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Three Theorems on Growth and Competition

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Abstract

This paper proves three theorems on growth and competition in a standard increasing variety endogenous growth model and draws conclusions for second-best competition policies. First, no growth may be better than some growth, since modest positive growth potentially requires sizeable static welfare losses. Second, the economy may converge to a steady state with zero growth, even though another (saddle-point stable) steady state with positive growth exists if the initial share of "cheap" competitive markets is sufficiently high, as this implies a relatively low demand for "expensive" innovative goods. Third, such a "no-growth trap" may happen in a world economy made up of several countries engaged in free trade with each other. As for competition policy, this implies that growth-enhancing policies maybe misguided and that quick deregulation as well as quick trade liberalization can lead to stagnation in the long term.

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1 Introduction

The most basic proposition about growth and competition, taught in introductory economics courses, is that there is a tradeoff between static welfare and long-term growth: perfect competition brings about static efficiency but undermines the incentives to invest in the innovation of new goods, services, and processes (see, e.g. Blanchard, 2006, p. 256). This paper highlights several important macroeconomic implications of this basic proposition, using a variant of Grossman and Helpman's (1991, Ch. 3) standard increasing product variety endogenous growth model. To address issues of growth and competition, this paper introduces erosion of monopoly power due to (exogenous) imitation in an otherwise standard Dixit-Stiglitz (1977) innovative intermediate goods sector and a non-innovative traditional sector (so that there are static distortions even if all the markets for innovative goods are monopolies).[1] We prove three theorems on the model's dynamics and welfare properties and derive several corollaries which characterize second-best competition policies.

The first theorem is a direct consequence of the tradeoff between static welfare and incentives to innovate. It states that no growth may be better than some growth. This result is due to the fact that, given the presence of a traditional sector, there is a static welfare loss (of non-infinitesimal size) due to monopoly pricing in the innovative sector.[2] If, given the economy's preferences and technologies, the incentives provided by monopoly profits bring about only a modest growth rate, it is preferable to dispense with growth altogether and implement static efficiency instead. The implication for competition policy is that so as to achieve a modest growth rate, it may not be worthwhile incurring the associated cost in terms of static welfare losses. While quite obvious, this finding has not been emphasized in the growth literature, which tends to stress the benefits of positive, fast growth. The second theorem says that the economy may get stuck in a "no-growth" trap (poverty trap): the unique perfect-foresight equilibrium possibly entails convergence to a steady state with zero growth, even though another saddle-point stable steady state with positive growth exists. This will happen if the initial share of competitive markets is sufficiently high.[3] The intuition is that, in that case, a potential innovator competes with many relatively cheap products, so that it does not pay to innovate. The implication

[1] These elements are not new to endogenous growth theory. Exogenous imitation is analyzed, for instance, by Arnold (1995), Barro and Sala-i-Martin (2004, Section 6.2, pp. 305 ff.), Kwan and Lai (2003), and Pelka (2005, Chapter 7). In Segerstrom (1991) and Walz (1995), imitation is endogenous. A traditional sector is contained, for instance, in Grossman and Helpman (1991, Section 5.3, pp. 130 ff.).

[2] In the absence of a traditional goods sector, this effect vanishes (see Grossman and Helpman, 1991, p. 70).

[3] The possibility of a no-growth trap is ignored in the papers with exogenous imitation mentioned in footnote 1 except in Pelka (2005), even though it arises for appropriate parameter values. The part of the present paper concerned with the possibility of a no-growth trap clarifies and extends Pelka's (2005) analysis.

of Theorem 2 for competition policy is that quick deregulation of monopolies may do more harm than good, as it makes it harder for potential innovators to compete with incumbent producers. This result is reminiscent of Tang and Wälde's (2001) finding that a two-country world economy may find itself in a no-growth trap if there are sufficiently many competitive markets due to a large initial overlap of products invented before trade is opened up between the countries. To relate our results to Tang and Wälde's (2001), our third theorem is concerned with the open-economy version of our model. Adapting the analysis in Arnold (2007) appropriately, we prove that, under certain conditions, the world economy made up of several (identical) countries replicates the equilibrium of the hypothetical integrated economy (that would prevail if national borders did not exist). Together with the second theorem, it follows immediately that if the model parameters are such that the no-growth trap occurs in the integrated economy, then the no-growth trap is also an equilibrium of the world economy if there are sufficiently many competitive markets due to a large initial overlap of products. From a policy point of view, it follows that, like quick deregulation in a closed economy, quick trade liberalization can lead to stagnation in the long term.

A cautionary note is in order: our focus is expressly not on competition *within* markets. Since we adopt the standard increasing variety growth model with a Dixit-Stiglitz (1977) intermediate goods sector and assume that markets are either monopolies or perfectly competitive, our approach is rudimentary in this respect. Rather, our theorems relate to the competition for the aggregate demand for goods by the different sectors, innovative and traditional, of the economy.[4]

Section 2 introduces the model. The growth equilibrium is derived in Section 3. Section 4 proves the three theorems on growth and competition. Section 5 concludes. Details of the derivations can be found in Bauer (2006).

2 Model

There is a continuum of mass one of identical households. Each household inelastically supplies L units of labor, the only primary factor of production. Their intertemporal utility is $U = \int_0^\infty e^{-\rho t}[\sigma \ln X + (1-\sigma)\ln Y]dt$ ($\rho > 0$, $0 < \sigma < 1$), where X and Y are the quantities consumed of two homogeneous goods, x and y.[5] Good x is produced using a set of intermediates, j, according to the production function

[4]To address competition within markets, quality upgrading models in the spirit of Aghion and Howitt (1992) are apparently better suited than product variety models. For example, Aghion at al. (2001) construct a model with innovation by both technological leaders and laggards that explains the inverted-U relationship between product market competition and innovation: some product market competition is conducive to growth as it induces firms to innovate so as to escape competition. Some imitation likewise intensifies escape competition and spurs growth.

[5]The time argument is suppressed here and in what follows whenever this does not cause confusion.

$X = [\int_0^n x(j)^\alpha dj]^{1/\alpha}$, where $x(j)$ is the input of intermediate j, n is the "number" of producible intermediates, and $0 < \alpha < 1$. Each producible intermediate, j, is obtained one-to-one from labor. The "traditional" good y is obtained one-to-one from labor: $Y = L_Y$, where L_Y is labor employed in the production of y (one may think of services with less scope for innovation than in manufacturing). Blueprints for new intermediates are invented in R&D according to $\dot{n} = nL_R/a$ $(a > 0)$, where L_R is employment in R&D (there are scale effects). As for market structure, we assume that all markets are perfectly competitive except for the markets for "new" intermediates. Immediately after the development of a new variety, the innovator is a monopolist (due to either patent protection or the fact that other agents are not yet able technologically to imitate the intermediate). Subsequently, in any short time interval dt the innovator loses his monopoly, and the market becomes perfectly competitive, with probability $\psi\, dt$, where ψ $(\psi \geq 0)$ is the rate of imitation (due to loss of patent protection or of technological leadership). Therefore, letting n_m and n_c denote the "numbers" of monopolistic and competitive markets for intermediate goods, respectively, we have

$$\dot{n_c} = \psi n_m, \quad n = n_c + n_m. \tag{1}$$

3 Equilibrium

Using aggregate expenditure as the numéraire, utility maximization yields

$$p_X X = \sigma,\ p_Y Y = 1 - \sigma,\ r = \rho, \tag{2}$$

where p_X and p_Y are the prices of goods x and y, respectively, and r is the interest rate. Cost minimization in the x-sector yields the input coefficient $a(j) = p(j)^{-\epsilon}[\int_0^n p(j')^{1-\epsilon}dj']^{-1/\alpha}$ for good j, where $p(j)$ is the price of intermediate j and $\epsilon \equiv 1/(1-\alpha)$. Consequently, the unit production cost and, because of perfect competition, the price of good x is $p_X = [\int_0^n p(j)^{1-\epsilon}dj]^{1/(1-\epsilon)}$. The x-sector's demand for intermediate j is $x(j) = a(j)X$. The price elasticity of demand is ϵ. Monopolists in the intermediate goods sector maximize profit, π, given these demand functions. Letting p_m and p_c denote the prices of monopolistic and competitive intermediate goods markets, respectively, and x_m the output of monopolistically supplied intermediates, we obtain the familiar pricing rules and ensuing profits:

$$p_m = \frac{w}{\alpha},\ p_c = w,\ \pi = (1-\alpha)p_m x_m. \tag{3}$$

Substituting the pricing rules into the expression for the input coefficients, $a(j)$, the demands $x(j) = a(j)X$ can be rewritten as

$$x_m = \alpha^\epsilon \left(n_c + \alpha^{\epsilon-1}n_m\right)^{-\frac{1}{\alpha}} X,\ x_c = \left(n_c + \alpha^{\epsilon-1}n_m\right)^{-\frac{1}{\alpha}} X, \tag{4}$$

where x_c denotes the output of competitively supplied intermediates. Moreover, substituting the pricing rules in (3) into the expression for p_X and using the fact that good y is obtained one-to-one from labor, we get the final goods' prices:

$$p_X = \left(n_c + \alpha^{\epsilon-1} n_m\right)^{-\frac{1}{\epsilon-1}} w, \;\; p_Y = w. \tag{5}$$

Using the fact that, as of time t, a monopolist's probability of still being a monopolist at τ ($\geq t$) is $e^{-\psi(\tau-t)}$, the value of a monopoly is

$$v(t) \equiv \int_t^\infty \exp\left\{-\int_t^\tau [r(s)+\psi]ds\right\} \pi(\tau)d\tau. \tag{6}$$

Imitation acts like additional discounting. Free entry into R&D requires

$$wa \geq nv, \;\text{ with equality if } \dot{n} > 0. \tag{7}$$

Finally, the labor market clearing condition reads:

$$L = a\frac{\dot{n}}{n} + n_c x_c + n_m x_m + Y. \tag{8}$$

Equations (1)-(8) comprise a system of 15 equations in 15 unknowns: n_c, n_m, n, p_X, X, p_Y, Y, r, p_m, w, p_c, π, x_m, x_c, and v.[6]

Let $\theta \equiv n_c/n$ and $g = \dot{n}/n$ denote the proportion of competitive intermediate goods markets and the growth rate of the "number" of intermediates, respectively, and $V \equiv 1/(nv)$. Using $Y = L_Y$, (2), (4), (5), (7), and these definitions, the labor market clearing condition (8) can be rewritten as

$$g = \max\left\{0, \; \frac{L}{a} - \sigma V\left[\frac{\theta(1-\alpha^\epsilon)+\alpha^\epsilon}{\theta(1-\alpha^{\epsilon-1})+\alpha^{\epsilon-1}} + \frac{1-\sigma}{\sigma}\right]\right\}. \tag{9}$$

Differentiating the definition of θ and using (1) yields

$$\frac{\dot{\theta}}{\theta} = \frac{1-\theta}{\theta}\psi - g. \tag{10}$$

From (2)-(5), we have

$$\pi = \frac{\sigma(1-\alpha)}{\left[1-\theta(1-\alpha^{1-\epsilon})\right]n}.$$

Differentiating the definition of V and (6) with respect to time, eliminating $\dot{v}$, and using (2) and the equation for monopoly profit above, we obtain

$$\frac{\dot{V}}{V} = \frac{(1-\alpha)\sigma}{1-\theta(1-\alpha^{1-\epsilon})}V - (\rho + \psi + g). \tag{11}$$

[6]The budget constraint $d(n_m v)/dt = rn_m v + wL - 1$ represents another equation in the same variables, but as usual in general equilibrium theory, one of the 16 equations can be obtained from the other 15, so that we have as many equations as unknowns.

Given (9), equations (10) and (11) comprise an autonomous system of ordinary differential equations in θ and V. The analysis of this system will bring forth our three theorems on growth and competition. From (9), $g > 0$ if, and only if,

$$V < \frac{L}{a} \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{1 - \theta[1 - \alpha^{1-\epsilon} - (1-\alpha)\sigma] - (1-\alpha)\sigma} \equiv \tilde{V}(\theta), \tag{12}$$

and $g = 0$ otherwise. $\tilde{V}(\theta)$ is continuous and strictly decreasing for $\theta \in [0, 1]$, with $\tilde{V}(0) = (L/a)/[1 - (1-\alpha)\sigma]$ and $\tilde{V}(1) = L/a$.

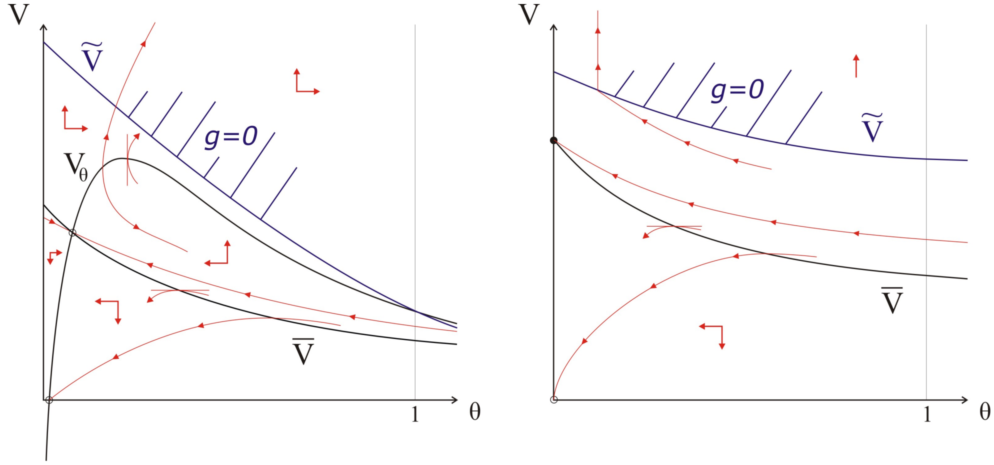

Figure 1: Dynamics in case 1

Consider first the $g = 0$-region. According to (11), V is constant for $V = 0$ and for

$$V = (\rho + \psi) \frac{1 - \theta(1 - \alpha^{1-\epsilon})}{(1-\alpha)\sigma} \equiv \bar{V}_0(\theta), \tag{13}$$

where $\bar{V}_0(0) = (\rho + \psi)/[(1-\alpha)\sigma]$, $\bar{V}_0(1) = (\rho + \psi)/[(1-\alpha)\alpha^{\epsilon - 1}\sigma]$, and $\bar{V}_0'(\theta) > 0$. $\dot{V}$ is positive for $V > \bar{V}_0(\theta)$ and negative for $V < \bar{V}_0(\theta)$. From (10), θ is constant if, and only if, $\theta = 1$. Otherwise, $\dot{\theta} > 0$. Here and in what follows, we distinguish three cases:

Case 1:

$$\frac{(1-\alpha)\alpha^{\epsilon-1}\sigma\frac{L}{a}}{\rho + \psi} > 1. \tag{14}$$

in this case, $\bar{V}_0(1) < \tilde{V}(1)$. As $\bar{V}_0'(\theta) > 0 > \tilde{V}'(\theta)$ for all $\theta \in [0, 1]$, it follows that $\bar{V}_0(\theta) < \tilde{V}(\theta)$ and, hence, $\dot{V}/V > 0$ for all $\theta \in [0, 1]$. That is, for $\psi > 0$, trajectories in the $g = 0$-region point to the north-east (see the left panel of Figure 1), while for $\psi = 0$, they are vertical (see the right panel of Figure 1).

Case 2:

$$\frac{(1-\alpha)\alpha^{\epsilon-1}\sigma\frac{L}{a}}{\rho + \psi} < 1 - (1-\alpha)\sigma. \tag{15}$$

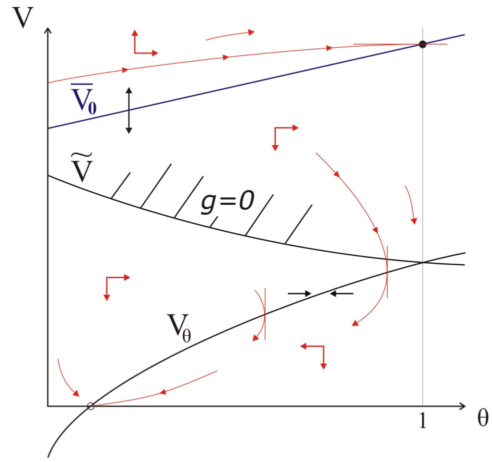

Figure 2: Dynamics in case 2

Here, $\bar{V}_0(0) > \tilde{V}(0)$, so that the curve $\bar{V}_0(\theta)$ is located above the curve $\tilde{V}(\theta)$. V rises above and falls below $\bar{V}_0(\theta)$. The point $(\theta, V) = (1, \bar{V}_0(1))$ is a steady state. As can be seen from Figure 2, for each $\theta \in [0,1]$, there exists a unique path converging to this steady state.

Case 3:

$$1 - (1-\alpha)\sigma < \frac{(1-\alpha)\alpha^{\epsilon-1}\sigma\frac{L}{a}}{\rho+\psi} < 1. \tag{16}$$

In this, intermediate, case the curves $\bar{V}_0(\theta)$ and $\tilde{V}(\theta)$ intersect for some $\theta \in (0,1)$. As in case 2, V rises above $\bar{V}_0(\theta)$ and falls below the curve (see Figure 3).

Next, consider the region with positive growth (i.e., $g > 0$). From (9) and (11), V is constant if $V = 0$ or if

$$V = \left(\rho + \psi + \frac{L}{a}\right)\frac{1-\theta(1-\alpha^{1-\epsilon})}{1-\theta[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma]} \equiv \bar{V}(\theta). \tag{17}$$

$\bar{V}(0) = \rho+\psi+L/a$, $\bar{V}(1) = (\rho+\psi+L/a)/[1+(1-\alpha)\alpha^{\epsilon-1}\sigma]$, and $\bar{V}'(\theta) < 0$ for all $\theta \in [0,1]$. V rises above the stationary locus and falls below it. From (10), $\dot{\theta} = 0$ for

$$V = \left(\frac{L}{a} - \frac{1-\theta}{\theta}\psi\right)\frac{1-\theta(1-\alpha^{1-\epsilon})}{1-\theta\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]-(1-\alpha)\sigma} \equiv V_\theta(\theta). \tag{18}$$

Using (12), we can rewrite $V_\theta(\theta)$ as

$$V_\theta(\theta) = \tilde{V}(\theta) - \frac{1-\theta}{\theta}\psi\frac{1-\theta(1-\alpha^{1-\epsilon})}{1-\theta\left[1-\alpha^{1-\epsilon}-(1-\alpha)\sigma\right]-(1-\alpha)\sigma}. \tag{19}$$

Inspection of (18) and (19) shows $V_\theta(1) = L/a = \tilde{V}(1)$ and $V_\theta(\theta) = 0$ for $(0 <)\ \theta = \psi/(\psi + L/a)\ (< 1)$ and for $\theta = 1/(1-\alpha^{1-\epsilon})\ (< 0)$. Moreover, $V_\theta(\theta) \to -\infty$ as $\theta \to 0$ from above, V_θ is continuous for $\theta \in (0,1]$, $V_\theta'(1) = \psi - (L/a)\alpha^{\epsilon-1}(1-\alpha)\sigma L/a$, and $V_\theta(\theta) < \tilde{V}(\theta)$ for all $\theta \in (0,1]$.

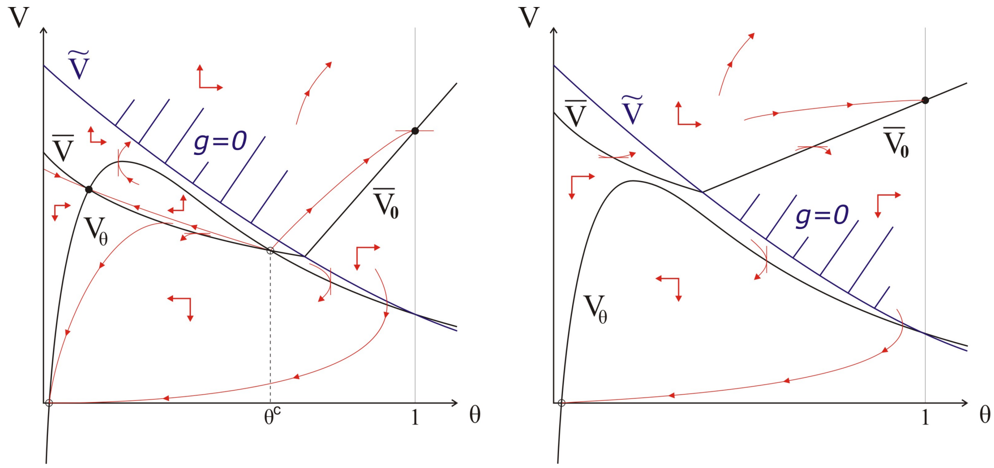

Figure 3: Dynamics in case 3

Case 1: Suppose $\psi > 0$. Then, the case distinction (14) implies $\bar{V}(\theta) < \tilde{V}(\theta)$ for all $\theta \in [0,1]$ (since $\bar{V}(0) < \tilde{V}(0)$ and $\bar{V}(\theta) = \tilde{V}(\theta)$ for some $\theta \in (0,1]$ contradicts the case distinction) and $V_\theta(\theta) < \tilde{V}(\theta)$ for all $\theta \in (0,1)$ and $V_\theta(1) = \tilde{V}(1)$ (i.e., $V_\theta(1) > \bar{V}(1)$). It follows that the stationary loci $\bar{V}(\theta)$ and $V_\theta(\theta)$ intersect an odd number of times on $(0,1)$. The fact that $V_\theta'(1) < 0$ implies that $V_\theta'(\theta)$ has an interior local maximum on $(0,1]$. From (17) and (18), $\bar{V}(\theta) = V_\theta(\theta)$ for $\theta = 1/(1-\alpha^{1-\epsilon})$ (< 0) and for those θ's which satisfy the equality $\bar{V}(\theta)/[1-\theta(1-\alpha^{1-\epsilon})] = V_\theta(\theta)/[1-\theta(1-\alpha^{1-\epsilon})]$. This is a quadratic equation, with an even number of real solutions. It follows that $\bar{V}(\theta)$ and $V_\theta(\theta)$ intersect exactly once in the interval $[0,1]$, which proves that a unique steady state exists in the $g>0$-region. As can be seen from the left panel of Figure 1, the steady state is a saddle point. For $\psi = 0$, we have $\dot{\theta}/\theta = -g < 0$. $(\theta, V) = (0, \bar{V}(0))$ is the unique steady state in the $g>0$-region (see the right panel of Figure 1). For each $\theta \in [0,1]$, there exists a unique trajectory converging to the steady state both for $\psi > 0$ and for $\psi = 0$. Divergent paths can be ruled out using the arguments put forward by Grossman and Helpman (1991, p. 61): paths starting above the saddle path yield $V \to \infty$ and $\theta \to \theta' > 0$, where $\theta' = 1$ if $\psi > 0$ (see Figure 1). However, once the economy is in the $g=0$-region, $\pi n = \sigma(1-\alpha)/[1-\theta(1-\alpha^{1-\epsilon})] \geq \sigma(1-\alpha)\alpha^{\epsilon-1}$ and, from (6), $vn \geq \sigma(1-\alpha)\alpha^{\epsilon-1}/(\rho+\psi)$. This contradicts $V \to \infty$. Paths starting below the saddle path converge to $(\theta, V) = (\psi/(\psi+L/a), 0)$ (see Figure 1). As $\pi \leq \sigma(1-\alpha)/n$, we have $nv \leq \sigma(1-\alpha)/(\rho+\psi)$, which contradicts $V \to 0$.

Case 2: In the $g=0$-region, $\dot{V}/V < 0$ below the curve $\bar{V}_0(\theta)$. A fortiori, from (11), $\dot{V}/V < 0$ in the $g>0$-region. As in case 1, the $\dot{\theta}=0$-locus converges to $-\infty$ as $\theta \to 0$ from above and satisfies $V_\theta(1) = \tilde{V}(\theta)$. As can be seen from Figure 2, all paths except that converging to $(\theta, V) = (1, \bar{V}_0(1))$ violate perfect foresight in the same way as the divergent paths in case 1.

Case 3: In the intermediate case, the curves $\bar{V}_0(\theta)$ and $\tilde{V}(\theta)$ intersect for some $\theta \in (0,1)$. From (11), the stationary locus for V is continuous on the border between positive and zero growth, $\tilde{V}(\theta)$. So, by the arguments put forward in case 2, the number of intersections of $\bar{V}(\theta)$ and $V_\theta(\theta)$ in the $g > 0$-region is two (see the left panel of Figure 3) or zero (see the right panel of Figure 3). In the former subcase (two intersections), let θ^c denote the abscissa value of the right intersection. Then, for each $\theta < \theta^c$, the unique trajectory consistent with perfect foresight converges to the left steady state, and for each $\theta > \theta^c$, the unique trajectory consistent with perfect foresight converges to $(\theta, V) = (1, \bar{V}_0(1))$. In the latter subcase (no intersection in the $g > 0$-region), the unique perfect-foresight equilibrium entails convergence towards $(\theta, V) = (1, \bar{V}_0(1))$.

4 Results

In this section, we use our findings about the model dynamics to prove the three main theorems about growth and competition and the corollaries addressing the issue of second-best competition policies mentioned in the introduction.

The first theorem states that "no growth may be better than some growth". To illustrate this, suppose patents effectively protect innovators forever: $\psi = 0$. Recall from Section 3 that if (14) holds (i.e., in case 1), $\psi = 0$, and $\theta(0) = 0$, then the economy is in a steady state with positive growth. The growth rate is obtained from (9), (11), $\psi = 0$, and $\theta = 0$:

$$g = (1-\alpha)\sigma\frac{L}{a} - [1-(1-\alpha)\sigma]\rho. \tag{20}$$

Let L_+ denote the value for L such that $g = 0$ for $L \leq L_+$ and $g > 0$ for $L > L_+$:

$$L_+ \equiv a\rho\left[\frac{1}{(1-\alpha)\sigma} - 1\right].$$

THEOREM 1 ("benefits of no growth"): *Suppose (14) holds (case 1). Then, there exists L_c ($> L_+$) such that for $L \in (L_+, L_c)$, intertemporal utility in an equilibrium with $\theta(t) = 1$ and $g = 0$ for all $t \geq 0$ is higher than in an equilibrium with $\theta(t) = 0$ and $g > 0$ for all $t \geq 0$.*

Proof: Intertemporal utility, U, in a steady state with θ and g constant satisfies

$$\rho U - \frac{1-\alpha}{\alpha}\sigma \ln n(0) = \frac{\sigma}{\alpha}\ln\left[(n_c x_c)^\alpha \theta^{1-\alpha} + (n_m x_m)^\alpha (1-\theta)^{1-\alpha}\right] + (1-\sigma)\ln Y + \frac{1-\alpha}{\alpha}\frac{\sigma}{\rho}g. \tag{21}$$

The important point to notice is that monopoly pricing in the intermediate goods sector causes the usual static welfare loss. To see this, notice that an allocation of labor across the intermediates and good y that maximizes static welfare requires symmetry across the intermediates (i.e., $x(j) = x$ for

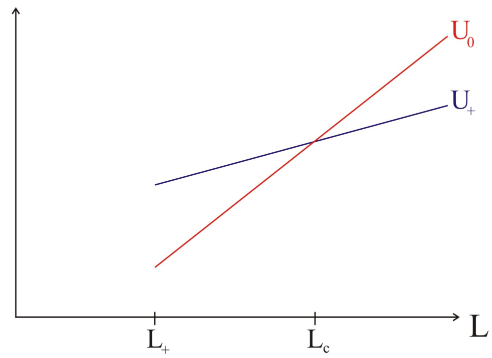

Figure 4: Comparing growth and no-growth equilibria

all $j \in [0, n]$) and $nx/Y = \sigma/(1-\sigma)$. Consider the allocation with $\theta(t) = 1$. As $n_c(t) = n(t)$, the symmetry condition is satisfied. And from zero profit in x- and y-production (i.e., $n_c x_c = \sigma/w$ and $Y = (1-\sigma)/w$, respectively), the allocation of labor is efficient. Next, consider the allocation with $\theta(t) = 0$ and $g > 0$. Let $L \to L_+$ from above, so that $g \to 0$. As $n_m x_m \to \sigma L\alpha/[1-(1-\alpha)\sigma]$ and $Y \to (1-\sigma)L/[1-(1-\alpha)\sigma]$, we have $nx/Y \to \alpha\sigma/(1-\sigma)$; markup pricing leads to too low a level of x-production. Let U_0 denote the intertemporal utility level in an equilibrium with $\theta(t) = 0$ and $g = 0$ and U_+ the utility level obtained in an equilibrium with $\theta(t) = 1$ and $g > 0$. It follows that $U_0 > U_+$ as $L \to L_+$ from above. As the right-hand side of (21) is concave in $n_c x_c$ and in $n_m x_m$ but linear in g (which is itself linear in L, see (20)), there is an $L_c > L_+$ such that $U_+ \geq U_0$ for $L \geq L_c$ (see Figure 4). Q.E.D.

A direct corollary of Theorem 1 is that no patent protection may be preferable to very strict patent protection. Suppose innovations can be protected from imitation perfectly (i.e., $\psi = 0$). By giving up patent protection, the policymaker can raise the imitation rate to $\psi = \bar{\psi}$ (> 0). Suppose superior technological knowledge of innovators is inessential, so that $\bar{\psi} \to \infty$.

COROLLARY 1 ("benefits of preventing growth"): *For L slightly greater than L_c, $\theta(0) = 0$, and $\bar{\psi} \to \infty$, intertemporal utility is higher with $\psi = \bar{\psi}$ than with $\psi = 0$.*

Proof: With $\psi = 0$, the economy settles down at a steady state with positive growth, $\theta = 0$, and, hence, with intertemporal utility U_+. For $\psi = \bar{\psi}$, zero growth prevails and, as $\bar{\psi} \to \infty$, θ quickly goes to unity, so that intertemporal utility is close to U_0. Since, by Theorem 1, $U_0 > U_+$ for L slightly greater than L_c, giving up patent patent protection raises welfare. Q.E.D.

Numerical results, admittedly, suggest that some growth is better than no growth. However, it is possible to construct counterexamples with parameter values that should not be deemed unrealistic a priori. For instance, let $\rho = 0.04$, $\alpha = 5/8$ (which gives rise to the standard 60% markup), $a = 1$, and $n(0) = 1$. We let $\sigma = 0.999$ or $\sigma = 0.5$ and choose L such that without imitation (i.e., if $\psi = 0$) $g = 0.5\%$, which implies 0.3% growth in the x-sector's output (i.e., $L = 0.0801$ or $L = 0.2$, respectively). For $\sigma = 0.999$, we get $U_0 = -63.3097 < -63.0515 = U_+$. For $\sigma = 0.5$, on the other hand, $U_0 = -57.5646 > -57.9441 = U_+$. So the economy with $\sigma = 0.5$ (but not the economy with σ close to unity) would be willing to give up long-term manufacturing output growth of 0.3% in exchange for static efficiency. That is, raising the imitation rate from zero to infinity is beneficial to this economy's representative consumer.

The second theorem states that the economy may get stuck in a "no-growth" trap (even though there exists a steady state with positive growth) due to too much competition initially.

THEOREM 2 ("no-growth trap"): *Suppose (16) holds (case 3) and $\bar{V}(\theta)$ and $V_\theta(\theta)$ intersect twice in the $g > 0$-region. Then, $g(t) > 0$ for all $t \geq 0$ if $\theta(0) < \theta_c$, while there is a $t_c > 0$ such that $g(t) = 0$ for all $t \geq t_c$ if $\theta(0) > \theta_c$.*

Proof: This simply rephrases the results of the analysis of case 3 with two intersections of $V_\theta(\theta)$ and $\bar{V}(\theta)$ in the previous section. t_c is the date at which the trajectory converging to $(\theta, V) = (1, \bar{V}_0(1))$ crosses the $\tilde{V}(\theta)$-curve (see the left panel of Figure 3). Q.E.D.

The intuition for Theorem 2 is: the lower its competitors' prices, the lower the share of aggregate demand that accrues to a potential innovator. If too many competitors supply at competitive prices, it does not pay to innovate, even though it would pay if the competitors' products were more expensive. As an example, let $\rho = 0.02$, $\psi = 0.01$, $\alpha = 0.6$, $a = 1$, $\sigma = 1$, and $L = 0.15$. This gives rise to case 3, and the stationary loci for V and θ intersect twice in the $g > 0$-region, at $\theta = 0.4352$ and $\theta = 0.7404 \equiv \theta_c$. The growth rate corresponding to the steady state with $\theta = 0.4352$ is $g = 4.94\%$, which implies 3.29% growth of the x-sector. So this economy fails to reach a steady state with 3.29% manufacturing output growth if the initial proportion of competitive markets exceeds 74.04%.

The implication of Theorem 2 for competition policy is that quick deregulation of monopolies may do more harm than good, as it makes it harder for a potential innovator to compete with incumbent producers. To illustrate this, consider an emerging economy with state monopolies in more than $1 - \theta_c$ intermediate goods markets (i.e., with $\theta(0) < \theta_c$ if no monopolies are deregulated). Suppose the government decides on how many markets to "deregulate", in which case they instantaneously become perfectly competitive. That is, $\theta(0)$ is considered as a policy parameter. From Theorem 2, we have:

COROLLARY 2 ("perils of quick liberalization"): *Suppose (16) holds (case 3) and $\bar{V}(\theta)$ and $V_\theta(\theta)$ intersect twice in the $g > 0$-region. If markets are deregulated such that $\theta(0) > \theta_c$, then there is a $t_c > 0$ such that $g(t) = 0$ for all $t \geq t_c$, while $g(t) > 0$ for all $t \geq 0$ without deregulation.*

In the example above, if in the absence of deregulation $\theta(0) = 74\%$, then deregulating a further 0.1% of the markets means giving up 3.29% long-term manufacturing growth.

Tang and Wälde (2001) show that a no-growth trap is possible in the two-country open economy version of our model with $\sigma = 1$ and $\psi = 0$. Theorem 2 is strongly reminiscent of this finding. We now turn to the m-country open economy version of our model. We show that under certain conditions the m-country economy behaves exactly identically to the hypothetical integrated economy that occurs in the absence of national borders (more precisely, the restrictions on labor movements they imply). The Tang-Wälde (2001) result is then obtained as a corollary to this replication theorem.[7] Consider a world economy made up of m (≥ 2) countries of the type introduced in Section 2 (i.e., with identical parameter values everywhere). Variables referring to individual countries, i, are distinguished by a superscript i ($= 1, \ldots, m$). We assume that knowledge spillovers in R&D are international in scope, so that the R&D technologies become $\dot{n}^i = nL_R^i/a$. An important issue is which products can be produced where. We start with the assumptions least conducive to the possibility of replication: non-imitated goods have to be produced where they were invented, and imitation is also "local", in that in short time intervals, dt, a fraction $(L^i/L)\psi\, dt$ of the n_m goods not yet imitated before becomes producible in country i: $\dot{n}_c^i = (L^i/L)\psi n_m$. We say that replication of the equilibrium of the integrated economy (ignoring national borders that inhibit movements of labor across borders) is feasible if this allocation is an equilibrium of the world economy (with national borders) as well.

THEOREM 3 ("replication"): *If*

$$L^i - n_c^i(t)x_c(t) - n_m^i(t)x_m(t) \geq 0, \quad \textit{for all } i = 1, \ldots, m, \ t \geq 0, \tag{22}$$

then replication is feasible.

Proof: Ignoring national borders, the equilibrium obeys equations (1)-(8), where $L \equiv \sum_{i=1}^m L^i$ is the world supply of labor. We have to show that this set of equations is also satisfied in the world economy with national borders. Equation (1) follows from adding up $\dot{n}_c^i = (L^i/L)\psi n_m$ for all $i = 1, \ldots, m$. The conditions for utility maximization in (2) are unaffected by the presence of national borders. Since the cost minimization problem is also unchanged, so are the input coefficients, $a(j)$, the zero profit condition $p_X = [\int_0^n p(j)^{1-\epsilon} dj]^{1/(1-\epsilon)}$, the demands for intermediates $x(j) = a(j)X$, and, therefore, the pricing rules and ensuing profit in (3) and the expressions for x_m and x_c in (4) (where X is the world production of good x, x_m is the output of any monopolistically supplied intermediate, and x_c is the output level of any competitively supplied intermediate). Evidently, the equations for pricing of the

[7] This follows Arnold (2007). As we allow for more than two countries, $\sigma > 0$, and $\psi > 0$, the analysis generalizes both Tang and Wälde (2001) and Arnold (2007).

final goods, the value of an innovation, and free entry into R&D, (i.e., (5)-(7), respectively) hold true in equilibrium. Finally, labor market clearing in country i requires

$$L^i = a\frac{\dot{n}^i}{n} + n_c^i x_c + n_m^i x_m + Y^i. \tag{23}$$

Assumption (22) ensures that for each country, i, given n_c^i and n_m^i, there exist $\dot{n}^i \geq 0$ and $Y^i \geq 0$ such that (23) is satisfied. Adding up (23) for all $i = 1, \ldots, m$ yields (8). Q.E.D.

Theorems 2 and 3 can be jointly used to prove a generalized version of the Tang-Wälde (2001) theorem on the existence of a no-growth trap due to the opening up of international trade between several countries. To do so, assume that at time $t = 0$, m countries with free international flows of knowledge between them engage in trade with each other. Suppose that, while still in autarky (i.e., before time $t = 0$), the producers do not take into account the possibility of future trade liberalization, so that they do not have an incentive to avoid the invention of identical intermediates in different countries ("duplication"). Let n and n_d denote the "numbers" of different and duplicated intermediates, respectively. From Theorems 2 and 3, we immediately obtain:

COROLLARY 3 (Tang and Wälde, 2001): *Suppose (16) holds (case 3), $\bar{V}(\theta)$ and $V_\theta(\theta)$ intersect twice in the $g > 0$-region, and (22) is satisfied. Then the no-growth trap described in Theorem 2 is an equilibrium of the world economy if $n_d/n > \theta_c$.*

From a policy point of view, this corollary implies that, like quick deregulation in a closed economy, quick trade liberalization can lead to stagnation in the long term.

Under the maintained assumptions, replication may fail due to the fact that a country, i, starts out with a disproportionately large number of blueprints. If, for instance, $n_m^i x_m > L^i$, then replication is not feasible. On the other hand, as $L^i - n_c^i(t)x_c(t) - n_m^i(t)x_m(t) = L_R^i + Y_i$, (22) is satisfied with strict equality in a steady state with L_R^i and Y_i positive, it follows that if the world economy is close to its steady state initially, then (22) will hold. Moreover, the problem vanishes altogether under assumptions more conducive to the possibility of replication. To see this, assume that intermediates invented in one country can be manufactured in a different country, i, either within multinational firms or via international patent licensing. Assume further that once imitation is possible in one country, i, it is possible in each country, $i = 1, \ldots, m$ ("simultaneous imitation"). Then, all production activities are "footloose". Equation (23) is satisfied, for instance, for $\dot{n}^i/\dot{n} = n_c^i/n_c = n_m^i/n_m = Y^i/Y = L^i/L$ (and for many other allocations of productive activities across countries as well). This proves:

COROLLARY 4 ("replication with footloose production"): *With simultaneous imitation and either multinational firms or international patent licensing, replication is feasible.*

5 Conclusion

This paper is concerned with the question of how competition with "cheap", old or traditional, goods affects the incentives to enter markets with new, innovative products. It shows that no growth may be better than some growth and that both a closed economy and a world economy made up of several countries engaged in free trade with each other may get stuck in a no-growth trap. As a result, growth-enhancing policies may be misguided, and quick deregulation as well as quick trade liberalization possibly lead to avoidable stagnation in the long term.

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