Konsistenz zwischen Kolonne Φ und babylonischen Aufzeichnungen der 'Lunar Four'

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Many of the Babylonian tablets with cuneiform texts contain records of the so-called 'lunar six' data. These are observable time differences between risings and settings of moon and sun, which the Babylonians regularly have recorded and analysed also at times long before the seleuchid era. We find that, in particular, the 'lunar four' data are accurate enough to allow the construction of column Φ. A detailed analysis of the data on tablets LBAT 1285 and Cambyses 400, and their comparison with calculations using a modern computer code for ephemerids valid at ancient times, strongly support our hypothesis that Φ has been constructed from the sum Σ of the 'lunar four'.

Anhand von überlieferten Keilschrifttexten zeigen wir, daß die babylonischen Aufzeichnungen der 'Lunar Four' unsere Hypothese stark unterstützen, daß Kolonne Φ aus deren Summe Σ konstruiert worden ist. Viele verschiedene Tafeln zeigen, daß die babylonischen Astronomen die 'Lunar Six' systematisch beobachtet und bearbeitet haben. Deren regelmäßig aufgezeichnete Werte gehen weit genug in der Zeit zurück und sind genügend genau, daß daraus die Kolonne Φ konstruiert werden konnte.

Einleitung

Dieser Artikel ist Teil eines größeren Projektes, das zum Ziel hat, die Entstehung der babylonischen mathematischen Astronomie zu erforschen und soweit wie möglich zu rekonstruieren. Es ist einleuchtend, daß eine Rekonstruktion nur auf solchen Erkentnissen und Beobachtungsdaten basieren darf, welche die Babylonier tatsächlich zur Verfügung hatten. Ihre Beobachtungsberichte, die sogenannten 'Diaries' [1], liefern uns die nötige Information darüber, welche Himmelsereignisse tatsächlich beobachtet wurden, sowie Information über die Genauigkeit der Beobachtungen. Im Laufe dieses Projektes wurde bereits gezeigt [2,3], daß es möglich ist, die fundamentale Funktion Φ aus Horizontbeobachtungen zu rekonstruieren, nämlich aus den sogenannten 'Lunar Four', deren Größen Monat für Monat in den Diaries notiert wurden. Wir wollen hier andere Keilschriftstexte daraufhin untersuchen, ob sie zusätzliche Evidenz für diese Rekonstruktion erbringen können.

1 Gefördert durch die Deutsche Forschungsgemeinschaft

Rekapitulation der vorgeschlagenen Rekonstruktion von Φ

Wir beginnen mit einer kurzen Rekapitulation unserer Rekonstruktion von Kolonne Φ. Die babylonische Zickzack-Funktion Φ tritt u.a. in den Mondephemeridentafeln (System A) auf. Sie ist die fundamentale Kolonne zur Berücksichtigung der variablen Mondgeschwindigkeit. Diese Rolle von Φ ergibt sich daraus, daß sämtliche anderen Größen, die von der Mondgeschwindigkeit abhängen, von Φ abgeleitet worden sind. Es ist gezeigt worden [2,3], daß es möglich ist, diese Funktion Φ aus genau solchen Beobachtungen zu konstruieren, wie sie die Babylonier über Jahrhunderte hinweg aufgezeichnet haben [1]. Es sind dies Beobachtungen der folgenden vier Zeitintervalle, die in den beiden Tagen vor und nach Vollmond (Opposition) gemessen werden können, der sogenannten 'Lunar Four':

ŠÚ = Zeit von letztem Monduntergang vor Opposition bis Sonnenaufgang
ME = Zeit von letztem Mondaufgang vor Opposition bis Sonnenuntergang
NA = Zeit von erstem Sonnenaufgang nach Opposition bis Monduntergang (1)
GE = Zeit von erstem Sonnenuntergang nach Opposition bis Mondaufgang

Um Neumond (Konjunktion) herum gibt es vier entsprechende Zeitintervalle, von denen aber nur zwei beobachtbar sind, da der Mond bei den anderen beiden unsichtbar ist: *KUR* = Zeit vom letzten sichtbaren Mondaufgang vor Konjunktion bis zum Sonnenaufgang, und *NA* = Zeit von Sonnenuntergang bis zum Monduntergang am ersten Abend, an dem der Neumond wieder sichtbar wird.

Diese sechs charakteristischen Zeitintervalle, insgesamt die 'Lunar Six' genannt, sind spektakulär und einfach zu messen - doch von einem theoretisch astronomischen Sichtpunkt aus sind es sehr komplizierte Größen, da sie je für sich von vielen verschiedenen Variablen abhängen. Jede der Lunar Four hängt ab vom Zeitpunkt der Opposition, von der Position des Mondes in der Ekliptik zu diesem Zeitpunkt, von der momentanen Geschwindigkeit des Mondes und von seiner momentanen Breite (d.h. seinem Abstand von der Ekliptik). Eben weil diese Größen so komplex sind, hat man lange nicht genau gewußt, wie und wozu sie verwendet werden konnten. Doch zeigen uns die Ephemeriden- und Lehrtexte, daß die babylonischen Astronomen imstande waren, jede dieser Größen durch sinnvolle Berücksichtigung aller Variablen zu berechnen (ACT I [4] und HAMA [5], SS. 474-555). In der von uns vorgeschlagenen Rekonstruktion von Φ werden die in (1) angegebenen Lunar Four zum ersten Mal systematisch ausgenützt. Die Rekonstruktion basiert darauf, daß etliche der Variablen durch geschickte Kombination der Lunar Four eliminiert oder deren Auswirkungen zumindest stark reduziert werden können.

Die bis in unsere Zeit überlieferten Tontafeln enthalten zwar viele Daten der Lunar Four - doch sind diese Daten zu sparsam und zu zerstreut, um daraus die Funktion Φ herzuleiten. In der vorgeschlagenen Rekonstruktion von Φ hatten wir deshalb anhand von Computerprogrammen die Lunar Four über einen großen Zeitraum hinweg berechnet und uns dadurch ein ideales (das heißt lückenloses) "babylonisches" Zahlenmaterial erstellt. Aus diesem konnten wir Φ konstruieren und dessen Periode $P_☾$ in folgender Weise bestimmen: Die Summe Σ der Lunar Four,

$$\Sigma = \check{S}\acute{U} + NA + ME + GE, \tag{2}$$

wurde als Funktion der Monatsnummer durch eine lineare Zickzackfunktion angenähert. Diese Zickzackfunktion, die wir $\hat{\Sigma}$ nennen, hat die gleiche Periode und die gleiche Amplitude wie Φ; nur ist ihr Mittelwert um 100° kleiner als der von Φ. Hiermit wurde gezeigt, daß es möglich und ganz einfach ist, Φ durch systematische Bearbeitung der Lunar Four zu erhalten.[2] Wir haben deshalb folgende Hypothese aufgestellt [2]:

$$\Phi = \hat{\Sigma} + 100^\circ. \tag{3}$$

Φ ist aus $\hat{\Sigma}$ entstanden, und zwar durch Addition einer Konstanten von 100°.

Die 'Lunar Six' in verschiedenen Texttypen

Die vorliegende Arbeit will nun untersuchen, ob auch andere babylonische Keilschrifttexte als die Diaries die obige Hypothese unterstützen. Daß die Babylonier sich schon zu einem sehr frühen Zeitpunkt für die Zeitintervalle der 'Lunar Six' interessiert haben, können wir daraus schließen, daß deren Größen schematisch in den großen alten Ominaserien Enuma Anu Enlil [7] und MUL.APIN [8] festgehalten wurden. Erstaunlicherweise wurden manchmal schon in den frühesten Diaries (d.h. ab 567 v.Chr.) Zahlenwerte für die Lunar Six angegeben, obwohl Bemerkungen wie "bewölkt, nicht gesehen" uns zeigen, daß diese nicht immer durch Beobachtung gefunden worden sind. Wie die Babylonier diese offensichtlich nicht beobachteten Werte gefunden haben und wie gut diese waren, wissen wir nicht. Dennoch zeigen diese Texte uns, daß die Babylonier sich schon sehr früh eingehend mit den Lunar Six beschäftigt haben; eine gute Voraussetzung dafür, eventuelle Regelmäßigkeiten zu finden, die dann zur Konstruktion von Φ führen konnten.

Wir wollen nun Texte aus den letzten 6 Jahrhunderten v.Chr. daraufhin untersuchen, ob sie Hinweise dafür enthalten, daß Φ aus den Lunar Four entstanden sei. Dabei gehen wir folgenden Fragen nach: Haben die Babylonier das Zahlenmaterial der Lunar Six systematisch bearbeitet? Sind die Daten genügend genau und zeigen sie diejenige Systematik auf, die zur Konstruktion von Φ notwendig und hinreichend ist? Und wenn ja, stimmt die Systematik des Beobachtungsmaterials tatsächlich mit Kolonne Φ überein? Alle diese Fragen werden wir mit "ja" beantworten können.

2 Maeyama [6] hat eine ganz andere Möglichkeit zur Rekonstruktion von Kolonne Φ vorgeschlagen, die u.a. darauf fundiert ist, daß Zeitpunkte von Vollmonden oder Mondfinsternissen, die 1 Saros auseinanderliegen, sehr genau gemessen wurden. Die dazu erforderliche Genauigkeit wird aber in den Diaries nicht dokumentiert.

Die erste Frage ist schnell beantwortet: Auf der Tafel B.M. 34075, die als LBAT 1431 von Sachs publiziert wurde ([9], S. xxxii) sind für mindestens fünf aufeinanderfolgende Jahre (323 - 319 v.Chr.) sämtliche Werte der Lunar Six Monat für Monat aufgezeichnet; ein Beweis dafür, daß diese Daten systematisch gesammelt wurden. Es gibt mehrere Tafeln, die nichts enthalten als eine Zusammenstellung dieser Größen. Solche Monddaten wurden schon ab dem 6. Jh. v.Chr. gesammelt: dies zeigt uns die Tafel Kambyses 400. Dieser Text ist von Kugler übersetzt und ausführlich behandelt worden (SSB I [10], SS. 61-73). Er enthält die Lunar Six für das 7.Regierungsjahr von Kambyses (d.h.von April 523 v.Chr. bis April 522 v.Chr.). Diese Daten werden wir in Abschnitt "Kambyses 400" untersuchen.

Eine andere Reihe von Tafeln, welche die Lunardaten systematisch sammeln, sind die sogenannten 'Goal-Year'-Tafeln. Sie sind für unsere Fragen von besonderer Wichtigkeit, weshalb wir nun etwas ausführlicher über dieser Texttyp berichten.

Die Goal-Year-Tafeln

Die Goal-Year-Tafeln enthalten Auszüge aus den Diaries und sind in ihrer Struktur alle gleich. Ihr Inhalt und Zweck ist von Kugler [10], Sachs [11], und van der Waerden [12] beschrieben worden. Sie dienten dazu, für ein bestimmtes Jahr - das 'Zieljahr' - Voraussagen über charakteristische Himmelsphänomene zu machen. Sachs nannte sie deshalb 'Goal-Year texts'. Sie bestehen aus einer Zusammenstellung von astronomischen Phänomenen aus früheren, jeweils eine Planeten- oder Mondperiode vor dem Zieljahr liegenden Jahren.

Es gibt ungefähr 150 Fragmente oder ganze Goal-Year-Tafeln. Davon sind 93 datierbar für 56 verschiedene Zieljahre zwischen 71 SÄ und 352 SÄ.[3] Diese Tafeln bieten uns somit ein großes Reservoir von babylonischen Beobachtungen, die über mehrere Jahrhunderte verteilt sind. Nur wenige dieser Tafeln sind aber übersetzt oder bearbeitet worden.

In dieser Arbeit werden wir nur eine Goal-Year-Tafel behandeln, nämlich die am besten erhaltene Tafel LBAT 1285. Wir werden hier ihre Rückseite übersetzen und kommentieren. Dann werden wir durch Analyse von neu berechneten Daten der Lunar Four herauszufinden versuchen, wie solche Tafeln zur Vorhersagen von Mondphasen benützt werden konnten. Schließlich werden wir die Monddaten auf unserer Tafel untersuchen und auf ihre Zuverläßigkeit hin kontrollieren.[4]

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3 SÄ = (Jahr der) Seleukidischen Ära

4 Andere einigermaßen gut erhaltene Goal-Year-Tafeln werden wir zu einem späteren Zeitpunkt behandeln. Sie sind von Sachs in LBAT [9] unter den Nummern 1214, 1220, 1225, 1230, 1251, 1252, 1257, 1265 und 1266 in Keilschrifttranskription veröffentlicht worden. Hier kann nur vorweg bemerkt werden, daß die Daten dieser Tafeln unsere hier gewonnenen Erkenntnisse unterstützen.

Jahr	175		Jahr	176							
VII	15	*ŠÚNA*	I 1			V 1	15 10	*NA*	IX 30	10 20	
	8	*MEGE*	14.	4 10	*ŠÚ*	14.	4	*ME*	13.	7 30	
VIII	15	*ŠÚNA*	15.	3 20	*ME*	14.	2 20	*ŠÚ*		6 30	*ŠÚ*
	9 40*	*MEGE*	15.	4	*NA*	15.	5	*GE*			
IX	14 50	*ŠÚNA*	16.	10 40	*GE*	15.	15	*NA*			
	12 10	*MEGE*	27.	15	*KUR*	27.	20	*KUR*			
X	14	*ŠÚNA*	II 30			VI 1	12	*NA*	X 1	20	*NA*
	13	*MEGE*	15.	10	[*ME*]	13.	7	*ŠÚ*	11.	13 40	*ŠÚ*
XI	9 30	*ŠÚNA*	15.		*ŠÚ*	14.	1 30	*ME*	12.	1 30	*NA*
	13 30	*MEGE*	16.	6	*GE*			*NA*	13.	5 30	*ME*
XII	8 40	*ŠÚNA*	16.	9	*NA*			*GE*	14.	7 30	*GE*
	14 30	*MEGE*	28.	9 50	*KUR*	27.	16	*KUR*	27.	10 10	*KUR*
...	...	...	III 1	23	*NA*	VII 1	11 30	*NA*	XI 30	17	*NA*
175 XII_2	Nacht 15	☾ - Ekl.		10	*ME*	12.	13	*ŠÚ*	12.	6	*ŠÚ*
175 XII_2	Tag 29	⊙ - Ekl.	14.	9	*ŠÚ*	13.	7	*ME*	13.	2 50	*NA*
176 VI	Nacht 14	☾ - Ekl.	15.	4 20	*GE*	13.	3 40	*NA*	14.	0*	*ME*
...	...	...	15.	5 30	*NA*	14.	1 10	?	15.	15	*GE*
Jahr	175		27.	20 30	*KUR*	27.	15 30	*KUR*	27.	11	*KUR*
XII_2 30	11	*NA*	IV 30	13	*NA*	VIII 1	15	*NA*	XII 30	13	*NA*
14.	3 50	*ŠÚ*	14.	12 20	*ME*	12.	2	*ŠÚ*	13.	4 30	*ŠÚ*
15.	7 40	*ME*	14.	15	*ŠÚ*	13.	5	*ME*	14.	7	*ME*
15.	3	*NA*	15.	0 40	*GE*	13.	13	*NA*	14.	2 30	*NA*
16.	6 20	*GE*	15.	2 20	*NA*	14.	4	*GE*	15.	6	*GE*
27.	16 30	*KUR*	27.	14 20	*KUR*	26.	22	*KUR*	27.	16	*KUR*
									I 30	10 40	*NA*

Tabelle 1: Goal-Year-Text LBAT 1285. Zwischen den Punkten (...) in der ersten Kolonne befinden sich auf der Tafel drei Eklipsenberichte, von denen wir hier nur die Daten wiedergeben. Ein Stern (*) bedeutet Kollation durch Sachs (H. Hunger, private Mitteilung).

Der Goal-Year-Text LBAT 1285

Die Goal-Year-Tafel LBAT 1285 enthält Mondbeobachtungen aus der Zeit von September 137 bis März 135 v.Chr.[5] In dem Kolophon zuunterst auf der Rückseite der Tafel steht: "Tage des Sichtbarwerdens, des Verschwindens und der Finsternisse, ermittelt für das Jahr 194, in dem Arsaka König war".

Hier ist offensichtlich der Zweck dieser Tafel bekundet, deren Zieljahr als das Jahr 194 SÄ angegeben wird. Auf der ganzen Vorderseite der Tafel sowie in den ersten drei Zeilen der Rückseite sind in verschiedenen Abschnitten Planetenbeobachtungen aufgezeichnet, die dazu dienten, Voraussagen für das Jahr 194 SÄ zu machen. So wurden z.B. die für den Planeten Jupiter im Jahre 123 SÄ (123=194-71) beobachteten "charakteristischen" Phänomene notiert. Die babylonischen Astronomen haben offensichtlich gewußt und ausgenützt, daß sich die charakteristischen Phänomene Jupiters nach 71 Jahren an ungefähr demselben Datum wiederholen.

Von den ersten drei Zeilen abgesehen, ist die ganze Rückseite dem Mond gewidmet. Diesen Mondtext wollen wir jetzt näher betrachten. Da dieser Goal-Year-Text nur in Keilschrifttranskription [9] vorliegt, bringen wir in Tabelle 1 eine Umschrift der Mondaufzeichnungen - doch ohne Mitnahme von Beobachtungsbemerkungen wie "Nebel, (trotzdem) gemessen", "der Mond stand tief" oder "bewölkt, nicht gesehen". In dieser ersten Bearbeitung werden wir sämtliche Daten verwenden und machen also keinen Unterschied zwischen sog. "beobachteten" und "nicht beobachteten" Daten.

Der Text ist in vier Kolonnen geteilt. In der oberen Hälfte von Kolonne 1 sind die Summen *ŠU+NA* und *ME+GE* für die Monate VII bis XII des Jahres 175 SÄ angegeben, also für die letzten 6 Monate des Jahres, das 19 Jahre vor unserem Zieljahr liegt. So steht z.B. in der 1. und 2. Zeile von Kolonne 1:

Jahr 175 (Monat) VII 15 *ŠÚNA*

Hiermit ist gemeint: [Für den Vollmond im] Monat VII [im] Jahr 175 [SÄ betrug die Summe von] *ŠÚ* [und] *NA* 15° [*uš*=Zeitgrad[6]]. Im alten Mesopotamien begann der neue Monat immer dann, wenn die Mondsichel (das Neulicht) zum ersten Mal nach Neumond sichtbar wurde. Dies bedeutet, daß der Vollmond immer ungefähr in die Mitte des babylonischen Monats, das Neulicht hingegen auf den 1. Tag des Monats fiel.

Weiter unten in Kolonne 1 befinden sich genaue Berichte zweier Mondfinsternisse sowie einer Sonnenfinsternis.[7] In Tabelle 1 sind nur die Daten dieser Finsternisse wiedergegeben. Wir weisen darauf hin, daß Monat I sowie Monat VII in

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5 Zur Umrechnung der Daten wurde "Babylonian Chronology" von Parker und Dubberstein [13] verwendet.

6 Diese Zeitdifferenzen wurden in *uš* (von den Griechen 'Zeitgrad' genannt) gemessen. 1 *uš* ist gleich 4 Minuten; dies ist die Dauer einer Drehung der Himmelskugel um 1° in ihrer täglichen Bewegung um $24^h = 1^d = 360°$.

7 Ich bin Professor H. Hunger in Wien zu Dank verpflichtet, der mir diesen Abschnitt übersetzt hat.

unserem Zieljahr 194 SÄ genau 223 Monate später als die erwähnten Finsternis-Monate liegen. Nun ist aber 223 synodische Monate eine altbekannte Finsternisperiode. Wir nennen sie 1 Saros, die Babylonier nannten sie "18 Jahre". Unser Goal-Year-Text bezeugt, daß die babylonischen Astronomen ihr Wissen über die Sarosperiode hier ausgenützt haben, um eventuelle Finsternisse im Zieljahr 194 SÄ vorauszusagen.

Der Rest von Tabelle 1 enthält sämtliche Lunar Six für die 13 Monate von XII_2 175 SÄ bis XII 176 SÄ.[8] Nach den Finsternisberichten steht:

Jahr 175 (Monat) XII_2 30 11 *NA*

Dies bedeutet: [Im] Jahr 175 [in der Seleukidischen Ära fiel der erste Tag vom] Monat XII_2 [auf den Tag] 30 [des vorhergehenden Monats XI.] 11 [*uš* ist der Wert von] *NA*.

In der Zeile darunter lesen wir:

14. 3 50 *ŠÚ*

[Im Monat XII_2 am Morgen des] 14. [Tages hatte] *ŠÚ* [den Wert] 3;50 [*uš*]. Die Lesung der übrigen Daten folgt ganz analog nach demselben Muster.

In allen vier Kolonnen wird also ganz links das Datum (d.h. Monat und Tagesnummer) angegeben. Ganz rechts steht der Name des Zeitintervalls aus den Lunar Six, und dessen beobachteter (oder berechneter) Wert wird durch die Zahl in der Mitte der Kolonne angegeben. Wir machen darauf aufmerksam, daß das Zieljahr 194 SÄ ein Schaltjahr war: es bestand aus den 13 Monaten I, II, ... , XI, XII und XII_2. Diejenigen Monate, die jeweils um 223 synodische Monate früher als diese 13 Monate lagen, sind die Monate XII_2 175 SÄ bis XII 176 SÄ, für die der Text die Werte der Lunar Six notiert.

Bemerkungen zu den Lunar Six

Daß die babylonischen Astronomen die Summen *ŠÚ+NA* und *ME+GE* berechnet und in Kolonne 1 aufgeschrieben haben, sehen wir als eine kräftige Unterstützung für unsere vorgeschlagene Rekonstruktion von Φ an. Wir meinen, daß Φ aus der Summe *ŠÚ+NA+ME+GE* entstanden ist. Diese Summe aller Lunar Four kommt zwar nicht vor, doch die Goal-Year-Tafeln zeigen uns, daß das Zahlenmaterial der Lunar Four systematisch bearbeitet wurde - und daß die Babylonier Summen bildeten und sich für *ŠÚ+NA* und *ME+GE* interessierten.

Es stellt sich jetzt die Frage: Wie und wozu genau wurden diese (teils beobachteten, teils berechneten) Lunar Six und deren Teilsummen verwendet? Schon Kugler (SSB II [10], S. 542) hat darauf hingewiesen, wie die Summen praktisch ausgenützt werden können, um eine fehlende Größe der Lunar Four zu ermitteln, die z.B. wegen Bewölkung nicht beobachtet werden konnte. So kann man mit

8 Im Originaltext steht der erste Abschnitt, der die Lunar Six für Monat XII_2 Jahr 175 SÄ angibt, zuoberst in Kolonne 2. Aus Platzgründen haben wir in Tabelle 1 diesen Abschnitt zuunterst in Kolonne 1 wiedergegeben.

genügender Genauigkeit etwa *ŠÚ* als Differenz zwischen der bekannten Summe *ŠÚ+NA* und einem beobachteten *NA* erhalten. Wie die Babylonier die Lunar Six verwendeten, um die Bewegung des Mondes vorherzusagen, wissen wir nicht. Vielleicht könnte ein Vergleich zwischen nicht beobachteten (d.h. berechneten) Werten aus den Diaries mit eventuellen Goal-Year-Daten, die um einen Saros früher liegen, hierüber Aufschluß geben. Die wenigen Fälle dieser Art, die wir bis jetzt gefunden haben, lassen jedoch keine Konklusionen zu.

Eine andere Frage läßt sich aber beantworten: Welche Gesetzmäßigkeiten weisen die Lunar Six auf? Die Wichtigkeit dieser Frage ist einleuchtend: wenn wir imstande sind, rein empirische Gesetzmäßigkeiten der Lunar Six oder ihrer Teilsummen *ŠÚ+NA* und *ME+GE* zu finden, dann können wir begründete Vermutungen über die Verwendung der Monddaten in den Goal-Year-Tafeln aufstellen.

Astronomische Überlegungen zeigen uns, daß die Struktur der einzelnen Lunar Six im Prinzip dieselbe sein muß auch die beiden Summen müssen strukturell gleich sein. Jede der Lunar Six mißt ja die Zeit, die es für einen kleinen Ekliptikbogen dauert, den Horizont zu passieren: *KUR* und *NA* messen Auf- und Untergangszeit des Ekliptikbogens zwischen Mond und Sonne; die übrigen Lunar Four messen die Auf- oder Untergangszeit des Ekliptikbogens zwischen Mond und Gegensonne (d.h. dem Punkt auf der Ekliptik, welcher der Sonne diametral gegenübersteht). Die Summen *ŠÚ+NA* und *ME+GE* messen Unter- und Aufgangszeit von einem Ekliptikbogen, dessen Länge gleich der täglichen Elongationsbewegung des Mondes ist [3]. Wir müssen deshalb nicht sämtliche Größen untersuchen, sondern können z.B. Erkenntnisse über eines der Lunar Six auf die anderen übertragen.

Wir erinnern daran, daß die Goal-Year-Tafel, die für das Zieljahr Y zusammengestellt war, folgendes angibt: die Summen *ŠÚ+NA* und *ME+GE* für die Monate VI bis XII des Jahres *Y* - 19, sowie die Lunar Six für die darauf folgenden 12 (oder, in einem Schaltjahr, 13) Monate bis zum Ende des Jahres *Y* - 18. Wir nehmen *NA* und *ŠÚ+NA* als Beispiel und untersuchen sie wie folgt: kann man unmittelbar die Größen *NA* und *ŠÚ+NA* für irgend einen Monat (Vollmond) voraussagen, wenn man ihre Größe in demjenigen Monat kennt, der genau 1 Jahr, 18 *Jahre (d.h. 223 Monate) oder 19 Jahre früher zu beobachten war?*

Systematische Untersuchung der Lunar Four

Moderne Computerberechnungen

In [2] wurde Kolonne Φ aus hypothetischen "Horizontbeobachtungen" unserer Zeit rekonstruiert. Wir hatten die Lunar Four über den Zeitraum von 1930 bis 2030 n. Chr. berechnet, so wie sie von Babylon aus zu sehen sind. Dies ist selbstverständlich erlaubt, denn die Bewegungen von Sonne, Erde und Mond sind dieselben wie zur Zeit der alten Babylonier. Die Bahnelemente haben sich zwar ein wenig in ihrer Größe geändert, doch Struktur und Systematik der zu beobachtenden Lunar Four ist heute genau dieselbe wie vor 2500 Jahren. Inzwischen haben wir uns ein präziseres Computerprogramm [14] erworben, das es erlaubt, Ephemeriden des Sonnensystems auch für archäologische Zeiten zu berechnen. Dies ist vom großem Vorteil, denn es setzt uns instand, die babylonischen Daten direkt zu kontrollieren, sowie ein echt altes "babylonisches Zahlenmaterial" zu konstruieren. Von diesen numerischen Daten werden wir im folgenden Gebrauch machen.

Die Goal-Year-Texte enthalten jeweils Monddaten über einen kurzen Zeitraum von anderthalb Jahren. Um dies zu illustrieren, haben wir die Lunar Four für eine Reihe von aufeinanderfolgenden Vollmonden berechnet, beginnend am 20. Juli 233 v.Chr. = JD[9] 1636521. Zuoberst in Abbildung 1 sind die berechneten Werte von *ŠÚ* und *NA* für 30 aufeinanderfolgende Monate durch Kreuze (×) dargestellt und mit Geraden verbunden. Darunter sind in gleicher Weise die Summen *ŠÚ+NA* und *ME+GE* wiedergegeben. Wir bemerken, daß die Größen *ŠÚ* und *NA* sehr unregelmäßig und scheinbar ohne System variieren; *ŠÚ+NA* und *ME+GE* hingegen bilden viel glattere Kurven. Dies läßt uns vermuten, daß die Größen *ŠÚ+NA* und *ME+GE* einfacher zu Voraussagen benützt werden können als die Lunar Four je für sich.

Unsere Vermutung wird bestätigt, wenn wir die Werte von *NA*, *ŠÚ+NA* und *ME+GE* mit denjenigen vergleichen, die jeweils 223 oder 235 synodische Monate früher berechnet wurden. Dieser Vergleich wird in Abbildung 2 durchgeführt. Die Kreuze markieren wieder dieselben Größen wie in Abb. 1. (Wir haben hier auf *ŠÚ* verzichtet, das genau so chaotisch aussieht wie *NA*.) Die um 223 synodische Monate (= 18 Jahre) früher zu messenden Werte sind mit Sternen (⋆) markiert, und diejenigen, die 235 Monate (= 19 Jahre) früher liegen, mit Kreisen (○). Wir sehen, daß *NA* keine offensichtliche Periodizität besitzt: seine Struktur wiederholt sich weder nach einem Jahr (⋆ und ○ variieren unabhängig voneinander) noch nach 18 Jahren (× und ⋆ variieren unabhängig voneinander), noch nach 19 Jahren. Hingegen sehen wir, daß die Struktur von *ŠÚ+NA* und *ME+GE* sich nach 223 Monaten = 1 Saros fast identisch wiederholt: die beiden Kurven, die durch × und ⋆ gehen, fallen fast zusammen. Dies gilt, wenn auch weniger genau, auch für die um 235 Monate verschobenen und im selben Grade für die um ein Jahr (= 12 Monate) verschobenen Daten.

Resultat der Untersuchung und dessen Folgen

Aus dieser Untersuchung schließen wir, daß es möglich ist, aus systematischen Beobachtungen der Summe *ŠÚ+NA* deren Werte vorauszusagen für Vollmonde, die 12 Monate, 223 Monate oder 235 Monate später stattfinden werden; dies gilt aber nicht unmittelbar für *ŠÚ* oder *NA* einzeln. Genau dasselbe können wir über die Zeitintervalle *ME* und *GE* sagen, d.h. daß sie je für sich keine brauchbare Systematik aufweisen, ihre Summe *ME+GE* hingegen regelmäßig variiert und vorausgesagt werden kann, wenn man deren Werte ein Jahr oder einen Saros früher durch Beobachtung bestimmt hat.

9 JD ('Julian Day') = Julianischer Tag in absoluter Zählung (JD 0 = 1. Januar 4713 v.Chr.)

All dies können auch die Babylonier leicht und rein empirisch gefunden oder erkannt haben, wenn sie bloß ihre Beobachtungen über große Zeiträume systematisch und einigermaßen lückenlos fetstgehalten haben; die Daten in Abb. 2, die uns diese Erkenntnisse gebracht haben, fundieren ja direkt auf möglichen Beobachtungen. Die Tatsache, daß die Goal-Year-Texte die Summen *ŠÚ+NA* sowie *ME+GE* enthalten, spricht dafür, daß die Babylonier deren Regelmäßigkeit bemerkt und zu Voraussagen praktisch ausgenützt haben.

Ausgehend von unserem astronomischen Verständnis der Lunar Four und ihrer Summen wissen wir, wie wir z.B. auch *ŠÚ* voraussagen können: 1 Saros ist in etwa gleich 6585 1/3 Tage. Findet eine Opposition unmittelbar nach Sonnenaufgang statt, so ist *ŠÚ*, die an diesem Morgen gemessene Zeit von Monduntergang bis Sonnenaufgang, ganz klein. Einen Saros später wird der Vollmond 1/3 Tag nach Sonnenaufgang stattfinden, weshalb *ŠÚ* größer wird, und zwar um etwa den Betrag 1/3 (*ŠÚ+NA*), während *NA* um denselben Betrag kleiner wird. Es kann nämlich gezeigt werden, daß um die Opposition herum *ŠÚ+NA* die tägliche Verminderung von *ŠÚ* (resp. Vergrößerung von *NA*) mißt.[10] Wegen dieser Verschiebung des Oppositionszeitpunktes im Verhältnis zum Sonnenaufgang um 1/3 Tag bei Vollmonden, die einen Saros auseinanderliegen, ändern sich *ŠÚ* und *NA* je für sich nach einem Saros; ihre Summe *ŠÚ+NA* ist aber unabhängig vom Zeitpunkt der Opposition und ändert sich deshalb nur sehr wenig. Ob die Babylonier dies bemerkt und ausgenützt haben, wissen wir nicht. Eine Untersuchung ihrer vorausgesagten Lunar Four mag hierüber Aufschluß geben.

Vergleich von Σ aus Seleukidischer Zeit mit Φ

Als wir in [2] demonstrierten, wie Kolonne Φ von Horizontbeobachtungen hergeleitet werden kann, hatten wir nur moderne Daten zur Verfügung. Deshalb konnten wir nur zeigen, daß Σ und Φ dieselben Perioden und Amplituden haben. Jetzt aber haben wir Σ für 260 Monate aus der Seleukidischen Zeit berechnet: Lunation 1 ist der Vollmond, der am 23. Januar 146 v.Chr. (JD 1668119) stattfand. In Abbildung 3 haben wir Σ für diese 260 Monate abgebildet (× mit dicken Strichen verbunden). Diese Kurve hat erwartungsgemäß exakt dieselbe Struktur wie diejenige aus unserer Zeit, die wir in [2] benützten. Sie hat aber den Vorteil, daß wir sie direkt mit Φ vergleichen können, um festzustellen, ob ihre Phasen wirklich auch so übereinstimmen, wie sie es müssen, wenn unsere Hypothese (3) richtig ist. Da die Lunationen in Abbildung 3 in die Seleukidische Ära fallen, können ihre Φ-Werte ermittelt werden. Die dünne gestrichelte Linie zeigt uns die lineare Zickzackfunktion Φ - 100 *uš* (in Minuten umgerechnet). Sie stellt eine optimale Über-

10 Van der Waerden hat ähnlich argumentiert [12]. Seine Deutung der Summe *ŠÚ+NA* ist dieselbe wie die unsrige; jedoch ist es nicht der Mondaufgang, der nach einem Saros um 1/3 Tag verschoben erscheint, sondern der Zeitpunkt der Opposition.

deckung der "beobachteten Kurve" Σ dar. Wie wir es hofften, haben Σ und Φ also exakt dieselbe Phase. Dies stellt eine weitere Bestätigung unserer Hypothese dar.

Die alten babylonischen Lunar Four

LBAT 1285

Wir wollen nun die auf LBAT 1285 vorkommenden Werte der Lunar Four untersuchen. Diese sind teils beobachtet, teils nicht - so besagt es jedenfalls der Text. Für diese erste Untersuchung machen wir aber hier keinen Unterschied und behandeln das ganze Zahlenmaterial. Der Text deckt den Zeitraum von Monat VII 175 SÄ bis XII 176 SÄ. Im Julianischen Kalender entspricht dies dem Zeitraum von September 137 v. Chr. bis März 135 v.Chr. Für die ersten 6 Monate VII bis XII 175 SÄ (wir nennen sie hier Lunation $L = 1,2,\ldots,6$) gibt der Text nur die Summen *ŠÚ+NA* und *ME+GE* an; für die 13 folgenden Monate ($L = 7,8,\ldots,19$) von XII_2 175 bis XII 176 sämtliche Lunar Six, aber keine Summen. Deshalb rechnen wir die Summen *ŠÚ+NA* und *ME+GE* für diese Monate aus. In Abbildung 4 links haben wir diese Daten mit Kreisen (O) markiert (analog zu Abb. 1.) In den beiden obersten Diagrammen haben wir *ŠÚ* und *NA* als Funktion der Monatsnummer L abgebildet (beide erst ab $L = 7$, da der Text in den ersten 6 Monaten nur die Summen, nicht aber die einzelnen Lunar Six angibt). Da der Text an einigen Stellen beschädigt war und Risse hat, sind nicht sämtliche Werte erhalten; trotzdem sind es so viele, daß wir feststellen können, daß *ŠÚ* und *NA* genau so chaotisch variieren wie in den berechneten Kurven von Abb. 1. In den beiden untersten Diagrammen links in Abb. 4 sehen wir *ŠÚ+NA* und *ME+GE*. Sie bilden relativ glatte Kurven mit den charakteristischen Formen, die wir von Abb. 1 kennen.

Wir können folgern: Die Babylonier haben uns gute Zahlen überliefert - solche, die die Natur gut beschreiben. Vielleicht ist dies nicht allzu verwunderlich, stammt dieser Text doch aus der Seleukidischen Ära, also aus der Zeit ihrer avanciertesten mathematischen Astronomie. Es gibt aber noch einen viel älteren Text, dessen Monddaten genau so gut sind und den wir im folgenden untersuchen wollen.

Kambyses 400

Dieser Text ist schon mehrmals von z.B. Epping und Kugler bearbeitet worden; am ausführlichsten von Kugler (SSB I [10], SS. 61-75). Der Text enthält Mond- und Planetendaten aus dem 7. Regierungsjahr von Kambyses. Im Julianischen Kalender ist dies die Zeit von April 523 v. Chr. bis April 522 v. Chr. Unser Text ist eine Abschrift eines originalen Textes aus Kambyses' Zeit. Epping und Kugler sind sich darüber einig, daß die Planetendaten beobachtet sein müssen; hingegen sind sie über die Herkunft der Monddaten uneinig: Epping hält sie für Beobachtungen [15], während Kugler sie für berechnet ansieht. Wir sind völlig davon überzeugt, daß die Monddaten aus Beobachtungen stammen: wie wir sehen werden, sind sie viel zu gut und detailliert, um von den Babyloniern zu einem so frühen Zeitpunkt berechnet worden zu sein.

Da Kugler diesen Text minutiös bearbeitet und die übersetzten Daten auf den Seiten 68 und 69 in [10] I publiziert hat, geben wir sie hier nur graphisch wieder, und zwar in der rechten Hälfte von Abbildung 4. Von oben nach unten haben wir dort die Werte für *ŠÚ*, *NA* und die von uns addierten Summen *ŠÚ+NA* und *ME+GE* für die 13 Monate, die in Kambyses 400 aufgezeichnet sind, durch Kreise (O) markiert. Die Struktur dieser Daten ist die richtige: *ŠÚ* und *NA* variieren unregelmäßig, während die beiden Summen *ŠÚ+NA* und *ME+GE* die erwarteten charakteristischen Kurven bilden.

Wie stimmen nun diese Daten mit Kolonne Φ überein?

Die babylonischen Lunar Four und Kolonne Φ

Wir haben oben gesehen, daß die babylonischen Daten die richtige Struktur haben; wir können sie aber auch direkt kontrollieren. Dazu haben wir die Summen *ŠÚ+NA*, *ME+GE* und Σ für die in den beiden Texten vorkommenden Zeitspannen direkt berechnet. In Abbildung 5 (Zeit von LBAT 1285) und Abbildung 6 (Zeit von Kambyses 400) sind sie abgebildet (Kreuze ×, durch dünne Linien verbunden) und um je 10 Lunationen vor und nach denen der überlieferten Daten ergänzt. In den gleichen Abbildungen haben wir auch die aus den Originalquellen stammenden Summen als Kreise eingezeichnet (O, verbunden durch dicke Linien). Die babylonischen Daten stimmen mit den von uns berechneten "theoretischen" erstaunlich gut überein - so gut, daß wir folgern: aus umfassenderen babylonischen Aufzeichnungen der Lunar Four würde man dieselben Schlüsse ziehen wie aus den von uns berechneten. Speziell würde man, wie in Abb. 3 demonstriert, Φ aus der Summe der beobachteten Lunar Four erhalten können - abgesehen von einer Konstanten von 100 *uš*. (In Abb. 3 ist die Zeitspanne, die von LBAT 1285 überdeckt wird, mit einem dicken horizonatalen Strich markiert.)

Zuunterst in Abb. 5 haben wir Φ-100 *uš* eingezeichnet und sehen, daß die babylonischen Daten recht gut damit übereinstimmen. Für die Kambyseszeit haben wir ebenfalls Φ - 100 *uš* berechnet und in Abb. 6 eingezeichnet. Auch hier finden wir eine gute Übereinstimmung. Wir wissen zwar nicht, wie früh die Babylonier die Kolonne Φ zur Verfügung hatten, doch müßte sie auch mit ganz frühen Werten der Lunar Four übereinstimmen, wenn Φ tatsächlich als Näherung der Kurve Σ entstanden ist.

Zusammenfassung

Die uns überlieferten Lunar Four können sehr wohl zur Konstruktion von Φ gedient haben: die Daten sind genügend genau und sind systematisch über viele hundert Jahre hinweg gesammelt worden. Wir haben die Summen *ŠÚ+NA*, *ME+GE und Σ mittels eines modernen Ephemeridenprogrammes berechnet und* finden, daß die Übereinstimmung zwischen Σ und Φ optimal ist. Nicht nur ihre Perioden und Amplituden stimmen überein (wie bereits in [2] gezeigt wurde), sondern auch ihre Phasen. Astronomisch bedeutet dies, daß die Information über die Position der Apsidenlinie, die Σ enthält, genau mit derjenigen übereinstimmt, die Kolonne Φ implizit für die betreffenden Daten angibt. Eine Analyse von Lunardaten in den Texten LBAT 1285 und Kambyses 400 und der Vergleich mit ihren berechneten Werten überzeugt uns von der Güte und Brauchbarkeit der babylonischen Aufzeichnungen.

Danksagungen

Ich möchte meinem Manne, Prof. M. Brack, ganz herzlich für seine stetige Unterstützung und Ermunterung danken; speziell aber für die große Hife, die er mir bei den Computerberechnungen und der Herstellung der Abbildungen geleistet hat. Ich bin auch meinem Lehrer, Prof. O. Schmidt, sowie Prof. H. Hunger und Prof. Y. Maeyama für wertvolle Diskussionen und Hinweise zu Dank verpflichtet.

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Abbildungen

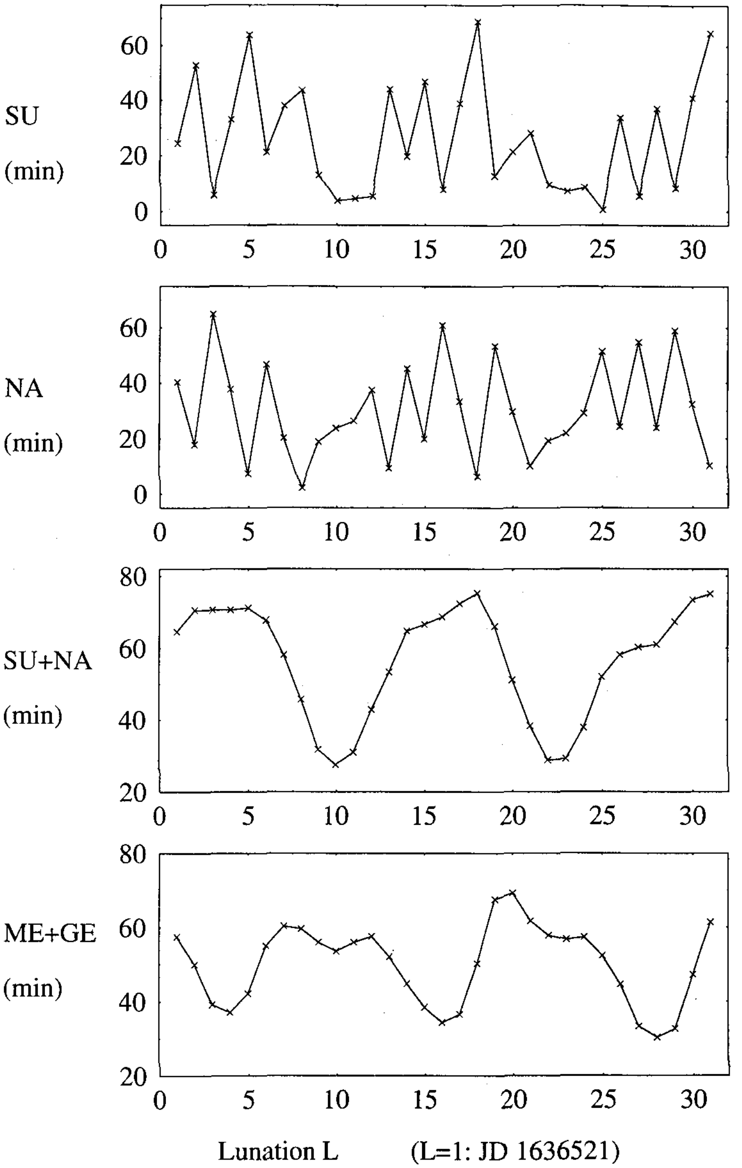

Abb. 1: Verschiedene Horizontphänomene aus den 'Lunar Four', abgebildet als Funktionen der Lunationsnummer *L*, berechnet für Babylon über eine Periode von 30 Monaten (ab JD 1636521 = 20. Juli 233 v.Chr.). Von oben nach unten: *ŠÚ*, *NA*, *ŠÚ+NA* und *ME+GE*. Die Größen sind in Minuten angegeben (4 min = 1 *uš*).

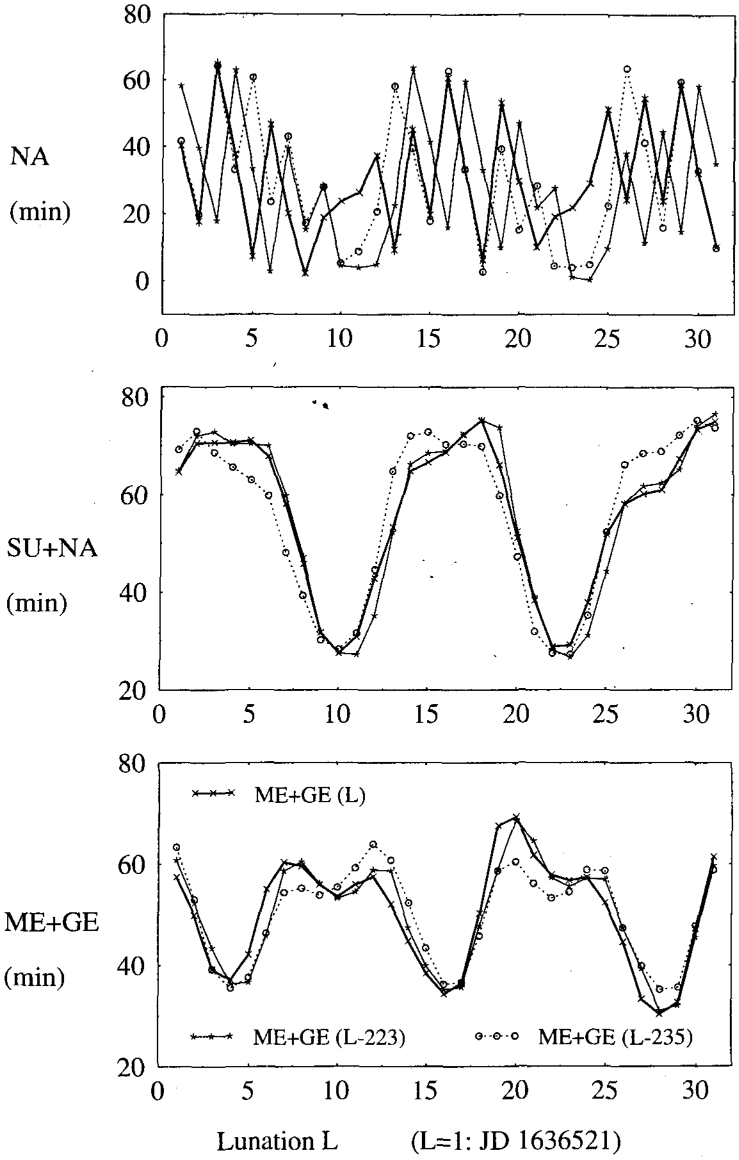

Abb. 2: Vergleich der Horizontphänomene aus Abb. 1 (durch Kreuze × markiert) mit denjenigen, die um 223 Monate früher (mit Sternen ⋆ markiert) oder 235 Monate früher (mit Kreisen O markiert) zu messen waren.

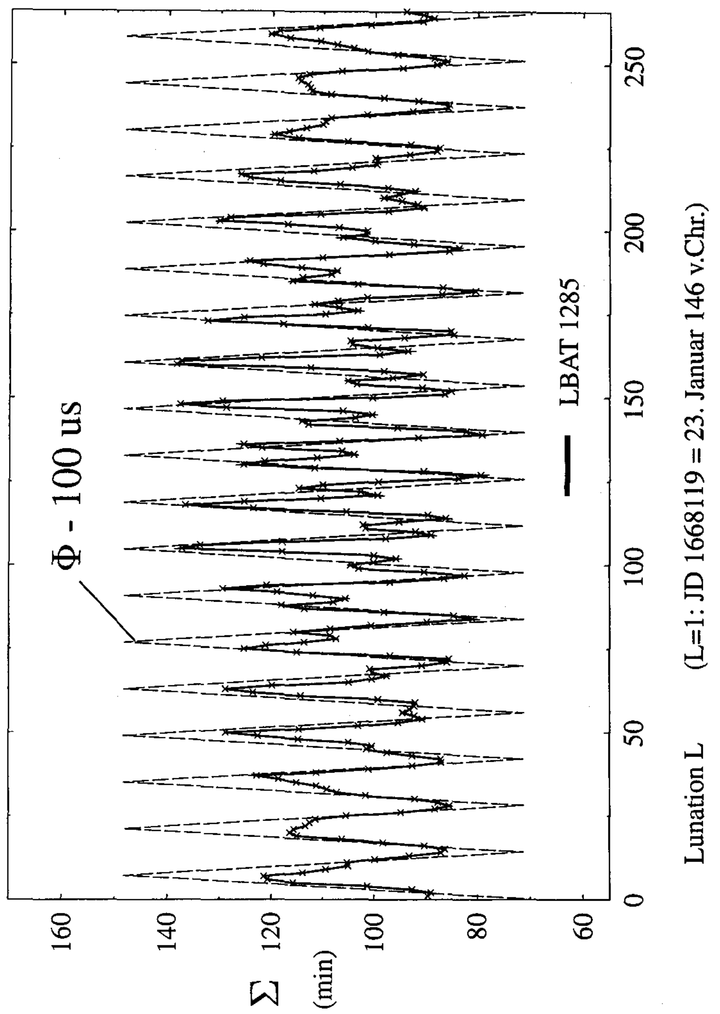

Abb.3: Die Summe $\Sigma = \check{S}\acute{U}+NA+ME+GE$ der 'Lunar Four' als Funktion der Lunationsnummer L (× mit dicken Strichen verbunden), berechnet für Babylon über eine Periode von 260 Monaten (ab JD 1668119 = 23. Januar 146 v. Chr.). Die dünne gestrichelte Linie zeigt die babylonische lineare Zickzackfunktion Φ - 100 *uš*, ermittelt für dieselbe Zeitspanne und in Minuten umgerechnet. Der horizontale Balken kennzeichnet Monddaten aus der Tafel LBAT 1285.

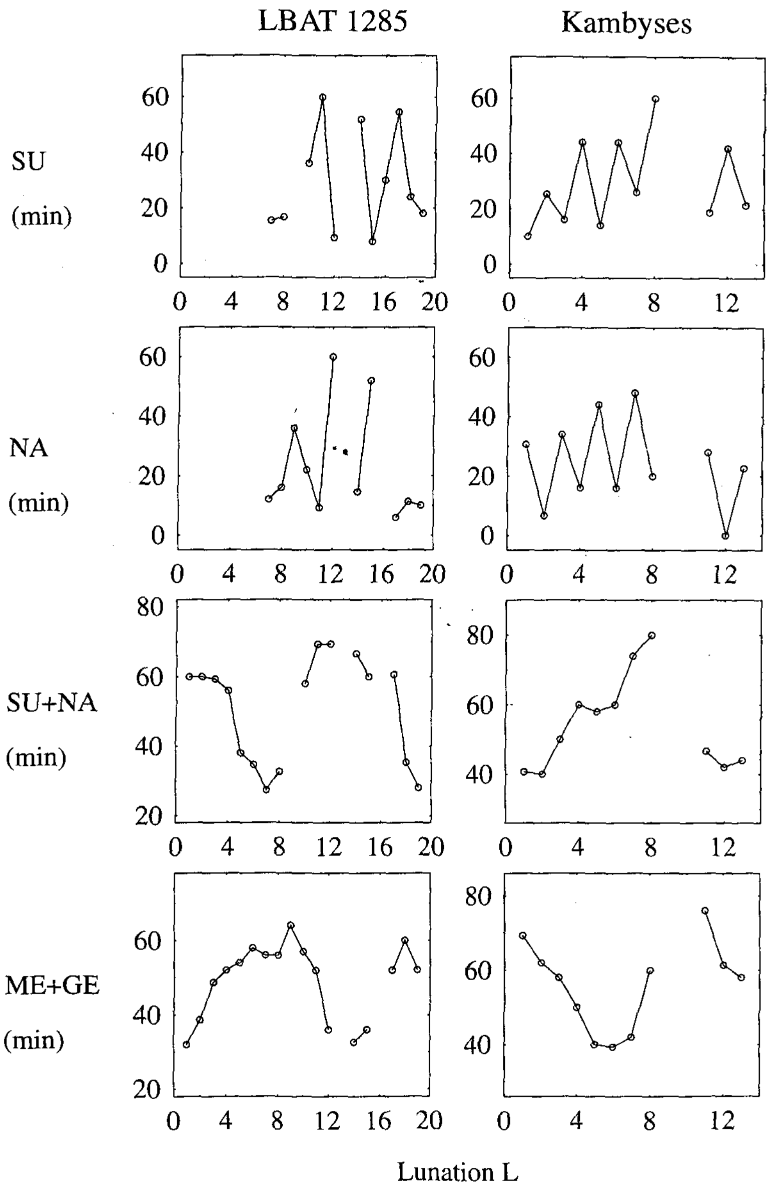

Abb. 4: Abbildung von 'Lunar Four'-Werten und ihren Teilsummen aus babylonischen Quelltexten. Links: Daten aus LBAT 1285. Rechts: Daten aus Kambyses 400. Hier, wie auch in Abb. 5 und 6, sind alle Größen in Minuten umgerechnet (1 *uš* =4 min).

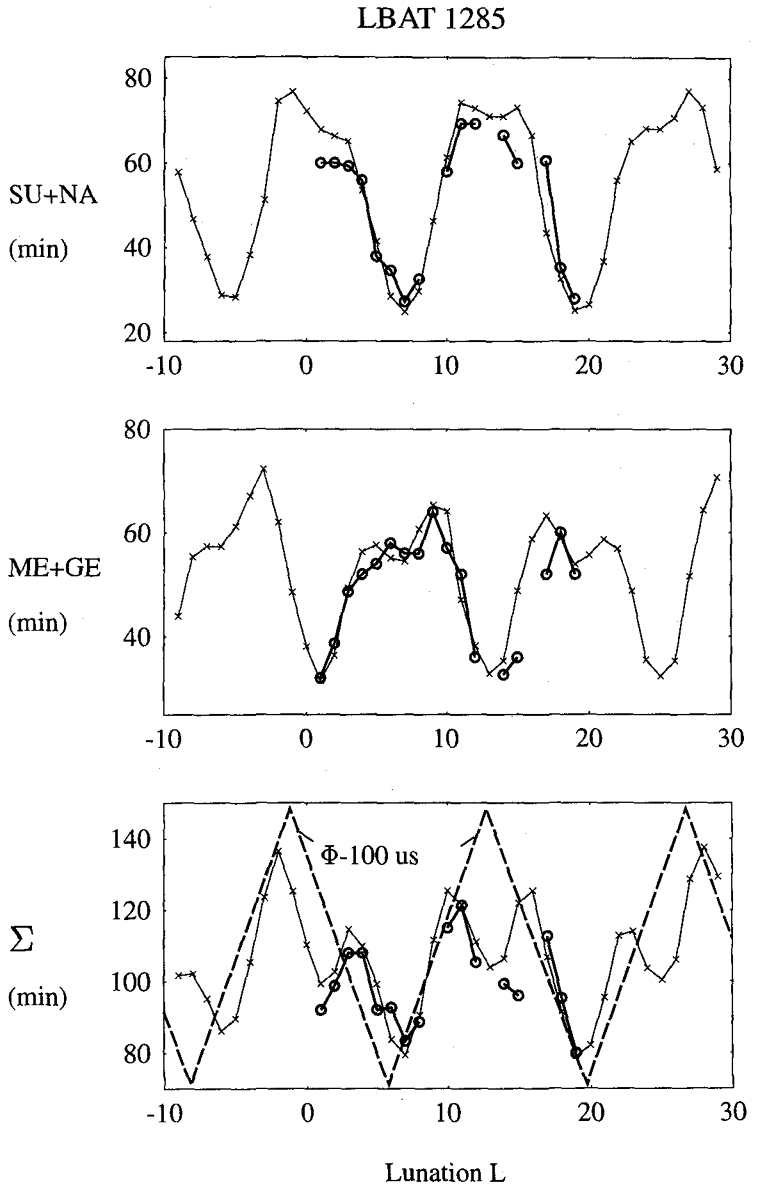

Abb. 5: Vergleich der Daten aus LBAT 1285 (O, mit dicken Linien verbunden) mit ihren berechneten Werten (×, mit dünnen Linien verbunden). Zuunterst ist neben Σ auch das babylonische Φ - 100 *uš* eingezeichnet (dick gestrichelt).

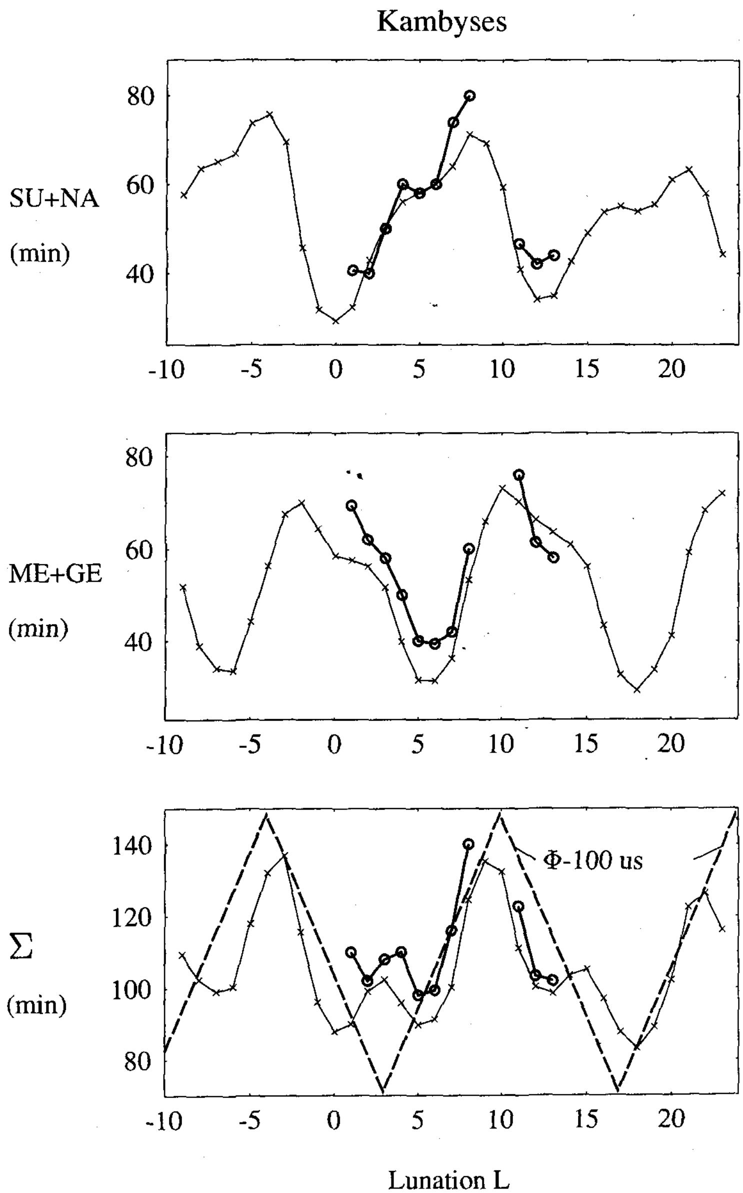

Abb. 6: Wie in Abb. 5: Vergleich zwischen Daten aus Kambyses 400 mit ihren berechneten Werten. Das babylonische Φ - 100 *uš* ist hier für die Zeit von Kambyses zurück extrapoliert worden.

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IN MEMORIAM

Olaf Schmidt
(1913–1996)

Olaf Henrik Schmidt was born in Sommersted, Denmark, on December 12, 1913. From 1929 to 1932 he attended the Gymnasium in Haderslev, and from 1932 to 1938 the University of Copenhagen. He received the degree of cand. mag. in mathematics (with physics, chemistry, and astronomy as minor subjects) with highest honors in 1938. During the year 1938–1939, he taught mathematics at Østre Borgerdyds Gymnasium, Copenhagen and did research in ancient mathematics and astronomy under the direction of Professor Otto Neugebauer.

In the summer of 1939, Olaf Schmidt followed Neugebauer to the United States and became an instructor in mathematics and a research assistant at Brown University. Because of the occupation of Denmark by the Germans, he was forced to stay in the United States much longer than he had planned. He studied Sanskrit and ancient astronomy, and in May 1943, he became a Doctor of Philosophy in the Department of Mathematics at Brown University, with a thesis *On the Relation between Ancient Mathematics and Spherical Astronomy*. In this thesis, he showed that the central problem in ancient astronomy originated in the observation of horizontal phenomena and consisted in finding the arc of the equator which was

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rising or setting during the same time as a given arc of the ecliptic. This treatise is written in Schmidt's characteristically clear and precise way. It has been used by many scholars in the field and, although written more than 50 years ago, is still the best extant work on this topic. The thesis is now being published by the Institute for Research in Classical Philosophy and Science of Princeton, New Jersey.

Although the years in the United States were scientifically fruitful, Olaf Schmidt considered them as lost or useless years since he was alone, having left his fiancée back in Denmark. After the end of the war, he took the first ship to Scandinavia and married. For four years, he worked as an amanuensis at Danmarks Tekniske Højskole, then in Copenhagen. After another stay at Brown University and at the Institute for Advanced Study in Princeton, he finally became a member of the Mathematical Institute of the University of Copenhagen in 1953. His duties were teaching and doing research in mathematics and history of the exact sciences. He became a full professor in 1965.

Olaf Schmidt was a careful, friendly, and very pedagogical teacher, and a brilliant and sharp thinker. He was also modest and unafraid of explaining things thoroughly. He admired and respected the ancient scientists, analyzing their problems and results by means of modern science and, at the same time, trying to understand them on their own terms. Since our modern mathematical notation implies much knowledge which was unknown to the ancient mathematicians, he never used modern notation in his lectures but invented a notation that came as close as possible to ancient mathematical thought. In his own words: "I want my representation to be such that an imaginative ancient Egyptian or Greek sitting in the back row of this room and listening to my lectures would nod his head as if saying: 'yes, that is right, that is fairly what I meant when I wrote it.'"

Many generations of mathematics students have benefited from Olaf Schmidt's lectures on Euclid's *Elements*, on Apollonius or Archimedes, or on Egyptian and Babylonian mathematics and Babylonian astronomy. These lectures were a treasure of new insights and exciting results of his own research. Here are some examples:

—In the old Egyptian mathematical texts, a rational number was written as an integer plus a sum of different unit fractions. Divisions performed by the Egyptians were therefore complicated. Olaf Schmidt showed that there are three standard methods for performing an "Egyptian" division, and that in principle every division of one rational number by another rational number can be carried out by the Egyptian methods.

—In connection with the $2/n$ Table of the Papyrus Rhind, Olaf Schmidt explained methods for finding "nice" expressions for the fractions $2/n$ in terms of unit fractions.

—Olaf Schmidt provided an axiomatic foundation of Euclid's geometry: On the basis of the first book of Euclid's *Elements*, he constructed a set of axioms which are rigorous according to modern mathematics and which suffice as a basis for all theorems in Books I–IV of the *Elements*.

—With reference to the theories of magnitude of Euclid and Archimedes, he pointed out that it is possible to prove the main theorem of Book XII of the

Elements, to the effect that circles are to one another as the squares of their diameters, within the framework of Euclid's axioms. Archimedes, however, introduced some additional axioms which enabled him to go beyond the limits of Euclid's *Elements* and prove that the surface of any sphere is four times its greatest circle.

Many students chose Olaf Schmidt as a master's degree supervisor because of his friendly and helpful attitude. Carefully and conscientiously, he met one hour a week with each of these students to discuss and guide their work. The themes he proposed covered a variety of topics ranging from Egyptian or Babylonian astronomy to Ptolemy's *Almagest* and Newton's *Principia*, and from Euclid, Apollonius, Archimedes, and Diophantus to Huygens and Fermat.

For many years, Olaf Schmidt was Secretary and Treasurer of Selskabet for de eksakte videnskabers historie—the Danish Society of the History of the Exact Sciences—and Secretary to the Danish National Committee for History and Philosophy of Science. He was Editor of *Centaurus* from vol. **7** (1960) to vol. **22** (1986). He hated polemics and struggles, but he did not keep his knowledge to himself. He was always willing to give advice to younger historians of science and to correct errors in articles submitted to *Centaurus*.

In 1956, Olaf Schmidt was among the first to travel to the camps of Hungarian refugees in Vienna. There, he selected students whom he could invite to study in Denmark, and he later supervised their education. Frequently invited to his home, many of these new citizens of Denmark maintained close relationships with him for many years.

When his beloved wife died very early, in 1967, he was left with his two young daughters, and the family grew even closer. Quickly he learned cooking and housekeeping. After a fine dinner served by Olaf, one of his former college friends admiringly exclaimed: "Behind this excellent food I suppose to have been deep literary studies." Family life always had a high priority. Each Sunday for many years—until the beginning of this year—Olaf invited his daughters with their growing families for dinner at his home.

Olaf Schmidt was not only humble and considerate, but also extremely self-critical. Often, when Neugebauer urged him to publish new results, Schmidt hesitated. He felt that he did not understand everything completely. Neugebauer used to shrug his shoulders and comment on his friend's attitude with the following quotation from Goethe's Faust: "Zwar weiss ich viel, doch möcht' ich alles wissen."

After his retirement in 1984, Olaf Schmidt lived a withdrawn life. He still received many historians of science at his home. I myself feel deeply indebted to my teacher whom I had the privilege to know for almost 30 years. Through his lectures and the interesting problems he posed to me, he introduced me to the exciting domain of the history of astronomy and transmitted his deep insight into the questions and working methods of ancient mathematics and astronomy. I gratefully remember countless occasions at his home, where I could also discuss my newest results with him.

Olaf Schmidt died on June 7, 1996. He often said: "All good things in life last too short." But his was a long and fulfilled life, and many of us will agree that knowing him was one of the good things in our lives.

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On the Babylonian Lunar Theory: A Construction of Column Φ from Horizontal Observations

by

LIS BRACK-BERNSEN*

Abstract

We demonstrate that column Φ in System A of the Babylonian moon ephemerides can be derived from such horizontal phenomena as were observed and recorded by the Babylonians. Combining four of the so-called 'Lunar Six' in such a way that the effects of the oblique ascension are eliminated, we obtain a curve which oscillates, indeed, with the exact period and the approximate amplitude of Φ. Our curve (which we call Σ) also contains oscillations with the approximate period of the Saros and allows us to find the period relation which is underlying column Φ. Herewith it has been shown for the first time that the length of the anomalistic month can be derived from horizontal observations.

1. *Introduction*

Babylonian astronomy is characteristic for its way of observing and calculating celestial phenomena. When we want to describe the movement of a planet or the moon, we derive formulae enabling us to find its position on the sky at any given time. The Babylonian astronomers concentrated on special characteristic events taking place at regular time intervals. They first observed and recorded these events over a long period of time, and then were somehow

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able to construct numerical schemes, mainly using arithmetic progression, for calculating and predicting these characteristic events. [For surveys of Babylonian astronomy, see Neugebauer 1955, 1975, Waerden 1974].

In case of the moon, the time interval was one synodic month. In the columns of the lunar ephemerides, a series of different quantities is recorded for each full moon or each new moon. The numerical algorithms for computing these columns have been known for a long time [Neugebauer 1955, 1975; Aaboe 1968; Aaboe and Hamilton 1979; Waerden 1966], and in most cases the astronomical meaning of the numbers has been understood. But *how* the Babylonians were able to derive these algorithms from their observations has so far largely remained unknown.

In this paper, we attempt to find a connection between observations and the methods of computation underlying column Φ in system A.

2. *Column Φ of system A*

2.1. *Common interpretation*

The predominant role of Φ is clear: it is the second column in the ephemerides of system A and is used as a basis for calculating all other columns and quantities related to the non-uniform velocity of the moon. The numerical values of Φ_i, where i refers to the consecutive lines (i.e., successive full moons) of the ephemerides, form a linear zig-zag function [ACT, 28, 44].

The common interpretation of column Φ is based upon its close connection to column G. G gives the length of the synodic month in the first approximation where only the variable moon velocity is taken into account; it also forms a linear zig-zag function. The structures of Φ and G are such that the difference between two values of G situated 1 Saros = 223 synodic months apart equals the difference of two successive values of Φ:

$$G_{i+223} - G_i = \Phi_{i-1} - \Phi_i . \qquad (1)$$

This led B. L. van der Waerden [1966] and A. Aaboe [1968] to interpret column Φ as the length of the Saros (up to a constant) in the first approximation where the sun velocity has a constant value equal to 30°/month:

$$223 \text{ synod. months} = 1 \text{ Saros} = 6585^d + \Phi^H. \qquad (2)$$

(Φ is measured in large hours H where $1^d = 6^H = 6{,}0^\circ$; $^\circ$ = time degrees.)

Inspired by this interpretation, we have in an earlier publication [Brack-Bernsen, 1980] calculated the length $\Delta^{223}t$ of consecutive Saroi as a function of the lunation number. We found that $\Delta^{223}t$ is varying with a (mean) period *different from that of column Φ.* This is so because the length of a Saros depends more strongly on the variable sun velocity than on the variable moon velocity. This means, however, that one would never arrive at column Φ by observing the durations of Saroi.

We are convinced that column Φ must be based *directly* on observations. Our aim is here to find out which kind of observations could possibly lead to column Φ – observations which connect Φ directly to the anomalistic month.

2.2. *Babylonian observations*

From the 'Astronomical Diaries from Babylonia' [Sachs & Hunger, 1988] we know which kind of observations the Babylonian astronomers made. They observed eclipses of the sun and of the moon and noticed, e.g., 'In the night of the 13th of month *XY*, 10° before sunrise lunar eclipse' [Sachs & Hunger, 1988, 243]. Thus we can conclude that they knew at least the exact time of observed eclipses. The diaries also remark the position of the moon with respect to fixed stars, e.g. 'month VII night of the 2nd, the moon was behind α Scorpii' [Sachs & Hunger, 1988, 173]. They also contain the dates on which equinoxes and solstices took place. These dates were, however, not observed but calculated according to a fixed scheme [Neugebauer, 1975, 357]. Nevertheless, this implies that the Babylonians at some earlier time must have observed the solstices and equinoxes.

Finally, the Babylonian astronomers observed moon and sun around each full moon and new moon, concentrating on six phenomena called the 'Lunar Six' by A. Sachs [Sachs & Hunger, 1988, 20]. These phenomena consist of six time intervals, two around new moon and four around full moon, which were regularly observed and recorded. In this paper, we confine ourselves to full moon phenomena and therefore only mention the latter four, calling them the 'Lunar Four'.

In order to obtain the 'Lunar Four', one has to observe the moonset on the western horizon the last morning before opposition and the next morning just after opposition; this gives the time intervals

$$\begin{aligned} \check{S}\acute{U} &= \text{time from last moonset to sunrise before opposition,} \quad (3)\\ NA &= \text{time from first sunrise after opposition to moonset.} \end{aligned}$$

Similarly, observations of the moonrise on the eastern horizon in the two evenings nearest to opposition will give the two intervals

$$\begin{aligned} ME &= \text{time from last moonrise to sunset before opposition,} \quad (4)\\ GE_6 &= \text{time from first sunset after opposition to moonrise.} \end{aligned}$$

These time differences were all measured in units of *uš*, also called 'time degrees' by the Greek. 1 *uš* equals 4 minutes, so that*) 6,0 *uš* $= 360° = 1^d = 24^h$.

Which kind of information can we get out of such observations? In order to answer this question, let us introduce a fictitious celestial body, denoted by $\overline{\odot}$, which is situated on the ecliptic directly opposite to the sun ($\odot$), such that $\lambda_{\overline{\odot}} = \lambda_{\odot} + 180°$. At the time of opposition, moon ($\bigcirc$) and $\overline{\odot}$ have the same length. The times observed and recorded by the Babylonians are the rising times of the little arc of ecliptic, $\Delta\lambda_{\bigcirc\overline{\odot}}$ lying between $\bigcirc$ and $\overline{\odot}$ at the four times where the observations are made.

At this point our trouble starts: horizontal observations of this kind are influenced by a variety of factors. First, the rising (or

*) We use throughout the Babylonian sexagesimal number system, such that $6{,}5 = 6 \cdot 60 + 5 = 365$, etc.

setting) times depend on the length of the ecliptic arc $\Delta\lambda_{\bigcirc\overline{\odot}}$, and this length in turn depends on the time difference between opposition and sunrise (or sunset) and also on the momentary velocities of the sun and the moon. Second, the rising time of $\Delta\lambda_{\bigcirc\overline{\odot}}$ depends on the angle between ecliptic and horizon; this angle varies between 30° and 80° in Babylon. (The rising time in Babylon of an ecliptic arc of 10° varies between 6°;45 and 13°;15, depending on its position on the ecliptic.) Finally, the observed time differences depend on the latitude of the moon.

We know from ephemerides and procedure texts [e.g., No 201, ACT, pp. 226–240] that the Babylonian astronomers knew of all these factors and were able to cope with them. In the different columns of their ephemerides, they had calculated for each full moon (amongst others) the momentaneous velocities of the sun and the moon, the longitude and latitude of the moon, the time of opposition and the corrections necessary in order to obtain the oblique ascension of a given ecliptic arc (i.e. the time it takes this arc to pass the horizon). Using all these quantities, the Babylonian astronomers were able to calculate the 'Lunar Four' *ŠÚ, NA, GE* (= GE_6) and *ME*.

Our working assumption is that the numerical methods developed for calculating the Babylonian ephemerides are based on observations such as found in the diaries: namely observations of eclipses, equinoxes, solstices and the 'Lunar Four'. In a way, we attempt to do the opposite of the procedure used in the procedure text: starting from observations of the 'Lunar Four', we try to reconstruct some of the numerical methods used in the ephemerides.

The following question now arises: How can we possibly filter out all the different variants hidden in *ŠÚ, NA, GE* and *ME*? In the first place, we search for ways of singling out the variable moon velocity of such observations, just as we believe it has been done in column Φ.

2.3. *A construction of column Φ from the 'Lunar Four'*

The basic idea of our reconstruction is to try to eliminate the effects of the oblique ascension. We remark that if a given little ecliptic arc

$\Delta\lambda_{\bigcirc}\lambda_{\overline{\odot}}$ passes the eastern horizon under a small angle (e.g., 30°), then the same ecliptic arc will pass the western horizon 12 hours later under a steep angle (80° in our example). This means that it should be possible to reduce the effects of the oblique ascension by combining morning observations and evening observations.

Unfortunately, the Babylonian observations left over to us are too scarce and incomplete to allow a systematic analysis. Therefore we had to produce the 'observed' material ourselves. We have used modern ephemerides and a computer code [Kreitmeier, 1990] for calculating rising and setting times of sun and moon. From these we have computed the 'Lunar Four', as seen from Babylon (32°;30 N, 45° W), for a series of successive oppositions O_i over a large time interval. (Of course, the Babylonians never could obtain such a complete series of observations due to weather conditions. We shall, nevertheless, perform our analysis from our complete data and show later on, how even a less complete and more scattered set of data still allows to find the same results.)

Investigating several combinations of morning and evening observations (see also Sect. 3), we found that the following procedure leads to a curve which oscillates with the period of the moon. Let us denote the 'Lunar Four', calculated or observed at the opposition (lunation) O_i, by $\check{S}\acute{U}_i$, NA_i, GE_i, and ME_i, respectively. We then build the sum

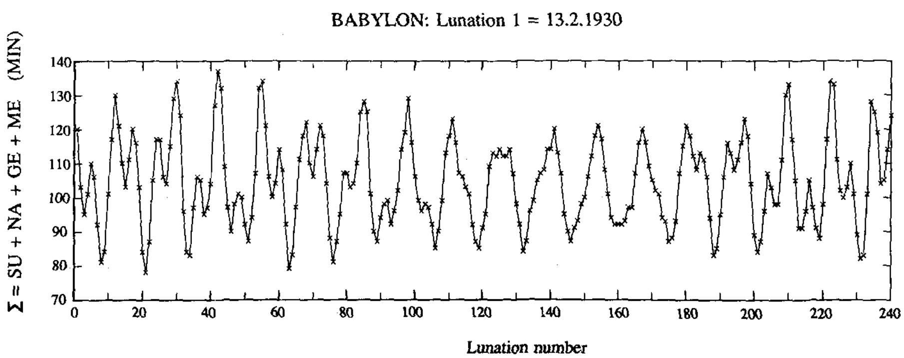

Figure 1: Sum Σ_i of the 'Lunar Four', (5), as function of the lunation number i, calculated for Babylon over a period of 240 lunations starting on the 13.2.1930 A.D. The values of Σ_i (in minutes) are shown by the crosses and connected by straight lines.

$$\Sigma_i = \check{S}\acute{U}_i + NA_i + ME_i + GE_i \qquad (5)$$

and plot it as a function of the lunation number i. The result obtained for 240 lunations starting on the 13.2.1930 A.D. is shown in Figure 1. We see that Σ_i, indeed, is oscillating with a period of approximately 14 lunations which is close to $P_\Phi = 13;56,39,6...$ synodic months. It is such a curve we looked for in order to reconstruct column Φ.

Can the function Σ_i be related to Φ_i? Σ_i is measured in minutes, while Φ_i in the common interpretation is measured in large hours H (see (2)). We translate both these units into time degrees ($1^h = 15°$, $1^H = 60°$) and compare the functions Φ_i and Σ_i in Figure 2. Σ_i is as in Fig. 1; Φ_i was reduced by a constant amount of 100° and is seen as the piecewise linear zig-zag curve; both are given in time degrees versus the lunation number i. The perfect agreement of the period leads us to the following hypothesis:

The linear zig-zag function Φ_i has been derived from the sum Σ_i, (5), of the 'Lunar Four'.

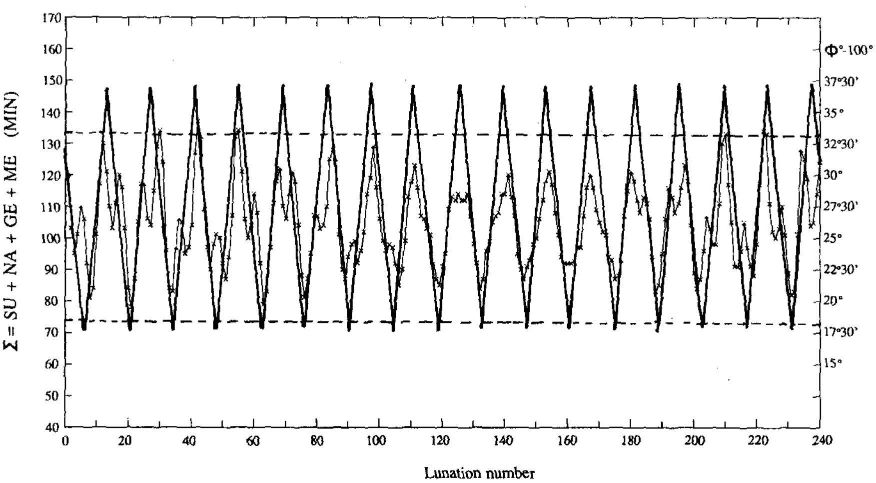

Figure 2: Σ_i, as in Fig. 1. The solid linear zig-zag line contains the values Φ_i of System A in the Babylonians' moon ephemerides, in time degrees ($1° = 4$ min), shifted by $K = 100°$ according to (6), (7). The horizontal dashed lines show where the Babylonians truncated the zig-zag function [Aaboe, 1969; Waerden, 1966].

If we neglect the small-amplitude variations in Σ_i, extending the longest straight sections to a linear zig-zag function $\hat{\Sigma}_i$, we find that $\hat{\Sigma}_i$ has a period $P_{\hat{\Sigma}} \simeq 13;57$ lunations (of course, the same as that of Σ_i) and an amplitude of 19° 12′, whereas the maximum amplitude of Σ_i is 14° 30′ = $0^h 58^m$. Thus, we can write

$$\Phi_i \simeq \hat{\Sigma}_i + K, \tag{6}$$

where the best fit of the two zig-zag curves is obtained for shifts K of the order

$$98^\circ \lesssim K \lesssim 102^\circ. \tag{7}$$

We shall discuss in a later publication, why the Babylonians have added this constant K to $\hat{\Sigma}_i$.

In conclusion, we have found that the 'Lunar Four' can yield a linear zig-zag function $\hat{\Sigma}_i$ through the sum in (5), which can be used exactly as Φ_i is used in the ephemerides, namely for each lunation to find the location of the moon within the anomalistic month. The fact that the dominating oscillations of Σ_i have the period $P_\bigcirc$ very strongly supports our assumption that the effects of the oblique ascension can be practically eliminated by taking the sum of the western ($\check{S}\acute{U}+NA$) and eastern ($GE+ME$) observations. A theoretical proof of this procedure and a deeper understanding of the astronomical significance of Σ, based upon O. Schmidt's excellent treatment of the oblique ascension [Schmidt 1943], will be the object of a forthcoming publication [Schmidt & Brack-Bernsen, 1991].

2.4. Σ and the Saros

Σ has to do with the movement in elongation of the moon, and since we interpret Φ according to (6), Φ is also closely related to the elongation movement of the moon. Now, from the 'Saros text' [Neugebauer, 1957] and from the calculational scheme connecting Φ with G and Λ [Aaboe, 1968], we know that the Babylonian zig-zag function Φ is closely related to the Saros, i.e. the period of 223 synodic months.

To our big surprise, our curve Σ does, indeed, reflect the Saros: If we look more closely at Σ, we note that its waves have rather varying and bizarre structures. However, these structures repeat themselves almost identically after 223 synodic months. This is clearly demonstrated in Figure 3, where two successive periods of 223 lunations of Σ are placed on top of each other for the sake of a better comparison.

Let us go back to column Φ. This column is constructed such that

$$1{,}44{,}7 \text{ syn. months} = (1{,}44{,}7 + 7{,}28) \text{ anom. months} = 7{,}28\ P_{\Phi} \quad (8a)$$

As Neugebauer noticed in his Saros paper [Neugebauer, 1957], the relation (8a) combined with the Saros S = 3,43 syn. months tells us that

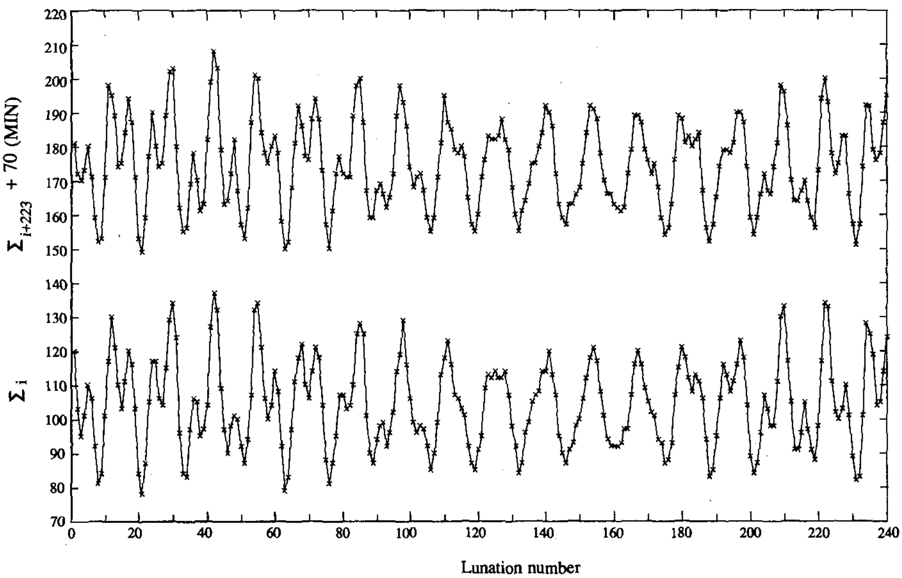

Figure 3: *Lower part*: Σ_i as in Fig. 1. *Upper part*: Σ_{i+223}, i.e. Σ over the next Saros period (1 Saros = 223 lunations), shifted by +70 minutes. Note how the fine structure repeats itself almost identically after one Saros.

$$1{,}44{,}7 \text{ syn. months} = 28 \cdot S + 3 \text{ syn. months} = 28 \cdot 16\, P_\Phi \,. \quad (8b)$$

In our analysis of the curve Σ, we would formulate these relations somewhat differently:

$$6247 \text{ lunations} = 448 \text{ `waves'} = 448\, P_\Sigma \,, \qquad (9a)$$

$$28 \cdot S + 3 \text{ lunations} = 28 \cdot 16 \text{ `waves'} = 448\, P_\Sigma \,. \qquad (9b)$$

From (8b) we see that 28 Saroi are almost equal to $28 \cdot 16\ P_\Phi$, being only 3 lunations shorter. (All this is, of course, well known – we just transform this knowledge into a form which allows us to compare Φ with our Σ.) (8b) tells us that if we were to look at the Φ curve 28 Saroi apart, we would have the situation depicted in Figure 4.

Do the lunation points on our curve Σ behave in a similar manner? The computer program we used is not accurate enough

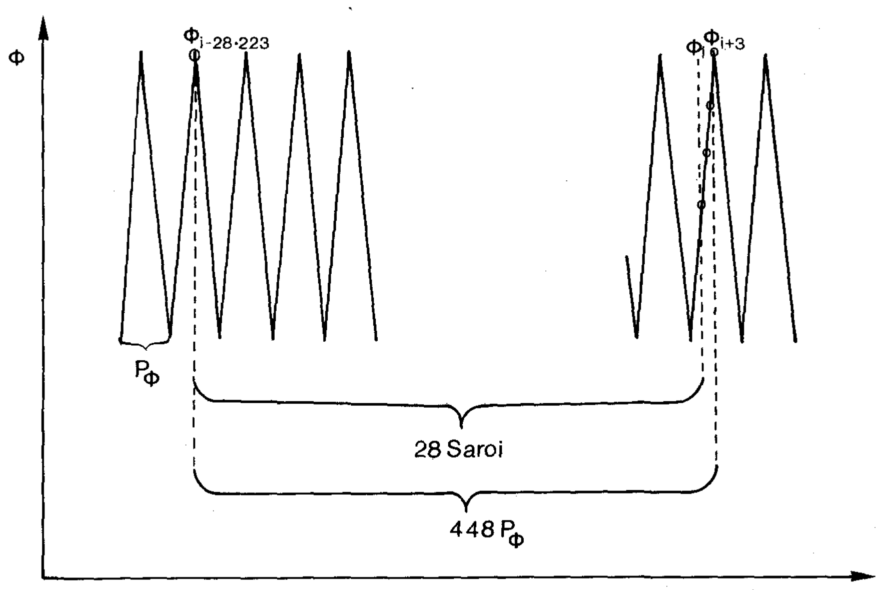

Figure 4: Two sections of the zig-zag function Φ_i, 28 Saroi = 28 · 223 lunations apart. Note how the maxima are shifted by 3 lunations after this period.

for extrapolation over a period of 28 Saroi, but it covers correctly some 10 Saroi. We therefore investigated the position of the lunation points on the curve Σ at time intervals of 4, 6, 7, 9 and $9\frac{1}{3}$ Saroi. We found that, indeed, the lunation points on Σ slide slowly backwards with respect to the extrema of Σ when going forward over several Saroi, exactly as they do on the zig-zag curve Φ.

If our interpretation of Φ given in (6) is correct, we can go one step further and try to find period relations by comparing carefully the lunation points on Σ over large time intervals. Choosing pairs of points which are situated analogously, but several Saroi apart, it is very easy to count the number of main periods P_Σ in between. As an example, we show in Figure 5 two sections of Σ_i, about $9\frac{1}{3}$ Saroi apart. The exact repetition of the fine structure of the lunation points after 2078 lunations leads to the period relation.

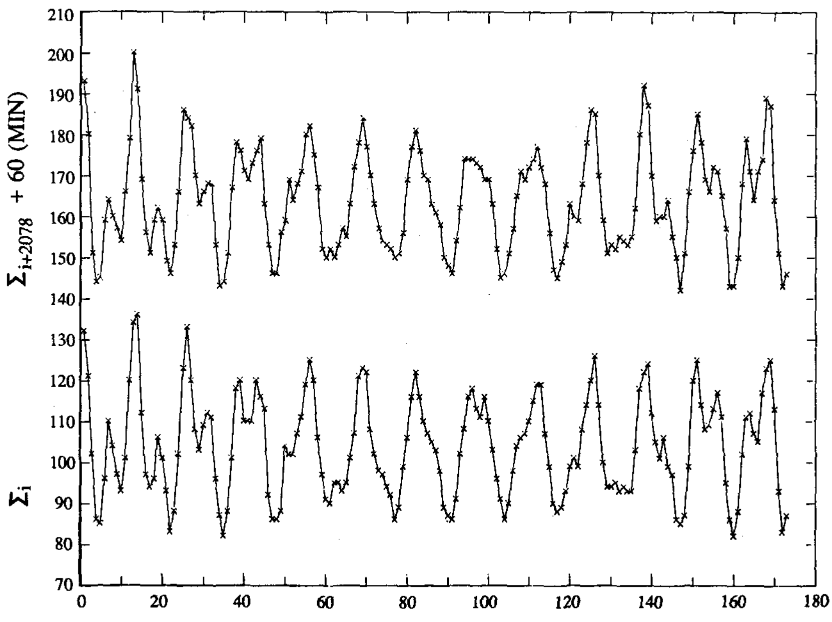

Figure 5: Two sections of the curve Σ_i, similarly as in Fig. 3. *Below*: First lunation on the 9.12.1821 A.D. *Above*: 2078 lunations $\simeq 9\frac{1}{3}$ Saroi later; this time interval encompasses exactly 149 periods P_Σ.

$$2078\,\text{lunations} = 149\,P_\Sigma\ . \tag{10a}$$

This corresponds to

$$448\,P_\Sigma = \frac{2078}{149}\cdot 448\,\text{syn. months} = 6247.9\,\text{syn. months}\,. \tag{10b}$$

One very characteristic structure on Σ, occurring exactly again between two lunation points some 4 Saroi apart, leads to

$$63\,P_\Sigma = 878.5\,\text{lunations}\ , \tag{11a}$$

giving

$$448\,P_\Sigma = 6247\,\text{syn. months}\ . \tag{11b}$$

Our curve Σ is, indeed, loaded with information!

In summary, the period of Σ is that of a function which tabulates the moon velocity once each full moon. Σ reflects the Saros, and the lunation points on the curve allow us to determine good period relations for the linear zig-zag function $\hat{\Sigma}$.

We are aware that the Babylonians cannot have made the curve analysis as we have done it so far. But they *did* have the same material of the 'Lunar Four', observed on consecutive full moons (whenever visible) over long periods of time, and we know that they were very skilled in handling numbers. We are therefore convinced that they, in some way or other, were able to extract the same kind of information from the sum Σ, (5), as we have done it above. This is possible, even if not all successive lunations have been seen without interruption: the lunation points on Σ lie quite densely, namely 14 on each period P_Σ. This means that only about one half of the points of a period would be sufficient to give the function Σ and, after averaging the monthly oscillations, the linear zig-zag function $\hat{\Sigma}$. Knowing that the Babylonians observed the 'Lunar Four' over many years, month after month, noting their magnitude if visible, but also noting when they were not visible to keep track of the lunation numbers, we feel sure that their material must have

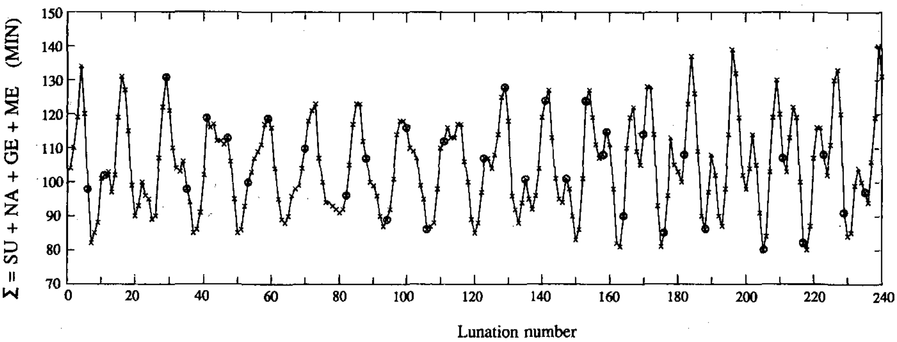

Figure 6: Σ_i as in Fig. 1 over 240 lunations, but starting from the 5.9.1971 A.D. (calculated for Babylon). The circles show the lunations on which a moon eclipse took place. (Only about half of the eclipses were visible in Babylon.)

been sufficiently comprehensive to enable them to construct the function $\hat{\Sigma}$.

Observations of the 'Lunar Four' might have been combined with accurate observations of moon eclipses (times and places), in order to find the rather precise period relation (11b) for $\hat{\Sigma}$ (and Φ). Eclipses of the moon alone would not suffice for this task. This will become evident from Figure 6 showing Σ over a period of 18 years, starting with lunation 1 on the 5.9.1971 A.D., with all moon eclipses marked by circles. They are so scarce that they will never allow to determine the period P_Σ of Σ. (Of course, only about one half of these eclipses would be visible from one and the same point on the earth.)

As a support for our conjecture that column Φ has been constructed from the sum, (5), of the 'Lunar Four', we point to the Goal-year texts mentioned by van der Waerden (1974, p. 108). We quote: "Lunar six and eclipses for the year X-18 and sums *ŠÚ+NA* and *ME+GE* for the second half of year X-19." We see that the Babylonians really were concerned with the sums *ŠÚ+NA* and *ME+GE*. They used these functions to produce astronomical predictions. In the following Section we shall see that more interesting information can be extracted from these two functions, besides what we just have gained from their sum Σ.

3. Further information found from the 'Lunar Four'

In Figure 7 we present a compilation of the following quantities as functions of the lunation number, taken over a period of 500 lunations: $\check{S}\acute{U}$, the sums $\check{S}\acute{U}+NA$ and $ME+GE$, their difference $\Delta = ME+GE-(\check{S}\acute{U}+NA)$, and their sum Σ, (5). We see that $\check{S}\acute{U}$ alone varies rather unpredictably; the monthly oscillations are so dominating that it is practically impossible to extract any information from $\check{S}\acute{U}$ by simple inspection of the curve. (The curves of NA, GE and ME look very similar to that of $\check{S}\acute{U}$.) However, taking the sums $\check{S}\acute{U}+NA$ and $ME+GE$, we get two very similar curves which oscillate rapidly with a mean period of ca. 12;22 syn. months $\simeq P_{\odot}$, while their amplitudes vary slowly with a period

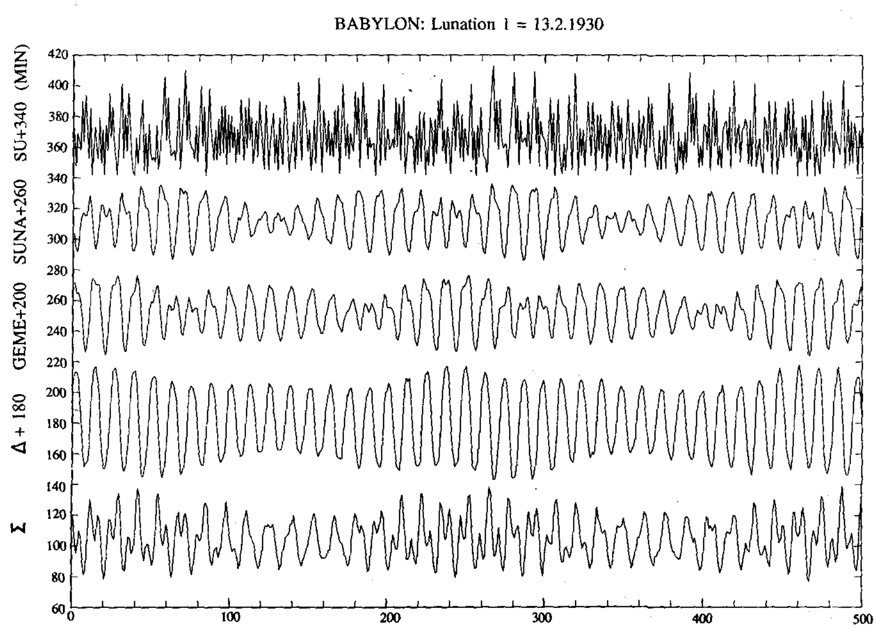

Figure 7: Graphs of different horizontal phenomena derived from the 'Lunar Four', over a period of 500 lunations. *From top to bottom*: $\check{S}\acute{U}$ (+340 min), $\check{S}\acute{U}+NA$ (+260 min), $GE+ME$ (+200 min), Δ as defined in (13) (+180 min), and Σ. Note how all amplitudes oscillate with the approximate period of a Saros (223 lunations).

$D \simeq 109.5$ syn. months which is the revolution time of the apside line. Oscillatory functions of this type have been discussed in our earlier publication [Brack-Bernsen, 1980]. They result from a superposition of two oscillating functions with the periods $P_\odot$ and $P_\bigcirc$, respectively. In the present case of $\check{S}\acute{U}+NA$ and $ME+GE$, the dominating term is the sun's influence giving the period $P_\odot$ of the rapid oscillations, the influence of the moon term being seen by the variation of the amplitudes with the period D:

$$D = \left(\frac{1}{P_\odot} - \frac{1}{P_\bigcirc}\right)^{-1}. \tag{12}$$

Of course, $\check{S}\acute{U}+NA$ is also influenced by other factors than those with the periods $P_\odot$ and $P_\bigcirc$. We also remark that both $\check{S}\acute{U}+NA$ and $ME+GE$, as their sum Σ, reflect the Saros. The astronomical meaning of $\check{S}\acute{U}+NA$ will be discussed in [Brack-Bernsen & Schmidt, 1991.]

Taking the *difference* of $ME+GE$ and $\check{S}\acute{U}+NA$, we obtain a curve which we shall call Δ:

$$\Delta = ME+GE-(\check{S}\acute{U}+NA). \tag{13}$$

This curve is very smooth, almost without any monthly disturbances, and oscillates with the period $P_\odot$. It reminds us very much of the zig-zag function in column A in the ephemerides of system B, determining the position of the sun (and the moon) at each lunation (i.e., at each full or at each new moon). The question therefore arises, if the lunation points on the curve Δ also slide slowly with respect to its extrema, as it is the case for the curve Σ. This would imply the possibility of determining a period relation from the curve Δ in exactly the same way as we did it above in the case of Σ. Indeed, by inspection of the lunation points about three Saroi apart on the curve Δ, we come to the following relation:

$$\begin{aligned} 3\ \text{Saroi} - 1\ \text{syn. month} &= 54\, P_\odot = 3\cdot 18\, P_\odot\,, \\ 668\ \text{syn. months} &= 54\, P_\odot\,. \end{aligned} \tag{14}$$

This period relation is identical to the one used in the abbreviated version of column A in system B [ACT, p. 71]:

$$\text{II syn. months.} = 5{,}34 \text{ syn. months} = 27\, P_{\odot}\,. \tag{15}$$

The original column A is based upon a more accurate period relation, namely

$$\begin{aligned} 2{,}46{,}59 \text{ syn. months} &= 13{,}30\, P_{\odot}\,, \\ 15 \cdot [3 \text{ Saroi} - (1 \text{ syn. month} + 2^r)] &= 15 \cdot 3 \cdot 18\, P_{\odot}\,, \end{aligned} \tag{16}$$

where $1^r = \frac{1}{30}$ syn. month $\simeq 1^d$.

It is possible that the Babylonian astronomers found their period relation (15) using the information hidden in the 'Lunar Four' in the combination Δ, similarly as we found (14). The relation (16), however, would require observations using smaller time units than the synodic month (or observations over about 45 Saroi which seems quite improbable). We rather see the relation (16) as a refinement of the relations (14) or (15) – and not (15) as an abbreviation of (16) found by rounding off, as suggested by O. Neugebauer [1975, p. 533]. We know that the Babylonians regularly calculated solstices and equinoxes; in their diaries they mentioned the days and months on which these events took place. It is exactly dates of this kind which enable us to add a minor correction to (14). (Note that here, we have to do with the day and not the month as a smallest unit.) Using exact dates of observed or calculated solstices, the Babylonian astronomers could easily have found that (14) should be corrected to (16).

4. Summary and Conclusions

We have shown that it is possible to construct column Φ from horizontal observations. The sum of the 'Lunar Four' *ŠÚ, NA, ME* and *GE* (calculated at successive lunations) defines us a function Σ which oscillates with the same period as Φ. The fact that also their amplitudes are closely the same allows us to derive Φ from Σ.

As we will show [Brack-Bernsen & Schmidt, 1991], Σ is closely connected to the movement of the moon relatively to the sun (i.e., to the moon's movement in elongation).

At this place, we remind the reader about O. Neugebauer's first surmise of the astronomical significance of column Φ [ACT, p. 44]: "Φ must describe a phenomenon very closely related to the lunar velocity", and later [ACT, p. 45]: "Perhaps Φ is obtained from the relative velocity". We are convinced that Neugebauer was right.

The occurrence of the Saros in Σ enables us to determine the period relation underlying Φ. In our interpretation of Φ, we therefore see its connection to the Saros rather as an *interior structure* – built into Φ as a consequence of the period relation (8b) – than as the origin of its construction.

Our new interpretation of column Φ as being derived from Σ will, of course, influence our understanding of the derivation of the quantities F, G, Λ, and W from column Φ. This will be discussed in detail in a forthcoming publication.

Acknowledgements

I am deeply indebted to my teacher, Prof. Olaf Schmidt, for his continuing interest and support during many years. To him I want to dedicate this work. I am grateful to Dr. S. Kreitmeier for leaving us his astronomical code, and to my husband, M. Brack, for computing the material to the figures.

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On the Construction of Column B in System A of the Astronomical Cuneiform Texts

by

LIS BERNSEN*

In the Astronomical Cuneiform Texts[1], vol. I, p. 45 a Column *B* is discussed. The difference column of this column is a step function the values of which are: 30° on the arc from ♍ 13 to ♓ 27 and 28; 7,30° on the remaining arc from ♓ 27 to ♍ 13. The interpretation which the ACT offers of Column *B* is primarily based on a theory of the annual movement of the sun in the ecliptic—without taking into consideration at all the movement of the moon.

O. Neugebauer, following Kugler, has pointed to a possible reconstruction of Column *B* by showing that the step function which is the "difference column" to Column *B* can be determined by means of the lengths of the yearly seasons, that is, by means of phenomena which are not at all dependent upon the movement of the moon but entirely of that of the sun.

In 1965 Aaboe[2] advanced the hypothesis that the step functions which occur in planetary texts that deal with horizon phenomena as e.g. heliacal risings have been produced as follows.

Successive heliacal risings have been marked down on the ecliptic for quite a few years. It then turns out that these phnomena are placed close together on some arcs and further apart on others; these are exactly the arcs into which the step function divides the ecliptic. And it is shown that from the distribution of these points one can reconstruct the step function.

We shall here examine whether the construction of column *B* can be explained on the basis of this hypothesis. To this end we mark on a circle, representing the ecliptic, the positions of new moons throughout many years; and the question is now whether these new-moon points (I will call them Q-points) are to be found close together on one part

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of the ecliptic but further apart on another part of the ecliptic. If this is so then it is possible to derive a step function from the distribution of the Q-points.

There are certain things which apparently—but, as we shall see, only apparently—contradict this hypothesis, that Column *B* should be based on observations of successive occurrences of new moons. The difference, $\Delta\lambda$, in longitude between consecutive occurrences of new moons is dependent upon the velocity of the sun as well as that of the moon. Here we must call attention to the fact that the motion of the sun is rather regular, in that it always assumes its maximal and minimal velocity in the same parts of the ecliptic, (in this connection we are justified in disregarding the slight motion of the apogee of the orbit of the sun) and the difference between the maximal and minimal velocity is "small". The motion of the moon is on the other hand rather irregular, in that it may assume its maximal and minimal velocity in any part of the ecliptic, and the difference between the maximal and minimal velocity is "not small". In other words the velocity of the sun is a function of its longitude, whereas the velocity of the moon is not a function of its longitude.

Intuitively one would think that the difference, $\Delta\lambda$, in longitude between consecutive new moons primarily would be determined by the variation of the velocity of the moon. If this indeed is the case the Aaboe hypothesis can not be applied to column *B*, because the distribution of the Q-points will then "follow" the velocity of the moon and hence not show a pattern which is determined by λ. From this we conclude that if the hypothesis can be applied to column *B*, then $\Delta\lambda$ must primarily depend upon the velocity of the sun.

We are interested then, in examining how the longitude λ of consecutive new moons depends upon the velocity of the sun and the moon. The first question to be put is whether it is the unevenness in the velocity of the sun or the unevenness in the velocity of the moon which primarily determines how far the sun has moved from one occurence of new moon to the next? In order to obtain an answer to this question I have drawn up two models.

Both models are geocentric; for the sake of simplicity the sun and the moon are assumed to move in circles the centre of which is the earth.

Model I: The velocity of the sun is variable whereas the velocity of the moon is constant.

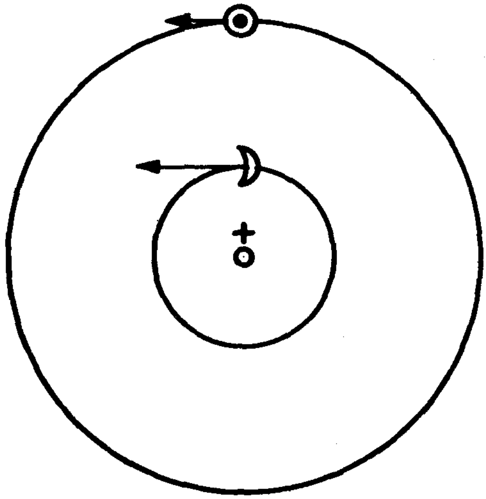

Fig. 1.

Model II: The velocity of the sun is constant whereas the velocity of the moon is variable.

By means of the Tuckermann tables I have found the maximum, the minimum and the mean velocities of the sun and the moon to be:

$v_{\odot}(\text{min}) = 0.95°/\text{day}$ $v_{\odot}(\text{max}) = 1.02°/\text{day}$ $v_{\odot}(\text{mean}) = 0.987°/\text{day}$

$v_{☾}(\text{min}) = 11.9°/\text{day}$ $v_{☾}(\text{max}) = 14.8°/\text{day}$ $v_{☾}(\text{mean}) = 13.26°/\text{day}$

In Model I where the moon moves at its mean velocity, the smallest difference in longitude between two consecutive new moons is to be found when we assume the sun to be moving at its lowest velocity. Assume t to be the length of the synodic month which will occur under this condition. The following then applies:

$$0.95\, t + 360 = 13.26\, t$$

thus

$$t = 29.2 \text{ days}$$

and therefore

$$\Delta\lambda(\text{min}) = 0.95 \cdot 29.2 = 27°.8$$

Similarly one finds

$$\Delta\lambda(\text{max}) \qquad = 30°.0$$

In Model II, where the sun moves at its mean velocity, the smallest difference in longitude between two consecutive new moons, $\Delta\lambda$(min), is to be found when the moon moves at its highest velocity. This, however, must be understood in the following way: The moon runs through its entire spectrum of velocities in the course of one anomalistic month, which is shorter than one synodic month. For this reason only that part of a synodic month which exceeds an anomalistic month should be taken into account, and in that part of a synodic month we assign to *the moon its greatest velocity.*

Assume t to be the length of the synodic month which occurs under this condition.

The following then applies:

$$0.987\,t + 360 = 27.55\,v_{☾}\,(\text{mean}) + (t - 27.55)\,14.82$$

hence

$$t = 29.13 \text{ days}$$

and therefore

$$\Delta\lambda(\text{min}) \qquad = 28°.8$$

Similarly one finds:

$$\Delta\lambda(\text{max}) \qquad = 29°.2$$

Thus we see that

In model I: $\Delta\lambda(\text{max}) - \Delta\lambda(\text{min}) = 2.2°$
In model II: $\Delta\lambda(\text{max}) - \Delta\lambda(\text{min}) = 0.4°$

Consequently it is the variation of the velocity of the sun which determines the variation of $\Delta\lambda$. In other words, the velocity of the moon is of no consequence for the calculation of the difference in longitude between consecutive new moons.

From an astronomical point of view Model I as well as Model II are incorrect since the velocities of the sun and the moon vary simultaneously and independently of each other. If one wishes more correct values of $\Delta\lambda$, one must find the longitude of real consecutive new

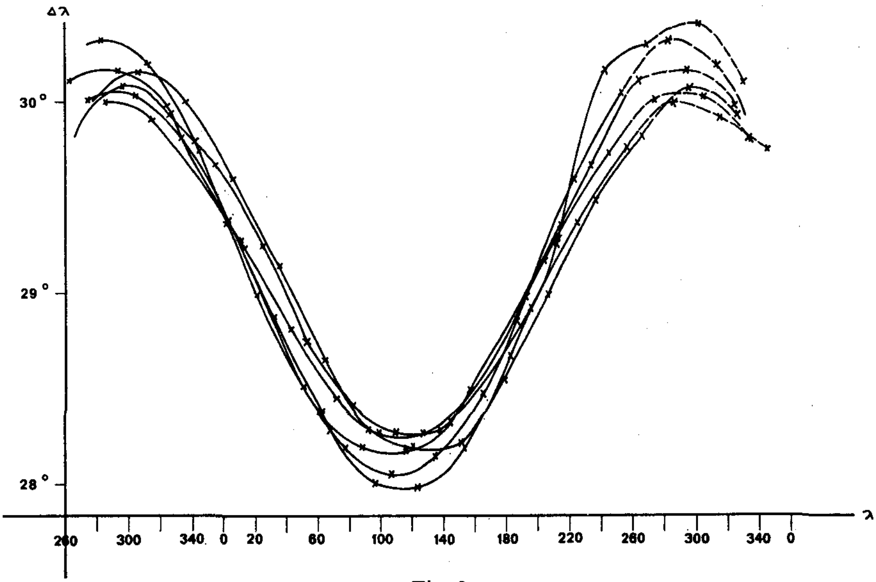

Fig. 2.

moons. By means of the National American Ephemeries I have found the longitudes

$$\lambda_0, \lambda_1, \ldots, \lambda_{256}$$

of consecutive new moons from the year 1940 to 1960, inclusive. From this we find

$$\Delta\lambda_i = \lambda_i - \lambda_{i-1}$$

We now plot the values $(\lambda_i, \Delta\lambda_i)$ in a coordinate system, and since the longitudes are reduced modulo 360 we get in this way 21 curves. For the sake of clarity we have drawn only 6 of these curves in fig. 2. This figure shows very clearly that the six curves are very close together and from this fact we conclude *that $\Delta\lambda$ is by and large a function of λ.*

This is surprising because it implies that $\Delta\lambda$ is a function of $v_{\odot}$ alone and is independent of $v_{☾}$. For $v_{\odot}$ is unambiguously derived from the longitude, λ, of the sun, while the velocity of the moon is not a function of its position on the ecliptic, as mentioned before.

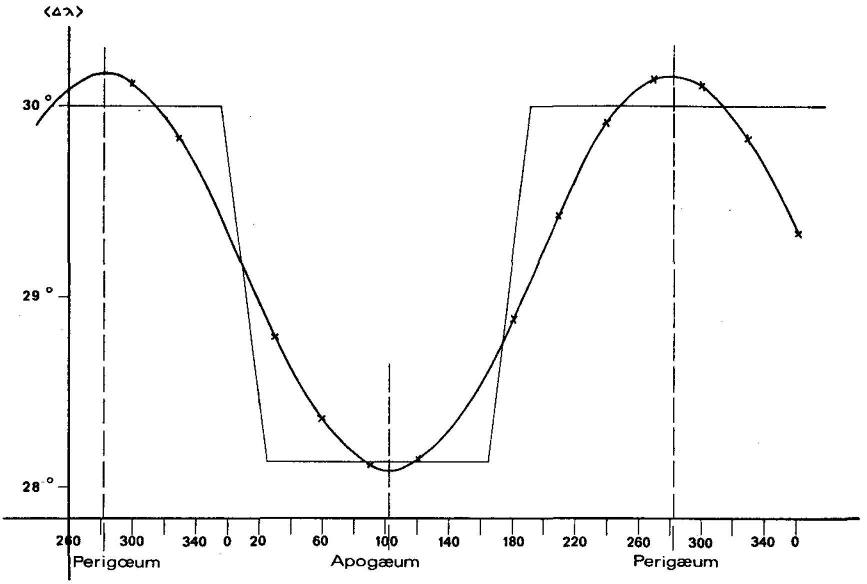

Fig. 3.

How do these results fit Column *B* in System *A*? The difference column for Column *B* is a function of λ, we call it $\sigma(\lambda)$, and the graph of $\sigma(\lambda)$ is shown on fig. 3. On fig. 3 I have also drawn a mean curve on the basis of the 21 curves mentioned above, taking into consideration the fact that in the year 1950 the apogee was at 102° whereas at the time of the text, i.e. 150 B.C. the apogee was at 66°.

This mean curve fits the graph of $\sigma(\lambda)$ so well that it is reasonable to assume that *observations of longitudes of new moon was precisely what served as the basis for Column B.*

NOTES

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On the Foundations of the Babylonian Column Φ: Astronomical Significance of Partial Sums of the Lunar Four

by

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Abstract:

A characteristic feature of the Babylonian mathematical astronomy is the use of periodically varying functions in the form of sequences of numbers (e.g. arithmetic progressions, zig-zag functions, or piecewise constant step functions) to describe periodically occurring astronomical phenomena. One major achievement of the Babylonian astronomers consists in a very precise determination of the periods of the number sequences used in their ephemeris texts. Any reconstruction of the Babylonian calculation schemes must explain how the fundamental periods or period relations can be determined empirically by such astronomical observations as were compiled in the Babylonian Diaries.

This paper is concerned with the Babylonian moon ephemerides. The fundamental periods used here are the length $P_{\odot}$ of the

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solar year, the period $P_{☾}$ of the lunar velocity, and the period P_{Ω} of the moon's movement in latitude. $P_{☾}$ and P_{Ω} are the periods of the moon velocity $v_{☾}$ and of the moon latitude $\beta_{☾}$, respectively, when these are measured once each synodic month, such as the Babylonians did it in their ephemeris texts.

We have recently shown that horizontal phenomena observed by the Babylonians, the so-called Lunar Four, contain information on these periods. E.g., the period $P_{☾}$ can be determined empirically by the sum of all Lunar Four, whereas partial sums oscillate with the period $P_{\odot}$. In the present paper we will explain why and how this works, through offering an astronomical interpretation and analysis of (partial) sums of the Lunar Four. In so doing, we will use the modern theory and ancient ideas on the oblique ascension of ecliptic arcs (i.e., the time it takes these arcs to pass the horizon). We also discuss the implication of this knowledge on our understanding of the development of the Babylonian astronomy.

1. Introduction on the Lunar Six

The 'Lunar Six' are some characteristic time intervals between sunrise or sunset and moonset or moonrise. These time intervals are very easy to observe: the Babylonians, indeed, recorded them regularly during the last six centuries B.C. as can be seen from the 'Diaries', the compilations of their observed data. In Figure 1 the phenomenon *KUR* is illustrated in detail as follows. The horizontal (thin) great circle is the horizon, the (thick) oblique circle is the celestial equator (as seen from Babylon), and the dotted great circle is the ecliptic. We consider a morning shortly before new moon. The sun and the moon are placed somewhere on the ecliptic near the eastern horizon; thus we have neglected the latitude of the moon. The arc of the ecliptic between moon and sun may be around 20°; the moon has thus risen visibly about $1\frac{1}{2}$ hours before sunrise. On the next morning, however, the moon will be so close to the sun that the moonrise is invisible. The time difference between this last visible moonrise (before conjunction) and the sunrise is called *KUR*. One might think that this time difference is measured by the arc of the ecliptic between moon and sun; but

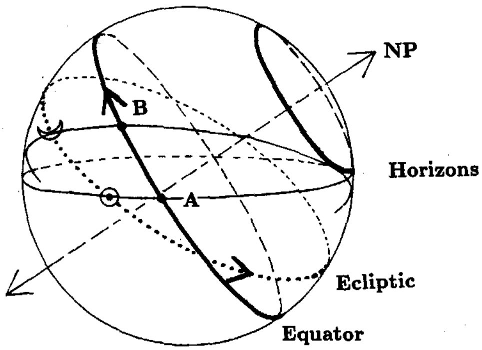

Figure 1. The celestial sphere for Babylon.

actually, times are measured on the equator and therefore we have to find the points *A* and *B* on the equator that rise at the same time as the sun and the moon, respectively. These points can be found by drawing great circles passing through the moon and the sun and being tangent to the greatest of the always visible small circles. (For drawing the celestial sphere in this way, see Olaf Schmidt [1994].) The observable *KUR* is the rising time of the elongation arc ☽☉. It is given by the length of the arc *AB* and depends on where the alongation arc ☽☉ is placed on the ecliptic and also upon the length of arc ☽☉.

KUR is one of the Lunar Six. The others of the Lunar Six, called *NA, ŠÚ, ME, NA,* and *GE* by the Babylonians,[1] are defined similarly. For completeness, let us repeat here their definitions.

Around conjunction, two characteristic time differences can be observed:

KUR=time interval between moonrise and sunrise measured on the last morning where the moon is still visible before conjunction.

NA_N=time interval between sunset and setting of the new moon crescent on the first evening where the moon is visible again after conjunction[1].

Around opposition, four characteristic time differences were regularly observed by the Babylonians. We call them in the following the 'Lunar Four':

$\check{S}\acute{U}$ = time interval from last moonset to sunrise before opposition,
NA = time interval from first sunrise after opposition to moonset[1],
ME = time interval from last moonrise to sunset before opposition,
GE = time interval from first sunset after opposition to moonrise.

Although spectacular and easy to observe, these time intervals are very complicated quantities from a theoretical point of view. Neugebauer [1957, pp. 107–109] describes in detail the factors which determine whether the new moon crescent can be seen after sunset on the evening after new moon (conjunction). The same factors also decide for how long time the new moon can be seen in the evening of its first visibility. This time interval is NA_N. The others of the Lunar Six are correspondingly determined by similar factors.

In Sections 5 and 6, the Lunar Four will be studied in more detail. Here we just mention that each of the Lunar Six strongly depends upon four variables:

1) the time interval Δt between syzygy (conjunction or opposition) and sunrise or sunset respectively:
2) the momentaneous moon velocity $v_☾$;
3) the moon's position $\lambda_☾$ in the ecliptic at the moment of the syzygy; and
4) the momentaneous latitude $\beta_☾$ of the moon.

In the case of the Lunar Four, Δt is the positive time interval between opposition (t_{op}) and sunrise (t_{sr}) or sunset (t_{ss}), respectively, so that we have

$$\Delta t = t_{op} - t_{sr} \quad \text{for} \quad \check{S}\acute{U}, \qquad \Delta t = t_{sr} - t_{op} \quad \text{for} \quad NA,$$
$$\Delta t = t_{op} - t_{ss} \quad \text{for} \quad ME, \qquad \Delta t = t_{ss} - t_{op} \quad \text{for} \quad GE\,. \qquad (1)$$

Taking $\check{S}\acute{U}$ as an example, we can thus write it as a function of four variables:

$$\check{S}\acute{U} = f(\Delta t, v_☾, \lambda_☾, \beta_☾) \qquad (2)$$

and similarly for *NA, GE,* and *ME*, using the corresponding definitions (1) of Δt. In Figure 1 we saw that *KUR* is the rising time of a small arc of the ecliptic, say **e**, and that the magnitude of *KUR* is determined by **e**'s position in the ecliptic and by the length of **e**. Similarly for the other Lunar Six. In all cases, the length of **e**, the relevant little ecliptic arc is determined by Δt and $v_{☾}$, whereas the position of **e** is given by our variable $\lambda_{☾}$.

Due to the complexity of these dependencies, it was silently assumed for a long time that the Lunar Six were of no practical or theoretical use – at least, nobody proposed how they could have been used by the Babylonians to develop their astronomical calculation schemes. The ephemeris and procedure texts stemming from the Seleucid era, however, demonstrate that during the last three centuries B.C., the Babylonians were able to calculate and predict the magnitude of the Lunar Six. We strongly agree with O. Neugebauer [1957] who writes: 'It is one of the most brilliant achievements in the exact sciences of antiquity to have recognized the independence of these influences and to develop a theory which permits the prediction of their combined effects.'

The fact that the Babylonian astronomers were able to calculate the Lunar Four[2] by a skillful combination of these influences gives us a hint that they might as well have been able to do the reverse, i.e. to separate out the different influences from the observations. We are convinced that they really did do so, namely by simple combination of the Lunar Four data, and that their column Φ was constructed from observations of the Lunar Four. In our search for genuine Babylonian observations which possibly could have been used for constructing the column Φ, we succeeded in showing that Φ can, indeed, be derived from the Lunar Four [Brack-Bernsen 1990]. This implies that the Babylonian function Φ was found empirically, and not deduced from theoretical considerations. It also provides for the first time a proposal how the Babylonians might have used their abundant observation material of the Lunar Four.

In this paper we will explain how this is possible. We will show that by simple addition of the Lunar Four, the influence of some of the variables can be eliminated and that of others strongly reduced. By taking the sum *ŠÚ*+*NA* or *ME*+*GE*, the dependence on $\triangle t$ is eliminated and the dependence on $\beta_☾$ is strongly reduced: *ŠÚ+NA* as well as *ME+GE* are independent of $\triangle t$ and almost independent of $\beta_☾$; they depend heavily upon $\lambda_☾$ and less upon $v_☾$. By adding *ŠÚ+NA* and *ME+GE*, the dependence on $\lambda_☾$ is drastically reduced: Σ=*ŠÚ+NA+ME+GE* depends mostly on $v_☾$ and less on $\lambda_☾$. This explains why the sum Σ of the Lunar Four as a function of the lunation number varies periodically concurrently with $v_☾$.

2. *Empirical information contained in the Lunar Four*

The uppermost curve of Figure 2 demonstrates graphically the complexity of one of the Lunar Four. We have chosen *ŠÚ* as an example and, inspired by the Babylonians who calculated or observed characteristic moon phenomena once each synodic month, we have calculated the observable *ŠÚ* during 60 consecutive synodic months. These values of *ŠÚ* are marked in Figure 2 by crosses ($\times$), connected by straight lines, and shown as function of the lunation number *L*. We remark how *ŠÚ* varies rather chaotically and seemingly without any regularity. It is practically impossible to extract any information from *ŠÚ* by simple inspection of this curve or, as the Babylonians might have done it, by inspection of the correspondingly tabulated values of *ŠÚ*. This is not surprising since we know that *ŠÚ* is a complicated function of four variables. Similar pictures would be obtained by plotting the other Lunar Four in the same way.

It turns out, however, that the influence of the different variables on which the Lunar Four depend can be effectively reduced if some of them are combined. In Figure 2 we also show the sums *ŠÚ+NA*, *ME+GE* and the sum of all of the Lunar Four: Σ= *ŠÚ+NA+ME+GE*. They all show a much more regular behaviour than *ŠÚ*, indicating that some of the dependencies on the variables $\triangle t$, $v_☾$, $\lambda_☾$ and $\beta_☾$ have been partially eliminated by simple addition of the Lunar Four values.

Figure 3 presents a compilation of the same four quantities *ŠÚ*, *ŠÚ+NA*, *ME+GE* and Σ, now taken over a period of 450 lunations. The curves of the sums *ŠÚ+NA* and *ME+GE* are very

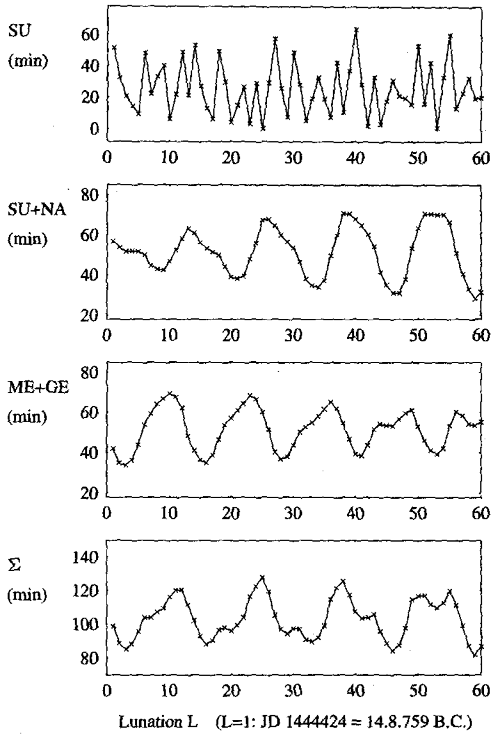

Figure 2. Different horizontal phenomena derived from the 'Lunar Four' as functions of the lunation number L, calculated for Babylon over a period of 60 months (starting on JD 1444424=Aug. 14, 759 BC). From top to bottom: *ŠÚ*, *ŠÚ+NA*, *ME+GE* and Σ.

similar. They oscillate rapidly with a mean period of ca. 12; 22 synodic months $\approx P_{\odot}$, the length of the solar year, and their amplitudes vary slowly with a period $D \approx 109.5$ synodic months, which is the revolution time of the moon's apside line in the ecliptic. Oscillating functions of this type have been discussed in an older publication [Brack-Bernsen 1980]. They result from a superposition of two oscillating functions with the periods $P_{☾}$ and $P_{\odot}$, respectively. In the case of *ŠÚ+NA* and *ME+GE*, the dominating term has the period $P_{\odot}$, reflecting the variable $\lambda_{☾}$ with this period; the influence of the variable $v_{☾}$ is being seen through the variation of the amplitudes.[3] The curve Σ, on the other hand, oscillates with

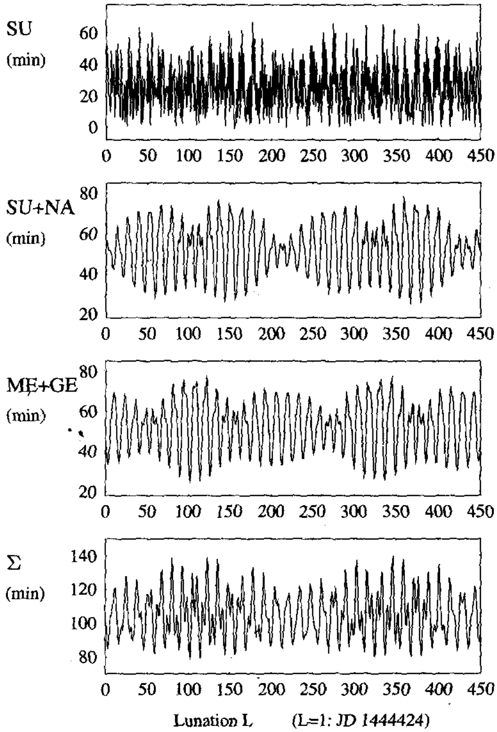

Figure 3. The same phenomena as in Figure 2, now calculated over a period of 450 synodic months.

a mean period of about 14 synodic months which is the period $P_☾$. This shows us that Σ depends most strongly on $v_☾$, while only a minor influence of $\lambda_☾$ is seen by the variation of its amplitude. The smaller irregularities in these three sum curves show that they are still influenced by other factors than those with the periods $P_\odot$ and $P_☾$.

3. *Reconstruction of column Φ*

The fundamental quality of column Φ consists of its period P_Φ which, with a surprisingly high accuracy, equals the period $P_☾$ of the moon velocity $v_{☾}$ as observed once each synodic month. Within system A of the Babylonian moon ephemerides, all calculated quantities depending on the moon velocity $v_{☾}$ are derived from column Φ, taking over the exact period P_{Φ} from column Φ. The fundamental question therefore was: from where did the Babylonians know the exact period relation which is built into their column Φ?

In searching for the origin of Φ, we had to look for observations which eventually contain information about $v_{☾}$ and concentrated on the Lunar Four. As already stated above, these are very complicated quantities. One of the disturbing factors stems from the fact that sun and moon both move along the ecliptic, while time and hence also the Lunar Four time intervals are measured along the equator. This fact is called the 'oblique ascension'. At the latitude of Babylon, the angle between the equator and the horizon equals 57.5°, whereas the angle between the ecliptic and the horizon varies between 34° and 81°. Therefore, depending upon its position in the ecliptic, the rising time of a 10° arc of the ecliptic varies between 6°;45 and 13°;15.

The basic idea behind the proposed reconstruction of column Φ was to reduce the influence of the oblique ascension (i.e., the dependence on $\lambda_{☾}$) by combining observations on the eastern with observations on the western horizon. This turned out to be possible: as we have seen in Figure 3, the sum Σ of the Lunar Four does indeed oscillate with the very period $P_{☾}$ we are looking for. (This indicates that Σ depends most strongly on $v_{☾}$ and less than each of the Lunar Four upon $\lambda_{☾}$, and also much less upon Δt and $\beta_{☾}$.) Indeed, it was shown [Brack-Bernsen, 1990] that the sum Σ of the Lunar Four:

$$\Sigma = \check{S}\acute{U}+NA+ME+GE \qquad (3)$$

varies with the same period and amplitude as Φ. We therefore proposed the hypothesis: *Column Φ is derived from the sum Σ of the Lunar Four.*

Recently, using a more accurate computer code enabling us to calculate lunar phenomena at ancient times [Moshier 1992], it was shown [Brack-Bernsen 1994] that also the phases of the calculated Σ and the Babylonian Column Φ were exactly the same in the

Seleucid time. This is illustrated in Figure 4 which shows Σ over a period of 260 synodic months starting January 23, 146 B.C., compared with the Babylonian zig-zag function Φ (dashed line) for the same time period. The accordance between the 'theoretical' curve and the Babylonian function Φ is optimal. Note, in particular, that the phase between Σ and Φ, i.e. their position along the time (L) axis, had *not* been adjusted. We take this as a further convincing support of our hypothesis that Φ is derived from Σ.

We have also demonstrated [Brack-Bernsen 1994] that old Babylonian observations, as found in the text Cambyses (523 B.C.) and in the Goal-Year texts (300–50 B.C.), show the right structure and accuracy which is necessary and sufficient for the construction of Σ and thus of Φ. The Goal-Year texts also explicitly list the sums *ŠÚ+NA* and *ME+GE:* a sign that the Babylonians, indeed, were interested in partial sums of the Lunar Four. We have found no tablets with the sum Σ of all the Lunar Four, but we think it is very probable that the old astronomers went one step further and also added *ŠÚ+NA* and *ME+GE*.

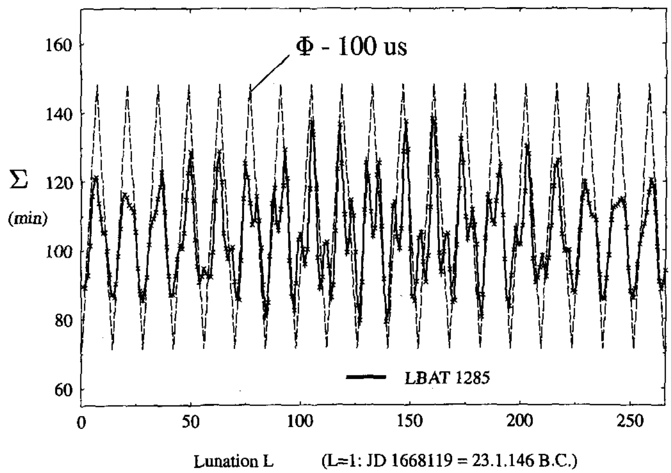

Figure 4. The sum Σ=*ŠÚ+NA+ME+GE* of the Lunar Four as function of the lunation number L (×connected by thick lines), calculated for Babylon over a period of 260 synodic months (starting on JD=1668119=Jan. 23, 146 BC). The thin dashed line shows the Babylonian zig-zag function $\Phi-100$ *uš* determined for this time interval.

In the remainder of this paper we shall examine the partial sums *ŠÚ*+*NA* and *ME*+*GE* and the sum Σ in more detail, interpret them astronomically and explain why the influence of some of the variables affecting the Lunar Four can be reduced by their addition.

4. Observation of the Lunar Four

In order to get a better understanding of the Lunar Four, let us reflect upon when, how and where the Lunar Four were observed (see also Neugebauer, ACT I, pp. 229–239). We are for a moment neglecting the latitude of the moon and assume it to move along the ecliptic. We consider one full moon and assume the opposition to take place some time after sunset of a day *N*.[4]

We then have the situation indicated along the time axis in Figure 5. In the morning of day *N*, *ŠÚ* can be observed: the moon sets at the western horizon and shortly afterwards the sun rises at the eastern horizon. *ŠÚ* is the time difference between these two events, measured in time degrees[5] *uš*. Similarly, *ME* can be observed in the evening of the day *N*. Towards the end of day *N*, the opposition takes place. Thereafter, *NA* and *GE* can be observed in the morning *N*+1 and in the evening *N*+1, respectively. We see that the observations of the Lunar Four take place within a time span of about one and a half day around full moon.

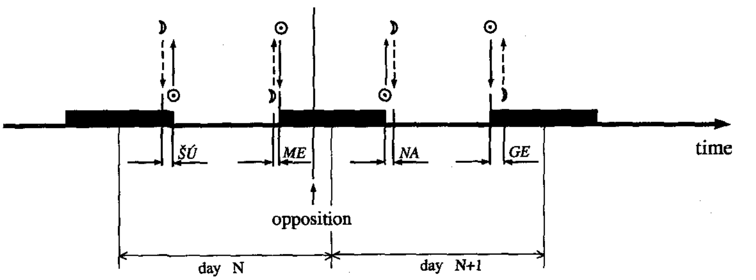

Figure 5. Times of opposition, sunrise, sunset, moonrise and moonset on the two days around opposition, marked along the time axis. Also indicated are the 'Lunar Four' time intervals *ŠÚ, ME, NA* and *GE.* (In this figure we have assumed the opposition to take place some time before midnight. In a case where the opposition takes place during daytime, the Lunar Four will occur in a different order, e.g. *ME, ŠÚ, GE, NA.*)

The Lunar Four are concerned with the sun at the horizon in one direction (east or west) and the moon in the opposite direction (west or east, respectively). In the following figures, we introduce for the sake of simplicity the symbol $\overline{\odot}$ for the 'antisun', which we define as the point on the ecliptic situated directly opposite the sun. At the very moment when the sun rises, $\overline{\odot}$ sets, and vice versa. With this definition, *ŠÚ* is the time difference between the setting of the moon ☽ and the antisun $\overline{\odot}$. Similarly, *ME* is the time difference between moonrise and antisun rise, *NA* is the time difference between antisun set and moonset, whereas *GE* is the time difference between antisun rise and moonrise.

Figure 6 illustrates the position of the moon relatively to the antisun $\overline{\odot}$ at these times (i.e., at moonsets and moonrises on the days N and $N+1$, respectively): it shows the distance (measured along the ecliptic) between the moon and $\overline{\odot}$ at the times when the Lunar Four are observed. (Remember that the opposition takes place at the moment when the moon passes the point $\overline{\odot}$.)

These distances or elongations are important for our understanding of the Lunar Four; we shall name them $\mathbf{e}_{SU}$, $\mathbf{e}_{ME}$, $\mathbf{e}_{NA}$ and $\mathbf{e}_{GE}$. They correspond to arc ☽$\odot$ which defines *KUR* in Figure 1 and tell us how far the moon has moved relatively to the sun in the time between opposition and the observation of the particular Lunar Four. Evidently, they depend upon the relative moon velocity $(v_{☾} - v_{\odot})$ and upon the time t_{op} at which the opposition takes place relatively to sunset t_{ss} or sunrise t_{sr}. We can take *NA* as an

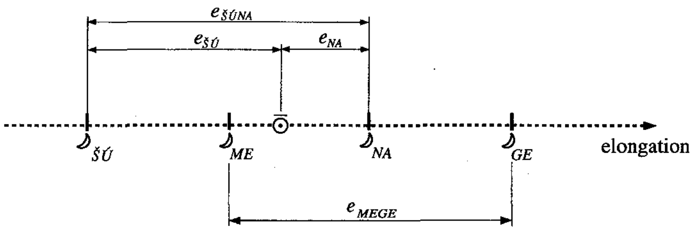

Figure 6. Positions of the moon ☽ relatively to the 'antisun' $\overline{\odot}$ at the times where the lunar four *ŠÚ, ME, NA* and *GE* are measured. ($\mathbf{e}_{SU}$, $\mathbf{e}_{ME}$, etc. denote the corresponding elongations of the moon.) The moon moves along the ecliptic from left to right; at opposition it passes the point $\overline{\odot}$.

example for all: knowing that *NA* is observed at the time of the sunrise, t_{sr}, we have:

$$\mathbf{e}_{NA} = (v_{☾} - v_{\odot}) \times (t_{sr} - t_{op}) \ .$$

The first term $(v_{☾} - v_{\odot})$ can vary between 10° per day and 14° per day; the variation of the second term Δt is much larger, namely from 0^h to 24^h, depending on whether the opposition takes place just after or before the sunrise. We can thus estimate $\mathbf{e}_{NA}$ to vary between 0° and 14°. Similarly, the other elongations are defined as

$$\mathbf{e}_{SU} = (v_{☾} - v_{\odot}) \times (t_{op} - t_{sr}) \ ,$$
$$\mathbf{e}_{ME} = (v_{☾} - v_{\odot}) \times (t_{op} - t_{ss}) \ ,$$
$$\mathbf{e}_{GE} = (v_{☾} - v_{\odot}) \times (t_{ss} - t_{op}) \ .$$

Let us now imagine what is going on at the western horizon when *ŠÚ* and *NA* are observed. The left half of Figure 7 shows us the western horizon (by the horizontal line) at sunrise on the last morning *N* before opposition (full moon), i.e. at the time when *ŠÚ* is measured. The dotted oblique line indicates the ecliptic at the western horizon on this morning; the arrow indicates the directions in which sun and moon are moving. In this figure, the ecliptic passes the horizon at a low angle of ~34°; this happens when the full moon occurs near the spring equinox. Had it taken place near the autumn equinox, the angle would have been steep, about 81°.

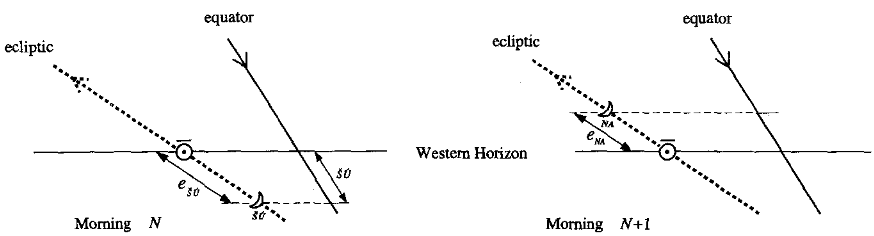

Figure 7. *Left half:* Position of moon and antisun on the western horizon at the moment of sunrise on the morning *N*. The moon has already set and the antisun is about to set. *Right half:* The situation on the next morning *N*+1: here the moon will set after the antisun.

The imaginary point $\overline{\odot}$ sets at the moment of sunrise, and through that very point on the ecliptic the moon passes at opposition. The solid oblique line indicates the equator at the western horizon, the arrow showing us the direction of the daily revolution of the sky. It is along this line that the times are measured.

The Babylonian observable *ŠÚ* is the time it takes the ecliptic arc $\mathbf{e}_{SU}$ between $☽_{SU}$ and $\overline{\odot}$ to set. This setting time is visualized in the figure, it depends on the length of $\mathbf{e}_{SU}$ and upon how steep the ecliptic stands at the western horizon. The angle between ecliptic and horizon at the moment when $\mathbf{e}_{SU}$ is setting depends on where in the ecliptic this arc is situated. Its position in the ecliptic is determined by the position $\lambda_{☾}$ in the ecliptic where the opposition takes place.

We have thus learned: *ŠÚ* is determined by $\Delta t = t_{op} - t_{sr}$, $v_{☾}$, and $\lambda_{☾}$ (and on the latitude $\beta_{☾}$ of the moon which we have neglected so far). Hereby Δt is the dominating variable.

The right part of Figure 7 shows the situation on the next morning $N+1$ when *NA* is observed. The moon now has passed the antisun $\overline{\odot}$, the opposition has taken place. *NA* is the time it takes the ecliptic arc $\overline{\odot}☽_{NA} = \mathbf{e}_{NA}$ to pass the western horizon. Analogously to the case of *ŠÚ*, we therefore get: *NA* is determined by $\Delta t = t_{sr} - t_{op}$, $v_{☾}$, and $\lambda_{☾}$ (and on the neglected lunar latitude $\beta_{☾}$).

Similar considerations of the rising full moon on the evenings before and after opposition will lead to an analogous understanding of *ME* and *GE* as the rising times of the ecliptic arcs $\mathbf{e}_{ME}$ and $\mathbf{e}_{GE}$ (again measured in time degrees).

We shall next examine the sums *ŠÚ+NA* and *ME+GE* in order to understand why they show a much more regular behaviour than each of the Lunar Four. Later, Figure 10 will show us how the Lunar Four depend heavily on $\beta_{☾}$, whereas its influence on the sums *ŠÚ+NA* and *ME+GE* is quite small.

5. *The astronomical significance of ŠÚ+NA and ME+GE*

We examine *ŠÚ+NA* as an example, since the structure of *ŠÚ+NA* and *ME+GE* in principle must be the same. Figure 8 summarizes, combining the left and the right halves of Figure 7, the situation on the western horizon on the two mornings *N* and $N+1$. Since we are concerned with the relative positions of sun

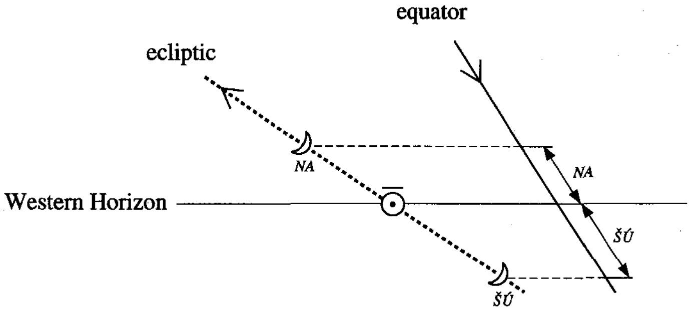

Figure 8. Position of moon and antisun at the western horizon on the two mornings N and $N+1$ around full moon at which $\check{S}\acute{U}$ and NA are measured. The opposition is assumed to take place before midnight of night N.

and moon, we have neglected the motion of the sun (and hence also of the antisun) during the time from morning N to morning $N+1$. The sum $\check{S}\acute{U}+NA$ is the setting time of the ecliptic arc between $☽_{SU}$ and $☽_{NA}$. This arc is, of course, the sum of $\mathbf{e}_{SU}$ and $\mathbf{e}_{NA}$, which we shall call $\mathbf{e}_{SUNA}=\mathbf{e}_{SU}+\mathbf{e}_{NA}$. Its length equals the elongation movement of the moon in the time between morning N and morning $N+1$. (By elongation movement we understand the movement of the moon relatively to the sun.) But this elongation movement does not depend on the time at which the opposition took place. We have thus seen that by the addition of $\check{S}\acute{U}$ and NA, the variable Δt is eliminated. The elongation movement of the moon, $\mathbf{e}_{SUNA}$, only depends on the relative moon velocity $(v_{☾}-v_{\odot})$ at the day of opposition. This relative velocity is felt over a time period of slightly more than a day. The observation of $\check{S}\acute{U}+NA$ starts $\check{S}\acute{U}$ time degrees before sunrise on day N and ends NA time degrees after sunrise on day $N+1$. This time interval equals thus $1+(\check{S}\acute{U}+NA)/360$ days. Hence we obtain:

$$\mathbf{e}_{SUNA} = (v_{☾}-v_{\odot})\,[1+(\check{S}\acute{U}+NA)/360] \ ,$$

where the velocity of moon and sun are measured in degrees per day. Since $v_{\odot}=1°/\text{day}$, we get:

$$\mathbf{e}_{SUNA} = (v_{☾}-1°/\text{day})\,[1+(\check{S}\acute{U}+NA)/360] \ .$$

The time which it takes $\mathbf{e}_{SUNA}$ to set only depends on its position along the ecliptic which is given by the longitude $\lambda_☾$ of the moon at opposition. We have thus seen that *ŠÚ+NA*, contrarily to *ŠÚ* or *NA*, is independent of the time of day when the opposition takes place.

Figure 9 shows the same phenomena as Figure 8, but for a situation where the opposition takes place in the morning shortly after sunrise whereas in Figure 8, it occurred before midnight. The comparison between Figures 8 and 9 clearly demonstrates that, although the single intervals *ŠÚ* and *NA* depend on the time of opposition, their sum *ŠÚ+NA* remains the same.

We now have to investigate the influence of the lunar latitude $\beta_☾$ which we have neglected so far. This is illustrated in Figure 10, where we show two situations. The first, drawn with thin lines, repeats the case of Figure 8 where the moon moves on the ecliptic (dotted line) with the latitude $\beta_☾=0$. The second case, drawn by the parallel solid line, shows the trajectory of the moon for a latitude of $\beta_☽=+3.3°$. We note that *ŠÚ* and *NA* (measured along the equator!) both change appreciably. However, their sum *ŠÚ+NA* remains the same for elementary geometrical reasons. Thus we have learned: Each single of the Lunar four depends strongly on the latitude $\beta_☾$ of the moon. This latitude varies between $+5°$ and $-5°$, so that each of the Lunar Four reflects a total latitude variation of 10°. By taking the sums *ŠÚ+NA* or *ME+GE*, the influ-

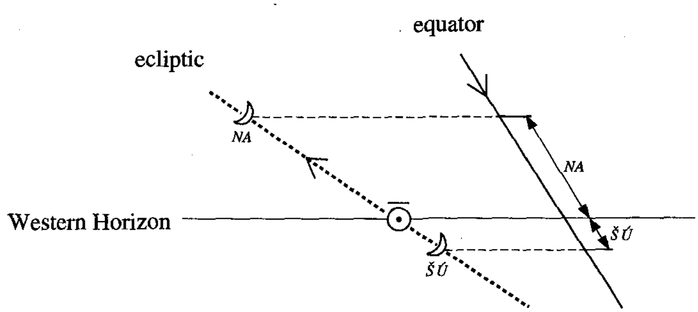

Figure 9. Position of moon and antisun at the western horizon on the two mornings *N* and *N*+1 around full moon at which *ŠÚ* and *NA* are measured. The opposition is assumed to take place shortly after sunrise on day *N*.

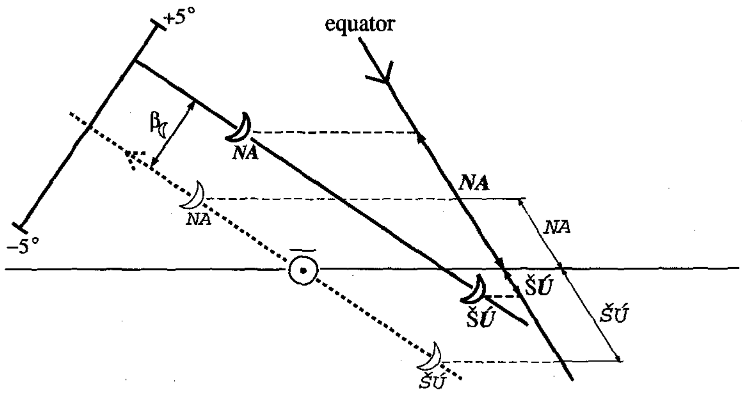

Figure 10. The same situation as in Figure 8 ($\beta_☾=0$) with a dotted line for the ecliptic, here in comparison to the situation where the moon has a positive latitude of $\beta_☾=3.3°$ and moves on the trajectory parallel to the ecliptic which is drawn by a solid line.

ence of the varying $\beta_☾$ is strongly reduced. In the time between the observations of *SU* and *NA* (or of *ME* and *GE*), the latitude of the moon cannot have changed by more than ~1°; only this variation of ~1° in latitude will contribute to the variation of *ŠÚ+NA* or *ME+GE*.

We summarize: *ŠÚ* and *NA* are strongly depending on Δt, $\beta_☾$, $\lambda_☾$ and $v_☾$. By addition of *ŠÚ* and *NA*, the variable Δt is eliminated and the influence of the variable $\beta_☾$ is strongly reduced. This explains the fact that the sum *ŠÚ+NA* in Figures 2 and 3 forms a nice and regular curve, whereas *ŠÚ* alone shows a very irregular and unpredictable behaviour.

The intervals *ME, GE* and their sum *ME+GE* can be treated completely analogously. We saw that *ŠÚ+NA* is the setting time of the ecliptic arc $\mathbf{e}_{SUNA}$. An investigation of the moon risings on the eastern horizon on the days *N* and *N*+1 will show that *ME+GE* is the rising time of $\mathbf{e}_{MEGE}$, an ecliptic arc situated around $\lambda_☾$, the length of which equals the elongation movement of the moon during the time between moonrise on day *N* and moonrise on day *N*+1.

6. The sum Σ of ŠÚ+NA and ME+GE

We have just seen: *ŠÚ+NA* is the setting time of $\mathbf{e}_{SUNA}$, an ecliptic arc situated around $\lambda_☾$ with a length equal to the elongation movement of the moon during the time between moonset on morning *N* and moonset on morning *N*+1. Analogously, *ME+GE* is the rising time of $\mathbf{e}_{MEGE}$, an ecliptic arc around $\lambda_☾$ corresponding to the elongation movement of the moon during the time between moonrise on evening *N* and moonrise on evening *N*+1. These two ecliptic arcs will, in a good approximation, have the same length and be situated almost at the same place of the ecliptic. Without committing an error worth mentioning, we can identify these two ecliptic arcs and replace them by $\mathbf{e}_☾$:

$$\mathbf{e}_☾ \simeq \mathbf{e}_{SUNA} \simeq \mathbf{e}_{MEGE} .$$

We have thus defined $\mathbf{e}_☾$ as the ecliptic arc situated symmetrically around $\lambda_☾$ with the length of the elongation movement of the moon on the day of opposition or, to be more precise, of the elongation movement of the moon during the time of one day plus (*ŠÚ+NA*)/360 ≃ one day plus (*ME+GE*)/360. In a good approximation, *ŠÚ+NA* and *ME+GE* are thus the setting and rising times of one and the same ecliptic arc $\mathbf{e}_☾$. Consequently, Σ is the sum of these times, i.e. the rising time of $\mathbf{e}_☾$ plus its setting time. This fact is crucial in our understanding of Σ in combination with our knowledge of the oblique ascension.

7. The problem of oblique ascension

Olaf Schmidt [1994, chapter III] treats the problem of the oblique ascension with modern methods. The interested reader who may want further detailed information is referred to this publication. For our present purpose we need the two important results mentioned in the following.

The first result is summarized in Figure 11, where the arc *AB* is part of the ecliptic and arc (*ADCE*) is part of the celestial equator. The yearly revolution in the ecliptic is indicated by an arrow, and

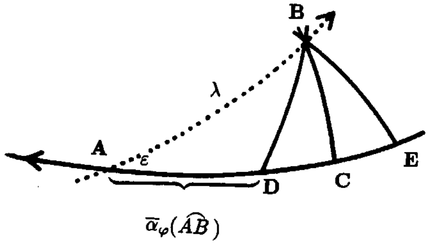

Figure 11. Enlarged part of the celestial sphere for a place whose geographical latitude equals φ=32.5°. It shows the rising and setting arcs *AD* and *AE*, respectively, of the ecliptic arc *AB*.

the daily revolution in the equator is similarly indicated by an arrow. The angle between the equator and the ecliptic is ε=24°. The geographical latitude of Babylon is φ=32.5° and therefore the angle between the equator and the horizon at Babylon is equal to 90°$-\varphi$=57.5°. In ancient astronomy, the geographic latitude of a place not on the terrestrial equator is called *sphaera obliqua.* We have drawn two positions of the horizon, namely the eastern and western horizon. Arc *BD* is part of the eastern horizon showing the situation when the ecliptic point *B* is about to rise. Arc *BE* is part of the western horizon. The arc *AB* of the eclitpic thus rises during the same time as the arc *AD* of the equator. We call arc *AD* the rising arc of arc *AB* and write:

$$AD = \bar{a}_\varphi(AB) \ ,$$

where φ is the geographic latitude of Babylon (see Figure 11). Similarly, we call arc *AE* the setting arc of arc *AB* and write

$$AE = \underline{a}_\varphi(AB) \ .$$

At the terrestrial equator, where the geographic latitude φ=0°, the angle between the horizon and the celestial equator is 90°, and this place is called *sphaera recta.* The horizon at *sphaera recta* is arc *BC* (see Figure 11). At *sphaera recta* the ecliptic arc *AB* rises at the same time as arc *AC*, and we write

$$AC = \overline{a}_0(AB) \ .$$

It also holds that AC is the setting arc of AB:

$$AC = \underline{a}_0(AB) \ .$$

We therefore write

$$AC = a(AB) \ .$$

In Figure 11 the spherical triangle DBE is equilateral because angle (CDB)=angle(BEC)=57.5°. Therefore

$$\text{arc } DC = \text{arc } CE \ .$$

and hence

$$\text{arc } AD + \text{arc } AE = 2\text{arc } AC$$

or

$$\overline{a}_\varphi(AB) + \underline{a}_\varphi(AB) = 2a(AB) \ .$$

Expressed in words: for any arc AB of the ecliptic with fixed end point in the vernal equinox A, the sum of the rising and setting times at *sphaera obliqua* equals twice its rising (or setting) time at *sphaera recta.* This is also true for any other arc B_1B_2 of the ecliptic. This can bee seen in the following way: the arbitrary arc B_1B_2 can by found as the difference between the two arcs of the ecliptic, AB_1 and AB_2, both having the vernal equinox A as end point.

$$B_1B_2 = AB_2 - AB_1$$

For these two arcs we know that:

$$\overline{a}_\varphi(AB_2) + \underline{a}_\varphi(AB_2) = 2a(AB_2) \ ,$$

$$\overline{a}_\varphi(AB_1)+\underline{a}_\varphi(AB_1) = 2a(AB_1) \ . \tag{4}$$

Now the rising time of arc B_1B_2 – let us call it $\overline{a}_\varphi(B_1B_2)$ – equals the rising time of AB_2 minus the rising time of arc AB_1. For the setting times, the analogous equation is true:

$$\begin{aligned} \overline{a}_\varphi(B_1B_2) &= \overline{a}_\varphi(AB_2)-\overline{a}_\varphi(AB_1) \\ \underline{a}_\varphi(B_1B_2) &= \underline{a}_\varphi(AB_2)-\underline{a}_\varphi(AB_1) \ . \end{aligned} \tag{5}$$

By addition of the two equations (5) we get:

$$\begin{aligned} \overline{a}_\varphi(B_1B_2)+\underline{a}_\varphi(B_1B_2) = {} & [\overline{a}_\varphi(AB_2)+\underline{a}_\varphi(AB_2)] \\ & - [\overline{a}_\varphi(AB_1)+\underline{a}_\varphi(AB_1)] \ . \end{aligned}$$

The combined use of Eq. (4) and Eq. (5) gives us the desired result:

$$\begin{aligned} \overline{a}_\varphi(B_1B_2)+\underline{a}_\varphi(B_1B_2) &= 2a(AB_2)-2a(AB_1) \\ &= 2a(B_1B_2) \ . \end{aligned} \tag{6}$$

In words: for any arc B_1B_2 of the ecliptic, the sum of rising and setting times at *sphaera obliqua* equals twice its rising (or setting) time at *sphaera recta*. In particular this is true for arc $\mathbf{e}_{☾}$, the elongation arc of the moon on the day of opposition. We therefore now know that Σ equals twice the rising time at *sphaera recta* of this arc $\mathbf{e}_{☾}$.

The second result in O. Schmidt [1994, chapter III] gives a handy graphic method, shown in Figure 12, for finding the rising time of a given arc of the ecliptic at *sphaera recta* (curve A) and at *sphaera obliqua* (curve C). The Curve C deals with the geographic latitude φ=32.5° of Babylon: We find the rising time of an arbitrary arc of the ecliptic B_1B_2 as the area bounded by the line segment B_1B_2, the vertical lines through B_1 and B_2 and the curve C. Thus the rising time of a minor arc of the ecliptic, say of e°, placed in ♈ is much smaller than the rising time of an ecliptic arc of e° located e.g. in ♋. In the same way, curve A gives the rising time of an arc of the ecliptic at *sphaera recta.* We notice that the variation in the rising time of an arc of the ecliptic is "smaller" at *sphaera recta* and "larger" at *sphaera obliqua* (cf. the amplitudes of A and C in Figure 12).

8. *The astronomical significance of Σ*

We have seen in Sect. 6 that Σ is the sum of the setting and rising times of $\mathbf{e}_{☾}$. (We remind the reader that $\mathbf{e}_{☾}$ is an ecliptic arc situ-

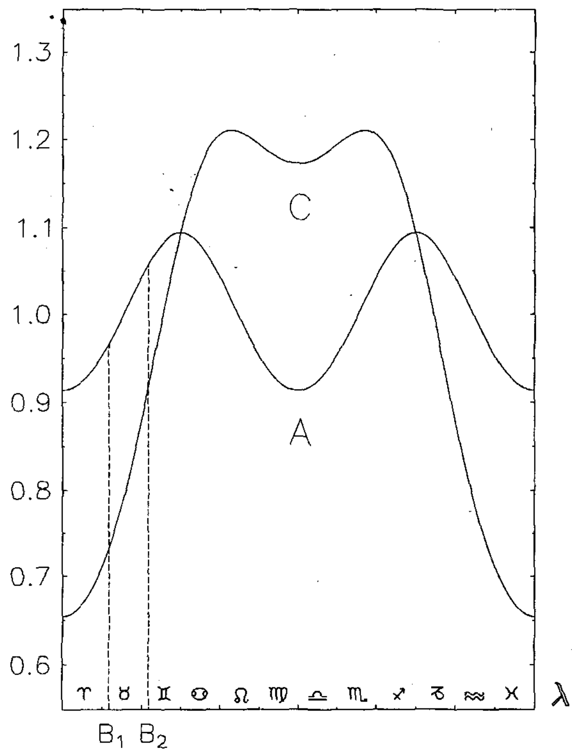

Figure 12. Graphical method of finding the rising time of a given arc of the ecliptic at *sphaera recta* (curve A) and at *sphaera obliqua* with φ=32.5° (curve C). The rising time of an arc B_1B_2 is equal to the area bounded by the line segment B_1B_2, the vertical lines through B_1 and B_2 and the curve A or C, respectively.

ated around the position $\lambda_{☾}$ in the ecliptic at which the opposition takes place, with a length that equals the elongation motion of the moon during the day of opposition). Let us briefly call this ecliptic arc $\mathbf{e}_{☾}$ the daily elongation arc of the moon. In Sect. 7 we have seen that the sum of the rising and setting times of an ecliptic arc equals twice its setting time at *sphaera recta.*

Combining these two findings, we now have found the astronomical significance of Σ: it equals twice the setting time at *sphaera recta* of the daily elongation arc of the moon on the day of opposition:

$$\Sigma = \overline{a}_{\varphi}(\mathbf{e}_{☾})+\underline{a}_{\varphi}(\mathbf{e}_{☾}) = 2a(\mathbf{e}_{☾}) \ . \tag{7}$$

Σ is twice the time it takes $\mathbf{e}_{☾}$ to rise (or to set) when measured from the earth's equator

Knowing the astronomical significance of Σ, we can show numerically that the lunar velocity $v_{☾}$ is the dominating variable in the sum Σ. The influence of $\lambda_{☾}$ is much smaller than the influence of $v_{☾}$. This explains why Σ oscillates with the same period as the lunar velocity. And we can show that in case of *ME+GE*, which is the time it takes $\mathbf{e}_{☾}$ to rise when measured from Babylon, $\lambda_{☾}$ is the dominating variable and $v_{☾}$ has much less influence. Of course, the same is true for *ŠÚ+NA*. Both *ME+GE* and *ŠÚ+NA* oscillate with the period $P_{\odot}$.

The lunar velocity $v_{☾}$ determines the length e of the ecliptic arc $\mathbf{e}_{☾}$, whereas its position in the ecliptic is determined by $\lambda_{☾}$.

The variation due to $v_{☾}$: The lunar velocity varies between 11.8°/day and 15.3°/day[6]; therefore the length e of $\mathbf{e}_{☾}$ will vary between 10.8° and 14.3°; a variation of 14.3°−10.8°=3.5°. Therefore, the variation of Σ due to $v_{☾}$ is 2×3.5°=7°, and the variation of *ME+GE* due to $v_{☾}$ is 3.5°.

The variation due to $\lambda_{☾}$: Let us assume $\mathbf{e}_{☾}$ to be of constant length: $\mathbf{e}_{☾}=12°$; this is the same as assuming the lunar velocity to be constant and equal its mean value. The curves A and C give us an estimate of the variation in rising time of $\mathbf{e}_{☾}$ depending on its position $\lambda_☾$ in the ecliptic. Curve A varies between 0.92 and 1.08; this means that at *sphaera recta* the shortest rising time of $\mathbf{e}_☾$ (of length 12°) is $0.92 \times 12°$ and the longest rising time of $\mathbf{e}_☾$ is $1.08 \times 12°$.

The variation of Σ due to $\lambda_☾$ equals twice the variation in rising time of $\mathbf{e}_☾$ at *sphaera recta:* $2 \times (1.08-0.92) \times 12° = 3.84°$. This variation is much smaller than 7°, the variation of Σ due to $v_☾$. The function Σ therefore oscillates with the mean period $P_☾$.

Curve C varies between 0.65 and 1.22. We can thus find the variation in rising time of $\mathbf{e}_☾$ at Babylon *sphaera obliqua:* namely $(1.22-0.65) \times 12° = 6.84°$. The variation of $ME+GE$ due to $\lambda_☾$ equals 6.84°. This variation is much larger than 3.5°, the variation of $ME+GE$ due to $v_☾$. The function $ME+GE$ therefore oscillates with the mean period $P_☾$.

We now understand why it is possible to find, purely empirically, a function varying with the period $P_☾$ from the observed Lunar Four, simply by calculating their sum.

9. *Concluding remarks*

We now know the astronomical significance of Σ. It is, however, so abstract and complicated that we must assume the Babylonians did not know it. If therefore our hypothesis is right, that Φ is directly derived from Σ, we must conclude: The Babylonians succeeded in finding a purely empirical function, which contained information on the elongation movement of the moon (on the day of opposition) and hence also on its momentaneous velocity on this day. Without knowing its astronomical significance, they derived all other quantities depending on $v_☾$ and hence of the period $P_☾$ from Φ.

Through systematic treatment of their observed data, so we think, the Babylonians observed the periodicity of different astronomical quantities. Quantities of the same period were coupled: Based on one known quantity, others of the same period were derived. Scientific research always uses procedures of this kind: The discovery of regularities leads to connections which can be used for predictions – also in cases where the fundamental natural laws are not known or only partially understood.

Many other scholars (O. Neugebauer, A. Aaboe, Y. Maeyama and B. L. van der Waerden) have pointed at the prevailing role of periodic functions in the Babylonian astronomy; however, not as radically as we do it in this paper. Therefore we mention two details from the cuneiform texts which clearly support this understanding of the development of the Babylonian astronomy:

The Goal-Year texts contain collections of characteristic phenomena for the moon and the five known planets. They were used for predicting astronomical events from known phenomena of the same kind which occurred some characteristic time period earlier. In case of the moon, the characteristic time interval was 223 synodic months = 1 Saros. Among the moon phenomena recorded on the Goal-Year tablets, we find the sums *ŠÚ+NA* and *ME+GE*. This shows us 1) that the Babylonians themselves, indeed, did calculate sums of the Lunar Four, and 2) that they probably used the sums *ŠÚ+NA* and *ME+GE* for prediction of Lunar Four to come one Saros later. But by so doing they have used empirically found periodic oscillations for lunar predictions.

In Figs. 2 and 3 we have seen that *ŠÚ+NA* and *ME+GE* as functions of the lunation number form nice curves, indicating that they might be easy to predict. In [Lis Brack-Bernsen 1994] it was demonstrated that the curve *ŠÚ+NA* as well as *ME+GE* was repeated almost exactly after one Saros. In the same paper, a short proposal was made about how some known values of the Lunar Four and their partial sums *ŠÚ+NA* and *ME+GE* might have been used in order to predict the Lunar Four one Saros later.

In the mean time this proposal has been confirmed by textual evidence: namely by the lines 35–38 on the back side of the text TU 11.[7] These lines show us that the Babylonians did, indeed, utilize the sum *ŠÚ+NA* for predicting *ŠÚ*. But more than that: in order to predict NA_N (new moon), they even used the sum *ŠÚ+NA* (full moon) as observed $5\frac{1}{2}$ months earlier. To us, this procedure can only be understood as an empirical utilization of periodic oscillations.[8]

We see this as a support for our reconstruction of *Φ* as a purely empirical function, derived from the sum of the Lunar Four. We therefore think that the Babylonian mathematical astronomy is more empirically founded and less theoretically than believed until now.

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NOTES

1. No difference is made in the Babylonian texts between the symbol for *NA* (new moon) observed around conjunction and that for *NA* (full moon) observed around opposition. In order to avoid confusion, we shall use the symbol NA_N for the time interval observed around conjunction.
2. Neugebauer [ACT I, pp. 229–239] has discussed the Lunar Four in detail and explained how the Babylonians calculated them.
3. Aaboe and Henderson [1975, p. 195] were the first to remark that the influence of $v_☾$ on a quantity $\Delta\lambda$ can be seen in the amplitude variation of $\Delta\lambda$ when plotted as a function of the lunation number. Here $\Delta\lambda$ denotes the ecliptic arc between the positions $\lambda_☾$ of consecutive full moons.
4. The days *N*, $N+1$ etc. here refer to the astronomical days starting at midnight – and not to the civil days of the Babylonian calendar in which a new day started in the evening at the moment of sunset.
5. The Lunar Four and all their combinations are measured in *uš*=time degrees: 1 *uš*=4 minutes, so that 360 *uš*=1 day (i.e. the time of a whole revolution of the sky about 360°).
6. The exact values $v_☾$ (max)=15.301°/day and $v_☾$ (min)=11.799°/day have been derived by Y. Maeyama. We thank him for this private communication.
7. H. Hunger, who has kindly given us a translation of this very difficult text, shall be warmly thanked at this place.
8. These lines of text TU 11 will be treated in more detail in a volume on "Ancient Astronomy and Celestial Divination", to be published under the auspices of the Dibner Institute of MIT.

Some Investigations on the Ephemerides of the Babylonian Moon Texts, System A

by

LIS BRACK-BERNSEN*

In the Babylonian lunar theory, system A, a series of variable time intervals were recorded: time intervals as 1, 6 and 12 synodic months. These time intervals depend on the velocities of both sun and moon. In the Babylonian approach they were calculated as a sum of two terms: one depending only on the lunar velocity and the other only depending on the solar anomaly.

In the present work we demonstrate how using the modern ephemerides one can separate these time differences in a good approximation into two such terms. By comparing those with the ones in the Babylonian tables, we can then check the Babylonian approach. In this approach the terms stemming from the moon anomaly are all calculated from column Φ. This column is normally interpreted as the lunar contribution to the duration of a Saros:

$$1 \text{ Saros} = 6585 \text{ days} + \Phi^H;$$

the velocity of the sun is assumed to be 30°/month. A supposed second term taking the solar anomaly into account has not been found in the Babylonian texts.

Our analysis of the time intervals gives the following results: In case of 1, 6 and 12 synodic months, the lunar term is the dominating one, the solar term just being a smaller correction. But in case of the Saros, the solar term is important while the lunar term plays a minor role. This throws doubt on the previous interpretation of column Φ.

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The Babylonians were among others concerned with some astronomical quantities $q(v_{☾}, v_{\odot})$ which depend on the irregularities in the movements of moon and sun. The duration of the synodic month can be mentioned as an example. According to system A (we are here using the terminology of A.C.T. [1]), this variable time interval was calculated as the sum of two independent contributions: one depending only on the variable moon velocity and the other only depending on the variable sun velocity. This method of separating the influence of the sun and moon anomalies into two additive terms was used by the Babylonians also in calculations of other interesting time intervals.

In the present work we will demonstrate a method to find out how quantities of the above mentioned type, $q(v_{☾}, v_{\odot})$, depend on the lunar and solar anomalies. In this connection, we will shortly recall some of the properties calculated in the ephemerides of the moon, system A, mentioning only the columns of interest for the present investigation and stating their traditional interpretations (i.e. the interpretations that are offered in A.C.T. [1] and Neugebauer [2]).

Column T: Dates (i.e. year and month) of successive new moons (or full moons; we will here concentrate on the new moon texts).

Column Φ: Under the assumption of a constant velocity of the sun of 30°/month, the length of a Saros is [3]:

$$223 \text{ synodic months} = 1 \text{ Saros} = 6585^d + \Phi^H. \qquad (1)$$

(Φ is measured in units of large hours, where $1^d = 6^H$ (large hours) = 6,0° (time degrees).)

Column B: Longitude of the moon at conjunction with the sun. (Hence the difference column to column B can be understood as the velocity function of the sun: $v_{\odot}$ given in °/synodic months. Its period, equal to 12;22,8 synodic months, we will call $P_{\odot}$.)

Column F: The moon velocity in units of °/day. But since $v_{☾}$ is only tabulated once each synodic month (namely at mean conjunction), the period P_F of this function equals $P_F = p_a/(1-p_a) = 13;56,40$ synodic months, see ref. [2], p. 476. Here p_a is the anomalistic month measured in units of synodic months.

Column G: $29^d + G^H$ is the length of the synodic month in the first approximation, where only the variation of the moon velocity is taken into account.

Column J: Correction of column G stemming from the variation of the solar velocity.

We recall that the periods P_Φ, P_G, and P_F of the three functions Φ, G and F are identical, and that function G is derived from Φ and not, as one could suspect, from function F (which represents the lunar velocity).

In the recent years, some other functions of the same type as G (with corrections of type J) have been found and identified in Babylonian sources. (The understanding of these functions is mainly due to the work of Aaboe [4–6].) Starting out with function Φ, the three functions G, W and Λ are derived such that

$$\begin{aligned} 1 \text{ synodic month} &= 29^d + G^H \\ 6 \text{ synodic months} &= 177^d + W^H \\ 12 \text{ synodic months} &= 354^d + \Lambda^H. \end{aligned} \tag{2}$$

As in the case with Φ, the functions G, W and Λ take only the anomaly of the moon into account. The relations (2) are thus only valid on the "fast arc" of the ecliptic where the velocity of the sun is constant and equal to 30°/synodic month. Parallel to the correction J, corrections Z and Y are applied on the "slow arc" of the ecliptic such that $177^d + W^H + Z^H = 6$ synodic months, while $354^d + \Lambda^H + Y^H = 12$ synodic months [5,6]. We see how these time intervals are calculated as the sum of two terms, one depending on the variable moon velocity and the other depending on the variable sun velocity only. We will now try to demonstrate how one can separate the influence of the sun and moon velocity variations on such functions $q(v_☾, v_\odot)$. As to the duration of the synodic month, Δt, and the length of the synodic arc, $\Delta\lambda$ (i.e. the difference in longitude between consecutive new moons), an old well-known method can give us an idea of their dependence upon $v_☾$ and $v_\odot$ [7].

We consider two geometric models in which (for the sake of simplicity) the sun and the moon are assumed to move in circles, the centre of which is the earth:

Model ☾: The velocity of the moon is variable whereas the velocity of the sun is constant.
Model ⊙: The velocity of the sun is variable whereas the velocity of the moon is constant.

Knowing the maximal, minimal and mean velocities of moon and sun, we can now with model ☾ calculate the variation of Δt and $\Delta\lambda$ due to the variation of $v_☾$. Results:

$$\delta_☾(\Delta t) = \Delta t\ (\max) - \Delta t\ (\min) = 0.41 \text{ days}$$
$$\delta_☾(\Delta\lambda) = \Delta\lambda\ (\max) - \Delta\lambda\ (\min) = 0.4°.$$

Similarly using model ☉, we get the variation of Δt and $\Delta\lambda$ due to the variable sun velocity $v_☉$:

$$\delta_☉(\Delta t) = \Delta t\ (\max) - \Delta t\ (\min) = 0.17 \text{ days}$$
$$\delta_☉(\Delta\lambda) = \Delta\lambda(\max) - \Delta\lambda(\min) = 2.2°.$$

From these results we can conclude that the irregularity of the sun movement is mainly determining the variation of $\Delta\lambda$. As to Δt, both $v_☾$ and $v_☉$ have quite an influence on its variation. Still, $v_☾$ has the largest influence on the function Δt.

This method gives only a rough qualitative estimate of the dependence of Δt and $\Delta\lambda$ on $v_☾$ and $v_☉$. It can only be used for these particular functions and not e.g. for the duration of 12 synodic months or a Saros. Therefore, we will now demonstrate another method to determine the dependence of a function of the type $q(v_☾, v_☉)$ on $v_☾$ and $v_☉$. To this purpose, we will introduce a new terminology. Let the times of consecutive new moons (i.e. conjunctions of sun and moon) be

$$t_0, t_1, t_2, \ldots \tag{3}$$

and the longitude of consecutive new moons be

$$\lambda_0, \lambda_1, \lambda_2, \ldots \tag{4}$$

These quantities can be found by means of Goldstine's tables for new and full moons [8].

From the t_i and λ_i we define

$$\Delta^n t_i = t_i - t_{i-n} \qquad i \in \{0, 1, \ldots\} \tag{5}$$

$$\Delta^n \lambda_i = \lambda_i - \lambda_{i-n} \qquad n \in \{1, 2, \ldots\} \tag{6}$$

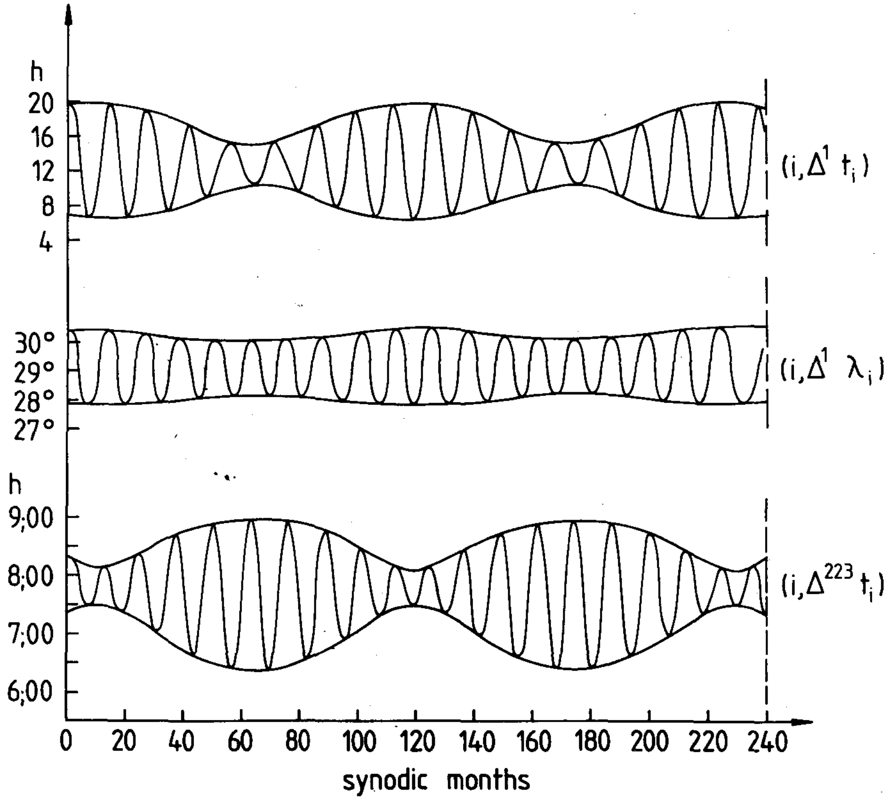

Figure 1: $\Delta^1 t$, $\Delta^1\lambda$ and $\Delta^{223}t$ plotted as functions of the lunation number.

$\Delta^n t_i$ is thus the total duration of n consecutive synodic months and $\Delta^n\lambda_i$ the length of the ecliptic arc covered by the sun during this time. For a fixed n, $\{\Delta^n t\}$ shall denote the sequence $\{\Delta^n t_1, \Delta^n t_2, \ldots\}$.

If we now plot $\Delta^1 t_i$ and $\Delta^1\lambda_i$ as functions of the number i of the lunation, we find an interesting result which is shown in figure 1. The curves $\Delta^1 t$ and $\Delta^1\lambda$ show a typical "beating" pattern, which results from a superposition of two periodic functions with slightly different periods. The curves oscillate rapidly with slowly varying amplitudes. Let us call these amplitudes (i.e. the differences between the two envelopes of the curves $\Delta^1 t$ and $\Delta^1\lambda$) $F^1(t)$ and $F^1(\lambda)$, respectively. We remark that they both seem to have the same period $D \approx 109.5$ synodic months. This we suspect to be the time of one revolution of the apside line in the ecliptic. (This is indeed the case, as shall be shown later.) Furthermore we see how $\Delta^1 t$ and $\Delta^1\lambda$ oscillate with variable periods. If we, however, abstract from this variation of the periods and calculate the mean period (of the rapid oscillations), taken over one large period D, we find an interesting result: The mean period of $\Delta^1 t$ is equal to 13.94 synodic months ($\approx P_F$), while the mean period of $\Delta^1\lambda$ is equal to 12.37 synodic months ($\approx P_\odot$).

We shall now try to understand why this is so and extract some more information from figure 1. The curves $\Delta^1 t$ and $\Delta^1\lambda$ reminded us of the sum of two periodic functions, say sine functions. Let us therefore briefly examine the behaviour of such a sum:

$$g(t) = \frac{A}{2} \sin \alpha t + \frac{B}{2} \sin \beta t. \tag{7}$$

In the simplest case, where $A = B$, we get:

$$g_0(t) = A \sin \left(\frac{\alpha + \beta}{2}\right) t \cos \left(\frac{\alpha - \beta}{2}\right) t. \tag{8}$$

The graph of such a function $g_0(t)$ is shown in the central part of figure 2. The typical pattern of the curve g_0 is immediately recognized from eq. (8) if α is not too different from β: It is a product of a sine function with a small period P_0 and a cosine function with a long period P_1 given by

$$\begin{aligned} \frac{1}{P_0} &= \frac{\frac{1}{2}(\alpha + \beta)}{2\pi} = \tfrac{1}{2}\left(\frac{1}{P_\alpha} + \frac{1}{P_\beta}\right) \\ \frac{1}{P_1} &= \frac{\frac{1}{2}(\alpha - \beta)}{2\pi} = \tfrac{1}{2}\left|\frac{1}{P_\alpha} - \frac{1}{P_\beta}\right| \end{aligned} \tag{9}$$

The section of the enveloping cosine from one zero to the next we will call one period (or one section) of the envelope. Its length, D, is of course equal to $\tfrac{1}{2} P_1$, i.e. half the period of the cosine in eq. (8):

$$\frac{1}{D} = \left|\frac{1}{P_\alpha} - \frac{1}{P_\beta}\right| \tag{10}$$

The period of the rapid oscillations is constant and equal to P_0. In the

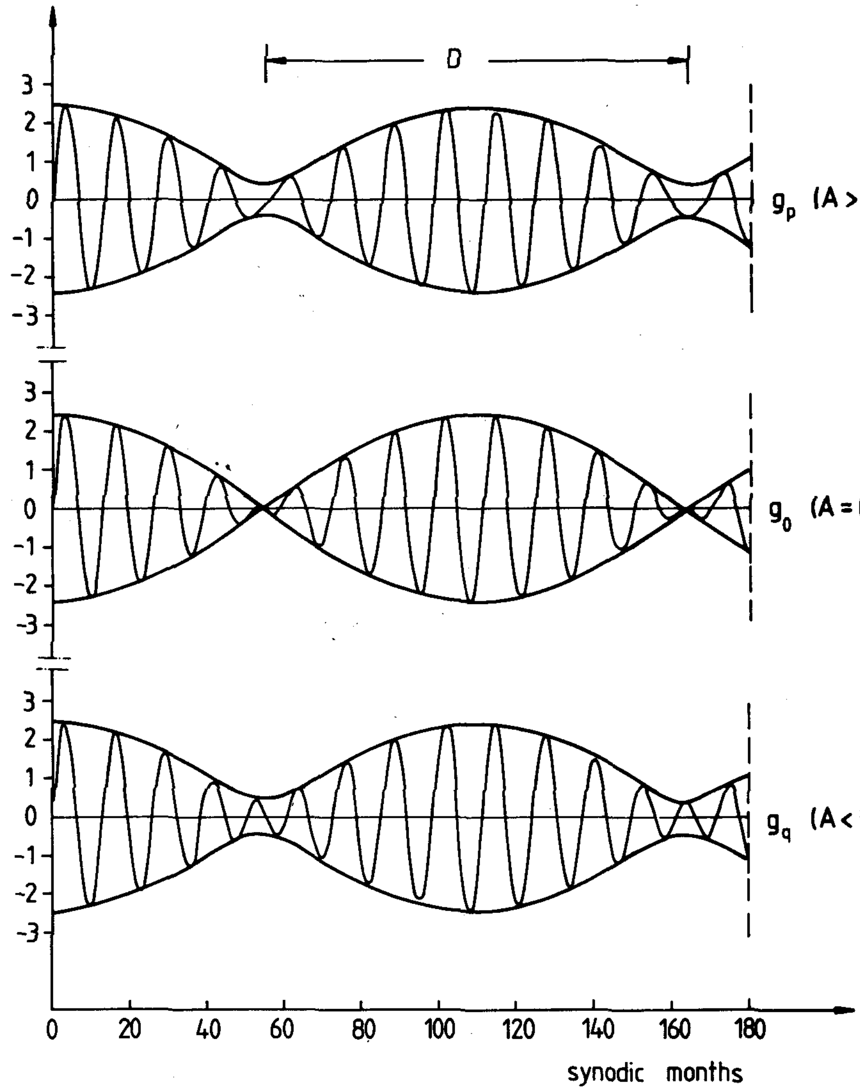

Figure 2: Graphs of functions of the type $g(t)$ of eq. (7).
The parameters in (7) are chosen as:
$P_\alpha = 2\pi/\alpha = P_\odot =$ 12;22,8 synodic months,
$P_\beta = 2\pi/\beta = P_☾ =$ 13;56,40 synodic months.
In the middle graph, g_o: $A/2 = B/2 = 1.2$.
For g_p: $A>B$, here $A/2 = 1.4$, $B/2 = 1$.
For g_q: $A<B$, here $A/2 = 1$, $B/2 = 1.4$.

case of $A \neq B$, $g(t)$ shows a slightly different pattern, see figure 2 ($g_p(t)$ for $A>B$ and $g_q1(t)$ for $A<B$.).

We notice that in these cases, the function $g(t)$ oscillates rapidly with a variable period while the period of the envelope is the same as in $g_0(t)$, namely D.

We will now point at three characteristics of the function $g(t)$ eq. (7), which are important for our understanding of $\Delta^1 t$ and $\Delta^1\lambda$, and which will be proved in detail in the appendix.

I) If $A>B$, the rapid and irregular oscillations of $g_p(t)$ will be such that their mean period, taken over one period D of the envelope, equals exactly P_α, i.e. the period of sin αt. (And, of course, correspondingly for the case $A<B$.)

II) The amplitudes of $g(t)$ vary between the limits of $V = A + B$ and $v = |A - B|$.

III) If the periods $P_\alpha = 2\pi/\alpha$ and $P_\beta = 2\pi/\beta$ of the sine functions in eq. (7) are chosen as $P_\odot \cong 12.37$ synodic months and $P_☾ \cong 13.94$ synodic months, the period D of the envelopes will be exactly equal to T, the time of revolution of the moon's apside line in the ecliptic.

We now can, of course, turn the argument around: Suppose a function $f(t)$ is known by its graph which shows a typical beating pattern. One can then, in a reasonably good approximation, write $f(t)$ as a sum of two sine functions as in eq. (7). The four parameters A, B, α and β can then be uniquely determined from the graph: The length D of one section of the envelopes can be directly measured. The mean period of the oscillation of $f(t)$, taken over one period D, can be deduced from the graph, too, by simple counting. It equals the period of the dominating term (corresponding to the larger of the amplitudes A and B), say P_α. From (10) one then calculates P_β. Finally, from the amplitude of the envelopes (measuring V and v from the graph), the amplitudes A and B themselves are determined from the rule II above.

We will now apply this method to the curves $\Delta^1 t$ and $\Delta^1\lambda$ in fig. 1. By comparing them with the curves on fig. 2, one can conclude that $\Delta^1 t$ and $\Delta^1\lambda$ in a good approximation can be written as functions of the type (7) (apart from an additive constant given by their mean values). We have already remarked that the mean rapid period of $\Delta^1 t$ equals $P_☾$, while that of $\Delta^1\lambda$ is $P_\odot$. We also stated above that the period D of their envelopes equals the revolution time of the apside line in the ecliptic. We now understand that this is so because both $\Delta^1 t$ and $\Delta^1\lambda$ are results of a superposition of two periodic functions with periods $P_{☾}$ and $P_{\odot}$, respectively. In the case of $\Delta^1 t$, the term with the period $P_{☾}$ is the dominating one, while for $\Delta^1\lambda$, the term with the period $P_{\odot}$ dominates. The amplitudes A and B of the oscillations in each curve we interpret as the variations of $\Delta^1 t$ and $\Delta^1\lambda$, caused by the sun and the moon. We therefore call them $\delta_{\odot}(\Delta^1 t)$ and $\delta_{☾}(\Delta^1 t)$, $\delta_{\odot}(\Delta^1\lambda)$, and $\delta_{☾}(\Delta^1\lambda)$ respectively. As we have demonstrated, they can be found from figure 1. The results are:

$$\delta_{☾}(\Delta^1 t) = 9^h\!.00 \qquad \delta_{\odot}(\Delta^1 t) = 4^h\!.15$$
$$\delta_{☾}(\Delta^1\lambda) = 0^\circ\!.38 \qquad \delta_{\odot}(\Delta^1\lambda) = 2^\circ\!.18$$

It is appropriate to compare these values with the corresponding values calculated with the geometric models ☾ and ⊙. We repeat those results:

$$\delta_{☾}(\Delta t) = 0^d\!.41 = 9^h\!.8 \qquad \delta_{\odot}(\Delta t) = 0^d\!.17 = 4^h\!.1$$
$$\delta_{☾}(\Delta\lambda) = 0^\circ\!.4 \qquad \delta_{\odot}(\Delta\lambda) = 2^\circ\!.2$$

Knowing that the geometric models only give a rough estimate, we can say that the agreement is excellent. The graphic analysis gives much more information and in addition, it has the advantage of being more precise and generally applicable.

Encouraged by these results, we will now go on to investigate other functions using the same graphic method. Inspired by the Babylonian columns W $(+Z)$, Λ $(+Y)$ and Φ, we have, using Goldstine's tables [8], calculated the following sequences: $\{\Delta^6 t\}$, $\{\Delta^{12} t\}$ and $\{\Delta^{223} t\}$. In figure 1, we show the graph of $\Delta^{223} t$; the others are similar to $\Delta^1 t$ in fig. 1. *A remarkable result for all these sequences is, that the* amplitudes vary with the same period of 109.5 synodic months, which is the revolution time of the moon's apside line in the ecliptic. Furthermore, in all cases the mean period of the rapid oscillations equals within an uncertainty of $\pm$ 0.02 to either $P_{\odot}$ = 12.37 synodic months or $P_{☾}$ = 13.94 synodic months. This justifies the approach of writing each of these functions as a sum of two contributions, one with the period $P_{\odot}$ of $v_{\odot}$, and the other with the period $P_{☾}$. We further notice that $\Delta^1\lambda$ and $\Delta^{223} t$ [*sic!*] have the same mean period, namely $P_{\odot}$, while

$\Delta^1 t$, $\Delta^6 t$ and $\Delta^{12} t$ all have the longer mean period $P_☾$. Hence, we can conclude that $\Delta^{223} t$ as well as $\Delta^1 \lambda$ mainly depend on the sun, the moon playing only a minor role, while $\Delta^6 t$ and $\Delta^{12} t$ as $\Delta^1 t$ mostly depend on $v_☾$. This means that our function $\Delta^{223} t$ and the Babylonian Φ do *not* alternate with the same (mean) period (column Φ being in phase with column F).

From what we have seen so far, it follows that 1 Saros approximately can be written as a constant plus two terms:

$$1 \text{ Saros} = \Delta^{223} t = \text{const.} + A_\odot + B_☾,$$

where $A_\odot$ is the dominating term. According to the traditional interpretation, however, column Φ then states the less important term, i.e. the correction $B_☾$. This raises an important question: Which kind of observation can have led the Babylonians to the function Φ? – It is hard to imagine. This may throw a little doubt on the previous interpretations of column Φ. A new interpretation, connecting column Φ more directly to the anomalistic month, would be much preferable. Remembering, that column F (i.e. the lunar velocity) as well as column G both are derived from column Φ, one should try to explain Φ as an astronomical quantity which the Babylonians could observe directly – and which contains information on the lunar velocity.

We have justified the approach of writing all the functions $\Delta^i t$, mentioned above, as sums of constants plus two periodic terms:

$$\Delta^i t = C_i + \frac{A_i}{2} \sin\left(\frac{2\pi}{P_\odot} \cdot t\right) + \frac{B_i}{2} \sin\left(\frac{2\pi}{P_☾} \cdot t\right)$$

The amplitudes A_i and B_i can, as we have demonstrated above, be found from the graphs of $\Delta^i t$. The constant C_i is of course easily determined as the mean value of $\Delta^i t$. *In the following we state the results of* this investigation:

	B The variation due to the anomaly of the moon	A The variation due to the anomaly of the sun	C Constant: mean value of $\Delta^i t$
$\Delta^1 t$	$9^h;00$	$4^h;10$	$29^d 12^h;44$
$\Delta^6 t$	$37^h;51$	$16^h;28$	$177^d\ 4^h;24$
$\Delta^{12} t$	$16^h;31$	$1^h;45$	$354^d\ 8^h;49$
$\Delta^{223} t$	$0^h;57$	$1^h;37$	$6585^d 7^h;42$

The Babylonian approach was also to calculate these time intervals as a constant plus the sum of two independent contributions, one depending on $v_☾$ and the other depending on $v_\odot$. For comparison, we have below stated the amplitudes of those terms:

amp. G:	$8^h;64$	amp. J:	$3^h;48$
amp. W:	$37^h;48$	amp. Z:	$22^h;22$
amp. Λ:	17^h	amp. Y:	$1^h;24$

We see that the agreement is quite good.

In case of Φ we get an interesting result: The amplitude of the zig-zag function Φ is $0^H;19,17 \approx 1^h;17$ which is far too much compared to $B = 0^h;57$. If we, however, look at the truncated Φ, the function which was first postulated by van der Waerden [3] (p. 148 ff.) and later indeed found by Aaboe [4] (p. 6 ff.), the agreement is much better: Φ was truncated by $2^H;13,20$ and $1^H;58,31,6,40$ which results in an amplitude of $\approx 0^H;14,49 \approx 0^h;59$. (This seems to support the common interpretation of column Φ.)

At this point we mention a difference between our and the Babylonian approach. In our splitting up $\Delta^i t$ we use a constant C_i plus two oscillating terms, the mean value of which is zero (i.e. the mean value of $\Delta^i t$ equals C_i). The Babylonians, however, split up these time intervals into two oscillating terms plus a constant which always equals an integer number of days, and therefore is slightly different from the mean value of $\Delta^i t$. To compensate for this, they must use oscillating terms with a mean value different from zero. Indeed, they are chosen such that the sum of their mean values plus the constant (the number of whole days) approximate the correct mean value quite well. An example:

$\langle \Delta^1 t \rangle = 29^d 53059 = 29^d + 12^h;44$
$29^d + \mu G + \mu J = 29^d + 14^h;32 - 1^h;45,16 = 29^d + 12^h;47.$

In the case of 1 Saros = $\Delta^{223}t$ one has in the texts only found the term Φ which corrects for the variable moon velocity, and no second term which could take $v_\odot$ into account. Aaboe [5] (pp. 11–15) remarked that 6585 days + μ_Φ does not approximate $\langle \Delta^{223}t \rangle$ well. He therefore constructed (parallel to the terms J, Z and Y) such a second oscillating term S, so that $6585^d + \mu_\Phi + \mu_s$ equals $\langle \Delta^{223}t \rangle$. This S is then the correction to Φ taking the variable sun velocity into account, and its amplitude is bigger than that of Φ. This is in complete agreement with our conclusions.

Appendix.

Proof of I, II and III.

We are concerned with functions of type

$$g(t) = \frac{A}{2} \sin \alpha t + \frac{B}{2} \sin \beta t \tag{7}$$

In the simplest case where $A = B$, the function $g(t)$ can be written as

$$g_0(t) = A \sin \left(\frac{\alpha + \beta}{2} \right) t \cos \left(\frac{\alpha - \beta}{2} \right) t \tag{8}$$

It has already been remarked that this function oscillates with the period

$$P_0 = \frac{2\pi}{\frac{\alpha + \beta}{2}}$$
$$\frac{1}{P_0} = \frac{1}{2} \left(\frac{1}{P_\alpha} + \frac{1}{P_\beta} \right) \tag{9}$$

while the amplitudes of g_0 vary periodically with the period D given by

$$\frac{1}{D} = \left| \frac{1}{P_\alpha} - \frac{1}{P_\beta} \right| \tag{10}$$

If $A \neq B$ we use the following identity

$$\alpha = \frac{\alpha+\beta}{2} + \frac{\alpha-\beta}{2}$$

$$\beta = \frac{\alpha+\beta}{2} - \frac{\alpha-\beta}{2}$$

to transform (7) into:

$$g(t) = \left(\frac{A+B}{2}\right) \sin\left(\frac{\alpha+\beta}{2}\right)t \cdot \cos\left(\frac{\alpha-\beta}{2}\right)t$$
$$+ \left(\frac{A-B}{2}\right) \cos\left(\frac{\alpha+\beta}{2}\right) t \cdot \sin\left(\frac{\alpha-\beta}{2}\right) t \qquad (11)$$

Hence we have written g as the sum of two functions g_1 and g_2 of the type g_0 discussed above. The rapid oscillation of g_1 and g_2

$$\left[\text{i.e.} \left(\frac{A+B}{2}\right)\sin\left(\frac{\alpha+\beta}{2}\right)t \text{ and } \left(\frac{A-B}{2}\right) \cos\left(\frac{\alpha+\beta}{2}\right)t\right]$$

have the same period, namely P_0; but they are out of phase by + or – $\pi/2$ (depending upon $A>B$ or $A<B$). The envelopes are also out of phase: at the times $t = n{\cdot}D$, $n = 0,1,2\ldots$ the difference of the envelopes of g_1 is maximal, namely $A+B$ while that of g_2 is zero and to the times $t = \frac{1}{2}D+nD$, the difference of the envelopes of g_1 is zero and the difference of the envelopes of g_2 is maximal, namely $|A-B|$.

Knowing this, it is easy to convince oneself of II: The amplitude of $g(t)$ will vary with the period D between the limits $v = |A-B|$ and $V = A + B$.

Also, we see that if $A>B$, then $g_p(t)$ will over the period D have fulfilled exactly 1/2 oscillation more than the function

$$\sin\left(\frac{\alpha+\beta}{2}\right) t,$$

which on the other hand will have fulfilled exactly 1/2 oscillation more than a function $g_q(t)$ with $A<B$. From (9) and (11) we get:

$$D = \frac{P_\alpha \cdot P_\beta}{P_\beta - P_\alpha} = P_\alpha\left(R+\frac{1}{2}\right) = P_o \cdot R = P_\beta\left(R-\frac{1}{2}\right) \qquad (12)$$

where

$$R = \frac{P_\alpha + P_\beta}{2(P_\beta - P_\alpha)} = \frac{P_\beta}{P_\beta - P_\alpha} - \frac{1}{2}$$

(R is the number of oscillations fulfilled by $\sin\left(\frac{\alpha+\beta}{2}\right)t$

during the time D. It is, of course, an irrational number in most cases.) We see now from (12) that $g_p(t)$ has fulfilled exactly as many oscillations as $\sin \alpha t$ during the time D, namely $R + 1/2$. Or with other words: If $A > B$, the rapid and irregular oscillations of $g_p(t)$ will be such that its mean period (taken over one period D of the envelopes) exactly equals P_α. Similarly: If $A < B$, the mean period of $g_q(t)$ is exactly equal to P_β. This proves I.

Proof of III: Let p_a be the duration of the anomalistic month and p_t the duration of the tropic month. A simple reasoning will show that the revolution time T of the apside line in the ecliptic is

$$T = \frac{p_t \cdot p_a}{p_a - p_t}$$

which also can be expressed as

$$\frac{1}{T} = \frac{1}{p_t} - \frac{1}{p_a} \qquad (13)$$

However, the time periods with which we are concerned, are $P_\odot$, the tropical year measured in synodic months, and $P_☾ = P_F$ which is the period of the function tabulating the lunar velocity each mean conjunction (of ☾ and ⊙). Let us assume p_t and p_a to be given in units of synodic months. We then have the following relation (see, e.g. [2], Vol. I, pp. 476 and 375):

$$\frac{1}{P_☾} = \frac{1}{P_F} = \frac{1}{p_a} - 1 \qquad (14)$$

Furthermore,

$$\frac{1}{P_\odot} = \frac{1}{p_t} - 1 \tag{15}$$

which can be demonstrated by use of period relations: Let A, B and C be integers so that (in a good approximation)

A tropic months = B synodic months = C tropic years.

Then $A = B + C$. But $P_\odot$, the length of the tropic year measured in units of synodic months, is B/C, while $p_t = B/A$. Using this, one easily gets (15).

Combining (13), (14) and (15) we get

$$\frac{1}{T} = \frac{1}{P_\odot} - \frac{1}{P_☾} \tag{16}$$

This formula reminds us of formula (10): If in (10) we replace P_α by $P_\odot$ and P_β by $P_☾$, we get: $1/D = 1/T$.

But this means that for a function $g(t)$ where $P_\alpha = P_\odot$ and $P_\beta = P_☾$, the period D of the envelopes will be exactly T, the revolution time of the apside line. *q.e.d.*

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