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Abstract

A new econometric approach to the analysis of downward nominal wage rigidity in micro data is proposed, the kernel-location approach. It combines kernel-density estimation and the principle of joint variation of location and shape of the distribution of per cent annual nominal wage changes. The approach provides partial estimates of the counterfactual and factual distributions, of the rigidity function and of the degree of downward nominal wage rigidity. It avoids problematic assumptions of other semi- or non-parametric approaches to downward nominal wage rigidity in micro data and allows a discussion of the type of downward nominal wage rigidity encountered in data.

Keywords: Downward Nominal Wage Rigidity; Kernel-Location Approach; Micro Data.

JEL-classification: E24; J30.

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1 Introduction

The global low inflation environment continues to stimulate the interest of economists in the working of economies at low inflation. The existence, extent and effects of downward nominal wage rigidity (DNWR) are one major aspect of this debate, since these may imply an optimum rate of inflation that is positive. The analysis of DNWR in micro data has been an important input to the discussion. Evidence from micro data has not been unambiguous, as has been noted repeatedly, recently for example by Rodríguez et al. (2003), one of the background studies in the examination by the European Central Bank of its monetary policy strategy. Further evidence is therefore clearly desirable.

Several nonparametric or semiparametric approaches to the analysis of DNWR in micro data have been proposed, the skewness-location approach, the symmetry approach and the histogram-location approach introduced by McLaughlin (1994), Card and Hyslop (1997) and Kahn (1997), respectively. The general advantage of these approaches is that they avoid functional assumptions with respect to the distributions of annual per cent wage changes used in the analysis and, with the exception of the histogram-location approach, also avoid assumptions with respect to the functional type of the nominal rigidity. However, some drawbacks of these approaches have also been pointed out in the literature. The skewness-location approach results in a rejection or non-rejection of the hypothesis of wage flexibility, but gives no quantitative information on the extent of DNWR in the data. The key assumption of the symmetry approach is the symmetry of the notional annual distributions of wage changes, an assumption that has been rejected by McLaughlin (1999) for US PSID data and by Beissinger and Knoppik (2001) for German IABS data, while it has been vindicated by Christofides and Stengos (2001) for Canadian union wage data. The histogram-location approach does quantify the extent of DNWR, and does not need the assumption of symmetry. It requires, however, a functional form for the type of rigidity (threshold, proportional, menu costs) and the formulation of a rather involved econometric system of equations which raises a number of estimation issues. In addition, the two step nature of the histogram-location approach leaves its overall statistical properties rather unclear.

In this paper a new approach to the analysis of downward nominal wage rigidity in micro data is proposed. It makes use of kernel density estimates to model the factual and counterfactual distributions of wage changes, and of the principle of joint variation of location and shape of annual distributions of per cent nominal wage changes for the identification of DNWR. Both aspects feature in the suggested name for the new approach, kernel-location approach

(KLA).[1] While the same principle of identification, 'joint variation of location and shape', is used in the skewness-location and the histogram-location approach, and kernel density estimation is used in the symmetry approach, it is the new combination of both features that leads to the advantages of the kernel-location approach. Sharing the general advantages of the non-parametric approaches, it provides a quantitative estimate of the degree of DNWR in the data without imposing a functional form of rigidity, it avoids the problematic symmetry assumption and the econometric complications, especially of the histogram-location approach.

In the following sections two and three the basic idea behind the kernel-location approach will be spelled out and the most helpful organization of data will be explained. Sections four and five present and explain the estimators for the counterfactual and factual distribution, the rigidity function and the average degree of rigidity. Section six shows an application of the kernel-location approach to artificial random data and the final section summarizes the findings and presents conclusions and an outlook to future applications of the approach.

2 Basic Idea

The kernel-location approach analyses annual cross-sectional distributions of per cent wage changes, as do other approaches to downward nominal wage rigidity (DNWR) in micro data. A counterfactual distribution that would prevail in the absence of DNWR and a factual distribution which may be influenced by DNWR are distinguished. DNWR is thought to lead to a thinning over the range of negative nominal wage changes, and a corresponding pile-up at zero, so that both distributions differ for non-positive wage changes if DNWR does exist. In principle, existence and extent of DNWR can therefore be examined by comparing factual and counterfactual distribution. Obviously, a key problem is that the counterfactual distribution cannot directly be observed for negative nominal wage changes. However, under the assumption that the counterfactual distribution has a shape that is invariant over time and only changes its location (reflecting changes in price inflation and other influences) the median-centered counterfactual distribution is identical over time. Therefore, changes in the location of the distribution can be exploited, to obtain information on the counterfactual over some range from some periods and on the factual over the same range from other periods.

This basic idea can be spelled out in more detail with the help of an example illustrated in column a) of FIGURE 1. Assume that there is a counterfactual distribution with time-invariant shape, that there is DNWR, and that there are $T = 10$ periods with different locations of the distribution, reflected by different medians m_t in ascending order. The lowest and highest median, $m_{\min}$ and $m_{\max}$, therefore have realized in period 1 and period 10. Panels a_{i}) to a_{iii}) of

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[1] The symmetry approach of Card and Hyslop (1997) uses KDEs and has been termed 'kernel method' by Lebow, Saks and Wilson (1999). It is unrelated to the proposed kernel-location approach.

FIGURE 1 show three median-centered “theoretical” observable distributions under this set up ($t = 1,5,10$). Note that in terms of median-centred changes, thinning occurs below $-m_t$. Information on the factual distribution is available only below $-m_{\min}$, whereas information on the counterfactual distribution is only available above $-m_T$. Therefore, for the range $-m_{\max} < x < -m_{\min}$ there is information on both distributions which can be used to analyse the effects of DNWR. Panels a_{iv}) and a_v) of FIGURE 1 show those parts of the counterfactual and the factual distributions that can be deducted from the period information. Varying thickness in plotting is used to mark the different amounts of information that are available in the subdivisions of $-m_{\max} < x < -m_{\min}$ defined by the m_t. For example, between $-m_2$ and $-m_1$ there is information on the factual only from period 1, but there is information on the counterfactual from periods 2 to 10.

FIGURE 1

The kernel-location approach uses kernel density estimates of distributions by period and in aggregate over time and exploits the overlap of the aggregate estimates of counterfactual and factual in order to reach conclusions with respect to the existence and extent of downward nominal wage rigidity.

3 Organization of data

The task of formulating estimators of the factual and counterfactual distribution is made quite a bit more transparent if one thinks of the data as organized in a specific way as in the example just outlined. Let $\tilde{x}_{it}$ denote the observed per cent wage changes in the data, where index i runs over the cross-section dimension and index t over the time dimension. N is the total number of observations, N_t is the number of observations from period t. The median of observations $\tilde{x}_{it}$ from period t is denoted by m_t and median-centered observations can then be defined as[2]

(1) $$X_{it} \equiv \tilde{x}_{it} - m_t \, .$$

As in the example, it is useful to re-index observations in the time dimension, such that periods with higher time-index τ have higher medians,

(2) $$m_\tau > m_{\tau'} \Leftrightarrow \tau > \tau' \, .$$

The re-indexed annual medians of uncentered observations can be used to define the following intervals I_2 to I_T.

(3) $$I_{\tau+1} = \left] -m_{\tau+1}, -m_\tau \right[, \ \tau = 1 \ldots T-1 \, .$$

[2] Alternative measures of location could be used, e.g. higher quantiles.

I_2 to I_T divide the 'core interval' between $-m_T$ and $-m_1$ that is defined by the lowest and the highest period-median in the sample; $I_C =]-m_T, -m_1[$. In addition, intervals to the right and to the left of the core interval can be defined, $I_1 =]-m_1, \infty[$ and $I_{T+1} =]-\infty, -m_T[$. While in any period there are only negative nominal changes in I_{T+1} and only positive nominal changes in I_1, signs of nominal wage changes do change over time in the intervals within the core interval I_c. Specifically, in period τ there are positive nominal changes in all I_s, $s \le \tau$ and negative nominal changes in all I_s, $s > \tau$.

4 Kernel density estimators of factual and counterfactual distribution

In order to fix the basic idea of the kernel-location approach, preliminary estimators will be formulated and discussed first, including a discussion of the variance of estimates. Then a more refined estimator will be proposed and discussed which deals with the complication of discontinuity bias.

Preliminary estimators

For each period τ, parts of the factual and counterfactual distribution can be estimated for the two disjunctive parts of the domain divided by $-m_\tau$:

(4) $$\hat{f}_\tau(x) = \frac{1}{N_\tau} \sum_{X_{i\tau} < -m_\tau} K_h(x - X_{i\tau}) \quad x < -m_\tau \quad \tau = 1 \ldots T \ ;$$

(5) $$\hat{g}_\tau(x) = \frac{1}{N_\tau} \sum_{X_{i\tau} > -m_\tau} K_h(x - X_{i\tau}) \quad x > -m_\tau \quad \tau = 1 \ldots T \ .$$

$K_h(.)$ denotes a kernel with bandwith h.

These partial estimates of period factual and period counterfactual distributions can be aggregated to yield estimates of the factual and counterfactual distributions over the core interval I_c. For each interval within the core interval, I_j, $j = 2 \ldots T$, aggregation is only over those periods that do provide information on the distribution in question. E.g., information on the factual distribution in interval I_2 comes only from period $\tau = 1$, wheras information on the counterfactual distribution in interval I_2 comes from periods $\tau = 2 \ldots T$. Appropriate weighing is also decisive and may be based on numbers of observations, as in equations (6) and (7), or may be based on numbers of periods.[3] The aggregate estimators are:

(6) $$\hat{f}(x) = \sum_{\tau=1}^{j-1} \frac{N_\tau}{Q_{j-1}} \hat{f}_\tau(x) \quad x \in I_j, \quad j = 2 \ldots T$$

[3] Period based weighing can be thought of as an approximation of the observations based weighing for panels where the N_τ do not change much over time. In such cases $N_s / Q_\tau \cong 1/\tau$ and $N_s / R_\tau \cong 1/(T - \tau + 1)$.

(7) $$\hat{g}(x)=\sum_{\tau=j}^{T}\frac{N_\tau}{R_j}\hat{g}_\tau(x) \quad x\in I_j, \quad j=2\ldots T$$

with

(8) $$Q_\tau=\sum_{s=1}^{\tau}N_s \text{ and } R_\tau=\sum_{s=\tau}^{T}N_s \text{ for } \tau=1\ldots T\,.$$

Q_τ is the number of all observations of periods that only have negative nominal observations in $I_{\tau+1}$, R_τ is the number of all observations of periods that only have positive nominal observations in I_τ.

Variance of the preliminary estimated distributions

The changing number of observations behind the aggregate estimates over the different intervals necessitates a modified way to compute the variance of the estimates. The variance of a standard kernel density estimate is given by[4]

(9) $$Var\left[\hat{f}_h(x)\right]\approx\frac{1}{Nh}\nu_2(K)f(x) \text{ with } \nu_2(K)=\int K(u)^2\,du\,,$$

The necessary modification is the use of the total number of observations from all periods involved in the estimation in the interval I_j in question. These numbers of observations are given by Q_{j-1} and R_j, leading to variances for the different parts of the factual of

$$Var\left[\hat{f}_h(x)\right]\approx\frac{1}{Q_{j-1}h}\nu_2(K)\hat{f}(x) \text{ for } x\in I_j,\ j=2\ldots T$$

and for the different parts of the counterfactual of

$$Var\left[\hat{g}_h(x)\right]\approx\frac{1}{R_jh}\nu_2(K)\hat{g}(x) \text{ for } x\in I_j,\ j=2\ldots T\,.$$

As usual, in applications the estimated distributions have to be used instead of the true distributions.

Discontinuity unbiased estimators

Because of the discontinuity of the distributions at nominal zero (i.e. at $-m_\tau$), in the application of estimators (4) to (7) some of the probability mass is spilled over the borders of the intervals and lost for estimation. It follows, that the estimates within the intervals are affected by a ‘discontinuity bias’, i.e. biased downward in the neighborhood of the interval borders which represent the discontinuities. This observation gives the choice of kernel and bandwidth a special role in the present context. The use of variable bandwidth is a sophisticated,

4 See Härdle and Linton (1994).

but also computationally costly strategy that might be used in order to deal with this type of bias.

However, for the question at hand it is in most cases sufficient to follow an alternative strategy, that consists of not using the kernel density estimates over ranges, where they are known to be affected by discontinuity bias. The discontinuity bias is only present within a distance of b from the discontinuity, where b is equal to half of the total width of the kernel used. For the uniform (or rectangular) kernel $b = h$, but this does not hold for other kernels.

The direct consequence of this solution to the discontinuity bias is that the analysis has to use 'effective intervals' for the aggregate estimation that are different from the intervals I_τ. Different effective intervals are needed for the estimation of the factual and the counterfactual.

$$I_{\tau+1}^{f} =]-m_{\tau+1} - b, -m_\tau - b[\,,\ \tau = 1\ldots T-1;$$

$$I_{\tau+1}^{g} =]-m_{\tau+1} + b, -m_\tau + b[\,,\ \tau = 1\ldots T-1.$$

Accordingly, the modified core interval $I_c^b = [-m_T + b, -m_1 - b]$ is now $2b$ smaller than the original core interval $I_c =]-m_T, -m_1[$. The overlap between the estimated counterfactual and factual is reduced to $m_T - m_1 - 2b$. The proposed pragmatic solution will work well enough, if this loss of overlap is not too large. Nevertheless, the dependency of the loss of overlap on bandwidth constitutes an argument for a tendency to under-smoothing, in addition to standard arguments to reduce bias in kernel density estimation.

The modified, discontinuity unbiased aggregate estimators for the counterfactual distribution $g(x)$ and the factual distribution $f(x)$ are:

(10) $$\hat{f}(x) = \sum_{\tau=1}^{j-1} \frac{N_\tau}{Q_{j-1}} \hat{f}_\tau(x) \quad x \in I_j^f, \quad j = 2\ldots T$$

(11) $$\hat{g}(x) = \sum_{\tau=j}^{T} \frac{N_\tau}{R_j} \hat{g}_\tau(x) \quad x \in I_j^g, \quad j = 2\ldots T$$

The computation of the variance of these estimates has to be adjusted accordingly. If the difference between the two estimates is significant, this indicates downward nominal wage rigidity; measures of the extent of downward nominal wage rigidity will be discussed next.

5 Form and Extent of Nominal Wage Rigidity

In this section two measures of downward nominal wage rigidity are proposed, the estimated rigidity function, and the average degree of DNWR on the other.

The rigidity function is a concept introduced in Beissinger and Knoppik (2001). It captures the possibly size-dependent thinning effect at wage reductions of different sizes as the per cent difference between the counterfactual and factual distributions. In the present context, the two estimated distributions are used to obtain an estimate of the rigidity function $\hat{\rho}(x)$ over the core interval.

(12) $$\hat{\rho}(x) = \frac{\hat{g}(x) - \hat{f}(x)}{\hat{g}(x)} \text{ for } -m_T < x < -m_1 .$$

By not imposing a functional form on the rigidity function $\rho(x)$, the kernel-location approach allows to shed some light on the ongoing debate on the type of rigidity, i.e. threshold, proportional, or menu cost, see Knoppik and Beissinger (2003).

The second measure proposed is the average degree of downward nominal wage rigidity. Two necessary integrals for its construction are:

$$\Delta G \equiv \int_{-m_T}^{-m_1} \hat{g}(z)dz, \qquad \Delta F \equiv \int_{-m_T}^{-m_1} \hat{f}(z)dz.$$

The estimated average degree of downward nominal wage rigidity can then be defined as

(13) $$\hat{\rho} = \frac{\Delta G - \Delta F}{\Delta G},$$

it measures the proportion of wage cuts prevented by downward nominal wage rigidity.

6 Example with artificial random data

In order to illustrate the workings of the proposed kernel-location approach and its potential application to real data, it has been applied to a set of artificial random data. The data was generated using the same specifications underlying the theoretical example discussed in section 2 and illustrated in Figure 1. With 500 observations in each of 10 periods the example roughly mimics the respective orders of magnitude of household panel surveys (PSID, GSOEP, BHPS) which have been used in previous analyses of DNWR. The estimation used the rectangular kernel. Its results are depicted in column b) of Figure 1. The underlying theoretical counterfactual and factual distributions are plotted in all panels as dotted curves. Panels b_i) to b_{iii}) show estimates of parts of the factual and counterfactual from three periods. In each case $-m_\tau$ and $-m_\tau \pm b$ are marked by vertical lines. Because of the relatively low number of observations per period the appearance of the estimates is rather ragged. Panels b_{iv}) and b_v) show aggregate estimates of the counterfactual and factual distributions. Solid vertical lines mark the original core interval (defined by the extreme medians), dotted vertical lines mark the effective intervals. The estimates are plotted with varying thickness reflecting the different number of underlying periods in the different intervals. The aggregate estimates are much smoother than the period estimates over most intervals, because of the much larger number of observations used in their estimation. They nicely recover the underlying true distributions over most of the effective core interval and correctly point to the high degree of DNWR built into the example. Overall, the example underlines the applicability of the kernel-location approach to the relevant data sets.

7 Summary, Conclusions, and Outlook

The new kernel-location approach to the analysis of downward nominal wage rigidity in micro data combines kernel density estimation and the identifying principle of joint variation of location and shape of the distribution of per cent annual nominal wage changes. The approach provides partial estimates of counterfactual and factual distributions of per cent annual nominal wage changes, of the rigidity function and of the degree of downward nominal wage rigidity. The proposed estimator for the distributions is based on a fairly straightforward basic idea, i.e. to suitably weigh the partial period-wise kernel density estimates of median-centered factual and counterfactual distributions in order to obtain overlapping partial estimates of the aggregate factual and counterfactual distributions. These aggregate estimates can then be used to construct measures of downward nominal wage rigidity.

It was shown that the proposed method shares the advantages of other semi- or non-parametric approaches to the analysis of downward nominal wage rigidity, but avoids a number of their problematic features. One specific advantage is that the rigidity function is estimated non-parametrically, i.e. without imposing a functional form. The approach therefore allows to shed some light on the ongoing debate on the type of rigidity, i.e. absolute rigidity threshold rigidity, proportional rigidity, or menu cost rigidity. The application to real data will be the topic of future papers.

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Figures

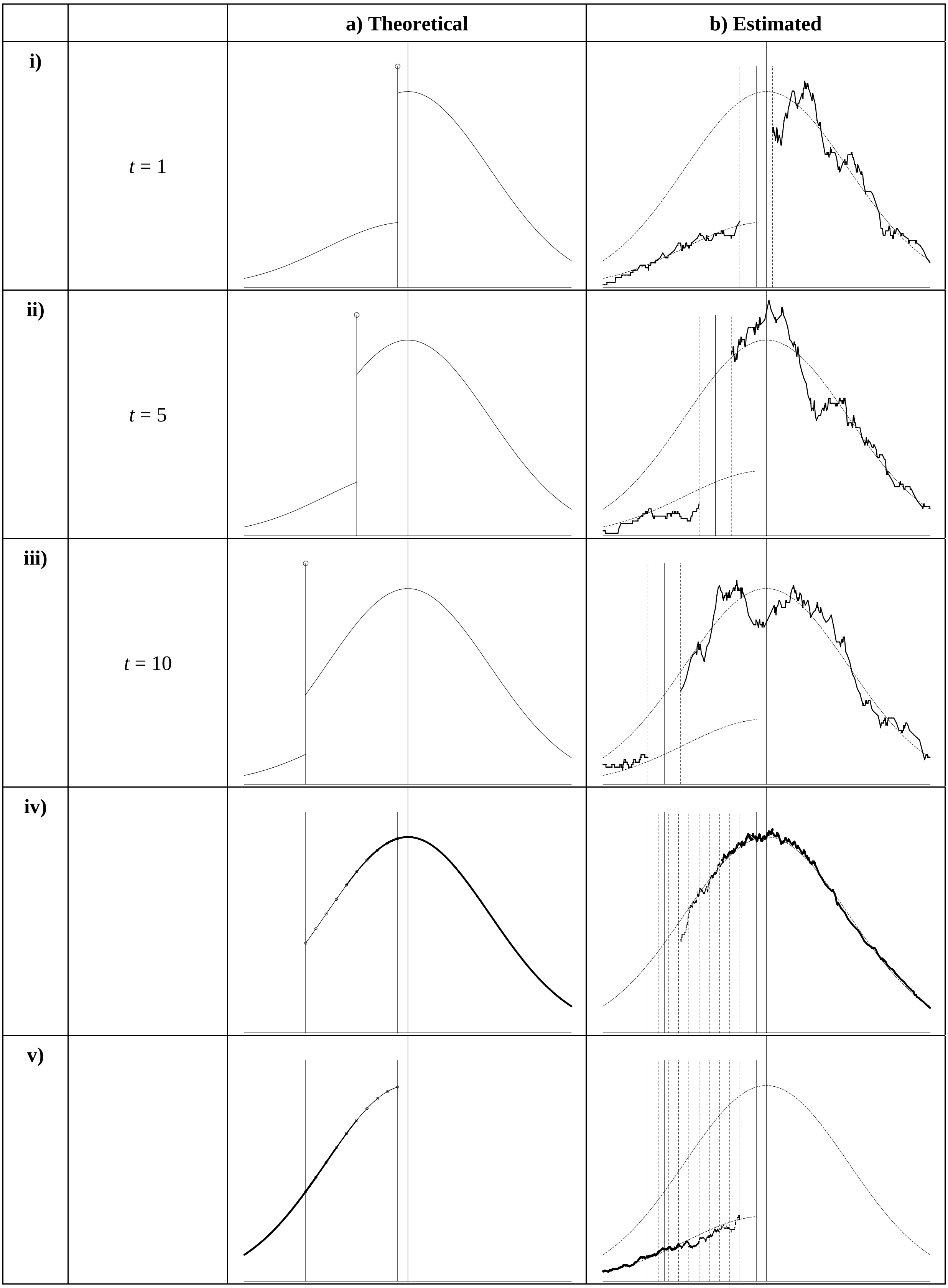

FIGURE 1 Theoretical and estimated distributions