Determination of the rapidity evolution kernel from Drell-Yan data at low transverse momenta

F. Hautmann[1,2], I. Scimemi[3] and A. Vladimirov[4]

1 Elementaire Deeltjes Fysica, Universiteit Antwerpen, B 2020 Antwerpen
2 Theoretical Physics Department, University of Oxford, Oxford OX1 3PU
3 Departamento de Física Teórica and IPARCOS, Facultad de Ciencias Físicas, Universidad Complutense Madrid, Plaza Ciencias 1, 28040 Madrid, Spain
4 Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany

Proceedings for the XXVIII International Workshop on Deep-Inelastic Scattering and Related Subjects, Stony Brook University, New York, USA, 12-16 April 2021

Abstract

We report recent results on the extraction of the rapidity evolution kernel for Sudakov processes from experimental measurements of Drell-Yan (DY) transverse momentum distributions. We discuss the role of high-precision Large Hadron Collider (LHC) data, and illustrate the interplay of this analysis with transverse momentum dependent and collinear parton density functions.

The experimental program of high-precision measurements in DY lepton-pair production at the LHC influences determinations of the rapidity evolution kernel for Sudakov processes in QCD, complementing the information which may be obtained from lower-energy DY experiments as well as semi-inclusive deep inelastic scattering. The kernel can be extracted from precise measurements of the DY transverse momentum distribution in the region of low transverse momenta. In this article we discuss recent results on this based on our work [1].

The rapidity evolution kernel was introduced by Collins and Soper [2, 3] in the context of transverse momentum dependent (TMD) factorization [4] for the DY differential cross section at low transverse momenta [5–7]. It controls the evolution in rapidity of partonic TMD distribution functions [8] as a function of mass μ and transverse coordinate b_T. Rapidity divergences are treated by cut-off methods [2,4] or by subtraction methods [9–12]. The kernel's rate of change with mass is given by the perturbatively-calculable cusp anomalous dimension (currently known numerically to four loops [13,14]). For large transverse distances $b_T \gtrsim \Lambda_{\rm QCD}^{-1}$ the kernel is non-perturbative.

Information on the non-perturbative rapidity evolution kernel may be obtained by lattice QCD methods. Studies in this direction have recently started [15–20]. Alternatively, it may be obtained by comparison of theory with experimental measurements. This has a long history,
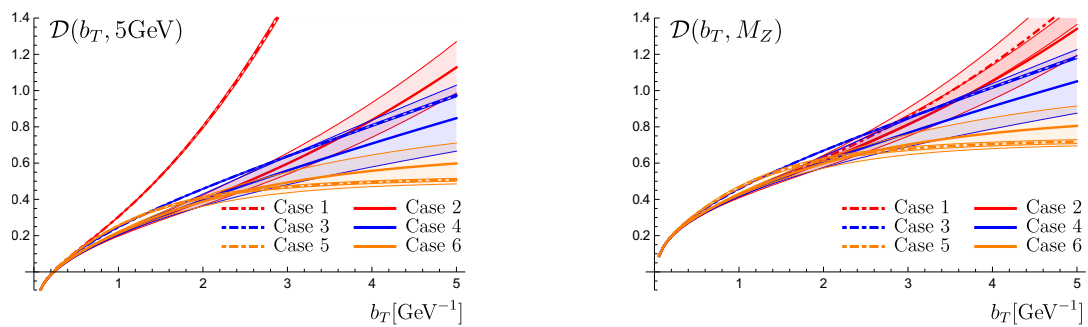
Figure 1: Rapidity evolution kernel [1] as a function of b_T for different values of mass, $\mu = 5$ GeV (left) and $\mu = M_Z$ (right), obtained from fits to LHC DY experimental data. The different curves correspond to the scenarios described in the text (see [1] for further details).

from the pioneering 'ResBos' fits [21–24] of Tevatron data to the recent 'artemide' [25] and 'NangaParbat' [26] global fits to LHC, Tevatron, RHIC and fixed-target data.

The large-distance behavior of the rapidity evolution kernel, currently subject of active investigation, has traditionally been modeled as a quadratic behavior $\propto b_T^2$. The fits [21–24], for instance, depend on such model. The analysis [1], emphasizing the bias due to this assumption, examines alternative scenarios. One of these scenarios has the perturbative quadratic behavior $\propto b_T^2$ at small b_T but a constant behavior $\propto b_T^0$ at large b_T, fulfilling the asymptotic condition $\partial \mathcal{D} / \partial \ln b_T^2 = 0$ for the kernel $\mathcal{D}$, in a similar spirit to parton saturation in the s-channel picture [27, 28] for partonic distribution functions. Another scenario, intermediate between the 'traditional' and the 'saturating' ones, has a linear behavior $\propto b_T^1$ at large b_T. We discuss below the results of fits based on these three different scenarios with, respectively, quadratic, linear or constant asymptotic large-b_T behaviors.

The determination of the rapidity evolution kernel is intertwined with the determination of the non-perturbative TMD parton distributions. While the evolution in mass of TMD distributions is controlled by perturbatively-calculable kernels, the TMD distributions at a fixed (low) mass scale are non-perturbative and may be determined from fits to experiment. This is analogous to the case of ordinary, collinear parton density functions (PDF) and for this reason, similarly to the PDF case, libraries of TMDs such as [29, 30], obtained from parameterizations and fits to data, are a necessary tool for phenomenology. The non-perturbative TMDs at low mass scales provide information about the 'intrinsic transverse momentum k_T' in the hadron and '3-dimensional' (3D) imaging of hadron structure. We discuss below the results of simultaneous fits of the non-perturbative rapidity evolution kernel and TMD distributions. In this respect, it is also interesting to ask how the determination of the rapidity evolution kernel is affected if the 3D structure is disregarded, that is, if the non-perturbative TMD distribution is simply taken to be a δ-function in k_T. Thus, besides the simultaneous fits, we also consider the alternative scenario of fitting the kernel while assuming δ-function TMD distributions.

Fig. 1 presents the results of the analysis [1] for the rapidity evolution kernel $\mathcal{D}$. The analysis is based on next-to-next-to-leading-logarithmic (NNLL) order predictions from the 'artemide' code [1, 25], fitted to experimental data for the DY transverse momentum q_T distribution from the LHC [31–37] in the low transverse momentum region specified by the cut $q_T/Q \leq \kappa$, where Q is the DY invariant mass and κ is taken to be 0.2.[1] The odd-numbered

---

[1]We have varied the cut κ on the data set in the range from 0.1 to 0.25 and verified that the results for the non-perturbative $\mathcal{D}$ and TMD parameters are stable in this range. The calculation cannot be used for κ near 1 as it does not include the matching of TMD-factorized contributions with next-to-leading and higher order contributions to

curves (dashed lines) in Fig. 1 correspond to fits performed in the three scenarios for the large-b_T asymptotics described above (1: quadratic; 3: linear; 5: constant) with δ-function TMDs, while the even-numbered curves (solid lines) correspond to simultaneous $\mathcal{D}$ and TMD fits in the three asymptotics scenarios (2: quadratic; 4: linear; 6: constant).

We see from Fig. 1 that while the kernel $\mathcal{D}$ is rather well determined for $b_T \lesssim 2$ GeV^{-1} using LHC DY q_T data, the sensitivity of current LHC measurements to the long-distance region of higher b_T is limited, which results into sizeable uncertainty bands for $b_T \gtrsim 2$ - 3 GeV^{-1}. It is shown in [1] that the scenarios with simultaneous $\mathcal{D}$ and TMD fits (even-numbered cases in Fig. 1) all give good χ^2, with comparable χ^2 values, while the corresponding results for $\mathcal{D}$ in Fig. 1 differ significantly.

We also see from Fig. 1 that the correlation between the kernel $\mathcal{D}$ and the non-perturbative TMD is significant at large b_T. This is measured by the difference between each of the even-numbered curves and its odd-numbered counterpart which has the same large-b_T asymptotic behavior but starts with a δ-distribution in k_T before TMD evolution sets in. The difference is especially large for the quadratic case in Fig. 1 (left) for $\mu = 5$ GeV, even exceeding the uncertainty bands. It is observed in [1] that it is possible to obtain good fits to LHC DY data with some of the δ-distribution scenarios, but inducing significant biases in the $\mathcal{D}$ determination.
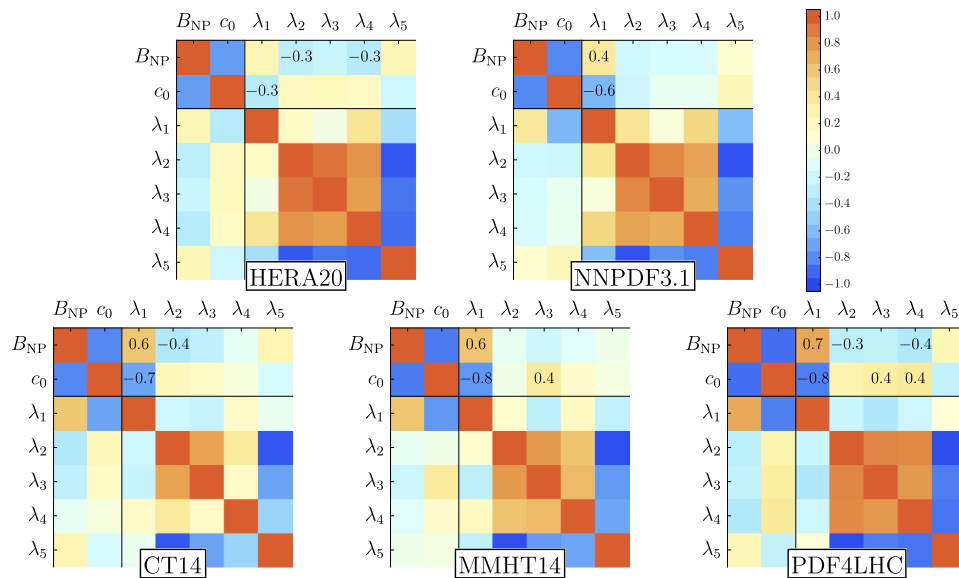
Figure 2: Correlations of $\mathcal{D}$ (B_{NP}, c_0) and TMD (λ_i) fit parameters [1] for different PDF sets. In the axes $1 = B_{NP}$, $2 = c_0$, $(3, 4, 5, 6, 7) = \lambda_{1,2,3,4,5}$. Light colors indicate low correlations; dark colors indicate high correlations. Shades of blue denote negative correlations; shades of brown denote positive correlations. (Diagonal entries are trivial.)

To improve the uncertainties seen at large b_T, one possibility is to use data from lower-energy DY experiments [42–50]. The sensitivity to large b_T here increases compared to LHC data, but the precision decreases. In [1] a global TMD fit is presented to DY data from LHC and lower-energy experiments. Useful constraints also come from low-energy semi-inclusive deep inelastic scattering (SIDIS) measurements [25].[2] A determination of the rapidity evolution

---

hard scattering matrix elements. See e.g. [38–40] for recent discussions of DY measurements with $q_T \sim Q$ in the context of analytic [4] and Monte Carlo [41] matching.

[2]This is however at the price of additional non-perturbative parameters from TMD fragmentation. See e.g. discussions in [51, 52] in the context of future lepton-hadron collider programs.

kernel from combined LHC + low-energy data, obtained by $\mathcal{D}$ and TMD fits and assuming linear large-b_T behavior, is found [1] to be lower than its LHC-only analogue, curve 4 in Fig. 1, and close to curve 6 within uncertainties. Another possibility is for future LHC experiments to extend the kinematic reach to regions of low DY masses in which the non-perturbative sensitivity increases, that is, measure the DY q_T, for low $q_T \ll Q$ and with fine binning $\lesssim 1$ GeV, in the so far unexplored region $Q < 40$ GeV by lowering lepton p_T thresholds.

At the same time, improving our knowledge of the rapidity evolution kernel will require that theoretical systematic uncertainties be under control. On one hand, these include perturbative uncertainties related to the expansions in the coupling α_s and the all-order logarithmic resummation. These can be evaluated by examining the dependence of the results on resummation scales as well as renormalization and factorization scales. On the other hand, uncertainties from PDFs, i.e., nonperturbative contributions of collinear origin, constitute a significant source of theoretical systematics as well. Fig. 2 illustrates the correlations among TMD parameters in the analysis [1] for different PDF sets [53–57]. We see for instance that the correlation between the parameters c_0 (controlling the long-distance behavior of the rapidity evolution kernel) and λ_1 (controlling the intrinsic transverse momentum distribution) is fairly low in the case of the HERAPDF set, but it increases in the NNPDF3.1 case, and is higher still in the CT14 and MMHT14 cases. Therefore, taking into account the role of collinear PDFs and their uncertainties systematically in TMD fits is one of the essential inputs to advances in the determination of the rapidity evolution kernel for Sudakov processes.

Acknowledgements

This study was supported by Deutsche Forschungsgemeinschaft (DFG) through the research Unit FOR 2926, "Next Generation pQCD for Hadron Structure: Preparing for the EIC", project number 30824754. I.S. is supported by the Spanish Ministry grant PID2019-106080GB-C21. This project has received funding from the European Union Horizon 2020 research and innovation program under grant agreement Num. 824093 (STRONG-2020).

References

[1] F. Hautmann, I. Scimemi and A. Vladimirov, *Non-perturbative contributions to vector-boson transverse momentum spectra in hadronic collisions*, Phys. Lett. B **806**, 135478 (2020), doi:10.1016/j.physletb.2020.135478.

[2] J. C. Collins and D. E. Soper, *Back-to-back jets in QCD*, Nucl. Phys. B **193**, 381 (1981), doi:10.1016/0550-3213(81)90339-4.

[3] J. C. Collins and D. E. Soper, *Back-to-back jets: Fourier transform from b to kT*, Nucl. Phys. B **197**, 446 (1982), doi:10.1016/0550-3213(82)90453-9.

[4] J. C. Collins, D. E. Soper and G. Sterman, *Transverse momentum distribution in Drell-Yan pair and W and Z boson production*, Nucl. Phys. B **250**, 199 (1985), doi:10.1016/0550-3213(85)90479-1.

[5] Yu. L. Dokshitzer, D. I. Dyakonov and S. I. Troyan, *Hard processes in quantum chromodynamics*, Phys. Rep. **58**, 269 (1980), doi:10.1016/0370-1573(80)90043-5.

[6] G. Parisi and R. Petronzio, *Small transverse momentum distributions in hard processes*, Nucl. Phys. B **154**, 427 (1979), doi:10.1016/0550-3213(79)90040-3.

[7] G. Curci, M. Greco and Y. Srivastava, *QCD jets from coherent states*, Nucl. Phys. B **159**, 451 (1979), doi:10.1016/0550-3213(79)90345-6.

[8] R. Angeles-Martinez et al., *Transverse Momentum Dependent (TMD) Parton Distribution Functions: Status and Prospects*, Acta Phys. Pol. B **46**, 2501 (2015), doi:10.5506/APhysPolB.46.2501.

[9] J. C. Collins and F. Hautmann, *Infrared divergences and non-lightlike eikonal lines in Sudakov processes*, Phys. Lett. B **472**, 129 (2000), doi:10.1016/S0370-2693(99)01384-2.

[10] J. C. Collins, *What exactly is a parton density?*, arXiv:hep-ph/0304122.

[11] F. Hautmann, *Endpoint singularities in unintegrated parton distributions*, Phys. Lett. B **655**, 26 (2007), doi:10.1016/j.physletb.2007.08.081.

[12] J. Collins, *Foundations of perturbative QCD*, Cambridge University Press, ISBN 9781107645257, 9781107645257, 9780521855334, 9781139097826 (2013).

[13] S. Moch, B. Ruijl, T. Ueda, J. A. M. Vermaseren and A. Vogt, *On quartic colour factors in splitting functions and the gluon cusp anomalous dimension*, Phys. Lett. B **782**, 627 (2018), doi:10.1016/j.physletb.2018.06.017.

[14] S. Moch, B. Ruijl, T. Ueda, J. A. M. Vermaseren and A. Vogt, *Four-loop non-singlet splitting functions in the planar limit and beyond*, J. High Energy Phys. **10**, 041 (2017), doi:10.1007/JHEP10(2017)041.

[15] M. A. Ebert, I. W. Stewart and Y. Zhao, *Renormalization and matching for the Collins-Soper kernel from lattice QCD*, J. High Energy Phys. **03**, 099 (2020), doi:10.1007/JHEP03(2020)099.

[16] M. A. Ebert, I. W. Stewart and Y. Zhao, *Determining the nonperturbative Collins-Soper kernel from lattice QCD*, Phys. Rev. D **99**, 034505 (2019), doi:10.1103/PhysRevD.99.034505.

[17] M. Schlemmer, A. Vladimirov, C. Zimmermann, M. Engelhardt and A. Schäfer, *Determination of the Collins-Soper Kernel from Lattice QCD*, J. High Energy Phys. **08**, 004 (2021), doi:10.1007/JHEP08(2021)004.

[18] Q. A. Zhang et al., *Lattice QCD Calculations of Transverse-Momentum-Dependent Soft Function through Large-Momentum Effective Theory*, Phys. Rev. Lett. **125**, 192001 (2020), doi:10.1103/PhysRevLett.125.192001.

[19] P. Shanahan, M. Wagman and Y. Zhao, *Lattice QCD calculation of the Collins-Soper kernel from quasi-TMDPDFs*, Phys. Rev. D **104**, 114502 (2021), doi:10.1103/PhysRevD.104.114502.

[20] P. Shanahan, M. Wagman and Y. Zhao, *Collins-Soper kernel for TMD evolution from lattice QCD*, Phys. Rev. D **102**, 014511 (2020), doi:10.1103/PhysRevD.102.014511.

[21] G. A. Ladinsky and C.-P. Yuan, *Nonperturbative regime in QCD resummation for gauge boson production at hadron colliders*, Phys. Rev. D **50**, R4239 (1994), doi:10.1103/PhysRevD.50.R4239.

[22] F. Landry, R. Brock, G. Ladinsky and C.-P. Yuan, *New fits for the non-perturbative parameters in the CSS resummation formalism*, Phys. Rev. D **63**, 013004 (2000), doi:10.1103/PhysRevD.63.013004.

[23] F. Landry, R. Brock, P. M. Nadolsky and C.-P. Yuan, *Fermilab Tevatron run-1 Z boson data and the Collins-Soper-Sterman resummation formalism*, Phys. Rev. D **67**, 073016 (2003), doi:10.1103/PhysRevD.67.073016.

[24] A. V. Konychev and P. M. Nadolsky, *Universality of the Collins–Soper–Sterman non-perturbative function in vector boson production*, Phys. Lett. B **633**, 710 (2006), doi:10.1016/j.physletb.2005.12.063.

[25] I. Scimemi and A. Vladimirov, *Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum*, J. High Energy Phys. **06**, 137 (2020), doi:10.1007/JHEP06(2020)137.

[26] A. Bacchetta, V. Bertone, C. Bissolotti, G. Bozzi, F. Delcarro, F. Piacenza and M. Radici, *Transverse-momentum-dependent parton distributions up to N3LL from Drell-Yan data*, J. High Energy Phys. **07**, 117 (2020), doi:10.1007/JHEP07(2020)117.

[27] F. Hautmann and D. E. Soper, *Parton distribution function for quarks in ans-channel approach*, Phys. Rev. D **75**, 074020 (2007), doi:10.1103/PhysRevD.75.074020.

[28] F. Hautmann and D. E. Soper, *Color transparency in deeply inelastic diffraction*, Phys. Rev. D **63**, 011501 (2000), doi:10.1103/PhysRevD.63.011501.

[29] N. A. Abdulov et al., *TMDlib2 and TMDplotter: a platform for 3D hadron structure studies*, Eur. Phys. J. C **81**, 752 (2021), doi:10.1140/epjc/s10052-021-09508-8.

[30] F. Hautmann, H. Jung, M. Krämer, P. J. Mulders, E. R. Nocera, T. C. Rogers and A. Signori, *TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions*, Eur. Phys. J. C **74**, 3220 (2014), doi:10.1140/epjc/s10052-014-3220-9.

[31] G. Aad et al., *Measurement of the Z/γ^* boson transverse momentum distribution in pp collisions at $\sqrt{s}$ TeV with the ATLAS detector*, J. High Energy Phys. **09**, 145 (2014), doi:10.1007/JHEP09(2014)145.

[32] G. Aad et al., *Measurement of the transverse momentum and ϕ^*_η distributions of Drell–Yan lepton pairs in proton–proton collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, Eur. Phys. J. C **76**, 291 (2016), doi:10.1140/epjc/s10052-016-4070-4.

[33] S. Chatrchyan et al., *Measurement of the rapidity and transverse momentum distributions of Z bosons in pp collisions at $\sqrt{s} = 7$ TeV*, Phys. Rev. D **85**, 032002 (2012), doi:10.1103/PhysRevD.85.032002.

[34] V. Khachatryan et al., *Measurement of the transverse momentum spectra of weak vector bosons produced in proton-proton collisions at $\sqrt{s} = 8$ TeV*, J. High Energy Phys. **02**, 096 (2017), doi:10.1007/JHEP02(2017)096.

[35] R. Aaij et al., *Measurement of the forward Z boson production cross-section in pp collisions at $\sqrt{s} = 7$ TeV*, J. High Energy Phys. **08**, 039 (2015), doi:10.1007/JHEP08(2015)039.

[36] R. Aaij et al., *Measurement of forward W and Z boson production in pp collisions at $\sqrt{s} = 8$ TeV*, J. High Energy Phys. **01**, 155 (2016), doi:10.1007/JHEP01(2016)155.

[37] R. Aaij et al., *Measurement of the forward Z boson production cross-section in pp collisions at $\sqrt{s} = 13$ TeV*, J. High Energy Phys. **09**, 136 (2016), doi:10.1007/JHEP09(2016)136.

[38] A. Bacchetta, G. Bozzi, M. Lambertsen, F. Piacenza, J. Steiglechner and W. Vogelsang, *Difficulties in the description of Drell-Yan processes at moderate invariant mass and high transverse momentum*, Phys. Rev. D **100**, 014018 (2019), doi:10.1103/PhysRevD.100.014018.

[39] A. Bermudez Martinez et al., *Production of Z bosons in the parton branching method*, Phys. Rev. D **100**, 074027 (2019), doi:10.1103/PhysRevD.100.074027.

[40] A. Bermudez Martinez et al., *The transverse momentum spectrum of low mass Drell–Yan production at next-to-leading order in the parton branching method*, Eur. Phys. J. C **80**, 598 (2020), doi:10.1140/epjc/s10052-020-8136-y.

[41] J. C. Collins and F. Hautmann, *Soft gluons and gauge-invariant subtractions in NLO parton-shower Monte Carlo event generators*, J. High Energy Phys. **03**, 016 (2001), doi:10.1088/1126-6708/2001/03/016.

[42] C. Aidala et al., *Measurements of $\mu\mu$ pairs from open heavy flavor and Drell-Yan in $p + p$ collisions at $\sqrt{s} = 200$ GeV*, Phys. Rev. D **99**, 072003 (2019), doi:10.1103/PhysRevD.99.072003.

[43] T. Aaltonen et al., *Transverse momentum cross section of e^+e^- pairs in the Z-boson region from $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV*, Phys. Rev. D **86**, 052010 (2012), doi:10.1103/PhysRevD.86.052010.

[44] T. Affolder et al., *Transverse Momentum and Total Cross Section of e^+e^- Pairs in the Z-Boson Region from $p\bar{p}$ Collisions at $\sqrt{s} = 1.8$ TeV*, Phys. Rev. Lett. **84**, 845 (2000), doi:10.1103/PhysRevLett.84.845.

[45] V. M. Abazov et al., *Measurement of the normalized $Z/\gamma^*- > \mu^+\mu^-$ transverse momentum distribution in $p\bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV*, Phys. Lett. B **693**, 522 (2010), doi:10.1016/j.physletb.2010.09.012.

[46] V. M. Abazov et al., *Measurement of the Shape of the Boson-Transverse Momentum Distribution in $p\bar{p} \rightarrow Z/\gamma^* \rightarrow e^+e^- + X$ events produced at $\sqrt{s}$=1.96-TeV*, Phys. Rev. Lett. **100**, 102002 (2008), doi:10.1103/PhysRevLett.100.102002.

[47] B. Abbott et al., *Measurement of the inclusive differential cross section for Z bosons as a function of transverse momentum in $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV*, Phys. Rev. D **61**, 032004 (2000), doi:10.1103/PhysRevD.61.032004.

[48] P. L. McGaughey et al., *Cross-sections for the production of high mass muon pairs from 800-GeV proton bombardment of ^2H*, Phys. Rev. D **50**, 3038 (1994), doi:10.1103/PhysRevD.50.3038.

[49] G. Moreno et al., *Dimuon production in proton-copper collisions at $\sqrt{s}$=38.8 GeV*, Phys. Rev. D **43**, 2815 (1991), doi:10.1103/PhysRevD.43.2815.

[50] A. S. Ito et al., *Measurement of the continuum of dimuons produced in high-energy proton-nucleus collisions*, Phys. Rev. D **23**, 604 (1981), doi:10.1103/PhysRevD.23.604.

[51] *Proceedings, Probing Nucleons and Nuclei in High Energy Collisions: Dedicated to the Physics of the Electron Ion Collider: Seattle (WA), United States, October 1 - November 16, 2018*. World Scientific Publishing, doi:10.1142/11684 (2020).

[52] P. Agostini et al., *The Large Hadron–Electron Collider at the HL-LHC*, J. Phys. G: Nucl. Part. Phys. **48**, 110501 (2021), doi:10.1088/1361-6471/abf3ba.

[53] R. D. Ball et al., *Parton distributions from high-precision collider data*, Eur. Phys. J. C **77**, 663 (2017), doi:10.1140/epjc/s10052-017-5199-5.

[54] H. Abramowicz et al., *Combination of measurements of inclusive deep inelastic $e^{\pm}p$ scattering cross sections and QCD analysis of HERA data*, Eur. Phys. J. C **75**, 580 (2015), doi:10.1140/epjc/s10052-015-3710-4.

[55] S. Dulat et al., *New parton distribution functions from a global analysis of quantum chromodynamics*, Phys. Rev. D **93**, 033006 (2016), doi:10.1103/PhysRevD.93.033006.

[56] L. A. Harland-Lang, A. D. Martin, P. Motylinski and R. S. Thorne, *Uncertainties on α_S in the MMHT2014 global PDF analysis and implications for SM predictions*, Eur. Phys. J. C **75**, 435 (2015), doi:10.1140/epjc/s10052-015-3630-3.

[57] J. Butterworth et al., *PDF4LHC recommendations for LHC Run II*, J. Phys. G: Nucl. Part. Phys. **43**, 023001 (2016), doi:10.1088/0954-3899/43/2/023001.