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Time matters: How default resolution times impact final loss rates

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[Correction added on 6th May 2021, after first online publication: The copyright line was changed.]

Abstract

Using access to a unique bank loss database, we find positive dependencies of default resolution times (DRTs) of defaulted bank loan contracts and final loan loss rates (losses given default, LGDs). Due to this interconnection, LGD predictions made at the time of default and during resolution are subject to censoring. Pure (standard) LGD models are not able to capture effects of censoring. Accordingly, their LGD predictions may be biased and underestimate loss rates of defaulted loans. In this paper, we develop a Bayesian hierarchical modelling framework for DRTs and LGDs. In comparison to previous approaches, we derive final DRT estimates for loans in default which enables consistent LGD predictions conditional on the time in default. Furthermore, adequate unconditional LGD predictions can be derived. The proposed method is applicable to duration processes in general where the final outcomes depend on the duration of the process and are affected by censoring. By this means, we avoid bias of parameter estimates to ensure adequate predictions.

KEYWORDS

default resolution time, Global Credit Data, loss given default, random effects

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1 | INTRODUCTION

One of the most important tasks for banks is the estimation and prediction of probabilities of default (PDs) and losses given default (LGDs) for loan contracts whereby the latter is a fraction of loss over the exposure at default. While PD estimation has a long history, LGD estimation is in the focus of more recent research and regulation. In contrast to default itself, final losses of defaulted loan contracts are not observable until the end of the resolution process. These processes might continue for many years, thus, their duration (default resolution time, DRT) is subject to censoring. Ignoring censoring is likely to generate biased parameter estimates. Positive dependence between LGDs and DRTs is found in the literature, and, accordingly, LGD predictions can be biased when censoring of default resolutions is ignored. Pure (standard) LGD models do not take this into account which may cause inadequate LGD predictions. The aim of this paper is to develop a joint modelling framework for DRTs and LGDs which takes into account censoring effects of unresolved loan contracts. This leads to unbiased LGD predictions and enables us to use the information of the time a loan contract has already spent in default for LGD prediction. Thus, consistent predictions conditional on the time in default can be made.

In general, LGD modelling is important to banks due to regulatory obligations for determining risk of credit losses, for internal risk monitoring and pricing of credit risk contracts. According to the Basel regulations, banks have to determine regulatory capital charges which are supposed to protect banks from unexpected credit losses. Hereby, a bank can determine these charges by standardized rules (standardized approach), by deriving its own PD estimates and by combining these with fixed values of LGDs and the exposure at default (foundation internal rating-based approach) or by using its own models for PDs, LGDs and exposures at default (advanced internal ratings based approach). The approaches are used to determine risk weighted assets which are needed for deriving capital charges. While the advanced ratings based approach has been introduced with the motivation to relieve banks with good and sensitive risk modelling abilities from high capital charges, it has also led to a high variability which is considered a disadvantage by regulators. As a consequence, internal modelling of LGDs and exposure at default will no longer be permitted for a certain range of asset classes in the near future (see Basel Committee on Banking Supervision, 2017). Nevertheless, the importance of LGD modelling remains due to its use for internal risk monitoring and pricing credit risk contracts. The latter is of special relevance for a bank's competitiveness. Banks with sensitive risk models are able to demand sensitive and appropriate risk premiums, thus, providing a high degree of differentiability between 'good' and 'bad' debtors.

Due to the importance of LGD modelling, different approaches have been developed and compared in the literature. A broad overview of regression-type models, neural networks, regression trees and similar approaches is provided in comparative studies such as Loterman et al. (2012) and Qi and Zhao (2011). Overall, these studies imply predictive superiority of models which are able to capture non-linear relationships between independent variables and the LGD. While most of these studies focus on predictions regarding expected loss given default, other papers rather focus on modelling the overall LGD distribution or at least certain quantiles of it. LGD distributions often exhibit bimodality with high probability masses at zero and one which is why mixture distributions seem like a natural choice for modelling the LGD distribution (see, e.g. Altman & Kalotay, 2014; Betz, Kellner, & Rösch, 2018; Calabrese, 2014; Kalotay & Altman, 2017). Alternatively, linear quantile regression is used by Krüger and Rösch (2017) for capturing specific levels of the LGD distribution. In general, a distinction should be made between market-based LGDs and workout LGDs. The former is typically defined as one minus the ratio of the market price 90 days after default over the outstanding amount. Hence, market-based LGDs are only available for traded securities such as bonds. Workout LGDs are based on actual recovery payments collected during the resolution process and are, thus, usually applied for loans. The distribution of workout LGDs can exhibit values below zero and above one because of administrative or legal costs and interest payments or high collateral recoveries, respectively. Typically this increases the challenging nature of LGD modelling, as market-based distributional models such as the beta distribution or models using inverse probability transformations of LGDs are not suited for this kind of data. One way to deal with this special feature lies in the application of multistage models. Common methods estimate separate models for LGD components (see Bellotti & Crook, 2012). Another reason for the application of multistage modelling may lie in the nature of default resolution which is an accumulation of different outcomes during resolution. For instance, Tanoue, Kawada and Yamashita (2017) distinguish between recovery and write-offs with a probabilistic model and LGDs being equal to one or not and afterwards use a regression type LGD model for the latter class. Further examples for multistage modelling are Bijak and Thomas (2015), Sun and Jin (2016), Tobback et al. (2014), or Yao, Crook and Andreeva (2015). Moreover, even though machine learning methods have previously been applied for LGD modelling, current publications seem to further develop their application in this area. Examples are Bellotti, Brigo and Gambetti (2019), Kaposty, Kriebel and Löderbusch (2020), Gambetti, Gauthier and Vrins (2019) or Gambetti (2020).

Besides the question concerning appropriate LGD models, it is also of great relevance which variables impact LGDs and are fit to generate valid LGD predictions. However, due to the different nature of loan contracts (e.g. retail, credit cards, mortgages, etc.), there is no clear picture regarding the impact of independent variables. The most common variables in LGD models include information of collateralization, seniority, guarantees, loan size or industry affiliation (Betz, Kellner & Rösch, 2018; Dermine & Neto de Carvalho, 2006; Grunert & Weber, 2009; Qi & Zhao, 2011). Furthermore, the systematic impact of the economic environment is discussed in various studies, whereby macroeconomic variables are often used as proxies. However, results are not consistent for certain variables. For instance, Grunert and Weber (2009) do not find the rate of unemployment to impact LGDs of commercial lending contracts, while a positive impact on LGDs from retail credit card contracts is found by Bellotti and Crook (2012). Capturing the systematic environment for LGDs can be challenging, especially in the case of workout LGDs with long lasting resolution which span different economic surroundings. Krüger and Rösch (2017) find indications for non-linear relationships between LGDs and the economy, Betz, Kellner and Rösch (2018) capture the economic environment with an auto-regressive random effect or Nazemi et al. (2017) and Nazemi, Heidenreich and Fabozzi (2018) approximate the systematic environment via principal components which stem from different macroeconomic variables.

The literature regarding DRTs is more sparse and mainly refers to the duration of Chapter 7 and Chapter 11 resolutions (see, e.g. Bris, Welch, & Zhu, 2006; Denis & Rodgers, 2007; Helwege, 1999; Partington & Russel, 2001; Wong et al., 2007). Chapter 7 and Chapter 11 refer to different default resolution mechanisms in the United States. The former aims at processing default resolution, while Chapter 11 concerns reorganization with the aim to return to a healthy (non-default) status for the firm. Betz, Kellner and Rösch (2016) and Betz et al. (2017) analyse DRTs of loan contracts and descriptively find impacts of DRTs on LGDs. The interconnection of DRTs and LGDs is also indicated in the related LGD literature. The LGD is censored during the DRT, which is why survival time analysis models can be applied during the collection process of recovery payments. In contrast to our approach, this does not take the time spent in default into account, but the amount of repayments until the final value of repaid debt is known. Examples for this approach can be found in Dermine and Neto de Carvalho (2006), Witzany, Rychnovsky and Charamza (2012) and Zhang and Thomas (2012). The latter additionally analyse the improvement in estimation when segmenting LGDs with decision trees before estimation, hereby, constructing a mixture of the LGD distribution. Gürtler and Hibbeln (2013) focus on the effects of censoring on LGD observations and the negative consequences when censoring is ignored (see Section 2 for further information on effects of censoring). They do not provide a methodical solution, but suggest to restrict the data set to avoid biased estimates. However, LGD data is sparse, so constraints might be unfavourable. Survival time models can also be used to determine probabilities of default in a certain time frame. Joubert, Verster and Raubenheimer (2019) use a Cox proportional hazards model to predict probabilities for loans to either get cured, be written-off or stay in an unresolved status. In comparison to these contributions, we model the resolution time with an accelerated failure time model in order to account for censoring of resolution time. With the estimation of final DRTs of unresolved loans, corresponding censoring effects for LGDs are substantially diminished.

In this paper, our focus is headed towards a modelling approach, which is able to take censoring of the default resolution time and its link to censored LGD observations during default resolution into account. In comparison to previous contributions, our approach enables us to include non-resolved loan contracts in the estimation process with the exact information of how long the loan has already lasted in the unresolved status. This allows us to generate unbiased estimators, to use the highest amount of data possible and to determine the indirect impact of the systematic environment of DRTs on LGDs. A useful by-product of our model is the conditional estimation of DRTs for unresolved loans. This is valuable to banks as, the time, how long defaulted loans are in their books, impacts their business decisions and possibilities. In more detail, we develop a hierarchical Bayesian modelling approach for joint estimation of DRTs and LGDs combining a finite mixture model (FMM)—which is well suited to capture the distributional features of workout LGDs (see Betz, Kellner & Rösch, 2018)—with a probabilistic substructure for the LGD and an accelerated failure time (AFT) model for the DRT. The inclusion of survival modelling techniques—in terms of the AFT model—in the LGD modelling context enables the consideration of censoring in LGDs. Thus, the hierarchical approach enables adequate unconditional LGD predictions for non-defaulted exposures and consistent LGD predictions conditional on the time in default for defaulted exposures within one modelling framework. In contrast to previous applications of survival time models for predicting the LGD distribution, our approach is different. Instead of directly applying these models to the process of recovery payments, we apply them to the DRT and hierarchically build on this to model the LGD distribution. By this means, the information of time spent in the resolution process after default is introduced in the LGD model, enabling us to predict conditional LGD distributions during the resolution process. In addition, due to the hierarchical approach, both models are estimated at the same time, allowing for interactions between parameter estimates. Other approaches estimate models for segmenting and predicting LGDs independently, hereby, accepting biased parameter estimates if the independence-assumption is not correct. Furthermore, correlated random effects are implemented in the hierarchical approach to allow for co-movements of DRTs and LGDs in the time line.

We apply the hierarchical approach to a unique European data set provided by Global Credit Data (GCD). GCD is a non-profit initiative which aims to support banks to measure credit risk by collecting and analysing historical loss data (see http://www.globalcreditdata.org/ for further information). Furthermore, we compare the hierarchical approach to a pure (standard) LGD model in terms of an FMM with probabilistic substructure and no inclusion of censoring. By this means, we contribute to the literature in three ways. First, we examine the dependency structure of DRTs and LGDs thoroughly allowing for a direct and an indirect channel. We find positive impacts of DRTs on LGD distributions (direct channel) which are even more pronounced in boom and crisis periods (indirect channel). Especially the latter has not been detected in previous literature, but is of special relevance for banks. Defaulted loans systematically exhibit higher losses and remain longer in the bank's book during crisis periods. In crisis periods, this burdens financial market liquidity as more loan contracts are stuck in the resolution process. On top of that, losses are even higher than indicated by the direct channel due to an intensified dependency (indirect channel). Second, we derive a new way to include censored data of the default resolution process. In contrast to the previous literature, we directly include censored default resolution times instead of LGDs. As single payments during default resolution are of discrete nature, the application of continuous survival time models to the LGD does not seem appropriate opposed to directly model the censoring of DRTs which is transmitted to the LGD side. With our model, it is possible to include the exact information of current DRT and to provide unbiased estimates of DRTs as well as LGDs. Both are valuable to banks which are not only interested in the loss amount, but also how long the defaulted loan lasts in their books. Furthermore, all (censored) data can be used in our approach which usually reduces variance in LGD predictions. This seems to be favourable in comparison to approaches which suggest to ignore more recent data which typically is most exposed to censoring.

In contrast, a standard LGD model which ignores censored non-finalized LGDs suffers from biased parameter estimates that can cause erroneous LGD predictions, and even censored LGD models might be exposed to biased parameter estimates by not being able to include the exact information of current resolution time during resolution which is positively linked to LGDs. In extreme cases when using non-censored LGD models, LGDs are underestimated in an unconditional perspective by up to 20 percentage points in our empirical study. Considering defaulted exposures, that is, LGD-in-default, this underestimation is intensified. Third, the hierarchical approach diminishes the bias of parameter estimates and, thus, leads to adequate unconditional LGD predictions. At first glance, the described application seems to be rather specific to credit risk management. However, the proposed modelling framework is applicable to duration processes in general in which dependency between time and some outcome at the end of the time horizon is present.

The remainder of this paper is structured as follows. Section 2 provides further information on the effects of censoring on LGDs and, thus, reasoning for the introduction of the EBA guidelines. Section 3 introduces the hierarchical modelling approach. Data and results are presented in Section 4. In Section 5, the model is further validated on an in-sample and out-of-sample perspective. Section 6 concludes.

2 | BACKGROUND

As stated in Section 1, *workout* LGDs are characterized by time-specific censoring, as they are indirectly affected by the DRT. This section focuses on the effects of censoring on LGDs and, thus, offers guidelines with respect to the treatment of defaulted exposures.

Generally, ignoring censoring of the default resolution process might result in biased parameter estimates. We illustrate this with a simplified example to demonstrate the effects of censoring. For instance, assume a Weibull distribution for DRTs ($T \sim$ Weibull) and—for simplicity—a fixed censoring time c. To ease transparency, we further assume that LGDs linearly depend on log DRTs, thus, follow the linear model

$$L_i = \alpha + \beta \ln(T_i) + e_i, \tag{1}$$

where L_i is the LGD of loan i and $e_i \sim N(0, \sigma^2)$. We randomly generate 10,000 pairs of DRTs and LGDs with a Monte-Carlo simulation based on this model. We choose the true values of the parameters to match location and scale of empirical DRTs and LGDs. The shape and scale of the Weibull distribution are set to one and two to ensure an average DRT of approximately two years. In Equation (1), we set $\alpha = 0.2$ and $\beta = 0.3$ resulting in an average LGD of approximately 0.25. It should be noted that the choice of parameters is arbitrary as resulting effects are independent of the specific values. Figure 1 illustrates the impact of censoring on the result variable LGD. The light grey bars illustrate the simulation result for the *true* distributions (which are not observable), whereas the dark grey bars are the observable censored distributions. The lines display the corresponding mean values, whereby the dotted lines indicate the absence of censoring. In the upper panel, the distribution of the time is displayed. Censoring limits the distribution to a certain value, which implies a slight underestimation of the average value if censoring is ignored. Due to the linear dependency between time and result in Equation (1), this phenomenon is transferred to the result variable (see lower left panel). Thus, average values are underestimated assuming a positive dependency of DRT and LGD, as the true mean marked by a dotted line is above the mean in the presence of censoring. In the conditional perspective, only censored cases are considered. Hereby, the effect of censoring and, thus, the underestimation, is intensified (see lower right panel) as the difference between true mean and mean in the presence of censoring increases. Ignoring censoring implies the same distribution for censored values as for non-censored ones. However, the true distribution is shifted towards higher values assuming a positive dependency, that is $\beta > 0$.

The unconditional perspective corresponds to unconditional LGDs for the non-defaulted exposure, whereas the conditional perspective reflects LGDs conditional on the time in default for the defaulted exposure. Thus, the consideration of post-default information—such as the time in default—is required. The hierarchical approach we develop is a joint modelling approach for DRTs and LGDs. It

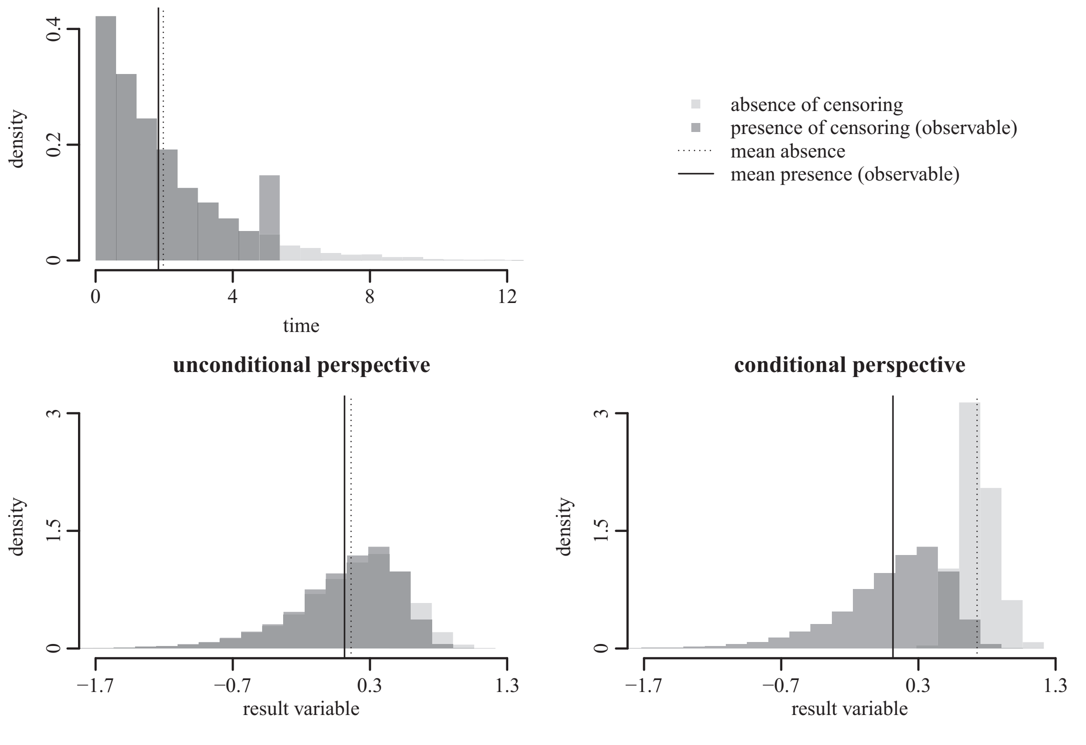

FIGURE 1 Impact of censoring on the result variable

Note: The figure illustrates the impact of censoring in the time variable on the result variable. The upper panel displays the distribution of time in the absence of censoring (light grey bars) and in the presence of censoring (dark grey bars). The distribution in the absence of censoring might be interpreted as the *true* distribution. The lower left panel illustrates the unconditional distribution of the result variable (dark grey indicates censoring, light grey no censoring). The lower right panel restricts the presentation to the censored cases. Means of the distributions are marked by lines, whereby the dotted line indicates absence of censoring considers censoring, dependencies between DRTs and LGDs and, thus, allows for adequate unconditional and consistent conditional LGD predictions within a single modelling framework.

3 | METHODS

This paper develops a hierarchical modelling approach for DRTs and LGDs and, thereby, analyses the dependency structure of DRTs and LGDs and reveals impacts of censoring in DRTs on LGDs. We combine a common LGD model from previous literature with survival analysis techniques—in terms of an accelerated failure time (AFT) model—in a hierarchical structure to consider censoring in an LGD modelling context. LGD distributions of workout LGDs are often bimodal with high probability masses at no and total loss. Furthermore, the two modes are characterized by bindings, that is, values which are exactly 0 or exactly 1. In addition, workout LGDs can be lower than 0 or higher than 1. We, therefore, extend the Bayesian FMM with a probabilistic substructure in terms of an ordered logit (OL) model developed by Altman and Kalotay (2014) and further extended by Betz, Kellner and Rösch (2018). This model seems to be well suited to capture the characteristic features of LGD distributions. To investigate direct dependencies of DRTs on LGDs, the DRT serves as an explanatory variable in the LGD model. Two correlated random effects are included to study co-movements of DRTs and LGDs in the time line (indirect dependencies). In the following, we briefly review the LGD model of Altman and Kalotay (2014), hereby, following the model description of Betz, Kellner and Rösch (2018) and discuss the extensions in the context of the hierarchical model.

3.1 | LGD model

We apply a normal FMM to model the distribution of LGDs. Generally, FMMs offer high flexibility in modelling distributions of unknown shape (see McLachlan & Peel, 2000). The dependent variable L is assumed to be divided into a finite number of K latent components. In each class k, L follows a Normal distribution with parameters θ_k depending on the latent class k. We assume normally distributed components to achieve computational transparency (see McLachlan & Peel, 2000). Thus, the probability density function (PDF) of an FMM $g(L \mid \theta_1, \ldots, \theta_K)$ is the p_k weighted sum of the component PDFs $f_k(L|\theta_k)$:

$$g\left(L|\theta_1, \ldots, \theta_K\right) = \sum_{k=1}^{K} p_k f_k\left(L|\theta_k\right). \tag{2}$$

To ensure the general properties of a PDF, that is $g(L) \geq 0$ for all $L \in \mathbb{R}$ and $\int_{-\infty}^{\infty} g(L) = 1$, $p_k \geq 0$ and $\sum_k p_k = 1$ must hold. Assuming conditional independence, the likelihood of a Normal FMM $\phi(L_1, \ldots, L_n \mid \mu, \sigma, p)$ is the product of the individual likelihood contributions:

$$\phi(L_1, \ldots, L_n \mid \mu, \sigma, p) = \frac{1}{(2\pi)^{\frac{n}{2}}} \prod_{i=1}^{n} \left(\sum_{k=1}^{K} \frac{p_k}{\sigma_k} \exp\left[-\frac{(L_i - \mu_k)^2}{2\sigma_k^2} \right] \right), \tag{3}$$

where μ_k and σ_k are the parameters of a normal distribution for the latent class k and n is the number of observations. To adapt data augmentation, the component weight p_k is replaced by an indicator variable d_{ik}, which takes the value one if L_i is a random draw of component k and zero, otherwise:

$$\phi(L_1, \ldots, L_n \mid \mu, \sigma, d) = \frac{1}{(2\pi)^{\frac{n}{2}}} \prod_{i=1}^{n} \left(\sum_{k=1}^{K} \frac{d_{ik}}{\sigma_k} \exp\left[-\frac{(L_i - \mu_k)^2}{2\sigma_k^2} \right] \right). \tag{4}$$

To identify loan contracts with no and total loss, we fix the parameters of the two outer components. The means are set to $\mu_1 = 0$ and $\mu_K = 1$ with small standard deviations ($\sigma_1 = \sigma_K = 0.0001$). Results are robust to different (small) values for σ_1 and σ_K. Note that this means we try to approximate estimates for discrete LGD observations 0 and 1 by means of continuous distributions. This could potentially result in biased parameters as well as (small) deviations of LGD forecasts. Overall, this is mainly determined by the choice for $\sigma_1 = \sigma_K$. We tried different (small) values in the course of our analysis and found our results to be robust with respect to this choice.

To estimate the probability of loan i belonging to the k-th component depending on covariates, a probabilistic substructure in terms of an ordered logit (OL) model is formulated. To rely on the classical formulation of the OL model, we define the component affiliation y_i:

$$y_i = k \quad \text{if } d_{ik} = 1, \tag{5}$$

where d_{ik} is the indicator in Equation (4). The component affiliation Y_i represents the component of the mixture distribution from which the LGD observation is drawn. It is categorically distributed and determined by the comparison of a metric latent variable Y_i^* with cut points c_k ($k \in \{1, \ldots, K-1\}$):

$$Y_i = \begin{cases} 1 & \text{if } Y_i^* \leq c_1 \\ 2 & \text{if } c_1 < Y_i^* \leq c_2 \\ \vdots & \\ K & \text{if } c_{K-1} < Y_i^*. \end{cases} \tag{6}$$

The latent variable Y_i^* follows a linear model:

$$Y_i^* = \boldsymbol{z_i \zeta} + F_{t(i)} + e_i, \qquad e_i \sim \text{logistic}, \tag{7}$$

where $\boldsymbol{z_i}$ is a $(1 \times J)$ vector of independent variables and $\boldsymbol{\zeta}$ is the $(J \times 1)$ vector of coefficients. The term e_i describes the errors. A random effect $F_{t(i)}$ is introduced into the modelling framework to control for co-movement in the time line. It originates from a normal distribution with mean zero and standard deviation σ:

$$F_{t(i)} \sim \mathrm{N}\left(0, \sigma^2\right). \tag{8}$$

The time stamp $t(i)$ in Equation (7) indicates the default time t of a loan i, expressed in quarters. Two loans i and i' which defaulted in the same quarter ($t(i) = t(i') = t$) share the same realization of the random effect ($f_{t(i)} = f_{t(i')} = f_t$). For $f_t > 0$ ($f_t < 0$), both loans exhibit higher (lower) values of y_i^* and, thus, higher (lower) probabilities of high component affiliations y_i. Higher component affiliations y_i correspond to higher loss rates and vice versa. Thus, the random effect displays the co-movement in time line, that is, higher or lower average loss rates in specific default quarters which cannot be explained by observable variables included in $\boldsymbol{z_i}$.

Betz, Kellner and Rösch (2018) additionally consider an autoregressive process of order 1, that is, AR(1), for the random effect to allow for cyclical movements in the realizations of the random effect. In this paper, we do not consider this specification due to simplicity, as the specification of the random effect seems to have negligible impact on its realizations. For conditional predictions (LGDs-in-default, EL_{BE}) in the hierarchical approach, we apply the realized value of the random effect in the corresponding time period, while we use the mean value 0 for unconditional predictions.

3.2 | Hierarchical model

In the hierarchical model, the pure (standard) LGD model of the previous subsection is extended by an additional hierarchical level in terms of an AFT model for the DRT to consider censoring and allow for LGD predictions conditional on the time in default (see, e.g. Wei, 1992). The logarithm of the resolution time $\ln(T_i)$ can be expressed by a linear model:

$$\ln(T_i) = \beta_0 + \boldsymbol{x_i}\boldsymbol{\beta} + F^T_{t(i)} + s\,\varepsilon^T_i, \qquad \varepsilon^T_i \sim \text{negative Gumbel}, \tag{9}$$

where $\boldsymbol{x_i}$ is a $(1 \times J_T)$ vector of independent variables, $\boldsymbol{\beta}$ is the $(J_T \times 1)$ vector of coefficients, β_0 is the intercept and s a scaling parameter. We assume the errors $\boldsymbol{\varepsilon}^T_i$ to follow a negative Gumbel distribution and thus the DRT to be Weibull distributed. Different distributional assumptions for the errors are possible with the normal, logistic, exponential and Weibull distribution being the most common ones in the AFT model. For our analysis, we compare the empirical distribution of the DRTs with posterior predictions of an AFT model considering a log normal, log logistic,and Weibull distribution. The comparison is based on density and quantile–quantile plots. With this respect, the Weibull distribution seems to have the best fit. A random effect $F^T_{t(i)}$ is introduced into the modelling framework to control for co-movement in the time line, where T in superscript stands for the resolution time specific random effect. Equation (9) applies to non-censored, that is final, observations. For censored observations, final realizations are estimated within the Bayesian modelling framework. By this means, we are able to predict final DRTs for censored data points, that is, unresolved loans.

In the hierarchical approach, the AFT model for the DRT is simultaneously estimated with an FMM for the LGD (see previous subsection). To develop an intuitive method to generate LGD predictions conditional on the time in default, the logarithm of the DRT is included as an explanatory variable. This accounts for direct dependencies of DRTs and LGDs. Equation (7) modifies to:

$$Y^*_i = z_i\boldsymbol{\gamma} + \ln(T_i)\gamma_T + F^L_{t(i)} + \epsilon_i, \qquad \epsilon_i \sim \text{logistic}, \tag{10}$$

where $\boldsymbol{z_i}$ is the $(1 \times J_L)$ vector of independent variables, $\boldsymbol{\gamma}$ is the $(J_L \times 1)$ vector of coefficients, γ_T is the coefficient of the logarithm of the DRT and errors ϵ_i with mean 0 and scale 1. Again, a random effect $F^L_{t(i)}$ is introduced into the modelling framework to control for co-movement in the time line where L in superscript stands for the LGD specific random effect. Equations (2), (3), (4), (5), and (6) apply in analogy to Y_i*.

The random effects $F^T_{t(i)}$ in Equation (9) and $F^L_{t(i)}$ in Equation (10) originate from a bivariate normal distribution:

$$\begin{pmatrix} F^T_t \\ F^L_t \end{pmatrix} \sim \mathrm{N}_2\left(\boldsymbol{0}_2, \Sigma\right), \tag{11}$$

where $\boldsymbol{0}_2$ is the two-dimensional zero vector $(0 \quad 0)^T$ and Σ is the (2×2) covariance matrix. The latter is based on individual standard deviations (σ_T and σ_L) and the (2×2) correlation matrix Ω:

$$\Sigma = \text{diag}\left(\sigma_T, \sigma_L\right)\ \Omega\ \text{diag}\left(\sigma_T, \sigma_L\right) = \begin{pmatrix} \sigma_T^2 & \sigma_T\sigma_L\omega_{L,T} \\ \sigma_T\sigma_L\omega_{T,L} & \sigma_L^2 \end{pmatrix}, \tag{12}$$

where $\omega_{T,L}\,(=\omega_{L,T})$ is the correlation of F_t^T and F_t^L. By the inclusion of the random effects, we control for joint co-movements of loss rates and resolution times in the time line. Two loans i and i' which defaulted in the same quarter $(t(i) = t(i') = t)$ share the same realizations of the random effects ($f_{t(i)}^T = f_{t(i')}^T = f_t^T$ and $f_{t(i)}^L = f_{t(i')}^L = f_t^L$, however, $f_t^T \neq f_t^L$ in most of the cases). For $f_t^T > 0$ ($f_t^T < 0$), average DRTs are higher (lower). Assuming a positive correlation between the random effects and a positive parameter estimate of the logarithm of the DRT in the LGD model ($\gamma_T > 0$), the corresponding LGDs are affected in two ways: Directly, as higher (lower) DRTs are inserted in the LGD model; indirectly, as positive (negative) realizations of f_t^T tend to imply positive (negative) realizations of f_t^L due to the positive correlation. Thus, LGDs are even higher. However, negative realizations of f_t^L remain possible which might reduce LGDs. Both scenarios are conceivable. Confronted with a tense economic surrounding, financial institutions might decide to follow a wait-and-see strategy and relocate resolution efforts in the future. This might provide benefits and reduce the LGD ($f_t^L < 0$). However, LGDs might be further increased ($f_t^L > 0$) if financial institutions are forced to resolve defaulted loans at a certain point in time.

3.2.1 | Estimation

The parameters of the LGD model and the hierarchical model are estimated via Bayesian inference. We use the Hamiltonian Monte Carlo (HMC) algorithm (see, e.g. Betancourt, 2017) for the simulation of posterior distributions. In comparison to a Gibbs sampler that we tried, the HMC algorithm exhibited less correlated posterior distributions and needed shorter burn-in periods. In addition, the HMC algorithm is efficiently implemented in Stan (see Stan Development Team, 2016, for further information on the implementation) and combines a Hamiltonian evolution with a Metropolis proposal to reduce the correlation in the chains. The parameters of the proposal distribution and the Hamiltonian evolution are tuned during the adaption phase. The LGD model and the hierarchical model are sampled with two HMC chains. Burn-in is set to 500. Posterior samples contain $N = 25,000$ iterations per chain with a thinning of 5. Metric dependent variables are standardized to ease convergence. Most of the model parameters are provided with weakly informative prior distributions. See Section 1 of the online companion of this paper for detailed information on the Bayesian model specifications. Common convergence diagnostics can be found in Section 2 of the online companion.

4 | DATA AND RESULTS

4.1 | Data

We use the unique loss database of GCD. The database includes detailed loss information on a transaction basis of 53 member banks all around the world. In the database, the LGD is determined by $L_i = 1 - \text{RR}_i$, whereby L_i is the loss rate of loan i and RR_i is the corresponding recovery rate. The recovery rate is calculated as the sum over the present values of all relevant transactions divided by the outstanding amount (see Betz, Kellner & Rösch, 2016, 2018).

We follow Höcht (2010) and Höcht, Kroneberg and Zagst (2011) and develop two selection criteria to eliminate loans with extraordinary payment structures. Both criteria are related to all relevant transactions including charge-offs (which are not included in the LGD calculation) to the outstanding amount. The first criterion, to which we refer as *pre-resolution criterion*, is related to transactions arising pre-resolution to the outstanding amount at default. We set the barriers of the pre-resolution criterion to $[90\%, 110\%]$ for resolved and $[-50\%, 400\%]$ for unresolved loans. Different barriers for the criteria of resolved and unresolved are applied because the transactions should come up to 100% for resolved loans as write-offs are already included if a loan is resolved. However, if a loan is still in the resolution process, write-downs do not yet exist in the records. Nevertheless, we aim to select loans with extraordinarily high or low individual payments. Thus, the corresponding barriers are much wider. In the second criterion, that is, *post-resolution criterion*, transactions occurring post resolution are related to a fictional outstanding amount at resolution. The barriers are set to $[-10\%, 110\%]$ for the post-resolution criterion. The post-resolution criterion applies for resolved loans only. Subsequently, loans with extremely low and high LGDs ($< -25\%$ and $> 125\%$) are eliminated. Overall, 0.50% of resolved loans are eliminated due to the pre-resolution criterion and 0.19% due to the post-resolution criterion, whereas 0.23% of unresolved loans are eliminated based on the pre-resolution criterion. Subsequently, 0.13% are removed due to extremely low and high LGD values. We consider a subsample from the original database which consists of defaulted European term loans and lines to small and medium sized enterprises (SMEs). We include the twelve most common European countries in the database—that is Great Britain, Germany, Denmark, Portugal, Ireland, France, Finland, Sweden, Norway, Latvia, Estonia and Poland. We further exclude loans which defaulted before 2004 and after 2016 (10.02% of subsample data). To estimate the random effects model in a stable way, we need a descent number of observations which is not the case in our database before 2004 and after 2016. A subsample of 38,165 loans remains.

Figure 2 illustrates the interconnection of the two parameters of the resolution process, that is, the DRT and the LGD. Therefore, the data set is divided into DRT buckets based on realized DRTs. The first bucket includes all loans with DRTs $\in [0, 0.5]$ years. The second bucket contains all loans with DRTs $\in (0.5, 1.0]$ years, and so on (see the horizontal axis of the left panel and the legend of the right panel). In the left panel, box plots of LGDs divided by DRT buckets are displayed. The thick black lines mark the medians, whereas the thick grey lines are the means. Considering the latter, average LGDs seem to linearly increase in the DRT buckets. To examine the origin of this increase, the right panel displays kernel density estimates for the DRT buckets. The LGD distribution of higher DRT buckets is shifted towards higher LGD values, that is, probability masses of lower LGD values decrease and probability masses of higher LGD values increase. Thus, average values increase.

Table 1 summarizes the descriptive statistics of the dependent and explanatory variables. Figures are stated for all loans (resolved, i.e. non-censored, and unresolved, i.e. censored, cases) and for resolved and unresolved loans separately. The upper panel of the table includes descriptive statistics for the LGD and the DRT. For unresolved cases, non-finalized LGDs are considered. Non-finalized LGDs are computed as the sum over the present values of all relevant transactions, which occurred up to the end of the observation period (end of 2016), divided by the outstanding amount. As the resolution process is not terminated, non-finalized LGDs are higher than final LGDs. DRTs for unresolved cases are censored to the end of the observation period (end of 2016), for example for unresolved loans defaulted at the end of 2015 a censored DRT of one year is assigned. Censored DRTs are lower than final DRTs as the resolution process is not terminated. In the table, average values of LGDs and DRTs for unresolved cases are higher compared to resolved cases as unresolved cases are shaped by loans exhibiting high DRTs and high LGDs. In the middle panels of the table, descriptive statistics of loan specific independent variables are stated. We use the EAD to control for the size of the loan. It

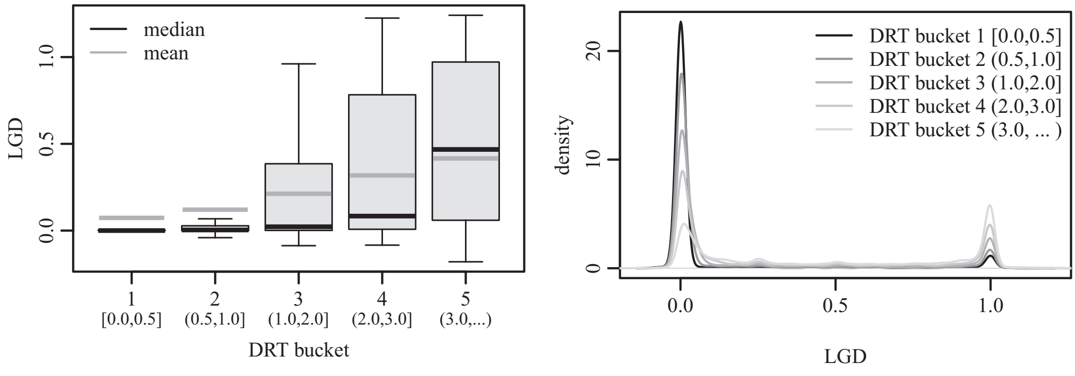

FIGURE 2 Relation of DRT and LGD

Note: The figure illustrates the relation of DRTs and LGDs. The data is divided into DRT buckets based on the realized DRTs. Thus, the first bucket includes all loans with DRTs $\in [0, 0.5]$ years. The second bucket contains all loans with DRTs $\in (0.5, 1.0]$ years, and so on (see the horizontal of the left panel and the legend of the right panel). In the left panel, box plots of LGDs for the DRT buckets are displayed. Outliers are hidden. The thick black lines mark the medians, whereas the thick gray lines are the means. In the right panel, kernel density estimates of LGDs for the DRT buckets are illustrated. The band width is fixed to 0.015 to ensure comparability

is further distinguished between term loans and lines, whether a loan is secured by collateral or guarantee or not, and whether the debtor has Finance, Insurance, Real Estate (FIRE) industry affiliation. Reference categories in the subsequent models are printed in italics in the table. The lower panel of the table contains descriptive statistics of the macroeconomic variables. The year-on-year (yoy) percentage change of weighted average real residential prices (Δ HPI) is employed as explanatory variable for the LGD, whereas we use the VSTOXX Volatility Index (VIX) for the DRT. We tested further macrovariables, for example the yoy percentage change of weighted average seasonally adjusted GDPs and the quarterly average yoy percentage change of weighted average equity indices. However, Δ HPI and VIX exhibit the highest statistical evidence. Furthermore, both variables have been used in previous analyses and exhibited potential impact on the LGDs (see, e.g. Qi & Zhao, 2011, Yao, Crook & Andreeva, 2017 or Tobback et al., 2014 for the impact of the HPI or Krüger & Rösch, 2017 for the impact of the VIX) as well as DRTs (see Betz et al., 2017 for the impact of the VIX).

Figure 3 illustrates the time patterns of average DRTs in the left panel and average LGDs in the right panel for resolved loans (thick black line) and all loans (resolved and unresolved loans, thin grey line). Regarding the latter, values for unresolved loans, that is censored observations, have to be calculated. Thus, DRTs are censored to the end of the observation period (end of 2016) and non-finalized LGDs are treated as unresolved cases. Non-finalized LGDs are computed as the sum over the present values of all relevant transactions which occurred up to the end of the observation period (end of 2016), divided by the outstanding amount. The relation of DRTs and LGDs (see Figure 2) might be partly driven by analogous time patterns. Both dependent variables sharply increase prior to the Great Financial Crisis (GFC, 2007) and reach their maximum during the climax of the GFC. The rebound in the aftermath of the crisis seems gradual. There are only minor deviations between resolved loans and all loans considering the average DRTs. The graph for all loans is shifted slightly upwards by the censored observations. Regarding average LGDs, this spread is particularly severe in the most recent time periods. This is mainly due to non-finalized LGDs, that is, LGDs based on transactions which occur up to the end of the observation period, in the averaging. Final LGDs will be lower as further payments will be received until resolution in most of the cases. However, final LGDs of all loans will still lie above the black line (final LGDs of resolved loans). Due to the effects of censoring, final LGDs are

TABLE 1 Descriptive statistics

		All	Resolved	Unresolved
n		38,165	35,272	2893
Dependent variables				
LGD	Mean	0.2534	0.2099	0.7839
	Median	0.0133	0.0082	0.9780
	Standard deviation	0.3810	0.3531	0.3017
DRT	Mean	1.9882	1.7342	5.0839
	Median	1.2621	1.1335	4.9090
	Standard deviation	2.0756	1.7509	3.0147
Loan specific (metric)				
EAD	Mean	533,118.89	516,582.05	734,739.20
	Median	102,987.29	100,237.48	155,097.53
	Standard deviation	3,624,711.52	3,610,978.60	3,782,983.43
Loan specific (categoric)				
Facility	*Term loan*	62.00%	60.39%	81.68%
	Line	38.00%	39.61%	18.32%
Secured	*No*	25.61%	26.03%	20.46%
	Yes	74.39%	73.97%	79.54%
Industry	*Non-FIRE*	83.13%	82.13%	95.30%
	FIRE	16.87%	17.87%	4.70%
Macrovariables				
Δ HPI	Mean	− 1.6966	− 1.7765	− 0.7229
	Median	0.1662	0.1662	0.8360
	Standard deviation	6.0221	5.9928	6.2878
VIX	Mean	24.4762	24.3681	25.7947
	Median	22.9249	22.6771	23.3451
	Standard deviation	9.4105	9.4699	8.5465

Note: The table summarizes descriptive statistics for dependent and independent variables in the data set. For metric variables, means, medians and standard deviations are stated. Proportions are presented for variables of categoric nature. The sample size is denoted by *n*. The abbreviation EAD stands for Exposure at default, FIRE stands for *Finance, Insurance, Real Estate* and denotes corporations of these industries. The macrovariable Δ HPI is the yoy percentage change of the *House Price Index*, whereas the VIX is the *Volatility Index*.

only observable for defaults with short DRTs in the more recent time periods (see Section 2). Due to the interconnection of DRTs and LGDs, these tend to be lower implying an underestimation of LGDs in more recent time periods.

In this paper, we analyse the effects of censoring on unconditional and conditional LGD predictions from in-sample and out-of-sample perspectives. Therefore, we divide the data set in Table 1 into subsamples. The first subsample serves as *estimation sample*. It includes all loans defaulted between 2004 Q1 and 2010 Q4. Thus, it comprises times of rather sound economic surrounding, the GFC, and parts of the rebound phase. We treat loans which are not resolved until 2010 Q4 as censored observations, that is, unresolved loans. The second subsample, to which we refer to as *validation sample I*, includes the final observations to the censored observations of the estimation sample. For instance, a

FIGURE 3 Time patterns of average DRTs and average LGDs
Note: The figure illustrates time patterns of average DRTs in the left panel and average LGDs in the right panel. The black lines display the average values for resolved loans, whereas the grey lines are average values for all loans, that is resolved and unresolved cases. Thus, the latter include censored values. Means over the entire time period are illustrated by dotted lines

loan defaulting in 2009 Q4 and being resolved in 2011 Q4 is included as a censored observation in the estimation sample whose final observation after resolution gets evaluated in validation sample I. We apply validation sample I to perform an *out-of-sample validation* of LGDs. The third sample, that is, *validation sample II*, includes all loans defaulted between 2011 Q1 and 2016 Q4. It is used to perform an *out-of-sample out-of-time validation* of LGDs. Table 2 summarizes the estimation sample and the validation samples. In the upper panel, the sample sizes are stated. Validation sample I consists of the 10,171 loans which are treated as unresolved cases in the estimation sample, that is, are unresolved until the end of 2010. Some of these loans (1724) are still unresolved at the end of 2016. However, the proportion of unresolved loans is lower in validation sample I compared to the estimation sample. In the lower panel, average values of LGDs and DRTs are stated. These are rather similar comparing the estimation sample and validation sample II, however, considerably higher in validation sample I. This is due to the fact that validation sample I contains final observations to censored cases in the estimation sample, thus, observations with higher DRTs and higher LGDs.

4.2 | Results

Under the Bayesian approach, we use highest probability density intervals (HPDIs) and posterior odds (po) to evaluate the effect of independent variables. As we are interested in the evidence of the signs, posterior odds are derived as the ratio of posterior mass favouring the sign of the posterior mean to posterior mass of the opposite sign:

$$\mathrm{po}\,(\mathrm{E}[\theta] < 0) = \frac{\mathbb{P}(\theta < 0 \mid \text{data})}{\mathbb{P}(\theta \geq 0 \mid \text{data})}$$
$$\mathrm{po}\,(\mathrm{E}[\theta] > 0) = \frac{\mathbb{P}(\theta > 0 \mid \text{data})}{\mathbb{P}(\theta \leq 0 \mid \text{data})},$$

where by θ denotes an arbitrary parameter. Prior odds are the corresponding ratio of the prior distribution. Assuming a symmetric prior distribution around zero, posterior odds are equivalent to the Bayes factor.

TABLE 2 Estimation sample and validation samples

		Estimation sample	Validation sample I (*out-of-sample*)	Validation sample II (*out-of-sample out-of-time*)
n	All	31,988	10,171	6177
	Resolved	21,817	8447	5008
	Unresolved	10,171	1724	1169
Dependent variables				
Average LGD	All	0.2586	0.4270	0.2267
	Resolved	0.1801	0.3511	0.1017
	Unresolved	0.4270	0.7987	0.7622
Average DRT	All	1.5763	4.2566	1.0495
	Resolved	1.1964	3.6851	0.7869
	Unresolved	2.3911	7.0568	2.1743

Note: The table summarizes the applied samples. The number n, the average LGD, and the average DRT of all loans, resolved loans and unresolved loans are presented for the estimation sample and the two validation samples. The models are estimated based on the estimation sample. This sample includes all loans defaulted between 2004 Q1 and 2010 Q4. Loans which are not resolved until 2010 Q4 are treated as censored observations, that is, unresolved cases, in the estimation. Validation sample I contains the final observations of these unresolved cases. However, observations exist which are still censored at the end of 2016 (unresolved cases in validation sample I). In validation sample II, loans which defaulted between 2011 Q1 and 2016 Q4 are included. Thus, validation sample I is applied for the *out-of-sample* validation, whereas the *out-of-sample out-of-time* validation is performed on validation sample II.

A Bayes factor exceeding 3.2 is deemed as substantial evidence. Values above 10 correspond to strong evidence, whereas values above 100 represent decisive evidence (see Kass and Raftery, 1995).

4.2.1 | LGD model

The LGD model is estimated based on the estimation sample (see Table 2) with five components of the FMM. However, it offers no possibility to include censored observations, that is unresolved loans, in the estimation process. Thus, the 21,817 resolved cases are included, whereas 10,171 unresolved defaults are ignored. As these unresolved loans tend to exhibit higher LGDs due to the resolution bias, parameter estimates are likely to be biased.

Table 3 summarizes the results of the LGD model. Parameters are stated in the first column, whereas the second column presents their posterior means. In the FMM within the LGD model, parameters of the outer components (μ_1 and σ_1 for the first component, μ_5 and σ_5 for the fifth component) are fixed to identify loans with no (LGD = 0) and total (LGD = 1) loss. The second and third component are located near the first component ($\mu_2 = 0.0067$ and $\mu_3 = 0.0290$) and have rather small standard deviations ($\sigma_2 = 0.0045$ and $\sigma_3 = 0.0249$), whereas the forth component seems to cover the range in between the extremes of no and total loss ($\mu_4 = 0.5114$ and $\sigma_4 = 0.3364$). The posterior distributions of the cut points (c_k for $k \in \{1, 2, 3, 4\}$) can not directly interpreted as they depend on the range of the latent variable (Y^*).

Component probabilities are derived based on the OL model within the LGD model. The parameter of the EAD (ζ_{EAD}) exhibits a negative posterior mean, indicating a lower value of the latent variable (Y^*) for higher EADs and, thus, lower LGDs. This impact is characterized by decisive evidence as the posterior odds tend towards infinity po$\left(\text{E}[\zeta_{\text{EAD}}] < 0\right) \to \infty$) and the HPDI$\left(\zeta_{\text{EAD}}\right) = [-0.14, -0.08]$ excludes zero. Reasons for the negative impact of the EAD might be found in higher resolution efforts

TABLE 3 Results of the LGD model

	Posterior mean	HPDI (95%)		Posterior odds	Naive standard error	Time series standard error
LGD model						
μ_1	0.0000	*Not estimated*				
μ_2	0.0067	0.0064	0.0070	∞	0.0000	0.0000
μ_3	0.0290	0.0277	0.0303	∞	0.0000	0.0000
μ_4	0.5114	0.5004	0.5229	∞	0.0000	0.0000
μ_5	1.0000	*Not estimated*				
σ_1	0.0010	*Not estimated*				
σ_2	0.0045	0.0042	0.0048	∞	0.0000	0.0000
σ_3	0.0249	0.0237	0.0261	∞	0.0000	0.0000
σ_4	0.3364	0.3295	0.3436	∞	0.0000	0.0000
σ_5	0.0010	*Not estimated*				
c_1	−0.6959	−0.9773	−0.4082	∞	0.0006	0.0012
c_2	−0.0349	−0.3203	0.2510	1.4857	0.0006	0.0012
c_3	0.8952	0.6087	1.1777	∞	0.0006	0.0012
c_4	2.7509	2.4649	3.0421	∞	0.0007	0.0012
ζ_{EAD}	−0.1099	−0.1357	−0.0824	∞	0.0001	0.0001
ζ_{Facility}	0.2038	0.1495	0.2584	∞	0.0001	0.0001
$\zeta_{\text{Protection}}$	−0.4147	−0.4751	−0.3559	∞	0.0001	0.0001
ζ_{Industry}	−0.2355	−0.3000	−0.1683	∞	0.0002	0.0002
ζ_{HPI}	0.0590	−0.2183	0.3311	2.0243	0.0006	0.0010
Random effect						
σ	0.8191	0.6200	1.0329	∞	0.0005	0.0005

Note: The table summarizes the results of the LGD model. Parameters are stated in the first column. Categorical variables are included via dummy coding. The reference categories are term loan for facility, no for protection, and non-FIRE for industry. The second column presents the posterior means. In the third and fourth columns, lower and upper bounds of the corresponding HPDIs to a credibility level of 95% are displayed. The fifth column contains the posterior odds. Naive and time series standard errors are shown in the last two columns. Time series standard errors are calculated based on the effective chain length (N^*) instead of the actual chain length (N), whereby, $N^* < N$ holds for autocorrelated chains.

and thus, lower loss rates, for loans of major size. The negative relation between EAD and LGD is also confirmed by Grunert and Weber (2009). However, this result should not be generalized because a positive relationship between EAD and LGD is found by Dermine and Neto de Carvalho (2006). The posterior mean of lines (ζ_{Facility}) is positive. Thus, lines are characterized by higher LGDs compared to term loans. This positive influence is decisively evident ($\text{po}\left(\text{E}[\zeta_{\text{Facility}}] > 0\right) \to \infty$ and $0 \notin \text{HPDI}\left(\zeta_{\text{Facility}}\right) = [0.15, 0.26]$). Protection ($\zeta_{\text{Protection}}$) exhibits a negative posterior mean with decisive evidence ($\text{po}\left(\text{E}[\zeta_{\text{Protection}}] < 0\right) \to \infty$ and $0 \notin \text{HPDI}\left(\zeta_{\text{Protection}}\right) = [-0.48, -0.36]$). This indicates lower loss rates for secured loans which correspond to the economic intuition. According to the negative sign of the industry FIRE (ζ_{Industry}), LGDs for loans of this industry affiliation are lower compared to other industries. This impact is decisively evident ($\text{po}\left(\text{E}[\zeta_{\text{Industry}}] < 0\right) \to \infty$ and $0 \notin \text{HPDI}\left(\zeta_{\text{Industry}}\right) = [-0.30, -0.17]$). The applied macrovariable, that is, the HPI (ζ_{HPI}), exhibits a positive sign indicating higher LGDs for higher values of the HPI. This contradicts the economic intuition and previous findings in Tobback et al. (2014) and Qi and Yang (2009), as a sound economic surrounding should be accompanied with lower loss rates. However, a positive sign is also reported in Yao, Crook and Andreeva (2017). Yet, our analysis does not indicate a positive impact which is statistically evident (po $\left(\mathrm{E}[\zeta_{\mathrm{HPI}}] > 0\right) = 2.02 < 3.2$ and $0 \in \mathrm{HPDI}\left(\zeta_{\mathrm{HPI}}\right) = [-0.22, 0.33]$). The last row of the table summarizes the posterior distribution of the random effect parameter.

Figure 4 illustrates the realizations of the random effect f_t in the LGD model. Higher realizations of the random effect ($f_t > 0$) indicate higher values of the latent variable Y^* for all loans defaulted in t and thus higher average LGDs in this quarter. The left panel of the figure presents the time patterns of f_t. The path of f_t seems to be related to the economic cycle. While the realizations of the random effect scatter around zero prior to the crisis, increased values occur after 2007 Q2. In the climax of the GFC, f_t reaches its maximum. The rebound in the aftermath of the crisis starts gradually. The right panel of the figure contrasts these time patterns of f_t to average LGDs in the time line in Figure 3. The latter include observations which are not considered in the estimation, that is, final LGDs of unresolved loans in validation sample I. Up to the more recent time periods, the random effect seems to mimic the path of average LGDs. The time series disperse afterwards, whereby the spread further increases in the time line. This deviation might be attributed to the exclusion of censored observations. The final realizations of censored observations tend to have higher LGDs. This leads to biased realizations of the random effect in the more recent time periods. The effect of censoring and the associated bias worsen in the time line, that is, the bias of f_t enlarges for higher t. Furthermore, a bias of the random effect parameter σ has to be considered, as the downward bias in the random effect realizations might erroneously increase the underlying standard deviation of the random effect. We will come back to this later on (see subsequent paragraph).

4.2.2 | Hierarchical Model

In analogy to the LGD model, the hierarchical approach is applied to the estimation sample (see Table 2). Due to the DRT model in the hierarchical approach, it is possible to include censored observations, that is, unresolved loans, in the estimation process. By this means, we are able to generate posterior predictive distributions for the DRT of unresolved cases and, thus, posterior predictive

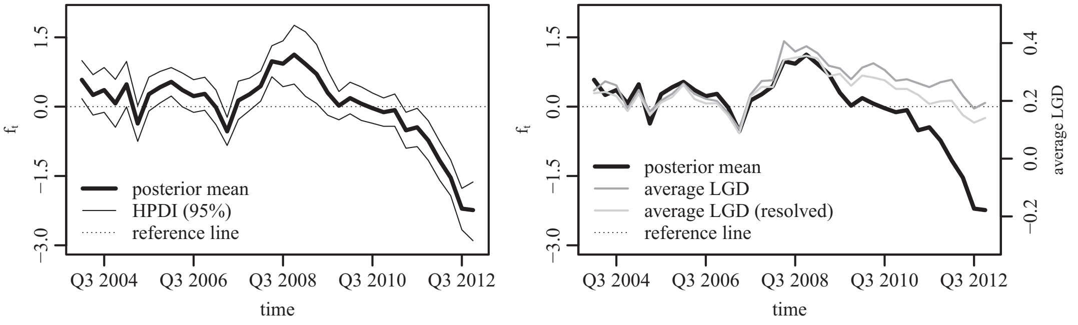

FIGURE 4 Random effect of the LGD model

Note: The figure illustrates the course of the random effect in the LGD model over time. In the left panel, the posterior means (thick line) and the HPDI (95%, thin lines) of the random effect realizations, that is, f_t, are displayed. In the right panel, the random effect (black line) is contrasted with the time patterns of average LGDs for all loans (dark grey line) and for resolved loans (light grey line). Final and non-finalized LGDs in validation sample I are included in the averaging. The dotted lines mark zero and serve as a reference line distributions for the LGD of unresolved loans. Furthermore, effects of the resolution bias as in the pure LGD model (see Figure 4) are diminished.

Table 4 summarizes the results of the hierarchical model. Parameters are stated in the first column, whereas the second column presents posterior means. Posterior distributions for the estimated component parameters (μ_k and σ_k for $k \in \{2,3,4\}$) and loan specific covariate parameters of the LGD model in the hierarchical approach ($\gamma_{\text{EAD}}, \gamma_{\text{Facility}}, \gamma_{\text{Protection}}$, and γ_{Industry}) are similar to their counterparts in the pure LGD model (see Table 3, μ_k and σ_k for $k \in \{2,3,4\}$ and $\zeta_{\text{EAD}}, \zeta_{\text{Facility}}, \zeta_{\text{Protection}}$, and ζ_{Industry}). A deviation arises for the parameter of the HPI (γ_{HPI}). In comparison with the corresponding parameter in the pure LGD model (ζ_{HPI}) it exhibits an intuitively negative sign, thus, indicating lower LGDs in sound economic surroundings which is displayed by an increasing HPI. However, the parameter of the macrovariable is still characterized by a lack of statistical evidence (po $\left(\text{E}[\gamma_{\text{HPI}}] < 0\right) = 1.18 < 3.2$ and $0 \in \text{HPDI}\left(\gamma_{\text{HPI}}\right) = [-0.13, 0.12]$). The sign switch of γ_{HPI} compared to ζ_{HPI} might be due to the inclusion of the logarithmized DRT as explanatory variable in the LGD model of the hierarchical approach (γ_T), as further systematic variables, that is, the VIX and the random effect of the DRT model, enter the LGD model through the DRT. The posterior mean of γ_T has a positive sign indicating higher LGDs for loans with higher DRTs. In Section 4.1 (see Figure 2), we determined this relation descriptively. The impact of the DRT is decisively evident (po $\left(\text{E}[\gamma_T] > 0\right) \rightarrow \infty$ and $0 \notin \text{HPDI}\left(\gamma_T\right) = [0.97, 1.03]$).

In the DRT model of the hierarchical approach, loan specific covariates and a macrovariable, that is the VIX, are included. The posterior mean of the EAD (β_{EAD}) exhibits a positive sign. Thus, loans of major size are accompanied with longer DRTs. This supports the thesis we stated in the previous subsection. Financial institutions might undertake higher resolution efforts for loans of major size. This might increase the DRTs and simultaneously lower LGDs. Decisive evidence can be stated for the positive impact of the EAD in the DRT model (po $\left(\text{E}[\beta_{\text{EAD}}] > 0\right) \rightarrow \infty$ and $0 \notin \text{HPDI}\left(\beta_{\text{EAD}}\right) = [0.03, 0.07]$). According to the negative posterior mean of lines (β_{Facility}), this facility type is accompanied with shorter DRTs compared to term loans. This impact is decisively evident (po $\left(\text{E}[\beta_{\text{Facility}}] < 0\right) \rightarrow \infty$ and $0 \notin \text{HPDI}\left(\beta_{\text{Facility}}\right) = [-0.12, -0.06]$). In analogy to the EAD, the impact of facility is reversed in the LGD and DRT model of the hierarchical approach. While lines are characterized by shorter DRTs, they result in higher LGDs. Reasons may be found in divergent resolution efforts related to the size of the loan and whether it is secured or not. The posterior mean of protection ($\beta_{\text{Protection}}$) exhibits a positive, decisively evident (po $\left(\text{E}[\beta_{\text{Protection}}] > 0\right) \rightarrow \infty$ and $0 \notin \text{HPDI}\left(\beta_{\text{Protection}}\right) = [0.10, 0.17]$) sign indicating longer DRTs for secured loans. The impact of protection is divergent among the models in the hierarchical approach ($\gamma_{\text{Protection}} < 0$ and $\beta_{\text{Protection}} > 0$). This might depend on the specific protection mechanism. If loans are secured either by collateral or guarantees, efforts have to be taken to liquidate collaterals or claim funds which are related to guarantees. This might extend DRTs, but can lead to lower LGDs if additional cash flows for the creditor are generated. The industry affiliation FIRE (β_{Industry}) reveals a negative posterior mean, thus, it is connected to shorter DRTs. The sign is decisively evident (po $\left(\text{E}[\beta_{\text{Industry}}] < 0\right) \rightarrow \infty$ and $0 \notin \text{HPDI}\left(\beta_{\text{Industry}}\right) = [-0.20, -0.11]$) and corresponds to the sign of the LGD model in the hierarchical approach ($\gamma_{\text{Industry}} < 0$ and $\beta_{\text{Industry}} < 0$). Resolution prospects in the FIRE industry might be limited compared to other industries due to less tangible assets. Thus, DRTs are short and LGDs low. To control for the impact of the macroeconomy, the VIX (β_{VIX}) is included in the DRT model of the hierarchical approach. Its posterior mean is positive and decisively evident ($\text{po}_{\text{E}[\beta_{\text{VIX}}]>0} = 7,141.86 > 100$ and $0 \notin \text{HPDI}_{\beta_{\text{VIX}}} = [0.15, 0.39]$). This entails longer DRTs in bad economic conditions which correspond to the economic intuition.

The parameters of the multivariate random effect in Equation (11) are stated in the lower panel of Table 4. As the DRT is included in the LGD model of the hierarchical approach, the random effect of

TABLE 4 Results of the hierarchical model

	Posterior mean	HPDI (95%)		Posterior odds	Naive standard error	Time series standard error
LGD model in the hierarchical approach						
μ_1	0.0000	*Not estimated*				
μ_2	0.0064	0.0062	0.0067	∞	0.0000	0.0000
μ_3	0.0279	0.0268	0.0290	∞	0.0000	0.0000
μ_4	0.5033	0.4923	0.5144	∞	0.0000	0.0000
μ_5	1.0000	*Not estimated*				
σ_1	0.0010	*Not estimated*				
σ_2	0.0043	0.0040	0.0045	∞	0.0000	0.0000
σ_3	0.0234	0.0223	0.0244	∞	0.0000	0.0000
σ_4	0.3384	0.3314	0.3453	∞	0.0000	0.0000
σ_5	0.0010	*Not estimated*				
c_1	−1.4391	−1.5803	−1.3004	∞	0.0003	0.0005
c_2	−0.5848	−0.7242	−0.4422	∞	0.0003	0.0006
c_3	0.5728	0.4306	0.7090	∞	0.0003	0.0005
c_4	2.6716	2.5262	2.8169	∞	0.0003	0.0005
γ_{EAD}	−0.1952	−0.2233	−0.1667	∞	0.0001	0.0001
γ_{Facility}	0.3259	0.2700	0.3840	∞	0.0001	0.0001
$\gamma_{\text{Protection}}$	−0.6291	−0.6932	−0.5676	∞	0.0001	0.0002
γ_{Industry}	−0.2736	−0.3437	−0.2036	∞	0.0002	0.0002
γ_{HPI}	−0.0061	−0.1287	0.1170	1.1847	0.0003	0.0005
γ_T	0.9996	0.9711	1.0280	∞	0.0001	0.0001
DRT model in the hierarchical approach						
β_0	0.7341	0.6112	0.8521	∞	0.0003	0.0006
β_{EAD}	0.0512	0.0343	0.0678	∞	0.0000	0.0000
β_{Facility}	−0.0903	−0.1238	−0.0555	∞	0.0001	0.0001
$\beta_{\text{Protection}}$	0.1345	0.0981	0.1718	∞	0.0001	0.0001
β_{Industry}	−0.1555	−0.1954	−0.1141	∞	0.0001	0.0001
β_{VIX}	0.2731	0.1514	0.3946	7141.8571	0.0003	0.0004
s	0.8488	0.8395	0.8583	∞	0.0000	0.0000
Random effect						
σ_T	0.3424	0.2627	0.4327	∞	0.0002	0.0002
σ_L	0.3615	0.2696	0.4634	∞	0.0002	0.0003
$\omega_{T,L}$	0.1863	−0.1398	0.5031	6.3057	0.0007	0.0008

Note: The table summarizes the results of the hierarchical model. Parameters are stated in the first column. Categorical variables are included via dummy coding. The reference categories are term loan for facility, no for protection, and non-FIRE for industry. The second column presents the posterior means. In the third and fourth column, lower and upper bounds of the corresponding HPDIs to a credibility level of 95% are displayed. The fifth column contains the posterior odds. Naive and time series standard errors are shown in the last two columns. Time series standard errors are calculated based on the effective chain length (N^*) instead of the actual chain length (N), whereby $N^* < N$ holds for autocorrelated chains.

the DRT model (F_t^T) enters the LGD model. Thus, the aggregated systematic impact of the random effects on LGDs ($\mathcal{F}_t$) is the linear combination of $\gamma_T F_t^T$ and F_t^L:

$$\begin{aligned}\mathcal{F}_t &= \gamma_T F_t^T + F_t^L \\ \sigma_{\mathcal{F}}^2 &= \gamma_T^2 \sigma_T^2 + \sigma_L^2 + 2\gamma_T \sigma_T \sigma_L \omega_{T,L},\end{aligned} \tag{13}$$

whereby $\sigma_{\mathcal{F}}^2$ is the variance of the aggregated systematic effect. Considering the results of Table 4, the standard deviation $\sigma_{\mathcal{F}}$ of $\mathcal{F}_t$ amounts to 0.54. This standard deviation is considerably smaller compared to the standard deviation of the random effect in the pure LGD model (see Table 3, $\sigma = 0.82$). As suspected in the previous subsection, the estimated standard deviation of the random effect in the pure LGD model seems to be biased due to the resolution bias. Ignoring censored observations, that is, unresolved loans, leads to biased realizations of the random effect (f_t) and, thus, subsequently to biased parameters (σ).

Figure 5 illustrates the realizations of the random effects of the DRT model f_t^T (upper left panel) and the LGD model f_t^L (lower left panel) in the hierarchical approach. Higher realizations of the random effect in the DRT model ($f_t^T > 0$) imply higher DRT for all loans defaulted in t, whereas higher realizations of the random effect in the LGD model ($f_t^L > 0$) lead to higher values of the latent variable $\mathcal{Y}^*$ for all loans defaulted in t and, thus, to higher average LGDs in this quarter. Hence, DRTs impact

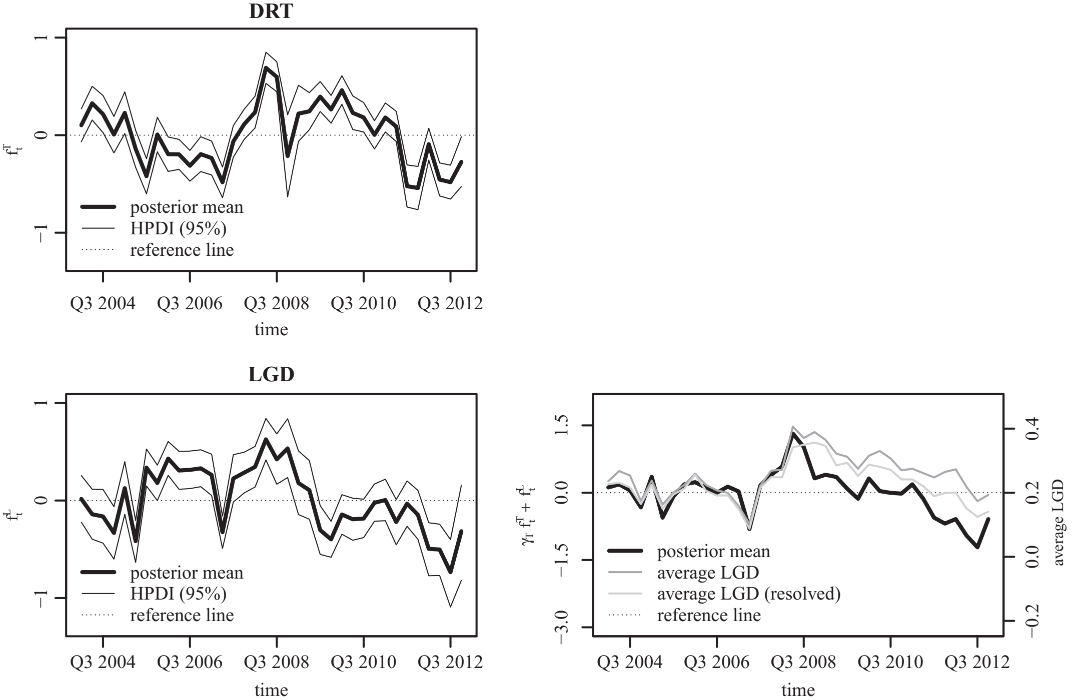

FIGURE 5 Random effect of the hierarchical model

Note: The figure illustrates the course of the random effects in the hierarchical model over time. In the left panels, the posterior means (thick lines) and the HPDI (95%, thin lines) of the random effect realizations, that is, f_t^T (DRT) and f_t^L (LGD), are displayed. In the right panel, the combined systematic effect on the LGDs according to the random effects of the hierarchical model ($\gamma_T f_t^T + f_t^L$, black line) is contrasted with the time patterns of average LGDs for all loans (dark gray line) and for resolved loans (light gray line). Final and non-finalized LGDs in validation sample I are included in the averaging. The dotted lines mark zero and serve as a reference line

LGDs in two ways (see Section 3): directly, as higher DRTs are inserted in the LGD model; indirectly, as positive realizations of f_t^T tend to imply positive realizations of f_t^L due to the positive correlation ($\omega_{T,L}$). However, the indirect channel might also weaken the impact of DRTs on LGDs as negative realizations of f_t^L are still possible. Considering the time patterns of the random effects in Figure 5, four settings of the indirect channel are apparent. In the first setting prior to the GFC, $f_t^T < 0$ and $f_t^L > 0$ are valid. Thus, average DRTs of loans defaulted in t are shorter. The positive realization of f_t^L, however, increases average LGDs. Resolutions of these loans at least partly take place during the crisis. This might depress recovery payments at the end of the resolution process and, thus, increase LGDs. The second setting in the climax of the GFC is characterized by positive realizations of both random effects ($f_t^T > 0$ and $f_t^L > 0$) indicating longer DRT and simultaneously higher LGDs of loans defaulted in t. In the third setting in the aftermath of the GFC, signs of the random effects are reversed ($f_t^T > 0$ and $f_t^L < 0$). Hence, average DRTs of loans defaulted in t are longer, whereas average LGDs are lower. This might be due to the time delay in the first setting. Analogously, parts of the recovery payments take place during the rebound period which favors recovery collection and decreases LGDs. The fourth setting is located in the most recent time period. The realizations of both random effects exhibit negative signs ($f_t^T < 0$ and $f_t^L < 0$) indicating shorter DRTs and simultaneously lower LGDs for loans defaulted in t. These settings illustrate the impacts of systematic effects in the resolution process. The positive correlation of the random effects ($\omega_{T,L}$) seems to be driven by extreme economic surroundings as synchronism appears in crises and boom periods. Furthermore, reasoning for the gradual rebound in the aftermath of the GFC can be provided (see Figure 3). While the random effect of the LGD model f_t^L indicates the rebound in the aftermath of the crisis (third setting), the random effect of the DRT model f_t^T remains at its high level. This might be due to the high stock of non-performing loans in the aftermath of the GFC which decelerated resolution proceedings. Average LGDs increase due to the direct channel.

The right panel of Figure 5 contrasts the aggregated systematic impact of the random effects ($\mathcal{F}_t$) to average LGDs in the time line. The latter include observations which are not considered in the estimation. The aggregated systematic effect seems to mimic the path of average LGDs. However, slight dispersions are apparent in the more recent time periods. Reasons might be found in a less accurate estimation of the random effect realizations of the LGD model (f_t^L) in the more recent time periods. Although censored observations are included through the DRT model, unresolved loans do not directly enter the LGD model in the hierarchical approach. Comparing the dispersions of the hierarchical model with the pure LGD model (see Figure 4), improvements are apparent. While the spread extremely increases in the time line for the pure LGD model, the deviation is considerably less pronounced in the hierarchical approach. Thus, the hierarchical approach succeeds in reducing bias of the estimated random effect.

5 | VALIDATION

As stated in Section 4.1 (see Table 2), the models are estimated based on the estimation sample. In the *in-sample* validation, the posterior predictive distributions based on the estimation sample are compared to the empirical distributions of completely resolved loans in the estimation sample. The *out-of-sample* validation examines the distributional fit for censored observations, that is, loans which have defaulted till the end of the estimation period but are still unresolved. Thus, posterior predictive distributions based on validation sample I are compared to the corresponding empirical distribution. The posterior predictive distributions are generated based on the estimated realizations of the random effect. In the *out-of-sample out-of-time* validation, loans which defaulted after the end of the estimation period are considered. As no random effect realizations are available for those loans, posterior predictive distributions are generated at the means of the random effects, that is, zero, and compared to the corresponding empirical distribution.

A detailed analysis of the distributional fit for both models can be found in the online companion of the paper. Overall, we find that both models exhibit good in-sample fits which speaks for the capability of the mixture model to capture the LGD distribution. In the out-of sample as well as out-of-time perspective, the hierarchical model shows a higher capability to capture the LGD distribution leading to more accurate LGD forecasts. A reason for this might be the inclusion of censored information which impacts LGD predictions. To understand the overall impact of censoring on LGDs in our model, we take a closer look at the validation in the time line of the posterior predictive means from both models.

5.1 | Validation in the time line

Figure 6 illustrates the time patterns of average LGD predictions based on the posterior predictive distributions for specific default quarters. The upper left panel contrasts average LGDs (thin black line) with average LGD predictions based on the LGD model (thick grey line) and the hierarchical model

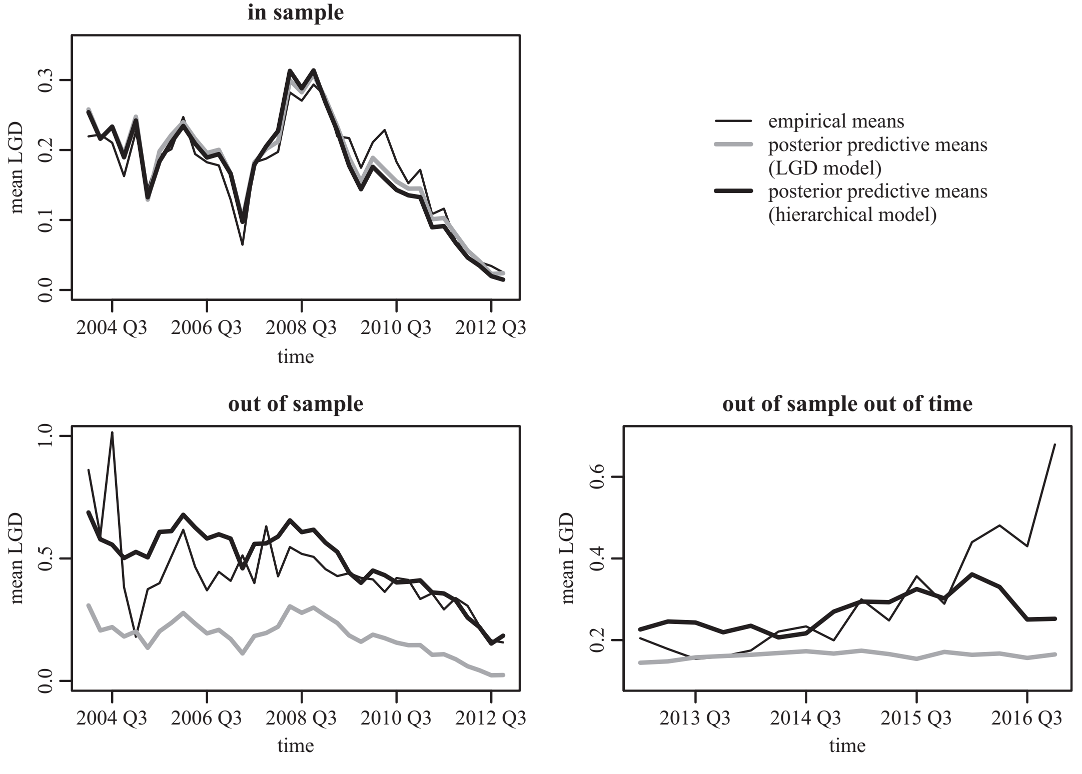

FIGURE 6 Validation in the time line

Note: The figure illustrates the validation in the time line. The means of the empirical distribution are displayed by a thin black line, whereas the means of the posterior predictive distributions are marked by a thick grey line for the LGD model and a thick black line for the hierarchical model, respectively. In the upper panel, the *in-sample* validation in the time line is presented (empirical means of resolved loans in estimation sample). The lower panels show the *out-of-sample* (empirical means of resolved loans in validation sample I) and *out-of sample out-of-time* validation (empirical means of all loans in validation sample II with non-finalized LGDs) in the time line (thick black line) for an in-sample perspective. A good in sample fit for both models can be observed. The lower left panel illustrates the time patterns of average LGDs and LGD predictions for an out-of sample perspective. Although the relative progressions of the LGD predictions based on the LGD model and the hierarchical model are similar, the predictions based on the LGD model are biased downward. Thus, average LGDs are underestimated by the LGD model in almost all quarters in validation sample I. This is not the case considering the predictions of the hierarchical model. The noisy behaviour of average LGDs at the beginning of the time period is due to a lack of data, as most loans defaulted in these quarters are resolved by the end of 2010 and, thus, not included in validation sample I. The lower right panel illustrates the time patterns of average LGDs and LGD predictions for an out-of sample out-of-time perspective. The predictions based on the LGD model seem to be constant through time, as the random effect is set to its mean, that is, zero, and the macrovariable is the only remaining systematic factor. However, the latter does not exhibit an impact which is statistically evident (see Table 3). Furthermore, LGD predictions based on the LGD model seem to be systematically too low. LGD predictions based on the hierarchical model better fit average LGDs. Deviations at the end of the time period might be attributed to the inclusion of non-finalized LGDs for unresolved cases (see Figure 3). Final LGDs will be lower and adjust the line downwards. In addition, LGD predictions based on the hierarchal model display systematic movement, as macrovariables with statically evident impact in the DRT model are included in the LGD model of the hierarchical approach (see Table 4).

6 | CONCLUSION

In this paper, we thoroughly examine the dependence-structure of DRTs and LGDs using a hierarchical modelling framework. Previous approaches do either not take censoring of LGD values into account or directly apply a censoring mechanism to accumulated recovery payments during default resolution. These payments are a discrete process of single events, while default resolution time is of continuous nature. This is why we model (censored) DRT first and hierarchically build on DRTs and DRT estimates from unresolved loans to generate unconditional as well as conditional LGD estimates for unresolved loan contracts. This substantially diminishes censoring effects for LGDs, reduces the bias of parameter estimates and leads to better out-of-sample and out-of-time LGD estimates.

Furthermore, we find direct and indirect dependencies among the credit risk parameters. First, LGDs seem to be directly impacted by DRTs, that is, longer resolution processes are accompanied with higher losses. Second, the parameters are characterized by common time patterns as correlation of the random effects in the individual models is positive. Due to the random nature of these effects, the dependency of DRTs and LGDs might be intensified or weakened in certain time periods. We find similar signs of the random effect realizations during the GFC and deviating signs pre- and post-crisis. Due to the consideration of direct dependency structures, we are able to generate intuitive LGD predictions for censored cases. These are of high practical relevance in the light of the recent EBA guidelines (see European Banking Authority, 2017). Besides LGD predictions for the non-defaulted exposure (unconditional predictions), financial institutions are required to predict LGDs for the defaulted exposure conditional on post-default information—such as the time in default. The hierarchical approach diminishes bias of parameter estimates due to the exclusion of censored observations in a pure (standard) LGD model and, thus, enables adequate unconditional LGD predictions for the non-defaulted exposures and consistent conditional LGD predictions for the defaulted exposures within one modelling framework.

Nevertheless, a number of limitations and possibilities for improvement accompanying our model approach. One challenge is the choice of independent variables for DRTs and LGDs. While a multitude of LGD studies exists whose findings can be used to select independent variables, we are only aware of the study by Betz, Kellner and Rösch (2016) which analyzes DRT drivers in detail. We select independent variables in our study based on their findings and the availability in our database. Previous findings suggest that country-specific laws and regulation might have a strong impact on DRTs, thus, including this kind of information could be a further step to refine the estimation and prediction of DRTs. This could also lead to additional improvements for LGD predictions. Furthermore, in contrast to researchers, banks should have access to detailed information regarding single payments during default resolution. Using this kind of information together with resolution time might also be of value for further model development. This could be subject to future research as it combines approaches which use accumulated payments as censored information and our approach which uses censored DRT.

Concluding, the consideration of censored observations is essential to generate suitable LGD predictions. The presented hierarchical model prevents the need of additional data constraints and provides fruitful insights into the dependency structure of DRTs and LGDs. Moreover, our approach might not only be relevant for DRT and LGD modelling. At first glance, the described setting seems to be a rather special case in credit risk management. However, possible applications are diverse. Generally, duration processes are subject to censoring. Whenever time-dependent result variables on a metric scale are of main interest, censoring should be considered to avoid underestimation (or overestimation respectively, if negative dependencies between time and result are present, i.e. $\beta < 0$). Examples might be found in business where complex negotiations lead to outcomes on a metric scale, for example, granting loans (negotiation process vs. granted amount). Moreover, applications in completely different fields are conceivable, for example, medicine and health science (healing process vs. resulting quality or strength). The developed hierarchical approach might be adjusted to different applications regarding the characteristics of the outcome variable. In this paper, we follow Betz, Kellner and Rösch (2018) and apply a FMM to model the distribution of LGDs which has been well proven in prior publications to capture the challenging shape exhibiting high probability masses at 0 and 1 and its typical bimodal form. In a setting in which the result variable follows a different distribution, the FMM can be replaced by other distributions, for instance (skewed) normal, Student or logistic distributions for unimodal or lognormal or gamma distributions for non-zero results variables.

REFERENCES

Altman, E. & Kalotay, E. (2014) Ultimate recovery mixtures. *Journal of Banking and Finance*, 40(1), 116–129.

Basel Committee on Banking Supervision. (2017) Basel III: Finalising post-crisis reforms. *Bank for International Settlements*.

Bellotti, A., Brigo, D. & Gambetti, P. (2019) *Forecasting recovery rates on non-performing loans with machine learning*. Social Science Research Network: SSRN Scholarly Paper.

Bellotti, T. & Crook, J. (2012) Loss given default models incorporating macroeconomic variables for credit cards. *International Journal of Forecasting*, 28(1), 171–182.

Betancourt, M. (2017) A conceptual introduction to Hamiltonian Monte Carlo. arXiv:1701.02434.

Betz, J., Kellner, R. & Rösch, D. (2016) What drives the time to resolution of defaulted bank loans? *Finance Research Letters*, 18(1), 7–31.

Betz, J., Kellner, R. & Rösch, D. (2018) Systematic effects among loss given defaults and their implications on downturn estimation. *European Journal of Operational Research*, 271(3), 1113–1144.

Betz, J., Krüger, S., Kellner, R. & Rösch, D. (2017) Macroeconomic effects and frailties in the resolution of non-performing loans. *Journal of Banking and Finance*, forthcoming.

Bijak, K. & Thomas, L. (2015) Modelling LGD for unsecured retail loans using Bayesian methods. *Journal of the Operational Research Society*, 66(2), 342–352.

Bris, A., Welch, I. & Zhu, N. (2006) The costs of bankruptcy: Chapter 7 liquidation versus Chapter 11 reorganization. *Journal of Finance*, 61(3), 1253–1303.

Calabrese, R. (2014) Downturn loss given default: Mixture distribution estimation. *European Journal of Operational Research*, 237(1), 271–277.

Denis, D. & Rodgers, K. (2007) Chapter 11: Duration, outcome, and postreorganization performance. *Journal of Financial and Quantitative Analysis*, 42(1), 101–118.

Dermine, J. & Neto de Carvalho, C. (2006) Bank loan losses-given-default: A case study. *Journal of Banking and Finance*, 30(4), 1219–1243.

European Banking Authority. (2017) Guidelines on PD estimation, LGD estimation and the treatment of defaulted exposure.

Gambetti, P., Gauthier, G. & Vrins, F. (2019) Recovery rates: Uncertainty certainly matters. *Journal of Banking & Finance*, 106, 371–383.

Gambetti, P. (2020) Meta-learning approaches for recovery rate prediction. Working Paper.

Grunert, J. & Weber, M. (2009) Recovery rates of commercial lending: Empirical evidence for German companies. *Journal of Banking and Finance*, 33(3), 505–513.

Gürtler, M. & Hibbeln, M. (2013) Improvements in loss given default forecasts for bank loans. *Journal of Banking and Finance*, 37(7), 2354–2366.

Helwege, J. (1999) How long do junk bonds spend in default? *Journal of Finance*, 54(1), 341–357.

Höcht, S., Kroneberg, A. & Zagst, R. (2011) Explaining aggregated recovery rates. *Working paper*.

Höcht, S. (2010) Loan recovery determinants—A Pan-European study. *Working paper*.

Joubert, M., Verster, T. & Raubenheimer, H. (2019) Making use of survival analysis to indirectly model loss given default. *Orion*, 34(2), 107–132.

Kalotay, E. & Altman, E. (2017) Intertemporal forecasts of defaulted bond recoveries and portfolio losses. *Review of Finance*, 21(1), 433–463.

Kaposty, F., Kriebel, J. & Löderbusch, M. (2020) Predicting loss given default in leasing: A closer look at models and variable selection. *International Journal of Forecasting*, 36(2), 248–266.

Kass, R. & Raftery, A. (1995) Bayes factors. *Journal of the American Statistical Association*, 90(430), 773–795.

Krüger, S. & Rösch, D. (2017) Downturn LGD modeling using quantile regression. *Journal of Banking and Finance*, 79(1), 42–56.

Loterman, G., Brown, I., Martens, D., Mues, C. & Baesens, B. (2012) Benchmarking regression algorithms for loss given default modeling. *International Journal of Forecasting*, 28(1), 161–170.

McLachlan, G. & Peel, D. (2000) *Finite mixture models*. Hoboken, NJ: Wiley.

Nazemi, A., Fatemi Pour, F., Heidenreich, K. & Fabozzi, F.J. (2017) Fuzzy decision fusion approach for loss-given-default modeling. *European Journal of Operational Research*, 262(2), 780–791.

Nazemi, A., Heidenreich, K. & Fabozzi, F.J. (2018) Improving corporate bond recovery rate prediction using multi-factor support vector regressions. *European Journal of Operational Research*, 271(2), 664–675.

Partington, G. & Russel, P. (2001) Predicting return outcomes to shareholders from companies entering Chapter 11 bankruptcy. *Managerial Finance*, 27(4), 78–96.

Qi, M. & Yang, X. (2009) Loss given default of high loan-to-value residential mortgages. *Journal of Banking and Finance*, 33(5), 788–799.

Qi, M. & Zhao, X. (2011) Comparison of modeling methods for loss given default. *Journal of Banking and Finance*, 35(11), 2842–2855.

Stan Development Team. (2016) Stan modeling language. User's Guide and Reference Manual.

Sun, H .S. & Jin, Z. (2016) Estimating credit risk parameters using ensemble learning methods: An empirical study on loss given default. *Journal of Credit Risk*.

Tanoue, Y., Kawada, A. & Yamashita, S. (2017) Forecasting loss given default of bank loans with multi-stage model. *International Journal of Forecasting*, 33, 513–522.

Tobback, E., Martens, D., Gestel, T.V. & Baesens, B. (2014) Forecasting loss given default models: Impact of account characteristics and the macroeconomic state. *Journal of the Operational Research Society*, 65(3), 376–392.

Wei, L.J. (1992) The accelerated failure time model: A useful alternative to the Cox regression model in survival analysis. *Statistics in Medicine*, 11(14–15), 1871–1879.

Witzany, J., Rychnovsky, M. & Charamza, P. (2012) Survival analysis in LGD modelling. *European Financial and Accounting Journal*, 7(1), 6–27.

Wong, B., Partington, G., Stevenson, M. & Torbey, V. (2007) Surviving chapter 11 bankruptcies: Duration and payoff? *Abacus*, 43(3), 363–387.

Yao, X., Crook, J. & Andreeva, G. (2015) Support vector regression for loss given default modelling. *European Journal of Operational Research*, 240(2), 528–538.

Yao, X., Crook, J. & Andreeva, G. (2017) Enhancing two-stage modelling methodology for loss given default with support vector machines. *European Journal of Operational Research*, 263(2), 679–689.

Zhang, J. & Thomas, L.C. (2012) Comparison of linear regression and survival analysis using single and mixture distributions approches in modelling lgd. *International Journal of Forecasting*, 28, 204–215.

SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section.

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