



Continuum extrapolation of Ward identities in $\mathcal{N} = 1$ supersymmetric SU(3) Yang–Mills theory

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Abstract In $\mathcal{N} = 1$ supersymmetric Yang–Mills theory, regularised on a space-time lattice, in addition to the breaking by the gluino mass term, supersymmetry is broken explicitly by the lattice regulator. In addition to the parameter tuning in the theory, the supersymmetric Ward identities can be used as a tool to investigate lattice artefacts as well as to check whether supersymmetry can be recovered in the chiral and continuum limits. In this paper we present the numerical results of an analysis of the supersymmetric Ward identities for our available gauge ensembles at different values of the inverse gauge coupling β and of the hopping parameter κ . The results clearly indicate that the lattice artefacts vanish in the continuum limit, confirming the restoration of supersymmetry.

1 Introduction

Supersymmetry (SUSY) is an elegant idea which relates fermions and bosons, whose spin differs by $1/2$, through supercharges [1]. SUSY provides dark matter candidates, arising from the lightest supersymmetric particles [2]. In addition to that, supersymmetric extensions of the Standard Model would resolve the hierarchy problem [3]. $\mathcal{N} = 1$ supersymmetric Yang–Mills (SYM) theory, which is being

considered in this article, provides an extension of the pure gluonic part of the Standard Model [4]. It describes the strong interactions between gluons and gluinos, the superpartners of the gluons. Gluinos are Majorana particles that transform under the adjoint representation of the gauge group. The on-shell Lagrangian of $\mathcal{N} = 1$ SYM theory, which consists of the gluon fields $A_\mu^a(x)$ and the gluino fields $\lambda^a(x)$, where $a = 1, \dots, N_c^2 - 1$, can be written in Minkowski space as

$$\mathcal{L}_{\text{SYM}} = -\frac{1}{4} F_{\mu\nu}^a F^{a,\mu\nu} + \frac{i}{2} \bar{\lambda}^a \gamma^\mu (\mathcal{D}_\mu \lambda)^a - \frac{m_{\tilde{g}}}{2} \bar{\lambda}^a \lambda^a, \quad (1)$$

where the first term, containing the field strength tensor $F_{\mu\nu}^a$, is the gauge part, and \mathcal{D}_μ in the second term is the covariant derivative in the adjoint representation of the gauge group $\text{SU}(N_c)$, N_c being the number of colors. The last part of the above Lagrangian is a gluino mass term which breaks SUSY softly for $m_{\tilde{g}} \neq 0$, which means that it does not affect the renormalisation properties of the theory and that the spectrum of the theory depends on the gluino mass in a continuous way. The physical spectrum of this theory is expected to consist of bound states of gluons and gluinos, arranged in mass degenerate supermultiplets if SUSY is not broken [5, 6].

In order to perform Monte-Carlo simulations of the theory, we discretise the Euclidean action and put it onto a four-dimensional hypercubic lattice. We use the Curci–Veneziano version [7] of the lattice action $S = S_g + S_f$, where the gauge part S_g is defined by the usual plaquette action

$$S_g = -\frac{\beta}{N_c} \sum_p \text{Re}[\text{tr}(U_p)], \quad (2)$$

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with the inverse gauge coupling given by $\beta = 2N_c/g^2$, and the fermionic part

$$S_f = \frac{1}{2} \sum_x \left\{ \bar{\lambda}_x^a \lambda_x^a - \kappa \sum_{\mu=1}^4 [\bar{\lambda}_{x+\hat{\mu}}^a V_{ab,x\mu} (1 + \gamma_\mu) \lambda_x^b + \bar{\lambda}_x^a V_{ab,x\mu}^T (1 - \gamma_\mu) \lambda_{x+\hat{\mu}}^b] \right\} \quad (3)$$

implements the gluinos as Wilson fermions. Here the adjoint link variables are defined by $V_{ab,x\mu} = 2 \text{tr} (U_{x\mu}^\dagger T_a U_{x\mu} T_b)$, where T_a are the generators of the gauge group, and the hopping parameter κ is related to the bare gluino mass $m_{\tilde{g}}$ by $\kappa = 1/(2m_{\tilde{g}} + 8)$. In order to approach the limit of vanishing gluino mass, the hopping parameter has to be tuned properly. In our numerical investigations the fermionic part is additionally $O(a)$ improved by adding the clover term $-(c_{sw}/4) \bar{\lambda}(x) \sigma_{\mu\nu} F^{\mu\nu} \lambda(x)$ [8].

In our previous investigations we have determined the low-lying mass spectrum of the theory with gauge group SU(2) and SU(3) non-perturbatively from first principles using Monte Carlo techniques [4,9–11], and obtained mass degenerate supermultiplets [12].

2 SUSY Ward identities

In classical physics, Noether’s theorem provides a relation between symmetries and conservation laws. In the case of quantum field theories, symmetries are translated to Ward identities, representing quantum versions of Noether’s theorem. In $\mathcal{N} = 1$ supersymmetric Yang–Mills theory a gluino mass term breaks SUSY softly. The soft breaking effects vanish in the chiral limit, a limit where theory is characterised by massless gluinos. In order to analyse this breaking of supersymmetry and to identify the chiral limit, we employ the Ward identities for supersymmetry. Moreover, on the lattice supersymmetry is broken explicitly due to the introduction of the discretisation of space-time lattice as a regulator of the theory. SUSY Ward identities can be used to check whether supersymmetry is restored in the continuum limit.

In the Euclidean continuum, on-shell supersymmetry transformations of the gauge and gluino fields are given by

$$\delta A_\mu^a = -2i \bar{\lambda}^a \gamma_\mu \varepsilon, \quad \delta \lambda^a = -\sigma_{\mu\nu} F_{\mu\nu}^a \varepsilon, \quad (4)$$

where the transformation parameter ε is an anticommuting Majorana spinor. From the variation of the action under a supersymmetry transformation with a space-time-dependent parameter $\varepsilon(x)$ one derives the SUSY Ward identities. For any suitable gauge invariant local operator $Q(y)$, they read

$$\langle \partial^\mu S_\mu(x) Q(y) \rangle = m_{\tilde{g}} \langle \chi(x) Q(y) \rangle - \left\langle \frac{\delta Q(y)}{\delta \bar{\varepsilon}(x)} \right\rangle, \quad (5)$$

where $S_\mu(x) = (S_\mu^\alpha(x))$ is the supercurrent of spin 3/2, and the term $m_{\tilde{g}} \langle \chi(x) Q(y) \rangle$ is due to the gluino mass in the action of the theory. In the continuum the supercurrent $S_\mu(x)$ and the operator $\chi(x)$ are given by

$$S_\mu(x) = -\frac{2i}{g} \text{tr}[F^{\nu\rho}(x) \sigma_{\nu\rho} \gamma_\mu \lambda(x)], \quad (6)$$

$$\chi(x) = +\frac{2i}{g} \text{tr}[F^{\mu\nu}(x) \sigma_{\mu\nu} \lambda(x)]. \quad (7)$$

The last term of Eq. (5) is a contact term, which contributes only if $x = y$, and it can be avoided if $Q(y)$ is not localised at x . Therefore the contact term is ignored in the following discussions.

The four-dimensional space-time lattice breaks SUSY explicitly. As a consequence, the lattice versions of the Ward identities differ from their continuum counter parts by an additional term $\langle X_S(x) Q(y) \rangle$. The explicit form of this term is known, but need not be displayed here. At tree level this term is proportional to the lattice spacing a and vanishes in the limit of zero lattice spacing. At higher orders in perturbation theory, nevertheless, the contribution of this term is finite in the continuum limit due to divergences proportional to $1/a$ that multiply the factor a . This plays a role for the renormalisation of the supercurrent and of the gluino mass [7,13]. In the renormalisation of SUSY Ward identities, operators of dimensions $\leq 11/2$ have to be taken into account. They lead to a modification of the gluino mass, and in addition a current T_μ , mixing with the supercurrent, appears, corresponding to an operator of dimension 9/2. Consequently, on the lattice the following Ward identities are obtained

$$Z_S \langle \nabla_\mu S_\mu(x) Q(y) \rangle + Z_T \langle \nabla_\mu T_\mu(x) Q(y) \rangle = m_S \langle \chi(x) Q(y) \rangle + O(a), \quad (8)$$

where Z_S and Z_T are renormalisation coefficients. The subtracted gluino mass is defined as $m_S = m_{\tilde{g}} - \bar{m}$, where \bar{m} is the mass subtraction coming from the operators of dimension 7/2. The mixing current is defined as

$$T_\mu(x) = \frac{2i}{g} \text{tr}[F_{\mu\nu}(x) \gamma_\nu \lambda(x)]. \quad (9)$$

Regarding the local insertion operator $Q(y)$, our choice is the spinor $Q(y) = \chi^{(\text{sp})}(y)$, with

$$\chi^{(\text{sp})}(y) = \sum_{i < j} \text{tr}[F_{ij}(y) \sigma_{ij} \lambda(y)], \quad (10)$$

where the indices $i, j \in \{1, 2, 3\}$. The reason behind this choice is that it gives the best signal [13].

3 Numerical analysis of SUSY Ward identities

We have analysed the SUSY Ward identities numerically, employing the configurations produced in our project on $\mathcal{N} = 1$ supersymmetric Yang–Mills theory with gauge group $SU(3)$. Numerically it is convenient to use integrated Ward identities where integration or sum is performed over all three spatial coordinates. The resulting identities will then hold for every time-slice distance t . In the analysis the data from all time-slice distances in an interval $t_{min} \leq t \leq t_{max}$ are included. The lower limit t_{min} is always taken to be larger or equal than 3 in order to avoid contamination from contact terms. The choice of t_{min} for the different ensembles of configurations is discussed below. Since the correlation functions are symmetric or antisymmetric in t , the upper limit t_{max} is chosen to be half of the time extent of the lattice. Each term in Eq. (8) is a 4×4 matrix in spin-space and can be expanded in the basis of 16 Dirac matrices, i.e. $\{\mathbf{1}, \gamma_5, \gamma_\mu, \gamma_\mu \gamma_5, i\sigma_{\mu\nu}\}$. It can be shown, with the help of discrete symmetries, that only the following two contributions are non-zero [13]:

$$\hat{x}_{b,t,1} + A\hat{x}_{b,t,2} = B\hat{x}_{b,t,3}, \quad \text{with } b = 1, 2, \quad (11)$$

where $A = Z_T Z_S^{-1}$, $B = am_S Z_S^{-1}$, and

$$\begin{aligned} \hat{x}_{1,t,1} &\equiv \sum_{\mathbf{x}} \langle \nabla_4 S_4(x) Q(0) \rangle, & \hat{x}_{2,t,1} &\equiv \sum_{\mathbf{x}} \langle \nabla_4 S_4(x) \gamma_4 Q(0) \rangle, \\ \hat{x}_{1,t,2} &\equiv \sum_{\mathbf{x}} \langle \nabla_4 T_4(x) Q(0) \rangle, & \hat{x}_{2,t,2} &\equiv \sum_{\mathbf{x}} \langle \nabla_4 T_4(x) \gamma_4 Q(0) \rangle, \\ \hat{x}_{1,t,3} &\equiv \sum_{\mathbf{x}} \langle \chi(x) Q(0) \rangle, & \hat{x}_{2,t,3} &\equiv \sum_{\mathbf{x}} \langle \chi(x) \gamma_4 Q(0) \rangle. \end{aligned} \quad (12)$$

In these equations the Dirac indices of $S_4(x)$, $T_4(x)$, $\chi(x)$ and of the insertion operator $Q(0)$ are not written, and sums over repeated (hidden) Dirac indices are implied. Also, $O(a)$ terms that vanish in the continuum limit are not written explicitly in these equations. Introducing a double index $i = (b, t)$, running over $2T$ values, where T is the time extent of the lattice, and denoting $A_1 = 1$, $A_2 = A$, $A_3 = -B$, Eq. (11) is written compactly

$$\sum_{\alpha=1}^3 A_\alpha \hat{x}_{i\alpha} = 0. \quad (13)$$

In these equations the $\hat{x}_{i\alpha} = \langle x_{i\alpha} \rangle$ are the expectation values of random variables $x_{i\alpha}$, which themselves are considered to be the results of a finite Markov chain. We compute the estimators $x_{i\alpha}$ for the correlation functions $\hat{x}_{i\alpha}$ numerically using high performance facilities. The Eq. (13), including all time-slice distances t from t_{min} to t_{max} , are solved simultaneously for A_α by means of minimal chi-squared methods. Two methods, namely the so-called local method and global method, have been used in the past by our collaboration [4, 13]. These methods, however, do not take properly into

account correlations between the different quantities appearing in Eq. (13). For this purpose we have developed a new method based on a generalised least squares fit, the so-called GLS method [14], based on the maximum likelihood. For fixed A_α ($\alpha = 1, 2, 3$) and given numerical data $x_{i\alpha}$, the probability distribution $P \sim \exp(-L)$ of the quantities $\hat{x}_{i\alpha}$, subject to the constraints (13), has its maximum at a point where $L = L_{min}$, with

$$L_{min} = \frac{1}{2} \sum_{i,\alpha,j,\beta} (A_\alpha x_{i\alpha})(D^{-1})_{ij}(A_\beta x_{j\beta}), \quad (14)$$

where

$$D_{ij} = \sum_{\alpha,\beta} A_\alpha A_\beta (\langle x_{i\alpha} x_{j\beta} \rangle - \langle x_{i\alpha} \rangle \langle x_{j\beta} \rangle). \quad (15)$$

Next, the desired coefficients A_α have to be found such that L_{min} as a function of A_2 and A_3 is minimised. This cannot be solved analytically, and we find A_α numerically such that the global minimum of $L_{min}(A_2, A_3)$ is reached; for details see Ref. [15]. In particular, owing to $A_3 = -am_S Z_S^{-1}$ this provides us with the subtracted gluino mass m_S up to the renormalisation factor. To estimate the statistical uncertainties we employ the standard Jackknife procedure.

3.1 Discretisation effects

All terms in the Ward identity (8), including the $O(a)$ term $\langle X_S(x) Q(y) \rangle$, are correlation functions of gauge invariant operators. In the corresponding Eq. (11) they are correlation functions of operators localised on time slices or pairs of adjacent time slices at distance t . As for any gauge invariant correlation function of this type, they decay exponentially in t , with a decay rate given by the mass gap of the theory. For very small t the contributions of higher masses will affect the impact of the $O(a)$ term on the Ward identities. Therefore we expect that the value of the obtained gluino mass will depend on the minimal time slice distance t_{min} . This effect should become negligible at sufficiently large t_{min} . On the other hand, if t_{min} is chosen too large, noise in the data will dominate. The behaviour that can be observed in Fig. 1 is compatible with these expectations.

An adequate choice of t_{min} is therefore important for the quality of the results. We cope with this in two ways.

In order to avoid perturbing effects at too small t_{min} and a poor signal-to-noise ratio at too large t_{min} , for each hopping parameter and inverse gauge coupling, the value of t_{min} is selected by finding an optimal starting point where a plateau in the subtracted gluino mass begins. The results are presented in Table 1.

In the second approach, we consider that our simulations of the theory are done at different values of the lattice spacing a , which leads to different $O(a)$ terms in the Ward identities. A fixed value of t_{min} in lattice units would mean a lower

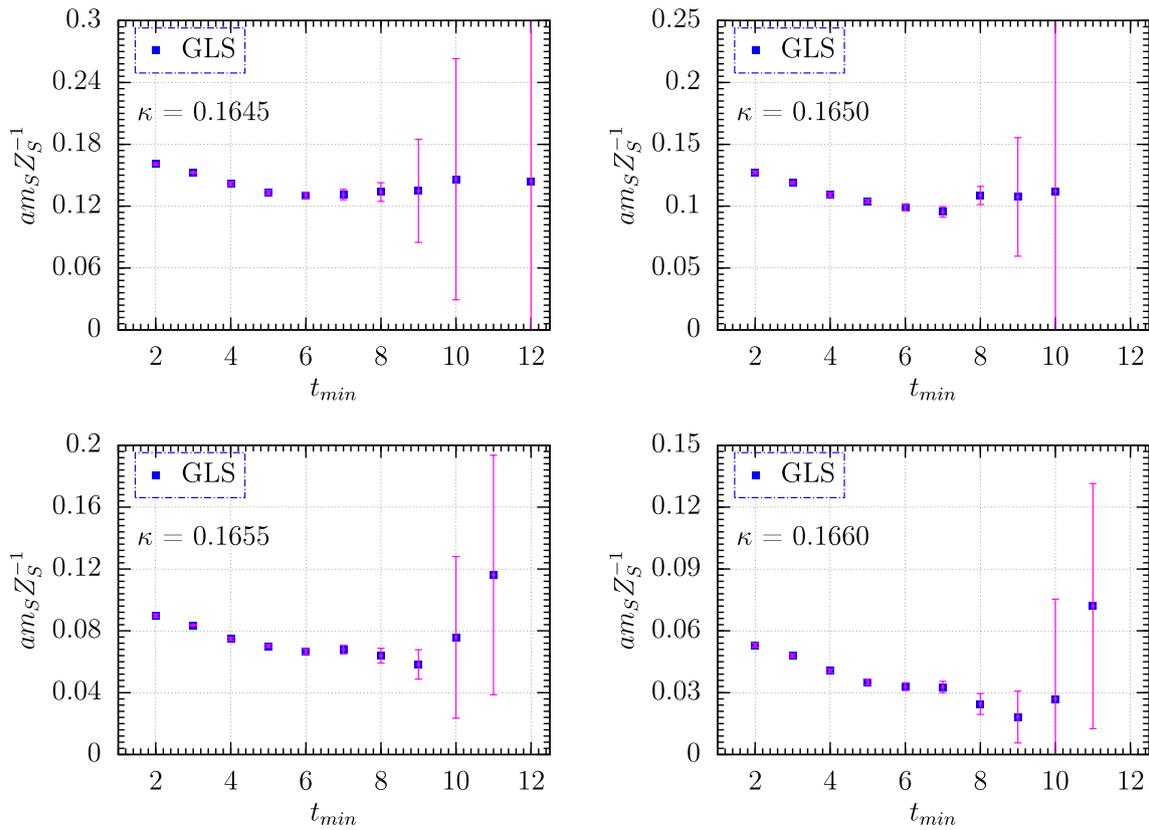


Fig. 1 The subtracted gluino mass $am_S Z_S^{-1}$ as a function of t_{min} calculated with the GLS Method at $\beta = 5.6$. At small values of t_{min} the subtracted gluino mass is affected by contact terms and by $O(a)$ terms. Data from $t_{min} = 2$ and $t_{min} = 3$ are shown, but do not enter our final analysis

Table 1 The values of t_{min} for all available gauge ensembles, chosen such that a plateau is formed

$\beta = 5.4$		$\beta = 5.4$		$\beta = 5.45$		$\beta = 5.5$		$\beta = 5.6$	
$V = 12^3 \times 24$		$V = 16^3 \times 32$		$V = 16^3 \times 32$		$V = 16^3 \times 32$		$V = 24^3 \times 48$	
κ	t_{min}								
0.1695	4	0.1692	4	0.1685	5	0.1667	5	0.1645	7
0.1700	4	0.1695	4	0.1687	5	0.1673	5	0.1650	7
0.1703	4	0.1697	4	0.1690	5	0.1678	5	0.1655	6
0.1705	4	0.1700	4	0.1692	5	0.1680	5	0.1660	7
–	–	0.1703	4	0.1693	4	0.1683	5	–	–
–	–	0.1705	4	–	–	–	–	–	–

limit on the time-slice distances in physical units, that is on the cutoff-scale and shrinks to zero in the continuum limit. Instead it would be more appropriate to consider t_{min} at constant physical distance for all gauge ensembles. This is done in the following way.

At the coarsest lattice spacing, at inverse gauge coupling β_0 , the value of t_{min} is selected according to the plateau criterion explained above. For finer lattice spacings at inverse gauge couplings β_i the corresponding t_{min} are then obtained by scaling with a physical scale. In order to determine the physical scale we use the mass $m_{g\tilde{g}}$ of the gluino-gluon parti-

cle and the Wilson flow parameter w_0 . Correspondingly, t_{min} is scaled according to

$$t_{min,\beta_i} = t_{min,\beta_0} \frac{m_{g\tilde{g},\beta_0}}{m_{g\tilde{g},\beta_i}}, \tag{16}$$

$$\text{or } t_{min,\beta_i} = t_{min,\beta_0} \frac{w_{0,\beta_i}}{w_{0,\beta_0}}, \tag{17}$$

where $\beta_0 = 5.4$, $\beta_1 = 5.45$, $\beta_2 = 5.5$, and $\beta_3 = 5.6$. The resulting t_{min} is rounded to the nearest integer value. The values obtained by this method are collected in Table 2. In most points they are equal or almost equal to those in Table 1.

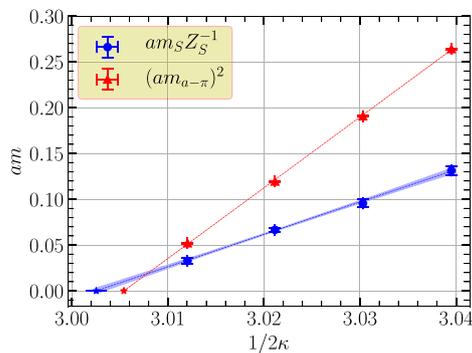
Table 2 The values of t_{min} at fixed physical temporal distance from scaling with the gluino-gluon mass $m_{g\tilde{g}}$ and with the Wilson flow parameter w_0

β	t_{min} from $m_{g\tilde{g}}$	t_{min} from w_0
5.4	4	4
5.45	5	5
5.5	5	6
5.6	7	7

3.2 Adjoint pion and remnant gluino mass

The chiral limit is defined by the vanishing of the subtracted gluino mass. Its measured values can therefore be employed for the tuning of the hopping parameter κ to the chiral limit. On the other hand, we can also use the vanishing of the adjoint pion mass $m_{a-\pi}$ for the tuning [16]. The adjoint pion $a-\pi$ is an unphysical particle in the SYM theory, that can be defined in partially quenched chiral perturbation theory [17]. In the numerical simulations its correlation function can be computed as the connected piece of the correlation function of the $a-\eta'$ particle. Similar to the Gell-Mann-Oakes-Renner relation of QCD [5], in the continuum limit there is a linear relation between the adjoint pion mass squared and the gluino mass: $m_{a-\pi}^2 \propto m_{\tilde{g}}$.

The numerical results for the subtracted gluino mass from the Ward identities and the adjoint pion mass squared in lattice units are shown for $\beta = 5.6$ in Fig. 2 together with their extrapolations towards the chiral limit.

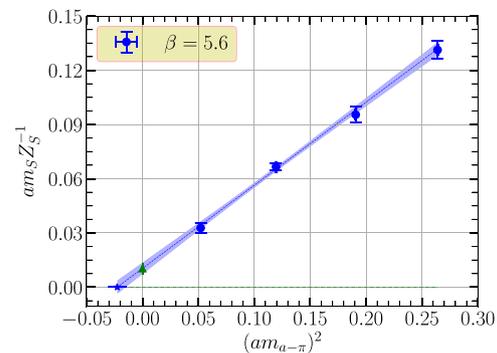


(a) The subtracted gluino mass $am_S Z_S^{-1}$ and the adjoint pion mass squared $(am_{a-\pi})^2$ as a function of $1/(2\kappa)$, and the corresponding extrapolations towards the chiral limit (κ_c).

In the continuum the subtracted gluino mass and the adjoint pion mass should vanish at the same point. On the lattice, however, this is not the case due to lattice artefacts. As an estimate for this discrepancy we determine the value of the subtracted gluino mass at vanishing adjoint pion mass. This quantity is called the remnant gluino mass $\Delta(am_S Z_S^{-1})$, and it is expected to vanish in the continuum limit. The values of the remnant gluino mass, obtained by taking an average of the values calculated using the procedures explained above, are presented in Table 3.

3.3 Continuum limit

The remnant gluino mass is a lattice artefact and should vanish in the continuum limit $a \rightarrow 0$. It is therefore a quantity to check on whether supersymmetry is recovered or not. Concerning the dependence of the remnant gluino mass on the lattice spacing, arguments based on partially quenched chiral perturbation theory suggest that the remnant gluino mass is of order a^2 at $m_{a-\pi}^2 = 0$ [13]. In order to investigate this relation, the remnant gluino mass has to be expressed in physical units. Our choice for the scale is the Wilson flow parameter w_0 , which is defined through the gradient flow [10]. We use its values extrapolated to the chiral limit, $w_{0,\chi}$. Similarly the lattice spacing is represented by $a/w_{0,\chi}$. Our numerical results for the remnant gluino mass as a function of the lattice spacing and its extrapolation towards the continuum limit are shown in Fig. 3. The data points in Fig. 3a show the results from separate chiral extrapolations for each

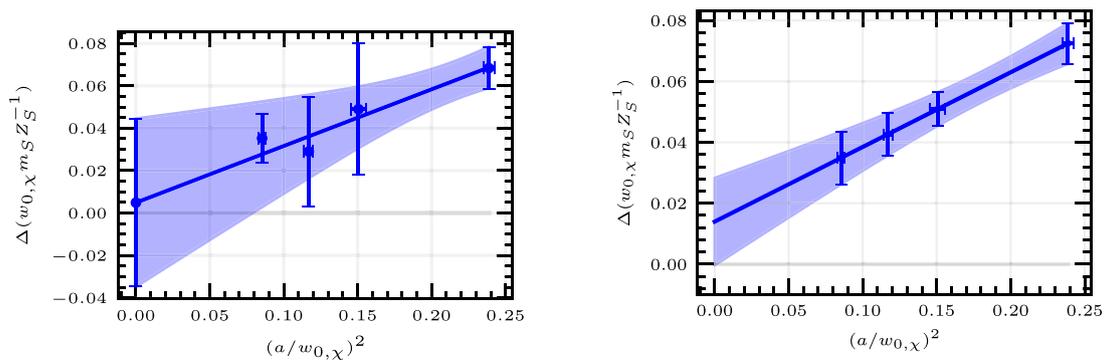


(b) The subtracted gluino mass $am_S Z_S^{-1}$ as a function of the adjoint pion mass squared $(am_{a-\pi})^2$ in order to obtain the remnant gluino mass $\Delta(am_S Z_S^{-1})$.

Fig. 2 Chiral limit and determination of the remnant gluino mass at $\beta = 5.6$. All quantities are in lattice units

Table 3 The values of the remnant gluino mass $\Delta(am_S Z_S^{-1})$ obtained at four different values of the inverse gauge coupling

β	5.4	5.45	5.5	5.6
$\Delta(am_S Z_S^{-1})$	0.0334(48)	0.019(12)	0.0099(88)	0.0103(33)



(a) The remnant gluino mass from separate extrapolations to the chiral limit where $m_{a-\pi}^2$ is zero, and the extrapolation to the continuum limit.

(b) The remnant gluino mass from a simultaneous chiral and continuum extrapolation. By construction, in this method the data points coincide with the error band.

Fig. 3 The remnant gluino mass $\Delta(w_0 m_S Z_S^{-1})$ in physical units w_0 as a function of the lattice spacing squared, and its linear extrapolation towards the continuum limit

lattice spacing and the corresponding extrapolation to the continuum limit. The extrapolation to the continuum and the error of this extrapolation are obtained by means of parametric bootstrap with linear fits. On the other hand, Fig. 3b is obtained by means of a simultaneous fit of the dependence on the hopping parameter and the lattice spacing [18].

The remnant gluino mass in the continuum limit is compatible with zero within one standard-deviation, confirming the preliminary results present in Ref. [15] with only two data points. Lattice artefacts vanish in the continuum limit as expected, and supersymmetry is recovered in the chiral and continuum limits, in agreement with our findings from the mass spectrum [12].

4 Conclusion

In this paper we have presented numerical results of an analysis of SUSY Ward identities in $\mathcal{N} = 1$ supersymmetric Yang–Mills theory on the lattice with gauge group SU(3). Contact terms and $O(a)$ lattice artefacts in the Ward identities have been controlled by suitable choices of time-slice distances. Ensembles of gauge configurations at four different values of the lattice spacing and various hopping parameters have been analysed, allowing us for the first time to perform an extrapolation to the continuum limit, where the lattice artefacts vanish. The remnant gluino mass has been extrapolated in two alternative ways, on the one hand by extrapolating to the chiral limit at each lattice spacing separately and then to the continuum limit, and on the other hand by means of a simultaneous extrapolation to the chiral and continuum limit. With both extrapolations the lattice artefacts in the subtracted gluino mass appear to scale to zero as of order a^2 in agreement with the theoretical expectations. Our

findings support the validity of SUSY Ward identities and the restoration of supersymmetry in the continuum limit.

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