The “days in excess” from MUL.APIN On the “first intercalation” and “water clock” schemes from MUL.APIN

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Introduction

MUL.APIN is used as name for an astronomical-astrological compendium which in its standard form was written (in cuneiform) on both sides of two clay tablets. There exist still several more or less damaged copies, the oldest of which was written in the 7th century B.C. (Hunger and Pingree, 1989). The series takes its name from its beginning words: MUL.APIN = "the Plow" which is the first in a list of stars and constellations. This star list is followed by a series of other astronomical data.

MUL.APIN collects information on stars, sun, moon, and the planets, ending with celestial omina. A short summary of its contents may indicate how the scribes coped with celestial phenomena at the time around 1000 B.C., when the series was composed – how they rationalized and systematized nature, observing some regularities and reducing the observed structure into lists and "ideal schemes".

In the first section, some 66 fixed stars and constellations (plus the 5 then known planets) are listed within three groups, according to their positions on the sky.

The second and fourth sections deal with heliacal risings of stars and constellations. The sun moves with respect to the fixed stars. Therefore the nightly sky

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varies with the period of a year and some special phenomena occur periodically each year. An annual event of special interest was the heliacal rising of the celestial bodies: a star which is above the horizon is only visible when the sky is sufficiently dark. During the time in which the sun is near a star, the star will not be visible. The first time such a star becomes visible again in the morning shortly before sunrise is its heliacal rising. Two sections of MUL.APIN are concerned with heliacal risings. The first lists some 20 dates throughout the year together with the constellations which are supposed to rise heliacally on those dates. The other section gives (accordingly) the differences in time between the heliacal risings of these constellations. Such data can be and have been used for astronomical dating.[1]

A further section gives a list of simultaneously rising and setting constellations, and another section lists the so-called *ziqpu* stars with the calendar dates on which they culminate simultaneously with the risings of certain constellations. Finally, the path of the moon is indicated by means of 17 constellations across which the moon passes, and it is also remarked that the sun and the planets travel through the same path as the moon. There follow sections on the sun (rising points throughout the year and daylength), the moon (visibility times), and the planets (if they became visible in east or west and for how long time).

The Babylonian calendar

The Babylonian calendar was a lunar calendar. The month started at sunset on the evening when the new crescent was visible for the first time after conjunction. It lasted either 29 or 30 days, the mean duration of a lunar month being about 29.5 days. The normal Babylonian year had twelve lunar months (I, II,..., XII) and lasted ca. 354.4 days. This is more than ten days short of the solar year. In order to keep months XII and I near the spring equinox, an extra month was added (intercalated) about every third year. Such an irregular astronomical calendar is quite awkward for book keeping and for finding the times between different dates – and it was a special task to determine the beginning of a new month and hence to know if the current month had 29 or 30 days. In cuneiform texts we also find an "ideal" or "schematic" year of 360 days consisting in 12 months of 30 days each. It was not used as an actual calendar but invented as a means to facilitate calculation. It is found in use on archaic tablets with book keeping (from the third millennium B.C.[2]) and it had many uses in astronomical contexts, e.g., to indicate approximate times of annual astronomical events. In MUL.APIN certain astronomical phenomena were arranged systematically within this schematic calendar. These phenomena were among various ways used to check if the year would be normal or a leap year: an astronomical event (e.g. the heliacal rising of some star) observed to have happened a month too late would indicate the need for intercalating a 13th month.

This paper focuses on two sections of MUL.APIN, known as "the first intercalation scheme" and the "water clock". They are mutually connected and concerned with 1) the change in rising point of the sun and daylength and 2) some time intervals called "the setting of the moon" calculated for the first day of each month and "the rising of the moon" calculated for the 15th – i.e. full moon – of each month. Concentrating on new moon and full moon is a characteristic feature of Babylonian astronomy. From its early beginning to its end, these lunar phases were of special importance: it is the phenomena around full moon and new moon that were observed, predicted and later calculated. According to the "ideal schemes" of EAE XIV (Al-Rawi and George, 1991/92) and MUL.APIN – both of which use the schematic year – full moon should occur at day 15 of a month and the months should have 30 days. Many omens testify that it was taken as a good omen if the moon was seen on the "right" day, e.g., if the new crescent was seen on day 1 of the new month (= on day "31" of the old month which therefore had had 30 days) or if the real full moon happened to occur on day 15 of the Babylonian month. Alternatively it was a bad omen if an event took place too early or late, say if full moon occurred on day 13–14 or 16. (See e.g. Beaulieu, 1993 and David Brown, 2000a, pp.146–153.) The fully developed ACT astronomy also concentrated on new moon and full moon. It aimed at calculating eclipses and "visibility times" of the moon, i.e., the time interval between the moon and the sun crossing the horizon in the days around conjunction and opposition.

The visibility of the moon and its daily retardation

Before starting the investigation, I shall introduce some astronomical quantities which were of special and continual interest to the scribes: the time intervals between rising and setting of sun and moon in the days around conjunction and opposition. These time intervals were measured in UŠ, which equal our time degrees. $360^\circ =$ 1 day; thus 1 UŠ = $1^\circ \simeq$ 4 minutes. The Babylonian month started at sunset on the evening when the new crescent was visible for the first time after conjunction. Let us look at an imagined but concrete example, assuming mean values. In this ideal case, the moon sets 12 UŠ (time degrees) after sunset on the first evening of an equinoctial month; next evening (having moved 12° in longitude relative to the sun) it sets 24 UŠ after the sun, the next night it sets 36 UŠ after sunset, and so on, until it is visible during the whole of the 15th night, setting 180 UŠ after sunset. Still in the ideal case, opposition is chosen to take place at sunset (= moonrise) on day 15. On the next evening the moon, rising 12 UŠ after sunset, will be visible for only (180 - 12 =) 168 UŠ, the time of visibility decreasing by 12 UŠ/day, until it disappears (is visible for 0 UŠ) on day 30. Tablet XIV of the astrological compendium Enuma Anu Enlil (henceforth EAE) gives in Scheme B in a similar way these (very schematic) times of lunar visibility during the equinoctial month. Note that in this ideal case, the "12" has a triple meaning: it is the time from sunset to moonset on the first day of the month, it is the time from sunset to moonrise on day 16, and it is the "daily retardation of the moon", i.e., the time by which the moon is retarded from night to night in comparison to the sun. The daily retardation of the moon, here 12 UŠ, equals 1/15 of the (equinoctial) night. For other months, the daily retardation was equally taken to be 1/15 of the supposed (or schematic) night length.[3]

In reality, it is rather an exception if conjunction and opposition take place at the moment of sunset or sunrise. One must therefore differentiate between daily retardation and visibility times. The daily retardation of the moon (measured at opposition) behaves rather regularly, it is roughly a function of the month. But the visibility times of the moon are very irregular – they depend (among other things) strongly on the time of opposition (or conjunction) with respect to sunset. In the special case where the moon rises at the moment of sunset, the time from sunset to moonrise measured on the next evening gives the retardation of the moon on the day of opposition. In the normal case the daily retardation of the moon splits into two time intervals: on the evening before opposition the moon may, e.g., rise 4° before sunset and on the next evening it will rise 8° after sunset, the opposition taking place sometime between these two events. Evidently, the sum of these two time intervals measures the daily retardation of the rising moon.

As we shall see below, the time intervals from MUL.APIN, given in the "water clock" scheme, equal 1/15 of the night, so they signify the daily retardation of the moon. The time intervals are called "setting of the moon" and "rising of the moon", respectively, while the common name of the two is the "visibility time of the moon" – perhaps indicating that the scheme presents the ideal case in which conjunction or opposition is taking place at sunset, in which special case the retardation of the moon equals its visibility time (the same is true for table D of EAE XIV).

The daily retardation of the moon is a practical quantity. It can be used, e.g., for finding the visibility time of the new crescent. Imagine the case where the sky was clouded on the 30th day of the month, making horizontal observations impossible, and that on the next evening moonset took place 28UŠ after sunset. Such a large value indicates that in case of good weather, the new crescent would have been visible the evening before. If the daily retardation R of the moon was known, one can find 28UŠ - R as its time of visibility, reconstructed of the 1st day on the new month.

Later, some time before 523 B.C., the daily retardation of the moon (calculated as the sum of visibility times) was utilized by the Babylonian scribes: they had found a very elegant and exact method, the "Goal-Year" method, for the prediction of lunar phases.[4]

Conflicting interpretations of MUL.APIN

There are quite different understandings of the purpose of MUL.APIN and its uses. David Brown (2000a, p. 120), writing on MUL.APIN, argues that *"the series was not 'an astronomical' compendium, but a divinatory collection of ideal schemes. ... Mul.Apin was concerned only with celestial omens."* In contrast, H. Hunger and D. Pingree in their joint edition (1989) of MUL.APIN call it *"An Astronomical Compendium in Cuneiform"*, adding some 17 pages of astronomical commentaries.

This paper shall take a new look into two specific tables, demonstrate their coherence and also investigate how these tables of MUL.APIN were used for calculation. That day length schemes, similar to the ones we find in MUL.APIN, were actually used by Babylonian scribes for astronomical calculation and prediction, is demonstrated in several sections of TU 11.[5] These sections were copied by Anu-uballit as late as around 213 B.C., but parallel tablets dating back to the 5th century B.C. have been found in the British Museum. At that time parts of MUL.APIN were evidently used for what we would call astronomical calculations. The series had, however, been compiled much earlier around 1000 B.C. How were they used at that time? D.Brown claims that they were exclusively used for divination; but even then, the ideal schemes have their roots in astronomical observations. Therefore, to me, it seems natural to assume that they were used for astronomy as well as divination from the beginning.

The following analysis of the inner structure of some of the tables and their connections will give us answers to some previously unanswered questions. It will be shown that MUL.APIN since its composition was designed to adjust the ideal schemes to nature, when it happened that occurrences were observed to take place at times or places different from the ideal ones given in the schemes. In other words, natural phenomena were modeled by ideal schemes, but these schemes were modified in order to fit them to nature. According to my understanding of science, I would call these methods astronomical modeling of nature.[6] In conclusion, MUL.APIN was used since the beginning for astronomical calculations, which on the other hand evidently were used for divination. It makes no sense to differentiate between the two at that time. Therefore I would modify David Brown's radical statement, and go no further than calling MUL.APIN an astronomical-astrological compendium.

The "first intercalation" scheme and the "water clock" scheme of MUL.APIN investigated in this paper have often been subject of investigations and comments, most recently by Hunger and Pingree 1999 where the two schemes are presented and discussed on pp. 75–83. In 2000a D. Brown argues that the numbers used for astronomical description were far off the precise values and hence chosen for other reasons than the wish of finding the best fit to nature. Below I shall present the two schemes again and explain them according to my understanding, especially by pointing at two details which I see differently from the authors up till now: what the "additional days" also might signify, and how I think that the interpolation within the "first intercalation" and "water clock" scheme was intended to be used.

Finally I shall take up an old idea, namely that the length of solar days (at some early times, e.g. the time of the composition of EAE) may have been measured by horizontal arcs. This hypothesis would explain the values 2, 3, and 4 for the length of day at winter solstice, at equinox and at summer solstice. If these values are assumed to be arces on the horizon they are rather precise – although very inaccurate if interpreted as equinoctial time units. This approach is supported by some Old Babylonian mathematical texts and by Pingree's convincing reading of the "Path of Ea, Anu and Enlil" as intervals along the horizon over which the celestial bodies rise.[7]

The first intercalation scheme

In the so-called "first intercalation scheme" from section g of MUL.APIN (Tablet II i9 – i21, see Hunger and Pingree 1989, pp. 72–77), the yearly movement of the sun is described through the movement of its rising point (along the eastern horizon) and is connected to the changing length of solar days. This scheme follows directly after the "Path of the Moon" (section f), listing the 17 constellations through which the moon passes and touches in the course of a month, affirming that the sun and the planets also travel through the same path. Below, the first part of the scheme is reproduced.[8]

> On the 15th of month IV the Arrow becomes visible, and 4 minas is the day, 2 minas the night. The sun which rose towards the North with the head of the Lion turns and keeps moving down towards the South. The days become shorter at a rate of 40 NINDA per day, the nights longer. On the 15th of month VII the sun rises in the scales in the East, and the Moon stands in front of the Stars behind the Hired Man, 3 minas is the day and 3 minas is the night. On the 15th of month X, the Arrow becomes visible in the evening, 2 minas is the day, 4 minas the night. The sun which rose toward the South with the head of the Great One turns and keeps coming up towards the North. At a rate of 40 NINDA per day the days become longer, the nights become shorter. On the 15th of month I the Moon stands in the evening in the Scales in the East, and the Sun in the West in front of the Stars behind the Hired Man. 3 minas is the day, 3 minas is the night.

We learn here that the Babylonian astronomers knew the positions on the eastern horizon where the sun rose at the cardinal points of the year, i.e., at equinoxes and solstices. These positions could be observed directly and were referred to as the rising points of stars or zodiacal constellations.[9] The length of day and night was also given for the four cardinal points of the year. The longest day is (correctly) connected to the northern rising point of the sun and the shortest to its southern rising point.

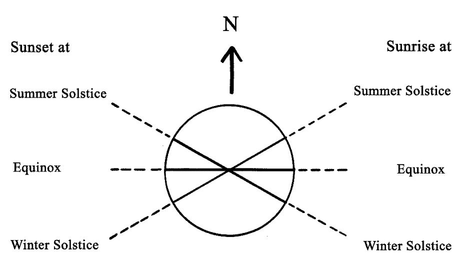

Fig. 1. The horizon at the latitude of Babylon. The directions to sunrise at summer solstice, SS, and to winter solstice, WS, were associated with those stars or constellations who rose in these directions.

Note that the dates of the cardinal points in this text are given within the so-called "ideal" or "schematic year" of 360 days = 12 months of 30 days, the solstices and equinoxes being linked to day 15 of months I, IV, VII, and X, respectively. In reality, the dates of these events will vary around the ideal dates. The text continues:

> On the 15th of month I, on the 15th of month IV, on the 15th of month VII, on the 15th on month X, you observe the risings of the Sun, the visibility time of the Moon, the appearances of the Arrow, and you will find how many days are in excess.

This passage tells us that the scribes knew that the real year deviated from the ideal one, and that they made observations for determining how many days the actual year deviated from the ideal dates. But what were these extra days used for?

Most scholars seem to have imagined only one use: the "days in excess" is the number of days by which the actual year was shifted in comparison to the ideal year. A sufficiently large number of days, found by observation, would indicate the need for an extra, intercalated, month. Therefore, this scheme is called an intercalation scheme. However, I see more information and possibility in the "additional days" and shall argue for the following interpretation: the Babylonian scribes knew that solstice did not necessarily take place at full moon so they counted how many days the real dates of the cardinal points deviated from the ideal dates, which were fixed to day 15, i.e., full moon of month I, IV, VII, and IX. Note that this number of days can be found by observing rising points of the sun on special days, as indicated in the text. Consequently the number "days in excess" could be used to correct the values given in the scheme, i.e., to calculate the appropriate lengths of night and day on the day of full moon in month I, IV, VII, and IX. And it could be used to calculate the lengths of day and night at the beginning and middle of each month. That this proposed usage makes sense within the Babylonian theory shall be demonstrated below. With this understanding I would rather call this part of MUL.APIN the "Cardinal Point" or "Day Length" scheme than the first intercalation scheme. It connects the position of the rising sun to the length of day and night.

What was the length of day and night used for?

One answer is given in the "water clock" scheme of MUL.APIN which uses the night length for finding the times of "lunar visibility".[10] This scheme is written further down on the second tablet of MUL.APIN than the "first intercalation" scheme, but they are clearly connected. In modern terms we would say that the "visibility" of the moon is a function of the night length. Several sections of the text TU 11 support the understanding that the schematic length of day and night was used for finding other astronomical quantities: in section 9–12 the time shift of eclipses one Saros apart is found as a function of the length of day or night.[11] Section 19 derives the daily retardation of the rising old moon as a function of the day length, and section 18 approximates the difference over a Saros in visibility times of the new crescent by 1/30 of the night (See TU 11, pp.70–75). In all these cases, the schematic length of day or night was used as an independent variable for finding other astronomical quantities.[12]

Therefore we concentrate on the indications of these times and reproduce the content of the "first intercalation scheme" in a condensed form below. Note that the scheme fixes the length of day and night of the four cardinal points and also their dates. However, the date of the cardinal points vary within the Babylonian lunar calendar. In the ideal calendar, the cardinal events were connected to day 15, i.e., full moon of the month in question; but in reality, it is the exception that the moon occurs in its full phase on the day of, e.g., Summer Solstice.

Cardinal point	Ideal Date	Sun rises	length of day	length of night
SS	IV day 15	towards North	4 minas	2 minas
Fall Eq	VII day 15	in the East	3 minas	3 minas
WS	X day 15	towards South	2 minas	4 minas
Spring Eq	I day 15	in the East	3 minas	3 minas

Confining itself to the cardinal points, this short scheme gives only the extrema and mean values of the lengths of day and night. From other schemes in MUL.APIN (and in EAE) we know that the Babylonian scribes used numerical functions with linear progression for specifying the duration of day and night. Accordingly we read the scheme as giving the corner points of a linear zig-zag function. Assuming linear progression within this scheme, 40 NINDA, the daily change in daylight can be found: 2 minas is the maximal change in daylight. It takes place of over a period of 6 months = 6 $\times$ 30 days = 180 days. Therefore, the daily change of daylight equals 2 minas divided by 180 = 0; 00, 40 minas = 0; 40 UŠ = 40 NINDA, exactly as the text says.[13] It can be used for interpolating in order to find day lengths between the cardinal dates. (Either to find the day length on any day of the ideal year, or to adjust the day length by full and new moon for each month of the real calendar.) The text seems to have used that

1 mina equals 1,00 UŠ = 60 UŠ,

an identity which is known from the two tables of EAE XIV, called A and B by Al-Rawi and George (1991/92 p.3-4.). The two different, but connected, units indicate times, minas by weight of water while UŠ and NINDA are measures of length.[14]

Although the 40 NINDA in the "first interpolation scheme" is written in connection with the rising point of the sun, it is now generally accepted that they signify the daily change in daylight and not the change in azimuth of the rising sun. In the "shadow table" and "water clock table" (written in the later sections k and l of MUL.APIN) the 40 NINDA occur again, here identified explicitly as "the difference for daytime and nighttime". In the "water clock" it is used to find the "daily change of (lunar) visibility". So, clearly, the interpolations within the two schemes are interconnected.[15] What is meant by the "daily change of visibility" in the "water clock scheme", and how it might have been used for interpolation, shall be presented in the next section. In anticipation of these results, one conclusion can be drawn. As mentioned above, within the "first intercalation scheme" there are two possible ways to use the 40 NINDA for interpolation. Since intercalation within the "waterclock" must aim at correcting the "visibility" at new and full moon, we can conclude backward that the intercalation in the "first intercalation scheme" was used in a similar way: to correct the day length on the first and fifteenth of each month, taking the "additional days" into account. (It may eventually also have been used to find the day length of, say, a day 4 in month M; but this usage does not utilize the extra days, and it is strange to all we know about lunar calculation and theory.)

Water clock: MUL.APIN, Table II ii 43–iii 15

The first part of the "water clock scheme" is reproduced below. To facilitate the reading, I give the units of weight in minas and 60ths thereof and write e.g. 3;50 minas instead of 3 5/6 minas and 3;10 minas instead of 3 minas 10 shekel. Similarly, I write 13;20 UŠ instead of 13 UŠ 20 NINDA.

Month	length of night on **day 1**	setting of the moon	length of night on **day 15**	rising of the moon
I	3;10 minas	12;40 UŠ	3 minas	12 UŠ
II	2;50 minas	11;20 UŠ	2;40 minas	10;40 UŠ
III	2;30 minas	10 UŠ	2;20 minas	9;20 UŠ
IV	2;10 minas	8;40 UŠ	2 minas	8 UŠ
V	2;10 minas	8;40 UŠ	2;20 minas	9;20 UŠ
VI	2;30 minas	10 UŠ	2;40 minas	10;40 UŠ
VII	2;50 minas	11;20 UŠ	3 minas	12 UŠ
VIII	3;10 minas	12;40 UŠ	3;20 minas	13;20 UŠ
IX	3;30 minas	14 UŠ	3;40 minas	14;40 UŠ
X	3;50 minas	15;20 UŠ	4 minas	16 UŠ
XI	3;50 minas	15;20 UŠ	3;40 minas	14;40 UŠ
XII	3;30 minas	14 UŠ	3;20 minas	13;20 UŠ

Note that 3;10 times 4 equals 12;40 – and that this is true for all pairs of data. Already van der Waerden (1951) has shown that it is the daily retardation of the moon which is calculated here (the text calls it the visibility of the moon). This daily retardation of the moon was calculated as a 15th of the night length: the length of the night, measured in minas, is multiplied by 4 which gives the retardation of the moon, measured in UŠ. Since 1 mina equals 60 UŠ, the result of the multiplication evidently equals the 15th of the night. The text expresses it as follows:

> 4 is the coefficient for the visibility of the Moon; you multiply 3 minas, a nighttime watch (3 MA.NA EN.NUN GI_6), by 4, and you find 12, the visibility of the Moon. You multiply 40 NINDA, the difference for daytime and nighttime, by 4, and you find 2,40, the difference of the visibility.

This final part of the text gives the rules for finding visibility times. The method is exemplified by saying "3 minas × 4 equal 12" (referring to day 15 of Month I or VII). In order to get the numbers of the whole scheme, we must understand that this rule is more general: "multiply the length of the night by four and you will find the visibility of the moon". Then the text introduces the "40 NINDA", identifying it as "the difference for daytime and nighttime" (*40 NINDA nap-pal-ti* u_4*-mi-u* GI_6).[16] We are told to multiply it with four to find the difference of the visibility. In both cases the factor four, called "the coefficient for the visibility", is used. We reproduce the calculations of the text by adding the units:

3 [minas] × 4 gives 12 [UŠ]

Utilizing that 1 mina = 1,00 UŠ, we prefer to render the recipe in the following form:

$$3{,}00\ \text{UŠ} \times 0;04 = 3{,}00\ \text{UŠ} \times 1/15 = 12\ \text{UŠ}$$

Accordingly we identify the last calculation described in the text as:

$$40\ \text{NINDA} \times 0;04 = 40\ \text{NINDA} \times 1/15 = 2;40\ \text{NINDA}$$

and interpret it as the daily change in visibility, to be used for interpolation.

We recapitulate: the "water clock scheme" calculates "the visibility" of the new crescent (on day 1) and of the full moon (on day 15 of the months) as a function of the night length. The visibility is proportional to the schematic night length. The text identifies the numbers calculated as *ŠÚ šá Sin* "setting of the moon" on days 1, and as *KUR šá Sin* "rising of the moon" on days 15. And it finds the daily change of the visibility, presumably used to facilitate interpolation. But how and what for? To find the lunar times on other days than the first and fifteenth (i.e. at full and new moon) within each month? Or to find the visibility times for new and full moon in cases where some "extra days" indicated that the real month deviated from the ideal calendar? I shall not go so far as to exclude the first possibility, but will argue strongly for the second one.

The visibility of the moon and its daily retardation

What the text calls the "visibility of the moon", or "rising of the moon (on day 15)", indicates an ideal interval between sunset and moonset (for new moons) or between sunset and moonrise (for full moons). As mentioned above, it equals 1/15 night and therefore, the numbers also signify the daily retardation of the moon. We understand them as such and shall sketch why these quantities might be equal.

The "rising of the moon" is normally understood as the time from sunset to the first moonrise after sunset; this time interval is generally not equal to the daily retardation of the moon. But "the water clock" scheme is idealized: the dates are given within the "ideal year" (12 months of 30 days) in which full moon falls on day 15 of each month. Implicitly, the scheme assumes conjunction to take place at sunset of day 30 and opposition to take place at sunset of day 15, exactly as the scheme from EAE XIV mentioned in the introduction. In this special case of opposition at the moment of sunset, the "rising of the moon" does, indeed, equal the retardation of the moon: if sun and moon set at the same time (are in conjunction) on the last day of a month, then on the next evening, the time from sunset to moonset indicates how much the moon has delayed its rising. In this case the "visibility time" of the moon measured on day 1 gives the retardation of the moon on that day.

Within the Babylonian astronomy, the daily retardation of the moon played an important role. In Section 19 of TU 11, a method for prediction is exemplified through three calculated examples. The calculations show how the daily retardation of the moon was used for finding the day of first visibility, i.e., the beginning of the new month.[17] The method uses the same value for the retardation of the moon at the eastern and western horizon. The same is true for the "water clock" scheme. It does not differentiate between the retardation of the rising and setting moon. Apparently, it has not yet been discovered that they are different.

Section 14 and 16 of TU 11 bear witness of the "Goal-Year" method, a very elegant empirical method for the prediction of visibility times of the moon by means of observed visibility times. These quantities are easy to observe, but as astronomical phenomena they are very irregular and complicated to calculate. The Goal-Year method is easy and astonishingly precise. It differentiates between retardation in the east and west and uses the daily retardation of the moon in the west for finding visibility times of the setting moon, and the daily retardation of the moon in the east for the visibility times of the rising moon.[18] In order to give a clearer discussion of the method, we will introduce the Babylonian names for the visibility times.

Starting at least back in the 6th century B.C., the Babylonians began regularly to observe the times between the risings and settings of sun and moon in the days around opposition and record the measured times (the earliest still existing Diary in which NA is measured comes from the year 568 B.C.). The following four special time intervals relating to the full moon (the Lunar Four) were identified and called:

$$\begin{aligned} \text{ŠÚ} &= \text{time from moonset to sunrise, measured at last moonset before sunrise,} \\ \text{NA} &= \text{time from sunrise to moonset, measured at first moonset after sunrise,} \\ \text{ME} &= \text{time from moonrise to sunset, measured at last moonrise before sunset,} \\ \text{GE}_6 &= \text{time from sunset to moonrise, measured at first moonrise after sunset.} \end{aligned}$$

For the daily retardation of the setting moon, the Goal-Year method uses the sum ŠÚ+NA measured 1 Saros earlier. For the daily retardation of the rising moon the sum ME+GE_6 is used. Evidently, at the time when the method was invented, it had been noticed and utilized that the retardations repeat themselves quite exactly after a Saros and that the retardation of the moon could be easily found as sum of the appropriate "visibility times". The Goal-Year method must have been developed not later than 550 B.C.: the tablet Cambyses 400 gives all Lunar Four data for the year 523 B.C. The values can not all have been observed, but still they are all so precise that we conclude that a very accurate method for prediction – the Goal-Year method – must have been used (Brack-Bernsen 1999b).

Normally, opposition does not take place at sunset so that the retardation of the rising moon is split up into the two intervals ME and GE_6. Therefore, in the general case, ME + GE_6, is a measure for the daily retardation of the rising moon.

Sometimes it happens that ME = 0, the sun is setting at the same time as the moon rises. This is the situation described in the ideal water clock scheme: ME equals 0 and GE_6 alone measures the lunar retardation. Therefore, in this ideal case, the "rising of the moon (GE_6)" has a double significance: it is the time from sunset to moonrise and it is the retardation of the moon on the day of opposition.

Each of the Lunar Four behaves extremely irregularly – they are all dependent of the time of the year (i.e. the position of the sun at opposition), of lunar and solar velocity, of lunar latitude (which we have ignored in this discussion) and of the time of opposition with respect to sunrise or sunset (see Brack-Bernsen and Schmidt 1994). We cannot be surprised that it took a long time until the Babylonian astronomers were able to handle such complicated quantities. Around 300 B.C., they had been able to separate out the different dependencies; by then they had developed a numerical model which could calculate visibility times, taking all the dependencies into account. What is surprising is that they were able to do it at all.

The "water clock" lets the daily retardation of the moon vary between 8 UŠ and 16 UŠ. In reality, the lunar retardation varies between 7.5° and 17.5°, so the old scheme yielded at least the variation quite correctly – although slightly out of phase. In the Goal-Year method, the retardation is not derived from a numerical model, but found directly by observations. At that time the scribes had realized that the values given in the MUL.APIN scheme were wrong and they had found a better way of determining the retardation. But still, they used – now with great success – the daily retardation of the moon for predictions.

The additional days

The last part of the "water clock" gives a rule for finding the daily change in the "visibility of the moon". According to the calculations, an amount of 2;40 NINDA = 0;02,40 UŠ is found. It is derived from the daily change 40 NINDA in day length (using 4, the coefficient for the visibility of the moon). The question is now: what was calculated by this interpolation number? Until now there seems to have been proposed only one out of two possibilities.

1) The "lunar visibility" is only given for the 1st and 15th day of the month (i.e., the ideal values for new moon and full moon); and not for the other days (or lunar phases). The number 2;40 NINDA (signifying the daily change of the lunar retardation) could have been used for calculating the retardation of the moon for each day in the month, i.e., for the dates between full and new moon. Or:

2) The daily change in "lunar visibility" 2;40 NINDA could have been used for correction in connection with some observed "extra days", i.e., in the cases where observations have shown that a cardinal point of an actual year takes place some days away from the ideal day 15 of the month. This possibility would explain the additional days mentioned in the "first intercalation scheme" above.[19] In the "Diviner's Manual" (Oppenheim, 1974 p.200 and 205.) some "extra days" are mentioned just after the advice to make horizontal observations. But the next sentence is about intercalation, so at this place these extra days were probably used as an indicator for intercalation. However, the diviner is also told to establish the months of the year (and) the days of the months (i.e., to determine if the month had 29 or 30 days) by observing the first appearances of the sun and moon in the months XII and VI. Note that these are the equinoctial months, according to the EAE scheme, and that observations of this kind are exactly what one needs for finding the displacement of the real months in comparison to the schematic year – as proposed in the imagined example below. That an intercalation – determined by astronomical criteria – could be challenged, in order to make a bad omen pass by, was discussed by C. Williams (2002, pp. 473–485). She argues that the advices given in the "Diviner's Manual" can be used for altering a bad omen by changing the month or the day to a more favourable one.

The method of interpolation, according to the second interpretation above, shall be illustrated by an example. Let observations have shown that in month I the sun will rise still somewhat south of East on day 15 while sunrise toward East takes place on day 28 (indicating that on this day, night and day both equal 3 mana). The ideal date of sunrise straight East is day 15, so one had to wait some 13 additional days before the event really took place. According to the sun's risingpoint, the length of night will be 3 minas on day 28, with a visibility of the moon of 12 UŠ. The "correct" value for the visibility of the (full) moon on day 15 could be found by linear interpolation. The corrected schematic day length on day 15 would be:

3 mana – 13 $\times$ 40 NINDA = 3,00 UŠ - 8,40 NINDA = 2,51 UŠ 20 NINDA.

And the corresponding visibility time:

12 UŠ – 13 $\times$ 2;40 NINDA = 12 UŠ - 36;40 NINDA = 11 UŠ 23;40 NINDA.

I prefer the second interpretation. It uses the observations called for in the "first interpolation scheme", and it gives a reasonable explanation for the "additional days" mentioned. It also explains why the 2;40 NINDA was derived from 40 NINDA, identified as the change in day and night. In addition, the special interest in conjunction, new crescent (day 1) and full moon (day 15), and not in the phases in between, is characteristic for Babylonian "lunar theory". Therefore it seems much more probable that the Babylonians corrected the value for the daily retardation of the moon on day 1 and 15 rather than to calculate its value on some odd days, e.g., on a day 17.

Daylength measured along the horizon

The two MUL.APIN schemes discussed above take 4 minas = 4$\times$ 60 UŠ = 240 UŠ for the longest day, and 2 minas = 2 $\times$ 60 UŠ = 120 UŠ for the shortest day of the year. The ratio of 4:2 between the longest and shortest day corresponds to the situation around a geographic latitude of 45 degrees and is a bad approximation for Babylon situated 32.5 degree north. A better value of 3:2 is found in later texts, e.g., in the ephemerides of the Seleucid time.

D. Brown (2000a) uses this inaccuracy, in combination with examples of number games for explaining astronomical data, to put forward his thesis that the parameters of EAE and MUL.APIN were not designed to give an accurate description of nature. According to him, they were chosen out of divinatory and number mystic reasons, and therefore these tables cannot be called astronomical.

It is true that there were other criteria than the wish of finding the best fit to nature when the parameters were chosen. But this does not necessarily exclude the possibility of choosing good numbers for several reasons. I can imagine that the choice of numbers in the numerical schemes was guided by a combination of the following aims: a good approximation, some nice (easy-to-handle) numbers, and some special "old" or "magic" numbers.

There might be a "natural explanation" for the 4:2 ratio. For sure, the ratio depends on how time was measured at that early time when it was first chosen. The "first intercalation" (or cardinal point) scheme connects the daylength directly with the rising points of the sun. The northern turning point of the sun has the longest day light, and after turning, the days becomes shorter by 40 NINDA each day. Therefore it seems natural to propose that the length of day and night at this time was determined by the sun's rising and setting points and measured along the horizon. This is not a new idea;[20] but I think that I have new and stronger arguments for its support.

Figure 1 showed the rising and setting points of the sun at the solstices and equinoxes. At the latitude of Babylon, the angle between risings of sun at SS and at WS happens to be nearly 60 degrees.[21] If the North-South line is drawn in figure 1, we get a circle divided up into 6 equal parts – or the regular hexagon. We shall therefore refer to Old Babylonian (hereafter OB) mathematical practices and take a closer look at the regular hexagon.

Such a configuration is known from the OB mathematical texts. Here for the regular polygons (both pentagon, hexagon, and heptagon) the side is put equal to 1 (see E. Robson 1999, pp. 48, 49).

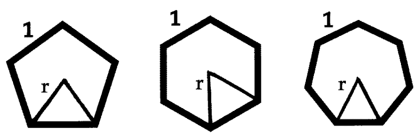

Fig. 2. Some OB geometrical figures and their measurements. The side of the regular polygons was the defining element and assumed to equal 1. As a consequence, the radii, r, had different sizes. They were given in the so called coefficient lists: for the regular pentagon r = 0;50, for the hexagon r = 1, and for the heptagon r = 1;10.

In the words of Eleanor Robson (1999, p.56) the *outer width* of the regular polygon is its *defining component*. This is similar to the case of the circle: It is the circumference (= 1) that is the defining component of the OB circle. But if the circle is divided into 3 equal parts, the third of the circumference is the defining component and put equal to 1 (e.g., in the "ox-eye" or in the "bow"). In case of geometrical figures which are connected to a division of the circumference into 4 equal parts, the defining component, equal to 1, is the quarter-circle arc (the "grain-field", "barge", or "apsamikkum"). In all these cases, the sexagesimal system is used to give the measures (in units of the defining element) of other parts of the figures in question.

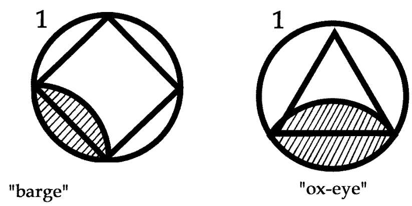

Fig. 3. Some OB geometrical figures and their defining elements: the quarter- circle arc and third-circle arc, both put equal to 1 in the standard figure.

Applying such mathematical usage to the situation in figure 1, we get figure 4 and its OB measurements: at SS the day measured from sunrise to sunset along the horizon (or measured along the sides of the hexagon) equals 4 parts or 4,00, the day at Eq equals 3 parts or 3,00 and the day length at WS lasts 2 parts or 2,00.

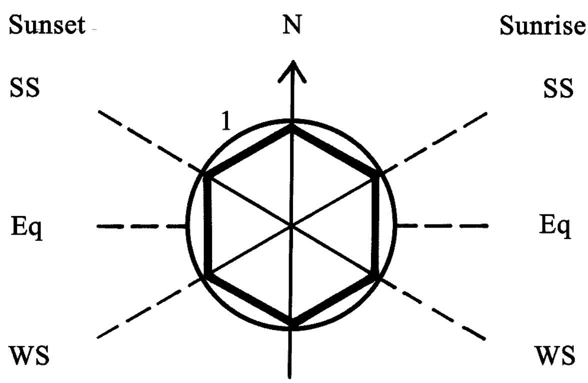

Fig. 4. The regular hexagon as realistic geometric approximation to the Babylonian horizon. The horizon is divided into sections by the North–South line and the points of sunrise and sunset on the solstices. At OB times, there was no difference considered between the sixth arc of the circle and its secant. In this figure they would both equal 1, corresponding to the "OB value of $\pi = 3$".

This way of measuring time can be explained without assuming a Babylonian measurement of angles; and this is very important. Since, according for instance to Jens Høyrup (2002 p.228), there existed no notion of the quantified angle within Babylonian mathematics, only so to speak a distinction between right or "wrong" angles. However, he emphasizes that "it is beyond doubt that a (probably intuitive) concept of similarity or ‘same shape’ was at hand". Therefore, time measurements equivalent to our measurement of time degrees can arise from the mathematical practices of the OB time. At this early time, problems connected to geometrical figures were given and solved by means of some standard figures and standard numbers. In the standard figure, the "defining element", (e.g., the side of the regular hexagon – which "equals" the sixth part of the circle circumference) was put equal to 1 or to 1,00 and all other lines, arcs or areas of that figure – measured in units of the defining element – were listed in the so–called coefficient lists. For a figure, similar in shape, but with the defining component of length p, the other linear parts were found as product of p and the appropriate coefficient (while area coefficients were multiplied by p square). See E. Robson (1999) for the latest treatment with reference to earlier works on OB mathematics.

We have here the concept of similar forms and proportionality. Techniques like these are exactly what may lead to something corresponding to our "angle measurements". The regular hexagon can be used to determine distances between directions by length measurements. Its secant may be called 1 or (in order to fit to the units used in MUL.APIN) 1,00 or 60 units.[22] Consequently the radius as well as the sixth part of the circle periphery will also be 60 units, independent of their actual size. Such a procedure is possible within the Old Babylonian mathematical techniques, and (in a good approximation) it is equivalent to measuring angles. This proposal would explain why "angles" or rather distances between directions were measured by units of lengths, and it would explain why already in the earliest texts, a unit of length occurs as a time measurement. In addition, we do not have to explain how the measurement of time suddenly also became a measure of distances in the sky, because they were just "born" that way.

As M.A. Powell has pointed out (1976, pp. 421-422), in Sumerian accounting practice, the term "shekel" (*gin*) is used to express "one-sixtieth" of a metrological unit. One text Powell refers to (p.421), uses "shekel" to signify "one-sixtieth" (0;1) and it has nothing to do with shekels used in weighing per se, for the number in question refers to bundles of reeds. Two additional texts counting workdays let Powell stress that Mana is the prime unit, and *gin* is used to express fractions of the unit. In EAE and MUL.APIN, the day length at the cardinal points is given as 2 mana, 3 mana, and 4 mana, respectively (1 mana = 60 shekel = 60UŠ). Is it possible, that the unit *mana* (a reminiscence of older times) here refers to the side of the regular hexagon and "shekel" = UŠ just refers to the sixtieth of this prime unit?

The retardation of the moon and the unit 1 UŠ

The point of view sketched above has the advantage of being able to explain how it was possible to calibrate instruments for measuring time to the unit of UŠ ($\simeq$ time degree) at a very early stage.[23]

The Diaries (containing recordings of Babylonian observations from 652 B.C. onwards) testify that measurements of short time intervals (e.g., the Lunar Six) were quite accurate; however measurements of longer intervals of time were inaccurate[24] – night and day could not be measured using water clocks with much accuracy. Therefore the unit UŠ can not come about by dividing the whole nychthemeron (or of an equinoctial day), measured with a water clock; but times measured in UŠ, i.e., in a unit of length ($\simeq 1°$), can be explained as a consequence of OB mathematical practices, which also leads to an equinoctial night of 180 = 3,00 UŠ = 3 Minas. It also opens a way of calibrating waterclocks to measure in units close to our time degrees.

Some of the Lunar Six can be measured directly by a unit of length – without using a water or sand clock: those phenomena by which the sun and the moon are visible at the same time, i.e., NA_N, ME, NA, and KUR. To illustrate how, I shall sketch an imagined scenery. Let two observers at a distance of a rope (of length R) collaborate in measuring ME, the time from last moonrise before sunset to sunset. Let R be divided into 60 equal parts r (60 r = R). We use R as prime unit, and r, its sixtieth part, is used to express sexagesimal fractions of the unit, i.e., in units of UŠ. The direction from the observer to his assistant is the direction to the rising moon. As seen from the observer, the rope is stretched in the direction of moonrise, the assistant standing at the other end. A yardstick (divided into units of r) can be used for measuring how far the full moon has moved from its rising to the moment of sunset. The assistant may place the beginning of the stick in the line of sight to moonrise and keep the stick in the direction of the rising moon so that it passes through the center of the moon as seen from the observer. At the moment of sunset, the upper edge of the moon is marked on the stick. The distance from bottom to this point – indicating the time from moonrise to sunset - can be measured. Read in units of r, it delivers a measurement of time which is quite close to our time degree. A measurement of, e.g., 14r, corresponds to an angle of $arctan(14/60) = 13.13°$.

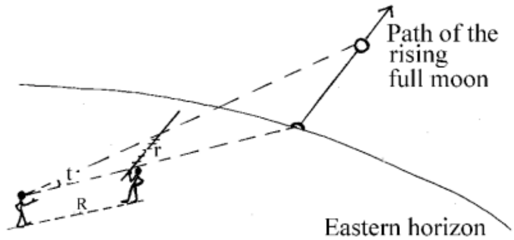

Fig. 5. A sketch of an imaginary setting by the observation of the time t = ME from moonrise to sunset. The assistant holds the stick parallel to the direction of the equator at the eastern horizon.

A water clock for short time measurements can have been calibrated in accordance with such measurements. Therefore, it was in principle possible to produce water clocks which could measure short time intervals in units equivalent to time degrees at a very early time.[25]

If the measurement of the length of daylight along the horizon gave rise to the ratio 4:2, we must admit that this ratio is a good approximation to nature – if it is supposed to measure horizontal arcs at a time where one had not yet figured out how times (as long as a whole day) could be measured accurately by other means. It is not a ratio chosen exclusively out of number-magical reasons.

Contrary to the duration of the whole day or night, the (short) retardation of the moon could be measured with a rather good accuracy. The "water clock scheme" of MUL.APIN, determined the retardation of the moon as proportional to the length of the night, ending up with numbers varying between 8 UŠ and 16 UŠ. And this is a good approximation for the variation of ŠÚ+NA or ME+GE_6. In average, ŠÚ+NA and ME+GE_6 does, indeed, vary between 8 and 16 UŠ. This first numerical approach to the retardation of the moon is not too bad. Therefore even after the time when it was noticed that 3:2 as ratio between longest and shortest day was better than the old ratio 4:2, the old day length schemes can still have been used as a good working generating function – to calculate the daily retardation of the moon or to calculate the shift in time between lunar eclipses 1 or 2 Saroi apart.[26]

We have other and indirect support for Babylonian measurements of (celestial) distances by means of yardsticks. Recently, Alexander Jones has analyzed and evaluated all Babylonian observations known so far, which measure distances in *kùš* (= cubit) between normal stars and passing planets.[27] Control calculations with modern computer codes enable him to compare the Babylonian measurements d_{kus} (given in units of cubit) with distances calculated in degrees, d_{deg}.

Jones' best fit to the data is a straight line, L, with the equation:

$$d_{\text{deg}} = L(d_{\text{kus}}) = (d_{\text{kus}} \times 2.27) + 0.13$$

Let us compare this distribution of data with distances observed with a length measuring device. Imagine, a rod of the length 2 nindan at the end of which, an other rod, divided in units of kùš is placed perpendicularly. Such an instrument, used as sighting device, would ideally deliver measurements along the curve D found as follows. The ratio between 2 nindan and 1 kùš is 24 to 1. Therefore the number d degrees, corresponding to a measurement of c in kùš, can be found:

$$\tan(d) = c \times 1/24. \quad \text{Or}\, d = \arctan(c/24)$$

The equation of D giving d as a function of c is:

$$d = D(c) = \arctan(c : 24)$$

Thus we can find the equivalent of 1 kùš, by putting c = 1:

$$D(1) = \arctan(1 : 24) = 2.3859^\circ$$

Note that 2.3859°($\simeq$ 1$\times$ 2.27 + 0.13), the value given by Jones line L for $c = 1$. Jones investigated what we call c: Babylonian observations of celestial distances $c = d_{\text{kus}}$ measured in units of kùš and fingers $= 1/24$ kùš. The measured values range from 1/24 kùš until 7 kùš. For this interval, c $\in$ [0,7], the arctan curve $D(c)$ follows line L very closely. It approximates the data as well as L. In other words, the Babylonian measurements d(kùš) plotted against their calculated value d(degree) are fitted optimally by that curve one ideally gets by measuring celestial distances with the sighting device proposed above.

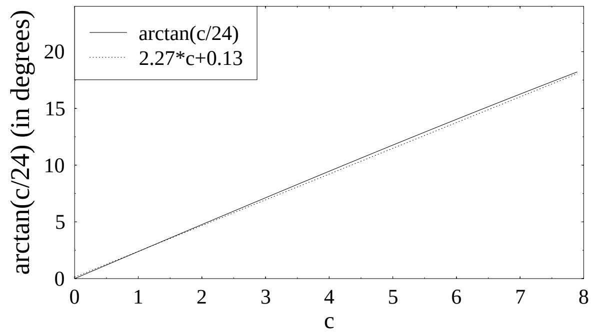

Fig. 6. Graphic comparison between line L(c) = 2.27 c + 0.13 and curve $D = \arctan(c/24)$, for $c \in [0,8]$.

This explains the line L and it suggests that celestial distances were measured by means of a "Jacob's staff" type of instrument. Independently, P. Huber has, in a comment to Jones' investigations, expressed very similar ideas on the use of a "Jacob's staff"; a mutual support for our supposition.[28]

The reason for the proposed size of the instrument, putting the rod equal to 2 nindan, is evident: By this choice will celestial distances (recorded in cubits = kùš in cuneiform texts), indeed, have been measured in units of the wellknown unit of length, the kùš. As further support for this proposal can be mentioned that ninda was the standard unit used in OB mathematical problems and that ninda and kùš were combined within the volume unit SAR. 1 SAR equals the square with the side 1 ninda raised to the hight of 1 kùš.

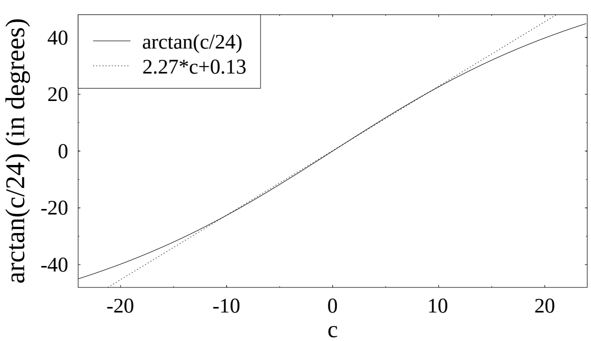

Fig. 7. Graphic comparison between line *L* (dotted) and curve *D* (solid line), for the values of *c* from -24 to 24. Within the range of interest, $c \in [0,8]$, the curves *D* and *L* are almost indistinguishable.

The sighting instruments described here are very large. However, the fact that OB mathematics clearly utilizes similar forms and proportionality, makes it quite possible that the Babylonian scribes constructed and used a smaller instrument with the same relative dimensions. For example, a little "Jacob's staff", or a set square with the length of 1 kùš on which the cross bar is divided in units of NB fingers, will as an observational tool deliver the same celestial distances as the large device described above. This is evident, since 1 kùš = 24 fingers, and R = 2 NINDA = 24 kùš. (At NB times the cubit was divided into 24 fingers, see Steele 2003).

The proposed use of a set square or "Jacob's staff" suggests how celestial distances have been measured in units of length. This supports indirectly the proposal that UŠ at early times also may have been measured in units of length. As further support, I shall finally mention Tablet BM 76738+, recently published by Christopher Walker (1999). It contains observations of Saturn from year 1 to 14 of Kandalanu (647 B.C - 634 B.C.). Contrary to later tablets which record observed celestial distances in units of kùš, this and a few other tablets, all of which are relatively early, measure the distance between Saturn and Normal stars in units of UŠ. This shows that the unit UŠ was not exclusively used for measuring times, in the 7th century B.C. it also measured celestial distances.

Conclusions

In this article I have argued for the fact that MUL.APIN since the time of its compilation can be seen as an early attempt of a systematical numerical description of astronomical phenomena. The lunar tables of EAE XIV and those of MUL.APIN are very similar. They are concerned with the same quantities and use the same methods. Therefore, I also consider both EAE XIV and MUL.APIN as consisting of tables to be used for early astronomical calculations in connection with divination. I have proposed that at early times, day and night length may have been measured along the horizon. An analysis of Babylonian data shows that celestial distances, measured in units of kùš, most probably were found by means of a Jacob's staff type of sighting instrument. I see this as an indirect support for my hypothesis that times given in UŠ similarly might have been measured in units of length by means of a sighting instrument. This kind of time measurement may have its origin in an old practice where night and day length were measured in units of length along the horizon.

Acknowledgements

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NOTES

1. Papke (1978) identified some of the stars and dated the scheme to fit the time around 2000 B.C., later J. Koch (1991/92) (and others) refuted this dating and showed convincingly that the time around 1000 B.C. is the most probable for the composition of the schemes of heliacal rising stars.
2. See Englund (1988).
3. See van der Waerden, 1950 with reference to earlier works.
4. See Brack-Bernsen 1999a and Brack-Bernsen and Hunger 2002, henceforth referred to as TU 11.
5. TU 11 is a cuneiform tablet from Uruk, concerned with lunar phenomena. See Brack-Bernsen and Hunger 2002 or Brack-Bernsen 2002.
6. Similar phenomenological approaches are used also in modern 20th and 21st century physics. A certain set of observations is followed by model building which in turn may lead, at a later stage, to a fundamental theory.
7. See Hunger and Pingree 1999 p.61 with relevant references to earlier publications.
8. I use the translation by H. Hunger (see Hunger and Pingree 1989, pp. 72–75); however some small corrections proposed in Hunger and Pingree 1999, pp. 75–76 are incorporated: the minas mentioned in the text give the length of the whole day or night and not just the length of a watch; 40 NINDA is the change of the day's length and not the change in the sun's rising position; and Pingree reads the head of the "Great One" instead of the head of the "Lion" by winter solstice. For simplicity I have rendered the months by roman numbers instead of their Babylonian names.
9. Another example of this practice, i.e., using constellations to indicate directions toward the horizon, may be found in the Babylonian texts LBAT 1494 and 1495 (Pinches 1955). These texts mention the morning shadow of Cancer and of Capricorn, which we understand as the shadow of the rising sun at summer solstice and at winter solstice (see Brack-Bernsen and Hunger 1999).
10. The text also calls it "setting of the moon" on day 1 (referring to the new crescent) or "rising of the moon" (on day 15, referring to the full moon). It is normally understood to be the ideal or schematic time interval from sunset to the setting of the new crescent or full moon, respectively. But as mentioned in the introduction, it has a double significance: within the Babylonian theory, these ideal (or schematic) times also equal the daily retardation of the moon.
11. See TU 11, pp.80–85. The Saros equals 223 synodic months, which is a wellknown eclipse cycle.

12. Admittedly, only in case of the water clock (and in similar schemes of EAE XIV) it is evident that such a method existed at the time when EAE and MUL.APIN were compiled. Tablet TU 11 is written much later, but the practices collected in Section 9–12, 18, and 19 are, with respect to methodology and concepts, very similar to the "water clock". This seems to indicate an early date of invention. The existence of several earlier parallel texts to TU 11 witness that these methods were in use at latest in the 5th centory B.C. In any case, at the time when EAE and MUL.APIN were compiled, the length of night was used for finding the visibility of the moon.
13. The numbersystem used by the Babylonian scribes was a place value system with the basis 60. The numbers calculated here are written according to the standard way of rendering sexagesimal numbers: e.g., 22,02;13,20 means $22\times60 + 2 + 13\times60^{-1} + 20\times60^{-2}$.
14. See the detailed discussion in Brown, Fermor, and Walker (1999).
15. The connection between these schemes of MUL.APIN is also discussed by Friberg et al 1990, pp. 496–499. Note that this paper still uses the "old" value, 3 times 3 minas for the equinoctial day. It results in a "wrong" connection between UŠ and minas, but should not cause problems. Since Al-Rawi and George's edition of EAE XIV (1991/92) we know that the equinoctial day had a duration of 3 minas, resulting in the equation: 1 mana = 1,00 UŠ.
16. We already know this quantity from the "first intercalation scheme", where it was specified as the "rate of 40 NINDA by which days become longer, the nights become shorter" for each day.
17. Starting with KUR, the time from last visible moonrise (before conjunction) to sunrise, the text uses 1/15 of the day length to extrapolate and calculate the times between moonrise and sunrise on the following days. In each example, the text stops at the moment when it is clear that the new crescent sets sufficiently late to be visible after sunset. See Brack-Bernsen and Hunger (2002, pp. 37–40 and 72–75).
18. See Brack-Bernsen 1999a and Brack-Bernsen and Hunger 2002.
19. To my knowledge, nobody has until now commented in this way on the days in excess. Brown (2000a, p.117) writes that the extra days measure how many days over 12 months the year has lasted, while Hunger and Pingree (1999, p.77) write that "The "risings of the Sun" with the stars mentioned for IV 15, VII 15, and X 15 will determine the actual day of a lunar month on which the summer solstice, fall equinox, and winter solstice occur; if that date in the real calendar is one month more than the given dates in the ideal calendar, intercalation is needed". In both comments, the extra days are just proposed used for indicating the need for an intercalated month.
20. See Bremner, 1993 and Gleßmer, 1996.
21. The angle between the risings of sun at SS and at WS depends on the geographic latitude. At the latitude of Babylon, this angle amounts to 57.2° while in Assur it equals 59.9°. According to Pingree and Hunger (1999, p. 58), the 1st, 2nd, 3rd, and 5th table of MUL.APIN were composed around 1000 B.C. in Assyria (at a Latitude of ca. 36°). If Assur also happens to be the town in which the lunar tables of MUL.APIN were composed, the fit to the regular hexagon is perfect. The hexagon approximation is, however, good for all of Mesopotamia. Another dating is proposed by T. de Jong, who excludes Nineveh as place of observation, since the star "NUN ki" is never visible in Nineveh. Having analyzed the dates of heliacal rising, using his extinction method, T. de Jong finds as epoch for the stellar lists of MUL.APIN the time 1300 B.C. ± 200 years. (Private communication, 2003.)
22. The unit 1 [mina] would be the choice to use for EAE XIV table B, where the duration of the equinoctial day is 3 mina. MUL.APIN as well as EAE XIV table A states the equinoctial day to last 3,00 UŠ. This we get, if the length of 1,00 UŠ is ascribed to the secant in Figure 4.

23. OB coefficient lists record: 12 ma-al-ta-ak-tum translated as "0;12 the water-clock" by E. Robson (1999, p.122*). We know this number 12 UŠ from EAE XIV, table B; it measures the "setting" or "rising" of the moon but also its "daily retardation". In the philological commentary, E. Robson (1999, p.123) writes that ma-al-ta-ak-tum should rather be translated as 12 the "testing instrument".
24. For the accuracy of the Lunar Six measurements, see Huber 2000 or Brack-Bernsen 1999b; for the accuracy of longer time intervals, e.g. the observed times of eclipses, see Steele 2000, pp. 57–68.
25. Later, from 750 B.C. onward, times were also indicated by means of culminating *Ziqpu* stars. A better and more precise way of measuring time had been found. From then on it was possible to calibrate water clocks to measure in units of UŠ by means of culminating *Ziqpu* stars, see e.g. Brown (2000a, p.259) and (2000b).
26. Time shifts between eclipses are calculated in TU 11 Section 9–12, see Brack-Bernsen and Hunger (2002, pp. 80–85).
27. The paper was presented by Alexander Jones at the Regensburger Workshop (2002). It will soon be published in the Archives for the History of Exact Sciences.
28. P. Huber kindly sent me a comment which he had written to A. Jones last summer. I quote a part of an e-mail: "Here are a few more thoughts on the cubit. The fact that the ratio between the observational and the theoretical cubit seems to stay constant over more than a few decades must mean there is a system behind it. In other words, the discrepancy must be due to instrumentation and/or data reduction. Now; I surmise that they measured chords rather than angles, even if we don't know how – perhaps with the help of strings (as you suggest), perhaps with a kind of Jacob's staff. If they reduced their data linearly (they would not have trigonometric tables!), they would get somewhat larger units, for example from subdividing a 60 degree chord into 30 equal parts, they might arrive at a cubit of 2.20 degrees."