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Z_b tetraquark channel from lattice QCD and Born-Oppenheimer approximation

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ABSTRACT

Two Z_b hadrons with exotic quark structure $\bar{b}b\bar{d}u$ were discovered by Belle experiment. We present a lattice QCD study of the $\bar{b}b\bar{d}u$ system in the approximation of static b quarks, where the total spin of heavy quarks is fixed to one. The energies of eigenstates are determined as a function of the separation r between b and $\bar{b}$. The lower eigenstates are related to a bottomonium and a pion. The eigenstate dominated by $B\bar{B}^*$ has energy significantly below $m_B + m_{B^*}$, which points to a sizable attraction for small r. The attractive potential $V(r)$ between B and $\bar{B}^*$ is extracted assuming that this eigenstate is related exclusively to $B\bar{B}^*$. The Schrödinger equation for $B\bar{B}^*$ within the extracted potential leads to a virtual bound state, whose mass depends on the parametrization of the lattice potential. For certain parametrizations, we find a virtual bound state slightly below $B\bar{B}^*$ threshold and a narrow peak in the $B\bar{B}^*$ rate above threshold - these features could be related to $Z_b(10610)$ in the experiment. We surprisingly find also a deep bound state within the undertaken approximations.

1. Introduction

The Belle experiment discovered two Z_b^+ states with exotic quark content $\bar{b}b\bar{d}u$, $J^P = 1^+$ and $I = 1$ in 2011 [1,2]. The lighter $Z_b(10610)$ lies slightly above $B\bar{B}^*$ threshold and the heavier $Z_b(10650)$ just above $B^*\bar{B}^*$. The observed decay modes are $\Upsilon(1S, 2S, 3S)\pi^+$, $h_b(1P, 2P)\pi^+$, $B\bar{B}^*$ and $B^*\bar{B}^*$ [1–3], where the $B\bar{B}^*$ and $B^*\bar{B}^*$ largely dominate $Z_b(10610)$ and $Z_b(10650)$ decays, respectively. Many phenomenological theoretical studies of these two states have been performed, for example [4–17], and the majority indicates that $B^{(*)}\bar{B}^*$ Fock component is important.

We explore this channel within the first-principle lattice QCD. The only preliminary lattice study of this channel has been reported in [18,19] and is reviewed below. No other lattice results are available since this channel presents a severe challenge. Scattering matrix would have to be determined using the Lüscher method for at least 7 coupled two-meson channels listed in the previous paragraph. Poles of the scattering matrix would render possible Z_b states. Following this path seems too challenging at present. Furthermore, the original Lüscher approach for two-particle scattering is not valid above the three-particle threshold.

In the present study we consider the Born-Oppenheimer approximation [20], inspired by the study of this system in [18,19]. It is applied in molecular physics since ions are much heavier than other degrees of freedom. It is valuable also for the Z_b system $\bar{b}b\bar{d}u$, where b and $\bar{b}$ represent heavy degrees of freedom (h), while the light quarks and gluons are light degrees of freedom (l), see for example [21,22]. The simplification comes from the fact that the heavy degrees of freedom have large mass and therefore small velocity and kinetic energy.

In the first step we treat b and $\bar{b}$ as static at fixed distance r (Fig. 1a) and the main purpose is to determine eigen-energies $E_n(r)$ of this system. This energy represents the total energy without the kinetic and rest energies of the b and $\bar{b}$, so $E_n(r)$ is related to the potential $V(r)$ felt by the heavy degrees of freedom. In the second step, we study the motion of the heavy degrees of freedom (with the physical masses) under the influence of the extracted potential $V(r)$. Solutions of the Schrödinger equation render information on possible (virtual) bound states Z_b, resonances and cross-sections.

The low-lying eigenstates of the system in Fig. 1a with quantum numbers (2) are related to two-hadron states in Figs. 1 (b-d)

$$B(0)\bar{B}^*(r),\ \Upsilon(r)\pi(\vec{p}=0),\ \Upsilon(r)\pi(\vec{p}\neq 0),\ \Upsilon(r)b_1(\vec{0}). \qquad (1)$$

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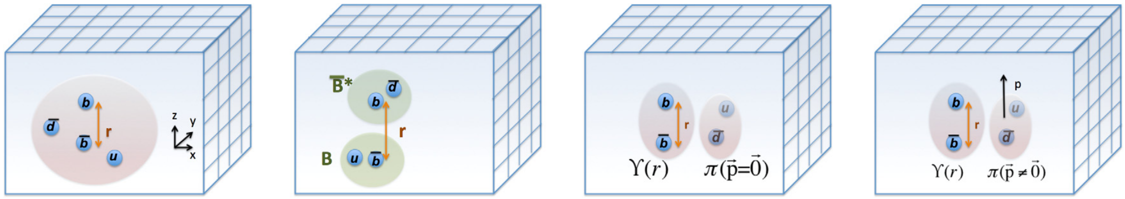

(a) (b) (c) (d)

Fig. 1. (a) The system considered. (b-d) Two-hadron Fock components in the system with quantum numbers (2).

The eigen-energy $E_n(r)$ related to $B\bar{B}^*$ in Fig. 1b is of major interest since Z_b lies near $B\bar{B}^*$ threshold [1]. The $\Upsilon(r)\pi(\vec{p})$ represent the ground state at small r. Here $\Upsilon(r)$ denotes the spin-one bottomonium where $\bar{b}$ and b are separated by r. Pion can have zero or non-zero momentum $\vec{p}=\vec{n}\frac{2\pi}{L}$ since the total momenta of light degrees of freedom is not conserved in the presence of static quarks, i.e. pion momentum can change when it scatters on an infinitely heavy Υ. Our task is to extract energies of all these eigenstates $E_n(r)$ as a function of r. The only previous lattice study of this system [18] presents preliminary results based on Fock components $B\bar{B}^*$ and $\Upsilon\pi(0)$; the presence $\Upsilon\pi(\vec{p}\neq 0)$ was mentioned in [19], but not included in the simulation.

2. Quantum numbers and operators

We consider Z_b^0 that has quantum numbers $I=1$, $I_3=0$, $J^{PC}=1^{+-}$ and $J_z=0$ in experiment. The list of conserved quantum numbers is slightly different in the systems with two static particles. We study the system in Fig. 1a with quantum numbers

$$I=1,\ I_3=0,\ \epsilon=-1,\ C\cdot P=-1 \tag{2}$$
$$S^h=1,\ S^h_z=0,\ J^l_z=0,\ (h=\text{heavy},\ l=\text{light})$$

where the neutral system is considered where C-conjugation can be applied (Fig. 1 shows the charged partner). Only the z-component of angular momenta for the light degrees of freedom (J_z^{light}) is conserved. The quantum number ϵ corresponds to the reflection over the yz plane. P refers to inversion with respect to mid-point between b and $\bar{b}$ and C is charge conjugation, where only their product is conserved. The quantum numbers in (2) are conventionally denoted by Σ_u^- using the conventions from [23].[1]

The spin of the infinitely heavy quark can not flip via the interaction with gluons, so spin S^h of $\bar{b}b$ is conserved. We choose to study the system with $S^h=1$, which can decay to Υ, while it can not decay to η_b and h_b since these carry $S^h=0$. Note that the physical Z_b and $B\bar{B}^*$ with finite m_b can be a linear combination of $S^h=1$ as well as $S^h=0$, and we study only $S^h=1$ component here. We have in mind this component, which includes $B\bar{B}^*$, $\bar{B}B^*$, $\bar{B}^*B^*$ (O_1 in Eq. (3)), when we refer to "$B\bar{B}^*$" throughout this paper.

The eigen-energies E_n of the system in Fig. 1a are determined from the correlation functions $\langle O_i(t)O_j^\dagger(0)\rangle$. We employ 6 operators O_i that create/annihilate the system with quantum numbers (2) and resemble Fock components (1) in Figs. 1 (b-d)

$$O_1=O^{B\bar{B}^*}\propto\sum_{a,b}\ \sum_{A,B,C,D}\Gamma_{BA}\tilde{\Gamma}_{CD}\ \bar{b}^a_C(0)q^a_A(0)\ \bar{q}^b_B(r)b^b_D(r)$$
$$\propto\big([\bar{b}(0)P_-\gamma_5 q(0)]\ [\bar{q}(r)\gamma_z P_+ b(r)]+\{\gamma_5\leftrightarrow\gamma_z\}\big)$$
$$+\big([\bar{b}(0)P_-\gamma_y q(0)]\ [\bar{q}(r)\gamma_x P_+ b(r)]-\{\gamma_y\leftrightarrow\gamma_x\}\big)$$

$$O_2=O^{B\bar{B}^*}$$
$$O_3=O^{\Upsilon\pi(0)}\propto[\bar{b}(0)U\gamma_z P_+ b(r)]\ [\bar{q}\gamma_5 q]_{\vec{p}=\vec{0}}$$
$$O_4=O^{\Upsilon\pi(1)}\propto[\bar{b}(0)U\gamma_z P_+ b(r)]\ \big([\bar{q}\gamma_5 q]_{\vec{p}=\vec{e}_z}+[\bar{q}\gamma_5 q]_{\vec{p}=-\vec{e}_z}\big)$$
$$O_5=O^{\Upsilon\pi(2)}\propto[\bar{b}(0)U\gamma_z P_+ b(r)]\ \big([\bar{q}\gamma_5 q]_{\vec{p}=2\vec{e}_z}+[\bar{q}\gamma_5 q]_{\vec{p}=-2\vec{e}_z}\big)$$
$$O_6=O^{\Upsilon b_1(0)}\propto[\bar{b}(0)U\gamma_z P_+ b(r)]\ [\bar{q}\gamma_x\gamma_y q]_{\vec{p}=\vec{0}}\,. \tag{3}$$

Here $\Gamma=P_-\gamma_5$, $\tilde{\Gamma}=\gamma_z P_+$, $[\bar{q}\Gamma' q]_{\vec{p}}\equiv\frac{1}{V}\sum_{\vec{x}}\bar{q}(\vec{x})\Gamma' q(\vec{x})e^{i\vec{p}\vec{x}}$, momenta is given in units of $2\pi/L$, capital (small) letters represent Dirac (color) indices, color singlets are denoted by [..] and U is a product of gauge links between 0 and r. First line in O_1 decouples spin indices of light and heavy quarks in order to make J_z^l and $S^h_{(z)}$ (2) are more transparent [18], while the second line is obtained via the Fierz transformation. O_2 is obtained from O_1 by replacing all $q(x)$ with $\nabla^2 q(x)$. $O_{4,5}$ have pion momenta in z direction due to $J_z^l=0$ and have two terms to ensure $C\cdot P=-1$. The Υb_1 is not a decay mode for finite m_b where C and P are separately conserved, but it is has quantum numbers (2) for $m_b\to\infty$. The pair $\bar{q}q$ indicates combination $\bar{u}u-\bar{d}d$ with $I=1$ and $I_3=0$. All light quarks $q(x)$ are smeared around the position x using the full distillation [24] with the radius about 0.3 fm, while the heavy quarks are point-like. Transformation properties of the operators are discussed in Section S1 of Supplemental Material.

We verified there are no other two-hadron states in addition to (1) with quantum numbers (2) and with non-interacting energies (4) below $m_B+m_{B^*}+0.2$ GeV.

3. Lattice details

Simulation is performed on an ensemble with dynamical Wilson-clover u/d quarks, $m_\pi\simeq 266(5)$ MeV, $a\simeq 0.1239(13)$ fm and 280 configurations [25,26]. We choose an ensemble with small $N_L=16$ and $L\simeq 2$ fm so that $\Upsilon\pi(p_z)$ with $p_z>2\frac{2\pi}{L}$ appear at $E>m_B+m_{B^*}+0.2$ GeV above our interest; larger L would require further operators like $O_{4,5}$ with higher $\vec{p}$. Small L restricts us to $r/a\leq\frac{1}{2}N_L=8$, but the statistical errors grow with r and the current precision prevents us from accurate results for $r/a>8$ anyway. Small L leads also to the usual exponentially-suppressed corrections related to the pion and a very mild effect on the light-quark cloud in a B-meson since $r_B\ll L$. The lattice temporal extent $N_T=32$ is effectively doubled by summing the light-quark propagators with periodic and anti-periodic boundary conditions in time [26].

4. Calculation of eigen-energies and overlaps

Correlation matrices $C_{ij}(t)=\langle O_i(t)O_j^\dagger(0)\rangle$ are evaluated using the full distillation method [24]. The $\bar{b}b$ annihilation Wick contraction is not present in the static limit considered here. C_{ij} are averaged over 8^3 or 16^3 space positions of $\bar{b}$, while sub-matrix for O_{3-6} is averaged over all source time slices to increase accuracy. Eigen-energies E_n and overlaps $\langle O_i|n\rangle$ are extracted from the 6×6 matrices $C_{ij}(t)=\sum_n\langle O_i|n\rangle e^{-E_n t}\langle n|O_j^\dagger\rangle$ using the widely used GEVP variational approach [27–29].

[1] This provides irreducible representation $(J_z^l)^\epsilon_{CP}=\Sigma_u^-$, where the notation here and in [23] is related by $J_z^l\to\Lambda$, $J_z^l=0\to\Sigma$, $CP\to\eta$, $CP=-1\to u$.

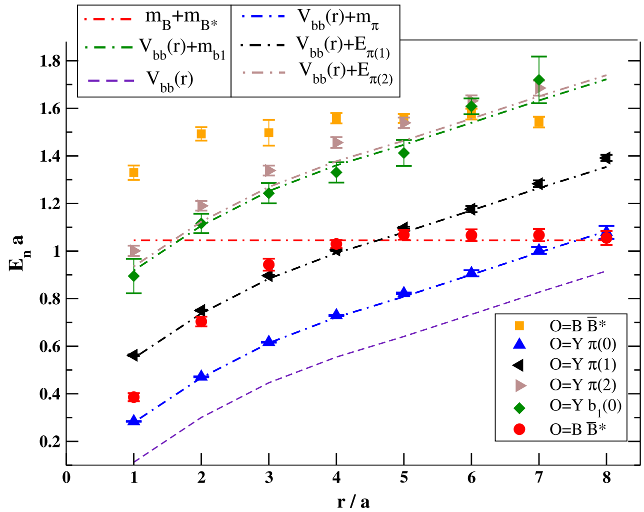

Fig. 2. Eigen-energies of $\bar{b}\bar{b}du$ system (Fig. 1a) for various separations r between static quarks b and $\bar{b}$ are shown by points. The label indicates which two-hadron component dominates each eigenstate. The dot-dashed lines represent related two-hadron energies $E^{n.i.}$ (4) when two hadrons (1) do not interact. The eigenstate dominated by $B\bar{B}^*$ (red circles) has energy significantly below $m_B+m_{B^*}$ and shows sizable attraction. Lattice spacing is $a\simeq 0.124$ fm.

5. Eigen-energies of $\bar{b}b\bar{d}u$ system as a function of r

The main result of our study are the eigen-energies of the $\bar{b}b\bar{d}u$ system (Fig. 1a) with static b and $\bar{b}$ separated by r. They are shown by points in Fig. 2. The colors of points indicate which Fock-component (1) dominates an eigenstate, as determined from the normalized overlaps of an eigenstate $|n\rangle$ to operators O_i. Normalized overlap $\tilde{Z}_i^n \equiv \langle O_i|n\rangle / \max_m \langle O_i|m\rangle$ is normalized so that its maximal value for given O_i across all eigenstates is equal to one. Effective masses and overlaps are shown in Section S2 of Supplemental Material.

The dashed lines in Fig. 2 provide the related non-interacting (n.i.) energies E_n of two-hadron states (1)

$$E^{n.i.}_{B\bar{B}^*} = 2m_B, \;\; E^{n.i.}_{\Upsilon\pi(\vec{p})} = V_{\bar{b}b}(r) + E_{\pi(\vec{p})}, \;\; E^{n.i.}_{\Upsilon b_1(0)} = V_{\bar{b}b}(r) + m_{b_1}, \tag{4}$$

where $\bar{b}b$ static potential $V_{\bar{b}b}(r)$, $E_{\pi(\vec{p})} \simeq \sqrt{m_\pi^2+\vec{p}^2}$, m_{b_1} and $m_B = m_{B^*} = 0.5224(14)$ (mass of $B^{(*)}$ for $m_b\to\infty$ without b rest mass) are determined on the same lattice.

The eigenstate dominated by $B\bar{B}^*$ has an energy close to $m_B+m_{B^*}$ for $r>0.5$ fm, but it has significantly lower energy for $r\simeq[0.1,0.4]$ fm (red circles in Fig. 2). This indicates sizable strong attraction between B and $\bar{B}^*$ in this system - something that might be related to the existence of Z_b tetraquarks. This is the most important and robust result of this lattice study.

Other eigenstates are dominated by $\Upsilon\pi(\vec{p})$ and Υb_1. Their energies E lie close to the non-interacting energies $E^{n.i.}$ (4) given by dot-dashed lines, so $E\simeq E^{n.i.}$. We point out that we can not claim nonzero energy shifts $E-E^{n.i.}$ for $\Upsilon\pi$ and Υb_1 states (although Fig. 2 shows small deviations from zero in some cases) since the statistical and systematic errors are not small enough.

6. Towards masses of Z_b states within certain approximations

Eigen-energies of $\bar{b}b\bar{d}u$ system in Fig. 1a indicate that eigenstate dominated by the $B\bar{B}^*$ has significantly lower energy than $m_B+m_{B^*}$ at small separation r between static b and $\bar{b}$. This suggests a possible existence of exotic hadrons (related to Z_b) and related peaks in the cross-section near $B\bar{B}^*$ threshold. Such physical observables require the study of the motion for the heavy degrees of freedom based on the energies $E_n(r)$ according to the Born-Oppenheimer approach. The precise prediction of such observables is not possible at present since lattice eigen-energies are not known for $r<a$. In addition, the accurate study would require the coupled-channel treatment of all Fock components (1) through the coupled-channel Schrödinger equation, which is a challenging task left for the future (this was recently elaborated in [30] for conventional $\bar{b}b$ with $I=0$).

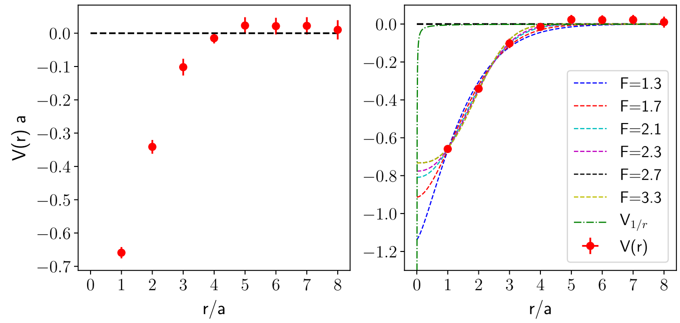

Fig. 3. (a) The extracted potential $V(r)$ between B and $\bar{B}^*$ from lattice. (b) Fits of $V(r)$ assuming the form of the regular potential V_{reg} (5) are presented by dashed lines for various values of parameter F. The singular potential $V_{1/r}(r)$ is shown by dot-dashed green line. Lattice spacing is $a\simeq 0.124$ fm.

We apply two simplifying approximations in order to shed light on the possible existence of Z_b based on energies in Fig. 2. The first assumption is that the eigenstate indicated by red circles in Fig. 2 is related exclusively to $B\bar{B}^*$ Fock component and does not contain other Fock components in (1). This is supported by our lattice results to a very good approximation, since this eigenstate couples almost exclusively to $O^{B\bar{B}^*}$ and has much smaller coupling to $O^{\Upsilon\pi}$ and $O^{\Upsilon b_1}$: the normalized overlap of this state to $O^{\Upsilon\pi,\Upsilon b_1}$ is $\tilde{Z}_{3-6}\le 0.05$ for $r\le 4$, while overlap to $O^{B\bar{B}^*}$ is $\tilde{Z}_{1,2}\simeq\mathcal{O}(1)$. In the reminder we explore the physics implications of this eigen-energy $E_{B\bar{B}^*}(r)$.

The energy $E_{B\bar{B}^*}(r)$ represents the total energy without the kinetic energy of heavy degrees of freedom. The difference $V(r) = E_{B\bar{B}^*}(r) - m_B - m_{B^*}$ therefore represents the potential felt by the heavy degrees of freedom, in this case between B and $\bar{B}^*$ mesons. The extracted potential is plotted in Fig. 3. The potential shows sizable attraction for $r=[0.1,0.4]$ fm and is compatible with zero for $r\ge 0.6$ fm within sizable errors. Lattice study that would probe whether one-pion exchange dominates at large r would need higher accuracy.

The problem is that the potential $V(r)$ is not determined from the lattice for $r<a$, it might be affected by discretization effects at $r\simeq a$ and the analytic form of r-dependence is not known apriori. This brings us to the second simplifying approximation

$$V(r) = V_{reg.}(r) + V_{1/r}(r), \;\; V_{reg.}(r) = -A\, e^{-(r/d)^F}, \tag{5}$$

where we assume a certain form of the regular potential $V_{reg.}(r)$ that has no singularity at $r\to 0$. The fits of the lattice potential for various choices of the parameter F (5) and range $r=[1,4]$ are shown in Fig. 3 (fits $r=[2,4]$ lead to similar conclusions, as shown in Section S5 of Supplemental Material). The question if the potential contains also a singular piece $1/r$ can be addressed perturbatively, giving $V^{\mathcal{O}(\alpha_s)}_{1/r}(r)=0$ and $V_{1/r}(r)=\frac{1}{9}[V_0(r)+8V_8(r)] = \frac{\delta a_2}{108\pi^2}\frac{\alpha_s^3}{r}$ [31] for very small r. This follows from the interaction of $\bar{b}$ and b within $B\bar{B}^*$, while other pairs among $\bar{b}b\bar{q}q$ are at average distance of the order of B-meson size and do not lead to singularity at $r\to 0$ (Section S4 of Supplemental Material). Results below are based on $V_{reg}+V_{1/r}$; we have verified that masses and cross-sections based solely on V_{reg} agree within the errors since $V_{1/r}$ is suppressed.

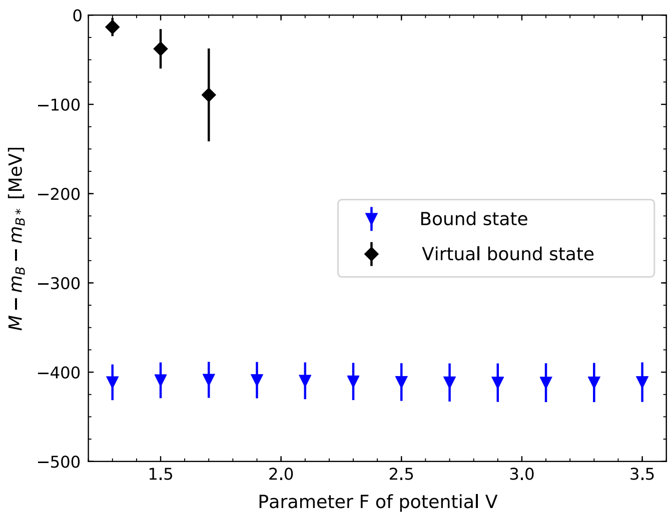

Fig. 4. Mass of the virtual bound state and the bound state for various choices of the parameter F in $V(r)$ (5).

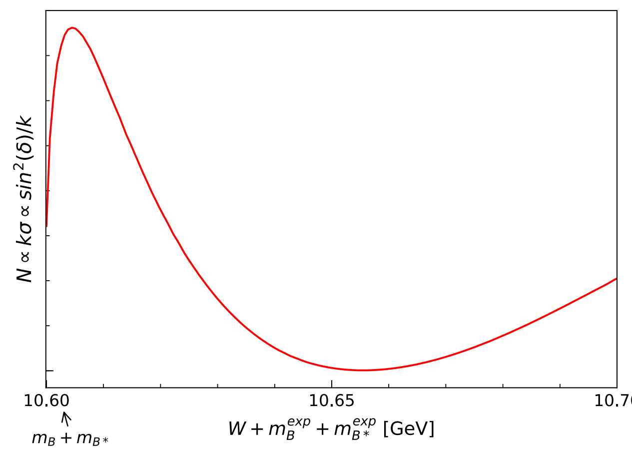

Fig. 5. The $B\bar{B}^*$ rate $N_{B\bar{B}^*} \propto k\sigma_{B\bar{B}^*}$ has a peak above threshold. The plotted rate is based on our lattice results and the choice of parameter $F = 1.3$ in $V(r)$ (5).

The motion of B and $\bar{B}^*$ within the extracted potential $V(r)$ is analyzed by solving the non-relativistic 3D Schrödinger equation $[-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)}{2\mu r^2} + V(r)]u(r) = Wu(r)$ for the experimentally measured $B^{(*)}$ meson masses and $1/\mu = 1/m_B^{exp} + 1/m_{B^*}^{exp}$. Here $W = E^{tot} - m_B - m_{B^*}$ is the energy with respect to $B\bar{B}^*$ threshold. The B and $\bar{B}^*$ can couple to Z_b channel with $J^P = 1^+$ in partial waves $l = 0, 2$. Below we extract (virtual) bound states and scattering rates for $l = 0$, while $l = 2$ is not discussed since $V(r) + \frac{l(l+1)}{2\mu r^2} > 0$ is repulsive for all r.

The wave functions of the Schrödnger equation render the phase shift $\delta_{l=0}(W)$ and $B\bar{B}^*$ scattering matrix $S(W) = e^{2i\delta_0(W)}$. Resonances above threshold do not occur for purely attractive s-wave potentials since there is no barrier to keep the state metastable, while (virtual) bound states below threshold may be present. Bound state (virtual bound state) corresponds to the pole of $S(W)$ for real $W < 0$ and imaginary momenta $k = i|k|$ ($k = -i|k|$) of B in the center of momentum frame.

We find a virtual bound state below threshold and its location is shown by diamonds in Fig. 4; this pole is present when the parameter F in V (5) is $F < 1.9$. If this pole is close below threshold, it enhances the $B\bar{B}^*$ cross-section above threshold. For example, we find a virtual bound state with mass M slightly below threshold

$$M - m_B - m_{B^*} = -13 \pm 10\ \text{MeV}\,, \qquad (6)$$

for the values of parameters $F = 1.3$, $A = 1.139(50)$, $d = 1.615(71)$ in V (5). This state is responsible for a peak in the $B\bar{B}^*$ rate $N_{B\bar{B}^*} \propto k\sigma \propto \sin^2\delta_0(W)/k$ above threshold, shown in Fig. 5 for the central value of parameters. The shape of the peak resembles the $Z_b(10610)$ peak in the $B\bar{B}^*$ rate observed by Belle (Fig. 2 of [3]).

The significantly attractive $B\bar{B}^*$ potential (Fig. 3) and the resulting virtual bound state (diamonds in Fig. 4) could be related to the existence of Z_b in experiment. The reliable relation between both will be possible only when simplifications employed here will be overcome in the future simulations. We note that $Z_b(10610)$ was found as a virtual bound state slightly below threshold also by the re-analysis of the experimental data [4] when the coupling to bottomonium light-meson channels was turned off [4] (the position of the pole is only slightly shifted when this small coupling is taken into account).

Surprisingly, the strongly attractive potential $V(r)$ (5) leads also to a deep bound state at $M - m_B - m_{B^*} = -411 \pm 20$ MeV. Such a state was never reported by experiments. If our approach can be trusted for such deep bound states and if such a bound state exists, it could be searched for in $Z_b \to \Upsilon(1S)\pi^+$ decays. The invariant mass distribution observed by Belle is indeed not flat (Fig. 4a of [2]) and it would be valuable to explore if some structure becomes significant at better statistics.

The exotic Z_b resonances were observed only by Belle, so their confirmation by another experiment would be highly welcome. LHCb could try to search for it in inclusive final state $B\bar{B}^*$.

7. Comparison with a previous lattice study

Only one preliminary lattice study [18] of this channel was reported up to now, considering heavier $m_\pi = 480$ MeV and twisted-mass fermions. It employed operators $O^{B\bar{B}^*}$ and $O^{\Upsilon\pi(0)}$, while $O^{\Upsilon\pi(p\neq 0)}$ and $O^{\Upsilon b_1(0)}$ were omitted. Two eigenstates are interpreted as $\Upsilon\pi(0)$ and $B\bar{B}^*$. The resulting potential $V(r) = E_{B\bar{B}^*}(r) - m_B - m_{B^*}$ (red line in Fig. 1 of [18]) is attractive, but it is weaker than our potential for small $r/a \simeq 1, 2$. This difference could be a consequence of a very different m_π or a choice of the fitting range for $E_{B\bar{B}^*}$ at small r. We note that the plateaus for $E_{B\bar{B}^*}$ form relatively late in time and large t_{min} is essential for the reliable extraction of the potential at small r in our study.[2] The weaker potential and its parametrization via $V(r) = -\frac{\alpha}{r}e^{-(r/d)^2}$ resulted in one bound state at $M - m_B - m_{B^*} = -58 \pm 71$ MeV in [18], while our binding energies are shown in Fig. 4 for various parametrizations. Both lattice studies agree on the main conclusion, i.e. that the existence Z_b is related to the attraction of B and $\bar{B}^*$ at small r. Further lattice studies, including the comparison of the potentials at similar m_π, are highly awaited.

8. Conclusions

We presented a lattice QCD study of a channel with quark structure $\bar{b}b\bar{d}u$, where Belle observed two exotic Z_b hadrons. We find significantly attractive potential $V(r)$ between B and $\bar{B}^*$ at small r when the total spin of the heavy quarks is equal to one. Dynamics of $B\bar{B}^*$ system within the extracted $V(r)$ leads to a virtual bound state, whose mass depends on the parametrization of V. Certain parametrizations give a virtual bound state slightly below $B\bar{B}^*$ threshold and a narrow peak in $B\bar{B}^*$ rate just above threshold, resembling Z_b in experiment.

[2] Our effective energies are shown in Fig. S2a of the supplemental material and require fits with $t_{min}/a \simeq 11$ for $r = 1, 2$. Effective energies of the study [18] are not available in the literature.

For quantitative comparison to experiment, future lattice studies need to explore how the dynamics of $B\bar{B}^*$ is influenced by the coupling to $\Upsilon\pi$ channels, and by the component where the total spin of the heavy quarks is equal to zero. Derivation of the appropriate analytic form for $V(r)$ would be very valuable.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.physletb.2020.135467.

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