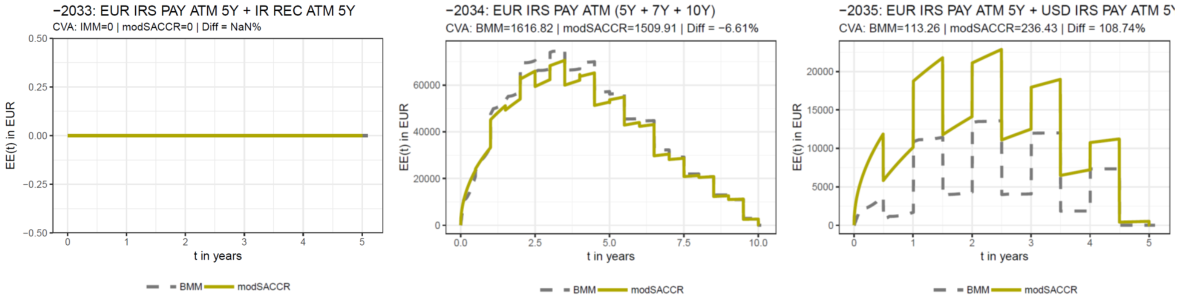


Figure 2.B.12: Expected exposure profile of IR multi-transaction netting sets. *Note* This figure provides the expected exposure profile and CVA results for the hypothetical netting sets 2033, 2034 and 2035. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

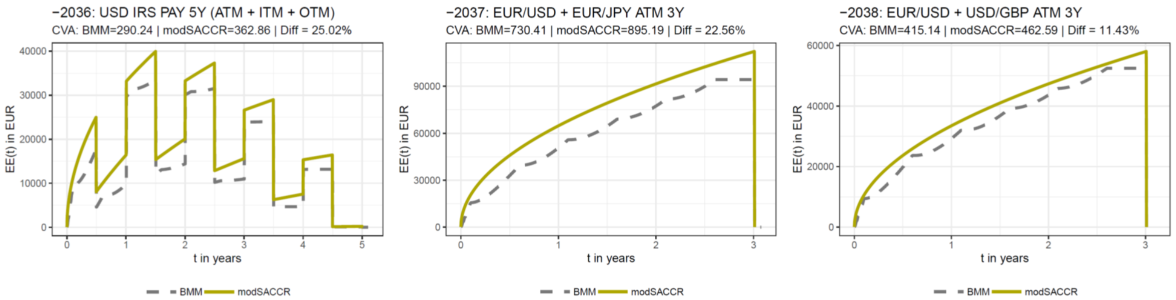


Figure 2.B.13: Expected exposure profile of multi-transaction netting sets. *Note* This figure provides the expected exposure profile and CVA results for the hypothetical netting sets 2036, 2037 and 2038. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

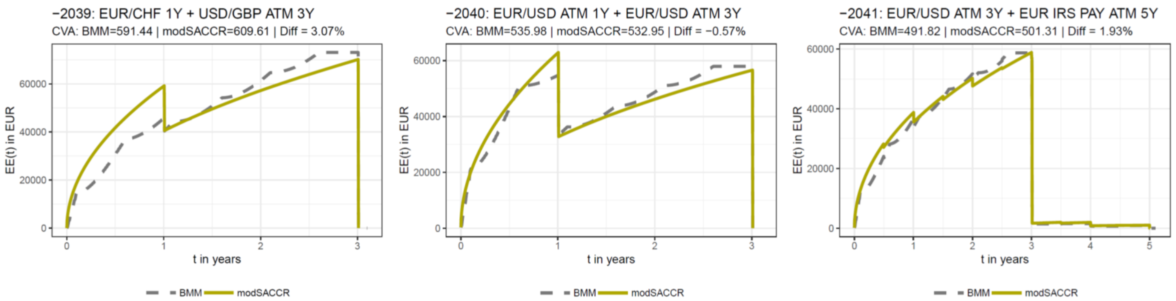


Figure 2.B.14: Expected exposure profile of IR multi-transaction netting sets. *Note* This figure provides the expected exposure profile and CVA results for the hypothetical netting sets 2039, 2040 and 2041. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

2.B.6 Output: Profiles for portfolios with perfect CSA

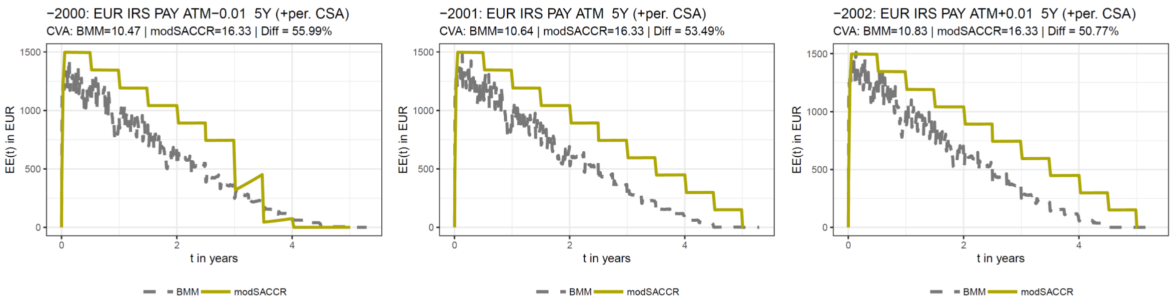


Figure 2.B.15: Expected exposure profile of IR swaps with perfect CSA. *Note* This figure provides the expected exposure profile and CVA results for the hypothetical netting sets 2000, 2001 and 2002 under the assumption of a perfect CSA. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

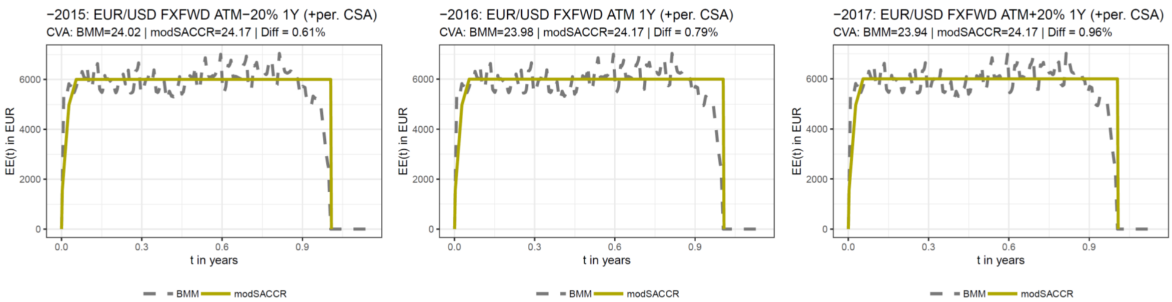


Figure 2.B.16: Expected exposure profile of FX forwards with perfect CSA. *Note* This figure provides the expected exposure profile and CVA results for the hypothetical netting sets 2015, 2016 and 2017 under the assumption of a perfect CSA. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

2.B.7 Output: Profiles for portfolios with imperfect CSA

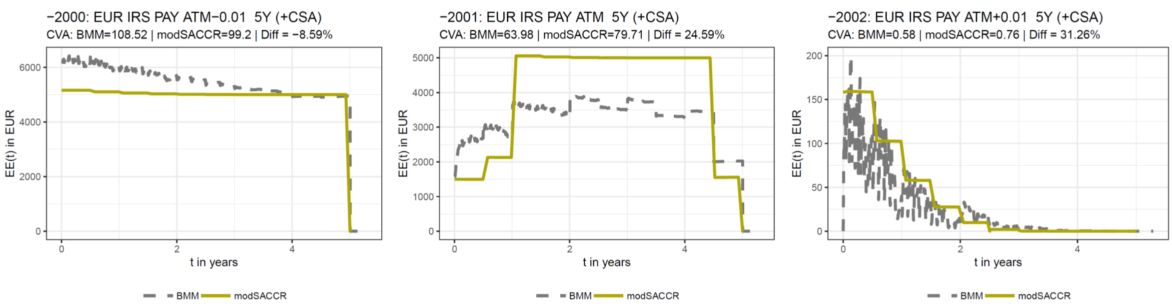


Figure 2.B.17: Expected exposure profile of IR swaps with a CSA (TH=5.000,MTA=1.000). *Note* This figure provides the expected exposure profile and CVA results for the hypothetical netting sets 2000, 2001 and 2002 under the assumption of an imperfect CSA. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

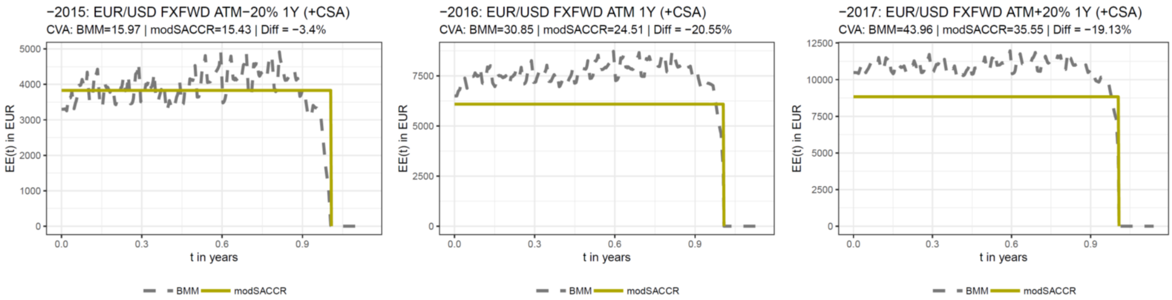


Figure 2.B.18: Expected exposure profile of FX forwards with a CSA (TH=5.000,MTA=1.000). *Note* This figure provides the expected exposure profile and CVA results for the hypothetical netting sets 2015, 2016 and 2017 under the assumption of an imperfect CSA. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

2.B.8 Output: Profiles for FX options

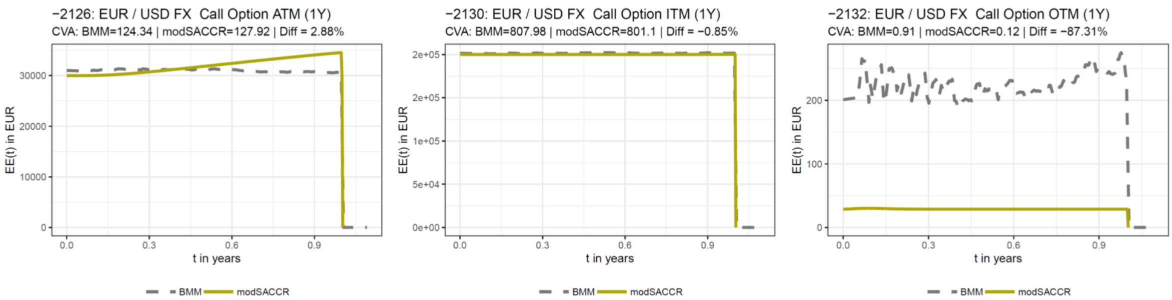


Figure 2.B.19: Expected exposure profile of FX call options. *Note* This figure provides the expected exposure profile and CVA results for the hypothetical netting sets 2126, 2130 and 2132. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

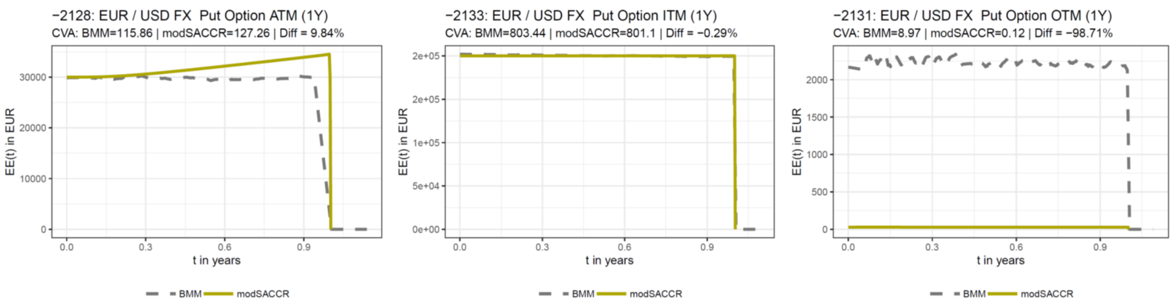


Figure 2.B.20: Expected exposure profile of FX put options. *Note* This figure provides the expected exposure profile and CVA results for the hypothetical netting sets 2128, 2131 and 2133. The solid line represents the result of the modified SA-CCR, while the dashed line shows the result based on the benchmark model.

Chapter 3

The KANBAN Approach
A new way to compute forward Initial Margin

This chapter is joint work with Patrick Büchel* and corresponds to a working paper with the same name (submitted to *Journal of Computational Finance*, currently under review).

Abstract
The implementation of regulatory requirements for the collateralization of non-centrally cleared derivatives has significantly increased the exchange of initial margin (IM). This has a substantial impact on counterparty credit risk and the profitability of derivatives transactions. Hence, current and especially future IM amounts have to be considered in the calculation of counterparty exposure, valuation adjustments (xVA) and capital requirements. This paper introduces a new approach for forecasting IM amounts under ISDA-SIMM™ based on forward sensitivities. The KANBAN approach utilizes cash flow and scenario information to efficiently calculate forward sensitivities within an existing Monte Carlo framework. We adopt elements of just-in-time manufacturing (Kanban) to calculate path- and time-dependent forward sensitivities and IM amounts ”on-the-fly”.

Keywords: Initial Margin; Forward sensitivities; Bilateral Margining; SIMM; xVA; Counterparty Credit Risk

JEL classification: G21, G23, G33

* Commerzbank AG, 60327 Frankfurt am Main, Germany, email: patrick.buechel@commerzbank.com.

3.1 Introduction

As a result of the Great Financial Crises (GFC), additional regulatory requirements were imposed to reduce the counterparty risk inherited in derivatives trading activities. This includes the obligation to clear certain derivatives products through central counterparties (CCPs) as well as additional requirements with respect to the collateralization of over-the-counter (OTC) derivative transactions. In 2016, the Basel Committee on Banking Supervision (BCBS) and the Board of the International Organization of Securities Commissions (IOSCO) issued a regulatory paper on new requirements with respect to the exchange of margining for OTC derivatives products (BCBS and IOSCO (2019)). The proposal was successively transferred into binding law in several jurisdictions. Hence, both cleared and most of the uncleared transactions will face initial margin requirements going forward.

According to the new regulatory specifications, certain financial institutions need to exchange variation and initial margin when trading derivatives bilaterally. While the exchange of variation margin (VM) is broadly practized, the requirement to calculate and exchange initial margin (IM) is considered a new challenge for the derivatives market. Counterparties have the option to choose between a regulatory prescribed standardized approach and an internally developed model when calculating IM amounts. According to BCBS and IOSCO (2019), an internal IM model may be developed by a single-counterparty, by two counterparties or by a third party. All internal IM models require the approval of the respective competent authority. The International Swaps and Derivatives Association (ISDA) has developed a standard IM model (ISDA-SIMM™), which adheres to the regulatory guidelines and provides market participants with a consistent and reliable methodology (ISDA (2019)). The introduction of a consistent and transparent IM model aims to reduce efforts for reconciliation and avoid disputes between counterparties. The model introduced by ISDA can basically be described as an analytical VaR approach with sensitivities and pre-calibrated correlations as the main input factors. From our point of view, ISDA-SIMM™ will become the market standard for the calculation of IM amounts for OTC derivatives.

The trade sensitivities used within ISDA's model, change over time due to market movements, payment of cash flows and aging as well as expiring positions. Consequently, the IM requirements calculated under ISDA-SIMM™ are time-dependent. Hence, it is necessary to forecast IM requirements for the proper assessment of counterparty exposures, valuation adjustments (xVA) and regulatory capital requirements. Furthermore, institutions should be able to forecast IM amounts, as the associated funding requirements drive the profitability of derivatives transactions. Under SIMM[1], the forecasting of future IM amounts requires the calculation of forward sensitivities. The estimation of forward sensitivities is a complex and laborious task, especially with respect to non-linear products, such as path-dependent options.

Our paper provides the following contributions. First, we develop an approach to dynamically calculate time- and path-dependent SIMM results "on-the-fly", within an established simulation model. Our methodology utilizes the representation of financial instruments as cash flows to calculate the required forward sensitivities. We use cash flows as central objects for steering the calculation process. This allows a lean manufacturing of forward sensitivities within an existing Monte Carlo (MC) framework. Second, we show that our methodology is applicable for all types of asset classes and risk factors in a unified way. We provide details on the calculation of delta, vertex delta and vega sensitivities required for ISDA-SIMM™. Third, we test our methodology for different products within a case study and show its practical use.

The remainder of the paper is structured as follows. Section 3.2 provides a methodological overview of the ISDA Standard Initial Margin Model (ISDA-SIMM™). A thorough understanding of the ISDA-SIMM™ methodology is important to derive the data requirements for its application. Readers acquainted with the methodological foundations and functionality of ISDA-SIMM™ might decide to skip this section. In section 3.3, we discuss different options and existing approaches for forecasting Initial Margin requirements. Our KANBAN methodology for the calculation of forward sensitivities is presented in section 3.4. A case study on the application of our approach for different financial products is shown in section 3.5. Section 3.6 concludes this paper.

3.2 ISDA-SIMM™

3.2.1 Methodological overview

ISDA-SIMM™ was developed to provide a transparent and consistent model framework for a wide range of counterparties. One main target was to identify and develop a model framework which is in line with the quantitative and qualitative requirements set by BCBS and IOSCO (2019). On the one hand, the model should be easy to replicate, fast to calculate and extensible.

[1] The terms SIMM and ISDA-SIMM™ are used are synonymously within this paper. Each time the term SIMM is used, we explicitly refer to the Standard Initial Margin Model developed and published by ISDA (2019).

On the other hand, the resulting margin requirements must be non-procyclical, predictable and appropriate for covering the underlying risk. ISDA was aiming to develop a standard initial margin model, which is accessible and applied by a wide range of financial institutions. Hence, the model needs to be transparent, easy to implement and associated with low costs as well as a strong governance framework (ISDA (2016)).

In general, the model needs to adhere to the quantitative principles set by BCBS and IOSCO (2019), where the initial margin requirement is defined as the Value-at-Risk for a confidence level of 99% and a holding period equal to the Margin Period of Risk (MPOR). After analysing a series of possible choices for an initial margin model, it was decided to build ISDA-SIMM™ based on a framework similar to the new sensitivity-based standardizd approach under the Fundamental Review of the Trading Book (SA-TB). Hence, key elements of the SA-TB can be found in the SIMM framework. The ISDA-SIMM™ can be described as a nested Variance / Co-Variance approach (ISDA (2016)). Sensitivities to a standardized set of risk factors are used as main input and are allocated to a predefined structure of buckets. The sensitivities are multiplied by risk-weights, which are calibrated to (stressed) historical market data to meet the regulatory requirements. A series of nested aggregation formulae are used to calculate the initial margin requirement (IMR) for a given netting set based on prescribed correlations.[2]

Figure 3.1 provides a high-level overview of the model's structure.[3] The model captures delta, vega, curvature risk and base correlation risk for CDO tranches on iTraxx and CDX credit indices. Inter-curve basis risk as well as concentration risk are implicitly covered within the calculation. A prerequisite for the calculation of IMR under SIMM is the assignment of each trade to one of four product classes (IRFX, EQ, CR, CO). For each trade within a given netting set, sensitivities to a set of risk factors are calculated. These trade sensitivities (s) are the main input parameters of the model. The sensitivities are scaled by risk-weights (RW) and a concentration risk factor (CR_b). The resulting weighted sensitivities (WS) are allocated to specific nodes, buckets and risk classes. Based on this allocation, aggregation formulae with predefined correlations (ψ,ρ,ϕ,γ) are used to calculate the Initial Margin Requirement (IMR) for each product class. The overall IMR is defined as the sum of the margin requirements for the four product classes.

When calculating forward IMR, we consider the risk-weights and correlations to be static as they are only recalibrated by the ISDA on an annual basis. In general, the parameters are calibrated based on stressed historic market data. Hence, they are not expected to change

[2] For a detailed discussion and justification of the application of nested aggregation formulae, please refer to ISDA (2016).

[3] For a comprehensive overview of ISDA-SIMM™, please refer to ISDA (2019).

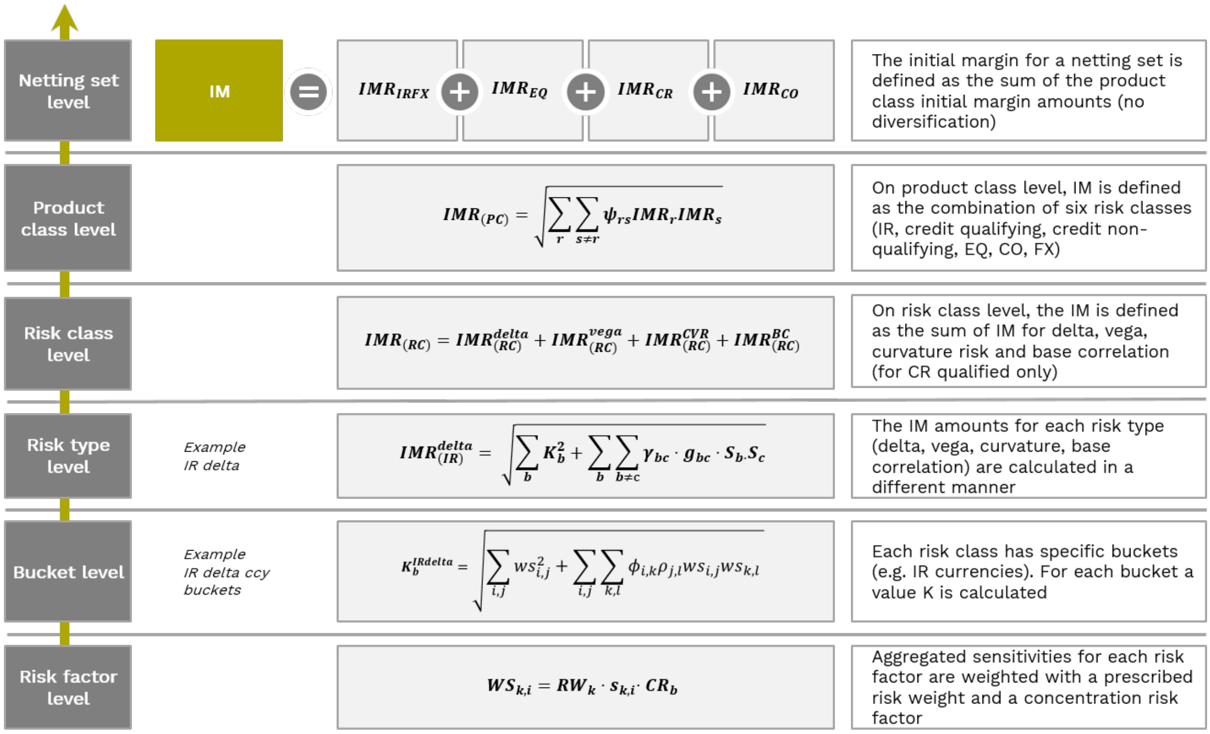


Figure 3.1: Structure of ISDA-SIMM™. *Note* The presented examples on risk type and bucket level are only relevant for IR delta. Additional details on the methodology are provided by ISDA (2019).

with high magnitudes over the life-time of a transaction. Following the assumption of static risk-weights and correlations, forward trade sensitivities are the only dynamic input required for the calculation of forward IMR results under ISDA-SIMM™. Consequently, we are focussing on the calculation of forward sensitivities within this paper. The following section provides an overview of the sensitivity requirements and risk factor definitions for the different SIMM risk classes. The understanding of these requirements is the foundation for the implementation of the KANBAN approach.

3.2.2 Sensitivity requirements

The following description of sensitivity requirements and risk factor definitions follows the guidelines set out in ISDA (2019). In general, the requirements for the calculation of sensitivities as well as the definition of risk factors differ by risk class. For the calculation of IMR under SIMM, three different types of sensitivities are required. **Delta sensitivities** are calculated for the risk classes EQ, CO and FX. The delta sensitivity (δ) of a transaction (i) to a risk factor (x) is generally defined as:[4]

$$\delta_{i,x} = V_i(x + 1\% \cdot x) - V_i(x) = V_i(1.01 \cdot x) - V_i(x) \tag{3.1}$$

where $V_i(x)$ represents the market value of the transaction given risk factor x. For the risk classes IR, CR (qualified) and CR (non-qualified) **vertex delta sensitivities** are calculated. The vertex delta sensitivity of an instrument (i) to an interest rate or credit spread (x) and vertex (t) is defined as:

$$\delta_{i,x_t} = V_i(x_t + 1bp) - V_i(x_t) \tag{3.2}$$

In addition to first order delta sensitivities, **vega sensitivities** are required for the calculation of the vega and curvature risk IMR. The vega risk sensitivity of a transaction (i) to an implied volatility risk factor (σ) is defined as:

$$\frac{\partial V_i}{\partial \sigma} = V_i\left(\sigma + \Delta_{(RC)}^{(vega)}\right) - V_i(\sigma) \tag{3.3}$$

where $V_i(\sigma)$ equals the market value of an instrument (i) given the implied volatility of the risk factor. When shifting σ, all other inputs (including skew and smile) remain constant. In general, the implied volatility is considered in the log-normal representation with an exemption for the interest rate risk class. The shift of the implied volatility ($\Delta_{(RC)}^{(vega)}$) needs to be in line with the implied volatility definition and is set to 1% for log-normal volatilities and 1 bp for normal volatilities.

ISDA (2019) provides a specific definition of risk factors for each risk class. For **interest rates**, the risk factors are defined as 12 yields for each sub-yield curve (e.g. OIS, Libor6m) at distinct vertices (2W, 1M, 3M, 6M, 1Y, 2Y, 3Y, 5Y, 10Y, 15Y, 20Y, 30Y). The sensitivities for each currency are assigned to a separate bucket. Risk factors for the risk class **Credit (qualifying)** are defined as credit spreads for each combination of issuer and seniority at five vertices (1Y, 2Y, 3Y, 5Y, 10Y). The same vertices are used to define the risk factors for the risk class **Credit (non-qualifying)**. For qualifying and non-qualifying credit, each sensitivity is allocated to a specific bucket based on the credit quality and industry / market sector. For the other risk classes **EQ, CO and FX**, the risk factors equal the respective spot price of the equity, commodity or exchange rate. EQ sensitivities are assigned to one of 12 buckets based on the market capitalization (size), region and industry sector. The allocation of commodity sensitivities is based on the commodity type

[4] Please note that there are additional possibilities to calculate delta, vertex delta and vega sensitivities in line with ISDA (2019), such as backward difference methods or the application of smaller shock sizes.

(e.g. Precious Metals). Within the risk class FX, each currency pair builds its own bucket. Table 3.1 provides some examples of sensitivity inputs for ISDA-SIMM™ in the standard data format (CRIF) introduced by ISDA (2017).

Table 3.1: ISDA-SIMM™ - Sensitivity inputs (examples)

ProductClass	RiskType	Qualifier	Bucket	Label1	Label2	Amount
Equity	Risk_Equity	FTSE100	11			84,496
Equity	Risk_EquityVol	FTSE100	11	1Y		59,578
RatesFX	Risk_IRCurve	USD	1	5y	Libor3m	-4,881
Credit	Risk_CreditQ	XS1081333921	3	5y		4,939
Commodity	Risk_Commodity	Precious Metals Gold	12			66,124
Commodity	Risk_CommodityVol	Precious Metals Gold	12	3m		23,754

Notes: This table provides examples for ISDA-SIMM™ input sensitivities in the Common Risk Interchange Format (CRIF) presented by ISDA (2017).

The preparation and processing of input sensitivities requires the following main steps. First, all relevant risk factors in a given netting set are identified. Second, the sensitivities to those risk factors are calculated in line with the model requirements. Third, the sensitivities are mapped to the ISDA-SIMM™ bucket structure. For the correct processing of input sensitivities the relevant variables presented in table 3.1 need to be determined. An approach for the calculation of forward ISDA-SIMM™ sensitivities must comprise all above steps to allow an "on-the-fly" calculation of forward IMR. Our approach offers an integrated framework for the identification of relevant risk factors as well as the calculation and allocation of forward sensitivities to the SIMM bucket structure.

3.3 Forecasting initial margin requirements

Driven by the regulatory and market development towards the collateralization of non-cleared derivatives, the forecast of initial margin requirements (IMR) gained interest and is heavily discussed in recent academic literature. Generally speaking, the initial margin amount equals a Value-at-Risk with a given confidence level (α) and a holding period equal to the Margin Period of Risk ($MPOR$):

$$IMR(t,n) = VaR_{\alpha}(t) = \inf\{l \in R : P(\Delta MtM(t, t+MPOR, n) > L) \leq \alpha\} \tag{3.4}$$

where $IMR(t,n)$ equals the Initial Margin Requirement at viewpoint (t) given path n. The confidence level is represented by α. For the calculation of this amount, the path-wise distribution of the portfolio loss at the end of MPOR is required. The generation of the local loss distribution would require nested Monte Carlo methods, which lead to high computational efforts. Hence, they are often not feasible in practice and institutions tend to apply semi-analytical methods involving assumptions on the properties of the loss distribution.

In the following consideration, we assume that ISDA-SIMM™ will become the market standard for the calculation of IMR for non-cleared derivatives. Hence, we believe that the task of forecasting IMR is not to develop an own dynamic initial margin model (DIM), but to estimate the future IM amounts under ISDA-SIMM™. This model equals a semi-analytical Value-at-Risk (VaR) based on assumptions with respect to the properties of the loss distribution and a calibration to stressed market data. As trade sensitivities are the only time-dependent input of the model[5], the problem of forecasting IMR diminishes to the calculation of forward sensitivities for each node[6] of the Monte Carlo simulation. In general, there are two options to estimate the forward IMR under a sensitivity based model: (i) the calculation of forward sensitivities as model input and (ii) the estimation of IMR based on an approximation methodology (e.g. regression).

The calculation of forward sensitivities is a complex and laborious task. As sensitivities are time- and path-dependent, a very large number of results needs to be produced. In particular, the determination of sensitivities for non-linear (path-dependent) instruments is complex and challenging, as there is often no analytical solution for valuing these products. The production of these sensitivities would in some cases again involve nested Monte Carlo, which requires a high amount of computational power and resources. Hence, classical "bump-and-run" approaches based on nested Monte Carlo are not feasible in practice. There has been recent academic research on the application of Adjoint Algorithmic Differentiation (AAD) for the calculation of forward sensitivities in the context of IM (Fries (2019), Fries et al. (2018), Antonov et al. (2017)) based on the work of Giles and Glasserman (2006) and Capriotti (2011). Their results show that AAD-based forward sensitivities are indeed very accurate and suitable for forecasting IMR. Nevertheless, the practical implementation is considered quite cumbersome and, in most cases, requires a major revamp of the existing Monte Carlo infrastructure. Additionally, there has been recent research on the calculation of forward IMR via the application of Chebyshev Spectral Decomposition. Zeron and Ruiz (2018) use this mathematical concept to calculate forward sensitivities based on pre-simulated sensitivity grids. They show that their concept is as accurate as AAD but requires much less computational efforts.

[5] In general, other parameters of the ISDA-SIMM™ could also vary over time due to recalibration (see section 3.2).
[6] Each combination of viewpoint and path is considered a node of the simulation.

Besides the calculation of forward sensitivities, various methods are discussed for the direct approximation of forward IMR. These methods do not require forward sensitivities and share the basic concept of providing an estimation of future IMR based on a dynamic initial margin model (DIM), which mirrors the properties of the ISDA-SIMM™ or any other IM model. Within an exposure simulation, the distribution of MtM values at default time (t) and close-out time ($t+MPOR$) is produced and available. Most literature on DIM models suggests utilizing this information by applying regression techniques. Andersen et al. (2017c) and Andersen et al. (2017a) use this information to determine the local distribution of $\Delta MtM(t, t+MPOR, n)$. They apply regression techniques where the MtM at default time is used as the independent variable. Anfuso et al. (2017) follow this approach and propose a DIM model, which applies an American Monte Carlo (AMC) methodology based on Longstaff and Schwartz (2001) to calculate the characteristics (moments) of the local distribution of ΔMtM. McWalter et al. (2018) implement a similar approach with an enhancement regarding the distribution assumption. They use a regression technique based on Johnson quantiles and are able to show that this leads to more accurate DIM results. Green and Kenyon (2015) also use AMC regression to calculate an expected initial margin profile. Chan et al. (2017) compare different parametric and non-parametric regression techniques and show that the models are conservative and suitable for the calculation of regulatory capital. Ma et al. (2019) present an alternative method for the direct approximation of future IMR based on deep neuronal networks (DNN). They show that DNNs are generally able to efficiently capture the high complexity and dependencies of future IMR.

In general, a methodology for forecasting IMR should be accurate, fast, computationally efficient and easy to implement. Furthermore, it should not add additional model uncertainty to the counterparty risk and pricing framework. For all of the existing methods, there is a significant trade-off between accuracy and simplicity. While the calculation of forward sensitivities for AAD is considered very accurate, its complexity hampers most institutions from its practical implementation. Furthermore, the implementation of AAD requires a major redesign of the Monte Carlo infrastructure. Zeron and Ruiz (2018) find that a method based on Chebyshev Sepectral Decomposition provides the same accuracy as ADD. Nevertheless, the concept is quite complex and requires significant implementation effort.

Regression approaches are usually easy to implement and computationally efficient, as they use existing information, but do not provide the same level of accuracy compared to AAD. Caspers et al. (2017) provide an overview and evaluation of different regression based approaches. They find that most regression-based models are suitable for single trades, but have difficulties coping with more complex situations (portfolios) and non-linear positions. In addition, there are usually deviations between ISDA-SIMM™ and internal counterparty risk / XVA models regarding the risk factor universe and calibration. Hence, the results from those approaches need to be adjusted to match the ISDA-SIMM™ specifications. According to Caspers et al. (2017), the application of a single (time-depending) scaling factor works well for short time horizons, but the calibration of regression-based dynamic IM models for longer time horizons is a challenging task. The application of DNNs seems to be an option worth considering. DNNs are able to capture the high dimensional relationships and dependencies. Nevertheless, a tremendous amount of data is required for the training of the algorithm. In addition, the application to exotic products is not yet analysed and open issues with respect to hyper-parameter optimization need to be analyzed in future research.

In summary, we consider the calculation of future IM amounts based on forward sensitivities to be more accurate compared to the direct approximation based on regression and machine learning techniques. Nevertheless, the existing methods for forecasting sensitivities are hardly implemented in practice. Within this paper, we introduce a new approach for the calculation of forward sensitivities within an existing Monte Carlo framework. Our approach is accurate, easy to implement and does not add additional model uncertainty as no further assumptions are required. We build our methodology based on the existing representation of financial instruments as a series of cash flows and use this information to provide forward sensitivities analytically according to the ISDA-SIMM™ specifications. Our approach can also be adapted to any kind of sensitivity-based initial margin model.

3.4 The KANBAN approach

3.4.1 Modeling framework and general aspects

For the development of our approach, we assume an existing Monte Carlo (MC) simulation model for counterparty credit risk which covers all asset classes and involves state-of-the-art stochastic processes. Our model framework involves a hybrid multi-asset class exposure model. For the modeling of interest rates, we apply a model based on Trolle and Schwartz (2009) calibrated to European swaptions considering a multi-curve environment and stochastic bases. For the asset classes Equity, Commodity and FX, we apply a model based on Heston (1993) calibrated to observable option prices. Credit spreads are modeled via a Cox-Ingersoll-Ross

(CIR) model (Cox et al. (1985)), where the model for the idiosyncratic component is based on an Ornstein-Uhlenbeck process (Uhlenbeck and Ornstein (1930)). The exposure model is calibrated under the risk-neutral measure.

In our model framework, all non-linear products are valued via the application of an **American Monte Carlo (AMC)** method. The AMC method is based on Longstaff and Schwartz (2001), but uses clungstering and local regression techniques instead of polynomial basis functions. By performing an AMC pre-simulation, we obtain fitting functions for each instrument at a discrete set of viewpoints. In the subsequent exposure simulation, these fitting functions are used to value non-linear products based on their inner value at each viewpoint. The AMC fitting functions are utilized in the KANBAN approach, for the calculation of vega and delta sensitivities for non-linear products.[7]

Our infrastructure includes a capsuled **Market Data Service (MDS)**, which stores all relevant market and static data required for the exposure simulation. This includes the future states of market data generated by the simulation as well as the AMC fitting functions and interim results stated above. The MDS transforms the simulated risk factors to path- and time-dependent market data objects and provides this information when needed within the calculation process. Furthermore, the MDS centrally stores information required for the calculation and allocation of sensitivities. Hence, this information is held only once and does not need to be propagated through the simulation model.

Cash flows are considered the central object for valuation in our framework. All trades are modeled by a collection of cash flows which are bundled in a cash flow container ($\mathcal{J}$). The container comprises all cash flows between a given viewpoint (t) and the maturity (M) of the trade.

$$CF(\mathcal{J}) = \{CF(t) \mid t \leq M\} \tag{3.5}$$

The modeling of financial instruments in a holistic cash flow based framework allows us to process a large number of different products without product-specific implementation. Hence, this provides a flexible approach for the treatment of exotic products with complex pay-off features. Each cash flow carries mandatory information, such as type (static, variable), payment date, notional amount and currency. In addition, asset class specific information is held, such as underlying risk factor (e.g. curve), fixing date or reference entity. The MDS is used to update

[7] Please refer to APPENDIX 3.A.2 for additional details on the AMC techniques implemented in the model framework.

the cash flows during the MC simulation. For each combination of path (n) and viewpoint (t), a new set of market data is generated. All variable cash flows, which are paid subsequent to the respective viewpoint, are updated according to the realized market data ($\mathcal{F}_t$). The update is limited to the cash flow amount. Other data (e.g. payment date) is not updated, but interpreted by the MDS in the respective context (path, viewpoint).[8]

Our approach adapts elements of just-in-time manufacturing. Each cash flow carries the required information for the production and assignment of its sensitivities. The MDS is called by each cash flow to interpreted this information in a certain context defined by the viewpoint and path of the MC simulation. The KANBAN approach uses this detailed information together with the data stored in the MDS to calculate the required sensitivities and to assign the results to the ISDA-SIMM™ buckets. Hence, the information associated with each single cash flow is similar to a KANBAN card in an industrial production environment. The required sensitivities are calculated based on the respective market data scenarios on a predefined viewpoint grid. The aim of the KANBAN approach is to calculate sensitivities "on-the-fly" for each combination of path (n) and viewpoint (t). Our approach utilizes information produced by the MC simulation as well as data inherited in the cash flow representation of financial instruments. Hence, a coherent and unified representation of financial instruments as series of cash flows is a key prerequisite for the application of the KANBAN method. We calculate all required sensitivities for a given MC node[9] simultaneously instead of successively, recognizing the path-wise information on the market scenario. Hence, our methodology is much faster compared to classical "bump-and-run" approaches.

In summary, the KANBAN approach is described by the following main steps. First, we perform AMC pre-simulations to obtain the required fitting functions based on un-shifted and shifted implied volatilities. Second, we assign each product to a specific ISDA-SIMM™ product class based on a predefined mapping algorithm. We assume, that the cash flow representation of each trade is already available. The third step of the process aims to identify all relevant risk factors of each cash flow. After this identification, each cash flow calls the MDS and the sensitivities for each combination of path and viewpoint are calculated. These sensitivities are mapped to ISDA-SIMM™ buckets by the MDS. Finally, the resulting sensitivity data is used as input for the calculation of the IM requirement (IMR) at each viewpoint. The subsequent paragraphs provide the specific calculation procedures for the main types of ISDA-SIMM™ sensitivities.[10]

[8] Please refer to APPENDIX 3.A.1 for additional information on the cash flow concept.
[9] We define a node of the MC simulation as a combination of path(n) and viewpoint (t).
[10] Within this paper, we focus on the most common risk factors. Hence, we do for example not address the calculation of base correlation sensitivities.

3.4.2 Linear products

Delta sensitivities

Delta sensitivities are calculated analytically based on the cash flow representation of financial instruments. The calculation procedure differs by risk class and depends on the type of sensitivities required by SIMM. The delta sensitivities for risk classes EQ, CO and FX are calculated without a vertex structure. In general, all delta sensitivities are calculated by the MDS. The MDS is called by each cash flow object for each combination of viewpoint and path. Based on the cash flow's information, such as payment date or underlying, the KANBAN approach assigns the calculated sensitivities to the correct SIMM buckets. The following paragraphs depict the calculation procedure for each ISDA-SIMM™ risk class.

Equity risk: Equity products are represented by a combination of payment cash flows ($CF^{(CASH)}$) and equity cash flows ($CF^{(EQ)}$). Each equity cash flow is exposed to EQ risk. The equity delta sensitivity (δ) for each equity cash flow (j) to its equity underlying (k) is calculated by:[11]

$$\delta_{j,k}^{(EQ)}(t,n) = CF_j^{(EQ_k)}(t,T \mid \mathcal{F}_t) \cdot \Delta_{(EQ)} \cdot DF(t,T \mid \mathcal{F}_t) \tag{3.6}$$

where $CF_j^{(EQ_k)}$ is interpreted as the amount of a cash flow, which depends on the market value of a specific equity underlying k. Hence, $CF_j^{(EQ_k)}(t,T \mid \mathcal{F}_t)$ equals the cash flow amount at payment date (T) given the path-dependent market realization ($\mathcal{F}$) at viewpoint (t). $\Delta_{(EQ)}$ represents the shift size in percent. The multiplication with $DF(t,T)$ is equal to discounting the sensitivity amount from payment date (T) to the viewpoint (t). For discounting, the respective risk-free rate of the cash flow‘s currency is used. If the IM calculation currency differs from the cash flow currency, the sensitivity amount needs to be converted to the calculation currency by multiplication with the FX rate. Please note that discounting and currency conversion depend on the path-dependent market realization ($\mathcal{F}_t$). The mapping of equity sensitivities to SIMM buckets is performed based on the information with respect to the underlying carried by the cash flow (e.g. ISIN) and the associated static data provided by the MDS (market capitalization, region and industry sector for the respective underlying).[12]

Commodity risk: Commodity products are represented by a combination of commodity and payment cash flows. The sensitivity to commodity underlyings is calculated by multiplying

[11] The derivation of equation (3.6) is presented in APPENDIX 3.B.1.
[12] For a detailed description of the definition of SIMM equity buckets, please refer to section G in ISDA (2019).

the cash flow amount at payment date with the commodity shift size and discounting to the viewpoint (t). Hence, equation (3.6) is also applied for calculation of delta sensitivities for commodity cash flows:

$$\delta_{j,k}^{(CO)}(t,n) = CF_j^{(CO_k)}(t,T \mid \mathcal{F}_t) \cdot \Delta_{(CO)} \cdot DF(t,T \mid \mathcal{F}_t) \tag{3.7}$$

The mapping of commodity sensitivities to SIMM buckets is based on information with respect to the underlying of the cash flow. Given a unique identifier for the underlying, a static commodity categorization is required to map sensitivities to the corresponding buckets by commodity type. The respective business logic and static data are part of the MDS.[13]

FX risk: In general, all cash flows denominated in any currency other than the calculation currency are exposed to FX risk. Cash flows denominated in the calculation currency are excluded from the calculation. The FX sensitivity of each cash flow denominated in a currency other than the calculation currency is calculated by:[14]

$$\delta_{j,u}^{(FX)}(t,n) = CF_j^{(FX_u)}(t,T \mid \mathcal{F}_t) \cdot \Delta_{(FX)} \cdot FX_u(t \mid \mathcal{F}_t) \cdot DF_u(t,T \mid \mathcal{F}_t) \tag{3.8}$$

For discounting, the risk-free rate of the cash flow's currency (u) is used. The resulting sensitivity amount is converted to calculation currency by multiplication with the corresponding FX rate. As each currency pair represents an FX risk factor within ISDA-SIMM™, the mapping is based on the cash flow's currency.

Vertex delta sensitivities

For the risk classes interest rate and credit (qualified and unqualified), delta sensitivities are required for a predefined vertex grid. **Interest rate (IR)** risk stems from discounting of cash flows as well as changes in the amount of floating IR cash flows. In general, any cash flow is exposed to IR risk due to **discounting**. Hence, we need to calculate the IR sensitivity to the respective discount curve for each cash flow. These discounting sensitivities are calculated for each cash flow (j) separately by applying the following formula:[15]

$$\delta_{j,k}^{(IR)}(t,n) = CF_j(t,T \mid \mathcal{F}_t) \cdot DF(t,T \mid \mathcal{F}_t) \cdot [\epsilon(t,T) - 1] \tag{3.9}$$

[13] For a detailed description of the definition of SIMM commodity buckets, please refer to section H in ISDA (2019).
[14] Please refer to APPENDIX 3.B.1 for a detailed derivation of equation (3.8).
[15] The derivation of this formula is provided in APPENDIX 3.B.2.

where $CF_j(t,T)$ equals a cash flow paid at T and $DF(t,T)$ stands for the discount factor derived from the risk-free curve of the cash flow's currency. $\epsilon(t,T)$ represents the shift of the discount factor and is defined as:

$$\epsilon(t,T) = e^{-1bp\cdot(T-t)} \tag{3.10}$$

The KANBAN approach applies equation (3.9) to all cash flows and calculates a discounting sensitivity for each node of the MC simulation. In addition to IR risk from discounting, variable IR cash flows (IR_FLOAT) are subject to additional IR risk, as their amount depends on a forward rate derived from the corresponding reference curve. The forward rate is defined as:

$$F(t,t_1,t_2) = \frac{1}{(t_2-t_1)} \cdot \left(\frac{DF^{(fwd)}(t,t_1)}{DF^{(fwd)}(t,t_2)} - 1 \right) \tag{3.11}$$

where t_1 and t_2 equal the start and end of the coupon period. $DF^{(fwd)}(t,t_x)$ represents the discount factor for tenor $(t_x - t)$ at viewpoint t derived from the cash flows reference curve. Each forward rate is defined based on two discount factors. Hence, we calculate two IR sensitivities for each IR_FLOAT cash flow with respect to the tenors $(t_1 - t) = t1$ and $(t_2 - t) = t2$. The calculation of these two different sensitivities is performed for each floating IR cash flow (j) by applying the following equations:[16]

$$\delta^{(IR)}_{i,r_{t1}}(t,n) = N_j \cdot DF(t,T \mid \mathcal{F}_t) \cdot \left[\frac{DF^{(fwd)}(t,t_1 \mid \mathcal{F}_t) \cdot (\epsilon(t,t_1) - 1)}{DF^{(fwd)}(t,t_2 \mid \mathcal{F}_t)} \right] \tag{3.12}$$

$$\delta^{(IR)}_{i,r_{t2}}(t,n) = N_j \cdot DF(t,T \mid \mathcal{F}_t) \cdot \left[\frac{DF^{(fwd)}(t,t_1 \mid \mathcal{F}_t) \cdot (1 - \epsilon(t,t_2))}{DF^{(fwd)}(t,t_2 \mid \mathcal{F}_t) \cdot \epsilon(t,t_2)} \right] \tag{3.13}$$

where $DF(t,T)$ equals the discount factor used for calculating the cash flow's value at t. $DF^{(fwd)}(t,t_1)$ and $DF^{(fwd)}(t,t_2)$ represent the discount factors derived from the cash flow's reference curve (e.g. 6m Euribor). $\epsilon(t,t_x)$ is defined in line with equation (3.10). In summary, we calculate three IR sensitivities for each IR_FLOAT cash flow. One sensitivity stems from IR discounting risk, while the other two sensitivities result from the forward rate risk.

The KANBAN approach calculates all relevant IR sensitivities for each cash flow. These IR sensitivities must be allocated to a sub-yield curve, currency and vertex. The assignment of IR sensitivities to currency and sub-yield curve is straight-forward, as the required information is stored in the cash flow object.[17] The allocation of IR sensitivities to the ISDA-SIMM™ vertex

[16] The derivation of equations (3.12) and (3.13) is provided in APPENDIX 3.B.2.
[17] For additional information on the cash flow concept and data, please refer to APPENDIX 3.A.1

grid is more complicated. Each cash flow has a specific payment date (T) and a tenor defined as the difference between the viewpoint and the payment date ($T - t$). Based on the cash flow information, we are able to assign a tenor to each IR sensitivity. In most cases, these tenors do not match the SIMM vertices. Hence, we implement a methodology which allocates each sensitivity to the two neighboring SIMM vertices based on linear interpolation. This is in line with the procedure proposed by ISDA (2017). The allocation is time-dependent as the tenors of the IR sensitivities shorten over time. Hence, the sensitivities "roll" into shorter ISDA-SIMM™ buckets during the simulation. Please note that the methodology for calculating IR delta sensitivities described above is equivalent to shifting zero rates. According to ISDA (2017), the SIMM IR sensitivities are to be calculated by shifting market spot rates (par rates). Hence, the resulting IR zero sensitivities need to be converted by Jacobi transformation.[18] According to ISDA (2019), there are additional risk factors in the IR risk class, such as inflation and cross-currency basis spreads. The KANBAN approach also calculates the required sensitivities to these risk factors analytically in line with the sensitivity definitions and guidelines given by ISDA-SIMM™. The general approach is congruent with the calculation of vertex delta sensitivities for interest rates.

Vertex delta sensitivities are also required for the risk classes **Credit (Qualifying)** and **Credit (non-Qualifying)**. Cash flows of credit instruments are categorized as contingent default and premium cash flows. Both cash flow types involve a coupon start (t_{j-1}) and end date (t_j). For the calculation of CR sensitivities, we shift survival probabilities instead of discount factors. The calculation of sensitivities differs by cash flow type: (1) For contingent premium cash flows one sensitivity to tenor ($t_j - t$) is generated, while for (2) contingent default cash flows two sensitivities for tenors ($t_j - t$) and ($t_{j-1} - t$) are calculated.[19] The KANBAN approach determines the required CR sensitivities for all contingent credit cash flows. For the risk class Credit (qualifying), each derived sensitivity has to be allocated to a credit curve (combination of issuer and seniority) and one of five vertices. The assignment to a credit curve is based on information stored in the cash flow object and the MDS. The sensitivities are allocated to two neighboring vertices based on linear interpolation.

[18] For additional information on the Jacobi transformation, please refer to Qu (2016).

[19] Please refer to APPENDIX 3.B.2 for additional details on the calculation of sensitivities for credit spread risk.

3.4.3 Non-linear products

Delta and vertex delta sensitivities

The delta and vertex delta sensitivities described in section 3.4.2 need to be calculated for non-linear products, too. In the prevailing model framework, the inner value of a non-linear instrument is described by a set of underlying cash flows. These cash flows are updated for each node of the MC simulation by the MDS. Hence, we are able to determine path- and time-dependent inner values for each non-linear instrument. As discussed in section 3.4.1, all types of non-linear instruments are valued during the exposure simulation based on AMC fitting functions. Trade- and time-specific AMC fitting functions ($\Theta_{i,t}[x]$) are used to calculate the option value (V_i) at viewpoints (t) based on its inner value (S_i):

$$V_i(t,n) = \Theta_{i,t}\left[S_i(t,n)\right] \tag{3.14}$$

The inner value of a non-linear trade (i) at viewpoint t is defined as the sum of the underlying cash flow values. The calculation of delta and vertex delta sensitivities for non-linear trades is based on the following main steps. First, we calculate the path- and time-dependent delta and vertex delta sensitivities of the inner value with respect to a specific risk factor (k) for each cash flow (j). These sensitivities are assessed in line with the methodological procedures laid down in section 3.4.2. Second, the resulting sensitivities are used to determine shifted inner values for each node of the MC simulation and each risk factor (k):

$$S_{i,j,k}^{(shift)}(t,n) = S_i(t,n) + \delta_{j,k}^{(RC_k)}(t,n) \tag{3.15}$$

Third, the sensitivity of $V_i(t,n)$ with respect to k is calculated by valuing the AMC fitting functions with the shifted and un-shifted inner value:[20]

$$\delta_{i,j,k}^{(RC)}(t,n) = \Theta_{i,t}\left[S_{i,j,k}^{(shift)}(t,n)\right] - \Theta_{i,t}\left[S_i(t,n)\right] \tag{3.16}$$

The KANBAN approach calculates delta and vertex delta sensitivities for each cash flow by applying the aforementioned steps. Thereby, most of the required sensitivities are determined based on a unified process. Nevertheless, there are two main exemptions. The calculation of sensitivities with respect to FX conversion and IR discounting risk is performed separately based on a different approach. After the time- and path-dependent market value of the trade is

[20] For additional details on the methodological background, please refer to APPENDIX 3.B.3 and 3.A.2.

calculated by application of equation (3.14), we generate a pseudo cash flow for each option paid at the exercise date (T) for further processing in the MC simulation. This pseudo cash flow $CF_{i,u}^{(EX)}$ is defined to fulfill the following relationship:

$$V_i(t,n) = CF_{i,u}^{(EX)}(t,T \mid \mathcal{F}_t) \cdot DF_u(t,T \mid \mathcal{F}_t) \cdot FX_u(t \mid \mathcal{F}_t) \tag{3.17}$$

We can use the relationship in equation (3.17) to calculate the sensitivities with respect to discounting and FX conversion risk of the non-linear product by applying the equations (3.8) and (3.9).[21] The allocation of the sensitivities from non-linear products to the ISDA-SIMM™ risk factor structure follows the risk class specific procedures for linear products described in section 3.4.2.

Vega sensitivities

Vega sensitivities must be calculated for all non-linear products. According to ISDA (2019), the vega sensitivity of a financial instrument (i) is defined as:

$$\frac{\partial V_i}{\partial \sigma} = V_i(\sigma + \Delta_{(RC)}^{(vega)}) - V_i(\sigma) \tag{3.18}$$

where $V_i(\sigma)$ is defined as the market value of an instrument (i) given the value of the implied volatility (σ). When shifting the implied volatility, all other inputs are kept constant. The definition of the shift size needs to be in line with the format of the implied volatility of the respective risk class (log-normal or normal). Calculating forward vega sensitivities is challenging, as implied volatilities at a future point in time t are not observable. Nevertheless, the forecasting of SIMM amounts requires the calculation of time- and path-dependent vega sensitivities.

For the calculation of these sensitivities, we utilize the AMC methodology already implemented in our model framework and construct vega fitting functions ($\Psi_{i,t}[x]$) for each non-linear product (i) at each viewpoint (t). A vega fitting function returns a vega sensitivity to an implied volatility based on the inner value of a transaction at viewpoint t. The construction of the vega fitting functions requires an additional pre-simulation run. This additional pre-simulation covers the same risk factors as the AMC pre-simulation and shares the same random number vectors. The only difference between both pre-simulations is the calibration of the stochastic parameters. While the AMC pre-simulation uses market implied volatilities for calibration, the additional

[21] For additional details on the methodological background, please refer to APPENDIX 3.B.3.

(vega) pre-simulation is based on shifted implied volatilities. For each pre-simulation, inner values are stored on a predefined viewpoint grid for each instrument. Based on shifted and un-shifted inner values, we are able to calculate the vega effect on the option value at each combination of viewpoint and path. Based on these results, we construct $\Psi_{i,t}[x]$ by fitting the inner values of the AMC pre-simulation to the vega effect across all paths at each viewpoint.[22] In the actual exposure simulation, we use the resulting vega fitting functions to obtain time- and path-dependent vega sensitivities:

$$\frac{\partial V_i(t,n)}{\partial \sigma(t,n)} = \Psi_{i,t}[S_i(t,n)] \tag{3.19}$$

where $S_i(t,n)$ is the inner value of a non-linear instrument (i) for a given combination of path and viewpoint. Based on vega fitting functions and the realized inner values in the exposure simulation, we are able to calculate time- and path-dependent vega sensitivities "on-the-fly".

3.5 Case study

3.5.1 Methodology and data

The case study aims to show the practical application and results of the KANBAN approach. We apply our methodology to a set of illustrative examples and calculate the forward sensitivities as well as the resulting IM requirement. Our examples comprise single instruments (FX forward, IR swap, IR swaption) as well as a portfolio of 108 IR swaps. We are aware that some of our examples are not relevant for the calculation of ISDA-SIMM™ in practice, as there is a clearing obligation in place or the respective products are exempt from the bilateral margin requirements. Nevertheless, the following examples are suitable for a comprehensible and transparent presentation of the KANBAN approach. The examples are hypothetical, but the calculation of sensitivities and SIMM results is based on real market data. In each part of the case study, we provide the following information and result types. First, the product / portfolio is described and the ISDA-SIMM™ input sensitivities at $t = 0$ are presented in the Common Risk Interchange Format (CRIF).[23] Second, we calculate the forward sensitivity for each risk factor. Third, we provide the distribution of IM values for a selected set of viewpoints. By calculating the expected IM amount for each viewpoint, we finally obtain the IM tenor profile.

[22] For more methodological details, please refer to APPENDIX 3.B.4.
[23] For detailed information on the Common Risk Interchange Format (CRIF), please refer to ISDA (2017).

In general, IM tenor profiles can be useful information for other risk management aspects, e.g. the modeling of collateral in- and outflows in the context of liquidity risk management.

3.5.2 FX forward

For this part of the case study, we assume a EUR/USD FX forward where we receive USD 10m and pay EUR 8m in 1 year. The set-up of the example is equivalent to the FX forward in ISDA (2017).[24] The transaction involves FX risk as well as IR risk from discounting. Hence, delta and vertex delta sensitivities are required as inputs for the SIMM calculation. We calculate our results based on market data as of 31st December 2019 and use EUR as the calculation currency. Hence, the results in this section are denominated in EUR. Based on OIS curves for EUR and USD as well as the EUR/USD spot rate, we are able to calculate the SIMM sensitivity inputs at $t = 0$ using the KANBAN methodology (see table 3.2). The trade produces one FX sensitivity as well as IR sensitivities at different vertices.

Table 3.2: SIMM - Sensitivity inputs (FX forward)

ProductClass	RiskType	Qualifier	Bucket	Label1	Label2	Amount
RatesFX	Risk_FX	USD				90,889.53
RatesFX	Risk_IRCurve	USD	1	1y	OIS	-908.84
RatesFX	Risk_IRCurve	USD	1	2y	OIS	-2.5
RatesFX	Risk_IRCurve	EUR	1	1y	OIS	803.62
RatesFX	Risk_IRCurve	EUR	1	2y	OIS	2.21

Notes: This table provides ISDA-SIMM™ input sensitivities for a EUR/USD 1Y FX forward at $t = 0$ in the Common Risk Interchange Format (CRIF).

Table 3.3 provides the IR sensitivity results at different viewpoints.[25] Over time, the sensitivity amount as well as the allocation of IR sensitivities to the SIMM vertices changes due to position aging. While the FX sensitivity remains more or less constant, the IR sensitivities change significantly over time. The results show that the KANBAN approach is capable of recognizing the decrease of sensitivity amounts over time as well as the change in allocation to the respective SIMM vertices.

By applying the KANBAN approach, we calculate the required SIMM sensitivities for each MC node. These sensitivities are used to calculate time- and path-dependent IM requirements. Hence, we obtain a distribution of IM values at each viewpoint based on all paths of the MC simulation. Figure 3.2 shows the distribution of IM amounts at selected viewpoints. Table 3.4 provides the associated descriptive statistics. The results show that the expected (mean) IM

[24] In ISDA (2017), this example is labelled *"3.7 FX forward #1"*.

[25] Please note that table 3.3 provides the expected value of the sensitivities at different viewpoints resulting from the Monte Carlo simulation.

Table 3.3: SIMM - Sensitivities over time (FX forward)

riskFactor	FX_USD	IR_EUR_OIS					IR_USD_OIS				
t		1m	3m	6m	1Y	2Y	1m	3m	6m	1Y	2Y
0.00	90,889.53				803.62	2.21				-908.84	-2.5
0.05	90,863.46			79.29	682.3				-89.67	-771.63	
0.10	90,828.96			142.82	581.2				-161.49	-657.18	
0.15	90,793.10			202.47	481.8				-228.89	-544.68	
0.20	90,757.28			254.28	390.25				-287.41	-441.1	
0.25	90,724.91			300.47	302.12				-339.58	-341.45	
0.30	90,693.23			337.94	222.73				-381.89	-251.7	
0.35	90,660.94			364.08	159.11				-411.37	-179.77	
0.40	90,639.97			384.16	99.35				-434.05	-112.25	
0.46	90,613.04			397.71	39.53				-449.35	-44.66	
0.50	90,573.54			400.9	1.1				-452.85	-1.24	
0.60	90,521.07		128.23	192.34				-144.83	-217.24		
0.71	90,474.67		193.52	43.51				-218.56	-49.14		
0.80	90,404.48	48.04	112.1				-54.24	-126.57			
0.90	90,337.65	72.35	8.78				-81.66	-9.91			

Notes: This table provides the expected values of FX and IR sensitivities for a EUR/USD 1Y FX forward at selected viewpoints (all values in EUR).

amount is relatively stable over time. The main risk driver of the FX forward is the EUR/USD exchange rate. The FX sensitivity does not change significantly, as the cash flows are paid at maturity. We observe that the width of the distribution as well as its standard deviation (σ) increase over time. This is driven by the increasing realized volatility of the underlying risk factors (diffusion effect).

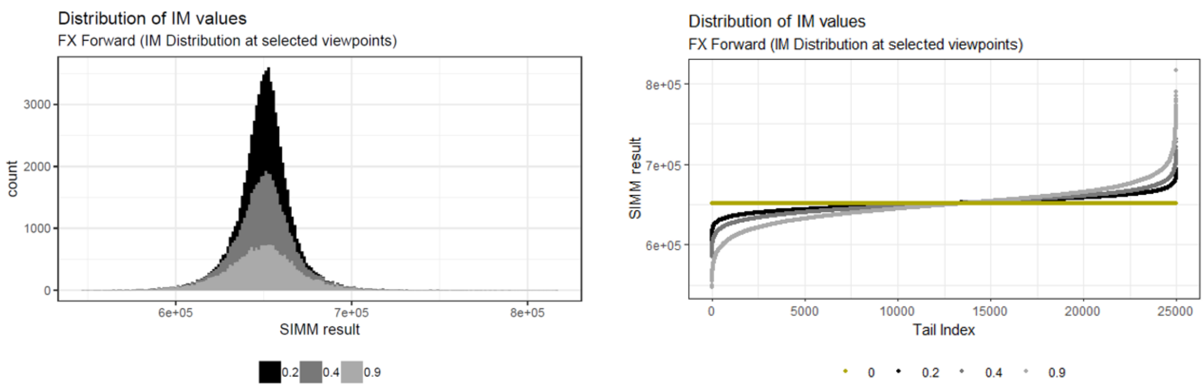


Figure 3.2: IM distribution at different viewpoints (FX forward). *Note* This figure provides the IM distribution at different viewpoints for a EUR/USD 1Y FX forward.

Table 3.4: IM distribution at different viewpoints (FX Forward)

t	Maximum IM	Mean IM	Minimum IM	σ
0.0	651,864.05	651,864.05	651,864.05	0.00
0.2	694,327.94	651,189.75	604,236.57	9,239.71
0.4	731,359.40	650,607.94	585,590.69	13,792.14
0.9	816,547.49	649,212.30	547,056.13	23,144.13

Notes: This table provides the descriptive statistics for the viewpoint specific IM distributions shown in figure 3.2.

The IM tenor profile is provided in figure 3.3 (left panel). The profile shows that the expected IM amount is relatively stable over the life-time of the transaction. This is in line with the observations discussed above. The sensitivity to the dominant risk factor (FX EUR/USD) does not change significantly over time. Hence, the evolution of the expected IM does not show significant changes until the expiry of the transaction.

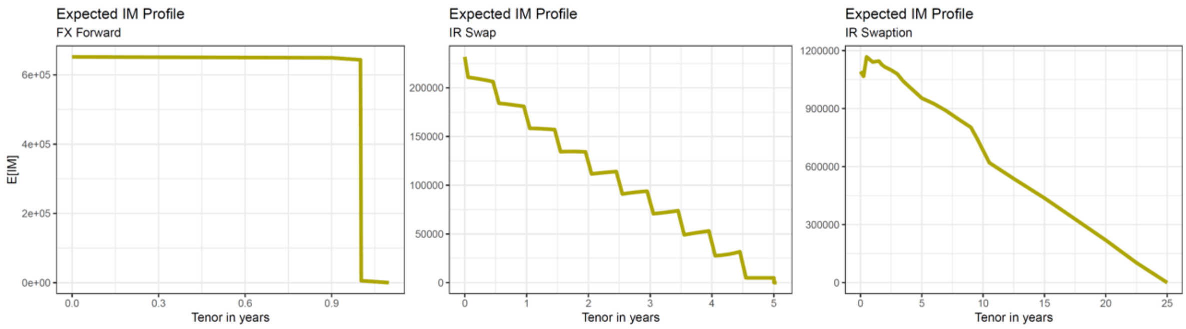


Figure 3.3: IM tenor profiles. *Note* This figure provides the IM tenor profile for EUR/USD 1Y FX forward (left panel), USD 5Y IR swap (mid panel) and EUR 10Y/15Y IR swaption (right panel).

3.5.3 Interest rate swap

For visualization of the KANBAN methodology for IR products, we assume an USD 5Y IR swap where we pay fixed cash flows on a notional of USD 10 million.[26] We use USD as the calculation currency and express all results in USD. For our calculations we use market data as of 31th December 2019. The swap's cash flows inherit IR risk to the USD OIS (discounting) and 3m LIBOR curve. Table 3.5 provides the SIMM sensitivity inputs at $t = 0$. The swap generates sensitivities to different sub-yield curves and vertices. The highest sensitivity amounts are allocated on the 5Y vertex for both sub-yield curves. The sensitivity amounts as well as their allocation to the SIMM vertices changes over time, due to changing market rates, payment of cash flows and aging of the trade. The KANBAN approach is capable of recognizing these effects. The sensitivities amounts over time are provided in tables 3.C.1 and 3.C.2 in APPENDIX 3.C.1. Figure 3.4 provides the distribution of SIMM results at different viewpoints. Table 3.6 provides the associated decriptive statistics. The results show that the expected (mean) IM amount decreases over time. During the life-time of the IR swap, cash flows are paid. This leads to decreasing sensitivities (amortization effect) and consequently to lower IM requirements. The IM standard deviation (σ) first increases (diffusion effect), but significantly decreases towards maturity driven by the payment of cash flows (amortization effect). These aspects affect the IM

[26] This example is loosely based on example *3.1 Interest-rate swap* in ISDA (2017).

Table 3.5: SIMM - Sensitivity inputs (IR swap)

ProductClass	RiskType	Qualifier	Bucket	Label1	Label2	Amount
RatesFX	Risk_IRCurve	USD	1	3m	OIS	-0.02
RatesFX	Risk_IRCurve	USD	1	6m	OIS	-3.94
RatesFX	Risk_IRCurve	USD	1	1Y	OIS	2.51
RatesFX	Risk_IRCurve	USD	1	2Y	OIS	2.12
RatesFX	Risk_IRCurve	USD	1	3Y	OIS	0.22
RatesFX	Risk_IRCurve	USD	1	5Y	OIS	14.53
RatesFX	Risk_IRCurve	USD	1	10Y	OIS	0.04
RatesFX	Risk_IRCurve	USD	1	6m	Libor3m	-0.1
RatesFX	Risk_IRCurve	USD	1	1Y	Libor3m	14.81
RatesFX	Risk_IRCurve	USD	1	2Y	Libor3m	15.08
RatesFX	Risk_IRCurve	USD	1	3Y	Libor3m	92.72
RatesFX	Risk_IRCurve	USD	1	5Y	Libor3m	4,791.21
RatesFX	Risk_IRCurve	USD	1	10Y	Libor3m	5.22

Notes: This table shows the SIMM sensitivity inputs for a USD 5Y IR swap (payer) with a notional of USD 10m at $t = 0$.

tenor profile in figure 3.3 (mid panel). The results clearly indicate that the KANBAN approach is capable of capturing the exposure dynamics of IR swaps as well as the associated effect on the IM requirement.

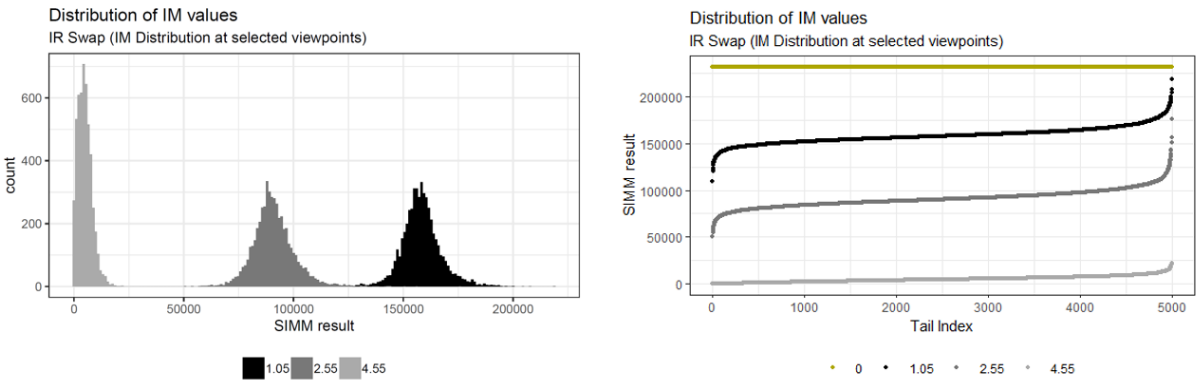


Figure 3.4: IM distribution at different viewpoints (IR swap). *Note* This figure provides the IM distribution at different viewpoints for a USD 5Y IR swap.

Table 3.6: IM distribution at different viewpoints (IR swap)

t	Maximum IM	Mean IM	Minimum IM	σ
0.00	231,761.27	231,761.27	231,761.27	0.00
1.05	218,793.10	158,413.16	109,637.29	8,890.83
2.55	176,230.06	90,935.52	502,48.89	9,494.14
4.55	21,843.16	4,898.53	1.94	3,168.77

Notes: This table provides the descriptive statistics for the viewpoint specific IM distributions shown in figure 3.4.

3.5.4 Interest rate swaption

The KANBAN approach also generates fast and accurate forward sensitivities for non-linear products. As an example, we assume a physically-settled EUR 10Y/15Y European swaption to pay fix at a swap rate of 0.284 % on a notional of EUR 12.5m. Additionally, we set EUR as the calculation currency. The start / trade date of the swaption is 2nd October 2019. The calculation is performed as of 31st December 2019 with the corresponding market data. Hence, the option has a maturity of about 9.75 years at the calculation date. Table 3.7 provides the SIMM input sensitivities at $t = 0$. The swaption generates sensitivities to the EUR OIS curve as well as the Euribor 6m curve in different ISDA-SIMM™ buckets. Additionally, we obtain an IR vega sensitivity which contributes to the 10Y and 5Y vertex, as the maturity of the swaption is 9.75 years at $t = 0$.

Table 3.7: SIMM - Sensitivity inputs (IR swaption)

ProductClass	RiskType	Qualifier	Bucket	Label1	Label2	Amount
RatesFX	Risk_IRCurve	EUR	1	5Y	OIS	-96.12
RatesFX	Risk_IRCurve	EUR	1	10Y	OIS	-1,971.39
RatesFX	Risk_IRCurve	EUR	1	15Y	OIS	-398.36
RatesFX	Risk_IRCurve	EUR	1	20Y	OIS	-521.47
RatesFX	Risk_IRCurve	EUR	1	30Y	OIS	-92.96
RatesFX	Risk_IRCurve	EUR	1	10Y	Euribor6m	-7,878.87
RatesFX	Risk_IRCurve	EUR	1	15Y	Euribor6m	307.05
RatesFX	Risk_IRCurve	EUR	1	20Y	Euribor6m	11,336.28
RatesFX	Risk_IRCurve	EUR	1	30Y	Euribor6m	9,485.38
RatesFX	Risk_IRVol	EUR	1	5Y		1,655.40
RatesFX	Risk_IRVol	EUR	1	10Y		31,543.58

Notes: This table shows the SIMM sensitivity inputs at $t = 0$ for a EUR 10/15Y European swaption (pay fix) with a notional of EUR 12.5m and a maturity of 9.75 years.

At $t = 0$, the majority of the IR delta sensitivity to EURIBOR is allocated to the 20Y and 30Y vertex, while the OIS sensitivity is concentrated on the 10Y vertex where the option expires. The sensitivity amounts as well as the allocation to the ISDA-SIMM™ vertices change over time. The IR delta sensitivities to OIS and EURIBOR roll into shorter vertices going forward. The results and allocation of these sensitivities are provided in APPENDIX 3.C.2. Table 3.8 shows the IR vega sensitivity at selected viewpoints. The total amount of IR vega decreases over time. Furthermore, the sensitivity rolls into shorter buckets as the allocation is based on the option tenor which reduces over the life-time of the swaption. The IR vega sensitivity vanishes after option expiry.

At the calculation date, the swaption is in-the money. Hence, the swaption generates significant sensitivities as well as IM requirements at $t = 0$. Nevertheless, the long time period until the exercise date facilitates that there is a significant number of MC paths, where the option gets

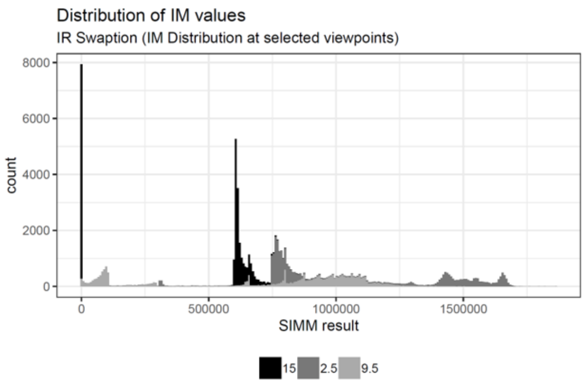
Table 3.8: IR vega sensitivity over time (IR swaption)

t	1m	3m	6m	1y	2y	3y	5y	10y
0.00							1,655.40	31,543.58
0.50							4,619.64	26,149.87
1.00							7,311.92	21,951.79
1.50							9,765.59	18,125.18
2.00							12,480.08	15,261.88
2.50							14,430.90	11,800.56
3.00							16,200.64	8,728.68
3.49							17,463.84	5,783.06
5.00						2,384.69	16,537.30	
6.00						10,024.51	6,023.50	
7.00					3,151.31	9,488.55		
8.00				2,166.32	6,522.75			
9.00			2,205.68	2,217.80				
9.50	6.15	1,490.31						

Notes: This table shows the expected value of IR vega sensitivities for a EUR 10/15Y swaption with a notional of EUR 12.5m and maturity of 9.75 years at selected viewpoints (t).

out-of-the-money (OTM) during the simulation and is not exercised at the expiry date of the option. Figure 3.5 provides the distribution of IM amounts at different viewpoints and table 3.9 the corresponding descriptive statistics. The IM tenor profile for the IR swaption presented in figure 3.3 (right panel). Until option expiry, the distribution of IM results widens and the standard deviation increases. This is driven by the increasing realized volatility in the Monte Carlo simulation over time (diffustion effect). There are no cash flows before the expiry of the option. Hence, the decrease in IM requirements until option maturity is mainly driven by decreasing IR vega sensitivity amounts. The expected IM amount generally decreases over the life-time of the swaption, driven by reducing IR vega and delta sensitivities. The transaction generates IM requirements until the expiry of the underlying swap at $t = 25$, as the swaption is physically-settled. At $t = 15$, there are scenarios, where the swaption has not been exercised. Hence, the IM distribution shows a significant amount of scenarios, where the IM requirement is 0.

Taking the results for the hypothetical IR swaption into consideration, we are certainly justified in saying that the KANBAN approach is able to cope with the complex nature of non-linear products. By applying our methodology based on the prevailing AMC implementation, we are able to calculate fast and accurate forward sensitivities for delta and vega risk. Please note that the staggered approach for the calculation of forward sensitivities involves multiple steps, such as the construction of fitting function and the shift of inner values. In particular, the application of clustering and local regression techniques for the construction of fitting functions strongly affects the path- and time-dependent sensitivities and the IM results for non-linear products. Hence, the analysis and explanation of results is not straight-forward and requires additional efforts as well as the storage of interim results.



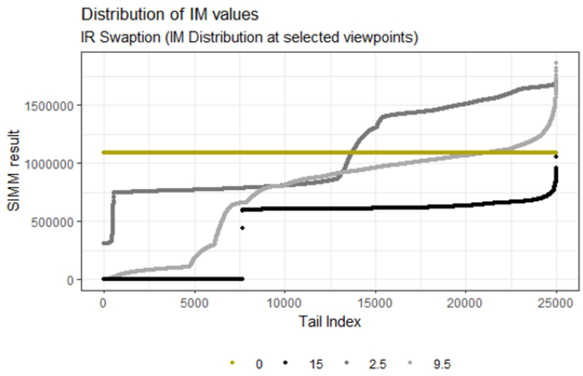


Figure 3.5: IM distribution at different viewpoints (IR swaption). *Note* This figure provides the IM distribution at different viewpoints for a EUR 10/15Y European swaption (pay fix) with a notional of EUR 12.5m and a maturity of 9.75 years.

Table 3.9: IM distribution at different viewpoints (IR swaption)

t	Maximum IM	Mean IM	Minimum IM	σ
0	1,091,836.64	1,091,836.64	1,091,836.64	0.00
1	1,516,333.09	1,140,770.64	404,364.99	241,638.45
2.5	1,722,164.02	1,099,281.62	307,778.74	371,098.20
9.5	1,863,632.11	745,787.13	15.42	402,261.43
15	1,094,110.54	437,168.95	0.00	291,696.40

Notes: This table provides the descriptive statistics for the viewpoint specific IM distributions shown in figure 3.5.

3.5.5 IR swap portfolio

The last example of the case study is a hypothetical client-clearing portfolio with a total of 108 standard IR swaps. Most positions in the portfolio are HUF IR swaps (105) referencing the HUF 6M forward curve with different maturities (up to 10 years). Additionally there are two EUR IR swaps and one USD IR swap in the portfolio. The notional weighted-average maturity of the portfolio equals 5.3 years. Given this portfolio composition, SIMM input sensitivities must be calculated for a multitude of risk factors in the IR and FX risk class. With respect to FX risk, the portfolio's main sensitivity is to the EUR/HUF exchange rate. Furthermore, a small FX sensitivity is generated by the USD IR swap. The EUR IR swaps do not carry any FX risk as their cash flows are denominated in the calculation currency (EUR). In the IR risk class, multiple sensitivities are generated to a variety of currencies, sub-yield curves and vertices.[27] The following calculations are based on market data as of 17th February 2020.

Figure 3.6 shows the distribution of IM amounts at selected viewpoints for the whole IR swap portfolio. Table 3.10 provides the corresponding descriptive statistics. The results show that

[27] For a detailed overview on the SIMM input sensitivities of the portfolio, please refer to table 3.C.5 in APPENDIX 3.C.3.

the expected (mean) IM amount decreases over time, while the standard deviation increases up to the viewpoint 5 years. In the first years, the distribution widens, driven by the diffusion effect. Over time, the amortization effect from expiry and aging of positions leads to a reduction in the expected IM as well as the standard deviation of the distribution. At $t = 9$, most of the positions have already expired and only about 20% of the initial notional remain. Furthermore, IR sensitivities decrease over time as the maturity of the cash flows shortens. These effects are captured by the KANBAN approach and lead to the IM tenor profile in figure 3.7 (right panel).

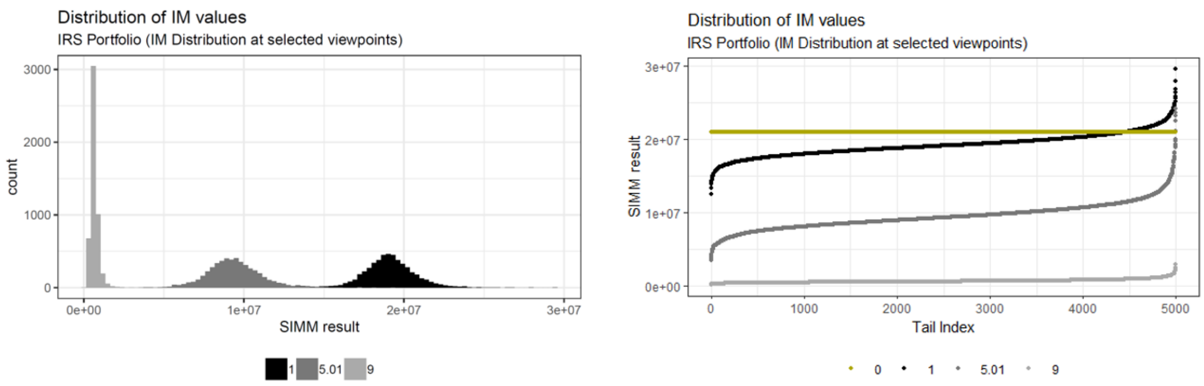


Figure 3.6: IM distribution at different viewpoints (IR swap portfolio). *Note* This figure provides the IM distribution at selected viewpoints for a hypothetical IR swap portfolio.

Table 3.10: IM distribution at different viewpoints (IR swap portfolio)

t	Maximum IM	Mean IM	Minimum IM	σ
0	20,935,343.60	20,935,343.60	20,935,343.60	0.00
1	29,559,393.07	19,167,124.97	12,475,979.17	1,543,464.37
5	24,163,812.36	9,440,757.67	3,511,105.44	1,821,634.45
9	2,851,633.90	652,914.39	126,266.27	230,929.08

Notes: This table provides the descriptive statistics for the viewpoint specific IM distributions shown in figure 3.6.

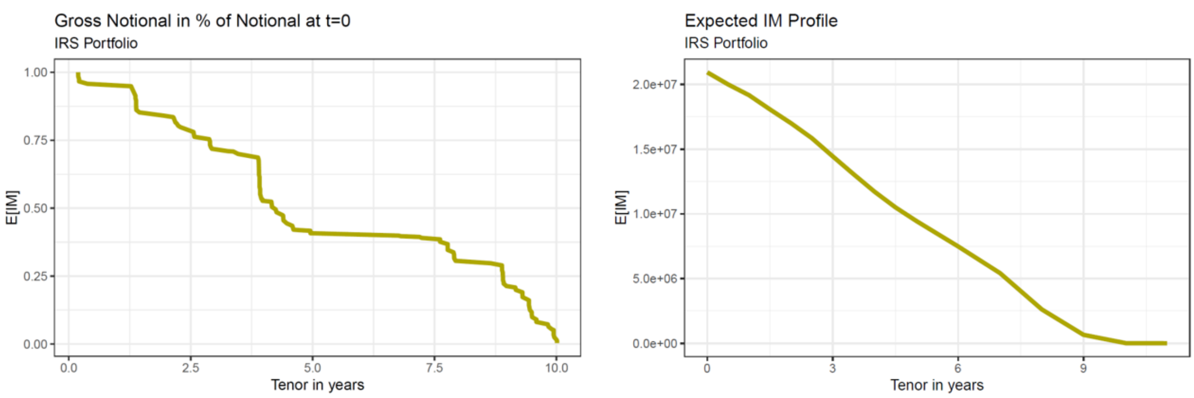


Figure 3.7: Notional and IM tenor profile for IR swap portfolio. *Note* This figure provides the notional profile (left panel) and the IM tenor profile (right panel) for a hypothetical IR swap portfolio.

Please note that the KANBAN approach calculates all of the aforementioned sensitivities on cash flow level. The aggregation and bucketing of sensitivities is performed in line with the guidelines set by ISDA (2019). Hence, our approach is capable of recognizing portfolio effects implicitly via the consideration of legal netting sets as well as the SIMM aggregation methodology.

3.6 Conclusion

This paper provides a new and innovative approach for the calculation of forward Initial Margin Requirements (IMR). Driven by emerging regulatory requirements, the majority of derivative trading activities will be supported by the exchange of initial margin going forward. We share the opinion that the standard initial margin model developed by ISDA (ISDA-SIMM™) will become the market standard for the calculation of IMR for OTC derivatives. Initial margin (IM) has a significant impact on counterparty risk exposures. Furthermore, the IM amounts and the associated funding requirements are main drivers for the profitability of derivative transactions. Hence, the exchange of initial margin needs to be considered in the calculation of counterparty risk capital and valuation adjustments for an appropriate assessment. For the consideration of IMR in an existing exposure model, future IM amounts need to be calculated. Under ISDA-SIMM™ the task of forecasting IMR reduces to the calculation of forward sensitivities.

Existing approaches for the calculation of forward sensitivities or the direct approximation of IM amounts usually involve a trade-off between accuracy and computational efficiency. We introduce a new approach for the analytical calculation of path- and time-dependent forward sensitivities in an existing Monte Carlo (MC) framework. The KANBAN approach utilizes existing information from the MC simulation as well as the representation of financial instruments as a series of cash flows for the efficient determination of SIMM forward sensitivities. Under the KANBAN approach, various sensitivities can be calculated simultaneously based on cash flows as central objects. By utilizing already availabe elements of the MC simulation, such as AMC pricing techniques, we leverage existing methodological concepts and do not need further methodological assumptions. Hence, we do not add additional model risk to the counterparty risk and valuation framework. The case study in this paper shows that the approach is capable of capturing main dynamics of sensitivities over time and leads to accurate results for linear and non-linear products. Given the efficiency and accuracy of our methodology, we believe that the KANBAN approach is of high practical relevance and offers a fast and efficient way to forecast IM requirements in an existing cash flow based MC implementation.

3.A APPENDIX | Background on modeling framework

3.A.1 Cash flow concept

Within the KANBAN approach, cash flows are the central objects for the calculation of sensitivities. Hence, the implementation of a cash flow concept is a prerequisite for the application of our approach. Within our model framework, cash flows are the atomic building blocks for the representation of financial instruments. Each trade is modeled by a series of cash flows which are used for valuation. The valuation of trades differs by trade type. Linear trades are valued directly based on their underlying cash flows. The valuation of non-linear products is performed by mapping their inner value, derived from underlying cash flows, to option values via American Monte Carlo (AMC) fitting functions.[28] Within our framework, trades are generally described by the set of attributes presented in table 3.A.1.

Table 3.A.1: Trade object

Attribute	Description	Example
ID	Unique identifier	99999
Type	Identifier for the asset class and type of trade	IR_InterestRateSwap
Notional	Notional of the trade in trade currency	10,000,000
Notional currency	Currency of the notional	USD
Maturity	Maturity of the trade	31/12/2029
Exercise dates	Collection of exercise dates for non-linear trades	31/12/2025
Strike Price	Strike price for options	0.568%
Option Type	Type of option	Long Call

Notes: This table describes the trade object used in our model framework. The first fields (above mid rule) are mandatory for all types of trades, while the last three attributes are only required for non-linear products.

For the representation of financial instruments, different types of cash flows are required. In general, we distinguish between static and variable cash flows. **Static cash flows** have a fixed amount which is not sensitive to fluctuations in market data. Hence, these cash flows do not change during the simulation and do not need to be updated for each node of the MC simulation. Nevertheless, the value of static cash flows fluctuates within the simulation due to changes in discount factors and exchange rates. Please note that this is taken into consideration when the cash flow is processed by the Market Data Service (MDS), but does not affect the representation of the cash flow itself. The payment amount of **variable cash flows** is sensitive to market data changes. Hence, the cash flow amount needs to be updated for each node of the MC simulation. A simple example is the payment amount of a floating IR cash flow which depends on the realized forward rate at each node.

[28] For additional information on the AMC implementation, please refer to APPENDIX 3.A.2.

Each cash flow is described by a set of attributes. There are attributes commonly required for all cash flows as well as specific attributes necessary for certain cash flow types. Common mandatory attributes are a unique identifier, the payment date, the notional as well as the currency of the cash flow. Table 3.A.2 provides the required data items to describe a floating interest rate cash flow.

Table 3.A.2: Example: IR floating cash flow

Attribute	Description	Category	Type	Example
Id	Unique identifier for each cash flow	Common	Mandatory	99999
cfType	Type of cash flow	Common	Mandatory	IR_Float
paymentDate	Date when cash flow is paid (T)	Common	Mandatory	31/12/2019
notional	Notional amount on which cash flow is paid	Common	Default = 0	10,000,000
currency	Currency of the cash flow	Common	Mandatory	EUR
fixingDate	Fixing date of the cash flow	Specific (IR_FLOAT)	Mandatory	01/09/2019
tenor	Period length for coupon payment	Specific (IR_FLOAT)	Default = 1	0.534
fwdCurve	Name of forward curve (reference)	Specific (IR_FLOAT)	Mandatory	EURIBOR
fwdType	Type of forward curve	Specific (IR_FLOAT)	Mandatory	6m

Notes: This table provides an example of a floating interest rate cash flow and the associated attributes.

Based on these attributes, each cash flow can be calculated and, if necessary, updated by the MDS. The MDS uses information provided by the cash flow and stored market data for the calculation of cash flow amounts and provides the respective valuation for a grid of viewpoints. Table 3.A.3 provides an example of a cash flow representation for a EUR 2Y IR Swap in which a fixed amount is payed and a floating amount received based on the 6m EURIBOR.

Table 3.A.3: Example: IR swap cash flows

Id	Type	paymentDate	notional	currency	fixingDate	tenor	rate	fwdCurve	fwdType
CF01	IR_FIX	31/12/2020	-1,000,000	EUR		1.0027	-0.003611		
CF02	IR_FIX	31/12/2021	-1,000,000	EUR		1.0000	-0.003611		
CF03	IR_FLOAT	30/06/2020	1,000,000	EUR	31/12/2019	0.5000	-0.003885	EURIBOR	6m
CF04	IR_FLOAT	31/12/2020	1,000,000	EUR	02/07/2020	0.5027	-0.003830	EURIBOR	6m
CF05	IR_FLOAT	30/06/2021	1,000,000	EUR	30/12/2020	0.4959	-0.003598	EURIBOR	6m
CF06	IR_FLOAT	31/12/2021	1,000,000	EUR	02/07/2021	0.5041	-0.003134	EURIBOR	6m

Notes: This table provides the cash flow representation of a EUR 2Y IR swap in which a fixed amount is payed and a floating amount received based on the 6m EURIBOR.

The cash flow data is utilized and interpreted by the MDS to update the cash flow (especially the rate) at each MC node. Based on the updated cash flow information, the corresponding cash flow amounts and values are calculated by the MDS and serve as the basis for the determination of SIMM sensitivities. The following paragraphs provide information on the cash flow representation for a selected set of common financial instruments.

FX Forward: FX Forwards involve a set of payment cash flows in different currencies. Both cash flows are static as the payment amounts do not depend on market data, but are fixed in the contractual terms of the underlying transaction.

EQ / CO Forward: Equity and Commodity Forwards are represented by two types of cash flows. First, there are payment cash flow which are static and based on the contractual terms of the transactions. Second, there are underlying EQ and CO cash flows which represent the delivery of the transaction's underlying.

Credit Default Swap (CDS): A CDS is decomposed into credit contingent premium and default cash flows. Both credit contingent cash flow types are variable cash flows which are updated during the simulation. The calculation of both cash flow types requires information with respect to the reference underlying as well as period start and end dates, which are stored in the cash flow object. Furthermore, the amount and value of those cash flows depend on survival probabilities derived from credit spread curves. This information is stored in the MDS.

Non-linear products: All non-linear financial instruments are first described as a set of underlying cash flows. These cash flows are used to calculate the inner value as well as the product specific pay-off profile. Hence, the inner value of a swaption is represented by the cash flows of the corresponding underling swap. The information with respect to path- and time-dependent inner values and pay-off profiles are utilized in the AMC framework (see APPENDIX 3.A.2). After valuation of the option for a combination of path and viewpoint, a pseudo cash flow is generated at each exercise date representing the expected exercise value of the trade. These cash flows are used for further processing in the MC simulation.

When running the simulation, risk factors are simulated for a given number of paths and a grid of viewpoints. The respective state variables of the risk factors are transformed into market data objects and stored in the MDS. The MDS is capable of providing the full set of required market data for each combination of path and viewpoint whenever needed during the simulation. In summary, our cash flow concept is defined by the following main aspects. First, all financial instruments are represented as a series of cash flows. Second, all cash flows carry information required for their calculation and valuation. Third, the required market data is stored in a central MDS. The MDS performs the update, calculation and valuation of cash flows by interpreting the information provided by the cash flow object in the respective context (path, time). Based on this information, we are able to calculate and allocate the ISDA-SIMM™ input sensitivities for all relevant products.

3.A.2 American Monte Carlo (AMC) implementation

Within our model framework, all non-linear products are priced by an American Monte Carlo (AMC) methodology based on Longstaff and Schwartz (2001). At each exercise date of an option, the holder needs to compare the pay-off from immediate exercise with the expected pay-off from not exercising the option to decide if the option is exercised. The calculation of conditional expectations would require a nested MC simulation, in which a new simulation is started at each exercised date for calculation of the expected pay-off under the condition that the option is not exercised. AMC techniques avoid the computational burden by using the information comprised in the existing MC paths.

In general, all non-linear product are represented by a series of cash flows which reproduce the inner value (S) of the trade (i) for each node (n,t). For example, the inner value of a swaption is given by the present value of the cash flows (CF_j) representing the underlying swap given the market realization for path n at t ($\mathcal{F}_t$).

$$S_i(t,n) = \sum_{j \in i} CF_j(t, t_{(CF)} \mid \mathcal{F}_t) \cdot DF(t, t_{(CF)} \mid \mathcal{F}_t) \tag{3.20}$$

The aim of AMC is to provide a methodology to assess values of non-linear products during the exposure simulation. This is done by generating approximation functions which assign a value to a non-linear product based on its inner value. Within this paper, we refer to these functions as fitting functions. The following paragraphs describe the AMC technique implemented within our model framework. Table 3.A.4 provides an overview of terms and variables used in this chapter.

Table 3.A.4: AMC variables

Term	Description	Symbol
Inner value	Value of the underlying	$S(t,n)$
Exercise value	Pay-off from immediate exercise at $t \in T_{EX}$	$V_i^{(ex)}(t,n)$
Continuation Value	Expected value when not exercising the option	$V_i^{(cont)}(t,n)$
Option value	Value of the option (i)	$V_i(t,n)$

Notes: This table explains different types of variables used in the AMC methodology.

To derive the respective fitting function for each trade (i) and viewpoint (t), a pre-MC simulation is performed previous to the actual exposure simulation. The pre-simulation equals a full MC simulation, in which all relevant risk factors are simulated. During the pre-simulation, the inner value for each trade is stored for each node of the pre-MC simulation. At each exercise date, the inner values are inserted in the pay-off function of the trade to calculate the exercise value.

$$V_i^{(ex)}(t_{(ex)}, n) = I\left[S(t_{(ex)}, n)\right] \tag{3.21}$$

where $I[x]$ equals the pay-off function of trade i. To derive the continuation values and exercise decisions, backward propagation is applied. While, the pre-simulation is performed for all trades together, the backward propagation needs to be done for each trade individually. The algorithm starts with the latest exercise date of the option (t_N). At t_N, no continuation value is required as the option value equals the pay-off from immediately exercising the option:

$$V_i(t_N, n) = I\left[S(t_N, n)\right] \tag{3.22}$$

In the next step, we take a look at the previous exercise date (t_{N-1}). At t_{N-1}, we know the inner value ($S(t_{N-1}, n)$) and are able to calculate the option value by discounting the exercise value at t_N:

$$V_i(t_{N-1}, n) = V_i(t_N, n) \cdot DF(t_{N-1}, t_N, n) \tag{3.23}$$

The AMC fitting functions (Θ) are calculated by fitting the inner values to option values over all paths (n) at a specific viewpoint (t). This step is done by applying regression techniques. We choose to use the inner value of the respective trade as the only explanatory variable. The inner value of a trade implicitly comprises all relevant risk factors for pricing the transaction. Hence, by using them as explanatory variables we take all risk factors affecting the transaction into account. The choice of the fitting function used for the regression of option values on inner values is a crucial issue when applying AMC techniques. Existing literature on this issue suggests using polynomial basis functions (e.g. Longstaff and Schwartz (2001), de Lima and Samanez (2016)). Within our model framework, we differ from the standard methodology. We apply a numerical method using local linear regression on clustered pairs of inner and option value. This allows us to process all kinds of non-linear products in a unified way. Furthermore, we are able to cope with complex non-monotonic pay-offs by piecewise linear fitting functions. Based on the resulting fitting functions (Θ), we are able to calculate the expected continuation values at t_{N-1}:

$$V_i^{(cont)}(t_{N-1}, n) = \Theta_{i,t}\left(S_i(t_{N-1}, n)\right) \tag{3.24}$$

The availability of the continuation value at each exercise date of the non-linear instrument allows the calculation of the corresponding option value at each exercise date. If the immediate exercise value exceeds the continuation value, the option is exercised and the value of the option at the exercise date equals the result of the pay-off function:

$$V_i(t_{N-1}, n) = I\left(S_i(t_{N-1}, n)\right) \tag{3.25}$$

If the expected continuation value is greater or equal to the immediate exercise value, the option is not exercised and the value of the option is defined as:

$$V_i(t_{N-1}, n) = V_i(t_N, n) \cdot DF(t_{N-1}, t_N, n) \tag{3.26}$$

The aforementioned procedure is performed backwards for each exercise date. The main output of the pre-simulation and backward propagation are trade- and time-specific fitting functions ($\Theta_{i,t}$), which can be used in the exposure simulation to assign values to non-linear products based on the realization of the inner value ($S_i(t,n)$). These fitting functions are utilized in the KANBAN approach for the calculation of delta and vega sensitivities for non-linear trades.

3.B APPENDIX | Derivation of sensitivity calculations

3.B.1 Delta sensitivities for linear products

According to ISDA (2019), the delta sensitivity for the risk classes Equity (EQ), Commodity (CO) and Foreign Exchange (FX) is defined as:

$$\delta_{i,x}^{(RC)} = V_i(x + \Delta_{(RC)} \cdot x) - V_i(x) \tag{3.27}$$

where V_i represents the market value of a transaction (i) as a function of the risk factor (x) and $\Delta_{(RC)}$ equals the shift size defined by ISDA (2019) for the respective risk class. For the risk class EQ, the formula is specified as:

$$\delta_{i,k}^{(EQ)} = V_i(EQ_k + \Delta_{(EQ)} \cdot EQ_k) - V_i(EQ_k) = V_i^{(shiftEQ_k)} - V_i \tag{3.28}$$

where EQ_k represents the market value of a specific equity risk factor. A prerequisite for applying the KANBAN approach is the representation of financial instruments as a series of cash flows. Hence, its value (V_i) can be expressed as a combination of cash flow values. As an example, we choose a long equity forward for one unit of a stock (k) with a maturity of T. We decompose the transaction in an equity cash flow $CF^{(EQ)}$ and a payment cash flow $(CF^{(CASH)})$. Both cash flows are paid at maturity (T) and are denominated in the calculation currency. Hence, the market value at time t is defined as the difference in value of both cash flows:

$$V_i(t) = CF^{(EQ)}(t,T) \cdot DF(t,T) - CF^{(CASH)}(T) \cdot DF(t,T) \tag{3.29}$$

where $CF^{(EQ)}(t,T)$ represents the amount of the equity cash flow and $DF(t,T)$ equals the discount factor from t to T derived from the risk-free curve of the cash flow currency. Based on our assumptions, we are able to replace $CF^{(EQ)}(t,T)$ with the forward price of the stock (k) at t and arrive at the following definition for the market value of the transaction:

$$V_i(t) = \left(F_k(t,T) - CF^{(CASH)}(T)\right) \cdot DF(t,T) \tag{3.30}$$

where the forward price $(F_k(t,T))$ is defined as:

$$F_k(t,T) = S_k(t) \cdot e^{(r-q)\cdot(T-t)} \tag{3.31}$$

In equation (3.31), r represents the risk-free rate and q equals the dividend yield. Based on the definition in equation (3.28), the shifted market value of the transaction is calculated by assuming a 1% shift of the spot price. All other risk factors (e.g. interest rates, dividend yield) remain constant. Under this assumption, shifting the spot price by 1% is equivalent to shifting the forward price by 1% (see equation (3.31)). Hence, we are able to calculate the shifted market value by:

$$V_i^{(shiftEQ_k)}(t) = \Big(1.01 \cdot F_k(t,T) - CF^{(CASH)}(T)\Big) \cdot DF(t,T) \tag{3.32}$$

Based on equation (3.30) and (3.32), the delta sensitivity of the trade (i) to its equity underlying (k) at viewpoint (t) solves for:

$$\begin{aligned}
\delta_{i,k}^{(EQ)}(t) &= V_i^{(shiftEQ_k)}(t) - V_i(t) \\
&= \Big(1.01 \cdot F_k(t,T) - CF^{(CASH)}(T) - F_k(t,T) + CF^{(CASH)}(T)\Big) \cdot DF(t,T) \\
&= 0.01 \cdot F_k(t,T) \cdot DF(t,T) \\
&= \Delta_{(EQ)} \cdot F_k(t,T) \cdot DF(t,T)
\end{aligned} \tag{3.33}$$

Equation (3.33) is equivalent to the multiplication of the Equity cash flow amount at payment date (T) with the shift size (0.01), discounted to the respective viewpoint (t):

$$\delta_{i,k}^{(EQ)}(t) = CF^{(EQ_k)}(t,T) \cdot \Delta_{(EQ)} \cdot DF(t,T) \tag{3.34}$$

This result shows that only equity cash flows ($CF^{(EQ)}$) are considered for the calculation of equity sensitivities. The payment cash flows ($CF^{(CASH)}$) do not carry any equity risk. This also holds true for the calculation of commodity sensitivities, in which only commodity cash flows have exposure to CO risk. The cash flow type information is stored in the cash flow object and interpreted by the KANBAN approach. When calculating path-and time-dependent forward sensitivities, we need to take the market realization ($\mathcal{F}$) at viewpoint (t) and path (n) into account:

$$\delta_{i,k}^{(EQ)}(t,n) = CF^{(EQ_k)}(t,T \mid \mathcal{F}_t) \cdot \Delta_{(EQ)} \cdot DF(t,T \mid \mathcal{F}_t) \tag{3.35}$$

The KANBAN approach calculates sensitivities on a cash flow basis. Hence, we aim for a more general and non-product specific formulation for the calculation of delta sensitivities on cash flow level:

$$\boxed{\delta_{j,k}^{(RC)}(t,n) = CF_j^{(RC_k)}(t,T \mid \mathcal{F}_t) \cdot \Delta_{(RC)} \cdot DF(t,T \mid \mathcal{F}_t)} \tag{3.36}$$

where $CF_j^{(RC_k)}$ is interpreted as a cash flow of type RC which is sensitive to a specific risk factor (k). By aggregating the sensitivities from all cash flows of a specific trade (i) which are sensitive to the same risk factor (k), we arrive at the trade level sensitivity:

$$\delta_{i,k}^{(RC)}(t,n) = \sum_{j \in i} CF_j^{(RC_k)}(t,T \mid \mathcal{F}_t) \cdot \Delta_{(RC)} \cdot DF(t,T \mid \mathcal{F}_t) \tag{3.37}$$

Equations (3.36) and (3.37) are used for the calculation of delta sensitivities for equity and commodity risk factors. For cash flows not denominated in the calculation currency, the resulting sensitivities need to be converted into calculation currency. In these cases, equation (3.36) extends to:

$$\delta_{j,k}^{(RC)}(t,n) = CF_{j,u}^{(RC_k)}(T \mid \mathcal{F}_t) \cdot \Delta_{(RC)} \cdot DF(t,T \mid \mathcal{F}_t) \cdot FX_u(t \mid \mathcal{F}_t) \tag{3.38}$$

where u equals a currency other than the calculation currency and $FX_u(t \mid \mathcal{F}_t)$ represents the foreign exchange rate between both currencies, given the path-dependent market realization at the respective viewpoint ($\mathcal{F}_t$).

Each cash flow denominated in a currency (u) other than the calculation currency bears FX risk. Considering the aforementioned example of an equity forward, in which both cash flows are paid in a currency other than the calculation currency, we arrive at the following valuation formula:

$$V_i(t) = \left[CF_u^{(EQ)}(t,T) - CF_u^{(CASH)}(T)\right] \cdot FX_u(t) \cdot DF_u(t,T) \tag{3.39}$$

When calculating the FX sensitivity, we shift the FX spot price at viewpoint t by $\Delta_{(FX)} = 1\%$:

$$V_i^{(shiftFX_u)}(t) = \left[CF_u^{(EQ)}(t,T) - CF_u^{(CASH)}(T)\right] \cdot (1.01 \cdot FX_u(t)) \cdot DF_u(t,T) \tag{3.40}$$

The sensitivity of the equity forward to FX_u solves for:

$$\begin{aligned}\delta_{i,u}^{(FX)}(t) &= V_i^{(shiftFX_u)}(t) - V_i(t) \\ &= \left(CF_u^{(EQ)}(t,T) \cdot 0.01 - CF_u^{(CASH)}(T) \cdot 0.01\right) \cdot DF_u(t,T) \cdot FX_u(t)\end{aligned} \tag{3.41}$$

Based on the representation of cash flows in equation (3.41), we can also calculate the time- and path-dependent FX sensitivity separately for each cash flow (j) as:

$$\delta_{j,u}^{(FX)}(t,n) = CF_j^{(FX_u)}(t,T \mid \mathcal{F}_t) \cdot 0.01 \cdot DF_u(t,T \mid \mathcal{F}_t) \cdot FX_u(t \mid \mathcal{F}_t) \tag{3.42}$$

The trade-level FX sensitivity is consequently defined as:

$$\delta_{i,u}^{(FX)}(t,n) = \sum_{j \in i} CF_j^{(FX_u)}(t,T \mid \mathcal{F}_t) \cdot 0.01 \cdot DF(t,T \mid \mathcal{F}_t) \cdot FX_u(t \mid \mathcal{F}_t) \tag{3.43}$$

Based on equations (3.36) and (3.42), we define a general formula for the calculation of delta sensitivities for a specific cash flow (j) and a risk factor of the risk classes EQ, CO and FX:

$$\boxed{\delta_{j,k}^{(RC)}(t,n) = CF_{j,u}^{(RC_k)}(t,T \mid \mathcal{F}_t) \cdot \Delta_{(RC)} \cdot DF(t,T \mid \mathcal{F}_t) \cdot FX_u(t \mid \mathcal{F}_t)} \tag{3.44}$$

3.B.2 Vertex delta sensitivities for linear products

For the risk classes interest rate (IR), credit (CR) qualified and credit non-qualified, delta sensitivities must be calculated for a prescribed set of vertices. According to ISDA (2019), the delta sensitivity for these risk classes is generally defined as:

$$\delta_{i,x} = V_i(x + 1bp) - V_i(x) \tag{3.45}$$

where x represents a single risk factor for the respective risk class. The corresponding risk factor is shifted by 1 bp. For each risk class, we apply a specific methodology for the calculation of vertex delta sensitivities, as different risk factors need to be captured by the KANBAN approach.

Interest rate risk

For interest rate (IR) risk, the vertex delta sensitivity is more specifically defined as:

$$\delta_{i,r_t} = V_i(r_t + 1bp) - V_i(r_t) \tag{3.46}$$

where r_t represents an interest rate at tenor t. Within our model framework, the series of cash flows describing a financial instrument are discounted to obtain its value. The value of a financial instrument is given by the sum of its discounted cash flows:

$$V_i(t) = \sum_{j \in i} CF_j(t,T) \cdot DF(t,T) = \sum_{j \in i} CF_j(t,T) \cdot e^{-r_{t,T} \cdot (T-t)} \tag{3.47}$$

where $CF_j(t,T)$ equals a cash flow paid at T. All cash flows are sensitive to changes in the risk-free rate of the respective cash flow currency and we need to calculate an IR sensitivity for each cash flow irrespective of its type. As the market value of the instrument is defined as the simple sum of the cash flow values, we are able to calculate the IR sensitivity from discounting separately for each cash flow. The IR sensitivity of a cash flow (j) to the risk-free rate from discounting is defined as:

$$\delta_{j,k}^{(IR)}(t) = CF_j(t,T) \cdot \left(DF^{(shiftIR_k)}(t,T) - DF(t,T) \right) \tag{3.48}$$

where k represents a specific IR risk factor $(r_{t,T})$ and $DF(t,T)$ equals the discount factor which gives the present value of cash flow (j) at viewpoint t. Consequently, the shifted discount factor is defined as:

$$DF^{(shiftIR)}(t,T) = e^{-(r_{t,T}+1bp) \cdot \Delta t} = e^{-r_{t,T} \cdot \Delta t} \cdot e^{-1bp \cdot \Delta t} = DF(t,T) \cdot e^{-1bp \cdot \Delta t} \tag{3.49}$$

where Δt is defined as the time difference in years between t and T. By inserting equation (3.49) into equation (3.48), we arrive at the following solution for the calculation of the IR sensitivity of a cash flow (j) from discounting:

$$\begin{aligned}\delta_{j,k}^{(IR)}(t) &= CF_j(t,T)\cdot\left(DF^{(shiftIR_k)}(t,T) - DF(t,T)\right)\\ &= CF_j(t,T)\cdot DF(t,T)\cdot e^{-1bp\cdot\Delta t}\ -\ CF_j(t,T)\cdot DF(t,T)\\ &= CF_j(t,T)\cdot DF(t,T)\cdot\left[e^{-1bp\cdot(T-t)}-1\right]\end{aligned} \tag{3.50}$$

We introduce a new function $\epsilon(t,T)$ representing the shift factor:

$$\epsilon(t,T) = e^{-1bp\cdot(T-t)} \tag{3.51}$$

When calculating path- and time-dependent sensitivities, the cash flow amount and discount factors have to be considered given the market realization of path n at time t. Hence, the IR sensitivity of a cash flow (j) at viewpoint (t) for a MC path (n) is defined as:

$$\boxed{\delta_{j,k}^{(IR)}(t,n) = CF_j(t,T\mid\mathcal{F}_t)\cdot DF(t,T\mid\mathcal{F}_t)\cdot[\epsilon(t,T)-1]} \tag{3.52}$$

Besides IR risk from discounting, floating IR cash flows are subject to additional IR risk. Shifting interest rates leads to changes in the forward rate and thereby to fluctuation in the amount paid on floating cash flows. We calculate the foward rate F at viewpoint t for a coupon period defined by the start (t_1) and end date (t_2) as:

$$F(t,t_1,t_2) = \frac{1}{(t_2-t_1)}\cdot\left(\frac{DF^{(fwd)}(t,t_1)}{DF^{(fwd)}(t,t_2)}-1\right) \tag{3.53}$$

where $DF^{(fwd)}$ represents the discount factor derived from the cash flow's reference curve. The interest rate sensitivity resulting from a change in the forward rate is equal to the change of the cash flow value and is defined as:

$$\delta_{j,k}^{(IR)}(t) = N_j\cdot\left[F^{(shift_k)}(t,t_1,t_2) - F(t,t_1,t_2)\right]\cdot DF(t,T)\cdot(t_2-t_1) \tag{3.54}$$

where N_j equals the notional of the cash flow (j) and $F^{(shift_k)}$ represents the shifted forward rate with respect to a specific risk factor (k). $DF(t,T)$ equals the discount factor which is used to calculate the value of the cash flow at t. According to equation (3.53), the forward rate $F(t,t_1,t_2)$ is defined based on two discount factors with different tenors. Hence, we calculate a sensitivity with respect to two discount factors for any IR_FLOAT cash flow. This calculation is based on two different shifted forward rates:

$$F_1^{(shift)}(t,t_1,t_2) = \frac{1}{(t_2-t_1)}\cdot\left(\frac{DF^{(fwd)}(t,t_1)\cdot e^{-1bp\cdot(t_1-t)}}{DF(t,t_2)}-1\right) \tag{3.55}$$

$$F_2^{(shift)}(t,t_1,t_2) = \frac{1}{(t_2-t_1)}\cdot\left(\frac{DF^{(fwd)}(t,t_1)}{DF^{(fwd)}(t,t_2)\cdot e^{-1bp\cdot(t_2-t)}}-1\right) \tag{3.56}$$

In equation (3.55), the "left" discount factor with tenor ($t_1 - t = t1$) is shifted, while the second shifted foward rate in equation (3.56) is calculated assuming a shift of the "right" discount factor with tenor ($t_2 - t$=t2). By inserting equations (3.55) and (3.53) into equation (3.54), we are able to derive the following solution for the calculation of the IR sensitivity with respect to the risk factor r_{t1}:

$$\delta^{(IR)}_{i,r_{t1}}(t) = N_j \cdot DF(t,T) \cdot \left[\frac{DF^{(fwd)}(t,t_1) \cdot (\epsilon(t,t_1) - 1)}{DF^{(fwd)}(t,t_2)} \right] \tag{3.57}$$

Taking path dependency into account, we arrive at:

$$\boxed{\delta^{(IR)}_{i,r_{t1}}(t,n) = N_j \cdot DF(t,T \mid \mathcal{F}_t) \cdot \left[\frac{DF^{(fwd)}(t,t_1 \mid \mathcal{F}_t) \cdot (\epsilon(t,t_1) - 1)}{DF^{(fwd)}(t,t_2 \mid \mathcal{F}_t)} \right]} \tag{3.58}$$

To calculate the IR sensitivity with respect to r_{t2} we need to insert equations (3.56) and (3.53) into equation (3.54). Thereby, the path- and time-dependent IR sensitivity for risk factor r_{t2} solves for:

$$\boxed{\delta^{(IR)}_{i,r_{t2}}(t,n) = N_j \cdot DF(t,T \mid \mathcal{F}_t) \cdot \left[\frac{DF^{(fwd)}(t,t_1 \mid \mathcal{F}_t) \cdot (1 - \epsilon(t,t_2))}{DF^{(fwd)}(t,t_2 \mid \mathcal{F}_t) \cdot \epsilon(t,t_2)} \right]} \tag{3.59}$$

Credit risk:

According to ISDA (2019), the sensitivity to credit (CR) risk factors is defined as:

$$\delta_{i,cs_t} = V_i(r_t, cs_t + 1bp) - V_i(r_t, cs_t) \tag{3.60}$$

where cs_t is the credit spread derived from the credit curve of the reference underlying at tenor t. In our modeling framework, credit products are represented by a series of contingent cash flows. For example, the building blocks of a standard credit default swap (CDS) are contingent premium and default cash flows. Before a credit event occurs, the market value V_i of a standard CDS is defined as the sum of the PV of the contingent premium leg and PV of the default leg:[29]

$$V^{(CDS)}_i(t) = V^{(P)}_i(t) + V^{(D)}_i(t) = \sum_{j \in i} CF^{(P)}_j(t,t_j) \cdot DF(t,t_j) + \sum_{j \in i} CF^{(D)}_j(t,t_j) \cdot DF(t,t_j) \tag{3.61}$$

where $CF^{(P)}_j(t_j)$ represents a contingent premium cash flow and $CF^{(D)}_j(t_j)$ equals a contingent default cash flow. A **contingent premium cash flow** is defined as:

$$CF^{(P)}_j(t,t_j) = N_j \cdot p \cdot (t_j - t_{j-1}) \cdot SP(t,t_j) \tag{3.62}$$

where N_j equals the notional on which the premium (p) is paid. $SP(t,t_j)$ represents the cumulative survival probability from t to t_j as defined by equation (3.64). A **contingent default cash**

[29] After a credit event has taken place, the CR sensitivities are zero. Hence, this case is not considered in the following contemplation.

flow is defined as:

$$CF_j^{(D)}(t,t_j) = N_j \cdot (1-RR) \cdot \Big[SP(t,t_{j-1}) - SP(t,t_j)\Big] \tag{3.63}$$

where RR equals the recovery rate. The cumulative survival probability is defined as:

$$SP(t,t_j) = exp\left(\frac{-cs_{t,t_j}}{1-RR} \cdot (t_j - t)\right) \tag{3.64}$$

As the value of a credit instrument is defined by the sum of its discounted cash flows (see equation (3.61)), we are able to calculate the CR sensitivity for each cash flow separately. For contingent premium cash flows, the sensitivity of a cash flow's value to a change in credit spread for tenor $(t_j - t = t1)$ is calculated by:

$$\delta_{j,cs_{t1}}^{(CR)}(t) = N_j \cdot p \cdot (t_j - t_{j-1}) \cdot \Big[SP^{(shift)}(t,t_j) - SP(t,t_j)\Big] \cdot DF(t,t_j) \tag{3.65}$$

where $SP^{(shift)}(t,t_j)$ is defined as:

$$SP^{(shift)}(t,t_j) = e^{\frac{-cs_{t,t_j}+1bp}{1-RR}\cdot(t_j-t)} = SP(t,t_j)\cdot e^{\frac{-1bp}{1-RR}\cdot(t_j-t)} = SP(t,t_j)\cdot\gamma(t,t_j) \tag{3.66}$$

By inserting equation (3.66) into equation (3.65), we arrive at the following solution for the CR sensitivity of contingent premium cash flows:

$$\begin{aligned}
\delta_{j,cs_{t_j}}^{(CR)}(t) &= N_j \cdot p \cdot (t_j - t_{j-1}) \cdot DF(t,t_j) \cdot \Big[SP(t,t_j)\cdot\gamma(t,t_j) - SP(t,t_j)\Big] \\
&= N_j \cdot p \cdot (t_j - t_{j-1}) \cdot DF(t,t_j) \cdot SP(t,t_j) \cdot \Big[\gamma(t,t_j) - 1\Big] \\
&= CF_j^{(P)}(t,t_j) \cdot DF(t,t_j) \cdot \Big[\gamma(t,t_j) - 1\Big]
\end{aligned} \tag{3.67}$$

Taking path dependency into account, we arrive at the final formulation for the CR sensitivity of a contingent premium cash flow:

$$\boxed{\delta_{j,cs_{t1}}^{(CR)}(t,n) = CF_j^{(P)}(t,t_j \mid \mathcal{F}_t) \cdot DF(t,t_j \mid \mathcal{F}_t) \cdot \Big[\gamma(t,t_j) - 1\Big]} \tag{3.68}$$

The amount and value of a contingent default cash flow depends on two tenors of the credit spread curve via the corresponding survival probabilities ($SP(t,t_j)$, $SP(t,t_{j-1})$). Hence, we calculate two CR sensitivities for each contingent default cash flow by shifting each involved survival probability separately. In the subsequent formulas we apply the definition for the shifted survival probability as presented by equation (3.66). Thereby, we arrive at the following solutions for the CR sensitivity at tenor $(t - t_j = T2)$ and $(t - t_{j-1} = T1)$ for a contingent default cash flow:

$$\boxed{\delta_{j,cs_{t1}}^{(CR)}(t,n) = N_j \cdot DF(t,t_j \mid \mathcal{F}_t) \cdot (1-RR) \cdot SP(t,t_{j-1} \mid \mathcal{F}_t) \cdot \Big[\gamma(t,t_{j-1}) - 1\Big]} \tag{3.69}$$

$$\boxed{\delta_{j,cs_{t2}}^{(CR)}(t,n) = N_j \cdot DF(t,t_j \mid \mathcal{F}_t) \cdot (1-RR) \cdot SP(t,t_j \mid \mathcal{F}_t) \cdot \Big[1 - \gamma(t,t_j)\Big]} \tag{3.70}$$

3.B.3 Delta sensitivities for non-linear products

When calculating delta and vertex delta sensitivities for non-linear products, we utilize the American Monte Carlo (AMC) implementation described in APPENDIX 3.A.2. In the pre-simulation step, AMC fitting functions are generated which are used for calculation of option values based on the inner values. This aspect is utilized when treating non-linear products in the actual exposure simulation. Within the exposure simulation, we aim for generating a cash flow at each exercise date of an option. These cash flows can be treated together with other cash flows in the exposure simulation. The amount of the cash flow at exercise date (T) equals the expected exercise value:

$$CF_i^{(EX)}(t,T) = \mathbb{E}\left[V_i(T) \mid \mathcal{F}_t\right] \tag{3.71}$$

To derive this cash flow within the exposure simulation, the following steps are performed. First, the inner value of the trade for a given viewpoint and path ($S_i(t,n)$) is calculated based on the underlying cash flow representation in line with equation (3.20). Second, the option value is calculated by evaluating the option's AMC fitting function for viewpoint t:

$$V_i(t,n) = \Theta_{i,t}\left[S_i(t,n)\right] \tag{3.72}$$

The amount of the exercise cash flow at the exercise date T is derived by compounding the option value from viewpoint t to the subsequent exercise date (T):

$$CF_i^{(EX)}(t,T,n) = V_i(t,n) \cdot \frac{1}{DF(t,T \mid \mathcal{F}_t)} \tag{3.73}$$

The generated exercise cash flow represents the expected exercise value under the market realization $\mathcal{F}_t$. Hence, the option value at viewpoint t is recursively defined as:

$$V_i(t,n) = CF_i^{(EX)}(t,T \mid \mathcal{F}_t) \cdot DF(t,T \mid \mathcal{F}_t) \tag{3.74}$$

The KANBAN method uses the relationship shown in equation (3.74) to calculate the sensitivity of the option value for discounting and FX conversion risk. The calculation of these sensitivities is based on equation (3.9) for discounting risk and equation (3.8) for FX conversion risk. This results in the following definitions for these types of sensitivities:

$$\delta_{i,r_t}^{(IR)} = CF_i^{(EX)}(t,T \mid \mathcal{F}_t) \cdot DF(t,T \mid \mathcal{F}_t) \cdot \left[\epsilon(t,T) - 1\right] \tag{3.75}$$

$$\delta_{i,u}^{(FX)} = CF_{i,u}^{(EX)}(t,T \mid \mathcal{F}_t) \cdot DF_u(t,T \mid \mathcal{F}_t) \cdot 0.01 \cdot FX_u(t \mid \mathcal{F}_t) \tag{3.76}$$

In addition to the FX conversion and IR discounting risk, the sensitivity of the option value to further risk factors needs to be calculated (e.g. EQ delta sensitivities). For the calculation of these sensitivities, we apply the following procedure. First, we determine a set of shifted inner values at viewpoint t based on sensitivities calculated in line with the methodology presented in APPENDIX 3.B.1 and APPENDIX 3.B.2. The inner value of a non-linear product at t is defined as:

$$S_i(t,n) = \sum_{j \in i} CF_j(t, t_{(CF)} \mid \mathcal{F}_t) \cdot DF(t, t_{(CF)} \mid \mathcal{F}_t) \tag{3.77}$$

Second, the KANBAN approach calculates the sensitivity for each cash flow to the corresponding risk factor. Based on these sensitivities, we are able to calculate the shifted inner value by:

$$S_{i,j}^{(shiftRC_k)}(t,n) = S_i(t,n) + \delta_{j,k}^{(RC)}(t,n) \tag{3.78}$$

Given the shifted inner value, we calculate a shifted option value by evaluating the AMC fitting function:

$$V_i^{(shiftRC_k)}(t,n) = \Theta_{i,t}\left[S_{i,j}^{(shiftRC_k)}(t,n)\right] \tag{3.79}$$

Third, we calculate the sensitivity of the non-linear instrument based on equations (3.72) and (3.79).

$$\delta_{i,k}^{(RC)}(t,n) = V_i^{(shiftRC_k)}(t,n) - V_i(t,n) = \Theta_{i,t}\left[S_{i,j}^{(shiftRC_k)}(t,n)\right] - \Theta_{i,t}\left[S_i(t,n)\right] \tag{3.80}$$

The KANBAN approach applies these steps to the whole set of cash flows representing the inner value of the option. Please note that IR sensitivities from discounting risk as well as FX sensitivities from conversion risk are not covered here, but handled by equations (3.75) and (3.76).

3.B.4 Vega sensitivities for non-linear products

Within SIMM, the definition of vega risk differs by risk class (ISDA (2019)). For **IR and CR risk**, the vega risk of a trade (i) with respect to a risk factor (k) at a specific viewpoint (t) is defined as:

$$VR_{ik}(t) = \sum_j \sigma_{kj}^{(RC)}(t) \cdot \frac{\partial V_i(t)}{\partial \sigma(t)} \tag{3.81}$$

where σ_{kj} equals the implied at-the-money volatility of a swaption with an expiry time equal to k and a swaption maturity of j. $\partial V_i / \partial \sigma$ equals the sensitivity of the price (V_i) to changes in the implied at-the-money volatility. For **EQ, CO and FX risk**, the vega risk formula is adjusted to:

$$VR_{ik}(t) = HVR_c \cdot \sum_j \sigma_{kj}^{(RC)} \cdot \frac{\partial V_i(t)}{\partial \sigma(t)} \tag{3.82}$$

where the historic volatility ratio HVR_c functions as a correction factor for the volatility estimate (σ_{kj}). The volatility (σ_{kj}) is approximated by the following formula:

$$\sigma_{kj} \approx \frac{RW_k \cdot \sqrt{365/14}}{\alpha} \tag{3.83}$$

where α equals the 99% percentile of the standard normal distribution. RW_k equals the delta risk-weight of risk factor (k) and j is interpreted as the volatility tenor equal to option expiry. Please note that the volatility (σ_{kj}) for risk classes EQ, CO and FX is not a function of time.

For the calculation of vega risk, we first need to calculate the corresponding vega risk sensitivities. According to ISDA (2019), vega risk sensitivities are defined as:

$$\frac{\partial V_i}{\partial \sigma} = V_i\left(\sigma + \Delta_{(RC)}^{(vega)}\right) - V_i(\sigma) \tag{3.84}$$

where $V_i(\sigma)$ is defined as the market value of a financial instrument given the implied volatility of the risk factor. The units of σ as well as the shift-size (Δ) differ by risk class. When calculating time-dependent sensitivities the formulation adapts to:

$$\frac{\partial V_i(t)}{\partial \sigma(t)} = V_i\left(\sigma(t) + \Delta_{(RC)}^{(vega)}\right) - V_i(\sigma(t)) \tag{3.85}$$

For the calculation of vega sensitivities, we utilize the AMC methodology already embedded in the prevailing model framework. The price of a non-linear transaction is a function of its implied volatility. In order to derive vega sensitivities, we need to apply a shift to the implied volatilities. In our model framework, each non-linear transaction is priced in the exposure simulation by AMC fitting functions. For the calculation of vega sensitivities, we construct a vega fitting function for each transaction at each viewpoint. A vega fitting function $\Psi[x]$ returns a vega sensitivity given an inner value (S) of a non-linear product (i) for a path (n) and a viewpoint (t). In the exposure simulation, we calculate the path- and time-dependent vega sensitivity by:

$$\frac{\partial V_i(t,n)}{\partial \sigma(t,n)} = \Psi_{i,t}\left[S_i(t,n)\right] \tag{3.86}$$

The vega fitting functions ($\Psi_{i,t}$) are constructed by conducting a further pre-simulation in addition to the AMC pre-simulation described in APPENDIX 3.A.2. Both pre-simulations share the same set of risk factors and random number vectors, but differ in the calibration of the model parameters. The calibration of the second pre-simulation is performed based on shifted implied volatilities. The shift of implied volatilities is performed in line with the guidelines provided in ISDA (2019). From both pre-simulations, we obtain a grid (path x viewpoint) of inner values for each non-linear product. Figure 3.B.1 shows an example of the distribution of shifted and un-shifted inner values of a European OTM 10Y/30Y swaption at the exercise date.

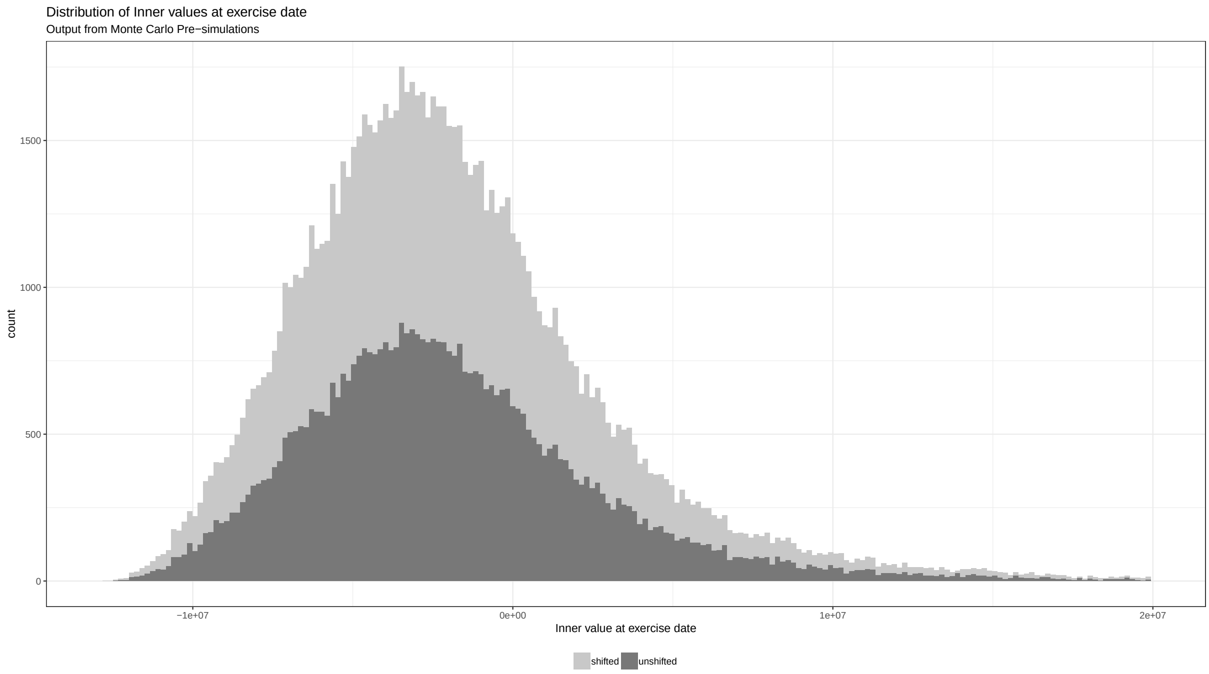


Figure 3.B.1: Output from Monte Carlo pre-simulations. *Note* This figures shows the distribution of shifted and un-shifted inner values at the exercise date for a EUR OTM swaption.

The distribution of the shifted pre-simulation is wider compared to the un-shifted pre-simulation. This can also be retrieved from the descriptive statistics in table 3.B.1.

Table 3.B.1: Descriptive statistics of pre-simulation results

Type	Min	Mean	Max	SDev
un-shifted	-12,451,343	-1,793,231	98,268,848	5,124,141
shifted	-12,463,091	-1,808,262	101,995,866	5,239,067

Notes: This table provides the descriptive statistics for the distributions shown in figure 3.B.1.

For constructing the vega fitting functions ($\Psi_{i,t}$), we need a value pair (inner value x vega effect) for each path and viewpoint. Based on the inner values at the exercise date(s), we are able to calculate the shifted and un-shifted expected pay-off for each MC path in both pre-simulations by applying the following equation:

$$V_i(t_{(ex)}, n) = I\left[S_i(t_{(ex)}, n)\right] \tag{3.87}$$

where $I[x]$ equals the pay-off function of trade i and $S_i(t_{(ex)}, n)$ represents the inner value at the exercise date for a given combination of viewpoint and path. In case of a European option with one exercise date at maturity (T), we discount the expected exercise value to viewpoint t to derive the option value at the viewpoint:[30]

$$V_i(t,n) = I\left[S(t_{(ex)},n)\right] \cdot DF(t,T) = V_i(T,n) \cdot DF(t,T) \tag{3.88}$$

Equation (3.88) also needs to be applied to shifted inner values at the exercise date. Due to the fact that the un-shifted (S_i) and shifted the inner values ($S_i^{(*)}$) at each viewpoint (t) are not equal for both pre-simulations ($S_i^*(t,n) \neq S_i(t,n)$), we have to adjust the inner value at exercise date in the shifted pre-simulation ($S_i^{(*adj)}(T,n)$) to account for this difference. Based on the adjusted inner value at the exercise date, we are able to calculate the path- and time-dependent market value of the option at viewpoint t and the associated vega effect:

$$\Delta V_i^{(vega)}(t,n) = V_i^{(*)}(t,n) - V_i(t,n) = I\left[S_i^{(*adj)}(T,n)\right] \cdot DF(t,T) - V_i(t,n) \tag{3.89}$$

By evaluating equation (3.89) for each path of the un-shifted AMC pre-simulation, we obtain a set of vega effects for each viewpoint and instrument (i). Based on these results, we fit the inner values from the un-shifted pre-simulation ($S_i(t,n)$) to the vega effects ($\Delta V_i^{(vega)}(t,n)$) over all paths of the un-shifted AMC pre-simulation at each viewpoint. The fitting process is equivalent to the procedure described in APPENDIX 3.A.2. This results in trade specific vega fitting functions ($\Psi_{i,t}[x]$) for each viewpoint of the simulation. Within the exposure simulation, these vega fitting functions are applied to calculate ISDA-SIMM™ vega senstivities for non-linear products based on the inner value realized for a combination of viewpoint and path. By generating and applying the vega fitting functions, the KANBAN approach provides time- and path-dependent vega sensitivities for the calculation of forward IM requirements.

For the calculation of vega risk according to equation (3.81) we have to scale the resulting vega sensitivities from equation (3.89) by $\sigma_{kj}(t)$. Implied volatilities at viewpoints other than the calculation date ($t \neq t_0$) are not observable. Hence, we use the implied at-the-money volatility for the corresponding option expiry at $t = t_0$ to scale vega sensitivities for the purpose of calculating ISDA-SIMM™ vega risk.

[30] The methodology for trades with multiple exercise dates follows the process described in APPENDIX 3.A.2.

3.C APPENDIX | Supplementary result data

3.C.1 Interest rate swap

The information in this chapter complements the results presented in section 3.5.3. Tables 3.C.1 and 3.C.2 provide expected values of SIMM input sensitivities for a USD 5Y IR swap with a notional of USD 10m. The sensitivities are presented for a selected set of viewpoints.

Table 3.C.1: LIBOR sensitivity over time (IR swap)

t	1m	3m	6m	1y	2y	3y	5y	10y
0.00			-0.10	14.81	15.08	92.72	4,791.21	5.22
0.55			-49.73	-389.57	32.19	1,221.42	3,124.93	
1.55			-48.61	-389.85	16.91	2,631.40	765.12	
2.05			-37.24	-400.72	164.79	2,764.23		
2.55			-48.60	-402.75	1,339.93	1,102.34		
3.05			-40.99	-305.93	1,853.89			
3.55			-42.21	383.17	662.38			
4.05			47.03	457.81				
4.45			453.65	54.27				

Notes: This table provides the expected values of IR sensitivities to the USD 3m LIBOR curve at selected viewpoints (t) for a USD 5Y IR swap (payer) with a notional of USD 10m.

Table 3.C.2: OIS sensitivity over time (IR swap)

t	2w	1m	3m	6m	1y	2y	3y	5y	10y
0.00			-0.02	-3.94	2.51	2.12	0.22	14.53	0.04
0.55			0.70	2.07	-0.01	1.53	-0.23	12.68	
1.55			0.72	2.36	-0.38	0.75	5.11	5.37	
2.05			-0.66	-1.78	1.83	-0.27	10.97		
2.55			0.62	2.33	-0.87	0.14	7.80		
3.05			-0.68	-1.85	1.04	8.13			
3.55			0.62	2.11	-1.07	4.67			
4.05			-0.68	-2.22	6.14				
4.45	-0.29	-0.12		3.53	0.42				

Notes: This table provides the expected values of IR sensitivities to the USD OIS curve at selected viewpoints (t) for a USD 5Y IR swap (payer) with a notional of USD 10m.

3.C.2 Interest rate swaption

The information in this chapter complements the results presented in section 3.5.4. Tables 3.C.3 and 3.C.4 provide expected values of SIMM input sensitivities for a EUR 10/15Y IR swaption with a notional of EUR 12.5m at selected viewpoints.

Table 3.C.3: EURIBOR sensitivity over time (IR swaption)

t	6m	1y	2y	3y	5y	10y	15y	20y	30y
0.00						-7,878.87	307.05	11,336.28	9,485.38
0.50					-449.14	-8,496.23	915.18	13,676.65	9,407.95
1.00					-1,318.50	-7,151.92	1,032.70	14,457.71	8,098.99
1.50					-2,169.04	-6,044.56	1,150.15	15,632.67	7,059.06
2.00					-2,810.02	-4,669.60	1,188.30	15,870.39	5,690.27
2.50					-3,431.35	-3,533.72	1,227.81	16,570.80	4,576.90
3.00					-3,935.02	-2,470.71	1,270.95	17,118.93	3,487.01
3.51					-4,223.11	-1,476.40	1,291.10	17,199.31	2,373.91
5.00					-4,540.16	712.10	2,084.79	16,541.69	
6.00				-1,514.30	-2,202.68	1,034.85	5,206.23	12,434.69	
7.00				-2,622.92	57.15	1,076.96	7,926.35	8,610.30	
8.00			-1,459.47	-432.06	507.62	1,096.00	10,245.41	5,156.86	
9.00		-642.03	-191.65	114.12	498.16	1,122.24	12,320.10	2,126.87	
9.50	-139.24	-133.63	27.64	133.38	473.48	1,064.78	12,748.70	698.32	
10.51	-155.94	-152.45	55.26	132.29	492.15	3,110.03	12,290.14		
12.50	-151.94	-155.72	49.80	118.18	516.37	7,543.18	5,706.64		
15.00	-152.59	-152.09	54.86	125.04	954.74	10,158.10			
17.51	-159.09	-138.75	56.71	130.92	4,683.69	3,690.89			
20.00	-153.64	-154.89	49.21	731.02	4,972.41				
22.50	-153.57	-164.74	2,045.83	756.73					

Notes: This table shows the expected values of IR sensitivities to the 6m EURIBOR curve at selected viewpoints (t) for a EUR 10/15Y swaption with a notional of EUR 12.5m and maturity of 9.75 years.

Table 3.C.4: OIS sensitivity over time (IR swaption)

t	1m	3m	6m	1y	2y	3y	5y	10y	15y	20y	30y
0.00							-96.12	-1,971.39	-398.36	-521.47	-92.96
0.50							-272.68	-1,748.51	-512.66	-626.35	-96.33
1.00							-427.93	-1,567.60	-621.13	-717.32	-94.63
1.50							-570.05	-1,420.33	-717.83	-782.01	-85.71
2.00							-698.82	-1,277.55	-764.49	-775.81	-67.23
2.50							-816.43	-1,152.19	-810.36	-762.24	-49.39
3.00							-918.11	-1,045.70	-859.66	-745.13	-34.08
3.51							-1,006.50	-930.01	-872.37	-682.35	-19.12
5.00						-113.70	-1,034.83	-749.50	-939.72	-477.03	
6.00						-474.77	-601.12	-803.51	-956.88	-322.90	
7.00					-137.25	-472.48	-370.49	-840.39	-922.08	-187.66	
8.00				-91.54	-295.75	-86.97	-397.77	-869.30	-840.25	-85.26	
9.00			-84.98	-97.31	-34.90	-96.88	-413.95	-882.31	-700.55	-21.25	
9.50	-0.23	-56.72	-3.38	-14.11	-35.14	-96.25	-401.16	-845.79	-585.19	-4.69	
10.51		-1.64	-3.89	-15.66	-39.32	-101.29	-414.63	-850.54	-398.02		
12.50		-2.25	-4.63	-18.60	-42.97	-105.87	-418.04	-707.95	-114.39		
15.00		-3.33	-3.72	-20.35	-45.67	-111.98	-405.95	-365.95			
17.51		-2.68	-5.28	-21.35	-48.16	-106.73	-310.57	-78.41			
20.00		-3.58	-3.85	-20.69	-45.88	-110.66	-91.77				
22.50		-2.74	-5.32	-21.38	-40.05	-6.40					

Notes: This table shows the expected values of IR sensitivities to the OIS curve at selected viewpoints (t) for a EUR 10/15Y swaption with a notional of EUR 12.5m and maturity of 9.75 years.

3.C.3 Interest rate swap portfolio

The information in this chapter complements the results presented in section 3.5.5. Table 3.C.5 provides the SIMM input sensitivities at $t = 0$ for the hypothetical IR swap portfolio containing 108 IR swaps.

Table 3.C.5: SIMM - Sensitivity inputs (IR swap portfolio)

ProductClass	RiskType	Qualifier	Bucket	Label1	Label2	Amount
RatesFX	Risk_FX	HUF				105,631.26
RatesFX	Risk_FX	USD				-3,954.42
RatesFX	Risk_IRCurve	EUR	1	3MForward	3m	84.14
RatesFX	Risk_IRCurve	EUR	1	3MForward	6m	324.99
RatesFX	Risk_IRCurve	EUR	1	3MForward	1y	-1,626.60
RatesFX	Risk_IRCurve	EUR	1	3MForward	2y	-1,325.53
RatesFX	Risk_IRCurve	EUR	1	6MForward	6m	-38.74
RatesFX	Risk_IRCurve	EUR	1	6MForward	1y	-168.71
RatesFX	Risk_IRCurve	EUR	1	6MForward	2y	-4.33
RatesFX	Risk_IRCurve	EUR	1	6MForward	3y	-16.77
RatesFX	Risk_IRCurve	EUR	1	6MForward	5y	589.63
RatesFX	Risk_IRCurve	EUR	1	6MForward	10y	4,321.92
RatesFX	Risk_IRCurve	EUR	1	OIS	3m	-0.29
RatesFX	Risk_IRCurve	EUR	1	OIS	6m	-2.51
RatesFX	Risk_IRCurve	EUR	1	OIS	1y	-6.31
RatesFX	Risk_IRCurve	EUR	1	OIS	2y	1.77
RatesFX	Risk_IRCurve	EUR	1	OIS	3y	11.27
RatesFX	Risk_IRCurve	EUR	1	OIS	5y	14.85
RatesFX	Risk_IRCurve	EUR	1	OIS	10y	-6.37
RatesFX	Risk_IRCurve	HUF	1	6MForward	3m	-0.00
RatesFX	Risk_IRCurve	HUF	1	6MForward	6m	1,291.90
RatesFX	Risk_IRCurve	HUF	1	6MForward	1y	3,721.47
RatesFX	Risk_IRCurve	HUF	1	6MForward	2y	6,379.05
RatesFX	Risk_IRCurve	HUF	1	6MForward	3y	12,175.85
RatesFX	Risk_IRCurve	HUF	1	6MForward	5y	-33,359.02
RatesFX	Risk_IRCurve	HUF	1	6MForward	10y	-188,378.68
RatesFX	Risk_IRCurve	HUF	1	6MForward	15y	-38.21
RatesFX	Risk_IRCurve	HUF	1	OIS	1m	-9.87
RatesFX	Risk_IRCurve	HUF	1	OIS	3m	-40.18
RatesFX	Risk_IRCurve	HUF	1	OIS	6m	42.97
RatesFX	Risk_IRCurve	HUF	1	OIS	1y	-425.02
RatesFX	Risk_IRCurve	HUF	1	OIS	2y	-702.86
RatesFX	Risk_IRCurve	HUF	1	OIS	3y	-873.76
RatesFX	Risk_IRCurve	HUF	1	OIS	5y	365.57
RatesFX	Risk_IRCurve	HUF	1	OIS	10y	1,028.61
RatesFX	Risk_IRCurve	HUF	1	OIS	15y	-0.17
RatesFX	Risk_IRCurve	USD	1	3MForward	3m	-114.89
RatesFX	Risk_IRCurve	USD	1	3MForward	6m	-166.09
RatesFX	Risk_IRCurve	USD	1	3MForward	1y	14.06
RatesFX	Risk_IRCurve	USD	1	3MForward	2y	43.15
RatesFX	Risk_IRCurve	USD	1	3MForward	3y	2,444.69
RatesFX	Risk_IRCurve	USD	1	3MForward	5y	5,539.69
RatesFX	Risk_IRCurve	USD	1	OIS	3m	1.41
RatesFX	Risk_IRCurve	USD	1	OIS	6m	0.54
RatesFX	Risk_IRCurve	USD	1	OIS	1y	6.62
RatesFX	Risk_IRCurve	USD	1	OIS	2y	17.87
RatesFX	Risk_IRCurve	USD	1	OIS	3y	41.59
RatesFX	Risk_IRCurve	USD	1	OIS	5y	29.82

Notes: This table shows the SIMM sensitivity inputs for a example portfolio of 108 IR swaps at $t = 0$. All values in calculation currency (EUR).

Conclusion

Summary

This thesis amends the literature on modeling counterparty credit risk exposures. The Global Financial Crisis (GFC) has led to a major reform of the regulatory framework for counterparty risk and the OTC derivatives market. The new regulatory requirements lead to challenges in the modeling of counterparty credit risk exposures (see Introduction). The independent research papers presented in this thesis analyze and tackle three selected issues resulting from the introduction of the new supervisory standardized approach for measuring CCR exposures (SA-CCR) and the mandatory exchange of initial margin (IM) for a multitude of non-centrally cleared derivatives. The first research paper *Credit Exposure under SA-CCR: Fixing the treatment of equity options* (see chapter 1) reveals weaknesses and flaws in the methodology and calibration of the SA-CCR regarding the treatment of equity options by performing a profound methodological and empirical analysis. Based on these findings, measures for improving the SA-CCR are proposed.

Within the second and third research paper, innovative approaches are developed which complement the toolbox for counterparty risk management. The second research paper *Computing valuation adjustments for CCR using a modified supervisory approach* (see chapter 2) utilizes and modifies the SA-CCR's model framework to develop a new semi-analytical method for the generation of exposure profiles for CVA calculation. The modified SA-CCR offers a fast and simple approach for the treatment of derivatives which are not covered by advanced exposure models. The third research paper *The KANBAN approach - A new way to compute forward Initial Margin* (see chapter 3) introduces an innovative method for forecasting future IM requirements under ISDA-SIMM™ based on forward sensitivities. The KANBAN approach utilizes information and methodological building blocks already available in a Monte Carlo framework and adopts principles from industrial just-in-time manufacturing. Hence, the approach is capable of generating fast and accurate forward sensitivities.

Discussion and outlook

This thesis addresses highly topical methodological issues regarding the measurement of counterparty credit risk exposures. The calculation of credit exposure for derivatives transactions is an important task for financial institutions for several reasons. First, exposure results are used for various applications, such as derivatives pricing, assessment of capital requirements and limitation of counterparty risk (see Introduction). Second, the global OTC derivatives market is still highly significant, comprising a global gross credit exposure (before collateralization) of USD 2.7tr (BCBS (2019c)). Third, methods for measuring counterparty credit risk are subject to ongoing development and innovation driven by changes in market conventions and regulatory reforms. There is vibrant academic research and industry discussion on a wide range of issues regarding the modeling of CCR exposure emerging from changes in the regulatory and market environment. This thesis covers selected issues with high practical relevance.

In particular, the introduction of the new regulatory standardized approach (SA-CCR) and its impacts is an important issue. The SA-CCR will be broadly applied and its results are used as inputs for other regulatory measures. Hence, weaknesses and flaws in the methodology of the SA-CCR are not an isolated issue, but propagate through the regulatory framework and significantly affect the majority of financial institutions. The results presented in the study on equity options indicate that the calibration of the SA-CCR is overly conservative and the treatment of non-linear products is flawed. Possibilities for the improvement of the SA-CCR are identified, which can be implemented without changing its underlying model foundations. An expansion of the empirical study presented in chapter 1 to further asset classes and types of non-linear products could help to further validate and shape the proposed measures for improvement. However, there is already clear evidence that regulators should consider a careful review of the SA-CCR calibration and methodology before applying the approach for the calculation of regulatory capital requirements.

The consideration of credit valuation adjustments (CVA) in the valuation of derivatives has become market standard. Hence, the calculation of exposure profiles as basis for the determination of valuation adjustments is an important task for financial institutions. In particular, small- and medium-sized market participants are not able to maintain advanced models for the calculation of CCR exposures. This leads to a demand for semi-analytical approaches (see Introduction). The utilization of the SA-CCR for the calculation of exposure profiles is an option worth considering for several reasons. First, the SA-CCR comprises various advantages and improvements compared to its predecessors. Second, the supervisory approach offers a holistic, transparent and accessible framework for the modeling of CCR exposures. Third, most banking institutions must implement the SA-CCR irrespectively of having an approved IMM model (see Introduction). After applying several modifications and performing a risk-neutral calibration, the resulting modified SA-CCR is capable of generating exposure profiles for the calculation of valuation adjustments. In summary, the modified SA-CCR offers a simple, flexible and fast alternative to existing semi-analytical models. An extension of the empirical study in research paper 2 to further asset classes and more complex financial instruments is recommended to ensure the appropriateness of the modified SA-CCR for its practical application. Furthermore, there should be additional work to improve the calibration for out-of-the money transactions.

The exchange of Initial Margin (IM) and its impact on capital and funding costs significantly affects the credit exposure and profitability of derivative transactions. Hence, the recognition of IM in the calculation of CCR exposures is a crucial prerequisite for the appropriate valuation of derivatives and correct monitoring of counterparty risk limits. However, the consideration of IM does not only require the IM amounts at the calculation date, but also at future points in time. Forecasting IM requirements is a highly complex and laborious task associated with a multitude of methodological issues. The KANBAN approach presented in chapter 3 offers a new and efficient method for forecasting IM requirements under ISDA-SIMM™. The approach is easy to implement in an existing Monte Carlo framework and enables a lean generation of path- and time-dependent forward sensitivities as well as associated IM requirements ”on-the-fly”. Given that ISDA’s standard model is expected to become market standard, the KANBAN approach is of high practical relevance. The empirical study in the respective research paper focusses on results for forward sensitivities, IM distribution and tenor profiles. An expected IM profile might provide valuable insights for the calculation of margin valuation adjustments (MVA) and liquidity risk management, where collateral in- and outflows are modeled. An extension of the KANBAN approach to these areas of applications might provide additional benefits for financial institutions.

In addition to the topics covered in this thesis, there are further prevailing issues and areas of research in the field of counterparty credit risk. As the exchange of initial margin (IM) for OTC derivatives is becoming mandatory for the majority of transactions, there are open questions regarding the actual economic impact of IM on credit exposure and the valuation of derivatives. For example, Andersen et al. (2017b) and Andersen et al. (2017a) analyze the impact of IM on credit exposure and CVA. Roberson (2018) takes a deeper look at the effects of IM on credit exposure under SA-CCR. Furthermore, there is a series of literature discussing the consideration of initial margin in pricing derivatives via the calculation of Margin Valuation Adjustments (MVA) (see, e.g. Green and Kenyon (2015), Antonov et al. (2017)). Given the increasing capital requirements for OTC derivatives, the recognition of cost of capital in the price of derivatives via capital valuation adjustments (KVA) has become an important and heavily discussed topic (see, e.g., Green et al. (2014), Green and Kenyon (2014)). In the last years, the rapid technological progress and sharp increase in computational performance has led to a renaissance of advanced statistical learning. This has resulted in research activities and studies on the application of machine learning techniques in the area of counterparty risk and derivatives valuation. In particular, the application of artificial neuronal networks for the calibration of pricing models is a focus of scientific research aiming to speed up the operational processes (see, e.g. Liu et al. (2019), Jian-Huang and Grecu. (2018), Horvath et al. (2019)). In addition, there is emerging literature on the calculation of valuation adjustments and initial margin based on advanced machine learning techniques (see, e.g., Henry-Labordere (2017), Welack (2019)). In general, the application of advanced statistical learning methods in the areas of counterparty risk and derivatives valuation is an interesting and promising field of research. However, further analysis and studies are required to identify use cases for a reasonable and beneficial practical application.

The measurement of counterparty risk exposures is an integral task in risk management and derivatives pricing. An accurate quantification of exposure requires the application of advanced statistical methods and is considered a complex task. Over the past decades, there has been a wide range of research leading to constant development and improvement of methods and approaches for CCR exposure modeling. New methodological challenges and opportunities in the area of counterparty risk arise constantly due to regulatory reforms, technological progress and changes in the behaviour of market participants. The new regulatory requirements imposed as a response to the Global Financial Crisis (GFC) provide a significant contribution to safeguard the financial system. Nevertheless, changes to the regulatory framework should be accompanied by a sound understanding of methodology, careful calibration of approaches as well as transparency regarding their results and impact. Profound academic research can provide a major contribution to the improvement of risk management practices and thereby help to ensure the stability and resilience of financial institutions and the economic system.

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