On the Babylonian Lunar Theory: A Construction of Column Φ from Horizontal Observations

by

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Abstract

We demonstrate that column Φ in System A of the Babylonian moon ephemerides can be derived from such horizontal phenomena as were observed and recorded by the Babylonians. Combining four of the so-called 'Lunar Six' in such a way that the effects of the oblique ascension are eliminated, we obtain a curve which oscillates, indeed, with the exact period and the approximate amplitude of Φ. Our curve (which we call Σ) also contains oscillations with the approximate period of the Saros and allows us to find the period relation which is underlying column Φ. Herewith it has been shown for the first time that the length of the anomalistic month can be derived from horizontal observations.

1. *Introduction*

Babylonian astronomy is characteristic for its way of observing and calculating celestial phenomena. When we want to describe the movement of a planet or the moon, we derive formulae enabling us to find its position on the sky at any given time. The Babylonian astronomers concentrated on special characteristic events taking place at regular time intervals. They first observed and recorded these events over a long period of time, and then were somehow

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able to construct numerical schemes, mainly using arithmetic progression, for calculating and predicting these characteristic events. [For surveys of Babylonian astronomy, see Neugebauer 1955, 1975, Waerden 1974].

In case of the moon, the time interval was one synodic month. In the columns of the lunar ephemerides, a series of different quantities is recorded for each full moon or each new moon. The numerical algorithms for computing these columns have been known for a long time [Neugebauer 1955, 1975; Aaboe 1968; Aaboe and Hamilton 1979; Waerden 1966], and in most cases the astronomical meaning of the numbers has been understood. But *how* the Babylonians were able to derive these algorithms from their observations has so far largely remained unknown.

In this paper, we attempt to find a connection between observations and the methods of computation underlying column Φ in system A.

2. *Column Φ of system A*

2.1. *Common interpretation*

The predominant role of Φ is clear: it is the second column in the ephemerides of system A and is used as a basis for calculating all other columns and quantities related to the non-uniform velocity of the moon. The numerical values of Φ_i, where i refers to the consecutive lines (i.e., successive full moons) of the ephemerides, form a linear zig-zag function [ACT, 28, 44].

The common interpretation of column Φ is based upon its close connection to column G. G gives the length of the synodic month in the first approximation where only the variable moon velocity is taken into account; it also forms a linear zig-zag function. The structures of Φ and G are such that the difference between two values of G situated 1 Saros = 223 synodic months apart equals the difference of two successive values of Φ:

$$G_{i+223} - G_i = \Phi_{i-1} - \Phi_i . \qquad (1)$$

This led B. L. van der Waerden [1966] and A. Aaboe [1968] to interpret column Φ as the length of the Saros (up to a constant) in the first approximation where the sun velocity has a constant value equal to 30°/month:

$$223 \text{ synod. months} = 1 \text{ Saros} = 6585^d + \Phi^H. \qquad (2)$$

(Φ is measured in large hours H where $1^d = 6^H = 6{,}0°$; ° = time degrees.)

Inspired by this interpretation, we have in an earlier publication [Brack-Bernsen, 1980] calculated the length $\Delta^{223}t$ of consecutive Saroi as a function of the lunation number. We found that $\Delta^{223}t$ is varying with a (mean) period *different from that of column* Φ. This is so because the length of a Saros depends more strongly on the variable sun velocity than on the variable moon velocity. This means, however, that one would never arrive at column Φ by observing the durations of Saroi.

We are convinced that column Φ must be based *directly* on observations. Our aim is here to find out which kind of observations could possibly lead to column Φ – observations which connect Φ directly to the anomalistic month.

2.2. *Babylonian observations*

From the 'Astronomical Diaries from Babylonia' [Sachs & Hunger, 1988] we know which kind of observations the Babylonian astronomers made. They observed eclipses of the sun and of the moon and noticed, e.g., 'In the night of the 13th of month *XY*, 10° before sunrise lunar eclipse' [Sachs & Hunger, 1988, 243]. Thus we can conclude that they knew at least the exact time of observed eclipses. The diaries also remark the position of the moon with respect to fixed stars, e.g. 'month VII night of the 2nd, the moon was behind α Scorpii' [Sachs & Hunger, 1988, 173]. They also contain the dates on which equinoxes and solstices took place. These dates were, however, not observed but calculated according to a fixed scheme [Neugebauer, 1975, 357]. Nevertheless, this implies that the Babylonians at some earlier time must have observed the solstices and equinoxes.

Finally, the Babylonian astronomers observed moon and sun around each full moon and new moon, concentrating on six phenomena called the 'Lunar Six' by A. Sachs [Sachs & Hunger, 1988, 20]. These phenomena consist of six time intervals, two around new moon and four around full moon, which were regularly observed and recorded. In this paper, we confine ourselves to full moon phenomena and therefore only mention the latter four, calling them the 'Lunar Four'.

In order to obtain the 'Lunar Four', one has to observe the moonset on the western horizon the last morning before opposition and the next morning just after opposition; this gives the time intervals

$$\begin{aligned} \check{S}\acute{U} &= \text{time from last moonset to sunrise before opposition,} \\ NA &= \text{time from first sunrise after opposition to moonset.} \end{aligned} \quad (3)$$

Similarly, observations of the moonrise on the eastern horizon in the two evenings nearest to opposition will give the two intervals

$$\begin{aligned} ME &= \text{time from last moonrise to sunset before opposition,} \\ GE_6 &= \text{time from first sunset after opposition to moonrise.} \end{aligned} \quad (4)$$

These time differences were all measured in units of *uš*, also called 'time degrees' by the Greek. 1 *uš* equals 4 minutes, so that*) 6,0 *uš* $= 360° = 1^d = 24^h$.

Which kind of information can we get out of such observations? In order to answer this question, let us introduce a fictitious celestial body, denoted by $\overline{\odot}$, which is situated on the ecliptic directly opposite to the sun ($\odot$), such that $\lambda_{\overline{\odot}} = \lambda_{\odot} + 180°$. At the time of opposition, moon ($\bigcirc$) and $\overline{\odot}$ have the same length. The times observed and recorded by the Babylonians are the rising times of the little arc of ecliptic, $\Delta\lambda_{\bigcirc\overline{\odot}}$ lying between $\bigcirc$ and $\overline{\odot}$ at the four times where the observations are made.

At this point our trouble starts: horizontal observations of this kind are influenced by a variety of factors. First, the rising (or

*) We use throughout the Babylonian sexagesimal number system, such that $6{,}5 = 6 \cdot 60 + 5 = 365$, etc.

setting) times depend on the length of the ecliptic arc $\Delta\lambda_{\bigcirc\overline{\odot}}$, and this length in turn depends on the time difference between opposition and sunrise (or sunset) and also on the momentary velocities of the sun and the moon. Second, the rising time of $\Delta\lambda_{\bigcirc\overline{\odot}}$ depends on the angle between ecliptic and horizon; this angle varies between 30° and 80° in Babylon. (The rising time in Babylon of an ecliptic arc of 10° varies between 6°;45 and 13°;15, depending on its position on the ecliptic.) Finally, the observed time differences depend on the latitude of the moon.

We know from ephemerides and procedure texts [e.g., No 201, ACT, pp. 226–240] that the Babylonian astronomers knew of all these factors and were able to cope with them. In the different columns of their ephemerides, they had calculated for each full moon (amongst others) the momentaneous velocities of the sun and the moon, the longitude and latitude of the moon, the time of opposition and the corrections necessary in order to obtain the oblique ascension of a given ecliptic arc (i.e. the time it takes this arc to pass the horizon). Using all these quantities, the Babylonian astronomers were able to calculate the 'Lunar Four' *ŠÚ, NA, GE* ($= GE_6$) and *ME*.

Our working assumption is that the numerical methods developed for calculating the Babylonian ephemerides are based on observations such as found in the diaries: namely observations of eclipses, equinoxes, solstices and the 'Lunar Four'. In a way, we attempt to do the opposite of the procedure used in the procedure text: starting from observations of the 'Lunar Four', we try to reconstruct some of the numerical methods used in the ephemerides.

The following question now arises: How can we possibly filter out all the different variants hidden in *ŠÚ, NA, GE* and *ME*? In the first place, we search for ways of singling out the variable moon velocity of such observations, just as we believe it has been done in column Φ.

2.3. *A construction of column Φ from the 'Lunar Four'*

The basic idea of our reconstruction is to try to eliminate the effects of the oblique ascension. We remark that if a given little ecliptic arc

$\Delta\lambda_{\bigcirc}\lambda_{\overline{\odot}}$ passes the eastern horizon under a small angle (e.g., 30°), then the same ecliptic arc will pass the western horizon 12 hours later under a steep angle (80° in our example). This means that it should be possible to reduce the effects of the oblique ascension by combining morning observations and evening observations.

Unfortunately, the Babylonian observations left over to us are too scarce and incomplete to allow a systematic analysis. Therefore we had to produce the 'observed' material ourselves. We have used modern ephemerides and a computer code [Kreitmeier, 1990] for calculating rising and setting times of sun and moon. From these we have computed the 'Lunar Four', as seen from Babylon (32°;30 N, 45° W), for a series of successive oppositions O_i over a large time interval. (Of course, the Babylonians never could obtain such a complete series of observations due to weather conditions. We shall, nevertheless, perform our analysis from our complete data and show later on, how even a less complete and more scattered set of data still allows to find the same results.)

Investigating several combinations of morning and evening observations (see also Sect. 3), we found that the following procedure leads to a curve which oscillates with the period of the moon. Let us denote the 'Lunar Four', calculated or observed at the opposition (lunation) O_i, by $\check{S}\acute{U}_i$, NA_i, GE_i, and ME_i, respectively. We then build the sum

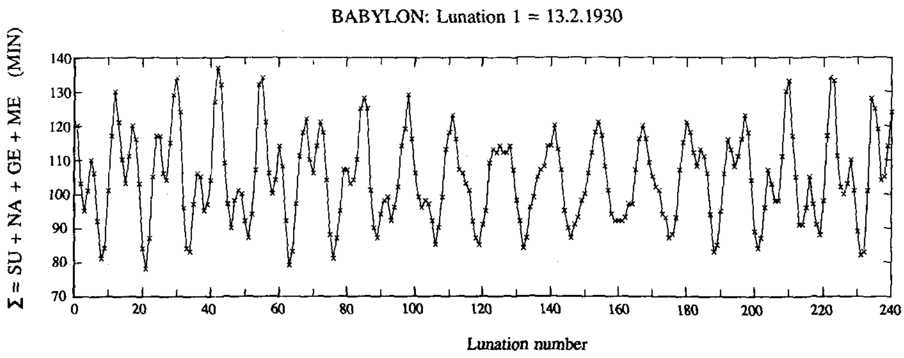

Figure 1: Sum Σ_i of the 'Lunar Four', (5), as function of the lunation number i, calculated for Babylon over a period of 240 lunations starting on the 13.2.1930 A.D. The values of Σ_i (in minutes) are shown by the crosses and connected by straight lines.

$$\Sigma_i = \check{S}\acute{U}_i + NA_i + ME_i + GE_i \tag{5}$$

and plot it as a function of the lunation number i. The result obtained for 240 lunations starting on the 13.2.1930 A.D. is shown in Figure 1. We see that Σ_i, indeed, is oscillating with a period of approximately 14 lunations which is close to P_Φ = 13;56,39,6... synodic months. It is such a curve we looked for in order to reconstruct column Φ.

Can the function Σ_i be related to Φ_i? Σ_i is measured in minutes, while Φ_i in the common interpretation is measured in large hours H (see (2)). We translate both these units into time degrees ($1^h = 15°$, $1^H = 60°$) and compare the functions Φ_i and Σ_i in Figure 2. Σ_i is as in Fig. 1; Φ_i was reduced by a constant amount of 100° and is seen as the piecewise linear zig-zag curve; both are given in time degrees versus the lunation number i. The perfect agreement of the period leads us to the following hypothesis:

The linear zig-zag function Φ_i has been derived from the sum Σ_i, (5), of the 'Lunar Four'.

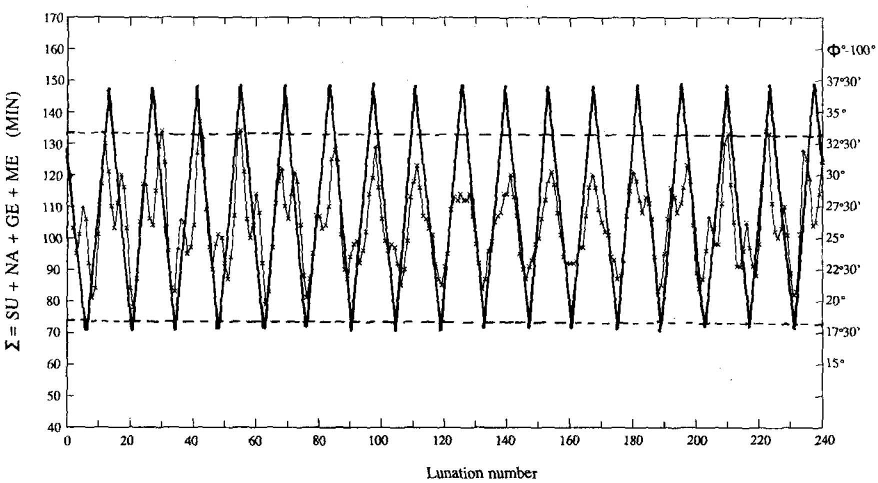

Figure 2: Σ_i, as in Fig. 1. The solid linear zig-zag line contains the values Φ_i of System A in the Babylonians' moon ephemerides, in time degrees ($1° = 4$ min), shifted by $K = 100°$ according to (6), (7). The horizontal dashed lines show where the Babylonians truncated the zig-zag function [Aaboe, 1969; Waerden, 1966].

If we neglect the small-amplitude variations in Σ_i, extending the longest straight sections to a linear zig-zag function $\hat{\Sigma}_i$, we find that $\hat{\Sigma}_i$ has a period $P_{\Sigma} \simeq 13;57$ lunations (of course, the same as that of Σ_i) and an amplitude of 19° 12′, whereas the maximum amplitude of Σ_i is 14° 30′ = 0^h58^m. Thus, we can write

$$\Phi_i \simeq \hat{\Sigma}_i + K, \qquad (6)$$

where the best fit of the two zig-zag curves is obtained for shifts K of the order

$$98° \lesssim K \lesssim 102°. \qquad (7)$$

We shall discuss in a later publication, why the Babylonians have added this constant K to $\hat{\Sigma}_i$.

In conclusion, we have found that the 'Lunar Four' can yield a linear zig-zag function $\hat{\Sigma}_i$ through the sum in (5), which can be used exactly as Φ_i is used in the ephemerides, namely for each lunation to find the location of the moon within the anomalistic month. The fact that the dominating oscillations of Σ_i have the period $P_{\bigcirc}$ very strongly supports our assumption that the effects of the oblique ascension can be practically eliminated by taking the sum of the western ($\check{S}\acute{U}+NA$) and eastern ($GE+ME$) observations. A theoretical proof of this procedure and a deeper understanding of the astronomical significance of Σ, based upon O. Schmidt's excellent treatment of the oblique ascension [Schmidt 1943], will be the object of a forthcoming publication [Schmidt & Brack-Bernsen, 1991].

2.4. Σ and the Saros

Σ has to do with the movement in elongation of the moon, and since we interpret Φ according to (6), Φ is also closely related to the elongation movement of the moon. Now, from the 'Saros text' [Neugebauer, 1957] and from the calculational scheme connecting Φ with G and Λ [Aaboe, 1968], we know that the Babylonian zig-zag function Φ is closely related to the Saros, i.e. the period of 223 synodic months.

To our big surprise, our curve Σ does, indeed, reflect the Saros: If we look more closely at Σ, we note that its waves have rather varying and bizarre structures. However, these structures repeat themselves almost identically after 223 synodic months. This is clearly demonstrated in Figure 3, where two successive periods of 223 lunations of Σ are placed on top of each other for the sake of a better comparison.

Let us go back to column Φ. This column is constructed such that

$$1{,}44{,}7 \text{ syn. months} = (1{,}44{,}7 + 7{,}28) \text{ anom. months} = 7{,}28\ P_\Phi \quad (8a)$$

As Neugebauer noticed in his Saros paper [Neugebauer, 1957], the relation (8a) combined with the Saros $S = 3{,}43$ syn. months tells us that

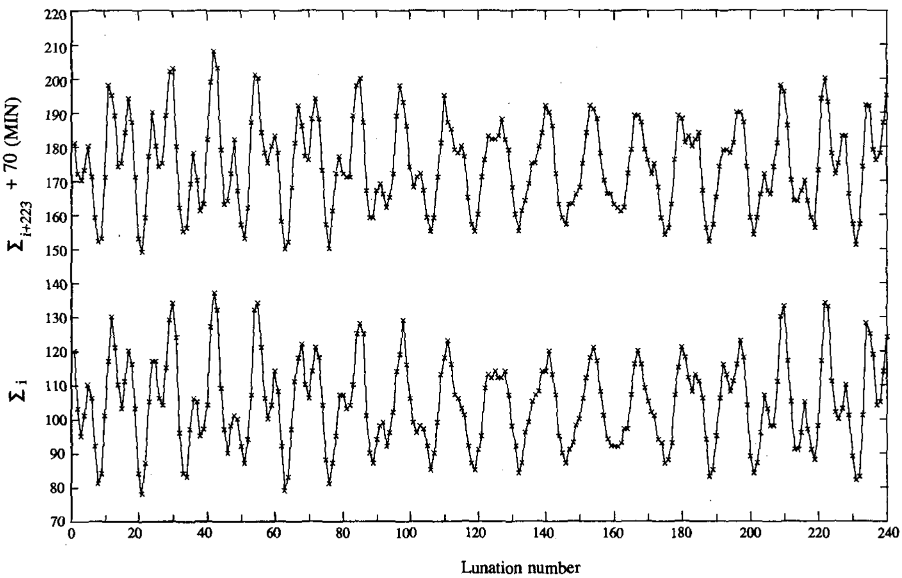

Figure 3: *Lower part*: Σ_i as in Fig. 1. *Upper part*: Σ_{i+223}, i.e. Σ over the next Saros period (1 Saros = 223 lunations), shifted by +70 minutes. Note how the fine structure repeats itself almost identically after one Saros.

$$1{,}44{,}7 \text{ syn. months} = 28 \cdot S + 3 \text{ syn. months} = 28 \cdot 16\, P_\Phi \;. \quad (8b)$$

In our analysis of the curve Σ, we would formulate these relations somewhat differently:

$$6247 \text{ lunations} = 448 \text{ 'waves'} = 448\, P_\Sigma \,, \quad (9a)$$

$$28 \cdot S + 3 \text{ lunations} = 28 \cdot 16 \text{ 'waves'} = 448\, P_\Sigma \;. \quad (9b)$$

From (8b) we see that 28 Saroi are almost equal to $28 \cdot 16\ P_\Phi$, being only 3 lunations shorter. (All this is, of course, well known – we just transform this knowledge into a form which allows us to compare Φ with our Σ.) (8b) tells us that if we were to look at the Φ curve 28 Saroi apart, we would have the situation depicted in Figure 4.

Do the lunation points on our curve Σ behave in a similar manner? The computer program we used is not accurate enough

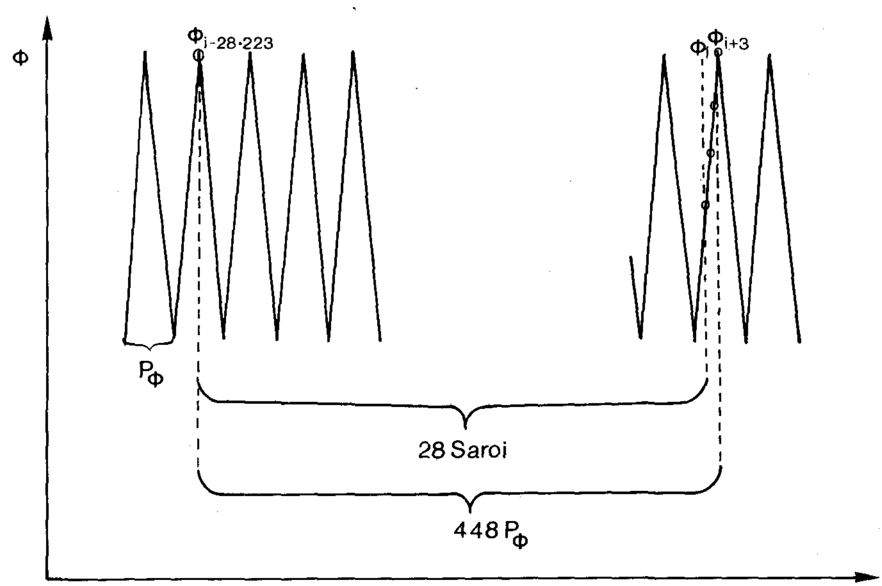

Figure 4: Two sections of the zig-zag function Φ_i, 28 Saroi = 28 · 223 lunations apart. Note how the maxima are shifted by 3 lunations after this period.

for extrapolation over a period of 28 Saroi, but it covers correctly some 10 Saroi. We therefore investigated the position of the lunation points on the curve Σ at time intervals of 4, 6, 7, 9 and $9\frac{1}{3}$ Saroi. We found that, indeed, the lunation points on Σ slide slowly backwards with respect to the extrema of Σ when going forward over several Saroi, exactly as they do on the zig-zag curve Φ.

If our interpretation of Φ given in (6) is correct, we can go one step further and try to find period relations by comparing carefully the lunation points on Σ over large time intervals. Choosing pairs of points which are situated analogously, but several Saroi apart, it is very easy to count the number of main periods P_Σ in between. As an example, we show in Figure 5 two sections of Σ_i, about $9\frac{1}{3}$ Saroi apart. The exact repetition of the fine structure of the lunation points after 2078 lunations leads to the period relation.

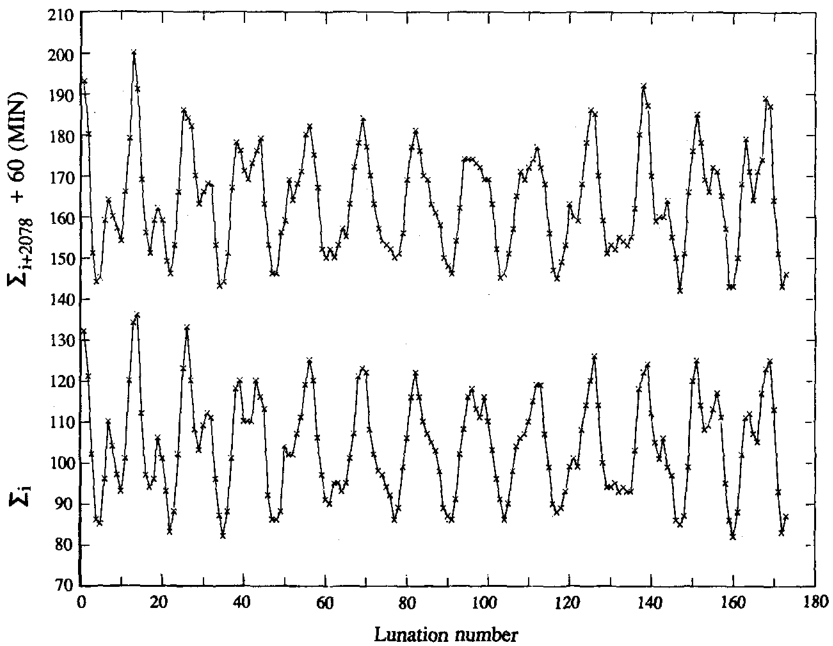

Figure 5: Two sections of the curve Σ_i, similarly as in Fig. 3. *Below*: First lunation on the 9.12.1821 A.D. *Above*: 2078 lunations $\simeq 9\frac{1}{3}$ Saroi later; this time interval encompasses exactly 149 periods P_Σ.

$$2078 \text{ lunations} = 149\, P_{\Sigma}\,. \tag{10a}$$

This corresponds to

$$448\, P_{\Sigma} = \frac{2078}{149} \cdot 448 \text{ syn. months} = 6247.9 \text{ syn. months}\,. \tag{10b}$$

One very characteristic structure on Σ, occurring exactly again between two lunation points some 4 Saroi apart, leads to

$$63\, P_{\Sigma} = 878.5 \text{ lunations}\,, \tag{11a}$$

giving

$$448\, P_{\Sigma} = 6247 \text{ syn. months}\,. \tag{11b}$$

Our curve Σ is, indeed, loaded with information!

In summary, the period of Σ is that of a function which tabulates the moon velocity once each full moon. Σ reflects the Saros, and the lunation points on the curve allow us to determine good period relations for the linear zig-zag function $\hat{\Sigma}$.

We are aware that the Babylonians cannot have made the curve analysis as we have done it so far. But they *did* have the same material of the 'Lunar Four', observed on consecutive full moons (whenever visible) over long periods of time, and we know that they were very skilled in handling numbers. We are therefore convinced that they, in some way or other, were able to extract the same kind of information from the sum Σ, (5), as we have done it above. This is possible, even if not all successive lunations have been seen without interruption: the lunation points on Σ lie quite densely, namely 14 on each period P_{Σ}. This means that only about one half of the points of a period would be sufficient to give the function Σ and, after averaging the monthly oscillations, the linear zig-zag function $\hat{\Sigma}$. Knowing that the Babylonians observed the 'Lunar Four' over many years, month after month, noting their magnitude if visible, but also noting when they were not visible to keep track of the lunation numbers, we feel sure that their material must have

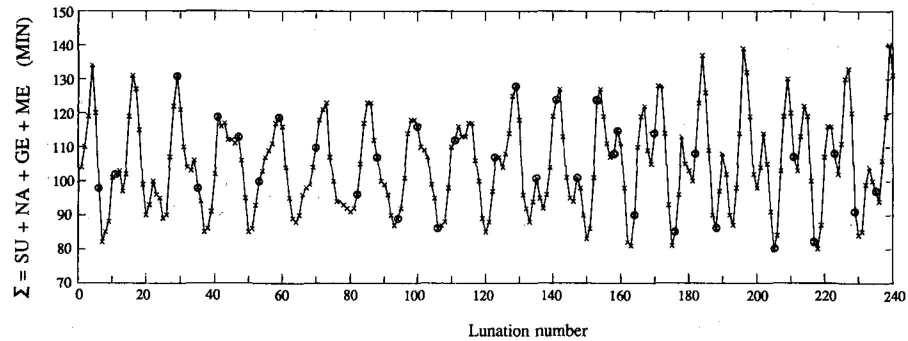

Figure 6: Σ_i as in Fig. 1 over 240 lunations, but starting from the 5.9.1971 A.D. (calculated for Babylon). The circles show the lunations on which a moon eclipse took place. (Only about half of the eclipses were visible in Babylon.)

been sufficiently comprehensive to enable them to construct the function $\hat{\Sigma}$.

Observations of the 'Lunar Four' might have been combined with accurate observations of moon eclipses (times and places), in order to find the rather precise period relation (11b) for $\hat{\Sigma}$ (and Φ). Eclipses of the moon alone would not suffice for this task. This will become evident from Figure 6 showing Σ over a period of 18 years, starting with lunation 1 on the 5.9.1971 A.D., with all moon eclipses marked by circles. They are so scarce that they will never allow to determine the period P_Σ of Σ. (Of course, only about one half of these eclipses would be visible from one and the same point on the earth.)

As a support for our conjecture that column Φ has been constructed from the sum, (5), of the 'Lunar Four', we point to the Goal-year texts mentioned by van der Waerden (1974, p. 108). We quote: "Lunar six and eclipses for the year X-18 and sums *ŠÚ*+*NA* and *ME*+*GE* for the second half of year X-19." We see that the Babylonians really were concerned with the sums *ŠÚ*+*NA* and *ME*+*GE*. They used these functions to produce astronomical predictions. In the following Section we shall see that more interesting information can be extracted from these two functions, besides what we just have gained from their sum Σ.

3. Further information found from the 'Lunar Four'

In Figure 7 we present a compilation of the following quantities as functions of the lunation number, taken over a period of 500 lunations: $\check{S}\acute{U}$, the sums $\check{S}\acute{U}+NA$ and $ME+GE$, their difference $\Delta = ME+GE-(\check{S}\acute{U}+NA)$, and their sum Σ, (5). We see that $\check{S}\acute{U}$ alone varies rather unpredictably; the monthly oscillations are so dominating that it is practically impossible to extract any information from $\check{S}\acute{U}$ by simple inspection of the curve. (The curves of NA, GE and ME look very similar to that of $\check{S}\acute{U}$.) However, taking the sums $\check{S}\acute{U}+NA$ and $ME+GE$, we get two very similar curves which oscillate rapidly with a mean period of ca. 12;22 syn. months $\simeq P_{\odot}$, while their amplitudes vary slowly with a period

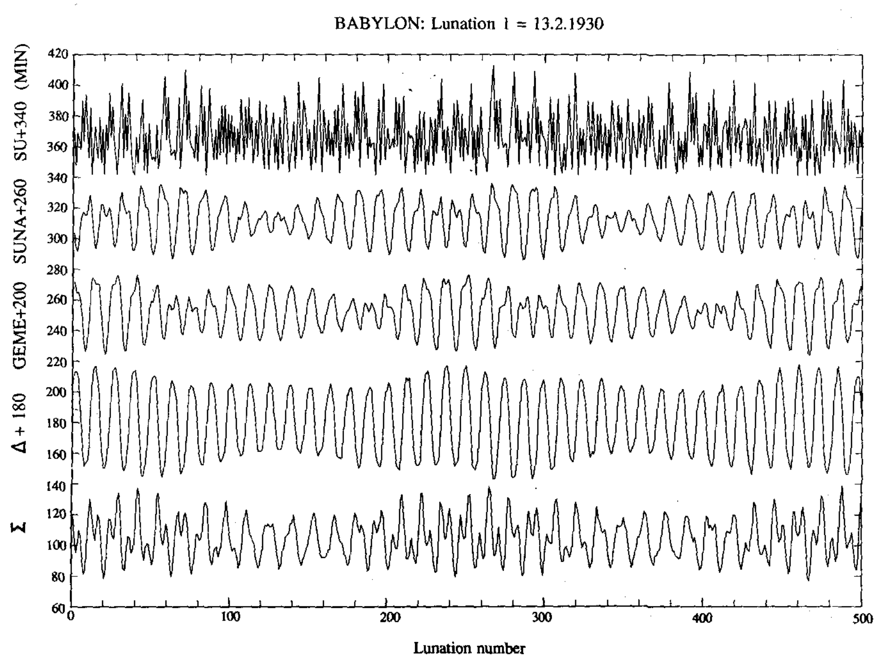

Figure 7: Graphs of different horizontal phenomena derived from the 'Lunar Four', over a period of 500 lunations. *From top to bottom*: $\check{S}\acute{U}$ (+340 min), $\check{S}\acute{U}+NA$ (+260 min), $GE+ME$ (+200 min), Δ as defined in (13) (+180 min), and Σ. Note how all amplitudes oscillate with the approximate period of a Saros (223 lunations).

$D \simeq 109.5$ syn. months which is the revolution time of the apside line. Oscillatory functions of this type have been discussed in our earlier publication [Brack-Bernsen, 1980]. They result from a superposition of two oscillating functions with the periods $P_{\odot}$ and $P_{\bigcirc}$, respectively. In the present case of $\check{S}\acute{U}+NA$ and $ME+GE$, the dominating term is the sun's influence giving the period $P_{\odot}$ of the rapid oscillations, the influence of the moon term being seen by the variation of the amplitudes with the period D:

$$D = \left(\frac{1}{P_{\odot}} - \frac{1}{P_{\bigcirc}}\right)^{-1}. \qquad (12)$$

Of course, $\check{S}\acute{U}+NA$ is also influenced by other factors than those with the periods $P_{\odot}$ and $P_{\bigcirc}$. We also remark that both $\check{S}\acute{U}+NA$ and $ME+GE$, as their sum Σ, reflect the Saros. The astronomical meaning of $\check{S}\acute{U}+NA$ will be discussed in [Brack-Bernsen & Schmidt, 1991.]

Taking the *difference* of $ME+GE$ and $\check{S}\acute{U}+NA$, we obtain a curve which we shall call Δ:

$$\Delta = ME+GE-(\check{S}\acute{U}+NA). \qquad (13)$$

This curve is very smooth, almost without any monthly disturbances, and oscillates with the period $P_{\odot}$. It reminds us very much of the zig-zag function in column A in the ephemerides of system B, determining the position of the sun (and the moon) at each lunation (i.e., at each full or at each new moon). The question therefore arises, if the lunation points on the curve Δ also slide slowly with respect to its extrema, as it is the case for the curve Σ. This would imply the possibility of determining a period relation from the curve Δ in exactly the same way as we did it above in the case of Σ. Indeed, by inspection of the lunation points about three Saroi apart on the curve Δ, we come to the following relation:

$$\begin{aligned} 3\text{ Saroi} - 1\text{ syn. month} &= 54\,P_{\odot} = 3\cdot 18\,P_{\odot}, \\ 668\text{ syn. months} &= 54\,P_{\odot}. \end{aligned} \qquad (14)$$

This period relation is identical to the one used in the abbreviated version of column A in system B [ACT, p. 71]:

$$\text{II syn. months.} = 5{,}34 \text{ syn. months} = 27\, P_{\odot}\,. \qquad (15)$$

The original column A is based upon a more accurate period relation, namely

$$\begin{aligned} 2{,}46{,}59 \text{ syn. months} &= 13{,}30\, P_{\odot}\,, \qquad (16)\\ 15\cdot[3 \text{ Saroi} - (1 \text{ syn. month} + 2^{r})] &= 15\cdot 3\cdot 18\, P_{\odot}\,, \end{aligned}$$

where $1^{r} = \frac{1}{30}$ syn. month $\simeq 1^{d}$.

It is possible that the Babylonian astronomers found their period relation (15) using the information hidden in the 'Lunar Four' in the combination Δ, similarly as we found (14). The relation (16), however, would require observations using smaller time units than the synodic month (or observations over about 45 Saroi which seems quite improbable). We rather see the relation (16) as a refinement of the relations (14) or (15) – and not (15) as an abbreviation of (16) found by rounding off, as suggested by O. Neugebauer [1975, p. 533]. We know that the Babylonians regularly calculated solstices and equinoxes; in their diaries they mentioned the days and months on which these events took place. It is exactly dates of this kind which enable us to add a minor correction to (14). (Note that here, we have to do with the day and not the month as a smallest unit.) Using exact dates of observed or calculated solstices, the Babylonian astronomers could easily have found that (14) should be corrected to (16).

4. Summary and Conclusions

We have shown that it is possible to construct column Φ from horizontal observations. The sum of the 'Lunar Four' *ŠÚ, NA, ME* and *GE* (calculated at successive lunations) defines us a function Σ which oscillates with the same period as Φ. The fact that also their amplitudes are closely the same allows us to derive Φ from Σ.

As we will show [Brack-Bernsen & Schmidt, 1991], Σ is closely connected to the movement of the moon relatively to the sun (i.e., to the moon's movement in elongation).

At this place, we remind the reader about O. Neugebauer's first surmise of the astronomical significance of column Φ [ACT, p. 44]: "Φ must describe a phenomenon very closely related to the lunar velocity", and later [ACT, p. 45]: "Perhaps Φ is obtained from the relative velocity". We are convinced that Neugebauer was right.

The occurrence of the Saros in Σ enables us to determine the period relation underlying Φ. In our interpretation of Φ, we therefore see its connection to the Saros rather as an *interior structure* – built into Φ as a consequence of the period relation (8b) – than as the origin of its construction.

Our new interpretation of column Φ as being derived from Σ will, of course, influence our understanding of the derivation of the quantities F, G, Λ, and W from column Φ. This will be discussed in detail in a forthcoming publication.

Acknowledgements

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