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Credit Risk:  
Evidence from Bankruptcies in Germany**

Daniel Rösch

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Dr. Daniel Rösch  
Department of Statistics  
Faculty of Business & Economics  
University of Regensburg  
93040 Regensburg (Germany)

Lehrstuhl für Statistik  
Wirtschaftswissenschaftliche Fakultät  
Universität Regensburg  
93040 Regensburg

Phone: +49 - 941 943 2752

FAX: +49 - 941 943 4936

Email: [Daniel.Roesch@wiwi.uni-regensburg.de](mailto:Daniel.Roesch@wiwi.uni-regensburg.de)

Internet: <http://www.danielroesch.de>

DANIEL RÖSCH

## **CORRELATIONS AND BUSINESS CYCLES OF CREDIT RISK: EVIDENCE FROM BANKRUPTCIES IN GERMANY**

Dr: Daniel Rösch (Daniel.Roesch@wiwi.uni-regensburg.de),

Department of Statistics, Faculty of Business and Economics, University of Regensburg, 93040 Regensburg, Germany, Phone: +49-941-943-2752, Fax : +49-941-943-4936

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### **1 Introduction**

In the last decade, credit risk modeling has become one of the most important topics in the field of finance and banking. The needs for a better understanding and dealing with default risky securities have been re-enforced by the BASEL COMMITTEE ON BANKING SUPERVISION (1999a, 1999b, 2000, 2001a, 2001b, 2003) who proposed a revision of the standards for banks' capital requirements.

One great challenge in analyzing credit risk is the issue of co-movements in default events. In many credit models, such as CreditMetrics, CreditRisk+, CreditPortfolioManager or CreditPortfolioView, not only default probabilities, but also correlations are important input parameters.[1] Value-at-Risk calculations are very sensitive due to changes in these items. Secondly, co-movements of obligors' credit qualities may stress a bank's liquidity, especially in times of economic downturns.[2] While much is known about default risk on individual basis [3], empirical studies on correlations between borrowers and cyclical patterns in default rates are rather scarce. The BASEL COMMITTEE ON BANKING SUPERVISION (1999a) states that – contrary to market risk models - data bases for analyzing credit risk are frequently insufficient, in particular for non-traded obligors.

By now, the main direction of modeling default correlations has its origin in the seminal model due to MERTON (1974, 1977) and BLACK/SCHOLES (1973). There it is assumed that the default event happens if the value of the assets of a firm falls short of the value of its liabilities. *Asset* correlations are introduced by modeling the common probability that two borrowers go into default simultaneously. The *default* correlation can be analytically determined by the threshold model.

It is possible to introduce a model which relates asset returns to one or more systematic risk factors. These factors are common to all obligors within a risk bucket and drive their credit risk into

the same direction. Thus, they are responsible for the co-movements or correlations. The factors may be assumed to be observable as in WILSON (1997a, 1997b) who uses macroeconomic sources or as in the CreditMetrics model which uses stock market indices, or they may be assumed to be latent themselves as in FINGER (1998, 1999), BELKIN/SUCHOWER/FOREST (1998a, 1998b), or CREDIT SUISSE FINANCIAL PRODUCTS (1997).

The framework of the New Basel Capital Accord is built on such kind of model which attributes these co-movements to one common random factor, whereas the factor itself remains unspecified. The exposure to the factor determines the asset correlation. GORDY (2000, 2003), FINGER (2001), or WILDE (2001) provide overviews of the model. The advantages of the model are its robustness and simplicity and the consequence of portfolio invariance of capital charges. Besides probability of default, the asset correlation becomes a key factor for regulatory capital requirements.

Empirical literature on asset correlations and the identification of common factors for credit risk is rather scarce, mainly due to data limitations. Regarding estimation of correlations broadly two approaches exist. Firstly, asset correlations may be estimated from observable market data as it is done by CreditMetrics or KMV who approximate asset return correlations by equity return correlations. This approach requires the subsistence of market data for the borrowers under consideration. The second class treats asset returns as latent variables and estimates asset correlations implicitly by observable default data of rating grades as it is done in LUCAS (1995), GORDY (2000), and GORDY/HEITFIELD (2000). This approach proves to be useful especially for non-marketable assets for which prices or returns cannot be observed.

Regarding risk factors, PEDROSA/ROLL (1998) analyse credit spread changes of index portfolios of bonds and find co-movements, COLLIN-DUFRESNE/GOLDSTEIN/MARTIN (2001) investigate the impact of several factors such as change of spot rates, change in slope of yield curve and changes in business climate on credit spread changes. After controlling for these factors a large part of co-movements still remains and seems to be dominant. Using principal component analysis they conclude that the omitted explanatory variables are likely to be not firm-specific, i.e. they are systematic. Some studies use rating transition matrices from Rating Agencies and analyze their dependence on the business cycle. Most of them, among others NICKELL/PERRAUDIN/VAROTTO (2000), find that rating transitions depend on macroeconomic conditions for which the authors use GDP growth as a proxy variable. They also provide a survey on related studies.

The present paper makes contributions to the existing research in three ways. First, further evidence on co-movements of default risk and the magnitudes of asset correlations is presented. We use a database of more than on average 2,000,000 German enterprises from year 1980 until 2001. We consider “pure” bankruptcy risk and analyze the numbers of firms which go bankrupt in each year, i.e. we use a default-mode model framework. Thus, effects of recovery rates which drive credit spreads are prevented. Moreover, the interpretation of modeling and estimation using actual defaults should be more straightforward than analyzing rating transitions since rating grades are not directly linked to default probabilities (see e.g. NICKELL/PERRAUDIN/VAROTTO, 2000). Then, we show how to find proxies for the systematic factors and present the results for various industries. We find that much of the co-movements can be explained by our business

cycle proxies. At last we demonstrate the consequences for Value at Risk calculations using a default-mode model framework.

The paper is organized as follows. The next section describes the model. Section 3 presents the results for the history of German bankruptcies and for several industry sectors. Section 4 discusses some practical implications and section 5 provides some remarks on potential directions for future research.

## 2 The Model

### 2.1 Model Outline

The model which we use is a variant of the two-state one factor return generating model from Credit Metrics which is employed in the framework of Basel II for calibrating risk weights. The two states are referred to as “default” vs. “non-default”. The discrete-time process for the normalized return  $R_{it}$  on a firm  $i$ 's assets at time  $t$  is assumed to follow a one-factor model of the form

$$R_{it} = \sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} \quad (1)$$

where

$$F_t \sim N(0,1), \quad U_{it} \sim N(0,1)$$

( $i=1,\dots,N_t$ ,  $t=1,\dots,T$ ) are normally distributed with mean zero and standard deviation one. Idiosyncratic shocks  $U_{it}$  are assumed to be independent from the systematic factor  $F_t$  (which influences all borrowers jointly) and independent for different borrowers. All random variables are serially independent. The exposure to the common factor is denoted by the square root of  $\rho$ . Under these assumption the correlation between the normalized asset returns of any two borrowers is  $\rho$ . [4] In the Basel II proposal as of January 2001 this correlation is set to 0.2. As in GORDY (2000) we assume that borrowers can be grouped into homogenous risk buckets, i.e. rating grades or industry sectors. In each bucket a borrower defaults at time  $t$  if his return falls short of some threshold  $\beta_0$ , i.e.

$$R_{it} < \beta_0 \Leftrightarrow Y_{it} = 1 \quad (2)$$

( $i=1,\dots,N_t$ ,  $t=1,\dots,T$ ), where  $Y_{it}$  is an indicator variable with

$$Y_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults at time } t \\ 0 & \text{else} \end{cases}$$

The probability of default at time  $t$  for borrower  $i$  within a given bucket is then [5]

$$\lambda = P(Y_{it} = 1) = P(R_{it} < \beta_0) = P(\sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} < \beta_0) = \Phi(\beta_0) \quad (3)$$

where  $\Phi(\cdot)$  denotes the cumulative standard normal distribution function. Conditional on a realization  $f_t$  of the common random factor at time  $t$  the default probability becomes

$$\lambda(f_t) = P\left( U_{it} < \frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right) = \Phi\left( \frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}} \right) \quad (4).$$

As described in FINGER (1998), the realization  $f_t$  of the factor can be interpreted as a kind of abstract “macroeconomic condition”. Thus, this factor can be seen as an aggregation of unobservable systematic forces which drive default probabilities and defaults jointly into the same direction. In “good years” - that is, a positive factor realization - the conditional default probabilities decrease whereas they increase in “bad years”. Conditional on the realization of the random factor defaults are independent between borrowers. The number of defaults  $D(f_t)$  at time  $t$  for a given number  $N_t$  of enterprises is (conditional) binomial with probability  $\lambda(f_t)$ , i.e.

$$D(f_t) \sim B(N_t, \lambda(f_t))$$

where  $B(\cdot)$  denotes the Binomial distribution, see e.g. GORDY/HEITFIELD (2000). The unconditional default probability can be obtained by

$$\int_{-\infty}^{+\infty} \lambda(f_t) \varphi(f_t) df_t$$

where  $\varphi(\cdot)$  denotes the density function of the standard normal distribution.

Model (4) assumes that there is a default threshold which is time invariant and, thus, that there is an unconditional default probability which is constant over the time period under consideration. [6] A more advanced specification is to model time-varying default probabilities. This is done by including systematic observable risk factors, i.e. macroeconomic key figures. [7] Let

$$\mathbf{z}_t = (z_{1t}, \dots, z_{Kt})'$$

denote a  $K$ -vector of risk factors and

$$\boldsymbol{\beta} = (\beta_1, \dots, \beta_K)'$$

a vector of sensitivities with regard to these factors. [8] Then within a segment the average probability of default, conditional on the observable risk factors is

$$\lambda(\mathbf{z}_t) = P(\sqrt{\rho} F_t + \sqrt{1-\rho} U_{it} < \beta_0 + \boldsymbol{\beta}' \mathbf{z}_t) = \Phi(\beta_0 + \boldsymbol{\beta}' \mathbf{z}_t) \quad (5).$$

Thus, the default probability depends on the state of the economy which is represented by the variables in the vector  $\mathbf{z}_t$ . A positive sensitivity with respect to a factor leads to a higher default probability when the factor increases et vice versa. Again, conditional on a realization  $f_t$  of the random factor the default probability is

$$\lambda(f_t, \mathbf{z}_t) = \Phi\left(\frac{\beta_0 + \boldsymbol{\beta}' \mathbf{z}_t - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right) \quad (6).$$

The realization of the random factor captures the effects of factors not included in the model, or the remaining asset correlation, respectively. The goal of including observable systematic factors is to make the state of the economy concisely interpretable. While the random F-factor only provides information about a “good” or “bad” somewhat abstract economic surrounding, the observable z-factors provide evidence on which risk factors may be responsible for the state of the credit cycle and finally on the fluctuations of default probabilities.

For a given time series of defaults and macroeconomic variables the parameters  $\boldsymbol{\beta}$ ,  $\beta_0$  and the asset correlation in model (4) and model (6) can be estimated by Maximum Likelihood as described below.

## 2.2 Estimation

Suppose one has observed a time series of defaults

$$D_t = \sum_{i=1}^{N_t} Y_{it}$$

and numbers  $N_t$  of borrowers ( $t=1, \dots, T$ ) for a given segment (for example an industry sector). For a given realization of the random factor in  $t$  the defaults are independent, that is, within a homogenous segment the number of defaults is conditional binomial distributed with  $N_t$  borrowers and conditional default probability  $\lambda(f_t)$ . To get the unconditional distribution one has to integrate over the random factor. Since the factor is independently and identically distributed over time, the marginal log-likelihood function for the observed time series  $(D_1, \dots, D_T)$  and  $(N_1, \dots, N_T)$  depends only on the parameters  $\beta_0$  and  $\rho$  and is

$$l(\beta_0, \rho) = \sum_{t=1}^T \ln \left\{ \int_{-\infty}^{\infty} \binom{N_t}{D_t} \Phi\left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)^{D_t} \cdot \left[1 - \Phi\left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)\right]^{(N_t - D_t)} \varphi(f_t) df_t \right\}$$

where

$$\Phi\left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)$$

is the conditional default probability given by (4). If covariates such as macroeconomic risk factors are included, the log-likelihood additionally depends on the parameter vector  $\boldsymbol{\beta}$  and becomes

$$l(\beta_0, \boldsymbol{\beta}, \rho) = \sum_{t=1}^T \ln \left\{ \int_{-\infty}^{\infty} \binom{N_t}{D_t} \Phi\left(\frac{\beta_0 + \boldsymbol{\beta}' \mathbf{z}_t - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)^{D_t} \cdot \left[ 1 - \Phi\left(\frac{\beta_0 + \boldsymbol{\beta}' \mathbf{z}_t - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right) \right]^{(N_t - D_t)} \varphi(f_t) df_t \right\}$$

where

$$\Phi\left(\frac{\beta_0 + \boldsymbol{\beta}' \mathbf{z}_t - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)$$

denotes the conditional default probability given by (6).

As an extension of common logit or probit models an important part of the method is the integral over the random effect. The integral approximation can for example be conducted by the adaptive Gaussian quadrature as it is described in PINHEIRO/BATES (1995). Usually this log-likelihood function is numerically optimized with respect to the unknown parameters for which several algorithms, such as the Newton-Raphson method, exist and are implemented in statistical software packages.

Note that the correlation and the betas are the relevant parameters which one is interested in to estimate. Though the realizations of the unobserved random factors  $f_t$  can be calculated via empirical Bayes estimates, in opposite to the  $z$ -factors they do not exhibit concrete economic content other than when a year was a “good year” (i.e. the realization was positive) or when a year was a “bad year” (i.e. the realization was negative).

Thus, the main difference between model (4) and model (6) is the following. Whereas in model (4) fluctuations of default probabilities are modeled via an unobservable systematic risk factor, model (6) asserts that default probabilities can be modeled time-dependent as functions of concrete observable covariates, i.e. the systematic observable macroeconomic risk factors. Thus, with model (6) observable credit cycles can be mapped.

Moreover, in model (4) the time-variation of the default probabilities is captured by the variation of the random effect. If cyclical patterns can be identified and proxied by macroeconomic variables as in model (6) the variation of the random effect, i.e. the asset correlation should be diminished.

In addition, we allow for the macroeconomic factors to be lagged. That means that some or all risk factors will affect defaults in the next period, or, the other way round, this years defaults are the result of macroeconomic conditions of past years. The advantage of including lagged factors

lies in the reduction of uncertainty when forecasts are established since the risk factors themselves do not have to be modeled or predicted. We will address this point in sections 3 and 4.

### 2.3 Forecasting Defaults

Given the parameters of the models a default distribution, i.e. the distribution of the potential numbers of defaulting companies for the next period  $T+1$  (e.g. one year), can be calculated as it is shown in VASICEK (1987). If model (4) with constant default probability is used the probability distribution for the number  $D_{T+1}$  of defaulting companies within a risk segment, given the number  $N_{T+1}$  of companies in this segment at the beginning of the period is

$$\begin{aligned}
 & P(D_{T+1}) \\
 = & \begin{cases} \binom{N_{T+1}}{D_{T+1}} \cdot \int_{-\infty}^{+\infty} [\lambda(f_{T+1})]^{D_{T+1}} \cdot [1 - \lambda(f_{T+1})]^{(N_{T+1} - D_{T+1})} \phi(f_{T+1}) df_{T+1} & D_{T+1} = 0, 1, 2, \dots, N_{T+1} \\ 0 & \text{else} \end{cases} \\
 & (7)
 \end{aligned}$$

where  $\lambda(f_{T+1})$  is defined analogously to (4). This distribution depends on the point of the credit cycle only by  $N_{T+1}$  since the distribution of the random factor is standard normal at each point in time. The cyclical variation is captured by the asset correlation and introduces some uncertainty and skewness into the default distribution.

On the other hand, if model (6) is assumed the probability distribution is

$$\begin{aligned}
 & P(D_{T+1}) \\
 = & \begin{cases} \binom{N_{T+1}}{D_{T+1}} \cdot \int_{-\infty}^{+\infty} [\lambda(f_{T+1}, \mathbf{z}_{T+1})]^{D_{T+1}} \cdot [1 - \lambda(f_{T+1}, \mathbf{z}_{T+1})]^{(N_{T+1} - D_{T+1})} \phi(f_{T+1}) df_{T+1} & D_{T+1} = 0, 1, 2, \dots, N_{T+1} \\ 0 & \text{else} \end{cases} \\
 & (8)
 \end{aligned}$$

where

$$\lambda(f_{T+1}, \mathbf{z}_{T+1})$$

is defined analogously to (6). The distribution (8) explicitly depends on the state of the economy by the macroeconomic factors.

Both default distributions can be extended to the case where the portfolio consists of an infinite number of borrowers and any idiosyncratic risk is eliminated. The formula is developed analo-



gously to (7) and (8). Firstly, note that in the case of infinitely many borrowers conditional on a realization of the common factor  $f_{T+1}$  the realized default rate, i.e. next year's percentage defaults  $\delta_{T+1}$ , equals the (conditional) default probability  $\lambda(f_{T+1})$  in the case of (4) and  $\lambda(f_{T+1}, \mathbf{z}_{T+1})$  in the case of (6). This is due to the fact that any idiosyncratic risk is diversified and the conditional binomial distribution of the defaults given the "state of the world"  $f_{T+1}$  shrinks to a single point.

In the second step - as in (7) and (8) - the unconditional distribution of defaults is obtained by "weighting" the conditional default probabilities with the respective probability of a given state of the world. Mathematically, one obtains the distribution of a function of a random variable. This function only depends on the unconditional default probability and the correlation.

The economic difference between the cases of finite and infinite numbers of borrowers is that in the latter case any non-systematic risk is eliminated. This risk is responsible for the "binomial effect" in the conditional default distribution and ceteris paribus widens the distribution in dependence of the numbers of borrowers in the portfolio. If only systematic risk is effective this binomial effect vanishes and c.p. only the asset correlation is responsible for the width and the skewness of the distribution. In the extreme case that the correlation is zero (with infinite many borrowers), there is no uncertainty about the default rate at all and the realized default rate equals the unconditional default probability.

VASICEK (1991) and KOYLUOGLU/HICKMAN (1997) mathematically derive the unconditional density of next year's percentage defaults  $\delta_{T+1}$  of the portfolio and show that the closed form solution is given by ( $\rho > 0$ )

$$f(\delta_{T+1}) = \begin{cases} \sqrt{\frac{1-\rho}{\rho}} \exp\left[-\frac{1}{2\rho} \left(\sqrt{1-\rho} \Phi^{-1}(\delta_{T+1}) - \Phi^{-1}(\lambda)\right)^2 + 0.5 \left(\Phi^{-1}(\delta_{T+1})\right)^2\right] & \delta_{T+1} \in [0,1] \\ 0 & \text{else} \end{cases}$$

(9)

in the case of the constant unconditional default probability  $\lambda$ . Its mean equals  $\lambda$ . In the case of a macroeconomic state dependent default probability,  $\lambda$  in (9) is replaced by  $\lambda(\mathbf{z}_{T+1})$  and the default distribution becomes dependent on the point of the business cycle. The effects of constructing state dependent default distributions will be addressed in section 4.

### 3 Results

#### 3.1 The Data

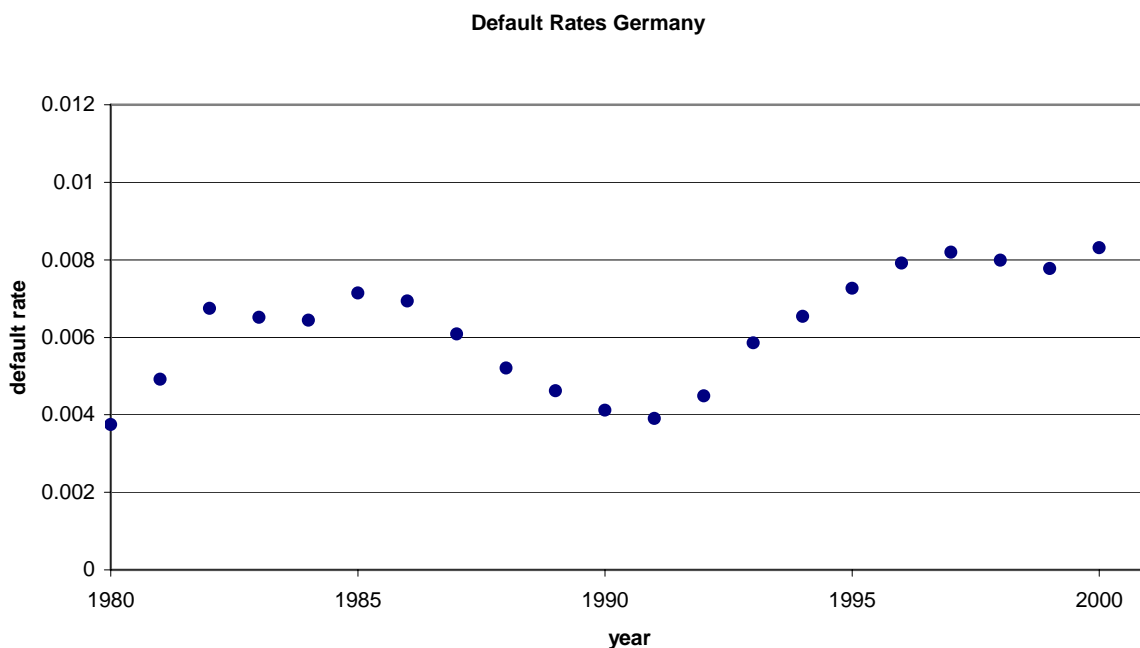
For this analysis a broad database from the Federal Statistical Office of Germany is used which covers all yearly collected numbers of German enterprises and bankruptcies from year 1980 until 2001. Figure 1 shows the default rates in Germany which are calculated as number of defaults in a given year divided by the number of enterprises at the beginning of the same year. For a more detailed analysis we divide the firms into 16 industry sectors and estimate the average default probabilities within these sectors. The sectors are composed due to the NACE code. Macroeco-

nomic key figures as proxies for the credit cycle are provided by the Federal Statistical Office of Germany and by the Monthly Reports from Deutsche Bundesbank. They cover the areas

- Money and capital market
- Labour market
- Foreign trade and payment
- Government activities
- Prices and wages
- Miscellaneous

Business Climate Indices which are provided by the German “IFO” Institute for Economic Research were used as additional indicators for the state of the business cycle.[9] More details on the risk factors used in this study and on the concrete selection procedure can be found in section 3.3 and Table 4.

**Figure 1: Default rates Germany, 1980-2001**



### 3.2 Constant Default Probabilities

Firstly, it is assumed that default probabilities are constant over time. We initially estimate the parameters of model (4) for Germany as a whole. Table 1 contains the results.

**Table 1: Parameter estimates for Germany as a whole using Maximum Likelihood with adaptive Gaussian quadrature**

Description	$\sqrt{\rho}$	$\beta_0$
Entire Germany	0.09257 *** (0.01385)	-2.4898 *** (0.02001)

\*\*\*: significant at 1% level, \*\*: significant at 5% level, \*: significant at 10% level  
Standard errors are in parentheses;

Model(4) is used:  $\lambda(f_t) = \Phi\left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)$

As can be seen from Table 1 the estimate for the constant is about -2.49 which corresponds to a default probability of about 0.64%, since latent asset returns are normalized random variables. The exposure to the random factor is 0.09257 which implies an asset correlation of about 0.86%. That means that the overall asset correlation is rather low. Both coefficients are highly significant.

In the next step we split our data into 16 industry sectors which we determine by the NACE-code and estimate model (4) for each sector. Our hypothesis is that default probabilities and correlations may differ between industries. The results are depicted in Table 2.

**Table 2: Parameter estimates for 16 industry sectors using Maximum Likelihood with adaptive Gaussian quadrature**

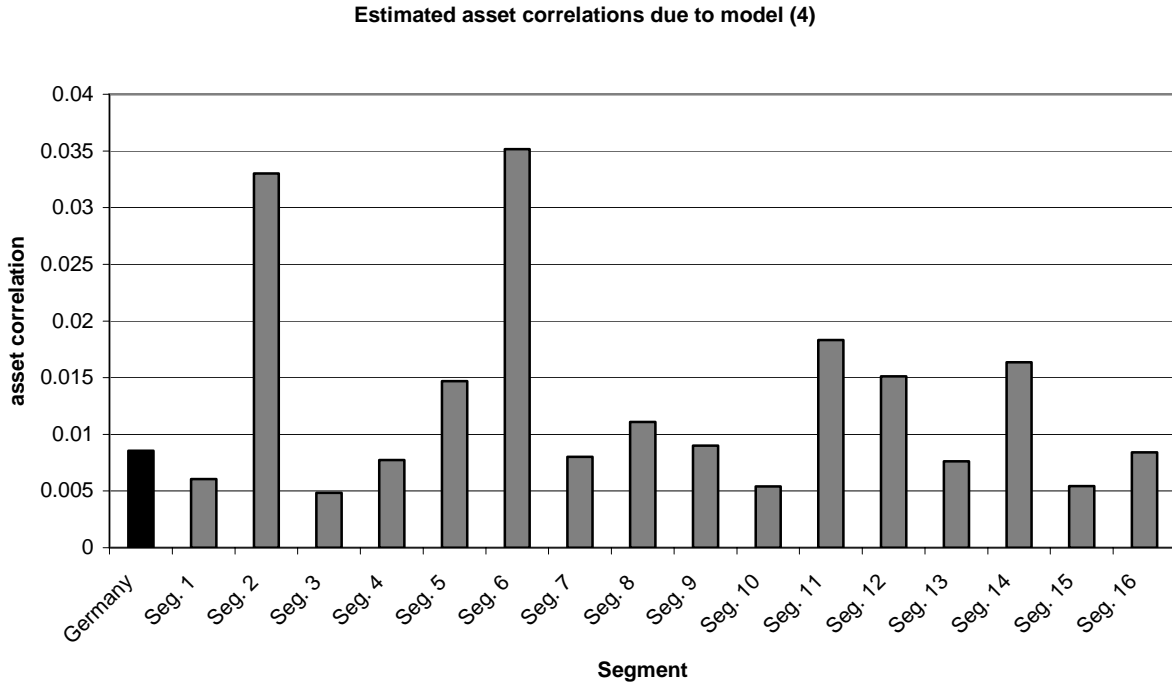
Segment	Description	$\sqrt{\rho}$	$\beta_0$
1	Agriculture, Fishing	0.07786 *** (0.01297)	-2.5573 *** (0.01753)
2	Mining, Energy	0.1817 *** (0.04012)	-3.0455 *** (0.04771)
3	Oil, Chemicals, Pharmaceuticals	0.06967 *** (0.02331)	-2.3457 *** (0.01995)
4	Gums, Plastics	0.08795 *** (0.01628)	-2.2163 *** (0.02081)
5	Glassware, Ceramics	0.1212 *** (0.02022)	-2.4915 *** (0.02770)
6	Metal	0.1875 *** (0.02779)	-2.4188 *** (0.04222)
7	Machinery, Engineering, Automobile, Transportation	0.08962 *** (0.01394)	-2.2357 *** (0.01968)
8	Software, Technology	0.1053 *** (0.01636)	-2.3878 *** (0.02326)
9	Wood, Paper	0.09489 *** (0.01477)	-2.4045 *** (0.02085)
10	Textiles	0.07356 *** (0.01238)	-2.3439 *** (0.01674)
11	Food, Tobacco	0.1354 *** (0.02070)	-2.7146 *** (0.03032)
12	Construction	0.1230 *** (0.01834)	-2.1934 *** (0.02675)
13	Consumer, Retail	0.08738 *** (0.01315)	-2.5397 *** (0.01890)
14	Traffic, Communications	0.1279 *** (0.01918)	-2.4027 *** (0.02805)
15	Banks, Financial Services, Insurances	0.07367 *** (0.01376)	-2.3773 *** (0.01763)
16	Consulting, Services	0.09181 *** (0.01378)	-2.6175 *** (0.01989)

\*\*\*: significant at 1% level, \*\*: significant at 5% level, \*: significant at 10% level  
Standard errors are in parentheses;

$$\text{Model(4) is used: } \lambda(f_t) = \Phi\left(\frac{\beta_0 - \sqrt{\rho} f_t}{\sqrt{1-\rho}}\right)$$

Table 2 shows that the constants – or the default probabilities, respectively - as well as the random factor exposures may vary between the sectors. The lowest default probability can be found in sector 2 (Mining, Energy) with about 0.11% whereas sector 12 (Construction) exhibits the highest probability with approximately 1.43%. Asset correlations are generally low and range between 0.5% (Oil, Chemicals, Pharmaceuticals) and 3.5% (Metal). All coefficients are highly significant. Figure 2 summarized the estimated correlations.

**Figure 2: Asset correlations for Germany and 16 industry sectors estimated due to model (4)**



### 3.3 Time-varying Default Probabilities

The model in section 3.2 assumes constant default probabilities over time. Any cyclical patterns are attributed to the asset correlations. Since it seems reasonable to assume that default probabilities change through the state of the economy, in the next step the parameters in (6) are estimated using observable risk factors described above. We choose the following selection proceeding. The realization of the random effects from model (4) – without covariates – were estimated and correlated to the available set of risk factors. Risk factors and one-year changes of the risk factors were incorporated with time-lags of one and two years, respectively. [10] Then, the parameters of model (4) were estimated with combinations of the risk factors which exhibit the (absolute) highest correlation with the random effect realizations. We do not claim that the presented risk factors are the unique risk drivers themselves. Rather they should be interpreted as potentially interchangeable proxies for the underlying risk factors or for a somewhat abstract “point of the credit cycle”. Furthermore, following DUFFIE/SINGLETON (1999) and DUFFEE (1999) who suggest an AR(1)-process for the default intensities, we include lagged default rates as regressors. We restrict our models to the inclusion of no more than four factors.

Firstly, Table 3 shows the estimation results of model (6) for Germany as a whole where we used the one year lagged overall default rate, the one year lagged average debit interest rate, the one-year lagged change of the business climate index for construction and the two year lagged change

of the index for order inflows as factors. A description of the variables and the abbreviations in Table 3 can be found in Table 4.

**Table 3: Parameter estimates for Germany as a whole using Maximum Likelihood with adaptive Gaussian quadrature**

Description	$\sqrt{\rho}$	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$
Entire Germany	0.02284*** (0.003585)	-2.9810*** (0.06081)	0.6222*** (0.04182) DR_GER_1	1.0158** (0.3959) DIR_1	-0.2362*** (0.07744) BCICS_chg_1	-0.1976* (0.1050) IOIC_chg_2

\*\*\*: significant at 1% level, \*\*: significant at 5% level, \*: significant at 10% level  
 \_1 means one year time lag, \_2 means two years time lag, \_chg means one year change of variable

Standard errors are in parentheses;

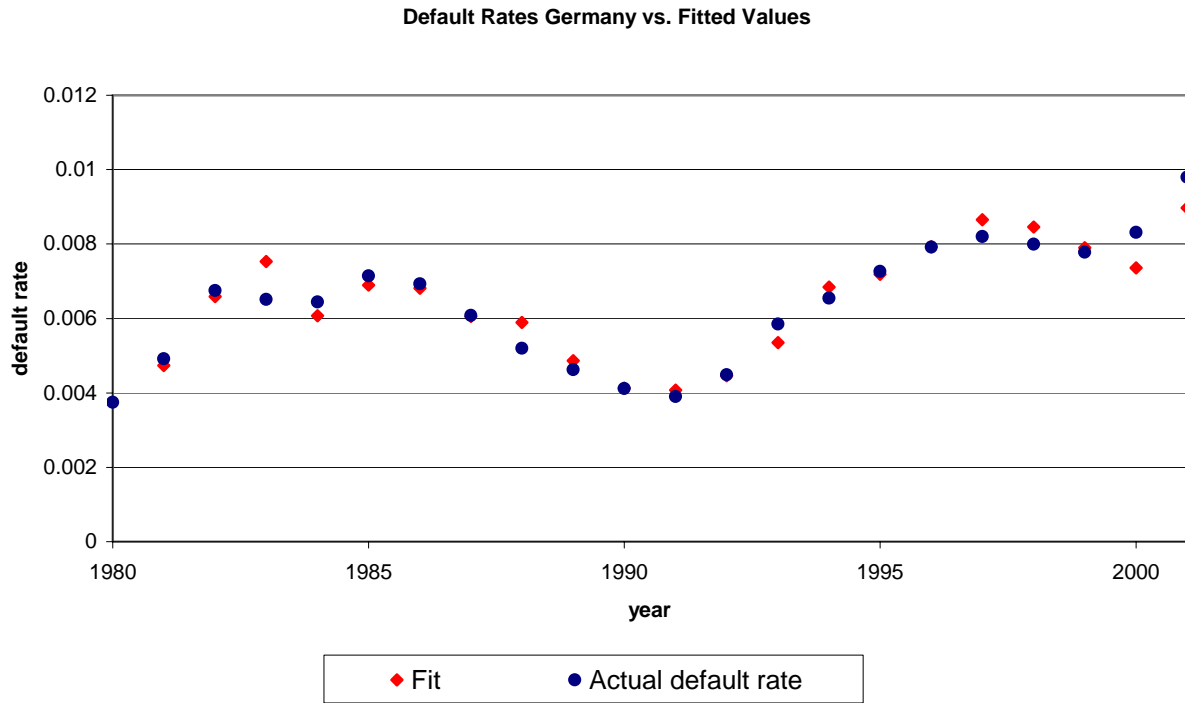
$$\text{Model (6) is used: } \lambda(f_t, z_t) = \Phi \left( \frac{\beta_0 + \boldsymbol{\beta}' z_t - \sqrt{\rho} f_t}{\sqrt{1 - \rho}} \right)$$

**Table 4: Variables which were used for whole Germany and the 16 industry sectors as proxies for the credit cycles**

<b>Variable</b>	<b>Code</b>
<i>Default Rates:</i>	
Default Rate Germany	DR_GER
Default Rate Sector 8	DR_S8
Default Rate Sector 9	DR_S9
Default Rate Sector 11	DR_S11
Default Rate Sector 14	DR_S14
<i>Economic Variables::</i>	
Overnight Money Rate	CMR
Debit Interest Rate	DIR
Real GDP	GDP
Private Households: Saving Ratio as a percentage of disposable income	SRPH
Index of Effective Exchange Rate of the U.S. Dollar	FX
Unemployment rate	UNEM
Employees (Construction)	EMPC
Index of Effective Exchange Rate of the U.S. Dollar	FX
Index of Order Inflow, Whole Industry	IOII
Index of Order Inflow, Capital Goods	IOIC
Index of Order Inflow, construction	IOICS
Index of Foreign Trade Prices for Imports (Crude Oil)	IPI
<i>Business Climate Indices:</i>	
Business Climate Index Retail	BCIR
Business Climate Index Wholesale	BCIW
Business Climate Index Consumption Goods	BCIC
Business Climate Index Construction	BCICS
Business Climate Index Commerce	BCICC
Consumption Climate Index	CCI
Default rates are provided by the Federal Statistical Office of Germany; economic variables are provided by the Federal Statistical Office of Germany and by the Monthly Reports from Deutsche Bundesbank, Business Climate Indices are obtained from the German IFO-Institute for Economic Research	

All exposures to the observable factors are statistically significant at the 1% to 10% level. Furthermore they are economically plausible. The signs of the coefficients for the lagged default rate and the lagged interest rate are positive. This means that higher values of the variables correspond with higher default probabilities *ceteris paribus*. Thus, last years default rate is a good predictor for this years default rate and a high interest rate is likely to lead to an increase in default probabilities. The negative signs for the other two lagged variables show that positive changes of business climate and order inflows correspond with lower default probabilities. Moreover, the asset correlation has considerably decreased in comparison with the model without risk factors from 0.85% to 0.052%. That is, the business cycle which is proxied by the four variables is responsible for a large part of co-movements. The patterns of default rates versus fitted values is displayed in Figure 3.

**Figure 3: Default rates Germany and fitted values and using model (6), 1980-2001**



In the next step the 16 German industry sectors are analyzed. Since it is reasonable to expect that different industries react differently on macroeconomic conditions we admit different risk factors. Table 5 contains the results for the parameter estimates of model (6).



**Table 5: Parameter estimates for 16 industry sectors using Maximum Likelihood with adaptive Gaussian quadrature**

Segment	Description	$\sqrt{\rho}$	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$
1	Agriculture, Fishing	0.01474 (0.00829)	-3.0310*** (0.06608)	0.6334*** (0.06995) DR_GER_1	1.3482*** (0.4250) CMR_1		
2	Mining, Energy	0.02179 (0.07368)	-5.0014*** (0.4264)	1.7604*** (0.2760) DR_GER_1	7.7889*** (2.6297) DIR_1	-0.5842* (0.2631) UNEM_chg_1	
3	Oil, Chemicals, Pharmaceuticals	0.00849 (0.07251)	-1.8312*** (0.2125)	-0.5829** (0.2405) BCIW_2			
4	Gums, Plastics	0.02212 (0.01531)	-0.9523*** (0.2025)	-1.5256*** (0.2267) BCIR_1	1.1793** (0.4261) CMR_1	-0.3316** (0.1177) EMPC_chg_2	-0.2163** (0.1004) UNEM_chg_2
5	Glassware, Ceramics	0.01099 (0.02321)	-2.8266*** (0.06419)	0.9692*** (0.08583) DR_GER_1	-0.3064*** (0.06670) IOII_1	0.07914 (0.04825) CMR_chg_1	
6	Metal	0.03946*** (0.008720)	-3.6415*** (0.4044)	1.2580*** (0.1267) DR_GER_1	6.3888*** (1.1911) DIR_1	0.2998*** (0.07239) FX_1	-0.5329* (0.2834) CCI_2
7	Machinery, Engineering, Automobile, Transportation	0.02399*** (0.005689)	-2.1915*** (0.2182)	0.3332*** (0.05776) DR_GER_1	0.2004*** (0.03695) FX_1	-0.6907*** (0.1654) BCIC_1	1.7227*** (0.5283) DIR_2
8	Software, Technology	0.02188*** (0.007222)	-1.6114*** (0.2435)	0.2593*** (0.03841) DR_S8_1	0.1462* (0.07481) DIR_chg_2	-0.9848*** (0.2179) CCI_2	-0.9439*** (0.1196) BCIC_1
9	Wood, Paper	0.01544** (0.006050)	-2.3391*** (0.1076)	0.3943*** (0.05315) DR_S9_1	2.6454*** (0.4492) CMR_1	-1.4731*** (0.4801) GDP_chg_1	-3.9129*** (0.6286) SRPH_1

\*\*\*: significant at 1% level, \*\*: significant at 5% level, \*: significant at 10% level  
 \_1 means one year time lag, \_2 means two years time lag, \_chg means one year change of variable  
 Standard errors are in parentheses

$$\text{Model (6) is used: } \lambda(f_t, z_t) = \Phi \left( \frac{\beta_0 + \beta' z_t - \sqrt{\rho} f_t}{\sqrt{1 - \rho}} \right)$$

**Table 5 (continued): Parameter estimates for 16 industry sectors using Maximum Likelihood with adaptive Gaussian quadrature**

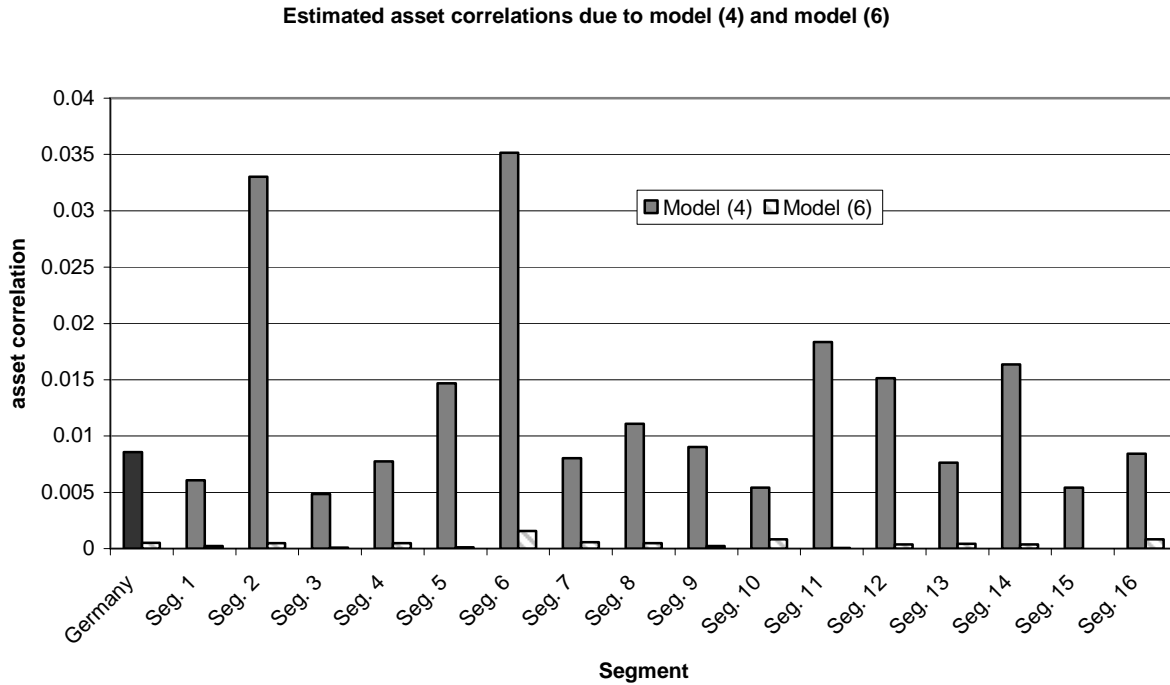
Segment	Description	$\sqrt{\rho}$	$\beta_0$	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$
10	Textiles	0.02866*** (0.008025)	-2.9448*** (0.08123)	0.3150*** (0.06432) DR_GER_1	2.2837*** (0.4494) DIR_2	0.1680*** (0.04705) FX_2	
11	Food, Tobacco	0.006538 (0.01798)	-2.1870*** (0.1269)	0.4919*** (0.08884) DR_S11_1	-5.3535*** (0.7994) SRPH_1	-0.5532*** (0.1421) BCICC_chg_1	
12	Construction	0.01897*** (0.003650)	-2.5394*** (0.1449)	0.6687*** (0.08806) DR_GER_1	1.6432*** (0.4125) DIR_1	-1.9121*** (0.6238) SRPH_1	-0.2825** (0.1092) IOICS_chg_1
13	Consumer, Retail	0.02081*** (0.003490)	-3.0810*** (0.05695)	0.5995*** (0.04149) DR_GER_1	1.6654*** (0.3330) DIR_1	-0.2359** (0.09662) IOIC_chg_2	
14	Traffic, Communications	0.01914*** (0.004610)	-2.2522*** (0.1078)	0.2855*** (0.03589) DR_S14_1	2.5393 *** (0.4501) DIR_1	-5.0260*** (0.7266) SRPH_1	-0.4323*** (0.1230) BCIW_chg_1
15	Banks, Financial Services, Insurances	0.002856 (0.06524)	-2.7969*** (0.05639)	0.4464*** (0.05751) DR_GER_1	0.1468*** (0.04604) FX_2		
16	Consulting, Services	0.02878*** (0.004617)	-3.1683*** (0.06527)	0.7356*** (0.06990) DR_GER_1	1.4939*** (0.4266) CMR_1	0.05794** (0.02220) IPI_chg_1	

\*\*\*: significant at 1% level, \*\*: significant at 5% level, \*: significant at 10% level  
 \_1 means one year time lag, \_2 means two years time lag, \_chg means one year change of variable  
 Standard errors are in parentheses

$$\text{Model (6) is used: } \lambda(f_t, \mathbf{z}_t) = \Phi \left( \frac{\beta_0 + \boldsymbol{\beta}' \mathbf{z}_t - \sqrt{\rho} f_t}{\sqrt{1 - \rho}} \right)$$

In each sector we found risk factors which influence default probabilities with a time lag of one or two years and which are statistically significant and economically plausible. In most sectors we used the lagged default rate and one or more additional variables as proxies for the credit cycle. These additional variables are mainly interest rates and business climate indices. They enter the equations with economically plausible signs. Moreover, in each sector the asset correlation can be considerably reduced by introducing these factors, for example from 3.5% to approximately 0.16% in the metal sector which exhibits the highest correlation, or from 1.5% to 0.036% in the construction sector. Note that in eight sectors the asset correlation is no longer significantly different from zero at the 10% level. A comparison of the asset correlations estimated via model (4) and via model (6) is summarized in Figure 4.

**Figure 4: Comparison of asset correlations for Germany and 16 industry sectors estimated due to model (4) and model (6)**



For each segment the in-sample fit is assessed by running regressions due to MIN-CER/ZARNOWITZ (1969)

$$DR_t = a + b \cdot DR_t^* + u_t \tag{10}$$

( $t=1, \dots, T$ ), where  $DR_t$  is the observed default rate,  $DR_t^*$  is the fitted default rate of the model,  $u_t$  is an error component,  $a$  and  $b$  are regression parameters. If the model fit is efficient then  $a=0$  and  $b=1$ . Furthermore, the errors should not exhibit autocorrelation. Table 6 contains the results for Germany and the 16 industry sectors. Columns 2 and 3 show the intercepts and slopes of the regressions, column 4 contains the F-statistic for a joint test of the intercept being equal to zero and the slope being equal to one, column 5 shows the R-square and column 6 shows the Durbin-Watson statistic for first-order autocorrelation. Although the model fit is evaluated in-sample rather than out-of-sample or out-of-time due to data limitations and, thus, we do not want to stress the results, all models seem to be efficient. Furthermore, except for segment 3, the model fit, measured by the R-square of the regression, is sufficient. [11] The values of the Durbin-Watson statistics range from 1.47 to 2.87. Thus, autocorrelation of the errors and systematic fitting errors of the model over time do not seem to be a severe problem.

**Table 6: Parameter estimates for Mincer-Zarnowitz regression  $DR_t = a + b \cdot DR_t^* + u_t$  of observed default rates on fitted values from model (6)**

Segment	a	b	F-value (p)	R <sup>2</sup>	DW
<b>Germany</b>	<b>0.00000784 (0.00047136)</b>	<b>0.99881 (0.07062)</b>	<b>0.00 (0.9999)</b>	<b>0.9132</b>	<b>1.574</b>
1	0.00008067 (0.00055940)	0.98409 (0.10221)	0.01 (0.9870)	0.8299	2.004
2	-0.00003960 (0.00017542)	0.98902 (0.12890)	0.16 (0.8510)	0.7560	1.581
3	0.00035383 (0.00414)	0.96498 (0.43520)	0.01 (0.9950)	0.2145	1.835
4	0.00023319 (0.00170)	0.98257 (0.12374)	0.01 (0.9901)	0.7684	2.433
5	-0.00007255 (0.00058937)	1.01136 (0.08771)	0.01 (0.9917)	0.8926	1.695
6	0.00005674 (0.00062799)	0.99248 (0.06762)	0.01 (0.9935)	0.9309	2.438
7	0.00038300 (0.00106)	0.97068 (0.08004)	0.07 (0.9352)	0.8910	1.748
8	-0.00000816 (0.00070221)	1.00361 (0.07694)	0.01 (0.9921)	0.9140	2.763
9	0.00014394 (0.00054729)	0.98230 (0.06514)	0.04 (0.9638)	0.9343	2.869
10	0.00005030 (0.00153)	0.99651 (0.15375)	0.00 (0.9977)	0.7000	1.609
11	0.00000223 (0.00016308)	1.00178 (0.04110)	0.01 (0.9899)	0.9738	1.682
12	0.00021198 (0.00063977)	0.98551 (0.04153)	0.06 (0.9411)	0.9724	1.928
13	0.00007249 (0.00040596)	0.98701 (0.07021)	0.02 (0.9830)	0.9123	1.467
14	-0.00000433 (0.00041219)	1.00046 (0.04508)	0.00 (0.9999)	0.9685	2.767
15	-0.00013522 (0.00113)	1.01185 (0.12382)	0.01 (0.9853)	0.7877	1.637
16	0.00006211 (0.00038809)	0.98608 (0.08368)	0.01 (0.9862)	0.8796	1.549

a is the intercept, b is the slope, standard errors are in parentheses; F-value is the result of an F-test of the null hypothesis a=0 and b=1, p-value is in parentheses, R<sup>2</sup> is the R-square of the regression; DW is the Durbin-Watson statistic for autocorrelation.

#### 4 Practical Implications for Risk Management

Distributions of potential defaults can be very sensitive with respect to the input parameters used. That is, one has to be very careful in specifying default probabilities, and correlations in particular. The internal ratings based approach of the New Basel Accord allows for estimates of default

probabilities which can be derived from banks' internal ratings models. The Accord as of January 2001 handles the problem of correlations by globally assuming a conservative value of the asset correlation  $\rho$  of 20% between any two borrowers and thereby tries to cover any uncertainty in these important parameter. [12]

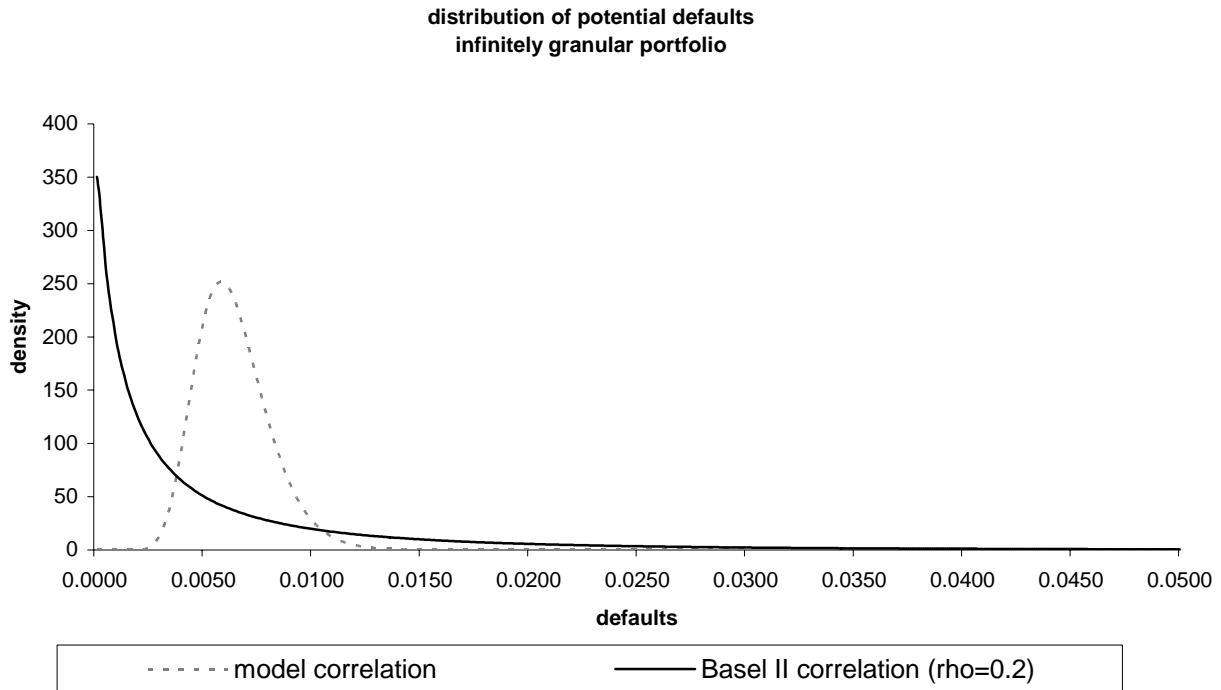
As it was shown in section 2 the Basel II model assumes two kinds of default probabilities, one which is unconditional and one which is conditional with respect to a realization of the random factor  $f_t$ . Since the realization of the random factor is contemporaneous to the realization of the default event the conditional PD is a random variable "ex-ante". As a result of the Basel II assumption of a high exposure to the random factor, i.e. the square root of 0.2 which is approximately 0.4472, and the assumption of a "bad-case" realization of the factor, the resulting conditional default probability is heavily catapulted upwards. Capital requirements are determined by this conditional default probability.

For the protection of the stability of our financial systems the Basel Accord well takes a conservative point of view. A bank may use methodologies for risk management and controlling which are consistent with the Basel Accord, but she may use her own parameters for her internal risk models and forecasting tools if it is convinced that they are more accurate than assumed under Basel II.

Suppose a bank with a very large homogenous and granular portfolio would use the Basel II model for calculating its distribution of potential defaults for the next year. That is, it differentiates only between default and non-default. [13] It needs estimates for the default probabilities and for the correlations. To keep things simple we take the parameter estimates from our model of Germany as whole. Under the model which assumes constant default probabilities an estimate for year 2002's average default probability is approximately 0.64%. Then we can calculate the distribution of potential future defaults under the Basel II asset correlation of 20% and alternatively under the estimate of model (4) of approximately 0.86% according to formula (9). The two forecasted distributions are shown in Figure 5.

### Figure 5: Forecasted distributions of potential defaults

Portfolio is infinitely granular, Defaults are given as a percentage of portfolio size, Estimates from model (4) for Germany as a whole are used for forecasting the default probability

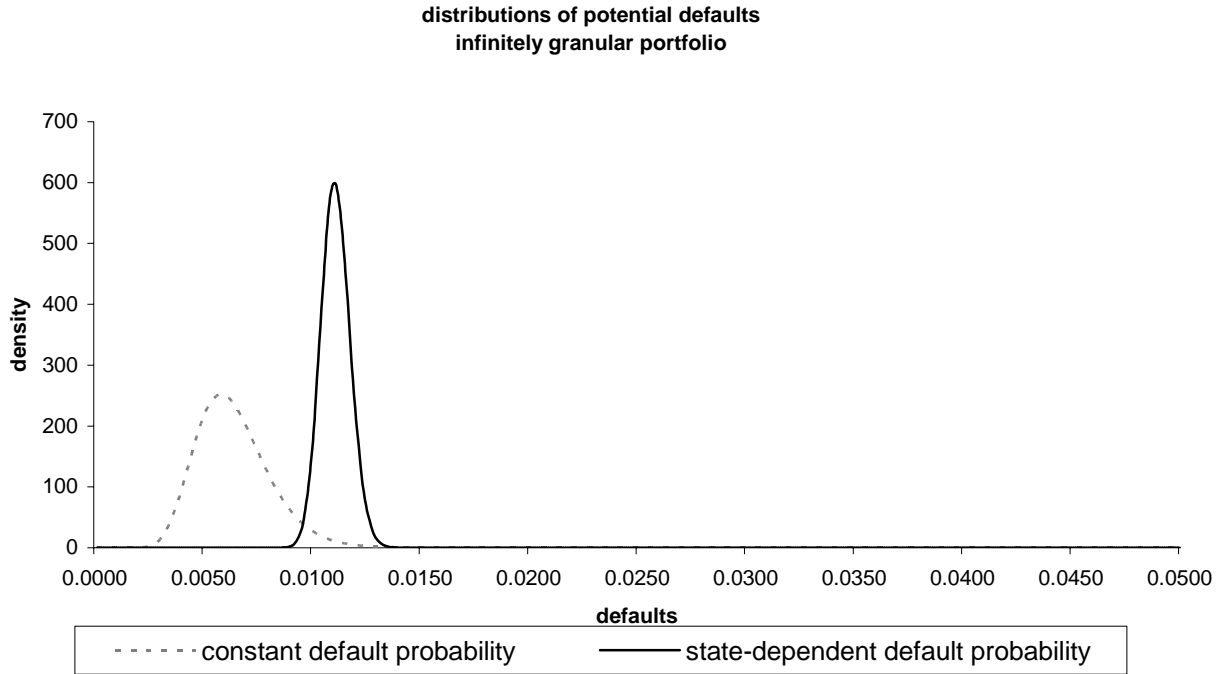


As can be seen the two forecasted distributions are quite different although the forecast for the (unconditional) default probability is actually the same. The reason for this lies in the assumed asset correlation which considerably expands the possible realizations of (conditional) default probabilities and, thus, possible defaults. For example, the 99.5% VaR-quantile under the Basel II asset correlation distribution is approximately 6.74% whereas it is only 1.19% under the estimated correlation. Thus, uncertainty about potential defaults under the internal model is substantially lower.

Note that the distribution is derived under the model which assumes constant default probabilities. However, the empirical evidence presented shows that default probabilities vary through time due to economic conditions. Then for the purpose of forecasting potential defaults the actual point of the credit cycle should be taken into account. Thus, calculating distributions of potential losses may be more accurate under model (6). Since the model is estimated using data until 2001 and the risk factors are lagged, we plug the values of the risk factors of 2001 into our model equation and forecast the default probability for year 2002 for given risk factors. Since 2001 was a rather “bad” year, the forecast for 2002 is approximately 1.11%. On the other hand the asset correlation is reduced to approximately 0.052%. Figure 6 compares the distributions under the constant and the point-of-the-credit-cycle default probability model according to formula (9) for a portfolio with infinitely many borrowers. In Figure 7 a portfolio with  $N=1000$  borrowers is assumed and the default distributions are constructed according to (7) and (8) respectively.

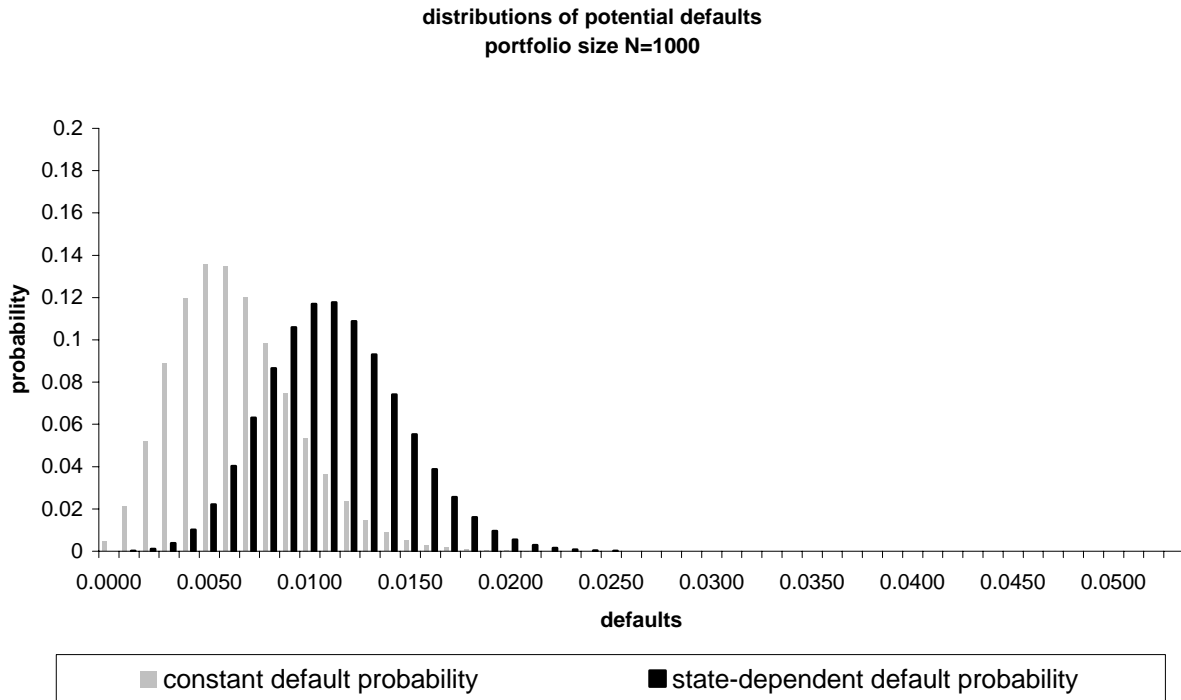
**Figure 6: Forecasted distributions of potential defaults**

Portfolio is infinitely granular, Defaults are given as a percentage of portfolio size; Estimates from model (4) and model (6) for Germany as a whole are used for forecasting the default probability



### Figure 7: Forecasted distributions of potential defaults

Portfolio consists of 1000 borrowers each; Defaults are given as a percentage of portfolio size; Estimates from model (4) and model (6) for Germany as a whole are used for forecasting the default probability



It can be seen that in both cases the distribution under the point-in-time model is shifted to the right since year 2002 is forecasted to be much worse than an average year. Moreover, the reduction of the asset correlation by model (6) lets the distribution be more narrow for large portfolios. For smaller portfolios the nature of the conditional binomial distribution leads to a wider shape. For the point-of-the-cycle model the distribution is almost symmetric.

Since interests in risk management often focus on the right tails of a distribution, the Value at Risk quantiles for confidence levels of 99%, 99.5%, and 99.9% are shown in Table 7 where the Value-at-Risk is defined as the quantile of the whole default distribution for convenience. As it can be seen, the differences may be substantial, especially for smaller portfolio size. Sometimes it is argued that banks cover part of their potential losses, in particular their Expected Loss by reserves and provisions. In this case economic capital must only buffer the difference between the Value-at-Risk and the Expected Loss, which is defined as Unexpected Loss. These Unexpected Losses are also given in Table 7 for the three distributions under consideration. Clearly, they are highest for the Basel II model, but note that although the distribution under the point-in-time model is shifted to the right its Unexpected Losses are considerably lower than under the constant probability model except for small portfolio size and lower confidence levels. Thus, the reduction of correlations not only reduces uncertainty of potential defaults, it may also reduce required risk capital in an internal model.



**Table 7: Value-at-Risk quantiles of forecasted default distributions for different portfolio size**

Portfolio size Confidence Level (%)	N=1000			N=5000			N=10000			IGP		
	99	99.5	99.9	99	99.5	99.9	99	99.5	99.9	99	99.5	99.9
VaR0	5.40	6.90	10.90	5.28	6.76	10.80	5.27	6.75	10.79	5.26	6.74	10.78
UL0	4.76	6.62	10.26	4.64	6.12	10.16	4.63	6.11	10.15	6.62	6.1	10.14
VaR1	1.50	1.60	1.90	1.20	1.28	1.46	1.16	1.24	1.41	1.12	1.19	1.35
UL1	0.86	0.96	1.26	0.56	0.64	0.82	0.52	0.60	0.77	0.48	0.55	0.71
VaR2	2.00	2.10	2.30	1.50	1.56	1.66	1.41	1.45	1.52	1.28	1.30	1.34
UL2	0.89	0.99	1.19	0.39	0.45	0.55	0.30	0.34	0.41	0.17	0.19	0.23

Model for Germany as whole is used; N is the number of borrowers in the portfolio, IGP is an infinitely granular portfolio; defaults are in % of portfolio size; the upper half shows the quantiles of the model which assumes constant probability of default (PD), both under Basel II and under model correlation, the lower half shows the quantiles of the model with state dependent probability of default (PD); Value-at-Risk is defined as a quantile of the default distribution; Unexpected Loss is defined as the difference between the Value-at-Risk quantile and the expected loss; VaR0 (UL0) is the Value-at-Risk (Unexpected Loss) with constant PD and Basel II asset correlation, VaR1(UL1) is the Value-at-Risk (Unexpected Loss) with constant PD and the estimated model correlation, VaR2(UL2) is the Value-at-Risk (Unexpected Loss) with time-varying PD and estimated model correlation.

In summary, a time-dependent view may have some certain advantages. Firstly, one may forecast default probabilities more accurately depending on the point of the credit cycle. If average default probabilities are used one implicitly assumes that the next year will be a random realization out of the population from which the historical defaults were drawn. The uncertainty around the mean is taken into account by the asset correlation. On the other hand, if one follows the time-dependent view one assumes that next year's defaults are randomly drawn conditional on the state of the economy. In summary, if the model is valid, forecasts for default probabilities may be more accurate and, in addition, asset correlations and probably risk capital within an internal model may be substantially reduced.

## 5 Concluding Remarks

Correlations are the main drivers for uncertainty in portfolio credit risk modeling. In the present paper a model is analyzed which attributes correlations to observable common risk factors. We use a database of German corporate bankruptcies and find that correlations are throughout lower than assumed in the New Basel Accord. By introducing proxy variables for the business cycle correlations can be significantly diminished. Thus, forecasts for default distributions can be generated more adequately when the current point of the business cycle is taken into account. This leads to less uncertainty in potential defaults and may reduce economic capital requirements if they were measured by an internal model.

Two directions for further research should be mentioned. Firstly, the present analysis focuses on aggregated data for industry sectors. Thus, we could map business cycles for each segment only on average. The analysis could also be carried out for individuals given the availability of individual firm data, for example from banks. Then, firm specific default probabilities or even correlations could be estimated more adequately.

Secondly, we did not consider estimation risk for the forecasts of potential defaults. Although the standard errors of the parameter estimates were rather low, the model produces additional uncertainty. This should also be taken into account when such forecasts are generated.

## FOOTNOTES

[1] For outlines of these models see GUPTON et al. (1997), CREDIT SUISSE FINANCIAL PRODUCTS (1997), CROSBIE (1998) and WILSON (1997a, 1997b).

[2] This effect of potential procyclicality is under active discussion. NICKELL/PERRAUDIN/VAROTTO (2000) find that ratings downgrades and defaults are more likely in economic downturns. CATARINEU-RABELL/JACKSON/TSOMOCOS (2002) show that through-the-cycle rating schemes would not produce procyclical capital requirements in opposite to point-in-time schemes, see also ALLEN/SAUNDERS (2002) for a review.

[3] See ALTMAN/SAUNDERS (1997) who provide a survey on developments over the past two decades.

[4] Denoting the square root of  $\rho$  as the factor exposure may seem somewhat unusually and is only due to parameter parsimony. We follow the model setup in BELKIN/SUCHOWER/FOREST (1998) who also provide further descriptions. In fact, specification (1) and the correlation can be re-parameterized from a normal distribution with any mean and variance. See HAMERLE/LIEBIG/RÖSCH (2003) for details.

[5] This probability is actually a conditional probability, given the borrower has survived until time  $t$ . We skip the condition  $Y_{it-1}=0$  for convenience.

[6] A related specification can be found in DAS/FREED/GENG/KAPADIA (2002), p.12.

[7] An analogous approach is also employed for equity data, see the seminal work by CHEN/ROLL/ROSS (1986).

[8] For a detailed description see HAMERLE/LIEBIG/RÖSCH (2003).

[9] A description can be found under [www.ifo.de](http://www.ifo.de): "The Ifo Business Climate Index is a widely observed early indicator for economic development in Germany. Every month the Ifo Institute surveys more than 7,000 enterprises for their appraisal of the business situation and their short-term planning. The confidence indicator, frequently referred to as the Ifo Index, is derived from these responses to the Ifo Business Survey. Twice a year the Ifo Institute publishes the Ifo Economic Forecast on the development of the German and the world economy for the current and the subsequent year."

[10] The reason for using time-lags lies in the advantage for making forecasts. If we used contemporaneous variables the forecasts in (7), (8) or (9) would require forecasts for the macroeconomic variables for period  $T+1$  which in turn requires a model for the variables, see for example WILSON (1997a, 1997b). In our time-lag approach the model assumptions for the macroeconomic variables are not necessary since their realizations are known in  $T$  and forecasts for default distributions for  $T+1$  can be made conditional on these realizations.

[11] Indeed, although for segment 3 one variable could be determined, the default rates are rather erroneous and do not seem to follow any systematic cycles. This segment also exhibits the lowest unconditional correlation, see Table 2.

[12] In the April 2003 proposal the asset correlation decreases with increasing default probability from 24% to 12%.

[13] The Basel Committee distinguishes between mark-to-market and default-mode models. Since our data only cover default events, we employ the latter. For mark-to-market models, see BAN-GIA/DIEBOLD/KRONIMUS/SCHAGEN/SCHUERMAN (2002).

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