# The Babylonian Dodecatemoria and Calendar Texts 

Inverse Schemes for Determining Position and Times for the Schematic Sun and Moon

by

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#### Abstract

The Dodecatemoria may be understood as a very simple reproduction of the Moon's movement: for each day in the schematic year, the Dodecatemoriascheme gives the position of the schematic Moon in the zodiacal circle. The Moon's angular velocity was calculated as $13^{\circ} /$ day and the Sun's, as $1 \%$ day. The scheme of the Calendar Texts may also be interpreted astronomically: for each position in the zodiacal circle, it gives the date at which the schematic Moon was in that position. We know that the schemes are inverse. A closer analysis of the texts accompanying the Calendar Text (LBAT 1586+1587) shows that the Babylonians also knew and utilized that fact as well.

\section*{About the Author}

LIS BRACK-BERNSEN'S research is in the history of mathematics and astronomy, especially the development of Babylonian astronomy and the use of computer simulations of ancient Babylonian observational data in systematic analyses. Her aim is to reconstruct the ancient rules governing prediction and to discover the concepts and methods behind early Babylonian astronomy.


The Dodecatemoria may be understood as a very simple reproduction of the Moon's movement: for each day in the schematic year, the Dodecatemoria gives the position of the schematic Moon in the zodiacal circle. ${ }^{1}$ The Moon's angular velocity was calculated as $13^{\circ} /$ day, and the Sun's as $1^{\circ} /$ day. The Calendar Text (Kalendertext) may also be interpreted astronomically: for each position in the zodiacal circle, it gives the date at which the schematic Moon [contra Wee 2016] was in that position. We know that the schemes are inverse. A closer analysis of the texts accompanying the Calendar Text (LBAT 1586+1587) shows that the Babylonians also knew and utilized that fact as well.

The following paper is part of a talk given at the conference "In Time: Astronomy and Calendars in the Ancient Near East", which was held in Jerusalem in June 2018. Its title was: "Babylonian Astronomy/Astrology, and the Role of (Modern and Babylonian) Mathematics in the Interpretation of Ancient Cuneiform Texts". The aim of the talk was to point to the effectiveness of modern mathematics for understanding old texts with numerical calculations or schemes, but also to warn against transferring too much modern knowledge into old texts. Our mathematics is a very rigorous tool which may give insights that the Babylonians did not have. Therefore, it is necessary to know how Babylonian mathematics worked and how calculations were performed in order to interpret the ancient texts as realistically as possible. (If we are solving an Old Babylonian mathematical exercise in "our way", and if this deviates from the directions given in the school text, we can be sure that we have not understood the method behind the exercise correctly.) In sum, mathematical and astronomical knowledge are important instruments for deciphering and understanding ancient technical cuneiform texts. We must, however, be aware of the danger of silently (and unconsciously) transferring too much modern knowledge and mathematical handicraft into these texts.

[^0]With this in mind, in an article on Dodecatemoria and Calendar Text schemes [Brack-Bernsen and Steele 2004], we mentioned merely that these were inverse schemes-knowledge which the Babylonians did not necessarily have. Therefore, at that time, we surmised that Calendar Texts were constructed from the Dodecatemoria by "number magics"; symmetries and play with numbers led to a strange scheme according to which, from line to line (i.e., for consecutive days), a position within the zodiac was shifted by $277^{\circ}$. In the Dodecatemoria, the lunar position shifted by $13^{\circ}$ from line to line. I have since found evidence, however, that the Babylonians indeed knew that the Calendar Text scheme, taken as a function, was the inverse of the Dodecatemoria function, and that it was constructed with this in mind. ${ }^{2}$
The Dodecatemoria scheme answers the question, Where is the schematic Moon $\left\langle D>^{3}\right.$ on day $d$ of month $M$ in the schematic calendar? I will now argue that the Babylonians constructed the Calendar Text scheme as an easy solution to the question, At which date will the schematic Moon be in sign $S$ at degree $d$ ? The answer to this question will give us a date which, at the same time, is the position of the schematic Sun on that day. Therefore, we can also formulate the task of the Calendar Text as follows: for each position of the schematic Moon within the zodiac, it gives us the corresponding date and position of the schematic Sun.

## 1. The Dodecatemoria scheme

The Babylonians assumed that the schematic Sun $<\odot>$ was at the beginning of Aries at the beginning of the schematic year, and that it moved $1^{\circ}$ per day, so that on day $d$ in month $M$ the $\langle\odot>$ would be situated at degree $d$ of sign $M .{ }^{4}$ As a result, the date "month $M$ day $d$ " was identified with "sign $M$ degree $d$ " in the zodiacal circle, which is the position of the schematic Sun on that day.

Month $M$ day $d \equiv$ zodiacal sign $M$ degree $d$, the position of $\langle\bigodot\rangle$.
The schematic Moon was supposed to move $13^{\circ}$ per day, i.e., $1^{\circ}$ as the Sun plus $12^{\circ}$ in elongation. When both start in conjunction at the beginning of

[^1]$\operatorname{sign} M$, then after 30 days the $\left\langle\odot>\right.$ will have moved $30^{\circ}$ and the $<D>390^{\circ}=$ $30^{\circ}+360^{\circ}$. At the beginning of the next month $M+1$, they are in conjunction again at the beginning of $\operatorname{sign} M+1$. The mean value of $12^{\circ}$ for the elongation movement (that is, the mean movement of the Moon with respect to the Sun) can be found in schemes B and D of Enūma Anu Enlil 14. ${ }^{5}$ At the time when $E A E$ was compiled (around 700 BC ), this $12^{\circ}$ was understood as the daily retardation of the Moon. Later, the number schemes from EAE and MUL.APIN ${ }^{6}$ were used for astronomical considerations and predictions. At that time, the scribes evidently knew that the daily retardation of the Moon is also a measure for the movement of the Moon with respect to the Sun [Brack-Bernsen and Hunger 2002, TU 11 §19: cf. Brack-Bernsen 1999, 152-154].
For each day of the schematic year, the Dodecatemoria lists simple meanvalue positions of the Sun and Moon within the newly invented zodiacal circle. The schematic year consists of 12 months of 30 days each; it is a practical approximation to the Babylonian lunisolar year of 12 synodic months, each of which having a duration of either 29 or 30 days. The length of the synodic month varies in a very irregular and (for the early Babylonians) unpredictable way. The real, observable Moon (D) moves irregularly, but such that its full phase always occurs close to day 15 of the Babylonian calendar. This is a consequence of the convention of letting the month begin on the evening when the waxing crescent was seen for the first time after conjunction. The schemes from MUL.APIN, EAE 14, and the Dodecatemoria do not give us precise numbers (times or positions) of the real Sun or Moon, but approximations which were utilized especially for astrological purposes. We call such astronomical investigations, based on the schematic calendar and mean movements of the Sun and Moon, schematic astronomy. MUL.APIN and EAE 14 are early examples of this astronomical tradition.
Table 1 [p. 48 below] lists a part of the Dodecatemoria scheme, specifically, that for the schematic months 1 and 6 . Reading the two first columns as a date (month and day) and the last two columns as the position (sign and degree) of $\langle D\rangle$, we see that the schematic Moon moves $13^{\circ}$ per day. Knowing that the schematic date equals the solar position, we can, therefore, also read the first two columns of each set of four columns as sign and degree

[^2](= solar position), and we have the position of the schematic Sun during the first 15 days of months 1 and 6.

The schematic Sun $\left\langle\odot>\right.$ moves $1^{\circ} /$ day.
The schematic Moon $<D>$ moves $13^{\circ} /$ day $=1^{\circ} /$ day $+12^{\circ} /$ day .
The numbers for the position of the Moon in the scheme in Table 1 seems to be calculated by adding $13^{\circ}$ per day, but its position can also be found from the solar position by adding $12^{\circ}$ per day. From a mathematical point of view, the two ways of calculating the position of the schematic Moon are equivalent.

| Date |  | $<D>$ <br> Position |  | Date |  | $<D>$ <br> Position |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Month | day | Sign | degree | Month | day | Sign | degree |
| 1 | 1 | 1 | 13 | 6 | 1 | 6 | 13 |
| 1 | 2 | 1 | 26 | 6 | 2 | 6 | 26 |
| 1 | 3 | 2 | 9 | 6 | 3 | 7 | 9 |
| 1 | 4 | 2 | 22 | 6 | 4 | 7 | 22 |
| 1 | 5 | 3 | 5 | 6 | 5 | 8 | 5 |
| 1 | 6 | 3 | 18 | 6 | 6 | 8 | 18 |
| 1 | 7 | 4 | 1 | 6 | 7 | 9 | 1 |
| 1 | 8 | 4 | 14 | 6 | 8 | 9 | 14 |
| 1 | 9 | 4 | 27 | 6 | 9 | 9 | 27 |
| 1 | 10 | 5 | 10 | 6 | 10 | 10 | 10 |
| 1 | 11 | 5 | 23 | 6 | 11 | 10 | 23 |
| 1 | 12 | 6 | 6 | 6 | 12 | 11 | 6 |
| 1 | 13 | 6 | 19 | 6 | 13 | 11 | 19 |
| 1 | 14 | 7 | 2 | 6 | 14 | 12 | 2 |
| 1 | 15 | 7 | 15 | 6 | 15 | 12 | 15 |
|  |  |  |  | 6 | 21 | 3 | 3 |
| 1 | 30 | 1 | 30 | 6 | 30 | 6 | 30 |

Table 1. Dodecatemoria schemes for the schematic Moon $<D>$ in months 1 and 6

Such schemes make it easy to find the lunar position $\langle D>$ for all days of the schematic year. In one month, the Sun has moved $30^{\circ}$ and the Moon $390^{\circ}$, so that they have reached each other at $30^{\circ}$, that is, the end of the sign on day 30 , which also is the beginning of the next sign on day o of the next month.

According to the scheme, the Moon moves $13^{\circ} /$ day $=1 \% /$ day $+12^{\circ} /$ day. The movement of the Moon with respect to the Sun is called the movement in elongation. Here the mean value of $12 \%$ day is used. The Babylonians had a correct understanding of elongation: they calculated the lunar movement in elongation as a function of the month and used it for finding the position of the Moon with respect to the Sun in the days around the New Moon (conjunction) [Brack-Bernsen 1999, 151-164; Brack-Bernsen and Hunger 2002, 72-75].
It is supposedly due to the connection

$$
13^{\circ} / \text { day }=1^{\circ} / \text { day }+12^{\circ} / \text { day },
$$

that the Babylonians subdivided each zodiacal sign (of $30^{\circ}$ ) into $12 \mathrm{mi}-$ cro-signs of $2^{1 / 2} 2^{\circ}$ each. Beginning with the name of the sign that is being subdivided, the micro-signs have the same names and sequences as the $30^{\circ}$-signs (see the example in Figure 1). Each micro-sign measures $2 \frac{1}{2} 2^{\circ}$, but it represents a real zodiacal sign of $30^{\circ}$. We have a scale factor of 12 : $2^{1 / 2} \times 12=30$. Thus, the movement of $1^{\circ}$ in the zodiacal-sign (ZS) scale corresponds to $12^{\circ}$ on the micro-sign ( $\mathrm{m}-\mathrm{s}$ ) scale. Let us imagine the schematic Sun $\langle\odot\rangle$ moving through ZS $S$ during month $S$. Its position in sign $S$ is $d^{\circ}$ on day $d$, but this position at the same time indicates the elongation of the Moon through its position in the micro-sign. In order to find the position of the Moon, one just has to add the number of days (= the number of degrees moved by the Sun) to the elongation, which is given through the solar position in the micro-sign scale. ${ }^{7}$
Figure 1 illustrates how this works. The whole sign Virgo (ZS 6) is divided into 12 micro-signs VI, VII, ...V, each of the length of $2^{1 / 2} 2^{\circ}$. The schematic Sun travels through Virgo during the 30 days of month 6, i.e., at one degree per day. On day 21 , the $\langle\odot\rangle$ will be in Virgo $21^{\circ}$, which is $1^{\circ}$ past the m -s II. This $1^{\circ}$ corresponds to $12^{\circ}$ in the m -s. Therefore, we can find the position of the schematic Moon as m-s II $12^{\circ}$ plus $21^{\circ}$ and we find the lunar position to

[^3]

Figure 1. Division of the zodiacal sign Virgo (= ZS 6)
This sign is divided into 12 micro-signs in which the positions of the schematic Sun and Moon on month 6 day 21 are indicated: $<\bigodot>$ is at $21^{\circ}$ of sign VI, which at the same time is $12^{\circ}$ in microsign II. $\langle\mathrm{D}\rangle$ is at $3^{\circ}$ in m-s III. Note that III $3^{\circ}=\mathrm{II} 12^{\circ}+21^{\circ}$.
be m-s III $3^{\circ}=$ II $12^{\circ}+21^{\circ}$. This is exactly the position given by the numbers 62133 in the Dodecatemoria scheme for month 6. We read it as follows: in month 6 on day 21, the schematic Sun is in position $21^{\circ}$ of Virgo ( $=$ ZS 6 $21^{\circ}$ ), the schematic Moon is situated in $3^{\circ}$ of micro-Gemini ( $=\mathrm{m}$-s III $3^{\circ}$ ).

## 2. Calendar Texts

The earliest Calendar Texts that we know about are BM 96258 and BM 96293, probably originating from the fifth century BC, shortly after the invention of the zodiacal circle. These texts only list four columns of numbers, without any indication as to how these numbers should be read. Table 2 [p. 51 below] reproduces on the left the Calendar Text scheme, and on the right the Dodecatemoria scheme for the first month 1, Nisan, of the schematic year. There exist schemes for all 12 months of the year. The texts display numbers, only. Other Calendar Texts identify the numbers in different ways. The columns marked by $\downarrow$ list the numbers $1,2,3, \ldots 12$, while the columns marked by $\downarrow$ display all the numbers from 1 to 30 . Thus, we may infer that the numbers in the first and third columns marked by § present either a month or a zodiacal sign, or both, while the numbers in columns 2 and 4, marked by $\downarrow$, present either the days in a month or the $30^{\circ}$ of a zodiacal sign, or both.

## 3. Inverse functions

Let me give an example of inverse functions: sine and arcsine. For each angle $\vartheta$, where $-90^{\circ}<\vartheta<90^{\circ}, \sin \vartheta$ equals a number $n$, where $-1<n<1$. And for each $n$ between -1 and $1, \arcsin n$ equals the number $\vartheta$, so that $\arcsin (\sin \vartheta)=\vartheta$.


Table 2. The Calendar Text and the
Dodecatemoria schemes for month 1
The position changes by $13^{\circ}$ from line to line in the last scheme, while the change of the number pair given in columns 3 and 4 of the Calendar Text changes by 277 from line to line. The pairs on bold red and magenta illustrate that the two schemes are inverse. The entry 11 (month 1 day 1) in the Dodecatemoria scheme gives 113 as the position of $\langle\mathcal{D}\rangle$. If in the Calendar Text scheme you look up what corresponds to 113 , you get back to 11 .

An inverse function brings us back to the starting point. That the two number schemes are inverse is illustrated by arrows in Table 2: the number quadruple (1 2126 ) from line 2 in the Dodecatemoria scheme is reversed by the number quadruple in line $26\left(\begin{array}{lll}1 & 26 & 1\end{array}\right)$ of the Calendar Text scheme. This relationthat the number schemes are mutually inverse-can also be expressed by two inverse functions $D$ and $K$. For each line in the schemes, we read the first number pair as an independent variable for finding the "result", namely, the number pair given in columns 3 and 4.
The relation between the number pairs $\left(M_{i}, m_{i}\right),\left(N_{i}, n_{i}\right)$ in line $i$ of the Dodecatemoria scheme can be expressed by a function $D$ :

$$
D\left(M_{i}, m_{i}\right)=\left(N_{i}, n_{i}\right)
$$

which is to be read
"On month $\mathrm{M}_{\mathrm{i}}$ day $\mathrm{m}_{\mathrm{i}}$, the schematic Moon will be at $\mathrm{n}_{\mathrm{i}}{ }^{\circ}$ of sign $\mathrm{N}_{\mathrm{i}}$ ".
Similarly, the relation between the number pairs $\left(N_{j}, n_{j}\right),\left(M_{j}, m_{j}\right)$ in line $j$ of the Calendar Text can be expressed by a function $K$ :

$$
K\left(N_{j}, n_{j}\right)=\left(M_{j}, m_{j}\right) .
$$

Here the generating pair is $\left(N_{j}, n_{j}\right)$ and the result is $\left(M_{j}, m_{j}\right)$.
The Calendar Text scheme is constructed such that when the generating pair ( $N_{j}, n_{j}$ ) in line $j$ of $K$ equals the result ( $N_{i}, n_{i}$ ) in line $i$ of $D$, then the result is:

$$
K\left(N_{j}, n_{j}\right)=\left(M_{j}, m_{j}\right)=\left(M_{i}, m_{i}\right) .
$$

And, inversely, if

$$
\left(M_{i}, m_{i}\right)=\left(M_{j}, m_{j}\right),
$$

then

$$
\left(N_{i}, n_{i}\right)=\left(N_{j}, n_{j}\right) .
$$

In conclusion, for all pairs of numbers $(N, n)$ and $(M, m)$, where $N$ and $M$ are two of the numbers $1,2,3, \ldots 12$, and $n$ and $m$ are two of the numbers 1 , $2,3,4 \ldots 30$, the following is true: if

$$
D(M, m)=(N, n),
$$

then

$$
K(N, n)=(M, m)
$$

and vice versa. Thus, it is always true that

$$
K(D(M, m))=K(N, n)=(M, m) \text { and } D(K(N, n))=D(M, m)=(N, n)
$$

The functions $D$ and $K$ are thus inverse.
We know that the Dodecatemoria function gives the position of $\langle\mathcal{D}\rangle$ for all days in the schematic year. Therefore, we conclude that the inverse Calendar Text function for all lunar positions in the zodiacal circle gives the date at
which the Moon occupies that position (according to the Dodecatemoria). Note that this date equals the position of $<\odot>$ at that day.

The Babylonians could have constructed the Calendar Text scheme from the Dodecatemoria by "number magic". First, they could have changed the order of the columns, letting columns 3 and 4 come first, followed by columns 1 and 2 . Then, they could have reorganized the lines so that the numbers in the first two columns come in "chronological order". Had it been constructed in this way, it would not now be evident that the Babylonians had any notion of inverse functions. Luckily, we have a text (LBAT 1586+1587) that gives us the position of $\langle\mathcal{D}\rangle$ for consecutive degrees in Gemini and the corresponding dates $(M, d)$, together with the comment that " $<D>$ is in sign $M$ at the position of Gemini". This clearly demonstrates that the Babylonians, indeed, had our understanding of these schemes: each degree of a zodiacal sign $(S, d)$, listed in the first two columns of the Calendar Text scheme, the date $(M, d)$ at which $<D>$ occupied that position is listed in the last two columns. The date indicates at the same time the position of $\langle\odot\rangle$. According to this interpretation, we can identify the number columns of the Calendar Text and Dodecatemoria schemes in Table 2 [p. 51 above]: see Table 3.

| Calendar Text <br> $<D>$ |  |  |  | Date |  | Dodecatemoria <br> Date |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 10 | 7 | 1 | 1 | 1 | 13 |  |
| 1 | 2 | 7 | 14 | 1 | 2 | 1 | 26 |  |
| 1 | 3 | 4 | 21 | 1 | 3 | 2 | 9 |  |
| 1 | 4 | 1 | 28 | 1 | 4 | 2 | 22 |  |
| 1 | 5 | 11 | 5 | 1 | 5 | 3 | 5 |  |
| 1 | 6 | 8 | 12 | 1 | 6 | 3 | 18 |  |
| 1 | 7 | 5 | 19 | 1 | 7 | 4 | 1 |  |
| 1 | 8 | 2 | 26 | 1 | 8 | 4 | 14 |  |

Table 3. Number columns of the two schemes
The numbers in the Calendar Text have been identified: columns 1 and 2 indicate the position of $\langle\mathcal{D}\rangle$, arranged so that it changes by $1^{\circ}$ from one line to the next. Columns 3 and 4 give the dates at which the schematic Moon was at that position according to Dodecatemoria. In the Dodecatemoria, the columns are identified as previously stated, that is, as date and corresponding position of the schematic Moon.
4. How the Calendar Texts were used for medicine

We have several texts which show that the Calendar Texts were used in medicine to determine the ingredients of ointments: three Calendar Texts, each covering one month, have been found; presumably they were part of a series of 12 tables covering a whole year. Some lines of the Calendar Text W22704 [von Weiher 1988] are reproduced below in a concentrated form. Maddalena Rumor [2021] has convincingly shown that Pliny the Elder, who criticized the medicine of the magics (Chaldeans), mentioned remedies which exactly match with the ones given in this Calendar Text covering month 4. Pliny's comments are very useful. They support our interpretation of the text and help clarify remedies and principles behind the scheme. Rumor concluded that ingredient names such as "eagle blood" were indeed Decknamen, which were referring to real medical herbs and plants.

| Date |  | ZS or Month |  |  | You anoint |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Month 4 day | 1 | (for Aries) | 1 | 7 | Sheep-blood, -fat, and -hair |
| - | 2 | Capricorn |  | 14 | Goat-blood, -fat, and-hair |
| - | 3 | Libra |  | 21 | "empty place"* |
| - | 4 | (for Cancer) | 4 | 28 | Crab-blood or crab-fat |
| - | 5 | Taurus |  | 5 | Bull-blood, -fat, or -hair |
| - | 6 | Aquarius |  | 12 | Eagle-head, wing, and blood |
| - | 12 | Virgo |  | 24 | Barley-flour, raven-head and -wing |
| - | 13 | (for Cancer) | 4 | 1 | Crab-blood or crab-fat |
| - | 14 | (for Aries) | 1 | 8 | Sheep-blood, -fat, and -hair |
| - | 15 | Capricorn |  | 15 | Goat-blood, -fat, and-hair |
| - | 16 | Libra |  | 22 | "empty place"* |
| - | 17 | (for Cancer) | 4 | 29 | Crab-blood or crab-fat |

* Wee 2016, 178 n109 proposes "Molted-skin of Scorpion".

Table 4. A section of the Calendar Text W22704
These lines show the close connection between the zodiacal sign and the ingredient written in the same line.

In Table 4, columns 1 and 2 list all 30 days of month 4, ŠU, which we also can read as all positions in Sign IV. Columns 3 and 4 seem to list positions in the zodiac, and column 5 lists ingredients which shall be used for ointments
to be produced for the day in question. ${ }^{8}$. As it has often been noticed, the ingredients listed in column 5 are closely connected to the zodiacal sign listed in column 3. But note also that for days 1 and 14, the name BAR of month 1 is written instead of Aries, and similarly, by the days 4,13 , and 17, the name ŠU of month 4 is written instead of the fourth zodiacal sign, Cancer. In cuneiform texts we often find that a date and the position of $\langle\odot>$ are identified or interchanged. Therefore, we may have a hint here that columns 3 and 4 list solar positions. That this really is the case, I show below. An astrological text (LBAT 1593), published by E. Reiner [2000] confirms that the Calendar Texts were used to produce medical remedies:

For the animal(s) of 13 and 4,37 (=277) you take one with the other, you salve, feed, and fumigate the patient with stone, herb, and wood (respectively).

We understand that the "animals" denotes the zodiacal signs while number 13 belongs to the Dodecatemoria and 277, to the Calendar Text.
5. The movement of the schematic Moon

There are three Late Babylonian tables with beautiful drawings of the zodiacal signs Taurus, Leo, and Virgo, where these signs are divided into 12 micro-signs [Weidner 1967, Tables 1, 2, 5-10]. In Plate 1 [p. 56 below], the text at the top describes a lunar eclipse. Below it, in many small squares, some remedies are listed: stones, temples, trees, plants, and other (medical) ingredients. I have added Roman numerals (V, VI,...) representing the zodiacal signs.
Let us imagine the Sun moving through Leo during the 30 days of month 5. On day 10 , it will have moved $10^{\circ}$ since the beginning of the month and reached the position Leo $10^{\circ}$, which is also the beginning of Micro Sagittarius, that is, m-s IX, a point which measures the Moon's movement in elongation during these 10 days. The position of the Moon on month 5 day 10 can be found by adding the $10^{\circ}$ (micro-degrees) to the solar position $0^{\circ}$ in m-s IX. Thus, we come to $10^{\circ}$ in m-s IX.
In short, on month 5 day 10 the position of $\langle\odot\rangle$ is Leo $10^{\circ}\left(\mathrm{m}-\mathrm{s}\right.$ IX $\left.0^{\circ}\right)$, and the position of $\left\langle D>\right.$ is found to be $10^{\circ}$ in m-s IX. In the Dodecatemoria, it is listed as 510910 [see Plate 1].

[^4]

Plate 1. The Dodecatemoria scheme for month 5
A visual representation of the numbers V 10 IX 10. It shows the position of the middle of $\langle\odot\rangle$, at Leo $10^{\circ} \mathrm{m}$-s IX $0^{\circ}$, and the position of the middle of $\left\langle D>\right.$ at $10^{\circ}$ in IX.

Plate 1 illustrates how the Dodecatemoria numbers, V 10 IX 10, or 5109 10 , can easily be found by means of the Babylonian concept of describing the movement of the schematic Sun and Moon. In the number schemes, $<\odot>$ moves $1^{\circ} /$ day and $\left\langle D>13^{\circ} /\right.$ day. That this model is equivalent to the calculation based on the daily movement of $\langle\odot\rangle$ plus the movement in elongation by $<\mathcal{D}>$ is evident, but it was also known to the Babylonians. The situation depicted in Plate 1 may be described as follows: the Sun is in $10^{\circ}$ of Leo and the Moon is in Leo at the position of $10^{\circ}$ Sagittarius.
We are in the lucky situation to have Calendar Texts in which special words describe exactly such situations: e.g., the Moon is in (=ina) the zodiacal sign $Z$ at the position (KI) of some micro-sign. This shows that the illustration in Plate 1 [p. 56 above] is not a modern speculation but rather one of the ways by which the Babylonians actually argued. On Table BM34452 +34738 (= LBAT 1586+1587 [see Hunger 1975]), we find number quadruples which we know from the Calendar Text schemes, with one difference: the columns have been interchanged so that the pairs of numbers, listed in the first two columns, rise by 277 from line to line, while the number-pairs in columns 3 and 4 rise by one from line to line. Therefore, according to our identification (in Table 3, p. 53 above), the two first columns of our Calendar Text shown in Plate 2 [p. 58 below], which increase by 277 from row to row, give us the date (that is, the position of $\langle\odot\rangle$ ) at which the schematic Moon $<$ D $\rangle$ was at the position given in columns 3 and 4 . We have now identified the Calendar Text numbers at the beginning of each section of Plate 2. Let us look closer at the accompanying text.
We have in Plate 2 a description similar to that of Plate 1. A translation, e.g., of lines $9-10$, might run as follows:

9153 15: The Moon stands in (ina) the back of Pabilsag (sign $9, \not{ }^{\wedge}$ ) at the position of $[\mathrm{KI}]$ the micro-sign Gemini, namely at $15^{\circ}$ within micro-sign 3 .
This text supports our new interpretation. In almost all legible cases of the whole text, the month given by the number in column 1 agrees with the zodiacal sign written after «ina», while the micro-sign within which the Moon is situated, written after «KI», always is Gemini = micro-sign III (which is also written as 3 in column 3). This is not just an accident; it must have been intended by the Babylonians. We now understand the structure of the text and are able to fill some holes and correct a few errors. Obv. 5-6 may be reconstructed to 6213 3. The Moon stands in the waist of Virgo within the micro-sign Gemini.

Rs.

| I) 2) | [8 173 II Sin ....] šá MÚL.PA? <br> [KI M]ÚL.SIP[A GUB] MARAD.DA.KI | KI: Micro sign always II = 3 |
| :---: | :---: | :---: |
| $\begin{aligned} & \text { 3) } \\ & \text { 4) } \end{aligned}$ | 5243 I2 Sin ina GÚ.MURGU MÚL.GU.LA <br> KI MÚL.SIPA GUB É d ${ }_{\text {GAŠAN.EDIN }}$ | $\delta=5$ |
| 5) <br> 6) | $\begin{aligned} & 3 \text { I } 3 \text { I3 } \operatorname{Sin} \text { ina MAŠ.SìL šá MÚL.MAŠ EG } \\ & \text { GUB GÚ.DU } \end{aligned}$ | $\mathrm{R}-i \text { II }=$ |
| $\begin{aligned} & \text { 7) } \\ & \text { 8) } \end{aligned}$ | $\begin{gathered} \text { I2 } 83 \text { I4 Sin ina MÚL.SIM.MAH KI } \\ \text { MÚL.SIPA GUB É d dNANŠE } \end{gathered}$ | $x=12$ |
| $9)$ 10) | $\begin{gathered} 9 \text { I5 } 3 \text { I5 } \operatorname{Sin} \text { ina EGIR MÚL.PA KI } \\ \text { MÚL.SIPA GUB ŠIR.BUR.[IA.KI] } \end{gathered}$ | $x^{7}=9$ |
| II) | šal-šú [ár $]$-hbu [....] Third Month |  |

Plate 2. The last readable lines of the Calendar Text LBAT 1586+1587 rev 10-11 in Hunger 1975

On the date given in the first two columns, the schematic Moon has reached the position given in columns 3 and 4 . The text after the numbers confirms this interpretation: The structure of all comments is alike; the Moon stands in (ina) some zodiacal sign (which in all cases is the same as the month of column 1) at the position (KI) of the micro-sign Gemini $=$ III, given in column 3.

Let me try to render the content of lines rev. 3-10:
rev. 3-4 524312 The Moon stands in (ina) the backbone of the Lion at the place (KI) the Shepherd = micro-Gemini....
rev. 5-6 31313 The Moon stands in (ina) the shoulder of the rear Twin (= micro-Gemini)....
rev. 7-8 128314 The Moon stands in (ina) the Swallow at the place of (KI) the Shepherd = micro-Gemini....
rev. 9-10 915315 The Moon stands in (ina) the back of
Pabilsag at the place of (KI) the Shepherd = micro-Gemini. ....
rev. 11 Third month.
In the last line, rev. 11, month 3 is clearly mentioned. This has led to the reading of all numbers 3 in column 3 as indicating month 3 . But when we
consider what this text might have been used for, we see that the identification is not compelling. I propose that we have an instruction here:

For consecutive days in month III you find in this text the zodiacal sign (of the Sun) and the micro-sign (of the Moon), given by the numbers in columns 1 and 3.
These signs were used to determine ingredients for ointments as indicated in LBAT 1593 and demonstrated in Calendar Text W22704 [see Table 4, p. 54 above], and they are repeated by words in the accompanying text. This means that we can read the pair of numbers (III, $n$ ) in columns 3 and 4 in two ways: as index, date (III, $n$ ), for finding remedies for month 3, day $n$. And at the same time, we understand (III, $n$ ) as the position of the $\langle\mathrm{D}\rangle$, namely, as $\mathrm{n}^{\circ}$ in ZS III. The date $(M, d)$ at which this happened, according to the Dodecatemoria, is listed in the columns 1 and 2. Here again, we have a double reading since date $(M, d)$ also indicates the position of $\langle\odot\rangle$ as $(M, d)$. The text following the numbers ( $M, d$, III, $n$ ) mentions the zodiacal sign $M$ of the Sun (on date $(M, d)$ ) and the micro-sign III of the Moon. They may both have served to determine remedies for ointments for month 3 day $n$.
This analysis shows us that the Babylonians, indeed, knew and utilized the fact that the Calendar Text and Dodecatemoria functions are inverse functions. It means that our first interpretation of the Calendar Text as a construction reached from playing around with numbers and symmetries, without any astronomical significance must be revised [Brack-Bernsen and Steele 2004, 118].
We have explained the fact that the Calendar Text scheme is inverse to the Dodecatemoria and we have also seen that the Babylonians knew and utilized this knowledge. Accordingly, we reject John Wee's interpretation [2016], in which the imaginary point that changes its position by 277 per day is a "virtual Moon". This must be replaced by the following:

After 277 days $=277^{\circ}$ movement of the schematic Sun $\langle\odot\rangle$ the position of the schematic Moon<D> has increased by $1^{\circ}$.

We do not have a "virtual Moon"; rather, we have the schematic Sun. We do agree, however, that each zodiacal sign was divided into 12 micro-signs.
In sum, the first two columns of the normal Calendar Text scheme, the rows of which increase by 1 , list consecutive positions of $\langle\mathcal{D}\rangle$ in the zodiacal circle; and the last two columns, the rows of which increase by 277 , give the date at which $<\mathrm{D}>$ was in the listed position. At the same time, the first two columns could be used as index (read as month day) for finding the
zodiacal signs (given by the month listed in column 3), which determined the remedies for ointments to be produced for that day.

Let us return to LBAT 1593, §2:
When the animal(s) of the 13 are before you, you take $1,1,1,13$ animal(s) of month V for? (month?) I, you take 1, 2, 1, 26 animal(s) of month XI for? (month?) I, you make the animal(s) of the trees, the date palm, the goose, the date palm for? The animal(s) of 13 , the writing board? of month VII for? (month) I for? (month) X , for the animal(s) of 4,37 the date palm, the goose, the date palm "you take away from" 9 the constellation Old Man...stone, herb, tree, the animal(s) of 13 and 4,37 you take one with the other, you salve, feed, and fumigate the patient with the stone, herb, and wood (respectively), the biblu (almanac?) of month I, from the 1st to the 30th.... [Reiner 2000, 424]
This text is damaged and very difficult to read—Reiner has inserted many question marks-but I think that we can still learn something from it. It mentions the numbers 13 and $4,37=277$, so that it is clearly referring to the Dodecatemoria and Calendar Text schemes. And it gives advice on how to determine medical ingredients for ointments by means of those schemes, starting with the first two lines ( $\begin{aligned} & 1 \\ & 1\end{aligned} 1113$ ) and ( $\begin{array}{ll}1 & 2\end{array} 126$ ) of the Dodecatemoria for month 1 . For these days, one has to take the animals of months 5 and 11, respectively.
But it is not evident where such months $\approx$ signs come from: Are they eventually determined by the micro-signs of 113 and 126 ? The text is too damaged and unclear to allow any reconstruction. The last line mentions month 1 and its 30 days from the 1 st to the 30th. This shows that month 1 was used as an example to illustrate the method for finding astral ingredients by means of the Dodecatemoria and Calendar Text schemes. In astronomical procedure texts, we find the same practice, namely, demonstration of a method by model calculations for month 1 , where the method was meant to be applied for all months. We do not know how the remedies were found by the Dodecatemoria scheme. But the Calendar Text procedure for finding the zodiacal sign, which determines the astral ingredients on an arbitrary date (month $N$ day $n$ ), is well known: read $(N, n)$ as a lunar position and find the date $(M, d)$ at which the Moon was at the position $(N, n) .(M d)$ is found in the Calendar Text scheme as partner to $(N, n) . M$ is the deciding sign.
Let the two first lines of the Calendar Text (1 $1 \mathbf{1} 1 \mathbf{1 0} 7$ 7) and ( $\begin{array}{lll}1 & 2 & 7\end{array} 14$ ) serve as an example:

[^5]On date ( $\mathrm{I}, 1$ ), the "animal" is indicated by the third number 10 (in bold dark blue), and similarly for date ( $\mathrm{I}, 2$ ), the "animal" for ointment is given by the third number 7 (in bold dark blue).

## 6. Conclusions

The Babylonians knew that the two astrological number schemes, the Dodecatemoria and the Calendar Text, were inverse schemes or functions, and used both in medicine. This conclusion is based on two Calendar Texts which give us two important insights.
(1) W22704 (treated in Table 4 [p. 54 above]) shows that Month $M=$ Sign $M$ given in column 3 determines the ingredients of ointments. The advice in column 5 , indicating the "animal" to be used, always mentions remedies connected to Sign $M$.
(2) LBAT $1586+1587$ (shown in Plate 2 [p. 58 above]) confirms the identification of the number pairs as lunar position and corresponding date. Note, however, that the order of the columns has changed. Here the last two numbers indicate the position of the schematic Moon for consecutive degrees in the zodiacal Sign $Z$, while the first two numbers give the date $(M, d)$ at which this happened, and which at the same time indicate the position of the schematic Sun on that day. For each quadruple of numbers, the accompanying text always describes the schematic Moon to be in Sign $M$ at the position of micro-sign $Z$.
(3) LBAT 1593 confirms that the ingredients to be used in medicine were indeed determined through the numbers from the CalendarText scheme (and the Dodecatemoria). Thus, when used for finding ingredients ("animals") for medicine for a special day, the Dodecatemoria might have indicated the "animals" through the position of $<D>$ in the zodiacal circle and micro-sign on that day. The Calendar Text found the ingredients for month $N$ day $n$ from the date (month $M$ day $d$ ), at which $\left\langle D>\right.$ was in $\operatorname{sign} N n^{\circ}$. Based on the identification of date and position of $\langle\odot\rangle$, we know that $\langle\odot\rangle$ was in zodiacal sign $M$ ("the animal") of that date, while the micro-sign was the same, namely, $N$, for all 30 lines of the Calendar Text of sign $N$.

Accordingly, we now understand the inner structure and, hence, the interplay between the two schemes and have shown that the Calendar Texts do not record positions of any "virtual Moon", but deliver the date at which $<D>$ was at the given position according to the Dodecatemoria.

## 7. Addendum

We observe the flexibility with which the Babylonians manipulated the two imaginative number schemes, that of the Dodecatemoria and of the Calendar Texts. Both concern numbers, which sometimes are identified as a date and position, or as a position with a date, or as a date with date, or as a position with a position.
The Calendar Text BM 47851, published in Hunger 1996, is an example:

| Line | Month | Day | Month | Day | Month | Day | Month | Day |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | V | 1 | V | 13 | V | 1 | II | 7 |
| 2 | V | 2 | V | 26 | V | 2 | XI | 14 |
| 3 | V | 3 | VI | 9 | V | 3 | VIII | 21 |
| 4 | V | 4 | VI | 22 | V | 4 | V | 28 |
| 5 | V | 5 | VII | 5 | V | 5 | III | 5 |
|  |  |  |  |  |  |  |  |  |
| 28 | V | 28 | V | 4 | V | 28 | XI | 16 |
| 29 | V | 29 | V | 17 | V | 29 | VIII | 23 |
| 30 | V | 30 | V | 30 | V | 30 | V | 30 |
| 31 | VI | 1 | VI | 13 | VI | 1 | III | 7 |
| 32 |  | I? | 1 | I | 11 | 18 | 19 |  |
| 33 |  |  | 14 |  |  | 26 |  |  |
| 34 |  |  | 21 |  |  | 27 |  |  |
| 35 |  |  | 22 |  |  | 28 |  |  |

Table 5. Calendar Text BM 47851

What the numbers in lines 32 to 35 stand for, I do not know. But maybe they are connected to the micro-sign indicated by $\langle\odot\rangle$ and $\langle D\rangle$. The microsign indicating the position of the Moon sometimes is the same as the one indicated by the Sun [see Plate 1, p. 56 above], and sometimes the lunar micro-sign is one position further than that of the Sun [see Figure 1, p. 50 above]. For all 12 months, this is the case for the days (5), ${ }^{10} 7,(10), 12,14$, (15), 17, 19, (20), 21, 22, 24, (25), 26, 27, 28, 29. But for days $1,2,3,4,6,8$, $9,11,13,16,18,23$, and 30, the micro-sign of the schematic Sun and Moon will be the same. We also find the numbers 1,11 , and 18 , in line 32 , in the list of "same micro-sign" dates, while 19 is also written in line 32 . All the

[^6]rest of the numbers (written in lines 33, 34, and 35) would be days at which the solar micro-sign was the one just before the lunar micro-sign. The tablet is, however, too damaged and too many relevant numbers are missing to prove that they were listed as indicators of days at which the micro-signs of the $\langle\odot\rangle$ and $\langle\mathcal{D}\rangle$ were the same or consecutive.

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[^0]:    ${ }^{1}$ The zodiacal circle is the division of the path of the Sun into 12 zodiacal signs of equal length. Before the introduction of this «lu-maš», the Babylonians used a number of constellations to locate bodies in the sky. The zodiac is a band consisting in constellations along the ecliptic through which the Moon, Sun, and planets move.

[^1]:    ${ }^{2}$ For a more recent discussion of Calendar Texts and their use and role within Late Babylonian culture, see Steele 2011 and 2017 as well as Wee 2016, which investigates the schemes thoroughly and gives many references.
    ${ }^{3}$ I use the angle brackets $<>$ to indicate that we are dealing with the schematic "mean value".
    ${ }^{4}$ The symbol $\langle n\rangle$, where $n$ is a number, denotes the mean value of a series of numbers $n$.

[^2]:    ${ }^{5}$ EAE 14 is the 14th of a series of astrological/astronomical tablets. See Al-Rawi and George 1991, 56 and 59 for schemes B and D.
    ${ }^{6}$ MUL.APIN is a Babylonian Astronomical compendium consisting in two cuneiform tablets. See Hunger and Pingree 1989 or Hunger and Steele 2019.

[^3]:    7 Thus, at the beginning of month 6 (day o), both Sun and Moon are at $o^{\circ}$ of sign VI. On day 1 , the Sun will be at $1^{\circ}$ of VI and 12 micro-degrees of micro-sign VI. Adding the $1^{\circ}$ to this position, we get $13^{\circ}$ in micro-sign VI, which gives us the schematic position of the Moon. On day 2 the Sun will be at ZSVI $2^{\circ}$ and at 24 micro-degrees in $\mathrm{m}-\mathrm{s}$ VI. On this day, the Moon is at $26^{\circ}$ of VI, which can be found from 24 of m-s VI by adding $2^{\circ}$. On day 3 , the Sun will be at VI $3^{\circ}$ and 6 micro-degrees of m -s VII. We find the position of the schematic Moon in the micro-zodiac by adding 3 to the 6 micro-degrees of VII, ending with m-s VII 9 micro-degrees, in agreement with the Dodecatemoria of month 6

[^4]:    ${ }^{8}$ For more details, see Rumor 2021, $\S 1.4$ and Steele 2011, where the latter argues convincingly for the idea that some of the ingredients (e.g., blood of lion or eagle) are used as synonyms for medical plants or herbs

[^5]:    ${ }^{9}$ Here I have used a reading proposed by Hermann Hunger, see Reiner 2000, 422 n 5.

[^6]:    ${ }^{10}$ The numbers $5,10,15,20$ and 25 all signify dates at which the Sun was at the end of one micro-sign, which at the same time is the beginning of the next micro-sign.

