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Abstract

The skewness-location approach to the analysis of downward nominal wage rigidity in micro data judges the existence of rigidity on the basis of estimated functional relationships between measures of location and of skewness of the distributions of individual rates of wage change in different years. Here it is shown that the properties of theoretical skewness-location relationships can deviate from those asserted by the skewness-location approach thus invalidating its test logic. Consequently, judgment based on the skewness-location approach is biased away from finding evidence for rigidity.

Keywords: Nominal Wage Rigidity; Skewness; Skewness-Location Approach; Nominal Price Rigidity; Downward Nominal Wage Rigidity; Wage Change Distribution.

JEL-classification: E24, J30.

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1 Introduction

Impediments to the adjustment of nominal prices and wages have far reaching consequences for the smooth functioning of the economy in most macroeconomic models. This explains the continuous interest of economists and policy makers alike, in whether these impediments do exist to a relevant extent in actual economies and might be responsible for phenomena like inflation persistence and excess unemployment at low rates of inflation.[1]

The earliest econometric approach to the analysis of downward nominal wage rigidity in micro data is due to McLaughlin (1994). His skewness-location approach asserts that rigidity skews the distribution of wage change rates to the right and that a negative relationship between location and skewness exists under nominal rigidity but not under flexibility. A measure of skewness of annual distributions of nominal wage changes is regressed on a measure of the location of these distributions in order to determine whether or not there is systematic joint variation of changes of shape and location caused by nominal rigidity.[2] If location indeed does explain skewness significantly, the null hypothesis of no rigidity is rejected and the existence of nominal rigidity has been proved, according to the skewness location approach. Recent applications of the approach to nominal wage rigidity include among others Lebow, Saks and Wilson (2003) and McLaughlin (1999a) for the US, Beissinger and Knoppik (2001) for Germany, Kuroda and Yamamoto (2003) for Japan, Christofides and Stengos (2002) for Canada, Dwyer and Leong (2003) for Australia, and Castellanos (2001) for Mexico.[3]

Despite some previous discussion of the relative merits of specific empirical measures of skewness and location for use in the approach (more below) there has been no systematic exploration of the validity of the basic assertions of the approach with respect to skewness. This paper demonstrates that these assertions are not in general correct. This invalidates the test logic sketched above and leads to failures to reject the null hypothesis of no nominal rigidity. Specifically and contrary to the assertions of the skewness-location approach, downward nominal wage rigidity can cause skewness to the left (rather than to the right) and shifts of the distribution to the right may increase skewness (rather than to reduce it). Explanations for this observation based on a rigorous concept of skewness are offered. These insights may contrib-

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1 See e.g. Rodríguez-Palenzuela, Camba-Mendez and Garcia (2003) and the documentation of ESCB's network on inflation persistence on http://www.ecb.int/ .

2 The term skewness-location approach used e.g. in Beissinger and Knoppik (2001) and Stiglbauer (2002) derives from this principle.

3 In addition, there are non-econometric assessments of downward nominal wage rigidity that rely on skewness. Skewness also matters in analyses of nominal price rigidity, e.g. Bryan and Cecchetti (1999) for the nominal rigidity of a wide array of prices and Genesove (2003) on the nominal rigidity of apartment rents. Comments on Bryan and Cecchetti (1999) and a rejoinder can be found in the same issue.

ute to a better understanding of rather mixed results on the existence of downward nominal wage rigidity in the literature.

The remainder of the paper is structured as follows. Section 2 explains the skewness-location approach in more detail. Section 3 gives several examples for theoretical skewness-location relationships with properties deviating from the assertions of the approach. Section 4 links these results to a rigorous concept of skewness based on convex transformations and Section 5 offers a brief summary and conclusions.

2 The Skewness-Location Approach to the Analysis of Downward Nominal Wage Rigidity

In empirical analyses of downward nominal wage rigidity in micro data, two distributions are distinguished. One is the unobservable notional distribution (alternatively: counterfactual distribution) of individual nominal wage change rates that would apply if there were no nominal rigidity. The second distribution is the observable actual (or factual) distribution of wage changes that is possibly influenced by the presence of downward nominal rigidity. In the recent empirical literature it is generally assumed that there are only direct effects of nominal rigidity, in the sense that rigidity only causes very specific, immediate deviations from the counterfactual distribution of wage changes, but no indirect effects operating through a transformation of the distribution of wage levels. In most cases these deviations are thought to consist of a 'thinning' of the left tail of the distribution for negative wage changes (some or all notional wage cuts do not take place) and a 'pile up' of the corresponding probability mass at zero (nominal wage freeze). Effects on small changes of either sign that might be caused by menu costs are taken into account less frequently.

In statistical terms this general framework can be interpreted as a model of censoring at zero. If rigidity were imperfect, certainly the more realistic case, it would have to be a model of random censoring, where censoring only occurs with a certain probability. This probability of censoring ρ can then be interpreted as the degree of rigidity with which notional wage cuts are prevented by downward nominal wage rigidity. In terms of random variables, the actual wage change rate X_t is derived from the underlying latent random variable X_t^*, the notional wage change rate that follows the counterfactual distribution. In the context of the skewness-location approach, the counterfactual distribution is assumed to be constant over time, except for a parameter of location L_t, which reflects the rate of inflation and other influences on the average wage change rate, in short

(1) $$X_t^* \sim G(x_t^*; L_t).$$

The actual wage change variable X_t follows the distribution function $F(.)$, which is defined as

(2) $$F(x_t; L_t, \rho) = \begin{cases} (1-\rho)G(x_t; L_t) & x_t < 0 \\ G(x_t; L_t) & x_t \geq 0. \end{cases}$$

Again, the parameter of location L_t shifts the distribution, but in addition the factual distribution depends on the degree of rigidity ρ, in short $X_t \sim F(x_t; L_t, \rho)$. Note that $F(.)$ is a mixed distribution with probability mass at $x_t = 0$ and not a continuous distribution like $G(.)$.

The skewness-location approach introduced by McLaughlin (1994) is based on the following two statements: First, downward nominal wage rigidity causes the factual distribution to be skewed to the right if the counterfactual is symmetric, or causes additional skewness (to the right) if the counterfactual is not symmetric. Second, shifts in location to the right make the factual distribution less skewed to the right. Since $F(.)$ is a function of the location and the degree of rigidity, so will be the skewness of random variable X_t denoted by S_{X_t},

(3) $$S_{X_t} = S(L_t, \rho).$$

Using this notation, the two assertions of the skewness-location approach can be formulated as

(A1) $$\partial S / \partial \rho > 0,$$

i.e. additional rigidity causes additional skewness, and

(A2a) $$\partial S / \partial L_t < 0 \text{ if } \rho > 0 \text{ and}$$

(A2b) $$\partial S / \partial L_t = 0 \text{ if } \rho = 0,$$

i.e. shifts of the distribution to the right reduce skewness under downward nominal wage rigidity but leave skewness unchanged under nominal flexibility. In other words, joint variation of location and shape of the factual distribution only occurs under downward nominal wage rigidity.

The empirical implementation of the skewness-location approach is by linear regressions of the type

(4) $$S_t = c + \beta L_t + \varepsilon_t,$$

where S and L are measures of skewness and location of the factual distribution, c is a constant and ε is the error term. The statistical test of "no downward nominal wage rigidity" is a one-sided test of $\beta = 0$. For a significant negative value of β, the null hypothesis is rejected, which amounts to a proof of the existence of downward nominal wage rigidity. If β is not significantly smaller than zero, the null hypothesis of nominal flexibility cannot be rejected. This test logic makes clear that the notion of a falling and approximately linear skewness-location relationship underlying equation (4) is at the core of the skewness-location approach.

In the literature, there is some discussion of the relative merits of specific empirical measures of skewness and location for use in the approach, but no exploration of the validity of the basic assertions of the approach with respect to skewness in the presence of downward nominal wage rigidity. One of the issues has been the sensitivity to outliers of measures of skewness. A frequently chosen pragmatic solution has been to eliminate outliers from the samples. Another issue was, whether skewness remains unchanged under nominal flexibility, assertion (A2b). McLaughlin (1999a) showed that for counterfactual distributions that are skewed to the right this requirement is violated by the thinness measure suggested by Lebow, Stockton and Washer (1995). The resulting bias causes too frequent rejections of the null hypothesis of no rigidity; McLaughlin (1999b) contains a correction for the bias. With respect to location, there is the question whether a measure of location of the distribution should be used directly in regression (4), or instead, more indirectly, one or more determinants of the location of the distribution. While indirect indicators like the rate of inflation have been used, there seems to be a consensus that it is preferable to use the median as a direct measure of location, e.g. Lebow et al. (1995). Note however that the median ceases to capture the shifts of location adequately if it is itself affected by rigidity, which may be the case in low inflation periods, see Knoppik and Beissinger (2006). An alternative is to use an unaffected higher percentile as the measure of location instead, e.g. the 60$^{\text{th}}$ percentile.

3 Skewness-Location Relationships with Unexpected Properties

While sounding plausible, the assertions of the skewness-location approach (A1) and (A2) are not in general correct. In the following, this is shown by studying the interaction of rigidity and location in the case of a normally distributed counterfactual distribution and their effect on the most widely known and used measure of skewness, the skewness coefficient s. The skewness coefficient is defined as,

$$s = \frac{m_3}{m_2^{3/2}}, \tag{5}$$

where m_3 and m_2 are the third and second moment about the mean. Since the normal distribution is symmetric, its third central moment is equal to zero, therefore $s = 0$. While there are some insights with respect to higher moments of censored normal distributions, see Johnson and Kotz (1970), the case of imperfect censoring with a degree of rigidity $0 < \rho < 1$ is more complicated. Therefore the skewness-location relationship is discussed using a numerical example that roughly resembles the orders of magnitude found in the relevant empirical studies of nominal rigidity, with a standard deviation of the counterfactual distribution of 5% and medians between 0 and 10 %.

Panel a) of FIGURE 1 shows the density of the counterfactual distribution $G'(.)$ with a median of zero. Since it is symmetric, its skewness coefficient is equal to zero. The remaining panels b) to d) of FIGURE 1 illustrate factual distribution functions $F(.)$ for different locations L_t by their derivatives $F'(.)$ for $x \neq 0$ and represent the probability mass at zero with an arrow shaped area.[4] The degree of rigidity ρ is set equal to one half. Panel b) plots the factual distribution corresponding to the counterfactual distribution in panel a). Half of the notional wage reductions do not take place. The skewness coefficient for the actual wage change rate is negative, rather than positive, contradicting (A1). Panel c) shows the factual distribution that arises from a shift by two percentage points further to the right. It presents a second surprise: The skewness coefficient has increased, instead of fallen; in other words, despite the shift to the right the distribution is more skewed to the right than before, rather than less, contradicting (A2a). Finally, panel d) shows that several percentage points further to the right a factual distribution can be found that exhibits the same skewness coefficient as in panel c), rather than one that is lower, again contradicting (A2a). Taken together, the skewness-location relationship in this example violates both assertions of the skewness-location approach about the nature of skewness-location relationships under downward nominal wage rigidity.

FIGURE 1

FIGURE 2 plots the skewness coefficient s against alternative locations L_t for the example underlying FIGURE 1 and additional degrees of rigidity ρ.

FIGURE 2

FIGURE 2 shows that the falling, approximately linear skewness-location relationship asserted by the skewness-location approach is rather the exception than the rule. The non-monotonous nature of the relationships for most degrees of rigidity is the most important observation. It implies that - depending on the range of medians for which observations are available - the location coefficient β in regression (4) could take values of either sign, despite the fact that substantial downward nominal rigidity exists by construction of the example. Therefore, failure to find a significantly negative location coefficient β must not be interpreted as proof of nominal wage flexibility and absence of downward nominal rigidity.

The influence of the shift in location on the mean of the factual distribution plays the most important role for the observed phenomena. For a given location, rigidity may cause *negative* skewness for moderate degrees of rigidity: While it does shift probability mass of the left tail inward (tendency for more positive measured skewness), at the same time this also increases the mean of the factual and thereby the deviations from the mean (opposite tendency for measured skewness).[5] The net effect of rigidity on skewness measured by the skewness coef-

[4] This is a useful extension of the usual visualization of censored distributions which only symbolize the probability mass a the censoring threshold by a circle or dot that does not reflect its size, e.g. in Greene (2003).

[5] Some mass in the left tail far from the mean (i.e. less than complete rigidity) is required for this opposing effect.

ficient is therefore unclear. In a similar vein a shift in location to the right not only spreads out probability mass from the zero spike in the left tail farther away from the mean (reducing measured skewness) but at the same time the mean moves less to the right than the underlying counterfactual distribution. The opposing effect may be stronger and lead to a locally upward sloping skewness-location relationship.

Since the preceding explanations equally apply to distributions other than the normal, the 'strange' behaviour of the skewness coefficient is not an artefact of the functional example chosen for illustration. That is not to say that there are no measures of skewness that are well behaved in the sense of the skewness-location approach. FIGURE 3 demonstrates that e.g. the mean-median difference has falling skewness-location relationships.

FIGURE 3

Taken together this means that different measures of skewness may yield opposing answers to the following two questions. First, is the factual distribution more skewed than the counterfactual and therefore skewed to the right if the counterfactual is symmetric; second, are factual distributions located more to the left more skewed to the right than those located more to the right. This diverging behaviour of different measures of skewness points to fundamental problems of the concept of skewness itself. „ ... at the root of the trouble lies the fact that these measures impose a simple ordering - i.e. an ordering where every pair of elements are comparable - on too large a class of probability distributions.“, see Zwet (1964), p. 433.

4 Applicability of a Rigorous Concept of Skewness to Factual Distributions under Downward Nominal Wage Rigidity

The difficulties with the skewness-location relationship appear more plausible after reminding one that the skewness coefficient and other measures of skewness are ad hoc indicators, but not definitions or formalizations of skewness. A much-used, rigorous formalization of the concept of skewness uses an ordering of distributions based on convex transformations. It was introduced by Zwet (1964); a brief exposition can be found in Oja (1982). According to this concept of skewness certain transformations can be used to classify distributions as skewed to either the right and to compare distributions with respect to their skewness. The ordering is partial, i.e. not all distributions can be compared and classified. It turns out that the factual distributions arising in the context of downward nominal wage rigidity are somewhat particular in the sense that they cannot be classified and ordered.

For purposes of classification of the distribution function $H(.)$ of a random variable X, a transformation $Q(x)$ is defined as

(6) $$Q(x) = H_X^{-1}(1 - H_X(x)).$$

If $Q(x)$ is convex, the distribution of *X* is said to be skewed to the right. The graphical representation of this transformation $Q(x)$ is related to a theoretical symmetry plot of the distribution of *X*.

For comparisons of the distributions $H_X(.)$ and $H_Y(.)$ of two random variables *X* and *Y,* a second transformation $R(x)$ is defined as:
(7) .

If $R(x)$ is convex on the support of then the distribution of *Y* is more skewed to the right than the distribution of *X*. Note that the graphical representation of the transformation $R(x)$ is equivalent to a theoretical qq-plot.

It has been shown that some measures of skewness, among them the skewness coefficient, preserve this ordering, while others, e.g. Pearson's skewness coefficients, do not. Note that for distributions outside the partial ordering the skewness coefficient can still be computed but then contains no information on the skewness in the sense of the ordering.

Definitions (6) and (7) can be applied to the counterfactual and factual distributions analysed in the context of downward nominal wage rigidity. In the following, the resulting transformations are analysed graphically in FIGURE 4, FIGURE 5 and FIGURE 6 in order to classify and compare the various factual distributions.

FIGURE 4

FIGURE 5

FIGURE 6

FIGURE 4 shows in a (theoretical) symmetry plot that the transformation defined with a factual distribution as in equation (6) is not convex, i.e. the factual distribution cannot be classified as being skewed to the right. FIGURE 5 shows in a (theoretical) qq-plot that the transformation defined with the counterfactual and its corresponding factual distribution as in equation (7) is not convex, therefore it cannot be said that the factual is more skewed to the right than the counterfactual. Finally, FIGURE 6 shows that the transformation defined with the two factual distributions that differ in location as in equation (7) is not convex, therefore it cannot be said that the more leftward factual distribution is more skewed to the right than the other distribution.

These negative results show that the specific changes in shape caused by nominal rigidity are not well related to a rigorous concept of skewness. It is therefore not surprising that skewness-location relationships do not necessarily possess the properties posited by the skewness-location approach. An analysis in the spirit of the skewness-location approach can, however, try to make use of specific indicators of shape that do not attempt to capture skewness in any rigorous sense; well-behaved indicator-location relationships are required in that case. An example with the required properties is the mean-median distance which is used in part of the literature cited in the introduction.[6]

5 Summary and Conclusion

The skewness-location approach has been widely used in the analysis of downward nominal wage rigidity in micro data. It tests the hypothesis of nominal wage flexibility based on the notion that under downward nominal wage rigidity (but not under flexibility) there is negative relationship between the location of distributions of wage changes and their skewness, i.e. a falling skewness-location relationship. The paper shows that this assertion is not in general correct and that the most widely known and used measure of skewness, the skewness coefficient has skewness-location relationships that are non monotonous. The result is explained by showing that the skewness of the specific distributions caused by nominal rigidity cannot be classified or ordered by a rigorous concept of skewness based on convex transformation. These findings undermine the test logic of the skewness-location approach that is based on a falling skewness-location relationship, thereby invalidating any conclusions about the existence of downward nominal rigidity based on notions of skewness. Any non-econometric assessments of the existence of downward nominal wage rigidity based on notions of skewness are similarly affected and should also be avoided. While skewness is too elusive a concept to be a reliable tool in the analysis of downward nominal wage rigidity, the spirit of the skewness-location approach might be saved by using specific indicators of shape that react in the necessary way to shifts in location of the distribution.

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[6] An earlier working paper version of the paper contains further suggestions along these lines and is available from the author upon request.

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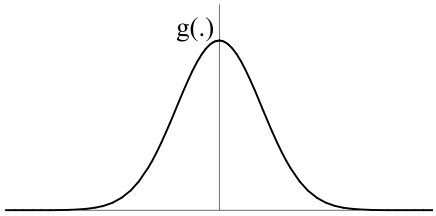
Figures

FIGURE 1: Unexpected Behavior of Skewness under Rigidity

a) Counterfactual distribution

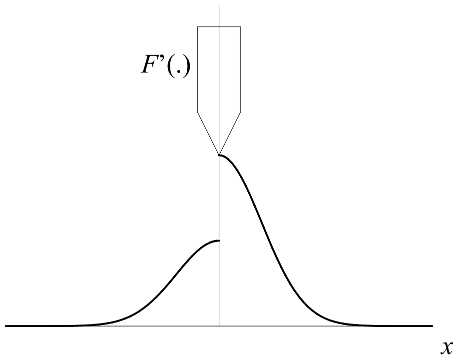
$L = x_{med} = 0.0$,
$s = 0.0$.



b) Factual distribution corresponding to panel a)

$L = x_{med} = 0.0$,
$s = -.058$.

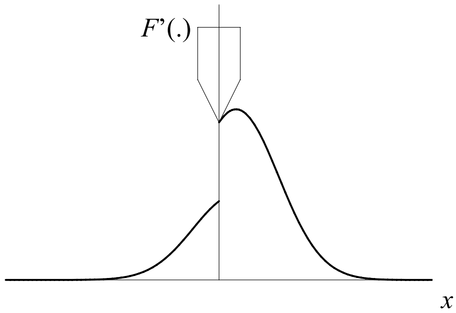
Rigidity causes a negative skewness coefficient s, contrary to (A1).



c) Factual distribution

$L = x_{med} = 0.02$,
$s = .19$.

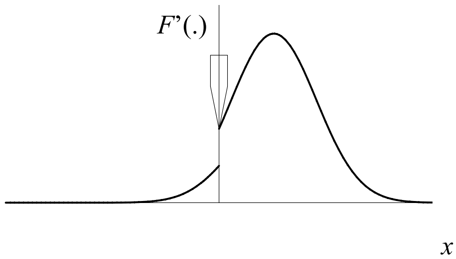
A shift of location L to the right increases measured skewness s, contrary to (A2).



d) Factual distribution

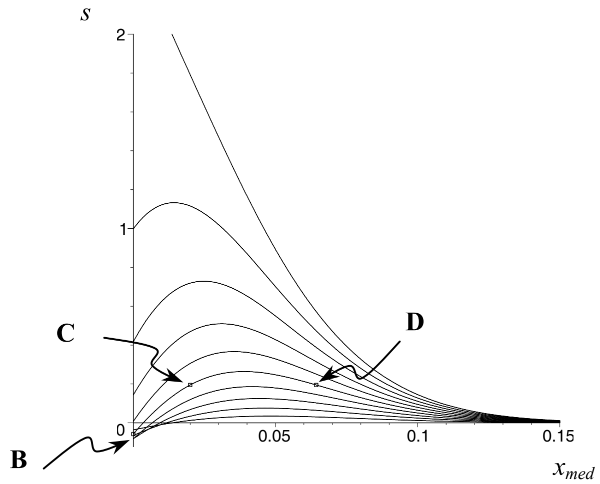
$L = x_{med} = 0.064$,
$s = .19$.

A further shift of location L to the right may leave measured skewness s unchanged, contrary to (A2).



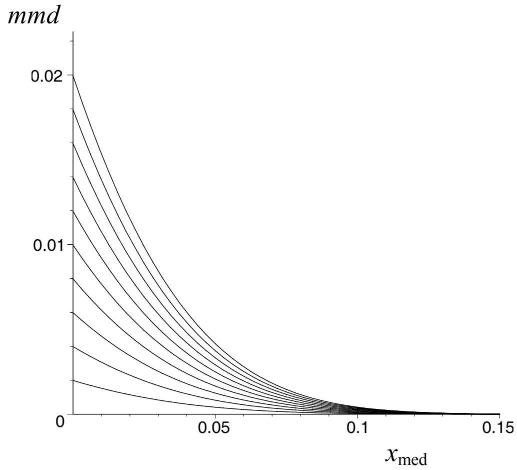
Note: Visualization and specification of example as discussed in Section 3 of the text, $(m_2)^{1/2} = \sigma = .05$.

FIGURE 2: Skewness-Location Relationships of the Skewness Coefficient for different degrees of rigidity



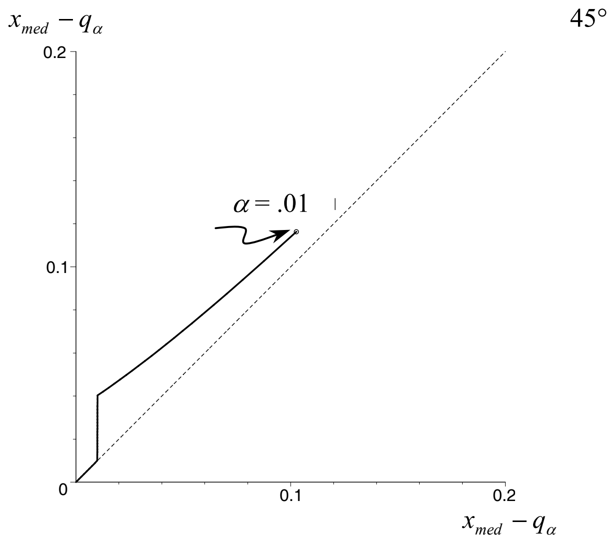
Notes: Skewness coefficient s plotted against location $L = x_{\text{med}}$ based on the numerical example described in the text. The ten curves are for values of the rigidity coefficient ρ from .1 to 1 in steps of .1 (bottom to top). $s(X)$ increases over some range of x_{med} for $\rho < 1$. Points **B**, **C** and **D** correspond to the scenarios depicted in panels b) c) and d) of FIGURE 1.

FIGURE 3: Skewness-Location Relationships of the Mean-Median Difference



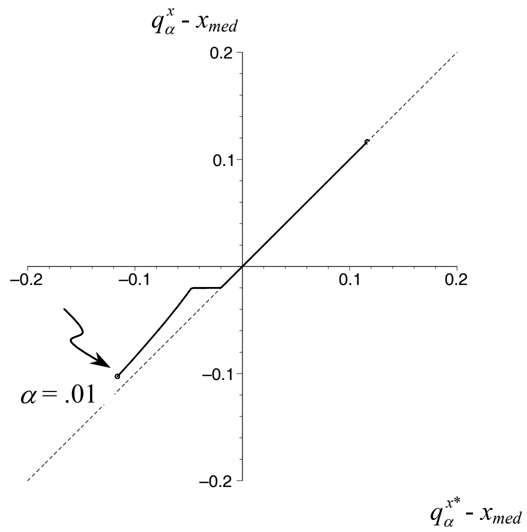
Notes: Mean-median difference *mmd* plotted against location $L = x_{\text{med}}$ based on the example described in the text. The ten curves are for rigidity coefficients ρ from .1 to 1 in steps of .1 (bottom to top). All skewness-location relationships are falling.

FIGURE 4: Symmetry Plot for Factual Distribution



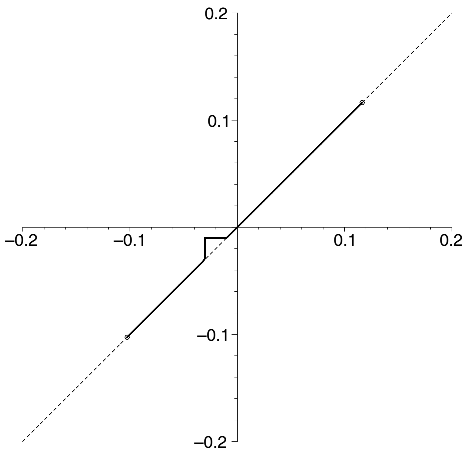
Notes: Plot of distances of and percentiles of the factual distribution $F(.)$ to the median x_{med} ($\alpha = .01 \ldots .50$). Left tail on horizontal, right tail on vertical axis. Convexity requirement of equation (6) violated by symmetric part reflected by part on 45° line. Specification: Functional and numerical specification as described in text; $L = x_{med} = 0.01$.

FIGURE 5: qq-Plot for Counterfactual and Factual Distributions



Notes: Percentiles of factual q_α^x plotted against percentiles of counterfactual $q_\alpha^{x^*}$, $\alpha = 0.01 \ldots .99$. Plot centred around respective medians. Since $R(.)$ is not convex, $F(.)$ and $G(.)$ can not be ordered. Specification: Functional and numerical specification as in FIGURE 4; $L = x_{med} = 0.02$.

FIGURE 6: qq-Plot for Factual Distributions at Different Locations



Notes: Factual distribution $F(L_1)$ further left than $F(L_2)$, $L_1 < L_2$. Percentiles of factual (vertical) plotted against percentiles of factual (horizontal); $\alpha = 0.01\ldots.99$. Note: Plot centred on respective means. Since transformation $R(.)$ is not convex, factual distributions $F(L_1)$ and $F(L_2)$ cannot be ordered. Specification: Functional and numerical specification as in FIGURE 4; $L_1 = x_{\text{med}} = 0.01$, $L_2 = x_{\text{med}} = 0.03$.