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Credit line exposure at default modelling using Bayesian mixed effect quantile regression

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Abstract

For banks, credit lines play an important role exposing both liquidity and credit risk. In the advanced internal ratings-based approach, banks are obliged to use their own estimates of exposure at default using credit conversion factors. For volatile segments, additional downturn estimates are required. Using the world's largest database of defaulted credit lines from the US and Europe and macroeconomic variables, we apply a Bayesian mixed effect quantile regression and find strongly varying covariate effects over the whole conditional distribution of credit conversion factors and especially between United States and Europe. If macroeconomic variables do not provide adequate downturn estimates, the model is enhanced by random effects. Results from European credit lines suggest that high conversion factors are driven by random effects rather than observable covariates. We further show that the impact of the economic surrounding highly depends on the level of utilization one year prior default, suggesting that credit lines with high drawdown potential are most affected by economic downturns and hence bear the highest risk in crisis periods.

KEYWORDS

credit conversion factor, credit risk, exposure at default, global credit data, quantile regression, random effects

1 | INTRODUCTION

Credit lines are the dominant funding source for companies all around the world (see Lins et al., 2010; Segura & Zeng, 2020). In the United States—a traditionally rather market-oriented country—80% of small- and medium-sized enterprises (SME) heavily rely on these funding instruments (see Sufi, 2009) and credit lines are the second most important debt financing category for listed companies (see Colla et al., 2013). Acharya et al. (2014), Acharya and Mora (2015) and Acharya et al. (2020) argue that credit lines are important for the economy in general as they provide (short-term) liquidity to corporations to sustain investments. Particularly in crisis periods when credit quality deteriorates, credit lines ensure that companies can maintain their operations and contribute to sustain investments and liquidity (see also Agarwal et al., 2006; Barraza & Civelli, 2020; Berrospide & Meisenzahl, 2015; Cornett et al., 2011; Gatev & Strahan, 2006). As a flip-side, they expose banks to both higher liquidity and credit risk. Ivashina and Scharfstein (2010) show that there was a bank run in the global financial crisis (GFC) inducing high liquidity risk. Following Acharya et al. (2013) and Acharya and Mora (2015), banks with undrawn lines become riskier due to this additional risk in times of increased aggregated volatility.

In addition to the well-documented liquidity risk, credit lines—such as loan contracts in general—also expose banks to credit risk. In this paper, we focus solely on defaulted credit lines, as we are interested in the dimensions of credit risk induced by the type of loan. In the advanced internal ratings-based (IRB) approach of the Basel regulations, banks are obliged to use their own estimates of the three central credit risk parameters—the probability of default (PD), the loss given default (LGD) and the exposure at default (EAD)—to calculate their capital requirements for loans. For credit lines, the EAD is particularly important because a bank's credit risk exposure is increased when a credit line is drawn and volatile over time.

While the literature on PD and LGD modelling has widened considerably during the last two decades, less attention has been paid to EAD modelling. Literature on EAD modelling can roughly be divided into direct and indirect approaches. Direct modelling of EAD usually involves multistage models (Hon & Bellotti, 2016; Leow & Crook, 2016; Thackham & Ma, 2019; Tong et al., 2016). In contrast, indirect approaches are based on conversion factors which can be interpreted as additional drawdowns on the credit line in a specific time period, for example one year prior to default (see Section 2). As this is also the approach required by Basel regulations (see Basel Committee on Banking Supervision, 2017, §241, §242), we follow this strand of literature. While indirect approaches allow for beneficial interpretations, they are challenging, that is conversion factors tend to exhibit extreme bimodal distributions—comparable to loss rate distributions—and are characterized by high amounts of outliers. Regardless, many studies use a classical linear OLS regression framework (see Araten & Jacobs, 2001; Moral, 2011; Qi, 2009). Barakova and Parthasarathy (2013) additionally apply median regression which is more robust to outliers. Although not recommended by the Basel regulations (see Basel Committee on Banking Supervision, 2017, §247), several studies trim or winsorize the data (see Araten & Jacobs, 2001; Barakova & Parthasarathy, 2013; Jacobs & Bag, 2011; Moral, 2011; Qi, 2009; Yang & Tkachenko, 2012). First suggestions to consider the distributional features of conversion factors are multi-stage models (see Valvonis, 2008) or beta regression (see Jacobs, 2010). Yang and Tkachenko (2012) find single-layer neural networks to be superior, indicating that conversion factors might not be linearly related to covariates. However, neural networks lack economic interpretability and transparency which hampers application for regulatory purposes.

The risky position of a bank is not only increased by higher exposures when credit lines are drawn, but also through a link between credit line usage and default that was found by several studies (see Araten & Jacobs, 2001; Jacobs, 2010; Jacobs & Bag, 2011; Jiménez et al., 2009; Qi, 2009; Valvonis, 2008; Zhao et al., 2014). Hence, obligors seem to draw heavier when tumbling towards default. In the literature, there is an ongoing debate regarding the impact of macroeconomic variables, and whether credit line-specific risk increases in economic downturns. Jiménez et al. (2009), Gatev and Strahan (2006), and Sufi (2009) find statistical evidence that firms tend to draw more lines in economic downturns, while Barakova and Parthasarathy (2013) report higher EADs in contraction (pre-crises) periods compared to crises. Zhao et al. (2014) find statistically significant higher conversion factors during recession periods. Thackham and Ma (2019) even state weak evidence of counter-cyclic patterns in the Global Financial Crisis, that is a negative relation of EADs and default rates. In general, the identification of meaningful and statistically evident macroeconomic variables is of high relevance with respect to modelling EAD and conversion factors. In analogy to loss rates, estimates of conversion factors for (economic) downturns are also mandatory for volatile segments in Basel regulations (see Basel Committee on Banking Supervision, 2017, §242) which is hampered by a lack of statistically evident systematic variables. With respect to the literature, conversion factors are almost exclusively estimated with mean-related methods (such as OLS), although the distribution is highly bimodal. Therefore, conclusion with respect to the mean, which is rarely observed, may not be representative for the whole distribution. Furthermore, the bimodality may lead to heterogeneous (varying) covariate effects for the different parts of the distribution. This may also be an explanation of the lack of statistically evident systematic variables. For a detailed discussion of heterogeneous covariate effects, we refer to Koenker (2005). Therefore, we argue that using a quantile regression may be more representative for this challenging setting. Additionally, individual quantile functions enable financial institutions to better differentiate between loans and their inherent risk profile.

Given the importance of credit lines and their relation to the macroeconomy, as well as the lack of clear evidence in the literature, this paper provides the following contributions. First, this paper is innovative by investigating the downturn, that is crisis periods, characteristics of credit lines for the first time and comparing two important regions, namely Europe and United States. Furthermore, our evidence is based on one of the world's largest international datasets with respect to defaulted credit lines. Second, we apply a novel approach to model conversion factors. Because of the regulatory requirements for conversion factors and their bimodal distribution which can hardly be tackled by linear OLS regression, we apply a Bayesian quantile regression (QR) approach. Therefore, this paper is—to the best of our knowledge—the first to model the full conditional distribution of credit conversion factors. We show that the QR approach yields an up to twice as good distributional fit, compared to the OLS regression in an out-of-time forecasting exercise. Additionally, we show that the impact of covariates strongly varies across quantiles, which cannot be captured by standard regression techniques. This suggests that there are severe differences in the determinants of low or high additional drawdowns and between regions, which is not documented in the literature so far. Third, we deeply investigate the impact of macroeconomic variables and their ability to generate sufficiently conservative downturn estimates, as required by Basel regulations. We find that evidence of macroeconomic variables seems to vanish in the tails of the distribution and for credit lines which exhibit high utilization, that is lines which are drawn heavily one year prior to default. Thus, credit lines with high risk (low utilization one year prior default) are particularly affected by the economic surrounding. Systematic variation which cannot be measured by macroeconomic variables is modelled via time-specific random effects. This allows us to create adequate downturn estimates, even in settings where the identification of meaningful and evident macroeconomic variables is unfeasible. Furthermore, it offers banks and regulators an approach to incorporate their individual margin of conservatism for capital requirements of credit lines in stressed periods.

The remainder of this paper is structured as follows. Section 2 presents the data of defaulted credit lines. In Section 3, Bayesian quantile regression—including the extension by time-specific random effects—is introduced. The main results are outlined in Section 4. Finally, Section 5 concludes.

2 | DATA

Summarizing the literature reviewed in Section 1, EADs might be modelled directly or indirectly by means of conversion factors. The latter represent additional drawdowns with respect to an observed limit, balance or difference at a specific time t. Hereby, a more complete picture of the drawdown behaviour of defaulted credit lines can be modelled. For example, (possible) different drivers for low and high additional drawdowns can be determined. Furthermore, the use of conversion factors is recommended by the Basel Accord (see Basel Committee on Banking Supervision, 2017, §241–§250).

Generally, conversion factors should be estimated with a fixed-horizon approach, that is all predictions should be linked to information 12 months prior to default (see Basel Committee on Banking Supervision, 2017, §245). Therefore, in the following the time stamp t refers to 12 months before the default in T. A rigorous discussion of advantages and disadvantages of various horizon approaches can be found in Gürtler et al. (2018). In general, the conversion factors consist of a composition of the following variables. Balance_t is the drawn amount of the credit line at time t, Limit_t is the available amount provided by the financial institution up to with the obligor can draw the line, and EAD_T is the drawn amount of the credit line at the time of default T. In the literature, four common conversion factors can be found: The loan equivalent exposure $\left(\text{LEQ, calculated by } \frac{\text{EAD}_T - \text{Balance}_t}{\text{Limit}_t - \text{Balance}_t}\right)$, the credit conversion factor $\left(\text{CCF, calculated by } \frac{\text{EAD}_T}{\text{Balance}_t}\right)$, the exposure at default factor $\left(\text{EADF, calculated by } \frac{\text{EAD}_T}{\text{Limit}_t}\right)$ and the additional utilization factor $\left(\text{AUF, calculated by } \frac{\text{EAD}_T - \text{Balance}_t}{\text{Limit}_t}\right)$. As the nomenclature of these factors is not universally defined, we follow the definitions of Leow and Crook (2016). A discussion about the drawbacks of the first three conversion factors can be found in Leow and Crook (2016) and Thackham and Ma (2019). The AUF is suggested by Yang and Tkachenko (2012) and found to be suitable for corporate credit lines by Barakova and Parthasarathy (2013) and the following analysis also focuses on AUF. While incorporating the limit as well as the balance at time t, it is stable for almost completely drawn lines. The AUF is undefined if the limit one year prior default is exactly zero. However, these credit lines are of minor concern in estimating credit risk due to their low potential of additional drawdowns. Furthermore, extreme values occur only if the limit one year prior default is extremely small compared to the additional drawdown.[1] Due to these benefits and the limited drawbacks,

[1]Note that an AUF of one indicates that the additional drawdown is equal to the limit one year prior default. This can only occur if there is no balance one year prior default.

we apply the AUF in the following analysis. For robustness, we also run our analysis using the EADF, but find no differences regarding our contributions.[2]

We use access to the world's largest loss and exposure database which is collected by Global Credit Data (GCD).[3] This cooperative consists of 55 globally acting member banks all around the world encompassing several systemically important institutions. The access to a unique sample of defaulted US American and European corporate credit lines provides exclusive insights accessing a large and important proportion of the banking universe. We use a sample from 2006 until the end of 2018. The database contains information about balance and limit at the time of default and one year prior to default. We use the fixed-horizon approach for calculating the AUF which is in line with the Basel Accord.

Imposing a materiality threshold of 500 Euro[4] and using only credit lines where all independent variables are available, we have 14,382 credit lines in Europe and 4432 credit lines in the United States. To reduce the problem of extreme values, we restrict the range of AUF values to $[-0.5, 1.5]$. By including negative AUFs, variables which impact balance reduction until default can be identified, whereas AUFs greater than one enable us to look deeper into the drivers of extreme additional drawdowns beyond the prearranged limits. These are possible due to accumulated interest or banks allowing borrowers to draw beyond their limits, resulting in values greater than 1. With respect to the interval, we delete 3466 credit lines in Europe and 390 in the United States, corresponding to 24.10% and 8.80% of the sample. In Europe, 2976 of the deleted credit lines have limits of zero one year prior default which implies a non-defined AUF.[5] As these credit lines have a low EAD potential, these observations are of minor economic concern. Values with limits greater than zero account for 3.34% in Europe.

Table 1 compares descriptive statistics of the AUF and applied covariables in the two regions. For metric variables the means and a range of quantiles are displayed. For each level of categorical variables, the means and quantiles of the AUF are shown.

Comparing the variable age, which represents the number of years from origination of the credit line until one year prior default, European lines are on average more than twice as old. This may be attributed to the fact, that in Europe it is much more common to have tight and long-lasting business relationships to banks with respect to funding, whereas in the United States, companies are usually more often funded by capital markets (see Antoniou et al., 2008). Furthermore, it is apparent that the AUF differs among regions—especially in higher quantiles as (positive) additional drawdowns are much more common in Europe. This is in line with the observation that Utilization, which represents the percentage of how much is already drawn one year prior default, is higher in the United States. In the first quartile, the lines are drawn up to 80%, whereas in Europe, only up to 48%. Due to the higher utilization in the United States, the potential of additional drawdowns is limited which might result in a lower AUF.

To control for the economic surrounding, we include the year-on-year growth of the Gross Domestic Product (GDP), labelled as ΔGDP in the final model. We also considered other macroeconomic variables, such as stock market growth, changes in house prices, volatility indexes,

[2]Rerunning our analysis using CCF would be counterintuitive, as we would have to omit the most risky credit lines, which are especially important in crisis periods. Furthermore, as the LEQ has these two severe drawbacks and is only weakly defined in our sample, an additional analysis would not add any robustness.

[3]GCD is a non-profit organization aiming to support its member banks in understanding and modelling credit risk parameters such as LGD and EAD by, inter alia, collecting and pooling detailed loss and exposure information of defaulted loan contracts including credit lines (for further information see https://www.globalcreditdata.org/).

[4]This is in line with the materiality threshold of the European Banking Authority (2016).

[5]In the US, only 34 lines have a limit of zero one year prior default.

TABLE 1 Descriptive statistics

Variable	Level	Quantiles 0.05	0.25	0.5	0.75	0.95	Mean	STD	Obs.
(a) USA									
AUF		−0.33	−0.07	0.00	0.03	0.59	0.03	0.26	4042
log(Limit)		9.69	11.63	12.96	14.51	16.72	12.86	2.92	4042
Age		0.10	0.84	1.99	3.79	7.92	2.73	2.63	4042
Utilization		0.15	0.80	1.00	1.00	1.00	0.84	0.28	4042
ΔGDP		−0.04	0.00	0.02	0.02	0.03	0.01	0.02	4042
Facility type	Medium term revolver	−0.31	−0.06	0.00	0.04	0.61	0.04	0.26	3250
	Short term revolver	−0.37	−0.11	0.00	0.00	0.45	−0.01	0.24	792
Seniority	Pari-passu	−0.40	−0.20	−0.02	0.00	0.61	−0.03	0.28	1010
	Super senior	−0.28	−0.05	0.00	0.06	0.54	0.04	0.25	1550
	Non senior	−0.36	−0.10	−0.02	0.01	0.77	0.03	0.31	150
	Unknown	−0.23	−0.03	0.00	0.06	0.59	0.06	0.25	1332
Industry	Finance, insurance, real estate (FIRE)	−0.28	−0.05	−0.02	0.00	0.42	−0.01	0.21	754
	Agriculture, forestry, fishing (AFF)	−0.34	−0.12	−0.01	0.02	0.38	−0.01	0.24	133
	Mining (MIN)	−0.39	−0.15	0.00	0.21	0.80	0.06	0.34	133
	Construction (CON)	−0.38	−0.05	0.00	0.11	0.55	0.04	0.27	428
	Manufacturing (MAN)	−0.34	−0.10	0.00	0.10	0.75	0.06	0.31	528
	Transp., commu., sanitary serv. (TCEGS)	−0.26	−0.04	0.00	0.06	0.75	0.06	0.28	223
	Wholesale and retail trade (WRT)	−0.35	−0.09	0.00	0.05	0.60	0.02	0.27	541
	Services (SER)	−0.32	−0.05	0.00	0.05	0.61	0.04	0.27	830
	Other (OTH)	−0.30	−0.08	0.00	0.00	0.30	−0.01	0.19	472

(Continues)

TABLE 1 (Continued)

		Quantiles							
Variable	**Level**	**0.05**	**0.25**	**0.5**	**0.75**	**0.95**	**Mean**	**STD**	**Obs.**
(b) Europe									
AUF		−0.31	−0.01	0.03	0.36	1.04	0.20	0.41	10, 916
log(Limit)		8.13	9.90	11.38	12.90	15.58	11.51	2.25	10, 916
Age		0.00	1.21	3.43	6.48	19.26	5.25	6.80	10, 916
Utilization		0.00	0.48	0.97	1.00	1.00	0.72	0.39	10, 916
ΔGDP		−0.05	−0.01	0.01	0.02	0.03	0.00	0.02	10, 916
Facility type	Medium term revolver	−0.32	−0.03	0.01	0.18	0.86	0.11	0.32	3206
	Short term revolver	−0.29	0.00	0.00	0.06	0.95	0.09	0.31	379
	Overdraft	−0.31	0.00	0.06	0.50	1.12	0.25	0.44	7331
Seniority	Pari-passu	−0.29	0.00	0.04	0.38	1.06	0.21	0.41	9835
	Super senior	−0.39	−0.08	0.00	0.17	0.94	0.08	0.34	981
	Non senior	−0.40	−0.16	0.04	0.46	0.88	0.14	0.41	100
Industry	Finance, insurance, real estate (FIRE)	−0.28	−0.01	0.01	0.26	1.03	0.18	0.40	2723
	Agriculture, forestry, fishing (AFF)	−0.29	−0.01	0.05	0.36	1.00	0.20	0.38	426
	Mining (MIN)	−0.20	−0.02	0.04	0.40	1.13	0.24	0.45	50
	Construction (CON)	−0.31	0.00	0.08	0.59	1.19	0.27	0.46	1138
	Manufacturing (MAN)	−0.36	−0.02	0.02	0.39	1.05	0.20	0.42	1069
	Transp., commu., sanitary serv. (TCEGS)	−0.34	−0.04	0.02	0.35	1.01	0.18	0.41	518
	Wholesale and retail trade (WRT)	−0.34	−0.03	0.05	0.37	1.06	0.20	0.42	2057
	Services (SER)	−0.31	0.00	0.13	0.62	1.18	0.30	0.46	979
	Other (OTH)	−0.24	0.00	0.03	0.22	0.96	0.15	0.34	1956

Note: The table shows means, standard deviations and quantiles for the AUF and the metric variables. For categorical variables, means, standard deviations and quantiles of the AUF for each level are displayed. The macro variable growth of the Gross Domestic Product (GDP) is lagged by one year. The variables log(Limit) and Utilization correspond to the logarithm of the limit respectively the utilization of the credit line one year prior to default.

interest rate spreads, unemployment rates and overall liquidity. ΔGDP has the highest and most evident impact among all tested variables. Following Betz et al. (2018), we use one macro variable in the final model, as they are highly correlated which might influence their statistical inference. Furthermore, our results in Section 4.2 show that the remaining systemic variation can be easily captured with the introduced random effect, avoiding issues with highly correlated macroeconomic variables. We further include line-specific variables. Facility-type controls for different revolving types of credit line and their maturity (overdraft,[6] short- and medium-term revolver). Additionally, the order of claims in the resolution process is included via different levels of Seniority.[7] Log(Limit) controls for the size of the credit line with respect to the available limit one year prior default. We also tested whether the size of the company is a driver of the AUF, but found no evident effect. The impact of the company size may be absorbed by the log(Limit) as larger firms usually require larger credit lines.[8] Furthermore, in the literature, the borrower rating is found to be suitable to model additional drawdowns for non-defaulted and defaulted credit lines. However, as we focus on *defaulted* credit lines using the fixed horizon approach, the ratings of the credit lines probably worsen for all defaulted lines one year prior default. To check this, we use a subsample of our data for which we have ratings, but find no difference between the rating categories in terms of the AUF distribution, and a very large part has a non-investmentgrade rating. This is similar to Thackham and Ma (2019), who do not include ratings in their final model for EAD prediction either.

The left panels of Figure 1 illustrate the kernel density estimates of the AUF. The probability mass around zero is more pronounced in the United States, whereas the probability mass around one is greater in Europe. The right panels of Figure 1 illustrate the time patterns of the average AUF (solid black line) and its 75% quantile (black dotted line). Hereby, differences among the regions occur. The average AUF is lower in the United States compared to Europe. The Global Financial Crisis and its aftermath is much more pronounced in Europe. This is especially true focusing on the 75% quantile where the values increased considerably in the GFC and the subsequent quarters. Summarizing, time varying behaviour is present in both regions, whereas it is more pronounced in Europe. This may be attributed to the fact of generally higher utilization one year prior default in the US American sample. To investigate this in more detail, we illustrate the distribution of Utilization depending on the realized AUF in Figure 2.

Lines with positive and negative AUFs seem to clearly differ in the level of utilization one year prior default. In Figure 2, the solid line illustrates the utilization of credit lines with positive AUFs and the dashed line represents credit lines with negative AUFs. Obligors with negative AUFs have more extensively drawn than obligors with positive AUFs. In Europe, there are many more credit lines with almost no and very high utilization one year prior to default, whereas in the United States, there is a more equal level of utilization for positive AUFs.

Overall, there is also evidence that the time varying behaviour is quantile-dependent. Usually, an explanation for different systematic behaviour may be different default definitions. In this study, all loans have the same default definition according to Basel Committee on Banking

[6]In general, the Basel Accord does not require banks to estimate credit conversion factors for non-revolving lines, like overdrafts. Instead, a comparatively low CCF of 10% is assigned. The descriptive statistics, however, show that these type of lines have a much greater potential of additional drawdowns. Hence, we include them in our sample to investigate their behaviour as well.

[7]Super senior refers to a priority order where only one creditor has prior claims. If there is at least another claimant on the same rank, the seniority is defined as pari-passu.

[8]We also tested other credit-line-specific characteristics such as collateral, but did not find an evident impact, similar to Thackham and Ma (2019).

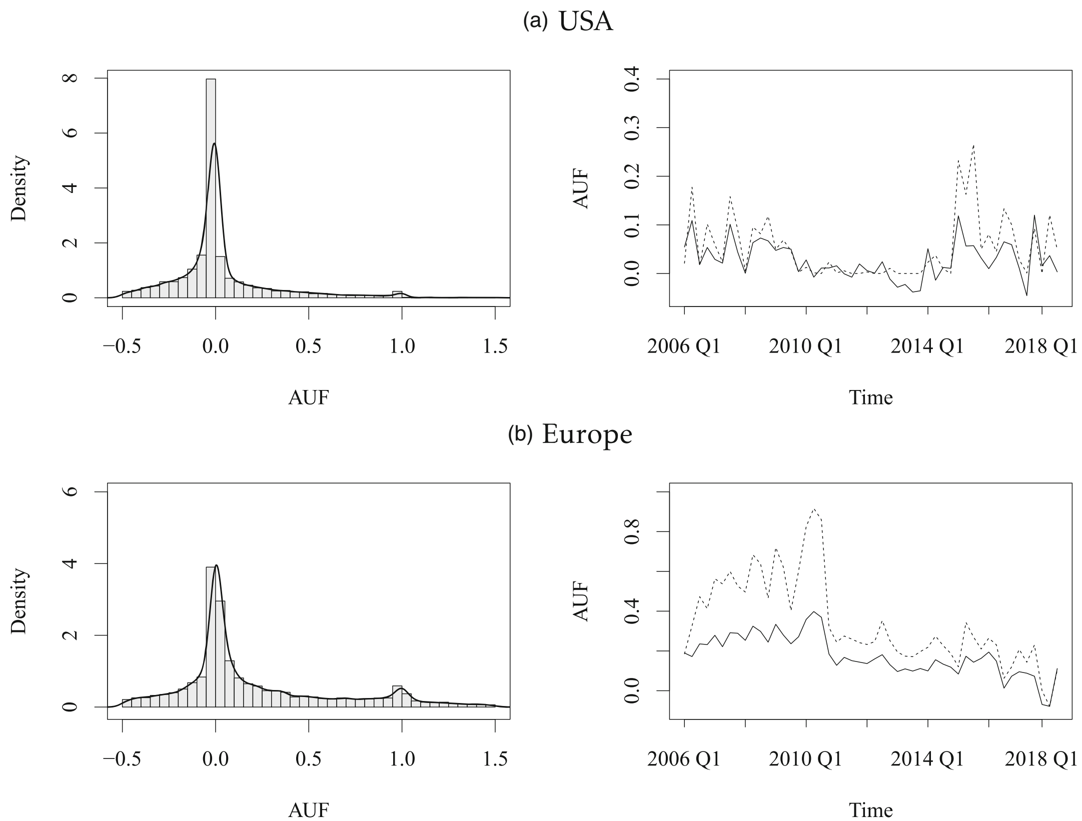

FIGURE 1 Distribution and time variation of additional utilization factor (AUF). *Note*: The left panels of the figure show the distribution of the AUF separated by regions. The black lines represent the kernel density estimates, whereas the grey bars illustrate the histograms. The right panels illustrate the time patterns of the AUF divided by regions. The solid lines represents the mean in the quarter of default and the dotted line is the 75% quantile.

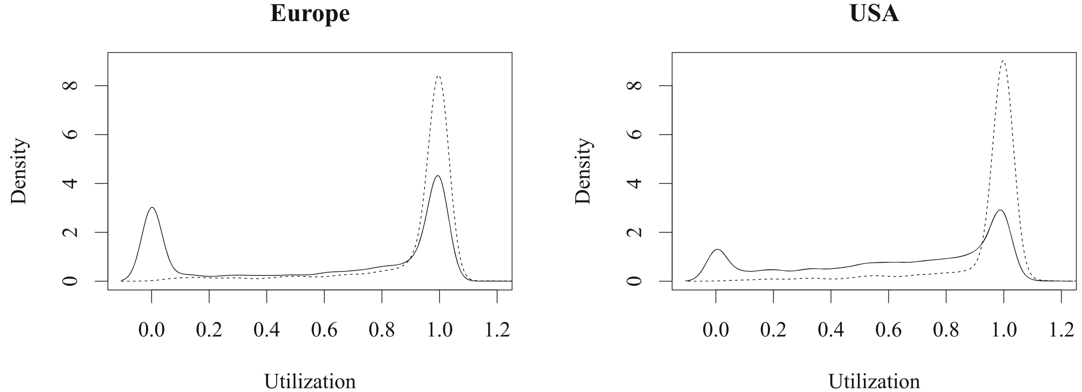

FIGURE 2 Distribution of utilization level 1 year prior to default. *Note*: The figure shows the distribution of the level of Utilization separated by positive and negative AUFs. The solid line represents the density of the Utilization for lines with a positive additional drawdown (positive AUF) and the dashed line illustrates the density of the Utilization with exposure reduction (negative AUF).

Supervision (2017). Hence, we can eliminate the possibility that different systematic behaviours are attributed to different default definitions.

3 | METHODOLOGY

With respect to the extreme bimodal distribution of the AUF (see left panels of Figure 1), analysis regarding the conditional mean of the distribution—such as a classical linear regression—may not be favourable as rigorously shown by Krüger and Rösch (2017). Modelling the entire distribution instead infers more comprehensive results. Furthermore, the impact of variables may differ over the distributional range. This is especially true in the existing setting as positive and negative AUFs are jointly modelled. Therefore, we analyse additional drawndowns using quantile regression introduced by Koenker and Bassett (1978) which allows us to model the full conditional distribution of the response variable.[9] As each quantile is modelled separately by a linear regression, a more comprehensive picture of the distribution is obtained. Additionally, it allows for varying impacts of covariates over the entire distributional range. This enables us to detect the (different) drivers of low and high additional drawdowns. These implications are important to financial institutions as they can adjust their line management and, hence, distinguish between low and high drawdowns more exactly.

In the quantile regression approach, each quantile τ of the dependent variable Y is modelled based on a linear function. The corresponding regression function is

$$y_i = x_i\beta(\tau) + \epsilon_i(\tau), \tag{1}$$

where y_i represents the ith observation of the response variable and x_i is the known covariate vector which includes a one for the τ-dependent intercept. The vector $\beta(\tau)$ contains the unknown parameters including the intercept and $\epsilon_i(\tau)$ is the quantile-specific error term. Assuming expectation $Q_\tau(\epsilon_i(\tau)) = 0$, the expected τ-quantile of the response variables is given by $Q_\tau(y_i|x_i) = x_i\beta(\tau)$ for $0 < \tau < 1$. The τ-specific estimates of $\beta(\tau)$ are obtained by minimizing the objective function with respect to $\beta(\tau)$:

$$\sum_{i=1}^{n}\rho_\tau\left(y_i - x_i\beta(\tau)\right)$$
$$\text{with } \rho_\tau(\omega) = \begin{cases} \tau\omega, & \text{if } \omega \geq 0, \\ (1-\tau)|\omega| & \text{else.} \end{cases} \tag{2}$$

According to Koenker and Bassett (1978), the minimization problem of Equation (2) is solved with simplex algorithms. Yu and Moyeed (2001) and Yu and Zhang (2005) linked the minimization to the maximum likelihood theory via the asymmetric Laplace distribution (ALD). This distribution is parametrized by μ, σ, and τ. The random variable ϵ follows the ALD as its probability density is:

$$f(\epsilon|\mu,\sigma,\tau) = \frac{\tau(1-\tau)}{\sigma}\exp\left\{-\rho_\tau\left(\frac{\epsilon-\mu}{\sigma}\right)\right\},$$
$$\text{with} -\infty < \mu < \infty,\ 0 < \tau < 1,\ \text{and } \sigma > 0, \tag{3}$$

[9] See Kellner et al. (2022) for a neural network version of the quantile regression.

where ρ_τ is the objective function defined in Equation (2). The parameter μ determines the location, τ controls the skewness, and σ is the variance. In general, σ can be considered as a nuisance parameter and the skewness parameter τ corresponds to the desired quantile. Therefore, maximizing Equation (3) with respect to μ is equivalent to solving the minimization problem in Equation (2). Yu and Moyeed (2001) argue that the resulting posterior is valid even if it is a misspecification of the true error and Sriram et al. (2013) provide a theoretical justification for posterior consistency under the ALD misspecification. The location parameter changes to $\mu_i = x_i\beta(\tau)$ and, for a fixed skewness parameter τ, the likelihood function—up to a proportional constant (see Luo et al., 2012)—results in

$$L(\beta(\tau), \sigma | y, \tau) \propto \sigma^{-1} \exp\left\{ -\sum_{i=1}^{n} \rho_\tau \left(\frac{\epsilon - \mu_i}{\sigma} \right) \right\}. \tag{4}$$

Geraci and Bottai (2007) extended this approach to include a mixed effects model by including a random effect. In this setting, we implement a time-specific random effect F to account for clustering in the time line.[10] According to Geraci and Bottai (2007), the regression function of Equation (1) (and, thus, the location parameter) changes to

$$y_i = x_i\beta(\tau) + F(\tau) + \epsilon_i(\tau), \tag{5}$$

where $\epsilon(\tau) \sim AL(0, \sigma_\epsilon)$ and $F(\tau) \sim N(0, \sigma_F(\tau))$. The realization of $F(\tau)$ corresponds to the quarter of default, for example, 2008 Q3, of the obligor. Therefore, obligors which default in the same quarter are exposed to the same τ-dependent realization of the random effect. The model in Equation (5) can be seen as a mixed effect model, where we treat impact of the covariates $\beta(\tau)$ as fixed and the impact of the time variation $F(\tau)$ as (additional) random intercept. Following Section 2, time patterns of AUFs vary among quantiles. Hence, it may be favourable to assign each quantile an individual impact of the random effect. Equations (2)–(4) apply to the model with random effects by analogy.[11]

The models (with and without random effects) are estimated via Bayesian inference as the likelihood in Equation (4) cannot be maximized analytically. The posterior distribution is

[10] Alternatively, one could use time-specific dummies to control for the remaining time variation. However, this might have at least two drawbacks. First, we want to use our model for predicting future conversion factors. Therefore, predicting an appropriate value for a future time-dummy is not straightforward. Second, with respect to the downturn estimates, the random effects structure gives financial institutions as well as prudential regulators a great flexibility to apply their margin of conservatism individually.

[11] Alternatively, we could have used finite mixture models as in Calabrese (2014), Altman and Kalotay (2014), Kalotay and Altman (2017), Betz et al. (2018) or Betz et al. (2021) for Losses Given Default (LGDs). These models assume a latent variable which describes the affiliation to individual components of the mixture model and use observable and unobservable covariates to model this latent variable. Some of these studies include a time-specific random intercept, as we did, and evaluate the impact of this time variation on the latent variable. However, the ordered logit or probit does not allow a direct link between changes in the latent variable and the resulting affiliation probabilities to the mixture components. An increase of the latent variable results in a higher probability of the highest component and a lower probability for the lowest component. However, the impact on intermediate components can not be inferred directly. Therefore, we think that the interpretation in terms of quantiles and the impact of the random effect on each quantile allows for a more direct interpretation. Moreover, one can think of fitting an unconditional mixture model on the conversion factor's distribution, following Tomarchio and Punzo (2019) for LGD estimation. As we observe different shapes of the conversion factor's distribution for different facility types or industries in our sample, we would have to redo the inference for many subsets of our data.

generated via Markov chain Monte Carlo (MCMC) procedure. By constructing reversible Markov chains, the algorithm samples from the posterior distribution which corresponds to the target distribution in the equilibrium. More details on the estimation and the specified prior distributions for every parameter in the model can be found in Appendix A.

Alternatively, frequentistic approaches could be used following, for example, Geraci and Bottai (2007), Chernozhukov et al. (2013), Galvao et al. (2013), Galvao and Kato (2017), Graham et al. (2018) or Galvao and Poirier (2019). However, the Bayesian framework has some favorable properties. Following the statements by Yu et al. (2005); Yue and Rue (2011) and Bernardi et al. (2015) the Bayesian quantile regression provides estimations and predictions which take into account parameter uncertainty. This is especially interesting if the sample size is not extensively large. Furthermore, inferring distributions instead of point estimates of the parameters contributes to a more comprehensive understanding, see, for example, Bernardi et al. (2015), and the interpretation of credibility intervals, for example highest posterior density intervals (HPDIs), is quite intuitive. Additionally, the convergence and stability for extreme quantiles can easily be assessed using the standard tools of Bayesian inference. With rising computational power, the estimation of Bayesian models is fairly efficient using standard software. Moreover, recent literature suggests that Bayesian quantile regressions are especially suitable for tail risk estimations, see for example Carriero et al. (2020), Clements et al. (2020) and Ferrara et al. (2021). Summarizing, we think that a Bayesian mixed effect quantile regression is a reasonable choice for modelling the challenging distribution of the AUF.

As we use a default database, there might be a concern regarding endogeneity in particular due to sample selection. Meaning, that our target variable is only observed after default and is censored otherwise. This could imply that the sample is not representative for the population. However, the endogeneity problem arises only if there is a dependence between the censoring event (i.e. the default) and the resulting AUF. This problem may be alleviated by including the time-to-default into the modelling framework. However, this metric is not known before default and, thus, it is difficult to estimate. An alternative solution might be the joint modelling of AUF and the probability of default and account for their dependencies via copulae, see, for example, Krüger et al. (2018). More specifically regarding the methods employed in this article, Arellano and Bonhomme (2017) propose a correction method for (frequentistic) quantile regressions in the case of sample selection by 'rotating' the check function by an amount that depends on the strength of selection. However, one has to quantify the strength of selection a priori. There is some evidence for sample selection regarding LGD, see, for example Rösch and Scheule (2014) or Krüger et al. (2018). To the best of our knowledge, there is no study which focuses on the dependence between probability of default and conversion factors and, thus, it is difficult to determine the potential impact of endogeneity in our empirical application. However, the question of sample selection in conversion factor models is certainly a interesting path of future research.[12]

We further include the ordinary-least-squares (OLS) regression as a benchmark for our novel approach. This model focuses on the conditional mean of the distribution by neglecting varying impacts through the bimodal distribution. However, it is the most common method in literature, see e.g. Barakova and Parthasarathy (2013), Jacobs (2010), Jacobs and Bag (2011), Qi (2009) and Zhao et al. (2014). We estimate this regression in a Bayesian framework using uninformed priors such that the posterior means coincide with the point estimates in the frequentistic framework.

[12]We would like to thank an anonymous associate editor for suggesting this discussion.

4 | EMPIRICAL RESULTS

In this section, we present the empirical results based on a subsample from 2006 to mid-2016. The remaining observations are used in an out-of-time validation at the end of this section. We start with the quantile regression without random effects—labelled as *Macro Only Model* (see Equation (1) and Section 4.1)—to investigate the impact of the independent variables on the AUF distribution in the United States and Europe. Afterwards, we look deeper in crisis periods and evaluate the model's ability to provide an AUF downturn distribution comparable to the one observed in the GFC. As the Macro Only Model only provides a sufficiently conservative downturn distribution in the United States, we include a time-specific random effect in the quantile regression for Europe. This model is labelled as *Random Effects Model* (see Equation (5) and Section 4.2). It provides sufficiently conservative downturn distributions for Europe.

To interpret the models in Bayesian terms, we follow two coherent concepts. The first is based on posterior odds which are used to quantify the statistical evidence of the posterior means' estimated signs. Posterior odds coincide with the Bayes factor if the prior odds are equal to one. This is true for any symmetric prior distribution with a mean of zero. Since we assume a normal distribution with a mean of zero as prior for each parameter in the β vector (see Appendix A), the posterior odds are equal to the Bayes factor.

They are defined as the ratio of the posterior probability that the parameter is negative and the posterior probability that the parameter is positive:

$$\text{Posterior odds}_{\beta(\tau)<0} = \frac{\mathbb{P}(\beta(\tau) < 0|\text{data})}{\mathbb{P}(\beta(\tau) \geq 0|\text{data})}$$
$$\text{Posterior odds}_{\beta(\tau)>0} = \frac{\mathbb{P}(\beta(\tau) > 0|\text{data})}{\mathbb{P}(\beta(\tau) \leq 0|\text{data})}$$

Therefore, we can directly quantify the evidence favouring the sign of the posterior means, for example posterior odds of 10 indicate that it is 10 times more likely that the sign of the posterior mean is true compared to the opposite sign. Based on Kass and Raftery (1995), posterior odds greater than 3.2 indicate substantial evidence, values exceeding 10 correspond to strong evidence and posterior odds larger than 100 to decisive evidence.

The second concept to evaluate the evidence of posterior means are HPDIs. These intervals quantify a range of the posterior distribution in which the unobservable parameter is located with a given probability, for example 95%. If zero is not included in the HPDI, statistical evidence for the sign of the posterior mean is assigned. For all model parameters, we assume non-informative priors as we do not impose a direction of impact. Nevertheless, due to the two coherent concepts, we are able to learn about the relation of covariates and AUF in a consecutive step.

4.1 | Macro Only Model

In this subsection, results of the Macro Only Model and OLS with all variables described in Table 1 plus an interaction between ΔGDP and Utilization, that is ΔGDP·Utilization, are presented. This interaction gives us insights, whether the impact of the macroeconomy depends on the level of Utilization. This could have important implications for risk management practice in general and for credit line exposure at default in particular. We choose for each categorical variable a reference category, which is indicated in brackets in the first column of Table 2. This table compares the posterior means of the parameter estimates for the 5%, 50%, 95% quantile and the OLS regression in the United States and Europe. Appendix D shows some conversion diagnostics of the estimated models.[13]

For interpretation, please note that the AUF distribution is negative for quantiles lower than the median and positive for quantiles greater than the median. Therefore, a negative posterior mean indicates a higher amount of exposure reduction for the left part of the distribution and a lower additional drawdown in the right part of the distribution. As there is a direct link between AUF and EAD in terms of lower or higher values, we can interpret the posterior means interchangeably for EAD and AUF. An increase of AUF results in an increase of EAD and vice versa. In Table 2, the coefficients vary over the quantiles and (in many cases) change their signs. This underpins the assumption that credit lines which reduce exposure are differently impacted by the independent variables than credit lines with positive additional drawdowns. This observation cannot be accounted for in the OLS model, where impacts are related only to the conditional mean. Hence, conclusions regarding positive or negative impacts of covariates for all levels of AUF are not possible. The applied quantile regression approach is well suited to consider this quantile-varying influence. Furthermore, setting AUFs outside the tolerated range back to the limits, for example, 0 or 1, which is common in the EAD literature, might distort the results gathered from these models. This can be seen by the different signs of coefficients for positive and negative additional drawdowns. Setting outliers back to the limits may also hamper the identification of significant drivers of credit conversion factors.

In the United States, we find decisive evidence that short-term revolving lines have lower additional drawdowns and larger exposure reductions compared to medium-term lines. These findings are valid in Europe for the positive part of the response distribution. Contrary, we find decisive evidence that another kind of short-term lines—so called overdrafts—have higher additional drawdowns compared to medium term lines. To summarize, short-term lines in the United States have lower EADs, whereas in Europe it depends on the type of credit line. A possible explanation may be that overdrafts are less in the focus of monitoring processes as they are unconditionally revocable. With respect to the results, we may see that these lines, however, also expose credit risk to banks.

With respect to seniority, we find decisive evidence that non-senior credit lines draw less, respectively, reduce more exposure than pari-passu in the United States. In Europe, we find decisive evidence that non-senior lines draw considerably more compared to pari-passu lines. The variable log(Limit) controls for the size of the credit line with respect to the limit one year prior to default. We find decisive evidence that larger lines reduce more or draw less additional exposure. This might be explained by the fact that banks monitor larger lines more tightly than smaller lines. The variable Age shows decisively evident negative signs for the quantiles

[13]The estimation of quantile regressions can be challenging in the tails of the distribution due to a very low number of observations, as for example outlined by Chernozhukov (2005). This is frequently the case if we think about distributions like normal, logit or Cauchy. However, considering the distribution of the conversion factors we can detect differences to the aforementioned distributions. We observer considerable more realizations in the tails of the distribution compared to the middle as both modes are at 0 and 1. Therefore, in our application, the tails of the distribution are well observed. Similar observations can be found in Krüger and Rösch (2017) and Kellner et al. (2022), who found no instability problems concerning LGD as target variable. Furthermore, we check for every estimated quantile regression the common convergence checks which were all satisfied as outlined in our Appendices. Alternative approaches for extreme quantiles can be found in Alhamzawi (2016), Huang and Chen (2015), Tian et al. (2017) or Hu et al. (2021).

TABLE 2 Results | Macro Only Model & OLS

Variable	Level	$\tau = 0.05$	$\tau = 0.50$	$\tau = 0.95$	OLS
(a) USA					
Intercept		0.128[c]	0.599[c]	1.125[c]	0.704[c]
Facility	Short term revolver	−0.042[c]	−0.010[b]	−0.019[c]	−0.042[c]
Industry (FIRE)	Agriculture	−0.120[c]	−0.008[a]	0.018[a]	−0.026
	Mining	−0.115[c]	−0.074[c]	−0.024[b]	−0.087[c]
	Construction	−0.075[c]	−0.017[c]	0.018[b]	−0.052[c]
	Manufacturing	−0.072[c]	−0.011[b]	0.115[c]	−0.023[a]
	Transportation	−0.008	0.001	0.076[c]	−0.009
	Wholesale	−0.088[c]	−0.016[c]	0.038[c]	−0.046[c]
	Service	−0.070[c]	−0.010[b]	0.041[c]	−0.027[a]
	Other	−0.070[c]	−0.008[a]	0.003	−0.050[c]
Seniority (pari-passu)	Super senior	0.080[c]	0.014[c]	−0.098[c]	−0.003
	Non senior	−0.037[c]	0.008[a]	−0.103[c]	−0.034[a]
	Unknown	0.133[c]	0.025[c]	−0.125[c]	−0.030[b]
log(Limit)		−0.016[c]	−0.011[c]	−0.007[c]	−0.017[c]
Age		−0.003[b]	−0.002[c]	−0.004[c]	−0.006[c]
ΔGDP		−0.129	−2.922[c]	−0.207	−1.100[b]
Utilization		−0.234[c]	−0.480[c]	−0.870[c]	−0.475[c]
Interaction		0.179	2.902[c]	−0.398[a]	1.076[a]
(b) Europe					
Intercept		0.132[c]	0.815[c]	1.099[c]	0.731[c]
Facility (medium term)	Short term revolver	0.017[a]	0.015[b]	−0.013[b]	0.027
	Overdraft	−0.029[c]	0.012[c]	0.220[c]	0.045[c]
Industry (FIRE)	Agriculture	−0.013[a]	0.004	0.117 [c]	0.044[a]
	Mining	0.029[a]	0.007	0.611[c]	0.110
	Construction	−0.050[c]	−0.007[a]	0.047[c]	−0.001
	Manufacturing	−0.053[c]	−0.019[c]	0.056[c]	−0.014
	Transportation	−0.065[c]	−0.021[c]	0.037[b]	−0.001
	Wholesale	−0.050[c]	−0.020[c]	0.019[a]	−0.027[b]
	Service	−0.043[c]	−0.009[b]	0.121[c]	0.011
	Other	−0.039[c]	−0.027[c]	0.015[a]	−0.054[c]

(Continues)

TABLE 2 (Continued)

Variable	Level	$\tau = 0.05$	$\tau = 0.50$	$\tau = 0.95$	OLS
Seniority (pari-passu)	Super senior	−0.040[c]	0.001	0.045[c]	−0.002
	Non senior	−0.045[c]	0.060[c]	0.461[c]	0.143[c]
log(Limit)		−0.013[c]	−0.010[c]	−0.037[c]	−0.028[c]
Age		−0.002[c]	0.000	0.004[c]	0.000
ΔGDP		−0.114[a]	−1.997[c]	−0.255[a]	−0.869[c]
Utilization		−0.269[c]	−0.687[c]	−0.295[c]	−0.382[c]
Interaction		0.393[b]	1.978[c]	2.569[c]	1.088[c]

Note: This table shows the estimated posterior means for several selected quantiles. The first column inherits the name of the different independent variables. If they are categorical, the reference group is indicated in brackets. The second column illustrates the different levels of categorical variables. Statistical evidence is indicated by the following [a], [b], [c]: [a]corresponds to substantial evidence (Odds >3.2), [b]corresponds to strong evidence (Odds > 10), [c]corresponds to decisive evidence (Odds >100). The quantiles are chosen as they roughly correspond to negative drawndowns, almost no drawdowns and very high drawdowns.

in the United States. Thus, obligors with a short business relationship draw more, respectively, reduce less. Banks may not know these obligors well and, hence, it is harder to foresee default and the drawdowns of the firm one year prior to default. In Europe, we find the same pattern for reductions, but the contrary sign for high additional draws. This might be explained by the fact that the overall business relationship is longer and, in some cases, longstanding obligors may be granted more financial leeway to draw their lines in the hope that default may be prevented.

Figure 3 illustrates the impact of the variables ΔGDP, Utilization and their interaction term over the full response distribution, based on the Macro Only Model illustrated in Table 2. In Appendix C, figures of all remaining independent variables are presented. We can clearly see that the posterior mean of all three variables varies considerably over the response distribution. The posterior mean (solid line) of ΔGDP is evidently negative for large parts of the distribution as the 95% HPDI (dotted line) does not include zero. The negative sign indicates an increase of the AUF in economic downturns, that is when ΔGDP is negative. This is in line with Figure 1 as quantiles of the AUF increase in the GFC. However, there is no statistically evident impact of the macroeconomic variable in the tails of the response distribution. This lack of evidence cannot be revealed by the OLS model, which underpins that our approach may be better suited to the non-linear impact of macroeconomic variables on the AUF and further reveals novel results to the literature of EAD modelling. This also suggests that the systematic of high additional drawdowns cannot be captured with the observable macrovariable and hence, downturn estimates may be difficult to obtain.

Regarding Utilization, we find a throughout evidently negative impact on the AUF distribution indicating that the exposure reduction increases and, respectively, the additional drawndowns decrease with increasing Utilization. The latter effect may be explained by the fact that the potential of additional drawdowns is limited with higher utilization one year prior to default. Furthermore, credit lines with exposure reductions are heavily drawn one year prior default (see Figure 2).

We include an interaction term between ΔGDP and Utilization to control for a different impact of the macroeconomic environment with respect to the available limit. The interaction term has an evidently positive posterior mean in large parts of the response distribution. The total impact of the macroeconomic variable with respect to the level of Utilization is:

$$\text{Total effect} = \beta^{(-)}_{\Delta\text{GDP}} + \beta^{(+)}_{\text{Interaction}} \cdot \text{Utilization}.$$

The overall negative impact of ΔGDP decreases with a higher Utilization as the interaction term is positive throughout the quantiles in both regions (see lower panel of Figure 3). For example, at the 50% quantile, the overall (negative) impact of the macroeconomic environment in Europe is reduced from −1.598 for 20% of utilization to −0.02 for 99% of utilization. Thus, the macroeconomic environment, especially in the inner quantiles, is more relevant for less drawn credit lines and less important for heavily drawn lines. This is plausible as less drawn lines have a higher drawdown potential which can be affected by economic downturns. Furthermore, the macroeconomic environment seems to be less important for credit lines with exposure reductions as they draw heavily one year prior default. This might have substantial consequences for credit risk management as crises affect those parts of the exposure distribution which bear higher risk—in terms of higher EADs.

4.1.1 | Downturn estimation based on Macro Only Model

In this paragraph, we investigate the ability of the Macro Only Model to produce appropriate downturn distributions—comparable to the one observed in the GFC. Hereby, we assume an adverse realization of the macroeconomic variable ΔGDP to adopt an economic downturn. The

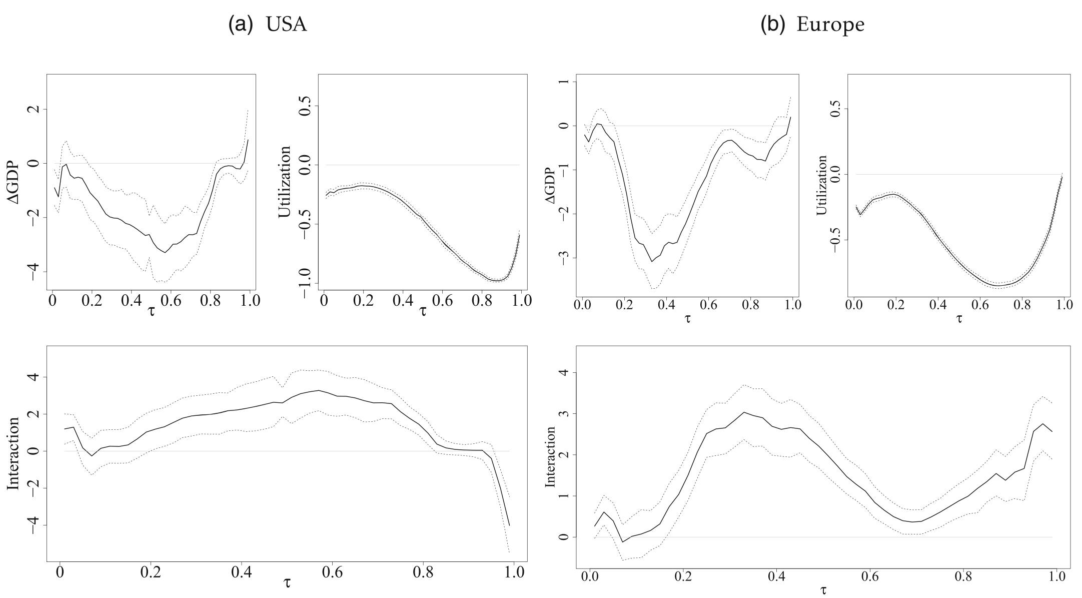

FIGURE 3 Results | Macro Only Model (coefficient plots). *Note*: The left three plots of the figure show the estimated coefficients for ΔGDP, Utilization and the interaction term over the whole distributional range in the United States. The black lines represent the posterior means, whereas the dotted lines illustrate 95% highest posterior density intervals. The right three plots illustrate the estimated coefficients in Europe.

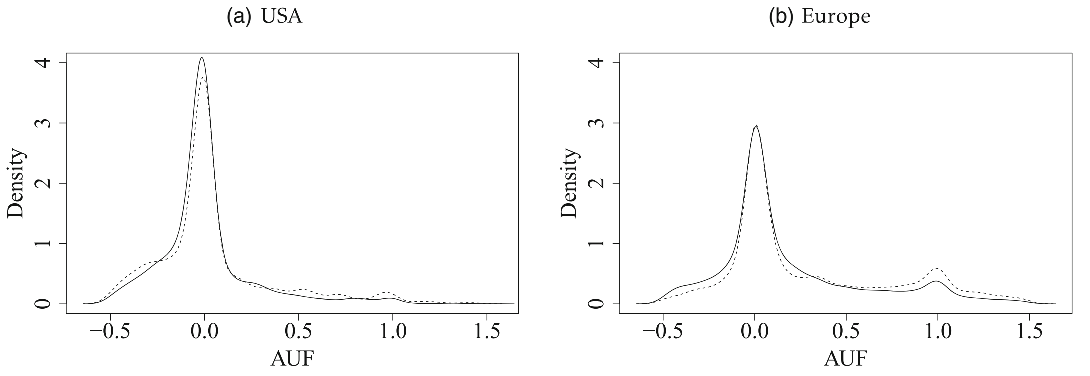

FIGURE 4 Distribution of AUF in the global financial crisis (GFC). *Note*: The figure illustrates kernel density estimates of the AUF during the GFC (grey line) and the remaining periods in the sample (black line). With respect to the comparability of the density estimates, the same bandwidth was applied to both regions.

adverse realization is set to −5.5% in Europe and −3.9% in the United States, corresponding to the 95% quantile of the observed growth rates in the sample period.

Figure 4 compares the density of the AUF during the GFC (crises distribution, dashed lines) and in the remaining time period (non-crises distribution, solid lines). According to the OECD,[14] the GFC lasts from 2007 Q4 to 2009 Q2 in the United States, whereas it is slightly shifted in Europe (2008 Q1 to 2009 Q3).

In the United States, the crises and non-crises distributions are very similar. This is in line with Figure 1 where only small variations of the AUF over time and slightly higher AUFs during the GFC arise. Contrary, there is less probability mass on exposure reduction ($AUF < 0$) and much more mass on higher additional drawdowns ($AUF \geq 1$) in Europe, indicating a substantial impact of the GFC.

To evaluate the fit of the posterior predictive distribution and the empirical distribution, we use probability–probability (PP) plots following Michael (1983). Hereby, the empirical and theoretical quantiles are compared. The empirical quantiles $p_{\text{empirical,i}}$ are generated via the posterior predictive distribution $\hat{F}(y_i)$, whereas the theoretical quantiles $p_{\text{theoretical},i}$ are calculated from the data:

$$p_{\text{empirical},i} = \hat{F}(y_i), \qquad \text{and} \qquad p_{\text{theoretical},i} = \frac{i - 0.5}{n} \tag{6}$$

where the credit lines $i = 1, \ldots, n$ are ordered by y_i to ensure monotone increasing quantiles $\hat{F}(y_i)$.[15] The compliance of all theoretical and empirical quantiles indicate perfect fit. Graphically, a perfect fit is obtained when the points in the PP plot lie on the bisecting line. If the points are above the bisecting line, the crisis distribution is underestimated, for example to little mass on high additional drawdowns, and vice versa. For the PP plot of the estimation sample, the points lie on the bisection line perfectly, thus, in-sample perfect fit is achieved for the *Macro Only Model*. Contrary, the OLS shows considerable deviations.[16]

[14]The recession indicators of the OECD are available at https://fred.stlouisfed.org/series/USARECDM for the US Area and available at https://fred.stlouisfed.org/series/EUROREC for the European Area.

[15]Having obtained the posterior distributions of the parameters after fitting the models, they can be used for predicting the desired quantile by sampling from the model using the information of the selected (new) observations.

[16]The corresponding figures for the estimation sample are available from the authors upon request.

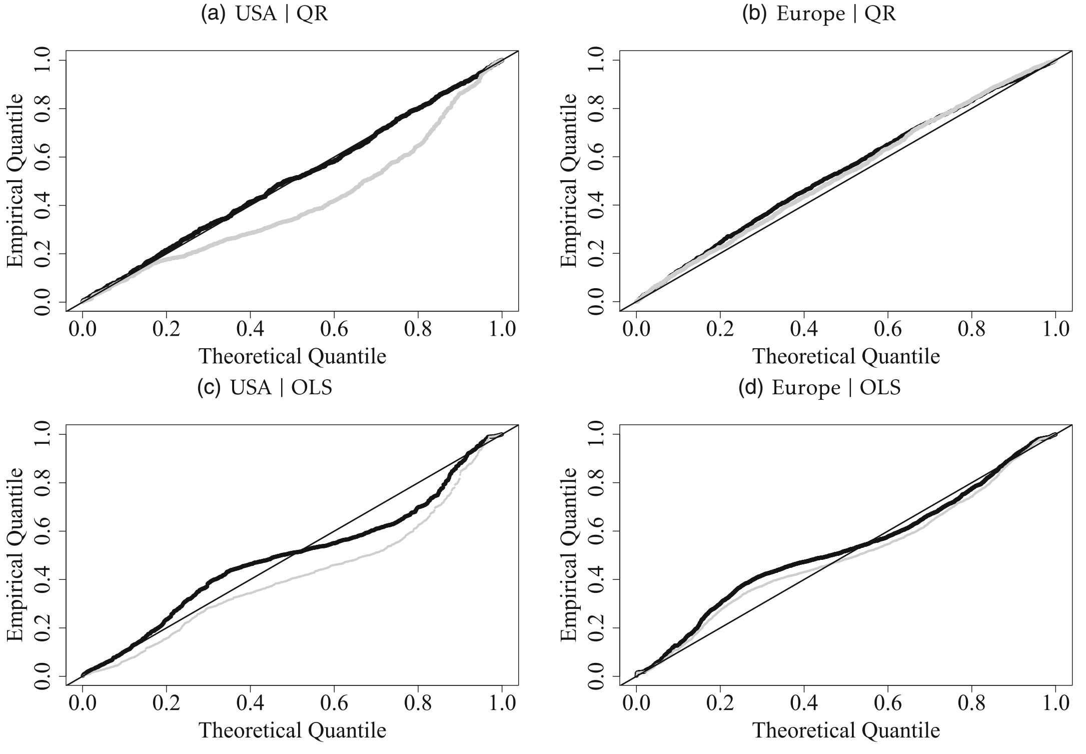

FIGURE 5 Distributional fit in downturn periods | macro only model & OLS. *Note*: The figure shows the distributional fit in the Global Financial Crisis separated by regions. The black lines indicate the fit of the posterior predicted distribution, whereas the grey lines illustrate the fit using a stress scenario. The stress scenario is generated by considering an extreme value of the macro variable ΔGDP for each obligor defaulting during the crisis period. We used the 95% quantile of ΔGDP during the whole sample period. For the US, the extreme value corresponds to −3.9% and to −5.5% for Europe. An underestimation of the empirical crisis distribution is indicated by a PP-line above the bisecting line. Contrary, overestimation, that is a too conservative posterior predictive distribution, is indicated by a line below the bisecting line.

Figure 5 illustrates the distributional fit in a downturn period, that is the GFC, for the United States (left panel) and Europe (right panel). The black points indicate the PP plot of the posterior predictive distribution. In the United States, the Macro Only Model produces an almost perfect fit. This might be expected as the crises and non-crises distribution do not substantially differ (see Figure 4). However, the linear model deviates strongly from the bisecting line, showing a rather poor distributional fit. In Europe, the empirical distribution is underestimated in the GFC as the points are above the bisecting line. Hence, the posterior predictive distribution is not sufficiently conservative. Again, the OLS provides a considerably lower fit.

To generate a stressed posterior predictive distribution, an extreme realization of ΔGDP is applied. We use the 95% quantile of ΔGDP which corresponds to −3.9% in the United Staets and −5.5% in Europe. According to the negative posterior mean of ΔGDP, a negative realization results in a higher AUF. In Figure 5, the grey dots correspond to the stressed predictive distribution. The stressed predictive distribution is too conservative in the United States which might have been expected as the posterior predictive distribution already delivers a perfect fit. Contrary, the stressed predictive distribution is still not conservative enough in Europe. This might be due to two reasons. First, ΔGDP does not have an evident impact on the tails of the distribution. Second, there are more credit lines with positive AUF and high utilization in Europe as shown in Figure 2. As we have seen, the negative impact of the macroeconomic environment is reduced with higher utilization, and hence the ability to stress the distribution via macroeconomic variables is limited.

To summarize, the Macro Only Model provides a good distributional fit in crises and non-crises periods in the United States, whereas the OLS does not. On the contrary, the macroeconomic variable does not seem to be able to capture the true systematic pattern in Europe. Therefore, we include a time-specific random effect in our quantile regression approach in the next step.

4.2 | Random effects model

The model set-up for the random effects model is similar to the Macro Only Model as the observable variables remain in the modelling framework. We extend the model by a time-specific random effect as stated in Equation (5). The realizations of the random effect refer to the quarter of default t. Obligors who default in the same quarter t, share the same realization of the random effect and, thus, their AUFs are either higher (positive realization of the random effect) or lower (negative realization of the random effect) on average. This enables us to capture the co-movement in the time dimension. As the coefficients of the independent variables are very similar to the ones obtained by the Macro Only Model, we focus only on the extension of this model. The coefficients for selected quantiles can be found in Table B.1 in Appendix B.

The main parameter of the random effect and, thus, the random effects model, is the standard deviation σ_F. It can be interpreted in terms of magnitude of the random effect's impact. The higher the standard deviation, the larger the impact of the random effect on the specific quantile. As an additional measure we use the inter cohort correlation (ICC) coefficient. It illustrates the proportion of variation in the quantile captured by the random effect. According to Geraci and Bottai (2007), the ICC is defined as:

$$\mathrm{ICC} = \frac{\sigma_F^2}{\sigma_F^2 + \sigma_\epsilon^2}, \tag{7}$$

where σ_F^2 is the variance of the random effect and σ_ϵ^2 is the variance of the error term in the quantile function (see Equation (5)). The higher the ICC, the more the random effect accounts for the variation in the quantile.[17]

Figure 6 illustrates the standard deviation σ_F of the random effect (left panel) and the ICC coefficient (right panel) for each quantile. The random effect has the highest impact in the tails of the distribution. This coincides with the lack of statistical evidence for the macroeconomic variable in this range (see right panels of Figure 3). From a credit risk management perspective, it is noteworthy that the impact of the random effect is stronger in the right tail of the distribution. Thus, unobservable systematic patterns are crucial for extreme positive additional drawdowns. According to the ICC, the random effect accounts for more than 60% of the variation in the far right tail. This has two major implications. First, modelling a quantile-dependent random effect is favourable as the impact differs along the response distribution. Second, the random effect

[17]As we estimate every time-specific random effect independently for every quantile regression, the problem of quantile crossing might be of concern. Overall, in less than 5% the quantiles cross and if we extend the distance of quantiles to be considered to two, that is. $\tau \in [0.01, 0.03, \dots, 0.97, 0.99]$, the proportion drops to slightly more than 1%. As a robustness check, we smooth the non-monotone quantiles and redo our analysis. The results are virtually unchanged. We thank an anonymous referee for raising this important point.

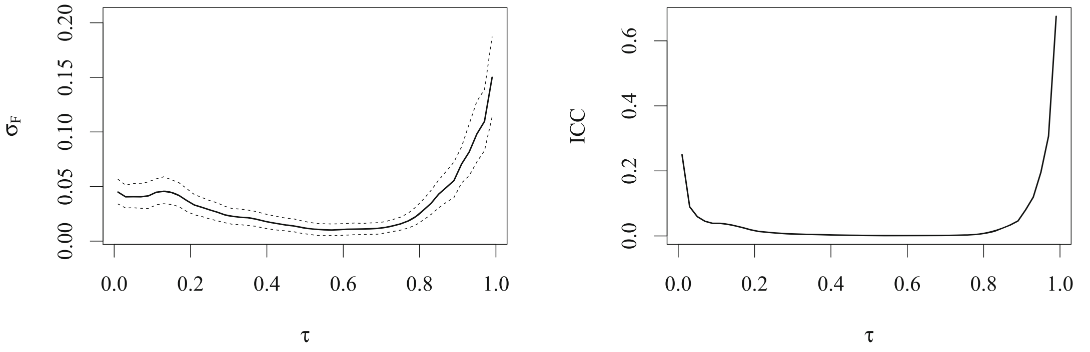

FIGURE 6 Results | random effects model (coefficients plots of σ_F and ICC). *Note*: The left panel of the figure illustrates the estimated posterior mean of σ_F in the Random Effects Model. The dashed lines indicate the 95% highest posterior density intervals. The standard deviation σ_F can be interpreted as the impact strength of the random effect in the corresponding quantile. The right part of the figure displays the posterior mean of the ICC coefficient (see Equation 7). It indicates how much of the variation in each quantile is due to the random effect compared to the fixed effects.

accounts for the true systematic variation in a value range where macroeconomic variables lack statistical evidence.

Figure 7 illustrates the posterior means (black solid line) and the HPDIs (black dashed line) of the random effect realizations for the 75% and 95% quantile. The dotted line marks the reference point of zero. As indicated by Figure 6, the magnitudes of the realizations substantially differ among the quantiles. Regarding the 95% quantile, the posterior means are up to ten times as high compared to the 75% quantile. In the GFC, large positive realizations indicating higher AUFs occur. So the question arises why the random effect accounts for systematic variation, especially in the early stages of the financial crisis and for higher quantiles? One reason may be that credit lines in general are among the first financial instruments that companies use to sustain their liquidity and financing duties when the economic condition deteriorates. This is in line with findings of Barakova and Parthasarathy (2013) who find that EAD of syndicated credit lines is especially high in pre- and early stages of crisis periods, where defaults are hard to anticipate for banks. Hence, finding an observable variable for very early stages of crisis periods may be tedious and largely portfolio dependent. The random effects approach provides a straightforward and tailor-made solution to this problem. Banks and regulators may use a *baseline* macroeconomic variable, like ΔGDP, to account for the overall economy and use the random effect to capture the remaining systematic variation of credit lines, as suitable variables are hard to find.

To underline the importance of the random effect, assume a short-term revolver, located in the FIRE industry, pari-passu in seniority, one year history of credit line and an available limit of 250,000. To forecast an adverse realization of the EAD, a bank may use the posterior means, displayed in Table B.1, of the Random Effects Model for the 95% quantile:

$$\begin{aligned} Q_{95th}\left(y_i|x_i\right) = 1.094 - 0.015 - 0.037 \cdot 250{,}000 + 1 \cdot 0.0004 - \Delta\text{GDP} \cdot 0.319 \\ - 0.287 \cdot \text{Utilization} + 2.221 \cdot \Delta\text{GDP} \cdot \text{Utilization} \end{aligned} \tag{8}$$

We can calculate the AUF based on observable variables in Equation (8) and subsequently estimate the EAD. To calculate the EAD with the random effect, its realizations can simply be added to the AUF based on Equation (8). For covering downturn characteristics, we use the realization in 2008 Q1 of 0.22 and 2009 Q1 of 0.10 with the corresponding values of ΔGDP. To assess the

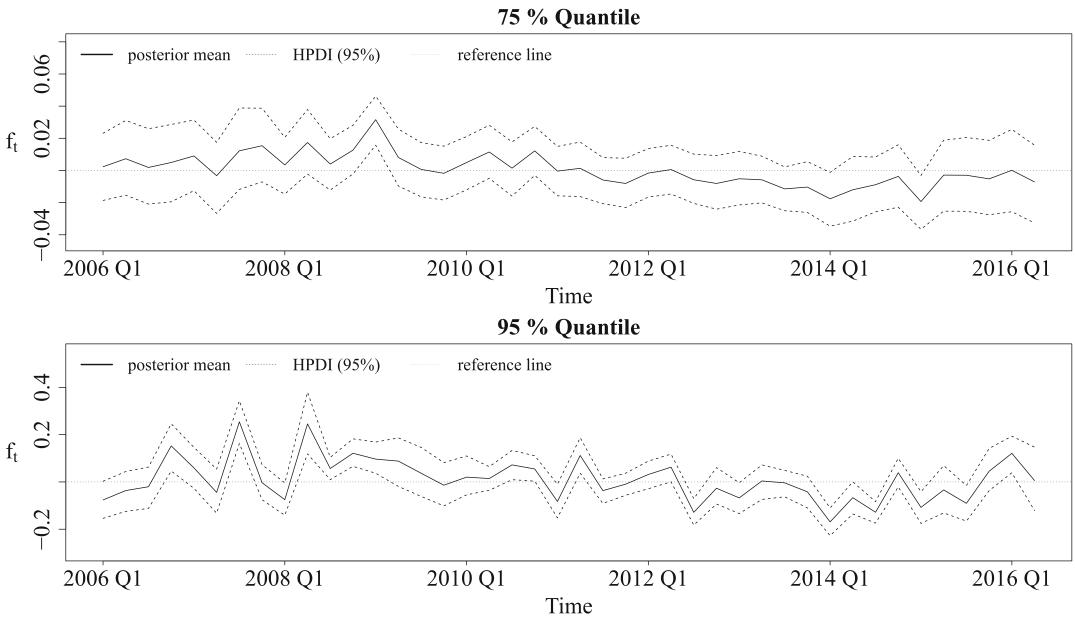

FIGURE 7 Results | random effects model (random effect realizations). *Note*: The figure illustrates the posterior means (solid grey line) of the random effect realizations for the 75% and 95% quantile. The dashed lines correspond to the 95% highest posterior density intervals. A positive posterior mean indicates a positive effect on the corresponding quantile function and, therefore, a higher AUF.

importance of the random effect, the relative difference[18] between the EAD estimate with random effect and the EAD estimate based on Equation (2), depending on the level of Utilization is shown in Figure 8.

We can obtain two important insights from this stylized example. First, the comparison of the two lines indicates that the realization of the random effects has a large impact on the EAD estimates, underlining the importance of this approach. The estimated EAD with the realization of the random effect is up to 35% higher than when neglecting the realization. Furthermore, we can see that the random effect, again, is most important for less drawn lines, which entail the greatest risk to banks. This clearly shows that the random effect accounts for a large and important share of systemic variation of credit lines, especially for higher quantiles of the AUF distribution.

4.2.1 | Downturn estimation based on Random Effects Model

In analogy to Section 4.1, we investigate the model's ability to produce sufficiently conservative downturn distributions. In Europe, the Macro Only Model underestimates the empirical AUF distribution—even if the macroeconomic variable is stressed to its 95% quantile. This might be due to its lack of statistical evidence in the tails of the AUF distribution. The downturn AUF distribution in the Random Effects Model is generated by applying an adverse realization of the random effect. As an adverse realization, we use the 95% quantile of each quantile-specific normal

[18]The relative difference is calculated by $\left(\frac{EAD_{\text{with random effect}}}{EAD_{\text{without random effect}}} - 1\right)$. Hence, a value greater than zero indicates a larger EAD estimate by using the realization of the random effect.

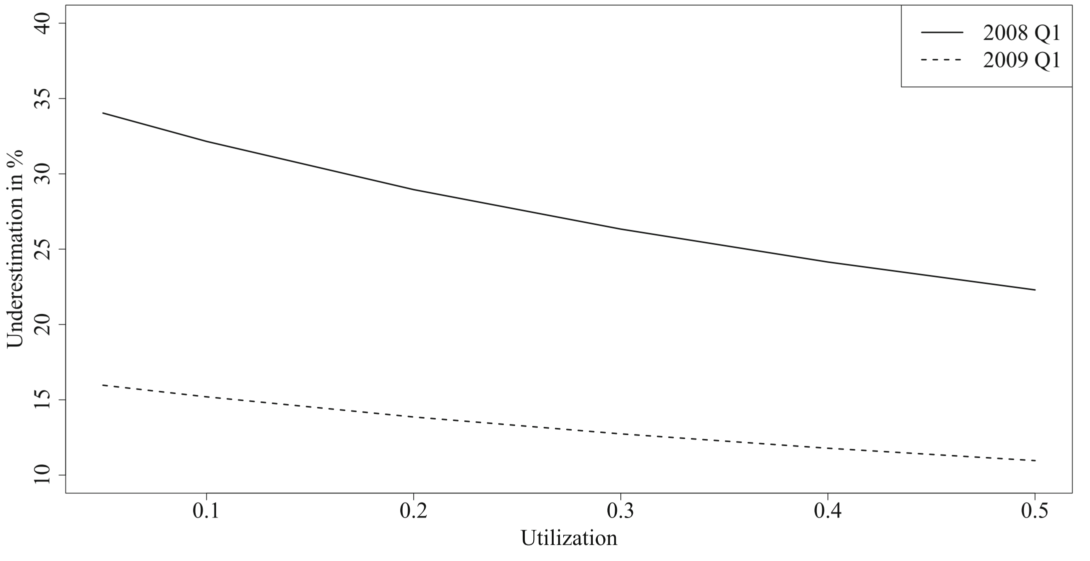

FIGURE 8 Results | impact of the random effect. The figure illustrates the relative difference of EAD estimates with and without considering the random effect. The black solid line represents the realization of 2008 Q1, whereas the dashed line illustrates the realization of 2009 Q1.

distribution with mean zero and standard deviation $\sigma_F(\tau)$. The posterior predictive distribution is generated by setting the random effect to its mean.

Figure 9 illustrates the PP plots of the posterior predictive distribution and downturn distribution based on the Random Effects Model in the GFC. The interpretation coincides to the one in Figure 5. The black points indicate the distributional fit of the posterior predictive distribution, whereas the grey dots illustrate the fit of the downturn distribution. The posterior predictive distribution underestimates the empirical AUF distribution as the black dots are above the bisecting line. However, the downturn distribution via the random effect delivers a sufficiently conservative distribution. Summarizing, the random effect accounts for systematic variation in the tails of the distribution where macroeconomic variables lack impact and statistical evidence. Therefore, sufficiently conservative downturn distributions can be generated based on the random effect in Europe.

4.2.2 | Out-of-time comparison[19]

The final part of this section focuses on the out-of-time performance of quantile regression and the benchmark model. In credit risk, we are usually interested in predicting the future. Hence, a model should be capable of predicting the EAD in unseen time periods. We use the hold-out sample ranging from mid-2016 to the end of 2018 to conduct this out-of-time validation. To provide a more broad picture, we sample 1000 portfolios including 200 credit lines each of the hold-out sample instead of comparing both methods only once. As the comparison of all PP plots is tedious, we summarize them using the Harmonic Mass Index (HMI). This measure averages the absolute deviations of empirical and theoretical quantiles which are plotted in the PP plot (Wagenvoort,

[19]We thank discussants of the CFE 2019 for suggesting this comparison.

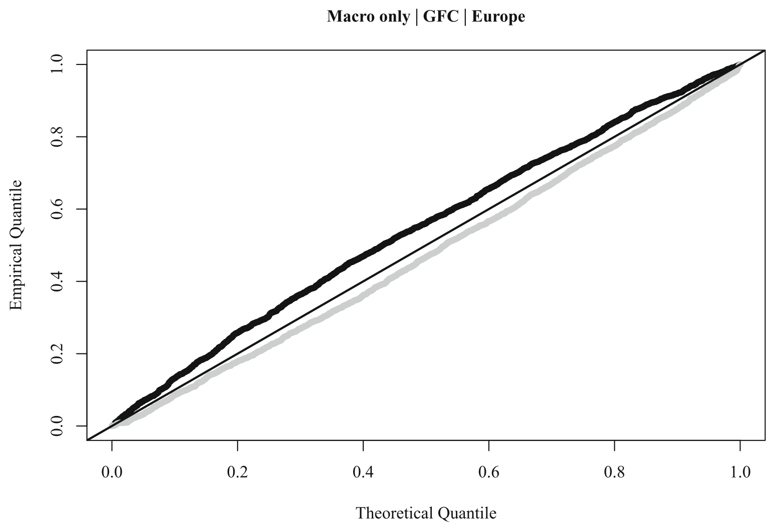

FIGURE 9 Distributional fit in downturn periods | random effects model. *Note*: The figure shows the distributional fit during the GFC for the random effects model. The black line indicates the fit of the posterior predictive distribution, whereas the grey line illustrates the fit using a stress scenario. The stress scenario is generated by considering an extreme realization of the random effect for each obligor defaulting during the global financial crisis (GFC). Recall that the quantile-specific random effect follows a normal distribution with mean zero and standard deviation σ_F. The 95% quantile of each quantile-specific random effect distribution is applied as extreme realization. An underestimation of the empirical crisis distribution is indicated by a PP-line above the bisecting line. Contrary, a too conservative posterior predictive distribution is indicated by a line below the bisecting line.

TABLE 3 Harmonic mass index

	Quantile regression	OLS
(a) USA		
Mean	0.0458	0.0823
Standard deviation	0.0080	0.0067
(b) Europe		
Mean	0.1216	0.1616
Standard deviation	0.0170	0.0130

Note: The table shows means, standard deviations of the HMI over the 1000 sampled portfolios in each region. The HMI summarizes the absolute deviations from the perfect fit. Hence, the lower the value, the better the distributional fit. For the European Data set, the random effects model is used, as it turned out to be superior. The random effects in the random effects model are set to their expectation for prediction. The Macro Only Model is used in the US American data set.

2006). Formally, it is defined as:

$$HMI = \frac{2}{n} \sum_{i=1}^{n} |p_{\text{empirical},i} - p_{\text{theoretical},i}| \tag{9}$$

The lower the calculated HMI, the better the distributional fit. A perfect fit results in an HMI of zero. Table 3 reports mean and standard deviation over the 1000 samples:

Regarding Table 3, the quantile regression performs much better over all samples and in both regions. In the US American sample, the HMI is almost cut by half and in Europe it decreased by 24.75%. The standard deviations across the 1000 portfolios in each region are similar. To underline the superiority of the quantile regression in each and every portfolio, we would like to stress the point that there is not a single portfolio in which our approach provides a worse fit than the linear model. This out-of-time validation clearly underpins the superior performance of our approach.

5 | CONCLUSION

By using access to one of the world's largest loss and exposure data bases, this paper sheds light onto the topic of modelling EADs and conversion factors and, thus, the drawdown behaviour of eventually defaulted credit lines. We apply Bayesian quantile regressions to model the full conditional distribution of conversion factors. If the identification of adequate (i.e. meaningful and statistically evident) macroeconomic variables is unfeasible, the quantile regression approach is extended by time-specific random effects to capture the unexplained systematic time patterns of conversion factors.

Quantile regression turns out to be a superior modelling technique in this setting as deviating effects among quantiles are captured. The most striking deviations throughout the quantile range refer to the impact of macroeconomic variables. We find statistically evident impacts on the inner quantiles, while evidence vanishes in the outer tails of the distribution. This is of special relevance in the light of the requirement for downturn estimates, that is estimates which reflect economic downturn conditions. Furthermore, macroeconomic effects on conversion factors vary for different utilization levels. Less drawn lines (low utilization) are affected to a higher extent by economic downturns. This entails tangible consequences for credit risk managements as these lines bear the highest risk in terms of an EAD increase. Credit lines which are already exhausted one year prior to default react less to economic decline.

With respect to downturn estimation, we reveal major differences among the two considered regions—the United States and Europe. In the United States, macroeconomic variables seem to capture wide parts of the systematic co-movement of conversion factors in the time line. Thus, sufficiently conservative downturn estimates are able to be generated via these observable systematic variables. This might be due to the fact that comovements are generally less pronounced compared to Europe.

In contrast to the Unitec States, macroeconomic variables do not seem to be suitable to produce adequate downturn estimates in Europe. Hence, time-specific random effects are included into the modelling framework. These unobservable systematic effects are able to capture the true systematic patterns in conversion factors. Indeed, the impact of the random effect is largest regarding the tails of the distribution where the impact of the macroeconomic variables vanishes. As a consequence, sufficiently conservative downturn estimations can be generated based on random effects for Europe. Comparing our approach with the most common method in literature, the OLS regression, we can provide evidence of superior fit and greater flexibility. Especially in the out-of-time forecasting exercise, our model provides an up to twice as good distributional fit compared to the benchmark model.

The results of this paper have three major implications for financial institutions and politics. First, less drawn credit lines not only bear the highest risk in terms of an EAD increase, but are also more severely affected by economic downturn. Second, systematic patterns in conversion factors might be of different kind and magnitude depending on the considered region. Thus, random effects might offer a reasonable option to generate sufficiently conservative downturn estimates if the identification of adequate macroeconomic variables is challenging. Furthermore, we can show that credit lines also induce higher credit risk besides the well documented liquidity risk in crisis periods, which is important for politics and regulators.

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APPENDIX A. BAYESIAN MODEL SPECIFICATION

The quantile regression and its extensions are estimated using Bayesian inference. Hence, for each parameter prior distributions have to be specified. Furthermore, to ensure a more efficient estimation, this paper uses the decomposition of the asymmetric Laplace distribution based on Yu and Stander (2007) and Luo et al. (2012). A random variable of the asymmetric Laplace distribution can be expressed as a mixture of a standard normal and an exponential random variable. Therefore, Equation (5) changes to:

$$y_i = x_i\beta(\tau) + F(\tau) + c_1 e_i + \sqrt{c_2\sigma e_i} z_i, \tag{A1}$$

where $c_1 = \frac{1-2\tau}{\tau(1-\tau)}$, $c_2 = \frac{2}{\tau(1-\tau)}$, $z_i \sim N(0,1)$ and $e_i \sim \exp\left(\frac{1}{\sigma}\right)$.

The Bayesian quantile regression and its priors can be formulated as follows:

$$\begin{aligned} f\left(y_i \mid \beta(\tau), F(\tau), \sigma_\epsilon, e_i, z_i\right) &= (2\pi c_2 \sigma_\epsilon e_i)^{-\frac{1}{2}} \exp\left\{-\frac{1}{2\pi c_2 e_i}(y_i - x_i\beta(\tau) - F(\tau) - c_1 e_i)^2\right\} \\ F(\tau) &\sim N(0, \sigma_F(\tau)) \\ \sigma_F(\tau) &\sim N(0, 10^5)[0, \infty] \\ \beta(\tau) &\sim N(0, 10^5) \\ \sigma_\epsilon &\sim N(0, 10^5)[0, \infty] \\ z_i &\sim N(0, 1) \\ e_i &\sim \exp\left(\frac{1}{\sigma_\epsilon}\right). \end{aligned} \tag{A2}$$

The squared brackets in the model specifications of the dispersion parameters indicate truncation. The prior specifications of model parameters are set to be uninformative assuming large values of their dispersion parameters. The random effect follows a Normal distribution with mean zero and the random effect specific standard deviation $\sigma_F(\tau)$. In this hierarchical setting, we also specified a truncated Normal distribution for this dispersion parameter as the prior distribution. The models are sampled using two MCMC chains each. We use a chain length of 10,000 for the European sample and 20,000 for the US sample due to a smaller sample size. Furthermore, the burn-in length was set to 2000 in Europe and 4000 in the United States.

APPENDIX B. RANDOM EFFECTS MODEL

TABLE B.1 Results | Macro Only Model (MOM) and Random Effects Model (REM) for Europe

		$\tau = 0.05$		$\tau = 0.50$		$\tau = 0.95$	
Variable	**Level**	**MOM**	**REM**	**MOM**	**REM**	**MOM**	**REM**
Intercept		0.132[c]	0.136[c]	0.815[c]	0.818[c]	1.099[c]	1.094[c]
Facility type (medium term)	Short term	0.017[a]	0.005[a]	0.015[b]	0.014[b]	−0.013	−0.015
	Overdraft	−0.029[c]	−0.035[c]	0.012[c]	0.008[c]	0.220[c]	0.189[c]
Industry (FIRE)	Agricult.	−0.013[a]	0.006	0.004	0.007	0.117[c]	0.148[c]
	Mining	0.029[a]	0.043[b]	0.007	0.004[a]	0.611[c]	0.543[c]
	Construct.	−0.050[c]	−0.051[c]	−0.007[a]	−0.008[a]	0.047[c]	0.071[c]
	Manufact.	−0.053[c]	−0.048[c]	−0.019[c]	−0.021[c]	0.056[c]	0.073[c]
	Transport	−0.065[c]	−0.074[c]	−0.021[c]	−0.020[c]	0.037[b]	0.067[c]
	Wholesale	−0.050[c]	−0.046[c]	−0.020[c]	−0.021[c]	0.019[a]	0.041[c]
	Service	−0.043[c]	−0.034[c]	−0.009[b]	−0.008[a]	0.121[c]	0.160[c]
	Other	−0.039[c]	−0.043[c]	−0.027[c]	−0.031[c]	0.015[a]	−0.010
Seniority (pari-passu)	Super sen.	−0.040[c]	−0.056[c]	0.001	−0.003	0.045[c]	0.015[c]
	Non sen.	−0.045[c]	−0.052[c]	0.060[c]	0.058[c]	0.461[c]	0.371[c]
log(Limit)		−0.013[c]	−0.013[c]	−0.010[c]	−0.011[c]	−0.037[c]	−0.037[c]
Age		−0.002[c]	−0.002[c]	0.000	0.000	0.004[c]	0.006[c]
ΔGDP		−0.114[a]	0.045	−1.997[c]	−1.952[c]	−0.255[a]	0.319
Utilization		−0.269[c]	−0.261[c]	−0.687[c]	−0.677[c]	−0.295[c]	−0.287[c]
Interaction		0.393[b]	0.483[a]	1.978[c]	1.960[c]	2.569[c]	2.221[c]
σ_F			0.041[c]		0.011[c]		0.098[c]

Note: This table shows the estimated posterior means for several selected quantiles and compares the Macro Only Model with the Random Effects Model. As one can see, the estimated posterior means do not differ much. The first column inherits the name of the different independent variables. If they are categorical, the reference group is indicated in brackets. The second column illustrates the different levels of categorical variables. Statistical evidence is indicated by the following [a], [b], [c]: [a]corresponds to substantial evidence (Odds > 3.2), [b]corresponds to strong evidence (Odds >10), [c]corresponds to decisive evidence (Odds >100).

APPENDIX C. COEFFICIENT PLOTS

The following figures show the estimated posterior means and the 95% HPDI for each parameter in the three different quantile regressions. Statistical evidence is indicated if zero is not included in the 95% HPDI (C1—C3).

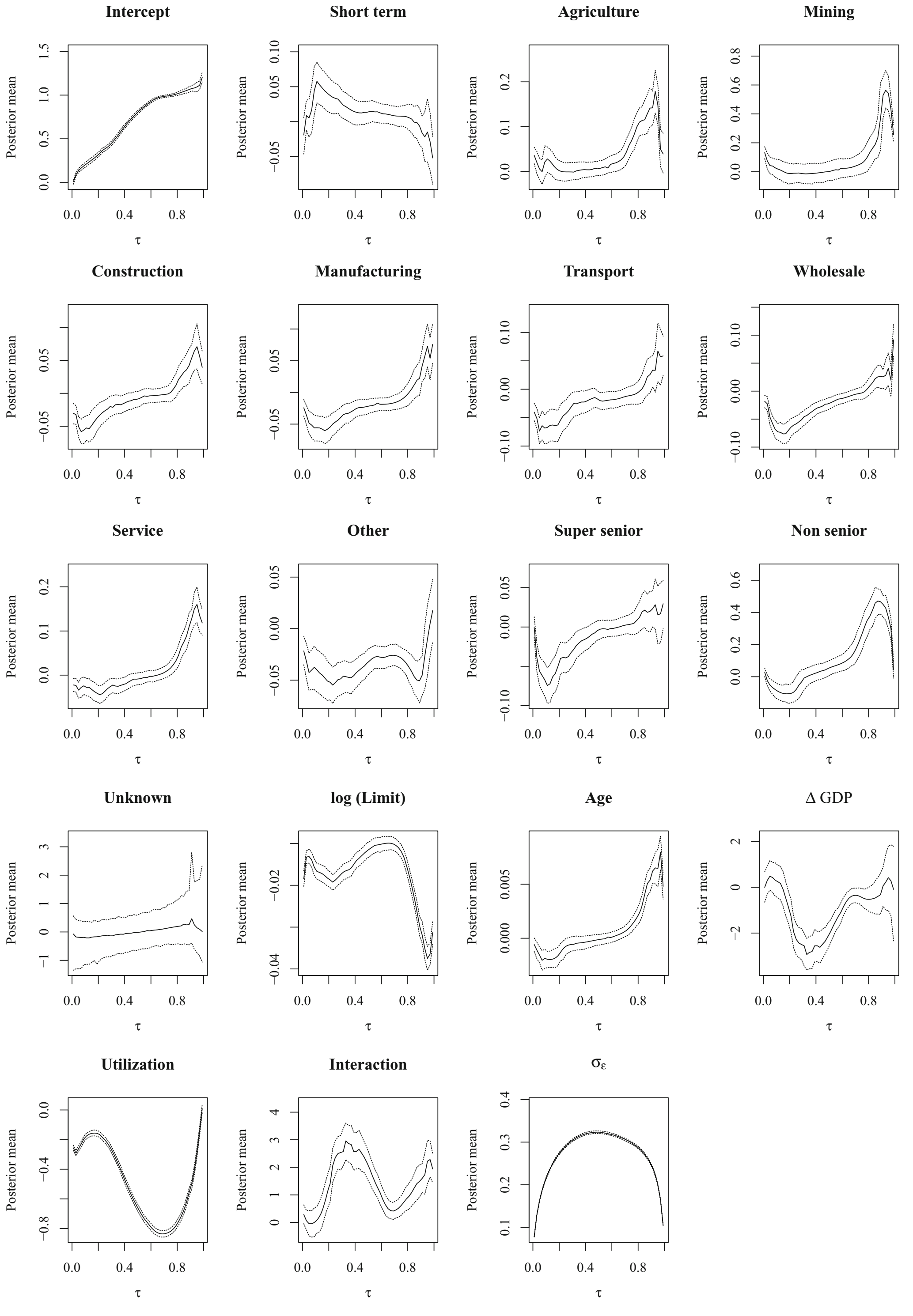

FIGURE C.1 Coefficients USA| Macro Only Model. *Note*: The figure shows the estimated coefficients and their 95% highest posterior density intervals (HPDIs) for all parameters in the whole distributional range in the United States. The black lines represent the posterior means, whereas the dotted lines illustrate 95% HPDIs.

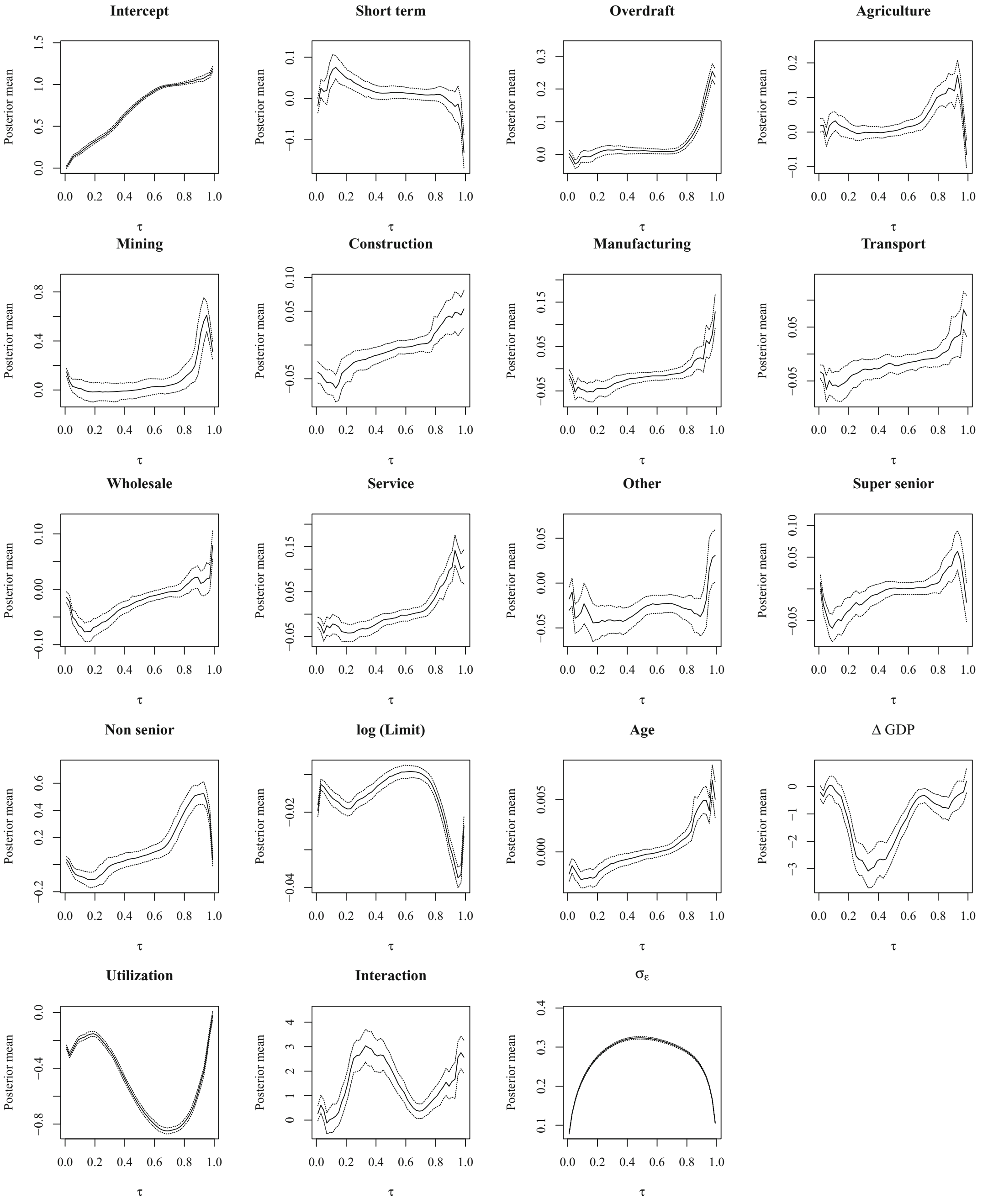

FIGURE C.2 Coefficients Europe| Macro Only Model. *Note*: The figure shows the estimated coefficients and their 95% highest posterior density interval (HPDI) for all parameters in the whole distributional range in the European sample. The black lines represent the posterior means, whereas the dotted lines illustrate 95% HPDIs.

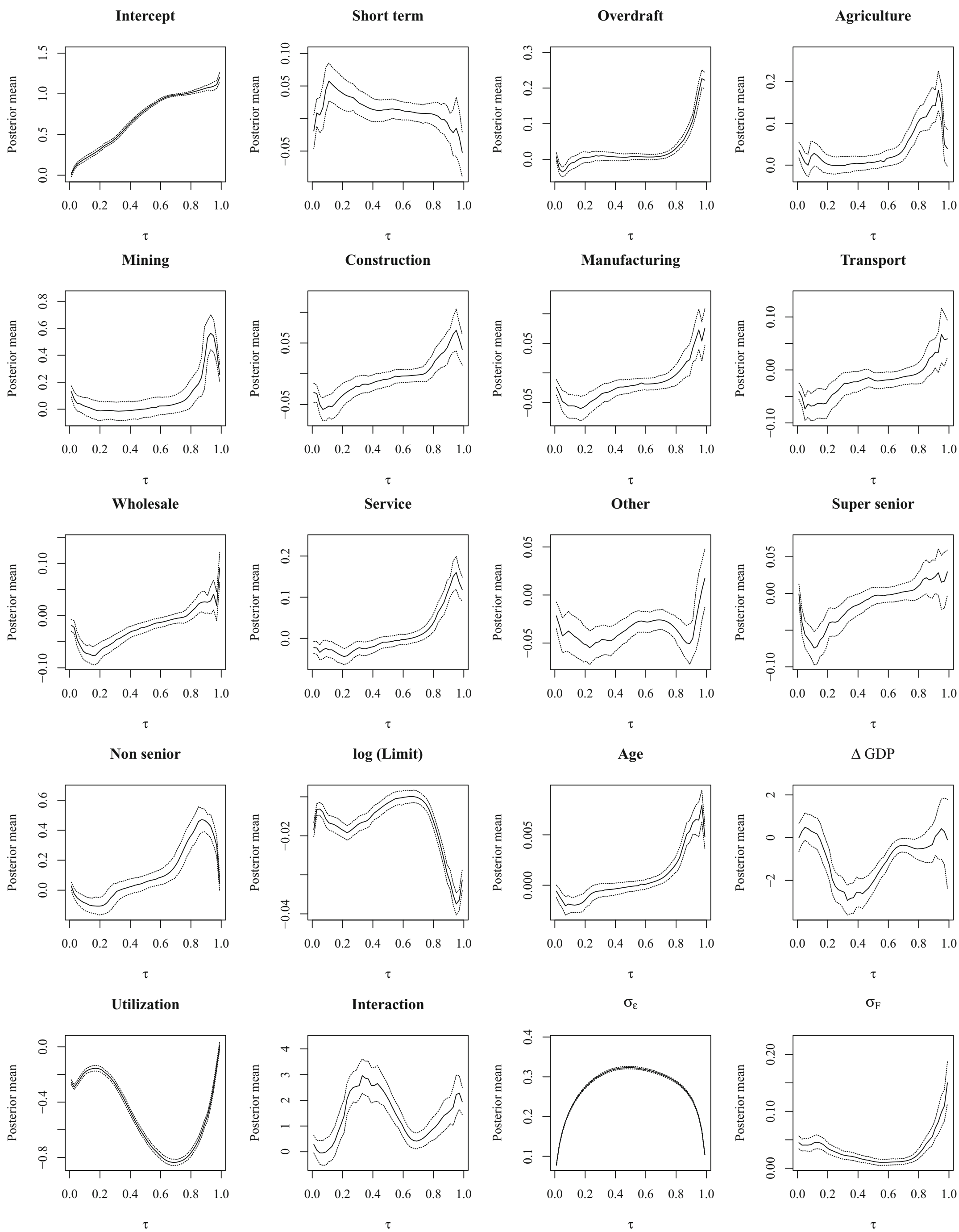

FIGURE C.3 Coefficients Europe| Random Effects Model. *Note*: The figure shows the estimated coefficients and their 95% highest posterior density intervals (HPDIs) for all parameters in the whole distributional range in the European sample. The black lines represent the posterior means, whereas the dotted lines illustrate 95% HPDIs.

APPENDIX D. CONVERGENCE DIAGNOSTICS

To evaluate the convergence of the estimated models, trace plots are the primary source of convergence diagnostics. Stable trace plots indicate that the chains converge to a steady state. Hence, priors are well calibrated and the burn-in is sufficient. Furthermore, we examine two well-known figures in Bayesian inference—the Gelman–Rubin and Heidelberger–Welch diagnostic. Both are hypotheses tests in frequentist terms, however, applied widely to evaluate the length of burn-in (Gelman–Rubin) and the length of chains (Heidelberger–Welch) (Tables D.1 and D.2). Furthermore, we display the diagnostic only for the median ($\tau = 0.5$). Please note that for all quantiles convergence is achieved (D.1—D.3).[20]

D.1 Traceplots

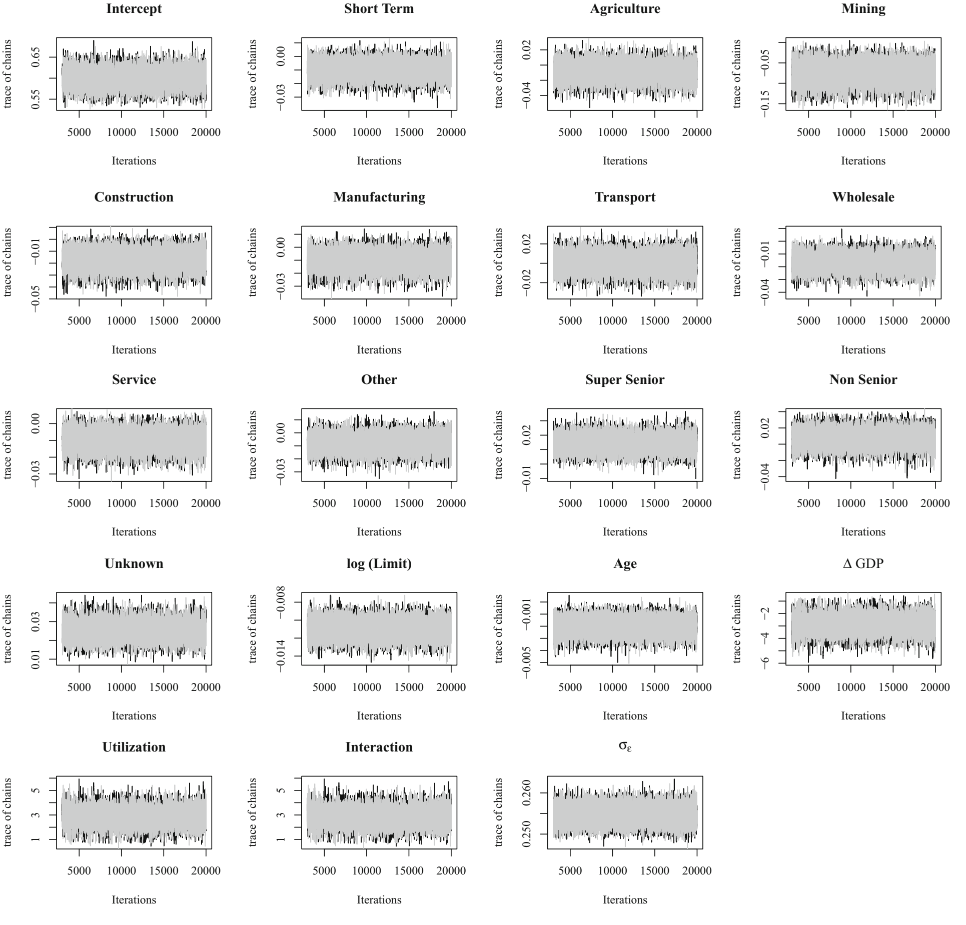

FIGURE D.1 Traceplot USA| Macro Only Model |$\tau = 0.5$. *Note*: The figure illustrates the MCMC chains for the Macro Only Model in the US American sample. The first chain is coloured in black, whereas the second one in grey.

[20]Traceplots, Gelman–Rubin and Heidelberger–Welch diagnostics for all quantiles are available from the authors upon request.

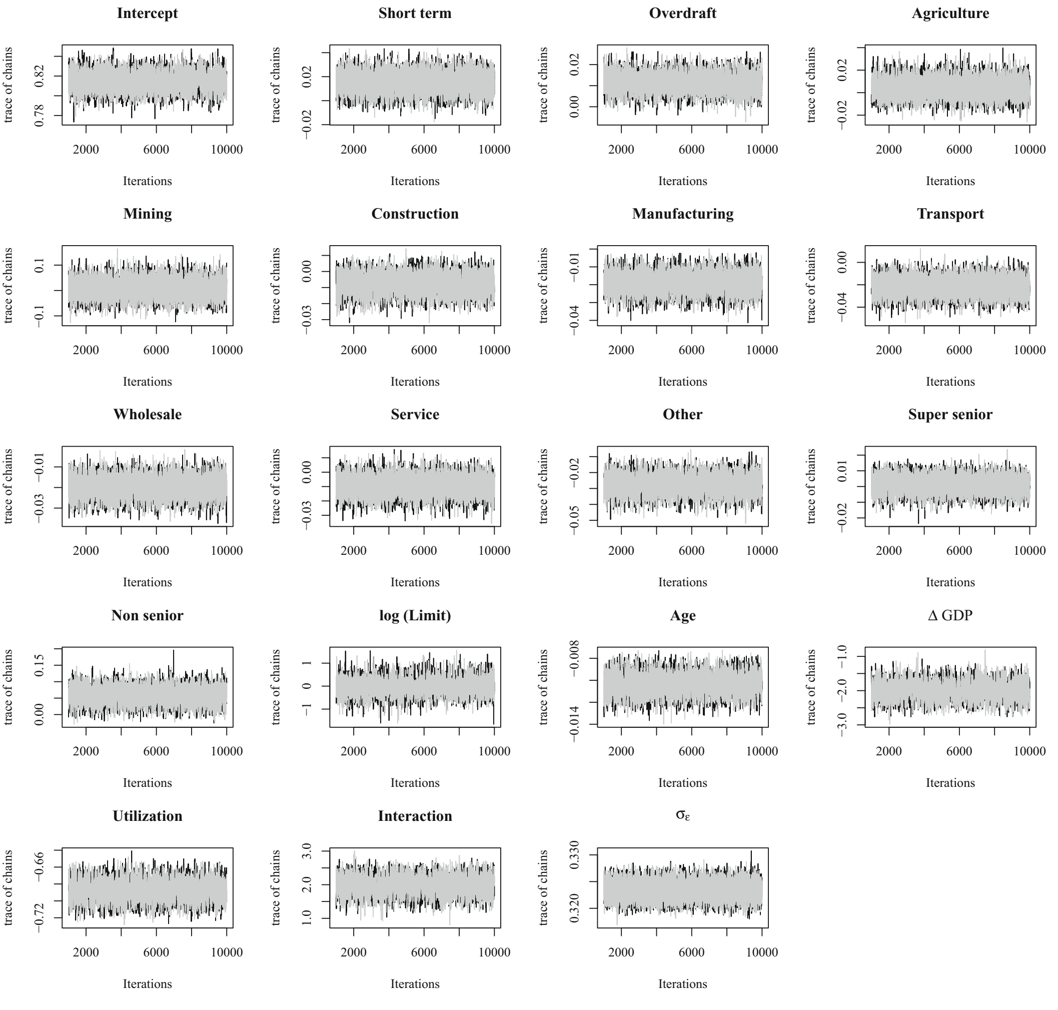

FIGURE D.2 Traceplot Europe| Macro Only Model $|\tau = 0.5$. *Note*: The figure illustrates the MCMC chains for the Macro Only Model in the European sample. The first chain is coloured in black, whereas the second one in grey.

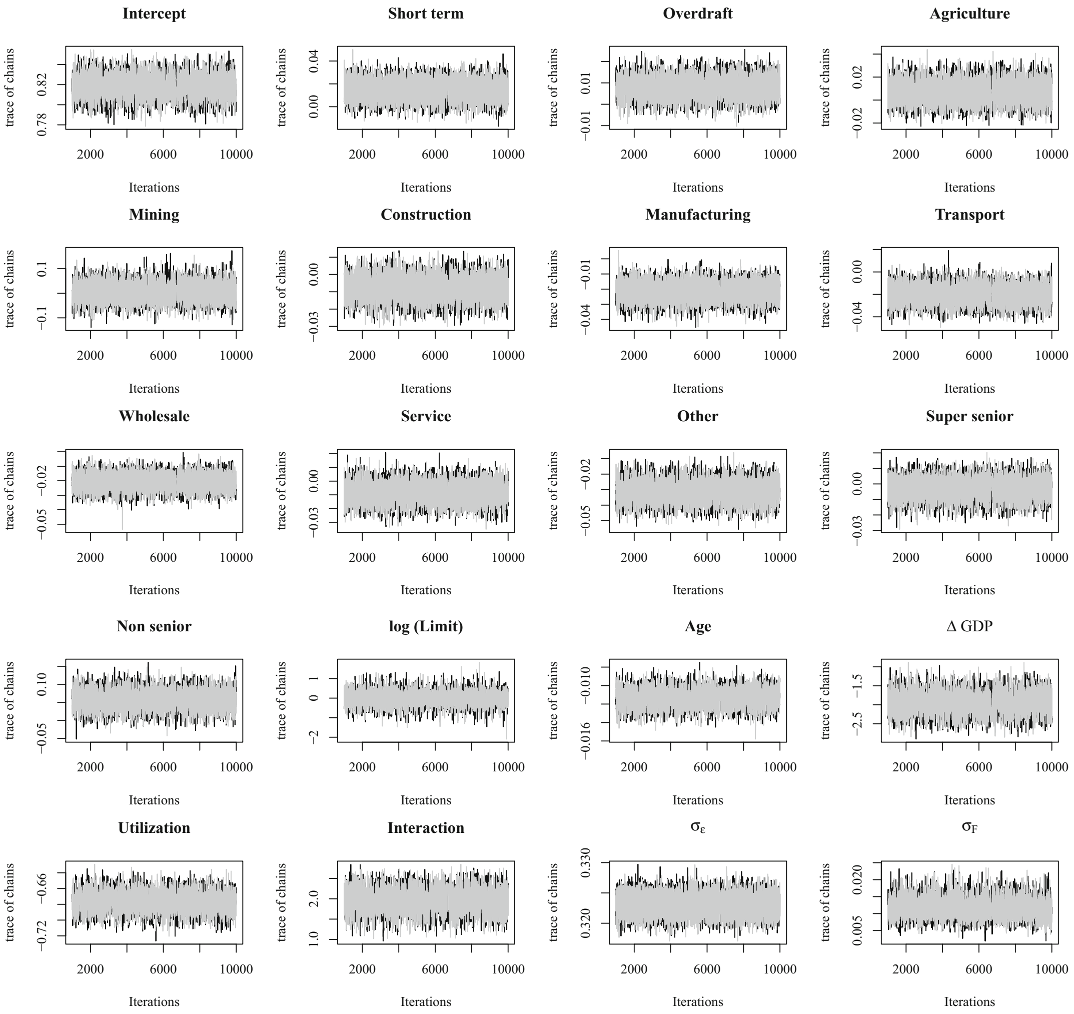

FIGURE D.3 Traceplot Europe| Random Effects Model $|\tau = 0.5$. *Note*: The figure illustrates the MCMC chains for the Macro Only Model in the European sample. The first chain is coloured in black, whereas the second one in grey.

D.2 Gelman–Rubin diagnostic

TABLE D.1 Results | Macro Only Model (MOM) and Random Effects Model (REM) for Europe | $\tau = 0.50$

Level	MOM | Europe		MOM | USA		REM Model | Europe	
	Point estimate	Upper confid. limits (90%)	Point estimate	Upper confid. limits (90%)	Point estimate	Upper confid. limits (90%)
$\beta_{Intercept}$	1.0016	1.0016	1.0008	1.0028	1.0007	1.0019
$\beta_{Shortterm}$	1.0000	1.0001	1.0010	1.0039	1.0000	1.0001
$\beta_{Overdraft}$	1.0011	1.0040			1.0009	1.0034
$\beta_{Agriculture}$	1.0003	1.0011	1.0027	1.0102	1.0003	1.0008
β_{Mining}	1.0003	1.0004	1.0010	1.0010	1.0001	1.0005
$\beta_{Construction}$	1.0014	1.0053	1.0036	1.0061	1.0003	1.0011
$\beta_{Manufact.}$	1.0015	1.0037	1.0005	1.0005	0.9999	1.0000
$\beta_{Transport}$	1.0011	1.0045	1.0014	1.0037	1.0001	1.0004
$\beta_{Wholesale}$	1.0002	1.0011	1.0000	1.0000	1.0001	1.0004
$\beta_{Service}$	1.0035	1.0136	1.0005	1.0016	1.0023	1.0074
β_{Other}	1.0005	1.0007	1.0003	1.0009	1.0014	1.0057
$\beta_{SuperSenior}$	1.0013	1.0028	1.0000	1.0001	1.0004	1.0012
$\beta_{NonSenior}$	1.0002	1.0005	1.0008	1.0032	1.0002	1.0004
$\beta_{Unknown}$			1.0004	1.0017		
$\beta_{\log(Limit)}$	1.0010	1.0037	1.0060	1.0165	0.9999	0.9999
β_{Age}	1.0008	1.0032	1.0008	1.0033	1.0012	1.0046
$\beta_{\Delta GDP}$	1.0011	1.0026	1.0007	1.0022	1.0002	1.0006
$\beta_{Utilization}$	1.0012	1.0021	1.0001	1.0002	0.9999	0.9999
$\beta_{Interaction}$	1.0012	1.0034	1.0007	1.0020	1.0003	1.0007
σ_{ϵ}	1.0000	1.0000	1.0006	1.0025	1.0002	1.0006
σ_F					1.0020	1.0073

Notes: The table summarizes the Gelman Rubin diagnostic for the different quantile regressions with $\tau = 0.5$. The first column indicates the estimated parameters. The Gelman-Rubin diagnostic examines the length of burn-in. The potential reduction factor and the upper confidence limit are displayed in this table. Convergence is achieved if chains do not depend on their initial values, that is for upper limits close to one (Gelman & Rubin, 1992). A rule of thumb assumes 1.1 as the critical value.

D.3 Heidelberger–Welch diagnostic

TABLE D.2 Results | Macro Only Model (MOM) and Random Effects Model (REM) for Europe | $\tau = 0.50$

Level	MOM | Europe Stationary test	Start	*p*-value	MOM | USA Stationary test	Start	*p*-value	REM Model | Europe Stationary test	Start	*p*-value
$\beta_{Intercept}$	Passed	1	0.8105	Passed	1	0.1476	Passed	1	0.1537
$\beta_{Shortterm}$	Passed	1	0.3552	Passed	1	0.2930	Passed	1	0.5847
$\beta_{Overdraft}$	Passed	1	0.1478				Passed	1	0.2819
$\beta_{Agriculture}$	Passed	1	0.6500	Passed	1	0.2812	Passed	1	0.1539
β_{Mining}	Passed	1	0.5665	Passed	1	0.1009	Passed	1	0.8425
$\beta_{Construction}$	Passed	1	0.6427	Passed	1	0.4143	Passed	1	0.7893
$\beta_{Manufact.}$	Passed	1	0.5964	Passed	8001	0.0791	Passed	1	0.8941
$\beta_{Transport}$	Passed	1	0.2271	Passed	1	0.5938	Passed	1	0.7341
$\beta_{Wholesale}$	Passed	1	0.1283	Passed	1	0.4641	Passed	1	0.3796
$\beta_{Service}$	Passed	5401	0.0705	Passed	1	0.5843	Passed	1	0.1254
β_{Other}	Passed	1	0.5231	Passed	1	0.5648	Passed	1	0.2908
$\beta_{SuperSenior}$	Passed	1	0.3019	Passed	1	0.2010	Passed	1	0.3966
$\beta_{NonSenior}$	Passed	1	0.3736	Passed	1	0.4174	Passed	1	0.6930
$\beta_{Unknown}$				Passed	1	0.2013			
$\beta_{log(Limit)}$	Passed	1	0.6185	Passed	1	0.0766	Passed	1	0.3555
β_{Age}	Passed	1	0.7987	Passed	1	0.7029	Passed	1	0.8754
$\beta_{\Delta GDP}$	Passed	1	0.3652	Passed	1	0.3879	Passed	1	0.2158
$\beta_{Utilization}$	Passed	1	0.1887	Passed	1	0.6711	Passed	1	0.4300
$\beta_{Interaction}$	Passed	1	0.5972	Passed	1	0.3807	Passed	1	0.5506
σ_{ϵ}	Passed	1	0.5997	Passed	1	0.1964	Passed	1	0.4853
σ_F							Passed	1	0.2112

Notes: The table summarizes the results of the Heidelberger-Welch diagnostic for the different quantile regression in the two samples. To evaluate whether the chain length is sufficiently long, both chains in each model are combined. In the Heidelberger-Welch diagnostic, a criterion of relative accuracy for the posterior means is calculated. The frequentistic stationary test uses the Cramer-von-Mises statistic to test the null hypotheses that the sampled values originate from a stationary process (see Gelman & Rubin, 1992).