The Taxation of Entrepreneurial Incomes: Effects on Job Creation and Occupational Choices

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Erstgutachter: Prof. Dr. Fabian Kindermann
Zweitgutachter: Prof. Dr. Lutz Arnold

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Johanna Karoline Röhrs

This dissertation is dedicated to my parents.

For their love, support and trust that I can always rely on

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Chapter 1

Introduction

"Trickle-down theory - the less than elegant metaphor that if one feeds the horse enough oats, some will pass through to the road for the sparrows."

John Kenneth Galbraith

In recent years, the debate about income inequality and its sharp rise since the 1970s in the US and Europe has attained major attention in the media, in politics, as well as in the academic world[1]. Accordingly, there were rising public claims for higher taxation of top incomes but also voiced concerns that higher tax rates for top income earners would have adverse effects on economic growth and productivity. When asked about the funding of his welfare spending plan, the American Families Plan, US president Joe Biden, for example, said in 2021:

> For too long we've had an economy that gives every break in the world to the folks who need it the least. It's time to grow the economy from the bottom up.[2]

On the plans of the Biden administration to raise corporate tax rates and federal income tax rates on top incomes to undo the tax cuts under the Trump presidency, Senate Minority Leader Mitch McConnel commented:

> This tax bill of 2017 undone would create an extensive loss of jobs in

[1] See, for example, Piketty and Saez (2003), Atkinson et al. (2011), and Piketty and Saez (2014).

[2] https://www.theguardian.com/us-news/2021/may/03/joe-biden-taxes-corporations-richest-americans.

> our country, do exactly the wrong thing, and move us in the wrong direction.[3]

As indicated by this recent political discussion, the taxation of top incomes is a highly debated and divisive topic, not only in the US. Especially in the aftermath of the COVID-19 pandemic, it must be discussed who pays for the debt that has been caused by the fight against the pandemic. Climate change is also another challenge that requires enormous investments in environmentally friendly technologies. Facing these challenges in line with high income inequality, left-wing politicians around the world opt for higher taxation of high income earners, which, in turn, might have negative effects on economic growth and welfare. If societies care about income inequality and observe that the top 1% income earners receive disproportionately large shares of total income, higher taxation of top income earners is indisputably reasonable from an equity perspective. If top income earners are mostly responsible for job creation, innovation, and economic growth, higher taxation of their incomes would have adverse effects on economic outcomes, and policy makers might even want to support these top earners by subsidising entrepreneurship. How should policy makers thus tax entrepreneurial incomes when they care about reducing inequality but also about promoting economic growth? In the economic literature, many papers can be found that discuss the taxation of top incomes to counteract inequality and a vast amount of papers can also be found on subsidising entrepreneurship to foster economic growth.[4] My dissertation adds a new and, hopefully, helpful contribution to the discussion by providing an efficiency argument for the higher taxation of entrepreneurial incomes. I argue that entrepreneurs might engage in rent-seeking, which increases their income at the expense of others and has detrimental effects on welfare. Hence, corrective taxation can restore efficiency.

An important question, which must be answered to satisfyingly examine the effects of taxation of top incomes on economic performance, is: who are the top income earners and how important are they for economic activity, e.g., for job creation, productivity etc.? If top incomes reflect that the earners are mostly entrepreneurs with innovative ideas who create a lot of jobs or efficient managers of successful firms,

[3]https://www.businessinsider.com/biden-infrastructure-corporate-tax-rate-trump-cuts-ceos-economic-growth-2021-4.

[4]For a discussion of the relevant literature, see Section 2.2.

higher taxation of their incomes could have negative effects on overall welfare, for example, it could result in job losses or even firm closures. On the other hand, if top incomes are mostly a result of rent-seeking behaviour, higher taxation at the top would be beneficial for welfare since it counteracts the inefficient striving of certain professions for rents that do not suitably reflect their economic contribution.[5] Put differently, the question is whether the trickle-down economy exists. If it exists, top income earners ought not to be taxed heavily as their economic activity benefits the whole economy by founding companies, providing jobs, having innovative ideas and the like. Thus, by supporting or at least not distorting the economic activity of these successful individuals, parts of their generated income or wealth will trickle-down to less economically successful individuals as described in the quote at the beginning of this chapter where the sparrows benefit from the oats that the horse is fed with. In the economic literature, multiple evidence can be found that entrepreneurial activity plays an important role in explaining the high income shares that top income earners receive.[6] Smith et al. (2019) pose the question whether the top income earners in the US are financial capital or human capital rich. The latter category would include entrepreneurs and wage earners who obtain high incomes because of their human capital. The authors show that for individuals at the 99 - 99.9th percentile of income earners, wage income accounts for a bit more than 60% and business income for approximately 30% of their total fiscal income[7] in 2014. For individuals in the top 0.1 percentile, wage income declines to 40% and business income raises to 48% of their fiscal income. If one looks closer at business income and decomposes it into income from pass-through businesses and from C-corporations, it becomes striking that pass-through income plays a major role for top income earners. The top 1% earners' incomes consist of 40% pass-through income in 2014. Pass-through income comprises S-corporation income, partnership income, and sole proprietor's income and is not taxed at the entity level, but the owners pay taxes on their share

[5]For the discussion on high incomes being caused by rent-seeking or rather being the result of high returns on certain rare skills, see Chapter 2.

[6]For more theoretical and empirical evidence on the role of entrepreneurial activity for income inequality, also see Chapter 2.

[7]Fiscal income in the Smith et al. (2019) paper is taken from observed tax return data. It is computed as total tax return income minus realised capital gains. Wages contain salaries, tips, pension distributions, and annuities. Business income includes pass-through income and C-corporation dividends.

of the firm's income. A typical individual in the 99 - 99.9th percentile is an owner of a single-establishment firm such as a lawyer or consultant. An individual in the 99.9 - 100th percentile rather is a regional business owner with approximately 100 employees. Moreover, the authors' data set allows them to distinguish whether pass-through income is caused by financial capital or human capital and they find that "approximately three-quarters of top pass-through profits are returns to owner human capital"(see Smith et al., 2019, p. 1698). They conclude that top income earners are mostly human capital-rich since 52% of their income (either wage or business income) arises from human capital. The data show that many individuals among the top income earners are, at least partly, business owners. Another group that is highly represented among top income earners consists of executives or managers, who are also responsible for the creation of jobs since their decisions influence a firm's success and growth. Bakija et al. (2012) use individual income tax return data from the Statistics of Income Division of the US Internal Revenue Service and calculate the percentages of primary taxpayers in the top 1% of income earners (excluding capital gains) that work in certain occupational groups. In 2005, 31% of these top earners were executives, managers or supervisors in non-financial industries, whereas 13.9% worked in a financial profession including management. In the top 0.1% of income earners, 42.5% were executives, managers or supervisors in non-financial industries and 18% were in the financial profession in 2005. The reported data above indicate that several top income earners are business owners or at least partly responsible for job creation with professions such as managers.

An obvious follow-up question that arises is how important entrepreneurs are for job creation. Therefore, empirical evidence is shown next that provides information on the number of entrepreneurs and start-ups in the US and their role for job creation. The Business Formation Statistics (BFS) from the US Census Bureau reports the monthly number of business applications in the US as indicated by applications for an employer identification number. For January 2022, the BFS reports 445,536 business applications.[8] The total number of business applications also includes potential non-employer firms that will not have any paid employees such as, e.g., real estate agents. Therefore, the number of high-propensity business

[8] In the following, the seasonally adjusted data from the BFS are used.

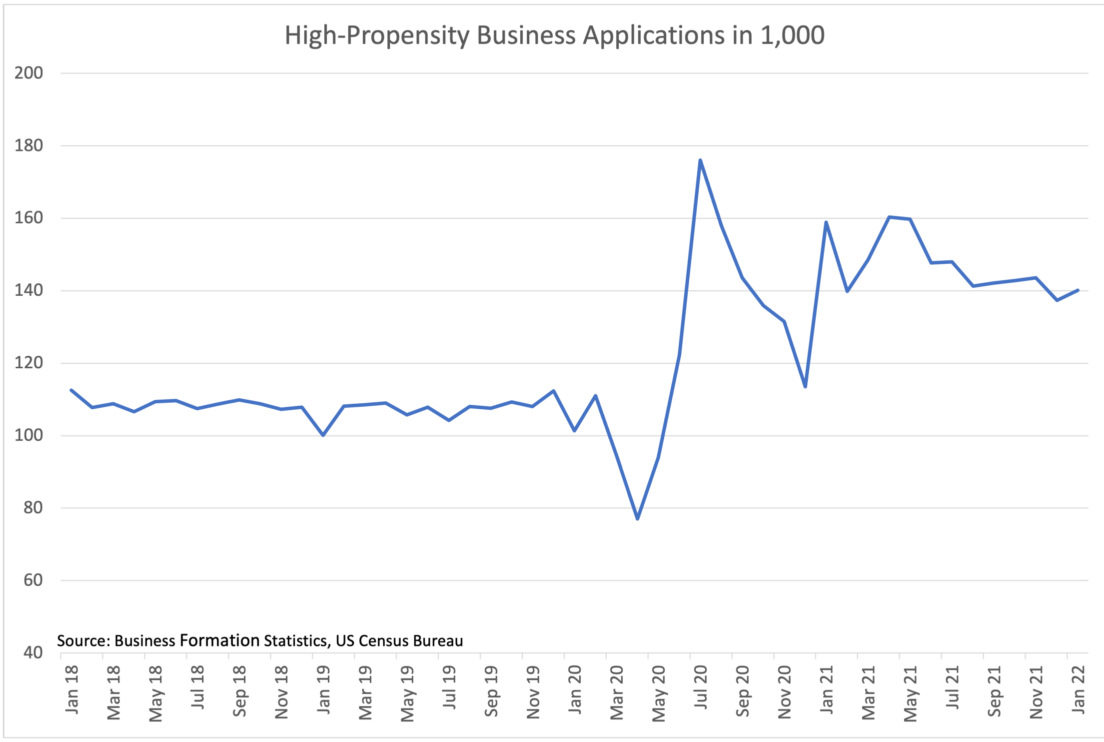

Figure 1: *Monthly number of HBAs (Seasonally adjusted)*

applications (HBA) might be more interesting to look at. HBAs are applications that have a high propensity of turning into businesses with payroll. For instance, this is indicated if the business application shows that employees are hired or reports planned wages. The provided number of HBAs in January 2022 is 140,174. Figure 1 shows the monthly number of HBAs from 2018 to January 2022. It was quit stable at around 110,000 until February 2020, then declined and rose sharply to 176,065 applications in July 2020. In 2021, the number of monthly HBAs fluctuated around 140,000, hence on a higher level than before the COVID-19 pandemic. Haltiwanger (2021) argues that this is indication for a restructuring of the economy induced by the pandemic.

Decker et al. (2014) mention that it is not trivial to measure entrepreneurial activity and to identify start-ups. The available data on US firms often contain information on the size and the age of firms. Using the age, which they assume is more important than the firm size for characterising start-ups, it is possible to identify new businesses. One then must distinguish between new firms and new establishments. The latter ones do not tend to indicate entrepreneurial activity because they are often replacing already existing old establishments or are caused by mergers and acquisitions. The next question that arises is how many jobs are created by these new firms. Decker et al. (2014) use the Business Dynamics Statistics from the US Census Bureau to estimate the importance of start-ups for job creation. They find that, between 1980 and 2010, an average of 2.9 million jobs per year was created by new firms, which accounted for 17.9% of average annual gross job creation. The average annual net job creation in the same time span was 1.4 million jobs[9]. One might conclude that all net job creation is caused by start-ups, but the authors argue that one must also investigate the performance of start-ups after their entry because they could fail. The authors show that 50% of the created jobs by young firms are destroyed due to exits until the age of five. Nevertheless, the jobs created by surviving firms of a given cohort largely offset the job loss caused by failing firms. Moreover, and in line with the former results, the growth rate of young firms is highly skewed to the right. Therefore, few young firms account for the largest part of net jobs created by new firms and these few start-ups have above average growth rates. Since the above mentioned statistics are not up-to-date, I calculate some important measures for 2019 using the Business Dynamics Statistics. It reports 437,926 firms in the year 2019 that are of age zero. This accounts for 8.2% of all counted firms within the same year. The overall number of jobs created in 2019 is given with 15,513,644 of which 15.6% comes from new firms with age zero. Figure 2 shows how many net jobs are created by firms within different age groups over time. One can see that the youngest firms account for the largest part of net job creation. Net job creation by new firms was at its lowest in 2010 and is slowly increasing since then, but has not reached the levels that could be observed prior to the global financial crisis. For older firms, net job creation fluctuates around zero since 2012.

In the 2019/2020 US Report from the Global Entrepreneurship Monitor (GEM), Kelly et al. (2020) estimate total entrepreneurial activity (or shortly TEA) among the US adult population to be 17.4%, which is the highest value measured for the US since the 21 years that GEM reports the data. TEA measures the percentage of the population aged 18 to 64 that is either a nascent entrepreneur or an owner-manager of a new business. Because of the COVID-19 pandemic, the TEA rate might have decreased since lastly measured. The established business ownership (EBO) rate in

[9]Note that for the firms with age zero, net job creation is equal to gross job creation since these new firms cannot destroy jobs which have not existed in the former period.

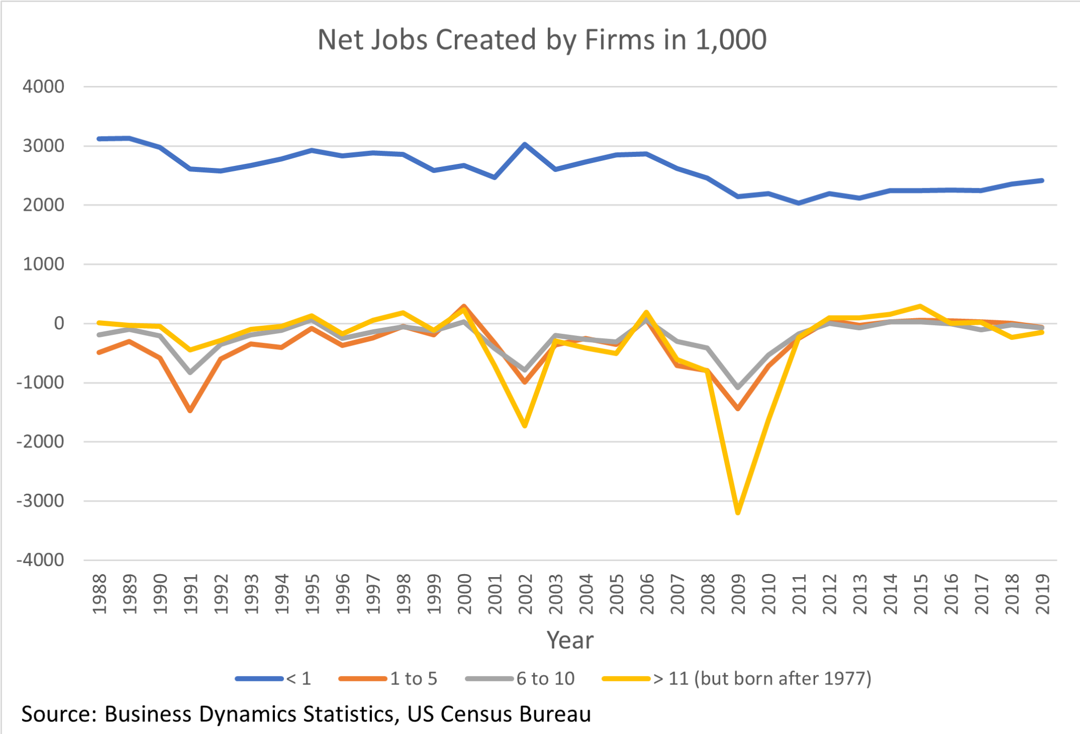

Figure 2: *Net jobs created by firms, sorted by age groups*

the US adult population for 2019 is displayed in the report as being 10.6%. EBO reports the percentage of the adult population that is currently an owner-manager of an established business. Established businesses are defined as having paid salaries, wages or other forms of payment to the owner for more than 42 months.

The empirical evidence shown above indicates that young firms and new entrepreneurs play an important role for job creation and economic growth. Moreover, significant parts of the US population engage in entrepreneurial activity. On the other hand, theoretical and empirical research indicates that entrepreneurial and managerial activity is important in understanding the unequal income distribution. Therefore, when talking about the taxation of entrepreneurial income, it is important to analyse the effects that the named taxation could have on the decision of individuals to become an entrepreneur/job creator, i.e., the extensive margin, and on welfare. Hence, some empirical results on the effects of taxation on employment, GDP, innovative activity and the location decision of innovators are presented next. Zidar (2019) estimates the effects of tax shocks for different income groups on employment growth and economic activity measured by GDP or net earnings. He uses US tax return data from 1950 to 2011 to construct a time series of income and payroll tax changes for different income groups. A 1% of state GDP tax cut for the bottom 90%

increases employment by approximately 3.4% over a 2-year period. Nominal state GDP decreases by about 8% following tax changes for the bottom 90 percent. Since prices decrease by 6%, the decline in real GDP is smaller. 1% of state GDP tax cuts for the top 10% have no significant effect on real and nominal GDP over the eight years around a tax change. An effect of tax changes affecting the top 10% of income earners on state employment is also not detectable. Zidar (2019) concludes that his results speak against the "trickle-down" effect and that policy makers who want to enhance economic activity in the short to medium run should consider tax cuts that address the bottom 90% of income earners. Nevertheless, tax cuts or increases for the top 10% could have an effect on economic activity in the long-run, for example, through human capital investment. Akcigit et al. (2021) use US state level data from 1940 to 2000 on patents, taxes, and inventors to estimate how personal income and corporate income tax rates affect inventors and therefore innovative activity. They distinguish between corporate inventors who work for a company and noncorporate inventors who do not work for firms. The authors find that the elasticity of inventors' patents with respect to the personal income net-of-tax rate is around 0.8. Noncorporate inventors' patenting activity is influenced more strongly by the personal income tax rate compared to the activity of inventors working for companies. The elasticity of corporate inventors' patents to the net-of-tax corporate rate is 0.49, whereas noncorporate inventors are not significantly influenced by the corporate income tax rate. The location choice of noncorporate inventors is also substantially affected by both tax rates. The location elasticity, more precisely the elasticity of the number of inventors residing in a state, for noncorporate inventors is 0.72 with respect to the personal income net-of-tax rate and 0.60 to the corporate income net-of-tax rate. Corporate inventors' location choices are only affected by the net-of-tax corporate rate with an elasticity of 1.25. Summing up, both personal and corporate income tax rates have a negative effect on the number of patents and the location choice of inventors. Therefore, there is mixed empirical evidence on the effects of income taxation on economic activity. In the short to medium run, income tax rate changes that affect the top 10% of income earners seem to have no effect on employment and GDP growth, whereas the latter mentioned study finds negative effects on innovation.

The information described in this introductory chapter suggests that entrepreneurs and managers account for a large part of the earners which can be found at the top of the income distribution. As incomes at the top and income inequality rose sharply in the last decades, many economists and politicians plead for higher taxation of top incomes and more redistribution. Nevertheless, if entrepreneurial incomes are taxed more heavily, this might have adverse effects since individuals are affected in their decision to start a firm, where to locate or, if they already are entrepreneurs, in their decision on how many jobs to create. As indicated by the mentioned empirical data, young firms play a major role in accounting for job creation and a significant fraction of the US population engages in entrepreneurial activity. Nevertheless, the empirical evidence on the consequences of higher taxation on economic activity is ambiguous. My dissertation thus tries to contribute answers from a theoretical perspective to the question how entrepreneurial incomes ought to be taxed and focuses on the efficiency perspective. For doing so, it is important to combine a model that features realistic assumptions about the labour market with heterogeneous agents who face an endogenous occupational choice. This way, I can first analyse whether the individuals' decisions, i.e., occupational choices and hiring decisions, are efficient. If this is not the case, taxation of entrepreneurial incomes and vacancy posting can lead to a more efficient outcome. If entrepreneurs engage in rent-seeking, progressive taxation might even be justified from an efficiency perspective. Second, in a more realistic, dynamic setting, I analyse the effects of taxation on occupational choices, the decisions of firms at different stages of their development and overall welfare.

The remainder of this dissertation is structured as follows. The next chapter provides an overview on the literature that closely relates to my dissertation. Chapter 3 presents a Diamond-Mortensen-Pissarides (DMP) model extended with an endogenous job decision that is based on heterogeneous job creation abilities. It is shown that it can be efficiency enhancing to tax the profits from job creation since entrepreneurs can appropriate inefficiently large parts from the surplus of a match in the decentralised market. Hence, the market equilibrium features an inefficiently large amount of entrepreneurs. Chapter 4 extends the model with intrafirm wage bargaining, and thus considers strategic behaviour within the firm. I demonstrate that the result of having too many entrepreneurs in the decentralised market can be reversed. If the negative effects of overemployment on the attractiveness of being an entrepreneur outweigh the positive effects of the possibility to acquire large parts of the surplus of a match, there might be too few firms in the market equilibrium compared to the first-best. Nevertheless, under a realistic parametrisation, the model also predicts that the laissez-faire market equilibrium features too many entrepreneurs. An extended and more dynamic version of the theoretical model described in the former chapter is presented in Chapter 5, as well as the results of a numerical simulation. If firm destruction is added to the model, subsidising vacancy postings and entrepreneurial incomes improves welfare. Chapter 6 offers concluding remarks. Appendix A and B provide mathematical derivations and further tables and figures.

Chapter 2

Literature Review

The theoretical models that I present in my dissertation are based on the DMP model, which describes a labour market with frictions. The first section within this chapter therefore summarises the main features of the DMP model, its potential and its shortcomings. An overview of the literature which closely relates to my dissertation can be found in the second section.

2.1 The DMP Model

The modelling of an incomplete labour market, caused by a matching process between firms looking for workers to fill their vacancies and currently unemployed workers looking for a job, was jointly developed by Peter Diamond, Dale Mortensen, and Christopher Pissarides. Therefore, it is known as the DMP model. The authors were awarded the Nobel Price in Economics in 2010 for this contribution. Mortensen (1982) describes and analyses the general matching process taking place when two agents come together to achieve a goal that they cannot achieve alone. When rents are created, one must decide how they are allocated to the members of the coalition. He analyses possible matching equilibria and shows that an equilibrium exists where unmatched agents search efficiently. Diamond (1982) extends this idea to the case of firms looking for employees and workers searching for employment. He assumes that a search process brings together vacancies and workers instead of a perfectly competitive market where frictional unemployment will not appear. When workers and employers are matched, they bargain about wages. Because of search externalities, the equilibrium in his model is in general not efficient. Mortensen and Pissarides (1994) focus on the endogenous creation and destruction of jobs in a matching framework and try to fit empirical facts concerning the cyclical behaviour of both. Pissarides (2000) outlines the DMP model along with its different versions and extensions in a widely used textbook. The basic model in continuous time and its general functioning, which I outline next, are based on the version described in his book.

The main idea behind the DMP model is that labour markets do not work perfectly, i.e., labour supply and labour demand do not directly adjust to changes in parameters. When firms post a vacant position, it takes some time until a worker is found and the search[1] for him or her is costly. Vice versa, when a worker wants to get employed, he or she must search for a vacant position, which consumes time and is costly as well. There is no coordinated mechanism that directly matches searching workers with firms. Because of that matching friction, there is unemployment. Moreover, matching is non-trivial when heterogeneities exist, e.g., workers differ in their skills, jobs have different characteristics, and firms offer varying amenities. Information asymmetries matter as well. All of this is captured by the matching function, which gives the aggregate number of contacts between workers looking for a job and firms wanting to fill a vacancy as a result of inputs. Pissarides writes about the matching function:

> It is a modelling device that captures the implications of the costly trading process without the need to make the heterogeneities [...] explicit. In this sense it occupies the same place [...] as other aggregate functions, such as the production function [...].[2]

In the basic model, firms are small and hire one worker who inelastically supplies labour and is risk-neutral. If I denote the labour force with L, the unemployment rate with u and the vacancy rate with v, the matching function depends on the

[1] The simplest version of the DMP model features undirected search. Workers and firms are matched randomly and do not search directly for a specific job, i.e., for workers with specific skills.

[2] See Pissarides (2000, p. 4).

number of unemployed workers and vacant positions:

$$m(uL, vL).$$

It gives us the number of matches between workers and vacancies per unit of time. The rate at which firms are able to fill their vacancies can then be calculated by dividing the matching function by the number of posted vacancies:

$$q(\theta) = m\left(\frac{u}{v}, 1\right).$$

$\theta = \frac{v}{u}$ denotes the tightness of the labour market and $q(\theta)$ thus yields the probability of a firm to be matched with a worker, which decreases in labour market tightness. When a firm and a worker meet randomly, a job is created and production takes place. This match does not have to last forever, it is destroyed with an exogenous Poisson rate. The worker hence becomes unemployed and the firm is either shut down or must advertise jobs again. The number of vacancies is determined by profit maximisation of firms. A firm, which enters the market, must post vacancies, which involves some costs per unit of time. Once a match occurred at rate $q(\theta)$, production takes place and the firm derives profits from the occupied job. In equilibrium, the present-discounted value of expected profits from posting a vacancy is zero. Using the firm's profit maximisation problem, I can derive the job creation condition, which features a negative relationship between the wage and θ. As the wage increases, posting vacancies gets less profitable since employing workers is more costly. This decreases labour market tightness. The job creation condition states that the equilibrium labour market tightness is determined such that the expected profit of hiring a worker is equal to the cost of hiring that worker. After a match occurred, Nash wage bargaining takes place to distribute the returns of that match. The wage w, resulting from Nash bargaining, maximises the product from the worker's net return of becoming employed and the firm's net return of filling a vacancy to the power of their respective wage bargaining power. The wage is hence given by the wage curve, which increases with θ. If the labour market is tighter, it is more difficult for firms to fill their vacancies and workers can thus increase their bargaining power, resulting in higher wages. In steady state, unemployment is constant, so the flows into unemployment have to equal the flows into employment. Therefore, one can derive the so-called Beveridge curve, which captures the relationship between unemployment and labour market tightness. Given θ and w, if there are few vacancies, unemployment is high, whereas if there are many vacant positions, unemployment is low because workers can find jobs easily.

The unique steady-state equilibrium is determined by the labour market tightness, the wage and the unemployment rate which fulfil the job creation condition, the wage curve and the Beveridge curve. The intersection of the wage curve and the job creation condition in a (θ, w)-space pins down the equilibrium wage and labour market tightness, whereas the intersection of the wage curve and the Beveridge curve in (u, v)-space determines equilibrium unemployment and vacancies. The outlined basic model can then be extended with, for example, capital, endogenous job destruction, large firms or on-the-job search.

The DMP model performs well in capturing long-run empirical facts, but it has some shortcomings in describing empirically observed cyclical phenomena. Shimer (2005) finds that the basic search and matching model predicts that the volatility of labour market tightness and average labour productivity is nearly the same, whereas US data show a much larger standard deviation for the former than for the latter. Thus, the DMP model underestimates the volatility of unemployment and vacancies as a response to a positive productivity shock since most of it is absorbed by an increase in wages. Therefore, firms nearly do not change their vacancy posting choices and the model cannot predict the observed fluctuations in unemployment. Shimer's critique is hence not a refusal of the labour search and matching structure per se but a criticism of the Nash wage-setting. Hall (2005) reacts to this critique by introducing wage stickiness to the DMP model. A small negative productivity shock in his setup results in a decrease in vacancy postings, lower job-finding rates for workers and therefore higher unemployment, which matches the empirically observed business cycle phenomena. In contrast, Pissarides (2009) argues that the volatility of wages in new matches should be preserved, since he finds that wages in new jobs are volatile, consistent with Nash wage bargaining, whereas wages in already existing jobs are more sticky. The author instead proposes decreasing marginal vacancy posting costs to solve the unemployment volatility puzzle. If firms pay proportional vacancy posting costs and fixed matching costs, which can be motivated as training or administrative costs, the job finding rate becomes more volatile while leaving the volatility in wages unchanged.

2.2 Related Literature

There are five strands of literature that I contribute to with my dissertation. First, by extending the DMP Model with heterogeneous firms and an endogenous job decision to analyse the effects of income taxation on efficiency, I contribute to the literature on taxation within the search and matching framework. The first part of the literature review will thus summarise the use of the DMP model for research on taxation and its influence on labour market characteristics. Second, I contribute to the literature on entrepreneurship and the creation of jobs and outline the literature that deals with the impact of taxation on outcomes such as the entry decision into entrepreneurship or firm size. In that part of the literature review, I also highlight the role of introducing entrepreneurial activity into macro models to generate realistic income or wealth distributions. Third, I focus on the literature on innovation, which mostly uses Schumpeterian growth models to analyse innovative activity and its impact on income distributions. I also briefly summarise papers on taxation and its effect on innovative activity. The fourth section mentions the discussion that deals with the questions whether high incomes are caused by rent-seeking behaviour or are returns to productive economic activity and how taxation could counteract rent-seeking. If economic rents occur, it must be discussed how these are shared between different agents. Therefore, the literature on rent-sharing is summarised last.

Extensions of the DMP Model The DMP model was developed to describe the labour market, unemployment, and its dynamics in a more realistic way compared to the perfectly competitive labour market.[3] It includes labour turnover since employees can separate from a job, become unemployed and then search (in a directed or undirected way) for a new job. Moreover, the model contains labour market frictions, such as time-consuming search, which lead to unemployment. Firms, who

[3] See, e.g., Diamond (1982), Pissarides (1985), Mortensen and Pissarides (1994), and Pissarides (2000).

create jobs by posting vacancies, are matched with workers looking for a job via a matching technology. This determines the labour market tightness and firms thus influence the probability to fill a vacancy with their recruiting effort. Furthermore, wages are determined via a wage bargaining process, which induces firms to create jobs since they can acquire parts of the surplus created by a worker-firm match.[4]
The version of the DMP model I use in my dissertation is based on Mortensen and Pissarides (1994) and Pissarides (2000) when it comes to modelling random search and the wage bargaining process. An overview about the different specifications of labour matching models can be found in Rogerson et al. (2005). Hosios (1990) shows that the matching model creates a Pareto efficient equilibrium allocation when firms' and workers' private gains from participating in the matching process are equal to the social gains of it. Shimer and Smith (2001) determine optimality in a match and search environment with heterogeneous agents and derive optimal search taxes that decentralise the social optimum.
There are several papers that deal with taxation in DMP models and its effects on labour market outcomes. One of the first contributions is Smith (1994) who identifies the effects of wage and profit taxes on unemployment in a two-sided search model. Mortensen and Pissarides (2002) explore the effects of taxes and subsidies on job creation, job destruction, employment, and wages. A further strand of the literature calculates tax rates in the search and matching framework and focuses either on optimal tax rates that correct for inefficiencies caused by, for example, information asymmetries or on optimal redistributive taxation. Boone and Bovenberg (2002) determine an optimal tax system in a DMP framework where entrepreneurial talent is scarce and focus on labour demand and labour supply elasticities. They find that if agents can arbitrage between the demand and supply side, optimal taxes should be proportional to rent shares. Domeij (2005) analyses how the introduction of search frictions into a neoclassical growth model alters the result of zero capital taxation. He finds a small but non-zero optimal capital tax. Hungerbühler et al. (2006) calculate optimal redistributive non-linear income tax rates in a DMP model and find positive marginal tax rates even at the top of the income distribution. In their approach, taxes are not used to correct for inefficiencies. If the government

[4] A more detailed description of the DMP model and its operating principle can be found in Section 2.1.

cannot observe the productivities of workers, taxation leads to lower wages and higher employment compared to the efficient level. Lehmann and van der Linden (2007) concentrate on the employees' side and their ability to extract rents in the bargaining process when their search efforts for a vacant job are observable and unobservable, respectively. Their optimal marginal tax rates are lower when search effort is unobservable compared to the case with perfect information. In a follow-up paper, Lehmann et al. (2011) introduce skill heterogeneity and focus on optimal redistributive taxation. They find positive marginal tax rates at the top of the skill distribution. A progressive tax schedule in their model lowers wages, unemployment, and participation rates compared to the efficient laissez-faire outcome. In contrast, Golosov et al. (2013) find that the optimal redistribution in a DMP model with directed search and heterogeneous firms can be obtained with an increasing and regressive labour income tax. The tax schedule is regressive because it ensures that the value of attracting one more applicant for the firm is equal to the value that applicant has to society. In a more recent paper, Dupuy et al. (2020) look at matching markets in general and map out the effects of taxation on match efficiency when agents have heterogeneous preferences over potential partners for a match.

Furthermore, the DMP model is increasingly used in macroeconomic analyses. Yashiv (2007) provides an overview of how the labour search and matching theory is used in macroeconomics. Garibaldi and Wasmer (2005) focus on the participation decision of workers in an imperfect labour market. They derive the effects of taxation and unemployment benefits on that decision, for instance that the introduction of a proportional tax on wages reduces entries into the labour market and increases exits. Cahuc et al. (2008) extend the DMP model with intrafirm wage bargaining, heterogeneous workers, and capital. They show that intrafirm bargaining can change the result of the standard matching model that an increase in the wage bargaining power of the worker decreases equilibrium employment. With intrafirm wage bargaining, firms increase the number of posted vacancies to lower the marginal product of workers and thus their wages. The model of Acemoglu and Hawkins (2014) is closely related to the model I use for the numerical simulation in Chapter 5. They augment the DMP model with intrafirm wage bargaining, a production technology with decreasing returns to scale, and heterogeneous firms, leading to firm size dispersion. Their model is therefore able to match several empirical facts concerning the distribution of firm size, firm growth, and wages in the US. Hagedorn et al. (2016) introduce high- and low-skilled workers into the DMP framework and endogenise productivity. They show that higher labour tax rates increase the productivity of low-skilled workers more than they decrease the productivity of the high-skilled, thereby reducing the skill premium and rising unemployment towards the latter group. I contribute to the summarised literature by extending the DMP model with an endogenous occupational choice and by analysing the impacts of taxation in such a framework.

Entrepreneurship and Job Creation There are some important papers demonstrating that the implementation of entrepreneurship into dynamic macro models is needed to generate realistic wealth and income distributions. Quadrini (1999) empirically shows that a large part of wealth is held by entrepreneurs and that they have higher upward mobility. He argues that the higher saving rate among entrepreneurs can explain these findings. In Quadrini (2000), the author develops a dynamic general equilibrium model with entrepreneurial choice and is able to generate the wealth concentration that is observed in the US economy. Cagetti and De Nardi (2006) build on this by including occupational choice and borrowing constraints in a life cycle model. They reveal that more restrictive borrowing constraints not only reduce the number of entrepreneurs and firm size, but also wealth inequality. The paper by Hurst and Lusardi (2004) describes the relationship between wealth and the decision to become an entrepreneur. The authors find a strong positive correlation between wealth and entry into entrepreneurship only for households at the very top of the wealth distribution. Hence, they argue that liquidity constraints do not play a major role in preventing small business formation. When it comes to taxation of entrepreneurial activity and the effects on outcomes such as the decision to become an entrepreneur, there are five papers exemplarily mentioned that relate to my research. Gentry and Hubbard (2000) analyse how progressive taxation affects the entry decision into entrepreneurship. Their finding is that a progressive tax system discourages entries if loss offsets are imperfect. Similarly, Jaimovich and Rebelo (2017) show that an increase in capital income tax rates reduces the entry of new entrepreneurs, but they argue that the effect on economic growth is small. The reason is that higher tax rates affect the marginal entrepreneur who is rather unproductive but do not affect the top entrepreneurs with high productivities. Scheuer (2013) argues in favour of regressive taxation of entrepreneurial profits compared to progressive taxation of labour incomes to counteract excessive entry of low-skilled individuals into entrepreneurship due to adverse selection in credit markets. Brueggemann (2021) calculates a welfare-maximising top marginal tax rate of 75% in a Bewley-Aiyagari model with entrepreneurs. She finds larger welfare gains for entrepreneurs than for workers because the tax increase leads to lower wages, and high-income entrepreneurs can thus increase their firms' sizes. Kitao (2008) focuses more on the effects of taxation on capital investment and distinguishes between different sources of income. He shows that in general equilibrium, entrepreneurial investment decreases as a result of the taxation of capital income. Takalo and Toivanen (2012) also analyse the role of capital constraints on the decision to become an entrepreneur. Individuals in their model can decide between being an entrepreneur or being a financier. They show that policy interventions, such as providing more capital to induce entrepreneurs with good ideas but a lack of financing possibilities to start a firm, might have adverse effects on the selection of individuals into entrepreneurs and financiers as being an entrepreneur becomes more attractive. Hence, the intervention makes individuals with low-quality projects to prefer to be an entrepreneur instead of financing the ideas of others. This is similar to the idea of my dissertation where subsidising entrepreneurship affects the occupational choice and induces individuals with low productivities to become an entrepreneur instead of a regular worker. Comparable to the model I describe in my dissertation, the before mentioned papers all feature an endogenous occupational choice where individuals have to decide whether they want to become an entrepreneur. However, it is assumed that labour markets are perfect (except for Takalo and Toivanen, 2012, who consider asymmetric information), whereas I aim to bring together endogenous occupational choices with a labour market with frictions.

A further strand of the literature on entrepreneurship covers the occurrence of start-ups and the thereby associated creation of jobs. In an empirical paper, Decker et al. (2014) demonstrate that the rate of business start-ups and the pace of job creation have recently declined in the US. Nevertheless, new and young businesses play an important role for job creation and productivity growth. The authors find that a small portion of young firms disproportionally create jobs, whereas many start-ups either fail or do not grow quickly post-entry. Sedláček and Sterk (2017) consider the macroeconomic conditions at the time the firms enter the market and show that they have long lasting effects on the number of jobs created by cohorts of firms. Therefore, it might be important to consider models of job creation in which young and smaller but growing firms coexist with large, mature firms. The model presented in Chapter 5 includes firm destruction and hence firms with different ages. As a result, entrepreneurial incomes and vacancy posting have to be subsidised to ensure that firms with very able entrepreneurs grow quickly and reach their target size more often.

Innovation Another strand of the literature on high income earners studies the relationship between income inequality and innovation. Aghion et al. (2018) claim that there is a positive and significant correlation between innovation and top income inequality and argue that it is at least partly a causal effect. Moreover, the authors construct a Schumpeterian growth model that explains how more innovation increases the top income share and social mobility. Likewise, Jones and Kim (2018) use a Schumpeterian growth model to generate a Pareto distribution for top incomes. Top income inequality in their model can be reduced when creative destruction is strengthened or high-growth entrepreneurs lose their status at higher rates. Frydman and Papanikolaou (2018) argue that the dispersion in compensation between workers and executives is driven by executives' heterogeneous skills to detect investment opportunities for the firm they are working for. In combination with technological progress, this leads to potential high incomes for executives.

Several papers analyse the effects that taxation exercises on innovative activity and mostly come to the conclusion that innovative activity should be subsidised and not taxed. Gersbach et al. (2019) use a model of creative destruction in which tax rates on labour income ought to be higher than on profits because this facilitates investment into basic research and thus fosters entry into entrepreneurship. The distributional consequences of the named tax policy and the trade-off between equity and efficiency could lead to stagnant growth. The impact of top tax rates on the mobility of inventors, especially superstar inventors, is examined in Akcigit et al. (2016). They show that the location decision of top inventors is significantly influenced by top tax rates. Akcigit et al. (2021) argue that higher personal and corporate income tax rates reduce the quantity and quality of innovations and lead to the shifting of businesses into other states within the US. In contrast to this, Bell et al. (2019) think that financial incentives have a minor impact on inventive activity. Instead, exposure to innovation can increase aggregate innovation. For a summary of the different channels through which tax policies can affect innovation, Akcigit and Stantcheva (2020) give a comprehensive overview. In contrast to the mentioned literature on innovation, my dissertation focuses more on an endogenous occupational choice, i.e., that individuals can decide between becoming an entrepreneur or a regular worker. Therefore, entrepreneurs are drawn from the same pool of individuals as the workers are. Whereas the literature on innovation concentrates on the entry decision of entrepreneurs, I focus on the relative attractiveness between being a worker or starting an own firm with the possibility of changing the occupation over time. Including the choice between these occupations is important when analysing the effects of taxation. Taxing entrepreneurial incomes affects entrepreneurs because they can also decide to not start a firm.

Rent-Seeking Piketty and Saez (2003) show that the top income shares in the US have risen substantially since the 1980s. Hence, the question occurs whether this surge is caused by an enforced rent-seeking behaviour of certain professions or whether these high incomes are “well-deserved” returns caused by economic factors such as skill-biased technological change, globalisation, or the superstar phenomenon. The superstar phenomenon, describing the fact that very few people earn outstanding amounts of money, was first described and analysed by Rosen (1981). Superstar earners can be found in several industries and occupations such as sports, entertainment and among managers or CEOs. In the model developed by Gabaix and Landier (2008), the best CEO manages the largest firm. Small differences in talent translate into huge compensation differentials because talent is enlarged with firm size. The models presented in this dissertation also have the feature that the most talented entrepreneurs own the largest firms and derive the highest profits. The ex-post consequences of the superstar status are studied in Malmendier and Tate (2009). They demonstrate that CEOs extract higher compensation but, at the same time, underperform after having received an award and having gained media attention. Another explanation for the high incomes of CEOs, which rather speaks in favour of rent-seeking mechanisms, can be found in Bertrand and Mullainathan (2001). They define several measures for luck, which capture shocks that are out of the CEO's control, and find that CEOs are partly paid for luck. Payment for luck is especially present in firms with bad governments. In contrast, Kaplan and Rauh (2013) argue that the rise in incomes for the top one percent is induced by skill-biased technological change and an increase in scale. Thus, top income earners have unique and scarce talents that allow them to obtain premiums in markets with increasing size. Their argumentation is based on the empirical observation that the top one percent income earners can be found in various occupations and are a heterogeneous group. On the other hand, Bivens and Mishel (2013) show that rent-seeking[5] indeed plays a role in explaining the increasing income shares of top earners. They argue that the rise in top incomes was not accompanied by a surge in economic performance, speaking for rent-seeking. Thus, policy measures which attack these rents would not have huge adverse effects on economic growth. Piketty et al. (2014) derive top tax rate formulas in a model where individuals could react to tax rates through three channels. When top tax rates are increased, top income earners can reduce their economic activity, they can try to avoid taxation or they are incentivised to bargain less aggressively for higher compensation. This third channel alludes to an institutional setting where high income earners engage in rent-seeking. Higher taxation at the top might then dampen the individuals' efforts to opt for inadequately high compensation. Likewise, Rothschild and Scheuer (2016) consider rent-seeking behaviour and its consequences on optimal tax rates. Individuals in their model can work either in a traditional sector, where private and social returns coincide, or in a rent-seeking sector which exercises externalities. Whether the optimal externality correction is larger or smaller than the Pigouvian correction depends on the desirability of rent-seeking relative to other activities. Edmans et al. (2017) summarise the literature on different explanations for the large increase in CEO compensation since the 1970's such as shareholder value maximisation or

[5]When referring to the term "rent" in the following, excess income above the marginal product is meant, as in Kaplan and Rauh (2013), who interpret rents as what individuals receive in excess of their marginal product. Bivens and Mishel (2013) describe rents as excessive income of what is needed to induce an individual to supply labour.

rent-extraction. They come to the conclusion that no single theory can fully explain the empirical evidence. My dissertation extends the literature on rent-seeking by providing a theoretical and micro-founded explanation for rent-seeking behaviour.

Rent-Sharing The literature on rent-sharing builds on the questions why firms pay different wages to workers with similar skills or in the same occupation and whether productivity differences at the firm-level transmit into wages. A possible explanation for observed wage inequality might be the role of wage bargaining between workers and firms in imperfect labour markets. Several studies try to estimate rent-sharing elasticities such as the wage-productivity elasticity, which measures how shocks to firm productivity impact workers' wages. Van Reenen (1996) analyses how rents from innovation are shared across firms and workers. He finds an elasticity for wages with respect to quasi rents of 0.2 to 0.3. Card et al. (2018) review different value-added based estimates and find that most wage-productivity elasticities lie in a range between 0.05 to 0.15, which means that wages increase by 0.5% to 1.5% if productivity, and therefore quasi rents, increase by 10%. Kline et al. (2019) use US patent data to examine how patent-induced shocks to labour productivity affect wages and find that workers capture on average 30% of the rents generated through patent allowances. Saez et al. (2019) use a payroll tax cut for young workers in Sweden to estimate how this decrease in firms' wage costs is shared with the workers. The authors do not provide an elasticity but show that young workers' wages do not grow disproportionally. The higher elasticity estimates from Van Reenen (1996) and Kline et al. (2019), which lie substantially above the estimates found by Card et al. (2018), might be explained by the focus on innovation-based estimates instead of estimates that are based on the value added. The values for the worker's wage bargaining power used in the numerical simulations in the following chapters draw on the estimates discussed above.

Chapter 3

A Theoretical Model of Income Taxation and Job Creation

The first two chapters of this dissertation are based on Röhrs (2021), which considers the taxation of entrepreneurial incomes in a DMP model extended with heterogeneous entrepreneurial abilities and an endogenous occupational choice. This chapter describes the basic model without within-firm strategic behaviour, whereas the next chapter extends the model with intrafirm wage bargaining.

When talking about the taxation of entrepreneurs or job creators, a common fear is that higher taxation might lead to job cuts or less business creation. Moreover, if entrepreneurs are very innovative, taxation might lead to less innovation and be harmful for growth. If the trickle-down effect matters, it is beneficial for the whole economy to not tax entrepreneurs heavily, since they create jobs and help reduce unemployment. The question that arises is whether entrepreneurs' profits suitably reflect their economic contribution or whether these profits might be too high because they can acquire a too large share of output. If the latter is the case, taxation of entrepreneurial profits is justified even from an efficiency perspective. To satisfyingly answer the question whether higher tax rates for entrepreneurs can be welfare enhancing, it is important to make realistic assumptions about the labour market. I therefore use the labour matching model developed by Diamond, Mortensen, and

Pissarides, hence the DMP Model,[1] augmented with heterogeneous agents and endogenous job decisions. Individuals are endowed with an entrepreneurial ability and can decide to become a regular worker or an entrepreneur. In equilibrium, every worker in regular employment earns a wage independent of her entrepreneurial ability, whereas entrepreneurs with higher ability employ more workers and earn higher profits.

In the first section, I describe the model setup, the firms' optimal hiring decisions and vacancy posting and the occupational choice that each individual must face. Next, the social planner's maximisation problem is outlined and the first-best allocation is derived. I obtain an equation stating how the planner sets the optimal threshold ability. Individuals with lower entrepreneurial ability become workers and individuals with an ability above the threshold become entrepreneurs. I then compare the outcome in the decentralised market with the social planner's solution and show that the market equilibrium is in general not efficient. Hereby and in the following sections, I focus on steady states to make the problem more tractable. Inefficiencies in the market can arise for the following two reasons: First, entrepreneurs post too many vacancies if their private return of a match exceeds the social return. The inefficiencies caused if the Hosios condition does not hold have been discussed extensively in the literature. Second, there are too many entrepreneurs if they can acquire a significant fraction of the surplus of a match. The occupational choice of individuals creates a new inefficiency that has not been treated by the literature so far. Even if the number of jobs created by entrepreneurs is efficient, the decision to become an entrepreneur and to start a firm can still be inefficient. If entrepreneurs and regular workers are drawn from the same pool of individuals and bargain about wages, occupational choices might be inefficient and exert externalities.

Therefore, when introducing taxation, it is important to consider these two margins of market failure simultaneously. If the Hosios condition does not hold, vacancy posting must be taxed or subsidised accordingly. The tax rate on entrepreneurial incomes should affect the decision to become an entrepreneur without distorting the vacancy posting choice of existing firms. I show that there is a tax on entrepreneurial profits that allows the restoration of the first-best with fewer entrepreneurs than in

[1] See, for example, Diamond (1982), Pissarides (1985), Mortensen and Pissarides (1994), and Pissarides (2000).

the market equilibrium. I calculate the tax rate on profits by comparing the equilibrium condition for the optimal number of entrepreneurs in a social planner setting with the condition in the decentralised market equilibrium. The marginal tax rate is determined first for the case where only entrepreneurs' profits are taxed and second for the case where workers' incomes are taxed as well. I find indication of a progressive tax system. In the laissez-faire market equilibrium, too many individuals decide to become entrepreneurs since they can acquire large parts of the surplus of a match. The taxation of profits from job creation is efficiency enhancing since it makes being an entrepreneur less attractive and reduces the number of individuals who decide to become an entrepreneur. Moreover, it crowds out only the least productive firms, whereas entrepreneurs with high entrepreneurial ability are not affected in their decision to create a firm. Efficiency increases if those entrepreneurs who are unproductive relative to others become workers. The entrepreneurs who opt out are the marginal, less productive ones and not the average ones. Hence, the potential adverse effect on growth is small. Furthermore, I demonstrate that the introduction of income taxation does not distort the vacancy posting and hiring decisions of the remaining more productive entrepreneurs. The numerical simulation of the model illustrates that, for a realistic parametrisation, the tax rates on entrepreneurial incomes that restore the first-best lie in a range between 8% and 21%.

To sum up, I find efficiency reasons that support a higher taxation of incomes from job creation, whereas most of the literature argues that entrepreneurship and job creation should be subsidised. When thinking about the taxation of entrepreneurial incomes, the argument is that it is also important to consider that rather low-productivity entrepreneurs might start a firm if they have the possibility to extract large parts of the surplus of a match. This leads to an inefficiently large number of "not so able" entrepreneurs who compete with more productive firms for workers. The analysis in this chapter can therefore also be understood as a word of caution not to exaggerate the support of entrepreneurship since this could have adverse effects on welfare.

This dissertation contributes to the literature that analyses taxation in search models. In a model setup similar to mine, with search and scarce entrepreneurial talent, Boone and Bovenberg (2002) analyse under which circumstances workers or entrepreneurs can reap surpluses and derive how different labour supply and labour demand elasticities influence the optimal tax system. They find that the labour market tightness should not be distorted by taxation. Nevertheless, if the government wants to raise revenues and its main source of revenues is wage taxation, the government might want to strengthen its tax base by increasing wage income relative to profit income, which can be achieved with higher tax rates on profits. This in turn makes being an entrepreneur less attractive and, consequently, affects the labour market tightness by influencing occupational choices. Hungerbühler et al. (2006) derive optimal tax rates in a matching model with directed search and workers with different skill levels but without endogenous occupational choices. They find positive marginal tax rates even at the top of the income distribution and larger marginal taxes compared to a Mirrlees setting. Lehmann et al. (2011) use the same model as Hungerbühler et al. (2006) but introduce individuals with heterogeneous values of nonmarket activities and study the influence of participation decisions on optimal redistributive taxation. They find a progressive tax schedule if the participation elasticities decrease with increasing skill levels. Furthermore, the calculated marginal tax rates are higher than in a framework with competitive labour markets without frictions. An overview of further literature on income taxation within the search and matching model framework can be found in Boadway and Tremblay (2012).

Moreover, the paper contributes to the literature dealing with the effects of taxation on innovation and entrepreneurship. There are several papers that find sizeable negative effects of high top tax rates on innovation activity and the location choice of inventors (see, e.g., Akcigit et al., 2016, Akcigit and Stantcheva, 2020, and Akcigit et al., 2021). Another strand considers the role of entrepreneurs in the top income distribution since entrepreneurs are highly represented in the group of top income earners. These papers include entrepreneurial activity in general equilibrium models to replicate the empirically observed income and wealth distributions with high concentrations at the top and analyse optimal taxation within these frameworks (see, e.g., Quadrini, 2000, Cagetti and De Nardi, 2006, or Brueggemann, 2021). In the mentioned papers, borrowing constraints limit the number of entrepreneurs whereas labour markets are complete in the sense that wages equal marginal products. The contribution of my dissertation is the focus on an endogenous occupational choice in imperfect labour markets where wages deviate from marginal products, which yields the possibility of rent-seeking and therefore an inefficient allocation of resources.

There is some evidence in the literature that parts of the incomes of top earners are caused by rent-seeking behaviour as already discussed in Chapter 2. Bivens and Mishel (2013), for example, argue that the increase in incomes of the top 1 percent since the 1980s is largely caused by the creation or redistribution of rents. In their opinion, very high incomes are not just efficient marginal returns to specific skills or high ability, as is claimed, for example, by Kaplan and Rauh (2013). If this holds true, higher taxes on high income earners might be justified and potentially welfare enhancing since they reduce the returns to rent-seeking. In contrast to the mentioned papers, I do not need to assume that individuals behave in some way of rent-seeking. The appropriation of surpluses is included in the model through the manner in which firms and workers match and bargain about wages. Therefore, the model provides some microfoundation for rent-seeking behaviour.

The remainder of this chapter is organised as follows. Section 3.1 describes the setup of the theoretical model, Section 3.2 examines the social planner's allocation, and Section 3.3 analyses the market equilibrium's inefficiency. Taxation and its effects on the efficient allocation is described in Section 3.4, and the results of a numerical simulation of the presented model can be found in Section 3.5. Finally, Section 3.6 provides a brief conclusion.

3.1 Model Setup

I consider a closed economy with a continuum of individuals who live forever. Their mass is normalised to one. Every individual has an entrepreneurial ability that is denoted by a. The cumulative distribution function of this ability is given by $\Phi(a)$ and the density by $\phi(a)$.

Individuals can decide at the beginning of each period whether they want to become entrepreneurs or workers. If they decide to become a worker, they can either be employed or unemployed. An employed worker earns the wage w_t, which is independent of the worker's talent for job creation, and an unemployed worker receives home production z.

If an individual instead decides to become an entrepreneur, she starts a firm and must decide how many workers l_t to hire and how many vacancies v_t to open in every period. The cost of posting a vacancy γ is constant over time. As an outcome, there is a cut-off level $\bar{a}_t$ for which all individuals with talent $a \geq \bar{a}_t$ become entrepreneurs and the less talented ones become workers. An entrepreneur with talent a posts $v_t(a)$ vacancies and hires $l_t(a)$ workers. The marginal entrepreneur with entrepreneurial talent $\bar{a}_t$ then opens $v_t(\bar{a}_t)$ vacancies and employs $l_t(\bar{a}_t)$ workers. The number of all vacancies in the economy in period t therefore is $V_t = \int_{\bar{a}_t}^{\infty} v_t(a) d\Phi(a)$.

In aggregate, there are $\Phi(\bar{a}_t)$ workers who can be divided into employed workers L_t and unemployed workers N_t. $1 - \Phi(\bar{a}_t)$ then gives the number of entrepreneurs because the number of individuals in the economy is normalised to one. There is a resource constraint on the supply of labour. The fraction engaged in creating jobs plus the fraction engaged as employees must be smaller or equal to one in each period : $1 - \Phi(\bar{a}_t) + L_t \leq 1$. This can also be written as $L_t \leq \Phi(\bar{a}_t)$, which means that labour demand must be smaller or equal to labour supply. Consequently, the aggregate number of employed workers in period t can be written as $L_t = \int_{\bar{a}_t}^{\infty} l_t(a) d\Phi(a) = \Phi(\bar{a}_t) - N_t$ and the unemployed are $N_t = \Phi(\bar{a}_t) - L_t$.

How firms and workers come together and form a match is described by the matching technology $m(N_t, V_t)$. It gives the number of aggregate contacts between the mass of vacancies and unemployed workers. The matching function is assumed to be homogeneous of degree one.

The tightness of the labour market is denoted with $\theta_t = \frac{V_t}{N_t}$. The probability to fill an open vacancy per unit of time can thus be written as $\frac{m(N_t,V_t)}{V_t} = q(\theta_t)$ with $q'(\theta_t) < 0$. If the labour market tightness rises, it gets more difficult for entrepreneurs to fill their vacancies. Moreover, the probability of finding a new job is $\frac{m(N_t,V_t)}{N_t} = \theta_t q(\theta_t)$. It increases if the labour market gets less tight for the unemployed, so $\frac{d[\theta_t q(\theta_t)]}{d\theta_t} > 0$. When workers and firms are matched, production y_t is taking place according to a production function $y_t = af(l_t)$ with $f'(l_t) > 0$ and $f''(l_t) < 0$. A match between a worker and a firm does not have to last forever. There is an exogenous job destruction rate s that is assumed to be constant over time. Once a match is destroyed, the worker becomes unemployed, and the entrepreneur must post vacancies to hire a new worker. The dynamics of unemployment are therefore described as follows: $N_{t+1} = (1-\theta_t q(\theta_t))N_t + s(\Phi(\bar{a}_t) - N_t)$. In every period, there are $\theta_t q(\theta_t)$ unemployed people who find a job and leave the unemployment pool, and there are s employees who lose their job and join the unemployed.

3.1.1 The Regular Worker

It is assumed that there is no storage technology and that individuals are risk neutral. An employee receives the wage w_t and an unemployed worker engages in home production z. The rate of time preference is denoted by β.

One can now set up the value equations for the different types of individuals. The value equation for an employed worker is the following:

$$W_t^e = w_t + \beta \left[sW_{t+1}^n + (1-s)W_{t+1}^e \right].$$

It depends on the current wage and the future value of being a worker. The employed worker becomes unemployed with probability s, in which case she would have the value of an unemployed worker, and stays employed with probability $1-s$.

The value of being an unemployed worker can be written as

$$W_t^n = z + \beta \left[\theta_t q(\theta_t) W_{t+1}^e + (1-\theta_t q(\theta_t))W_{t+1}^n \right].$$

The unemployed worker engages in home production in the current period and knows that with probability $\theta_t q(\theta_t)$ she will be matched with a firm and become employed in the next period. Otherwise, she stays in unemployment.

3.1.2 The Firm

If an individual decides to become an entrepreneur, her value equation depends on the firm's profit, which is maximised by choosing the optimal number of workers and vacancies. After the vacancies are posted and matched with workers, wage bargaining takes place. Since multiple workers are bargaining with a firm, the wage-setting is more complex than in the standard DMP model where firms bargain with only one worker. I assume that entrepreneurs do not try to decrease wages with their hiring decisions and show that a common wage is paid across firms. The assumption is based on Westermark (2003), who demonstrates that, if contracts are binding, a stationary subgame perfect equilibrium exists where each worker receives a share of her marginal product and wages are in this sense competitive. Because I have a production function with decreasing returns to scale, a worker who bargains with the firm receives a fraction of her marginal product because all other workers who bargain with the firm as well are treated as employed. This contrasts with the solution of Stole and Zwiebel (1996). They show that with nonbinding contracts and continuous bargaining, workers receive a weighted average of the inframarginal contributions, which leads to wage dispersion when firms are heterogeneous. Since I want to analyse the effects of an endogenous job decision and the potential inefficiencies resulting from it in the simplest and clearest form without introducing an additional channel to depress wages, I rely on the first approach when it comes to the wage bargaining process.[2]

The firm's maximisation problem for given w_t and $q(\theta_t)$ is the following:

$$W_t^f(a, l_t) = \max_{l_{t+1}, v_t} \left\{ af(l_t) - w_t l_t - \gamma v_t + \beta W_{t+1}^f(a, l_{t+1}) \right\}$$

$$\text{s.t. } l_{t+1} = (1-s)l_t + q(\theta_t)v_t.$$

The corresponding Lagrangian function is thus given with

$$\mathcal{L} = af(l_t) - w_t l_t - \gamma v_t + \beta W_{t+1}^f(a, l_{t+1}) + \lambda_t \left[(1-s)l_t + q(\theta_t)v_t - l_{t+1} \right]$$

where λ_t denotes the multiplier on the equality constraint. The first order conditions are derived as

$$\frac{\partial \mathcal{L}}{\partial v_t} = -\gamma + \lambda_t q(\theta_t) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial l_{t+1}} = \beta \frac{\partial W_{t+1}^f(a, l_{t+1})}{\partial l_{t+1}} - \lambda_t = 0.$$

[2] The model with intrafirm wage bargaining is discussed in Chapter 4.

Solving them for λ_t and equalising them, one receives

$$\frac{\gamma}{q(\theta_t)} = \beta \frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}},$$

which states that the cost of hiring an additional worker must be equal to the discounted change in the entrepreneur's value equation when an additional worker joins the workforce. The latter term can be determined using the envelope theorem:

$$\frac{\partial W^f_t(a, l_t)}{\partial l_t} = af'(l_t) - w_t + \lambda_t(1 - s).$$

An additionally hired worker increases the entrepreneur's current value function by the marginal product of labour minus the wage that must be paid to the worker. One also must consider that the match is not permanent. Using the envelope condition, one obtains the job creation condition:[3]

$$\frac{\gamma}{q(\theta_t)} = \beta \left[af'(l_{t+1}) - w_{t+1} + (1 - s)\frac{\gamma}{q(\theta_{t+1})} \right]. \tag{3.1}$$

It states that the expected costs of hiring a worker must be equal to the value generated by employing an additional worker. A hired worker expands the firm's production by the marginal product of labour multiplied by the entrepreneur's ability minus the wage that is paid to him or her. With probability $1-s$, the worker stays at the firm in the next period as well, and the continuation value must be added. If one rearranges the job creation condition and defines the surplus of having an additional worker as P_t, so that $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$ holds, it can be written as

$$P_t = af'(l_t) - w_t + \beta(1 - s)P_{t+1}.$$

P_t is the value of having a match for the firm or the value of an occupied job. Since the first order conditions describing the entrepreneur's optimal choice must hold for every firm, P_t is the same for each firm no matter how high the entrepreneurial talent a is. Hence, there is no wage dispersion across firms. If there are no differences in wages and hiring costs across firms, more able entrepreneurs hire more workers so that the marginal product of labour is the same in each firm.

[3]For a more detailed derivation of the job creation condition, see Appendix A.1.1.

After having posted their vacancies according to the job creation condition, firms and workers are matched randomly according to the matching technology, and they bargain about wages. Any wage-setting process is consistent with the model as long as the wage is not too low, so that the present value of being unemployed is not higher than the present value of being employed, or too high, which makes the match unprofitable for the entrepreneur. It is assumed that wages are determined by the generalised Nash bargaining solution. Workers and entrepreneurs bargain about the surplus of the match. The worker's surplus of being employed consists of the difference between the value equations of being employed and unemployed. The difference is given with

$$W_t^e - W_t^n = w_t - z + \beta(1 - s - \theta_t q(\theta_t))(W_{t+1}^e - W_{t+1}^n).$$

The entrepreneur instead compares the value of having an additional worker P_t to the option of not employing that worker, which is zero since there is free entry of firms. The wage is therefore determined as

$$w_t = \arg\max \ (W_t^e - W_t^n)^{\xi} P_t^{1-\xi}$$

with the worker's bargaining power $\xi \in [0, 1]$. The wage thus must satisfy the first order condition from wage bargaining:

$$\xi P_t = (1 - \xi)(W_t^e - W_t^n). \tag{3.2}$$

Plugging in for the respective surpluses and solving for the wage rate, it results in the wage curve[4]

$$w_t = \xi \left[a f'(l_t) + \gamma \theta_t\right] + (1 - \xi) z. \tag{3.3}$$

The wage consists of a fraction ξ of the worker's marginal product and the hiring costs plus a fraction $1 - \xi$ of home production. Workers are therefore rewarded for the saving of vacancy posting costs since the firm does not have to pay them

[4]For a more detailed derivation, see Appendix A.1.2.

anymore after a match is formed.[5] I can show that the wage is constant across firms. The first order condition from wage bargaining can be written as

$$\frac{\xi}{1-\xi}\left[af'(l_t) - w_t + \beta(1-s)P_{t+1}\right] = w_t - z + \beta\left[1 - s - \theta_t q(\theta_t)\right]\frac{\xi}{1-\xi}P_{t+1}.$$

The term on the left-hand side is the same for each firm, as can be deducted from (3.1). Therefore, the term on the right-hand side also must be constant. Since z, β, s, θ, ξ, and $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$ are the same for each firm, the wage must be constant across firms for the above equation to hold.

3.1.3 Occupational Choice

The marginal entrepreneur is indifferent between being a worker and becoming an entrepreneur. The profit of the marginal entrepreneur with talent $\bar{a}_t$ must equal the outside opportunity of being an employed worker in period t:[6]

$$W_t^f(l_t, \bar{a}_t) = W_t^e.$$

Plugging in for the respective value equations leads to the indifference equation

$$\begin{aligned} &\bar{a}_t f(l_t(\bar{a}_t)) - w_t l_t(\bar{a}_t) - \gamma v_t(\bar{a}_t) + \beta W_{t+1}^f(\bar{a}_t, l_{t+1}(\bar{a}_t)) \\ &\qquad = \quad w_t + \beta\left[sW_{t+1}^n + (1-s)W_{t+1}^e\right]. \end{aligned} \tag{3.4}$$

The profits of the marginal entrepreneur plus the future value of being an entrepreneur must be equal to the wage this very individual would earn as an employed worker plus the future value of being a worker, which comes with some uncertainty because she can lose her job with probability s. Figure 3 illustrates the value functions for regular workers and for entrepreneurs depending on talent a. If an individual has a talent above a certain threshold $\bar{a}$, it is optimal for her to become an entrepreneur because the value function exceeds the value function of being a regular employed worker. Therefore, all individuals with an entrepreneurial

[5] The average hiring costs for unemployed workers are $\gamma\theta_t = \frac{\gamma V_t}{N_t}$.

[6] Here, I assume that the entrepreneur directly becomes an employed worker when the threshold shifts.

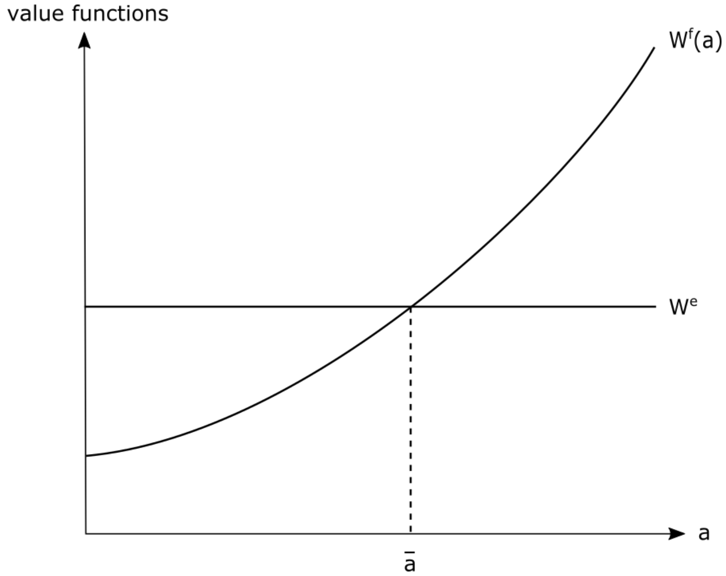

Figure 3: *Decision to become an entrepreneur*

talent below $\bar{a}$ prefer to become workers, whereas the others create their own firm. If a is bounded above, at least one individual must decide to become an entrepreneur and employ workers. Therefore, the upper bound must be sufficiently large so that it is more profitable for at least one individual to be an entrepreneur instead of being a regular worker. Hence, I assume that the upper bound $\hat{a}$ is sufficiently large so that at least one individual creates a firm and hires workers. It must hold that

$$\begin{aligned}
&\hat{a}f(l_t(\hat{a})) - w_t l_t(\hat{a}) - \gamma v_t(\hat{a}) + \beta W^f_{t+1}(\hat{a}, l_{t+1}(\hat{a})) \\
&\qquad\qquad\qquad\qquad \geq \; w_t + \beta\left[sW^n_{t+1} + (1-s)W^e_{t+1}\right].
\end{aligned}$$

If the above equation holds, there will be at least one entrepreneur, and I can rule out an equilibrium where every individual wants to be a worker.

3.1.4 Market Equilibrium

After having described the model setup and the individuals' optimisation problems, I can define the market equilibrium.

Definition 1. *$W^f_t(a, l_t)$, W^e_t, W^n_t, $v_t(a)$, θ_t and $\bar{a}_t$ define a market equilibrium if the following conditions hold for all t:*

- *W^e_t, W^n_t and $W^f_t(a, l_t)$ fulfil the value equations stated above and satisfy* (3.2),

- *optimal vacancy posting* $v_t(a)$ *takes place according to the job creation condition* (3.1),
- *the threshold* $\bar{a}_t$ *is set in line with Equation* (3.4),
- *the labour market tightness* θ_t *is given with* $\frac{V_t}{N_t}$.

The next section introduces the social planner's optimisation problem to characterise the efficient allocation as a benchmark. The market outcome described in the former sections can then be compared to the social optimal allocation.

3.2 The Social Planner

The social planner wants to maximise the aggregate sum of the utilities of employees, unemployed workers and entrepreneurs. The utility functions depend only on consumption and are assumed to be linear. Therefore, they are of the form $u(c_t^i) = c_t^i$ for $i = e, f, n$. An individual consumes c_t^f if she becomes an entrepreneur and starts a firm. Employed workers consume c_t^e and unemployed workers c_t^n. In aggregate, there are $\Phi(\bar{a}_t) - N_t$ employed workers, N_t unemployed workers, and $1 - \Phi(\bar{a}_t)$ entrepreneurs. The social planner is constrained by a resource constraint, a labour supply constraint and the law of motion for unemployment. His maximisation problem therefore reads

$$\max_{c_t^e, c_t^n, c_t^f, V_t, N_{t+1}, \bar{a}_t, l_t(a)} \sum_{t=0}^{\infty} \beta^t \left[(\Phi(\bar{a}_t) - N_t) u(c_t^e) + N_t u(c_t^n) + \int_{\bar{a}_t}^{\infty} u(c_t^f(a)) d\Phi(a) \right]$$

$$\text{s.t. } (\Phi(\bar{a}_t) - N_t) c_t^e + N_t c_t^n + \int_{\bar{a}_t}^{\infty} c_t^f(a) d\Phi(a) + \gamma V_t = \int_{\bar{a}_t}^{\infty} a f(l_t(a)) d\Phi(a) + N_t z,$$

$$N_{t+1} = N_t - m(N_t, V_t) + s(\Phi(\bar{a}_t) - N_t),$$

$$\Phi(\bar{a}_t) - N_t = \int_{\bar{a}_t}^{\infty} l_t(a) d\Phi(a).$$

The matching function is assumed to be Cobb-Douglas:

$$m(N_t, V_t) = N_t^{\alpha} V_t^{1-\alpha}.$$

Since the social planner still faces the matching function and cannot directly match workers with firms without vacancy posting, the outcome of his maximisation problem is a constrained first-best outcome. Combining the first order conditions from the maximisation problem and simplifying them, one obtains two central equations that describe the social planner's optimal choice.[7] First, it must hold that

$$\frac{\gamma}{\beta q(\theta_t)} = (1-\alpha)\left[af'(l_{t+1}(a)) - z\right] - \alpha\gamma\theta_{t+1} + \beta(1-s)\frac{\gamma}{\beta q(\theta_{t+1})} \tag{3.5}$$

and second,

$$\bar{a}_t f(l_t(\bar{a}_t)) + \frac{s\gamma}{(1-\alpha)q(\theta_t)} - \bar{a}_t f'(l_t(\bar{a}_t))(1 + l_t(\bar{a}_t)) = 0 \tag{3.6}$$

must be fulfilled.

Equation (3.5) states that today's discounted costs of having an additional employed worker must equal the social benefit of having that additional worker in the firm in the next period plus her future value.

Equation (3.6) instead describes the optimal choice of $\bar{a}_t$, the threshold above which every individual becomes an entrepreneur. Rearranging Equation (3.6) to

$$\bar{a}_t f\left[l_t(\bar{a}_t)\right] - \bar{a}_t f'\left[l_t(\bar{a}_t)\right] l_t(\bar{a}_t) = \bar{a}_t f'\left[l_t(\bar{a}_t)\right] - \frac{s\gamma}{(1-\alpha)q(\theta_t)},$$

one can see that the threshold must be set such that the value added to GDP that is attributable to the marginal entrepreneur is equal to her contribution if she had been an employed worker. The left-hand side describes the production in the marginal entrepreneur's firm minus the part of production that is assignable to the workers working at that firm. The entrepreneur provides only her technology $\bar{a}$, but she does not provide any labour herself. The contribution as a worker on the right-hand side of the equation consists of production attributable to that worker reduced by the hiring costs that must be paid in case the worker loses the job. In contrast, an additional entrepreneur means that there is one additional firm in the economy that opens vacancies, employs workers, and contributes to aggregate production. Nevertheless, that entrepreneur is not an employed worker anymore and

[7] Detailed derivations can be found in Appendix A.1.3.

is not available for production within a firm. Moreover, moving an individual from being a worker to being an entrepreneur affects the tightness of the labour market. It gets harder for the firms to fill vacancies, since there is increased competition for fewer workers.

In the following, I will focus on steady states. In a steady state, (3.5) can be reformulated to

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\alpha)}{1-\beta(1-s)+\alpha\beta\theta q(\theta)}\left[af'(l)-z\right]. \tag{3.7}$$

Using this equation, the optimal marginal product of labour multiplied with total factor productivity can be calculated as

$$af'(l(a)) = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{[1-\beta(1-s)+\alpha\beta\theta q(\theta)]}{(1-\alpha)}. \tag{3.8}$$

Since $af'(l(a))$ in steady state depends only on exogenously given parameters and θ, which is the same for all entrepreneurs, the marginal products of labour multiplied with total factor productivities are constant across firms. Thus, wages are constant across firms as well. The steady state version of (3.6) is

$$\bar{a}f(l(\bar{a})) + \frac{s\gamma}{(1-\alpha)q(\theta)} - \bar{a}f'(l(\bar{a}))(1+l(\bar{a})) = 0. \tag{3.9}$$

Equation (3.7) and Equation (3.9) describe the equilibrium resulting from the social planner's optimisation problem. If I combine (3.8) and (3.9) by plugging in for the marginal product, I receive an equation that describes the optimal threshold $\bar{a}$:[8]

$$\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\alpha)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{[1-\beta+\alpha\beta\theta q(\theta)]}{(1-\alpha)}. \tag{3.10}$$

In the next section, I describe the steady state market equilibrium and analyse its efficiency properties.

[8]A detailed discussion on the efficient threshold and the interpretation of Equation (3.10) can be found in Section 3.3.2.

3.3 Efficiency in Market Equilibrium

The market equilibrium in steady state is characterised by the job creation condition, the wage curve, the Beveridge curve, the indifference equation, and the value equations for employees, unemployed workers. and entrepreneurs.
The job creation condition that is derived from the firm's maximisation problem determines labour demand in steady state:

$$af'(l(a)) = w + (1 - \beta(1 - s))\frac{\gamma\theta}{\beta\theta q(\theta)}.$$

The wage curve in steady state resulting from Nash wage bargaining after a match is formed is given with

$$w = \xi\left[af'(l(a)) + \gamma\theta\right] + (1 - \xi)z.$$

Combining both leads to

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1 - \xi)}{1 - \beta(1 - s) + \xi\beta\theta q(\theta)}\left[af'(l(a)) - z\right] \tag{3.11}$$

which states that the costs of employing an additional worker must be equal to the returns of employing that additional worker.
The Beveridge curve defines that the aggregate flows into unemployment must equal the aggregate flows out of unemployment in steady state:

$$s\int_{\bar{a}}^{\infty} l(a)d\Phi(a) = \theta q(\theta)\left[\Phi(\bar{a}) - \int_{\bar{a}}^{\infty} l(a)d\Phi(a)\right].$$

Using the fact that the number of employed workers is given with the number of all workers minus the unemployed workers, so

$$\int_{\bar{a}}^{\infty} l(a)d\Phi(a) = L = \Phi(\bar{a}) - N,$$

the Beveridge curve can be written as

$$
\begin{aligned}
sL &= \theta q(\theta)\left[\Phi(\bar{a}) - L\right] \\
\Leftrightarrow \quad s\left[\Phi(\bar{a}) - N\right] &= \theta q(\theta) N \\
\Leftrightarrow \quad N &= \frac{s\Phi(\bar{a})}{s + \theta q(\theta)}
\end{aligned}
$$

and determines the unemployment rate in steady state. The indifference equation given with (3.4) in steady state takes the form

$$
\begin{aligned}
\bar{a} f l(\bar{a}) - \gamma v(\bar{a}) - w l(\bar{a}) &= w + \beta s(W_n - W_e) \\
&= (1-\beta)W_e.
\end{aligned}
$$

The marginal entrepreneur is indifferent between being an entrepreneur and receiving the immediate profits of the firm and being an employed worker who earns the wage w but might be unemployed in the future with probability s. Combining the wage curve and the job creation condition, the wage in steady state can be written as

$$
w = z + \frac{\xi}{(1-\xi)} \frac{\gamma}{\beta q(\theta)} \left[1 - \beta(1-s) + \beta\theta q(\theta)\right]. \tag{3.12}
$$

Using the steady state wage, the value equation for being an employee is

$$
(1-\beta)W_e = z + \frac{\xi}{(1-\xi)} \frac{\gamma\theta}{\beta\theta q(\theta)} \left[1 - \beta + \beta\theta q(\theta)\right] \tag{3.13}
$$

and the value equation for being an unemployed worker is

$$
(1-\beta)W_n = z + \frac{\xi}{1-\xi}\gamma\theta. \tag{3.14}
$$

By inserting (3.12), (3.13), and (3.14) into the indifference equation in steady state, the marginal entrepreneur's profit in market equilibrium is calculated:[9]

$$
\begin{aligned}
\bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a}) &= \frac{\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right]+z \\
&= w - \beta s\frac{\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta)}.
\end{aligned}
$$

In the market equilibrium, the marginal entrepreneur's profit is equal to the wage she would earn as an employed worker minus the discounted difference between the value of being employed and being unemployed[10] multiplied by the probability of losing the job s. The marginal entrepreneur's profits are lower than an employed worker's instantaneous income, but as an employed worker there is always the risk of becoming unemployed, in which case the worker would earn less than the marginal entrepreneur. To sum up, the following proposition describes the steady state market equilibrium.

Proposition 1. *The steady state market equilibrium is characterised by w, θ, N, and $\bar{a}$ which fulfil*

- *the wage curve:* $w = \xi\left[af'(l(a)) + \gamma\theta\right] + (1-\xi)z$
- *the job creation condition:* $\frac{\gamma}{q(\theta)} = \frac{\beta}{1-\beta(1-s)}\left[af'(l) - w\right]$
- *the Beveridge curve:* $N = \frac{s\Phi(\bar{a})}{s+\theta q(\theta)}$
- *the indifference equation:*
  $\bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a}) = \frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right]+z$

The wage curve and the job creation condition have a unique intersection in a (θ, w)-space and thus pin down the unique equilibrium for wages and labour market tightness, as can be seen in Figure 4. The wage rises linearly in tightness, whereas the job creation condition is convex and decreases in θ. If wages are higher, firms create fewer jobs, and hence there are fewer vacancies per worker. The equilibrium for vacancies and unemployment is determined by the job creation condition and the Beveridge curve. In a (u, v)-space, the job creation condition is upward sloping

[9] For the derivation of the steady state wage, the value equations, and entrepreneurial profits, see Appendix A.1.4

[10] Using (3.13) and (3.14), one can calculate $W_e - W_n = \frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}$.

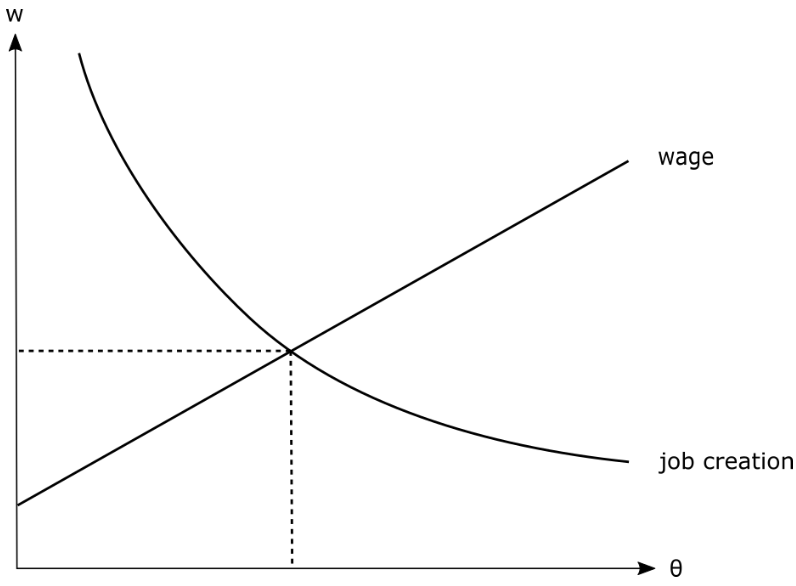

Figure 4: *Equilibrium wages and labour market tightness*

and uniquely intersects with the Beveridge curve, which is convex and downward sloping in u. As the number of posted vacancies increases, unemployment shrinks because it is easier to be matched with a firm. The unique equilibrium threshold $\bar{a}$ is determined by the indifference equation. For a further discussion, see Section 3.3.2, which describes the optimal threshold $\bar{a}$.

3.3.1 Efficient Hiring

To compare the market equilibrium with the social planner's solution, it is convenient to reformulate the indifference equation[11] to

$$\frac{\bar{a}f(l(\bar{a}))}{1+l(\bar{a})} - \frac{l(\bar{a})}{1+l(\bar{a})}\frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)}\frac{[1-\beta+\xi\beta\theta q(\theta)]}{(1-\xi)} - \frac{1-\beta}{\beta}\frac{\gamma}{q(\theta)}. \tag{3.15}$$

Together with Equation (3.11), it describes the equilibrium in the decentralised market. In the following, I compare the conditions describing the steady state market equilibrium to the conditions for an optimal allocation in the social planner's setting.

First, if one compares (3.11) and (3.7), it is obvious that $\xi = \alpha$ for efficient job creation if I assume that the threshold $\bar{a}$ is the same in the market as in the efficient

[11] The derivation can be found in Appendix A.1.4.

case. This result is the so-called Hosios condition,[12] which is a common outcome in the literature when analysing the efficiency of DMP models. The market solution is efficient if the private returns of a match ξ are equal to the social returns α. If, for example, $\alpha > \xi$, entrepreneurs will create too many vacancies compared to the efficient situation because their returns of a match would exceed the socially optimal returns. Thus, equilibrium unemployment is too low. The entrepreneurs in decentralised markets do not consider that their creation of jobs poses a negative externality on other entrepreneurs, since it gets harder for them to fill their own vacancies.

Second, if I assume that the Hosios conditions holds, I can determine whether the allocation of individuals into workers and entrepreneurs who create a firm is efficient. This assumption can easily be made for simplicity, since by using taxes or subsidies it can be assured that the Hosios condition holds. Comparing the indifference equation that conditions the threshold $\bar{a}$ in market equilibrium (3.15) to Equation (3.10), which determines the optimal threshold, an extra term is subtracted on the right-hand side of the equation for the market equilibrium. It suggests that the production of the marginal entrepreneur's firm is smaller in the decentralised market, indicating that the marginal entrepreneur is of lower ability than what would be optimal. Moreover, a conjecture is that entrepreneurs in the decentralised market capture larger shares of the production within their firm due to wage bargaining compared to the amount that the social planner would allocate to them. Therefore, too many individuals in the market decide to start a firm. The next sections deal with the comparison of the threshold ability $\bar{a}$ in the social planning setting and the decentralised market equilibrium.

3.3.2 The Optimal Threshold

In this section, I will first describe how the optimal number of entrepreneurs is set by the social planner and then compare it to the number of entrepreneurs in the market equilibrium.

Equation (3.8) and Equation(3.9) pin down the constrained first-best equilibrium.

[12] See Hosios (1990).

Combining them delivers Equation (3.10), which is stated here again for visibility:

$$\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\alpha)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{[1-\beta+\alpha\beta\theta q(\theta)]}{(1-\alpha)}.$$

The left-hand side of this condition can be interpreted as the value of the marginal firm relative to employment, whereas the right-hand side is the value of a worker. If an individual decides to be an entrepreneur, he or she cannot be a worker anymore. The marginal firm contributes the production $\bar{a}f(l(\bar{a}))$ to total output. This production value is distributed to all workers plus the entrepreneur. But in the case that workers lose their jobs, vacancies must be posted, which is costly. Vacancy posting costs are sunk costs and therefore lower overall welfare. As a worker instead, the individual would at least produce z and would have an influence on the labour market tightness, which potentially saves vacancy posting costs. If more individuals decide to become workers, the labour market tightness decreases, which makes it easier for entrepreneurs to fill their vacancies. Therefore, they do not have to post so many vacancies in excess only to make sure that some workers are hired and save costs. Moreover, if more individuals become workers, there are fewer entrepreneurs per se, which lowers the competition among them for workers. In equilibrium, the value of the marginal firm must be equal to the value of a worker, which determines the optimal number of entrepreneurs. If I define the left-hand side of Equation (3.10) as function $g(\bar{a})$ and the right-hand side as function $h(\bar{a})$, the intersection of these two functions pins down the unique threshold $\bar{a}$.

First, I focus on a partial equilibrium analysis. For given l, $g(\bar{a})$ rises in $\bar{a}$, whereas $h(\bar{a})$ decreases. This is depicted in Figure 5, where the value of the marginal firm is an increasing function of $\bar{a}$, and the value of a worker shrinks in $\bar{a}$. As $\bar{a}$ grows, the production of the marginal firm increases as well. A higher $\bar{a}$ means that there is one fewer entrepreneur but one more worker available in the economy. Therefore, the hiring costs diminish ceteris paribus, since it gets easier for the remaining firms to fill their vacancies. Put differently, labour supply rises if the former marginal entrepreneur decides to become a worker. The market tightness shrinks since there are more available workers and fewer firms looking for a worker. Hiring costs decrease. With lower hiring costs and unchanged policy functions, labour demand becomes larger and a new equilibrium is established.

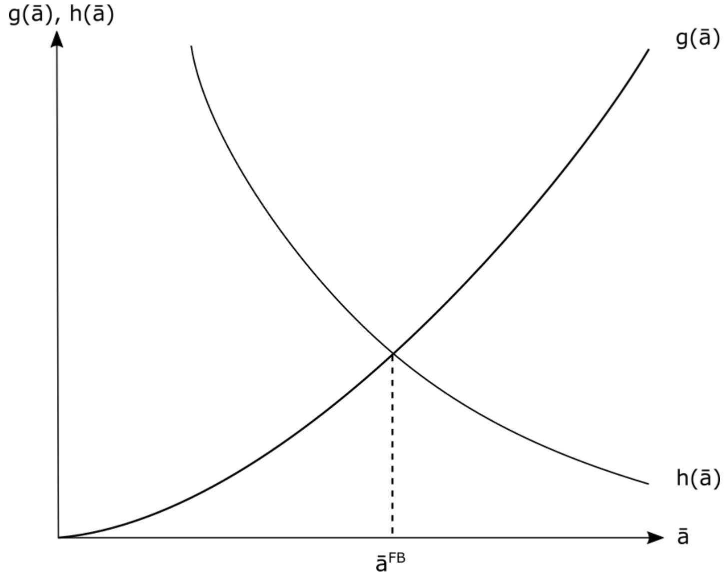

Figure 5: *The optimal threshold* $\bar{a}^{FB}$

In general equilibrium, if $\bar{a}$ increases by a small amount, Equation (3.10) shows that θ must grow since the left-hand side is clearly increasing in $\bar{a}$ and the right-hand side can only rise if θ gets larger. The tightness of the labour market is therefore increasing in $\bar{a}$. A higher $\bar{a}$ means that there are fewer rather unproductive firms. Therefore, it is easier for the remaining more productive firms to fill their vacancies, which enlarges overall production. Wages increase but not as much as productivity itself. At higher productivity, the profit from job creation is higher because the wages do not fully absorb the rise in productivity. Therefore, entrepreneurs post more vacancies, which increases labour market tightness. To sum up, the optimal allocation of individuals into workers and entrepreneurs must ensure that the value of the marginal firm for the economy equals the value of that firm's founder being a worker.

3.3.3 The Threshold in Market Equilibrium

The threshold in the market equilibrium is determined according to Equation (3.15), which is stated here again for comparability:

$$\frac{\bar{a}f(l(\bar{a}))}{1+l(\bar{a})} - \frac{l(\bar{a})}{1+l(\bar{a})}\frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)}\frac{[1-\beta+\xi\beta\theta q(\theta)]}{(1-\xi)} - \frac{1-\beta}{\beta}\frac{\gamma}{q(\theta)}.$$

The term on the left side is the value of the marginal entrepreneur, which is denoted as $g^{ME}(\bar{a})$ in the following, and the right side defines the value of a worker in market equilibrium, denoted as $h^{ME}(\bar{a})$. This indifference equation differs from (3.10) only with respect to ξ, instead of α, and the term $\frac{(1-\beta)}{\beta}\frac{\gamma}{q(\theta)}$, which is subtracted on the right side and clearly larger than one for $\beta \in]0,1[$. The left-hand sides are equal to each other if the Hosios condition holds and the thresholds in the first-best and the market equilibrium align. Assuming that $\xi = \alpha$ holds in the following, the threshold $\bar{a}$ in the market equilibrium is clearly smaller than the constrained first-best threshold. For a given θ, the right-hand side is smaller than in the first-best, and thus the left-hand side must decrease, which is possible if $\bar{a}^{ME}$ is smaller. Firms efficiently post vacancies, but there are too many firms that post vacancies, leading to a deviation of the threshold ability and labour market tightness from the first-best.

Proposition 2. *If the Hosios condition holds, $\gamma > 0$, $0 < \beta < 1$, and ξ is not too large, then*

$$\bar{a}^{ME} < \bar{a}^{FB},$$

which means that there are more entrepreneurs in the market equilibrium compared to the social planner's allocation.

Figure 6 demonstrates that the threshold $\bar{a}^{ME}$ in market equilibrium is smaller than in the social planning problem. There are too many firms compared to the efficient situation, since the value of being a worker is too low in the market equilibrium, which makes being an entrepreneur more profitable. Because of the wage bargaining process, workers in the decentralised market receive a smaller part of the surplus of a match than what is allocated to them by the social planner. This is depicted in $h^{ME}(\bar{a})$ lying below $h^{FB}(\bar{a})$. Put differently, entrepreneurs can acquire an inefficiently large part of the surplus of a match. Moreover, entrepreneurs with lower ability do not consider the negative effect that they exercise on the marginal product of more efficient firms. For an individual with a low a, it might be optimal to create a firm, but it is inefficient for the whole economy since that entrepreneur competes for workers with other firms. Within these other firms with more able entrepreneurs,

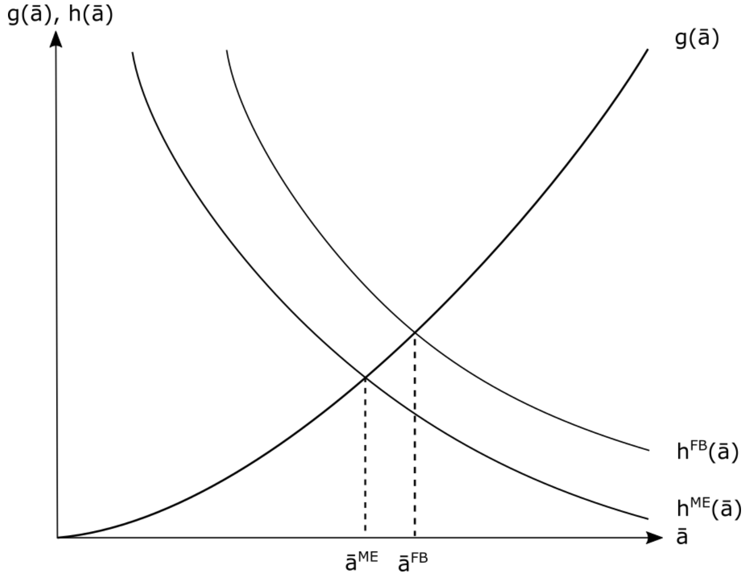

Figure 6: *Threshold $\bar{a}^{ME}$ in market equilibrium compared to $\bar{a}^{FB}$*

an employed worker would contribute more to overall production.
If the wage bargaining power of a worker ξ increases, being an entrepreneur becomes less attractive because they cannot acquire that much of the worker's marginal product and the saved vacancy posting costs. The number of entrepreneurs in market equilibrium decreases. With fewer entrepreneurs, the probability to fill a vacancy rises, which lowers the term that is subtracted on the right-hand side of Equation (3.15), and the equation becomes more similar to (3.10). Assuming the Hosios condition holds, the threshold $\bar{a}^{ME}$ gets closer to the efficient threshold the larger ξ gets. Nevertheless, for all values of $0 < \xi < 1$, there are always more firms in the market equilibrium compared to the first-best. Figure 7 shows that $h^{ME}(\bar{a})$ shifts upwards as ξ grows because the value of a worker rises with her bargaining power. As a result, the intersection of $h^{ME}(\bar{a})$ with $g(\bar{a})$ gets closer to the first-best intersection but still stays on the left side of it.

To sum up, an efficient market equilibrium does not exist. Comparing (3.7), (3.9), (3.11), and (3.15), it is not possible that the threshold $\bar{a}$ in the social planner problem equals the one in the market equilibrium and at the same time $\xi = \alpha$ holds.[13] If $\xi = \alpha$ holds, the market equilibrium is still inefficient, since too many individuals decide to become entrepreneurs. If instead the Hosios condition does not hold but

[13]The market equilibrium is of course efficient when $\xi = \alpha = 1$. This is the trivial case when I have efficient matching, e.g., when every unemployed worker directly finds a job. Then there are no labour market frictions and no unemployment.

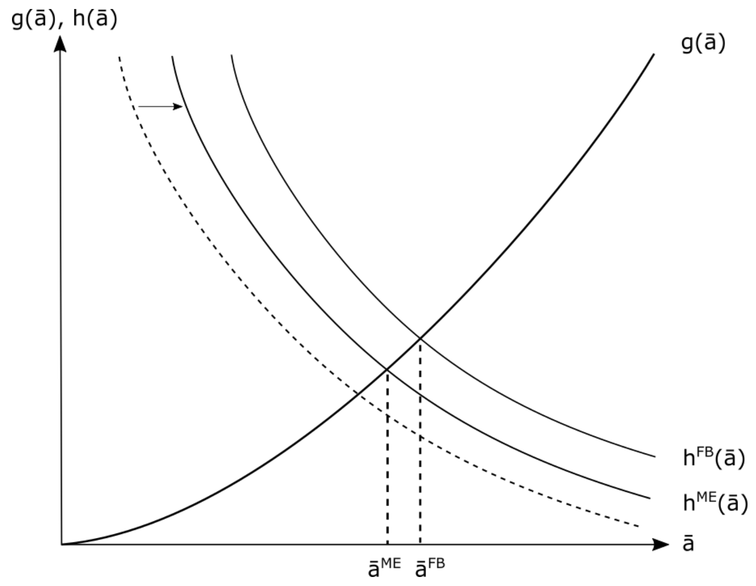

Figure 7: *Threshold in market equilibrium as ξ increases*

the threshold $\bar{a}$ is the same in the market as in the social planner's allocation, the number of firms in the market is efficient, but the entrepreneurs inefficiently post too many or too few vacancies. Therefore, the market equilibrium is in general inefficient, and taxation might be useful to restore efficiency.

3.4 Taxation

This section analyses how taxation can be used to restore the constrained first-best allocation. Entrepreneurs' incomes are taxed with a marginal tax rate τ_t^f, workers' incomes with the marginal tax rate τ_t^e, and the tax revenue is used for wasteful government spending.[14]

The firm's optimisation problem thus becomes

$$W_t^f(a, l_t) = \max_{l_{t+1}, v_t} \left\{ (1 - \tau_t^f) \left[af(l_t) - w_t l_t - \gamma v_t \right] + \beta W_{t+1}^f(a, l_{t+1}) \right\}$$

$$\text{s.t. } l_{t+1} = (1 - s)l_t + q(\theta_t)v_t.$$

[14]The entrepreneur's taxable income is the net profit from the firm, whereas the worker's taxable income is her labour income. I abstract from corporate taxation. For the numerical simulation described later on, I assume that the tax revenue is redistributed via a lump-sum transfer to all individuals. Since all individuals receive the lump-sum transfer, its introduction does not alter the individuals' decision problems.

Setting up the Lagrange function, deriving the first order conditions and using the envelope theorem, the condition for optimal job creation is[15]

$$\frac{(1-\tau_t^f)\gamma}{\beta q(\theta_t)} = (1-\tau_{t+1}^f)\left[af'(l_{t+1}) - w_{t+1}\right] + (1-s)\frac{\beta(1-\tau_{t+1}^f)\gamma}{\beta q(\theta_{t+1})}.$$

Assuming that the tax rate is fixed over time, the job creation condition is the same as in the case without taxation, because the tax rates cancel out. Job creation is therefore not distorted since wages and vacancy posting costs can be deducted from taxed gross profits. Defining $P_{t+1} := \frac{\gamma}{\beta q(\theta_t)}$ as above, the condition can be written as

$$P_t = af'(l_t) - w_t + \beta(1-s)P_{t+1},$$

which is the same formulation as in Section 3.1.2. The workers' value equations instead change when I introduce the taxation of incomes. The value equation for an employed worker changes to

$$W_t^e = (1-\tau_t^e)w_t + \beta\left[sW_{t+1}^n + (1-s)W_{t+1}^e\right]$$

and the value of being an unemployed worker stays

$$W_t^n = z + \beta\left[\theta_t q(\theta_t)W_{t+1}^e + (1-\theta_t q(\theta_t))W_{t+1}^n\right]$$

because it is assumed that home production is not taxed. Using the above value equations and the surplus of a match for the entrepreneur, the Nash wage bargaining result[16] is different compared to the case without taxes since vacancy creation is not distorted, but employed and unemployed workers are affected differently by taxation. The wage curve is therefore given with

$$w_t = \xi\left[af'(l_t) + \gamma\theta_t\right] + (1-\xi)\frac{z}{(1-\tau^e)}$$

[15] For a more detailed derivation of the job creation condition, see Appendix A.1.5.

[16] See Appendix A.1.5 for a derivation. I assume that workers and entrepreneurs bargain about gross wages.

and increases in τ^e. If employed workers' incomes are taxed more heavily, the outside option becomes more attractive, and therefore the wage must rise as well. In the following, I again focus on steady states to calculate the marginal tax rate on entrepreneurial incomes that restores the social planner's allocation. The indifference equation for the marginal entrepreneur becomes

$$
\begin{aligned}
& (1-\tau^f)\left[\bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a})\right] + \beta W^f(\bar{a}) \\
= \;& (1-\tau^e)w + \beta\left[sW^n + (1-s)W^e\right] \\
\Leftrightarrow \;& (1-\tau^f)\left[\bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a})\right] = (1-\tau^e)w - \frac{\beta s\left[(1-\tau^e)w - z\right]}{1-\beta(1-s)+\beta\theta q(\theta)}.
\end{aligned}
$$

Plugging in for w and using the job creation condition, it can be rearranged to

$$
\begin{aligned}
& (1-\tau^f)\left[\bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a})\right] \\
& \qquad = (1-\tau^e)\frac{\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right] + z
\end{aligned}
$$

and gives us the net income of the marginal entrepreneur depending only on exogenously given parameters and labour market tightness. Substituting further for the wage and $v(\bar{a})$ on the left-hand side, the indifference equation can also be written as

$$
\begin{aligned}
\frac{\bar{a}f(l)}{(1+l)} - \frac{l}{(1+l)}\frac{s\gamma}{(1-\xi)q(\theta)} &= \frac{(1-\tau^e)\frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]+z}{(1-\tau^f)(1+l)} \\
&+ \left[\frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right] + \frac{z}{(1-\tau^e)}\right]\frac{l}{1+l}, \qquad (3.16)
\end{aligned}
$$

which is easily comparable to Equation (3.10), which gives us the efficient number of entrepreneurs. I first assume that $\tau^e = 0$, so that only entrepreneurial incomes are taxed. The τ^f that restores the first-best allocation under the assumption that the Hosios condition holds therefore is[17]

$$
\tau^f = 1 - \left[\frac{\frac{\xi(1-\beta)}{(1-\xi)}\frac{\gamma}{\beta q(\theta^M)} + \frac{\xi}{(1-\xi)}\gamma\theta^M + z}{\frac{(1-\beta)}{(1-\xi)}\frac{\gamma}{\beta q(\theta^S)} + \frac{(1-\beta)}{\beta}\frac{\gamma}{q(\theta^M)}l(\bar{a}) + \frac{\xi}{(1-\xi)}\gamma\theta^S + z}\right].
$$

[17]For the derivation, also see Appendix A.1.5.

θ^M denotes the labour market tightness in the market equilibrium, whereas θ^S is the market tightness in the first-best setting. If they are identical,[18] one receives

$$\tau^f = \frac{(1-\beta)(1-\xi)\gamma\left[1+l(\bar{a})\right]}{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right]+\beta q(\theta)\left[\xi\gamma\theta+(1-\xi)z\right]}. \tag{3.17}$$

The tax rate acts as a Pigouvian tax and restores the first-best allocation. There are too many entrepreneurs in the market equilibrium without correcting taxation. A tax that makes being an entrepreneur less attractive relative to being a worker is therefore efficiency enhancing. Profit taxation makes it unprofitable for rather unproductive entrepreneurs to stay entrepreneurs. For them it is now better to become a worker, and the number of firms decreases. If there are fewer entrepreneurs and more workers, the labour market tightness shrinks, and it gets easier for the remaining firms with better technologies to fill their vacant positions.

The above tax rate is clearly increasing in $l(\bar{a})$, so it is increasing in the number of workers who are hired by the marginal entrepreneur. It also surges in θ if home production is sufficiently high. The tighter the labour market is, the higher the tax must be to induce more entrepreneurs to become workers, which in turn relaxes the labour market. Moreover, the tax rate decreases in ξ. A higher ξ means that workers have a higher bargaining power in wage negotiations. Entrepreneurs therefore can acquire only a smaller part of the worker's marginal product, and thus the tax rate on the firms' profits diminishes.

The total tax payment that the marginal entrepreneur must make is given with

$$\begin{aligned} T(\bar{a}) &= \frac{\tau^f}{(1-\tau^f)}\left[\frac{\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right]+z\right] \\ &= \frac{1-\beta}{\beta}\frac{\gamma}{q(\theta)}(1+l(\bar{a})). \end{aligned}$$

The gross income of the marginal entrepreneur can be calculated as

$$\frac{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right]}{\beta(1-\xi)q(\theta)}+\frac{\xi}{(1-\xi)}\gamma\theta+z.$$

[18] If $\xi=\alpha$, and if $\bar{a}$ is the same in the social planner's allocation and the market equilibrium, which is the aim of the introduction of taxation, conditions (3.7) and (3.11) tell us that $\theta^M=\theta^S=\theta$.

It is increasing in $l(\bar{a})$, hence a more able entrepreneur has a higher gross income. She also must make a larger total tax payment, which is increasing in the number of hired workers as well, as can be seen above. Introducing taxation does not change the net income of the marginal entrepreneur but reduces the number of firms in the economy. The marginal entrepreneur who receives the same net income as the marginal entrepreneur in the case without taxation is now an individual with a higher ability.

Now, I assume that workers' incomes are taxed as well with a given fixed tax rate τ^e. Given any $\tau^e \in [0, 1]$, the tax rate on entrepreneurial profits that restores the first-best allocation can be calculated[19] by comparing Equation (3.10) to Equation (3.16).

Proposition 3. *Given any $\tau^e \in [0, 1]$, the constrained first-best allocation can be achieved with the marginal tax rate on entrepreneurial incomes*

$$\tau^f = \frac{(1-\beta)(1-\xi)\gamma\left[1+l(\bar{a})\right] + \tau^e\xi\gamma\left[1-\beta+\beta\theta q(\theta)\right] - \frac{\beta(1-\xi)q(\theta)\tau^e lz}{(1-\tau^e)}}{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right] + \beta q(\theta)\left[\xi\gamma\theta+(1-\xi)z\right] - \frac{\beta(1-\xi)q(\theta)\tau^e lz}{(1-\tau^e)}}. \quad (3.18)$$

It holds that $\tau^f \geq \tau^e$ for

$$\tau^e \leq \frac{(1-\beta)\gamma}{(1-\beta)\gamma + \beta q(\theta)z}.$$

The tax rate τ^f is larger than τ^e if τ^e is smaller than the threshold reported above. Since the threshold is positive and τ^f is positive for $\tau^e = 0$, the tax rate on entrepreneurial income lies above the tax rate on workers' incomes for $\tau^e \in \left[0, \frac{(1-\beta)\gamma}{(1-\beta)\gamma+\beta q(\theta)z}\right]$. If workers' incomes are taxed additionally, becoming an entrepreneur gets relatively more attractive, and therefore τ^f must be larger than τ^e to counteract this effect. The point at which τ^f becomes smaller than τ^e decreases as z or $q(\theta)$ become larger. A larger z makes being a worker more attractive, since the outside option of not finding a job is larger and thus so are the wages paid to workers. If γ instead increases, the point at which τ^e exceeds τ^f also soars. As γ rises, the costs for posting a vacancy increase, and thus entrepreneurs post fewer vacancies. It becomes more difficult for workers to find a job. Therefore, the rela-

[19] A detailed derivation can be found in Appendix A.1.5.

tive attractiveness of being a worker declines, which results in taxing entrepreneurs more heavily than workers. The calculated τ^f restores the first-best allocation for any given τ^e since the labour market and vacancy postings are not distorted. Since the above formula shows us that τ^f is larger than τ^e for not too high values of τ^e, it is evidence that progressive taxation is needed within this framework to establish efficiency.

The above derivation of τ^f was done under the assumption that the Hosios condition holds.[20] If the Hosios condition does not hold, I would of course need a second tax policy that aims at correcting the vacancy posting decisions of the single firms. If, for example, $\alpha > \xi$, so the social returns of a match are larger than the private returns for the worker, entrepreneurs post too many vacancies and unemployment is lower than in the efficient situation. Entrepreneurs do not consider the negative externality that they exercise on the labour market. Taxing vacancy posting would then be a way to correct for that inefficiency. By taxing or subsidising vacancy creation, it can be assured that the Hosios condition holds. If I have two channels that cause inefficiencies, I will need two distinct tax instruments to correct for them.

To sum up, I calculate tax rates on entrepreneurial profits that restore the constrained first-best allocation with and without including the taxation of workers' incomes as well. Moreover, I find some indication for a progressive tax system since Equation (3.18) demonstrates that $\tau^f \geq \tau^e$ for τ^e not being too large. A progressive tax system in the described model can be justified by pure efficiency arguments. It corrects for the private decision of too many individuals to become an entrepreneur, which causes a loss in welfare because of inefficiently high vacancy posting costs caused by a tight labour market. Moreover, a second tax policy can correct for inefficient vacancy posting by subsidising or taxing the named and directly targeting the originator of the positive or negative externality.

[20] In Appendix A.1.5, I calculate the marginal tax rate when the Hosios condition is violated. Whether the progressivity result still holds depends on α, ξ, the socially optimal labour market tightness, and the labour market tightness in the market equilibrium. I argue that a progressive tax system still arises when the difference between α and ξ is not too large.

Parameter	Value	
$\alpha = \xi$	0.15	Hosios condition
β	0.9879	Annual discount factor of 0.95
γ	4.0	
η	0.7	Short-term labour share
μ_a	4.0	
σ_a^2	0.15	
s	0.1	Shimer (2005)
z	1	

Table 1: *Parameter values used for the numerical simulation in Chapter 3*

3.5 Numerical Simulation

In this section, I use a numerical simulation of the described model in steady state to briefly describe the magnitude and impact of the tax rates calculated above. The unit of time is considered to be a quarter of the year. For the parameters, I draw on the values that are usually used in the literature. Table 1 summarises the parametrisation and reports the evidence for the parameter values. I set the exponent of the matching function equal to the worker's wage bargaining power for the Hosios condition to hold.[21] For the production function, I assume $f(l) = l^\eta$ with $\eta = 0.7$, which is approximately in line with the short-term labour share.[22] Entrepreneurial ability a is distributed according to a log-normal distribution $\ln \mathcal{N}(\mu_a, \sigma_a^2)$ with $\mu_a = 4.0$ and $\sigma_a^2 = 0.15$. The monthly separation rate is estimated by Shimer (2005) to be on average equal to 0.034, which leads us to a quarterly probability of losing the job of 10 percent, and therefore $s = 0.1$. I use $\beta = 0.9879$, consistent with an annual discount factor of 0.95. I set $\gamma = 4.0$ and normalise $z = 1$ to receive an unemployment rate of 5.12% in the laissez-faire market equilibrium, which is in line with the long-term unemployment rate in the US. The resulting threshold entrepreneurial

[21] Petrongolo and Pissarides (2001) analyse estimates for the exponent of a Cobb-Douglas matching function. Relying on their survey the matching elasticity approximately takes on the value $\alpha = 0.5$. Card et al. (2018) argue that the worker's wage bargaining power lies in a range between 5% and 15%. Therefore, if one relies on the empirical estimates, the Hosios condition is violated. In this chapter, I assume that the Hosios condition holds and thus set $\alpha = \xi = 0.15$ to make a compromise between the different values. The numerical simulation in the Chapter 4 focuses on a more realistic parametrisation where the Hosios condition is violated.

[22] For the derivation of the equations used for the numerical simulation, see Appendix A.1.6.

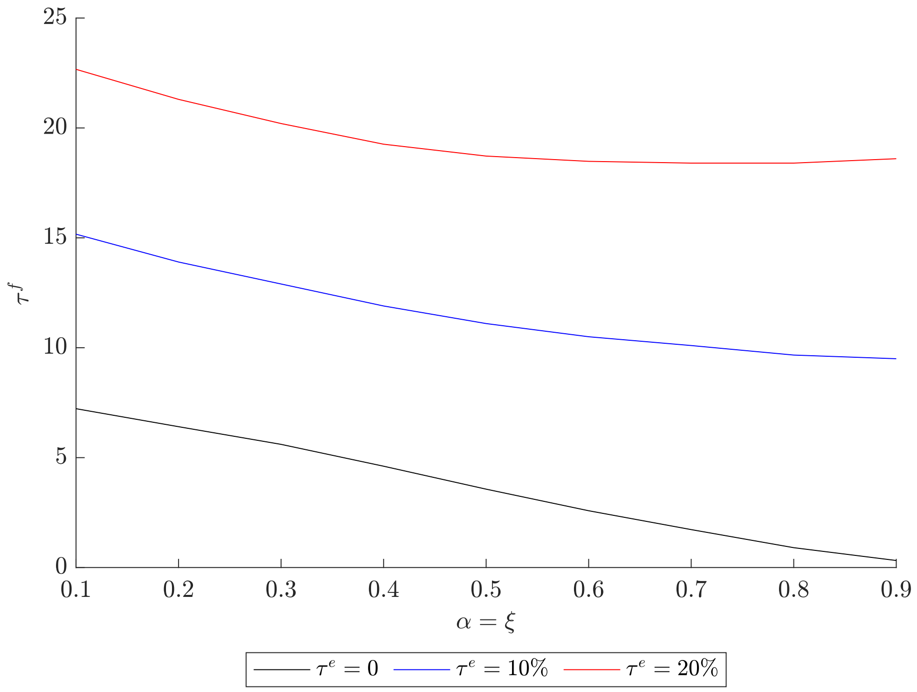

Figure 8: *Tax rate on entrepreneurial income for varying* τ^e

ability is $\bar{a}^{ME} = 4.18$. If I compare it to the first-best outcome $\bar{a}^{FB} = 4.21$, it is obvious that there are too many entrepreneurs in the market equilibrium, and thus entrepreneurial incomes must be taxed. The corresponding tax rate that maximises welfare under the assumption that workers' incomes are not taxed is given with $\tau^f = 7.16\%$. If, for example, $\tau^e = 20\%$, the tax rate on entrepreneurial income that restores the first-best is given with $\tau^f = 22.8\%$.

Figure 8 illustrates the tax rates on entrepreneurial incomes that maximise welfare for different tax rates τ^e on labour income. As α and ξ increase, the tax rate on entrepreneurial incomes decreases but always stays positive. Therefore, under the assumption that the Hosios condition holds, there are always too many entrepreneurs in the decentralised market without correcting taxation compared to the first-best. Thus, being an entrepreneur must be taxed to make it less attractive. The tax rate on entrepreneurial incomes is lower the lower τ^e is and the larger $\xi = \alpha$ is. The larger the match efficiency is and the larger the worker's wage bargaining power is, the lower the tax on entrepreneurial incomes. The larger wage bargaining power makes being an entrepreneur less attractive, whereas higher matching efficiency increases the attractiveness of owning a firm. Nevertheless, the effect of the higher wage bargaining power outweighs the effect of increased matching efficiency, as tax rates decrease in $\alpha = \xi$. This indicates that the attractiveness of becoming an entrepreneur in the laissez-faire equilibrium declines as both soar. If the taxation of labour income is included as well, the tax rates on entrepreneurial incomes rise by approximately 7 percentage points. For lower values of $\alpha = \xi$, τ^f exceeds τ^e which speaks in favour of a progressive tax system. If $\tau^e = 10\%$, the marginal tax rate on entrepreneurial incomes becomes smaller than τ^e for $\alpha = \xi > 0.74$. Nevertheless, such high values for the matching elasticity and the worker's wage bargaining power seem to be empirically implausible. For $\tau^e = 20\%$ and high values of $\alpha = \xi$, the tax rate nearly stays constant and even seems to increase a bit for $\alpha = \xi > 0.8$.

3.6 Conclusion

The former sections show that the market equilibrium can feature an inefficiently high number of entrepreneurs. Since entrepreneurs can acquire a large part of the surplus from a match with a worker, it is optimal for an individual to become an entrepreneur in the decentralised market, whereas the social planner would assign that individual to become a regular worker. In an economy with an imperfect labour market, entrepreneurs can acquire a disproportionately large part of the firms' revenue. When facing an occupational choice, individuals who become entrepreneurs do not consider their effect on overall labour market tightness. If there is an additional entrepreneur, there is one fewer worker available in the workforce, making it more difficult for other entrepreneurs to hire the remaining workers. If an individual with a mediocre talent for entrepreneurship decides to become an entrepreneur, she competes with more productive firms for the available workers who would contribute more to production in a more productive firm with a better entrepreneur. This overall effect on total production is not considered in individual utility maximisation. Inefficiencies in the market may also arise because entrepreneurs do not consider the negative effect that their vacancy posting choice is exerting on other entrepreneurs. Throughout the analytical part of this chapter, I assume that the Hosios condition holds and job creation thus is efficient. To sum up, it is shown that the value of being an entrepreneur is too high in the market equilibrium compared to the efficient situation.

Having described the above problem, I calculate a Pigouvian tax on entrepreneurial profits that corrects for the externalities and increases the costs of engaging in job creation. It restores the first-best allocation without distorting the labour demand and vacancy posting choices of individual firms. Moreover, the additional introduction of taxation of workers' incomes leads to a higher tax rate on profits. The marginal tax rate for entrepreneurs is increasing in the tax rate on workers' incomes, since the taxation of workers' labour incomes makes it less attractive to become a worker relative to being an entrepreneur. The tax rate on profits therefore must rise to restore the efficient relative attractiveness between both professions. Additionally, I can demonstrate that the marginal tax rate on entrepreneurial incomes is larger than the one on workers' incomes for not unrealistically high values of τ^e. This is a first hint on the effectiveness of a progressive tax system in the described setting. Since the analysis in this paper is limited to a steady state analysis, future work should concentrate on characterising complete policy functions and transition paths that occur from tax reforms. Then, it must be possible to investigate the progressiveness of a complete tax schedule in more detail. Moreover, the model can be extended, with various features influencing the taxation of entrepreneurs. One can argue that entrepreneurs might face a higher risk when creating a firm than regular workers. Additionally, potential entrepreneurs might be exposed to financial frictions and restricted by collateral constraints. Allowing for an open economy and the mobility of entrepreneurs might also be an interesting extension of the above model to include international tax competition. All these mentioned expansions speak in favour of lowering taxes on incomes from job creation. Therefore, it might be interesting to study the trade-offs of these different channels influencing the optimal taxation of entrepreneurial incomes.

In this chapter, I assume that the Hosios condition holds and I rely on the assumption that firms do not strategically try to depress wages by hiring more workers. Both assumptions were made to show, in the most transparent way, that entrepreneurs engage in rent-seeking, and thus being an entrepreneur can be too attractive in the laissez-faire market equilibrium. The next chapter relaxes these two assumptions and focuses on wage bargaining between workers and firms as described in Stole and

Zwiebel (1996). Moreover, the parametrisation used for the numerical simulation of the model considers that the wage bargaining power is empirically proven to be lower than the exponent of the matching function.

Chapter 4

A Theoretical Model with Intrafirm Wage Bargaining

If the production function exhibits decreasing returns in labour productivity, entrepreneurs can exploit the diminishing returns to manipulate wages as shown by Stole and Zwiebel (1996). In their static model, wage-setting is an ongoing process within the firm because contracts are nonbinding and workers can quit the firm at any time. Therefore, it is optimal for firms to overhire workers and to decrease the marginal product of labour to lower the wages of the incumbent workers and acquire larger rents. Cahuc and Wasmer (2001) were the first ones to include strategic intrafirm wage bargaining in the style of Stole and Zwiebel (1996) into the DMP model with large firms. They show that the standard predictions of the labour matching model hold under two assumptions: a perfect capital market and constant returns to scale in all production factors. Cahuc et al. (2008) extend the dynamic search and matching model with intrafirm wage bargaining. They argue that both the DMP model and the Stole and Zwiebel (1996) model lack important features. The DMP model without intrafirm wage bargaining does not include any within firm strategic interactions, whereas the latter one is static and only delivers partial equilibrium outcomes. Therefore, by providing a synthetic model, the implications of the overemployment result can be analysed. The authors find that an increasing bargaining power of workers does not necessarily lead to lower equilibrium employment, which would be the prediction of neo-classical labour market models.

Since the model described in the former chapter does not feature strategic behaviour within the firm, this chapter extends the model with intrafirm wage bargaining. Hence, this chapter expands the literature on intrafirm wage bargaining in matching models by adding heterogeneous entrepreneurial abilities and taxation. I find that introducing the Stole and Zwiebel wage-setting alters the theoretical result of always having too many entrepreneurs in the laissez-faire equilibrium compared to the first-best. The overemployment of workers makes being an entrepreneur less attractive because there is more competition for workers among firms. Thus, the number of entrepreneurs in the decentralised market equilibrium with intrafirm wage bargaining is smaller c.p. than in the setup without intrafirm bargaining. Whether the number of entrepreneurs exceeds or falls below the first-best number depends largely on the worker's wage bargaining power and the matching elasticity. Nevertheless, for realistic parameter values, the result of having too many entrepreneurs prevails. Moreover, the overhiring behaviour of firms leads to inefficient job creation even if the Hosios condition holds. Therefore, I impose a tax rate on vacancy posting that ensures efficient hiring. Having derived this tax rate, I calculate the marginal tax rate on entrepreneurial incomes that leads to the efficient quantity of firms in equilibrium.

The remainder of this chapter is structured as follows. The next section derives the setup of the model with intrafirm wage bargaining. Section 4.2 compares the equilibrium outcome to the first-best. Section 4.3 analyses the two mentioned tax rates, whereas Section 4.4 describes the numerical simulation of the model and the magnitudes of the derived tax rates. The last section within this chapter provides a brief conclusion.

4.1 Model Setup

The model analysed in the former chapter is extended with intrafirm wage bargaining while the general setup stays the same. The indifference equations of employed and unemployed workers do not change if I introduce intrafirm bargaining, except that intrafirm wage bargaining affects the wages, which are paid to the regular workers. In contrast, the firm's maximisation problem is altered.

4.1.1 The Firm

If an individual decides to become an entrepreneur, her value equation depends on the firm's profit, which is maximised by choosing the optimal number of workers and vacancies. After having posted the vacancies and being matched with workers, wage bargaining takes place. Since multiple workers bargain with a firm, the wage-setting is more complex than in the standard DMP model. If the production function exhibits decreasing returns in labour inputs, entrepreneurs can exploit the diminishing returns to manipulate wages as is shown by Stole and Zwiebel (1996). Therefore, it is optimal for firms to overhire workers and to decrease the marginal product of labour to lower the wages of the incumbent workers and acquire larger rents. In Chapter 3, I assumed that contracts are binding and firms do not engage in strategic wage-setting.
In this chapter, the firm's maximisation problem for given $w_t(a, l_t)$ and $q(\theta_t)$ is the following:

$$W_t^f(a, l_t) = \max_{l_{t+1}, v_t} \left\{ af(l_t) - w_t(a, l_t)l_t - \gamma v_t + \beta W_{t+1}^f(a, l_{t+1}) \right\}$$

$$\text{s.t. } l_{t+1} = (1-s)l_t + q(\theta_t)v_t.$$

In comparison to the maximisation problem in Chapter 3, the wage now depends on l_t, which captures the firm's possibility to depress wages by hiring more workers. Solving the maximisation problem by using a Lagrangian function,[1] one obtains the job creation condition:

$$\frac{\gamma}{q(\theta_t)} = \beta \left[af'(l_{t+1}) - w_{t+1}(a, l_{t+1}) - \frac{\partial w_{t+1}(a, l_{t+1})}{\partial l_{t+1}} l_{t+1} + (1-s)\frac{\gamma}{q(\theta_{t+1})} \right]. \tag{4.1}$$

It states that the expected costs of hiring a worker have to be equal to the value generated by having an additional worker. A hired worker increases the firm's production by the marginal product of labour multiplied by the entrepreneur's ability minus the wage that is paid to him or her. The derivative of the wage with respect to labour multiplied by labour input is subtracted and reflects the benefits of over-

[1] For the derivation of the job creation condition, see Appendix A.2.1.

hiring. With probability $1-s$, the worker stays at the firm in the next period and the continuation value must be added. The job creation condition can be rearranged to

$$\gamma\left[\frac{1}{\beta q(\theta_t)}-\frac{(1-s)}{q(\theta_{t+1})}\right]=af'(l_{t+1})-\frac{\partial w_{t+1}(a,l_{t+1})}{\partial l_{t+1}}l_{t+1}-w_{t+1}(a,l_{t+1}).$$

The term on the left-hand side does not depend on any of the choice variables of the firm. Therefore, the right-hand side must be equal across all firms in equilibrium and the following equation must hold:

$$\begin{aligned} af'(l_{t+1}(a))-\frac{\partial w_{t+1}(a)}{\partial l_{t+1}}l_{t+1}(a)-w_{t+1}(a) \\ =\bar{a}f'(l_{t+1}(\bar{a}))-\frac{\partial w_{t+1}(\bar{a})}{\partial l_{t+1}}l_{t+1}(\bar{a})-w_{t+1}(\bar{a}). \end{aligned}$$

The instantaneous marginal value of employing an additional worker is constant across firms because the average costs of employing an additional worker are the same for all entrepreneurs. It implies that the wage paid by all firms is the same in equilibrium and does not vary with entrepreneurial ability. There is no wage dispersion across firms.[2] If one rearranges the job creation condition and defines the surplus of having an additional worker as P_t, so that $P_{t+1}=\frac{\gamma}{\beta q(\theta_t)}$ holds, one obtains

$$P_t=af'(l_t)-w_t(a,l_t)-\frac{\partial w_t(a,l_t)}{\partial l_l}l_t+\beta(1-s)P_{t+1}.$$

P_t is the value of having a match for the firm or the value of an occupied job. If there are no differences in wages and hiring costs across firms, more able entrepreneurs hire more workers so that the marginal product of labour is the same in each firm. After having posted their vacancies according to the job creation condition, firms and workers are matched randomly according to the matching technology and they bargain about wages. It is assumed that wages are determined by the generalised Nash bargaining solution. Workers and entrepreneurs bargain about the surplus of the match. The worker's surplus of being employed consists of the difference between the value equations of being employed and unemployed. The entrepreneur bargains

[2] For a proof, see Appendix A.2.2.

with the marginal worker and thus compares the value of having an additional worker P_t to the option of not employing that worker, which is zero. The wage is therefore determined as

$$w_t(a, l_t) = \arg\max \ (W_t^e - W_t^n)^{\xi} P_t^{1-\xi},$$

with the worker's bargaining power $\xi \in [0, 1]$. The wage must thus satisfy the first order condition

$$\xi P_t = (1 - \xi)(W_t^e - W_t^n). \tag{4.2}$$

As a result, the solution to the Nash wage bargaining[3] is the wage curve

$$w_t(a, l_t) = \xi \left[a f'(l_t) - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + \gamma \theta_t \right] + (1 - \xi) z. \tag{4.3}$$

The wage consists of the fraction ξ of the worker's marginal product, the derivative of the wage with respect to labour input, and the hiring costs plus a fraction of home production. Workers are therefore rewarded for the saving of vacancy posting costs since the firm does not have to pay it anymore after a match is formed.[4] The wage curve also includes the effect that the hiring of an additional worker exerts on wages. For given employment, wages are higher in the DMP model with intrafirm wage bargaining than in the standard model[5] because hiring an additional worker has a higher value for the firm since it decreases wages for the already employed workers within the firm. Nevertheless, since increasing employment reduces the wage bill, firms post more vacancies compared to a standard DMP model without intrafirm wage bargaining. The marginal product of workers decreases with increased hiring, which in turn lowers the wages paid to them.

[3] For the derivation, see Appendix A.2.2.

[4] The average hiring costs for unemployed workers are $\gamma\theta_t = \frac{\gamma V_t}{N_t}$.

[5] The standard DMP model assumes that firms are small and hire only one worker. In the standard model, firms thus cannot strategically influence wages since they only consist of one worker. The model described in Chapter 3 features large firms, but I assumed that contracts are binding and entrepreneurs do not have the possibility to decrease the wages of workers who are already employed at their firms.

4.1.2 Occupational Choice

The marginal entrepreneur is indifferent between being a worker and becoming an entrepreneur as already described in the former chapter. The indifference equation for the marginal entrepreneur thus has the form

$$\begin{aligned}
\bar{a}_t f(l_t(\bar{a}_t)) - w_t l_t(\bar{a}_t) - \gamma v_t(\bar{a}_t) + \beta W^f_{t+1}(\bar{a}_t, l_{t+1}(\bar{a}_t)) \\
= \quad w_t + \beta \left[s W^n_{t+1} + (1-s) W^e_{t+1} \right]. \quad (4.4)
\end{aligned}$$

The profits of the marginal entrepreneur plus the future value of being an entrepreneur have to be equal to the wage this very individual would earn as an employed worker plus the future value of being a worker, which comes with some uncertainty because she can lose her job with probability s. In contrast to the model without intrafirm wage bargaining, it is even more profitable to become an entrepreneur because entrepreneurs can lower the wages paid to workers and increase their profits.

As before, if a is bounded above, it must be made sure that at least one individual decides to become an entrepreneur. Therefore, the upper bound must be sufficiently large so that it is more profitable for at least one individual to be an entrepreneur instead of being a regular worker. The upper bound $\hat{a}$ therefore must fulfil the following equation:

$$\begin{aligned}
\hat{a}_t f(l_t(\hat{a}_t)) - w_t l_t(\hat{a}_t) - \gamma v_t(\hat{a}_t) + \beta W^f_{t+1}(\hat{a}_t, l_{t+1}(\hat{a}_t)) \\
\geq w_t + \beta \left[s W^n_{t+1} + (1-s) W^e_{t+1} \right].
\end{aligned}$$

If the equation holds, there will be at least one entrepreneur who opens up a firm.

4.1.3 Market Equilibrium

The market equilibrium with intrafirm wage bargaining is defined as follows:

Definition 2. *$W^f_t(a, l_t)$, W^e_t, W^n_t, $v_t(a)$, θ_t, and $\bar{a}_t$ define a market equilibrium if the following conditions hold for all t:*

- *W_t^e, W_t^n, and $W_t^f(a, l_t)$ fulfil the value equations stated above and satisfy* (4.2),
- *optimal vacancy posting $v_t(a)$ takes place according to the job creation condition* (4.1),
- *the threshold $\bar{a}_t$ is set in line with Equation* (4.4),
- *the labour market tightness θ_t is given with $\frac{V_t}{N_t}$.*

The next section compares the market equilibrium featuring intrafirm wage bargaining with the social optimal allocation that was derived in Section 3.2.

4.2 Efficiency in Market Equilibrium

The market equilibrium in steady state is characterised by the job creation condition, the wage curve, the Beveridge curve, the indifference equation, and the value equations for employees, unemployed workers, and entrepreneurs.
The first order condition for hiring in steady state is

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l) - \frac{\partial w}{\partial l}l - z\right]. \tag{4.5}$$

The costs of employing an additional worker have to be equal to the returns of employing that additional worker. The wage curve in steady state resulting from Nash wage bargaining after a match is formed is given with

$$w = \xi\left[af'(l) - \frac{\partial w}{\partial l}l + \gamma\theta\right] + (1-\xi)z.$$

Plugging Equation (4.5) into the wage equation, the steady state wage reads

$$w = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right] \tag{4.6}$$

and depends only on exogenously given parameters and the endogenously determined labour market tightness in equilibrium. The marginal product of labour multiplied with total factor productivity a can be derived as

$$af'(l) = z + \frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right]}{(1-\xi)} + \frac{\partial w}{\partial l}l. \tag{4.7}$$

The Beveridge curve states that, in steady state, the aggregate flows into unemployment have to equal the aggregate flows out of unemployment:

$$s\int_{\bar{a}}^{\infty} l(a)d\Phi(a) = \theta q(\theta)\left[\Phi(\bar{a}) - \int_{\bar{a}}^{\infty} l(a)d\Phi(a)\right].$$

Therefore, unemployment in steady state can be derived as

$$N = \frac{s\Phi(\bar{a})}{s + \theta q(\theta)}.$$

The indifference equation given with (4.4) in steady state takes the form

$$\begin{aligned}\bar{a}fl(\bar{a}) - \gamma v(\bar{a}) - wl(\bar{a}) &= w + \beta s(W_n - W_e)\\ &= (1-\beta)W_e.\end{aligned}$$

The marginal entrepreneur is indifferent between being an entrepreneur and receiving the immediate profits of the firm and being an employed worker, who earns the wage w but might get unemployed in the future with probability s. Using the steady state wage, the value equation for being an employee is

$$(1-\beta)W_e = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1 - \beta + \beta\theta q(\theta)\right] \tag{4.8}$$

and the value equation for being an unemployed worker is

$$(1-\beta)W_n = z + \frac{\xi}{1-\xi}\gamma\theta. \tag{4.9}$$

By inserting Equation (4.8) into the indifference equation in steady state, the marginal entrepreneur's profit in the market equilibrium is calculated:[6]

$$\begin{aligned}\bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a}) &= z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1 - \beta + \beta\theta q(\theta)\right]\\ &= w - \beta s\frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}.\end{aligned}$$

[6] For the derivation of the steady state wage, the value equations, and the entrepreneur's profits, see Appendix A.2.3.

In the market equilibrium, the marginal entrepreneur's profit is equal to the wage she would earn as an employed worker minus the discounted difference between the value of being employed and being unemployed[7] multiplied by the probability of losing the job s. The marginal entrepreneur's profits are lower than an employed worker's instantaneous income, but as an employed worker, there is always the risk of becoming unemployed, in which case the worker would earn less than the marginal entrepreneur. The following proposition describes the steady state market equilibrium.

Proposition 4. *The steady state market equilibrium is characterised by w, θ, N, and $\bar{a}$ that fulfil*

- *the wage curve:* $w = \xi\left[af'(l) - \frac{\partial w}{\partial l}l + \gamma\theta\right] + (1-\xi)z$,
- *the job creation condition:* $\frac{\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l) - \frac{\partial w}{\partial l}l - z\right]$,
- *the Beveridge curve:* $N = \frac{s\Phi(\bar{a})}{s+\theta q(\theta)}$,
- *the indifference equation:*
  $\bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a}) = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]$.

It can be shown that the equilibrium is unique following the same argumentation as in the chapter before.

4.2.1 Efficient Hiring

The indifference equation together with Equation (4.5) describes the equilibrium in the decentralised market. In the following, I compare the conditions describing the steady state market equilibrium to the conditions for an optimal allocation, which are derived in Section 3.2.

First, if one compares Equation (4.5) and Equation (3.7), it is obvious that the Hosios condition $\xi = \alpha$ is not sufficient for efficient job creation if I assume that the threshold $\bar{a}$ is the same in the market as in the efficient case. The Hosios condition states that the market solution in the standard DMP model is efficient if the private returns of a match ξ are equal to the social returns α. This result was proven to hold in the model without intrafirm wage bargaining described in the former chap-

[7] Using Equation (4.8) and Equation (4.9), one can calculate $W_e - W_n = \frac{\xi}{(1-\xi)}\frac{\gamma\theta}{\beta\theta q(\theta)}$.

ter. However, in the extended model with intrafirm wage bargaining, job creation is inefficient even if the Hosios condition holds. The reason is that entrepreneurs hire too many workers since this suppresses wages. They consider that hiring an additional worker lowers the wage for all workers who are already employed at the firm. These additional returns for an entrepreneur caused by a match are not taken into account in the Hosios condition, which only compares the social returns of a match to the bargaining power. Nevertheless, by taxing or subsidising vacancy creation, it can be assured that job creation is efficient.

As for the model without intrafirm bargaining, I can determine whether the allocation of individuals into workers and entrepreneurs who start a firm is efficient by comparing the threshold $\bar{a}$ in the market equilibrium to the threshold described in Section 3.3.2.

4.2.2 The Threshold in Market Equilibrium

To compare the market equilibrium with the social planner's solution, it is convenient to reformulate the indifference equation[8] to

$$\frac{\bar{a}f(l(\bar{a}))}{1+l(\bar{a})} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{[1-\beta+\xi\beta\theta q(\theta)]}{(1-\xi)} - \frac{(1-\beta)}{\beta} \cdot \frac{\gamma}{q(\theta)}. \quad (4.10)$$

As before, the term on the left side is the value of the marginal entrepreneur, which is denoted as $g^{ME}(\bar{a})$, and the right side defines the value of a worker in market equilibrium, denoted as $h^{ME}(\bar{a})$. If intrafirm wage bargaining did not take place, this equation would only differ from (3.10) with respect to ξ and with respect to the term $\frac{(1-\beta)}{\beta}\frac{\gamma}{q(\theta)}$ that is subtracted on the right side. The left-hand sides would be equal to each other if threshold abilities would align and $\alpha = \xi$. In the following, it is assumed that $\alpha = \xi$ for simplicity. Without intrafirm wage bargaining, the entrepreneurial threshold ability in the market equilibrium $\bar{a}^{ME}$ thus would be smaller than $\bar{a}^{FB}$, which is shown in Section 3.3.3. With intrafirm wage bargaining, which is introduced in this chapter, the left-hand side of the above equation would differ from the left-

[8] The derivation can be found in Appendix A.2.3.

hand side of (3.10) because job creation is inefficient even if the Hosios condition holds. By comparing the labour demand in the market equilibrium given with Equation (4.7) to the first-best labour demand (3.8), it is obvious that, for a given $\bar{a}$, the labour demand in the decentralised market is smaller. The threshold $\bar{a}^{ME}$ and θ^{ME} are thus smaller or larger in the market equilibrium than in the constrained first-best depending on the specification of parameter values.

Proposition 5. *If the Hosios condition holds,* $l(\bar{a}) \geq 0$*, and* ξ *is not too large, then*

$$\bar{a}^{ME} < \bar{a}^{FB},$$

which means that there are more entrepreneurs in the market equilibrium compared to the social planner's allocation.

Whereas Stole and Zwiebel (1996) describe the over-creation of jobs at the intensive margin, the channel here is rather the over-creation of jobs at the extensive margin. Figure 9 shows that the threshold $\bar{a}$ in market equilibrium can be smaller than in the social planning problem. There are too many firms compared to the efficient situation, since the value of being a worker is too low in the market equilibrium, which makes being an entrepreneur more profitable. Because of the wage bargaining process, workers in the decentralised market receive a smaller part of the surplus of a match than what is allocated to them by the social planner. This is reflected by $h^{ME}(\bar{a})$ lying below $h^{FB}(\bar{a})$. Put differently, entrepreneurs can acquire an inefficiently large part of the surplus. For a given $\bar{a}$, $g^{FB}(\bar{a})$ is larger than $g^{ME}(\bar{a})$. The reason is that entrepreneurs overhire workers to decrease wages. This in turn increases the competition for workers among firms and thus labour market tightness, which makes being an entrepreneur less attractive. Depending on the relative effect sizes, I can have too few or too many entrepreneurs. Nevertheless, the equilibrium illustrated in Figure 9 features too many entrepreneurs compared to the first-best. Thus, the acquisition of large shares of the surplus of a match outweighs the negative effects of overhiring on being an entrepreneur, and overall it is too attractive to start a firm. In the shown case, entrepreneurs with lower ability do not consider the negative effect that they have on the marginal product of more efficient firms. Without intrafirm bargaining, $g^{ME}(\bar{a})$ would always be equal to $g^{FB}(\bar{a})$ be-

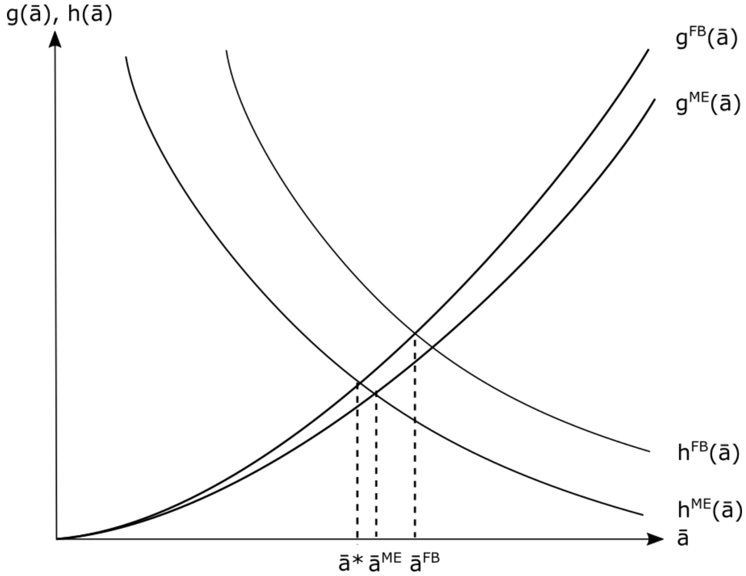

Figure 9: *Threshold in market equilibrium with intra-firm bargaining*

cause hiring is efficient if the Hosios condition holds. The threshold ability in the decentralised market would be equal to $\bar{a}^*$ and thus further away from the optimal value than $\bar{a}^{ME}$, which gives the threshold in the market equilibrium with intrafirm wage bargaining. In the model without intrafirm bargaining, overhiring does not occur and the competition among firms for workers is less severe, which induces entrepreneurs with even lower abilities to start a firm. Therefore, if I compare the market equilibrium without intrafirm bargaining to the equilibrium with this kind of bargaining, there are inefficiently more entrepreneurs in the former one.

The larger ξ, the closer the threshold $\bar{a}^{ME}$ gets to the efficient one. If the worker's bargaining power increases, the entrepreneur cannot acquire that much of the worker's marginal product and the saved vacancy posting costs. If ξ rises further, $h^{ME}(\bar{a})$ moves upwards because θ decreases. With intrafirm wage bargaining, the slope of $g^{ME}(\bar{a})$ changes, because, as $\alpha = \xi$ increases, entrepreneurs inefficiently post too many vacancies. The intersection of $g^{ME}(\bar{a})$ and $h^{ME}(\bar{a})$ can thus lie on the right side of the first-best intersection point depicted in Figure 10. In that case, the former results would be reversed, and there would be too few entrepreneurs and too many workers. Being an entrepreneur in that scenario would be too unattractive compared to the first-best. Without intrafirm wage bargaining, the result of having too many entrepreneurs in the market equilibrium would prevail as shown in Section 3.3.3. The counteracting effect of overhiring on the attractiveness of being an

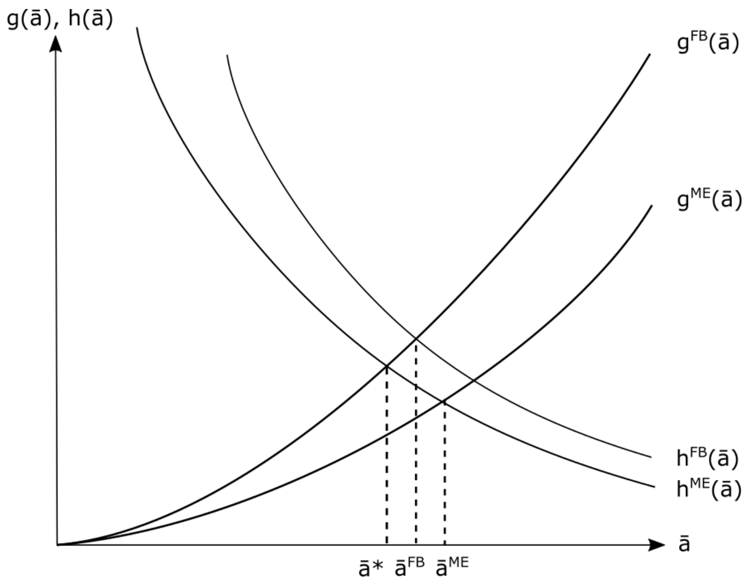

Figure 10: *Market equilibrium with fewer entrepreneurs than in first-best*

entrepreneur does not exist in that model version, and thus $\bar{a}^*$ still lies on the left side of $\bar{a}^{FB}$, even if α and ξ become larger.

To summarise, an efficient market equilibrium does not exist in the model with intrafirm wage bargaining either. Comparing Equations (3.7), (4.5), (3.10), and (4.10), it is not possible that the threshold $\bar{a}$ in the social planner's problem equals the one in the market equilibrium and job creation is efficient at the same time.[9] If job creation is efficient, the market equilibrium is still inefficient since too many or too few individuals decide to become entrepreneurs. If, on the other hand, the threshold $\bar{a}$ is the same in the market as in the social planner's allocation, the number of entrepreneurs in the market is efficient, but these entrepreneurs inefficiently post too many or too few vacancies. Therefore, the market equilibrium is in general inefficient and taxation might be useful to restore efficiency.

4.3 Taxation

This section analyses how taxation can be used to restore the constrained first-best allocation. Since I am confronted with two channels for inefficiencies, I need two tax instrument to correct for them. Entrepreneurs' incomes are taxed with marginal

[9]The market equilibrium is of course efficient when $\xi = \alpha = 1$. This is the trivial case when I have efficient matching, e.g., when every unemployed worker directly finds a job. Then there are no labour market frictions and no unemployment.

tax rate τ_t^f and vacancy posting is either taxed or subsidised with marginal tax rate τ_t^v.[10]

The firm's optimisation problem thus becomes

$$W_t^f(a, l_t) = \max_{l_{t+1}, v_t} \left\{(1-\tau_t^f)\left[af(l_t) - w_t(a, l_t)l_t - (1+\tau_t^v)\gamma v_t\right] + \beta W_{t+1}^f(a, l_{t+1})\right\}$$

$$\text{s.t. } l_{t+1} = (1-s)l_t + q(\theta_t)v_t.$$

Wages and vacancy posting costs can be deducted from taxed gross profits. Setting up the Lagrange function, deriving the first order conditions, and using the envelope theorem, the condition for optimal job creation is[11]

$$\frac{(1-\tau_t^f)(1+\tau_t^v)\gamma}{\beta q(\theta_t)} = (1-\tau_{t+1}^f)\left[af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1}\right] + (1-s)\frac{\beta(1-\tau_{t+1}^f)(1+\tau_{t+1}^v)\gamma}{\beta q(\theta_{t+1})}.$$

Assuming that the tax rate on entrepreneurial incomes is fixed over time, τ^f disappears from the above equation. Therefore, income taxation does not distort the vacancy posting behaviour of firms. Defining $P_{t+1} := \frac{(1+\tau_t^v)\gamma}{\beta q(\theta_t)}$, the condition can be written as

$$P_t = af'(l_t) - w_t - \frac{\partial w_t}{\partial l_t} l_t + \beta(1-s)P_{t+1}.$$

The workers' value equations are not affected by the introduction of taxation except that both the employed and unemployed workers receive the lump sum transfer in each period. Using the value equations and the surplus of a match for the entrepreneur, the Nash wage bargaining result[12] is

$$w_t = \xi\left[af'(l_t) - \frac{\partial w_t}{\partial l_t} l_t + (1+\tau_t^v)\gamma\theta_t\right] + (1-\xi)z,$$

[10] The entrepreneur's taxable income is the net profit from the firm. I abstract from corporate taxation. Moreover, I assume that the tax revenue is redistributed via a lump sum transfer to each individual. Since a lump sum transfer does not alter the results, it does not appear in the respective equations for simplicity.

[11] For a detailed derivation, see Appendix A.2.4.

[12] I assume that workers and entrepreneurs bargain about gross wages. For a more detailed derivation, see Appendix A.2.4.

which differs from the wage curve in the former sections. The average vacancy posting costs are now multiplied by $(1+\tau_t^v)$. If the tax on vacancy postings increases, the wage rises as well because a hired worker saves the firm higher vacancy posting costs. In the following, I again focus on steady states to calculate the marginal tax rates that recreate the social planner's allocation. Moreover, for simplicity, I assume that $\xi = \alpha$ holds.

First, the focus lies on the taxation or subsidisation of vacancy posting. If I plug in for the wage, the condition for job creation in steady state becomes

$$\frac{(1+\tau^v)\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l) - \frac{\partial w}{\partial l}l - z\right].$$

Comparing this first order condition to Equation (3.7), the tax rate τ^v can be calculated that leads to efficient job creation. It is thus given with[13]

$$\tau^v = \frac{-\frac{\partial w}{\partial l}l}{af'(l) - z}$$

and ensures that entrepreneurs efficiently post vacancies. Since the derivative of the wage with respect to labour input is negative, the above tax rate is always positive if z is not too large. Now, I can analyse the entrepreneurial decision under efficient hiring, by setting τ^v as above, and $\tau^f = 0$. The indifference equation becomes

$$\begin{aligned}\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} = \;& z + \frac{\gamma}{\beta q(\theta)}\frac{[1-\beta+\alpha\theta q(\theta)]}{(1-\alpha)} \\ & - \frac{\gamma}{\beta q(\theta)}\left\{(1-\beta)(1+\tau^v) - \frac{\tau^v}{(1-\alpha)}\left[1-\beta\left(1-\alpha\theta q(\theta) - \frac{sl(\bar{a})}{1+l(\bar{a})}\right)\right]\right\}.\end{aligned}$$

If I compare this equation to the first-best indifference equation, I can calculate the threshold τ^v below which there are still too many entrepreneurs in the decentralised market equilibrium. If the term in curly brackets is larger than one, too many individuals decide to create a firm, even if job creation itself is efficient. This corresponds to[14]

$$\tau^v < \frac{(1-\alpha)(1-\beta)}{\alpha\left[1-\beta+\beta\theta q(\theta)\right] + \frac{\beta sl(\bar{a})}{1+l(\bar{a})}}.$$

[13]See Appendix A.2.4 for a derivation of the result.

[14]See the end of Appendix A.2.4 for a derivation.

Whenever the tax rate that restores efficient hiring is smaller than the fraction on the right, entrepreneurial profits have to be taxed to ensure an efficient number of firms and workers.

Having derived τ^v, I concentrate on the taxation of entrepreneurial incomes under efficient hiring, i.e., it is assumed that τ^v is set as described above.[15] The indifference equation for the marginal entrepreneur becomes

$$(1-\tau^f)\left[\bar{a}f(l(\bar{a})) - wl(\bar{a}) - (1+\tau^v)\gamma v(\bar{a})\right]$$
$$= z + (1+\tau^v)\frac{\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right].$$

It yields the net income of the marginal entrepreneur depending only on exogenously given parameters and labour market tightness. Substituting further for the wage on the left-hand side of the equation, it can be rearranged to

$$\begin{aligned}
&\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} \\
&= \frac{\frac{1}{1-\tau^f}+l(\bar{a})}{(1+l(\bar{a}))}\left\{\frac{(1+\tau^v)\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\xi\beta\theta q(\theta)\right]}{(1-\xi)} - \frac{(1-\beta)}{\beta}\frac{(1+\tau^v)\gamma}{q(\theta)}+z\right\} \\
&+\ \tau^v\frac{l(\bar{a})}{(1+l(\bar{a}))}\frac{s\gamma}{(1-\xi)q(\theta)},
\end{aligned} \tag{4.11}$$

which is easily comparable to Equation (3.10) which gives us the efficient number of entrepreneurs. The τ^f that restores the first-best allocation therefore is

$$\tau^f = \frac{A(1+l) - B\tau^v(1+l) - \beta\gamma\tau^v sl}{Al + B(1-\tau^v l) - \beta\gamma\tau^v sl + (1-\alpha)\beta q(\theta)z},$$

with

$$A \equiv (1-\alpha)(1-\beta)(1+\tau^v)\gamma \quad \text{and} \quad B \equiv \left[1-\beta+\alpha\beta\theta q(\theta)\right]\gamma.$$

The tax rate acts as a Pigouvian tax and reconstitutes the first-best. If ξ is not too large, there are too many entrepreneurs in the market equilibrium without taxation. A tax that makes being an entrepreneur less attractive relative to being a worker

[15]For a more detailed derivation of the following results, see Appendix A.2.4.

is therefore efficiency enhancing. Profit taxation makes it unprofitable for rather unproductive entrepreneurs to stay in that occupation. For them, it is now better to become a worker, and the number of entrepreneurs decreases. If there are fewer entrepreneurs and more workers, the labour market tightness diminishes and it gets easier for the remaining firms with better technologies to fill their vacant positions. The above tax rate rises in $l(\bar{a})$, so it increases in the number of workers who are hired by the marginal entrepreneur. It also grows with θ if the unemployment benefit is sufficiently high. The tighter the labour market is, the higher the tax will be, since it induces more entrepreneurs to become workers, which in turn relaxes the labour market. Moreover, the tax rate decreases in α or ξ. A higher ξ means that workers have higher bargaining power in the wage negotiations. Entrepreneurs can therefore only appropriate a smaller part of the worker's marginal product, and the tax rate on the firms' profits decreases. The tax rate can thus become negative if the workers' bargaining power is high. In that case, being a worker is too attractive and there is an inefficiently low number of entrepreneurs in the decentralised market equilibrium. Being an entrepreneur thus must be subsidised.

Proposition 6. *The constrained first-best allocation can be achieved with two distinct marginal tax rates on vacancy posting and entrepreneurial income:*

$$\tau^v = \frac{-\frac{\partial w}{\partial l} l}{af'(l) - z}$$

and

$$\tau^f = \frac{A(1+l) - B\tau^v(1+l) - \beta\gamma\tau^v sl}{Al + B(1-\tau^v l) - \beta\gamma\tau^v sl + (1-\alpha)\beta q(\theta)z}$$

with

$$A \equiv (1-\alpha)(1-\beta)(1+\tau^v)\gamma \quad \textit{and} \quad B \equiv [1-\beta+\alpha\beta\theta q(\theta)]\,\gamma.$$

The first tax rate ensures that vacancy posting is efficient, whereas the second one ensures that the occupational choice to become an entrepreneur is efficient.

To sum up, I calculate tax rates on vacancy posting and on entrepreneurs' profits that restore the constrained first-best allocation. The taxation of entrepreneurial

Parameter	Value	
α	0.5	Petrongolo and Pissarides (2001)
β	0.9879	Annual discount factor of 0.95
γ	2.5	
η	0.7	Short-term labour share
ξ	0.1	Card et al. (2018)
μ_a	8.5	
σ_a^2	0.15	
s	0.1	Shimer (2005)
z	1	

Table 2: *Parameter values used for the numerical simulation in Chapter 4*

profits can be justified by pure efficiency arguments. It corrects for the private decision of too many individuals to become an entrepreneur, which causes a loss in welfare because of inefficiently high vacancy posting costs that are caused by a tight labour market. Moreover, a second tax policy can correct for inefficient vacancy posting by subsidising or taxing the named and directly targeting the originator of the positive or negative externality.

4.4 Numerical Simulation

In this section, I use a numerical simulation of the described model in steady state to briefly describe the magnitude and impact of the tax rates calculated above. The unit of time is considered to be a quarter of the year. The production function used for the numerical simulation takes the form $f(l) = l^{\eta}$.[16] For the parameters, I draw on values that are usually used in the literature. A summary of the parameter values is provided in Table 2. Petrongolo and Pissarides (2001) analyse estimates for the exponent of a Cobb-Douglas matching function. Relying on their survey of estimates, which mostly lie between 0.5 and 0.7, the exponent is set to $\alpha = 0.5$. As Card et al. (2018) estimate the worker's bargaining power to be in a range of 5% to 15%, I set $\xi = 0.1$. Compared to the previous chapter, α is not equal to ξ

[16]For the derivation of the equations used for the numerical simulation, see Appendix A.2.5.

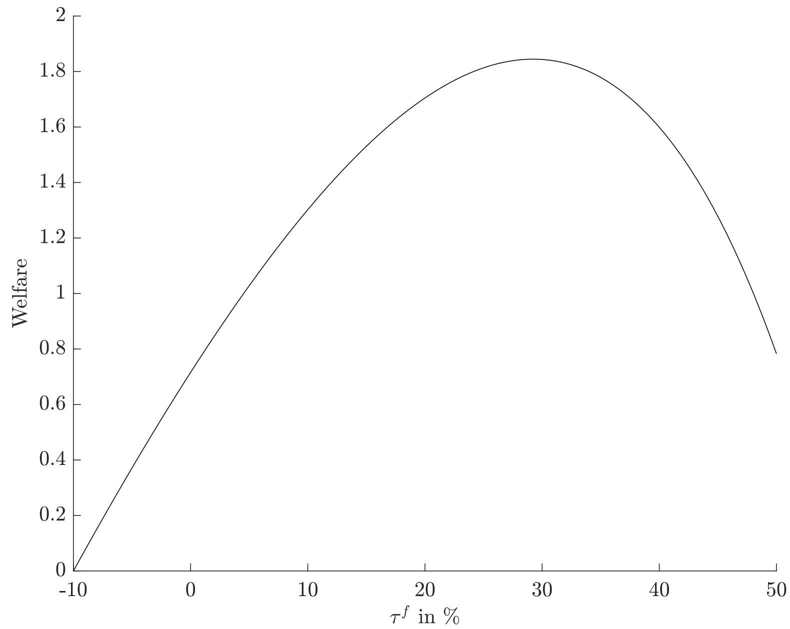

Figure 11: *Welfare for different* τ^f

since this is the more realistic parametrisation supported by the empirical literature. Entrepreneurial ability is distributed according to a log-normal distribution $\ln \mathcal{N}(\mu_a, \sigma_a^2)$ with $\mu_a = 8.5$ and $\sigma_a^2 = 0.15$. The monthly separation rate is estimated by Shimer (2005) to be on average equal to 0.034, which leads us to a quarterly probability of losing the job of 10%. Therefore, $s = 0.1$. I use $\beta = 0.9879$, consistent with an annual discount factor of 0.95. Home production z is normalised to 1. I set $\gamma = 2.5$ to receive an unemployment rate of 5.32% in the market equilibrium, which is approximately in line with the observed long-term unemployment rate in the US. The tax revenue is assumed to be redistributed as a lump sum transfer to every individual. The tax rates that maximise welfare are thus $\tau^v = 400\%$ and $\tau^f = 28.8\%$. Figure 11 depicts overall welfare under the described parametrisation for different values of τ^f. The marginal tax rate τ^v is set to correct for the overhiring behaviour of firms. If $\tau^f = 0\%$, welfare increases from 0 to approximately 0.6. This increase is purely caused by correcting vacancy posting through τ^v. Increasing the marginal tax rate on entrepreneurial incomes to $\tau^f = 28.8\%$ raises welfare to 1.8, the maximum. Figure 12 depicts the marginal tax rate τ^f depending on μ_a while holding everything else constant. For a very low μ_a, the tax rate is negative. It decreases as μ_a grows, peaks at around 30% and then declines.

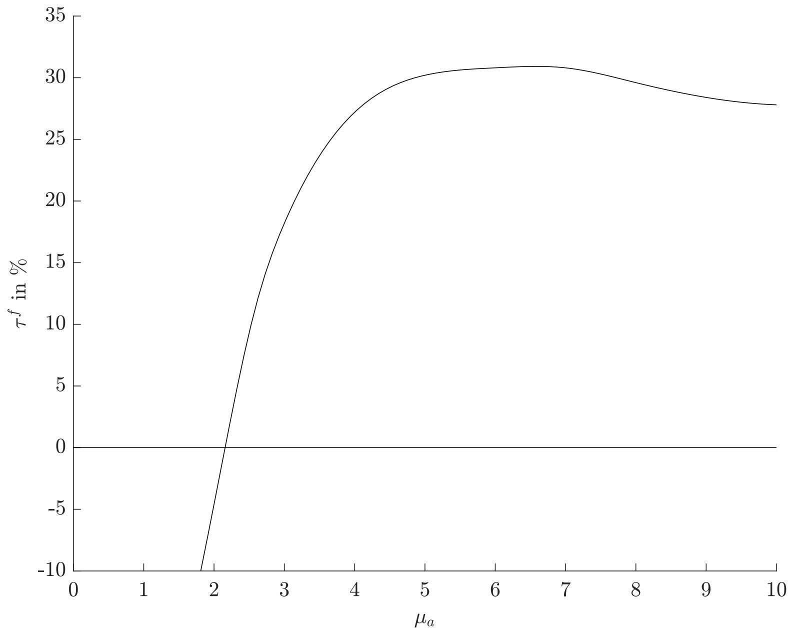

Figure 12: *τ^f for varying levels of μ_a*

Figure 13 shows the tax rates for different values of α and ξ if the respective other parameter is fixed. Panels a and b depict the marginal tax rates on entrepreneurial incomes and vacancy posting if $\alpha = 0.5$ and ξ varies. For small values of ξ, which are empirically relevant as Card et al. (2018) argue, entrepreneurial income is taxed because otherwise there would be too many entrepreneurs in the market equilibrium. If ξ is low, they can acquire a large share of the surplus of a match. The tax on entrepreneurs shrinks with ξ as a larger wage bargaining power makes being an entrepreneur less attractive. On the other hand, the tax rate on vacancy posting decreases as ξ rises. If ξ is small and matching is quite efficient with an $\alpha = 0.5$, overhiring workers comes with a small cost for the firms. They engage in a lot of overhiring, and thus τ^v must be large to correct for that. In contrast, Panels c and d illustrate τ^f and τ^v depending on α if ξ is fixed at 0.1. The matching elasticity and τ^f are positively correlated. If the matching efficiency is high and the wage bargaining power is low, being an entrepreneur is very attractive because wages are low and vacancies are filled in a short amount of time. Thus, as matching becomes more efficient and vacancy posting costs decrease, the tax rate on entrepreneurial income must increase to lower the attractiveness of being an entrepreneur. The marginal tax rate τ^v increases in α. A large α means that matching is quite efficient, which

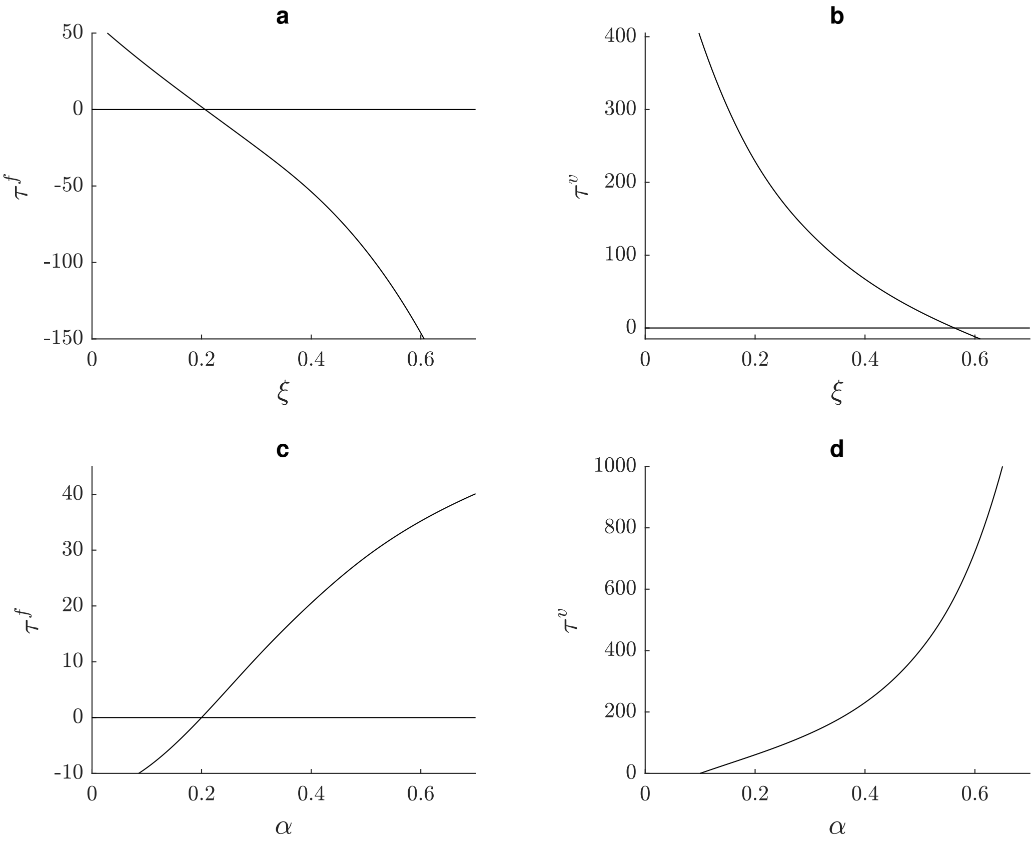

Figure 13: *Tax rates for varying ξ and α*

makes overhiring more attractive for firms. To counteract the increasing overhiring behaviour of firms, the tax on vacancy posting must become larger. Moreover, Panels b and d point out that τ^v takes more extreme values the larger the gap between α and ξ is. Nevertheless, if α is equal to ξ, the tax on vacancy posting is still positive because of the overhiring behaviour of firms. Overall, if α and ξ both increase proportionally, the marginal tax rate on entrepreneurial incomes declines, whereas the tax on vacancy posting increases. For example, if $\alpha = \xi = 0.2$, the tax rates from the numerical simulation are $\tau^f = -15.6\%$ and $\tau^v = 8.0\%$, with the other parameters being as reported in Table 2. For $\alpha = \xi = 0.7$, they are $\tau^f = -54.7\%$ and $\tau^v = 27.9\%$. Hence, the effect of the worker's higher wage bargaining power outweighs the effect of an increase in match efficiency with regard to the attractiveness of being an entrepreneur. As α and ξ are equal and increase, being an entrepreneur becomes less attractive and must be taxed with a low rate or even subsidised. On the other hand, τ^v increases in α and ξ being equal. The higher matching efficiency induces firms to engage massively in overhiring, which cannot be outweighed by higher wages caused by a higher ξ.

4.5 Conclusion

In this chapter, I have shown that the market equilibrium with intrafirm wage bargaining can feature an inefficiently high number of entrepreneurs. Since entrepreneurs can acquire a large part of the surplus from a match with a worker, it is optimal for an individual to become an entrepreneur in the decentralised market, whereas the social planner would assign that individual to become a regular worker. If an individual with a mediocre entrepreneurial talent decides to become an entrepreneur, she competes with more productive firms for the available workers who would contribute more to production in a more productive firm with a better entrepreneur. This overall effect on total production and labour market tightness is not considered in individual utility maximisation. Moreover, inefficiencies also arise because of intrafirm wage bargaining as described in Stole and Zwiebel (1996). Entrepreneurs post too many vacancies so that they can overhire workers to depress the wages paid to them.

Having outlined the above problem, I calculate two Pigouvian tax rates. The first tax rate ensures that vacancy posting, and thus hiring, is efficient because firms tend to overhire workers to decrease wages. The tax on entrepreneurial profits corrects for the externalities caused by a too large number of entrepreneurs and increases the costs of engaging in job creation. It restores the first-best allocation without distorting labour demand and the vacancy posting choice of individual firms. Without intrafirm wage bargaining, the market equilibrium would always feature too many entrepreneurs compared to the first-best. With intrafirm bargaining, the result depends on the worker's bargaining power and match efficiency. If matching efficiency is high, overhiring is severe and the negative effect of overhiring on the attractiveness to open up a firm outweighs the positive effect of the entrepreneur's ability to acquire parts of the surplus of a match.

A further aspect for future work is the introduction of on-the-job search into the analysed model. On-the-job search might lead to wage dispersion with more able entrepreneurs paying higher wages. This would make setting up a firm less attractive for individuals with lower abilities, and thus counteracts, at least to some extent, the effects this chapter describes. The welfare effects of on-the-job search are ambiguous and left for future work.

The following chapter extends the model with quadratic vacancy posting costs and firm death. By adding these features, firms grow slowly over time and do not directly jump to their optimal size from one period to the next one as in this chapter. Moreover, since firms can die, entrepreneurs can lose their occupation as well. Occupational choices hence become more dynamic.

Chapter 5

Firm Dynamics

This chapter analyses an augmented version of the theoretical model described in the previous chapter with the aim of modelling more realistic firm dynamics. More specifically, a production function with decreasing returns to scale, convex vacancy posting costs, and a more elaborate way of changing the occupation are added. The extended model is therefore quite similar to that of Acemoglu and Hawkins (2014), who specify a labour matching model with heterogeneous agents, large firms, a production function with decreasing returns to labour inputs, and convex vacancy posting costs in continuous time. Notwithstanding, the version I outline in the next sections includes an endogenous occupational choice, which alters the model substantially, and time is assumed to be discrete. The basic models described in Chapters 3 and 4 need to be augmented for several reasons. First, they predict that wages are constant across firms even if the Nash wage bargaining process described in Stole and Zwiebel (1996) is considered. The extended version takes intrafirm wage bargaining into account. The wage a certain firm pays thus depends on the talent of the firm's entrepreneur and the size of that company. Moreover, entrepreneurs engage in the overhiring of workers to influence wages. Second, the basic model predicts that all firms post their optimal number of vacancies when they are established and then directly jump to their optimal size in the next period. After a firm has reached the optimal size, vacancies are posted to keep the number of employees constant, which means that the number of newly posted vacancies ensures that the workers losing their job with a certain probability are replaced. In this chapter, I introduce convex vacancy posting costs. Combined with a production function with decreasing returns to scale, they lead to firms growing over time and not directly jumping to their optimal target size. Third, a firm exists forever in the basic model. To relax this assumption, a probability that the whole firm dies is implemented in this chapter. If the whole firm is shut down, all workers employed at the company and the entrepreneur who started that firm become unemployed. Even in steady state, I then have firms at different stages of their development. Young start-up firms with few employees and older firms, which have reached their optimal target size, exist concurrently. To have firms with different ages and sizes is especially important for analysing the effects of taxation on welfare. As Schumpeter (1934) argues, young firms are the driving force for creative destruction and thus important for economic growth. Baumol (2002) claims that the most revolutionary innovations are made by single entrepreneurs, who eventually build their own firm, and not by large companies. The dependence between firm age and job creation is explored by Haltiwanger et al. (2013). They find that business start-ups and young firms are a main driver of gross and net job creation in the US. However, as the exit rate is especially high among young firms, job destruction is also high. Nevertheless, conditional on surviving, young firms grow faster than more mature ones. Coad et al. (2018) provide an extensive survey of the literature on firm performance and firm age. In the introductory chapter of this dissertation, more data are shown that underline the importance of newly founded companies for job creation in the US. As young firms seem to be important for innovation and job creation, it is essential that a model aiming to analyse the effects caused by the taxation of entrepreneurial incomes features firms with different ages and sizes. If income taxation has negative impacts on entrepreneurs with mature, slowly growing, and small firms, this would not be as detrimental as negatively affecting the very innovative entrepreneur with a start-up that has high potential for growth.

However, the dynamic model with endogenous occupational choices raises a problem that is not of concern in the standard DMP model and the quasi-static models from Chapters 3 and 4. Instead of two separate pools, which ensure that there are always enough workers available for the firms, workers and entrepreneurs in this and the former chapters' models are drawn from one pool. Adding firm dynamics leads to individuals recurrently entering the decision state and deciding which occupation to take. As a consequence, there might be the problem that everyone decides to become an entrepreneur and no one wants to be a worker. Hence, firms cannot produce output as there are no workers to hire. Therefore, further assumptions are needed to ensure an equilibrium with workers and firms that follow a stationary distribution. This chapter's model and its numerical simulation thus constitute a relevant contribution to the literature on DMP models by providing a tractable and solvable dynamic matching model with endogenous occupational choices. It can be used to analyse, for example, policy measures that influence occupational choices between entrepreneurship and being a salaried employee and the policy's potential welfare implications.

The remainder of this chapter is organised as follows. Section 5.1 describes the setup of the dynamic model and focuses on the steady state equilibrium. A description of the numerical simulation and several results from it are presented in Section 5.2. Section 5.3 introduces taxation into the theoretical model and demonstrates its effects on several measures. Section 5.4 concludes.

5.1 Theoretical Model

There is a unit measure of risk-neutral individuals which can decide whether to become a regular worker or an entrepreneur who founds a firm. Their discount rate is given with β. In period zero, every individual draws a level of idiosyncratic entrepreneurial ability $a > 0$ from the distribution $\Phi(a)$, learns about her ability a, and finds herself in the decision state where she receives the value D_t. In the decision state, everyone can decide whether to become an entrepreneur or a regular worker. In contrast to the previous chapters' models, the individual faces heterogeneous entry costs ϵ when deciding for entrepreneurship. The costs are distributed according to the normal distribution $\mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$. As an entrepreneur, the individual starts her own firm and posts vacancies to hire workers. As a regular worker, she is unemployed and waits until a match with a firm occurs. Then, she becomes employed. Therefore, when facing the occupational choice in the decision state, individuals evaluate the value of starting a firm minus entry costs against the value of being an unemployed worker. Individuals know their ability a and the profits they would make as an entrepreneur given the labour market tightness θ. The value of being an unemployed worker is based on the expected value of employment because individuals do not know which firm they will eventually be matched with. Once an unemployed worker is matched with an entrepreneur and becomes employed, the worker must work at the firm until the match is destroyed. The employed worker earns the wage $w_t(l_t, a)$ with a denoting the entrepreneurial talent of the firm's founder she works for and l_t being the number of workers employed at that firm. The unemployed worker engages in home production and produces z. As an entrepreneur, the individual starts a firm, posts vacancies v_t, and receives the profits of her firm. With Poisson rate $\delta > 0$, an active firm is destroyed and the entrepreneur as well as all employed workers at the firm return to the decision state. They draw a new a and can again decide whether to become an entrepreneur or a worker based on their new entrepreneurial talent. With Poisson rate $s > 0$, a worker is separated from the firm and also enters the decision state. Production within a firm takes place according to a production function $f(l_t, a)$ that features decreasing returns to scale with respect to labour input. The entrepreneur faces convex vacancy posting costs $c(v_t)$ with $c'(v_t) > 0$ and $c''(v_t) > 0$ and pays the wage $w_t(l_t, a)$ to the employed workers at her firm. The wage is determined via a Nash wage bargaining process, which is described in more detail in Section 5.1.5.

The aggregate number of employed workers in period t is denoted with L_t and the aggregate number of unemployed individuals with N_t. Hence, the total number of regular workers is $L_t + N_t$. The number of firms (or entrepreneurs) is given with $1 - L_t - N_t$. Firms and workers looking for a job are matched according to the matching technology $m(N_t, V_t) = \omega N_t^{\alpha} V_t^{1-\alpha}$ with V_t denoting the aggregate number of vacancies and a matching function parameter ω. As firms post more vacancies or unemployment decreases, it gets more difficult for the entrepreneurs to fill their vacancies. The tightness of the labour market is given with $\theta_t = \frac{V_t}{N_t}$ and reports the number of vacant positions per unemployed worker. The probability to fill a vacancy in period t is denoted with $q(\theta_t) = \frac{m(N_t,V_t)}{V_t} = \omega\theta^{-\eta}$, which declines in labour market tightness. The tighter the labour market, the less likely firms are to find a worker.

From the worker's perspective, it is easier to be matched with a firm when the labour market tightness is high. The probability to find a job is $\frac{m(N_t,V_t)}{N_t} = \theta_t q(\theta_t) = \omega\theta^{1-\eta}$, which increases in θ_t. In every period, $\theta_t q(\theta_t) N_t$ individuals leave the unemployment pool and become employed. $[\delta + (1-\delta)s]\, L_t + \delta(1 - L_t - N_t)$ individuals lose their occupation and enter the decision state. The next sections describe the timing of events and the Bellman equations for the entrepreneurs and the regular workers, who are either employed or unemployed.

5.1.1 Timing

In every time period, there are four different states. An individual is either an entrepreneur with a firm, an employed worker, an unemployed worker, or finds herself in the decision state. How the individual's transition between the different states within a period takes place is explained in the following. Figure 14 illustrates the process. The states that individuals leave or enter are marked in red. At the beginning of a time period, all individuals are either entrepreneurs, employed workers, or unemployed workers. The firms with all their employed workers produce output. Next, firm destruction takes place. With probability δ, a firm is destroyed and the entrepreneur as well as all the workers employed at that firm lose their occupation and enter the decision state. The red arrows in the figure show the movements of a fraction of entrepreneurs and employed workers who leave their current state and enter the decision state. After firm destruction, dismissals or new hirings occur. Conditional on the firm not being destroyed before, workers can still lose their job with probability s and return to the decision state. Meanwhile, unemployed workers are matched with a firm and become employed if they agree to the match. If they did not agree to work at the firm, they would enter the decision state. The last step within the period encompasses the occupational choice of all the individuals who are in the decision state. Every individual in the decision state draws a new entrepreneurial ability a and then decides whether to become an entrepreneur and to start a firm or whether to become a regular worker. If an individual decides to become a worker, she starts in unemployment and might get matched with a firm in the next period. Otherwise, as an entrepreneur, she starts a firm, posts vacancies, and hires workers in the next period. In steady state, the distribution of firms must

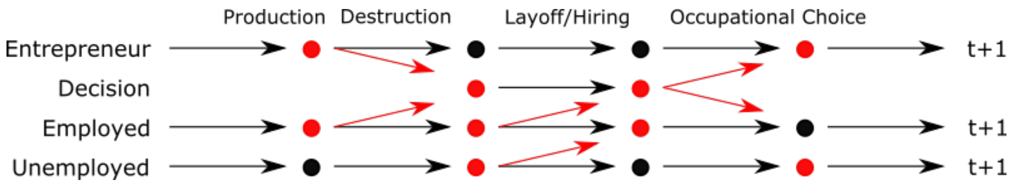

Figure 14: *Movements between different states*

be stationary. Therefore, as δ firms are destroyed every period, only δ firms are allowed to be founded in each period. Hence, the probability for the individuals in the decision state to draw an a that is large enough to make them become an entrepreneur must be adjusted. More specifically, the entry probability is multiplied by the probability of firm death divided by the number of individuals in the decision state.

5.1.2 The Workers' Bellman Equations

A worker who is employed at a firm receives the wage that is paid at that firm. Once a worker is matched with a firm and obtains a certain wage, the contract is binding meaning that the worker cannot terminate the contract to wait for a match with a firm that probably pays a higher wage. The job match is only destroyed with exogenously given probabilities. With probability s, the worker loses her job. With probability δ, the whole firm is destroyed and all workers employed at that firm along with the entrepreneur enter the decision state. With probability $(1-\delta)(1-s)$, the worker stays employed at the firm in the next period. The Bellman equation of an employed worker at a firm with l_t employees and an entrepreneur with talent a is therefore given with

$$W_t^e(l_t, a) = w_t(l_t, a) + \beta\left[\delta + (1-\delta)s\right] D_{t+1} + \beta(1-\delta)(1-s)W_{t+1}^e(l_{t+1}, a).$$

Note that a stands for the ability of the entrepreneur at whose firm the worker is employed and not for the worker's own a. The own entrepreneurial talent is irrelevant once an individual decides to become a regular worker. The worker loses her job with probability $\delta + (1-\delta)s$ because the firm is either destroyed with probability δ or, if the firm is not destroyed before, she loses her job with probability s. If the worker is dismissed, she enters the decision state where she draws a new a and must decide, conditional on a, whether she wants to become a worker or open up her own firm. If she decides to be a worker, she is unemployed until a match with a firm occurs. An unemployed worker engages in home production and produces z in every period during unemployment. The Bellman equation of an unemployed worker is thus given with

$$W_t^u = z + \beta \left[1 - \theta_t q(\theta_t)\right] W_{t+1}^u + \beta \theta_t q(\theta_t) \mathbf{E}_t \left[W_{t+1}^e(l_{t+1}, a)\right].$$

With probability $1 - \theta_t q(\theta_t)$, the worker is not matched with a firm and stays unemployed in the next period. With probability $\theta_t q(\theta_t)$, she finds a job at the end of the current period and is employed in the next period. In period t, she forms expectations about the match that might occur because the value of being an employed workers depends on the talent of the respective entrepreneur and the size of the firm the match occurs with.

5.1.3 The Firm's Maximisation Problem

The entrepreneur with entrepreneurial ability a faces the following maximisation problem:

$$\begin{aligned} J_t(l_t, a) = \max_{l_{t+1}, v_t \geq 0} & \Big\{ a f(l_t) - w_t(l_t, a) l_t - c_t(v_t) + \beta(1-\delta) J_{t+1}(l_{t+1}, a) \\ & + \beta \delta D_{t+1} \Big\} \\ \text{s.t. } & l_{t+1} = (1-s) l_t + q(\theta_t) v_t. \end{aligned}$$

She maximises her profits subject to the law of motion for employment and considers that employing an additional worker has an impact on the wages that are paid within the firm. Wage bargaining therefore proceeds in the style of Stole and Zwiebel (1996). The wage, which a firm with an entrepreneur with talent a pays, depends on the number of employees the firm has and on the entrepreneur's ability. With probability δ, the firm is destroyed and the entrepreneur as well as all the employees enter the decision state. Otherwise, the firm remains in operation. For the following calculations, I resort to specific functional forms for the production function and vacancy posting costs. Output is produced according to the Cobb-Douglas production function

$$f(l_t) = l_t^{\eta},$$

which features decreasing returns to scale. The cost function is assumed to be quadratic, so

$$c(v_t) = \frac{1}{2}\gamma v_t^2,$$

which ensures that entrepreneurs hire their optimal workforce not immediately but rather optimally over time. To solve the maximisation problem, the Lagrangian function

$$\begin{aligned}\mathcal{L} = a l_t^{\eta} - w_t(l_t, a) l_t - \frac{1}{2}\gamma v_t^2 + \beta\left[(1-\delta) J_{t+1}(l_{t+1}, a) + \delta D_{t+1}\right] \\ + \lambda_t\left[(1-s) l_t + q(\theta_t) v_t - l_{t+1}\right]\end{aligned}$$

is set up, which delivers the first order conditions

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial v_t} &= -\gamma v_t + \lambda_t q(\theta_t) = 0, \\ \frac{\partial \mathcal{L}}{\partial l_{t+1}} &= \beta(1-\delta)\frac{\partial J_{t+1}(l_{t+1}, a)}{\partial l_{t+1}} - \lambda_t = 0.\end{aligned}$$

Combining the first order conditions, the optimal vacancy posting strategy of a firm can be determined as

$$\gamma v_t = \beta(1-\delta) q(\theta_t)\frac{\partial J_{t+1}(l_{t+1}, a)}{\partial l_{t+1}}.$$

The marginal costs of posting a new vacancy on the left-hand side have to be equal to the term on the right-hand side. The latter term represents the discounted value of employing an additional worker, which happens with probability $q(\theta_t)$ multiplied by $(1-\delta)$, which is the probability that the firm still exists in the next period. Using the envelope condition

$$\begin{aligned}\frac{\partial J_t(l_t,a)}{\partial l_t} &= \eta a l_t^{\eta-1} - \frac{\partial w_t(l_t,a)}{\partial l_t} l_t - w_t(l_t,a) + \lambda_t(1-s) \\ &= \eta a l_t^{\eta-1} - \frac{\partial w_t(l_t,a)}{\partial l_t} l_t - w_t(l_t,a) + (1-s)\frac{\gamma v_t}{q(\theta_t)},\end{aligned}$$

the above equation can be rearranged to

$$\frac{\gamma v_t}{q(\theta_t)} = \beta(1-\delta)\left[\eta a l_{t+1}^{\eta-1} - \frac{\partial w_{t+1}(l_{t+1},a)}{\partial l_{t+1}} l_{t+1} - w_{t+1} + (1-s)\frac{\gamma v_{t+1}}{q(\theta_{t+1})}\right]. \quad (5.1)$$

Equation (5.1) is the job creation condition. It states that the costs of employing an additional worker have to be equal to the surplus generated by employing that additional worker. In contrast to the job creation conditions delivered by the former chapters' theoretical models, the vacancies still appear in the condition because of the quadratic vacancy posting costs. The more vacancies an entrepreneur wants to post, the costlier it becomes. As opposed to the models of Chapters 3 and 4, the firm does not reach its optimal size after one period. Instead, it grows gradually over time until the optimal size is reached, conditional on not being destroyed before.
If an individual in the decision state decides to become an entrepreneur, she must first post vacancies before the firm can produce output. The entrepreneur's Bellman equation in the first period of owning a firm thus is

$$J_t(0,a) = -0.5\gamma v_t^2 + \beta(1-\delta)J_{t+1}(l_{t+1},a) + \beta\delta D_{t+1}.$$

5.1.4 Decision State

When individuals are in the decision state, they draw their entrepreneurial ability a from the distribution $\Phi_t(a)$ and observe it. They then face the occupational choice. It is optimal for an individual to start a firm if

$$\begin{aligned}& J_t(0,a) - \epsilon > W_t^u \\ \Leftrightarrow\ & J_t(0,a) - \epsilon > z + \beta\left[1-\theta_t q(\theta_t)\right] W_{t+1}^u + \beta\theta_t q(\theta_t)\mathbf{E}_t\left[W_{t+1}^e(l_{t+1},a)\right].\end{aligned}$$

If the value of starting a firm with $l_t = 0$ workers at the beginning minus entry costs ϵ is larger than the value of being an unemployed worker in the next period, the individual decides to become an entrepreneur. Entry costs ϵ, which can also be interpreted as unobserved heterogeneity, are distributed according to the normal distribution $\mathcal{N}(\mu_\epsilon, \sigma_\epsilon^2)$. The value of being unemployed on the right-hand side depends on the expected value of being an employed worker, because with probability $\theta_t q(\theta_t)$ the now unemployed worker gets matched with a firm and becomes employed. The date-t expected value of being employed in period $t+1$ is given with

$$\begin{aligned}\mathbf{E}_t\left[W^e_{t+1}(l_{t+1}, a)\right] = \mathbf{E}_t\left[w_{t+1}(l_{t+1}, a)\right] + \beta\left[\delta + (1-\delta)s\right] D_{t+2} \\ + \beta(1-\delta)(1-s)\mathbf{E}_t\left[W^e_{t+2}(l_{t+2}, a)\right].\end{aligned}$$

Hence, the decision to become an entrepreneur depends on the expected wage, which is the same for each individual. They do not know in the decision state with which firm they will potentially match in the future and, therefore, form expectations about the wage that is paid to them. Expectations are also formed about the value of being employed two periods ahead, which happens with probability $(1-\delta)(1-s)$. If instead the match gets destroyed, the worker enters the decision state. The value equation for being in this state is known in every period.

The probability for an individual with talent a to enter as an entrepreneur in period t is denoted with $P^{entry}_t(a)$ and can be written as

$$\begin{aligned}P^{entry}_t(a) &= P\Big(\epsilon < J_t(0, a) - W^u_t\Big) \\ &= \mathcal{N}\Big(J_t(0, a) - W^u_t\Big).\end{aligned}$$

If entry costs are smaller than the gain of owning a firm compared to being an unemployed worker, it is optimal to become an entrepreneur. The utility level for individuals in the decision state can then be calculated as follows:

$$D_t = \int_0^\infty \left[P^{entry}_t(a) J_t(0, a) + (1 - P^{entry}_t(a))W^u_t\right] d\Phi(a).$$

Individuals draw their entrepreneurial ability a from the distribution $\Phi(a)$. It is either optimal for them to become entrepreneurs with probability $P_t^{entry}(a)$ or, with the counter probability, to become workers. The value equation is thus the ex-ante expectation of the occupational choice to be made in the decision state because individuals do not know which a they will draw once they enter this state.

5.1.5 Wage Bargaining

After a worker is matched with a firm, she bargains about the wage with the entrepreneur who started that firm. Once a worker has accepted the bargained wage, she cannot terminate the contract by herself. The worker-firm match gets destroyed only with the exogenously given probabilities s and δ. Workers and firms bargain about the surplus of a match. The firm's surplus is given with

$$\frac{\partial J_t(l_t, a)}{\partial l_t} = \eta a l_t^{\eta-1} - w_t(l_t, a) - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + (1-s)\frac{\gamma v_t}{q(\theta_t)}$$

and the surplus of a match for the worker with

$$\begin{aligned} W_t^e(l_t, a) - D_t = {} & w_t(l_t, a) + \beta\left[\delta + (1-\delta)s\right] D_{t+1} \\ & + \beta(1-\delta)(1-s) W_{t+1}^e(l_{t+1}, a) - D_t. \end{aligned}$$

So, the outside option for the worker is to enter the decision state and to face the occupational choice again. The wage is determined via a Nash wage bargaining process and solves

$$w_t(l_t, a) = \arg\max \left[W_t^e(l_t, a) - D_t\right]^{\xi} \left[\frac{\partial J_t(l_t, a)}{\partial l_t}\right]^{1-\xi}.$$

Solving the first order condition from the wage bargaining process, one obtains for the wage[1]

$$w_t(l_t, a) = \xi\left[\frac{\eta a l_t^{\eta-1}}{1-\xi(1-\eta)} + (1-s)\frac{\gamma v_t}{q(\theta_t)}\right] + (1-\xi) X(l_{t+1}, a)$$

[1]For a detailed derivation of the result, see Appendix A.3.1.

with

$$X(l_{t+1}, a) = D_t - \beta \left[\delta + (1-\delta)s\right] D_{t+1} - \beta(1-\delta)(1-s)W^e_{t+1}(l_{t+1}, a),$$

which measures the difference between what the individual would receive in the decision state today and what she would get in the future period if she accepts the match and starts working at the firm she bargains with. Accepting the match and staying at the firm entails the risk of losing the job in the next period, either because the whole firm is destroyed or because the single worker is fired. In that case, the individual would be in the decision state again. Therefore, $X(l_{t+1}, a)$ can be interpreted as the outside option for the worker.[2] Compared to the models described in Chapters 3 and 4, in which the outside option for the worker in the wage bargaining process was to become unemployed and receive z, the above outside option is more complex. If a worker rejected the firm match and entered the decision state, she would draw a new entrepreneurial ability a and would become a worker or an entrepreneur depending on this ability.
Having derived the wage, one can plug it into the job creation condition (5.1) and solve for v_t:

$$v_t = \frac{\beta(1-\delta)(1-\xi)q(\theta_t)}{\gamma}\left[\frac{\eta a l_{t+1}^{\eta-1}}{1-\xi(1-\eta)} - X(l_{t+2}, a) + (1-s)\frac{\gamma v_{t+1}}{q(\theta_{t+1})}\right].$$

The number of vacancies a firm advertises today depends positively on the talent of the entrepreneur, the number of workers it wants to employ tomorrow, and the number of vacancies it must post tomorrow. The worker's outside option one period ahead negatively influences vacancy postings. As the wage increases in the outside option, hiring workers becomes costlier.

5.1.6 Steady State Equilibrium

The value functions for the four different states in which an individual can be in steady state are described in the following. The value function for an unemployed

[2] The subscripts denoting the time periods can be a bit confusing. $X(l_{t+1}, a)$ describes the worker's outside option at the time of negotiation, here period t. It depends on the value of being employed in the next period, and hence on the number of workers at the firm in period $t+1$.

worker is given with

$$W^u = z + \beta\left[1-\theta q(\theta)\right]W^u + \beta\theta q(\theta)\mathbf{E}\left[W^e\right].$$

For an employed worker who works at a firm with l workers and an entrepreneur with ability a, I have

$$W^e(l,a) = w(l,a) + \beta\left[\delta + (1-\delta)s\right]D + \beta(1-\delta)(1-s)W^e(l^+,a).$$

The variable l^+ denotes the number of employed workers in the next period because firms can still grow in steady state if they have not reached their optimal size yet. The value function for an entrepreneur with talent a is

$$J(l,a) = al^\eta - w(l,a)l - 0.5\gamma v^2 + \beta(1-\delta)J(l^+,a) + \beta\delta D.$$

The entrepreneur employs l workers in the current period and l^+ workers in the next period until the optimal size is reached, conditional on the firm not being destroyed before. If the firm is at its optimal size, l equals l^+ and the entrepreneur posts exactly enough vacancies to replace the workers who lose their job with probability s.

When it comes to the decision state in steady state, one must adjust the individual entry probability. For a stationary firm distribution, only δ firms can enter as δ firms are destroyed. Therefore, the probability to become an entrepreneur must be adjusted for the number of individuals who are in the decision state. The number of individuals in the decision state can be calculated as

$$S = \int_{\bar{a}}^{\infty}\left[\delta(1+l(a)) + (1-\delta)sl(a)\right]d\Phi(a).$$

With probability δ, a firm dies and all the workers employed at that firm plus the entrepreneur enter the decision state. Conditional on the firm not being destroyed before, s workers additionally lose their job in every period. Having derived the number of individuals in the decision state, the adjusted entry probability for an individual with talent a is given with

$$P^{adj}(a) = \frac{\delta}{S} \cdot P^{entry}(a) = \frac{\delta}{S} \cdot \mathcal{N}\Big(J(0,a) - W^u\Big).$$

The value function for individuals in the decision state in steady state thus factors in that only the fraction $\frac{\delta}{S}$ of all individuals who want to start a firm can actually start one:

$$D = \int_0^\infty \Big[P^{adj}(a)J(0,a) + (1 - P^{adj}(a))W^u\Big]\, d\Phi(a).$$

For the marginal entrepreneur, it must hold that

$$J(0,\bar{a}) - \epsilon = W^u,$$

because she is indifferent between becoming an entrepreneur and being a regular worker. The steady state wage is given by

$$w(l,a) = \xi \left[\frac{\eta a l^{\eta-1}}{1-\xi(1-\eta)} + (1-s)\frac{\gamma v(l,a)}{q(\theta)}\right] + (1-\xi)X(l^+,a)$$

with the outside option

$$X(l^+,a) = \Big\{1 - \beta\left[\delta + (1-\delta)s\right]\Big\}D - \beta(1-\delta)(1-s)W^e(l^+,a).$$

The job creation condition in steady state boils down to

$$\frac{\gamma v(l,a)}{q(\theta)} = \beta(1-\delta)\left[\eta a l^{+\eta-1} - \frac{\partial w(l^+,a)}{\partial l^+} l^+ - w(l^+,a) + (1-s)\frac{\gamma v(l^+,a)}{q(\theta)}\right]. \quad (5.2)$$

Even in steady state, there are firms that have not reached their target size yet and thus l is not equal to l^+. Because of the probability δ that a firm is destroyed, there are always new and still growing firms, which have not reached their target size yet. Plugging in for the wage in Equation (5.2) and solving for v, these firms post their vacancies according to the following policy:[3]

$$v(l,a) = \beta(1-\xi)(1-\delta)\left\{\frac{q(\theta)}{\gamma}\left[\frac{\eta a l^{+\eta-1}}{1-\xi(1-\eta)} - X(l^{++},a)\right] + (1-s)v(l^{+},a)\right\}.$$

At one point in time, the firm has reached its target size meaning that it is not optimal to expand the workforce any further. The entrepreneur wants to keep employment at her firm constant and only posts vacancies to replace the dismissed workers. Denote the number of employees a firm has at its target size as $l^*(a)$. The optimal number of vacancies, which a firm with $l^*(a)$ workers and an entrepreneur with ability a advertises, is

$$v(l^*,a) = \frac{\beta(1-\xi)(1-\delta)q(\theta)}{\gamma\left[1-\beta(1-\xi)(1-\delta)(1-s)\right]}\left[\frac{\eta a l^*(a)^{\eta-1}}{1-\xi(1-\eta)} - X(l^*,a)\right].$$

In these firms, $l(a) = l^+(a) = l^*(a)$ because vacancies are posted to keep the workforce constant. The number of vacancies a firm advertises at its target size can also be calculated as

$$v(l^*,a) = \frac{s l^*(a)}{q(\theta)}.$$

The number of vacancies a firm that has reached its target size posts must be equal to the number of the firm's workers who lose their job divided by the probability to fill the vacancies. Therefore, the optimal firm size $l^*(a)$ must solve the following equation:

$$\frac{s l^*(a)}{q(\theta)} = \frac{\beta(1-\xi)(1-\delta)q(\theta)}{\gamma\left[1-\beta(1-\xi)(1-\delta)(1-s)\right]}\left[\frac{\eta a l^*(a)^{\eta-1}}{1-\xi(1-\eta)} - X(l^*,a)\right].$$

By plugging in for $X(l^*,a)$, this equation can be written as[4]

$$\begin{aligned}\frac{s l^*}{q(\theta)} &= \frac{\beta(1-\xi)(1-\delta)q(\theta)}{\gamma\left[1-\beta(1-\xi)(1-\delta)(1-s)\right]} \\ &\cdot\left\{\frac{\eta a l^{*\eta-1}}{1-\xi(1-\eta)} - \frac{(1-\beta)}{[1-\beta(1-\delta)(1-s)]}D + \frac{\beta(1-\delta)(1-s)}{[1-\beta(1-\delta)(1-s)]}w(l^*,a)\right\},\end{aligned}$$

[3]For a more detailed derivation, see Appendix A.3.2.
[4]For a derivation, see Appendix A.3.2.

which implicitly delivers the respective target sizes $l^*(a)$ for all levels of entrepreneurial ability a. The Beveridge curve in steady state is given with

$$N = \frac{S \int_0^\infty \left[1 - P^{adj}(a)\right] d\Phi(a)}{\theta q(\theta)}.$$

It ensures that the flows into unemployment equal those out of unemployment.

5.2 Numerical Simulation

When solving the firm's maximisation problem described in the former section, one faces a dynamic programming problem with two state variables: productivity a and the number of employed workers l. To determine the optimal vacancy posting policy, I use policy function iteration with the method of endogenous gridpoints. After deriving the optimal policy, I iterate the value functions, resolve the firms' entry decisions, analyse occupational choices, and finally solve for the aggregate values in equilibrium. The procedure is outlined in more detail in the following.

For the numerical simulation, I first calculate the ability distribution. For the distribution over a, I use an exponentially modified Gaussian distribution with a fat tail. Then, I set up the labour supply distribution by creating gridpoints with growing distances. I initialise vacancy postings, wages, and the value of being employed to their steady state values. For θ, the value equation for an unemployed worker, the decision state, and total tax revenue, I use reasonable starting values. The next step is to derive the firm's optimal policy function. I use the method of endogenous grid points to solve for the optimal vacancy posting choice described in Section 5.1.3. I first iterate over all productivity levels and calculate vacancy postings and labour supply levels in the current period for all labour supply levels in the next period. Since I calculated optimal vacancy postings based on the endogenous gridpoints, I have to stretch it on the original grid by using linear interpolation. Having derived the optimal policy for vacancy posting, I can then calculate the wages that firms with differing employment levels and with entrepreneurs of different abilities pay. Moreover, I can calculate the next period's labour supply and interpolate it to arrive at the employed worker's value function and the outside option. In the next step, I solve for the firms' value functions by determining the functions at the gridpoints

Parameter	Value	
α	0.5	Petrongolo and Pissarides (2001)
β	0.9879	Annual discount factor of 0.95
γ	0.06	
δ	0.002	Shimer (2005)
η	0.7	Short-term labour share
ξ	0.1	Card et al. (2018)
μ_a	0.75	
σ_a^2	0.1	
μ_ϵ	0	
σ_ϵ^2	0.5	
s	0.0982	Shimer (2005)
z	0.1	

Table 3: *Parameter values used for the numerical simulation in Chapter 5*

and iterating backwards over all productivity and labour supply levels until the value function converges. Having computed the firms' value functions, I calculate the entry share of firms according to the occupational choice as described in Section 5.1.4. Then I can solve for the firm distribution by using forward iteration. I initialise the variable that measures the distribution of firms, i.e., the number of firms with a certain a and l, iterate over all productivity and labour supply levels, and subtract all firms that are destroyed with probability δ from the distribution measure. The entrepreneurs and all the workers from these non-surviving firms are added to the variable that measures the number of individuals who are in the decision state. Moreover, the workers who lose their job with probability s also enter the decision state. Then, I have to ensure that, as δ firms are destroyed, only δ firms enter the market. After deriving the firm distribution, I determine the distribution of the unemployed and the employed workers. Next, I calculate aggregate macroeconomic variables such as labour supply, labour demand, aggregate unemployment, the total number of vacancies, aggregate production, and total tax revenue. After that, I solve for the household's value of being unemployed, for which I first have to derive the expectation of the value function for an employed worker. Lastly, the value

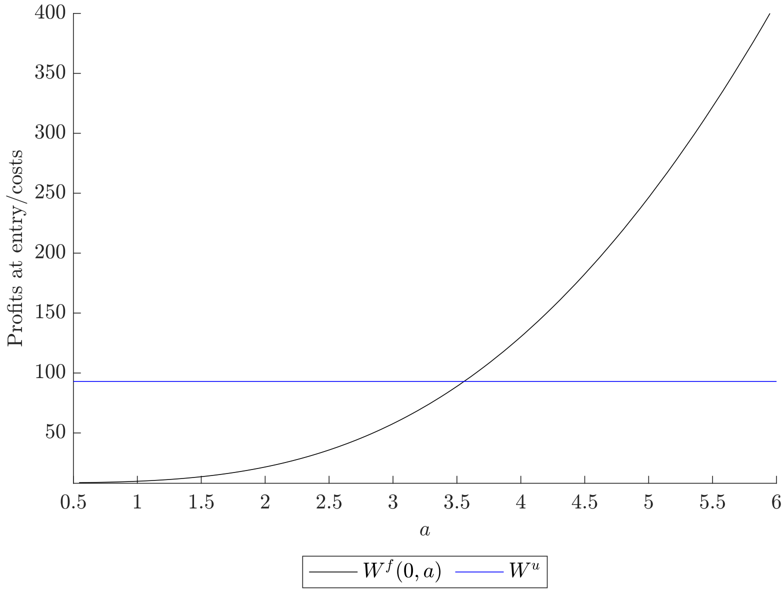

Figure 15: *Occupational choice*

equation for individuals being in the decision state can be calculated. By doing so, I have to consider that the probability for starting a firm is adjusted for the number of individuals in the decision state. Finally, I search for the macroeconomic equilibrium that is pinned down by labour market tightness θ, the value of unemployment W^u, the value equation for the decision state D, and total tax revenue T. For this, I use a rootfinding process.

The following section describes the equilibrium outcomes of the model without correcting taxation, i.e., the laissez-faire market equilibrium, for a realistic parametrisation.

5.2.1 Results

This section presents the results of the numerical simulation. A time period is assumed to be a quarter of the year and the respective parameter values are drawn from the empirical literature as in Section 4.4. Table 3 summarises the parametrisation used in this chapter. I set $\alpha = 0.5$ and $\xi = 0.1$ as in the previous chapters. The time discount factor β is set to 0.9879, which aligns with an annual discount factor of 0.95. The elasticity of output with respect to labour input is $\eta = 0.7$. Shimer (2005) estimates the quarterly probability of losing a job to be 10%. I choose $\delta = 0.002$

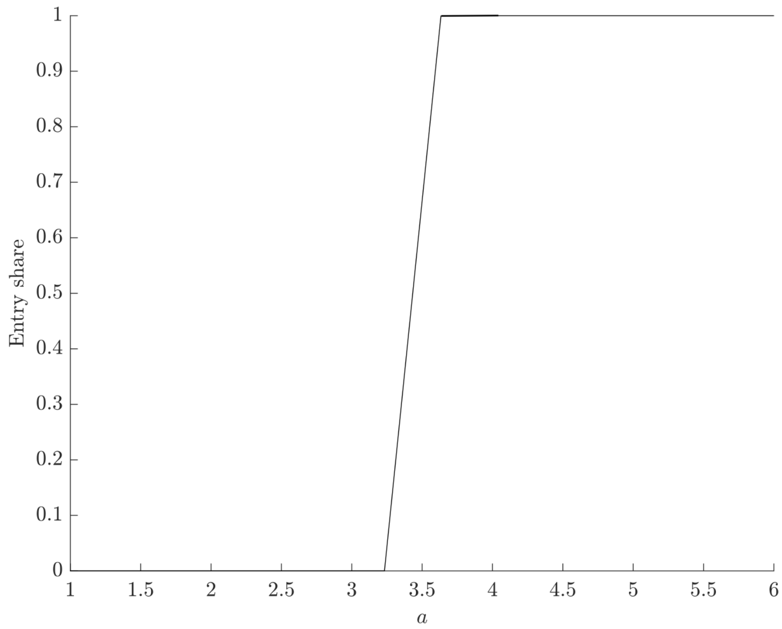

Figure 16: *Entry share*

and, accordingly, set $s = \frac{0.1-\delta}{1-\delta} \approx 0.0982$. Entrepreneurial ability a is log normally distributed with $\mu_a = 0.75$ and $\sigma_a^2 = 0.1$. Entry costs ϵ are assumed to be normally distributed with mean $\mu_\epsilon = 0$ and variance $\sigma_\epsilon^2 = 0.5$. Lastly, home production $z = 0.1$ and vacancy posting costs $\gamma = 0.6$ are set such that the unemployment rate in equilibrium lies at 5.04%. As an outcome, 83.36% of the population are regular workers, whereas 11.59% engage in entrepreneurship. The labour market tightness in equilibrium is given with $\theta = 10.97$, and thus the probability to fill a vacancy is $q(\theta) = 15.09\%$.

Figure 15 illustrates the value functions that an individual faces when she is in the decision state and must determine whether to start a firm or to become a worker. The figure thus depicts the occupational choice each individual in the decision state makes. As an entrepreneur, the individual starts with zero workers and must post vacancies before production can take place resulting in the value equation $W^f(0, a)$. As a worker, she would be unemployed until a match with a firm occurs, therefore her value is given with W^u. The value of being an unemployed worker does not depend on the individual's a, whereas the value of entrepreneurship increases in ability as more able entrepreneurs make higher profits. If ability a is approximately larger than 3.5, it is optimal to become an entrepreneur and start a firm (neglecting

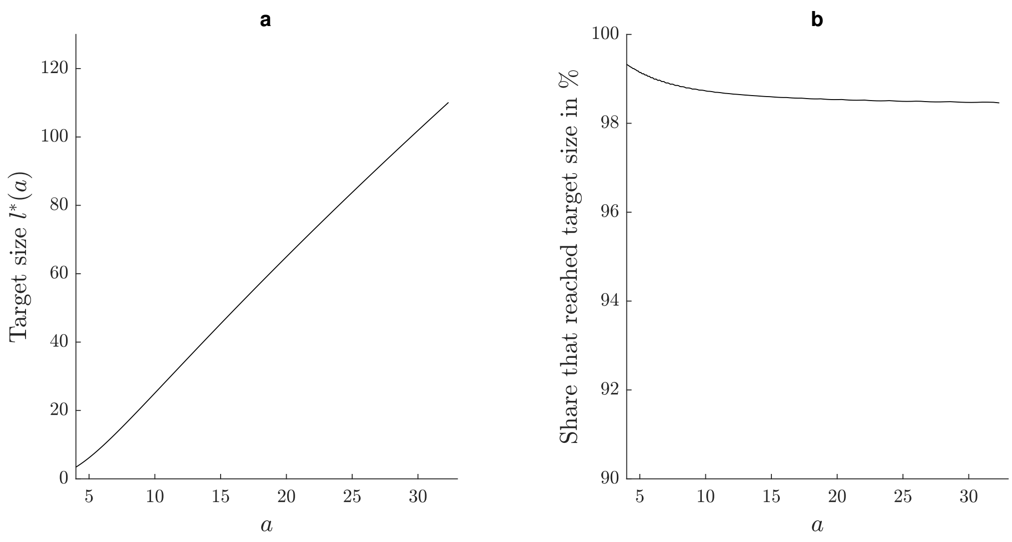

Figure 17: *Target size and share of firms that reached target size*

entry costs ϵ). The entry share, i.e., the share of firms that are newly founded, is depicted in Figure 16. For low values of a, there is no firm entry because being an entrepreneur is not optimal for individuals with a low ability. As a reaches approximately 3.2, a small share of individuals with that ability start a firm. The entry share gradually goes up with a until every individual decides to become an entrepreneur. In contrast to the model in Chapters 3 and 4, the entry share does not directly jump from 0 to 1 at a certain threshold $\bar{a}$. The reason lies in the entry costs ϵ that act as an entry shock. Without heterogeneous entry costs, all individuals above a certain threshold would become entrepreneurs. With entry costs, some individuals, who otherwise would have decided to create a firm, suddenly face very high costs and refrain from being an entrepreneur. After a firm is founded, vacancies are posted and employees are hired. The firm grows over time and eventually reaches its optimal size if it is not destroyed with probability δ before.

The firms' target sizes are depicted in Figure 17a. It can be seen that the target size increases in entrepreneurial ability a. Firms run by an entrepreneur with a very low ability, for example $a = 5$, have an optimal size of approximately six employees, whereas firms with very able entrepreneurs, e.g., with $a = 30$, have a target size of around 110 workers. Because of the hazard rate δ, not all firms in steady state have reached their target size. As firms die with a certain probability and new firms enter, there are always firms with different ages and sizes, which are at different

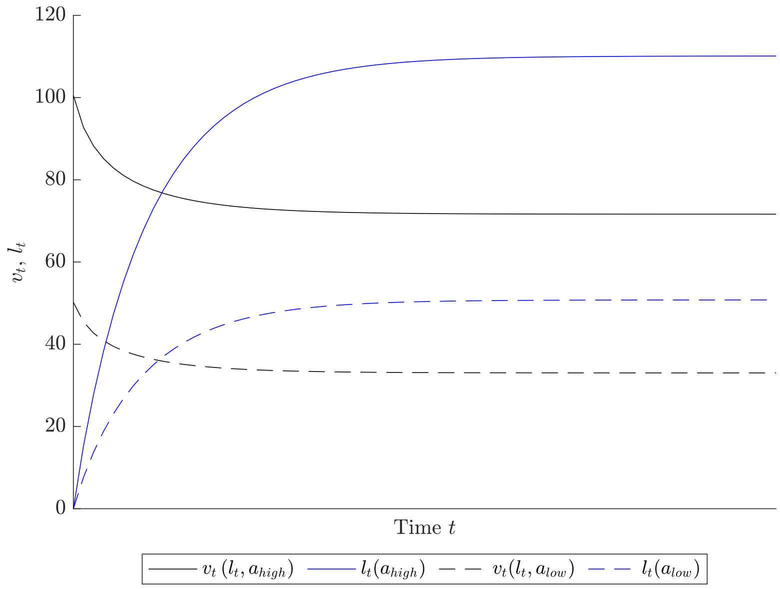

Figure 18: *Time paths for v_t and l_t*

stages of their growth paths. Figure 17b shows the percentage of firms that have reached their optimal size for differing entrepreneurial abilities a. More firms with a less able entrepreneur have reached their target size compared to firms of more able entrepreneurs. When inspecting the entrepreneur with the lowest ability, 99.33% of all firms with such an entrepreneur have reached their optimal size. Of all firms founded by an entrepreneur with the highest ability, 98.46% have reached their target size. As firms with an entrepreneur with a higher ability are much larger, it is more difficult for these firms to reach the optimal size.

The policy functions for firms with entrepreneurs which have different talents a are shown in Figure 18. The solid lines illustrate the time paths for vacancies posted and the number of hired employees for an entrepreneur with the highest ability. The dashed lines depict the time paths for a firm run by an entrepreneur with a lower ability.[5] When an entrepreneur opens up a firm, she starts with zero employees, posts many vacancies, and starts to hire workers. The number of advertised vacancies gradually decreases over time until the target size is reached. Then, the number of posted vacancies stays constant and is set such that only the workers who lose their job are replaced. The number of hired employees increases over time

[5]The time paths depicted in Figure 18 are of course derived under the assumption that the firms are not destroyed during the shown time periods.

	0-5%	5-10%	90-95%%	99-100%
Model	0.0	3.4	12.7	19.5
Data	0.30	0.60	16.7	19.2

Table 4: *Shares of total income at lower and upper end of the income distribution*

and the rise is largest at the beginning, because firms post the most vacancies in the first periods of their existence. As the number of posted vacancies shrinks, the increase in the number of hired workers becomes smaller. The number of employees stays constant once the target size is reached. One can see that firms with more able entrepreneurs post more vacancies, hire more workers, and grow faster at the beginning.

Table 4 compares the lower and upper end of the income distribution generated by the model to US data from the Survey of Consumer Finances for 2019. It reports the shares of total income for the bottom and the top 10% of the income distribution. The model is able to replicate the observed shares quite well at the top and the bottom of the distribution. Whereas the top 1% of income earners receive 19.2% of total income in the US in 2019, the top earners in the model, who are highly able entrepreneurs, get 19.5% of total income. The data show that individuals in the lowest decile earn 0.90% of total income. The share generated by the model is a bit too large with 3.4%. The unemployed individuals in the model engage in home production, which is their only source of income as there are no unemployment benefits considered in the model. Accordingly, there is no variation in the income of the non-working population at the lower end of the distribution.

5.3 Taxation

I assume that an employed worker's income is taxed with marginal tax rate τ^e and that of an entrepreneur with marginal tax rate τ^f. The tax rates are assumed to be constant over time. Since an unemployed worker engages only in home production, she does not have to pay any taxes. Vacancy posting in period t must be taxed with the marginal tax rate τ_t^v to correct for the strategic overhiring of firms as described in Chapter 4. The tax revenue in period t is redistributed via a lump sum transfer

T_t that every individual receives. The Bellman equation for an employed worker thus becomes

$$W_t^e(l_t, a) = (1 - \tau^e)w_t(l_t, a) + T_t + \beta(\delta + (1 - \delta)s)D_{t+1} \\ + \beta(1 - \delta)(1 - s)W_{t+1}^e(l_{t+1}, a).$$

For the unemployed worker, it is given with

$$W_t^u = z + T_t + \beta\left[1 - \theta_t q(\theta_t)\right] W_{t+1}^u + \beta\theta_t q(\theta_t)\mathbf{E}_t\left[W_{t+1}^e(l_{t+1}, a)\right].$$

Compared to the model without taxation, the employed worker pays the marginal tax rate τ^e on her wage income and receives the lump sum transfer T_t in each period. The unemployed worker engages in home production and also obtains the transfer. The entrepreneur can deduct labour and vacancy posting costs from taxable income and receives the lump sum transfer. The firm's maximisation problem therefore becomes

$$J_t(l_t, a) = \max_{l_{t+1}, v_t \geq 0} \Big\{(1 - \tau^f)\left[af(l_t) - w_t(l_t, a)l_t - (1 + \tau_t^v)c_t(v_t)\right] + T_t \\ + \beta(1 - \delta)J_{t+1}(l_{t+1}, a) + \beta\delta D_{t+1}\Big\}$$

$$\text{s.t. } l_{t+1} = (1 - s)l_t + q(\theta_t)v_t.$$

In the following, the explicit functional forms of the production function and the vacancy posting costs from Section 5.1.3 are used. Setting up the Lagrangian function to compute the first order conditions and using the envelope theorem, the job creation condition can be derived as[6]

$$\frac{(1 + \tau_t^v)\gamma v_t}{q(\theta_t)} = \beta(1 - \delta)\left[\eta a l_{t+1}^{\eta - 1} - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1} + (1 - s)\frac{(1 + \tau_{t+1}^v)\gamma v_{t+1}}{q(\theta_{t+1})}\right].$$

The marginal tax rate on entrepreneurial income does not appear in the above condition because, if this tax rate is constant over time, it cancels out. As before, the wage

[6]For the derivation of the job creation condition and the wage curve, see Appendix A.3.3.

is determined via Nash wage bargaining and maximises the following expression:

$$w_t(l_t, a) = \arg\max \left\{ [W_t^e(l_t, a) - D_t]^{\xi} \left[\frac{\partial J_t(l_t, a)}{\partial l_t} \right]^{1-\xi} \right\}.$$

The first order condition from wage bargaining is

$$\xi(1-\tau^e) \left\{ (1-\tau^f) \left[\eta a l_t^{\eta-1} - w_t - \frac{\partial w_t}{\partial l_t} l_t \right] + (1-s) \frac{(1-\tau^f)(1+\tau_t^v)\gamma v_t}{q(\theta_t)} \right\}$$
$$= (1-\xi)(1-\tau^f) \left[(1-\tau^e) w_t - (1-\tau^e) X(l_{t+1}, a) \right],$$

which can be solved for the wage:

$$w_t(l_t, a) = \xi \left[\frac{\eta a l_t^{\eta-1}}{1-\xi(1-\eta)} + (1-s) \frac{(1+\tau_t^v)\gamma v_t}{q(\theta_t)} \right] + (1-\xi) X(l_{t+1}, a).$$

The outside option $X(l_{t+1}, a)$ is defined as

$$X(l_{t+1}, a) = \frac{D_t - \beta [\delta + (1-\delta)s] D_{t+1} - \beta(1-\delta)(1-s) W_{t+1}^e(l_{t+1}, a) - T_t}{(1-\tau^e)}.$$

Compared to the wage equation from the former section, where taxation was not considered, the wage now increases c.p. in τ^e. For a given level of employment and a given ability, the entrepreneur pays a higher wage since the taxation of labour income makes employment less attractive relative to being in the decision state. Moreover, the wage depends positively on τ^v. If vacancy posting is taxed, the firm saves higher vacancy posting costs once a worker is hired. Therefore, the wage increases because the worker is rewarded for the larger costs saved. Next, I plug the wage into the job creation condition and solve for v_t:

$$v_t(l_t, a) = \frac{\beta(1-\delta)(1-\xi) q(\theta_t)}{(1+\tau_t^v)\gamma} \left[\frac{\eta a l_{t+1}^{\eta-1}}{1-\xi(1-\eta)} - X(l_{t+2}, a) + (1-s) \frac{(1+\tau_{t+1}^v)\gamma v_{t+1}}{q(\theta_{t+1})} \right].$$

The number of posted vacancies in period t declines with the tax on vacancy posting τ_t^v but rises with the next period's tax rate τ_{t+1}^v. If it becomes more costly to post vacancies in the next period, entrepreneurs shift vacancy postings to the current period. The next section describes the steady state equilibrium with taxation, which is then numerically simulated in the subsequent section.

5.3.1 Steady State Equilibrium

As already pointed out in Section 5.1.6, the firm distribution in the steady state equilibrium must be stationary and θ is constant over time. It is assumed that all three marginal tax rates do not change over time either. The value equations for the individuals in one of the four possible states are written down in the following. An unemployed worker receives

$$W^u = z + T + \beta\left[1 - \theta q(\theta) W^u\right] + \beta\theta q(\theta)\mathbf{E}\left[W^e\right],$$

whereas an employed worker obtains

$$W^e(l,a) = (1-\tau^e)w + T + \beta\left[\delta + (1-\delta)s\right]D + \beta(1-\delta)(1-s)W^e(l^+,a).$$

The value equation for an entrepreneur in steady state is given with

$$J(l,a) = (1-\tau^f)\left[al^\eta - w(l,a)l - (1+\tau^v)0.5\gamma v^2\right] + T + \\ \beta(1-\delta)J(l^+,a) + \beta\delta D.$$

Lastly, if an individual is in the decision state, her value equation is

$$D = \int_0^\infty \left[P^{adj}(a)J(0,a) + (1-P^{adj}(a))W^u\right] d\Phi(a).$$

A firm run by an entrepreneur with talent a and l workers pays the wage

$$w(l,a) = \xi\left[\frac{\eta a l^{\eta-1}}{1-\xi(1-\eta)} + (1-s)(1+\tau^v)\frac{\gamma v(l,a)}{q(\theta)}\right] + (1-\xi)X(l^+,a)$$

with the outside option

$$X(l,a) = \frac{\left\{1-\beta\left[\delta+(1-\delta)s\right]\right\}D - \beta(1-\delta)(1-s)W^e(l,a) - T}{1-\tau^e}.$$

The number of vacancies the firm posts is given with

$$v(l,a) = \beta(1-\delta)(1-\xi)\left\{\frac{q(\theta)}{(1+\tau^v)\gamma}\left[\frac{\eta a(l^+)^{\eta-1}}{1-\xi(1-\eta)} - X(l^{++},a)\right] + (1-s)v(l^+,a)\right\}.$$

For a given labour market tightness, the number of vacancies decreases in τ^v. As posting vacancies becomes more costly, firms post fewer ones. The equation that determines the target size $l^*(a)$ in steady state is the following:

$$\frac{sl^*}{q(\theta)} = \frac{\beta(1-\delta)(1-\xi)q(\theta)}{(1+\tau^v)\gamma\left[1-\beta(1-\delta)(1-\xi)(1-s)\right]}\left\{\frac{\eta a l^{*\eta-1}}{1-\xi(1-\eta)}\right.$$
$$-\frac{(1-\beta)}{\left[1-\beta(1-\delta)(1-s)\right](1-\tau^e)}D+\frac{\beta(1-\delta)(1-s)}{1-\beta(1-\delta)(1-s)}w(l^*,a)$$
$$\left.+\frac{T}{\left[1-\beta(1-\delta)(1-s)\right](1-\tau^e)}\right\}.$$

The firms' target sizes are altered by the introduction of taxation. A positive tax on vacancy posting, for example, has a direct negative impact on the target sizes taking wages and the labour market tightness as given. As vacancy posting becomes more costly, firms post fewer vacancies in each period, hire fewer workers, and their optimal size is reduced. As before, the steady state equilibrium with taxation features firms having reached their target size already and younger firms, which still grow over time until they reach their target size, conditional on not being destroyed before.

Next, it must be determined whether taxation or subsidisation of entrepreneurial incomes and vacancy posting is welfare-enhancing. The effects of taxing vacancy posting are ambiguous in this dynamic context with growing and dying firms. Taxing vacancy posting counteracts the overhiring behaviour of firms, which leads to a relaxation of the labour market, but makes it more costly for young firms to hire employees, slows down their growth, and reduces their size. Moreover, in a world where firms can die, it is per se more difficult to reach the optimal firm size. The taxation of entrepreneurial incomes might also have negative effects. If incomes from entrepreneurship are taxed, rent-extracting firms with low productivities are forced out of the market, but, on the downside, taxing entrepreneurial incomes might also lead to a too small number of entrepreneurs in equilibrium. Since growing to the optimal size is already difficult, which makes becoming an entrepreneur less attractive, taxing entrepreneurial incomes further reduces the attractiveness of starting a business. Whether it is welfare enhancing to tax or to subsidise vacancy posting and entrepreneurial incomes is identified with the help of a numerical simulation of the model. A numerical solution is needed since it is difficult to analytically derive the welfare implications of taxation, which also effects the labour market tightness. The next section builds on the description of the steady state equilibrium and analyses the results from the numerical simulation of the model with taxation to answer the questions raised above.

5.3.2 Results of the Numerical Simulation

For the numerical simulation, all parameter values are set as put down in Table 3, except for δ and s.[7] The marginal tax rate on labour income τ^e is assumed to be zero for simplicity. Later on, when I outline the outcomes under correcting taxation, the tax rates on entrepreneurial incomes and vacancy posting are derived by searching for the tax rates that maximise total welfare, measured as the sum of total entrepreneurial income, total labour income, and aggregate home production.[8]

First, I describe the equilibrium outcomes if $\delta = 0$, meaning that firms are not destroyed. Thus, in steady state, all firms will have reached their target size because there is no firm destruction and firms can grow over time until they reach their optimal size. This version of the dynamic model relates closely to the theoretical model dealt with in Chapter 4, which features infinitely living firms and a clear cut-off entrepreneurial ability that divides individuals into workers and entrepreneurs. In the first period, every individual draws an a, faces the occupational choice, and eventually becomes an entrepreneur if a is large enough. Once the individual starts the firm, she will stay an entrepreneur forever. Firms post vacancies and hire workers until they reach their target size. In steady state, all firms have reached this size and post vacancies to replace the workers who have lost their job. It is assumed that these workers, who enter the decision state, are only allowed to become workers again. This steady state assumption is needed because no entrepreneur enters the decision state, and thus no individual is allowed to become an entrepreneur. The number of firms and workers in steady state is constant. Therefore, if $\delta = 0$, the model described in this chapter also features a threshold ability that divides individuals into regular workers and entrepreneurs. Moreover, in steady state, the value

[7] In the following, I set $\delta = 0$ or $\delta = 0.002$ and $s = \frac{0.1-\delta}{1-\delta}$.

[8] The equations used for the numerical simulation can be found in Appendix A.3.3.

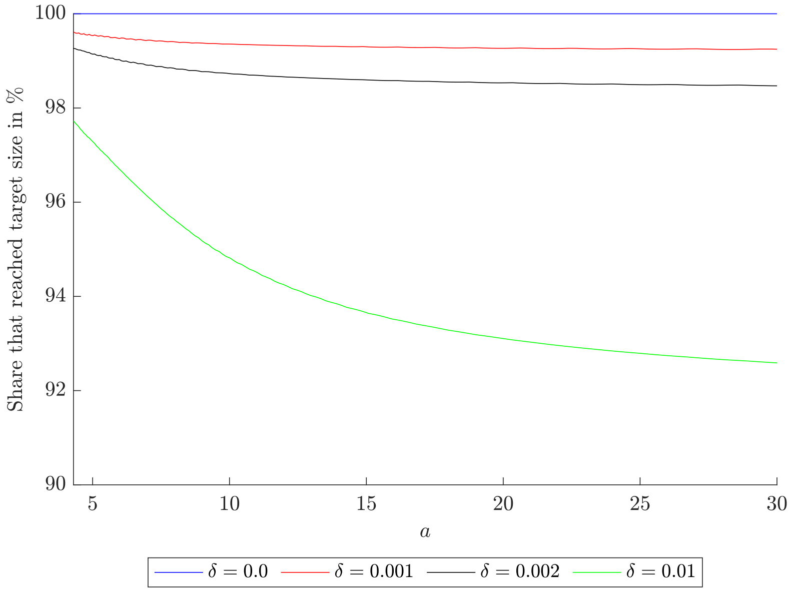

Figure 19: *Share at target size for different* δ

equation for an individual who is in the decision state is equal to the unemployed worker's value equation. Workers lose their job with probability s and, accordingly, enter the decision state. They are only allowed to become a regular worker and hence are unemployed until a match with a firm occurs. Entrepreneurs cannot enter the decision state since they cannot lose their occupation. Hence, they stay an entrepreneur forever and keep their firm. Thus, all individuals who enter the decision state in steady state are workers and decide to become workers again. They become employed in the next period with probability $\theta q(\theta)$ and stay unemployed with the counter-probability. Therefore, the value equation for an individual in the decision state is equal to that of an unemployed worker, i.e., $D = W^u$.

Without taxation and $\delta = 0$, unemployment in equilibrium is equal to 3.27% and 15.21% of the individuals are entrepreneurs. The probability to fill a vacancy is given with $q(\theta) = 10.04\%$. As all other parameters, except δ and s, are set as in Section 5.2, $\delta = 0$ leads to a lower unemployment rate and more people being an entrepreneur. If firms are not destroyed, individuals can be sure that once they started a firm, the firm will always exist and they can acquire high profits forever. This makes becoming an entrepreneur more attractive compared to the case with a positive δ. In that case, entrepreneurs eventually enter the decision state, draw a new a, and potentially end up being unemployed for some time. Unemployment is lower compared to the case with $\delta = 0.002$ because I hold the worker's probability to lose the job constant at 10%.[9] With a positive probability that the whole firm dies and the entrepreneur and all workers at the firm lose their current occupation, unemployment is of course higher compared to the case where only single workers lose their jobs with probability s. Without the positive hazard rate, all firms indeed have reached their target size, as shown in Figure 19. The blue line depicts the share of firms that reached their optimal size for the case that $\delta = 0$. For positive values of δ instead, not all firms reach the target size in steady state as some of them die before. As δ increases, it becomes more difficult for firms to grow towards their optimal size.

The welfare-maximising tax rates in the economy with $\delta = 0$ lie at $\tau^v = 257\%$ and $\tau^f = 81\%$. With these tax rates, unemployment rises to 7.34% and the number of entrepreneurs decreases, so that 6.41% of all individuals engage in entrepreneurship. The probability to fill a vacancy thus increases to 21.28% because fewer entrepreneurs post fewer vacancies and there are more unemployed workers available. As I receive positive tax rates on vacancy posting and entrepreneurial incomes, the decentralised market equilibrium without correcting taxation features a too large number of entrepreneurs who engage in overhiring, which makes the labour market inefficiently tight and makes it difficult for firms to fill their vacancies. By introducing a positive marginal tax rate on entrepreneurial incomes, being an entrepreneur becomes less attractive, which induces more individuals to become workers. This relaxes the labour market as there are fewer entrepreneurs who compete for more available workers. Thus, the simulated model with quadratic vacancy posting costs and without firm destruction delivers similar results to the model from Chapter 4. Figure 20 shows that the introduction of the correcting tax rates moves the threshold ability upwards. In the laissez-faire market equilibrium, all individuals with an a approximately greater than 3.2 are entrepreneurs. The implementation of τ^v and τ^f raises the threshold to $\bar{a} = 4.25$. Now, all individuals with an ability greater than 4.25 have decided to open up a firm.

[9] For the case that $\delta = 0$, s is set to 0.1 as in Sections 3.5 and 4.4. If $\delta = 0.002$, s is calculated such that the worker's probability to lose the job is still 10%, so $\delta + (1 - \delta)s = 0.1$.

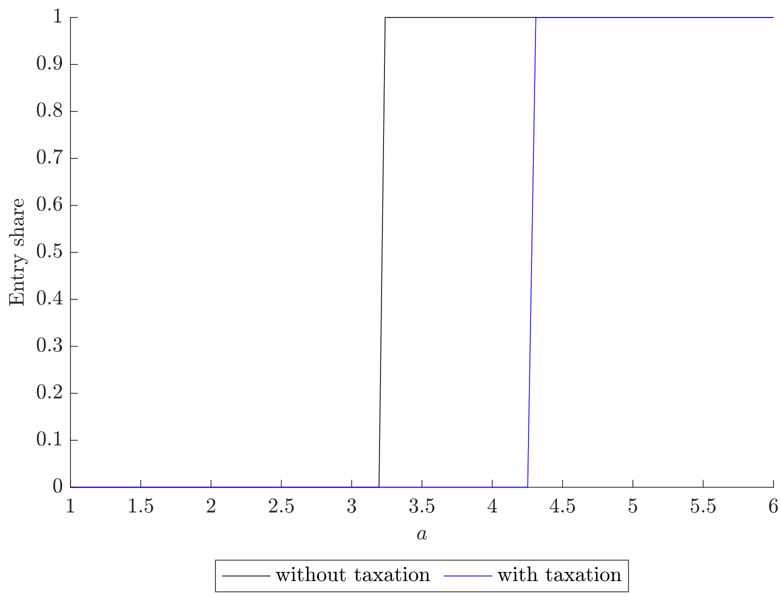

Figure 20: *Threshold with and without correcting taxation*

Considering the model with $\delta = 0.002$ described in Section 5.2.1 and introducing taxation, the welfare-maximising tax rates will be quite different to those in the model without firm destruction. It gets more difficult for firms to reach their optimal size, as can be seen in Figure 19, because there is the probability that the firm is destroyed on its path of growth towards the target size. By introducing firm destruction, I establish an additional inefficiency as there are newly entered firms, which have not reached their target size, and thus operate below their optimal marginal product of labour. Since I assume that the probability of firm death is independent of a, the firms of very able entrepreneurs might get destroyed quite soon and cannot grow to large firms with high productivities.[10] A tax on vacancy posting therefore might have detrimental effects on welfare as it makes it even more difficult for young firms with a highly productive entrepreneur to reach the target size. The welfare-maximising tax rates in the model with $\delta = 0.002$, and other parameters set as shown in Table 3, are given with $\tau^v = -42\%$ and $\tau^f = -152\%$. Indeed, both vacancy posting and entrepreneurship have to be subsidised for welfare maximisation. By correcting for inefficiencies, the unemployment rate increases

[10] There is some empirical evidence that the hazard rate for young firms is larger than for old firms, for example, Phillips and Kirchhoff (1989), Ejermo and Xiao (2014), and Calvino et al. (2022).

	Laissez-faire	With τ^v	With τ^v, τ^f
Unemployment rate	5.04%	4.73%	7.87%
Share of entrepreneurs	11.59%	10.30%	13.83%
Aggregate vacancies	0.5534	0.6130	0.3127
$q(\theta)$	15.09%	13.89%	25.09%

Table 5: *Comparison of labour market outcomes*

from 5.04% to 7.87% and more individuals, 13.83% compared to 11.59%, decide to become entrepreneurs. The probability to fill a vacancy advances to 25.09%. This indicates that the economy without correcting taxation described in Section 5.2.1 features an inefficiently low number of firms that inefficiently post vacancies. As a result, the labour market is too tight. The aggregate number of vacancies in the equilibrium with taxation is lower than in the laissez-faire equilibrium, as can be seen in Table 5.[11]

To understand the operating mechanisms, I first describe the outcomes if only vacancy posting is subsidised with $\tau^v = -42\%$, and τ^f is assumed to be zero. Compared to the laissez-faire equilibrium, unemployment declines to 4.73%, the number of entrepreneurs is lower with 10.30%, the number of aggregate vacancies increases, and the probability to fill a vacancy falls to 13.89%, as summarised in Table 5. If vacancy posting is subsidised, firms post more vacancies and intensify their overhiring behaviour. This leads to a more tight labour market and makes it less attractive to become an entrepreneur because hiring employees gets more difficult. Therefore, many individuals decide against starting a firm. The firms that are in the market are hence owned, on average, by entrepreneurs with higher abilities. Figure B.1 in Appendix B shows that indeed not so many more firms reach their target size if only vacancy posting is subsidised because the labour market becomes inefficiently tight for the entrepreneurs. Unemployment is lower compared to the laissez-faire since there are many vacant positions and the probability for the worker to be matched with a firm rises. Compared to the case without taxation, welfare is lower if only vacancy posting is subsidised.[12] Even though aggregate production is slightly larger,

[11]For a comparison of several other aggregate measures between the laissez-faire equilibrium and the efficient equilibrium, also see Table B.1 in Appendix B.

[12]See again Table B.1 in Appendix B.

because firms are on average more productive, aggregate vacancy posting costs are higher since the labour market is too tight, which decreases welfare. A very tight labour market with small probabilities for the firms to fill their vacancies makes being an entrepreneur less attractive. As too many firms opt out, a subsidy on entrepreneurial incomes is needed to increase the attractiveness of starting a firm. If $\tau^f = -152\%$ is introduced additionally, the number of entrepreneurs, unemployment, and the probability to fill a vacancy rise, whereas the number of aggregate vacancies declines compared to the situation where only vacancy posting is subsidised. The subsidy on entrepreneurial incomes induces more individuals to become an entrepreneur and dampens the overhiring behaviour of firms. Entrepreneurs post fewer vacancies in aggregate, which relaxes the labour market tightness and decreases overall vacancy posting costs. Nevertheless, compared to the laissez-faire, highly able entrepreneurs individually post more vacancies because of the subsidy on vacancy posting. To sum up, the subsidy on vacancy posting ensures that firms post more vacancies to counteract the negative effects of potential firm destruction. Only subsidising vacancy posting would lead to a too tight labour market with too few firms. Consequently, entrepreneurial incomes have to be subsidised as well.

Next, I compare the economy with both correcting subsidies to the laissez-faire outcomes to further explain the implications and mechanisms of the welfare-maximising subsidies. Figure 21 compares the occupational choice with correcting tax rates to that in the laissez-faire equilibrium. The dashed lines, depicting the value equation for an unemployed worker and an entrepreneur without correcting taxation, represent therefore the same value equations as in Figure 15. The solid lines show the value equations with correcting subsidisation. W^u is smaller and $W^f(0, a)$ takes on lower, even negative, values for low values of a but higher values for high values of a. The unemployed worker's value equation decreases as unemployed individuals have to pay a lump-sum transfer to finance the subsidies on entrepreneurial incomes and vacancy posting. The negative lump-sum transfer does of course not affect the occupational choice because unemployed workers and entrepreneurs both pay it. Only the tax rate on entrepreneurial incomes alters the choice between the two occupations. The intersection point of the two solid curves pins down the threshold ability above which it is optimal to become an entrepreneur. The intersection

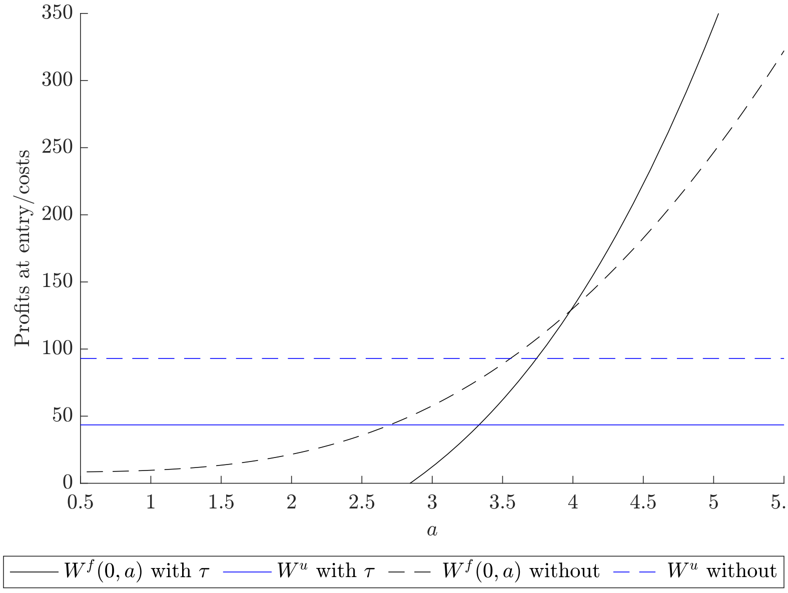

Figure 21: *Occupational choice with correcting taxation*

point with correcting taxation lies on the left-hand side of the intersection point of the dashed curves, which proves that the subsidy on entrepreneurial profits indeed makes it attractive for more individuals to start a firm. Entrepreneurs with a higher entrepreneurial ability (a approximately larger than 3.9) profit more from the subsidies as their value of being an entrepreneur becomes larger than in the laissez-faire market equilibrium.

Figure 22 shows the target size and the share of firms that reached this size in the steady state with and without correcting taxation. The blue line in Panel a indicates that the target size $l^*(a)$ becomes larger for most firms if I introduce the welfare-maximising subsidies on vacancy posting and entrepreneurial incomes. An exception are firms owned by entrepreneurs with low abilities. Since vacancy posting is subsidised and the labour market tightness decreases, it becomes less costly for more able firms to post vacancies. Thus, their optimal size surges. For less able entrepreneurs, the target size slightly declines. Panel b demonstrates that more firms reach their target size with both correcting tax rates. Without taxation, a single firm posts too few vacancies, grows slower, and therefore more firms are destroyed before reaching the optimal firm size.

For a firm with a certain entrepreneurial ability (here $a = 16.36$), the time paths for

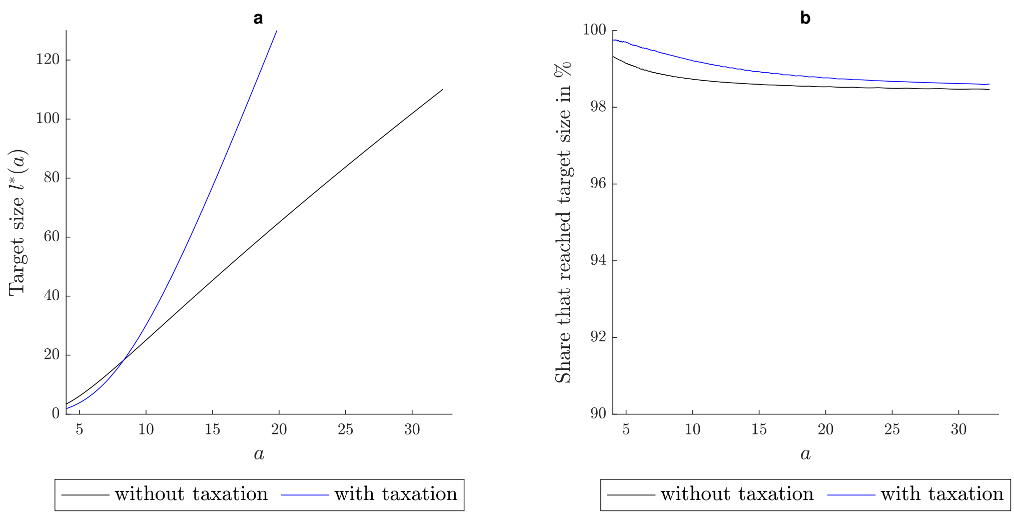

Figure 22: *Target size and share that reached target size with correcting taxation*

vacancies and hired workers are displayed in Figure 23. The solid lines depict the policy functions for the firm if there is welfare-maximising subsidisation, whereas the dashed lines show v_t and l_t without it. One can see that the firm posts only slightly more vacancies because vacancy posting is subsidised, but the number of hired workers soars. This is caused by the less tight labour market and the resulting higher probability of filling a vacant position. Moreover, the firm grows faster at the beginning of its life-cycle compared to the laissez-faire economy. If a goes up, the increase in vacancy postings generated by the subsidisation becomes larger. This means that the correcting subsidies initiate more able firms to post more vacancies, whereas less able entrepreneurs post fewer vacancies compared to the equilibrium without correcting tax rates.[13]

Overall, the labour market in the laissez-faire equilibrium is too tight. Less able entrepreneurs post too many vacancies and more able entrepreneurs post too few. Consequently, the aggregate number of vacancies is smaller in the equilibrium with correcting subsidisation. In total, there are more firms and the more able firms post more vacancies, whereas less able entrepreneurs post fewer vacancies. By subsidising entrepreneurship and vacancy posting, vacancy creation and the allocation of workers across firms becomes more efficient. More able entrepreneurs, who have more productive firms, hire more employees at lower costs. The workers hired by these

[13]For the time paths for an entrepreneur with low ability ($a = 4.8$), see Figure B.2 in Appendix B.

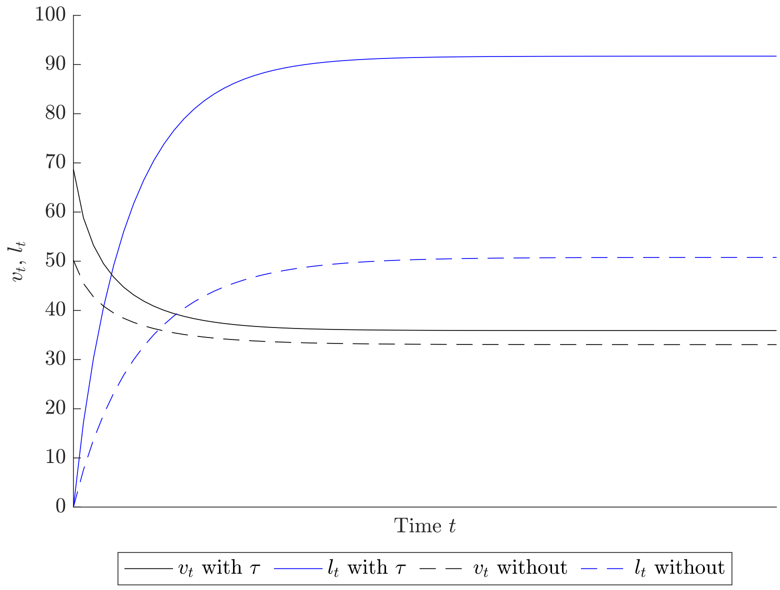

Figure 23: *Policy functions with correcting taxation*

firms contribute more to overall production than they would have contributed in a firm with a less able entrepreneur since total factor productivity at this firm is quite low. Compared to the laissez-faire equilibrium, average productivity increases. The subsidy on vacancy posting hence aims at inducing high able entrepreneurs to post more vacancies and helps them to reach their target size. It therefore corrects for the negative effects that firm destruction exerts on able entrepreneurs who otherwise, without the subsidy, would post too few vacancies as the firm can be destroyed in the next period and they would not be able to profit from the increased firm size. As a result, the subsidy on vacancy posting makes more able firms post more vacancies and become bigger. Less able firms hire fewer workers and become smaller. Total production in the economy increases and a worker is on average employed at a firm with higher productivity. More firms have reached their target size. As the wage that firms pay decreases on their path of growth towards the target size, being an employed worker becomes less attractive relative to being an unemployed worker, who receives fixed home production z.[14] Therefore, unemployment increases, which relaxes the labour market tightness. This in turn makes it easier for firms to fill

[14] Here, one could extend the model with z being dependent on average productivity and wages in the economy. This then would correct for the shift in the relative attractiveness between being employed or unemployed.

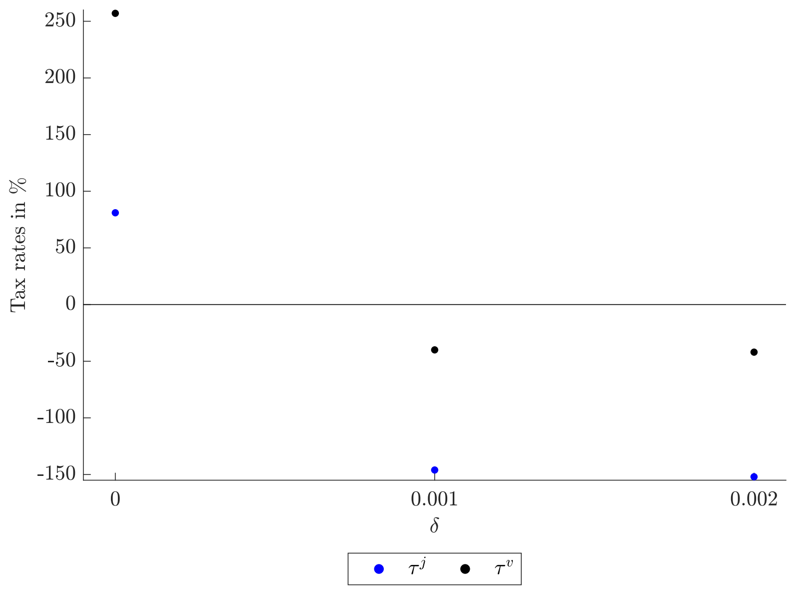

Figure 24: *Welfare maximising tax rates for different values of δ*

their vacancies. If the labour market is less tight, firms have to post fewer vacancies because these are filled with a higher probability. As a result, the number of aggregated vacancies and posting costs are lower than in the laissez-faire.

5.4 Conclusion

To conclude this chapter, Figure 24 shows the welfare-maximising tax rates τ^v and τ^f for different values of δ. If $\delta = 0$, entrepreneurial incomes and vacancy posting are taxed because otherwise too many individuals would decide to become entrepreneurs and would engage in overhiring. The model without firm death described in this chapter therefore replicates the findings from the theoretical model analysed in Chapter 4. In the laissez-faire equilibrium, too many individuals decide to start a firm because they can acquire a too large share of the surplus of a match and they hire too many workers to suppress the wages. Using a realistic parametrisation, these findings are confirmed. If the model is extended with exogenous firm destruction, it is optimal to subsidise vacancy posting and being an entrepreneur. As firm destruction makes it more difficult for firms to reach their target size and lowers the number of vacancies that entrepreneurs advertise, vacancy posting must be subsidised. A subsidy on vacancy posting leads to increased overhiring behaviour of firms, raises labour market tightness, and makes being an entrepreneur less attractive. Therefore, entrepreneurial incomes have to be subsidised as well to improve the attractiveness of opening up a firm. Moreover, the probability of firm death encourages individuals from being an entrepreneur per se, because owning a firm becomes more risky. The welfare-maximising policy thus consists of subsidising entrepreneurial incomes and vacancy posting. This induces more able entrepreneurs to post more vacancies compared to the laissez-faire, whereas entrepreneurs with lower abilities post fewer vacancies. The result is a less tight labour market with more productive firms since more workers work for highly productive firms. Figure 24 also depicts the tax rates on entrepreneurial incomes and vacancy posting for a δ lying between the values discussed in this chapter. For $\delta = 0.001$, the welfare-maximising subsidies are $\tau^v = -40\%$ and $\tau^f = -146\%$ and thus very close to those when δ is a bit larger. The large differences in the tax rates between the models with and without firm destruction mainly stem from the fact that it gets more difficult for firms to reach their optimal size. Therefore, a subsidy that aims at able entrepreneurs to increase vacancy posting is welfare improving. At the same time, entrepreneurial incomes have to be subsidised as well to raise the attractiveness of being an entrepreneur. Otherwise, firms would engage massively in overhiring and too many individuals would decide against starting a business.

Chapter 6

Conclusion

The literature on the taxation of entrepreneurial income so far has revealed that entrepreneurial activity ought to be subsidised since entrepreneurs are important for economic growth. They have innovative ideas and, if their firms are successful, create jobs. Moreover, for creative destruction to happen, it is important that individuals with the best ideas and highest entrepreneurial talent can start their own businesses. Taxation of entrepreneurial incomes might dampen the incentives to become an entrepreneur and might distort decisions such as where to locate a firm or how many jobs to create. In the innovation literature summarised in Chapter 2, Schumpeterian growth models are used to analyse the top income distribution. These models feature entrepreneurs who are monopolists, and thus are able to gain high profits, but do not consider endogenous occupational choices where individuals have to decide between becoming an entrepreneur or a regular worker. The literature on entrepreneurship instead focuses on the decision to become an entrepreneur but assumes that labour markets are perfectly competitive in the sense that wages equal marginal products. The aim of this dissertation is combining an imperfect labour market with endogenous occupational choices. This dissertation merges important features of the two literature strands mentioned above by introducing occupational choice into the DMP model. Coming back to this dissertation's research question whether and how entrepreneurial incomes ought to be taxed if heterogeneity in entrepreneurial ability and endogenous occupational choices are considered, the theoretical models described in Chapters 3 and 4 deliver an efficiency reason for the taxation of entrepreneurial incomes. Chapter 3 even finds an argument for progressive taxation. Taxing these incomes decreases the attractiveness of becoming an entrepreneur for individuals with low entrepreneurial abilities. In the absence of taxation, these individuals would otherwise decide to become an entrepreneur because they can acquire a too large share of the surplus of a match. Therefore, this dissertation produces an argument in favour of the taxation of entrepreneurial incomes, even without taking equity concerns into account. In a model with convex vacancy posting costs and firm destruction, the result is altered and entrepreneurial incomes must be subsidised, which is in line with much of the literature.

Chapter 3 lines out the basic theoretical model, which extends the DMP model with individuals with heterogeneous entrepreneurial abilities who face an endogenous occupational choice. They can decide to either become a regular worker or an entrepreneur. In equilibrium, there will be a threshold entrepreneurial ability. Individuals with a talent below the threshold are workers and individuals with an ability above are entrepreneurs. I demonstrate that the decentralised market equilibrium features too many entrepreneurs compared to the first-best allocation. Because of wage bargaining, entrepreneurs are able to acquire an inefficiently large part of the surplus of a match, and thus entrepreneurship is too attractive. By introducing taxation of entrepreneurial incomes, the first-best can be restored. Moreover, when looking at the marginal tax rates on entrepreneurial profits and wage incomes simultaneously, evidence on progressive taxation can be found, meaning that entrepreneurial income is taxed more heavily than labour income.

In Chapter 4, intrafirm wage bargaining is added to the theoretical model. For entrepreneurs, it is thus optimal to hire too many workers to depress the wages paid to them. Hence, taxation of vacancy posting is needed to correct for the negative externality caused by the overhiring behaviour of firms. The comparison of the decentralised market equilibrium to the first-best outcome delivers ambiguous results, at least theoretically. Whether there are too many or too few entrepreneurs in the market equilibrium depends on the workers' wage bargaining power ξ and on match efficiency α. As in the former model, entrepreneurs can acquire a too large share of the surplus of a match if the workers' wage bargaining power is low. Additionally, firms hire too many worker to decrease wages, which intensifies the labour market tightness. The overhiring behaviour lowers the attractiveness of entrepreneurship and hence outweighs, at least partly, the positive effect of rent-seeking on the relative attractiveness of being an entrepreneur compared to being a worker. Which effect prevails depends on the size of α and ξ. The wage bargaining power and the exponent of the matching function do not take on the same value if one takes the empirically estimated values from the literature seriously. Using these estimates, the numerically simulated results show that the laissez-faire market equilibrium features too many entrepreneurs. Therefore, a positive marginal tax rate on entrepreneurial incomes and an additional positive tax rate on vacancy posting restore the first-best. Taxation now corrects for two inefficiencies. First, vacancy posting must be taxed to revise the overhiring behaviour of firms. Second, entrepreneurial income must be taxed if there are too many entrepreneurs or subsidised if there are too few entrepreneurs in the decentralised market equilibrium. Hence, the introduction of intrafirm wage bargaining has a substantial effect on the model's predictions because it creates a channel for entrepreneurs to suppress wages, which influences their hiring behaviour and, consequently, aggregate labour market tightness. By maximising their private profits, firm owners do not consider the negative effects that overhiring exerts on other, potentially more able, entrepreneurs. For a realistic parametrisation, the main result from Chapter 3 prevails, namely that there are too many entrepreneurs in the laissez-faire market equilibrium.

Chapter 5 concentrates on firm dynamics. The theoretical model from the former two chapters is extended with quadratic vacancy posting costs, an exogenously given probability that a firm is destroyed, and a decision state. These extensions result in firms growing over time until they reach their target size. Moreover, because of exogenous firm destruction, there are firms at different ages in steady state. Young firms, which still grow over time, and old firms, which have already reached their target size, coexist, even in steady state. Firm destruction has a negative effect on total welfare as it gets more difficult for firms to reach their optimal size and thus prevents them from operating at full capacity. The numerical simulation of this more complex model reveals that both entrepreneurial incomes and vacancy posting have to be subsidised to maximise welfare. By subsidising vacancy posting, firms are induced to post more vacancies, which allows them to grow faster. Subsidising only vacancy posting would increase the labour market tightness and would therefore decrease the number of firms. In return, entrepreneurial incomes have to be subsidised to improve the attractiveness of starting a business. The combination of both subsidies leads to more firms reaching their target size and very able entrepreneurs possessing larger firms. As a consequence, aggregate production rises. At the same time, as less able entrepreneurs post fewer vacancies, the labour market becomes less tight, which decreases aggregate vacancy posting costs. Introducing firm destruction therefore alters the DMP model with endogenous occupational choices substantially and leads to an optimal policy of subsidising entrepreneurship.

There are several fruitful directions for future research. One can, for example, add workers with heterogeneous skills and introduce direct search, which means that workers and firms search for specific partners they want to be matched with. This would make wage bargaining much more complex because the decision to proceed with a match depends on one's own skills and entrepreneurial abilities and those of the partner. Implementing other policy instruments, such as a minimum wage, might also be interesting. A minimum wage would make hiring more expensive for firms and would limit the entrepreneur's ability to acquire large rents, at least to some extent. Moreover, adding capital constraints, different levels of risk aversion, or international mobility of entrepreneurs to the model and analysing their impact on taxation would be promising.

Appendix A

Mathematical Derivations

A.1 Chapter 3

A.1.1 Derivation of the Job Creation Condition

The Lagrangian function for the firm's maximisation problem is given with

$$\mathcal{L} = af(l_t) - w_t l_t - \gamma v_t + \beta W^f_{t+1}(a, l_{t+1}) + \lambda_t \left[(1-s)l_t + q(\theta_t)v_t - l_{t+1}\right].$$

The first order conditions are the following:

$$\frac{\partial \mathcal{L}}{\partial v_t} = -\gamma + \lambda_t q(\theta_t) = 0 \quad \Leftrightarrow \quad \lambda_t = \frac{\gamma}{q(\theta_t)} \tag{A.1}$$

and

$$\frac{\partial \mathcal{L}}{\partial l_{t+1}} = \beta \frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}} - \lambda_t = 0 \quad \Leftrightarrow \quad \lambda_t = \beta \frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}}. \tag{A.2}$$

Setting (A.1) equal to (A.2), one obtains

$$\frac{\gamma}{q(\theta_t)} = \beta \frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}}. \tag{A.3}$$

The envelope theorem states that

$$\frac{\partial W_t^f(a, l_t)}{\partial l_t} = af'(l_t) - w_t + \lambda_t(1-s)$$

and, therefore,

$$\begin{aligned}\frac{\partial W_{t+1}^f(a, l_{t+1})}{\partial l_{t+1}} &= af'(l_{t+1}) - w_{t+1} + \lambda_{t+1}(1-s) \\ &= af'(l_{t+1}) - w_{t+1} + (1-s)\frac{\gamma}{q(\theta_{t+1})}. \end{aligned} \tag{A.4}$$

Plugging (A.4) into (A.3), one receives the job creation condition

$$\frac{\gamma}{q(\theta_t)} = \beta\left[af'(l_{t+1}) - w_{t+1} + (1-s)\frac{\gamma}{q(\theta_{t+1})}\right].$$

A.1.2 Nash Wage Bargaining

The surplus of a match for the firm is given with

$$P_t = af'(l_t) - w_t + \beta(1-s)P_{t+1}$$

and for the worker it is

$$W_t^e - W_t^n = w_t - z + \beta(1 - s - \theta_t q(\theta_t))(W_{t+1}^e - W_{t+1}^n).$$

The wage in a Nash wage bargaining process solves

$$w_t = \arg\max\ (W_t^e - W_t^n)^{\xi} P_t^{1-\xi}$$

with ξ being the bargaining power of the worker. The resulting FOC from the maximisation of surpluses is

$$\xi P_t \frac{\partial(W_t^e - W_t^n)}{\partial w_t} + (1-\xi)(W_t^e - W_t^n)\frac{\partial P_t}{\partial w_t} = 0$$

$$\Leftrightarrow\ \xi P_t - (1-\xi)(W_t^e - W_t^n) = 0.$$

Rearranging terms, the FOC becomes

$$\xi P_t = (1-\xi)(W_t^e - W_t^n). \tag{A.5}$$

Using (A.5), I can write the surplus of a match for the worker as

$$W_{t+1}^e - W_{t+1}^n = \frac{\xi}{1-\xi} P_{t+1}.$$

Plugging in for P_t and $W_t^e - W_t^n$, (A.5) becomes

$$\xi \left[af'(l_t) - w_t + \beta(1-s)P_{t+1} \right]$$
$$= (1-\xi) \left[w_t - z + \beta(1-s-\theta_t q(\theta_t)) \frac{\xi}{(1-\xi)} P_{t+1} \right],$$

which can be rearranged to

$$w_t = \xi af'(l_t) + \xi\beta\theta_t q(\theta_t) P_{t+1} + (1-\xi)z.$$

Using $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$ from the job creation condition, the wage curve can be derived as

$$w_t = \xi \left[af'(l_t) + \gamma\theta_t \right] + (1-\xi)z.$$

A.1.3 Social Planner's Maximisation Problem

The Lagrangian function for the social planner's maximisation problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ \Phi(\bar{a}_t) - N_t) u(c_t^e) + N_t u(c_t^n) + \int_{\bar{a}_t}^{\infty} u(c_t^f(a)) d\Phi(a)$$

$$+ \lambda_t \left[\int_{\bar{a}_t}^{\infty} a f(l_t(a)) d\Phi(a) + N_t z - (\Phi(\bar{a}_t) - N_t) c_t^e - N_t c_t^n \right.$$

$$\left. - \int_{\bar{a}_t}^{\infty} c_t^f(a) d\Phi(a) - \gamma V_t \right]$$

$$+ \mu_t \left[N_{t+1} - N_t - s(\Phi(\bar{a}_t) - N_t) + m(N_t, V_t) \right]$$

$$+ \nu_t \left[\Phi(\bar{a}_t) - N_t - \int_{\bar{a}_t}^{\infty} l_t(a) d\Phi(a) \right] \Bigg\}$$

with λ_t being the Lagrange multiplier on the resource constraint, μ_t being the multiplier for the law of motion of unemployment and ν_t being the multiplier on the labour supply constraint.

The FOCs are:

$$\frac{\partial \mathcal{L}}{\partial c_t^e} = 1 - \lambda_t = 0,$$

$$\frac{\partial \mathcal{L}}{\partial c_t^n} = 1 - \lambda_t = 0,$$

$$\frac{\partial \mathcal{L}}{\partial c_t^f(a)} = 1 - \lambda_t = 0,$$

$$\frac{\partial \mathcal{L}}{\partial V_t} = -\lambda_t \gamma + \mu_t \frac{\partial m(N_t, V_t)}{\partial V_t} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial l_t(a)} = \lambda_t a f'(l_t(a)) - \nu_t = 0,$$

$$\frac{\partial \mathcal{L}}{\partial N_{t+1}} = \mu_t + \beta \left[u(c_{t+1}^n) - u(c_{t+1}^e) \right] + \beta \lambda_{t+1} \left[z + c_{t+1}^e - c_{t+1}^n \right]$$

$$- \beta \mu_{t+1} \left[1 - s - \frac{\partial m(N_{t+1}, V_{t+1})}{\partial N_{t+1}} \right] - \beta \nu_{t+1} = 0,$$

$$\frac{\partial \mathcal{L}}{\partial \bar{a}_t} = \phi(\bar{a}_t)u(c_t^e) - \phi(\bar{a}_t)u(c_t^f(\bar{a}_t))$$

$$+ \lambda_t \left[-\phi(\bar{a}_t)\bar{a}_t f(l_t(\bar{a}_t)) - \phi(\bar{a}_t)c_t^e + \phi(\bar{a}_t)c_t^f(\bar{a}_t) + \phi(\bar{a}_t)\gamma v_t(\bar{a}_t)\right]$$

$$+ \mu_t \left[-\phi(\bar{a}_t)\frac{\partial m(N_t, V_t)}{\partial V_t}v_t(\bar{a}_t) - \phi(\bar{a}_t)s\right] + \nu_t \left[\phi(\bar{a}_t) + \phi(\bar{a}_t)l_t(\bar{a}_t)\right] = 0.$$

The matching function is assumed to be of Cobb-Douglas form: $m(N_t, V_t) = N_t^{\alpha}V_t^{1-\alpha}$. The derivatives of the matching function with respect to N_t and V_t are therefore

$$\frac{\partial m(N_t, V_t)}{\partial V_t} = (1-\alpha)N_t^{\alpha}V_t^{-\alpha} = (1-\alpha)\left(\frac{m(N_t, V_t)}{V_t}\right) = (1-\alpha)q(\theta_t)$$

and

$$\frac{\partial m(N_t, V_t)}{\partial N_t} = \alpha N_t^{\alpha-1}V_t^{1-\alpha} = \alpha\left(\frac{m(N_t, V_t)}{N_t}\right) = \alpha\theta_t q(\theta_t).$$

The elasticity of the Cobb-Douglas matching function with respect to N_t is

$$\frac{\partial m(N_t, V_t)}{\partial N_t}\frac{N_t}{m(N_t, V_t)} = \alpha.$$

The utility function is assumed to be linear, therefore $u(c_t^i) = c_t^i$ for $i = e, f, n$. Since the social planner weighs employed workers, unemployed workers, and entrepreneurs with their share of total population, in equilibrium it has to hold that $c_t^e = c_t^n = c_t^f(\bar{a})$.

From $\frac{\partial \mathcal{L}}{\partial l_t(a)}$ it is clear that the marginal product of labour must be the same for each individual, no matter how high a is. Therefore, instead of $af'(l_t(a))$, one can always use $\bar{a}_t f'(l_t(\bar{a}_t))$.

Using the respective FOCs, the multipliers are derived as

$$\lambda_t = 1,$$

$$\mu_t = \frac{\gamma}{(1-\alpha)q(\theta_t)},$$

$$\nu_t = af'(l_t(a)).$$

In the following, the Lagrange multipliers are eliminated from the derivatives of the Lagrangian with respect to N_t and $\bar{a}_t$.

Derivative with Respect to $\boldsymbol{N_t}$:
First, $\frac{\partial \mathcal{L}}{\partial N_{t+1}}$ is rearranged to

$$\mu_t = \beta \left[u(c^e_{t+1}) - u(c^n_{t+1}) \right] + \beta \lambda_{t+1} \left[c^n_{t+1} - c^e_{t+1} - z \right] + \beta \mu_{t+1} \left[1 - s - \frac{\partial m(N_{t+1}, V_{t+1})}{\partial N_{t+1}} \right] + \beta \nu_{t+1},$$

which becomes

$$\mu_t = \beta \lambda_{t+1}(-z) + \beta \mu_{t+1} \left[1 - s - \alpha \theta_{t+1} q(\theta_{t+1}) \right] + \beta \nu_{t+1}.$$

Plugging in for μ_t, μ_{t+1}, λ_{t+1}, and ν_{t+1} one obtains

$$\frac{\gamma}{\beta q(\theta_t)} = (1-\alpha) \left[a f'(l_{t+1}(a)) - z \right] - \alpha \gamma \theta_{t+1} + \beta (1-s) \frac{\gamma}{\beta q(\theta_{t+1})}. \tag{A.6}$$

Derivative with Respect to $\boldsymbol{\bar{a}_t}$:
Divide the derivative $\frac{\partial \mathcal{L}}{\partial \bar{a}_t}$ by $\phi(\bar{a})$ and use the fact that $c^e_t = c^f_t(\bar{a}_t)$. The derivative simplifies to

$$\lambda_t \left[-\bar{a}_t f(l_t(\bar{a}_t)) + \gamma v_t(\bar{a}_t) \right] + \mu_t \left[-(1-\alpha) q(\theta_t) v_t(\bar{a}_t) - s \right] + \nu_t \left[1 + l_t(\bar{a}_t) \right] = 0.$$

Replacing λ_t, μ_t, and ν_t, one obtains

$$\bar{a}_t f(l_t(\bar{a}_t)) + \frac{s\gamma}{(1-\alpha) q(\theta_t)} - \bar{a}_t f'(l_t(\bar{a}_t))(1 + l_t(\bar{a}_t)) = 0.$$

A.1.4 Efficiency in Market Equilibrium

The steady state wage in the market equilibrium is

$$w = z + \xi \left[a f'(l) - z \right] + \xi \gamma \theta. \tag{A.7}$$

The job creation condition in steady state is

$$\frac{\gamma}{q(\theta)} = \beta\left[af'(l) - w + (1-s)\frac{\gamma}{q(\theta)}\right]$$

and can be rearranged to

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l) - z\right]$$

by plugging in (A.7). It can be rewritten as

$$af'(l) - z = \frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right]}{(1-\xi)}.$$

If I plug this into the steady state wage (A.7), the wage becomes

$$\begin{aligned} w &= z + \frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right] + \xi\gamma\theta \\ &= z + \frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\xi\beta\theta q(\theta)+(1-\xi)\beta\theta q(\theta)\right] \\ &= z + \frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right], \end{aligned} \tag{A.8}$$

which depends only on exogenously given parameter values and the labour market tightness θ.

The value equations for an employed and an unemployed worker in steady state are

$$W_e = w + \beta\left[sW_n + (1-s)W_e\right] \tag{A.9}$$

and

$$W_n = z + \beta\left[\theta q(\theta)W_e + (1-\theta q(\theta))W_n\right]. \tag{A.10}$$

Rearranging W_n gives

$$W_n = \frac{z+\beta\theta q(\theta)W_e}{1-\beta+\beta\theta q(\theta)}.$$

Inserting this into (A.9), one can solve for W_e:

$$W_e = w + \beta s \left(\frac{z + \beta\theta q(\theta) W_e}{1 - \beta + \beta\theta q(\theta)} \right) + \beta(1-s) W_e$$

$$\Leftrightarrow \quad (1-\beta) W_e = w + \frac{\beta s z}{1 - \beta + \beta\theta q(\theta)} - \frac{\beta s (1-\beta) W^e}{1 - \beta + \beta\theta q(\theta)}$$

$$\Leftrightarrow \quad (1-\beta) W_e = w - \frac{\beta s (w - z)}{1 - \beta(1-s) + \beta\theta q(\theta)}$$

using

$$W_e = \frac{w + \beta s \left[\frac{z + \beta\theta q(\theta) W_e}{1 - \beta + \beta\theta q(\theta)} \right]}{1 - \beta(1-s)}$$

$$\Leftrightarrow \quad W_e = \frac{w \left[1 - \beta + \beta\theta q(\theta) \right] + \beta s z}{(1-\beta) \left[1 - \beta(1-s) + \beta\theta q(\theta) \right]}.$$

Analogously, rearranging (A.9) gives

$$W_e = \frac{w + \beta s W_n}{1 - \beta(1-s)}. \tag{A.11}$$

By using (A.11), W_e can be eliminated from Equation (A.10) and one obtains

$$(1-\beta) W_n = z + \frac{\beta\theta q(\theta)(w - z)}{1 - \beta(1-s) + \beta\theta q(\theta)}.$$

For w one can insert (A.8) into both reformulated value equations and receives

$$(1-\beta) W_e = z + \frac{\xi}{(1-\xi)} \frac{\gamma\theta}{\beta\theta q(\theta)} \left[1 - \beta + \beta\theta q(\theta) \right] \tag{A.12}$$

and

$$(1-\beta) W_n = z + \frac{\xi}{1-\xi} \gamma\theta. \tag{A.13}$$

Reformulating the Indifference Equation

The indifference equation in steady state is given with

$$\begin{aligned}
\bar{a}f(l(\bar{a})) - wl(\bar{a}) - \gamma v(\bar{a}) &= (1-\beta)W_e \\
&= w - \frac{\beta s(w-z)}{1-\beta(1-s)+\beta\theta q(\theta)}
\end{aligned}$$

Inserting (A.8) and $v(\bar{a}) = \frac{sl(\bar{a})}{q(\theta)}$, one arrives at

$$\begin{aligned}
&\bar{a}f(l) - \frac{s\gamma l}{q(\theta)} - \left[z + \frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right]\right] l \\
&\qquad = z + \frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right] \\
\Leftrightarrow\ &\bar{a}f(l) - \frac{s\gamma l}{q(\theta)} - \frac{s\gamma l}{q(\theta)}\frac{\xi}{(1-\xi)} = \left[z + \frac{\gamma}{\beta q(\theta)}\frac{\xi\left[1-\beta+\beta\theta q(\theta)\right]}{(1-\xi)}\right](1+l) \\
\Leftrightarrow\ &\frac{\bar{a}f(l)}{1+l} - \frac{l}{(1+l)}\frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right] \\
\Leftrightarrow\ &\frac{\bar{a}f(l)}{1+l} - \frac{l}{(1+l)}\frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\xi\beta\theta q(\theta)\right]}{(1-\xi)} - \frac{(1-\beta)}{\beta}\frac{\gamma}{q(\theta)}
\end{aligned}$$

with $l = l(\bar{a})$.

A.1.5 Taxation

The Lagrangian function for the firm's maximisation problem is given with

$$\begin{aligned}
\mathcal{L} = (1-\tau_t^f)\left[af(l_t) - w_t l_t - \gamma v_t\right] + \beta W_{t+1}^f(a, l_{t+1}) \\
+ \lambda_t\left[(1-s)l_t + q(\theta_t)v_t - l_{t+1}\right].
\end{aligned}$$

It delivers the following FOCs:

$$\frac{\partial\mathcal{L}}{\partial v_t} = -(1-\tau_t^f)\gamma + \lambda_t q(\theta_t) = 0,$$

$$\frac{\partial\mathcal{L}}{\partial l_{t+1}} = \beta\frac{\partial W_{t+1}^f(a, l_{t+1})}{\partial l_{t+1}} - \lambda_t = 0.$$

Combining both as before leads to

$$\frac{(1-\tau_t^f)\gamma}{q(\theta_t)} = \beta \frac{\partial W_{t+1}^f(a, l_{t+1})}{\partial l_{t+1}}.$$

Using the envelope condition

$$\frac{\partial W_t^f(a, l_t)}{\partial l_t} = (1-\tau_t^f)\left[af'(l_t) - w_t\right] + \lambda_t(1-s),$$

one derives the job creation condition as

$$\frac{(1-\tau_t^f)\gamma}{q(\theta_t)} = \beta \left\{ (1-\tau_{t+1}^f)\left[af'(l_{t+1}) - w_{t+1}\right] + (1-s)\frac{(1-\tau_{t+1}^f)\gamma}{q(\theta_{t+1})} \right\}.$$

Assuming that $\tau_t^f = \tau_{t+1}^f = \tau^f$, I arrive at the same job creation condition derived in Section A.1.1:

$$\frac{\gamma}{q(\theta_t)} = \beta \left[af'(l_{t+1}) - w_{t+1} + (1-s)\frac{\gamma}{q(\theta_{t+1})} \right].$$

Wage Bargaining

Workers and firms bargain about gross wages and the FOC from Nash wage bargaining is

$$\xi P_t \frac{\partial(W_t^e - W_t^n)}{\partial w_t} + (1-\xi)(W_t^e - W_t^n)\frac{\partial P_t}{\partial w_t} = 0$$

$$\Leftrightarrow \; \xi P_t(1-\tau^e) = (1-\xi)(W_t^e - W_t^n).$$

The surplus of the match for the worker is

$$W_t^e - W_t^n = (1-\tau^e)w_t - z + \beta(1 - s - \theta_t q(\theta_t))(W_{t+1}^e - W_{t+1}^n)$$

and for the entrepreneur it is

$$P_t = af'(l_t) - w_t + \beta(1-s)P_{t+1}.$$

Using these equations, plugging them into the FOC and substituting

$$W^e_{t+1} - W^n_{t+1} = \frac{\xi}{1-\xi}(1-\tau^e)P_{t+1},$$

one receives

$$\begin{aligned}
&\xi(1-\tau^e)\left[af'(l_t) - w_t\right] + \beta\xi(1-s)(1-\tau^e)P_{t+1} \\
&\qquad = (1-\xi)\left[(1-\tau^e)w_t - z + \beta(1-s-\theta_t q(\theta_t))\frac{\xi}{(1-\xi)}(1-\tau^e)P_{t+1}\right].
\end{aligned}$$

Inserting $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$ and rearranging then leads to

$$\begin{aligned}
w_t &= \xi\left[af'(l_t) + \gamma\theta_t\right] + (1-\xi)\frac{z}{(1-\tau^e)} \\
&= \frac{z}{(1-\tau^e)} + \xi\left[af'(l_t) - \frac{z}{(1-\tau^e)}\right] + \xi\gamma\theta_t.
\end{aligned}$$

In steady state, the wage becomes

$$w = \frac{z}{(1-\tau^e)} + \xi\left[af'(l) - \frac{z}{(1-\tau^e)}\right] + \xi\gamma\theta.$$

The first order condition for hiring is

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l) - \frac{z}{(1-\tau^e)}\right],$$

which I can rewrite as

$$af'(l) - \frac{z}{(1-\tau^e)} = \frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right]}{(1-\xi)}.$$

If I plug this into the steady state wage, it becomes

$$\begin{aligned}
w &= \frac{z}{(1-\tau^e)} + \frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right] + \xi\gamma\theta \\
&= \frac{z}{(1-\tau^e)} + \frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right].
\end{aligned}$$

Deriving the Tax Rate

The indifference equation for the marginal entrepreneur in steady state is given with

$$
\begin{aligned}
&(1-\tau^f)\left[\bar{a}f(l(\bar{a}))-wl(\bar{a})-\gamma v(\bar{a})\right]+\beta W^f(\bar{a},l) \\
&\qquad\qquad = \quad (1-\tau^e)w+\beta\left[sW^n+(1-s)W^e\right].
\end{aligned}
$$

Inserting for W^e, W^n, w, it can be written as

$$
(1-\tau^f)\left[\bar{a}f(l(\bar{a}))-wl(\bar{a})-\gamma v(\bar{a})\right]=z+\frac{(1-\tau^e)}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right].
$$

If I plug in the steady state wage on the left-hand side and insert $v(\bar{a})=\frac{sl(\bar{a})}{q(\theta)}$, it becomes[1]

$$
\begin{aligned}
&(1-\tau^f)\left\{\bar{a}f(l)-\left[\frac{z}{(1-\tau^e)}+\frac{\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right]\right]l-\frac{s\gamma l}{q(\theta)}\right\}= \\
&\qquad\qquad z+\frac{(1-\tau^e)}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right],
\end{aligned}
$$

which, if I divide both sides of the equation with $1+l$, can be rearranged to

$$
\begin{aligned}
&\frac{\bar{a}f(l)}{(1+l)}-\frac{l}{(1+l)}\frac{s\gamma}{(1-\xi)q(\theta)}= \\
&\left[\frac{1}{(1-\tau^f)(1+l)}+\frac{l}{(1-\tau^e)(1+l)}\right]\left\{z+\frac{(1-\tau^e)\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]\right\}.
\end{aligned}
$$

If I set this equation equal to the indifference equation from the social planner's maximisation problem

$$
\frac{\bar{a}^{FB}f(l)}{(1+l)}-\frac{l}{(1+l)}\frac{s\gamma}{(1-\alpha)q(\theta^{FB})}=z+\frac{\gamma}{\beta q(\theta^{FB})}\frac{\left[1-\beta+\alpha\beta\theta^{FB}q(\theta^{FB})\right]}{(1-\alpha)},
$$

[1] In the following I use l instead of $l(\bar{a})$ for clarity.

I obtain

$$\left[\frac{1}{(1-\tau^f)(1+l)}+\frac{l}{(1-\tau^e)(1+l)}\right]\left\{z+\frac{(1-\tau^e)\gamma}{\beta q(\theta)}\frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]\right\}$$
$$=z+\frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\xi\beta\theta q(\theta)\right]}{(1-\xi)} \quad \text{(A.14)}$$

under the assumption that the Hosios condition holds. This can be solved for τ^f in the following steps:

$$\frac{1}{(1-\tau^f)}\left\{z+\frac{(1-\tau^e)\gamma}{\beta q(\theta)}\frac{\xi\left[1-\beta+\beta\theta q(\theta)\right]}{(1-\xi)}\right\}$$
$$=\left\{z+\frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\xi\beta\theta q(\theta)\right]}{(1-\xi)}\right\}(1+l)-\left\{z+\frac{(1-\tau^e)\gamma}{\beta q(\theta)}\frac{\xi\left[1-\beta+\beta\theta q(\theta)\right]}{(1-\xi)}\right\}\frac{l}{(1-\tau^e)}$$

$$\Leftrightarrow\ (1-\tau^f)=\frac{z+\frac{(1-\tau^e)\gamma}{\beta q(\theta)}\frac{\xi[1-\beta+\beta\theta q(\theta)]}{(1-\xi)}}{\left\{z+\frac{\gamma}{\beta q(\theta)}\frac{[1-\beta+\xi\beta\theta q(\theta)]}{(1-\xi)}\right\}(1+l)-\left\{z+\frac{(1-\tau^e)\gamma}{\beta q(\theta)}\frac{\xi[1-\beta+\beta\theta q(\theta)]}{(1-\xi)}\right\}\frac{l}{(1-\tau^e)}}$$

$$\Leftrightarrow\ \tau^f=1-\frac{z+\frac{(1-\tau^e)\gamma}{\beta q(\theta)}\frac{\xi[1-\beta+\beta\theta q(\theta)]}{(1-\xi)}}{\left\{z+\frac{\gamma}{\beta q(\theta)}\frac{[1-\beta+\xi\beta\theta q(\theta)]}{(1-\xi)}\right\}(1+l)-\left\{z+\frac{(1-\tau^e)\gamma}{\beta q(\theta)}\frac{\xi[1-\beta+\beta\theta q(\theta)]}{(1-\xi)}\right\}\frac{l}{(1-\tau^e)}}.$$

Expanding and simplifying the fraction hence leads to

$$\tau^f=\frac{(1-\beta)(1-\xi)\gamma\left[1+l(\bar{a})\right]+\tau^e\xi\gamma\left[1-\beta+\beta\theta q(\theta)\right]-\frac{\beta(1-\xi)q(\theta)\tau^e lz}{(1-\tau^e)}}{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right]+\beta q(\theta)\left[\xi\gamma\theta+(1-\xi)z\right]-\frac{\beta(1-\xi)q(\theta)\tau^e lz}{(1-\tau^e)}}.$$

It $\tau^e=0$, the marginal tax rate on entrepreneurial incomes simplifies and becomes

$$\tau^f=\frac{(1-\beta)(1-\xi)\gamma\left[1+l(\bar{a})\right]}{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right]+\beta q(\theta)\left[\xi\gamma\theta+(1-\xi)z\right]}.$$

Comparing $\boldsymbol{\tau^e}$ and $\boldsymbol{\tau^f}$

To find out, until which point the tax on entrepreneurial incomes is larger than the marginal tax rate on labour incomes, I calculate the intersection point of both tax rates. If $\tau^f = \tau^e = \hat{\tau}$, it holds that

$$\hat{\tau} = \frac{(1-\beta)(1-\xi)\gamma\left[1+l(\bar{a})\right] + \hat{\tau}\xi\gamma\left[1-\beta+\beta\theta q(\theta)\right] - \frac{\beta(1-\xi)q(\theta)\hat{\tau}lz}{(1-\hat{\tau})}}{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right] + \beta q(\theta)\left[\xi\gamma\theta+(1-\xi)z\right] - \frac{\beta(1-\xi)q(\theta)\hat{\tau}lz}{(1-\hat{\tau})}}.$$

This can be solved for $\hat{\tau}$:

$$\hat{\tau}\left\{(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right] + \beta q(\theta)\left[\xi\gamma\theta+(1-\xi)z\right] - \frac{\beta(1-\xi)q(\theta)\hat{\tau}lz}{(1-\hat{\tau})}\right\}$$

$$= (1-\beta)(1-\xi)\gamma\left[1+l(\bar{a})\right] + \hat{\tau}\xi\gamma\left[1-\beta+\beta\theta q(\theta)\right] - \frac{\beta(1-\xi)q(\theta)\hat{\tau}lz}{(1-\hat{\tau})}$$

$$\Leftrightarrow\ \hat{\tau}(1-\beta)\gamma\left[1+(1-\xi)l(\bar{a})\right] + \hat{\tau}\beta(1-\xi)q(\theta)z - \hat{\tau}(1-\beta)\xi\gamma + \hat{\tau}\beta(1-\xi)q(\theta)l(\bar{a})z$$

$$= (1-\beta)(1-\xi)\gamma(1+l(\bar{a}))$$

$$\Leftrightarrow\ \hat{\tau}(1-\beta)(1-\xi)\gamma(1+l(\bar{a})) + \hat{\tau}\beta(1-\xi)q(\theta)(1+l(\bar{a})) = (1-\beta)(1-\xi)\gamma(1+l(\bar{a}))$$

$$\Leftrightarrow\ \hat{\tau}\left[(1-\beta)\gamma+\beta q(\theta)z\right] = (1-\beta)\gamma$$

$$\Leftrightarrow\ \hat{\tau} = \frac{(1-\beta)\gamma}{(1-\beta)\gamma+\beta q(\theta)z}.$$

Since $\tau^f > 0$ for $\tau^e = 0$, $\tau^f > \tau^e$ until $\tau^e = \hat{\tau}$. If $\tau^e > \hat{\tau}$, τ^e becomes larger than τ^f.

If $\boldsymbol{\xi \neq \alpha}$

Assume that the Hosios condition does not hold. By setting (A.14) equal to the social planner's indifference equation, one obtains the following equation that can be solved for τ^f:

$$\left[\frac{1}{(1-\tau^f)(1+l^{ME})} + \frac{l^{ME}}{(1-\tau^e)(1+l^{ME})}\right]\left\{z + \frac{(1-\tau^e)\gamma}{\beta q(\theta^{ME})}\frac{\xi\left[1-\beta+\beta\theta^{ME}q(\theta^{ME})\right]}{(1-\xi)}\right\} =$$

$$z + \frac{\gamma}{\beta q(\theta^{FB})}\frac{\left[1-\beta+\alpha\beta\theta^{FB}q(\theta^{FB})\right]}{(1-\alpha)}$$

with θ^{ME}, $l^{ME} = l(\bar{a}^{ME})$ denoting the labour market tightness and the labour demand of the marginal entrepreneur in the market equilibrium and θ^{FB} denoting the first-best labour market tightness. The tax rate τ^f that restores the first-best allocation is thus given with

$$\tau^f = 1 - \frac{z + \frac{(1-\tau^e)\gamma}{\beta q(\theta^{ME})} \frac{\xi}{(1-\xi)} \left[1 - \beta + \beta\theta^{ME} q(\theta^{ME})\right]}{\begin{array}{c}\left\{z + \frac{\gamma}{\beta q(\theta^{FB})} \frac{\left[1-\beta+\alpha\beta\theta^{FB} q(\theta^{FB})\right]}{(1-\alpha)}\right\}(1+l^{ME}) \\ - \left\{z + \frac{(1-\tau^e)\gamma}{\beta q(\theta^{ME})} \frac{\xi\left[1-\beta+\beta\theta^{ME} q(\theta^{ME})\right]}{(1-\xi)}\right\} \frac{l^{ME}}{(1-\tau^e)}\end{array}},$$

which is

$$\tau^f = \frac{\begin{array}{c}\left\{z + \frac{\gamma}{\beta q(\theta^{FB})} \frac{\left[1-\beta+\alpha\beta\theta^{FB} q(\theta^{FB})\right]}{(1-\alpha)}\right\}(1+l^{ME}) \\ - \left\{z + \frac{(1-\tau^e)\gamma}{\beta q(\theta^{ME})} \frac{\xi\left[1-\beta+\beta\theta^{ME} q(\theta^{ME})\right]}{(1-\xi)}\right\} \left[1 + \frac{l^{ME}}{(1-\tau^e)}\right]\end{array}}{\begin{array}{c}\left\{z + \frac{\gamma}{\beta q(\theta^{FB})} \frac{\left[1-\beta+\alpha\beta\theta^{FB} q(\theta^{FB})\right]}{(1-\alpha)}\right\}(1+l^{ME}) \\ - \left\{z + \frac{(1-\tau^e)\gamma}{\beta q(\theta^{ME})} \frac{\xi\left[1-\beta+\beta\theta^{ME} q(\theta^{ME})\right]}{(1-\xi)}\right\} \frac{l^{ME}}{(1-\tau^e)}\end{array}}.$$

Consider the case where $\alpha > \xi$, meaning that the social return of a match is larger than the private return of a match for the worker or lower than the private return for the entrepreneur. If $\tau^e = 0$, τ^f is still positive and larger than zero if

$$\frac{1 - \beta + \alpha\beta\theta^{FB} q(\theta^{FB})}{(1-\alpha) q(\theta^{FB})} > \frac{\xi\left[1 - \beta + \beta\theta^{ME} q(\theta^{ME})\right]}{(1-\xi) q(\theta^{ME})}.$$

If $\alpha > \xi$, the labour market tightness in the market equilibrium would be larger than the first-best tightness so $\theta^{FB} < \theta^{ME}$ and $q(\theta^{FB}) > q(\theta)^{ME}$.

A.1.6 Numerical Simulation

For the numerical simulation, I use the explicit production function $f(l) = l^\eta$. I assume that entrepreneurial ability a is distributed according to a log normal CDF. The tax revenue is redistributed via a lump-sum transfer T to each individual. I construct a grid for the as and calculate the following firm decisions for each grid point. First, I calculate labour demand for all as. Using the wage curve and

$q(\theta) = \theta^{-\alpha}$, the job creation condition in steady state can be rearranged:

$$\frac{\gamma}{\beta\theta^{-\alpha}} = af'(l) - w + \beta(1-s)\frac{\gamma}{\beta q(\theta)}$$

$$\Leftrightarrow \quad [1-\beta(1-s)]\frac{\gamma}{\beta\theta^{-\alpha}} = af'(l) - w$$

$$\Leftrightarrow \quad [1-\beta(1-s)]\frac{\gamma}{\beta\theta^{-\alpha}} = (1-\xi)af'(l) - \xi\gamma\theta - (1-\xi)\frac{z}{(1-\tau^e)}.$$

Using $af'(l) = \eta a l^{\eta-1}$, the labour demand for an entrepreneur with ability a can be calculated as

$$(1-\xi)\eta a l(a)^{\eta-1} = [1-\beta(1-s)]\frac{\gamma}{\beta\theta^{-\alpha}} + \xi\gamma\theta + (1-\xi)\frac{z}{(1-\tau^e)}$$

$$\Leftrightarrow \quad l(a) = \left\{\frac{[1-\beta(1-s)]\frac{\gamma}{\beta\theta^{-\alpha}} + \xi\gamma\theta + (1-\xi)\frac{z}{(1-\tau^e)}}{(1-\xi)\eta a}\right\}^{\frac{1}{\eta-1}}.$$

Having derived $l(a)$, I can calculate the number of vacancies for each a by using the law of motion for employment:

$$v(a) = \frac{sl(a)}{\theta^{-\alpha}}.$$

The wage is given with

$$w(a) = \xi\left[a\eta l(a)^{\eta-1} + \gamma\theta\right] + (1-\xi)\frac{z}{(1-\tau^e)}.$$

I then calculate the firms' profits and value equations as

$$\pi(a) = al(a)^{\eta} - w(a)l(a) - \gamma v(a),$$

$$W^f(a) = \frac{(1-\tau^f)\pi(a) + T}{1-\beta}.$$

The value equations for employed and unemployed workers are determined as

$$W^n = \frac{1}{1-\beta}\left[z + T + (1-\tau^e)\frac{\xi}{(1-\xi)}\gamma\theta\right],$$

$$W^e = \frac{1}{1-\beta}\left[(1-\tau^e)\frac{\xi}{(1-\xi)}\frac{\gamma}{\beta\theta^{-\alpha}}\left[1-\beta+\beta\theta^{1-\alpha}\right] + z + T\right].$$

To determine the equilibrium, I search for θ, $\bar{a}$, and T that fulfil the following three equations:

$$\Phi(\bar{a}) - N - \int_{\bar{a}}^{\hat{a}} l(a)d\Phi(a) = 0,$$

$$W^f(\bar{a}) - W^e = 0,$$

$$T - \tau^f \int_{\bar{a}}^{\hat{a}} \pi(a)d\Phi(a) = 0,$$

where $\hat{a}$ denotes the upper bound of the distribution over a. The three equations ensure that labour supply is equal to labour demand, that the indifference equation holds, and that the tax revenue equals the sum of lump-sum transfers paid to all individuals.

A.2 Chapter 4

A.2.1 The Firm's Maximisation Problem

The Lagrangian function from the maximisation problem of the firm has the form

$$\mathcal{L} = af(l_t) - w_t(a, l_t)l_t - \gamma v_t + \beta W^f_{t+1}(a, l_{t+1}) + \lambda_t\left[(1-s)l_t + q(\theta_t)v_t - l_{t+1}\right]$$

The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial v_t} = -\gamma + \lambda_t q(\theta_t) = 0 \quad \Leftrightarrow \quad \lambda_t = \frac{\gamma}{q(\theta_t)},$$

$$\frac{\partial \mathcal{L}}{\partial l_{t+1}} = \beta\frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}} - \lambda_t = 0 \quad \Leftrightarrow \quad \lambda_t = \beta\frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}}.$$

Combining the first order conditions, one receives

$$\frac{\gamma}{q(\theta_t)} = \beta \frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}}.$$

Finally, using the envelope condition

$$\frac{\partial W^f_t(a, l_t)}{\partial l_t} = af'(l_t) - w_t - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + \lambda_t(1-s),$$

which implies that

$$\begin{aligned}\frac{\partial W^f_{t+1}(a, l_{t+1})}{\partial l_{t+1}} &= af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}(a, l_{t+1})}{\partial l_{t+1}} l_{t+1} + \lambda_{t+1}(1-s) \\ &= af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}(a, l_{t+1})}{\partial l_{t+1}} l_{t+1} + (1-s)\frac{\gamma}{q(\theta_{t+1})},\end{aligned}$$

one gets to the job creation condition.

A.2.2 Nash Wage Bargaining

The wage in a Nash wage bargaining process solves $w_t = \arg\max \ (W^e_t - W^n_t)^\xi P_t^{1-\xi}$. The FOC is

$$\xi P_t \frac{\partial (W^e_t - W^n_t)}{\partial w_t} + (1-\xi)(W^e_t - W^n_t)\frac{\partial P_t}{\partial w_t} = 0.$$

Calculating the derivatives and rearranging, the FOC becomes

$$\xi P_t = (1-\xi)(W^e_t - W^n_t) \tag{A.15}$$

The surplus of a match for the firm is

$$P_t = af'(l_t) - w_t - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + \beta(1-s)P_{t+1}$$

and for the worker it is

$$W^e_t - W^n_t = w_t - z + \beta(1 - s - \theta_t q(\theta_t))(W^e_{t+1} - W^n_{t+1}).$$

Using (A.15), I can write the surplus of a match for the worker also as

$$W^e_{t+1} - W^n_{t+1} = \frac{\xi}{1-\xi} P_{t+1}.$$

If I plug in the surplus of a match for the entrepreneur and the worker, the FOC from wage bargaining becomes

$$\begin{aligned}
& \frac{\xi}{1-\xi}\left[af'(l_t) - \frac{\partial w_t(a,l_t)}{\partial l_t} l_t - w_t(a,l_t) + \beta(1-s)P_{t+1}\right] \\
= \; & w_t(a,l_t) - z + \beta\left[1 - s - \theta_t q(\theta_t)\right]\frac{\xi}{1-\xi}P_{t+1} \\
\Leftrightarrow \; & \frac{\xi}{1-\xi}\left[af'(l_t) - \frac{\partial w_t(a,l_t)}{\partial l_t} l_t - w_t(a,l_t)\right] = w_t(a,l_t) - z - \beta\theta_t q(\theta_t)\frac{\xi}{1-\xi}P_{t+1}.
\end{aligned}$$

Using $P_{t+1} = \frac{\gamma}{\beta q(\theta_t)}$ from the job creation condition, it becomes

$$\Leftrightarrow \; \frac{\xi}{1-\xi}\left[af'(l_t) - \frac{\partial w_t(a,l_t)}{\partial l_t} l_t - w_t(a,l_t)\right] = w_t(a,l_t) - z - \theta_t \frac{\xi}{1-\xi}\gamma. \quad \text{(A.16)}$$

The left-hand side of (A.16) needs to be constant across all firms because

$$af'(l_{t+1}) - \frac{\partial w_{t+1}(a,l_{t+1})}{\partial l_{t+1}} l_{t+1} - w_{t+1}(a,l_{t+1}) = \gamma\left[\frac{1}{\beta q(\theta_t)} - \frac{(1-s)}{q(\theta_{t+1})}\right]$$

must hold. Therefore, the right-hand side of (A.16) also must be constant across firms, which implies that $w_t(a,l_t)$ is the same for all firms. Using Equation (A.16) and rearranging terms, the wage curve is derived as

$$w_t(a,l_t) = \xi\left[af'(l_t) - \frac{\partial w_t(a,l_t)}{\partial l_t} l_t + \gamma\theta_t\right] + (1-\xi)z.$$

A.2.3 Efficiency in Market Equilibrium

In the market equilibrium, I have

$$\begin{aligned}
w &= \xi\left[af'(l) - \frac{\partial w}{\partial l} l + \gamma\theta\right] + (1-\xi)z \\
&= z + \xi\left[af'(l) - \frac{\partial w}{\partial l} l - z\right] + \xi\gamma\theta.
\end{aligned}$$

The optimality condition for hiring is

$$\frac{\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l) - \frac{\partial w}{\partial l}l - z\right], \tag{A.17}$$

which I can rewrite as

$$af'(l) - \frac{\partial w}{\partial l}l - z = \frac{\gamma}{\beta q(\theta)} \cdot \frac{[1-\beta(1-s)+\xi\beta\theta q(\theta)]}{(1-\xi)}.$$

Hence, the wage equation reads

$$\begin{aligned} w &= z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right] + \xi\gamma\theta \\ &= z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\xi\beta\theta q(\theta) + (1-\xi)\beta\theta q(\theta)\right] \\ &= z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right]. \end{aligned} \tag{A.18}$$

The wage in steady state depends only on exogenously given parameters and θ, which is the same for each firm. Therefore, each firm pays the same wage. From (A.17), I also know that the total factor productivity multiplied with the marginal product of labour is

$$af'(l) = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{[1-\beta(1-s)+\xi\beta\theta q(\theta)]}{(1-\xi)} + \frac{\partial w}{\partial l}.$$

The value equations for an employed and an unemployed worker in steady state are

$$W_e = w + \beta\left[sW_n + (1-s)W_e\right] \tag{A.19}$$

and

$$W_n = z + \beta\left[\theta q(\theta)W_e + (1-\theta q(\theta))W_n\right]. \tag{A.20}$$

Rearranging W_n gives

$$W_n = \frac{z + \beta\theta q(\theta)W_e}{1-\beta+\beta\theta q(\theta)}.$$

Inserting this into (A.19), one can solve for W_e:

$$W_e = w + \beta s \left(\frac{z + \beta\theta q(\theta) W_e}{1 - \beta + \beta\theta q(\theta)} \right) + \beta(1-s)W_e$$

$$\Leftrightarrow \quad (1-\beta)W_e = w - \frac{\beta s(w-z)}{1 - \beta(1-s) + \beta\theta q(\theta)}.$$

Analogously, rearranging (A.19) gives

$$W_e = \frac{w + \beta s W_n}{1 - \beta(1-s)}. \tag{A.21}$$

By using (A.21), W_e can be eliminated from Equation (A.20) and one obtains

$$(1-\beta)W_n = z + \frac{\beta\theta q(\theta)(w-z)}{1 - \beta(1-s) + \beta\theta q(\theta)}.$$

For w one can insert (A.18) into both reformulated value equations and receives

$$(1-\beta)W_e = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)} \left[1 - \beta + \beta\theta q(\theta)\right] \tag{A.22}$$

and

$$(1-\beta)W_n = z + \frac{\xi}{1-\xi}\gamma\theta. \tag{A.23}$$

Reformulating the Indifference Equation

The indifference equation for the entrepreneur is

$$\bar{a}(f(l(\bar{a}))) - \gamma v(\bar{a}) - w l(\bar{a}) = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)} \left[1 - \beta + \beta\theta q(\theta)\right].$$

For the firm's steady state, I get

$$v(\bar{a}) = \frac{s l(\bar{a})}{q(\theta)}.$$

Plugging this, as well as (A.18), into the above indifference equation, I receive

$$\bar{a}(f(l(\bar{a}))) - \frac{s\gamma l(\bar{a})}{q(\theta)} - \left[z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right]\right] l(\bar{a})$$

$$= z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]$$

$$\Leftrightarrow \quad \bar{a}(f(l(\bar{a}))) - \frac{s\gamma l(\bar{a})}{q(\theta)} - \frac{s\gamma l(\bar{a})}{q(\theta)} \cdot \frac{\xi}{(1-\xi)}$$

$$= \left[z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]\right](1+l(\bar{a}))$$

$$\Leftrightarrow \quad \frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\xi)q(\theta)} = z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{\xi}{(1-\xi)}\left[1-\beta+\beta\theta q(\theta)\right]$$

$$\Leftrightarrow \quad \frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\xi)q(\theta)}$$

$$= z + \frac{\gamma}{\beta q(\theta)} \frac{\left[1-\beta+\xi\beta\theta q(\theta)\right]}{(1-\xi)} - \frac{(1-\beta)}{\beta} \cdot \frac{\gamma}{q(\theta)}$$

A.2.4 Tax Rates

The maximisation problem of the entrepreneur is

$$\max_{v_t, l_{t+1}} \; W_t^f(a, l_t) = (1-\tau_t^f)\left[af(l_t) - w_t l_t - (1+\tau_t^v)\gamma v_t\right] + \beta W_{t+1}(a, l_{t+1})$$

$$\text{s.t.} \quad l_{t+1} = (1-s)l_t + q(\theta_t)v_t.$$

The related Lagrangian function is

$$\mathcal{L} = (1-\tau_t^f)\left[af(l_t) - w_t l_t - (1+\tau_t^v)\gamma v_t\right] + \beta W_{t+1}(a, l_{t+1})$$

$$+ \lambda_t\left[(1-s)l_t + q(\theta_t)v_t - l_{t+1}\right]$$

The FOC thus are

$$\frac{\partial \mathcal{L}}{\partial v_t} = -(1-\tau_t^f)(1+\tau_t^v)\gamma + \lambda_t q(\theta_t) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial l_{t+1}} = \beta\frac{\partial W_{t+1}(a, l_{t+1})}{\partial l_{t+1}} - \lambda_t = 0,$$

which can be combined to

$$\frac{(1-\tau_t^f)(1+\tau_t^v)\gamma}{q(\theta_t)} = \beta \frac{\partial W_{t+1}(a, l_{t+1})}{\partial l_{t+1}}.$$

Using the envelope condition

$$\frac{\partial W_t(a, l_t)}{\partial l_t} = (1-\tau_t^f)\left[af'(l_t) - w_t - \frac{\partial w_t}{\partial l_t} l_t\right] + \lambda_t(1-s),$$

I derive the job creation condition

$$\begin{aligned}\frac{(1-\tau_t^f)(1+\tau_t^v)\gamma}{\beta q(\theta_t)} &= (1-\tau_{t+1}^f)\left[af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1}\right] + \lambda_{t+1}(1-s) \\ &= (1-\tau_{t+1}^f)\left[af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1}\right] \\ &\quad + \beta(1-s)\frac{(1-\tau_{t+1}^f)(1+\tau_{t+1}^v)\gamma}{\beta q(\theta_{t+1})}.\end{aligned}$$

If I assume that τ_t^f is constant over time, I can divide by $(1-\tau^f)$ and receive

$$\frac{(1+\tau_t^v)\gamma}{\beta q(\theta_t)} = af'(l_{t+1}) - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1} + \beta(1-s)\frac{(1+\tau_{t+1}^v)\gamma}{\beta q(\theta_{t+1})}.$$

Nash wage bargaining

For Nash wage bargaining, I define $P_{t+1} := \frac{(1+\tau_t^v)\gamma}{\beta q(\theta_t)}$. The wage thus solves

$$w_t = \arg\max \left(W_t^e - W_t^n\right)^{\xi} P_t^{1-\xi},$$

which results in the FOC

$$\xi P_t = (1-\xi)\left[W_t^e - W_t^n\right].$$

As

$$P_t = af'(l_t) - w_t - \frac{\partial w_t}{\partial l_t} l_t + \beta(1-s) P_{t+1}$$

and

$$W_t^e - W_t^n = w_t - z + \beta\left[1 - s - \theta_t q(\theta_t)\right]\left(W_{t+1}^e - W_{t+1}^n\right)$$
$$= w_t - z + \beta\left[1 - s - \theta_t q(\theta_t)\right]\frac{\xi}{(1-\xi)}P_{t+1},$$

the FOC from wage bargaining becomes

$$\xi\left[af'(l_t) - w_t - \frac{\partial w_t}{\partial l_t}l_t + \beta(1-s)P_{t+1}\right]$$
$$= (1-\xi)\left[w_t - z + \beta\left[1 - s - \theta_t q(\theta_t)\right]\frac{\xi}{(1-\xi)}P_{t+1}\right].$$

If I plug in $P_{t+1} := \frac{(1+\tau_t^v)\gamma}{\beta q(\theta_t)}$ and solve for the wage, it becomes

$$w_t = \xi\left[af'(l_t) - \frac{\partial w_t}{\partial l_t}l_t + (1+\tau_t^v)\gamma\theta_t\right] + (1-\xi)z.$$

Deriving $\boldsymbol{\tau^v}$

The following derivations are done in steady state. The wage in steady state can be written as

$$w = z + \xi\left[af'(l) - \frac{\partial w}{\partial l}l - z\right] + (1+\tau^v)\xi\gamma\theta$$

and the job creation condition in steady state is given with

$$\frac{(1+\tau^v)\gamma}{\beta q(\theta)} = af'(l) - w - \frac{\partial w}{\partial l}l + \beta(1-s)\frac{(1+\tau^v)\gamma}{\beta q(\theta)}.$$

By plugging in the wage, the condition for job creation is

$$\frac{(1+\tau^v)\gamma}{q(\theta)} = \frac{\beta(1-\xi)}{1-\beta(1-s)+\xi\beta\theta q(\theta)}\left[af'(l) - \frac{\partial w}{\partial l}l - z\right]. \tag{A.24}$$

I then compare this equation to the first-best job creation condition to calculate τ^v under the assumption that $\xi = \alpha$. The tax rate τ^v thus solves

$$\frac{\beta(1-\alpha)}{1-\beta(1-s)+\alpha\beta\theta q(\theta)}\left[af'(l)-z\right]$$
$$=\frac{\beta(1-\alpha)}{\left[1-\beta(1-s)+\alpha\beta\theta q(\theta)\right](1+\tau^v)}\left[af'(l)-\frac{\partial w}{\partial l}l-z\right],$$

which can be simplified to

$$(1+\tau^v)\left[af'(l)-z\right]=af'(l)-\frac{\partial w}{\partial l}l-z.$$

Solving for τ^v, one obtains

$$\tau^v=\frac{-\frac{\partial w}{\partial l}l}{af'(l)-z}.$$

Deriving $\boldsymbol{\tau^f}$

Rewriting (A.24) as

$$af'(l)-\frac{\partial w}{\partial l}l-z=\frac{(1+\tau^v)\gamma}{\beta q(\theta)}\frac{\left[1-\beta(1-s)+\xi\beta\theta q(\theta)\right]}{(1-\xi)}$$

and plugging it into the steady state wage equation, the wage becomes

$$w=z+\frac{(1+\tau^v)\gamma}{\beta q(\theta)}\cdot\frac{\xi}{(1-\xi)}\left[1-\beta(1-s)+\beta\theta q(\theta)\right].$$

The indifference equation for the entrepreneur is

$$(1-\tau^f)\left[\bar{a}f(l)-wl-(1+\tau^v)\gamma v\right]=w-\beta s\frac{(w-z)}{\left[1-\beta(1-s)+\beta\theta q(\theta)\right]}$$
$$=z+(1+\tau^v)\frac{\xi}{(1-\xi)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right].$$

Plugging in for the wage and $v(\bar{a})$ on the left-hand side and rearranging, I receive

$$\begin{aligned}
&\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))} \cdot \frac{s\gamma}{(1-\xi)q(\theta)} \\
&= \frac{\frac{1}{(1-\tau^f)} + l(\bar{a})}{1+l(\bar{a})} \cdot \left\{ \frac{(1+\tau^v)\gamma}{\beta q(\theta)} \frac{[1-\beta+\xi\beta\theta q(\theta)]}{(1-\xi)} - \frac{(1-\beta)}{\beta}\frac{(1+\tau^v)\gamma}{q(\theta)} + z \right\} \\
&+ \tau^v \frac{l(\bar{a})}{(1+l(\bar{a}))} \frac{s\gamma}{(1-\xi)q(\theta)}.
\end{aligned}$$

If I set this equation equal to the first-best indifference equation given by (3.10) from Section 3.2 and assume that $\alpha = \xi$, I obtain

$$\begin{aligned}
&z + \frac{\gamma}{\beta q(\theta)} \cdot \frac{[1-\beta+\alpha\beta\theta q(\theta)]}{(1-\alpha)} \\
&= \frac{\frac{1}{(1-\tau^f)} + l(\bar{a})}{1+l(\bar{a})} \cdot \left\{ \frac{(1+\tau^v)\gamma}{\beta q(\theta)} \frac{[1-\beta+\alpha\beta\theta q(\theta)]}{(1-\alpha)} - \frac{(1-\beta)}{\beta}\frac{(1+\tau^v)\gamma}{q(\theta)} + z \right\} \\
&+ \tau^v \frac{l(\bar{a})}{(1+l(\bar{a}))} \frac{s\gamma}{(1-\alpha)q(\theta)}.
\end{aligned}$$

Solving this for τ^f, one receives

$$\tau^f = \frac{(1-\alpha)(1-\beta)(1+\tau^v)\gamma(1+l) - [1-\beta+\alpha\beta\theta q(\theta)]\,\gamma\tau^v(1+l) - \tau^v\beta\gamma s l}{(1-\alpha)(1-\beta)(1+\tau^v)\gamma l + [1-\beta+\alpha\beta\theta q(\theta)]\,\gamma(1-\tau^v l) + (1-\alpha)\beta q(\theta)z - \tau^v\beta s\gamma l}$$

or

$$\tau^f = \frac{A(1+l) - B\tau^v(1+l) - \beta\gamma\tau^v s l}{Al + B(1-\tau^v l) - \beta\gamma\tau^v s l + (1-\alpha)\beta q(\theta)z}$$

with

$$A = (1-\alpha)(1-\beta)(1+\tau^v)\gamma \quad \text{and} \quad B = [1-\beta+\alpha\beta\theta q(\theta)]\,\gamma.$$

Entrepreneurial decision under efficient hiring

I set $\tau^f = 0$, use $\alpha = \xi$, and assume that τ^v is set as calculated above. Hence, the indifference equation for the marginal entrepreneur is

$$\bar{a}f(l) - wl - (1+\tau^v)\gamma v(\bar{a}) = z + (1+\tau^v)\frac{\alpha}{(1-\alpha)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right].$$

Plugging in for the wage and $v(\bar{a})$ on the left-hand side and rearranging, the indifference equation becomes

$$\begin{aligned}
&\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} - \tau^v\frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} \\
=\ & z + \frac{\alpha}{(1-\alpha)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right] + \tau^v\frac{\alpha}{(1-\alpha)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right] \\
=\ & z + \frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\alpha\theta q(\theta)\right]}{(1-\alpha)} - \frac{(1-\beta)}{\beta}\frac{\gamma}{q(\theta)} + \tau^v\frac{\alpha}{(1-\alpha)}\frac{\gamma}{\beta q(\theta)}\left[1-\beta+\beta\theta q(\theta)\right].
\end{aligned}$$

Rearranging further, it can be written as

$$\begin{aligned}
\frac{\bar{a}f(l(\bar{a}))}{(1+l(\bar{a}))} - \frac{l(\bar{a})}{(1+l(\bar{a}))}\cdot\frac{s\gamma}{(1-\xi)q(\theta)} &= z + \frac{\gamma}{\beta q(\theta)}\frac{\left[1-\beta+\alpha\theta q(\theta)\right]}{(1-\alpha)} \\
- \ \frac{\gamma}{\beta q(\theta)}&\left\{(1-\beta)(1+\tau^v) - \frac{\tau^v}{(1-\alpha)}\left[1-\beta\left(1-\alpha\theta q(\theta)-\frac{sl(\bar{a})}{1+l(\bar{a})}\right)\right]\right\}.
\end{aligned}$$

I then compare this indifference equation to the first-best one. There are too many entrepreneurs in the decentralised market equilibrium if the term in curly brackets is larger than one:

$$\begin{aligned}
&(1-\beta)(1+\tau^v) - \frac{\tau^v}{(1-\alpha)}\left[1-\beta\left(1-\alpha\theta q(\theta)-\frac{sl(\bar{a})}{1+l(\bar{a})}\right)\right] > 0 \\
\Leftrightarrow\ & \frac{1+\tau^v}{\tau^v} > \frac{1-\beta\left[1-\alpha\theta q(\theta)-\frac{sl(\bar{a})}{1+l(\bar{a})}\right]}{(1-\alpha)(1-\beta)} \\
\Leftrightarrow\ & \tau^v < \frac{(1-\alpha)(1-\beta)}{\alpha\left[1-\beta+\beta\theta q(\theta)\right]+\frac{\beta sl(\bar{a})}{1+l(\bar{a})}}.
\end{aligned}$$

A.2.5 Numerical Simulation

For the numerical simulation, I use the explicit production function

$$af(l) = al^{\eta}.$$

The wage curve is given with

$$w(a,l) = \xi \left[af'(l) - \frac{\partial w}{\partial l} l + (1+\tau^v)\gamma\theta \right] + (1-\xi)z.$$

From the first order condition from Nash wage bargaining, I know that

$$w(a,l) + \xi \frac{\partial w(a,l)}{\partial l} l = \xi\eta a l^{\eta-1} + \xi(1+\tau^v)\gamma\theta + (1-\xi)z. \tag{A.25}$$

I conjecture that

$$w(a,l) = c\eta a l^{\eta-1} + \xi(1+\tau^v)\gamma\theta + (1-\xi)z.$$

Plugging this into A.25, it becomes

$$\begin{aligned} &c\eta a l^{\eta-1} + \xi(1+\tau^v)\gamma\theta + (1-\xi)z + \xi c(\eta-1)\eta a l^{\eta-1} \\ &\qquad = \xi\eta a l^{\eta-1} + \xi(1+\tau^v)\gamma\theta + (1-\xi)z, \end{aligned}$$

which can be simplified to

$$\begin{aligned} & c + \xi c(\eta - 1) = \xi \\ \Leftrightarrow\ & c = \frac{\xi}{1+\xi(\eta-1)} = \frac{\xi}{1-\xi(1-\eta)}. \end{aligned}$$

The wage is thus

$$w(a,l) = \xi \left[\frac{\eta a l^{\eta-1}}{1-\xi(1-\eta)} + (1+\tau^v)\gamma\theta \right] + (1-\xi)z.$$

Abstracting from taxation, it deviates from the wage in the standard DMP model only in the fraction $\frac{1}{1-\xi(1-\eta)} > 1$ that the marginal product is multiplied with.[2]
As for the numerical simulation described in Chapter 3, I first construct the grid for the entrepreneurial ability a and a is assumed to be log normally distributed. The firm's decisions at each grid point are calculated first and described in the following. The labour demand in steady state can be derived by using the vacancy posting condition and the wage curve derived above. Labour demand at each grid point a can be calculated as

$$l(a) = \left\{ \frac{[1-\xi(1-\eta)]\,[1-\beta(1-s)]\,(1+\tau^v)\frac{\gamma}{\beta\theta^{-\alpha}} + (1+\tau^v)\xi\gamma\theta + (1-\xi)z}{(1-\xi)\eta a} \right\}^{\frac{1}{\eta-1}}.$$

Having determined $l(a)$, I can determine vacancy postings, wages, gross profits, and the entrepreneur's value equation at each grid point a:

$$v(a) = \frac{sl(a)}{\theta^{-\alpha}},$$

$$w(a) = \xi\left[\frac{\eta a l(a)^{\eta-1}}{1-\xi(1-\eta)} + (1+\tau^v)\gamma\theta\right] + (1-\xi)z,$$

$$\pi(a) = al(a)^{\eta} - w(a)l(a) - (1+\tau^v)\gamma v(a),$$

$$W^f(a) = \frac{(1-\tau^f)\pi(a) + T}{1-\beta}.$$

The value equation for the employed and the unemployed worker are calculated as

$$W^n = \frac{1}{1-\beta}\left[\frac{z + T + \xi(1+\tau^v)\gamma\theta}{1-\xi}\right],$$

$$W^e = \frac{1}{1-\beta}\left[\frac{\xi}{(1-\xi)}\frac{(1+\tau^v\gamma)}{\beta\theta^{-\alpha}}\left[1-\beta+\beta\theta^{1-\alpha}\right] + z + T\right].$$

[2] This result is in line with Cahuc et al. (2008) who argue that the CRS model in a matching framework converges to the Pissarides solution. If instead the production function is of Cobb-Douglas form and without CRS, the marginal product in the wage equation is multiplied by the fraction shown above, which includes the worker's bargaining power and the returns to scale parameter.

I then search for θ, $\bar{a}$, T, and τ^v that solve the following four equations:

$$\Phi(\bar{a}) - N - \int_{\bar{a}}^{\hat{a}} l(a) d\Phi(a) = 0,$$

$$W^f(\bar{a}) - W^e = 0,$$

$$T - \int_{\bar{a}}^{\hat{a}} \left[(1-\tau^f)\pi(a) + \tau^v \gamma v(a)\right] d\Phi(a) = 0,$$

$$\left\{ \frac{\beta(1-\xi)\left[\frac{\eta a l^{\eta-1}}{1-\xi(1-\eta)} - z\right]\left[1-\beta(1-s)+\alpha\beta\theta^{-\alpha}\right]}{\beta(1-\alpha)\left[\eta a l^{\eta-1} - z\right]\left[1-\beta(1-s)+\xi\beta\theta^{-\alpha}\right]} \right\} - 1 - \tau^v = 0$$

with $\hat{a}$ being the upper bound of the entrepreneurial ability.[3] These equations ensure that the labour market clears, the indifference equation for the marginal entrepreneur holds, the government runs a balanced budget, and that τ^v is set to restore the first-best vacancy posting. The tax rate τ^f is determined by searching for the marginal tax rate that maximizes welfare. Social welfare is calculated as

$$SW = \left[\Phi(\bar{a}) - N\right](w+T) + N(z+T) + \int_{\bar{a}}^{\hat{a}} \left[(1-\tau^f)\pi(a) + T\right] d\Phi(a).$$

A.3 Chapter 5

A.3.1 Wage Bargaining

The surplus of a match for the firm is given with

$$\frac{\partial J_t(l_t, a)}{\partial l_t} = \eta a l_t^{\eta-1} - w_t(l_t, a) - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + (1-s)\frac{\gamma v_t}{q(\theta_t)}$$

and the surplus of a match for the worker is given with

$$W_t^e(l_t, a) - D_t = w_t(l_t, a) + \beta(\delta + (1-\delta)s)D_{t+1}$$
$$+ \beta(1-\delta)(1-s)W_{t+1}^e(l_{t+1}, a) - D_t.$$

[3] For $al^{\eta-1}$, I can use any gridpoint a because the marginal product of labour is constant across firms.

The wage is determined via a Nash wage bargaining process and solves

$$w_t(l_t, a) = \arg\max \left[W_t^e(l_t, a) - D_t\right]^{\xi} \left[\frac{\partial J_t(l_t, a)}{\partial l_t}\right]^{1-\xi}.$$

The first order condition from wage bargaining hence is

$$\xi \left[W_t^e - D_t\right]^{\xi-1} \frac{\partial \left[W_t^e - D_t\right]}{\partial w_t} \left[\frac{\partial J_t}{\partial l_t}\right]^{1-\xi} + (1-\xi)\left[W_t^e - D_t\right]^{\xi} \left[\frac{\partial J_t}{\partial l_t}\right]^{-\xi} \frac{\partial \left(\frac{\partial J_t}{\partial l_t}\right)}{\partial w_t}$$
$$= 0,$$

which simplifies to

$$\xi \left[\frac{\partial J_t(l_t, a)}{\partial l_t}\right] = (1-\xi)\left[W_t^e(l_t, a) - D_t\right].$$

If I plug in for the respective surplus, I obtain

$$\xi \left[\eta a l_t^{\eta-1} - w_t(l_t, a) - \frac{\partial w_t(a, l_t)}{\partial l_t} l_t + (1-s)\frac{\gamma v_t}{q(\theta_t)}\right]$$
$$= (1-\xi)\left[w_t(l_t, a) - X(l_{t+1}, a)\right]$$

with

$$X(l_{t+1}, a) = D_t - \beta(\delta + (1-\delta)s)D_{t+1} - \beta(1-\delta)(1-s)W_{t+1}^e(l_{t+1}, a).$$

Rearranging terms, one receives

$$w_t(l_t, a) + \xi \frac{\partial w_t(a, l_t)}{\partial l_t} l_t$$
$$= \xi \eta a l_t^{\eta-1} + \xi(1-s)\frac{\gamma v_t}{q(\theta_t)} + (1-\xi)X(l_{t+1}, a). \quad \text{(A.26)}$$

To solve further for the wage, the conjecture is that

$$w_t(l_t, a) = c\eta a l_t^{\eta-1} + \xi(1-s)\frac{\gamma v_t}{q(\theta_t)} + (1-\xi)X(l_{t+1}, a),$$

and thus

$$\frac{\partial w_t(a, l_t)}{\partial l_t} = c\eta(\eta - 1)al_t^{\eta-2}.$$

Inserting this into (A.26), one obtains

$$c\eta al_t^{\eta-1} + \xi(1-s)\frac{\gamma v_t}{q(\theta_t)} + (1-\xi)X_{t+1}(l_{t+1}, a) + \xi c\eta(\eta-1)al_t^{\eta-1}$$
$$= \xi\eta al_t^{\eta-1} + \xi(1-s)\frac{\gamma v_t}{q(\theta_t)} + (1-\xi)X(l_{t+1}, a).$$

This simplifies to

$$c + \xi c(\eta - 1) = \xi \quad \Leftrightarrow \quad c = \frac{\xi}{1 - \xi(1-\eta)}$$

and the wage therefore becomes

$$w_t(l_t, a) = \xi\left[\frac{\eta al_t^{\eta-1}}{1-\xi(1-\eta)} + (1-s)\frac{\gamma v_t}{q(\theta_t)}\right] + (1-\xi)X(l_{t+1}, a).$$

A.3.2 Steady State Equilibrium

The job creation condition in steady state is

$$\frac{\gamma v(l,a)}{q(\theta)} = \beta(1-\delta)\left[\eta al^{+\eta-1} - \frac{\partial w(l^+, a)}{\partial l^+}l^+ - w(l^+, a) + (1-s)\frac{\gamma v(l^+, a)}{q(\theta)}\right]$$

and the wage is derived as

$$w(l, a) = \xi\left[\frac{\eta al^{\eta-1}}{1-\xi(1-\eta)} + (1-s)\frac{\gamma v(l,a)}{q(\theta)}\right] + (1-\xi)X(l^+, a).$$

To solve for $v(l, a)$, I plug the wage and its derivative with respect to l into the job creation condition:

$$\frac{\gamma v(l,a)}{q(\theta)} = \beta(1-\delta)\Bigg[\eta al^{+\eta-1} - \frac{\xi\eta(\eta-1)al^{+\eta-1}}{1-\xi(1-\eta)} - \frac{\xi\eta al^{+\eta-1}}{1-\xi(1-\eta)} - \xi(1-s)\frac{\gamma v(l^+, a)}{q(\theta)}$$
$$- (1-\xi)X(l^{++}, a) + (1-s)\frac{\gamma v(l^+, a)}{q(\theta)}\Bigg].$$

This simplifies to

$$\frac{\gamma v(l,a)}{q(\theta)} = \beta(1-\xi)(1-\delta)\left[\frac{\eta a l^{+\eta-1}}{1-\xi(1-\eta)} - X(l^{++},a) + (1-s)\frac{\gamma v(l^{+},a)}{q(\theta)}\right].$$

Dividing by $\frac{\gamma}{q(\theta)}$, one obtains

$$v(l,a) = \beta(1-\xi)(1-\delta)\frac{q(\theta)}{\gamma}\left[\frac{\eta a l^{+\eta-1}}{1-\xi(1-\eta)} - X(l^{++},a) + (1-s)\frac{\gamma v(l^{+},a)}{q(\theta)}\right].$$

Target Size

In steady state, the value for $X(l^*,a)$ is given with

$$X(l^*,a) = \left[1-\beta(\delta+(1-\delta)s)\right]D - \beta(1-\delta)(1-s)W^e.$$

If the firm at which a worker is employed has reached its target size, the value equation for that worker can be written as

$$W^e = \frac{w(l^*,a) + \beta\left[\delta+(1-\delta)s\right]D}{1-\beta(1-\delta)(1-s)}.$$

Plugging this into $X(l^*,a)$, it becomes

$$\begin{aligned} X(l^*,a) = {} & \left[1-\beta(\delta+(1-\delta)s)\right]D \\ & - \frac{\beta(1-\delta)(1-s)}{1-\beta(1-\delta)(1-s)}\Big[w(l^*,a) + \beta\left[\delta+(1-\delta)s\right]D\Big], \end{aligned}$$

which can be simplified to

$$X(l^*,a) = \frac{(1-\beta)}{1-\beta(1-\delta)(1-s)}D - \frac{\beta(1-\delta)(1-s)}{1-\beta(1-\delta)(1-s)}w(l^*,a).$$

Hence, the equation that determines $l^*(a)$ can be written as

$$
\begin{aligned}
\frac{sl^*(a)}{q(\theta)} &= \frac{\beta(1-\xi)(1-\delta)q(\theta)}{\gamma\left[1-\beta(1-\xi)(1-\delta)(1-s)\right]}\left[\frac{\eta a l^*(a)^{\eta-1}}{1-\xi(1-\eta)} - X(l^*,a)\right] \\
&= \frac{\beta(1-\xi)(1-\delta)q(\theta)}{\gamma\left[1-\beta(1-\xi)(1-\delta)(1-s)\right]} \\
&\quad \cdot \left[\frac{\eta a l^*(a)^{\eta-1}}{1-\xi(1-\eta)} - \frac{(1-\beta)}{1-\beta(1-\delta)(1-s)}D + \frac{\beta(1-\delta)(1-s)}{1-\beta(1-\delta)(1-s)}w(l^*,a)\right].
\end{aligned}
$$

A.3.3 Taxation

The Lagrangian function for the entrepreneur's maximisation problem is

$$
\begin{aligned}
\mathcal{L} = (1-\tau^f)\left[a l_t^\eta - w_t(l_t,a)l_t - 0.5(1+\tau_t^v)\gamma v_t^2\right] + T_t + \beta(1-\delta)J_{t+1}(l_{t+1},a) \\
+ \beta\delta D_{t+1} + \lambda_t\left[(1-s)l_t + q(\theta_t)v_t - l_{t+1}\right].
\end{aligned}
$$

One receives the following first order conditions:

$$
\begin{aligned}
\frac{\partial\mathcal{L}}{\partial v_t} &= -(1-\tau^f)(1+\tau_t^v)\gamma v_t + \lambda_t q(\theta_t) = 0, \\
\frac{\partial\mathcal{L}}{\partial l_{t+1}} &= \beta(1-\delta)\frac{\partial J_{t+1}}{\partial l_{t+1}} - \lambda_t = 0,
\end{aligned}
$$

which can be combined to

$$
\frac{(1-\tau^f)(1+\tau_t^v)\gamma v_t}{q(\theta_t)} = \beta(1-\delta)\frac{\partial J_{t+1}}{\partial l_{t+1}}.
$$

The Envelope condition is

$$
\begin{aligned}
\frac{\partial J_t}{\partial l_t} &= (1-\tau^f)\left[\eta a l_t^{\eta-1} - w_t - \frac{\partial w_t}{\partial l_t}l_t\right] + \lambda_t(1-s) \\
&= (1-\tau^f)\left[\eta a l_t^{\eta-1} - w_t - \frac{\partial w_t}{\partial l_t}l_t\right] + (1-s)\frac{(1-\tau^f)(1+\tau_t^v)\gamma v_t}{q(\theta_t)}.
\end{aligned}
$$

The job creation condition thus becomes

$$\frac{(1-\tau^f)(1+\tau_t^v)\gamma v_t}{q(\theta_t)} = \beta(1-\delta)(1-\tau^f) \cdot \left[\eta a l_{t+1}^{\eta-1} - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1} + (1-s)\frac{(1+\tau_{t+1}^v)\gamma v_{t+1}}{q(\theta_{t+1})}\right]$$

$$\Leftrightarrow \frac{(1+\tau_t^v)\gamma v_t}{q(\theta_t)} = \beta(1-\delta)\left[\eta a l_{t+1}^{\eta-1} - w_{t+1} - \frac{\partial w_{t+1}}{\partial l_{t+1}} l_{t+1} + (1-s)\frac{(1+\tau_{t+1}^v)\gamma v_{t+1}}{q(\theta_{t+1})}\right].$$

Wage Bargaining

The wage maximises

$$w_t(l_t,a) = \arg\max\left\{[W_t^e(l_t,a)-D_t]^{\xi}\left[\frac{\partial J_t(l_t,a)}{\partial l_t}\right]^{1-\xi}\right\}.$$

Hence, the first-order condition from wage bargaining is

$$\xi\,[W_t^e(l_t,a)-D_t]^{\xi-1}\,\frac{\partial\,[W_t^e(l_t,a)-D_t]}{\partial w_t}\left[\frac{\partial J_t(l_t,a)}{\partial l_t}\right]^{1-\xi} + [W_t^e(l_t,a)-D_t]^{\xi}\,(1-\xi)\left[\frac{\partial J_t(l_t,a)}{\partial l_t}\right]^{-\xi}\frac{\partial\left[\frac{\partial J_t(l_t,a)}{\partial l_t}\right]}{\partial w_t} = 0,$$

which simplifies to

$$\xi\frac{\partial\,[W_t^e(l_t,a)-D_t]}{\partial w_t}\left[\frac{\partial J_t(l_t,a)}{\partial l_t}\right] + [W_t^e(l_t,a)-D_t]\,(1-\xi)\frac{\partial\left[\frac{\partial J_t(l_t,a)}{\partial l_t}\right]}{\partial w_t} = 0.$$

Plugging in for the derivatives with respect to w_t, the first-order condition becomes

$$\xi(1-\tau^e)\left[\frac{\partial J_t(l_t,a)}{\partial l_t}\right] - [W_t^e(l_t,a)-D_t]\,(1-\xi)(1-\tau^f) = 0$$

$$\Leftrightarrow \xi(1-\tau^e)\left[\frac{\partial J_t(l_t,a)}{\partial l_t}\right] = (1-\xi)(1-\tau^f)\,[W_t^e(l_t,a)-D_t]\,.$$

Plugging in for the surpluses, it becomes

$$\xi(1-\tau^e)\left\{(1-\tau^f)\left[\eta a l_t^{\eta-1} - w_t - \frac{\partial w_t}{\partial l_t} l_t\right] + (1-s)\frac{(1-\tau^f)(1+\tau_t^v)\gamma v_t}{q(\theta_t)}\right\}$$

$$= (1-\xi)(1-\tau^f)\left[(1-\tau^e)w_t - (1-\tau^e)X(l_{t+1}, a)\right]$$

with

$$X(l_t, a) = \frac{D_t - \beta(\delta + (1-\delta)s)D_{t+1} - \beta(1-\delta)(1-s)W_{t+1}^e(l_{t+1}, a) - T_t}{(1-\tau^e)}.$$

Solving this for the wage, one obtains

$$w_t = \xi\left[\frac{\eta a l_t^{\eta-1}}{1-\xi(1-\eta)} + (1-s)\frac{(1+\tau_t^v)\gamma v_t}{q(\theta_t)}\right] + (1-\xi)X(l_{t+1}, a).$$

Plugging the wage into the job creation condition, the job creation condition becomes

$$\frac{(1+\tau_t^v)\gamma v_t}{q(\theta_t)} = \beta(1-\xi)(1-\delta)\left[\frac{\eta a l_{t+1}^{\eta-1}}{1-\xi(1-\eta)} - X(l_{t+2}, a) + (1-s)\frac{(1+\tau_{t+1}^v)\gamma v_{t+1}}{q(\theta_{t+1})}\right]$$

and can be solved for v_t:

$$v_t = \frac{\beta(1-\xi)(1-\delta)q(\theta_t)}{(1+\tau_t^v)\gamma}\left[\frac{\eta a l_{t+1}^{\eta-1}}{1-\xi(1-\eta)} - X(l_{t+2}, a) + (1-s)\frac{(1+\tau_{t+1}^v)\gamma v_{t+1}}{q(\theta_{t+1})}\right].$$

Steady State Equations for Numerical Simulation

The wage in steady state is given with

$$w(l, a) = \xi\left[\frac{\eta a l^{\eta-1}}{1-\xi(1-\eta)} + (1-s)\frac{(1+\tau^v)\gamma v}{q(\theta)}\right] + (1-\xi)X(l^+, a)$$

with

$$X(l^+, a) = \frac{\left[1-\beta(\delta + (1-\delta)s)\right]D - T - \beta(1-s)(1-\delta)W^e(l^+, a)}{1-\tau^e}.$$

The number of vacancies that a firm with l workers and an entrepreneur with ability a posts is

$$v(l, a) = \beta(1-\delta)(1-\xi)\left\{\frac{q(\theta)}{(1+\tau^v)\gamma}\left[\frac{\eta a (l^+)^{\eta-1}}{1-\xi(1-\eta)} - X(l^{++}, a)\right] + (1-s)v(l^+, a)\right\}.$$

The Bellman equation for an employed worker becomes

$$W^e = (1-\tau^e)w + T + \beta(\delta + (1-\delta)s)D + \beta(1-\delta)(1-s)W^e(l^+, a).$$

The target size is determined as follows: $X(l^*, a)$ is given as

$$X(l^*, a) = \frac{\left[1 - \beta(\delta + (1-\delta)s)\right] D - \beta(1-\delta)(1-s)W^e - T}{1-\tau^e}$$

and W^e for a workers employed at a firm that has reached the target size is

$$W^e = \frac{(1-\tau^e)w(l^*, a) + T + \beta(\delta + (1-\delta)s)D}{1 - \beta(1-\delta)(1-s)}.$$

Therefore,

$$X(l^*, a) =$$

$$\frac{\left[1 - \beta(\delta + (1-\delta)s)\right] D - \frac{\beta(1-\delta)(1-s)}{1-\beta(1-\delta)(1-s)} \left[(1-\tau^e)w(l^*, a) + T + \beta(\delta + (1-\delta)s)D\right] - T}{1-\tau^e}$$

which can be simplified to

$$\begin{aligned} X(l^*, a) = {} & \frac{(1-\beta)}{\left[1 - \beta(1-\delta)(1-s)\right](1-\tau^e)} D \\ & - \frac{\beta(1-\delta)(1-s)}{1-\beta(1-\delta)(1-s)} w(l^*, a) - \frac{T}{\left[1 - \beta(1-\delta)(1-s)\right](1-\tau^e)}. \end{aligned}$$

Total welfare is equal to

$$\begin{aligned} & \int_{\bar{a}}^{\hat{a}} \left\{ (1-\tau^f) \left[a l^\eta - wl - (1+\tau^v) 0.5 \gamma v^2 \right] + T \right\} d\Phi(a) \\ & \qquad + \int_{\bar{a}}^{\hat{a}} l \left[(1-\tau^e)w + T\right] d\Phi(a) + N(z+T) \end{aligned}$$

with $\hat{a}$ being the upper bound of the entrepreneurial ability.

Appendix B

Additional Tables and Figures

	Laissez-faire	With τ^v	With τ^v, τ^f
Welfare	2.6587	2.6214	2.7526
Production	2.8345	2.8992	2.8911
Vacancies	0.5534	0.6130	0.3127
Net income employed	1.6116	1.5983	0.7372
Income unemployed	0.0050	-0.0009	-0.1074
Net profits	1.0420	1.0239	2.1227
Wage payments	1.6116	1.6991	1.8830
Sunk costs	0.0092	0.0113	0.0029

Table B.1: *Aggregate values*

Table B.1 compares aggregate values in the laissez-faire equilibrium to the values in the equilibrium with only τ^v and the equilibrium with welfare-maximising τ^v and τ^f. Total welfare is measured as aggregate firm profits plus the employed workers' aggregate net incomes (net wage income plus the lump sum transfer) plus the unemployed workers' aggregate incomes. Production only includes the production within the firms and does not include home production. The income of the unemployed consists of home production and the negative lump sum transfer. Net profits include the lump sum transfer as well. The total wage payments reported above are the aggregate gross wage payments that the firms make to the workers and the sunk costs are calculated as 0.5γ multiplied by squared aggregate vacancies.

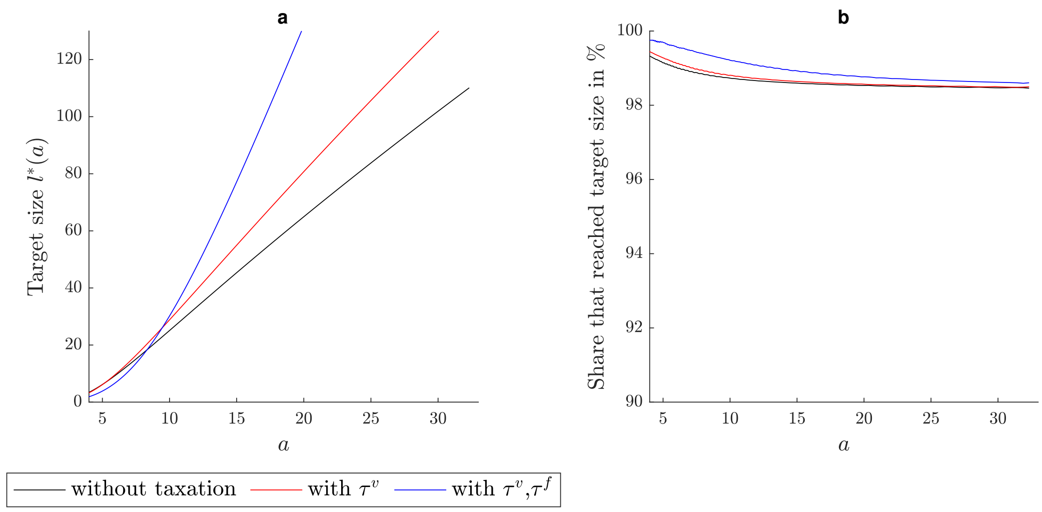

Figure B.1: *Target size and share of firms that reached that size*

Figure B.1 displays the target size and the share of firms that reached their target size in dependence on a. The black lines depict the results without correcting taxation and the red lines show the results if only vacancy posting is subsidised with $\tau^v = -42\%$. The blue lines present the respective outcomes if entrepreneurial incomes are taxed as well with the marginal tax rate $\tau^f = -152\%$.

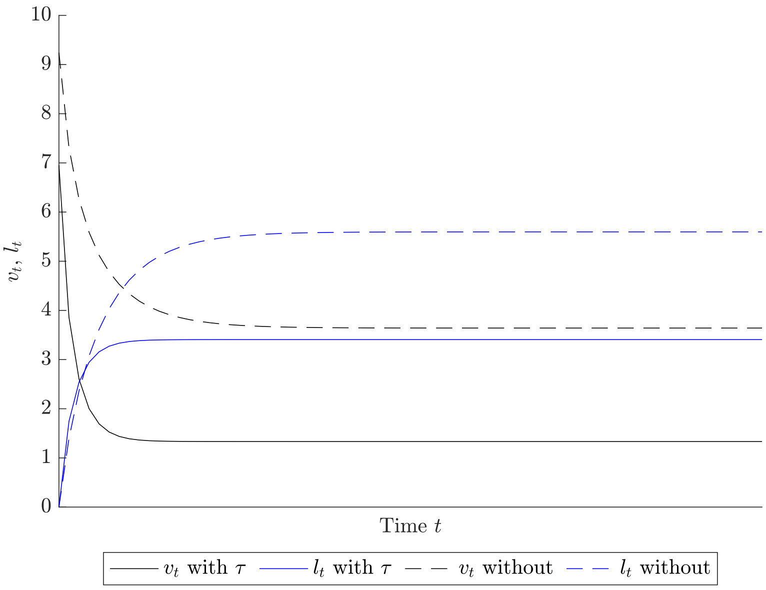

Figure B.2: *Time paths for an entrepreneur with low ability*

Figure B.2 shows the time paths of vacancies and hired employees for an entrepreneur with low entrepreneurial ability ($a = 4.8$). With correcting taxation (solid lines), the entrepreneur posts fewer vacancies than in the laissez-faire equilibrium (dashed lines). Therefore, the number of employees is lower compared to the number of workers who the firm would have hired in the laissez-faire.

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