On the Construction of Column B in System A of the Astronomical Cuneiform Texts

by

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In the Astronomical Cuneiform Texts[1], vol. I, p. 45 a Column *B* is discussed. The difference column of this column is a step function the values of which are: 30° on the arc from ♍ 13 to ♓ 27 and 28; 7,30° on the remaining arc from ♓ 27 to ♍ 13. The interpretation which the ACT offers of Column *B* is primarily based on a theory of the annual movement of the sun in the ecliptic—without taking into consideration at all the movement of the moon.

O. Neugebauer, following Kugler, has pointed to a possible reconstruction of Column *B* by showing that the step function which is the "difference column" to Column *B* can be determined by means of the lengths of the yearly seasons, that is, by means of phenomena which are not at all dependent upon the movement of the moon but entirely of that of the sun.

In 1965 Aaboe[2] advanced the hypothesis that the step functions which occur in planetary texts that deal with horizon phenomena as e.g. heliacal risings have been produced as follows.

Successive heliacal risings have been marked down on the ecliptic for quite a few years. It then turns out that these phnomena are placed close together on some arcs and further apart on others; these are exactly the arcs into which the step function divides the ecliptic. And it is shown that from the distribution of these points one can reconstruct the step function.

We shall here examine whether the construction of column *B* can be explained on the basis of this hypothesis. To this end we mark on a circle, representing the ecliptic, the positions of new moons throughout many years; and the question is now whether these new-moon points (I will call them Q-points) are to be found close together on one part

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of the ecliptic but further apart on another part of the ecliptic. If this is so then it is possible to derive a step function from the distribution of the Q-points.

There are certain things which apparently—but, as we shall see, only apparently—contradict this hypothesis, that Column *B* should be based on observations of successive occurrences of new moons. The difference, $\Delta\lambda$, in longitude between consecutive occurrences of new moons is dependent upon the velocity of the sun as well as that of the moon. Here we must call attention to the fact that the motion of the sun is rather regular, in that it always assumes its maximal and minimal velocity in the same parts of the ecliptic, (in this connection we are justified in disregarding the slight motion of the apogee of the orbit of the sun) and the difference between the maximal and minimal velocity is "small". The motion of the moon is on the other hand rather irregular, in that it may assume its maximal and minimal velocity in any part of the ecliptic, and the difference between the maximal and minimal velocity is "not small". In other words the velocity of the sun is a function of its longitude, whereas the velocity of the moon is not a function of its longitude.

Intuitively one would think that the difference, $\Delta\lambda$, in longitude between consecutive new moons primarily would be determined by the variation of the velocity of the moon. If this indeed is the case the Aaboe hypothesis can not be applied to column *B*, because the distribution of the Q-points will then "follow" the velocity of the moon and hence not show a pattern which is determined by λ. From this we conclude that if the hypothesis can be applied to column *B*, then $\Delta\lambda$ must primarily depend upon the velocity of the sun.

We are interested then, in examining how the longitude λ of consecutive new moons depends upon the velocity of the sun and the moon. The first question to be put is whether it is the unevenness in the velocity of the sun or the unevenness in the velocity of the moon which primarily determines how far the sun has moved from one occurence of new moon to the next? In order to obtain an answer to this question I have drawn up two models.

Both models are geocentric; for the sake of simplicity the sun and the moon are assumed to move in circles the centre of which is the earth.

Model I: The velocity of the sun is variable whereas the velocity of the moon is constant.

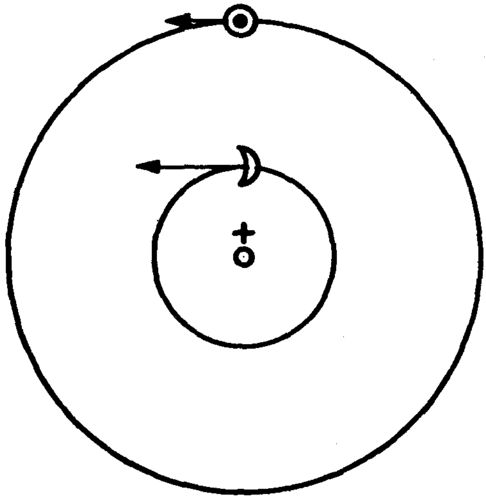

Fig. 1.

Model II: The velocity of the sun is constant whereas the velocity of the moon is variable.

By means of the Tuckermann tables I have found the maximum, the minimum and the mean velocities of the sun and the moon to be:

$v_{\odot}(\text{min}) = 0.95°/\text{day}$ $v_{\odot}(\text{max}) = 1.02°/\text{day}$ $v_{\odot}(\text{mean}) = 0.987°/\text{day}$

$v_{☾}(\text{min}) = 11.9°/\text{day}$ $v_{☾}(\text{max}) = 14.8°/\text{day}$ $v_{☾}(\text{mean}) = 13.26°/\text{day}$

In Model I where the moon moves at its mean velocity, the smallest difference in longitude between two consecutive new moons is to be found when we assume the sun to be moving at its lowest velocity. Assume t to be the length of the synodic month which will occur under this condition. The following then applies:

$$0.95\, t + 360 = 13.26\, t$$

thus

$$t = 29.2 \text{ days}$$

and therefore

$$\Delta\lambda(\text{min}) = 0.95 \cdot 29.2 = 27°.8$$

Similarly one finds

$$\Delta\lambda(\max) \qquad = 30°.0$$

In Model II, where the sun moves at its mean velocity, the smallest difference in longitude between two consecutive new moons, $\Delta\lambda$(min), is to be found when the moon moves at its highest velocity. This, however, must be understood in the following way: The moon runs through its entire spectrum of velocities in the course of one anomalistic month, which is shorter than one synodic month. For this reason only that part of a synodic month which exceeds an anomalistic month should be taken into account, and in that part of a synodic month we assign to *the moon its greatest velocity.*

Assume t to be the length of the synodic month which occurs under this condition.

The following then applies:

$$0.987\,t + 360 = 27.55\,v_{☾}\,(\text{mean}) + (t - 27.55)\,14.82$$

hence

$$t = 29.13 \text{ days}$$

and therefore

$$\Delta\lambda(\min) \qquad = 28°.8$$

Similarly one finds:

$$\Delta\lambda(\max) \qquad = 29°.2$$

Thus we see that

In model I: $\Delta\lambda(\max) - \Delta\lambda(\min) = 2.2°$
In model II: $\Delta\lambda(\max) - \Delta\lambda(\min) = 0.4°$

Consequently it is the variation of the velocity of the sun which determines the variation of $\Delta\lambda$. In other words, the velocity of the moon is of no consequence for the calculation of the difference in longitude between consecutive new moons.

From an astronomical point of view Model I as well as Model II are incorrect since the velocities of the sun and the moon vary simultaneously and independently of each other. If one wishes more correct values of $\Delta\lambda$, one must find the longitude of real consecutive new

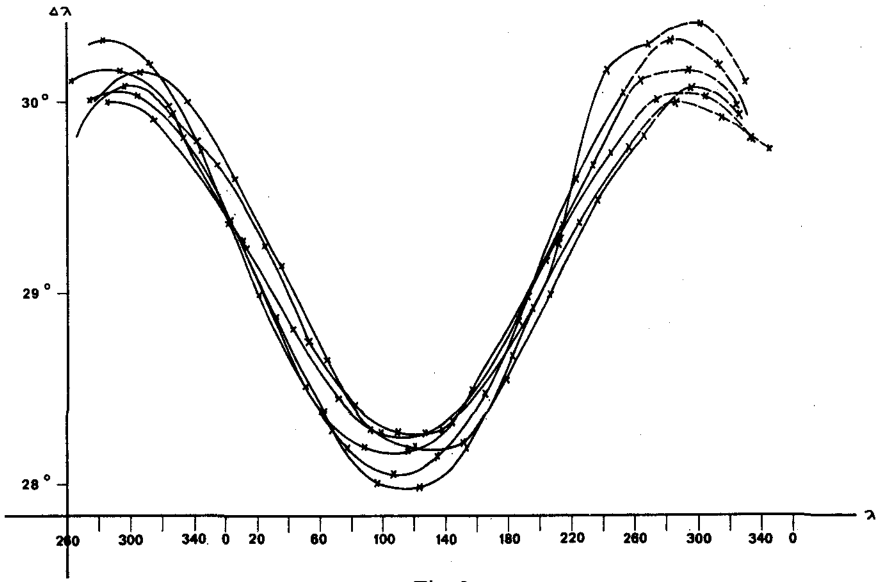

Fig. 2.

moons. By means of the National American Ephemeries I have found the longitudes

$$\lambda_0, \lambda_1, \ldots, \lambda_{256}$$

of consecutive new moons from the year 1940 to 1960, inclusive. From this we find

$$\Delta\lambda_i = \lambda_i - \lambda_{i-1}$$

We now plot the values $(\lambda_i, \Delta\lambda_i)$ in a coordinate system, and since the longitudes are reduced modulo 360 we get in this way 21 curves. For the sake of clarity we have drawn only 6 of these curves in fig. 2. This figure shows very clearly that the six curves are very close together and from this fact we conclude *that $\Delta\lambda$ is by and large a function of λ.*

This is surprising because it implies that $\Delta\lambda$ is a function of $v_{\odot}$ alone and is independent of $v_{☾}$. For $v_{\odot}$ is unambiguously derived from the longitude, λ, of the sun, while the velocity of the moon is not a function of its position on the ecliptic, as mentioned before.

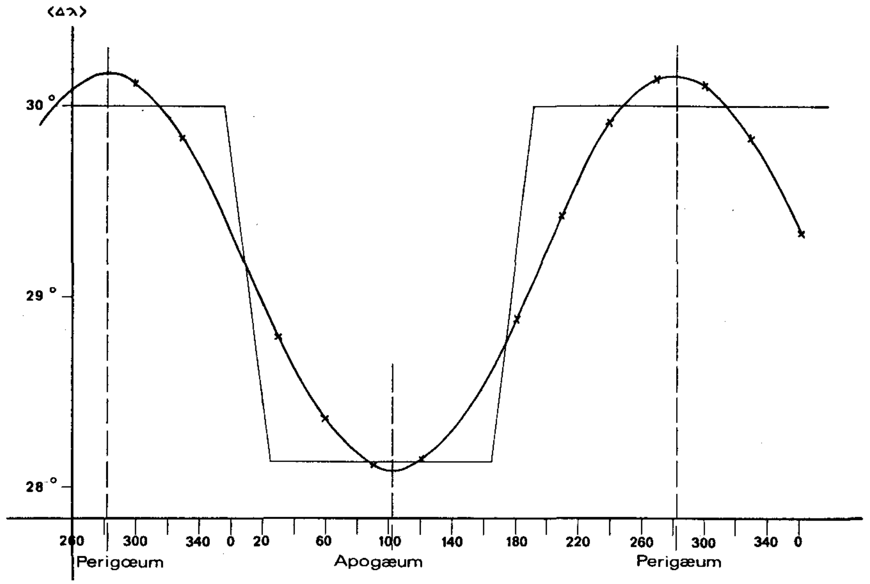

Fig. 3.

How do these results fit Column *B* in System *A*? The difference column for Column *B* is a function of λ, we call it $\sigma(\lambda)$, and the graph of $\sigma(\lambda)$ is shown on fig. 3. On fig. 3 I have also drawn a mean curve on the basis of the 21 curves mentioned above, taking into consideration the fact that in the year 1950 the apogee was at 102° whereas at the time of the text, i.e. 150 B.C. the apogee was at 66°.

This mean curve fits the graph of $\sigma(\lambda)$ so well that it is reasonable to assume that *observations of longitudes of new moon was precisely what served as the basis for Column B.*

NOTES

1. O. Neugebauer: Astronomical Cuneiform Texts. Published for the Institute for Advanced Study, Princeton, New Jersey. By Lund Humphries, 12 Bedford Square, W C 1, London, England. Here called ACT.
2. Asger Aaboe: On Period Relations in Babylonian Astronomy Centaurus 10 (1965), p. 213–231.