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Ancient Astronomy and Celestial Divination

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The MIT Press
Cambridge, Massachusetts
London, England

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Goal-Year Tablets: Lunar Data and Predictions

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This paper deals with lunar observations and predictions of the ancient Babylonians. Apart from lunar and solar eclipses (which are not dealt with here), the Babylonians regularly observed some characteristic time intervals in the days around conjunction and opposition, the so-called "Lunar Six." We elucidate these horizontal phenomena and mention the Babylonian sources in which they occur. We explain how the lunar data collected on the Goal-Year tablets can be used for predicting the Lunar Six, and we demonstrate through textual evidence that the Babylonians, indeed, predicted the phenomena in exactly this way. More precisely, we show that the "Lunar Four" time intervals around opposition can be predicted very accurately for an arbitrary full moon by a very simple calculation, using their values observed one Saros earlier than the full moon in question. In case of the new-moon phenomena *KUR* and *NA*, one has to know not only their values one Saros earlier but also the sums $SU + NA$ and $ME + GE$, respectively, occurring one Saros plus 6 months earlier. This fact explains for the first time why the Goal-Year text composed for a year Y contain not only all the Lunar Six of the year $Y - 18$, but also the sums $SU + NA$ and $ME + GE$ for the last 6 months of year $Y - 19$.

1 Introduction

The "Lunar Six" are some characteristic time intervals between sunrise or sunset and moonset or moonrise observed on the days around conjunction (*KUR* and *NA*) and opposition (*ŠÚ*, *NA*, *ME*, and *GE*, which we call the "Lunar Four").[1] Although spectacular and easy to observe, these time intervals are very complicated quantities from a theoretical point of view. Neugebauer (1957), 107–109, describes in detail the factors that determine whether the new moon crescent can be seen after sunset on the evening after new moon (conjunction). The same factors also decide how long the new moon can be seen in the evening of its first visibility: The time interval between sunset and the first visible moonset after conjunction is called NA_N. The others of the Lunar Six are correspondingly determined by similar factors.

The Lunar Six time intervals were of great interest for the Babylonians. The Diaries [Sachs and Hunger (1988, 1989, 1996)] contain reports of some of these horizontal phenomena as early as 568 B.C., indicating that the Babylonians observed them month by month during a time span of at least 500 years (568–62 B.C.).

During the last three centuries B.C. (Seleucid time), ephemerides for the moon were calculated with the aim of determining the different Lunar Six, whose calculated magnitudes were recorded in the last column of the text. In the preceding columns leading to this goal, the parameters that determine this value were taken into account through periodically varying functions. The fundamental periods used here are the length $P_{\odot}$ of the solar year, the period $P_{☾}$ of the lunar velocity, and the period P_{Ω} of the moon's movement in latitude. ($P_{☾}$ and P_{Ω} are the periods of the moon velocity $v_{☾}$ and of the moon latitude $\beta_{☾}$, respectively, when these are computed once each synodic month, such as the Babylonians did in their ephemeris texts.) See ACT I, in which Neugebauer has published the lunar ephemerides, discussed the Lunar Six in detail, and explained how the Babylonians calculated them by a skillful combination of the relevant influences. See also HAMA, 474–555, and van der Waerden (1974), 205–249.

We have recently shown that the Lunar Four contain information on the fundamental periods mentioned above. For example, the period ($P_{\Phi} = P_{☾}$) of the second column of a System A lunar ephemeris tablet can be determined empirically by the sum of all Lunar Four, whereas partial sums oscillate with the period $P_{\odot}$ (Brack-Bernsen 1990, 1994). In Brack-Bernsen and Schmidt (1994), we have explained why and how this works, through offering an astronomical interpretation and analysis of (partial) sums of the Lunar Four.

The present paper concentrates on the Babylonian treatment of these lunar observables. We shall see how they predicted the Lunar Six using collections of earlier lunar data recorded on the so-called Goal-Year tablets. Before doing so, we give a short overview of the variety of Babylonian texts concerned with the Lunar Six time intervals.

2 Lunar Six Data in Babylonian Texts

The interest of the Babylonians in the Lunar Six is documented through their occurrence in a variety of different types of texts (Sachs 1948; LBAT).

Over a period of more than 500 years, Lunar Six time intervals were regularly observed and recorded in the Diaries. There, however, we often find remarks about cloudy weather and that it was not possible to observe the moon. But still, the text gives the relevant lunar data. Clearly, such Lunar Six values must have been predicted somehow. Until now it has been a mystery how the Babylonians were able to predict such complicated phenomena, even as early as 568 B.C., and it has not been known how good their predictions were.

The *Almanacs* and *NS (normal star) Almanacs* record, among other information, Lunar Six data. The Babylonians even collected series of Lunar Six data over at least 60 consecutive months. For example, this is documented by a table (*LBAT 1431*)[2] containing a compilation of all Lunar Six data for a period of more than five years. It gives us a hint that they probably used these data for theoretical or empirical purposes.

Still another type of tablets contains compilations of Lunar Six data, excerpted from the Diaries, the so-called *Goal-Year tablets*. The Goal-Year tablets also contain observed characteristic phenomena for all the five planets known by the Babylonians. It is well known how the Goal-Year texts were used for predicting astronomical events for the planets, namely by using events of the same kind from some earlier characteristic time interval (see Sachs 1948 and *LBAT*, xxv).

Finally, the *Ephemerides* of the moon aimed at calculating the value of the different Lunar Six, whereas some *Procedure Texts* gave brief instructions on how to proceed. These lunar ephemerides attest to the most advanced part of the Babylonian astronomy. Throughout the columns leading to the calculated value of, for example, NA_N, all the variables influencing NA_N were correctly taken into account.

In this paper we shall present a detailed analysis of the lunar data collected on a Goal-Year tablet and explain how they could be used for predicting the Lunar Four. We will then demonstrate through textual evidence that the Babylonians predicted the phenomena in exactly this way. Indeed, a tablet from Uruk, TU 11, contains a short remark about finding *NA* and *GE*. This remark is so concentrated that it only became intelligible after we had reconstructed how the lunar data could possibly be used for predictions (Brack-Bernsen, 1994, section 4b). But then it reveals a procedure identical to the one we had reconstructed. In addition, the text TU 11 tells us how to calculate (and hence predict) the new moon phenomenon NA_N, another of the Lunar Six, by a very easy and precise method that we shall explain in detail. This text, also known as AO 6455, was published in cuneiform transcription by Thureau-Dangin (1922) and has recently been translated by Hunger. A small part of this text, namely, section 19 (rev. 8–15), was translated and published by Neugebauer (1947). Neugebauer just states that TU 11 contains a collection of rules for lunar and planetary phenomena and that it would lead far beyond the scope of his article to analyze all the relevant passages of the text. He writes that the translation of section 19 will show the general direction of these rules.

The rules demonstrated in section 19 through calculated examples are, indeed, very important. They reveal to us one (although rather primitive) method to extrapolate from one observed value of *KUR*. We therefore asked Hunger to translate the rest of the tablet TU 11.

Van der Waerden (1949) explained the rules of section 19 and connected them to older texts. Van der Waerden (1951), 29, must also have been in possession of at least a partial translation of other parts of the text TU 11. He surmises that in this text, some rules are given for calculating risings and settings of the Moon from observed values either a few days or 18, 36, or 54 years earlier. How these rules worked in detail, he does not tell; only that some indications about the methods might be drawn from TU 11. Our present result confirms his surmise and demonstrates how some of the rules worked.

Before presenting our systematic analysis and the new results from section 7 on, we shall give in sections 3–5 some basic knowledge and understanding of the Lunar Six phenomena. In section 6, we give a short survey of the Goal-Year tablets and summarize what so far has been known about them and their use.

3 The Phenomenon *KUR*

In figure 1 the phenomenon *KUR* is illustrated in detail. The horizontal (thin) great circle is the horizon, the (thick) oblique circle is the celestial equator (as seen from Babylon), and the dotted great circle is the ecliptic. We consider a morning shortly before new moon. The sun $\odot$ and the moon $☾_{KUR}$ are placed somewhere on the ecliptic near the eastern horizon; we have neglected the latitude of the moon. The arc of the ecliptic between moon and sun may be around 20°; the moon has thus risen visibly about $1\frac{1}{2}$ hour before sunrise. On the next morning, however, the moon will be so close to the sun that the moonrise is invisible. The time difference between the last visible moonrise (before conjunction) and the sunrise is called *KUR*. It is the time it takes the elongation arc($☾_{KUR}$, $\odot$) to rise.

The moon rises at the same time as point *B* of the equator, while the sun rises simultaneously with point *A*. Therefore, *KUR* is given by the length of arc(*A*, *B*) and depends on where the elongation arc($☾_{KUR}$, $\odot$) is

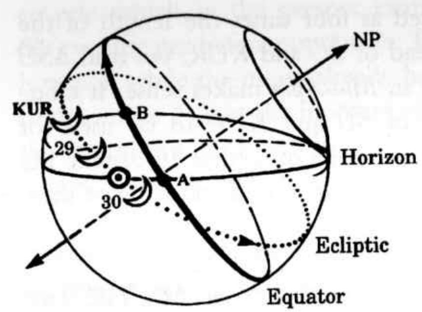

Figure 1
The celestial sphere for Babylon. Position of the moon near the eastern horizon at the moment of *sunrise*, shown on three consecutive mornings around conjunction (new moon). In the positions 29 and 30, the moon is too close to the sun to be observable.

placed on the ecliptic and also upon the length of arc($☾_{KUR}$, $\odot$).[3] For measuring arc(A, B) = *KUR*, we here use the Babylonian unit time degrees *uš*.[4] For further details, see Brack-Bernsen and Schmidt (1994).

Let us visualize the movement of the moon relative to the sun in the days around conjunction. On the following mornings, the rising moon will not be visible. Still, we have in figure 1 marked the position of the moon relative to the sun at the moment of sunrise. In our example we have assumed that the phenomenon *KUR* was measured on the 28th day of the Babylonian month. The next morning (of day 29), the invisible moon will be at the position $☾_{29}$ when the sun rises, while it will be at position $☾_{30}$ with the next sunrise. Sometimes between mornings 29 and 30, the moon will pass the sun: the conjunction has taken place.

Note that, if one knew the time (say, *x uš*) it takes arc($☾_{KUR}$, $☾_{29}$) to rise, it would be possible to determine the time difference between moonrise and sunrise on the morning of day 29. This nonobservable "KUR_{29}" is, of course, equal to *KUR* − *x uš*. We will call this rising time, *x uš*, of arc($☾_{KUR}$, $☾_{29}$) the "daily change of *KUR*" and denote it by Δ*KUR*.

In this connection, we note that section 19 of TU 11, translated by Neugebauer as mentioned above, is very important. It shows us by some examples how the Babylonians, starting from the known value of *KUR*, found through extrapolation the estimated value of *KUR* for following mornings, when the moon was invisible. For Δ*KUR*, the daily change of *KUR*, they used four times the length of daylight measured in minas. In *Mul-Apin*, tablet II, ii 43–iii 15, *ŠÚ* (the setting of the moon) and *KUR* (the rising of the moon) are calculated as four times the length of the nighttime measured in minas. If, instead of *ŠÚ* and *KUR*, we read Δ*ŠÚ* and Δ*KUR*, respectively, this scheme in *Mul-Apin* makes sense: It tabulates the values of the daily change of *ŠÚ* and *KUR* to be used for extrapolations, as demonstrated in the examples of section 19 of text TU 11. Van der Waerden (1949) has exactly the same understanding of these texts.

4 LUNAR PHENOMENA AT THE EASTERN HORIZON: *ME*, *GE*, AND *ME* + *GE*

We now concentrate on the Lunar Four, the characteristic time intervals observable in the days around the opposition. As an example, we consider the opposition that occurs half a month later than the conjunction dealt with in figure 1. The time difference *ME* between the last moonrise before opposition and the sunset is based on the moon at the eastern horizon and the sun in the opposite direction (i.e., on the western horizon). In the following figures, we introduce for the sake of simplicity the symbol $\bar{\odot}$ for the "anti-sun," which we define as the point on the ecliptic situated directly opposite the sun. At the very moment when the sun rises, $\bar{\odot}$ sets, and vice versa. With this definition, *ME* is the time difference between the risings of the full moon and the anti-Sun $\bar{\odot}$. In figure 2 the phenomenon *ME* is illustrated analogously to *KUR* in figure 1. The horizontal (thin) great circle is the horizon, the (thick) oblique circle is the celestial equator (as seen from Babylon), and the dotted great circle is the

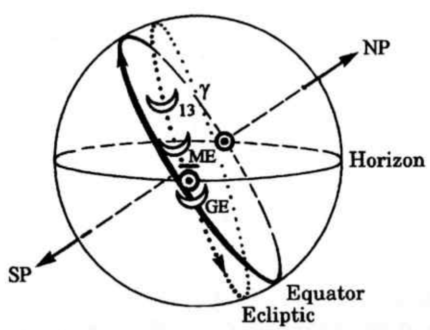

Figure 2
The celestial sphere for Babylon. The position of the moon near the eastern horizon at the moment of *sunset* is shown on three consecutive evenings around the opposition (full moon), which takes place half a month later than in figure 1.

ecliptic which in the present example stands very steep. We consider the evening just before opposition. The sun is about to set (i.e., the anti-sun is rising), while the moon already has risen. We have marked the position of the moon $\leftmoon_{ME}$ at the moment of anti-Sun rise. *ME* is the time it takes the elongation arc($\leftmoon_{ME}, \bar{\odot}$) to rise. This rising time is determined by the length of the arc and by its position on the ecliptic.

As in figure 1, we mark also here the positions of the moon at the moment of anti-sun rise on some consecutive days around opposition. Let us assume *ME* to be observed on the evening of the 14th day in the Babylonian month. On the previous evening, the moon was at the position $\leftmoon_{13}$ at the moment of anti-sun rise. Sometime during the day 14, at the moment of opposition, the moon will pass the anti-sun $\bar{\odot}$. On the following evening (15) at anti-sun rise, the moon will be in position $\leftmoon_{GE}$; still under the horizon. The time it takes the arc($\bar{\odot}, \leftmoon_{GE}$) to rise is the Babylonian observable *GE*. In short:

ME is the time from last moonrise to sunset before opposition and *GE* is the time from the first sunset after opposition to moonrise.

Both quantities denote actually the same time interval that changes sign between the two days around opposition; their definitions are chosen such that they always are positive. The sum *ME* + *GE* can be interpreted as the decrease of *ME* during the day of opposition, or as the increase of *GE* during the same day. Now, *ME* + *GE* is the rising time of arc($\leftmoon_{ME}, \leftmoon_{GE}$). It is determined by its length (1 day × $(v_{\leftmoon} - v_{\odot})$) and by its position $\lambda_{\leftmoon}$ on the ecliptic. Here the velocities ($v_{\leftmoon}$ and $v_{\odot}$) are measured in degrees per day (°/day) and $\lambda_{\leftmoon}$ (the position of $\bar{\odot}$) is the longitude on the ecliptic at which the opposition takes place. The length (1 day × $(v_{\leftmoon} - v_{\odot})$) of arc($\leftmoon_{ME}, \leftmoon_{GE}$) tells us how far the moon has moved relative to the sun during the day of opposition.

The sum *ME* + *GE* is a rather simple quantity: It depends mainly on the two variables $v_{\leftmoon}$ and $\lambda_{\leftmoon}$:

$$ME + GE = (ME + GE)(v_{\leftmoon}, \lambda_{\leftmoon}).$$

Each single of the Lunar Four, however, is a much more complicated quantity, depending on four different astronomical variables. Let us, as an example, consider *GE*, which is the time it takes the arc($\bar{\odot}, \leftmoon_{GE}$) to rise. This rising time depends on $\lambda_{\leftmoon}$ (i.e., the position on the ecliptic at which the opposition takes place), on the lunar latitude $\beta_{\leftmoon}$ at opposition, and on the length of arc($\bar{\odot}, \leftmoon_{GE}$). This length equals the product of Δt and $(v_{\leftmoon} - v_{\odot})$, the lunar velocity relative to the sun. Here Δt is the time interval between opposition and anti-sun rise on the next morning. Hence *GE is also strongly depending on when the opposition takes place with respect to sunset.* In summary, we must consider it as function of all these four variables:

$$GE = GE(\Delta t, v_☾, \lambda_☾, \beta_☾).$$

The same holds for all the Lunar Four. The crucial point is now that through addition of *ME* and *GE*, the dependence on Δt and $\beta_☾$ is practically eliminated; for further details, see Brack-Bernsen and Schmidt (1994).

5 LUNAR PHENOMENA AT THE WESTERN HORIZON: *ŠÚ*, *NA*, AND *ŠÚ* + *NA*

We are now concerned with the *western* horizon in the days around the same opposition as in figure 2. The phenomena *ŠÚ* and *NA* are visualized in figure 3. Here we have drawn the celestial sphere (as seen from Babylon) at the moment of sunrise. In the same way as in figure 2, we have marked the positions of the (full) moon relative to the anti-sun—now at the moment of anti-sun set—in the mornings around opposition. The position of the ecliptic has the same inclination to the horizon as in figure 1.

ŠÚ is the setting time of arc($☾_{SU}$, $\bar{\odot}$), and *NA* is the setting time of arc($\bar{\odot}$, $☾_{NA}$). These observables are both strongly dependent upon the variable Δt, which equals the time from sunrise to opposition for *ŠÚ*, and

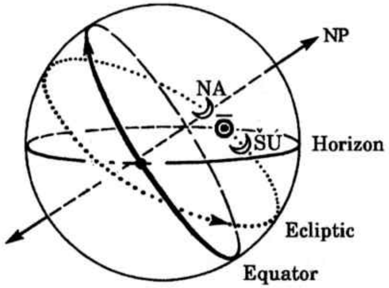

Figure 3
The celestial sphere for Babylon. Position of the moon near the western horizon at the moment of *sunrise* on two consecutive mornings around the same opposition (full moon) as in figure 2.

the time from opposition to next sunrise for *NA*. They also depend on $\beta_☾$, $v_☾$, and $\lambda_☾$:

$$NA = NA(\Delta t, v_☾, \lambda_☾, \beta_☾).$$

The sum *ŠÚ* + *NA*, finally, is the setting time of arc($☾_{SU}$, $☾_{NA}$) and depends only on $v_☾$ and $\lambda_☾$. The length of this arc is, of course, (1 day × $(v_☾ - v_\odot)$) as in the case of *ME* + *GE*, namely, the distance which the moon traveled relatively to the sun during the day of opposition.

6 THE GOAL-YEAR TABLETS

There exist about 150 Goal-Year texts, counting both fragments and whole tablets. They are registered, and most of them also published, as numbers 1213 through 1367 by Sachs in *LBAT*. They contain observations excerpted from the Diaries: raw material for the prediction of planetary and lunar phenomena for a given year *Y*, named the "goal year" by Sachs. (See also his discussion of the Goal-Year tablets in Sachs [1948], 282–285.) Of these tablets, 93 contain data concerning 56 different goal years in the time between 71 S.E. and 352 S.E.

The structure of all the Goal-Year tablets is the same. On the front side, planetary observations are recorded in a strict order while most of the reverse side is covered with lunar data, always written in a schematic and similar way. After the lunar section, a colophon-title usually follows. The title states the contents of the table and its purpose, and typically reads (in Sachs's translation): "the first day, appearances, passings, and eclipses which have been established for the year *Y*," where *Y* is the goal year. The tablet is covered with observations of appearances, passings, first days of visibility, etc., from different specific years prior to year *Y*.

For each of the five known planets, the characteristic phenomena, observed in a year that precedes the actual goal year *Y* by a number of years specific for each planet, are recorded in different sections. Obviously, the Babylonians were aware that a planet returned to the same characteristic appearance after the lapse of a relevant period of time; and obviously they used this knowledge for making predictions.

A concrete example may elucidate how this works in practice. In the first paragraph of the Jupiter section, the Greek-letter phenomena observed throughout year $Y - 71$ are recorded. The text utilizes the fact that after 71 years, namely in the year *Y*, these phenomena repeat themselves. However, Jupiter reaches the same position on the sky only after

83 years. Therefore, Jupiter's conjunctions with normal stars occurring during the year $Y - 83$ are collected in a second paragraph.

All this is well known, as well as the type of data that were put together in the lunar section: lunar and solar eclipses occurring one Saros (= 18 years) earlier than the goal year *Y*; the Lunar Six, month by month, during this whole year $Y - 18$; and their partial sums $\check{S}\acute{U} + NA$ and $ME + GE$ for the last 6 months of year $Y - 19$. We know that lunar eclipses repeat after one Saros; hence we understand how the Babylonians could utilize the recorded eclipse data. However, the way in which the Lunar Six data were used for predictions has not been investigated yet. (A part of our new results has been published in German in Brack-Bernsen (1994). Van der Waerden (1974, 110) made some conjectures about the use of these lunar data; we will comment on his remarks at the appropriate places.)

Let us therefore take a closer look at the Lunar Six data occurring on the Goal-Year tablets. Knowing that the structure is the same for all tablets, we take *LBAT* 1285 as an example.

7 THE GOAL-YEAR TABLET LBAT 1285

The colophon states the goal year to be 194 SE. The lunar data collected for predictions for this year are written in four columns on the reverse of the tablet. We know that year 194 SE had 13 months: I, II, . . . , XII, and XII_2, and that the months one Saros = 223 synodic months prior to these were month XII_2 175 SE and I–XII 176 SE. Correspondingly, the values of the Lunar Six as well as data from five (observed or expected) eclipses from this period of time were recorded. Table 1 contains a transcription of the text. Here we have, however, omitted observational remarks and only reproduced the values of the Lunar Six and their sums as well as the existing dates of the eclipses.

The first column records the values of the sums $\check{S}\acute{U} + NA$ and $ME + GE$ for months VII–XII of year 175 SE. Our reproduction is as short as that of Babylonians; lines 1 and 2 of column (1) state:

year 175 (month) VII 15 *ŠÚ NA*.

The meaning is the following: "In the year 175 (of the Seleucid era) for month VII the sum of *ŠÚ* and *NA* was 15 *uš*."

After the remarks on eclipses, the Lunar Six data for all the months from SE 175 XII_2 to SE 176 XII follow in thirteen entries, one for each month. The dates within the month on which the phenomenon was

Table 1
Transcription of the Goal-Year text LBAT 1285

year	175		year	176							
VII	15	*ŠÚNA*	I 1			V 1	15 10	*NA*	IX 30	10 20	
	8	$MEGE_6$	14.	4 10	*ŠÚ*	14.	4	*ME*	13.	7 30	*ŠÚ*
VIII	15	*ŠÚNA*	15.	3 20	*ME*	14.	2 20	*ŠÚ*		6 30	
	9 40*	$MEGE_6$	15.	4	*NA*	15.	5	GE_6			
IX	14 50	*ŠÚNA*	16.	10 40	GE_6	15.	15	*NA*			
	12 10	$MEGE_6$	27.	15	*KUR*	27.	20	*KUR*			
X	14	*ŠÚNA*	II 30			VI 1	12	*NA*	X 1	20	*NA*
	13	$MEGE_6$	15.	10	[*ME*]	13.	7	*ŠÚ*	11.	13 40	*ŠÚ*
XI	9 30	*ŠÚNA*	15.		*ŠÚ*	14.	1 30	*ME*	12.	1 30	*NA*
	13 30	$MEGE_6$	16.	6	GE_6			*NA*	13.	5 30	*ME*
XII	8 40	*ŠÚNA*	16.	9	*NA*			GE_6	14.	7 30	GE_6
	14 30	$MEGE_6$	28.	9 50	*KUR*	27.	16	*KUR*	27.	10 10	*KUR*
. . .	. . .	. . .	III 1	23	*NA*	VII 1	11 30	*NA*	XI 30	17	*NA*
175 XII_2	night 15	☾ ecl.		10	*ME*	12.	13	*ŠÚ*	12.	6	*ŠÚ*
175 XII_2	day 29	⊙ ecl.	14.	9	*ŠÚ*	13.	7	*ME*	13.	2 50	*NA*
176 VI	night 14	☾ ecl.	15.	4 20	GE_6	13.	3 40	*NA*	14.	0*	*ME*
. . .	. . .	. . .	15.	5 30	*NA*	14.	1 10	?	15.	15	GE_6
year	175		27.	20 30	*KUR*	27.	15 30	*KUR*	27.	11	*KUR*
XII_2 30	11	*NA*	IV 30	13	*NA*	VIII 1	15	*NA*	XII 30	13	*NA*
14.	3 50	*ŠÚ*	14.	12 20	*ME*	12.	2	*ŠÚ*	13.	4 30	*ŠÚ*
15.	7 40	*ME*	14.	15	*ŠÚ*	13.	5	*ME*	14.	7	*ME*
15.	3	*NA*	15.	0 40	GE_6	13.	13	*NA*	14.	2 30	*NA*
16.	6 20	GE_6	15.	2 20	*NA*	14.	4	GE_6	15.	6	GE_6
27.	16 30	*KUR*	27.	14 20	*KUR*	26.	22	*KUR*	27.	16	*KUR*
									I 30	10 40	*NA*

Between the dots (. . .) in the first column, the table contains three reports on eclipses of which we here only render the dates. An asterisk (*) signifies collation by Sachs (H. Hunger, private communication).

observed are written as ordinal numbers in the left side of the column, to the right is the name of the phenomenon, and in the middle of the column is its ascertained value.

As an example, we look closer at the first entry for the month following after the eclipse dates:

year 175 (month) XII_2 30 11 *NA*.

Here we are told that "in the year SE 175 the first day of month XII_2 (which was identical with) the 30th (of the preceding month XII), NA_N was 11 *uš*." The next line states:

14 3 50 *ŠÚ*,

which means "(in the morning of) day 14 (of month XII_2 year 175 SE), *ŠÚ* (the time from moonset to sunrise) amounted to 3;50 *uš*."

Our task is now to find out if and how these lunar data can be used for predicting the Lunar Six for the 13 months of our goal year SE 194. Or more generally: How can the Lunar Six data collected on a Goal-Year tablet be used to predict Lunar Six phenomena of the goal year?

We know that the planetary phenomena and the eclipses collected on the table are expected to repeat themselves in year SE 194. Is the same true for the Lunar Six or for their sums? In order to answer this question we can, for example, compare corresponding values of the same phenomena for lunations 223 synodic months (= 1 Saros) apart.

8 Systematic Analysis of Lunar Four Data

We know that the structure of each of the Lunar Six in principle must be the same. They are all either the rising or setting time of a little arc of the ecliptic: In case of *KUR* and NA_N this is the elongation arc between the moon and the sun, while for the Lunar Four (*ŠÚ*, *NA*, *ME*, and *GE*), it is an elongation arc between the moon and the anti-sun.

In our search for an eventual regular behavior of these quantities, it therefore suffices to examine one representative of the Lunar Six and the sums *ŠÚ* + *NA* and *ME* + *GE*. We look for regularities and for empirical information contained in the Lunar Four that the Babylonians might have found and used for predictions.

For this investigation we have used a recent computer program for lunar ephemerides valid at ancient times (Moshier 1992). Starting at the full moon (lunation 1) that occurred on 20 July 233 B.C., we have calculated[5] the Lunar Four for a series of lunations $1, 2, \ldots, i, \ldots$. At the top of

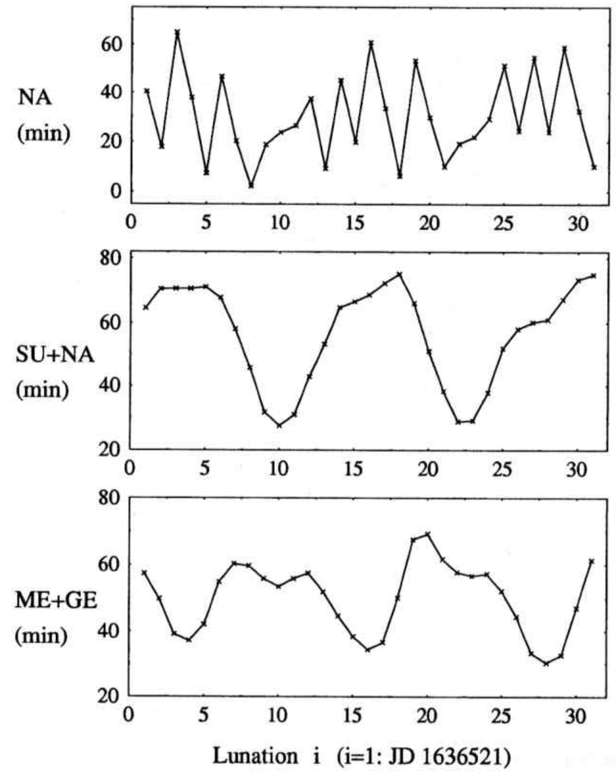

Figure 4
Calculated horizontal phenomena (from top to bottom) *NA*, *ŠÚ* + *NA* and *ME* + *GE*, as seen from Babylon, over a period of 30 months starting (for $i = 1$) from JD 1636521 = July 20, 233 B.C.).

figure 4, the calculated values of *NA* for thirty consecutive full moons are marked by crosses (×), connected by straight lines, and shown as functions of the lunation number *i*. Below *NA*, the sums *ŠÚ* + *NA* and *ME* + *GE* are shown in the same way. They exhibit a much more regular behavior than *NA*, demonstrating that some of the dependencies on the variables Δt, $v_{☾}$, $\lambda_{☾}$, and $\beta_{☾}$ have been partially eliminated by simple addition of the Lunar Four values.

We now investigate if *NA* or *ŠÚ* + *NA* and *ME* + *GE* repeat themselves after one Saros. Figure 5 compares corresponding values of these phenomena for lunations 223 synodic months (= 1 Saros) apart. The crosses mark the same quantities as in figure 4; their values measured 223 synodic months earlier are marked by small circles (○). The curves

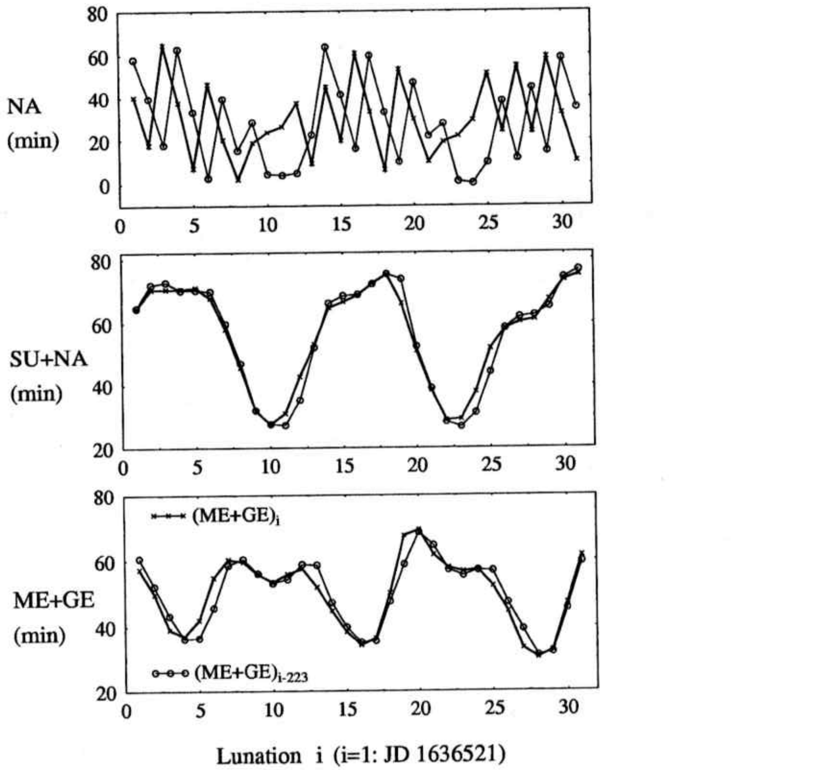

Figure 5
Comparison of the horizontal phenomena from figure 4 (crosses ×, connected by heavy lines), with those that could be observed 223 months (= 1 Saros) earlier (circles 0, connected by thin lines).

$\check{S}\acute{U} + NA$ and $ME + GE$ obviously repeat themselves after one saros, but NA does not. Denoting the values of $\check{S}\acute{U}$, NA, and $\check{S}\acute{U} + NA$ established for the i^{th} lunation as $\check{S}\acute{U}_i$, NA_i, and $(\check{S}\acute{U} + NA)_i$, respectively, we can express our knowledge in the following way:

$$(\check{S}\acute{U} + NA)_i = (\check{S}\acute{U} + NA)_{i-223},$$

$$(ME + GE)_i = (ME + GE)_{i-223}.$$

In other words, the sums $\check{S}\acute{U} + NA$ and $ME + GE$ are known for each of the months of year Y (plus for the preceding six months), if a Goal-Year tablet for this year exists. This can be used in the following way: In a case where the Babylonians were able to observe, say $\check{S}\acute{U}$, but clouds made the

moon invisible the next morning, the missing NA could of course be established by means of a Goal-Year table composed for the actual year. Let the missing NA be NA_i. In the corresponding Goal-Year tablet, the values for $\check{S}\acute{U}_{i-223}$ and NA_{i-223} can be found and their sum calculated, and hence NA_i is found:

$$NA_i = (\check{S}\acute{U} + NA)_i - \check{S}\acute{U}_i = (\check{S}\acute{U} + NA)_{i-223} - \check{S}\acute{U}_i.$$

Or, inversely: If NA is known, $\check{S}\acute{U}$ can be found. Correspondingly, one can deal with ME and GE: If one of them is measured, the other can be found using the data on an appropriate Goal-Year tablet.

This way of using the sums $\check{S}\acute{U} + NA$ and $ME + GE$ for finding one missing observation was also proposed by van der Waerden (1974, 110). The difference to our interpretation is that he suggested the use of $\check{S}\acute{U} + NA$ from the lunation 19 years earlier instead of using $(\check{S}\acute{U} + NA)_{i-223}$, that is, the value 18 years earlier. The sum $\check{S}\acute{U} + NA$ repeats itself very accurately after 1 Saros (as we have seen in figure 5), but less exactly after 19 years (Brack-Bernsen 1994, figure 2). The sums $\check{S}\acute{U} + NA$ and $ME + GE$ were only recorded during 6 months of the year $Y - 19$, and not during the whole of the year. We are therefore convinced that the Babylonians used the Lunar Six observations 1 Saros before the month in question, and not 19 years before. For the purpose of the data recorded from the year $Y - 19$, we have another proposal (see section 11 below).

In the Diaries, however, there are lunations where both values ($\check{S}\acute{U}$ and NA or ME and GE) are recorded, in spite of remarks saying that in none of the cases had the moon been visible. In the next section, we will investigate how such Lunar Four eventually can be calculated by means of the lunar data observed one Saros earlier.

9 Comparing $\check{S}\acute{U}_i$ and $\check{S}\acute{U}_{i-223}$

In sections 4 and 5 we saw that each of the Lunar Four is a function of four variables. Taking $\check{S}\acute{U}$ as an example, we write:

$$\check{S}\acute{U} = \check{S}\acute{U}(\Delta t, v_{☾}, \lambda_{☾}, \beta_{☾}).$$

We concentrate on $\check{S}\acute{U}_i$ and $\check{S}\acute{U}_{i-223}$. The variable Δt is the time difference between the last sunrise before opposition and the opposition itself in the months i and $i - 223$, respectively. The other variables, $v_{☾}$, $\lambda_{☾}$, and $\beta_{☾}$, are the velocity of the moon, its longitude, and its latitude at opposition O_i or O_{i-223}, respectively.

One Saros is defined as 223 synodic months; but in a good approximation it also equals an integer number of anomalistic, sidereal or draconitic months:

223 syn.m. ≈ 239 anom.m. ≈ 241 sid.m. ≈ 242 drac.m. ≈ 18 years.

Therefore the three variables $v_{☾}$, $\lambda_{☾}$, and $\beta_{☾}$ will have approximately the same magnitudes at oppositions O_i and O_{i-223} situated one Saros apart. The only variable determining $\check{S}\acute{U}$ that might have changed is Δt. We therefore try to find the difference between Δt_{i-223} and Δt_i. Now:

1 Saros = 223 syn.m. = $6585 + \frac{1}{3}$ day.

Therefore, in comparison to sunrise, the opposition O_i will take place $\frac{1}{3}$ day later than was the case at opposition O_{i-223}. Our variable Δt_i, the time difference between the last sunrise before opposition i and the opposition itself, is hence equal to $\Delta t_{i-223} + \frac{1}{3}$ day:

$\Delta t_i = \Delta t_{i-223} + \frac{1}{3}$ days for $\check{S}\acute{U}$.

We remember that $\check{S}\acute{U}$ is the setting time of the ecliptic arc between $☾_{SU}$ and $\bar{\odot}$, the length of which equals $\Delta t(v_{☾} - v_{\odot})$. The velocities of the sun and moon are the same at the oppositions O_{i-223} and O_i; only the factor Δt has changed by $\frac{1}{3}$ day. Therefore, at lunation i, arc($☾_{SU}, \bar{\odot}$) will be $\frac{1}{3}$ day × $(v_{☾} - v_{\odot})$ larger than at lunation $i - 223$. The time it takes this little arc of length $\frac{1}{3}$ day × $(v_{☾} - v_{\odot})$ to set is the difference between $\check{S}\acute{U}_i$ and $\check{S}\acute{U}_{i-223}$. But this setting time is just one third of $\check{S}\acute{U} + NA$. As we saw, $\check{S}\acute{U} + NA$ is the setting time of arc($☾_{SU}, ☾_{NA}$) of length 1 day × $(v_{☾} - v_{\odot})$. We therefore now have:

$$\check{S}\acute{U}_i - \check{S}\acute{U}_{i-223} = \tfrac{1}{3}\,(\check{S}\acute{U} + NA)_i = \tfrac{1}{3}\,(\check{S}\acute{U} + NA)_{i-223}. \qquad (1)$$

This can also be demonstrated in a figure. In figure 3, the position of the moon relative to the anti-sun in the days around opposition had been visualized: On a celestial sphere we had drawn the anti-sun at the horizon and for consecutive mornings the position the moon at the moment of anti-sun set (= sunrise). In figure 6, we have twice drawn an enlarged part of the celestial sphere, namely, the situation on the western horizon at the moment of anti-sun set, for the two oppositions O_{i-223} and O_i taking place 1 Saros apart. The movement of the moon relatively to the anti-sun is illustrated. On the morning just before opposition, the moon is in the position $☾_{SU}$, moving slowly along the ecliptic in the direction indicated by the arrow. At opposition, the moon passes the anti-sun and on the next morning, it has reached the position $☾_{NA}$.

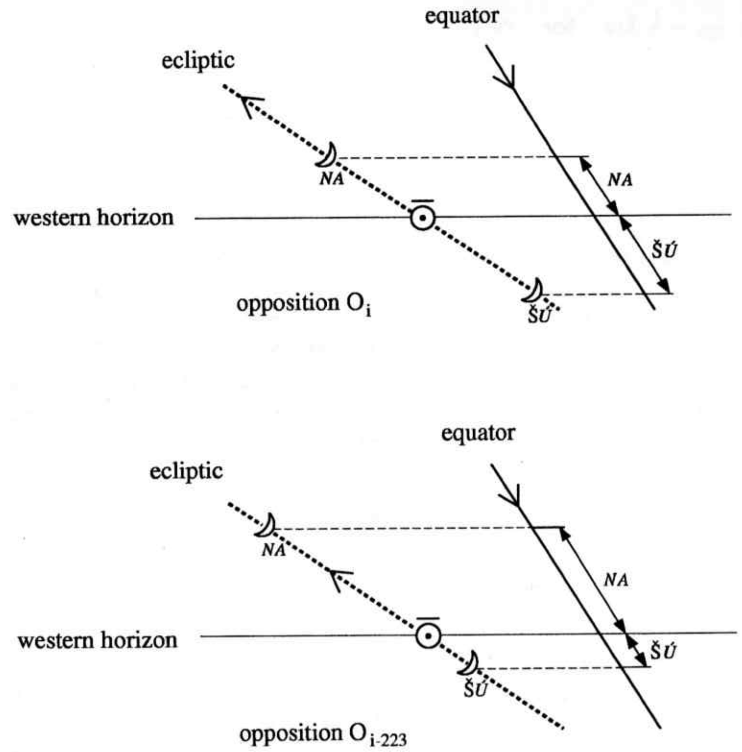

Figure 6
Lower part: Position of the moon and "anti-sun" at the western horizon on two mornings around opposition O_{i-223} at which $\check{S}\acute{U}$ and NA are measured. *Upper part*: The situation at the western horizon one Saros later, that is, at opposition O_i.

The two situations in figure 6 are almost identical: The angles between the ecliptic and the horizon are the same and so are the lengths of arc($☾_{SU}, ☾_{NA}$), (i.e., the relative displacement of the moon with respect to the anti-Sun during the 24 hours considered). Its rising time in both situations is the same: $(\check{S}\acute{U} + NA)_i = (\check{S}\acute{U} + NA)_{i-223}$. The only difference is the position of the anti-sun $\bar{\odot}$ with respect to $☾_{SU}$ and $☾_{NA}$: At lunation $i - 223$, the moon passes the anti-sun shortly after measuring $\check{S}\acute{U}$ (Δt_{i-223} is small), whereas at lunation i, the opposition takes place 8 hours later ($\Delta t_i = \Delta t_{i-223} + \frac{1}{3}$ day).

In case of NA, the situation is reversed: The distance between $\bar{\odot}$ and $☾_{NA}$ is large at lunation $i - 223$ and smaller at lunation i, by an amount equal to one third of the arc between $☾_{SU}$ and $☾_{NA}$. Therefore, in case of NA the parameter Δt will satisfy the following relation:

$$\Delta t_i = \Delta t_{i-223} - \tfrac{1}{3}\text{ day} \quad \text{for} \quad NA.$$

We thus conclude: NA_i must be smaller than NA_{i-223} by one third of the amount $(\check{S}\acute{U}+NA)_{i-223}$. (The sum $\check{S}\acute{U}+NA$, however, will remain the same, as we already know.)

We can also explain $\check{S}\acute{U}+NA$ in another way. It is the daily change (i.e., decrease) of $\check{S}\acute{U}$:

$$\Delta\check{S}\acute{U} = -(\check{S}\acute{U}+NA).$$

The time difference between moonset and sunrise measured one day too early, i.e. one day before $\check{S}\acute{U}$, would amount to (approximately) $\check{S}\acute{U}+(\check{S}\acute{U}+NA)$. The same time difference measured one day too late is of course $-NA$ (NA being the time difference between sunrise and moonset): $-NA = \check{S}\acute{U}-(\check{S}\acute{U}+NA)$. The observable NA, measured one day too late, would be roughly $NA+(\check{S}\acute{U}+NA)$. For that very reason, $\check{S}\acute{U}+NA$ is the setting time of the moon's daily elongation arc($☾_{SU}$, $☾_{NA}$). This elongation arc represents the displacement of the moon relative to the sun on the day of opposition. Without committing grave errors, we can use the same amount for the moon's displacement in the days just before and after opposition.

We summarize: The sum $\check{S}\acute{U}+NA$ can be interpreted as the daily decrease of $\check{S}\acute{U}$ or as the daily increase of NA. Van der Waerden (1974), 110 has the same understanding of $\check{S}\acute{U}+NA$ as the daily retardation of the moon's rising. Concerning the Lunar Six, he proposes to add $\frac{1}{3}$ day to every rising or setting of the moon. As we have shown, however, it is rather the moment of opposition in comparison to sunrise (and to sunset) that is shifted by $\frac{1}{3}$ day.

10 Prediction Rules for Calculating the Lunar Four

We have seen—equation (1)—how it is possible to calculate $\check{S}\acute{U}$ for an arbitrary month i by means of $\check{S}\acute{U}_{i-223}$ and $(\check{S}\acute{U}+NA)_{i-223}$, quantities written on the relevant Goal-Year tablet. Arguments completely analogous to the ones above will in case of the observables NA, ME, and GE lead to formulae similar to equation (1). We therefore conclude: Whenever we have a Goal-Year tablet for a year Y, we are able to calculate and thus predict all Lunar Four occurring in year Y. The following four equations state this mathematically:

$$\check{S}\acute{U}_i = \check{S}\acute{U}_{i-223} + \tfrac{1}{3}(\check{S}\acute{U}+NA)_{i-223}, \tag{2}$$

$$NA_i = NA_{i-223} - \tfrac{1}{3}(\check{S}\acute{U}+NA)_{i-223}, \tag{3}$$

$$ME_i = ME_{i-223} + \tfrac{1}{3}(ME+GE)_{i-223}, \tag{4}$$

$$GE_i = GE_{i-223} - \tfrac{1}{3}(ME+GE)_{i-223}. \tag{5}$$

These equations are actually correct only if their right-hand sides are positive for equations (3) and (5), not larger than $\check{S}\acute{U}+NA$ for equation (2), and not larger than $ME+GE$ for equation (4), respectively. Otherwise they would correspond to the phenomena observed one day too early and therefore have to be corrected. For example, if $\check{S}\acute{U}_i$ according to equation (2) becomes larger than $(\check{S}\acute{U}+NA)_{i-223} = (\check{S}\acute{U}+NA)_i$, it gives the value of $\check{S}\acute{U}$ observed one morning too early. On the next morning, the Moon would still set before sunrise; the correct time interval $\check{S}\acute{U}_i$ then is obtained by subtracting $(\check{S}\acute{U}+NA)_{i-223}$ from the right-hand side of equation (2). The analogous corrections for the other three quantities are obvious.

In Brack-Bernsen (1994), section 4b, we proposed that it is possible to calculate $\check{S}\acute{U}$ and NA in this way. At the time, we thought that it probably would never be possible to find out if the Babylonians, too, had discovered and used these simple rules. We furthermore had not the slightest idea how the values of KUR and NA_N could be determined.

In the mean time, Hunger has provided us with a translation of the very difficult and important text TU 11. Passages in this text tell us that the Babylonians, indeed, did know and use these rules! But more than that: Other passages contain the procedures for calculating NA_N in an easy and precise way, as we shall see. The procedures are written so briefly that it is doubtful that we would have understood the meaning without knowing the rules expressed in equations (2)–(5).

11 Passages of Text *TU* 11

The passages relevant for this analysis are written in the lines 29, 30, and 36–38 on the front side of *TU* 11. In Hunger's preliminary translation we read:[6]

29) For you to see lack and fullness (of a month). If in an 18(-year period), month I, the 1st day (following a month of 30 days)—there is no addition added to it; month II, which is after it, is full—one-third

30) of $\check{S}\acute{U}+NA$ is 6: you subtract this from NA of the 1st day of month II, and if it is less than in month I, which is before it, then month II of your new year is full

...

36) For you to make the counterpart(?) of the moon in the west. You go back from month I of the 36(-year-period) by 6 months I, and you take(?) 0;40 of $\check{S}\acute{U}+NA$ of month VII and subtract it from NA of the 1st day

37) of month I of the 36(-year-period). And if it is less than 10 uš, you add ŠÚ + NA completely(?) to it. 0;40 of ŠÚ + NA you subtract from NA of the middle of the month.

38) 0;40 of ME + GE you subtract from GE.

These lines give rules for calculating NA_N (explicitly called "*NA* of the first day"), *NA* (identified as "*NA* in the middle of the month"), and *GE*; the sums *ŠÚ* + *NA* and *ME* + *GE* are used, and so is a 36-year period plus the coefficient $0;40 = \frac{2}{3}$. The Saros is, as often in cuneiform texts, just called "18." This justifies the reading: "36 [year period] as 2 Saroi = 446 synodic month."

We first concentrate on the comments on the Lunar Four. Having mentioned 2 Saroi, the text goes on and requests the reader in the last third of line 37 and in line 38 to calculate the following difference:

$$NA - \tfrac{2}{3}(\check{S}\acute{U} + NA) \quad \text{and} \quad GE - \tfrac{2}{3}(ME + GE).$$

The text does not tell clearly from which month or opposition these values shall be taken. But the passages make sense when we read them as: "In order to find *NA* one has to go 2 Saroi back and then to subtract $\frac{2}{3}$ of *ŠÚ* + *NA* from *NA*" (both values stemming from the lunation two Saroi earlier than the one, say number *i*, we are concerned with):

$$NA_i = NA_{i-446} - \tfrac{2}{3}(\check{S}\acute{U} + NA)_{i-446},$$

and analogously for *GE*:

$$GE_i = GE_{i-446} - \tfrac{2}{3}(ME + GE)_{i-446};$$

but these formulae are simply the equations (3) and (5) used twice, namely for two consecutive Saroi.

We consider this as a strong support for our reading. Furthermore, it is a clear confirmation that the Babylonians really did know the formulae (3) and (5), and presumably also formulae (2) and (4).

Lines 36 and 37 deal with *NA* of the first day, which we call NA_N. The text, here more clearly formulated than the passages mentioned above, gives a procedure how to estimate (calculate) NA_N for the first month (*I*) in a year *Y*.

We understand the text, aiming at finding $(NA_N)_I$, as follows: "From month *I* [of your] 36 [year period] (i.e., month $I - 446$); you go 6 months backward (to month $I - 452$) and 0;40 of *ŠÚ* + *NA* [i.e., $\frac{2}{3}$ of $(\check{S}\acute{U} + NA)_{I-452}$] you subtract from *NA* of the first day of the 36 [year period] [i.e., from $(NA_N)_{I-446}$]."[7]

We write this instruction as an equation:

$$(NA_N)_I = (NA_N)_{I-446} - \tfrac{2}{3}(\check{S}\acute{U} + NA)_{I-452}.$$

This formula calculates NA_N from the NA_N observed 2 Saroi earlier and uses for the daily change of NA_N (which we call ΔNA_N) the quantity *ŠÚ* + *NA* stemming from a full moon $5\frac{1}{2}$ months further back in time. The daily change of NA_N cannot be determined by observation: The setting moon is invisible before conjunction. Using *ŠÚ* + *NA* for ΔNA_N is a very clever and, as we shall see later, also a very precise method.

The text continues: "... and if it is less than 10 *uš*, you add the whole *ŠÚ* + *NA* to it." The Babylonians knew and utilized that $(NA_N)_I$, when not observable on the first evening after conjunction, would be equal to the expected amount plus the sum *ŠÚ* + *NA* observed 6 months earlier. We know, and they evidently knew too, that the tiny new moon cannot be seen if it is too close to the sun and sets less than, say, 10 *uš* = 40 minutes after sunset. The new moon will in that case first be visible in the next evening, and its amount will then be enlarged by ΔNA_N. The text really tells us to add the whole *ŠÚ* + *NA*. The Babylonians evidently used *ŠÚ* + *NA* as the daily change of NA_N.

We derived the above formula for the specific case of the first month (*I*) of a year, as it occurs in the text *TU* 11. Obviously, the recipe works equally well for an arbitrary month of the year. We shall therefore continue using the general index *i* for a month.

If we now instead of 2 Saroi only go back by 1 Saros, the formula would be:

$$(NA_N)_i = (NA_N)_{i-223} - \tfrac{1}{3}(\check{S}\acute{U} + NA)_{i-229}. \quad (6)$$

The correspondingly reconstructed formula for calculating *KUR* must be:

$$KUR_i = KUR_{i-223} + \tfrac{1}{3}(ME + GE)_{i-229}. \quad (7)$$

The lines 29 and 30 confirm for us that the recipe expressed in equation (6) really was in use by the Babylonians of the Seleucid era. The concern in these lines is to predict whether a month has 29 or 30 days. We are, however, at this place, only interested in the calculation of NA_N. The 18-year period (*one* Saros) is mentioned, and so is NA_N of a month II, and the third of *ŠÚ* + *NA* is said to be 6. At spring time (month II), *ŠÚ* + *NA* always is minimum, ranging between 8 and 10 *uš*; but here is *ŠÚ* + *NA* indirectly said to be 18. We hence know that this value must stem from fall time, where *ŠÚ* + *NA* assumes its maximum value. The whole text is consistent, and that brief remark makes sense if we read it as follows:

"Subtract the third of $\check{S}\acute{U}+NA$ (six months earlier than II) from NA_N (of month II)." But this is exactly the rule contained in equation (6).

We saw above that the data necessary for calculating the Lunar Four for the whole year Y are written in the entries for the months of year $Y-18$ (on the relevant Goal-Year tablet). These data do, however, not suffice for calculating the new moon phenomena NA_N and KUR for the year Y. In order to calculate, for instance, NA_N in month 1 of year Y, one needs the NA_N from the first month's entry of the Goal-Year tablet; but also the sum $\check{S}\acute{U}+NA$ from the full moon six months further back. This sum is, indeed, also recorded in the upper left corner of the corresponding Goal-Year tablet!

We have hereby for the first time understood and explained why at the beginning of each Goal-Year tablet for a year Y, the sums $\check{S}\acute{U}+NA$ and $ME+GE$ valid for the last 6 months of year $Y-19$ are recorded.

12 THE DAILY CHANGE OF $(NA_N)_i$ AND ITS RELATION TO $(\check{S}\acute{U}+NA)_{(i-6)}$

The setting of the moon is not visible in the days before conjunction, and the moonrise is invisible during the first days after conjunction. The daily changes of NA_N and KUR around conjunction therefore cannot be observed. Nevertheless, the new moon data corresponding to $\check{S}\acute{U}+NA$ and $ME+GE$ can be computed. Using the ephemeris code of Moshier (1992), we have computed NA_N, as well as the time differences between sunset and moonset the evening just before, as seen from Babylon in ancient times. The difference of these two gives us ΔNA_N, the daily change of NA_N. In this way it is possible to test the Babylonian approach using $(\check{S}\acute{U}+NA)_{i-6}$ instead of $\Delta(NA_N)_i$.

In figure 7 we have plotted the calculated ΔNA_N as function of the lunation number i (thick line). For comparison, we have in the same figure also plotted the computed values of $\check{S}\acute{U}+NA$ observable $5\frac{1}{2}$ months earlier (dotted line). The two curves are almost identical. This shows that the use of $(\check{S}\acute{U}+NA)_{i-6}$ instead of $\Delta(NA_N)_i$ is, indeed, well justified and very precise.

We can also explain why this Babylonian approach works by comparing figures 1, 2, and 3. In order not to draw more figures we argue with KUR instead of NA_N. Figure 1 shows the situation near the eastern horizon at *sunrise* in the mornings around conjunction. What we would call ΔKUR (corresponding to ΔNA_N) is the rising time of arc($☾_{KUR}, ☾_{29}$). Half a month later, that is, around opposition, the situation at the eastern

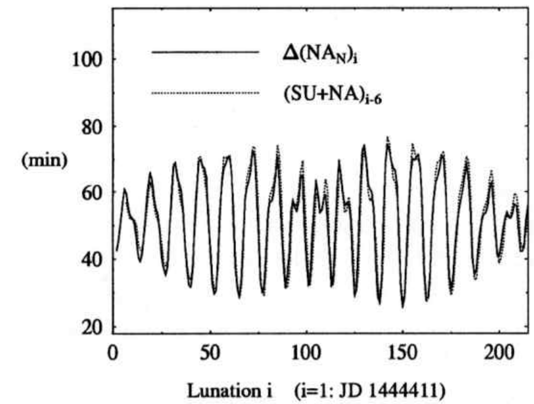

Figure 7
Comparison of calculated values of ΔNA_N (thick line) and of the $\check{S}\acute{U}+NA$ observable 6 months earlier (dotted line), both plotted versus the lunation number i (starting for $i=1$ on JD 1444411 = August 1, 759 B.C.).

horizon at *sunset* will be as in figure 2. The sun having moved not more than 15° along the ecliptic is about to set; the half of the ecliptic that is above the horizon in figure 1 will now be under the horizon. (For our present concern the small displacement of the sun can be neglected.)

In our example in figure 1 the sun was near the spring equinox, so that the ecliptic at sunrise had a small inclination to the horizon. At sunset half a month later, the fall equinox will be near the eastern horizon and the ecliptic will stand very steep. This illustrates how the parameter $\lambda_☾$ has changed from figure 1 to figure 2. Another parameter, the lunar velocity $v_☾$, which is crucial for the magnitude of KUR and GE, may also have changed considerably. In case of KUR, $v_☾$ is the velocity of the Moon at conjunction; in case of ME and GE it is the velocity at opposition half a month later. Now, half a synodic month is only about one day longer than one half of the anomalistic month, the period of the variable lunar velocity. If $v_☾$ is maximum in figure 1 it will be minimum in figure 2, and vice versa.

We see that the situations in figures 1 and 2 are very different: there is no hope of determining ΔKUR from $ME+GE$. But if we go 6 months backward from this month and imagine what the conditions for $ME+GE$ then will be, we realize that now we will have a situation almost identical with the one in figure 1.

We now are concerned with the opposition taking place 6 months earlier than in figure 2 and consider how the celestial sphere might have looked like at that time. In the course of 6 months, the sun and the anti-sun have moved about 180° and hence have exchanged places. Therefore, at the moment of *anti-Sun rise*, the position of the ecliptic will be as in figure 3, where the anti-sun was setting at the time of figure 2. In order not to draw another figure, we just look at figure 3, now being interested in the front side, imagining the anti-sun placed at the eastern horizon. The elongation arc($☾_{ME}$, $☾_{GE}$) is placed in some way around $\bar{\odot}$. It is the rising time of this elongation arc which the Babylonians used as ΔKUR.

We now understand why that is a good approach: The rising time of arc($☾_{ME}$, $☾_{GE}$) must be approximately the same as the rising time of arc($☾_{29}☾_{30}$) in figure 1: The angles between the horizon and the ecliptic will be the same in the two figures as will the lengths of the two ecliptic arcs. This length is, in fact, determined by the lunar velocity, which is approximately the same in the two cases. The time difference of $5\frac{1}{2}$ months between figure 1 and figure 3 is, indeed, approximately equal to 6 anomalistic months.

13 Test of Babylonian Prediction Rules by Modern Computations

Similarly as we have checked the equality of $\Delta(NA_N)_i$ and $(\check{S}\acute{U} + NA)_{i-6}$ in the section above, we now shall test some of the prediction rules discussed in Sects. 10 and 11 by modern computer calculations, in order to control their accuracy.

In figure 8, we show by solid lines the quantities $(NA_N)_i$ *(upper part)* and $\check{S}\acute{U}_i$ (lower part) for 50 successive lunations between 236 and 232 B.C. Their predicted values based on earlier data, according to the right-hand sides of equations (6) and (2), respectively, are shown by the dashed lines. Again, the agreement is excellent, demonstrating that these Babylonian procedures for predicting NA_N and $\check{S}\acute{U}$ were not only clever, but also very precise.

14 Test of Lunar Four Data on Some Goal-Year Tablets by Modern Computations

We now know how to predict each single one of the Lunar Six by means of an appropriate Goal-Year table, and we understand the meaning of the lines treated above from TU 11, which we see as a kind of procedure text that confirms our method.

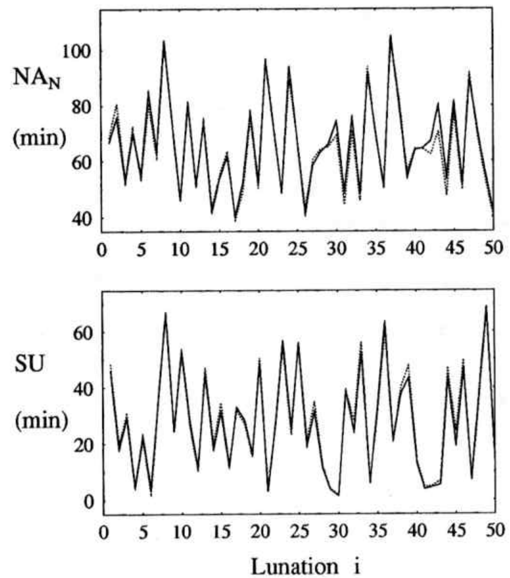

Figure 8
Numerical test of the prediction rules equation (6) for NA_N (upper part) and equation (2) for $\check{S}\acute{U}$ (lower part) for 50 successive lunations between 236 and 232 B.C. The quantities $\check{S}\acute{U}_i$ and $(NA_N)_i$ to be predicted are shown by solid lines; their predictions based on the earlier date, according to the right-hand sides of these equations, are shown by the dashed lines.

If we were to write a "User's Manual" for the Goal-Year tables, we would use more explicit words than the Babylonians, or else we would just give equations (2)–(7) above. This Babylonian method for calculating the Lunar Four must be very precise, provided the data collected on the Goal-Year tablets are sufficiently accurate. In Brack-Bernsen (1994), the data of two texts were analyzed and shown to be surprisingly good: Each of the Lunar Four, drawn as functions of the lunation number, shows the expected irregular behavior, whereas the sums $\check{S}\acute{U} + NA$ and $ME + GE$ form smooth and periodic curves. These curves were tested by modern computations. The sums $\check{S}\acute{U} + NA$ and $ME + GE$ were computed for a long period of time and plotted as functions of the lunation number. Comparison of the Babylonian and the calculated data demonstrated that the sums $\check{S}\acute{U} + NA$ and $ME + GE$ originating from the texts Cambyses 400 (523 B.C.) and the Goal-Year text LBAT 1285 (136 B.C.) were very accurate. This is not always the case. In a forthcoming paper, all the

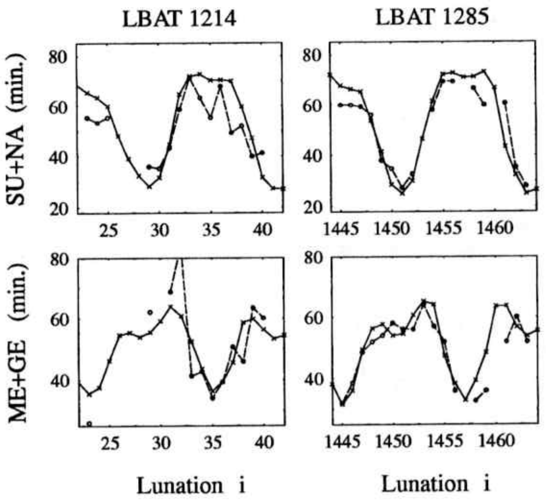

Figure 9
Comparison of the calculated lunar observables $\check{S}\acute{U} + NA$ and $ME + GE$ (crosses, connected by solid lines, as in figure 4) with the corresponding data recorded on the Babylonian Goal-Year tablets LBAT 1214 and LBAT 1285 (circles, for consecutive months connected by dashed lines).

existing Goal-Year data will be examined in this way. Here we only show the comparison of data from two Goal-Year texts: from LBAT 1214, the one with the worst data, and from LBAT 1285 which contains the best data and which we treated above in section 7.

In figure 9 we have drawn the values of $\check{S}\acute{U} + NA$ and $ME + GE$, computed with the modern ephemerides code (Moshier 1992), as functions of the lunation number (crosses connected by straight lines), while the values of these sums recorded on the relevant Goal-Year tablets are shown by small circles. The Babylonian data, indeed, form curves with maxima and minima at the same places as the theoretical curves. The data from LBAT 1214 deviate considerably from the computed curves, whereas the data from LBAT 1285 follow the computed curve and even show the same fine structures.

15 Concluding Remarks

We have analyzed the data collected by the Babylonians for predicting lunar phenomena and have seen that it is possible to predict the Lunar Four by means of such data. The text TU 11 proves that the Babylonians

did indeed predict the Lunar Four exactly in this way. Furthermore, it also shows us how to predict NA_N (and *KUR*), the phenomena occurring around conjunction. This Babylonian way of prediction is not only very easy and elegant, but also precise. How far back in time these rules were known, we do not know.

But what we do know is that the Lunar Six are complicated functions depending on four different astronomical variables. It has been pointed out as "one of the most brilliant achievements in the exact sciences of antiquity to have recognized the independence of these influences and to develop a theory which permits the prediction of their combined effects" (Neugebauer 1957, pp. 108, 109).

Our present new findings are almost as impressive. We see them as further evidence that the ancient Babylonians were extremely clever at handling observed data. Their easy and elegant way of predicting the Lunar Six deserves our greatest respect.

Acknowledgment

This research was supported by Deutsche Forschungsgemeinschaft.

Notes

1. Normally no difference is made in the Babylonian texts between the symbol for *NA* (new moon) observed after conjunction and that for *NA* (full moon) observed shortly after opposition. In order to avoid confusion, we shall use the symbol NA_N for the time interval observed after conjunction (new moon).

2. The observed data collected on this tablet, LBAT 1431 (also known as BM 34075), have been evaluated by F. R. Stephenson (1974), who judged them to be rather exact.

3. The position of arc($☾_{KUR}$, ⊙) is given by $\lambda_☾$, the position in the ecliptic at which the conjunction will take place. The length of arc($☾_{KUR}$, ⊙) is determined by the relative lunar velocity and the time difference between the sunrise we are concerned with and the conjunction.

4. The Lunar Six and all their combinations are measured in *uš* = time degrees: 1 *uš* = 4 minutes, so that 360 *uš* = 1 day (i.e., the time of a whole revolution of the sky, about 360°).

5. I thank my husband, M. Brack, for his help in computing the Lunar Six in the following figures.

6. We warmly thank Prof. H. Hunger for the translation of this very difficult text.

7. The indices i, I, and $I-452$ refer to the Babylonian months starting on the evening when NA_N is observed. The magnitude of $(NA_N)_{I-446}$ is measured on the first day of month $I-446$, whereas $(\check{S}\acute{U}+NA)_{I-452}$ is measured $5\frac{1}{2}$ months earlier, namely in the middle of month $I-452$.

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