

Constraining Weil-Petersson volumes by universal random matrix correlations in low-dimensional quantum gravity

Torsten Weber¹, Fabian Haneder¹, Klaus Richter¹, and Juan Diego Urbina¹

¹Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

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Abstract

Based on the discovery of the duality between Jackiw-Teitelboim quantum gravity and a double-scaled matrix ensemble by Saad, Shenker and Stanford in 2019, we show how consistency between the two theories in the universal Random Matrix Theory (RMT) limit imposes a set of constraints on the volumes of moduli spaces of Riemannian manifolds. These volumes are given in terms of polynomial functions, the Weil-Petersson volumes, solving a celebrated nonlinear recursion formula that is notoriously difficult to analyze. Since our results imply *linear* relations between the coefficients of the Weil-Petersson volumes, they therefore provide both a stringent test for their symbolic calculation and a possible way of simplifying their construction. In this way, we propose a long-term program to improve the understanding of mathematically hard aspects concerning moduli spaces of hyperbolic manifolds by using universal RMT results as input.

1. Introduction

While initially, the methods and concepts of quantum chaos attempted to explain how chaos in a classical system finds its way into the observed universality of short-range spectral fluctuations in the corresponding quantised version [1–3], since its precise formulation in the 1980's [4] the connection between dynamical chaos and Random Matrix Theory (RMT) has also offered some deep insights into mathematical questions. A paradigmatic example of how such an approach works is number theory. Here, one starts from the conjectured RMT-like statistical distribution of the non-trivial zeroes of the Riemann zeta function [5], supported by a huge amount of numerical evidence [6], and uses it to obtain number-theoretical results [7], even defining a whole research program.

The rationale of this approach can be exported to any field and regime of parameters where fidelity to RMT is expected to hold. Then, the universal – and usually tractable – RMT results can be considered as constraints, imposing relations between physical objects of the theory, similar to how the statistical correlations of prime numbers are constrained by RMT. This is particularly the case for theories where the microscopic mechanism responsible for classical chaotic dynamics is not well understood, and therefore the well-developed machinery of periodic orbit theory [1, 8–10] cannot be invoked to explain RMT-like features.

Remarkably, certain aspects of quantum gravity fall into this category. This is because, although the precise connection between periodic orbit theory and the conjectured chaotic character of important quantum gravitational models [11–16] is still an open problem, an exact mapping between 2D dilaton gravity (so-called Jackiw-Teitelboim gravity [17, 18]) and a matrix model [19] has recently been discovered, creating an explosion of interest [20–24].

The present paper (and its companion [25]) follows the route depicted above. We invoke the well justified *assumption* that in a certain precise limit, spectral correlations computed from the exact solution of JT gravity are identically given by the universal RMT results. Imposing such an equivalence then constrains the objects appearing on the JT side, which in this case turn out to be related to the moduli space of two-dimensional manifolds. Our objective is to make these constraints explicit, and to initiate the study of their structure and consequences. Doing so, we provide further evidence for the equivalence of JT gravity and universal RMT, and in particular the operational meaning of the “universal limit” as proposed in [21] and further developed in [26].

To begin with, we now briefly present the main features of Jackiw-Teitelboim gravity, which is a two-dimensional theory of gravity coupled to a dilaton ϕ , described by the action

$$S_{\text{JT}} = -\frac{S_0}{2\pi}\chi(\mathcal{M}) - \frac{1}{2}\int_{\mathcal{M}}\sqrt{g}\phi(R+2) - \int_{\partial\mathcal{M}}\sqrt{h}\phi(K-1). \quad (1.1)$$

The first term is the Euler characteristic of the manifold \mathcal{M} , multiplied by a constant $S_0 \sim \frac{1}{G_N}$, and causes the path integral over (1.1) to decompose into a genus expansion of the form

$$\langle Z(\beta_1)\dots Z(\beta_n)\rangle = \sum_{g=0}^{\infty} e^{(2-2g-n)S_0} \int_0^{\infty} b_1 db_1 \dots b_n db_n Z_{\text{tr}}(\beta_1, b_1)\dots Z_{\text{tr}}(\beta_n, b_n) V_{g,n}(b_1, \dots, b_n), \quad (1.2)$$

where $Z_{\text{tr}}(\beta, b)$ is called the trumpet partition function (the precise form of which is not important here), and $V_{g,n}(b_1, \dots, b_n)$ are the Weil-Petersson volumes, detailed further below. Eq. (1.2) holds except for special cases¹, the genus 0 contributions for $n = 1, 2$.

This theory has received a lot of attention in recent years due to its utility in studying black holes [27–33], but also owing to the convenient computability of n -point functions, and therein the appearance of wormhole geometries. The latter are of central importance to the factorisation problem [34–43]. Several limits, deformations, generalisations and applications have also been studied [44–63] after a duality between JT gravity and a certain double-scaled Hermitian matrix model was established in [19]. To be more precise, the authors of Ref. [19] found a matrix model defined by some potential $V(H)$, and performed the limit $\dim H = N \rightarrow \infty$ in such a way that the leading spectral density remained normalised, yielding

$$\rho_0^t(E) = e^{S_0} \frac{\gamma}{2\pi^2} \sinh\left(2\pi\sqrt{2\gamma E}\right). \quad (1.3)$$

In the course of taking this limit, the usual perturbative $1/N$ expansion of the matrix model is replaced by an expansion in e^{-S_0} , e.g. for correlators of the partition function $Z(\beta) = \text{tr} e^{-\beta H}$,

$$\langle Z(\beta_1)\dots Z(\beta_n)\rangle = \sum_{g=0}^{\infty} e^{(2-2g-n)S_0} Z_{g,n}(\beta_1, \dots, \beta_2). \quad (1.4)$$

It is this genus expansion that has been shown to exactly compute (1.2). However, establishing the translation of nonperturbative aspects of the matrix model to JT gravity has proven more challenging (see however [64–67]). A key reason for this is that the matrix model of [19] suffers

¹The $n = 1$ result is the disk $Z_{\text{disk}}(\beta) = e^{S_0} \frac{2^{\frac{3}{2}} e^{\frac{2\pi^2\gamma}{\beta}}}{(2\pi)^{\frac{1}{2}} \beta^{\frac{3}{2}}}$, while for $n = 2$, one has the “double-trumpet”, which can be written as part of (1.2) by formally defining $V_{0,2}(b_1, b_2) = \delta(b_1^2 - b_2^2)$.

from a nonperturbative instability, meaning that the integration contour of the matrix integral must be deformed. This process is not unique, however, and thus leads to an ambiguity in the nonperturbative completion. Much work has been done on trying to give JT gravity a rigorous nonperturbative definition and to study its features, particularly by Johnson [68–77], and more recently, a nonperturbative completion in terms of Kodaira-Spencer theory has been given [78, 79].

In light of the amount of interest particularly the nonperturbative sector of the JT matrix model has received, it seems useful to look at concrete nonperturbative features of the matrix model and study how they are realised in JT gravity. Particularly, we want to focus on what we call the universal limit of the matrix model² (see e.g. [80]). This is the limit of the matrix model in which correlation functions are given by the ones obtained in the appropriate Altland-Zirnbauer ensemble (in the present case, the Gaussian unitary ensemble). We will describe how to access this limit in JT gravity below, but for now it is sufficient to recall that in this limit, correlation functions of e.g. the level density are described by universal, *finite* functions. From this latter property alone, we will be able to derive very nontrivial identities between coefficients of the *Weil-Petersson volumes*, i.e. the polynomials $V_{g,n}$ appearing in (1.2).

The Weil-Petersson (WP) volumes [81] describe the volume of the moduli space of hyperbolic surfaces with genus g and n geodesic boundaries with lengths b_1, \dots, b_n . In principle, they are computable individually from scratch by performing all possible decompositions of the manifold in terms of three-holed spheres, parametrising these decompositions by Fenchel-Nielsen coordinates (l_i, τ_i) and integrating the Weil-Petersson form in these coordinates $\omega_{\text{WP}} = dl_i \wedge d\tau_i$ over the moduli space, while modding out the mapping class group of the surface to account for overcounting the decompositions.

In practice, however, this is not feasible and one uses Mirzakhani’s recursion relation [81] satisfied by the $V_{g,n}$ instead. While this is doable, the effort required to determine the volumes particularly for high g or n is still substantial, and a method of relating the different terms in the WP volumes might be useful to simplify their calculation.

In this paper then, we will show that the existence of a finite universal limit of the JT matrix model implies constraints on the coefficients appearing in the WP volumes $V_{g,2}(b_1, b_2)$ determining the JT gravity 2-point function. To this end, we will begin in section 2 by computing the spectral form factor in the universal limit, using the full, non-perturbative result for the spectral two-point function. Then, we will redo the calculation in section 3 using the genus expansion, and compare the two results. Demanding that they are at least compatible (i.e. that the perturbative result does not diverge more strongly than the nonperturbative one) will yield constraints relating certain coefficients appearing in a given $V_{g,2}$. Finally, in section 4, we discuss the possible relation of our results to other areas of research. As a last note, the same cancellations as the ones we report have been found independently using intersection theory computations in [25].

2. The late time spectral form factor from random matrix universality

In this section, we will compute the late time spectral form factor (SFF) of the matrix model of [19]. To do so, recall that the two-point function of $Z(\beta) = \text{tr } e^{-\beta H}$ can be expressed as a Laplace transform,

$$\langle Z(\beta_1, \beta_2) \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \rho^t(E_1) \rho^t(E_2) \rangle e^{-\beta_1 E_1 - \beta_2 E_2} dE_1 dE_2, \quad (2.1)$$

²We sometimes refer to this limit as the RMT limit, or particularly when talking about JT, the τ -scaling limit.

where $\rho^t(E)$ is the *total* spectral density in the convention of [19], i.e. it is of order e^{S_0} . This yields a useful form for the SFF,

$$\kappa_\beta(t) = \int_{-\infty}^{\infty} dE e^{-2\beta E} \int_{-\infty}^{\infty} d\Delta e^{-it\Delta} \left\langle \rho^t\left(E + \frac{\Delta}{2}\right) \rho^t\left(E - \frac{\Delta}{2}\right) \right\rangle. \quad (2.2)$$

Next, we study the universal behaviour of $\kappa_\beta(t)$ by evaluating it at times scaling with the Heisenberg time e^{S_0} . We define a rescaled time

$$\tau := e^{-S_0} t, \quad (2.3)$$

and obtain

$$\begin{aligned} \kappa_\beta(\tau) &= \int_{-\infty}^{\infty} dE e^{-2\beta E} \int_{-\infty}^{\infty} d\Delta e^{-ie^{S_0}\tau\Delta} \left\langle \rho^t\left(E + \frac{\Delta}{2}\right) \rho^t\left(E - \frac{\Delta}{2}\right) \right\rangle \\ &= e^{2S_0} \int_{-\infty}^{\infty} dE e^{-2\beta E} \int_{-\infty}^{\infty} d\Delta e^{-ie^{S_0}\tau\Delta} \left\langle \rho\left(E + \frac{\Delta}{2}\right) \rho\left(E - \frac{\Delta}{2}\right) \right\rangle, \end{aligned} \quad (2.4)$$

making the dependence on e^{S_0} explicit in the second line. The rapidly oscillating factor $e^{-ie^{S_0}\tau\Delta}$ localises the integral near small Δ (more precisely, of order e^{-S_0}), making it sufficient to evaluate $\langle \rho(E + \frac{\Delta}{2}) \rho(E - \frac{\Delta}{2}) \rangle$ for small differences in the arguments.

To find an expression for this, we utilise the universal RMT limit of the matrix model. Note that we leave the firm ground established in [19] here, since the universal limit of RMT correlation functions is a nonperturbative result, while the duality uncovered in [19] is only at the perturbative level³. To finally express the universal limit, we introduce a different, useful notation [80],⁴

$$R_1(E) = e^{S_0} \langle \rho(E) \rangle \quad (2.5)$$

$$R_2(E_1, E_2) = e^{2S_0} \langle \rho(E_1) \rho(E_2) \rangle - e^{S_0} \delta(E_1 - E_2) \langle \rho(E_1) \rangle. \quad (2.6)$$

Accordingly, the connected two-point correlation function is given by

$$C_2(E_1, E_2) = R_1(E_1) R_2(E_2) - R_2(E_1, E_2) \quad (2.7)$$

$$= e^{2S_0} \langle \rho(E_1) \rangle \langle \rho(E_2) \rangle + e^{S_0} \delta(E_1 - E_2) \langle \rho(E_1) \rangle - e^{2S_0} \langle \rho(E_1) \rho(E_2) \rangle. \quad (2.8)$$

The universal limit, which is valid for all matrix models of unitary symmetry class, now takes the form

$$\lim_{e^{S_0} \rightarrow \infty} \frac{C_2(E_1, E_2)}{R_1(E_1) R_2(E_2)} = \frac{\sin^2(\pi \langle \rho(E) \rangle e^{S_0} \Delta)}{\pi^2 \langle \rho(E) \rangle^2 e^{2S_0} \Delta^2}, \quad (2.9)$$

where $e^{S_0} \Delta := \tilde{\Delta}$ is kept finite in accordance with the above requirement of evaluating the correlation function for energy difference of the order of the inverse Heisenberg time⁵. This also

³A nonperturbative completion of this kind can however be interpreted in the context of minimal string theory in terms of D-brane insertions [19, 82]. In particular, this is consistent with the nonperturbative completion in terms of Kodaira-Spencer universe field theory [78, 79]. However, our computation does not rely on a particular (stringy) UV completion of JT gravity or a specific nonperturbative matrix model, but only on the existence of a perturbatively dual matrix model of GUE symmetry class.

⁴Note that it is not possible to take the “usual” universal limit $N \rightarrow \infty$, since this has already been performed in the course of implementing the double scaling limit, where N is replaced by e^{S_0} . Hence, we identify the appropriate universal limit for the double scaled theory as $e^{S_0} \rightarrow \infty$, and adapt the expressions in [80] accordingly.

⁵Note that the same limit is considered, and the same universal result used in [83]. However, the cancellation they observe for the 2-point function between perturbative and nonperturbative divergent pieces is not the same as the cancellation we will observe. In particular, the disconnected term in (2.6) vanishes on its own in the SFF when taking the limit.

justifies the replacement $\langle \rho(E_1) \rangle \approx \langle \rho(E_2) \rangle \approx \langle \rho(E) \rangle$. Using the explicit expressions and taking the universal limit, we can solve for the desired correlation function,

$$\left\langle \rho\left(E + \frac{\Delta}{2}\right) \rho\left(E - \frac{\Delta}{2}\right) \right\rangle = \langle \rho(E_1) \rangle \langle \rho(E_2) \rangle + \delta(\tilde{\Delta}) \langle \rho(E) \rangle - \frac{\sin^2\left(\pi \langle \rho(E) \rangle \tilde{\Delta}\right)}{\pi^2 \tilde{\Delta}^2}. \quad (2.10)$$

We can now plug this back into the SFF eq. (2.2) to find

$$\begin{aligned} \kappa_\beta(\tau) &= \langle Z(\beta + ie^{S_0}\tau) \rangle \langle Z(\beta - ie^{S_0}\tau) \rangle \\ &+ e^{S_0} \int_{-\infty}^{\infty} dE e^{-2\beta E} \int_{-\infty}^{\infty} d\tilde{\Delta} e^{-i\tau\tilde{\Delta}} \left[\delta(\tilde{\Delta}) \langle \rho(E) \rangle - \frac{\sin^2\left(\pi \langle \rho(E) \rangle \tilde{\Delta}\right)}{\pi^2 \tilde{\Delta}^2} \right] \\ &= \langle Z(\beta + ie^{S_0}\tau) \rangle \langle Z(\beta - ie^{S_0}\tau) \rangle + e^{S_0} \int_{E_0}^{\infty} dE e^{-2\beta E} \min\left\{\frac{\tau}{2\pi}, \langle \rho(E) \rangle\right\}, \end{aligned} \quad (2.11)$$

with E_0 the left edge of the spectrum of $\rho(E)$. At this point, a comment on our notion of taking the limit is in order. Although taken to infinity in the strict universal limit, e^{S_0} is still around. This can be understood in the sense of isolating the most strongly divergent term in the limit $e^{S_0} \rightarrow \infty$. By the same argument, one immediately concludes that the disconnected term is subleading compared to the integral in (2.11). Likewise, it is enough to consider the genus 0 part of $\langle \rho(E) \rangle$. With these simplifications, it is possible to give an explicit result for the JT gravity SFF:

$$\kappa_\beta(\tau) = e^{S_0} \int_0^{\infty} dE e^{-2\beta E} \min\left\{\frac{\tau}{2\pi}, \rho_0(E)\right\}. \quad (2.12)$$

This result has been first reported in [21] and evaluated in [26]. Important for us is that $\kappa_\beta(\tau)$ is finite at finite τ .

3. Scaling: Power counting considerations

In the previous section, we derived an expression for the late time SFF of JT gravity from non-perturbative information about the dual matrix model. Another way to compute this quantity would be to use the JT genus expansion (which is equivalent to the perturbative expansion of the matrix model) and then perform the universal and late time (“ τ -scaling”) limit $e^{S_0} \rightarrow \infty$, $t \rightarrow \infty$, $\tau = e^{-S_0}t = \text{const}$.

In this limit, the SFF (as calculated from the genus expansion) takes the form

$$e^{S_0} (a_1\tau + a_3\tau^3 + a_5(\beta)\tau^5 + \dots), \quad (3.1)$$

with coefficients independent of e^{S_0} . The $a_i(\beta)$ specify a convergent series⁶ which, importantly, agrees with the Taylor expansion of the result (2.12) around $\tau = 0$ and moreover gives the exact SFF for JT gravity in the τ -scaling limit, including the plateau [26].

In order to systematically compute the contributions to (3.1), we employ the polynomial structure of the WP volumes [81]:

$$V_{g,2}(b_1, b_2) = \sum_{\vec{\alpha}}^{|\vec{\alpha}| \leq 3g-1} C_{\vec{\alpha}}^{(g)} b_1^{2\alpha_1} b_2^{2\alpha_2} \quad (3.2)$$

⁶Technically, a series with finite radius of convergence, which can be smoothly continued to $\tau \rightarrow \infty$.

with non-negative constants⁷ $C_{\vec{\alpha}} \in \pi^{6g-2-2|\vec{\alpha}|} \cdot \mathbb{Q}$. Here, $\vec{\alpha} = (n, m) \in \mathbb{N}$ with $|\vec{\alpha}| := n + m$. The contributions to the SFF from each term appearing in (3.2) can be straightforwardly evaluated (all one has to do is integrate a monomial against a Gaussian), yielding contributions to the SFF of the form

$$C_{n,m} \left(\kappa_{\beta}^g(t) \right)^{n,m} = C_{n,m} \frac{\sqrt{\beta^2 + t^2}}{\pi} \frac{n!m!2^{n+m-1}}{\gamma^{n+m+1}} \sum_{k=0}^n \sum_{j=0}^m \binom{m}{j} \binom{n}{k} i^{j+k} (-1)^j \beta^{m+n-k-j} t^{j+k}. \quad (3.3)$$

We now consider these contributions for some fixed g in the τ -scaling limit. For given (n, m) , take for example the leading term in the sum (3.3), scaling like

$$t^{n+m+1} = e^{(n+m+1)S_0} \tau^{n+m+1}. \quad (3.4)$$

Meanwhile, every $\left(\kappa_{\beta}^g(t) \right)^{n,m}$ is suppressed by a factor e^{-2gS_0} due to the genus expansion. As an example, the highest order term coming from $n + m = 3g - 1$ contributes to the SFF to the order

$$e^{-2gS_0} e^{3gS_0} \tau^{3g} = e^{gS_0} \tau^{3g}. \quad (3.5)$$

The universal part of the SFF emerges by performing the τ -scaling limit and extracting the leading term, which should be of order e^{S_0} . Clearly however, there are contributions to the $g > 1$ SFF (or rather, to e^{-S_0} SFF, for given n, m) that diverge in this limit. From this observation, we pose the central claim of this work:

*The finiteness of the universal part of the SFF, identified by the τ -scaling limit, requires that all terms scaling as $e^{(1+n)S_0}$ mutually cancel.*⁸

Let us elaborate on this claim: In order for different terms of (3.3) to be able to cancel, they must have the same order in e^{S_0} , β and τ . It is easy to see that this can only happen for terms with common g and $n + m \equiv |\vec{\alpha}|$. Hence, we need to sum up the contributions from all partitions (n, m) of $|\vec{\alpha}|$ for fixed relevant g , and indeed we need to do this separately for each relevant⁹ power of β and τ . Doing so will provide constraint equations that some set of coefficients $C_{n,m}$ of the WP volumes need to satisfy.

Consider for example any relevant fixed $g > 1$ and $|\vec{\alpha}|$. The first equation we obtain from the above procedure (taking $j + k = m + n$) is

$$\sum_{\substack{n,m=0 \\ n+m=|\vec{\alpha}|}} C_{n,m} n!m! (-1)^m = \sum_{\substack{n \geq m \\ n+m=|\vec{\alpha}|}} \frac{1}{2} \delta^{(n,m)} C_{n,m} n!m! [(-1)^n + (-1)^m] = 0. \quad (3.6)$$

A well-known property of the WP volumes is that the coefficients are symmetric, $C_{n,m} = C_{m,n}$. We can thus immediately conclude that (3.6) is always satisfied for odd $n + m$, as then the (n, m) term automatically cancels with (m, n) .

However, for even $n + m$, this trivial cancellation will not occur, and (3.6) is a nontrivial constraint on the WP coefficients. To check this claim for plausibility, let us construct the first example of such a nontrivial cancellation. For $g = 2$, the leading equation for maximal $n + m = 3g - 1 = 5$

⁷Here and thereafter, we will suppress the dependence of the constants on g for readability.

⁸It is conceivable that nonperturbative effects cancel perturbative divergences instead. However, none of the nonperturbative corrections known to the authors are simple powers of the expansion parameter, but rather terms of order e.g. $e^{-e^{S_0}}$. Corrections of this type could not cancel perturbative divergences (which are $\mathcal{O}(e^{-nS_0})$). This suggests that our prediction still holds, and indeed that the perturbative and nonperturbative parts of the SFF are individually finite. All results we have obtained until now are consistent with this claim.

⁹Relevant in this context means $g > 1$ and $j + k > 2g$ in the sum (3.3). Note also that terms with $j + k < 2g$ are suppressed in the τ -scaling limit, while $j + k = 2g$ provides precisely the universal part.

is trivial, while the next-to-leading equations will contribute at order $e^{(5-2g)S_0} = e^{S_0}$, and hence provide the universal part.

The first nontrivial example thus requires $g = 3$, and again maximal $n + m = 3g - 1 = 8$. Taking the contributions from $j + k = 8$ also maximal, we find the following constraint equation:

$$280C_{8,0} - 35C_{7,1} + 10C_{6,2} - 5C_{5,3} + 2C_{4,4} = 0 \quad (3.7)$$

We can check this by plugging in the coefficients given by [84]

$$\begin{aligned} C_{8,0} &= \frac{1}{856141332480}, & C_{7,1} &= \frac{1}{21403533312}, & C_{6,2} &= \frac{77}{152882380800} \\ C_{5,3} &= \frac{503}{267544166400}, & C_{4,4} &= \frac{607}{214035333120}, \end{aligned} \quad (3.8)$$

which indeed solve (3.7). One can now leave g fixed and vary $n + m$, obtaining a similar equation for a different set of coefficients. In the example at hand however, we would only find the trivial cancellation for $n + m = 7$, as well as the result $n + m = 6$, which provides the β -independent part of the universal result.

However, we could also leave e.g. $n + m = 8$ fixed, and simply take terms with $j + k < 8$, i.e. contributing at a lower – but still divergent – order in e^{S_0} . In this way, we can find additional equations for the same set of coefficients as before, taking the form

$$\sum_{\substack{n,m=0 \\ n+m=|\bar{\alpha}|}} C_{n,m} n! m! (-1)^m (n-m) = 0 \quad (3.9)$$

$$\sum_{\substack{n,m=0 \\ n+m=|\bar{\alpha}|}} C_{n,m} n! m! (-1)^m (n-m)^2 = 0 \quad (3.10)$$

⋮

$$\sum_{\substack{n,m=0 \\ n+m=|\bar{\alpha}|}} C_{n,m} n! m! (-1)^m (n-m)^l = 0 \quad (3.11)$$

⋮

If the leading order equation is nontrivial, it follows that (3.9) is trivially satisfied by cancellation of the (n, m) contribution with the (m, n) one, reminiscent of what happens for odd $n + m$ at leading order. However, we can also see that (3.10) is a nontrivial constraint equation which is linearly independent from (3.6)¹⁰.

For $g = 3$, the combination in (3.10) provides the β^2 part of the universal result. Likewise, the analogue of (3.9) for $n + m = 7$ gives the β -linear part. Generally however, we will obtain a hierarchy of constraints for the coefficients belonging to a given $n + m$, requiring alternately trivial and nontrivial cancellations, until we arrive at a combination that contributes in the universal limit. The general form (3.11) of these constraints is readily determined by summing up the contributions to a given $n + m$, yielding

$$\sum_{\substack{n,m=0 \\ n+m=|\bar{\alpha}|}} C_{n,m} n! m! \sum_{\substack{j,k=0 \\ j+k=|\bar{\alpha}|-l}} \binom{m}{j} \binom{n}{k} (-1)^j = 0, \quad (3.12)$$

¹⁰In particular, $C_{n,n}$ is not involved in (3.10).

and then cleverly subtracting combinations of the constraints appearing before, i.e. for smaller l . For a more rigorous proof, see appendix A.

The first example of another constraint than (3.6) appearing is for $g = 4$, where the leading order term vanishes by symmetry. The first (and only) nontrivial constraint for the coefficients with $n + m = 3g - 1 = 11$ is given by (3.9), which can be simplified to

$$3630C_{11,0} - 270C_{10,1} + 42C_{9,2} - 10C_{8,2} + 3C_{7,4} - \frac{5}{7}C_{6,5} = 0. \quad (3.13)$$

This equation, as expected, is satisfied by the coefficients obtained by solving the topological recursion,

$$C_{11,0} = \frac{1}{650941377911193600}, \quad C_{10,1} = \frac{1}{8453784128716800}, \quad C_{9,2} = \frac{149}{59176488901017600}$$

$$C_{8,3} = \frac{947}{46026158034124800}, \quad C_{6,5} = \frac{487}{3287582716723200}. \quad (3.14)$$

Furthermore, for $g = 4$, we find the first constraint on coefficients associated to a non-maximal value of $n + m = 3g - 2 = 10$. The constraint is of the type (3.6) and reads

$$-\frac{5}{56}C_{5,5} + \frac{3}{14}C_{6,4} - \frac{3}{8}C_{7,3} + C_{8,2} - \frac{9}{2}C_{9,1} + 45C_{10,0} = 0, \quad (3.15)$$

which is indeed satisfied by the relevant coefficients,

$$C_{5,5} = \frac{533\pi^2}{71345111104000}, \quad C_{4,6} = \frac{1081\pi^2}{20547391979520}, \quad C_{3,7} = \frac{16243\pi^2}{898948399104000}$$

$$C_{2,8} = \frac{53\pi^2}{19025362944000}, \quad C_{1,9} = \frac{149\pi^2}{924632639078400}, \quad C_{0,10} = \frac{23\pi^2}{9246326390784000}. \quad (3.16)$$

It would be instructive to go to $g = 5$ or higher to test these predictions further, however the required coefficients prove very difficult to find both by direct computation and in the literature. Let us endeavour to make some general claims about the constraint hierarchies regardless.

First, in order for $n + m = |\vec{\alpha}|$ to admit a nontrivial constraint, we must have

$$2g + 1 < |\vec{\alpha}| \leq 3g - 1. \quad (3.17)$$

Hence, there are in total $3g - 1 - 2g = g - 1$ different sets of constrained coefficients, i.e. $g - 1$ different constraint hierarchies. In these hierarchies, we will find in principle $n + m - 2g$ linearly independent constraints, but every other one of those will be trivially satisfied. This leaves the number of nontrivial constraints for given $n + m$ as

$$\frac{n + m}{2} - g \quad : n + m \text{ even}$$

$$\frac{n + m - 1}{2} - g \quad : n + m \text{ odd}. \quad (3.18)$$

An interesting question might be how strongly these equations constrain the WP coefficients. The answer, unfortunately, is ‘not very’. To this end, note that the number of constrained coefficients in a given hierarchy is equivalent to the number of (unordered) pairwise partitions of an integer $n + m = |\vec{\alpha}|$. This number is clearly $\frac{n+m}{2} + 1$ for even $|\vec{\alpha}|$ and $\frac{n+m+1}{2}$ for odd $|\vec{\alpha}|$. Hence, we observe that there are

$$\# \text{ variables} - \# \text{ constraints} = 1 + g \quad (3.19)$$

coefficients more per hierarchy than can be determined by the constraints. Notably, this number is independent of $n + m$, but it still tells us that there will never be enough equations to actually determine any of the WP coefficients unambiguously, and indeed the problem gets worse as you increase g .

4. Discussion

To summarise, we have shown that the Weil-Petersson volumes, which arise as natural objects in the JT gravity path integral, obey some rather nontrivial constraints on their coefficients. These constraints were predicted using nonperturbative, universal information from the dual matrix model. It is this universality that is one of the strengths of our result: we did not need to rely on a particular nonperturbative completion either on the matrix model or on the gravity side. All that was required is the universal limit of the matrix model correlation function we studied, the spectral form factor.

The striking success of applying RMT universality in the JT/RMT context suggests several further research directions. An obvious and interesting generalisation of our results would be to compute higher n -point functions and see if there are similar constraints on the WP volumes appearing therein. However, identifying the correct analogue of the τ -scaling limit that we used for the SFF has proven somewhat difficult so far. A possible resolution would be that the genus expansion for $n > 2$ simply cannot capture the universal limit, as it does in the $n = 2$ case; a result that would be curious since there does not seem to be anything special about the 2-point function that makes our prediction hold there and not elsewhere.

Assuming that it is possible to generalise our results to higher n , another obvious target would be to try and prove the cancellations directly from the Mirzakhani recursion or (perhaps more simply, and certainly more generally) the topological recursion. As evident from the coefficients cited above, the WP volumes encode information about hyperbolic surfaces in a highly nontrivial manner, and the resulting coefficients need to conspire very precisely to be able to produce a finite result in the τ -scaling limit. It is not far-fetched then to suspect there to be a mechanism *at the level of the recursion* to ensure that the coefficients take values that are compatible with a finite universal limit.

Possibly useful tools to investigate such a mechanism could be provided by intersection theory, which has been used in [25] to identify the same cancellations as we have reported in this work. Some attention has also been directed towards resonance and resurgence of the JT gravity genus expansion [85, 86], though mainly at the level of the free energy, and it is not immediately clear whether such considerations could be profitably used for the questions at hand.

Finally, since the physical argument that led us to identifying the constraint relations – the existence of a finite universal limit of the matrix model dual – isn’t particularly fine-tuned to JT gravity (see e.g. [46]), it seems expedient to try to apply our reasoning to other gravitational models that are often studied in conjunction with JT, for example Liouville quantum gravity or minimal string models. Doing so might help better elucidate the relation of universality and (quantum) gravity if indeed there is any. Another option would be to simply work directly with matrix models defined by the topological recursion, e.g. the one constructed from the spectral density $\tanh \sqrt{E}$. This one is particularly interesting because it seems like the more natural choice for JT gravity when using its description as an $SL(2, \mathbb{R})$ BF -theory¹¹. Here, the metric information is encoded in a flat $SL(2, \mathbb{R})$ connection, for which the natural Plancherel density is the hyperbolic tangent, rather than the hyperbolic sine. However, restricting to smooth geometries (cf. [87]), one selects only one component of the connection, exchanging the \tanh for a \sinh in the process (i.e. going over to a description as a $SL^+(2, \mathbb{R})$ BF -theory) [88]. A similar issue appears in the “quantum particle in AdS_2 ” description of [89], where summing over different $SL(2, \mathbb{R})$ representations, the spectral density tracks a kind of winding number that would correspond to singular geometries in JT gravity, hence requiring regularisation in the form of an infinite imaginary magnetic field in the quantum mechanical system to arrive at the JT result.

¹¹We thank Thomas Mertens for explaining this point to us.

5. Conclusion

We have here proposed the systematic use of universal RMT results to uncover new relations among functions defined over the moduli space of 2-dimensional manifolds. The essence of this program is the conjectured existence – well motivated on physical grounds – of a regime where a strict equivalence between low-dimensional quantum gravity models and the universal correlations given by RMT holds.

As a first key step in this direction, a particular form of this equivalence has been made explicit in [26] by identifying the regime of parameters where the 2-point functions of Jackiw-Teitelboim gravity are conjectured to be given by the corresponding RMT correlators for the GUE ensemble. Assuming the validity of this conjecture, that includes both perturbative and non-perturbative contributions, we found hitherto unknown identities among the numerical factors of the polynomial Weil-Petersson volumes on the JT side.

Since the non-trivial correctness of the identities for $n = 2$ is now firmly established and checked against exact results, the possible extension of this program for higher order functions (and the corresponding Weil-Petersson volumes) as well as to other types of models as discussed here is a promising route to merge RMT with the theory of hyperbolic manifolds.

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A. Proof of the form of the constraint hierarchies

As stated above, for a chosen value of genus g and $|\vec{\alpha}| := n + m$ one finds a hierarchy of constraints, given by

$$C_l(|\vec{\alpha}|) := \sum_{\substack{n,m=0 \\ n+m=|\vec{\alpha}|}} C_{n,m} n! m! \sum_{\substack{j,k=0 \\ j+k=|\vec{\alpha}|-l}} \binom{m}{j} \binom{n}{k} (-1)^j \quad (\text{A.1})$$

$$=: \sum_{\substack{n,m=0 \\ n+m=|\vec{\alpha}|}} C_{n,m} n! m! P_l(n, m), \quad (\text{A.2})$$

for values of l such that $j + k > 2g$. We proceed to prove that a linearly independent combination of these constraints takes the form

$$C_l^s(|\vec{\alpha}|) := \sum_{\substack{n,m=0 \\ n+m=|\vec{\alpha}|}} C_{n,m} n! m! (-1)^m (n - m)^l. \quad (\text{A.3})$$

As a first step by using the property $\binom{n}{m} = \binom{n}{n-m}$ of the binomial coefficient it is useful to rewrite the $P_l(n, m)$ into the more convenient form

$$P_l(n, m) = \sum_{\substack{j, k=0 \\ j+k=|\bar{\alpha}|-l}} \binom{m}{m-j} \binom{n}{n-k} (-1)^j \quad (\text{A.4})$$

$$= (-1)^m \sum_{\substack{\delta, \gamma=0 \\ \delta+\gamma=l}} (-1)^\gamma \binom{m}{\gamma} \binom{n}{\delta}, \quad (\text{A.5})$$

where $\gamma = m - j$ and $\delta = n - k$.

Next, we show that $P_l(n, m)$ is a polynomial of degree l in m , as well as in n . To do so, it suffices to show that each individual term in $P_l(n, m)$ is a polynomial. For $a < b$, it holds that

$$\binom{b}{a} = \frac{b!}{a!(b-a)!} = \frac{1}{n!} \prod_{j=0}^{a-1} (b-j), \quad (\text{A.6})$$

which is a polynomial of degree a in b . Hence, both binomial coefficients appearing in $P_l(n, m)$ are polynomials in n or m , and so is their product. Evidently, the degree of these polynomials is l , whence

$$\begin{aligned} P_l(n, m) &= (-1)^m \left[\binom{m}{0} \binom{n}{l} + (-1)^l \binom{m}{l} \binom{n}{0} + (\text{lower order in } n, m) \right] \\ &= (-1)^m \left[\frac{1}{l!} n^l + \frac{1}{l!} (-m)^l + (\text{lower order in } n, m) \right] \\ &= (-1)^m \left[\frac{1}{l!} (n-m)^l + (\text{lower order in } n, m) \right]. \end{aligned} \quad (\text{A.7})$$

From here, (A.3) follows immediately upon realising that the lower order terms, plugged into (A.2), all reduce to linear combinations of constraints for smaller l (notice that $n + m$ is a global constant for each term and can be pulled out of the sum). If the previous constraints are satisfied, they can be safely subtracted, leaving the desired expression (A.3).

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