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Heuristic pricing rules not requiring knowledge of the price response function

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Abstract

Heuristic rules are appropriate, if a decision maker wants to set the price of a new product or of a product, whose past price variation is low, and budget limitations prevent the use of marketing experiments or customer surveys. Whereas such rules are not guaranteed to provide the optimal price, generated profits should be as close as possible to their optimal values. We investigate eleven pricing rules that do not require that a decision maker knows the price response function and its parameters. We consider monopolistic market situations, in which sales depend on the price of the respective product only. A Monte Carlo simulation that is more comprehensive than extant attempts found in the literature, serves to evaluate these rules. The best performing rules either hold price changes between periods constant or make them dependent on the previous absolute price difference. These rules also outperform purely random price setting, which we use as benchmark. On the other hand, rules based on arc elasticities or on a loglinear approximation to sales and prices, turn out to be even worse than random price setting. In the conclusion, we discuss how heuristic pricing rules may be extended to deal with product line pricing, additional marketing variables (e.g., advertising, sales promotion, and sales force) and a duopolistic market situation.

Keywords Marketing · Pricing · Heuristics · Market response

JEL Classification M30 · M2

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1 Introduction

Most pricing methods require knowledge of the underlying price response function. This knowledge can be acquired by econometric methods, market experiments, customer surveys or expert interviews (Maurice et al. 1992; Simon and Fassnacht 2019). Econometric methods need at least a moderate number of sales and price observations. In the case of new products this condition is never fulfilled. In addition, observed prices should show sufficient variation.

Though market experiments are ideal in terms of external validity, decision makers often avoid them. High costs constitute one possible reason for such behavior. Though experimentation costs have decreased in online settings, high costs remain relevant in offline environments. Moreover, offline retailing remains dominant in many consumer product categories. According to a recent survey, between 58 and 93 % of customers in Germany mostly buy offline across sixteen categories (Kunst 2021). Because of high costs, firms make relatively little use of market experiments. Surveys of over 11,000 firms in nine euro-area countries show that the majority set prices by markup rules (Fabiani et al. 2006). In Germany, for example, about 73% of firms fix prices this way.

Recent studies demonstrate that even in online situations price experiments are not very frequent. According to an extensive empirical study based on daily U.S. and U.K. price listings across a broad range of product categories, firms often do not use experimentation fearing that it "may alienate customers and harm firm's reputation" (Gorodnichenko et al. 2014). In a similar manner, Wood et al. (2020) show that the managers of internationally operating online fashion retailers in the U.K. rely on markup rules. In fact, these managers not only bypass experimentation, they do not take any consumer behavior data into account.

Customer surveys to obtain estimates of elasticities (e.g., by conjoint analysis) entail high costs. Expert interviews are less costly because of the low number of people involved. The external validity of both customer surveys and expert interviews may be low. In surveys customers are exposed to an artificial situation, often with a focus on price. Therefore, their responses may differ from those of customers in the market place (East et al. 2013). It is well known that expert judgments as a rule perform worse than even very simple statistical models (Camerer and Johnson 1991). In addition the assessment of customers' price response by experts may be motivationally biased (e.g., sales managers tend to be overconfident and therefore to underestimate the price sensitivity of customers (Markovitch et al. 2015).

As an alternative approach we investigate pricing rules which do not require that a decision maker knows the price response function and its parameters. These pricing rules are heuristic, which means that they are not guaranteed to provide the optimal price. Nonetheless, profits generated by a rule should be as close as possible to profits that result at the unknown optimal price.

Such pricing rules are especially appropriate if a decision maker faces a budget limitation and wants to set the price of a new product or the price of a product whose price variation in the past is low. Costs involved are much lower compared to experiments, customer surveys and expert interviews. Contrary to customer surveys and expert interviews, these rules have high external validity as they process responses of customers in the market place.

We notice a striking lack of research on the performance of heuristic pricing rules. In the following we describe previous relevant studies and explain how we contribute over this research. Baumol and Quandt (1964) investigate two heuristic rules, random sampling and price setting based on a linear approximation of sales and prices, for two periods only. We use the random sampling rule as well. We also rely on a linear approximation, but in contrast to Baumol and Quandt (1964) we refer to sales and prices for a maximum of 30 periods. We also investigate modified versions of three rules developed by Thore (1964) that provide price changes between two subsequent periods. In addition, we look at six rules, which have not been investigated before. Of these six rules three comprise price elasticities that are constant across periods, two depend on price elasticities that change between periods, and one rule is based on a loglinear approximation.

Baumol and Quandt (1964) generate sales and prices for two periods by deterministic simulation from linear, multiplicative, and exponential price response functions to which they approximate a linear function. The Monte Carlo simulation of Billström and Thore (1964) continues the work of Thore (1964). Billström and Thore (1964) compute prices for a fixed number of periods. These authors only consider the linear response function with additive errors. Moreover, their simulation is based on one set of parameters only, i.e., one coefficient vector and one residual variance.

Compared to Billström and Thore (1964) our Monte Carlo simulation is more comprehensive. We consider five functions by adding semi-logarithmic and logistic functions to the forms found in Baumol and Quandt (1964). For each functional form, we have 36 different coefficients vectors at our disposal as we distinguish three minimum sales values, three maximum sales values, and four elasticity values. In addition, the Monte Carlo simulation uses three different residual variances and three different values for the number of periods.

In the concluding section we discuss how several investigated rules may be extended to deal with product line pricing, effects of additional marketing variables on sales (e.g. of advertising budgets, sales promotion budgets, sales force budgets) and pricing in a duopolistic market situation. To the best of our knowledge, the previous literature has not treated such extensions.

We investigate eleven heuristic pricing rules with respect to their capability to get close to optimal profit. Decision makers may resort to such rules if they know neither the form of the price response function nor its parameters. We evaluate these rules by two performance measures, average forgone profit and average asymmetric foregone profit, both over planning horizons of different lengths. Foregone profit in a period is defined as difference between the observed profit achieved by a pricing rule and the expected optimal profit. We determine foregone profits by Monte Carlo simulation that generates unit sales (in the following briefly called sales) from static aggregate price response functions. Let us emphasize that the information about both form and parameters of the functions is only available for the Monte Carlo simulation, but remains hidden for each of the evaluated pricing rules.

We proceed as follows: In Sect. 2 we prepare the simulation experiment. We start this section by detailing the decision problem and the two performance measures used to evaluate pricing rules. We present the eleven investigated pricing rules and the other factors of the simulation experiment (maximum and minimum sales values, price response functions, variable unit costs, error terms, planning horizon). We also show how to determine optimal prices and their point elasticities for the various price response functions. In Sect. 3 we analyze the simulation experiment by heteroscedastic regression models. In Sect. 4 we summarize results, discuss limitations of our study and indicate possible approaches to overcome these.

2 Preparing the simulation

2.1 Decision problem and objective function

A static aggregate price response function $Q(p)$ links the price of a product to its sales. We regard a monopolistic market in which prices of other products do not affect sales. The dominance of markup rules for price setting noticed in Fabiani et al. (2006) indicates that many firms operate in such markets. A static price response function considers sales for an aggregate of costumers (e.g., all costumers residing in a certain region) and not sales for individual consumers. The function is static as it does not include dynamic effects (e.g., lagged prices or lagged sales) and its parameters are constant across time.

We assume throughout that the cost function is linear with known constant variable unit costs c. Therefore profit $\Pi(p)$ as function of price p can be written as:

$$\Pi(p) = (p - c)\, Q(p) \tag{1}$$

We do not include fixed cost in expression (1) because it is irrelevant for the optimal solution. If the price response function and its parameter values are known, the profit-maximizing price p^* can be determined by solving the monopolistic pricing rule (also known as Amoroso-Robinson condition) which includes elasticities (Hanssens et al. 2001; Hirschey et al. 1993):

$$p^* = \frac{\varepsilon}{\varepsilon + 1}\, c \tag{2}$$

For a differentiable response function price elasticities ε can be determined as point elasticities in the following way:

$$\varepsilon = \frac{\partial Q(p)}{\partial p} \frac{p}{Q(p)} \tag{3}$$

Except for the multiplicative response function point elasticities vary with price (see Online Resource 2).

Let $Q : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ be a static aggregate price response function, that maps the price in a period t to sales in the same period. Q itself is continuous, monotonically decreasing, and may have concave and convex areas, to allow for an inverse S-shape. Furthermore, Q is not dependent on the period t.

In accordance with the marketing science literature we take into account that response functions are not deterministic (Hanssens et al. 2001; Leeflang et al. 2015). Therefore we add an error term to obtain a stochastic response function $Q_s(p) = Q(p) + e$. Errors, whose construction will be explained in Sect. 2.7, are normally distributed as $e \sim \mathcal{N}(0, \sigma^2)$ for a given variance σ^2 .

For a known price response function, it would be sufficient to consider expected profit because variable unit cost is constant (Jagpal 1999). Expected profit can be written as

$$\Pi : \mathbb{R}^+ \to \mathbb{R}^+, p \mapsto (p - c)\, Q(p). \tag{4}$$

$\Pi(p)$ is continuous, and hence attains its maximum Π^* at a price p^* due to Theorem 4.16 in Rudin (1976) since p is limited to a compact interval.

We define the forgone profit $F(p)$ at a price p as difference between the observed profit $\Pi_s(p) = (p - c)\, Q_s(p)$ at a price p and the expected profit at the optimal price $\Pi(p^*)$:

$$F(p) := \Pi_s(p) - \Pi(p^*) \tag{5}$$

As $\Pi(p^*)$ is the global maximum of Π, F is non-positive, and $F(p) = 0 \Leftrightarrow p = p^*$. Therefore the global maximum of F is identical to the global maximum of Π and maximizing F is equivalent to maximizing Π. Furthermore, this maximum is unique for all price response functions that we consider in the simulation (see Sect. 2.4).

The decision problem that we investigate is different, because there is no information on the true price response function and its parameters. The decision maker only observes sales Q_s as responses to prices she or he has set in previous periods. From observed sales the decision maker can, of course, determine the corresponding observed profit. Note that the optimal price and its corresponding expected profit are only determined during our Monte Carlo simulation, they are unknown to the decision maker and consequently not processed by any of the investigated rules.

We use two alternative performance measures based on the forgone profit just defined over an entire planning horizon to evaluate pricing rules. The higher any of these measures turns out for a pricing rule, the better we evaluate the latter. The first measure considers that the profit due to a price set by a rule should be as close as possible to the profit which results for the unknown optimal price p^*. This performance measure equals the average forgone profit across a given planning horizon of T periods:

$$\frac{1}{T} \sum_{t=1}^{T} F(p_t). \tag{6}$$

Specifying the second performance measure we follow the suggestion of one anonymous reviewer to allow for an asymmetry regarding the deviation from the optimal price, namely that “overpricing”is more costly than “underpricing”. We call this performance measure average asymmetric foregone profit. It is based on the generalized loss function of Elliot et al. (2005) and can be written as follows:

$$\frac{1}{T}\sum_{t=1}^{T}[\omega + (1-2\omega)\,\mathbb{1}(p_t > p*)]\,F(p_t) \tag{7}$$

For $0.0 < \omega < 0.5$ this measure puts more weight on a forgone profit $F(p_t)$ if the price set is greater than the optimal price because of the indicator function $\mathbb{1}(p_t > p*)$. ω equals 0.4 in our simulation study. Consequently, foregone profits are weighted by $0.6 = 0.4 + (1 - 2 \times 0.4)$ if the price is greater than the optimal price and by only 0.4 if the price is less equal the optimal price. This way we put more emphasis on foregone profits in the case of "overpricing". On the other hand, we still consider "underpricing" in a sufficient manner.

Seven of the investigated eleven rules require data of previous periods. Therefore sales and prices of the first two periods $t = 1, 2$ are equal and deterministic for each rule. An equivalent perspective is that $p_1, p_2, Q_s(p_1)$ and $Q_s(p_2)$ are common knowledge. We will explain these values at the beginning of Sect. 3.

2.2 Pricing rules

We investigate eleven pricing rules. Five rules assume either constant or changing elasticities that they insert into the Amoroso-Robinson condition (2) to determine price. Two rules start by approximating price responses by a linear or loglinear function. These two rules and the five rules based on elasticities can be seen as variants of the markup rule which dominates price setting in practice according to the survey of Fabiani et al. (2006) already mentioned in Sect. 1. In addition, we take three rules from a publication of Thore (1964) and finally consider random pricing as benchmark.

The first three rules assume that price elasticity is constant. Due to restrictions on time, resources or data a decision maker may simply rely on average elasticities published in a meta-analytic study (Bijmolt et al. 2005; Tellis 1988) or known for the respective product category and market. These three rules differ with respect to the value of the constant elasticity:

(1). a low elasticity of -1.5
(2). a medium elasticity -2.5
(3). a high elasticity of -3.5

The next two rules allow for changing elasticities:

(4). Arc elasticities

As the price response function is unknown, we estimate the arc elasticity ε_t from observed sales and prices of the previous two periods as (Monroe 1999):

$$\varepsilon_t = \frac{Q_s(p_{t-1}) - Q_s(p_{t-2})}{p_{t-1} - p_{t-2}} \frac{p_{t-2}}{Q_s(p_{t-2})} \tag{8}$$

(5). Smoothed arc elasticities

We apply first order exponential smoothing to reduce fluctuations of estimated elasticities. In period $t = 3$ we define $\tilde{\varepsilon}_3 := \varepsilon_3$ as in (8), and for $t > 3$ we define

$$\tilde{\varepsilon}_t := \beta\, \tilde{\varepsilon}_{t-1} + (1 - \beta)\, \varepsilon_{t-1} \quad \text{with} \quad \beta \in (0;1) \tag{9}$$

A global optimization search provides $\beta = 0.4$ as best value (see Appendix A).

The next two rules approximate the unknown response function by either a linear or a loglinear function using observed sales and prices up to period t for $t = 3, 4, \ldots, T$. Note that coefficients of the approximating model usually change from period to period.

(6). Linear approximation

We modify Rule 3 presented in Baumol and Quandt (1964) which is based on estimating a linear function for sales and prices of only two periods. If the true price response function is stochastic, using only two periods may easily run into difficulties implying a positive price elasticity. We approximate sales by a linear model $Q_s = a_0 - a_1 p$ and compute

$$p_t = (a_0/a_1 + c)/2 \tag{10}$$

from Baumol and Quandt (1964). This expression is equivalent to the static monopolistic pricing rule if both price response and cost functions are linear and known.

(7). Loglinear approximation

Log sales are approximated by estimating the loglinear model $\log(Q_s) = b_0 - b_1 \log(p)$ by OLS. Taking its antilog we obtain the multiplicative model $Q_s = \exp(b_0)p^{-b_1}$. If the multiplicative model is the true response function, $-b_1$ equals the constant elasticity. As the response function is unknown, we consider $-b_1$ an approximation to the unknown true elasticity. Consequently, we insert this elasticity estimate into the Amoroso-Robinson condition (2) to compute the price in period t.

The next three rules are based on a publication of Thore (1964) and look inter alia at observed profits $\Pi_s(p)$ which are computed from observed sales by $(p - c)\, Q_s(p)$. Trying to find the profit maximizing solution they determine the price in period t as $p_t = p_{t-1} + \Delta p_t$. These rules differ from each other with respect to the computation of the price change Δp_t.

(8). Constant changes

We extend Equation (4) in Thore (1964) which only outputs the sign of a price change by setting the absolute value of a price change to a constant value α_1:

$$\Delta p_t = \alpha_1 sign((\Pi_s(p_{t-1}) - \Pi_s(p_{t-2}))\,(p_{t-1} - p_{t-2})). \tag{11}$$

The rule builds upon the idea that after a price increase which has led to higher (lower) profits the decision maker is inclined to increase (decrease) next period's price. According to the same reasoning, if a price decrease has led to lower (higher) profits, the decision maker increases (decreases) next period's price. This simple mechanism is reproduced by

$$sign((\Pi_s(p_{t-1}) - \Pi_s(p_{t-2}))\,(p_{t-1} - p_{t-2})). \tag{12}$$

If this expression equals one (minus one) price is increased (decreased) by the constant amount α_1. A global optimization determines $\alpha_1 = 0.3$ as best value (see Appendix A).

(9). Changes dependent on the previous absolute price difference

This rule is similar to the previous one, but allows price changes to vary. We use the modification of the rule given by Equation (5) in Thore (1964) defined as:

$$\Delta p_t = \alpha_2 \sqrt{|(p_{t-1} - p_{t-2})|}\, sign((\Pi_s(p_{t-1}) - \Pi_s(p_{t-2}))\,(p_{t-1} - p_{t-2})). \tag{13}$$

We decide to insert price differences under the square root instead of the profit differences of the original formulation to get similar orders of magnitude. An independent simulation confirms that our modification works better than the original rule from Thore (1964). Price changes are not constant because $\alpha_2 \sqrt{|(p_{t-1} - p_{t-2})|}$ acts as stretch factor to the constant step size given by rule 8. This factor stabilizes the absolute amount of price changes, because higher (lower) previous price changes favor higher (low) current price changes in absolute terms. The best value for α_2 is found to be 0.4 by the method presented in Appendix A.

(10). Slope dependent changes

This rule is given by Equation (6) in Thore (1964):

$$\Delta p_t = \alpha_3 \frac{\Pi_s(p_{t-1}) - \Pi_s(p_{t-2})}{p_{t-1} - p_{t-2}}. \tag{14}$$

The rule is similar to gradient ascent of the profit function, but replaces the first derivative by the slope of the straight line connecting observed profits $\Pi_s(p_{t-1})$ and $\Pi_s(p_{t-2})$ (Chiang 1984). For the price response functions investigated in the Monte Carlo simulation gradient ascent with first derivative is known to find the optimal solution. Alas, here the derivative cannot be computed because it requires knowledge of the price response function. In our study slopes of observed profits take very high absolute values. We therefore decide to cushion the rule by the ratio of the interval centers p_{center}/Q_{center}

($p_{center} = 5, Q_{center} = Sat/2$ with Sat as saturation level of the response function). Introducing this factor leaves us with

$$\Delta p_t = \alpha_3 \left(p_{t-1} - c\right) \frac{Q_s(p_{t-1}) - Q_s(p_{t-2})}{Q_{center}} \frac{p_{center}}{p_{t-1} - p_{t-2}}, \tag{15}$$

ensuring that α_3 has the same order of magnitude as α_1 and α_2. The optimal value for α_3 (see Appendix A) was found to be 0.1. Notice further that using this rule requires a vague knowledge of the function's saturation level.

(11). Random pricing.

Taken from Baumol and Quandt (1964), we sample the price from a uniform distribution whose lower bound equals unit cost c and whose upper bound equals 9. Random pricing serves as benchmark in the evaluation, because only pricing rules that perform better deserve further scrutiny.

2.3 Restrictions on variables, parameters and elasticities

In our simulation the price range is [1; 9], the sales range [0; 1,000,000]. As we do not round off to integers, the entire simulation is scalable. In other words, these intervals can be transformed to different ranges and orders of magnitude yielding, as a rule, identical results.

As we want all of our functions to be comparable, we need them to have similar properties. As a rule, we set the highest possible value to $Q(1) = Max$, and the lowest possible value to $Q(9) = Min$.

We assume that price response is elastic and do not allow for elasticities outside the interval $[-7; -1.25]$. If a computed elasticity lies outside the interval, it is projected to the nearest boundary before determining the price p_t in period t. We show how elasticities are computed in Sect. 2.6. This way we cover a large part of the distribution of elasticity values determined in the meta-analysis of Bijmolt et al. (2005).

Moreover, we are in a position to investigate pricing rules using the Amoroso-Robinson relation, i.e., rules drawing upon elasticities as well as rules involving a linear or loglinear approximation. Obviously, these rules do not work for elasticities greater than – 1.0, no matter whether they lie in the interval $[-1.0, 0.9]$ or are positive > 0.0. In the meta-analysis of Bijmolt et al. (2005) the relative frequencies of these two value ranges amount to 16.5 % and 2.2 %, respectively. On the other hand, the two rules *constant changes* and *changes dependent on the previous absolute price difference* do not require elasticities lower than – 1.0 and may work under such circumstances.

We do not allow prices to be smaller than variable unit costs or larger than the maximum price. Should that happen, the price is set to equal to variable unit costs or the maximum price of 9, respectively.

As we add a normally distributed error term to Q in order to obtain Q_s, it is possible to obtain non-positive sales values. To prevent non-positive values we set the absolute minimum sales to 10 (an order of magnitude below the low minimum level). Therefore we use the value max $(10, Q_s)$ in the simulation. This might lead to the fraction in expression (15) for the rule *slope dependent changes* being ill-defined, in which case Δp_t is randomly set to either $+\alpha_3$ or $-\alpha_3$.

2.4 Functional forms, sales levels and coefficients

We consider the following functional forms of price response functions (Hanssens et al. 2001):

(1). The linear function $Q_{lin}(p) = a_0 - a_1 p$
(2). The multiplicative function $Q_{mult}(P) = b_0 p^{-b_1}$
(3). The exponential function $Q_{exp}(p) = e^{c_0 - c_1 p}$
(4). The semi-logarithmic function $Q_{semlog}(p) = d_0 - d_1 \, log(p)$
(5). The logistic function $Q_{log}(p) = \frac{Q_{max}}{1+e^{-(f_0 - f_1 p)}}$

with positive parameters $a_0, a_1, b_0, b_1, c_0, c_1, d_0, d_1, f_0, f_1 > 0$ (see Appendix B).

Please note that we restrict our attention to widespread functional forms. Therefore, we do not include the Gutenberg price response function (Simon and Fassnacht 2019). In addition, the fact that this price response function usually requires knowledge of the average price of competitors justifies this decision.

The properties of the five investigated functional forms depend on their coefficients. We therefore start with a table of properties for the functions and choose coefficients such that the functions adhere to these properties. We define a low, medium and high level for each of the two properties ‘Maximum’, ($Q(1)$, the highest possible sales value) and ‘Minimum’ (Q(9), the lowest possible sales value). The values can be seen in Table 1.

Given values of maximum and minimum sales, we can uniquely solve for the coefficients of these functions (for a proof, see Appendix B):

(1). for the linear function:
$$a_0 = \frac{9\,Max - Min}{8}, a_1 = \frac{Max - Min}{8}$$
(2). for the multiplicative function
$$b_0 = Max, b_1 = \frac{log(b_0/Min)}{log(9)}$$
(3). for the exponential function
$$c_0 = \frac{9\,log(Max) - log(Min)}{8}, c_1 = \frac{log(Max) - log(Min)}{8}$$

Table 1 Properties

Variable name	Maximum Sales	Minimum Sales
Level	*Max*	*Min*
Low	500,000	1000
Medium	750,000	5000
High	1,000,000	10,000

(4). for the semi-logarithmic function
$d_0 = Max, d_1 = \frac{d_0 - Min}{log(9)}$
(5). and for the logistic function
$$Q_{max} = 1.005\, Max, f_0 = \frac{9\, log\left(\frac{Max}{Q_{max}-Max}\right) + log\left(\frac{Q_{max}-Min}{Min}\right)}{8}, f_1 = \frac{log\left(\frac{Max}{Q_{max}-Max}\right) + log\left(\frac{Q_{max}-Min}{Min}\right)}{8}.$$
To obtain unique coefficient values for the logistic function we add a third restriction that makes Q_{max} 0.5% higher than the respective maximum sales value.

2.5 Optimal prices and variable unit costs

As explained in Sect. 2.1, we need to know the price maximizing the expected profit. So, for each function type, given its coefficients and variable unit cost c we need an exact formula for the optimal price p^*. The derivation of the following formulae can be found in Online Resource 1:

(1). for the linear function:
$p^* = 1/2(\frac{a_0}{a_1} + c)$
(2). for the multiplicative function
$p^* = \frac{-b_1 c}{-b_1 + 1}$
this coincides with the Amoroso-Robinson condition (2) for a constant elasticity.
(3). For the exponential function:
$p^* = \frac{1 + c_1 c}{c_1}$
(4). for the semi-logarithmic function:
$p^* = \frac{d_1 c}{W(d_1\, c\, e^{1-d_0})}$
(5). and for the logistic function:
$p^* = \frac{W(exp(f_0 - 1 - cf_1)) + 1 + cf_1}{f_1}$

Variable unit costs take one of the three values 2, 3 and 4 in our simulation. We explain the rationale for these values, which also applies to the other functional forms, for the linear price response function. For the linear function the contribution margin $p - c$ is $(a_0/a_1 - c)/2$ at the optimal price. Contribution margins amount to 3.52, 3.02 and 2.55 for the three cost levels at the medium sales level, i.e., they differ from each other by about 0.50. Contribution margins at low and high sales levels turn out to be similar. Regression analyses presented in Sect. 3 support these values as the performance of pricing rules differs significantly between the three cost levels.

2.6 Point elasticities

To analyze the results of the Monte Carlo simulation we need to know the point elasticities at the optimum price p^* (see Sect. 3) which for a differentiable price response function $Q(p)$ are defined by expression (3). The point elasticities at p^* can be calculated as follows (for the derivation see Online Resource 2):

(1). for the linear function:
$$\varepsilon = \frac{-a_1 p^*}{a_0 - a_1 p^*}$$
(2). for the multiplicative function:
$$\varepsilon = -b_1$$
(3). for the exponential function:
$$\varepsilon = -c_1 p^*$$
(4). for the semi-logarithmic function:
$$\varepsilon = \frac{-d_1}{d_0 - d_1\, log(p^*)}$$
(5) and for the logistic function:
$$\varepsilon = -f_1 p^* \left(1 - \frac{Q_{log}(p^*)}{Q_{max}}\right)$$

2.7 Error terms

As mentioned in Sect. 2.1, we add a normally distributed error term $\varepsilon \sim \mathcal{N}(0, \sigma^2)$ to the deterministic part of the response function to obtain the stochastic response function Q_s as $Q_s(p) := Q(p) + \varepsilon$. Again, we wish to be able to compare different functions while keeping the error level constant. A high (low) error level corresponds to a low (high) coefficient of determination, R-squared, which measures the percentage of variance of the dependent variable (in our case: sales) explained by a regression model. We use R-squared values because in contrast to residual variances σ^2 they can be compared across studies even if the dependent variable sales is scaled differently.

We choose a value for the coefficient of determination R-squared from $\{0.9, 0.7, 0.5\}$ corresponding to a low, medium and high error-term level, respectively. Then we perform an iterative search over values of σ^2 in the interval between 10^6 and 10^{15}. For each value of σ^2 we input 5,000 prices uniformly distributed in the interval [1; 9] into the deterministic response function to obtain $Q(p)$ to which we add errors from $\mathcal{N}(0, \sigma^2)$ giving stochastic sales $Q_s(p)$. For the values of prices and sales obtained this way we then estimate the parameters of the response function by nonlinear least squares and compute its R-squared value. We stop the iterative search over σ^2 once the absolute difference to the desired R-squared value is less than 0.0001.

2.8 Planning horizon

We also wish to vary the planning horizon and distinguish short, medium and long planning horizons with 10, 20 and 30 periods, respectively. Each of these planning horizons includes the first two predetermined periods.

3 Analysis of simulation results

Our simulation experiment uses a complete factorial design, i.e., it considers all factor level combinations. For eleven pricing rules, five functional forms, three maximum sales and three minimum sales values, three variable unit cost levels, three R-squared values and three planning horizons we obtain $11 \times 5 \times 3^5 = 13,365$ combinations. The simulation generates one value for the sum of forgone profits for each combination. In other words, we have 13,365 observations at our disposal, a number that should be more than sufficient for regression models with 29 coefficients, which we use to analyze simulation results. Power analyses (Cohen 1988) providing values close to 1.0 even for regression models with small R^2 values confirm this assumption.

Across all combinations point elasticities at the optimal price attain rounded relative frequencies of $5\%, 16\%, 47\%$ and 31% for the intervals $[-6; -4], (-4; -3], (-3; -2], (-2; -1]$, respectively. We note that this distribution of elasticities is very similar to the one documented in the meta-analysis of price elasticities (Bijmolt et al. 2005) with relative frequencies of $16\%, 19\%, 32\%$ and 31%. A Battacharyya coefficient of 0.96 provides evidence to this high similarity (Aherne et al. 1997).

We analyze the simulation results by means of two linear regression models with multiplicative heteroscedasticity specifying residuals as function of the investigated pricing rules. The regression models are estimated by the iterative scoring algorithm described in Greene (2003). The two regression models differ with respect to the dependent variable, namely average foregone profit and average asymmetric foregone profit. The independent variables of the regression models consist of binary dummy variables that are related to levels of the experimental factors and to four class intervals of the point elasticities at the optimum price. We define binary dummy variables of each factor or elasticity classification with respect to a base category (pricing rules: random pricing, functional form: linear, minimum sales: 1000, maximum sales: 500,000, variable unit cost: 2, planning horizon: 10 periods, R-squared value: 0.5, elasticity interval : $[-6; -4]$).

The heteroscedastic regression models allow that residual variances differ between pricing rules. The homoscedastic regression models with their restriction of equal residual variances are rejected for both average foregone profits and average asymmetric foregone profits due to likelihood ratio tests amounting to 3182.55 and 3003.90, respectively.

Table 2 contains the estimation results. Let us remind you that the higher foregone profits are, the closer a rule gets to the optimum (see Sect. 2.1). Four pricing rules are worse than *random pricing* (i.e., they lead to lower forgone profits) as their negative coefficients show (*low constant elasticity*, *arc* and *smoothed arc elasticities*, *loglinear approximation*). A positive coefficient that implies higher forgone profits on the other hand indicates that the respective rule does better than *random pricing*. *Constant changes*, *changes dependent on the previous absolute price difference*, and *medium constant elasticity* turn out as the three best performing rules. Their coefficients amount to 99.80, 94.46 and 90.44 for average

Table 2 Heteroscedastic regressions for the two performance measures

Independent Variable	Average Foregone Profit			Average Asymmetric Foregone Profit		
	Coefficient	Standard error	t-value	Coefficient	Standard error	t-value
(Intercept)	– 212.35	8.76	– 24.24	– 109.97	4.27	– 25.75
Pricing Rule						
Constant low elasticity	– 70.34	6.94	– 10.14	– 58.11	4.26	– 13.64
Constant medium elasticity	90.44	3.94	22.95	51.13	2.05	24.94
Constant high elasticity	27.08	4.94	5.49	28.21	2.30	12.27
Arc elasticities	– 148.29	6.94	– 21.37	– 44.84	2.98	– 15.05
Smoothed arc elasticities	– 79.47	5.81	– 13.68	– 20.67	2.63	– 7.86
Linear approximation	87.93	4.46	19.70	39.71	2.41	16.48
Loglinear approximation	– 122.58	8.30	– 14.77	– 34.57	3.49	– 9.91
Constant changes	99.80	3.97	25.17	52.75	2.15	4.53
Changes dependent on the						
Previous absolute price difference	94.46	3.96	23.85	53.50	2.11	25.36
Slope dependent changes	5.05	4.06	1.24	3.59	2.37	1.51
Functional form						
Multiplicative	203.45	3.76	54.04	90.45	1.78	50.77
Exponential	156.82	4.28	36.66	70.49	2.02	34.82
Semilogarithmic	108.35	3.54	30.64	46.82	1.67	27.98
Logistic	– 57.64	3.89	– 14.81	– 32.37	1.84	– 17.58
Minimum Sales						
5,000	– 10.38	2.78	– 3.74	– 6.32	1.31	– 4.81
10,000	– 0.31	2.89	– 0.11	– 2.82	1.37	– 2.06
Maximum Sales						
750,000	– 46.94	2.63	– 17.84	– 20.94	1.25	– 16.82
1,000,000	– 96.15	2.65	– 36.32	– 43.21	1.25	– 34.49
Variable unit cost						
3	68.95	2.99	23.06	33.9	1.42	23.95
4	81.00	3.71	21.84	37.14	1.76	21.16
Planning horizon						
20 periods	– 2.00	2.60	– 0.77	– 0.94	1.23	– 0.76
30 periods	– 0.84	2.60	– 0.32	– 1.66	1.23	– 1.35
R-squared value						
0.7	9.08	2.60	3.50	2.86	1.23	2.33
0.9	20.37	2.60	7.84	7.96	1.23	6.47
Elasticity range						
(– 4,– 3]	– 13.85	5.75	– 2.41	– 1.56	2.72	– 0.57
(– 3,– 2]	– 26.31	6.11	– 4.30	– 2.40	2.89	– 0.83
(– 2,– 1]	– 136.27	7.28	– 18.72	– 49.69	3.45	– 14.42

Performance Measures in Thousands

forgone profit and to 52.75, 53.50 and 51.50 for average asymmetric foregone profit. We explain lower coefficient values for average asymmetric foregone profits by the fact that foregone profits are multiplied by either 0.6 or 0.4 to compute this performance measure (see Sect. 2.1).

For average foregone profit the rule *constant changes* attains the highest coefficient, *changes dependent on the previous absolute price difference* is second best. For average asymmetric foregone profit, these two rules change places. Therefore, we recommend decision makers to choose one of these two price setting rules.

In the following, we discuss regression results for the remaining factors, which, as a rule, are similar for both performance measures. We see positive coefficients for multiplicative, exponential and semi-logarithmic sales response functions. The multiplicative function attains the highest coefficient, which may be explained by the constant elasticity property in contrast to the other functions. We obtain the lowest coefficients for the logistic function. This result indicates that heuristics are more likely to deviate from the optimum as the expression of the optimal price is more complex compared to the other investigated functions. Length of the planning horizon has no effect on foregone profits, as coefficients of its values are not significant. Higher values of maximum sales and higher elasticities (lower price sensitivity) allow higher profits, but also imply that foregone profits (profit losses) are lower. With respect to average asymmetric forgone profits, we get such a result only for the highest elasticity range $(-2, -1]$. For the other elasticity ranges we obtain no significant differences. On the other hand, higher unit variable costs, which lead to lower profits, increase foregone profits. Higher R-squared values that are equivalent to lower errors of the response functions make it easier for rules to get close to the optimum solution. That is why the regressions provide higher and positive coefficients for higher R-squared values.

4 Conclusion

We conduct a simulation experiment to evaluate eleven rules of thumb for setting the price in a monopolistic market for one product based on their capability to approximate optimal profit. Decision makers may resort to such rules if they know neither the form of the price response function nor its coefficients. Functional forms, maximum sales and minimum sales values, unit cost levels, R-squared values and planning horizons constitute additional factors of the simulation experiment. We evaluate pricing rules with respect to average foregone profit and average asymmetric foregone profit across a time horizon of several periods. Foregone profits measure how close profits achieved by determined prices get to optimal profits.

We analyze the simulation data by two heteroscedastic regression models with average foregone profit or average asymmetric foregone profit as dependent variable. Factor levels constitute the independent variables of these regression models. Random pricing serves as benchmark for the other ten rules. Rules based on arc elasticities and a rule based on approximating a loglinear function perform worse than random pricing. Based on these regression models we recommend decision makers to choose one of the two price setting rules, *constant changes* or *changes dependent on the previous absolute price difference*.

As suggested by two anonymous reviewers we discuss the limitations of our study in the following. If deemed possible we also give tentative hints how these limitations may be overcome by extending the investigated pricing rules. Of course, further research is needed to assess these hints.

We base the simulation on aggregate price response functions and linear cost functions, which imply that the optimal price is uniform. We admit that our approach is unable to deal with nonlinear tariffs such as quantity discounts or quantity surcharges (Iyengar and Gupta 2009), because nonlinear tariffs require information on the price response of individual consumers as shown in Allenby et al. (2004).

4.1 Extensions for product line pricing

We start by indicating how our approach may be extended to tackle product line pricing, i.e., pricing of several substitutive products that belong to the same category. One could extend the *linear approximation rule* to consider products of the same category. To this end we specify a linear approximation of sales for each product i of the category which also includes prices of the other products $j \neq i$:

$$Q_i = a_{i0} - a_{ii}p_i + \sum_{j \neq i} a_{ij}p_j \tag{16}$$

Assuming linear cost functions we get an Ersatz optimization problem consisting of linear equations (Simon and Fassnacht 2019). For each product of the category we have one equation of the following form:

$$Q_i + (p_{it} - c_i)(-a_{ii}) + \sum_{j \neq i} (p_{jt} - c_j)\, a_{ji} = 0 \tag{17}$$

c_i are the variable unit costs of product i. a_{ji} denotes the price coefficient of product i in the linear approximation of sales for product j.

Extending the *constant elasticity rule* by taking cross-price elasticities into account constitutes another possibility. In this case, optimization leads to a system of nonlinear equations. Usually, in absolute size cross-price elasticities are much smaller than elasticities. For example, in Draganska and Jain (2006) cross-price elasticities of yogurt brands assume values between 0.06 and 0.27, whereas elasticities lie between -2.45 and -6.25. A starting point could be the average cross-price elasticity of 0.26 that Auer and Papies (2020) determine in their meta-analysis. Decision makers may modify this average value in accordance with Sethuraman et al. (1999) who show for 280 brands in 19 different grocery categories that cross-price elasticities are higher if brand i has a higher price and for pairs of brands with similar prices. Of course, we need further investigations to evaluate the performance of such an extended constant elasticity rule.

We could also apply the rules *constant changes* and *changes dependent on the previous absolute price difference* to each product as these rules try to approximate the unknown gradient of the profit function with respect to the price of a product. Both rules would erroneously attribute the marginal price effects of other products to the marginal price effect of each considered product, which could deteriorate performance.

4.2 Extensions for additional marketing variables

We now deal with the question how effects of additional marketing variables, e.g. advertising, promotion and sales force budgets, may be taken into account. At first, let us assume that these budgets are constant. We can justify this assumption by the fact that optimal budgets are constant for most (even for most dynamic) sales response functions (Leeflang et al. 2015). Under this assumption, all rules are appropriate that attain a better performance in our simulation. For the *constant medium elasticity* rule, the decision maker may increase the elasticity by adding the respective positive values of coefficients for such marketing variables that Bijmolt et al. (2005) determine in their meta-analysis (0.68 for distribution, 0.84 for advertising, and 0.79 for promotion). These additions lead to higher elasticities, i.e., less elastic demand.

If one allows for changing budgets , the rules *constant changes* and *changes dependent on the previous absolute price difference* should be avoided because they cannot separate between the price effect and the effects of budgets. On the other hand, one may add budgets (which may be transformed to reflect decreasing returns to scale (Hanssens et al. 2001)), to the *linear approximation*. For several budget variables x_k this leads to a linear model $Q_s = a_0 - a_1 p + \sum_k a_{k+1} x_k$. The corresponding pricing rule can be written as:

$$p_t = ((a_0 + \sum_k a_{k+1} x_k)/a_1 + c)/2 \tag{18}$$

4.3 Extensions for the duopolistic situation

So far we have only dealt with the monopolistic situation. Although many firms operate in such markets (Fabiani et al. 2006), we now sketch several possibilities to specify pricing rules that take a duopolistic situation into account. In the following competition might refer either to an individual competitor or to an average of competitive firms. One possibility consists in extending the *constant elasticity* rule by using a corrected price elasticity that considers the cross-price elasticity of sales with respect to the price of competitor ϵ_c and the reaction elasticity of the competitor's price with respect to the price of the firm ρ_c as follows (Simon and Fassnacht 2019):

$$\epsilon^{corr} = \epsilon + \rho_c \epsilon_c \tag{19}$$

We see from expression (19) that we are back to the monopoly situation if either price reaction elasticity or cross-price effects are zero. In addition, if cross-price elasticities or reaction elasticities are low, decision makers may safely set prices like monopolists.

As a rule, both cross-price and reaction elasticities are positive. The former, because a higher competitive price leads to higher sales of the product. The latter, because competitors react to a price increase (decrease) if the firm by a price increase (decrease). These properties make demand less elastic, i.e., the corrected elasticity is higher than the original price elasticity ϵ.

To determine the corrected price elasticity a decision maker could take the average cross-price elasticity from the meta-analysis of Auer and Papies (2020), which is 0.26. Alas, we are not aware of meta-analytic studies on reaction elasticities. Published competitive reaction elasticities vary strongly (e.g., greater than 0.50 in Lambin (1976) and mostly below 0.20 in Cotterill et al. (2000)).

Extending the *linear approximation rule* constitutes another possibility to deal with the duopolistic situation. Now the estimated linear function $Q_s = a_0 - a_1 p + a_2 p_c$ also includes the competitor's price p_c, which must be known. This extended linear approximation rule corresponds to the optimal price reaction to a competitive price for a linear price response function (Simon and Fassnacht 2019). It can be written as:

$$p_t = ((a_0 + a_2 p_c)/a_1 + c)/2 \tag{20}$$

The two rules *constant changes* and *changes dependent on the previous absolute price difference* may be applied to the oligopoly situation as they try to approximate the unknown gradient of the profit function with respect to the price of the product. They do not need information on the effect of competitive prices, competitive price reactions or competitive prices. Future research may investigate if this advantage is outweighed by lower performance of these rules.

4.4 Additional extensions

Finally, let us mention a few other extensions of our study. Future research may investigate whether modifications of the response functions underlying the Monte Carlo simulation have an effect on the performance of pricing rules. Allowing discrete or continuous changes of parameters of price related to different preferences or price sensitivities of customers constitutes one extension. It is also possible to consider switches of the functional form, e.g., from a multiplicative to an exponential price response function. Moreover, one could use diffusion models including a price decision variable. Diffusion models are appropriate to analyze the demand for durable goods. Generalized variants of the Bass model are obvious examples of diffusion models of this kind (Mahajan et al. 1990).

A Optimization of rule parameters

To find an optimal value for a parameter of a certain rule, we proceed as follows:

For a parameter $\mu \in \mathbb{R}$ and a rule $R = R(\mu)$ which depends on the parameter μ, let $p_t(R(\mu))$ be the price determined by rule R given parameter μ. For $t = 1, 2$ this is the common-knowledge price from the first two periods, and for $t > 2$ it is whatever rule R calculates, based on previous periods' data. We have four rules that depend on such a parameter:

(1). *smoothed arc elasticities* need a smoothing constant $\mu = \beta$,
(2). *constant changes* need a constant step size $\mu = \alpha_1$,
(3). *changes dependent on the previous absolute price difference* need a step size factor $\mu = \alpha_2$, and
(4). *slope dependent changes* likewise need a step size factor $\mu = \alpha_3$.

We then determine a reasonable range for μ and then discretize this range to a number of fixed values $\mu_1, ..., \mu_9$.

For β the range is already (0; 1), and we choose the nine discrete values $\mu_1 = 0.1, \mu_2 = 0.2, ..., \mu_9 = 0.9$ with a distance of 0.1 between consecutive values.

For both α_1 and α_2 the most useful range is (0; 1) as well, so we also choose the same discrete values.

Based on further analysis, any value of α_3 larger that 0.5 is useless, so from the range (0; 0.5) we choose the values $\mu_1 = 0.05, \mu_2 = 0.1, ..., \mu_8 = 0.4, \mu_9 = 0.45$ with a distance of 0.05 between consecutive values.

Given a rule R, for each possible index $i = 1, .., 9$ we perform the price finding optimization from the simulation study for the parameter μ_i by setting the price to be $p_t(R(\mu_i))$ for period t and obtain an average performance value of the sum of forgone profits and proximity to the optimal price. Based on the rankings of those two performances compared among all values $\mu_1, ..., \mu_9$, we chose the parameter μ_i with the overall best ranking.

We obtained the values of $\beta = 0.4, \alpha_1 = 0.3, \alpha_2 = 0.4, \alpha_3 = 0.1$. For β we repeated the process with a finer step-size of 0.05, which led to the same result.

B Derivations of coefficients

In this section, we derive the coefficients of the five functions, given their properties. As mentioned earlier, we want all deterministic functions to achieve their predetermined maximum value *Sat* at a price of $p = 1$, and the predetermined minimal value *Min* at $p = 9$. For purpose of readability, we call the maximum *Sat* instead of *Max* throughout this Appendix. Remember that our functions have two parameters each, leading to two degrees of freedom, with one exception. The logistic function has a third parameter Q_{max}, which is the asymptotic maximum, or supremum. So we set $Q_{max} := 1.005 \cdot Sat$, so the supremum is 0.5% above the value at $p = 1$. All we need to do is start with the equations $Q(1) = Sat, Q(9) = Min$ and possibly

$Q_{max} := 1.005 \cdot Sat$ and solve for the parameters of the function. At some point we use the fact, that the maximal attained value of Min is 1, 000, and the minimal attained value of Sat is 500, 000.

(1). The linear function $Q_{lin}(p) = a_0 - a_1 \cdot p$

$$Q(1) = a_0 - a_1 = Sat \tag{B.1}$$

$$Q(9) = a_0 - 9a_1 = Min \tag{B.2}$$

This is just an ordinary system of two linear equations, so define

$$\mathbf{M} := \begin{pmatrix} 1 & -1 \\ 1 & -9 \end{pmatrix}, \mathbf{a} := \begin{pmatrix} a_0 \\ a_1 \end{pmatrix}, \mathbf{V} := \begin{pmatrix} Sat \\ Min \end{pmatrix}, \tag{B.3}$$

and all that's left is $\mathbf{Ma} = \mathbf{V}$, with $\mathbf{M}^{-1} := \begin{pmatrix} 9/8 & -1/8 \\ 1/8 & -1/8 \end{pmatrix}$ and hence $\mathbf{a} = \mathbf{M^{-1}V}$, in particular

$$a_0 = \frac{9Sat - Min}{8}, \tag{B.4}$$

$$a_1 = \frac{Sat - Min}{8}, \tag{B.5}$$

both of which are positive, as $Sat > Min$.

(2). The multiplicative function $Q_{mult}(p) = b_0 \cdot p^{-b_1}$
$Q(1) = b_0 = Sat > 0$ is fairly straightforward, and then

$$Q(9) = Sat \cdot 9^{-b_1} = Min \Rightarrow -b_1 log(9) = log(Min/Sat) \tag{B.6}$$

$$\Rightarrow b_1 = \frac{log(Sat/Min)}{log(9)}. \tag{B.7}$$

Again, $b_1 > 0$ as $Sat > Min$.

(3). The exponential function $Q_{exp}(p) = e^{c_0 - c_1 \cdot p}$
taking the logarithm on both sides of the equations $Q(1) = Sat$ and $Q(9) = Min$ again leads to a system of linear equations with the same matrix $\mathbf{M}$ as in (B.3). We can therefore proceed as in the linear case to obtain.

$$c_0 = \frac{9log(Sat) - log(Min)}{8} \tag{B.8}$$

$$c_1 = \frac{log(Sat) - log(Min)}{8} = \frac{log(Sat/Min)}{8}. \tag{B.9}$$

Again, since $Sat > Min$, both c_0 and c_1 are positive.

(4). The semi-logarithmic function $Q_{semlog}(p) = d_0 - d_1 \cdot log(p)$

$$Q(1) = d_0 = Sat > 0 \text{ is again straightforward, followed by} \tag{B.10}$$

$$Q(9) = Sat - d_1 log(9) = Min \Rightarrow d_1 log(9) = Sat - Min \Rightarrow d_1 = \frac{Sat - Min}{log(9)} > 0. \tag{B.11}$$

(5). The logistic function $Q_{log}(p) = \frac{Q_{max}}{1+e^{-(f_0 - f_1 \cdot p)}}$
The equations

$$Q(1) = Sat = \frac{Q_{max}}{1 + exp(f_1 - f_0)}, \text{ and} \tag{B.12}$$

$$Q(9) = Min = \frac{Q_{max}}{1 + exp(9f_1 - f_0)} \tag{B.13}$$

can be transformed into

$$f_1 - f_0 = log\left(\frac{Q_{max} - Sat}{Sat}\right) \tag{B.14}$$

$$9f_1 - f_0 = log\left(\frac{Q_{max} - Min}{Min}\right). \tag{B.15}$$

We rename the right hand side of (B.14) and (B.15) as $\hat{S} := log(\frac{Q_{max}-Sat}{Sat})$ and $\hat{M} := log(\frac{Q_{max}-Min}{Min})$. and again summarize the equations as

$$-\mathbf{M}\begin{pmatrix} f_0 \\ f_1 \end{pmatrix} = \begin{pmatrix} \hat{S} \\ \hat{M} \end{pmatrix} \tag{B.16}$$

and solve my multiplying with $-\mathbf{M}^{-1}$ from the left to obtain

$$f_0 = \frac{-9\hat{S} + \hat{M}}{8} \tag{B.17}$$

$$f_1 = \frac{-\hat{S} + \hat{M}}{8}. \tag{B.18}$$

Remains to show positivity. Note that

$$\hat{M} = log\left(\frac{Q_{max} - Min}{Min}\right) = log(Q_{max} - Min) - log(Min) \tag{B.19}$$

and since

$$Min \le 1000, Q_{max} > S \ge 500.000 \tag{B.20}$$

we have

$$Q_{max} - Min > Sat - Min \geq 500.000 - 1000 = 499.000 \tag{B.21}$$

and

$$\hat{M} = log(Q_{max} - Min) - log(Min) > log(499.000) - log(1000) > 6. \tag{B.22}$$

Furthermore

$$\hat{S} = log\left(\frac{Q_{max} - Sat}{Sat}\right) = log\left(\frac{1.005Sat - Sat}{Sat}\right) = log(0.005) < -5, \tag{B.23}$$

and hence

$$f_0 = \frac{-9\hat{S} + \hat{M}}{8} \geq 51/8 > 0 \tag{B.24}$$

$$f_1 = \frac{-\hat{S} + \hat{M}}{8} \geq 11/8 > 0 \tag{B.25}$$

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