*to be publihed by:*
International Journal of Modern Physics E


# ANALYZING SHELL STRUCTURE FROM BABYLONIAN AND MODERN TIMES

LIS BRACK-BERNSEN
*Institut für Philosophie, Wissenschaftsgeschichte, Universität Regensburg*
*D-93040 Regensburg, Germany*
*e-mail: lis.brack-bernsen@psk.uni-regensburg.de*

MATTHIAS BRACK
*Institut für Theoretische Physik, Universität Regensburg*
*D-93040 Regensburg, Germany*
*e-mail: matthias.brack@physik.uni-regensburg.de*

We investigate "shell structure" from Babylonian times: periodicities and beats in computer-simulated lunar data corresponding to those observed by Babylonian scribes some 2500 years ago. We discuss the mathematical similarity between the Babylonians' recently reconstructed method of determining one of the periods of the moon with modern Fourier analysis and the interpretation of shell structure in finite fermion systems (nuclei, metal clusters, quantum dots) in terms of classical closed or periodic orbits.

## 1. Introduction

Beats are an ubiquitous phenomenon arising from the interference of waves oscillating with different frequencies. In classical physics, they occur, e.g., in water or sound waves. In microscopic physics, they appear as quantum oscillations occurring, e.g., in the form of (super-) shell structure in finite fermion systems such as atomic nuclei, metallic clusters or semiconductor quantum dots. But beating amplitudes occur also in coupled mechanical systems when the uncoupled subsystems have nearly commensurable periods. One example – and perhaps the first studied by mankind – is our planetary system including the earth's moon. In fact, the system sun-earth-moon represents the oldest three-body problem,[1] which has occupied astronomers since more than 3000 years and until today, despite all our modern mathematical knowledge, is not exactly solvable.

As an illustration of the similarity of beats occurring on astronomical and microscopic scales, we juxtapose in Fig. 1 some lunar data, computer simulated for the time around 500 B.C. in Babylon, with shell structure of a modern mesoscopic quantum-mechanical system. On the left side, we show the quantity $\Sigma =$ ŠÚ+NA+ME+GE$_6$, from which the Babylonians are thought[2] to have derived one of the periods of the moon (as discussed in Sect. 3.1 below), calculated as a function

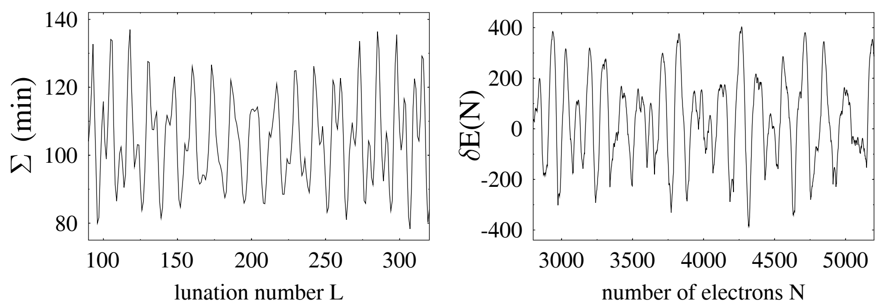


Fig. 1. *Left:* Sum of calculated "Lunar Four" $\Sigma = \check{\text{S}}\acute{\text{U}}+\text{NA}+\text{ME}+\text{GE}_6$, as observed by the Babylons since 2500 years ago, plotted versus lunation number $L$. *Right:* Energy shell correction of the electrons in a two-dimensional semiconductor quantum dot, plotted versus electron number $N$.

of the lunation number $L$ that counts the oppositions of moon and sun. On the right side of Fig. 1, we show the energy shell correction $\delta E$ of a two-dimensional semiconductor quantum dot with a radius of a few hundred nanometers,[3] calculated as a function of the number $N$ of conduction electrons contained in the dot (which was modeled here by a circular billiard).

In Sect. 2 we briefly discuss those lunar phenomena which were observed by the Babylonians and recorded on clay tablets. In Sect. 3 we give a short account of our present understanding, making use of modern computer simulations of the lunar data, of some of the methods by which the Babylonians arrived at their precise knowledge of the periods of the sun and the moon. In Sect. 4 we analyse their determination of the lunar period by applying modern Fourier analysis. This technique is also successfully used in the semiclassical description of quantum oscillations in terms of classical closed (periodic) orbits,[4,5,6,7] which has had a substantial impact on recent research on quantum chaos.[8]

## 2. Observation and theory of lunar phenomena from Babylon

The Babylonian astronomy focused on special phenomena. In case of the moon, these were eclipses and some ocurrences around conjunction (new moon, i.e., sun and moon have the same ecliptical longitude) and opposition (full moon: the elongation of the moon from the sun equals 180°). In case of the planets they were, e.g., the first and last visibility.

The so-called Babylonian mathematical astronomy, which was fully developed around 300 B.C., enabled the scribes to calculate positions and times of these special phases. They did, however, not consider them as functions of a continuous time variable, but they calculated a series of discrete phases along the path of the celestial body in question. For the moon they recorded times and positions of consecutive conjunctions or oppositions, labeled by the names of the months during which they occurred.

### 2.1. *The period of the lunar velocity*

The new Babylonian month started on the evening when the crescent moon became visible for the first time after conjunction. The time interval $\mathrm{NA}_N$ from sunset until moonset on that evening was observed. This quantity is quite obvious and easy to observe. However, from a theoretical point of view, it is very complicated. It depends on the velocity $v_☾$ of the moon, its longitude $\lambda_☾$ (position in the ecliptic), its latitude $\beta_☾$ (distance from the ecliptic), and on the time interval $\Delta t$ from conjunction to sunset. It is an impressing achievement of the Babylonian scribes that they were able to develop very elegant numerical systems capable of calculating $\mathrm{NA}_N$ for consecutive new moons, taking all the variables into account. The values of $\mathrm{NA}_N$ were listed in so-called lunar ephemerides for each month. (Similarly, in the modern computer simulations discussed in Sect. 3 below, we present the data as functions of the lunation number $L$ counting the full moons.) At least since O. Neugebauer[9] we know how the ephemerides were calculated; but we have still rather little knowledge about how this theory was derived from observations.

One big question was how the period $T_☾$ of the lunar velocity was found. The movement of the moon is very irregular: it keeps changing phase and latitude and can have its maximal velocity anywhere on the ecliptic. Which kind of observations did the Babylonians use? In lunar ephemerides concerned with conjunctions, the calculated values of the lunar velocity for successive new moons are given in a column called F. But these values are all derived from a linear zig-zag function, given in a column $\Phi$ appearing directly after the first column containing the names of the months. Therefore we must assume that $\Phi$, rather than F, was constructed from observations. Their common period is $\mathrm{P}_\Phi = 6247/448 = 13.94420$ synodic months, which is surprisingly accurate. $\mathrm{P}_\Phi$ is the mean period of the lunar velocity measured on the days of conjunction (or at oppositions for the full-moon ephemerides). Let $\mathrm{P}_☾$ be the period of the lunar velocity, measured from day to day. The value of $\mathrm{P}_\Phi$ corresponds to the period $\mathrm{P}_☾ = 6247/6695$ synodic months $= 27.55453$ days. The presently known value, calculated for Babylonian times, is 27.55455 days.

Column $\Phi$ was long supposed to be based on lunar eclipses which were of great importance in Mesopotamia and had been observed since early times. However, the Babylonian observational records of moon eclipses were too inaccurate to allow for their accurate value of $\mathrm{P}_\Phi$. Therefore it has been postulated[2] that some other observations were used to construct $\Phi$ – namely those of some short time intervals around full moons.

### 2.2. *The Lunar Four*

Since 747 B.C., celestial phenomena were observed regularly and recorded month after month in the so-called Diaries.[10] The astronomical observations conducted in Mesopotamia may be called the longest scientific project ever. Diaries were produced continuously during a period of almost 700 years - the latest Diary found so far stems from the year 61 B.C. They were written in cuneiform on clay tablets: for each month of the year lunar phases, eclipses and planetary phases were recorded together with market prices, weather observations and historical events. In the earliest Diaries only the days of the special lunar phases were recorded.

However, starting at least back in the 6th century B.C., the Babylonians began to regularly observe the times between the risings and settings of sun and moon in the days around opposition and record the measured times. (The oldest preserved Diary in which NA is mentioned stems from the year 568 B.C.) The following four special time intervals relating to the full moon, the so-called "Lunar Four", were observed and recorded:

ŠÚ = time from moonset to sunrise measured at last moonset before sunrise,
NA = time from sunrise to moonset measured at first moonset after sunrise,
ME = time from moonrise to sunset measured at last moonrise before sunset,
$\mathrm{GE}_6$ = time from sunset to moonrise measured at first moonrise after sunset.

These time intervals were measured in uš = time degrees (1 day = 380°), and since they were rather short ($< 20° = 80$ minutes), they could be measured much more accurately than the times of eclipses. Therefore, Lunar Four data may be much better candidates for the reconstruction of $\Phi$ than eclipse observations. However, these intervals – all of them being similar to $\mathrm{NA}_N$ – are very complicated functions of the lunar velocity, its longitude and latitude, and of the time from opposition to sunrise: ŠÚ = ŠÚ($v_{☾}$,$\lambda_{☾}$,$\beta_{☾}$,$\Delta t$), etc. Was it possible for the Babylonians to extract information on $v_{☾}$ from these beating functions, i.e., to find $P_\Phi$?

From cuneiform tablets it is known that the Babylonians did observe the Lunar Four with quite some accuracy. But since only about 5% of all diaries have been found until now, there are large gaps in the recorded data. It is not possible to extract a sufficient amount of Lunar Four data to check exactly what information they contain. Therefore, it is of great help to simulate the Lunar Four data by means of a computer code for lunar ephemerides, as will be discussed in the following section. Since we thereby are concerned mainly with the partial sums ŠÚ+NA and ME+$\mathrm{GE}_6$ of the Lunar Four, it is necessary to briefly mention their astronomical significance.[11,12] On the last morning before opposition, the moon sets ŠÚ degrees before sunrise and on the next morning, it sets NA degrees after sunrise. We see that during the day of opposition, in comparison to sunrise, the setting moon on the western horizon is retarded by the amount ŠÚ+NA. Similar arguments show that ME+$\mathrm{GE}_6$, observed on the eastern horizon, is the retardation of the rising moon during the day of opposition.

## 3. Modern simulations of old observations

For our computer simulations of lunar observables at ancient Babylonian times, we used a code developed by S. Moshier,[13] which employes a semi-analytical lunar ephemeris adequate for historical times.[14] From the risings and settings of sun and moon, evaluated at the days around the oppositions, we computed the Lunar Four and tabulated them as functions of the lunation number $L$.

### 3.1. *The origin of column Φ*

Using such calculated Lunar Four data and the simple idea that observations in the east should be combined with observations in the west, it was found[2] that the sum of all Lunar Four, $\Sigma =$ ŠÚ+NA+ME+$GE_6$, yields oscillations with the period $T_☾$ that can be fitted by the linear zig-zag function recorded in column Φ. In Fig. 2, we show the curve $\Sigma(L)$ by crosses, connected with thick lines. The thin dashed line is the function $\Phi(L)$, shifted by an amount $-100°$. It yields an optimal fit to the calculated function $\Sigma(L)$. It overshoots the extrema of $\Sigma(L)$ but reproduces its main oscillations yielding, in particular the correct period $P_\Phi \simeq T_☾$. Note that the phase of $\Sigma(L)$ (i.e., its position along the $L$ axis) was not adjusted, but obtained directly from the calculated ephemerides appropriate for the time span covered by the data on the cuneiform tablet LBAT 1285 (indicated by a horizontal bar in the figure). The only adjusted parameter here is the vertical shift of $-100°$, whose origin and significance have remained unclear so far.

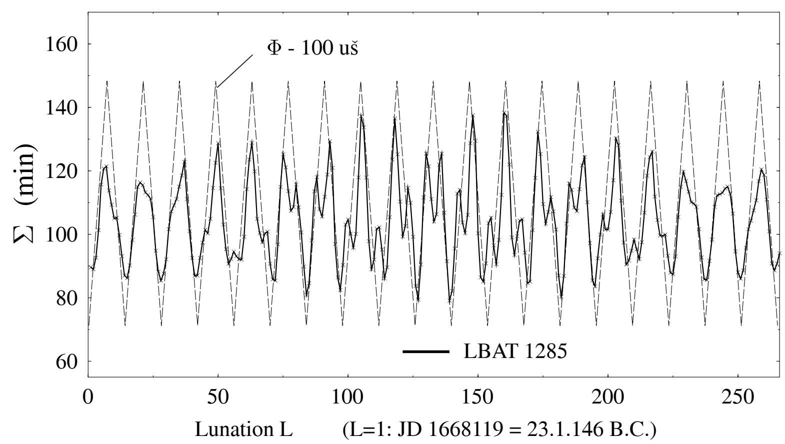


Fig. 2. The sum $\Sigma =$ ŠÚ+NA+ME+$GE_6$ of the Lunar Four, plotted versus lunation number $L$ (crosses, connected by thick lines). The thin dashed line shows the Babylonian zig-zag function $\Phi - 100°$. The horizontal bar covers the time span of the data on the table LBAT 1285.

In Sect. 4.2 we will use Fourier analysis to illustrate that the period $P_\Phi$ can be extracted from a fit to the function $\Sigma(L)$, but not to any of the single Lunar Four, nor to the partial sums (ŠÚ+NA)$(L)$ or (ME+$GE_6$)$(L)$.

The conclusion is therefore nearlying that we have found the observational origin of the column Φ from which all data related to the lunar velocity were derived. The hypothesis that Φ was constructed from the combination Σ of lunar observables is theoretically well supported[11,12] by the astronomical significance of Σ.

In order to support this hypothesis with historical evidence, it must be shown that the Babylonians really did collect Lunar Four data of consecutive months, and that the accuracy of these data was sufficient.

### 3.2. *Ancient observations compared with computer-simulated data*

A special type of tablets, the Goal-Year tablets,[15] collect lunar and planetary data to be used for astronomical predictions in a special year, the "goal year". A Goal-Year tablet for the year $Y$ records all Lunar Four intervals from the year $Y-18$ together with the eclipses that took place (or were expected to occur) during the year $Y-18$. It also lists the sums ŠÚ+NA and ME+GE$_6$ for the last six months of year $Y-19$. The Babylonians have thus, indeed, recorded the Lunar Four data continuously, and we can test their accuracy by modern simulations.

In Fig. 3 we plot the Babylonian data recorded on the Goal-Year tablet LBAT 1285 (circles and dashed lines) and compare them with the computer-simulated values (crosses and solid lines). Although the single Lunar Four behave rather irregularly, the agreement between old observations and modern calculations is very good, considering the fact that no adjustable parameter has been used. Especially important is the fact that the recorded values of the partial sums ŠÚ+NA and ME+GE$_6$ lie very close to the computer-simulated curves and reflect even some of their fine structure. Similar agreement could be found with data recorded on many other tablets, one of them dating back to the times of Cambyses (523 B.C.).[16]

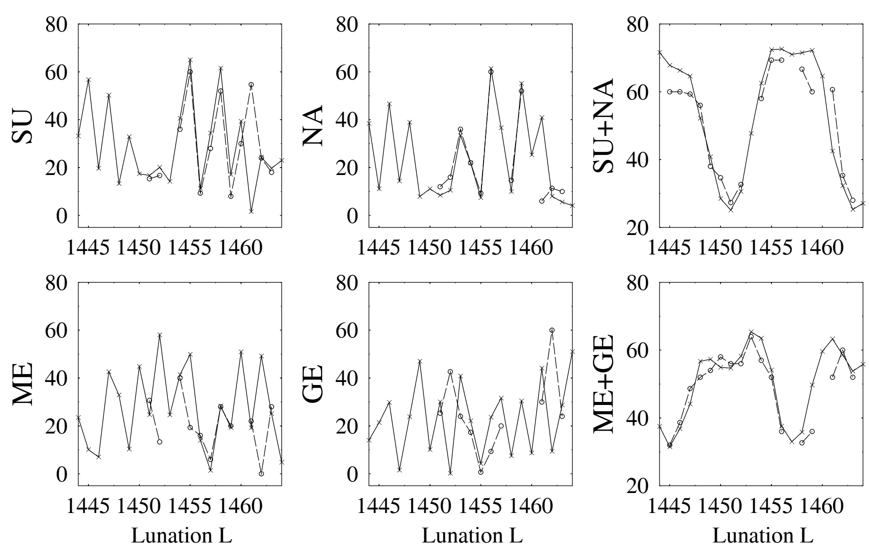


Fig. 3. Comparison of Babylonian lunar data recorded on the tablet LBAT 1285 (circles and dashed lines) with computer-simulated data (crosses and solid lines).[12]

We have thus clear historical evidence that the Babylonians did record the Lunar Four regularly over several hundred years and, in particular, paid attention also to their partial sums ŠÚ+NA and ME+GE$_6$. The accuracy of these recorded data is sufficient to support the hypothesis that the lunar period $P_\Phi$ could be extracted from their sum $\Sigma$.

### 3.3. *The Goal-Year method for predictions*

What if some Lunar Four data could not be observed because of bad weather or some other reason? There must have been ways to reconstruct or predict them.

One Saros = 223 synodic months $\simeq$ 18 years is a well-known eclipse cycle. A lunar eclipse observed in the year $Y-18$ will occur again in the year $Y$, and it will be visible in Babylon if it takes place during the night when the moon is above the horizon. We can thus easily understand that the eclipse data on the tablet for the Goal-Year $Y$ could be used for predicting eclipses in the year $Y$. But the question arises: how and to what purpose were the recorded Lunar Four data of 1 Saros ago used? Fig. 4 helps us to answer this question. Over a period of 30 months, the functions NA($L$), (ŠÚ+NA)($L$), and (ME+GE$_6$)($L$) are compared here with their respective values 223 months earlier. The sums ŠÚ+NA and ME+GE$_6$ repeat themselves nicely after 1 Saros. This is not too surprising since 1 Saros, in a good approximation, equals an integer number of periods of $v_☾$, $\lambda_☾$ and $\beta_☾$. However, NA($L$) (taken as a representative of all single Lunar Four) behaves very irregularly and there seems to be no simple connection between NA($L$) and NA($L-223$).

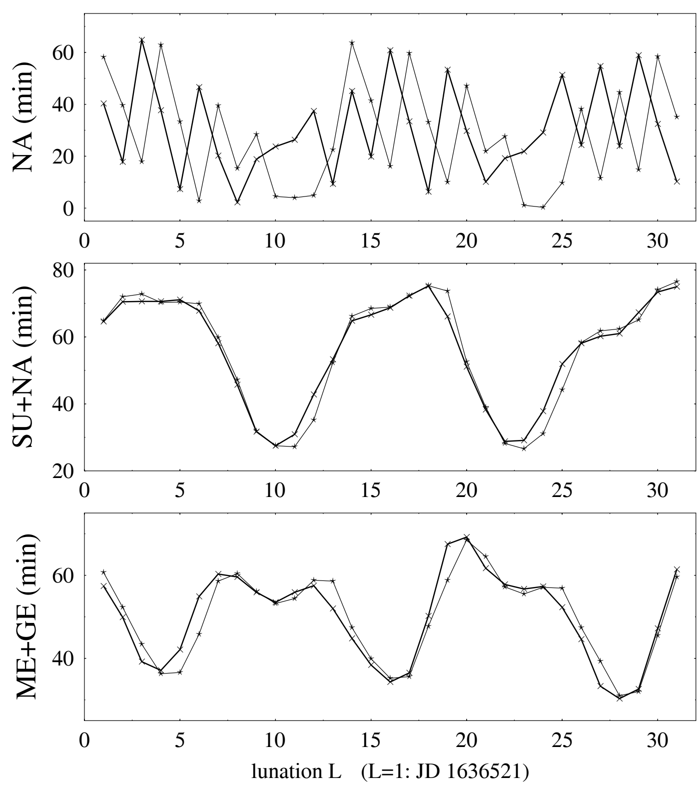


Fig. 4. Comparison of Lunar Four data 1 Saros = 223 months apart. Crosses and heavy lines: data evaluated at lunations $L$; stars and thin lines: data evaluated at lunations $L-223$.

It is, nevertheless, possible to predict single Lunar Four data by means of Goal-Year data. The knowledge that (ŠÚ+NA)$(L)$ = (ŠÚ+NA)$(L-223)$ and (ME+GE$_6$)$(L)$ = (ME+GE$_6$)$(L-223)$ can be combined with our knowledge that the three variables $v_☾$, $\lambda_☾$, and $\beta_☾$ will have approximately the same magnitudes at two oppositions $O_L$ and $O_{L\text{–}223}$, situated one Saros apart. The only variable determining the Lunar Four which has changed after one Saros is the time at which opposition takes place: 1 Saros = 223 synodic months = 6585 + 1/3 days. The time of opposition, compared to sunrise and sunset, is shifted by 1/3 day. These considerations led to the following proposal[17] how the Goal-Year tablets could be used for predicting the Lunar Four by what we call the "Goal-Year method":

$$\check{S}\acute{U}(L) = \check{S}\acute{U}(L-223) + 1/3\,(\check{S}\acute{U} + NA)(L-223)\,,$$
$$NA(L) = NA(L-223) - 1/3\,(\check{S}\acute{U} + NA)(L-223)\,,$$
$$ME(L) = ME(L-223) + 1/3\,(ME + GE_6)(L-223)\,,$$
$$GE_6(L) = GE_6(L-223) - 1/3\,(ME + GE_6)(L-223)\,.$$

The shift of 1/3 day in the time of opposition lets ŠÚ$(L)$ become 1/3 (ŠÚ+NA) larger than ŠÚ$(L-223)$, while NA is reduced by the same amount. The quantity ŠÚ+NA measures the retardation of the setting moon during the day of opposition. The correction of ŠÚ and NA by one third of this quantity therefore takes into account the retardation of the moon after 1 Saros.

The cuneiform tablet TU 11, which contains astrological as well as astronomical sections (at the time the two were not distinguished), nicely comfirms the Goal-Year method. In section 16 of TU 11 we find parts of the equations above spelled out in words.[18] This proves that the Babylonians had, indeed, found and used the above relations for the prediction of Lunar Four time intervals.

What is impressing with the Goal-Year method is that it is easy, elegant and surprisingly precise. In Fig. 5 we illustrate the accuracy of the method by comparing calculated values of ŠÚ$(L)$ with those predicted according to the right-hand side of the first equation above.

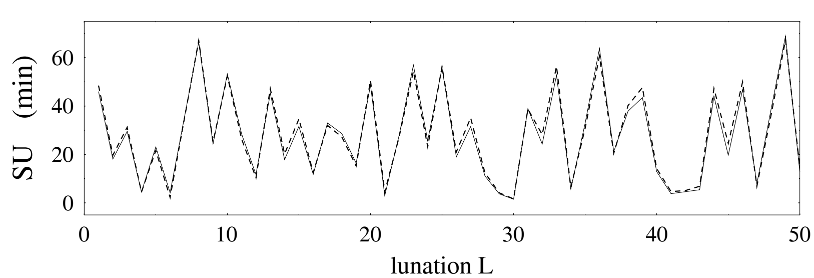


Fig. 5. Numerical test of the Goal-Year method for predicting ŠÚ$(L)$ for 50 successive lunations between 236 and 232 B.C. The quantity ŠÚ$(L)$ to be predicted is shown by the solid line; its prediction based on earlier data is shown by the dashed line.

## 4. Fourier analysis of shell structure

To unravel the origin of beats, one can apply the technique of Fourier analysis: by Fourier transforming the ocillating data with respect to a suitable variable (e.g. energy or wave number), one obtains a spectrum (e.g. of periods or orbit lenghts) of the interfering sources. As mentioned in the introduction, this tool is employed also in the study of quantum chaos in order to establish the semiclassical correspondence of quantum mechanics and classical mechanics. We first give two examples of such analyses from atomic and cluster physics and then apply the Fourier analysis to the Babylonian lunar observables. (Further examples relevant for nuclear physics are given by Arita in this volume.[19])

### 4.1. *Examples of modern spectra*

When hydrogen atoms are put into strong magnetic fields, their electronic motion becomes classically chaotic. A – by now famous – semiclassical analysis of photoionization energy spectra of hydrogen in magnetic fields by Holle *et al.*,[20] using an extension of the periodic orbit theory of Gutzwiller,[4] established the one-to-one correspondence of strong Fourier peaks with closed classical orbits of the electron, as shown in Fig. 6. (See Ref.[20] for the scaling variable $\gamma$ which depends on both energy and magnetic field strength.)

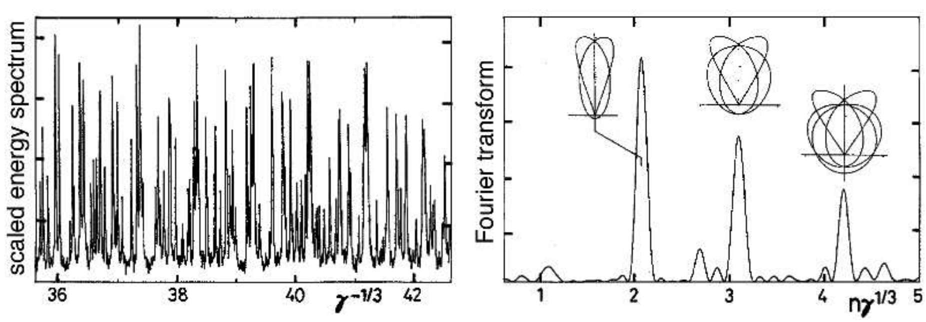


Fig. 6. *Left:* Scaled photoionization energy spectrum of hydrogen in a strong magnetic field. *Right:* Its Fourier transform. The peaks correspond to the shown classical closed orbits of the electron.[20]

A textbook example[6] of a classically integrable system without chaos is shown in Fig. 7, where we display the coarse-grained level density of a three-dimensional spherical quantum billiard as a function of the wave number $k = \sqrt{2mE}/\hbar$, Gaussian-averaged over a range $\Delta k = 0.4/R$ in order to emphasize the gross-shell structure. Its Fourier transform exhibits the length spectrum of the shortest classical periodic orbits contributing at this level of resolution, whose shapes are polygons with $n$ corners (the number $n$ is given near the Fourier peaks in the figure). We see that the quantum beats in $\delta g(k)$ are mainly due to the triangles and squares (and, with less weight, pentagons etc.; see Ref.[6] for details). Although this appears to be a rather naive toy model, it is realistic enough to describe the supershell structure

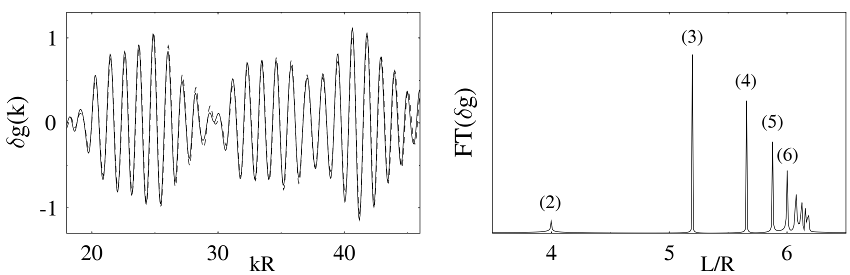


Fig. 7. *Left:* Coarse-grained level density $\delta g(k)$ of a spherical billiard (radius $R$), plotted versus wave number $kR$ (solid line: using the semiclassical trace formula;[5,6] dashed line: using the exact quantum spectrum). *Right:* Fourier transform of $\delta g(k)$ (absolute value in arbitrary units) versus orbit length $L/R$. In parentheses are given the numbers of corners of the periodic orbits (polygons).

that has been observed in the abundance spectra of metal clusters.[21]

### 4.2. *Fourier Analysis of Babylonian observables*

The three-body system sun-earth-moon is not integrable, but it is – luckily – not chaotic. We refer to a recent review by Gutzwiller[1] for an account of the various levels of sophistication at which it has been treated over the last two millenia, and for the basic periods of the sun and the moon (as observed from the earth) which govern its observables. As we have already mentioned in Sect. 2, the Babylonians were able to extract the period $T_{☾}$ of the lunar velocity by a suitable combination of observations on the western and eastern horizon. In the following we shall illustrate their procedure with the help of Fourier transforms.

In Fig. 8 we show on the upper left side two characteristic observables recorded by the Babylonians, ŠÚ and ŠÚ+NA, plotted versus the lunation number $L$ counting the successive oppositions of sun and moon. On the right side we present their Fourier transforms with respect to $L$, which yield the spectra of periods $T$ responsible for the oscillations and beats in these observables. All of the "Lunar Four" ŠÚ, NA, GE$_6$ and ME defined in Sect. 2.2 appear as rather erratic functions of $L$ yielding similar, relatively noisy Fourier spectra (we show here only the quantity ŠÚ at the top.) The spectra are dominated by the periods of the moon, $T_{☾} = 13.944$ months, and of the sun, $T_{\odot} = 12.368$ months = 1 year, but a large number of smaller peaks demonstrate the complexity of the system. Next from above we show the sum of the two quantities observed on the western horizon, ŠÚ+NA. This quantity – and similarly the sum ME+GE$_6$ observed on the eastern horizon – is a much smoother and more regular function of $L$. As its Fourier spectrum reveals, it is mainly a beat due to the two periods $T_{\odot}$ and $T_{☾}$. The two small components with $T \simeq 6$ months are responsible for the fine structure in (ŠÚ+NA)($L$) and do not affect the mean spacing of the "shells" nor the period of the beating amplitude. The function (ME+GE$_6$)($L$) has an almost identical Fourier spectrum, but its oscillations as functions of $L$ are phase shifted with respect to those in (ŠÚ+NA)($L$).

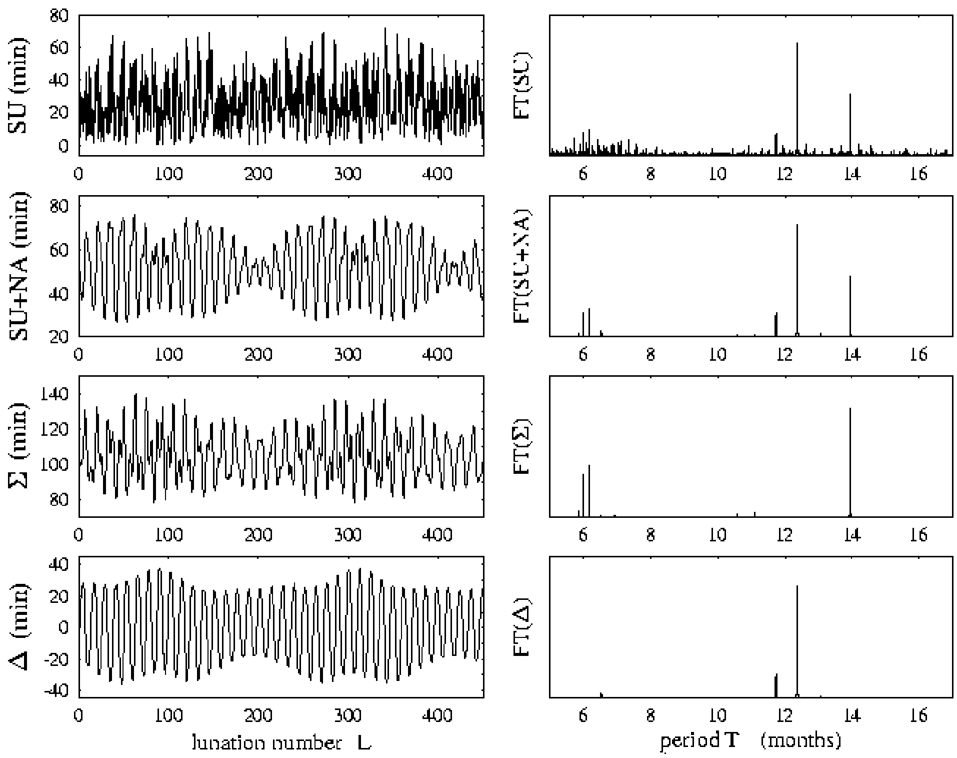


Fig. 8. Calculated lunar data (from top to bottom) SU, ŠÚ+NA, $\Sigma$ = (ŠÚ+NA)+(ME+GE$_6$) and $\Delta$ = (ŠÚ+NA)$-$(ME+GE$_6$). *Left:* values as functions of the lunation number $L$. *Right:* their Fourier transforms (absolute values in arbitrary units) versus period $T$ in months.

This behaviour is explained if one writes (ŠÚ+NA)($L$) and (ME+GE$_6$)($L$) as a sum and a difference, respectively, of two periodic functions:

$$(\check{S}\acute{U}+NA)(L) = f_{☾}(L) + g_{\odot}(L)\,, \qquad (ME + GE_6)(L) = f_{☾}(L) - g_{\odot}(L)\,,$$

whereby $f_{☾}(L)$ does not depend on $T_{\odot}$ and $g_{\odot}(L)$ does not depend on $T_{☾}$ (see Ref.[11] for the astronomical justification of this statement). Hence, by constructing the sum $\Sigma$ = (ŠÚ+NA)+(ME+GE$_6$) one can eliminate the component with the period $T_{\odot}$ of the sun and obtain a curve that is dominated by the lunar period $T_{☾}$. Alternatively, the difference $\Delta$ = (ŠÚ+NA)$-$(ME+GE$_6$) yields a curve dominated by the solar period $T_{\odot}$. (There is no evidence, however, that the Babylonians were interested in $\Delta$.) These facts are clearly revealed by the Fourier spectra of $\Sigma(L)$ and $\Delta(L)$ shown in the lower half of Fig. 8. Since for both sums ŠÚ+NA and ME+GE$_6$ the Fourier peak corresponding to the solar period $T_{\odot}$ is much stronger than that belonging to the lunar period $T_{☾}$, both these functions oscillate mainly with the period of the sun. It is only the function $\Sigma(L)$ that can be fitted by the zig-zag function $\Phi(L)$ with the period of the moon, as demonstrated in Fig. 2.

## 5. Summary and conclusions

This paper has been stimulated by the close similarity between beating oscillations appearing in the shell structure of nuclei and other many-fermion systems, and in the computer-simulated lunar observables from Babylonian times. We have focused on the method of Fourier analysis for extracting the dominating periods behind beating oscillations. For the quantum oscillations in fermionic systems this method is successfully employed in their semiclassical interpretation in terms of classical closed or periodic orbits through trace formulae, for which we have presented some examples.

We have given a short introduction into observation and theory of lunar phenomena from ancient Mesopotamia. We have then shown how the use of modern computer simulations of the lunar and solar ephemerides at ancient times allowed for a partial reconstruction of the methods by which the Babylonians have arrived at their nmerical lunar theory and their empirical schemes for prediciting lunar phases. There is no evidence at all that they had any theoretical understanding of the dynamics of the planetary system, nor any geometrical model for it. However, they were excellent numerical calculators and based their schemes on the collection and analysis of observational data over hundreds of years.

For a physicist it is a rather breath-taking experience to see old "experimental" data, recorded over 2500 years ago, reproduced by calculations requiring the best present numerical knowledge of our planetary system. Of course, the computer code of Moshier makes use of some old astronomical data, without which the extrapolation of presently valid lunar ephemerides back to ancient times would not be possible. E.g., there are long-term variations in the earth's rotational velocity which result in a clock error $\Delta$T. For the time around 300 B.C., $\Delta$T could be determined from a solar eclipse recorded on the tablet LBAT 1285.[22] However, the lunar data reproduced in Fig. 3 and on many other tablets have not been used as an input. Therefore the agreement seen in this figure, as well as in Fig. 2, gives us confidence into the accuracy of Moshier's code. At the same time it confirms the consistency of our present understanding of the empirical origins of the Babylonian lunar theory.

With the help of Fourier analysis we have illustrated the way by which the Babylonians could have determined the period $P_\Phi$ of the lunar velocity, contained in the column $\Phi$ of their lunar tables, from the sum $\Sigma$ of all Lunar Four. If this hypothesis is correct, then they have – without knowing it – performed a Fourier decomposition of their observed data.

In any case – the mere fact that the Babylonians some 2400 years ago were able to determine the length of the synodic month with an accuracy of six digits must be considered as one of the greatest scientific achievements of history.

Lis Brack-Bernsen is grateful to the Deutsche Forschungsgemeinschaft for financial support.

## References

1. M. C. Gutzwiller: *Moon-Earth-Sun: The oldest three-body problem*, Rev. Mod. Phys. **70**, 589 (1998).
2. L. Brack-Bernsen: *On the Babylonian lunar theory: A construction of column Φ from horizontal observations*, Centaurus **33**, 39 (1990).
3. S. M. Reimann, M. Persson, P. E. Lindelof and M. Brack, Z. Phys. B **101**, 377 (1996).
4. M. C. Gutzwiller, J. Math. Phys. **12**, 343 (1971) and earlier references quoted therein.
5. R. Balian, and C. Bloch, Ann. Phys. (N. Y.) **69**, 76 (1972).
6. M. Brack and R. K. Bhaduri: *Semiclassical Physics*, Frontiers in Physics Vol. 96 (Addison-Wesley, Reading, USA, 1997); revised paperback edition (Westview Press, Boulder, USA, 2003).
7. M. Brack, Ch. Amann, M. Pletyukhov and O. Zaitsev, these proceedings.
8. M. C. Gutzwiller: *Chaos in classical and quantum mechanics* (Springer, New York, 1990).
   *Quantum Chaos Y2K*, Proceedings of Nobel Symposium 116, Eds. K.-F. Berggren and S. Åberg, Physica Scripta Vol. **T90** (2001).
9. O. Neugebauer: *Astronomical Cuneiform Texts*, Vols. I - III (Lund Humphries, London, 1955).
   O. Neugebauer: *A History of Ancient Mathematical Astronomy*, Vols. I - III (Springer Verlag, New York, 1975).
10. A. J. Sachs and H. Hunger: *Astronomical diaries and related texts from Babyloia* (Österreichische Akademie der Wissenschaften, Vienna, Vol. I: 1988, Vol. II: 1989, Vol. III: 1996).
11. L. Brack-Bernsen and O. Schmidt: *On the foundations of the Babylonian column Φ: Astronomical significance of partial sums of the Lunar Four*, Centaurus **37**, 183 (1994).
12. L. Brack-Bernsen: *Zur Entstehung der Babylonischen Mondtheorie*, Boethius-Reihe Bd. 40 (Franz Steiner Verlag, Stuttgart, 1997).
13. S. S. Moshier: Computer code AA (v. 5.4, public domain, 1996) for ephemerides of the solar system, using algorithms published in the *Astronomical Almanac (AA)* of the U.S. Government Printing Office. Version 5.4 and more recent versions of the code AA are vailable at ⟨http://www.moshier.net⟩.
14. M. Chapront-Touzé and J. Chapront: *ELP2000-85: A semi-analytical lunar ephemeris adequate for historical times*, Astronomy and Astrophysics **190**, 342 (1988).
15. A. J. Sachs: *A classification of the Babylonian astronomical tablets of the Seleucid period*, Journal of Cuneiform Studies **2**, 271 (1948).
16. L. Brack-Bernsen: *Ancient and modern utilization of the lunar data recorded on the Babylonian Goal-Year tablets*, published in: *Actes de la Vème Conférence Annuelle de la SEAC, Gdansk 1997* (Warszawa – Gdansk, 1999), p. 13.
17. L. Brack-Bernsen: *Goal-Year Tablets: Lunar Data and Predictions*, Ancient Astronomy and Celestial Divination, Ed. N. Swerdlow (MIT Press, Boston, 1999), p. 149.
18. L. Brack-Bernsen and H. Hunger: *TU 11, A Collection of Rules for the Prediction of Lunar Phases and of Month Lenghts*, SCIAMVS **3**, 3 (2002).
19. K. Arita, these proceedings.
20. A. Holle, J. Main, G. Wiebusch, H. Rottke, and K. H. Welge, Phys. Rev. Lett. **61**, 161 (1988).
21. see, e.g., M. Brack: *Metal clusters and magic numbers*, The Scientific American, December 1997, Vol. 277, No. 6, p. 50, and the literature quoted therein.
22. F. R. Stephensen: *Historical Eclipses and Earth's Rotation* (Cambridge Univ. Press, Cambridge, 1997).