Low Fertility of Highly Educated Women: The Impact of Child Care Infrastructure

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Abstract

Most studies of the negative correlation between fertility and education treat education as exogenously raising wages and the cost of child rearing, thus reducing fertility. I relax these assumptions in two respects. First, child costs don’t increase with the value of time when external child care is used. Second, over a lifetime, education is endogenous. I model women’s choice of education, fertility, and form of child care, allowing for economies of scale in parental child care. Compatibility between work and family duties increases labor supply, the demand for children of educated women, and enhances incentives for obtaining education.

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1 Introduction

Empirical studies show a persistent negative correlation between women’s educational achievement and fertility. The negative relationship is found in aggregate data for total fertility rates and average years of female schooling across different countries (Barro/Lee 1994, UN 1995), in micro-level data for developed countries (e.g. Sander 1992, Mathews/Ventura 1997, Björklund 2006, Hoem et al. 2006a) and also for developing countries (UN 1995, Bongaarts 2003). There are few exceptions to this regularity: In the least developed countries, receiving some education can raise a woman’s fertility compared to women who never go to school, probably due to higher fecundity, and across OECD countries, total fertility rates have recently been higher where a larger percentage of women acquire tertiary education (d’Addio/Mira d’Ercole 2005).

In many European societies there are concerns about below-replacement fertility rates because demographic changes lead to a shortage of working age population, which could challenge the economy’s productivity and destabilize the social security systems. But fertility of more educated women being lower than the average fertility rate is of particular importance because of the strong correlation between children’s academic success and their families’ educational background (Wössmann 2004, Chevalier et al. 2005). The average number of children born in families with an academic background is low, but these children have good chances of obtaining a good education themselves. Parents with little education on the other hand tend to have more children, who in turn have low prospects of receiving an extended education. If fertility differences persist and intergenerational educational mobility remains low, the population’s composition changes, higher education is limited to a shrinking part of society, human capital becomes more concentrated and the average education of the population could decrease.

1Related issues are the tendency of more educated women to postpone maternity (Gustafsson/Kalwij 2006, Billari/Philipov 2004), their higher incidence of childlessness (Hoem et al. 2006b, Schmitt/Winkelmann 2005), and the positive correlation between women’s education and their labor market participation (OECD 2002). Several studies (surveyed in Del Boca/Locatelli 2006) have also found a negative correlation between a woman’s number of children and her labor supply.
This affects the income distribution, economic growth, and carries negative fiscal externalities (Schäfer 2005, de la Croix/Doepke 2003, Werding/Hofmann 2005). As de la Croix and Doepke (2003, p. 1109) put it, "it is not overall population growth, but the distribution of fertility within the population which is important. In other words, who is having the children matters more than how many children there are overall."

Economic explanations of the negative correlation between women’s education and fertility are often based on the New Home Economics theory (Becker 1981). They reason that more educated, better earning women have higher opportunity cost of time than women with less education. Child raising is a time intensive task, so the cost of children increases with the wage rate if child care is done as part of household production, using parents’ time.\(^2\) The higher cost exerts a negative substitution effect which reduces the demand for children, and this effect is usually assumed to dominate the ambiguous income effect.\(^3\)

A related strand of research using dynamic models of fertility timing finds that longer educational enrollment leads to postponement of a first birth (Kravdal 1994, Gustafsson/Kalwij 2006). A faster transition of more educated women to higher birth orders, as found by Kravdal (1992) and Kreyenfeld (2002), could allow for "catching up" to the completed fertility of their less educated counterparts. The total number of children, conditional on having a first birth, would not depend upon a woman’s education, and the demographic effects of postponement would only be temporary. The empirically low average fertility of highly educated women would then have to be driven by the higher-than-average fraction of these women remaining childless (Schmitt/Winkelmann 2005, Hoem et al. 2006 a). This might be attributed to continued postponement of a

\(^2\)The value of time as a reason for fertility differences between more and less educated women has been assumed, for example, in models of economic growth with endogenous fertility (Becker et al. 1990, Galor/Weil 1996, de la Croix/Doepke 2003, Schäfer 2005, Kimura/Yasui 2007).

\(^3\)The income effect on the demand for children is ambiguous if parents are altruistic: If child related expenditures raise parents’ utility, a rise in income would be used to "buy" more children and to spend more on each of them. But higher expenses per child increase the cost of children, reducing demand for them – possibly more than the direct positive income effect, so the total income effect could be negative without implying that children are an inferior good (Becker/Lewis 1973).
first birth by women who suffer high opportunity costs when interrupting their career.  
But these explanations have some shortcomings. One is, they assume that the value of the parents’ time enters child costs because child raising comes at the expense of time for other purposes. This assumption is valid in an institutional environment where external child care is unavailable, not affordable or of poor quality, but not if purchasing time in child care facilities or from other external providers is an option. In that case the cost of child rearing includes the child care fee instead of the parent’s wage rate, and the latter no longer explains different fertility rates. Accounting for the availability or price of external child care is standard in empirical analyses of women’s labor supply, but has been neglected in theoretical and empirical work that explores the fertility choices of women with different education.

Second, the opportunity cost argument typically assumes women’s wages to be exogenous, which is not an innocuous simplification. Most women complete their education before considering the transition to motherhood, so their education and wage rate is given at the time fertility choices are realized. But over a lifetime, the amount of education a woman obtains is not exogenous. She chooses a field and level of education, just as she chooses how many children she wants to have, and these decisions mutually interact. When planning her career, a woman has to take into account her plans and options for family formation and whether these are compatible with the vocational plans that she chooses from.

Schmitt/Winkelmann (2005) find another connection between education and childlessness in German data. The share of university graduates not living with a partner is higher than the population average, and the rate of childlessness is higher for women living alone. Models of partner choice (e.g. Cigno 1991) offer an explanation for the large incidence of singles among academic women: They are less inclined to specialize in housework, which reduces the possible gains from labor division, and as their outside options are generally better than those of less educated women, they can extract a higher share of cooperation gains in a partnership. This leaves them with a lower number of potential matches to start a family with, and contributes to low average fertility.

Ermisch (1989), Lundholm/Ohlsson (2002) and Apps/Rees (2004) model the effect of external child care on fertility choices, and how these vary with income and female wages. Fertility rates are predicted to decrease when wages rise for those women who have earnings that are low compared to external child care costs. If wages are high enough to make external child care affordable, then further wage increases would have a positive income effect on the demand for children.
In designing policies that are meant to increase fertility of more educated women, these two points need to be taken into consideration. In fact, improving child care infrastructure is one of the most discussed measures meant to raise fertility in general and especially that of educated women. The present paper uses a novel approach to model how individual choices on both education and fertility are driven by policies regarding public child care. Availability of affordable, high quality child care is of crucial importance for compatibility of motherhood and labor market participation. It allows mothers to supply more labor, thus it changes the cost of having children and consequently changes fertility incentives at a stage where education is given. In addition, it also changes the incentives for obtaining education, because having children does not require prolonged absences from the labor market if external child care is available. Thus, for a given number of children, being able to use day care extends the amount of time during which the returns on educational investment (in terms of higher wages) can be reaped.

Another feature that distinguishes the model set up in this paper from previous literature is that it explores economies of scale in parental child care and how they modify the effect of external child care. The absence of such economies of scale is a common assumption in the theoretical literature, but it does not stand up well to empirical evidence, which finds economies of scale when births are not timed at long intervals (Robinson 1987, Gustafsson/Kjulin 1994). The main part of this paper will investigate two versions of the model of endogenous fertility and education, one without and one with economies of scale present in home-based child care.

2 The effect of day care on fertility and education

The model used for analyzing a woman’s choice of education, fertility, and child care is a static model, focussing on aggregate variables over a lifetime. Regarding parental child care, two specifications are discussed: section 2.1 assumes no economies of scale,
i.e. each additional child requires the same amount of parental time if cared for at home. Section 2.2 makes the contrarian assumption that parents can look after all their children at the same time. Reality is probably somewhere in between the two extremes; closer to the first when children are born at intervals of several years, and more like the second in the case of multiple births. Section 2.3 captures this by adding a minimum amount of parents’ time that is needed per child, regardless of whether siblings are cared for together or whether the child is in day care most of the time.

The model differs from other models of fertility choice in that individuals choose their own education, instead of parents choosing how much education to buy for their children. This is a plausible assumption for decisions on post-secondary education, so the model is more applicable to countries where lower secondary schooling is near universal than to countries where the duration of primary or secondary schooling depends upon parental resources. Academic ability acts as the source of heterogeneity.

Compatibility between family duties and labor market participation is crucial to fertility and educational decisions. Different degrees of compatibility are captured by the expected price of full time external child care, where external care is a perfect substitute for parental care. The expected price incorporates the effect of subsidies to public child care facilities, of limited hours of operation (because part time public care needs to be supplemented with additional, more expensive means of care), and of rationing of child care slots (since a low chance of getting a subsidized place in a day care facility increases the expected cost of external child care).

2.1 Household choices with no economies of scale in child care at home

2.1.1 Fertility and child care choice

The decision maker (whom I call, synonymously, parent or mother, though it could of course be a father as well) derives utility from consumption of a numeraire good $c$ and
from having \( n \) children,

\[ u = \alpha \ln c + (1 - \alpha) \ln n \tag{1} \]

with \( 0 < \alpha < 1 \) and \( n \) continuous for the sake of differentiability. Note that this utility function precludes voluntary childlessness, which will only be approximated by low fertility. The parent maximizes utility subject to the following constraints on the monetary budget and time use:

\[ \begin{align*}
wl & \geq c + n \gamma + np(1 - h) \tag{2} \\
T & \geq e + l + nh \tag{3}
\end{align*} \]

In addition to their own consumption, parents spend a fixed amount \( \gamma \) on consumption for every child they raise. \( p \) is the net fee for external child care, which is used during \((1 - h)\) time units for every child in the family. Children need to be looked after by adults for an amount of time that is normalized to one, so external care time and home care time \( h \) have to add up to one per child and \( h \) can also be regarded as the fraction of child care that is done at home. Both \( h \) and \((1 - h)\) are bounded between 0 and 1. All expenditures are financed from the net wage \( w \) earned per unit of working time \( l \); an exogenous income, e.g. from a working spouse, is neglected for simplicity. The total time endowment \( T \) is divided between \( e \) time units in education, \( l \) in paid employment, and child care time \( h \) for each of the \( n \) children.

Combining constraints (2) and (3) yields the full income constraint

\[ wT \geq c + n [\gamma + (1 - h)p + hw] + we \tag{4} \]

where the term in square brackets is the cost of a child, consisting of monetary costs and the opportunity cost of time the mother looks after her child. The nonnegativity
restriction for child care time at home and in day care is

\[ 0 \leq h \leq 1 . \]  

(5)

Assuming for the moment that education and wages are given, the first order conditions for an extremum of (1) with respect to \( c, n \) and \( h \), subject to (4) and (5), yield demand functions

\[ c = \alpha w (T - e) \]  

(6)

\[ n = \frac{(1 - \alpha) w (T - e)}{\gamma + (1 - h) p + h w} \]  

(7)

and a fraction of home care \( h \) that is chosen to minimize the cost of child supervision

\[ h \begin{cases}  
= 0 & \text{for } p \leq w \\
= 1 & \text{for } p > w 
\end{cases} \]  

(8)

The price of external child care determines which form of child care a mother chooses for her child. Thus, the cost of a child is either \( \gamma + p \) (with external care) or \( \gamma + w \) (with home care). Figure 1 illustrates that if \( p \) is less than the mother’s wage, then \( h = 0 \) and her demand for children is falling in \( p \), while it is constant in \( p \) for \( h = 1 \). Lowering the child care fee from some value \( p_0 \) to \( p_1 < p_0 \) would coax women with high earnings that already use external care (i.e. women whose wage exceeds the original fee, \( p_0 < w \), so that they are in the left part of the diagram) to increase their fertility, since their children become relatively cheaper compared to \( c \). In addition, those women with a wage rate such that \( p_0 > w > p_1 \) decide to switch to external care, increase their labor supply by \( n \) units of time no longer needed at home, and have more children in response to the fee reduction. Women with wages lower than \( p_1 \) are unaffected by the price change; they are in the right part of figure 1 before and after, and their choices are not changed.
Figure 1: No economies of scale: Demand for children and child care choice at different day care fees

Thus, a favorite instrument of family policy – lowering child care facilities’ fees, extending hours of operation and increasing the number of child care slots – increases average fertility, $\partial n / \partial p \leq 0$. Moreover, it does so by increasing fertility of those women where low fertility has been identified as most critical: the most educated women, with high earnings capacities.

What about the effect of education on fertility, i.e. how does the level of $n$ change for a woman when her time in education is exogenously varied? Taking into account that education raises the wage rate,

$$\frac{\partial n}{\partial e} = \frac{1}{\gamma + (1 - h) p + h w} \left[ (1 - \alpha) \left( \frac{\partial w}{\partial e} (T - e) - w \right) - n h \frac{\partial w}{\partial e} \right]$$

If education is exogenously determined, as we assume in this section, the sign of $\partial n / \partial e$ is ambiguous. Substituting $h = 0$ into (9), one finds that for a woman who uses external child care, more education implies higher fertility if $(T - e) \frac{\partial w}{\partial e} > w$, i.e. if income during her working time rises more than the opportunity cost of the extended education. This corresponds to the rising section of the curve for $h = 0$ in figure 2. With the reverse inequality, lifetime income is reduced by marginally increasing education, which reduces
the demand for children (note that with $h = 0$, the wage rate does not enter the cost of a child, so there is only a positive endowment effect of education on the demand for children).

Figure 2: No economies of scale: Demand for children with exogenous variations in education, using day care (if $w(e) > p$) or child care at home (if $w(e) < p$)

A woman looking after her children at home, $h = 1$, has to make up for the opportunity costs of education during a shorter time of labor market participation if income is to increase with education, and has to consider that the cost of a child increases with the wage rate. In this case, there is a positive endowment effect of $\frac{1-\alpha}{\gamma+w} \left( \frac{\partial w}{\partial e} (T-e) - w \right)$, which increases the demand for children as long as education increases the value of the total time left after finishing education. But in addition, there is a negative income and substitution effect from the higher opportunity costs of time spent with children when the wage increases with education. These effects add to $-n \frac{\partial w}{\partial e}$ and reduce the demand for children. The inequality for rising fertility with education is more strict then with external child care, $(T-e) \frac{\partial w}{\partial e} > w$ is now required. Figure 2 sketches this for the case where with no education a woman prefers to look after her children at home, while she would use day care if enough education raises her wage rate above $p$.

Figure 2 again shows that the option of using external day care increases the fertility

\[ n^* = \frac{1-\alpha}{\alpha} \frac{\alpha}{\gamma+w} \exp(\bar{u}) \]

From the compensated demand function for children, $n^* = \frac{1-\alpha}{\alpha} (\gamma+w)^{-\alpha} \exp(\bar{u})$, the substitution effect is found to be responsible for a fraction $\alpha$ of this negative effect, the income effect for the rest.
of educated women compared to a scenario with home care only, as the line for \( h = 0 \) is higher than that for \( h = 1 \) whenever \( e \) is so large that \( w(e) > p \) and external care is chosen. Reducing \( p \) shifts the line for external child care upwards and its intersection with the line for parental care to the left. Thus, a lower \( p \) makes day care attractive for more women and increases \( n \) for those using it, i.e. those with enough education to have a wage rate above the critical level.

### 2.1.2 Educational choice

Now consider a woman’s choice of educational investment. If she decides against education, \( e \) takes on a value of 0 and her earnings will be the wage rate for unskilled labor times a labor supply of \( T - nh \) time units. By choosing \( e > 0 \) she increases her wage rate, where the gain depends on her ability and \( \frac{\partial w}{\partial e} > 0, \frac{\partial w^2}{\partial^2 e} \leq 0 \).

Substitute demand functions (6) and (7) into (1) to find utility as a function of education and child care choice, with optimal fertility and consumption choices anticipated:

\[
u = a + \ln[w(T-e)] - (1 - \alpha) \ln[\gamma + (1 - h) p + h w] (10)\]

with \( a = [\alpha \ln \alpha + (1 - \alpha) \ln(1 - \alpha)] \). Maximization with respect to \( e \) yields the marginal condition for optimal education

\[
w(e) = \frac{\partial w}{\partial e} (T - e) \left( \frac{\gamma + (1 - h) p + \alpha h w(e)}{\gamma + (1 - h) p + h w(e)} \right) = \frac{\partial w}{\partial e} (T - e - h n) (11)\]

for an interior solution. A corner solution \( e = 0 \) is possible if \( w > \frac{\partial w}{\partial e} (T - h n) \) holds for uneducated women, i.e. the opportunity cost of spending even a marginal amount of time in education is too high to be made up for by higher wages during the time spent working. The last term in (11), which is derived making use of (7), shows that reducing \( h \) increases the amount of time spent in the labor market and thus raises the returns to education: The right hand side of equation (11) increases if \( h \) equals 0 rather than
1. The option of using external day care for children thus tends to make educational investment more attractive. By a similar argument, the planned number of children has a negative effect on educational choices if $h > 0$, but not if $h = 0$, because only in the former case children take time that has to be saved up elsewhere.

To find whether $h = 1$ and little education or $h = 0$ and more education is actually chosen, consider a woman planning her utility maximizing education for different forms of child care:

- If she is planning on using external child care ($h = 0$), then $w(e^0) = w'(e^0) (T - e^0)$ is the optimality condition for education. This is equivalent to maximizing lifetime income $w(T - e)$, and corresponds to the level of education where $n(e)$ given $h = 0$ is maximized (see figure 2).

But external child care is only chosen later in life if $w(e^0) > p$. If the woman’s ability and consequently her wage gain from education is too low to fulfill this requirement, the initial assumption of $h = 0$ will be violated and she has to plan on looking after her children at home.

- If planning on $h = 1$, (11) is at the same time the solution to maximization of $w(T - e)/(\gamma + w)^{1-\alpha}$, i.e. lifetime income adjusted for the effect of increasing child costs by more education. In figure 2, this corresponds to a point on the downward sloping part of the $n(e)$-curve for $h = 1$, since that curve has its maximum where $w(e) = \frac{\partial w}{\partial e} (T - e) \frac{T}{\gamma + w}$ (from equation (9) set equal to 0 for $h = 1$), and optimal education requires $w(e) = \frac{\partial w}{\partial e} (T - e) \frac{2 + \alpha w}{\gamma + w}$ (from (11)), which is satisfied at larger values of $e$.

For high ability women, $w(e^1) > p$ would make this plan inconsistent, so they would unambiguously opt for $h = 0$.

- For women with medium ability, so that $w(e^1) < p < w(e^0)$, either plan is an option. This is the case illustrated in figure 2. The woman chooses between
external child care and more education on the one hand and less education and parental child care on the other hand by comparing the utility for both options:

\[ u = \begin{cases} 
  a + \ln[w(e^0) (T - e^0)] - (1 - \alpha) \ln[\gamma + p] & \text{for } h = 0 , \ e = e^0 \\
  a + \ln[w(e^1) (T - e^1)] - (1 - \alpha) \ln[\gamma + w(e^1)] & \text{for } h = 1 , \ e = e^1 
\end{cases} \quad (12) \]

Since \( p \) reduces utility with external child care but not with parental care, \( h = 0 \) and \( e = e^0 \) is preferred for values of \( p < (\gamma + w(e^1)) \left( \frac{w(e^0) (T - e^0)}{w(e^1) (T - e^1)} \right)^{1/(\alpha - 1)} - \gamma \), and \( h = 1, e = e^1 \) is chosen for larger values of \( p \).

Summing up, the effects of increasing the supply of child care or making it cheaper are the following: The most educated women already use external child care. They do not change their education or the form of child care in response to a reduction of \( p \), but will bear more children after the cost reduction since \( \partial n/\partial p < 0 \) for \( h = 0 \). Some women with medium ability will be convinced to switch from own to external child care. This frees up some of their time, and they spend part of this time on longer education and part on additional labor market time. The number of children increases for these women. Women who don’t reach a market wage high enough to afford child care facilities are unaffected by the price change. So in total, improving child care not only leads more women to enter the labor market, it also increases fertility of the more educated, and it increases the proportion of women deciding on higher education.

### 2.2 Household choices with economies of scale in child care at home

#### 2.2.1 Fertility and child care choice

Now consider the case where parents can exploit economies of scale by looking after several children at the same time. The time constraint (3) changes to

\[ T \geq e + l + h \quad (13) \]
and, substituting into the monetary constraint (2), the full income constraint now reads

\[ wT \geq c + n[\gamma + (1 - h)p] + w(h + e) \]  \hspace{1cm} (14)

Equation (14) makes it clear in which sense this version of the model assumes economies of scale in parental child care: Parents can watch over several children at a time and only incur the opportunity cost of time \( w h \) once. The fee for external care in contrast is charged on a per-child basis. One would thus suspect that mothers of larger families are more likely to stay at home with their children than those with a smaller number of children. We will see in the following that this is not necessarily true, because the relationship between the number of children and form of care chosen is determined also by the amount of education and the institutional framework as represented by the cost of external child care. A woman might choose to have more children in external care than she would if she had to stay at home to look after them.

The first order conditions for the utility maximization problem yield demand functions

\[ c = \alpha w(T - e - h) \]  \hspace{1cm} (15)
\[ n = (1 - \alpha) \frac{w(T - e - h)}{\gamma + (1 - h)p} \]  \hspace{1cm} (16)

and, given \( n \), a fraction of home care

\[ h \begin{cases} 
= 0 & \text{for } pn < w \\
\in (0, 1) & \text{for } pn = w \\
= 1 & \text{for } pn > w
\end{cases} \]  \hspace{1cm} (17)

If having all children in day care is cheaper than the wage foregone when staying at home, children will be in external care full time and vice versa. Part time external care is a possibility if its cost just equals the parent’s wage. But clearly, the choice of \( c, n \)
and $h$ is interdependent:

- If $h = 0$, then the utility-maximizing demand for children is $n = \frac{(1-\alpha)w(T-e)}{\gamma+p}$.
  In order to satisfy $pn < w$, as required for $h = 0$, the inequality $p < \bar{p} := \gamma/[(1-\alpha)(T-e-1) - \alpha]$ has to hold, i.e. child care fees may not be too high.\(^7\)

- By similar argument, $h = 1$ leads to demand $n = \frac{(1-\alpha)w(T-e-1)}{\gamma}$ and is compatible with child care fees exceeding $\hat{p} := \gamma/[(1-\alpha)(T-e-1)]$.

- An interior solution with respect to $h$ results in demand functions (15) and (16). From (17), it is only compatible with $n = w/p$. Setting this equal to (16) one finds that $h$ has to be chosen to equal $h = \frac{\gamma}{\alpha p} + \frac{1}{\alpha}[1-(1-\alpha)(T-e)]$, which is strictly decreasing in $p$ and falls into the $(0; 1)$ range for values of $p$ between $\hat{p}$ and $\bar{p}$.

\(^7\)Since attention is limited to positive values for $n$ and $w$, $pn < w$ is always satisfied if $p$ is set to zero, i.e. all women would use external child care if the fee were subsidized at a rate of 100 per cent.

Figure 3: Economies of Scale: Demand for children and child care choice at different day care fees I

Assume for now that $\bar{p} > 0$ (I will discuss the case that $\bar{p} < 0$ in section 2.2.2). Since $\hat{p} < \bar{p}$, the regions for different child care regimes and the corresponding demand for children are partially overlapping, as depicted in figure 3. At child care fees below $\hat{p}$,
a woman unambiguously chooses external care for her children and dedicates her entire time endowment (after education) to the labor market. In this region, the number of children decreases with $p$ because the fee enters the price of a child. At fees exceeding $\bar{p}$, a woman does all child care at home and only works during the residual time net of education, i.e., before having children and after they are grown. Since external child care is not used in this case, its price does not influence the demand for children.

The middle region, where $\hat{p} \leq p \leq \bar{p}$, needs closer inspection, as two corner solutions and an interior solution are candidates for optimal behavior: full-time, part-time or no external care are possible. To find which of these strategies yields the greatest utility, substitute (15) and (16) into (1):

$$u = a + \ln[w(T - e - h)] - (1 - \alpha) \ln[\gamma + (1 - h)p]$$  \hspace{1cm} (18)

where $a = [\alpha \ln \alpha + (1 - \alpha) \ln(1 - \alpha)]$ and the last conversion makes use of (16). The first derivative with respect to $h$,

$$\frac{\partial u}{\partial h} = \frac{(1 - \alpha)p}{\gamma + (1 - h)p} - \frac{1}{T - e - h} = \frac{pn - w}{w(T - e - h)}$$  \hspace{1cm} (19)

confirms that the interior solution, with $pn = w$, is an extremum. But from the second derivative and again using (16),

$$\frac{\partial^2 u}{\partial h^2} = \frac{(1 - \alpha)p^2}{(\gamma + (1 - h)p)^2} - \frac{1}{(T - e - h)^2} = \frac{(pn)^2 - w^2 + \alpha w^2}{(1 - \alpha)w^2(T - e - h)^2}$$  \hspace{1cm} (20)

$u$ can be found to be convex in $h$ at $pn = w$, so the extremum is actually a minimum and the interior solution can be ruled out as a candidate for optimal behavior (see figure 5 below for an illustration of $u(h)$ for different child care fees).

Parents will choose a corner solution with $h$ equal to 0 or 1, depending on which yields
higher utility. Comparing the two realizations

$$u = \begin{cases} 
  a + \ln[w(T - e)] - (1 - \alpha) \ln[\gamma + p] & \text{for } h = 0 \\
  a + \ln[w(T - e - 1)] - (1 - \alpha) \ln(\gamma) & \text{for } h = 1
\end{cases}$$  \hspace{1cm} (21)

one finds the critical price level \(p^*\) at which the household is indifferent between both forms of child care

$$p^* = \gamma \left( \frac{T - e}{T - e - 1} \right)^{\frac{1}{1 - \alpha}} - 1$$  \hspace{1cm} (22)

At day care fees above \(p^*\), looking after one’s children at home is preferred, and at lower price levels external child care will be chosen, as shown in figure 4.

![Figure 4: Economies of Scale: Utility maximizing demand for children and child care choice at different day care fees](image-url)

To see that \(p^*\) is between \(\hat{p}\) and \(\bar{p}\) and thus actually divides the region of interest, notice that the interior solution for \(h\) from (17) gives the minimum of \(u(h)\), \(\arg\min u(h) = \frac{\gamma}{\alpha p} - \frac{(1 - \alpha)(T - e - 1)}{\alpha} .\) This is decreasing in \(p\), so in figure 5 the minimum of the curve \(u(h)\) moves to the left with increasing \(p\). The minimum of \(u(\hat{p})\) is at \(h = 1\), and the minimum of \(u(\bar{p})\) at \(h = 0\). By construction, \(u(p^*)\) must have its minimum on the interval \(h = [0, 1]\), so by monotony of \(\arg\min\{u(h)\}\) in \(p\), \(\hat{p} < p^* < \bar{p}\) can be deduced.
Figure 4 illustrates that improving external child care or making it cheaper does not unambiguously have the desired effect of increasing educated women’s fertility when there are economies of scale in parental child care. The discontinuity of $n(p)$ is further to the right for more educated women, $\partial p^*/\partial e > 0$, so the more education a woman has obtained the more likely she will use external care for her children. Reducing $p$ from an original value of $p_0$ to $p_1 < p_0$ has, as in section 2.1, no effect on the fertility decision of the least educated women – they will still stay at home with their children as long as $p^*(e) < p_1$, so the price change does not affect their choices. The price reduction increases fertility for those women with enough education to make their $p^*(e) > p_0$, since the cost of a child in external care is reduced. But for women of an educational level where $p^*(e)$ is lower than $p_0$ but a little larger than $p_1$, the demand for children is actually reduced as they switch to external child care. If $p$ is reduced further until it is below $\gamma/(T - e - 1)$, these women will decide on having more children in external care than the number of children they would have had if looking after them at home.

Similar to the case with no economies of scale in child care, exogenous variations in education could have a positive or negative effect on a woman’s demand for children,
depending on whether additional education increases or decreases her lifetime income for a given form of child care:

\[
\frac{\partial n}{\partial e} = \frac{1 - \alpha}{\gamma + (1 - h)p} \left[ \frac{\partial w}{\partial e} (T - e - h) - w \right]
\]  

(23)

In contrast to section 2.1, more education does not increase the cost of children while they are cared for at home. It only does so if education is increased such that it raises \( p^*(e) \) over the actual child care fee, so that the mother switches to external care. Fertility then drops sharply although lifetime income rises, because the cost of a child jumps from \( \gamma \) to \( \gamma + p \).

Figure 6: Economies of scale: Demand for children with exogenous variations in education, using day care (if \( p^*(e) > p \)) or child care at home (if \( p^*(e) < p \))

Figure 9 visualizes that the option of using external care as a substitute for one’s own time reduces fertility for some educational levels and increases it for others, relative to the number of children with \( h = 1 \) (the curve for \( h = 0 \) is partly below, partly above that for \( h = 1 \)). A reduction of \( p \) makes it more attractive to use child care facilities, and the point of discontinuity moves to the left. This reduces the fertility of women switching the child care regime. But at the same time, the curve for \( h = 0 \) moves upwards when \( p \) is lowered, which means that fertility increases for women already using external care.
The effect on average fertility rates is ambiguous.

If economies of scale in parental child care time are important – and data suggests they are – it takes much greater care than without economies of scale to choose the level of $p$ so that it increases the fertility of the more educated women, not just their labor market participation. If $p$ is set too high, few women will choose external care – and some of those will even decide for lower fertility than without external child care.

2.2.2 What if $\bar{p} < 0$?

It was assumed in section 2.2.1 that $\bar{p} > 0$, which was used in constructing figure 3 to delimit the different regions for $p$ that had to be considered in choosing the form of child care. If the household has spent a long time in education and has a strong preference for children, such that $(T - e - 1) < \alpha/(1 - \alpha)$, then $\bar{p}$ will turn negative and there is no price level where the medium region (with an interior and two corner solutions) ends. This does not, however, affect any of the conclusions found above.

The utility maximizing form of child care is again found by analyzing equation (18). The extremum of $u(h)$ is still a minimum, so the mother will choose a corner solution $h = 0$ or $h = 1$ to maximize her utility. At a child care fee of $p^*$ she is indifferent between the two options, at lower fees she prefers external care and vice versa, as illustrated in figure 4. The only change from figure 5 to 7 is that the minimum of the curve $u(h)$ is at $h = 0$ for $p = \bar{p}$ and at negative values of $h$ for larger $p$ in the former, whereas it remains at positive values of $h$ for all $p \geq 0$ when $\bar{p} < 0$.

2.2.3 Educational choice

Differentiating (18) with respect to $e$, the utility maximizing duration of education is determined by

$$\frac{\partial w}{\partial e} (T - e - h) = w(e)$$  \hspace{1cm} (24)
This marginal condition states that the wage forgone during the last marginal unit of time spent in schooling has to be regained in terms of the wage increase during working time. Since $e$ is restricted to non-negative values, a corner solution of $e = 0$ is chosen by women with ability so low that $\frac{\partial w}{\partial e}(T - h) < w$ holds at zero education.

Equation (24) is equivalent to saying that education is always chosen to maximize total lifetime income (and to maximize the number of children, which is a constant fraction of income), anticipating whether $h$ will be chosen equal to 0 or 1. Choosing $e$ to maximize income is natural in this version of the model, as additional education does not affect the marginal cost of children. Again it turns out that a woman planning to use external child care has an incentive to use more time for her education, as the extra time invested pays off during a longer working time than if she were to stay at home with her children. The number of children does not affect education, since at the margin another child does not change time use.

Which combination of $e$ and $h$ a woman decides for depends upon her ability to increase her lifetime income through educational investment, and upon the child care fee. If the amount of education $e^0$ that maximizes $w(e)(T - e)$ leads to a critical value
\( p^*(e^0) < p \), this woman will not choose external child care and will spend less time on her education. Along the same line, if optimal education with external child care \( e^1 \) makes \( p^*(e^1) > p \), then the woman will surely choose more education and have her children cared for externally. Finally, if \( p^*(e^1) < p < p^*(e^0) \) – as depicted in figure 6 – then utility for both combinations has to be compared. It turns out that the larger amount of education is chosen for \( p < \gamma \left[ \left( \frac{w(e^0)}{w(e^1)} \frac{(T-e^0)}{(T-e^1-1)} \right)^{1/(1-\alpha)} - 1 \right] \).

Comparing the results of the model with no economies of scale in parental child care to the model with economies of scale, the following is found: In both cases, making external child care more attractive has a positive effect on women’s labor market participation as more women opt for external child care, which frees up some of their time. They use the time saved for working and for obtaining more education. But the latter model cautions that the fertility results of the former may be misleading: a reduction of \( p \) could increase fertility for the most educated women, but if there are economies of scale in child care at home, the fee reduction will decrease the demand for children from some women whose critical price level \( p^* \) is not too far above the reduced price.

2.3 Adding realism: Minimum parental time requirement

The model used for deriving the above results assumed that parents can delegate all child care to external providers. Further, if there are economies of scale in parental care, even a child cared for at home did not cost its parents any time at the margin, since an arbitrarily large number of siblings could be looked after at the same time. Therefore, a negative substitution effect of additional education and higher wages on the demand for children was only found for mothers staying at home with no economies of scale. This was also the only case where the number of children had a negative influence on educational decisions.

The assumption that children in certain care arrangements don’t cost parents’ time is of course oversimplified. It is more realistic to assume that in addition to the time where
other adults can substitute parents or where a parent can look after several children, every child needs a parent’s full attention for some minimum amount of time. Mothers are usually on maternity leave for some weeks before and after confinement, and later on, even a child in day care needs some parental attention for healthy development. In addition, mothers may only be able to find part time external care and have to stay with their children for the rest of the day. This aspect can be added to the model as follows.

The total amount of child care time, normalized to 1, includes a fraction \( t \) that needs to be done by parents. The rest can be divided between parents’ time \( h \) and \( 1 - t - h \) units of time in external child care, where \( 0 \leq h \leq (1 - t) \). This reduces the degrees of freedom for optimizing the allocation of time, at least for those mothers that chose \( h = 0 \) when \( t \) was neglected. If there are no economies of scale in parental child care, demand for children is modified to

\[
 n = \frac{(1 - \alpha) w (T - e)}{\gamma + (1 - t - h) p + (t + h) w} \tag{25}
\]

For women staying at home with their children there is no change, but high earning women who use child care reduce their fertility, and their demand for children is somewhat less responsive to changes of \( p \) compared to equation (7).

An exogenous increase in education now has the following effect on fertility:

\[
 \frac{\partial n}{\partial e} = \frac{1}{\gamma + (t + h) w + (1 - t - h) p} \left[ (1 - \alpha) \left( \frac{\partial w}{\partial e} (T - e) - w \right) - n(t + h) \frac{\partial w}{\partial e} \right] \tag{26}
\]

The ambiguous endowment effect of education on the demand for children is the same as when \( t \) was neglected. But the negative income and substitution effect of the higher wage rate, which amount to \( \frac{-n(t + h) \partial w / \partial e}{\gamma + (t + h) w + (1 - t - h) p} \), are now present even if \( h = 0 \), and their effect is stronger the more children the household has, the more time \( (t + h) \) is spent

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\(^{8}\)In Western Germany for example, about three quarters of child care slots for 3-6 year old children are limited to the morning hours, and the supply of nannies, au pairs etc. to supplement this is insufficient. As a consequence, many mothers can only work part time.
with them, and the stronger the wage increase. The optimal amount of education is also
affected by the minimum time requirement; the optimality condition is modified to

\[ w(e) = \frac{\partial w}{\partial e} (T - e - (h + t) n) \] (27)

The planned number of children reduces the optimal investment in education because
children take time that has to be cut down on elsewhere, in education and labor market
participation.

For the case with economies of scale in caring for children at home, the analysis runs
along the same lines as above. If \( t \) units of time are used by parents for each of their
children, \( h \) can be divided among all siblings, and \( 1 - t - h \) bought in external care, then
the demand for children is

\[ n = \frac{(1 - \alpha) w (T - e - h)}{\gamma + (1 - t - h) p + t w} \] (28)

The effect of education on fertility is

\[ \frac{\partial n}{\partial e} = \frac{1}{\gamma + t w + (1 - t - h) p} \left[ (1 - \alpha) \left( \frac{\partial w}{\partial e} (T - e - h) - w \right) - n t \frac{\partial w}{\partial e} \right] \] (29)

and the negative income and substitution effects \( \frac{-n (\partial w/\partial e)}{\gamma + t w + (1 - t - h) p} \) now derive from the
fact that a higher wage makes the time required per child more expensive. Optimal
education is determined by

\[ w(e) = \frac{\partial w}{\partial e} (T - e - h - t n) \] (30)

and is reduced if more children are planned, whether they will be cared for at home or
externally.

The effect of good and affordable child care infrastructure, though it can not substitute
for all of parents’ time, is only slightly changed: Improvements lead some mothers to
switch to external child care and use the time this frees up for them on additional education and labor supply, just as they would without \( t \). Women who have used external child care even before the improvement respond to a decrease in \( p \) with higher fertility rates. But now, with \( t > 0 \), more children need more time at home, so labor market participation and education decline.

### 3 Conclusion and implications

Underlying the low fertility of educated women are the conflicting demands of children, work and education on women’s allocation of time. Parents’ expenditures on children tend to rise with income. But the increase is attenuated by the fact that a higher wage is reached by spending more time on education, which reduces the time available for children and work; in addition, since children cost parents’ time, a wage increase makes children more expensive, which reduces demand for them via a negative income and substitution effect. The option of using child care infrastructure can alleviate this conflict by reducing the time required for child rearing, thus reducing the negative effect of wages on fertility. Further, the time saved offers the opportunity of spending a longer time in education, and to make use of this education in the labor market for longer – which increases income and consequently the demand for children via the positive endowment effect of higher income.

Improving external child care increases the demand for children in many situations. But the second version of the model developed in the present paper cautions that this is not always true. Cheaper, better or more child care leads to a strong reduction in fertility if the expected price is lowered slightly below the reservation price of women who exploited economies of scale in child care at home before the reduction. If external child care is improved just enough to convince these women to use external child care instead, where they have to pay a fee on a per child basis, they will plan on having less children then they would without day care. But if child care infrastructure is improved further,
their demand for children increases again, and it can eventually exceed the number of children without external care when its expected price is low enough. The positive effect of day care supply on educational investment is also present in this case.

An important aspect I have neglected so far is how to finance child care subsidies. Apps/Rees (2004) show that revenue neutral shifts from public transfers to families towards more child care subsidies can increase fertility. Mothers entering the labor force increase the tax base and contribute to the public budget. The present paper strengthens this point: Not only the increase in the participation rate increases tax revenues, but also the higher wage rates earned by women who obtain more education if they can rely on external child care. In a dynamic context, the argument is even stronger, because shorter career interruptions imply less depreciation of human capital. Mothers who return to the labor market soon after having a child face higher wages during the rest of their working life than if they had staid at home for longer, so they pay more taxes.

Other measures that improve compatibility between work and family duties can complement the effects of improving child care. An example is offering more flexibility of work timing, so that mothers can more easily adjust working hours and external child care hours. Another promising approach is a wage dependent child raising allowance that replaces a parent’s wage for a limited amount of time while he or she stays at home with a child, as common in Scandinavian countries and recently introduced in Germany. Of course measures to ease the combination of having a family and career are only useful to the extent that women aspire to both. It is possible that low fertility of educated women is due to unobserved heterogeneity, i.e. they are less family oriented, while other women have a stronger preference for children and are less interested in a career and thus have less use for an extended education. If this were true, then to prevent a decline of average human capital in society, raising the educational performance of children from an uneducated family background would have to be the topmost priority. To this aim, high quality child care can also contribute, but it would have to be cheap enough to
attract children even from mothers with very low earnings capacities.

But if it is not heterogeneity of preferences but the difficulties of combining a job and a family that prevents educated women from having more children, and if family policy is intended to allow them to do so, then improving the supply and quality of child care and keeping its fees affordable can yield the desired results in more than one respect. Fertility tends to increase, and it does so especially for the most educated women. Further, the prospect of having good child care options will make education more attractive for an increasing fraction of women, and they put this education to use in the labor market – and their taxes even contribute to paying for child care subsidies.

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