Prediction of Days and Pattern of the Babylonian Lunar Six

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1. Abstract

One of the current questions in Babylonian astronomy is how the Babylonians were able to find and agree on the length of lunar months. Another question is how *the days* of the four full-moon observations and the last visibility of the old moon were predicted.

This paper shall show how the days of each of the Lunar Six, i.e., the days of each of the six special lunar phases observed by the Babylonians, can be found easily. The days of the Lunar Six are interdependent and rather difficult to determine by means of modern astronomical considerations. But a detail in the Babylonian Goal-Year method for predicting the values of the Lunar Six gives the clue for finding the days: is the normal Goal-Year method utilized, all days will remain the same as they were one Saros earlier - if the Goal-Year method needs corrections, then the day of the corresponding phenomena will be displaced by one day. This means that if one knows the values and days of all Lunar Six for one month (plus some values found six months earlier), then one can find the *values and days* for all the Lunar Six to come one Saros later by means of the Goal-Year method; particularly the day of the NA_N. This day (30 or 31), at which the first of the Lunar Six occurs, indicates the length of the previous month. Therefore, evidently, this reconstructed method also gives easy means for finding - and agreeing on - the lengths of months to come.[1]

2. Preliminaries

The Babylonian months began at sunset in the evening on which the new crescent was visible for the first time after a conjunction. The next first visibility of the new crescent could occur on day 30 of that month or on the next day "31". If the crescent became visible on day 30, then this day was "turned back" and identified with the first day of the next month [MN 30]. If the first visibility took place on the day after day 30, it was noted under the day number 1 of the next month [MN 1]. A (synodic) Babylonian month had either 29 or 30 days; a month of 29 days is called hollow while a 30 days month is full.[2]

TU 11, an important procedure text concerned with lunar phenomena, gives several methods for determining month lengths (see Brack-Bernsen and Hunger 2002), but nothing is said about how to determine the days of the other lunar phases. The best method on TU 11 for predicting month lengths gives the correct answer in 97 per cent of all cases. It is based on the Goal-Year method for predicting the time, NA_N, from sunset to setting of the new crescent, and it utilizes among other criteria the actual procedure used: if the predicted $NA_N \geq 10$ u¡ or if $NA_N < 10$ u¡ (their limit of visibility). In the latter case it was assumed that the new crescent would not yet be visible so that one had to wait one day and correct the calculated NA_N by adding the daily retardation of the setting crescent (see Brack-Bernsen and Hunger 2002, p. 40-54). The method, which we shall present here, is similar and can be seen as an improved and extended version (of the reconstructed rule), which predicts not only the day of the new crescent but also the days of all the other Lunar Six.

The Goal-Year Texts (Hunger 2006), a special type of astronomical cuneiform tables, collect specific data of lunar and planetary phenomena. These selected data can be used to predict the same phenomena for a year Y to come. This coming year of interest was called the "Goal-Year". For instance were the "Greek-letter" phases of Jupiter from year Y-71 recorded and Jupiter's passings by Normal Stars from year Y-83. After 71 or 83 years, respectively, these phenomena repeat around the same date in (the Babylonian) year Y.

The Lunar Data collected in a Goal-Year text (for year Y) comes from year Y-18, which is the year situated 1 Saros = 223 synodic months ≈ 18 years prior to year Y. A Goal-Year tablet lists those lunar and solar eclipses (or eclipse possibilities) which had taken place in the year Y-18. Then comes for each of the 12 (or 13) lunar months of year Y-18 some 6 characteristic time intervals - the Lunar Six, listed under the day number at which they had occurred. These intervals measure the time between risings and settings of sun and moon in the days around conjunction and opposition.

And finally there were given some sums of intervals stemming from the last 6 months of year Y-19.

[1]) The content of this paper was presented during the "Third Regensburg Workshop on Babylonian Astronomy: Goal-Year Astronomy" held in Durham in May 2008.

[2]) Note that if month is full then the first day of month M+1 equals the 31st day of month M: in this paper we shall use both descriptions and call the day which ends a full month M either day 1 of month +1 or day 31 of month M.

These sums measure the daily retardation of the setting crescent or of the rising old moon, respectively.[3] How such lunar data can be used for predicting all the Lunar Six for year Y was shown in Brack-Bernsen 1999. I have called this method the "Goal-Year" Method (for the prediction of Lunar Six). That the Babylonians knew and used this method was confirmed by TU 11.

The Lunar Six

Before discussing the Lunar Six, let us present them: NA_N was observed on the evening when the new crescent was visible for the first time after conjunction, indicating the first day of the month:

NA_N = time between sunset and first visible setting of the new moon.[4]

At sunrise and sunset in the days (12 to 16) around opposition, the following "Lunar Four" time intervals were regularly measured:

ŠÚ = time from moonset to sunrise, measured at last moonset before sunrise.

NA = time from sunrise to moonset, measured at first moonset after sunrise.

ME = time from moonrise to sunset, measured at last moonrise before sunset.

GE_6 = time from sunset to moonrise, measured at first moonrise after sunset.

Toward the end of the month, the event *KUR* took place and could be measured:

KUR = time from last visible moonrise before conjunction to sunrise.

These time intervals are obvious and easy to observe. From our theoretical-astronomical point of view, however, the intervals are quite complicated quantities. I refer to O. Neugebauer's thorough discussion (1969, pp. 107-109) on which factors determine when the new crescent will become visible for the first time - and for how long a time. At this place we just sum up and give the result of such considerations: the Lunar Six depend on the time Δt from conjunction (or the opposition) to sunset. They also depend on $\lambda_☾$, the position of the full (or new) moon in the ecliptic, and on the lunar velocity $v_☾$, and the latitude $\beta_☾$:

$NA = NA(\Delta t, \lambda_☾, v_☾, \beta_☾)$

is a complex and complicated function.

Therefore it was a big surprise that the Babylonians found an elegant, easy, and astonishingly precise predicting rule, a "short cut" leading directly from known Lunar Six to Lunar Six to come a Saros later; this is what the Goal-Year method delivers.

3. Dead ends, or useless approaches

Using modern computers, it is possible to calculate all Lunar Six and to find the days on which they occurred, including the length of all months.[5] It is, however, very hard to find an easy way to predict the days by means of astronomical theoretical considerations alone. Several persons including myself have tried hard to do it; but without much success.

The time of opposition with respect to sunrise or sunset is determining for the Lunar Four; but it gives no direct clue to days (or patterns of days) of the Lunar Four, nor does the lunar velocity. In case of NA_N and *KUR* it is the time of conjunction with respect to sunrise or sunset which is important.

Also the lunar latitude, $\beta_☾$, influences the Lunar Six - especially around the equinoxes (where the ecliptic makes a small angle to the horizon at morning- or evening-time, respectively). But extreme latitude around the equinoxes gives no direct clue to the days or pattern of the Lunar Six either.

A few figures may help to illustrate how the lunar latitude influences the size and days of the Lunar Four. There is an asymmetry with respect to the influence on the Lunar Six of $\beta_☾$, the lunar latitude:

$\beta_☾$ plays a major role for *ŠÚ* and *NA* around spring-equinox: in the mornings when these phenomena are measured the ecliptic makes a small angle with the horizon so that variation in lunar latitude will have a large impact on their respective magnitudes. But $\beta_☾$ has only little influence on *ŠÚ* and *NA* around fall equinox, where the ecliptic makes a large angle with the horizon in the morning.

For *ME* and GE_6 and also for NA_N it is the other way around:

$\beta_☾$ plays a major role for NA_N, *ME* and GE_6 around fall-equinox, where the ecliptic makes a small angle to the horizon in the evenings when NA_N, *ME* and GE_6 are measured. But $\beta_☾$ has only little influence on these time intervals around spring equinox, where the ecliptic makes a large angle to the horizon at evening time.

[3]) In average and with respect to sunrise (or sunset) the moon is delayed by 48 minutes from one day to the next; in reality the delay of the moon varies roughly between 32 and 64 minutes: if e.g. the full moon sets at sunrise one day, then it may set 50 minutes after sunset the next day. These 50 minutes are the daily retardation of the setting moon.

[4]) In the texts with which we are working, this interval is called *NA*, but it occurs always together with an indication that it is the *NA* of the first day or the *NA* at the beginning of the month. I put this identification into the name, calling it *NA*(of the new crescent), or NA_N. I do this in order to be as precise as the Babylonian texts. There the term *NA* is also used for a time interval in the middle of the month, but always identified by calling it the *NA* of day 14 or the *NA* opposite the sun.

[5]) Peter Huber's Creslong.dat or Files computed by M. Brack using Moshier's program.

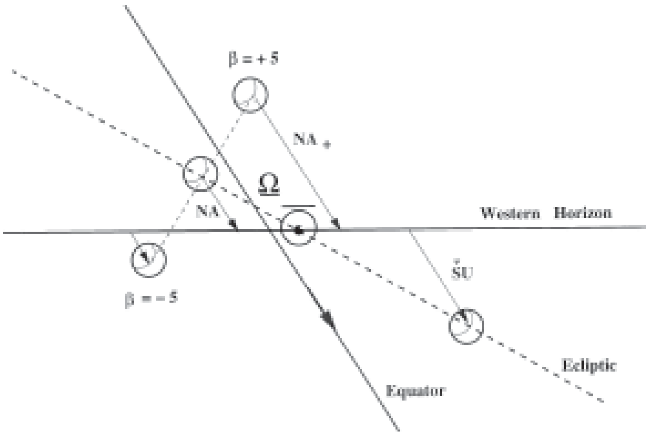

Fig. 1: The situation at the western horizon on a morning just after opposition and shortly before spring equinox. The sun is rising - here shown as the setting of an "antisun" (i.e. $\overline{2}$, the point on the ecliptic situated 180° from the sun). The full moon is shown in the three positions with zero, maximum and minimum latitude, respectively. It will set soon. We see that the time *NA* between antisun set and moon set is strongly influenced by the lunar latitude. The position of the full moon on the the day before, where *ŠÚ* was measured, is also indicated on the ecliptic below the horizon.

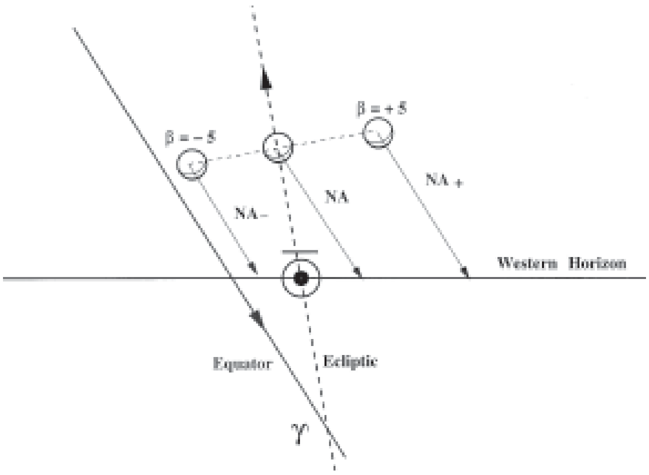

Fig. 2: The situation at the western horizon on a morning just after opposition some days after fall equinox. The sun is rising - here shown through the setting "antisun" (i.e. $\overline{2}$, the point on the ecliptic situated 180° from the sun). The full moon is shown in three positions with zero, maximum and minimum latitude, respectively. It will set soon. We see that the time *NA* between antisun set and moon set is only little influenced by the lunar latitude.

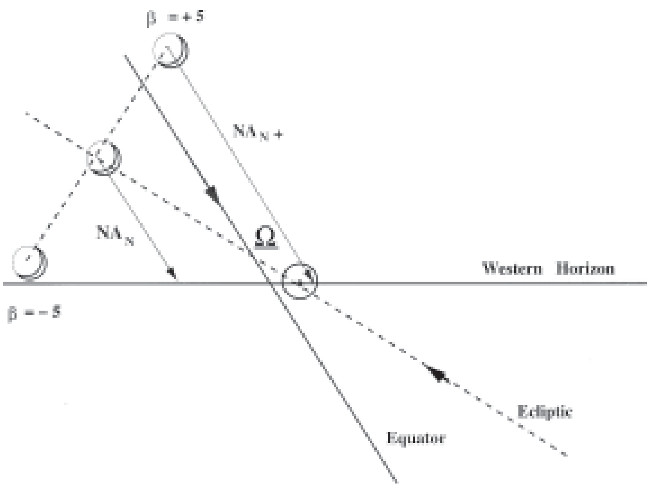

Fig. 3: The situation at the western horizon on an evening shortly after conjunction and just before fall equinox. The sun is setting and the new crescent (with $\beta_{☾} = 0$) is visible for the first time after conjunction; it will set soon. The positions of the new moon in case of maximum and minimum latitude are also shown. We see that the time NA_N between sunset and the setting of the new crescent is strongly influenced by the lunar latitude. For $\beta_{☾} = -5°$, the moon is invisible; the new crescent will only be visible on the next day.

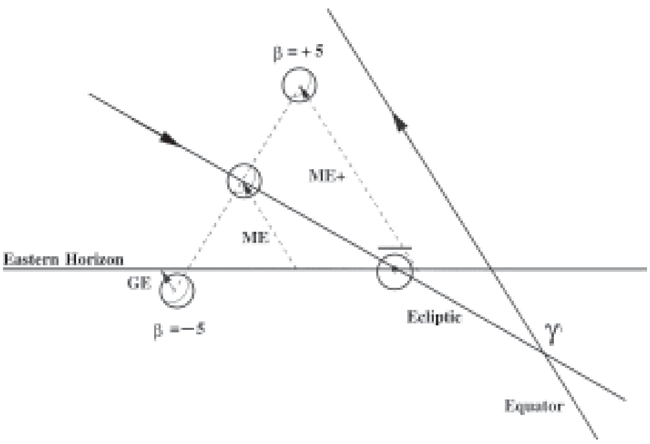

Fig. 4: The situation at the eastern horizon at sunset, half a month later than in figure 3 (i.e., around opposition and some days after fall equinox). The moon (with $\beta_{☾} = 0$, or +5) has already risen and the antisun is rising. We see that the time *ME* between moon rise and antisun rise is strongly influenced by the lunar latitude. In case of $\beta_{☾} = -5$, the moon will rise after sunset. In this case *ME* and GE_6 will take place a day earlier that in the case of zero latitude. Figure 2 shows the situation at the western horizon half a day later.

If for example the lunar latitude is very low, β☾ ≈ -5°, at the beginning of a new month, then NA_N becomes much smaller than it would have been by β☾ = 0°, so small that the new crescent may become visible a day later than in the "mean" case where its latitude is zero. The Lunar Six phenomena are, however, interdependent: when the lunar latitude is minimal at new moon (β☾ ≈ 5°), then it will be maximal at the next full moon (β☾ ≈ +5°), and *vice versa*. Also the days of the Lunar Six are inter-dependent: a new crescent seen a day later than "expected" will reduce the day number of the Lunar Four by one. (This is so since the Babylonians in each month started counting the days on that evening when the new crescent was seen for the first time).

A high latitude at full moon may let *ME* and GE_6 be observable a day later than they would have been if β☾ = 0°. Whereas a high latitude at full moon will hardly ever give reason to change the day of *ŠÚ* or *NA* to what it would have been with zero latitude.

But the matter is even more complicated, since the day number of each of the Lunar Six also depends on their respective values and day number the month before - and upon when the month in question began (i.e., upon the day of the preceding NA_N).

It is not easy to find a simple way or argument for determining the days of the Lunar Six; until now modern scholars have only been able to find their values by direct calculation and thereafter to determine the days on which they occur.

4. New "Babylonian" approach

The next approach (or question) is to investigate if the prediction of days of the Lunar Six is possible by means of Babylonian methods. We know that the Babylonians were able to determine the *values* of the Lunar Six by means of the surprisingly exact Goal-Year method. And we know that *also the days* of these phenomena were somehow predicted. In many texts we find remarks like the one below taken from Goal-Year Text SE 194.[6]

> Year 176. Month I, the 1st (of which followed the 30th of the preceding month),
> (NA_N =) sunset to moonset: 21°; the moon(?) could be seen; clouds, I did not watch.
> The 14th, (*ŠÚ* =) moonset to sunrise: 4° 10'; clouds, I did not watch.
> The 15th, (*ME* =) moonrise to sunset: 3° 20', clouds, I did not watch.
> The 15th, (*NA* =) sunrise to moonset: 4°; measured (despite) clouds.
> The 16th, (GE_6 =) sunset to moonrise: 10° 40'; clouds, I did not watch.
> The 27th, (*KUR* =) moonrise to sunrise: 15°; clouds, I did not watch.

It is generally agreed upon, that such precise - but not observed - Lunar Six values were found by means of the Goal-Year method. The question is, if this method (which is based on regularities in connection with the Saros period) somehow also could deliver the correct days of these phenomena. Before going into this investigation, let us summarize how the Lunar Six values for a month could be found by the Goal-Year method.

The Goal-Year formulae

The Goal-Year procedure is a very easy, elegant and quite precise method. We have found textual evidence for all the six "normal" formulae as well as for their corrected versions - so we know that the Babylonians knew and used them all. At this place we shall just give the formulae and refer to Brack-Bernsen and Hunger 2002 and 2008 for further details.

In the normal case, the Lunar Six of a month M to come (= the "new" month) are found in the following way from their respective values in the "old" month, i.e., in the month M-223, which came 223 months = 1 Saros before the "new" month M: in case of NA_N, *NA*, and GE_6, one third of the daily retardation (of the setting or rising moon, respectively) was subtracted from the "old" values - while for *ŠÚ*, *ME*, and *KUR*, one third of the daily retardation had to be added to the "old" values.

$$
\begin{array}{llll}
(NA_N)_{new} & = (NA_N)_{old} & - \ 1/3(ŠÚ+NA)_{old\text{-}6} & (1)\\
ŠÚ_{new} & = ŠÚ_{old} & + \ 1/3(ŠÚ+NA)_{old} & (2)\\
NA_{new} & = NA_{old} & - \ 1/3(ŠÚ+NA)_{old} & (3)\\
ME_{new} & = ME_{old} & + \ 1/3(ME+GE_6)_{old} & (4)\\
GE_{6\ new} & = GE_{6\ old} & - \ 1/3(ME+GE_6)_{old} & (5)\\
KUR_{new} & = KUR_{old} & + \ 1/3(ME+GE_6)_{old} & (6)
\end{array}
$$

If $NA_{N\ new}$ happened to become smaller than 10 u¡, TU 11 tells us to add the whole *ŠÚ+NA*. The idea behind such a correction is that, when the moon sets only 10 u¡ after sunset, the new crescent is supposed not to be visible. In the next evening, the new crescent will be visible for the time $NA_{N\ newC} = NA_{N\ new} + (ŠÚ+NA)_{old\text{-}6} = NA_{N\ old} + 2/3(ŠÚ+NA)_{old\text{-}6}$. The correction is based on the fact that the new crescent, in a good approximation, is delayed by $(ŠÚ+NA)_{old\text{-}6}$ from one evening to the next. The corrected procedure is given in (1C):

$$(NA_N)_{newC} = (NA_N)_{old} + 2/3(ŠÚ+NA)_{old\text{-}6} \qquad (1C)$$

Similarly, the calculated Lunar Four values might need a correction: if NA_{old} is smaller than 1/3(*ŠÚ+NA*), which should be subtracted from it according to (3), then *ŠÚ* and *NA* for the new month were found by procedures, reproduced in formulae 2C and 3C, respectively:

[6]) Hunger 2006, p. 271.

$$\check{S}\acute{U}_{newC} = 1/3(\check{S}\acute{U}+NA)_{old} - NA_{old} \quad (2C)$$
$$NA_{newC} = (\check{S}\acute{U}+NA)_{old} - \check{S}\acute{U}_{new} \quad (3C)$$

And analogously, if $GE_{6\ old} < 1/3(ME+GE_6)$, then procedures (4C) and (5C) were used.

$$ME_{newC} = 1/3(ME+GE_6)_{old} - GE_{6\ old} \quad (4C)$$
$$GE_{6\ newC} = (ME+GE_6)_{old} - ME_{new} \quad (5C)$$

Sometimes, the calculated KUR_{new} happened to become so large that the old moon would have been visible also the next day, in which case the corrected procedure was used:

$$KUR_{newC} = KUR_{old} - 2/3\ (ME+GE_6)_{old\text{-}6} \quad (6C)$$

In all cases where the corrected formulae were needed, the event would take place one day later than if the normal procedure had been applied. Evidently, the days of the Lunar Six are somehow connected to which Goal-Year procedure was to be used: the normal one (n) or the corrected one (nC).

The question still is, if and how the Goal-Year method (which is based on regularities in connection with the Saros period) also could deliver the *correct days* of the Lunar Six. In order to answer this question, Lunar Six data situated 1 Saros apart were compared.

Analysis of days of the Lunar Six

For this investigation we have used Peter Huber's computer-simulated "Babylonian" data. In his output (called Creslong.dat) Peter Huber has among others calculated the Lunar Six for all the Babylonian years from 758 to -68, giving also the month lengths and the days of the Lunar Six. Starting with Lunation GN 8537[7] (the first month of the first year of the Seleucid Era), the series of the next 13 lunations occurring at steps of 1 Saros = 223 synodic months were chosen; i.e., lunations with the Goldstine numbers GN = 8537 + n × 223, for n = 0, 1, 2, ..., 13. For this "Saros series" of 14 lunations, the values and days of the Lunar Six were extracted from the data base. The lunar latitude, β☾, at new and full moon for these lunations was found by means of Moshier's ephemerides program.[8] These Data are collected in table 1 where the Lunar Four values are listed in columns according to the night or day on which they occurred. *ME* and GE_6 were observed around sunset - so they are listed under N = night, while *ŠÚ* and *NA*, observable around sunrise, are listed under D = day.

In order to find eventual regularities with respect to the days at which each of the Lunar Four occur, more data are needed - and the eventual influence of the Lunar latitude should also be examined. Therefore we chose 10 consecutive lunations, GN 8537 to GN 8546, each of which serves as starting lunation for a series of 14 "Saros lunations". For each of these 10 series of 14 "Saros lunations" the data NA_N, *ŠÚ* and *NA*, *ME* and GE_6 have been extracted into tables and listed in columns according to the night or day on which they occurred.

We know that the Saros (≈ 18 years) is an important time interval for the moon, because in a good approximation it equals an integer number of anom(alistic), sid(ereal) and drac(onitic) m(onths). Therefore, the lunar latitude will not change much within the lunations of each Saros series:

1 Saros = 223 syn. m. ≈ 239 anom. m. ≈ 241 sid. m. ≈ 242 drac. m. ≈ 18 years.

The latitude at opposition for the 10 starting lunations GN 8537 - GN 8546 is given below. They cover the whole spectrum of the variation in the lunar latitude.

GN	β☾	GN	β☾
8537	3.1	8542	-1.0
8538	4.8	8543	-3.4
8539	5.0	8544	-4.8
8540	3.9	8545	-4.8
8541	1.6	8546	-3.3

Presentation of the data

The scheme below serves as a model for how the lunar data of the 10 Saros series are presented in Table 1 to 10. It shows three lunations from the first Saros series: the 10th, 11th, and the 12th lunations.

Bm	GN	full or hollow	12		13		14	
		NA_N	N	D	N	D	N	D
IV	10544	1/17.6 - 30						
					-5.2		4.5	
				- 8.0		**6.8**		
IV	10767	30/12.8 - 1						
					-8.0		1.3	
				-13.1		**2.2**		
V	10990	1/17.3 - 30						

Column 1 records the data at the beginning of the months under consideration. In the left part of column 1 are the Babylonian month name (Bm) represented by Roman numbers, then (in the middle) the number (GN) of the month according to Goldstine and the day (30 or 1) at which the new crescent became visible, then (in the right part) the magnitude of NA_N followed by the starting day (30 or 1) of the next month. This day indicates the length of the month in question. The 11th lunation may serve as example:

Bm	GN	NA_N
IV	10767	30 / 12.8 - 1.

[7]) We identify the lunations by means of the numbers used by Goldstine 1973 listing calculated New and Full Moons.

[8]) Moshier 1996, Computer code "AA", public domain.

BM	GN	full or hollow NA_N		12 N	12 D	13 N	13 D	14 N	14 D	15 N	15 D	16 N	16 D	β(moon) Opp
I	8537	1/23.7	-30		**-5.7**	-10.6	**2.9**	2.8						3.1
I	8760	30/18.3	-30		**-8.7**		**0.2**	-1.0		12.3				3.1
II	8983	30/13.0	-1				**-2.8**	-4.9	**6.8**	8.5				3.2
II	9206	1/24.1	-30		**-6.2**	-8.9	**3.9**	4.7						3.1
II	9429	30/18.6	-30		**-10.1**	-12.8	**0.6**	0.7						3.1
III	9652	30/13.3	-1				**-3.1**	-3.2	**9.0**	9.4				3.1
III	9875	1/22.3	-30		**-7.4**	-6.9	**5.4**	5.4						3.0
III	10098	30/16.7	-1		**-12.2**	-10.4	**1.3**	1.5						3.0
IV	10321	30/11.5	-1				**-3.2**	-2.0	**11.2**	8.2				2.9
IV	10544	1/17.6	-30		**-8.0**	-5.2	**6.8**	4.5						2.9
IV	10767	30/12.8	-1		**-13.1**	-8.0	**2.2**	1.3						2.9
V	10990	1/17.3	-30		**-2.6**	-1.6	**12.4**	6.3						2.8
V	11213	1/13.3	-1		**-7.5**	-4.2	**7.6**	3.5						2.8
VI	11436	30/9.8	-1		**-12.5**	-6.6	**2.8**	0.9						2.8

Table 1: The data from the first Saros Series, starting with lunation GN 8537.

The lunation with the Goldstine-number 10767 was month IV in the Babylonian calendar. It began on day 30 of month III with the first visibility of the new crescent (30/12.8); the new crescent being visible for 12.8 u¡. This day became the first of month IV, so that month III only had 29 days: it was hollow. The "- 1" after 12.8 tells that the next month V started on the 31st day of month IV, indicating that month IV was full - having 30 days. Had month IV been hollow, -30 had come after 12.8.

The next columns 2, 3, ... indicate the Babylonian days (12, 13, ... 16) on which the Lunar Four occurred, the days being separated into two parts, N and D. N stands for night or evening (sunset was the beginning of a new day) and D for day or morning (the second part of the Babylonian day started at sunrise). The "evening observables" *ME* and GE_6, respectively, are recorded under the index of N and the day numbers at which these phenomena took place. And the respective values of the "morning observables" *ŠÚ* and *NA* are recorded under their day number and D. The Lunar Four values are presented in the same way as in Huber's creslong.dat: the time intervals are given in units of u¡ = time degrees. In order to save place, the values of the phenomena *ME* and *ŠÚ*, where the moon is seen before sunset or sunrise, respectively, are given as negative numbers, while the values of GE_6 and *NA* (after sunrise or sunset) are positive.[9] For each month,

[9]) This convention is a modern one, introduced by Peter Huber. It reflects the insight that e.g. the time from moonset to sunrise equals minus the time from sunrise to moonset. In our schemes a minus before a bold number indicates that the number gives the value of *ŠÚ*. The Babylonians, of course,

BM	GN	full or hollow NA_N		12 N	12 D	13 N	13 D	14 N	14 D	15 N	15 D	16 N	16 D	β(moon) Opp
I	8537	1/23.7	-30		**-5.7**	-10.6	**2,9**	2,8						3.1
I	8760	30/18.3	-30		**-8.7**		**0,2**	-1.0	C↑	12.3				3.1
II	8983	30/13.0	-1			C↓	**-2.8**	-4.9	**6.8**	8.5				3.2
II	9206	C 1/24.1	-30		**-6.2**	-8.9	**3.9**	4,7						3.1
II	9429	30/18.6	-30		**-10.1**	-12.8	**0.6**	0.7						3.1
III	9652	30/13.3	-1			C↓	**-3.1**	-3.2	C↑ **9.0**	9.4				3.1
III	9875	C 1/22.3	-30		**-7.4**	-6.9	**5,4**	5,4						3.0
III	10098	30/16.7	-1		**-12.2**	-10.4	**1,3**	1,5						3.0
IV	10321	30/11.5	-1			C↓	**-3.2**	-2.0	C↑ **11,2**	8,2				2.9
IV	10544	C 1/17.6	-30		**-8.0**	-5.2	**6,8**	4,5						2.9
IV	10767	30/12.8	-1		**-13.1**	-8.0	**2,2**	1,3						2.9
V	10990	C 1/17.3	-30		C↓ **-2.6**	-1.6	**12,4**	C↑ 6,3						2.8
V	11213	1/13.3	-1		**-7.5**	-4.2	**7,6**	3,5						2.8
VI	11436	30/9.8	-1		**-12.5**	-6.6	**2,8**	0,9						2.8

Table 1a: The first Saros Series with lines visualizing the changes in days and with indications C, C↓ and C↑ of corrected procedures.

the "evening values" *ME* and GE_6 are listed first, and underneath comes in *bold* the *values* of *SÚ* and *NA*. The Lunar Four of a month, e.g., of month 10767, are listed in the lines between NA_N of month 10767 and NA_N of the next month 10990 (= 10767 + 223).

Our example shows that *ŠÚ* = **13.1** u¡ in month 10767, registered under 12 D, could have been measured in the morning of day 12 and *NA* = **2.2** u¡ the next morning (D of day 13). The phenomenon *ME* = 8.0 u¡ occurred near the beginning (N) of day 13, and GE_6 = 1.3 u¡ one day later (N of day 14).[10]

Table 1 gives all the data of the first Saros-series, starting with lunation GN 8537. Note that in this Saros-series (where the lunar latitude $\beta_☾ \approx 3.0°$) all of the Lunar Four were measured at an early day: the *ŠÚ*-phenomena took place in the mornings of either the 12th or the 13th day (but not of 14D or 15D), while GE_6 could be observed in the evenings of day 14 or 15, respectively (but never at the beginning of day 16). Note also that the day of each of the Lunar Four may change by one day from one month to the next (the time between two consecutive months being one Saros). This means that each of the Lunar Four can occur on

gave all numbers as positive numbers and identified a number by writing the name of the phenomenon behind its numerical value.

[10]) Here we use the same convention as the Babylonians. To be strict, one should say that *ME*, with the full moon rising before sunset, was measured at the end of day 12, while GE_6 with moonrise after sunset was measured at the beginning of day 14. There are about 25 hours between the two time measurements, therefore it makes sense to record *ME* under the beginning of day 13.

BM	GN	full or hollow NA_N		12 N	12 D	13 N	13 D	14 N	14 D	15 N	15 D	16 N	16 D	β(moon) Opp
II	8538	30/18.3	- 30				**-1.6**	-6.4	**7.5**	6.7				4.8
II	8761	30/12.5	-1				**-4.9**	-10.3	**4.8**	2.7				4.8
III	8984	C 1/23.1	-30		**-8.6**		**1.6**	-1.2		11.2	C↑			4.7
III	9207	30/17.0	-1			**C↓**	**-2.0**	-5.0	**9.5**	7.0				4.7
III	9430	30/11.6	-1				**-6.1**	-8.5	**6.1**	3.0				4.6
IV	9653	C 1/19.0	-30		**-10.6**		**2.1**	-0.6		9.5	C↑			4.6
IV	9876	30/13.2	-1			**C↓**	**-2.1**	-4.0	**11.3**	5.6				4.6
IV	10099	C 1/18.9	-30		**-6.6**	-6.9	**7.1**	2.2						4.5
V	10322	1/13.7	-30		**-11.2**		**2.8**	-0.8		C↑ 7.2				4.5
V	10545	30/9.1	-1			**C↓**	**-1.7**	-3.5	**12.1**	4.2				4.5
V	10768	C 1/14.2	-30		**-6.1**	-5.9	**7.7**	1.5						4.5
VI	10991	30/10.4	-1		**-10.6**		**3.4**	-1.0		C↑ 6.2				4.5
VI	11214	C 1/15.9	-30		**C↓ -1.1**	-3.4	**12.9**	3.8						4.4
VII	11437	1/12.7	-30		**-5.6**	-5.8	**8.5**	1.5						4.4

Table 2: The second Saros Series with lines visualizing the changes in days and with indications C, C↓ and C↑ of corrected procedures.

the same day (within the Babylonian month) as it did one Saros earlier - or it can take place one day earlier or later. The problem is to find out when a displacement takes place and when not. In the next table 1a the *ŠÚ*-values from table 1 have been connected by lines which help to visualize the change in days of the event *ŠÚ*, and similarly also the change in days of GE_6 is visualized by connecting lines.We shall now show how the changes in days are related to the Goal-Year method; just imagine all the Lunar Four values in Table 1 had been predicted by the Goal-Year method. We remind the reader that the Goal-Year method calculates the values of *ŠÚ* and *NA* for a "new" month M by means of the values they had one Saros earlier, i.e., from their values in the "old" month, which are written in lines above those of month M. Therefore the actual values of *ŠÚ* and *NA* in one line can help to find the appropriate Goal-Year procedure for predicting the values of *ŠÚ* and *NA* in the next line. A predicted *ŠÚ* has been found either by formula (2) or by (2C). We know that in the cases, where the corrected procedure (2C) had to be used, *ŠÚ* (and *NA*) would take place a day later than it would have done according to the normal procedure (2). Procedure (2) (and (3) for finding *NA*) were used when $NA_{old} \geq 1/3(ŠÚ+NA)_{old}$ while the corrected procedures (2C) and (3C) were utilized when $NA_{old} < 1/3(ŠÚ+NA)_{old}$. According to (2) will $ŠÚ_{new} > ŠÚ_{old}$, while $ŠÚ_{new\mathrm{C}}$ found by (2C) will be smaller than $ŠÚ_{old}$. Therefore, the 14 *ŠÚ*-values in Table 1 enable us to determine which procedure, (2) or (2C), should have been used if the *ŠÚ*-values had been predicted by the Goal-Year method. Is the *ŠÚ*-value of

Babylonian month	GN	full or hollow NA_N	12 N	12 D	13 N	13 D	14 N	14 D	15 N	15 D	16 N	16 D	β(moon) Opposition
III	8539	30/ 12.1 - 1											
									-2.7		9.8		
		C				**- 9.3**		**0.7**					5.0
III	8762	1 / 21.8 - 30				**C↓**							
							-6.5		5.5				
						- 2.9		**8.4**					4.9
IV	8985	30/ 15.4 - 1											
							-10.0		1.5				
		C				**- 6.9**		**4.9**					4.9
									C↑				
IV	9208	1 / 22.6 - 30											
							-2.1		8.0				
				- 11.3		**1.0**							4.9
IV	9431	1 / 16.5 - 30			C↓								
							-5.3		4.2				
						- 3.1		**9.8**					4.9
V	9654	30/ 12.1 - 1											
							-8.1		0.9				
		C				**- 7.3**		**5.7**					5.0
									C↑				
V	9877	1 / 16.7 - 30											
							-2.0		6.1				
				- 11.6		**1.6**							5.0
V	10100	30/ 12.0 - 30			C↓								
							-4.7		3.2				
						- 2.6		**10.6**					5.0
VI	10323	30/ 7.7 - 1											
							-7.1		0.5				
		C				**- 6.7**		**6.4**					5.0
									C↑				
VI	10546	1 / 13.6 - 30											
							-2.0		5.6				
				- 10.9		**2.3**							5.0
VI	10769	30/ 9.9 - 1			C↓								
							-4.5		3.1				
		C				**- 1.8**		**11.6**					5.0
VII	10992	1 / 17.1 - 30											
					7.1		-0.7						
				- 6.0		**7.6**							5.0
							C↑						
VII	11215	30/ 13.8 - 30											
							-1.9		6.8				
				- 10.3		**3.4**							5.0
VIII	11438	30/ 10.5 - 1			C↓								
							-4.6		4.5				5.0
						- 0.8		**13.4**					

Table 3: Data from the third Saros Series with lines visualizing the changes in days and with indications C, C↓ and C↑ of corrected procedures.

month M larger than that above (i.e., of the month a Saros earlier), then procedure (2) applies; is, however, a *ŠÚ*-value smaller than that above, then procedure (2C) had to be used. The difference between $ŠÚ_{new}$ (found by the normal procedure) and $ŠÚ_{newC}$ (found by the corrected procedure) is large, it equals (*ŠÚ*+*NA*), the daily retardation of the moon.[11] Therefore, the appropriate Goal-Year procedure can be determined unambiguously just by comparing the *ŠÚ* values in consecutive lines. Test calculations have verified this criterion for finding the appropriate Goal-Year procedure. *ŠÚ* and *NA* indicate the time of the setting (↓) full moon, while *ME* and GE_6 give the time of the rising (↑) full moon.

In Table 1a we have marked by C↓ each transition from a *ŠÚ* to the one below (one Saros later) which needs the corrected procedure (2C), if calculated by means of the Goal-Year method. All other *ŠÚ* values would come as result of the normal procedure (2). Analogously, the values of the observable GE_6 allow us to find the appropriate procedure (5) or (5C) which

[11]) $ŠÚ_{newC} = 1/3(ŠÚ+NA)_{old} - NA_{old} = ŠÚ_{old} - 2/3(ŠÚ+NA)_{old} = ŠÚ_{new} - (ŠÚ+NA)_{old}$.

Babylonian month	GN	full or hollow NA_N	12		13		14		15		16		β(moon) Opposition
			N	D	N	D	N	D	N	D	N	D	
IV	8540	1/20.0 -30											
							-10.2		1.6				
						-9.3		**2.9**					3.8
									C↑				
IV	8763	30/14.0 -1					**C↓**						
									-2.1		8.4		
		C						**-1.1**		**12.0**			3.8
V	8986	1/20.4 -30											
							-5.3		4.6				
						-5.2		**7.9**					3.9
V	9209	30/15.1 -30											
							-8.2		1.2				
						-9.3		**3.8**					3.9
									C↑				
V	9432	30/10.2 -1					**C↓**						
									-1.9		6.8		
		C						**-0.3**		**12.9**			3.9
VI	9655	1/16.2 -30											
							-4.6		3.7				
						-4.4		**8.7**					3.9
VI	9878	30/12.0 -30											
							-7.2		0.9				
						-8.5		**4.6**					4.0
									C↑				
VI	10101	30/ 8.1 -1											
									-1.9		6.4		
		C				**-12.5**		**0.6**					4.0
VII	10324	1/15.1 -30				**C↓**							
							-4.6		3.8				
						-3.5		**9.7**					4.0
VII	10547	30/11.5 -30											
							-7.4		1.1				
		C				**-7.7**		**5.7**					4.0
									C↑				
VII	10770	1/20.5 -30											
							-1.7		7.9				
				-11.9		**1.6**							4.0
VIII	10993	30/17.2 -30			**C↓**								
							-4.8		5.2				
						-2.5		**11.3**					4.0
VIII	11216	30/13.6 -30											
							-8.2		2.3				
		C				**-6.5**		**7.2**					4.0
									C↑				
IX	11439	1/25.8 -30											
							-1.0		11.3				
				-10.5		**3.3**							4.0

Table 4: Data from the fourth Saros Series with lines visualizing the changes in days and with indications C, C↓ and C↑ of corrected procedures.

would apply, had all GE_6 been predicted by the Goal-Year method. Is $GE_{6\ new} \leq GE_{6\ old}$ then the normal procedure (5) had been used, while in case of $GE_{6\ new} > GE_{6\ old}$ procedure (5C) was the appropriate one. In Table 1a each transition form a GE_6 to the one below (one Saros later), which is found by (5C), is marked by a C↑, while all "normal" transitions have no marking.

A glance at Table 1a shows that we have a corrected procedure C↑ each time the new GE_6 was measured one day later than the old GE_6 a Saros earlier: a GE_6-line sloping from left to right always coincides with C↑. The corresponding is true for the *ŠÚ* values: each time the event of *ŠÚ* is one day later than in the Saros before, the transition is marked by C↓.

Sometimes *ŠÚ* or GE_6 occurs one day earlier than in the previous Saros: the connecting line is sloping backwards toward left from one month to the one below. These backward shifts can be connected to, and explained by, the actual procedure utilized for finding NA_N, the event that announced the beginning of the new month. The shift backward (in day number of *ŠÚ* or of GE_6, respectively) always occurs when the corrected formula (1C) applies for finding $NA_{N\ new}$. This does not surprise since we know that (1C) was used each time when $NA_{N\ new}$ < 10 u¡, indicating that the

Babylonian month	GN	full or hollow NA_N	12 N	12 D	13 N	13 D	14 N	14 D	15 N	15 D	16 N	16 D	β(moon) Opposition
V	8541	30/12.9 -1											
									-3.5		6.9		
		C						**-7.2**		**6.8**			1.6
V	8764	1/18.9 -30											
							-6.6		3.3				
						-11.4		**2.4**					1.7
										C↑			
VI	8987	30/14.5 -30					**C↓**						
									-0.2		9.2		
								-1.9		**-12.1**			1.7
VI	9210	30/10.3 -1											
									-3.1		5.9		
		C						**-6.2**		**7.6**			1.8
VI	9433	1/16.7 -30											
							-6.0		2.8				
						-10.4		**3.2**					1.8
										C↑			
VII	9656	30/13.1 -30					**C↓**						
									-0.2		9.0		
								-1.1		**12.6**			1.8
VII	9879	30/ 9.5 -1											
									-3.3		6.0		
		C						**-5.5**		**8.2**			1.8
VII	10102	1/17.6 -30											
							-6.5		2.9				
						-9.8		**3.8**					1.8
										C↑			
VIII	10325	30/14.2 -30					**C↓**						
									-0.3		10.4		
								-0.5		**13.3**			1.8
VIII	10548	30/10.5 -1											
									-3.9		7.6		
		C						**-4.9**		**8.8**			1.8
VIII	10771	30/21.3 -30											
							-7.8		3.8				
						-9.1		**4.5**					1.8
										C↑			
IX	10994	30/17.3 -30											
									-0.1		13.2		
						-13.0		**0.3**					1.8
IX	11217	30/12.7 -1					**C↓**						
									-4.3		9.4		
		C						**-3.5**		**8.6**			1.8
X	11440	30/25.6 -30											
							-8.8		5.2				
						-6.8		**4.7**					1.8

Table 5: The fifth Saros Series starting with lunations GN 8541; most of the corrections C↓ and C↑ occur simultaneously.

new crescent could not yet be seen. One should wait another day and add the daily retardation of the setting moon, getting $NA_{N\ new\mathrm{C}} = NA_{N\ new} + (\check{S}\acute{U}+NA)_{old\text{-}6}$. But this means that the new crescent in such a month was seen a day later than expected in the first place. Such a month would start on day 31 of the previous month, instead of beginning on day 30, as it was the case one Saros earlier. The beginning of such a month has been pushed one day forward - with the result that the day number of each of the Lunar Four should be reduced by 1 as compared to the days one Saros earlier. Therefore the corrected procedures for finding NA_N are also indicated in the tables. In the first column of Table 1a a C written above a NA_N value indicates that this $NA_{N\ new}$ should be predicted by means of the corrected procedure (1C). Where no C occurs, the normal procedure applies for finding $NA_{N\ new}$. Note that a correction C (for finding NA_N) indeed has influence on the days of *ŠÚ* and GE_6, as argued above: there is a C each time connecting lines jump a step to the left. The actual Goal-Year procedure used for finding the Lunar Six seems to give the clue for also finding their days.

Babylonian month	GN	full or hollow NA_N	12	13	14		15		16		β(moon) Opposition
					N	D	N	D	N	D	
VI	8542	1/ 17.7 - 30			-6.6	-2.0	3.4 C↑	13.1			- 1.0
VI	8765	30/ 14.1 - 1				-6.7	-0.15	8.1	9.6		- 1.0
VI2	8988	30/ 10.6 - 1 C				-11.4	-3.3	3.2	6.2		- 1.0
VII	9211	1 / 17.3 - 30			-6.7	C↓ -1.7	2.8 C↑	13.0			- 1.0
VII	9434	30/ 14.2 - 1				-6.6	-0.7	7.9	9.7		- 0.9
VIII	9657	30/ 11.0 - 1 C				-11.5	-4.4	2.9	6.2		- 0.9
VIII	9880	1 / 19.7 - 30			-8.4	C↓ -2.1	2.4 C↑	12.3			- 0.9
VIII	10103	30/ 16.4 - 1				-6.9	-1.8	7.2	10.7		- 0.9
IX	10326	30/ 12.6 - 1 C				-11.5	-6.3	2.2	6.5		- 0.9
IX	10549	1 / 23.7 - 30			-11.3	C↓ -2.4	1.9 C↑	10.2			- 0.9
IX	10772	30/ 19.3 - 1				-6.4	-3.0	5.5	11.6		- 0.9
X	10995	30/ 14.2 - 1 C				- 9.8	-8.1	1.4	6.6		- 0.9
X	11218	1/ 25.9 - 30			-13.2	C↓ -2.1	1.6 C↑	7.3			- 0.9
XI	11441	30/ 20.2 - 1				-4.9	-3.4	3.9	11.5		- 0.9

Table 6: The sixth Saros Series. In all cases the corrected procedures C and C↓ take place in the same months, therefore ŠÚ stays on day 14 (*NA* on day 15).

Goal-Year Method to determine the days of the Lunar Four

The day of a new GE_6 is determined by the procedure for finding GE_6 as well as by the procedure for finding NA_N. When in a month NA_N as well as GE_6 would need a correction, then the days of *ME* and GE_6 will be the same as in the month above. The changes in days, due to the two corrected procedures, neutralize each other. This is often the case in Table 2, and in Table 1a it can also be seen: from lunation 10767 to lunation 10990 all three procedures need a correction, and *ŠÚ* and *NA* as well as *ME* and GE_6 are placed on the same days in month 10990 as they were in month 10767 a Saros earlier. This means that we have found a method for predicting the days of the Lunar Four by means of their values and days one Saros earlier. Is for instance *ŠÚ* determined by the normal procedure (2) (and consequently NA_N found by (1)), then *ŠÚ* will occur on the same day as in the previous Saros. Is the corrected procedure (2C) needed for *ŠÚ*, then it takes place a day later, while it will take place one day earlier if only NA_N (1C) but not *ŠÚ* is found by the corrected procedure. Scheme 1 below summarizes the

Babylonian month	GN	full or hollow NA_N	12	13	14 N	14 D	15 N	15 D	16 N	16 D	β(moon) Opposition
VII	8543	30/13.3 - 1									
					-9.6		0.3				
		C				**-10.6**		**5.3**			- 3.4
							C↑				
VII	8766	1/19.8 - 30				**C↓**					
					-3.4		6.9				
						-0.4		**15.6**			- 3.4
VII	8989	1/ 17.2 - 30									
					-7.5		3.0				
						- 6.1		**9.7**			- 3.4
							C↑				
VIII	9212	30/ 14.1 - 1									
							-1.2		10.5		
		C				**-11.7**		**3.9**			- 3.4
VIII	9435	1/ 23.0 - 30				**C↓**					
					- 5.8		6.2				
						-2.0		**13.4**			- 3.4
IX	9658	1/ 20.2 - 30									
					-10.9		1.5				
						- 7.6		**7.3**			- 3.4
							C↑				
IX	9881	30/ 16.8 - 1									
							-3.8		10.4		
		C				**-12.7**		**1.6**			- 3.4
IX	10104	1/ 27.1 - 30				**C↓**					
					- 9.4		5.1				
						- 3.6		**9.3**			- 3.4
							C↑				
X	10327	1/ 22.9 - 30									
							-0.6		15.1		
						-8.1		**4.1**			- 3.4
X	10550	30/ 18.1 - 1					**C↓**				
							-6.3		9.3		
		C						**-0.4**		**10.0**	- 3.4
X	10773	1/ 28.5 - 30									
					-12.0		3.6				
						-4.2		**5.6**			- 3.4
							C↑				
XI	10996	1/ 23.1 - 30									
							-1.9		13.7		
						- 7.3		**1.8**			- 3.3
XI	11219	30/ 17.6 - 1					**C↓**				
							-7.1		8.2		
		C						**-1.3**		**6.8**	- 3.3
XII	11442	1/ 27.7 - 30									
					-12.0		3.1				-3.3
						-4.0		**3.7**			

Table 7: The seventh Saros Series with lines visualizing the changes in days and with indications C, C↓ and C↑ of the corrected procedures.

rules for predicting the days of the Lunar Four, utilizing the Goal-Year method. In the scheme, a “0” means same day, a “+1” means one day later, and a “-1” means one day earlier than in the previous Saros:

Goal-Year formula used for month(i)	*ŠÚ*(i)		*NA*(i)		*ME*(i)		GE_6(i)	
	(2)	(2C)	(3)	(3C)	(4)	(4C)	(5)	(5C)
NA_N (1)	0	+1	0	+1	0	+1	0	+1
NA_N (1C)	- 1	0	- 1	0	- 1	0	- 1	0

Scheme 1 gives the displacement of the days of the Lunar Four, as compared to the days at which they took place one Saros earlier. Evidently the displacements for *ŠÚ* and *NA* are identical and so are those for *ME* and GE_6. Note that the 0 in the lower right corner of each box comes as the sum of two corrections which neutralize each other: -1 +1 = 0.

Goal-Year Method to determine the days of the Lunar Six

The regularities given in scheme 1 are generally true. In the 10 Saros series investigated in this paper

month	GN	full or hollow NA_N	12	13	14 N	14 D	15 N	15 D	16 N	16 D	β(moon) Opposition
VIII	8544	1/ 18.1 - 30			- 4.1	**- 3.2**	7.0	**13.8**			-4.8
VIII	8767	30/ 15.7 - 1			- 9.0	**- 9.6**	2.5	**7.1** C↑			-4.8
VIII	8990	30/ 13.0 - 1 C				**-15.9**	- 2.5	**0.5**	10.7		-4.8
IX	9213	1/ 21.3 - 30			- 8.1	**C↓** **- 5.8**	5.5	**9.9** C↑			-4.8
IX	9436	30/ 18.1 - 1				**-11.6**	-0.2	**3.4**	15.2		-4.8
X	9659	30/ 14.3 - 1 C					**C↓** -6.4	**- 2.4**	9.3	**11.0**	-4.8
X	9882	1/ 23.6 - 30			-12.8	**- 7.4**	3.1	**5.2** C↑			-4.8
X	10105	30/ 19.0 - 1				**-11.6**	-3.1	**0.2**	13.5		-4.8
XI	10328	30/ 14.1 - 1					**C↓** - 9.3	**- 4.0**	7.3	**6.4**	-4.8
XI	10551	1/ 23.4 - 30 C			-15.1	**- 7.5**	1.3	**2.3** C↑			-4.8
XI	10774	30/ 18.5 - 1					**C↓** -4.5	**-1.3**	11.9	**7.8**	-4.8
XII	10997	30/ 13.6 - 1 C					-9.9	**- 4.4**	6.3	**4.4**	-4.8
XII	11220	1/ 23.5 - 30			-14.9	**- 7.2**	1.0	**1.3** C↑			-4.7
XII_2	11443	30/ 19.0 - 1					**C↓** -4.0	**-1.6**	12.3	**7.3**	-4.7

Table 8: The eighth Saros Series with lines visualizing the changes in days and with indications C, C↓ and C↑ of corrected procedures.

(see tables 1 to 10), there is no single exception from the rules to determine the days of the Lunar Four. A correction C (for finding NA_N) causes the events *ŠÚ*, *NA*, *ME*, and GE_6 to take place a day earlier than expected. A correction C↑ delays *ME* and GE_6 one day while the correction C↓ will delay *ŠÚ* and *NA* one day in comparison to their days in the month one Saros earlier (which is the month in the line above the month in question). As we shall see below, something similar is also true for *KUR* and the next NA_N, observed at the end of a month. A correction for finding NA_N(i) at the beginning of month(i) will let *KUR*(i) and NA_N(i+1) be seen one day earlier than expected. A correction C_K for finding *KUR* pushes the event (last visible moonrise before sunrise) to become one day later than in the month one Saros earlier. The same is true if the corrected procedure (1C) is appropriate for finding NA_N(i+1), then the new crescent will be seen a day later that it was the case a Saros earlier. Remember that this first visibility, which always occurs on day 30 or day "31" of month(i), indicates the end of month(i) and the beginning of month(i+1) whereby it *deter-*

Babylonian month	GN	full or hollow NA_N	12	13	14 N	14 D	15 N	15 D	16 N	16 D	β(moon) Opposition
IX	8545	30/ 12.3 - 1 C			-11.8	**-12.0**	0.9	**5.2** C↑			-4.8
IX	8768	1/ 20.3 - 1			- 5.0	C↓ **- 1.6**	9.7	**14.4**			-4.8
IX	8991	1/ 17.1 - 1			-11.4	**- 7.8**	3.8	**7.6** C↑			-4.8
X	9214	30/ 13.4 - 1 C				**-13.4**	- 2.5	**1.4**	14.1		-4.8
X	9437	1/ 22.0 - 1			- 9.1	C↓ **- 4.0**	7.7	**8.9**			-4.8
XI	9660	1/ 17.7- 1			-15.7	**- 8.7**	1.3	**3.6** C↑			-4.8
XI	9883	30/ 13.2 - 1 C					C↓ - 5.1	**- 1.0**	12.1	**10.0**	-4.8
XI	10106	1/ 21.9 - 1			-11.2	**- 5.1**	5.9	**5.5** C↑			-4.8
XII	10329	1/ 17.5 - 1				**- 8.6**	- 0.15	**1.6**	17.1		-4.8
XII	10552	30/ 13.2 - 1 C					C↓ - 5.8	**- 2.1**	11.4	**8.0**	-4.9
XII	10775	1/ 22.7 - 30			-11.3	**- 5.4**	5.8	**4.5**			-4.9
I	10998	1/ 18.9 - 1			-16.6	**- 8.7**	0.4	**1.1** C↑			-4.9
I	11221	30/ 15.2 - 1 C					C↓ - 4.8	**- 2.2**	12.7	**8.8**	-4.9
I	11444	1/ 26.1 - 30			-10.1	**- 5.5**	7.6	**5.6**			-4.9

Table 9: The ninth Saros Series with lines visualizing the changes in days and with indications C, C↓ and C↑ of corrected procedures.

mines the length of month(*i*). Therefore, the reconstructed rule for finding the days of the Lunar Six includes a rule for predicting the lengths of consecutive months: the length of every month in a year, for which a Goal-Year table is composed, can easily be determined in advance. The rules to determine the days of all the Lunar Six are summarized in scheme 2 below:

Goal-Year formula used for month(i)	*ŠÚ*(i) (2)	(2C)	GE_6(i) (5)	(5C)	*KUR*(i) (6)	(6C)	NA_N(i+1) (1)	(1C)
NA_N (1)	0	+1	0	+1	0	+1	0	+1
NA_N (1C)	- 1	0	- 1	0	- 1	0	- 1	0

Scheme 2 gives the "Goal-Year" rule for finding the days of the Lunar Six in a month(i). The events *NA* and *ME* have been left out since their days obey the same rules as *ŠÚ* and GE_6, respectively. A "0" means that the event in question will take place at the same day as in the month one Saros earlier, a "-1" indicates that it takes place a day earlier, while "+1" indicates that it takes place a day later than in the "old" month. Note that the 0 in the lower right corner of each box comes as the sum of two corrections which neutralize each other: -1 +1 = 0.

With other words: the days of the six_{new} time intervals *ŠÚ*, *NA*, *ME*, GE_6, *KUR*, and $NA_{N\ new+1}$, which all take place during the month starting by the event $NA_{N\ new}$, obey the same rules. If no corrections are needed, the days will be the same as in the Saros

Babylonian month	GN	full or hollow NA_N	12	13		14		15		16	β(moon) Opposition
				N	D	N	D	N	D		
X	8546	1/ 14.7 - 1									
						- 8.3		7.5			
		C					**- 2.0**		**12.9**		-3.3
X	8769	1/ 23.4 - 30									
				-14.9		1.4					
					- 7.6		**6.7**				-3.4
							C↑				
X	8992	1/ 19.0 - 1									
						- 5.0		12.2			
					-12.6		**1.3**				-3.4
XI	9215	1/ 15.2 - 1				**C↓**					
						-11.3		5.9			
		C					**- 3.6**		**8.6**		-3.4
									C↑		
XI	9438	1/ 23.8 - 30									
						- 0.2		17.0			
					- 7.9		**3.9**				-3.4
XII	9661	1/ 19.6 - 1				**C↓**					
						- 6.2		11.1			
							- 0.3		**10.7**		-3.4
XII	9884	1/ 15.5 - 1									
						-12.1		5.2			
		C					**- 4.2**		**6.6**		-3.5
									C↑		
XII	10107	1/ 24.6 - 30									
						- 0.5		17.0			
					- 7.9		**2.8**				-3.5
XII2	10330	1/ 20.9 - 1				**C↓**					
						- 6.1		11.5			
							- 0.9		**10.3**		-3.6
I	10553	1/ 17.4 - 1									
						-11.3		6.1			
							- 4.6		**6.8**		-3.6
I	10776	30/ 14.0 - 1									
						-17.1		0.7			
		C					**- 8.4**		**3.1**		-3.7
									C↑		
I	10999	1/ 24.5 - 30					**C↓**				
						- 4.6		13.5			
							- 0.7		**12.7**		-3.7
II	11222	1/ 21.3 - 1									
						- 9.9		8.3			
		C					**- 4.8**		**9.1**		-3.8
II	11445	30/ 18.0 - 1									
						-15.0		3.1			
							- 9.2		**5.1**		-3.9

Table 10: The tenth Saros Series with lines visualizing the changes in days and with indications C, C↓ and C↑ of corrected procedures.

before. Is a correction needed for finding $NA_{N\,new}$, the days of the six following phenomena will be reduced by one, while the corrected procedure for finding one of these six time intervals indicates that the day number of that event will be enlarged by one.

The correctness of the rule for finding the days of NA_N is illustrated in Table 11. It presents the NA_N(i+1) data (of the 9th Saros series) in the same way as the Lunar Four data were given in Table 1 to 10, namely through their actual values recorded in the columns of the days at which they occurred. Similarly, in Table 11 the time intervals NA_N(i+1) are listed under day 30 or "31" of month(i) (of the 8th Saros series), and the NA_N values (of the 9th Saros series) are connected with lines. A C(i) or C(i+1) indicates when the corrected procedure was necessary. We find the same regularity as we saw in Table 1 to 10: a correction C(i+1) pushes the event to occur one day later than in the line above (= the Saros before) while a correction C(i) lets NA_N(i+1) be visible a day earlier than in the line above. Evidently, the day-finding rule, given in scheme 2, is correct for the 9th Saros series. It is always correct. Table 12 compares the NA_N data of three consecutive Saros series - also there the rule for the days of NA_N can easily be checked to be true. That the rule is of universal validity can be shown by combining the first columns of Table 1 to 10. Or, as we shall see below, even more easily by means of Table 13 and 14.

Babylonian month(i)	GN	full or hollow NA_N(i)	beginning of month(i+1)	on day 30 of month i "30, NA_N"	on day "31" month i " 1, NA_N"
VIII	8544	1/ 18.1 - 30	8545 IX	12.3	
VIII	8767	30/ 15.7 - 1	8768 IX		C(i+1) 20.3
VIII	8990	30/ 13.0 - 1	8991 IX		17.1
IX	9213	C(i) 1/ 21.3 - 30	9214 X	13.4	
IX	9436	30/ 18.1 - 1	9437 X		C(i+1) 22.0
X	9659	30/ 14.3 - 1	9660 XI		17.7
X	9882	C(i) 1/ 23.6 - 30	9883 XI	13.2	
X	10105	30/ 19.0 - 1	10106 XI		C(i+1) 21.9
XI	10328	30/ 14.1 - 1	10329 XII		17.5
XI	10551	C(i) 1/ 23.4 - 30	10552 XII	13.2	
XI	10774	30/ 18.5 - 1	10775 XII		C(i+1) 22.7
XII	10997	30/ 13.6 - 1	10998 I		18.9
XII	11220	C(i) 1/ 23.5 - 30	11221 I	15.2	
XII_2	11443	30/ 19.0 - 1	11444 I		C(i+1) 26.1

Table 11: The corrections C(i) and C(i+1) determine the beginning of month(i+1).

Conclusions

We have reconstructed a very easy, elegant, and powerful rule for finding the days of the Lunar Six by means of the Goal-Year method. Especially the days of NA_N(i+1) are interesting, since they deliver the length of month(i).

There has not been found textual evidence for this reconstructed rule, predicting the days of the Lunar Six. But we know that the days were (well) predicted by the Babylonians and that the texts generally agreed on month lengths. In addition it can be mentioned that TU 11 confirms some of the details of the reconstructed day-finding rule. In TU 11 Section 14 we read:

In order for you to see a hollow or full (*month*). *If in the 18*(*th year preceding*) *month I* (*begins on*) *the 1st* (*day*), *and an addition is not added to it, month II,*

Babylonian month(i-1)	GN	full or hollow NA_N(i-1)	Babylonian month(i)	GN	full or hollow NA_N(i)	Babylonian month(i+1)	GN	full or hollow NA_N(i+1)
VIII	8544	1/ 18.1 - 30	IX	8545	30/ 12.3 - 1 C(i)	X	8546	1/ 14.7 - 1 C(i+1)
VIII	8767	30/ 15.7 - 1	IX	8768	1/ 20.3 - 1	X	8769	1/ 23.4 - 30
VIII	8990	30/ 13.0 - 1 C(i-1)	IX	8991	1/ 17.1 - 1	X	8992	1/ 19.0 - 1
IX	9213	1/ 21.3 - 30	X	9214	30/ 13.4 - 1 C(i)	XI	9215	1/ 15.2 - 1 C(i+1)
IX	9436	30/ 18.1 - 1	X	9437	1/ 22.0 - 1	XI	9438	1/ 23.8 - 30
X	9659	30/ 14.3 - 1 C(i-1)	XI	9660	1/ 17.7- 1	XII	9661	1/ 19.6 - 1
X	9882	1/ 23.6 - 30	XI	9883	30/ 13.2 - 1 C(i)	XII	9884	1/ 15.5 - 1 C(i+1)
X	10105	30/ 19.0 - 1	XI	10106	1/ 21.9 - 1	XII	10107	1/ 24.6 - 30
XI	10328	30/ 14.1 - 1 C(i-1)	XII	10329	1/ 17.5 - 1	XII2	10330	1/ 20.9 - 1
XI	10551	1/ 23.4 - 30	XII	10552	30/ 13.2 - 1 C(i)	I	10553	1/ 17.4 - 1
XI	10774	30/ 18.5 - 1	XII	10775	1/ 22.7 - 30	I	10776	30/ 14.0 - 1 C(i+1)
XII	10997	30/ 13.6 - 1 C(i-1)	I	10998	1/ 18.9 - 1	I	10999	1/ 24.5 - 30
XII	11220	1/ 23.5 - 30	I	11221	30/ 15.2 - 1 C(i)	II	11222	1/ 21.3 - 1 C(i+1)
XII2	11443	30/ 19.0 - 1	I	11444	1/ 26.1 - 30	II	11445	30/ 18.0 - 1

Table 12: The beginning, 30 or 1, of month(i+1) is determined by the corrections C(i) and C(i+1).

which is after it, is full. One-third of ŠÚ+NA is 6: you subtract this(?) from NA of the 1st day of month II, and (if) it is less than in month I, which is before it, then month II of your new year is full. Whatever (month) in your 18(th year preceding) is full, and to which there is no addition added, and a subtraction is subtracted from it, and which is less than the month preceding it, you declare as full. If in your 18(th year preceding) (a month) is full, and an addition is not added to it, and a subtraction is subtracted from it, and it is less (sic!) than 10 UŠ, you declare (your month) as full. If in your 18(th year preceding) (a month) is full, and an addition is added to it, you declare (your month) as hollow.

For the discussion of this text we refer to Brack-Bernsen and Hunger 2002, pp. 40-54. At this place only a few comments shall be added: in procedure texts, as here in TU 11, the different methods (of calculation or for predicting) are often presented through numerical examples, whereby one month (mostly month I) is taken as an example for all months. TU 11, Section 14 is concerned with the prediction of month lengths by means of the Goal-Year method. In this case the text gives hints for finding the length of month II. [Note that the Goal-Year method was, indeed, used to predict the length of a month to come!] In TU 11, Section 14 we do not find the rules reconstructed in this paper (the text, sometimes corrupt, presents another rule) - but still we find traces of our rule: note that month I as well as month II of the "old year" are taken into consideration - and also the question if an addition is added or not. That an addition is added means in our terminology, that the corrected procedure (1C) was applied for finding $NA_{N\ new}$. We see here that the text combines the two months I and II, and that the length of month II_{new} also is dependent on how NA_N (I) was found. The passage presupposes that NA_N (I) was found without addition, i.e., by procedure (1). Therefore the text implies that the Babylonians had realized that the length of a month could only be predicted when it was known which procedure was adequate for finding NA_N of two consecutive months. This is parallel to our predicting rules for the days summarized in Scheme 2. Predictions are based on the combined information: correction or not for NA_N(i) and correction or not for the relevant phase in month(i+1), e.g., NA_N(i+1). What differs from "our rule" is that TU 11, Section 14 predicts the length of month II and not of month I, and that the values of NA_N(I) and NA_N(II) are compared. We shall now examine the days of the Lunar Four and their patterns more closely.

The pattern of Lunar Four days represented by "forks"

The information on the days of the Lunar Six, collected in Table 1 to 10, shall now be condensed into Tables 13 and 14 in the following way. From now on we are only interested in the days of the Lunar Four and not in their respective numerical values. Therefore we ignore the values and concentrate on the day patterns, representing them by different forks. In Table 1 to 10 we have met only 3 different patterns of the Lunar Four days. Figure 5 shows these three different patterns of days together with the forks by which we shall substitute them.

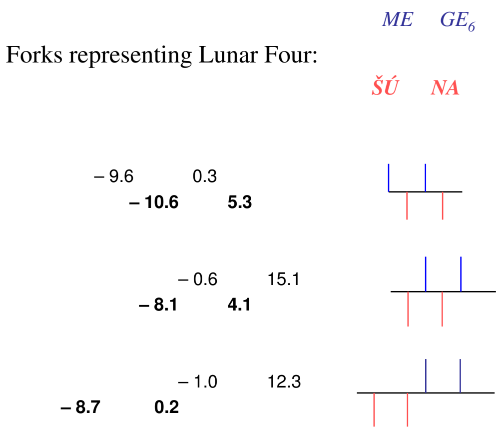

Fig. 5: The days of *ME*, GE_6, *ŠÚ*, *NA* are represented as forks: the horizontal axis is the time, the two vertical lines pointing upwards represent the moon rise events *ME* and GE_6, and the two vertical lines going downwards represent the setting moon at *ŠÚ* and *NA*.

At the top of Figure 5 we have an example where *ME* (-9.6) is observed first, at the beginning of day J, then comes *ŠÚ* (-10.6) in the middle of day J: ("J+1/2"), GE_6 (0.3) at the beginning of day J+1 and finally *NA* (5.3) at the time "J+1+1/2". In the middle of Figure 5, a case is exemplified where *ŠÚ* is observed first and *ME* half a day later. At the bottom a case is shown where *ŠÚ* is observed first (on day "J - 1/2"), then *NA* a day later ("J + 1/2"), while *ME* is observed at the beginning of day "J + 1" and GE_6 on day "J + 2". In our data there was no single case where the events were observed in the inverse order: *ME* on day "J", GE_6 on day "J + 1", *ŠÚ* on day "J + 1 + 1/2" and *NA* on day "J + 2 + 1/2". This combination is apparently very rare.

Table 13 shows the days of the Lunar Four for the first five Saros series in which the lunar latitude was positive, and Table 14 shows the days for the last five Saros series where the moon had negative latitude. Within each column (Saros series) the distance from line to line (i.e., from fork to fork) is one Saros = 223 syn. months. The forks in a horizontal line represent consecutive full moons. The numbers 1 to 14 in the first column of Table 13 and 14 count the lunations of the Saros series, which are separated by narrow vertical boxes. A "1" in the box between two forks (representing two lunations J and J+1) tells us that month J+1 started on the "31st" day of month J; month J was full. Where nothing is noted in the box between two forks, a "30" is meant: month J+1 started on the 30th day of month J - month J was hollow. The C's in the columns give information about when a correction was needed for finding a Lunar Six by means of the Goal-Year method: a C in front of and between two forks (representing month(i-223) and month(i)) indicates that the corrected procedure (1C) leads from NA_N(i-223) = $NA_{N\ old}$ to NA_N(i) = $NA_{N\ new}$. The day-rules from Scheme 1 can easily be controlled in Table 13 and 14: a C↓ refers to a corrected procedure for finding $ŠÚ_{new}$ and NA_{new}, and a C↑ behind and between two forks indicates that ME_{new} and $GE_{6\ new}$ should be predicted by the corrected procedures. Where no C's are written, the normal procedures apply. A C alone lets the forks be pushed a day to the left: all Lunar Four take place a day earlier than in the line above (= in the Saros before). A C↑ pushes the upward lines (= the days of *ME* and GE_6) one day to the right, while a C↓ lets *ŠÚ* and *NA* occur one day later than in the line above. If two corrections e.g., C and C↑ (but not C↓) occur in the same line and column, then the phenomena *ME* and GE_6 of the rising full moon will take place on the

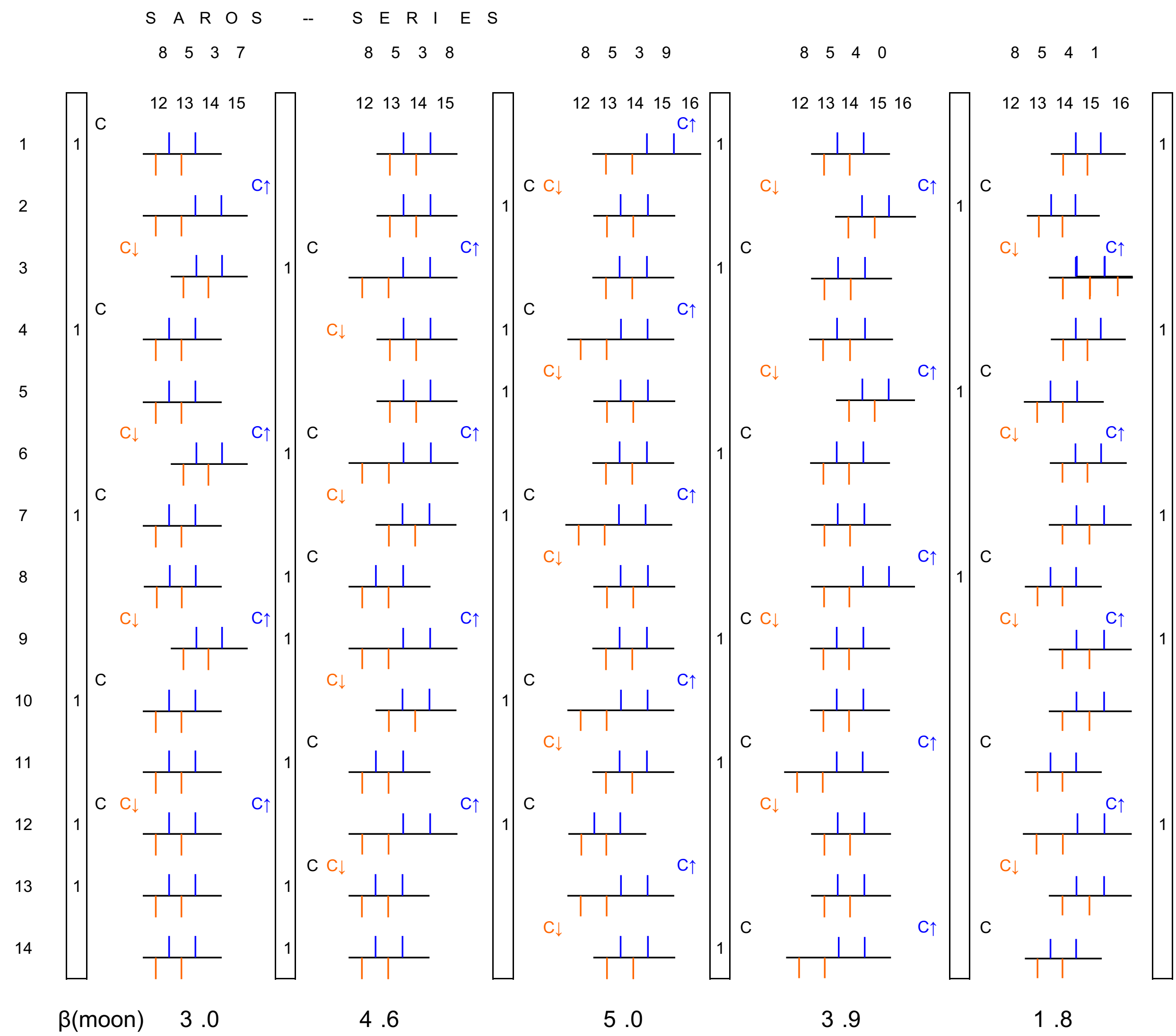

Table 13: The day-pattern of the Lunar Four through the five first Saros series, by which the moon was above the ecliptic: it had positive latitude. For further explanation of table see *ad* Fig. 5.

same days as in the line above, while the rising full moon, *ŠÚ* and *NA*, will take place a day earlier. The day-rules from Scheme 1 are confirmed by Table 13 and 14.

But also the rule (in Scheme 2) for the day of NA_N can be checked to be true: the change from a blank place in the box (representing "30") to a 1 in the next line occurs only then when a C indicates that the corrected procedure (1C) was demanded for finding $NA_{N\ new}$.

The change from a "1" in the vertical box to a blank (= "30") in the next line, can be explained by a corrected procedure for the beginning of the month before: a C in the column before a fork (of month i) indicates that NA_N(i-1) was found by (1C). It changes the starting day of month(i) from 1 to 30. But in all cases, where there are corrections in two consecutive months i-1 and i, the first day of month i will stay the same (1 or blank) as it was in the Saros before.

We have here seen that the data from the 10 Saros series confirm the rule for finding the day of NA_N. The days of *KUR* can similarly be found by means of the Goal-Year method.

Summary

The method for finding the days of the Lunar Six, reconstructed in this paper, may give the answer to our question: how did the Babylonian astronomers find and agree on the length of their months. At least we have here - for the first time - found an easy and elegant "Babylonian" method, which determines the days of the Lunar Six and hence also in advance gives the length of Babylonian months to come.

The method would enable the Babylonians to agree on the day at which their months began, and to determine the days of the other Lunar Six. We know through analysis of different tablets with information on beginning and length of Babylonians months that in the overwhelming number of cases, the Babylonian texts really did agree.[12] Now we eventually know how

[12]) See Steele 2007.

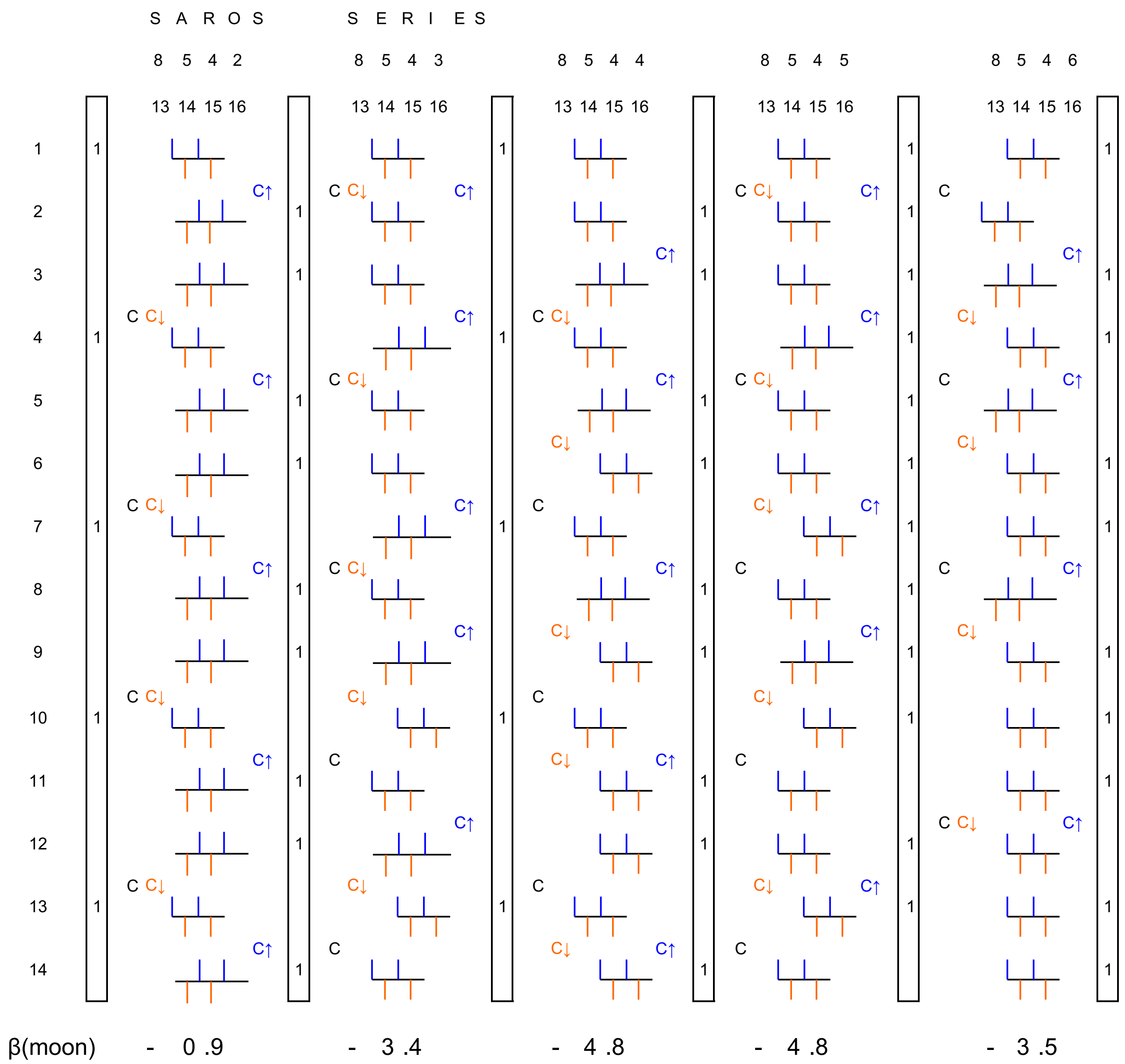

Table 14: The day-pattern of the Lunar Four through the five last Saros series, by which the moon was below the ecliptic: it had negative latitude. For further explanation of table see *ad* Fig. 5.

they managed to agree in advance on full and hollow months and on the days of the Lunar Four.

Outlook: *lunar latitude and the days of the Lunar Four*

The survey of the days of the Lunar Four in Table 13 and 14 delivers even more: when the lunar latitude is positive and larger than +1°, then *ŠÚ* will always be observed first: sometimes 1/2 day before *ME*, and other times 1 1/2 days before *ME*. (See Figure 6, the two lowest patterns) When the lunar latitude is negative and smaller than -1°, then sometimes *ŠÚ* is seen before *ME*, and other times is *ME* seen before *ŠÚ* (See Figure 6, the two upper patterns). The day-patterns change each time one of the corrections C↓ or C↑ occurs alone.

It has often been stated that the time of opposition (night or day) decides the pattern of the Lunar Four days, lately in the following way: "*The sequence in which the four intervals occur depends on the lunar*

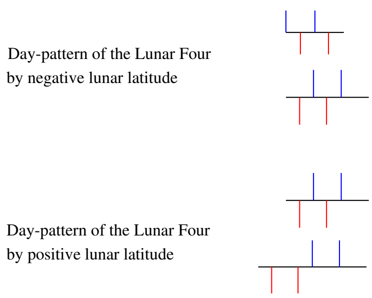

Fig. 6: The days of the Lunar Four are shown as forks: there are different patterns by positive as compared to negative lunar latitude.

position relative to the sun, and in particular on whether opposition falls into daytime or nighttime.[13] We know now that the lunar latitude plays the prevailing role for the sequence in which the Lunar Four intervals occur. For the first few rows of Table 13 and 14, we have compared the time of opposition (at night time or day time) with the patterns of Lunar Four days. Even when Table 13 (with positive lunar latitude) and Table 14 (where the moon's latitude is negative) are analysed separately, we can find no direct dependency of the time of opposition on the sequence in which the Lunar Four occur. The dominating factor is the changing lunar latitude.

We return to Table 13 and 14. The lunar latitude determines not only the day pattern of the Lunar Four but also to some extent how early or late in the month they can occur. Note that observations of *ŠÚ* on a day 12 of the Babylonian month only occur when the lunar latitude is high and positive (β☾ > 3.9 u¡). The graphic "fork" representation of the Lunar Four days in dependency of lunar latitude have helped us to understand more of Section 15 than we did at the time when we published TU 11. These new findings shall be published in a forthcoming paper.[14]

Addendum

Sacha Stern has kindly drawn my attention to a paper in which he has investigated new moon data from the Diaries: "The Babylonian Month and the New Moon: Sighting and Prediction," in *Journal for the History of Astronomy* XXXIX (2008), pp. 19-41. In this important paper, Stern has compiled 331 sightings and 110 predictions of the new crescent and checked the days (30 or 1) of the new crescent against modern astronomical data and visibility criteria. There were several incidences of first visibility at which the time NA_N from sunset to moonset was correctly measured to be less than 10 u¡, which is the visibility limit mentioned in TU 11. However, the data collection of predicted new moons is particularly interesting for the question of how the beginning of the Babylonian month was found. Stern could show that 7 (out of 110) predicted new crescents were still invisible: the new moon could not have been sighted until the next day. Therefore he questioned the visibility limit of 10 u¡ for the new crescent.

In order to answer this question, I have analysed the beginning of Babylonian months for a series of 669 months (= 3 Saroi). Among these 669 months, there were 19 cases where the new crescent would be visible for less than 10 u¡, and all these cases took place at the beginning of month V, VI or VII (and sometimes in A years in month VIII). These are the autumn months in which the ecliptic is very low at the western horizon at the time of sunset, as shown in Figure 3. This means that the new crescent can be far away from the sun and still set quite soon after sunset - so that we can understand why at this time of the year the visibility limit of NA_N can be reduced.

TU 11 contains the visibility criterion of 10 u¡ for NA_N. Maybe the Babylonians have found a better criterion, depending on the month - or maybe they have only recognized that sometimes the new crescent could set sooner than 10 u¡ after sunset - and then utilized a lower visibility limit for all months. This could explain the cases where the Babylonian month begins before the crescent was visible according to modern calculations. The Goal-Year method combined with the TU 11 visibility limit of 10 u¡ gives the correct month length in 97 % of all cases. A more detailed limit of visibility, depending on the month, could deliver an even better rate of prediction. But until we have textual evidence for such knowledge we can only guess.

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Bibliography

Brack-Bernsen, L. 1999: "Goal-Year Tablets: Lunar Data and Predictions," in N. M. Swerdlow (ed.), *Ancient Astronomy and Celestial Divination* (*Publications of the Dibner Institute for the History of Science and Technology*, MIT Press, Cambridge, Mass.), pp. 149-177.

Brack-Bernsen, L. and Hunger, H. 2002: "TU 11, A Collection of Rules for the Prediction of Lunar phases and of Month Lengths," SCIAMVS 3, pp. 3-90.

Brack-Bernsen, L. and Hunger, H. 2008: "BM 42282+42294 and the Goal-Year Method," SCIAMVS 9, pp. 3-33.

Brack-Bernsen, L. and Hunger, H. (forthcoming): "Further comments to Section 15 of TU 11."

Goldstine, H. H. 1973: "New and Full Moons 1001 B.C. to A.D. 1651," *Memoirs of the American Philosophical Society* 94.

Huber, P. J. and Steele, J. M. 2007: "Babylonian Lunar Six Tablets," SCIAMVS 8, pp. 3-36.

Hunger, H. 2006: *Astronomical Diaries and Related Texts from Babylonia*. Volume VI: *Goal Year Texts*. Österreichische Akademie der Wissenschaften, Wien.

Moshier, S. L. 1996: Computer code "AA" (v. 5.4, public domain) for ephemerides of the solar system, using algorithms published in the "Astronomical Almanac" (AA) of the U.S. Government Printing Office. For the moon, special use is made of: M. Chapront-Touzé and J. Chapront, "ELP2000-85: a semi-analytical lunar ephem-

[13]) See Huber and Steele 2007, p.3.

[14]) Brack-Bernsen and Hunger (forthcoming), "Further comments to Section 15 of TU 11."

eris adequate for historical times," *Astronomy and Astrophysics* 190 (1988) pp. 342-352.

Neugebauer, O. 1957: *The Exact Sciences in Antiquity*. 2nd ed., Brown University Press, Providence (also Dover 1969).

Steele, J. M. [2007] "The Length of the Month in Mesopotamian Calendars of the First Millennium BC" in J. M. Steele (ed.), *Calendars and Years*: *Astronomy and Time in the Ancient Near East*, Oxbow Books, Oxford, pp. 133-148.