Multi-Country Endogenous Growth Models

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Abstract

The transitional dynamics of open-economy endogenous growth models are largely unexplored. The present paper fills this gap in the literature. By applying the familiar Dixit-Norman (1980) approach to a general class of growth models, it provides original results on the transitional dynamics of the multi-country open-economy versions of several prominent special cases, including the models of Romer (1986, 1990), Lucas (1988), Grossman and Helpman (1991a, Chapters 3 and 4, 1991b), Jones (1995a), and Segerstrom (1998). This approach also shows that, in the class of models considered, the question of whether or not international economic integration accelerates growth in the long run is equivalent to the question of whether or not scale effects prevail.

JEL classification: F12, F15, O41

Key words: international trade, international knowledge spillovers, multinational corporations, international patent licensing, economic growth

1 Introduction

One of the most interesting branches of new growth theory, initiated by the seminal work of Grossman and Helpman (1991a), is the analysis of open-economy endogenous growth models. Such models are used to study the growth and welfare effects of international trade in goods and financial capital, the implications of trade policy and international knowledge spillovers, the role of multinational firms and international patent licensing, the consequences of low-wage competition and imitation, and many other aspects of the growth process in open economies. From a theoretical point of view, a drawback of the voluminous literature on international trade and economic growth is that it is mostly concerned with balanced-growth paths, while little is known about transitional dynamics.[1] This is problematic for several reasons. First and foremost, in the absence of an analysis of transitional dynamics, one is uncertain about whether the balanced-growth path is in fact the long-term solution of the model considered. Second, the transitional dynamics may be of interest on their own. Third, a full dynamic analysis is needed in order to simulate the models. Another notable feature of the trade and growth literature is the heterogeneity of the models used. This makes it difficult to judge which assumptions are responsible for differing implications, for instance with regard to the question of whether international economic integration boosts long-run growth.

The present paper serves two purposes. First, it provides an analytical framework within which we produce new results on the transitional dynamics of multi-country endogenous growth models. In particular, we give a complete characterization of the dynamics of the multi-country open-economy versions of the R&D growth models of Romer (1990), Grossman and Helpman (1991a, Chapters 3 and 4, 1991b), Jones (1995a), Segerstrom (1998), and Arnold (1998) as well as the physical and human capital models of Solow (1956), Arrow (1962), Uzawa (1965), Sheshinski (1967), Romer (1986), and Lucas (1988). To do so, rather than addressing the problem directly, we formulate a general growth model that nests all these models as special cases and prove the validity of the "Dixit-Norman theorem" (Dixit and Norman, 1980) for this general model: under certain conditions (which, as in the static trade theory, tend to be satisfied in the presence of multinationals or international patent licensing and with similar relative factor endowments or physical-capital mobility), factor prices equalize and the world economy behaves exactly like a hypothetical integrated economy without national borders.[2]

[1] There are a few exceptions to this rule. Grossman and Helpman (1991a, Subsection 8.1) and Wälde (1996), for instance, analyze the dynamics of the two-country Grossman and Helpman (1991a, Chapter 3) model. Devereux and Lapham (1994) prove an interesting instability result for the two-country Romer (1990) model with national spillovers (first analyzed by Rivera-Batiz and Romer, 1991). And Helpman (1993) and Arnold (2002a, 2003) analyze the dynamics of North-South R&D models.

[2] This approach was initiated by Travis (1964, Chapter 2). Since Dixit and Norman (1980) (Chapter 4) made it

To characterize the dynamics of the multi-country world economy completely, all we have to do, then, is show that the conditions for replication are satisfied and apply existing stability results for the integrated economy. The special cases of our general model mentioned above satisfy the conditions for replication and possess a unique equilibrium growth path.[3] From our Dixit-Norman theorem, it thus follows that the same holds true for the integrated world economy. Interestingly, we thus come up with several original stability results without having to solve a single differential equation. Furthermore, it turns out that (similar to static trade models with more goods than factors) there are several indeterminacies regarding the division of aggregate economic activity across countries. The second purpose of the present paper is to shed light on the relation between international economic integration and the pace of long-term growth. In this regard, our Dixit-Norman theorem implies that the question of whether or not international economic integration accelerates growth in the long run is equivalent to the question of whether or not scale effects prevail (i.e., "larger size means faster growth").

The idea of applying the Dixit-Norman approach to open-economy growth models is not new. It has been applied to R&D growth models by Grossman and Helpman (1989, 1991a, Chapter 7), to models with physical-capital accumulation by Ventura (1997), Nishimura and Shimomura (2002), and Cuñat and Maffezzoli (2004),[4] and to a model with human-capital accumulation by Bond, Trask, and Wang (2003).[5] However, although these papers focus explicitly on model dynamics, the potential of

popular, it seems justified to name it after them. They remark that Samuelson (1953) already "saw through the whole problem, and we think that if he had filled out some of the asides and terse remarks he makes, he would have developed the argument much as we have done here" (Dixit and Norman, 1980, p. 125). Helpman and Krugman (1985) helped popularize the approach further with their work on imperfect competition in product and factor markets.

[3] A qualification is in order here: Benhabib and Perli (1994) demonstrate that in the Lucas (1988) model, there are parameter combinations such that the equilibrium growth path is indeterminate. We will focus on the parameters that give rise to a unique equilibrium.

[4] Ventura (1997) uses the Dixit-Norman approach to show that a fairly standard growth model with physical capital as the only source of growth is sufficient to make fast conditional convergence (as in the East Asian "growth miracles") consistent with complete economic integration. Nishimura and Shimomura (2002) demonstrate that introducing sector-specific externalities to the model opens up the possibility of equilibrium indeterminacy. Cuñat and Maffezzoli (2004) show that identically parameterized countries do not necessarily reach the same steady state if factor supplies are so dissimilar that complete specialization occurs. The authoritative survey of early open-economy models with physical-capital accumulation, such as Stiglitz (1970), is Smith (1984). Interestingly for our purposes, Smith (1984) takes the steady-state assumption as the distinguishing feature of one of two classes of models and warns: "let us be wary of steady-state analysis" (Smith, 1984, p. 290).

[5] Bond, Trask, and Wang (2003) apply the Dixit-Norman method to the open-economy human capital model of Uzawa (1965) and Lucas (1988). They emphasize an indeterminacy as regards individual human-capital profiles (which we will encounter several times in this paper), which requires rethinking of both the static and a dynamic version of the

the Dixit-Norman approach for systematically investigating the transitional dynamics of the multi-country versions of a broad class of existing endogenous growth models has gone unnoticed. Moreover, our approach to growth and trade is more general than the existing models in two respects. For one thing, it contains both R&D and physical and human capital. For another, contrary to Grossman and Helpman (1989, 1991a, Chapter 7), the R&D sector may or may not be characterized by scale effects, may or may not use physical capital, and labor and human capital inputs may or may not grow.

The paper is organized as follows. Section 2 analyzes the integrated economy. Section 3 turns to the replication of the integrated economy's equilibrium in the world economy with national borders. Section 4 shifts the focus to the special cases of the model mentioned above. The main original results concerning the dynamics of the open-economy versions of these models are derived in Section 5. Section 6 deals with the question of whether economic integration boosts long-run growth. Section 7 concludes.

2 Integrated economy

In this section, we state the assumptions about technologies, tastes, and market structure underlying our general model and derive the equations that characterize the model's general equilibrium. For now, we ignore the presence of national borders which inhibit factor movements, so that our focus is on the hypothetical integrated economy.

General assumptions

The backbone of the model is formed by production functions for final output, intermediate products, R&D, and human capital (we allow for the special case that total factor productivities (TFPs) are identically zero, so that the corresponding economic activities are not performed in equilibrium). Three important assumptions are necessary to apply the Dixit-Norman approach. First, returns to scale in the private factors of production are constant in all sectors.[6] By contrast, we do not put a restriction on social returns to scale in final-goods production and R&D.[7] Second, countries have identical tastes and technologies. They differ only with respect to factor endowments.[8] Third, as is usual in the static international trade theory (cf. Ethier, 1979, and Helpman and Krugman, 1985,

Heckscher-Ohlin theorem.

[6]See Kortum (1993) and Stokey (1995) for the implications of decreasing returns in the private factors of production in R&D.

[7]Eicher and Turnovsky (1999) carefully investigate the requirements which have to be placed on social returns in order for balanced growth to be possible.

[8]Grossman and Helpman (1991a, Chapters 11 and 12), Helpman (1993), and Arnold (2002a, 2003), among others, discuss "North-South" R&D growth models with differences in technological sophistication (the North innovates, the South imitates) and without factor price equalization.

Chapter 3), we assume that spillover effects, if present, are global in scope. With regard to R&D spillovers, empirical support is provided by Coe and Helpman (1995) (see also Lichtenberg and van Pottelsberghe de la Potterie, 1998).[9] We make four further simplifying assumptions which do not conflict with our aim to develop an endogenous growth model that is sufficiently general so that it nests the endogenous growth models mentioned in the Introduction as special cases. First, there is only one final good. Since it is well understood that the inclusion of additional goods (as in Grossman and Helpman, 1991a, Chapter 7), of different factor intensities in the production of consumption and investment goods, or of intermediate goods with differing factor intensities makes factor price equalization and the replication of the hypothetical integrated equilibrium more likely, we refrain from this complication. Second, there is only one R&D activity: either product innovation or quality upgrading. Recent work by Li (2000) shows that two-R&D-sector models tend to behave similarly to one-R&D-sector models without scale effects, which are covered as special cases of our model.[10] Third, the final good is homogeneous. Product innovation or quality upgrading take place in the intermediate-goods sector. It is well known from Grossman and Helpman (1991a, Chapters 3 and 4) that slight modifications are sufficient to interpret the model as one with innovation in the final-goods sector. Fourth, labor is homogeneous. The outcome of human-capital accumulation is not a different kind of high-skilled labor supply, but an additional supply of the homogeneous labor.[11]

Model

A homogeneous final good, which can be used for consumption or investment (depreciation of capital is ignored), is produced by perfectly competitive firms according to the neoclassical constant-returns-to-scale production function F_Y:[12]

$$Y = F_Y(K_Y, B_Y L_Y, D_Y), \tag{1}$$

where Y is aggregate output, K_Y and L_Y are capital and labor input, respectively, B_Y is a labor-augmenting productivity parameter common to all firms, and D_Y is an index of intermediate goods inputs explained below. The productivity parameter, B_Y, obeys

$$B_Y = \frac{K_Y^{\eta}}{L_Y^{\varepsilon}}, \tag{2}$$

[9]Grossman and Helpman (1991a, Chapters 8 and 9) and Devereux and Lapham (1994) show that with national spillovers, qualitatively different dynamics may occur.

[10]Models with both kinds of R&D originate from Young (1998).

[11]Following Findlay and Kierszkowski (1983), Grossman and Helpman (1991a, Section 5.2, Chapter 7) and Dinopoulos and Segerstrom (1999) consider labor and human capital as distinct factors production.

[12]We ignore the time argument wherever this does not cause confusion.

where η and ε are constants. This is the usual way of capturing learning-by-doing effects in the production of final goods. In the main text, as is usual in the literature on R&D growth models, we ignore these externalities, i.e., set $\eta = \varepsilon = 0$. Accordingly, we discuss alternative specifications later, when we turn to models without R&D. We ignore learning-by-doing effects in the other sectors of the economy.

The model comprises a *product variety (PV)* variant and a *quality upgrading (QU)* variant, which differ with regard to how D_Y is produced from a set of intermediates. The input of quality ω of intermediate j is denoted $X_\omega(j)$. In the PV case, the number of producible intermediates is denoted as A, and only one quality ω of each product exists, so we can drop index ω and write $X(j)$ for the input of intermediate j. More generally, we adopt the convention that subscript ω is dropped in formulas which apply to the PV variant of the model. The production function reads:

$$D_Y = \left[\int_0^A X(j)^\alpha dj \right]^{\frac{1}{\alpha}}. \tag{3}$$

Returns to scale are constant. The constant elasticity of substitution between any pair, j and j', of intermediates is $-1/(1-\alpha)$ $(0 < \alpha < 1)$. In the QU model, the number of producible intermediates is equal to unity, but different qualities, ω, of the given set of intermediates, j, are available. The highest quality producible of intermediate j is denoted $\Omega(j)$. The production function for D_Y is:

$$D_Y = \exp\left\{ \int_0^1 \log\left[\sum_{\omega=1}^{\Omega(j)} \lambda^\omega X_\omega(j) \right] dj \right\}. \tag{4}$$

Returns to scale are constant. For all intermediates j, one unit of quality $\omega + 1$ is a perfect substitute for λ (> 1) units of quality ω. The elasticity of substitution between two intermediates, j and j', of given qualities is -1. For the sake of simplicity, we assume that the intermediates are not used (directly) in the other sectors of the economy.[13] Both in the PV, and the QU, variants of the model, the intermediates are produced from capital $k_{X_\omega}(j)$ and labor $l_{X_\omega}(j)$ according to the neoclassical constant-returns-to-scale production function

$$X_\omega(j) = F_X[k_{X_\omega}(j), l_{X_\omega}(j)]. \tag{5}$$

R&D increases the number of producible intermediates, A, in the PV model and upgrades the highest qualities producible $\Omega(j)$ in the QU model. Let K_A and L_A denote the aggregate capital and labor inputs in R&D and $F_A(K_A, L_A)$ a neoclassical constant-returns-to-scale production function. In the

[13]They are used indirectly because, as will be explained below, other sectors use physical capital produced in the final-goods sector. In Steger (2004), intermediate goods (produced using physical capital alone) are used as an input in R&D.

PV model, the number of new intermediates, $\dot{A}$, is $F_A(K_A, L_A)A^{\chi}$ ($\chi \leq 1$). The presence of the aggregate number of currently producible intermediates, A, in the R&D production function reflects the presence of knowledge spillovers. In the QU model, let $I(j)\,dt$ denote the probability of a quality jump ($I(j)$ is called the rate of innovation) in industry j in the short time interval dt. We assume that the amount of R&D targeted at each market j is the same ($I(j) \equiv I$), so the number of markets with a quality improvement in a short time interval dt is $d[\int_0^1 \Omega(j)dj] = I\,dt$. The rate of innovation is $I = F_A(K_A, L_A)A^{-(1-\chi)}$, where $A(t) = \exp[\int_{-\infty}^{t} I(\tau)d\tau]$. The presence of the cumulated past aggregate innovation rates, $I(\tau)$, captures the effect that successes in R&D become harder and harder to accomplish (cf. Segerstrom, 1998, p. 1297). Differentiating the expression for $A(t)$ with respect to time and inserting the equation for the innovation rate, I, gives $\dot{A} = AI = F_A(K_A, L_A)A^{\chi}$. So

$$\dot{A} = F_A(K_A, L_A)A^{\chi} \tag{6}$$

holds true both in the PV model and in the QU model. In the latter, we have, from $d[\int_0^1 \Omega(j)dj] = I\,dt$ and $\dot{A} = AI$,

$$\frac{d}{dt}\left[\int_0^1 \Omega(j)dj\right] = I = \frac{\dot{A}}{A}. \tag{7}$$

Due to technological leadership or protection of intellectual property rights, the innovator of a new product variety or of a new quality is the only supplier of the respective variety or quality, respectively. It is assumed that at the outset there is also only one supplier of each intermediate in the PV model and only one supplier of the highest quality producible of each intermediate in the QU case.

The economy is populated by a continuum of unit length of identical Barrovian families which share the total family consumption equally among the family members. Letting c denote per-capita consumption, the utility function is $\int_t^{\infty}[c(\tau)^{1-\sigma}/(1-\sigma)]e^{-\rho(\tau-t)}d\tau$ ($\rho, \sigma > 0$)[14]. The total population size is denoted N and grows at rate g_N (≥ 0). The economy is said to display scale effects if the level population, N, positively affects the growth rate of per-capita output and consumption on a balanced-growth path. Each agent supplies l units of labor, so aggregate labor supply is $Nl \equiv L$ (we use the terms "labor" and "human capital" interchangeably). Following Uzawa (1965) and Lucas (1988), individuals can increase their effective per-capita supply of labor by acquiring human capital in education according to the constant-returns-to-scale production function

$$\dot{l} = F_l(k_l, l_l), \tag{8}$$

where k_l and l_l denote the individual's capital and labor input in human-capital accumulation, respectively.[15]

[14] It is understood that instantaneous utility is logarithmic if $\sigma = 1$.

[15] Note that in the presence of population growth ($g_N > 0$), the Uzawa-Lucas education technology implies that newly

All markets clear in equilibrium. All markets except those for the intermediate goods are perfectly competitive. In the PV model, the producers of the intermediates are monopolistic competitors. In the QU model, the intermediate-goods producers are engaged in price competition. There is free entry into R&D.

Equilibrium conditions and outlook

The equations characterizing the equilibrium of the integrated economy (also called the *integrated equilibrium* in what follows) describe cost minimization and the pricing behavior of firms, free entry into R&D, the choice of an optimal consumption profile and optimal investments in human capital by consumers, and market clearing. Since the derivation of the equations is straightforward, it is delegated to Appendix A. An important property of the equilibrium of the QU version is that the producer of the maximum-quality intermediate, $\Omega(j)$, prices all competitors out of the market for intermediate j. Since innovators are protected from imitation, it follows that both in the PV and in the QU versions of the model, there is a single active producer in each intermediate-goods market. Both for the PV model and for the QU model one obtains a system of equations, whose validity is not confined to balanced-growth paths, in which, as usual in general-equilibrium theory, the number of equations exceeds the number of unknowns by one. Since one of the equations can be derived from the other ones, the system contains as many independent equations as unknowns. An important property of the equilibrium is pointed out by Bond, Trask, and Wang (2003, p. 1046): because of constant returns in the education technology (8), even if the aggregate investment in human capital in general equilibrium is determinate, the individual investments in human capital are not (see Appendix A).

Of course, a solution to this system of equations does not in general exist. It would be hard to give a general characterization of the conditions required for the existence of an equilibrium and of the equilibrium itself. However, the model nests the R&D growth models of Romer (1990), Grossman and Helpman (1991a, Chapters 3 and 4, 1991b), Jones (1995a), Segerstrom (1998), and Arnold (1998), as well as the growth models of Solow (1956), Arrow (1962), Uzawa (1965), Sheshinski (1967), Romer (1986), and Lucas (1988) as special cases. So the literature has identified several combinations of functional forms and parameter restrictions which guarantee the existence of an equilibrium in the general model considered here. What we want to show that, for the non-empty set of specifications of our general model for which an equilibrium exists (including the models just mentioned), the world economy with national borders replicates the integrated equilibrium under certain conditions. Fortunately, to do so, we need not *solve* the equations that characterize the integrated equilibrium. All we have to do is show that these equations are valid in the world economy with national borders

born generations "inherit" their parents' current human capital.

as well. This is the object of the next section.

3 World economy

Model

Suppose now that the world economy is divided into M (≥ 2) countries with identical technologies, tastes, and market structures everywhere. The countries are distinguished by a superscript $m \in \{1, \ldots, M\}$. Inputs and outputs in a given country, m, are denoted by lower-case letters. Upper-case letters denote world aggregates. Three remarks are in order. First, as an exception from the general rule, the output of a monopolistic intermediate-goods producer is denoted X (since he serves the entire world-wide demand). Second, $k_{X_\omega}(j)$ and $l_{X_\omega}(j)$, on the one hand, and k_l and l_l, on the other hand, have been defined as per-firm and per-capita variables, respectively. Third, human capital per capita, l^m, may differ across countries (within each country, it is assumed to be distributed uniformly across agents). Labor supply in country m is $n^m l^m$. $l \equiv (1/N) \sum_{m=1}^{M} n^m l^m$ now denotes *average* human capital in the world economy, and $L \equiv Nl$. Analogously, c^m is per-capita consumption in country m, and c is *average* consumption in the world economy. Following Ethier (1979), spillover effects are assumed to be international in scope, so that the parameters B_Y in the production function for final goods and A in the R&D technology are also the same in each country (with the assumption $\eta = \varepsilon = 0$, maintained in the R&D growth models, only the latter externalities are relevant). For example, aggregate production in country m is $y^m = F_Y(k_Y^m, B_Y l_Y^m, d_Y^m)$, where $B_Y = K_Y^\eta / L_Y^\varepsilon$, $K_Y = \sum_{m=1}^{M} k_Y^m$, and $L_Y = \sum_{m=1}^{M} l_Y^m$. Let a^m denote the number of intermediates invented in country m in the PV model and the number of intermediate-goods markets with a quality leader from country m in the QU model. Then the number of new intermediates invented in country m in the PV model is $\dot{a}^m = F_A(k_A^m, l_A^m) A^\chi$, where $A = \sum_{m=1}^{M} a^m$. In the QU model, the rate of innovation in m is $i^m = F_A(k_A^m, l_A^m) A^{-(1-\chi)}$, where $A(t) = \exp[\int_{-\infty}^{t} I(\tau) d\tau]$ and $I = \sum_{m=1}^{M} i^m$. We maintain the assumption that at the outset there is only one supplier of each intermediate in the PV model and only one supplier of the highest quality producible of each intermediate in the QU case.[16] There is free trade in the final good and the intermediate goods. As a consequence, the prices of the final good (unity) and the intermediates, $p_{X_\omega}(j)$, are the same in each country. Financial capital flows freely internationally. So one country can finance

[16] Tang and Wälde (2001) investigate the implications of an initial overlap of intermediate goods in the two-country Grossman-Helpman (1991a, Chapter 3) model. Their main result, the possible existence of a "no-growth trap" (i.e., stagnation despite the existence of a balanced-growth equilibrium with a positive rate because of unfavorable initial conditions), can be proved by showing that it holds for a closed economy and then demonstrating that the the world equilibrium replicates the integrated equilibrium. Since the equilibrium loses its symmetry properties with regard to the different intermediates, the analysis becomes much more tedious however, so we refrain from a formal exposition.

spending on consumption, physical capital, or R&D by incurring debt elsewhere in the world economy, and there is a unique world interest rate, r. Labor is immobile internationally.

Factor mobility

Two case distinctions are necessary in the analysis below. The first one refers to the mobility of physical capital. Let k^m denote physical capital *owned by* residents of country m and k'^m capital *used in* country m. The fact that the final good is traded internationally and that it can be used as investment in physical capital implies that *new* physical capital can be accumulated by final-goods imports. We must also specify, however, if it is possible to import ("old") physical capital already installed in foreign factories.[17] Here we allow for two different cases. On the one hand, as in the static Heckscher-Ohlin theory, (1) physical capital, once installed, is immobile internationally, so that $k'^m = k^m$ is a state variable. Alternatively, as is usual in growth theory, we can assume that (2) physical capital is perfectly mobile internationally. It can be de-installed and transferred abroad, and the distribution of aggregate capital, $K = \sum_{m=1}^{M} k^m = \sum_{m=1}^{M} k'^m$, across countries can change instantaneously. The model also covers special cases with (3) no physical capital. As labor is immobile and intermediates are tradable, we conclude that *(1) the number of internationally immobile factors of production is two (labor and capital) if physical capital is immobile, whereas (2) labor is the only immobile factor of production if physical capital is mobile or (3) not contained in the model.*

Mobility of economic activities

There are four productive economic activities in our model: the production of final goods and of intermediate goods, R&D, and human-capital accumulation. The second case distinction relates to the mobility of these economic activities. We say that an activity is *internationally immobile* if there is a technological restriction that pins down which parts of the integrated-equilibrium production levels are produced where, and *internationally mobile* otherwise. Final-goods production and R&D are internationally mobile activities. Nothing pins down where the integrated-equilibrium output levels are produced. As for human-capital formation, it has been noted in Section 2 that, because of constant returns to scale in (8), individual investments in human capital are not determinate. Applied to the world economy, this means that it is indeterminate which portion of the world-wide growth in human capital takes place where. So human-capital formation, though conducted domestically, is an internationally mobile activity (cf. Bond, Trask, and Wang, 2003, Section 3). Turning to intermediate-goods production, following Grossman and Helpman (1991a, Subsections 7.3 and 7.4), we allow for two possibilities. While a^m denotes the number of intermediates *invented* in country m, let a'^m the number of intermediates *produced* in country m. On the one hand, we assume that (a) intermediates have to be

[17] Cf. the discussion in Smith (1984, p. 298).

produced where they have been invented, so that $a'^m = a^m$ in each country m. In this case, $(k^m, l^m)' - (a_{k_X}, a_{l_X})' a^m X$ gives the factor supplies net of the resources used in the internationally immobile production of intermediates (briefly: *net factor supplies*). Alternatively, we allow for the existence of (b) multinational corporations or costless licensing contracts (with an enforceable commitment by the innovator not to compete with the licensee): a'^m need not equal a^m. In this case, innovators can reap the benefits of an innovation even though by producing intermediates abroad.[18] In sum, *(a) the number of internationally mobile activities is three (final-goods production, R&D, human-capital formation) if a'^m must equal a^m; with (b) a'^m not necessarily equal to a^m (intermediate-goods production also mobile), there are four internationally mobile activities* (recall that some of these activities may have a zero TFP).

Replication of the integrated equilibrium

We now investigate the conditions under which the replication of the integrated equilibrium in the world economy with immobile factors of production is possible.[19] Let w denote the wage rate, p_D the minimum price of one unit of D_Y, and a_{z_Z} the input coefficient of factor z in the production of good Z. $\mathcal{D}$ is a dummy variable which equals zero for the PV model and unity for the QU model. As usual in the static international trade literature inspired by Dixit and Norman (1980), the question of whether replication is possible is approached in two steps. First, we check if the equations that describe the integrated equilibrium hold true in the world economy with national borders as well. If so, factor prices equalize internationally and the input coefficients, a_{z_Z}, for all factors, z, in the production of all goods, Z, are the same as in the integrated equilibrium everywhere, i.e., the national factor input vectors (with z-th component $a_{z_Z} Z$) are parallel. Second, we turn to the question of whether the integrated-equilibrium factor input vectors can be divided across countries in such a way that, in each country, all factor input vectors are non-negative and factor markets clear.

The first step is quite mechanical. The conditions for cost minimization by firms, pricing, free entry into R&D, an optimal consumption profile, and optimal investments in human capital are the same as in the integrated equilibrium. And adding up the market clearing conditions for the M countries yields the integrated-equilibrium market clearing conditions. The formal treatment is given in Appendix B. Here we focus only the equilibrium conditions for the markets for labor and physical capital, since

[18] Apparently, the notion of internationally immobile versus mobile activities is related to the distinction between non-traded and traded goods, respectively. The analogy is not complete, however. Human capital is a non-traded good, but human-capital formation is an internationally mobile activity. And the production of intermediate goods is an internationally immobile activity in case (a) even though the intermediates are tradable.

[19] We do not examine the question of whether other equilibria of the world economy, without replication, exist.

these will prove crucial in the second step:

$$\begin{pmatrix} n^m l^m \\ k'^m \end{pmatrix} = \begin{pmatrix} a_{L_Y}(r,w,p_D,B_Y) & a_{L_A}(r,w) & a_{l_l}(r,w) & a_{l_X}(r,w) \\ a_{K_Y}(r,w,p_D,B_Y) & a_{K_A}(r,w) & a_{k_l}(r,w) & a_{k_X}(r,w) \end{pmatrix} \begin{pmatrix} y^m \\ A^{1-\chi}\left(\frac{\dot{a}^m}{A}\right)^{1-\mathcal{D}}(i^m)^{\mathcal{D}} \\ n^m \dot{l}^m \\ a'^m X \end{pmatrix}. \tag{9}$$

The four terms on the right-hand sides of these equations are labor and physical capital, respectively, in their four different uses, final-goods production, R&D, human-capital accumulation, and intermediate-goods production, in country m. Adding up these market clearing conditions for the M countries leads to the integrated-equilibrium market clearing conditions.[20]

Since the equations that characterize the integrated equilibrium hold true in an equilibrium of the world economy with national borders as well, replication is feasible if the integrated-equilibrium factor input vectors can be divided across countries so that in each country, all factor input vectors are non-negative and factor markets clear (step two):

Theorem 1: *Suppose an integrated equilibrium with $Y(t) \geq 0$, $\dot{A}(t)^{1-\mathcal{D}} I(t)^{\mathcal{D}} \geq 0$, $X \geq 0$, and $\dot{l}(t) \geq 0$ for all t exists. (1) Suppose further physical capital is immobile. (a) If $a'^m = a^m$, the replication of the integrated equilibrium is possible if, and only if, for all t and for all $m \in \{1,\ldots,M\}$, (a) there exist $(y^m, (\dot{a}^m)^{1-\mathcal{D}}(i^m)^{\mathcal{D}}, \dot{l}^m)' \geq \mathbf{0}$ such that*

$$\sum_{m=1}^{M} \begin{pmatrix} y^m \\ (\dot{a}^m)^{1-\mathcal{D}}(i^m)^{\mathcal{D}} \\ n^m \dot{l}^m \end{pmatrix} = \begin{pmatrix} Y \\ \dot{A}^{1-\mathcal{D}} I^{\mathcal{D}} \\ N\dot{l} \end{pmatrix}$$

and (9) holds with $k'^m = k^m$ and $a'^m = a^m$. (b) If a'^m does not necessarily equal a^m, the replication of the integrated equilibrium is possible if, and only if, for all t and for all $m \in \{1,\ldots,M\}$, (a) there exist $(y^m, (\dot{a}^m)^{1-\mathcal{D}}(i^m)^{\mathcal{D}}, \dot{l}^m, a'^m)' \geq \mathbf{0}$ such that

$$\sum_{m=1}^{M} \begin{pmatrix} y^m \\ (\dot{a}^m)^{1-\mathcal{D}}(i^m)^{\mathcal{D}} \\ n^m \dot{l}^m \\ a'^m \end{pmatrix} = \begin{pmatrix} Y \\ \dot{A}^{1-\mathcal{D}} I^{\mathcal{D}} \\ N\dot{l} \\ A^{1-\mathcal{D}} \end{pmatrix}$$

and (9) holds with $k'^m = k^m$. With physical capital (2) mobile or (3) absent from the model, the conditions are the same except that only the first equality in (9) has to be satisfied.

Proof: (1) For the case of immobile physical capital ($k'^m = k^m$), Theorem 1 merely restates the requirement that the national input vectors have to be non-negative. (2) Suppose physical capital is

[20] In what follows, we suppress the arguments of the input-coefficient functions.

mobile and the first equality in (9) is satisfied (i.e., the labor market clears) in country m. Inserting the solution, (a) $(y^m, (\dot{a}^m)^{1-\mathcal{D}}(i^m)^{\mathcal{D}}, \dot{l}^m)'$ ($\geq \mathbf{0}$) or (b) $(y^m, (\dot{a}^m)^{1-\mathcal{D}}(i^m)^{\mathcal{D}}, \dot{l}^m, a'^m)'$ ($\geq \mathbf{0}$), into the first line yields the non-negative amount of capital, k'^m, used in country m. (3) If the model does not contain physical capital, the second equality in (9) drops out. *q.e.d.*

Theorem 1 has three immediate implications, which will prove useful below. Let θ^m denote country m's share in the world-wide net supply of labor: $\theta^m \equiv (n^m l^m - a_{l_X} a^m X)/(Nl - a_{l_X} a^m X)$ in case (a) and $\theta^m \equiv n^m l^m/(Nl)$ in case (b).

Corollary 1: *In case (a), a necessary condition for the replicability of the integrated equilibrium is the non-negativity of the net factor supplies, i.e.,* $(k^m, n^m l^m)' - (a_{l_X}, a_{k_X})' a^m X \geq \mathbf{0}$.

Corollary 2: *(1) Suppose physical capital is immobile. In case (a), assume further that the net factor supplies are non-negative and that the relative net factor supplies are uniform across countries in that* $(k^m - a_{k_X} a^m X)/(K - a_{k_X} A^{1-\mathcal{D}} X) = \theta^m$ *holds for all* $m \in \{1, \ldots, M\}$. *In case (b), assume that the relative net factor supplies are uniform across countries in that* $k^m/K = \theta^m$ *for all* $m \in \{1, \ldots, M\}$. *Then replication is feasible, with each country conducting a proportion* θ^m *of each internationally mobile economic activity.*

Corollary 3: *Suppose physical capital is (2) mobile or (3) not contained in the model and the net supply of labor is non-negative in case (a). Then replication is feasible, with each country conducting a proportion* θ^m *of each internationally mobile economic activity.*

Thus, similar as in the static trade theory, three factors make the replication of the integrated equilibrium more likely: the presence of multinationals or international patent licensing (in which case the necessary condition in Corollary 1 becomes obsolete), the similarity of the relative factor endowments (according to Corollary 2, identical relative factor endowments are sufficient for replicability if physical capital is immobile), and physical-capital mobility (which, according to Corollary 3, together with non-negative net labor supply in case (a) is sufficient for replicability).[21] The allocation of physical capital and labor to their different uses consistent with replication of the integrated equilibrium may not be unique. Examples of this sort of indeterminacy are encountered frequently below. Moreover, even if it is unique, the allocation of *new* physical capital across countries may be indeterminate (see Fischer and Frenkel, 1972, and Smith, 1984, p. 307).[22]

[21]Replication would also become more likely if one added additional consumption goods with linearly independent integrated-equilibrium input vectors (cf. Grossman and Helpman, 1991a, Chapter 7), different factor intensities in the production of consumption and investment goods (cf. Stiglitz, 1970, and Bond, Trask, and Wang, 2003), or intermediate goods with different factor intensities (cf. Ventura, 1997, and Cuñat and Maffezzoli, 2004).

[22]Notice that we have to distinguish four different kinds of indeterminacy: first, indeterminacy of the world equilibrium

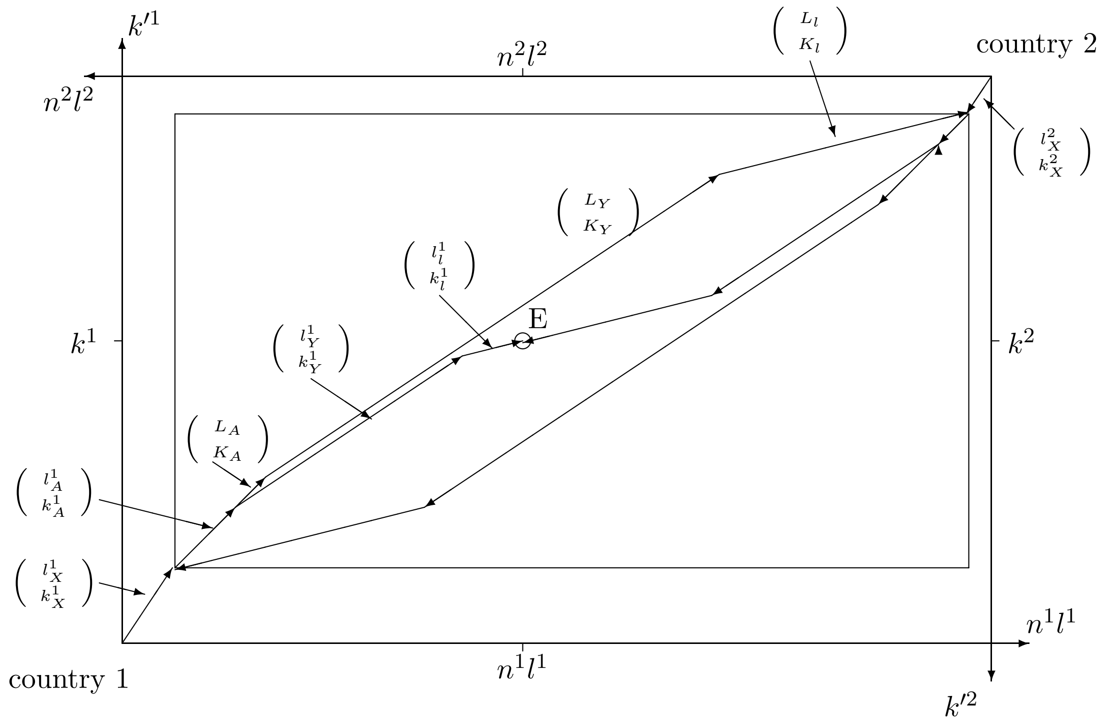

Figure 1: Two countries (a) without multinational firms or international patent licensing

Two countries

To get an intuition of the content of Theorem 1 and the subsequent corollaries, we now give the familiar graphical illustration for the case of two countries ($M = 2$) (cf., e.g., Dixit and Norman, 1980, Section 4.4, Helpman and Krugman, 1985, Chapter 1, Grossman and Helpman, 1991a, Chapter 7, and Bond, Trask, and Wang, 2003, Section 3, Cuñat and Maffezzoli, 2004, Subsection 2.2). In the static two-country trade model, replication is possible with sufficiently similar factor endowments if the number of internationally mobile activities (traded goods) is no less than the number of immobile factors of production (e.g., Dixit and Norman, 1980, p. 111).[23] Our model contains (a) three or (b) four internationally mobile productive activities and (2), (3) one or (1) two immobile factors of production. This suggests that replication is possible with sufficiently similar factor endowments.

Consider Figures 1 and 2. The lower left and upper right corners are country 1's and country 2's origin,

due to more stable eigenvalues than state variables in the system of differential equations that describes the integrated equilibrium (as in Benhabib and Perli, 1994); second, the indeterminacy of where new human capital is produced (pointed out by Bond, Trask, and Wang, 2003); third, indeterminacy of the static factor allocation; fourth, the indeterminacy of the allocation of new physical capital across countries (Fischer and Frenkel, 1972).

[23] A set of differentiated goods is counted as one activity in the imperfect-competition extension by Helpman and Krugman (1985).

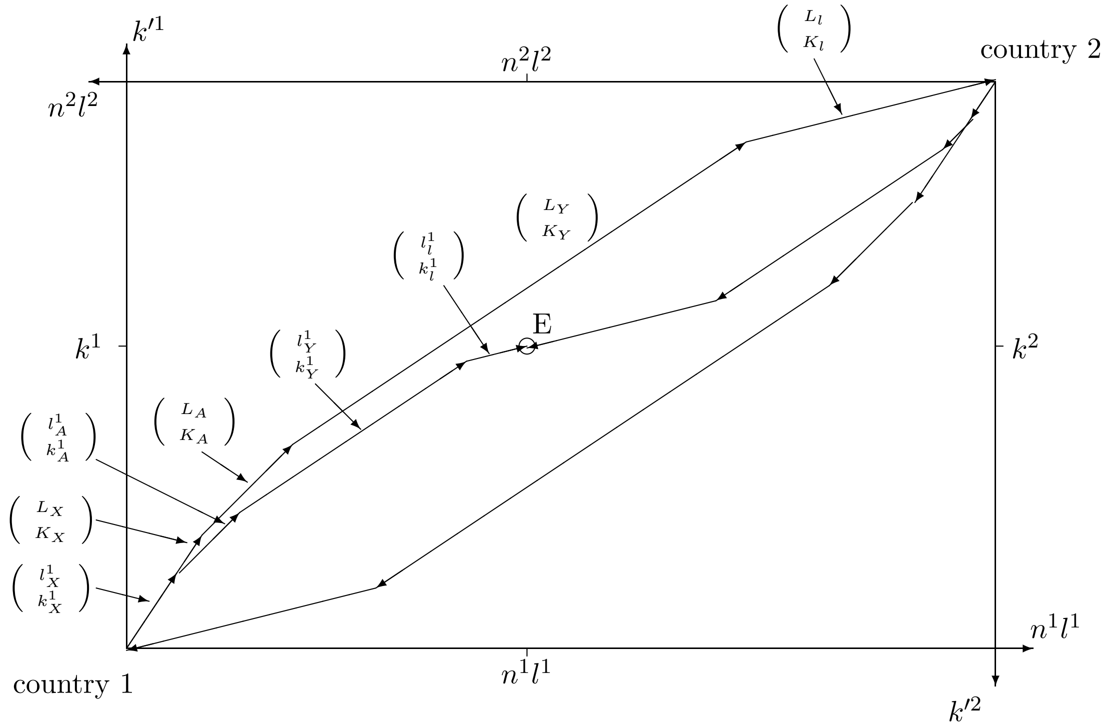

Figure 2: Two countries (b) with multinational firms or international patent licensing

respectively. Labor and capital inputs, l^m and k'^m, are measured along the horizontal and vertical axes, respectively. The width and the height of the boxes are equal to the world supplies of labor, $n^1l^1+n^2l^2 = Nl$, and physical capital, $k^1+k^2 = K$, respectively. Figure 1 applies to case (a) with $a'^m = a^m$. It depicts the input vectors for the immobile activity intermediate-goods production, $(l_X^m, k_X^m)'$ $(= (l_X, k_X)'a^m)$, starting from the two countries' respective origins. The end points of these vectors determine the lower left and upper right corners of a smaller rectangle, whose dimensions represent the net factor supplies, available for the internationally mobile activities. Figure 2 applies to case (b), with a'^m possibly different from a^m. Starting from the countries' origins in the boxes (the smaller one in Figure 1), the figures depict the integrated-equilibrium input vectors for the internationally mobile activities, i.e., $(L_A, K_A)'$, $(L_Y, K_Y)'$, and $(L_l, K_l)'$ $(= N(l_l, k_l)')$, plus $(L_X, K_X)'$ $(= (l_X, k_X)'A^{1-\mathcal{D}})$ in case (b). These input vectors are assumed linearly independent. Suppose, to begin with, that (1) physical capital is immobile. Then the factor supplies, $(l^1, k^1)'$ and $(l^2, k^2)'$, determine an endowment point, E, in the rectangles. The integrated-equilibrium input vectors for the internationally mobile activities form a hexagon in Figure 1 and an octagon in Figure 2. The replication of the integrated equilibrium is feasible if the endowment point, E, is located inside this polygon. Obviously, if E is located outside the smaller box in Figure 1 (case (a)), then the replication of the integrated equilibrium is not feasible.

This illustrates the necessity of non-negative net factor supplies for replication (cf. Corollary 1).[24] Since (a) the hexagon covers the diagonal of the box in Figure 1 and (b) the octagon covers the diagonal of the smaller rectangle in Figure 2, uniformity of the relative net factor endowments ensures the replicability of the integrated equilibrium (this illustrates the assertion made by Corollary 2). The division of the integrated-equilibrium input vectors between countries is determinate or indeterminate, depending on whether there are exactly two or more than two internationally mobile activities (with non-zero TFPs). (2) Next consider the case of physical-capital mobility. In Figures 1 and 2, the labor inputs, $n^1 l^1$ and $n^2 l^2$, are determinate. By contrast, the amounts of physical capital used in the two countries, k'^1 and k'^2, are not determined by the endowments k^1 and k^2 respectively. So the economies may operate on any point on the vertical line through $(n^1 l^1, 0)'$ and $(n^2 l^2, 0)'$. Any point on this line located inside (a) the hexagon in Figure 1 or (b) the octagon in Figure 2 is consistent with replication of the integrated equilibrium. The non-negativity of the net labor supply, $n^m l^m - a_{l_X} a^m X \geq 0$, remains a necessary condition in case (a) (as required by Corollary 1). If the equilibrium is determinate with (1) physical-capital immobility, it becomes indeterminate with (2) physical-capital mobility. If it is indeterminate with (1) physical capital-immobility, (2) there it an additional degree of freedom in the division of the integrated-equilibrium production vectors between countries. Finally, if (3) the model does not contain physical capital, the rectangles in Figures 1 and 2 degenerate to horizontal lines. As in the case of (2) physical-capital mobility, the non-negativity of the net factor supplies is necessary and sufficient for the possibility of replication (cf. Corollaries 1 and 3).

4 Examples

Before turning to the implications for transitional dynamics (in Section 5) and the relationship between economic integration and long-run growth (in Section 6), we now present several prominent special cases of our general model. Ignoring open-economy issues for the moment, we state for each special case considered the dynamic properties of the balanced-growth equilibrium (to be cited in Section 5) and the growth rate of aggregate output, $g_Y \equiv \dot{Y}/Y$, in a balanced-growth equilibrium of the integrated economy (to be cited in Section 6). For the sake of brevity, we focus on R&D growth models (with $\varepsilon = \eta = 0$) in the main text. Appendix C shows how our general model can also be applied to models without R&D. Throughout we assume that parameter values are such that the equilibrium growth rate is positive and leads to bounded intertemporal utility. The analysis applies for $M \geq 2$, there

[24] It will turn out in Section 6 that this is a genuine possibility in R&D growth models. A country m with a small resource base, a sufficiently large a^m, and (a) no possibility to manufacture domestically invented intermediates abroad will not be able to realize the input vector $(l_X^m, k_X^m)'$.

is no need to restrict attention to the (graphically tractable) two-country case. We let $\mu \equiv 1/\alpha$ and $\gamma \equiv (1-\alpha)/\alpha$ in the PV model and $\mu \equiv \lambda$ and $\gamma \equiv \log \lambda$ in the QU model.

Models with constant returns to knowledge in R&D

With non-diminishing returns to knowledge in R&D ($\chi = 1$), we have, from (6), $\dot{A}/A = F_A(K_A, L_A)$ in the PV model and $I = F_A(K_A, L_A)$ in the QU model. In order for $\dot{A}/A$ or I, respectively, to be constant in equilibrium, capital must not be an argument of the R&D production function and the supply of labor must not rise due to human-capital accumulation nor because of population growth. The absence of physical capital in (6) together with the assumption of constant returns to scale implies $F_A(K_A, L_A) = L_A/a_{L_A}$, where a_{L_A} is exogenous. Moreover, the TFP in human-capital accumulation is identically zero and $g_N = 0$. This class of growth models contains three of the most prominent ones.

Example 1: Grossman and Helpman (1991a, Chapter 3) consider a PV model with constant returns to knowledge in R&D. Labor is the only input in the production of intermediates, but is not used in final-goods production. In a balanced-growth equilibrium,

$$g_Y = \frac{(\mu - 1)\frac{L}{a_{L_A}} - \rho}{\sigma - 1 + \frac{\mu}{\lambda}}.$$

Grossman and Helpman (1991a, Chapter 3) assume logarithmic utility ($\sigma = 1$). Since growth is due to increasing product variety, $\mu = 1/\alpha$. So the expression for the balanced growth rate simplifies to $g_Y = (1-\alpha)[(1-\alpha)L/a_{L_A} - \alpha\rho]/\alpha$. The economy jumps on its balanced-growth path (see Grossman and Helpman, 1991a, p. 61).

Example 2: Grossman and Helpman (1991a, Chapter 4, 1991b) also analyze the QU variant of Example 1. In this case the formula for output growth in a balanced-growth equilibrium simplifies to $g_Y = \log \lambda[(\lambda - 1)L/a_{L_A} - \rho]/\lambda$. As in Example 1, the economy jumps on its balanced-growth path (Grossman and Helpman, 1991a, p. 96).

Example 3: In Romer's (1990) model, final-goods production obeys $Y = L_Y^{1-\alpha} D_Y^{\alpha}$ and physical capital is the only input in the production of the intermediates. Therefore,

$$g_Y = \frac{\left(1 - \frac{1}{\mu}\right) \frac{\alpha}{1-\alpha} \frac{L}{a_{L_A}} - \rho}{\sigma - 1 + \frac{1-\alpha}{\alpha\gamma} + \left(1 - \frac{1}{\mu}\right) \frac{1}{\gamma}}.$$

As Romer (1990) considers the PV variant of the model (with $\mu = 1/\alpha$), this boils down to $g_Y = (\alpha L/a_{L_A} - \rho)/(\sigma + \alpha)$. Arnold (2000, Theorem 1, p. 74) proves that there is (locally) a unique trajectory converging to the balanced-growth equilibrium.

R&D models with population growth

Suppose the returns to existing knowledge in R&D (cf. (6)) are diminishing: $\chi < 1$. Assume further that the TFP in human-capital formation is identically zero.

Example 4: Segerstrom (1998) analyzes a QU model without capital. Labor is the only input in the production of intermediates and in R&D and is not used in final-goods production. In a balanced-growth equilibrium, $I = g_N/(1-\chi)$ and

$$g_Y = \left(1 + \frac{\gamma}{1-\chi}\right) g_N.$$

There is a unique trajectory converging to the balanced-growth equilibrium (see Segerstrom, 1998, p. 1300).

Example 5: The formula above for the growth rate of output in a balanced-growth equilibrium of the Segerstrom (1998) model applies to the PV version of the model as well. It is straightforward to show that in this case, too, there exists a unique trajectory converging to the balanced-growth path.

Example 6: Jones (1995a) considers a PV model in which, as in Example 3, final goods are produced using labor and intermediates according to $Y = L_Y^{1-\alpha} D_Y^{\alpha}$. Physical capital is the only input in the production of the intermediates, and R&D does not require capital. In a balanced-growth equilibrium, the growth rate of aggregate output is

$$g_Y = \left(1 + \frac{\alpha}{1-\alpha}\frac{\gamma}{1-\chi}\right) g_N.$$

In the PV model considered by Jones (1995a), $\gamma \equiv (1-\alpha)/\alpha$ and, hence, $g_Y = [1 + 1/(1-\chi)]g_N$. In a related paper (Arnold, 2005, Theorem 1, p. 4), we show that, locally, there is a unique convergent growth path.

R&D models with human-capital accumulation

In this class of models, $\chi < 1$ and the TFP in human-capital accumulation is not set equal to zero.

Example 7: In Arnold's (1998) PV model, as in Examples 1 and 5, there is no physical capital, the final good is produced from the intermediates alone, labor is the only input in the production of intermediates and in R&D, and population is constant ($g_N = 0$). Physical capital is not used in education: $F_l(k_l, l_l) = l_l/a_{l_l}$, where a_{l_l} is exogenous. In a balanced-growth equilibrium,

$$g_Y = \frac{\frac{1}{a_{l_l}} - \rho}{\sigma - 1 + \frac{1}{1+\frac{\gamma}{1-\chi}}}.$$

For $\sigma = 1$, there is a unique trajectory converging to the balanced-growth equilibrium (see Arnold, 1998, Propositions 2, p. 91, and 9, p. 103).

Models without R&D

Models without R&D are obtained as special cases of our general model by assuming that the TFP in R&D is identically zero. Appendix C considers several prominent models as Examples 11-17.

5 Transitional dynamics

The dynamics of the closed-economy versions of some of the most prominent endogenous growth models are by now well understood. The central results have been cited in the previous section. By contrast, little is known about the dynamics of the multi-country open-economy versions of the same models. We can now address this issue with the help of Theorem 1 in conjunction with the following obvious fact:

Theorem 2: *Suppose the world economy replicates the integrated equilibrium. Then the dynamics of the world economy replicate the dynamics of the integrated economy.*

In order to characterize the dynamics of the world economy, we merely have to show that the conditions for replication are satisfied and apply the stability results for the integrated economy cited in the previous section. As announced above, we will not have to solve a single differential equation. In the main text, we focus on the R&D growth models in Examples 1-7. The analysis can be applied straightforwardly to other special cases of our model. Results on the transitional dynamics of multi-country growth models without R&D (Examples 11-17) are delegated to Appendix C. The non-negativity constraint for the supply of labor net of employment in intermediate-goods production (a) without multinationals or patent licensing (see Corollary 1), $n^m l^m - a_{l_X} a^m X \geq 0$, can be written as

$$\frac{a^m}{A^{1-\mathcal{D}}} \leq \frac{n^m l^m}{L_X} \tag{10}$$

for all t and all m.

Models with constant returns to knowledge in R&D

Example 1: In the Grossman-Helpman (1991a, Chapter 3) model, the location of final-goods production is indeterminate ("footloose") because it requires intermediates only. Labor is the only (immobile) factor of production (case (3)). (a) With $a'^m = a^m$, intermediate-goods production is immobile, while R&D is an internationally mobile activity. Given that (10) with $\mathcal{D} = 0$ is satisfied, replication is feasible and the division of the world input vectors across countries is determinate because the number of internationally mobile activities (R&D) is equal to the number of immobile factors of production (labor). (b) With a'^m not necessarily equal to a^m, there is no internationally immobile activity and the number of internationally mobile activities (R&D and intermediate-goods production) exceeds the number of immobile factors of production (labor) by one. Since $a_{l_X} = 0$, (10) is satisfied: the net supply of labor is non-negative. So replication is feasible with one degree of indeterminacy: any exhaustive allocation of non-negative portions of the integrated-equilibrium input vectors to countries constitutes an equilibrium of the world economy. These findings generalize the results of Grossman and

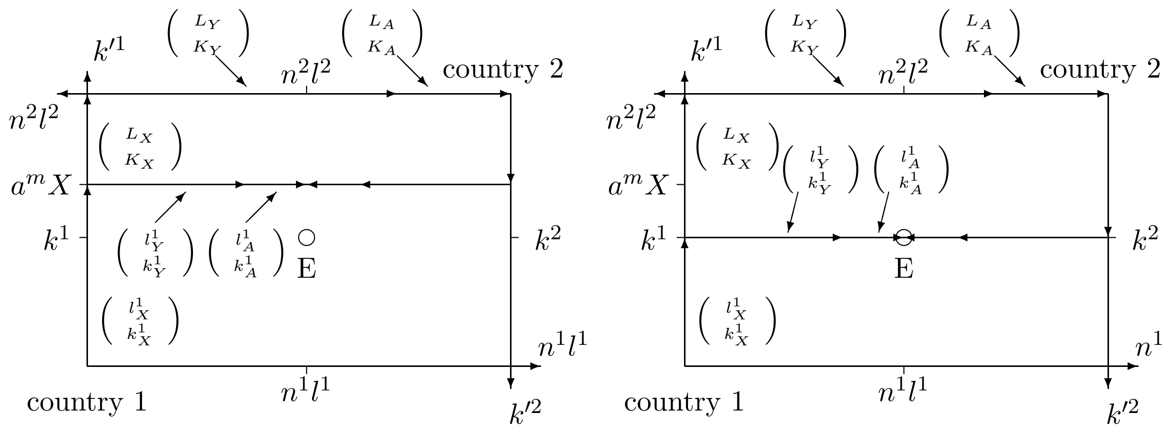

Figure 3: Two-country Romer (1990) model (a) without (left panel) and (b) with (right panel) multinational firms or international patent licensing

Helpman (1991a, Subsection 9.2) and Wälde (1996), who analyze the two-country ($M = 2$) version of the model.[25] The results presented subsequently are novel.

Example 2: The analysis of the Grossman-Helpman (1991a, Chapter 4, 1991b) QU model proceeds analogously to Example 1. (a) With $a'^m = a^m$, intermediate-goods production is immobile. Non-negativity of the supply of labor net of employment in intermediate-goods production is ensured by (10) with $\mathcal{D} = 1$. The number of internationally mobile activities (R&D) is equal to the number of immobile factors of production (labor). So given (10), replication is feasible, and the division of the integrated-equilibrium input vectors between countries is determinate. (b) With a'^m not necessarily equal to a^m, there is no internationally immobile activity. The number of internationally mobile activities (R&D and intermediate-goods production) exceeds the number of immobile factors of production (labor) by one. Since $a_{l_X} = 0$, (10) is satisfied. So replication is feasible, with one degree of indeterminacy: any division of non-negative portions of the integrated-equilibrium input vectors across countries is an equilibrium.

Example 3: In Romer's (1990) PV model, $a_{K_Y} = a_{K_A} = 0$, $a_{k_X} = 1$, and $a_{l_X} = 0$. Since $a_{L_Y} > 0$, production is not "footloose". (9) becomes

$$\begin{pmatrix} n^m l \\ k'^m \end{pmatrix} = \begin{pmatrix} a_{L_Y} & a_{L_A} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y^m \\ A^{1-\chi} \frac{\dot{a}^m}{A} \\ a'^m X \end{pmatrix} \tag{11}$$

[25] Since more consumption goods do not add state variables, the approach taken here is also applicable with two or more consumption goods (as in Grossman and Helpman, 1991a, Chapter 7). The integrated economy still jumps to its balanced-growth path, and the national proportions of the intermediate goods, a^m/a, converge. Each additional good adds an additional degree of indeterminacy to the static factor allocation.

with $\chi = 1$. Suppose, to begin with, that (a) $a'^m = a^m$. All three productive activities (final-goods production, R&D, and intermediate-goods production) are internationally mobile. $a_{l_X} = 0$ implies that condition (10) is satisfied. Nonetheless, with (1) immobile physical capital ($k'^m = k^m$), replication is feasible if, and only if, the economy happens to be endowed with the amount of capital needed to produce the integrated-equilibrium quantities of those intermediates for which it has a monopoly: $k^m = a^m X$. Given that intermediate-goods production is the only use of physical capital, this is necessary for non-negative net supply of capital in all countries m (cf. Corollary 1). The l_Y^m's and l_A^m's are indeterminate. (2) If physical capital is mobile, the replication of the integrated equilibrium is possible (cf. Corollary 3). k'^m can adjust such that $k'^m = a^m X$.[26] Again, the division of the two horizontal input vectors $(L_Y, 0)$ and $(L_A, 0)$ is indeterminate. This is illustrated for the two-country case in the left panel of Figure 3. Since the input vectors for the internationally mobile activities (final-goods production and R&D) are horizontal, the "smaller rectangle" encountered in Figure 1 has height zero here. (1) With immobile physical capital, replication is not possible unless the endowment point happens to be located on this horizontal line. (2) With mobile physical capital, replication is possible because the vertical line through $(n^1 l^1, 0)'$ and $(n^2 l^2, 0)'$ intersects the horizontal line at height $a^m X$. (b) With multinationals or patent licensing and with (1) immobile physical capital, a'^m is free to adjust such that the equality in second line in (11) holds for $k'^m = k^m$: $a'^m = k^m/X$. The first line represents one equation in two unknowns, y^m and $\dot{a}^m$. Replication is feasible, with one degree of indeterminacy. If (2) physical capital is mobile, there is another degree of indeterminacy. In terms of the right panel of Figure 3, the length of the production vectors (l_X^m, k_X^m) is determined by the location of the endowment point, E. As before, the division of the two horizontal input vectors is indeterminate.

R&D models with population growth

Example 4: Segerstrom's (1998) model can be analyzed following the lines pursued in in Example 2. (a) With $a'^m = a^m$, intermediate-goods production is immobile. Non-negativity of the supply of labor net of employment in intermediate-goods production requires (10) with $\mathcal{D} = 1$. The number of internationally mobile activities (R&D) is equal to the number of immobile factors of production (labor). So given (10), replication is feasible, and the equilibrium is determinate. (b) With a'^m not necessarily equal to a^m, there is no internationally immobile activity. The number of internationally mobile activities (R&D and intermediate-goods production) exceeds the number of immobile factors of production (labor) by one. Replication is feasible, with one degree of indeterminacy.

Example 5: The PV version of Segerstrom (1998) can be treated analogously.

[26] This equation determines what part of the new physical capital, $\dot{K}$, is installed in country m, so the Fischer-Frenkel (1972) indeterminacy does not arise here.

Example 6: In the Jones (1995a) model, (11) holds with $\chi < 1$. By the same reasoning as in Example 3, if (a) $a'^m = a^m$, $a_{l_X} = 0$ implies the validity of (10), and the possibility of replication requires (1) mobility of physical capital. (b) The presence of multinationals or patent licensing is an alternative sufficient condition for replication. In both cases, the division of $(L_Y, K_Y)'$ and $(L_A, K_A)'$ across countries is indeterminate. If both conditions are satisfied, there are two degrees of indeterminacy.

R&D models with human-capital accumulation

Example 7: In Arnold's (1998) model, as (3) physical capital is absent, replication is possible if the labor-market clearing condition in (9) has a non-negative solution for each country m (see Corollary 3). Condition (10) with $\mathcal{D} = 0$ ensures the non-negativity of the supply of labor net of employment in intermediate-goods production in case (a) ($a'^m = a^m$). In this case, there are two internationally mobile activities (R&D and human-capital accumulation). Replication is feasible, with one degree of indeterminacy. (b) With a'^m not necessarily equal to a^m, there is another degree of indeterminacy.

Dynamics

Equipped with the results stated above, we can now characterize the evolution of country m's share in the total number of intermediate-goods markets. For the sake of simplicity, we assume that $l^m = l$ is uniform across countries in Examples 1-6. Rewrite the labor-market clearing condition in (9) as

$$\left(\frac{\dot{a}^m}{A}\right)^{1-\mathcal{D}} (i^m)^{\mathcal{D}} = \frac{n^m l^m - a_{L_Y} y^m - a_{l_l} n^m \dot{l}^m - a_{l_X} a'^m X}{a_{L_A} A^{1-\chi}}.$$

Summing over all m and using $l \equiv (1/N) \sum_{m=1}^{M} n^m l^m$, $\sum_{m=1}^{M} n^m \dot{l}^m = N\dot{l}$, and $\sum_{m=1}^{M} a'^m = A^{1-\mathcal{D}}$ yields:

$$\left(\frac{\dot{A}}{A}\right)^{1-\mathcal{D}} I^{\mathcal{D}} = \frac{Nl - a_{L_Y} Y - a_{l_l} N\dot{l} - a_{l_X} A^{1-\mathcal{D}} X}{a_{L_A} A^{1-\chi}}.$$

In the PV version ($\mathcal{D} = 0$), $d(a^m/A)/dt = \dot{a}^m/A - (a^m/A)(\dot{A}/A)$. In the QU variant ($\mathcal{D} = 1$), country m makes $i^m dt$ innovations, which add to the number of intermediate-goods markets in which it has a temporary monopoly, a^m, in a short time interval, dt. On the other hand, maintaining the assumption that the amount of R&D targeted at each market j is the same, $Ia^m dt$ domestic monopolists lose their position as a quality leader. So $da^m = i^m dt - Ia^m dt$ or $\dot{a}^m = i^m - Ia^m$. Using $Nl = L$, $a_{L_Y} Y = L_Y$, $a_{l_X} A^{1-\mathcal{D}} X = L_X$, and $a_{l_l} N\dot{l} = L_l$, it follows from the above two equations that

$$\begin{aligned} \frac{d}{dt}\left(\frac{a^m}{A^{1-\mathcal{D}}}\right) &= \frac{L}{a_{L_A} A^{1-\chi}} \left(\frac{n^m}{N}\frac{l^m}{l} - \frac{a^m}{A^{1-\mathcal{D}}}\right) + \frac{L_Y}{a_{L_A} A^{1-\chi}} \left(\frac{a^m}{A^{1-\mathcal{D}}} - \frac{y^m}{Y}\right) \\ &\quad + \frac{L_l}{a_{L_A} A^{1-\chi}} \left(\frac{a^m}{A^{1-\mathcal{D}}} - \frac{n^m}{N}\frac{\dot{l}^m}{\dot{l}}\right) + \frac{L_X}{a_{L_A} A^{1-\chi}} \left(\frac{a^m}{A^{1-\mathcal{D}}} - \frac{a'^m}{A^{1-\mathcal{D}}}\right). \end{aligned} \tag{12}$$

Notice that the terms in front of the parantheses are constants in a balanced-growth equilibrium. In models with constant returns to knowledge in R&D, $\chi = 1$ and the supply of labor, L, as well as the portions employed in its different uses are constant. So both the numerators and the denominators are constant. In models with population growth or human-capital accumulation (and $\chi < 1$), $\dot{A}/A = g_N/(1-\chi)$ in a balanced-growth equilibrium and constant proportions of labor supply are devoted to its different uses. So the numerators and denominators grow at the same rate.

Example 1: The only state variables are the proportions of the intermediates invented in the respective countries, a^m. With $\mathcal{D} = L_Y = L_l = 0$, $l^m = l$, and $\chi = 1$, (12) becomes:

$$\frac{d}{dt}\left(\frac{a^m}{A}\right) = \frac{L}{a_{L_A}}\left(\frac{n^m}{N} - \frac{a^m}{A}\right) + \frac{L_X}{a_{L_A}}\left(\frac{a^m}{A} - \frac{a'^m}{A}\right).$$

Since the world economy jumps on its balanced growth path, $L_X = \alpha(L + a_{L_A}\alpha\rho)$ is constant. So (a) if $a'^m = a^m$, this is an autonomous differential equation in a^m/A. Each country's share in the number of intermediate-goods markets, a^m/A, converges to its share in the world-wide population, n^m/N. Since L_X is constant and convergence is monotonic, if (10) is satisfied initially (i.e., $a^m/A \leq n^m l/[\alpha(L + a_{L_A}\alpha\rho)]$), it is always satisfied. (b) With a'^m and a^m not necessarily identical, a country that produces some of the products it has invented abroad ($a'^m < a^m$) can capture a higher proportion, a^m/A, of the intermediate-goods-market.

Example 2: Setting $\mathcal{D} = 1$, $L_Y = L_l = 0$, and $l^m = l$ in (12) gives:

$$\dot{a}^m = \frac{L}{a_{L_A}A^{1-\chi}}\left(\frac{n^m}{N} - a^m\right) + \frac{L_X}{a_{L_A}A^{1-\chi}}(a^m - a'^m) \tag{13}$$

with $\chi = 1$. As in Example 1, the world economy as a whole jumps to its balanced-growth path with $L_X = [L + (\lambda - 1)a_{L_A}\rho]/\lambda$ constant. (a) With $a'^m = a^m$, a^m converges monotonically to n^m/N. If (10) is satisfied initially (i.e., $a^m/A \leq n^m l\lambda/[L + (\lambda-1)a_{L_A}\rho]$), it continues to be satisfied during the transition. (b) With $a'^m \neq a^m$, a country can get a higher share in the aggregate number of intermediate goods markets by manufacturing some of its products abroad or selling the right to do so.

Example 3: Suppose one of the conditions for replicabillity is satisfied. That is, (2) physical capital is mobile and/or (b) a'^m is allowed to differ from a^m. Then the transitional dynamics of the world economy looks as follows. Locally, the world economy converges to its balanced-growth equilibrium with L_Y constant. From (12), with $\mathcal{D} = L_l = L_X = 0$ and $l^m = l$,

$$\frac{d}{dt}\left(\frac{a^m}{A}\right) = \frac{L}{a_{L_A}A^{1-\chi}}\left(\frac{n^m}{N} - \frac{a^m}{A}\right) + \frac{L_Y}{a_{L_A}A^{1-\chi}}\left(\frac{a^m}{A} - \frac{y^m}{Y}\right) \tag{14}$$

with $\chi = 1$. Recall that y^m is indeterminate if replication is feasible. So we may assume that $y^m/Y = a^m/A$. Then, as in Example 1, country m's share in the total number of intermediate goods, a^m/A, converges to its share in world-wide population, n^m/N. If it produces more (y^m/Y is higher), the proportion of markets it monopolizes is smaller (a^m/A is smaller), and vice versa.

Example 4: The world economy as a whole converges to its balanced-growth equilibrium. With $\mathcal{D} = 1$, $L_Y = L_l = 0$, and $l^m = l$, (12) becomes (13) (with $\chi < 1$). (a) If $a'^m = a^m$, country m's share in the total number of intermediate-goods monopolies, a^m, converges to its share in world-wide population, n^m/N. If one confines attention to the model's local dynamic behavior (i.e., if L_X/L is close to its balanced-growth level already), then the validity of (10) initially ensures its validity subsequently. (b) If $a'^m \neq a^m$, country m can get a higher share in the aggregate number of intermediate goods markets by manufacturing some of the intermediates it has invented abroad.

Example 6: Locally, the integrated economy converges to its balanced-growth path. Given that either (3) capital is mobile or (b) domestically invented intermediates can be produced abroad, the world economy behaves like the integrated economy. With $\mathcal{D} = L_l = L_X = 0$ and $l^m = l$, (12) becomes (14) (with $\chi < 1$). Since y^m is indeterminate if replication is feasible, there is an equilibrium growth path with $y^m/Y = a^m/A$, on which a^m/A converges to n^m/N. If y^m/Y is higher, a^m/A is smaller, and vice versa.

Example 7: Here we allow for $l^m \neq l$. The world economy converges to its balanced-growth path. From (12), with $\mathcal{D} = L_Y = 0$, we have

$$\frac{d}{dt}\left(\frac{a^m}{A}\right) = \frac{L}{a_{L_A}A^{1-\chi}}\left(\frac{n^m}{N}\frac{l^m}{l} - \frac{a^m}{A}\right) + \frac{L_X}{a_{L_A}A^{1-\chi}}\left(\frac{a^m}{A} - \frac{a'^m}{A}\right) + \frac{L_l}{a_{L_A}A^{1-\chi}}\left(\frac{a^m}{A} - \frac{n^m}{N}\frac{\dot{l}^m}{\dot{l}}\right).$$

(a) For $a'^m = a^m$, in one equilibrium, human capital grows at the same rate everywhere ($\dot{l}^m/l^m = \dot{l}/l$). a^m/A then converges to $(n^m/N)(l^m/l)$. If the economy is already close to its balanced-growth path, so that L/L_X is close to its balanced-growth level, then if (10) holds initially, it will be satisfied during the local transition to the balanced growth path. Since $\dot{l}^m$ is indeterminate, other equilibrium growth paths exist. (b) If a'^m is allowed to differ from a^m, the trajectories which are equilibria in case (a) are equilibria. Further equilibria with uneven human-capital growth and with $a'^m \neq a^m$ exist.

6 International economic integration and long-run growth

Theorem 1 is also helpful in order to deal with the question of whether international economic integration boosts long-run growth.

Theorem 3: *Suppose the world economy replicates the integrated equilibrium. Then an in increase in the size of the world economy leads to faster output growth in a balanced-growth equilibrium if, and only if, scale effects prevail.*

According to Theorem 3, if replication occurs, the question of whether or not international economic integration boosts long-run growth boils down to the question of whether or not scale effects prevail, and what we have to do is check if the growth rate, g_Y, in a balanced-growth equilibrium of the integrated economy depends on N. In doing so, we consider three further examples (we introduce these examples here because their balanced-growth equilibria are well-known, but their transitional dynamics are not):

Example 8: The QU version of the Romer (1990) model (Example 3) .

Example 9: The QU version of the Jones (1995a) model (Example 6).

Example 10: The QU version of Arnold's (1998) model (Example 7) is analyzed in Arnold (2002b). In each case, the formula for g_Y reported in Section 4 applies with $\mu \equiv \lambda$ and $\gamma \equiv \log \lambda$.

Unfortunately, the general model does not, of course, provide an unambiguous answer to the question of whether larger scale means faster long-run growth. Models with constant returns to knowledge in R&D (Examples 1-3, 8) display scale effects: g_Y depends positively on $L = Nl$. The other R&D growth models (Examples 4-7, 9, 10) do not display scale effects: g_Y does not depend on the level of population, N. The models without R&D in Appendix C do not provide an unambiguous answer either. The models of Solow (1956), Arrow (1962), Uzawa (1965), Sheshinski (1967), and Lucas (1988) do not feature scale effects. However, Romer's (1986) model does.

In view of the ambiguity of these theoretical results, the question becomes an empirical one. The empirical evidence on scale effects is also controversial. Backus, Kehoe, and Kehoe (1992) present several regressions which cast doubt on the statistical and economic significance of scale effects. For instance, in a cross-section of countries, they find that the effect of GDP on the growth rate of GDP per capita is insignificant. To give an impression of the size of the effect, they reckon that a hundred-fold increase in total GDP is associated with an increase in per capita growth of less than one percentage point (Backus, Kehoe, and Kehoe, 1992, p. 387). Similar results obtain with other scale variables. Kremer (1993) objects, in line with our general model, that the relevant unit of analysis is not individual countries but geographical areas which share a common pool of technology. He argues that world-wide population growth as well as population growth in technologically separate regions from 1,000,000 B.C. to 1990 is consistent with a model in which technical change is proportional to the level of population (i.e., with scale effects) if one also adopts the Malthusian assumption that population is limited by technology. The latter assumption implies, however, that the model features constant GDP per capita in the long run. Moreover, the implied positive link between population and population growth has broken down in the more recent past. Jones (1995b, Section IV) launched the most forceful attack on the scale effects hypothesis by pointing out that employment in R&D increased several-fold in the industrial nations in the post-war period without an accompanying boost in total factor productivity. Segerstrom (1998, Section I) provides similar evidence. Barro and Sala-i-Martin (2004, p. 537) use the log of population as their measure of the economy's scale in their cross-country regression and report a positive but insignificant coefficient. Recently, Todo and Miyamoto (2002) have argued that a careful look at the data may bring about “the revival of scale effects”. As yet, it seems fair to say that the presence of scale effects is at best controversial. This casts doubt on the proposition that international economic integration has an impact on the long-run growth rate.

7 Conclusion

In this paper, we have analyzed a fairly general multi-country endogenous growth model with or without scale effects, with or without population growth, with or without human-capital accumulation, with or without physical capital, and with growth in product variety or with quality upgrading. We have shown that, under certain conditions, the world economy replicates the equilibrium of the hypothetical integrated economy. This result allows it to analyze the so far largely unexplored transitional dynamics of several prominent endogenous growth models. This approach has the advantage that one can make use of existing results on the dynamics of closed economies. One does not have to solve a single differential equation in order to come up with original results on multi-country dynamics of several important growth models. This assures us that balanced-growth analyses are indeed concerned with the models’ long-term solutions. Moreover, it offers guidelines for performing simulations, which recognize the various indeterminacies that emerge in our analysis. In sum, the growth model analyzed here allows a systematic investigation of the transitional dynamics of a broad variety of growth models. The analysis also sheds light on the question of whether international economic integration boosts long-run growth: if the world economy replicates the integrated economy, then this boils down to the question of whether scale effects prevail, and we can consult the empirical literature on scale effects in order to come up with an answer.

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Appendix A: Integrated equilibrium

Producers

In the final-goods sector, price equals unit cost due to perfect competition. The firms' cost minimization problem can be split into two stages. Stage one: minimize the cost, p_D, of producing one unit of D_Y given (3) or (4). Stage two: minimize total cost given (1) and p_D. For the PV model, the first stage consists of minimizing the cost, $\int_0^A p(j)X(j)dj$, of producing one unit of D_Y subject to (3), where $p(j)$ is the price of intermediate j (cf. Grossman and Helpman, 1991a, Chapter 3). The solution to this problem yields the input coefficients

$$a_{X(j)} = \frac{p(j)^{-\frac{1}{1-\alpha}}}{\left[\int_0^A p(j')^{-\frac{\alpha}{1-\alpha}} dj'\right]^{\frac{1}{\alpha}}}$$

and p_D is the price of one unit of D_Y:

$$p_D = \left[\int_0^A p(j)^{-\frac{\alpha}{1-\alpha}} dj\right]^{-\frac{1-\alpha}{\alpha}}.$$

In the QU model, the first stage of the cost minimization problem entails minimizing $\int_0^1 \sum_{\omega=1}^{\Omega(j)} p_\omega(j)X_\omega(j)dj$ subject to (4) with $D_Y = 1$, where $p_\omega(j)$ is the price of quality ω of intermediate j (cf. Grossman and Helpman, 1991a, Chapter 4). The solution to this problem entails the input coefficients

$$a_{X_\omega(j)} = \begin{cases} \frac{p_D}{p_\omega(j)}; & \text{for } \omega = \tilde{\omega}(j) \\ 0; & \text{for } \omega \neq \tilde{\omega}(j) \end{cases},$$

where $\tilde{\omega}(j) \equiv \arg\min_\omega \{p_\omega(j)/\lambda^\omega\}$, and the price of one unit of D_Y,

$$p_D = \exp\left\{\int_0^1 \left[\log p_{\tilde{\omega}(j)}(j) - \tilde{\omega}(j)\log\lambda\right] dj\right\}.$$

Turning to the second stage of the cost minimization problem, let w and r denote the wage rate and the interest rate, respectively. The cost, $rK_Y + wL_Y + p_D D_Y$, of producing $Y = 1$ is minimized subject to (1). This gives the input coefficients $a_{K_Y}(r, w, p_D, B_Y)$, $a_{L_Y}(r, w, p_D, B_Y)$, and $a_{D_Y}(r, w, p_D, B_Y)$ of capital, labor, and D_Y, respectively, and the unit cost function $c_Y(r, w, p_D, B_Y)$. Choosing the final good as the numeraire, competitive pricing implies

$$1 = c_Y(r, w, p_D, B_Y). \tag{A.1}$$

The final-goods sector's demand for intermediates is $a_{X_\omega(j)}a_{D_Y}(r, w, p_D, B_Y)Y$. (2) can be rewritten as

$$B_Y = \frac{[a_{K_Y}(r, w, p_D, B_Y)Y]^\eta}{[a_{L_Y}(r, w, p_D, B_Y)Y]^\varepsilon}. \tag{A.2}$$

Next, consider the producers of intermediate goods. Minimizing the cost, $rk_{X_\omega}(j) + wl_{X_\omega}(j)$, of producing one unit of $X_\omega(j)$ subject to (5) yields the input coefficients $a_{k_{X_\omega}(j)}(r, w)$ and $a_{l_{X_\omega}(j)}(r, w)$ and the unit cost function $c_x(r, w)$. The single producer of an intermediate good j (PV) or a given quality ω of an intermediate good j (QU) maximizes his monopoly profit $\pi_\omega(j) \equiv [p_\omega(j) - c_x(r, w)]X_\omega(j)$ given the final-goods sector's demand. In the PV model, the price elasticity of demand is $-1/(1-\alpha)$. So intermediate-goods monopolists maximize profit with the markup price $c_x(r, w)/\alpha$. In the QU model, for each good j, only the producer $\tilde{\omega}(j)$ with the lowest quality-adjusted price, $p_\omega(j)/\lambda^\omega$, faces a positive demand. This producer's price elasticity of demand is -1, so his profits increase as he raises his price. In equilibrium, the producer of the maximum-quality intermediate, $\Omega(j)$, prices the lower-quality producers out of the market ($\tilde{\omega}(j) = \Omega(j)$) with the limit price $\lambda c_x(r, w)$. So both in the PV variant and in the QU variant of the model, each active producer charges the same monopoly price,

$$p = \mu c_x(r, w), \tag{A.3}$$

where $\mu \equiv 1/\alpha$ in the PV model and $\mu \equiv \lambda$ in the QU model. This has several important consequences. Since the demand curves, $a_{X_\omega(j)} a_{D_Y}(r, w, p_D, B_Y)Y$, are also the same for each producer, so are the quantities brought out, X, and monopoly profits, π:

$$\pi = \left(1 - \frac{1}{\mu}\right) pX. \tag{A.4}$$

For the sake of notational convenience, we introduce the dummy variable $\mathcal{D}$ which equals zero for the PV model and unity for the QU model.[27]. Then p_D can be rewritten as

$$p_D = p\left(A^{-\frac{1-\alpha}{\alpha}}\right)^{1-\mathcal{D}} \left\{\exp\left[-\log\lambda \int_0^1 \Omega(j)dj\right]\right\}^{\mathcal{D}} \tag{A.5}$$

and the input coefficients $a_{X_\omega(j)} \equiv a_X$ as

$$a_X = \frac{p_D}{A^{1-\mathcal{D}}p}.$$

Finally, consider firms engaged in R&D. The reward to investments in R&D is the expected present value of the ensuing monopoly profits,

$$v(t) \equiv \int_t^\infty e^{-\int_t^\tau [r(\vartheta)+\mathcal{D}I(\vartheta)]d\vartheta} \pi(\tau)d\tau. \tag{A.6}$$

In the QU model, the instantaneous probability of losing a monopoly, I, acts like an additional discount factor. Let $c_A(r, w)$ denote the cost of producing $F(K_A, L_A) = 1$ and $a_{K_A}(r, w)$ and $a_{L_A}(r, w)$ the

[27] So if a term Z_{PV} appears in the PV model but not in the QU model, writing $Z_{PV}^{1-\mathcal{D}}$ covers both cases. If Z_{QU} appears in the QU model where a term Z_{PV} appears in the PV model, we can write $Z_{PV}^{1-\mathcal{D}} Z_{QU}^{\mathcal{D}}$

corresponding input coefficients. Suppose a firm uses $a_{K_A}(r,w)$ units of capital and employs $a_{L_A}(r,w)$ workers in R&D. This costs $c_A(r,w)$. In the PV model, the result is $\dot{A} = A^{\chi}$ new intermediates, each worth v. So free entry into R&D implies $A^{\chi}v = c_A(r,w)$. In the QU model, the result is the innovation rate $I = A^{-(1-\chi)}$, and free entry implies $A^{-(1-\chi)}v = c_A(r,w)$. Hence,

$$A^{\chi-\mathcal{D}}v = c_A(r,w). \tag{A.7}$$

Households

The households choose their investments in education, k_l and l_l, and per-capita consumption, c, so as to maximize the household's members' utility. This problem can be solved in two stages. Stage one: the cost, $rk_l + wl_l$, of producing one unit of $F_l(k_l, l_l)$ is minimized. This yields the cost function $c_l(r,w)$ and the input coefficients $a_{k_l}(r,w)$ and $a_{l_l}(r,w)$. Stage two: each household solves

$$\max_{c,\dot{l}} : \int_t^{\infty} \frac{c(\tau)^{1-\sigma}}{1-\sigma} e^{-\rho(\tau-t)} d\tau$$

$$\text{s.t.: } \dot{\nu} = (r - g_N)\nu + wl - c_l(r,w)\dot{l} - c,$$

where ν is the household's per-capita financial wealth. For future reference, note that adding up the households' budget constraints yields

$$\frac{d(N\nu)}{dt} = rN\nu + wL - c_l(r,w)N\dot{l} - Nc. \tag{A.8}$$

The current-value Hamiltonian for the households' problem is

$$\mathcal{H} \equiv \frac{c^{1-\sigma}}{1-\sigma} + \zeta_\nu[(r - g_N)\nu + wl - c_l(r,w)\dot{l} - c] + \zeta_l \dot{l}$$

with ζ_ν and ζ_l as co-state variables. The necessary and sufficient conditions for an interior optimum are:

$$\frac{\partial \mathcal{H}}{\partial c} = c^{-\sigma} - \zeta_\nu = 0$$

$$\dot{\zeta}_\nu = \rho\zeta_\nu - \frac{\partial \mathcal{H}}{\partial \nu} = \rho\zeta_\nu - (r - g_N)\zeta_\nu$$

$$\frac{\partial \mathcal{H}}{\partial \dot{l}} = -\zeta_\nu c_l(r,w) + \zeta_l = 0$$

$$\dot{\zeta}_l = \rho\zeta_l - \frac{\partial \mathcal{H}}{\partial l} = \rho\zeta_l - \zeta_\nu w$$

plus two transversality conditions. The former two conditions yield $-\sigma\dot{c}/c = \dot{\zeta}_\nu/\zeta_\nu$, $\dot{\zeta}_\nu/\zeta_\nu = \rho - r + g_N$ and, hence, the Ramsey rule:

$$\frac{\dot{c}}{c} = \frac{r - \rho - g_N}{\sigma}. \tag{A.9}$$

The third condition implies $\dot{c}_l/c_l = \dot{\zeta}_l/\zeta_l - \dot{\zeta}_\nu/\zeta_\nu$. Substituting $\dot{\zeta}_l/\zeta_l = \rho - \zeta_\nu w/\zeta_l = \rho - w/c_l$ (from the third and fourth conditions) and $\dot{\zeta}_\nu/\zeta_\nu = \rho - r + g_N$ yields

$$\dot{c}_l(r, w) = (r - g_N)c_l(r, w) - w. \tag{A.10}$$

Notice that the Hamiltonian is linear in $\dot{l}$. So the third condition ($\partial\mathcal{H}/\partial\dot{l} = 0$) is necessary for an interior optimum. As emphasized by Bond, Trask, and Wang (2003, p. 1046), the individual investments in human capital are not determinate, however, even if aggregate human-capital accumulation in general equilibrium is.

Market clearing

It remains for us to formulate the market clearing conditions. Equality of supply and demand in the market for final goods requires

$$Y = \dot{K} + Nc. \tag{A.11}$$

The demand for intermediates is $a_X a_{D_Y}(r, w, p_D, B_Y)Y$. Using the fact that $a_X = p_D/(A^{1-\mathcal{D}}p)$, the condition for an equilibrium in the markets for the intermediates can be written as:

$$X = \frac{p_D}{A^{1-\mathcal{D}}p} a_{D_Y}(r, w, p_D, B_Y)Y. \tag{A.12}$$

The markets for physical capital and labor clear if

$$\begin{pmatrix} Nl \\ K \end{pmatrix} = \begin{pmatrix} a_{L_Y}(r, w, p_D, B_Y) & a_{L_A}(r, w) & a_{l_l}(r, w) & a_{l_X}(r, w) \\ a_{K_Y}(r, w, p_D, B_Y) & a_{K_A}(r, w) & a_{k_l}(r, w) & a_{k_X}(r, w) \end{pmatrix} \begin{pmatrix} Y \\ A^{1-\chi}\left(\frac{\dot{A}}{A}\right)^{1-\mathcal{D}} I^{\mathcal{D}} \\ N\dot{l} \\ A^{1-\mathcal{D}}X \end{pmatrix}. \tag{A.13}$$

The four terms on the right-hand side of these two equations are labor and capital, respectively, in their four different uses, final-goods production, R&D, education, and intermediate-goods production. For instance, capital in R&D is $K_A = a_{K_A}(r, w)\dot{A}/A^\chi$ in the PV model (i.e. for $\mathcal{D} = 0$) and $K_A = a_{K_A}(r, w)A^{1-\chi}I$ in the QU model (i.e. for $\mathcal{D} = 1$). Finally, equating the demand ($N\nu$) and supply ($K + A^{1-\mathcal{D}}v$) of financial assets gives the condition for an equilibrium in the market for financial capital:

$$N\nu = K + A^{1-\mathcal{D}}v. \tag{A.14}$$

Equilibrium

For the PV model (i.e., for $\mathcal{D} = 0$), (A.1)-(A.14) form a system of 15 equations (as (A.13) contains two equations) in the 14 variables B_Y, r, w, p_D, p, X, π, A, v, c, ν, Y, K, and l. For the QU model ($\mathcal{D} = 1$), the additional variable $\int_0^1 \Omega(j)dj$ appears in (A.5) and the innovation rate, I, in (A.6) and (A.13), and

(7) provides two additional equations. As usual in general equilibrium, the budget constraints and the market-clearing conditions are not independent. One of the equations can be derived using the other ones.[28] So we have a determinate system of equations. The validity of the system is not confined to balanced-growth paths. If a solution exists, (A.1)-(A.14) determine the evolution of the integrated economy through time.

Appendix B: Replication of the integrated equilibrium

In this appendix, we show that integrated-equilibrium conditions (A.1)-(A.14) hold true in the world economy with national borders as well.

Eq. (A.1): Producers solve the same two-stage cost minimization problem as before. Minimization of the cost of producing one unit of d_Y (stage one) leads to the same input coefficients, $a_{X_\omega(j)}$, and to the same price, p_D, of one unit of d_Y. The second-stage problem is also unchanged and leads to the same input coefficients, $a_{K_Y}(r, w, p_D, B_Y)$, $a_{L_Y}(r, w, p_D, B_Y)$, and $a_{D_Y}(r, w, p_D, B_Y)$, and to the same unit cost function, $c_Y(r, w, p_D, B_Y)$. Competitive pricing implies the validity of (A.1).

Eq. (A.2): Capital used in final-goods production in country m is $k_Y^m = a_{K_Y} y^m$. Using $Y = \sum_{m=1}^M y^m$, it follows that $K_Y = \sum_{m=1}^M k_Y^m = a_{K_Y} Y$. Similarly, $L_Y = a_{L_Y} Y$. Inserting this into (2) proves (A.2).

Eq. (A.3): The intermediate-goods producers' cost minimization problem is the same as before. So the input coefficients, $a_{k_{X_\omega(j)}}(r, w)$ and $a_{l_{X_\omega(j)}}(r, w)$, and the unit cost function, $c_x(r, w)$, are unaltered. The (homothetic) demands for the intermediates by firms in the final-goods sector in country m are $a_{X_\omega(j)} a_{D_Y}(r, w, p_D, B_Y) y^m$. As in the integrated economy, world-wide demand is $a_{X_\omega(j)} a_{D_Y}(r, w, p_D, B_Y) Y$. Since the unit cost function and the world-wide demand, $a_{X_\omega(j)} a_{D_Y}(r, w, p_D, B_Y) Y$, for intermediates as well as the market structure are the same as in the integrated economy, so is the monopoly price, p, in (A.3).

Eq. (A.4): It follows immediately that monopoly profits, π, obey (A.4).

Eq. (A.5): Analogously to Section 2, p_D can be rewritten as in (A.5) and the input coefficients satisfy $a_X = p_D/(A^{1-\mathcal{D}} p)$.

Eq. (A.6): The definition of the value of a patent remains unchanged.

Eq. (A.7): Cost minimization in the R&D sector yields the same input coefficients, $a_{K_A}(r, w)$ and $a_{L_A}(r, w)$, and the same cost function, $c_A(r, w)$, as before. Since $F(k_A^m, l_A^m) = 1$ is worth $A^{\chi-\mathcal{D}} v$ and costs $c_A(r, w)$, free entry into R&D implies (A.7).

[28] Using (A.6) and (A.7), $d(A^{1-\mathcal{D}} v)/dt$ can be written as $wL_A + rK_A + rAv - A\pi$. From (A.1), (A.3), (A.4), and (A.11), we have $\dot{K} = w(L_Y + L_X) + r(K_Y + K_X) + A\pi - Nc$. Together with (A.14) and $c_l N \dot{l} = wL_l + rK_l$, (A.8) follows.

Eq. (A.8): The representative consumer's budget constraint in country m is

$$\dot{\nu}^m = (r - g_N)\nu^m + wl^m - c_l \dot{l}^m - c^m.$$

Adding the constraints for all countries after multiplying by n^m yields

$$\sum_{m=1}^{M} n^m \dot{\nu}^m = (r - g_N) \sum_{m=1}^{M} n^m \nu^m + w \sum_{m=1}^{M} n^m l^m - c_l \sum_{m=1}^{M} n^m \dot{l}^m - \sum_{m=1}^{M} n^m c^m.$$

Notice that $l \equiv \sum_{m=1}^{M} n^m l^m / N$ and $c \equiv \sum_{m=1}^{M} n^m c^m / N$. Let ν denote average per-capita wealth: $\nu \equiv \sum_{m=1}^{M} n^m / N$. From the definition of l and $\dot{n}^m / n^m = g_N$, it follows that $\sum_{m=1}^{M} n^m \dot{l}^m = \dot{N} l + N \dot{l} - \sum_{m=1}^{M} \dot{n}^m l^m = g_N N l + N \dot{l} - g_N \sum_{m=1}^{M} n^m l^m = N \dot{l}$. Similarly, $\sum_{m=1}^{M} n^m \dot{\nu}^m = N \dot{\nu} = d(N\nu)/dt - g_N N \nu$. Inserting these results into the equation above yields (A.8).

Eqs. (A.9) and (A.10): Since the households' maximization problem remains unchanged, (A.9) and (A.10) follow. Clearly, the transversality conditions are also identical. Notice that, although $\dot{l}^m$ is indeterminate, different human-capital investments lead to identical income profiles and, hence, identical and determinate consumption profiles, c^m.

Eq. (A.11): Let s^m denote country m's net exports of the final good. The world's net exports as a whole must be zero: $\sum_{m=1}^{M} s^m = 0$. The supply of final goods equals demand in country m if $y^m = \dot{k}^m + n^m c^m + s^m$. Summing over m, using $\sum_{m=1}^{M} \dot{k}^m = \dot{K}$, $\sum_{m=1}^{M} c^m = Nc$, and $\sum_{m=1}^{M} s^m = 0$, yields (A.11).

Eq. (A.12): It has already been shown that the demand for intermediates, $a_X a_{D_Y}(r, w, p_D, B_Y)Y$, and the input coefficients, $a_X = p_D/p$, are the same as in the integrated economy. So (A.12) continues to give the condition for an equilibrium in the markets for intermediate goods.

Eqs. (A.13): The conditions for equality of the supply and demand in the markets for physical capital and labor in country m are stated in (9) in the main text. For instance, capital in R&D is $k_A^m = a_{K_A}(r, w)\dot{a}^m / A^\chi$ in the PV model (i.e. for $\mathcal{D} = 0$) and $k_A^m = a_{K_A}(r, w) A^{1-\chi} i^m$ in the QU model. This follows from the R&D technologies, $\dot{a}^m = F(k_A^m, l_A^m) A^\chi$ and $i^m = F_A(k_A^m, l_A^m) A^{-(1-\chi)}$, respectively. $\sum_{m=1}^{M} a'^m$ equals A in the PV model and unity in the QU model. Adding equations (9) for all M countries, using $\sum_{m=1}^{M} n^m \dot{l}^m = N\dot{l}$ (see the proof of the validity of (A.8) above), proves the validity of (A.13).

Eq. (7): As for the additional two equations for the QU model, adding the national innovation rates, $i^m = F_A(k_A^m, l_A^m) A^{-(1-\chi)}$, making use of identical factor intensities and constant returns to scale, we have $I = \sum_{m=1}^{M} i^m = F_A(K_A, L_A) A^{-(1-\chi)}$, where $A(t) = \exp[\int_{-\infty}^{t} I(\tau) d\tau]$. As in Section 2, differentiating A gives $I = \dot{A}/A$. Together with the definition $d[\int_0^1 \Omega(j) dj] = I\, dt$, (7) follows.

Appendix C: Models without R&D

This appendix focuses on several prominent special cases of our general model without R&D activity. The TFP in R&D is assumed to be identically zero. There is no intermediate-goods sector. So we can delete argument D_Y from the production function, which becomes $Y = F_Y(K_Y, B_Y L_Y)$. Various cases of externalities in final-goods production are allowed for. First, there is no learning by doing: $\eta = \varepsilon = 0$, $B_Y = 1$. Second, externalities emanate from the capital stock with diminishing returns: $0 < \eta < 1$ and $\varepsilon = 0$, so that $B_Y = K_Y^\eta$. Third, externalities emanate from the capital stock with non-diminishing returns: $\eta = 1$, $\varepsilon = 0$, $B_Y = K_Y$. Fourth, externalities emanate from the capital intensity with non-diminishing returns: $\eta = \varepsilon = 1$, $B_Y = K_Y/L_Y$. Fifth, positive externalities emanate from human capital in production: $\eta = 0$, $\varepsilon < 0$, and $B_Y = L_Y^{varepsilon}$.

Models without human-capital accumulation

Suppose the TFP in human-capital accumulation is also identically zero. We consider four prominent special cases, which correspond to the first four admissible specifications for η and ε mentioned above.

Example 11 (Solow, 1956): $\eta = \varepsilon = 0$. Then,

$$g_Y = g_N.$$

Productivity growth, $g_Y - g_N$, is zero. There is a unique trajectory converging to the balanced-growth equilibrium (Cass, 1965, p. 236).

Example 12 (Arrow, 1962, Sheshinski, 1967): $0 < \eta < 1$, $\varepsilon = 0$. Then,

$$g_Y = \frac{1}{1-\eta} g_N.$$

If the population grows ($g_N > 0$), so does labor productivity: $g_Y - g_N = \eta g_N/(1-\eta) > 0$. There is a unique trajectory converging to the balanced-growth equilibrium.

Example 13 (Romer, 1986): $\eta = 1$, $\varepsilon = 0$, $g_N = 0$. Letting $D_z f$ denote the partial derivative of a function f with respect to its z-th argument, we get

$$g_Y = \frac{D_1 F_Y(1, L) - \rho}{\sigma}.$$

In contrast to Examples 11 and 12, scale effects prevail. The model does not display transitional dynamics. It enters its balanced-growth path immediately (Barro and Sala-i-Martin, 2004, p. 216).

Example 14: $\eta = \varepsilon = 1$, g_N. Here we have

$$g_Y = \frac{D_1 F_Y(1, 1) - \rho}{\sigma}.$$

Like the previous one, this model has no transitional dynamics (Barro and Sala-i-Martin, 2004, p. 220).

Models with human-capital accumulation

Now suppose the TFP in human-capital accumulation is not zero.

Example 15: Uzawa (1965) makes the additional assumptions that learning-by-doing effects are absent and that education does not require physical capital: $\eta = \varepsilon = 0$, $F_l(k_l, l_l) = l_l / a_{l_l}$ with a_{l_l} exogenous. One obtains:

$$g_Y = g_N + \frac{\frac{1}{a_{l_l}} - \rho - g_N}{\sigma}.$$

There is a unique trajectory converging to the balanced-growth equilibrium (Caballé and Santos, 1993, Theorem 1, p. 1056, Faig, 1995, Section 3, Arnold, 1997, Sections 4 and 5, Barro and Sala-i-Martin, 2004, Subsection 5.2.2).

Example 16 (Lucas, 1988): For the sake of simplicity, ignore population growth here and set $N = 1$ such that L_Y is human capital per capita in production. Assume that positive externalities emanate from L_Y: $\eta = 0$ and $\varepsilon < 0$. All other assumptions of Example 15 are maintained. Then:

$$g_Y = \frac{\frac{1}{a_{l_l}} - \rho}{\sigma + \frac{\varepsilon}{1-\varepsilon}}.$$

Benhabib and Perli (1994) analyze the model's transitional dynamics for the Cobb-Douglas special case. Suppose $1/a_{l_l} > \rho$. Assume further $\sigma > 1 - \rho a_{l_l}(1-\beta)/(1-\beta-\varepsilon)$, where β is the production elasticity of capital in final-goods production. Then there is (locally) a unique trajectory converging to the balanced-growth equilibrium (Benhabib and Perli, 1994, Proposition 1, p. 123).[29]

Example 17 (based on Mulligan and Sala-i-Martin, 1993, and others): As in Example 15, there are no externalities ($\eta = \varepsilon = 0$). However, capital is used in human-capital formation. The interest rate, r, and r/w are determined by

$$r = \frac{1}{c_l\left(\frac{r}{w}, 1\right)}$$

and

$$r = D_1 F_Y \left[\frac{a_{K_Y}\left(\frac{r}{w}, 1\right)}{a_{L_Y}\left(\frac{r}{w}, 1\right)}, 1\right].$$

g_Y is obtained by inserting the solution for r into the Ramsey rule (A.9). There is a unique trajectory converging to the balanced-growth equilibrium (Mulligan and Sala-i-Martin, 1993, Interesting Result 6, p. 759, Bond, Wang, and Yip, 1996, Proposition 2, p. 160, Mino, 1996, Subsection 3.3, Ladrón-de-Guevara, Ortiguera, and Santos, 1997, Subsection 3.1).

[29] Our $1/a_{l_l}$ is Benhabib and Perli's (1994) δ, and our $-\varepsilon$ is their γ. Our analysis also applies when the parameters are such that indeterminacy arises.

Replication of the integrated equilibrium

Since all the examples listed above are special cases of our general model, we can apply Theorem 1 to investigate the models' dynamics. As the models do not contain intermediate-goods production, the net factor supplies are non-negative (the necessary condition for replication in Corollary 1 is satisfied).

Models without human-capital accumulation

Examples 11-14: Without R&D, intermediate-goods production, and human-capital accumulation, (9) becomes

$$\begin{pmatrix} n^m l^m \\ k'^m \end{pmatrix} = \begin{pmatrix} a_{L_Y} \\ a_{K_Y} \end{pmatrix} y^m.$$

(1) If capital is immobile ($k'^m = k^m$), this is an over-determinate system of equations. A solution, y^m, exists only if the countries' endowments of physical capital per worker, $k^m/(n^m l^m)$ are uniform (cf. Corollary 2).[30] (2) If physical capital is mobile, $y^m = n^m l^m / a_{L_Y}$, and capital imports or exports lead to $k'^m = a_{K_Y} n^m l^m / a_{L_Y}$ (cf. Corollary 3). The equilibrium is determinate, as the world economy possesses a unique equilibrium growth path and $k'^m = a_{K_Y} n^m l^m / a_{L_Y}$ (with n^m and l^m exogenous) pins down the allocation of factors of production across countries.

The two-country special case is illustrated in the left panel of Figure 4. There is only one integrated-equilibrium input vector, $(L_Y, K_Y)'$. (1) Without physical-capital mobility, replication is not feasible unless E happens to be located on the diagonal of the rectangle. (2) With physical-capital mobility, replication is feasible because the vertical line through $(n^1 l^1, 0)'$ and $(n^2 l^2, 0)'$ intersects the diagonal.

Models with human-capital accumulation

Examples 15-17

Without R&D and intermediate-goods production, but with human-capital accumulation (as in Examples 15-17), (9) becomes

$$\begin{pmatrix} n^m l^m \\ k'^m \end{pmatrix} = \begin{pmatrix} a_{L_Y} & a_{l_l} \\ a_{K_Y} & a_{k_l} \end{pmatrix} \begin{pmatrix} y^m \\ n^m \dot{l}^m \end{pmatrix}.$$

(1) Suppose physical capital is immobile internationally ($k'^m = k^m$). From Corollary 2, a sufficient condition for replication is that relative factor endowments are uniform across countries ($k^m/K = \theta^m$). Each country can, then, conduct a fraction $\theta^m \equiv (n^m l^m)/(Nl)$ of world-wide final-goods production and human-capital accumulation. Suppose further the integrated-equilibrium production vectors, (L_Y, K_Y) and (L_l, K_l), are linearly independent. Then, replication is feasible if the relative factor endowments are sufficiently similar. While the division of the integrated-equilibrium production vectors

[30] As noted above, adding additional consumption goods, different factor intensities in the production of consumption and investment goods, or intermediate goods with different factor intensities generates scope for replication.

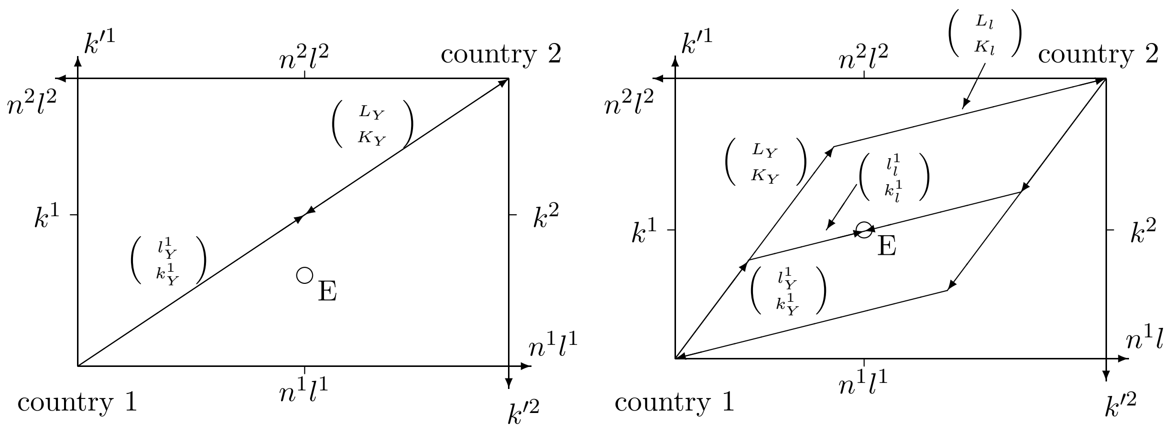

Figure 4: Two countries without R&D activity without (left panel) and with (right panel) human-capital accumulation

between countries is determinate, the distribution of new physical capital, $\dot{K}$, across the M countries is not (the Fischer-Frenkel, 1972, indeterminacy). (2) An alternative sufficient condition for replication is physical-capital mobility (cf. Corollary 3). In one equilibrium, country m uses $k'^m = \theta^m K$ units of physical capital and conducts a fraction θ^m of world-wide final-goods production and human-capital accumulation. If the vectors (L_Y, K_Y) and (L_l, K_l) are linearly independent, there are other equilibria. For instance, in the models without physical capital in the education technology (Examples 15 and 16), a country m can accumulate more human capital ($n^m l_l^m > \theta^m L_l$, $l_Y^m > \theta^m L_Y$), attract less physical capital ($k_Y'^m < \theta^m K_Y$) and produce less ($y^m < \theta^m Y$). The Fischer-Frenkel (1972) indeterminacy is also present.

The case of $M = 2$ is illustrated in the right panel of Figure 4. The two integrated-equilibrium input vectors, $(L_Y, K_Y)'$ and $(L_l, K_l)'$, form a parallelogram. If physical capital is not used in education (as in Examples 15 and 16), the input vectors for education are horizontal. (1) If the endowment point, E, is located in this parallelogram, replication is feasible even if physical capital is immobile. (2) With mobile physical capital, replication is feasible, because the vertical line through $(n^1l^1, 0)'$ and $(n^2l^2, 0)'$ passes through the parallelogram.