The 360-Day Year in Mesopotamia

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The Cultic or Civil Calendar and the "Administrative Calendar"

From ancient Mesopotamia we know two different "calendars": the cultic or civil calendar and the "administrative calendar".[1] In both calendars, the year was the largest unit. In the cultic calendar, the year was divided into lunar months of 29 or 30 days. In the administrative calendar each month was put equal to 30 days. As we shall see, the administrative calendar was only used in the context of making calculations and never for dating. This latter use was taken care of by the cultic calendar which was defined by the solar year and by lunar phases: the first day of a new month began on the evening when the new crescent became visible for the first time after conjunction. First visibility takes place either 29 or 30 days after the previous first visibility. Since the sequence of 29- and 30-day months is highly irregular the cultic calendar was troublesome to handle. The mean value of the synodic month (the time between two consecutive conjunctions or oppositions) is approximately 29.53 days, so that 12 lunar months equals about 354.37 days, some 10.88 days shorter than the solar year. Therefore, on the average, an intercalated month was necessary every three years or so in order to keep the months in tune with the seasons.

This paper shall present textual evidence for the claim that both systems of time measurements were used simultaneously from the early dynastic time (*ca*. 2600 BC) until around 1000 BC when the astronomical-astrological compendium MUL.APIN was composed. Prior to the early dynastic period there is insufficient evidence to say for certain whether the lunar calendar was used; the protoliterate texts (from *ca*. 3200 BC) have not been understood sufficiently—one can only speculate, that some signs in Uruk III texts may refer to the kind of cultic lunar month known from later periods. However, the number systems in these very early texts are sufficiently well understood to enable us to understand the calculations they contain and to establish that rations in bureaucratic texts were calculated of the basis of the administrative year. The paper shall also clarify how (i.e., for what or in which way) the two calendars were used: the cultic calendar for dating and the administrative one for centralized bookkeeping and later also for astronomy. For example, in MUL.APIN, a canonical astronomical/astrological text perhaps dating from around 1000 BC, we find astronomical schemes in which the ideal year of 360 days is used, together with indications of how to adjust schematic values according to observations.

I start with Ur III texts, which are understood profoundly, and then go backwards in time to the earlier texts. The knowledge gained from the Ur III texts makes it is easier to understand and explain older texts. For textual evidence from earliest times until the end of Ur III times (2000 BC), I rely mostly on R. K. Englund's investigations in "Administrative

Timekeeping in Ancient Mesopotamia" (1988), a careful and systematic analysis of all available texts of interest together with many useful references. The conclusions: Bureaucratic texts from the Ur III period show unambiguously that and how the artificial administrative year was used—namely as a fixed grid opposed upon the varying lunar calendar. In bureaucratic calculations the duration of each month (independent of its actual length) was simply always taken to be 30 days. The same accounting practice is documented for pre-Sargonic Girsu (2400 BC) by feed texts (i.e., texts listing the daily and monthly amount of grain necessary for feeding different types of animals). The tradition can, however, be followed further back. Texts from the protoliterate period use a system of time notation which is very similar to that of the texts from the pre-Sargonic period. And archaic fodder and ration texts firmly establish the correctness of archaic time notation as proposed by Vaiman in the early 1970s.[2] Englund has thus been able to conclude "that in the protoliterate period the same system of administrative time recording was employed as was the notational basis 1000 years later". This means that also at these early times, a month was always calculated as having 30 days and a year as 360 days in administrative records.

There is, hence, clear textual evidence for the administrative calendar from protoliterate time and onwards. This is not the case for the cultic calendar. The number symbols and calculations on protoliterate texts have been understood; but not the other signs which may refer to cultic events connected to the year or lunar phases. The understanding of archaic texts is too limited. But still, most scholars (including myself) tend to believe that daily life from the earliest beginning, i.e., from proto-literate time, was regulated by the sun and by lunar phases, while at the same time the artificial time measuring system was used in centralized bookkeeping. One argument for this claim is the continuity of cult and tradition in Mesopotamia, and the cultic year is evident from the earliest tablets (from 2600 BC), where also the text can be deciphered, and onwards. Another argument is that the administrative calendar is an approximation to the lunisolar calendar. But we have no textual evidence for this conviction. Another possibility would be that the administrative year was used as a calendar in archaic times. This would mean that each month had 30 days, so that the lunar phases were not in tune with the calendar month. New- and full-moon could take place on every day of the month. It is possible, but to me not very plausible, that in archaic times a schematic month and not direct observations, was the basis of time regulation. At least from 2600 BC onwards, we know that cultic events took place on days with special lunar phases and that the civil astronomical lunisolar calendar was used for timekeeping.

In *Der kultische Kalender der Ur III-Zeit*, Sallaberger (1993) analyses Ur III-texts concerning cyclical cult festivals. He comes to the same conclusion that the cultic calendar regulated life while the 360 day year was used for accounting. On p.11 he writes:

> Ein Mondmonat (synodischer Monat) dauert 29d 12h 44' 2,9'', es wären also etwa abwechselnd Monate zu 29 oder 30 Tagen zu erwarten. In Ur III-Urkunden über Arbeiter, Rationen, Futter oder in Bilanzrechnungen wird dagegen ein normierter 30-Tage-Monate verwendet. Dieser 30-tägige Normmonat dient offenkundig nur der Berechnung im administrativen Bereich und gibt keine Auskunft über den allgemein gebrauchten "bürgerlichen" Kalender.

I refer to the work of Eleanor Robson (1999) and of Jöran Friberg *et al.* (1987–90) in order to demonstrate the continuous tradition of the artificial administrative year (of 12 × 30 days) until it appears in astronomical tables in the astronomical/astrological compendium

MUL.APIN. Eleanor Robson has shown that the training of Old Babylonian scribes demonstrates a continuous tradition leading back to Ur III administrative practices and mathematics: work rates known from Ur III times show up in Old Babylonian school mathematics and coefficient lists. And Jöran Friberg has been able to document that an unbroken tradition ran from the well-documented mathematics of the Old Babylonian period to the mathematics of the Late Babylonian period more than a millennium later.

Since the argumentation on the archaic texts depends heavily on the archaic systems of number notation, I shall start with a short outline of early counting.

Early Accounting Techniques and Writing

Tokens (and counting symbols) have been used since the 8th millennium BC. D. Schmandt-Besserat (1977) noted that many decorated tokens from the 4th millennium bear a striking resemblance to signs on the earliest tablets, so she interpreted them as three-dimensional precursors of the two-dimensional proto-cuneiform signs. Whilst some of her conclusions remain controversial, what has been generally accepted is her argument that simple tokens served as precursors of number signs.

Writing was invented toward the end of the fourth millennium BC within a growing society with an urgent need of economic administration: the purpose of the earliest clay tablets with proto-writing was to record numerical information.

In archaic administrative texts one finds a variety of number symbols and counting systems. The number symbols were produced by pressing round sticks of different diameters perpendicular or slanted into clay (see figure 1). A. Vaiman and J. Friberg were the first to investigate the archaic accounting texts.[3] Friberg showed that the number symbols were dependent on the context: the •, when used for counting items or animals, meant 10 while it was used for the number 6 within the recording of quantities of grain. Later a group in Berlin, analyzing a large amount of archaic Texts from Uruk, was able to identify 60 different number signs (N_1, N_2, ... N_{60}), which were used in five basic and a further five derived number sign systems. The symbols show a great variation as if they were slightly altered for different purposes. Analysis of texts of archaic bookkeeping, on which the balance, i.e., the sum of all contributions is recorded on the back, led to the recognition of all the different counting systems.[4]

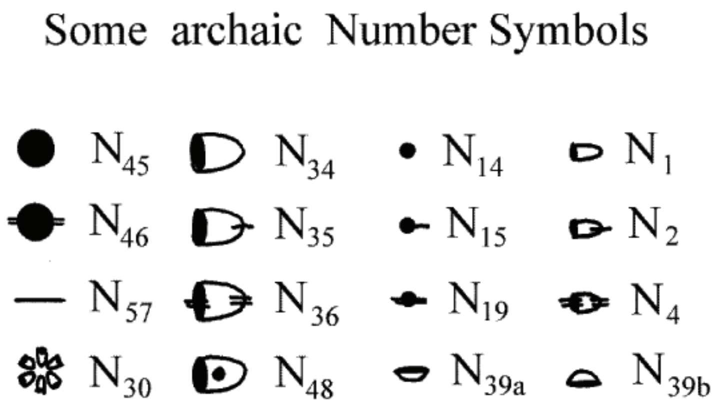

FIGURE 1. Some of the sixty different number symbols found on archaic tablets.

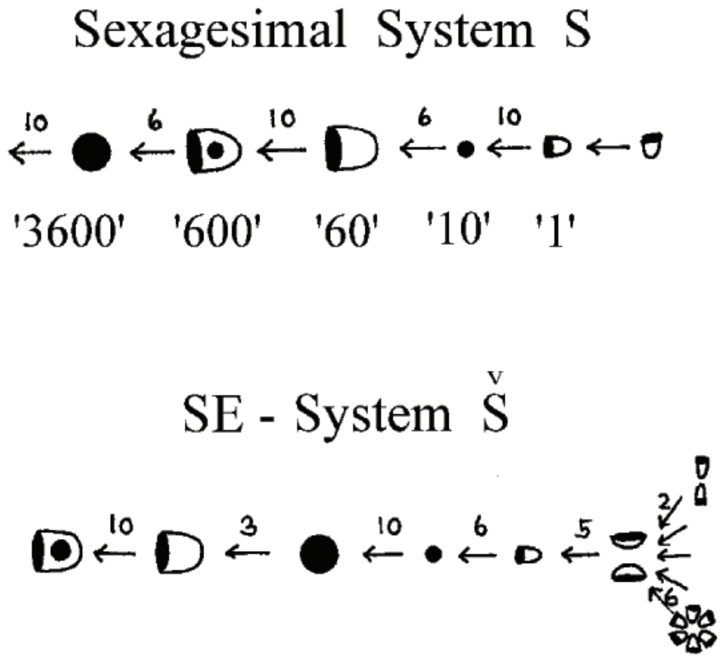

FIGURE 2. Two of the counting systems: the sexagesimal system S (used for counting discrere objects) and the system Š (used for the recording of volumes of grains, mostly barley).

The order in which the number symbols occurred on the tablets helped identify higher counting units, and since the sum of all contributions was recorded, the factor between the different number symbols could be determined. At this place we only show two of the archaic numbersign systems: the sexagesimal system S, which was used for counting discrete objects, and the ŠE-system Š, which was used for recording volumes of grains, mostly barley (see figure 2). Note the different interpretation of the symbol • = N_{14}: In system S, • equals 10 units (• = 10 N_1) while in system Š it must be read as 6 basic units (• = 6 N_1). We see that number symbols were dependent on the context. This has been discovered only rather recently by Friberg (1978).

The archaic system S was later developed into the well-known sexagesimal place value system used in all cuneiform texts from Old Babylonian times onwards. Here all numbers from 1 to 59 were written by means of the two signs 𒁹 and 𒌋. The cone, 𒁹, which is the later version of N_1 for one, and the angle, 𒌋, which is the later version of N_{14} = • for 10. For instance, the number twenty-three would be written as 𒎙𒐈. That all numbers could be written by means of only the sign for 1 and that for 10, led earlier scholars to the incorrect conclusion that at early times, a decimal and a sexagesimal number system have coexisted. However, the postulate of two competing systems, the sexagesimal and the decimal system, originates in the incorrect reading of • = 10. We now know that • can be read as 6 in case of measuring grain, and as 10 when used for counting things, and also as 18 when • was used for notations on the sizes of fields.

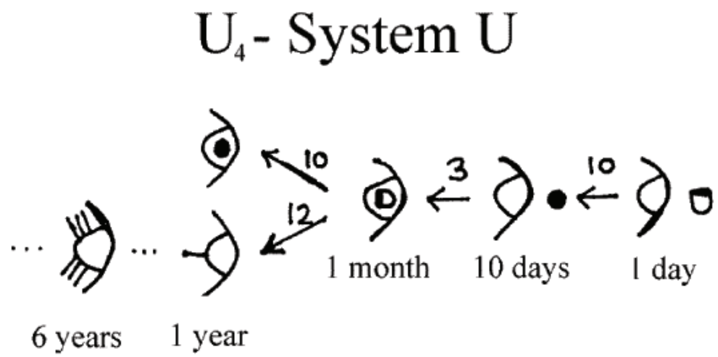

FIGURE 3. System U was used for recording time intervals. The common sign U_4 is assumed to represent the sun rising among the eastern mountains of Mesopotamia. Note that the sexagesimal system S was used for counting months and days. Vaiman proposed that numbers inside U_4 counted months and numbers after the sign U_4 counted days, while strokes in front of the sign U_4 counted years.

Archaic administrative texts exhibit a variety of counting systems. Throughout the third millennium BC, concurrently with the increasing demands of large-scale state bureaucracies, the counting and measuring systems were gradually revised and the way of writing numbers was changed. At the end the sexagesimal place value system, as we know it from astronomical and mathematical texts, was derived. In the early Ur III period, the king Šulgi introduced major bureaucratic reforms, the sexagesimal system sometimes being used in calculations although the results were given in the traditional metrological system. At the same time mathematical methods for accounting were developed and standardized, for example for managing labour and resources, or for the calculation of expected (or needed) labour, but the calculations were still based on the administrative year of 12 months of 30 days. Scribes were also trained in mathematical methods, enabling them to fulfil duties in accounting and administrating. From the earliest phase and throughout the history of cuneiform, tablets were produced for the education of scribes. Many such tables happen to survive from the Old Babylonian period, the Old Babylonian mathematical texts being a rich source of early mathematics. Mathematics was an integral part of Mesopotamian scribal culture.[5]

Next I shall present the textual evidence for the claim that the administrative and civil calendar were used simultaneously.

Time Notations in Ur III Texts

The administrative calendar and the cultic or civil calendar are both clearly attested in administrative texts from Ur III. More than that, many texts from Ur III times (*ca*. 2100 BC) exhibit simultaneously the use of these two parallel systems of time division. But as we shall see, daily life was regulated by the lunar calendar while the administrative calendar was used only for bookkeeping.

During the Ur III period cultic festivals are attested at significant times during the agricultural year and on notable days in the cycle of lunar phases. As a result the cultic year was naturally divided into 12 (or 13) synodic months, where the beginning of a new month could be found by observing the moon. The astronomical lunisolar calendar at this period was quite irregular as no easy system for organising the intercalation of the 13th month had been found. Furthermore, the duration of the synodic month, which can be either 29 or 30 days, varies in a very irregular pattern. Nevertheless, life and cultic events were structured by this cultic calendar.

At the same time, scribal computations were carried out by an administrative system of time division in which the year consisted of 12 months of 30 days each. In contrast to the irregular lunisolar calendar, the 360-day year is very practical for bureaucratic calculations—it simplifies calculations and at the same time it increases the state's demands on labour. Independent of its actual length, each synodic month was simply calculated as having 30 days. As an example of this practice we can consider an Ur III text published by Englund (1988), p.126. The text calculates the number of workdays which a group of 36 female workers should deliver during a year of 12 (lunar) months.

> 36 female workers
> from the "harvest(-festival)" (month 1, Umma calendar)
> through "Tammuz(-festival)" (month12),
> performance involved: 12,960 days

The total number of workdays, 12,960 days, equals 36 × 12 × 30. We see that the expected labour was calculated as the product of the number of workers and the period recorded. The text also illustrates that the accounting year (of 12 months from month 1 through to month 12) was calculated as having 360 days.

By similar texts, Englund (1988), p.128 has shown, that a diri year (a year of 13 months) was calculated as having 390 days. For example:

> 20 ^{gis}gid$_2$.da–workers,
> ...,
> and 4 porters
> from "Harvest(-festival)" (month 11, Girsu calendar)
> of the year "The throne of Enlil was constructed" (Amar-Zu'en 3)
> through "Amar.a.a.si" (month 10)
> of the year "Enmahgalana was installed as priestess of Nanna" (Amar-Zu'en 4)
> performance involved: 9360 workdays, male workers,
> it is (a period of) 13 months,
> including one extra month.

Again the text covers the period of a year, staring with the beginning of month 11 and running until the end of month 10 the following year. But the text indicates (by "13 months") that in this case, an extra month has been inserted. Since 24 × 30 × 13 = 9360, we can conclude that this civil diri year was reckoned as having 30 × 13 = 390 days.

Note the month name "Harvest(-festival)" of the civil Girsu calendar (agricultural festivals were celebrated by sowing and harvest). The civil/agricultural calendar was evidently used for recording dates on this administrative tablet. Englund provides other examples of bookkeeping tablets where the civil/agricultural calendar and the parallel administrative calendar were used simultaneously within the same text.

The division of the year into 12 lunar months in the cultic or civil calendar was dictated by the moon cult: Ur III texts tell that cultic offerings took place on the 7th, 15th and new moon days. Evidently, the moon phases first quarter, full moon and new moon are meant here. It is highly improbable that the new moon celebration should not take place at real new moon but, instead, on the first day of an administrative 30-day month. Had the 360-day year been utilized as a calendar, then the first day of the administrative month could fall on every lunar phase, since the 30 day month shifts 0.5 day per month in comparison to the lunar month. In addition, Ur III deliveries of offerings for the 15th (full moon) are received at the end of the 12th day, while new moon offerings are recorded only on the 27th to the 30th days of particular months. Here, evidently, the civil calendar is used: within the administrative month of 30 days, the full and new moon would wander and could take place at every date throughout the month.[6]

In his dissertation from 1993, Sallaberger analyses Ur III archives in the search for evidence for the civil lunisolar calendar. In texts connected to agricultural and lunar cults or festivals, he was able to find many examples of evidence for use of the lunar/agricultural calendar, i.e., the civil/cultic calendar. Let me just summarize some of the evidence in the following.

Each month in Gula either 29 or 30 carcasses of dead cattle were delivered, a clear hint to the changing length 29 or 30 days of the lunar month. The sum of delivered carcasses over some whole years were recorded to be 384, 354 or 353, respectively. Thirteen lunar months have 384 days, while 12 months encompass 354 days. Evidently, the civil calendar was used here and not the administrative calendar. Finally, texts on cultic actions testify that the month was regulated by the real moon: some purifying bath rites were performed by the queen on the "day at which the moon rests" (= "Schwarzer Mondtag" = day of conjunction). Obviously, these cultic actions must have been performed at days where the moon was absent. Lists shows that carcasses of small cattle for these rites were always delivered during the days from the 24th to the 27th of the month, indicating that the rites took place at the end of the lunar month.

Further evidence for the fact that the cultic year and not the administrative year was used for regulating civil life can be found by the frequency of "diri" years with a 13th intercalated month. The civil calendar requires an intercalation every three years, while the administrative year of 360 days would only require an additional month every six years. A text from the time of Šulgi writes that six intercalations had taken place within 16 years which is only possible in the cultic calendar:

> Tablet basket:
> accounts
> of Nalu
> from the "Gazelle-eating (festival)" (month1, Drehem calendar)
> of the year "Simurum was destroyed for the second time" (= Šulgi 26)
> through the extra month "Harvest(-festival)" (month 13)
> of the year following the year following "The house/temple of Drehem was built" (= Šulgi 41),
> It is (a period of) 16 years, including 6 extra months

This and other Ur III texts confirm the pattern of an intercalation (on the average) every three years, indicating that the cultic calendar was used.[7] We see that daily life and cultic events were organized by the synodic calendar, while at the same time bureaucratic calculations utilized the administrative calendar. In their dependence on the parallel lunisolar year, Ur III administrators inserted into the administrative year an intercalary month of 30 days, again in principle, every three years, in tune with the intercalations for the cultic year. In other words:

each lunar month was just reckoned as having 30 days independently of its actual duration.

Finally, the administrative bakery text TUT 102 (originating from Telloh and dated to Ur III times) can be mentioned.[8] It records 59 (60 lal 1) days in months 3–4 of the Lagash calendar. Obviously, in this administrative text the cultic calendar is attested.

To conclude: Ur III texts provide clear evidence for the simultaneous use of the two calendars. The administrative calendar facilitated calculations and increased the state's demands. I would describe the artificial year (with months of 30 days) as a fixed grid opposed to the real and variable year and adjusted to it by adding an extra month in tune with the intercalations necessary for the civil year. For dating and normal/cultic life, the lunisolar calendar alone was used. The 360-day year was only used in connection with accounting.

Presargonic Texts from Girsu

The use of the 360 day year can be demonstrated also for the presargonic period (*ca*. 2500 BC – 2350 BC) through ration and feed texts, which calculate the food needed for fattening swine of different ages. Knowing the pre-Sargonic Girsu gur system, Englund (1988) pp. 141–142 could show how, for example, the monthly amount grain needed to feed the largest wild boar was found by multiplying the daily need by 30. The animals are fed according to their age which is given in years (e.g., šah$_2$ .. mu.2 "swine in its second year"). The older the animal, the more food it was given. The barley needed for feeding N animals during a month is found by multiplying the daily ration R_i (for age$_i$ swine) by N times 30:

$$\text{Monthly need for N (ration } R_i\text{)-swine} = N \times 30 \times R_i$$

The yearly need equals 12 × 30 × the daily need. From texts such as this we can conclude that the year of 12 × 30 days is well attested as an administrative time unit in pre-Sargonic Girsu. In other words, in texts calculating the food for animals, the administrative calendar was used.

However, we also have evidence for the use of the lunisolar calendar in texts from pre-Sargonic Girsu. Many of the 24th century texts, recording quantities of feed for animals, show a mixture of the two calendar systems within the text.[9] For example, the colophon of an administrative text mentions the "Malt-eating festival" month of the cultic calendar. This month is administratively reckoned as the ninth 30-day rationing period. Obviously, the "malt festival" month of the cultic calendar has here been identified with the ninth schematic month. Other presargonic texts show that the artificial administrative and the traditional agricultural year have existed side by side, and that the administrative year sometimes would have an extra 13th month of 30 days. As it was the case for the Ur III period, it is also true for the presargonic period that the normal year of 12 month was reckoned as 12 × 30 days = 360 days, while the intercalary years would have 13 months of 30 days which equals 390 days.

The pre-Sargonic texts are important for the understanding of archaic texts. The cuneiform signs for one month and one day (see figure 4), used in ration and feed texts from pre-Sargonic Girsu, are so similar to those proposed by Vaiman for the Uruk IV and III periods, that his reading is clearly established.

Archaic Protoliterate Texts

How was the situation in even earlier times? Archaic protoliterate texts may contain information. However, from such tables only the numbers and time notations can be read with certainty,

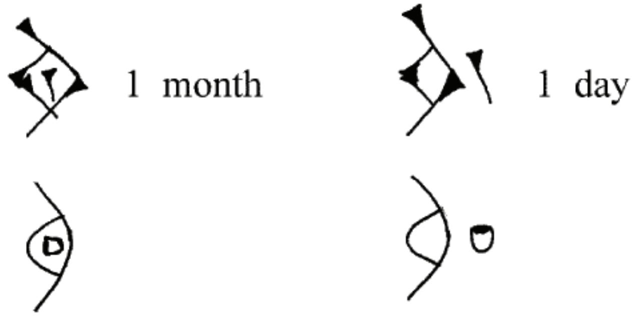

FIGURE 4. The signs (top line) for 1 month and for 1 day used in feed and ration texts from pre-Sargonic Girsu. These signs are very similar to the archaic time symbols (shown underneath), which Vaiman proposed to mean 1 month and 1 day respectively.

while it is much more difficult to understand the text. Archaic texts from Uruk (3200 BC – 3000 BC) concern centralized bookkeeping. They document the same artificial division of the year into 12 months of 30 days each that we know from later periods. Analyses of archaic accounting texts have led to the conclusion that the same system of administrative time reckoning was employed to that used 1000 years later. Exactly as we know it from later times, the archaic ration or fodder texts are calculated on the basis of the 30-day month. The rations for, say, M months are always calculated as M × 30 times the daily ration. The key for understanding the archaic administrative texts is the knowledge of archaic number-sign systems and archaic time notation. This knowledge allows us to find the factor between rations for different time intervals and to establish the great similarity with accounting methods found in later texts (see figure 5).

Note that, according to the number system Š (see fig. 2), 4 N_{14} equals $4 \times 6N_1$ (= 24 monthly rations), which again equals $24 \times 30\ N_{30} = 720\ N_{30}$ = 720 daily rations. Many texts make it apparent that the NINDA was the daily grain ration for a worker. It represented a grain quantity equal to the sign $N_{30} = \frac{1}{30}N_1$.

A group in Berlin working on archaic administrative texts has proposed that the bevelled-rim bowl with an average capacity of 0.8 litre was used as a model for the pictogram NINDA, which represented one day's grain ration for a worker. They propose to read the head with a NINDA bowl in front of it as the sign for ration.[10]

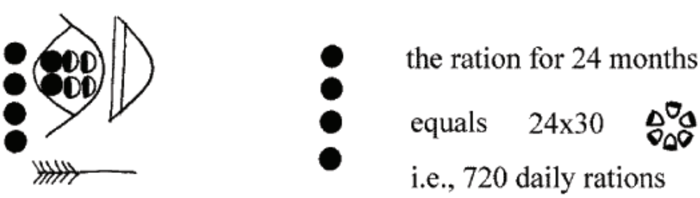

FIGURE 5. Excerpt from an archaic text (shown on the left side). We see the gran quantity of 4 N_{14}, the sign for 24 months and the NINDA bowl. The ration corresponding to 24 months is, indeed, 4 N_{14}: according to the SE-system S (figure 2), 1 N_{14} = 6 N_1, 1 N_1 = 5 N_{39}, and 1 N_{39} = 6 N_{30}. Here the administrative calendar has evidently been used for calculating rations.

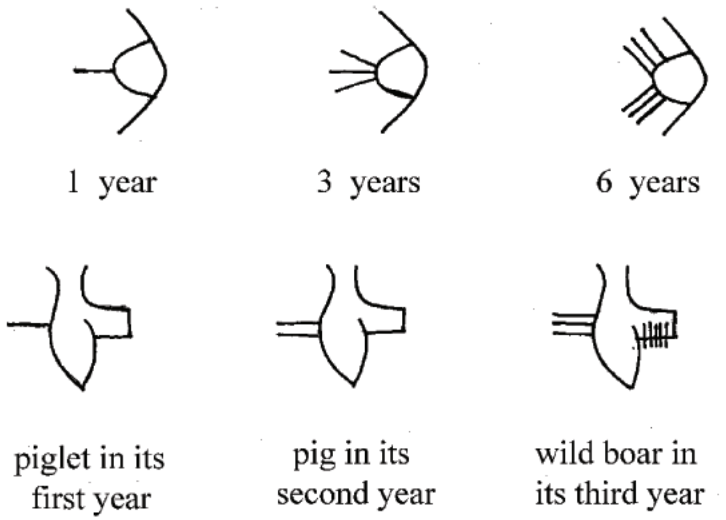

FIGURE 6. Below, the signs for piglets in their first and second years plus wild boar in the third year. The dash in front of the sign for pig is here used for indicating the age. In agreement with Vaiman's proposal, the upper signs can be read as one year, three years and six years.

Due to the still limited understanding of the archaic texts, Englund is still not able to offer solid evidence of the cultic/agricultural calendar being used in the protoliterate period. This would presuppose a higher linguistic level of decipherment than afforded by the now solid clarification of the arithmetical operations behind administrative time keeping in archaic sources. However, both J. Friberg as well as Englund see textual hints to the cultic calendar in archaic times although they are currently unable to give a semantic justification.[11] For archaic texts which might contain evidence of a cultic calendar, see Englund (1998), p. 127 and (2001), pp. 20–21). Based on other texts, Englund presents reasonable arguments for the reading of a colophon as a date, referring to the month "new growth festival".

If we accept the judgment of Englund and Friberg that the archaic texts refer to lunar festival-months, then we must conclude that the lunisolar calendar was the primary time counting system and must have been used first. This would indicate that in archaic times, as in later periods, the 360 days year consisting in 12 months of 30 days was just a practical approximation to the year of 12 synodic months. It seems plausible, since one cannot construct an approximation out of the blue.

The final proof for the correctness of Vaiman's reconstruction of archaic time notation can be found in texts on feeding pigs. As was the case with ration texts from presargonic Girsu, the archaic texts list pigs according to their age in years. Englund has shown that the accounting system, known from Girsu texts, was already used in archaic texts, so that the signs for 1-, 2-, and 3-year old pigs could be identified. The dash, which Vaiman proposed for indicating years in front of the "sun at horizon sign", is here used in front of the sign for pig to indicate its age. Piglets in their first year were listed by the sign for pig with one stroke in front of it, and pigs in their second (or third) year were written by two (or three) strokes in front of the pig sign (see figure 6).

We conclude that the continuous and parallel use of the two calendar has been shown all way through from 2600 BC to 2100 BC, while in the archaic texts, only the administrative calendar is clearly demonstrated. We shall now look for evidence for a continuation of this tradition and show that the scribes sometimes used the approximate year of 12 × 30 days in later periods as well.

The Calendar in the Old Babylonian Period and After

Most scholars assume that the lunisolar calendar was used by the Babylonians all way through until the end of the Seleucid Era. The reason for this is that the real month was basis of the civil calendar in Ur III times and also of the calendar used in the Late Babylonian Astronomical Diaries to record astronomical observations and predictions, historical events, and prices. Whilst there is no direct Old Babylonia evidence for the 29-day month it seems very improbable that during the intermediate period between Ur III and the last few centuries BC a switch would have been made to using the schematic year as the civil calender.

A hint to the use of the lunisolar calender during the Old Babylonian period was found by Landsberger (1949) in the text CT 2, 18, an administrative doument calculating wages and other disbursements. Some ud.da.gid.da days, recorded in this text in connection with time intervals, were interpreted by Landsberger as representing the correction between the schematic and the true (lunisolar) calendar. This would seem to be a nice confirmation of the parallel use of the two calendars. However, it must be noted that the text is corrupt. Michel Tanret (2004) offered a new interpretation of the text CT 2, 18. Referring to inconsistencies in the text, he came to the conclusion that the calendar year had 360 days, so he questions the use of a real lunar calendar in Old Babylonian times. However, Peter Huber (private communication) has given several strong arguments for a lunisolar calendar during the Old Babylonian period: for example, during this time (as well as for Ur III times), an intercalary month was inserted, on the average, every three years, as we would expect in a lunisolar calendar. The schematic calendar of 360 days would only need an intercalation every six years.

The text CT 2, 18 is too corrupt to settle the matter, but Landsberger's interpretation of the ud.da.gid.da days seems to me to be correct. In any case, the evidence for the real month and the lunisolar year to be the basis of the calendar in Ur III and in Old Babylonian times is strong. Therefore, I take it for granted, that the civil calendar as means of dating was used continuously from Ur III through Old Babylonian times to the end of the Seleucid Era. We shall now look for textual evidence for parallel utilization of the approximate year of 12 × 30 days during these later periods.

The Archaic Administrative Calendar and the "Ideal Year"

The civil calendar as means of dating is attested continuously from Old Babylonian times to the end of the Seleucid Era. In the astronomical/astrological compendia *Enūma Anu Enlil* tablet XIV and MUL.APIN, a so-called "ideal" or "schematic" year of 12 × 30 days is used. Omen texts, letters and reports show that it was interpreted as a good omen when a new month started on day 1 (which means that the month before had 30 days). And it was a bad omen when a new month started on day 30 (indicating that the former month had only 29 days). Obviously, it was taken as a good sign when nature behaved as the "ideal" 360-day year and a bad sign when nature deviated from the "ideal calendar".[12] Great efforts were made for finding rules enabling the astronomers to predict the length of a month to come. The Late Babylonian procedure text TU11 gives several methods for the prediction of "full" months

Month	Day	Zodiacal Sign			Degree	Advice: "... you brush"
IV	1	1	=	♈	7	sheep-blood, sheep-fat and sheep-hair you brush
IV	2	10	=	♑	14	goat-blood, goat-fat and goat-hair you brush
IV	3	7	=	♎	21	"empty place"
IV	4	4	=	♋	28	crab-blood and crab-fat you brush
IV	5	2	=	♉	5	bull-blood, bull-fat and bull-hair your brush
.	.	.	.	.	.	.
.	.	.	.	.	.	.
IV	30	4	=	♋	30	crab-blood and crab-fat you brush
V	1	2	=	♉	7	bull-blood, bull-fat and bull-hair you brush

TABLE 1. The *Kalendertext* scheme for Month IV: through columns one to four each day of Month IV is associated with a position in the zodiac, while column five mentions blod, fat, hair or other body parts of the animal that corresponds to the zodiacal sign relevant to the day in question.

(i.e., of 30 days) or "hollow" (29 days) months.[13] Many mathematical astronomical tablets of the Seleucid period aim at calculating the day of new crescent and the time NA_N from sunset to the setting of the crescent.[14] In the following I will show that the lunisolar calendar and the "ideal" calendar coexisted all way through from Old Babylonian times (1800 BC) to around 300 BC, and that the "ideal" calendar just is a continuation of the artificial year of accounting.

The so-called *Kalendertexts* may give a good illustration for the coexistence of the two calendars. The different types of *Kalendertext* are explained in Brack-Bernsen and Steele (2004). One type of *Kalendertext*, relates dates to positions in the zodiac. The text, W.22704,[15] from the end of the 4th century BC, may serve as an example of this type. The text has the form of a scheme consisting in 5 columns. In the first column, the name of Month IV (ŠU) is written (or the repetition sign MIN after line 1). Column two records the day numbers from 1 to 30. The entries in columns three and four record a zodiacal signs and a number of degrees within that sign, and finally the last column list for each entry ingredients that are usually linked to the zodiacal sign given in column three. For example, when the sign is Taurus, then blood, fat, and hair of a bull are mentioned (see table 1).[16]

For each day of Month IV the scheme indicates the ingredients which may be used in some ritual actions. But in the *Kalendertext*-schemes, the months are always 30 days long, so we are not dealing here with a real lunar calendar, but instead with the schematic or "ideal calendar" of twelve 30-day months. The *Kalendertext*-schemes are constructed on the basis of the practical "ideal year" and the zodiacal positions relating to dates were given for 30 days of each month. But we know that the calendar used at this time was the well-known astronomical (civil/cultic) lunisolar calendar, with months of 29 or 30 days length. Sometimes, the scheme would have a day more than the real month; but of course, the *Kalendertext*-scheme could still be used to determine a zodiacal position for each day of the real calendar.

As mentioned above, one encounters the so-called "ideal year" in *Enūma Anu Enlil* tablet XIV and MUL.APIN. In idealized schemes, the lengths of day and night are given as functions of the date within the schematic year consisting of 12 months of 30 days each. In these compendia, the length of night is then used as a generating function for finding the "visibility times" for the moon. The Late Babylonian procedure text TU11 testifies to many more astronomical quantities derived from the length of day or night. We see that the ideal year was utilized heavily in the early formation of numerical astronomical theory.[17] Even in the so-called ACT texts of mathematical astronomy from the Seleucid and Parthian periods we find traces of the "ideal year" in the useful unit *tithi* = 1/30 synodic month. I have no doubt that the "ideal year" was simply a continuation of the old accounting practice from the Ur III period and the *tithis* of Late Babylonian astronomy a practical reminiscence of the 360-day year.

Only a few non-astronomical texts from later times prove that the schematic 360-day-year was still considered by the Babylonian scribes. A text with a prayer from the time of Ammizaduga refers to the year (from 20th of Nisan to 20th of Nisan) as having 6 times 60 days and nights.[18] Similarly, some mathematical exercises and coefficients of work are based on the practical approximation that each lunar month was counted as 30 days, independently of its actual length. There has not, however, been found any single administrative text from the later time of the kind which could prove that the schematic 360-day year was used in everyday practice. From the Old Babylonian period and onwards here exists no evidence of such advance planning that gives the ration for the whole year and also the daily ration. Maybe the way of registration and control changed after the Ur III period? We know that Šulgi had introduced major changes into the accounting system (e.g., by forcing through that only the sexagesimal number system be used by calculations). Indeed, E. Robson (1999), p. 171 concludes that "scribal students in the Ur III period were taught to perform their calculations in sexagesimal notation" (see also *ibid* p. 169). This new practice may explain the "missing texts".

We do, however, have evidence of the 360-day year being used in the education of scribes. Some mathematical texts used in scribal schools state a problem and give a model solution. Such texts show how Babylonian mathematics functioned and what pupils were to learn. The subject of many problem texts is clearly connected to bureaucracy and administrative accounting, as is described in the Neo-Assyrian "Examenstext A".[19]

> Do you know multiplication, how to find reciprocals and coefficients, bookkeeping, administrative accounting, how to make all kinds of pay allotments, (can you) divide property and delimit shares of fields?

One interesting problem text, the Old Babylonian mathematical text BM 85196 (xiii), is discussed by Robson (1999), pp. 80–81. The text makes use of the 360-day year. The problem consists in finding how much time it will take one man to carry 6,00 (= 360) sheaves over a long distance: Three hundred and sixty sheaves are arranged in a line, each 5 nindan ($\simeq$ 5 $\times$ 6 metre) apart, so that the furthest is 30,00 nindan ($\simeq$ 10.8 km) and the closest 5 nindan away. A worker has to carry them, one at the time, to the same place. The problem is to find out how long a time it will take the carrier to do it. The text calculates the mean distance to be

$$(30,00 + 5) \div 2 = 15,02;30$$

and by multiplying it by 6,00 (the number of sheaves), the sum of the distances, over which the sheaves have to be carried, is correctly found to be[20]

$$6,00 \times 15,02;30 = 1,30,15,00$$

i.e., 324,900 nindan ($\simeq$ 1950 km). This total distance is then divided by 45,00 nindan, which is the daily distance a worker is expected to carry the goods

1,30,15,00 ÷ 45,00 = 2,00;20

Note that the daily distance 45,00 nindan gives only the length over which the sheaves are carried. Therefore the worker going forth and back is expected to walk twice as long, i.e., 1,30,00 nindan (≃ 32.5 km) a day.

The text has shown how to solve the problem and ends with the following solution:

> You will see 2,00;20. He carried here in 4 months and ⅓ day.

This last remark attests evidently the use of the 360-day year: Since 2,00;20 = 120⅓ we see that each of the four months was calculated as 30 days. My point is, that the scribes or pupils working with such texts were aware of the of the practical approximation of 30 days to the lunar month—in other words, they knew about the artificial year of 12 × 30 days.

The Coefficient Lists

From Old Babylonian times and later we have coefficient lists and mathematical problem texts using coefficients. These coefficient lists also indicate that the administrative 360-day year was used in Old Babylonian scribal training. A coefficient, called IGI.GUB in the cuneiform texts, is a parameter which is not given in the statement of a problem but is essential for its solution (for example, 45,00 the daily rate of carrying sheaves). The scribes were supposed to know the coefficients or where to find them in order to solve the problem.

We use π for finding the area of a circle. The Old Babylonian method for finding the area of a circle from its circumference goes as follows: one calculated the square of the circumference and multiplied the result by 0;05. The coefficient 0;05 = 1⁄12 used for finding the area of the circle was given in many coefficient lists: "0;05, the coefficient of a circle" or "0;05, of the area of a circle". Geometrical coefficients were used for finding areas or volumes while other coefficients were used for Old Babylonian administrative calculations in mathematical problem texts.

Daily working rates were given by coefficients: a worker was expected to carry the load of 540 standardized bricks over 30 nindan in a day, or to demolish the volume of 0;20 sar_v of a wall per day. The working rate for a month was just 30 times the daily working rate. Evidently, these coefficients and the calculations in mathematical texts utilize the artificial year of 360 days, whilst daily life was regulated by the lunisolar calendar.

That these Old Babylonian coefficients and calculations, indeed, go back to the Ur III period has been shown by Robson (1999), p. 160. Here she has published an Ur III text that gives a detailed record of demolition work. A wall of length 22 nindan, average width 2 cubits (= 2 × 1⁄12 nindan), and height ½ nindan 2 cubit shall be demolished, the total volume being 29 ⅓ sar_v. The text states: "the wages per worker are ⅓ sar_v each". Then it gives the correct answer: "Its workers are 1,28 for 1 day" (29⅓ × 3 = 1,28). The Ur III wage per worker = ⅓ sar_v each, equals the coefficient 0;20 sar_v of wall per day known from Old Babylonian texts. Robson concludes: "In other words, this [Ur III] document provides unequivocal evidence that the daily rate of demolition used in Ur III quantity surveying calculations was later used in OB scribal training".[21]

Analysis of Ur III work rates, Old Babylonian coefficient lists, and Old Babylonian mathematical texts have thus shown that accounting practices involving the artificial year of 12 × 30 days in either administrative or scribal training contexts are found throughout the time from Ur III to the Old Babylonian period. And from this time onwards, we have evidence for the utilization of the artificial 360-day year in "astronomical" texts.

Already during Old Babylonian times, the schematic year of 12 months of 30 days was used for recording astronomical regularities. The Old Babylonian text BM 17175+17284 contains a scheme that connects day length and season (time within the year).[22] The text places the solstices and equinoxes on the 15th of Months XII, III, VI and IX and the day length varies linearly between 2 minas and 4 minas. This text is a forerunner for the "first intercalation scheme" in MUL.APIN. Having thus shown that the "ideal year" of MUL.APIN can be seen as part of a old tradition, originating in the 360 days accounting year from archaic times, I shall briefly mention how this "regular grid" was used in the early formation of astronomical theory.

Within the astronomical schemes of MUL.APIN, the ideal year can again be interpreted as a grid imposed upon and adjusted to nature. This can be seen by the fact that the "first intercalation scheme" indicates some observations aiming at finding some "extra days" which tell how far, for example, the real spring equinox is from full moon (day 15) of Month I. In the ideal case, solstices and equinoxes were assumed to take place in the middle of Month I, IV, VII, and X, and the length of the night was found through linear interpolation between the extreme values, 2 minas on day 15 of Month IV and 4 minas on day 15 of Month X. Therefore, in the ideal case, the duration of day and night would be 3 minas on day 15 in Month I and VII, and the sun would rise straight east. Imagine a grid with bars, representing the time axis with intersections in months: the bars being situated 30 days apart, each of them marking the beginning of a month. In the ideal case, sunrise straight east would happen at full moon (day 15) of Month I, and the schematic length of day and night would be as given in the ideal scheme. In the normal case, however, sunrise straight east did not take place at full moon. The grid, being dependent of the real month, would have to be adjusted—pushed in such a way—that for the specific month in question, the bar would coincide with the time of first visibility of the new crescent.

In this case, sunrise straight east would not take place on day 15; the bars would not be situated symmetrically around this sunrise which tells that day and night will be equal. The shift in days between the ideal place of the grid and its actual place could be observed, and the length of the night at full moon (on day 15) would have to be corrected accordingly. In MUL.APIN there are, indeed, indications for such an interpolation, necessary for finding the actual length of the night at full moon of Month I. In Brack-Bernsen (2005), I have analyzed two schemes of MUL.APIN and argued for the fact that "ideal values" were adjusted. In the so-called "first intercalation scheme", the lengths of day and night were given for the middle of Month I, IV, VII, and IX, together with indications where on the eastern horizon the sun would rise. The "water-clock scheme" was derived from the "first intercalation scheme": the length of the night (given in the "first intercalation scheme") was used as a generating function for finding the "visibility of the moon" which was recorded for the 1st (new crescent) and the 15th (full moon) of 12 ideal months. Equinoxes and solstices do normally not take place at full moon, but some "extra days" away. Therefore the value of visibility was corrected by interpolation according to the shift in night length imposed by the "extra days".[23] That the "ideal year" of 360 days is meant as an approximation to the real lunisolar year becomes also clear, when MUL.APIN writes about intercalations every three years and calculates the effect of such intercalations (Tablet II ii 11–17).

The concepts behind the two schematic calendars (the archaic administrative calendar and the ideal year) are quite similar, and so are their dependences on the real lunar calendar. Both serve as a means of simplifying calculations, and both are coupled to the real lunar calendar and adjusted accordingly. The archaic calendar was adjusted by adding an extra month in tune with the intercalations necessary (on average every three years) within the lunisolar calendar. This means that each lunar month was just calculated as having 30 days independently of its actual length. The "bars" of the grid did always coincide with the beginning of a new lunar month. The same is true for the ideal calendar when used much later in astronomical schemes—the ideal year can be seen as the continuation of the artificial year of accounting. Therefore, it is just the logical consequence, that the daylength (together with all derived quantities) would have to be corrected when the "bars" were displaced with respect to their ideal places symmetrically around solstices and equinoxes.

Conclusions

We have seen how the administrative and the cultic/civil calendars coexisted all way through from the earliest literate time (*ca*. 2600 BC) until *ca*. 300 BC. The astronomical lunisolar calendar regulated life, while the artificial accounting or ideal calendar (which I see as a practical and regular approximation to nature) could be used for calculation.

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Notes

1. For lack of a better expression the word "calendar" is (in the artificial "administrative calendar") applied for denoting the practical way of reckoning time, where each month (independently of its real duration) is approximated by 30 days.
2. See Vaiman (1989) and (1990), where many of Vaiman's earlier papers, written in Russian, have been slightly revised and translated into German.
3. Vaiman (1989 and 1990), Friberg (1978).
4. See Green *et al.* (1987, pp. 117–166) and Nissen *et al.* (1990, pp. 61–65).
5. The Sexagesimal system is attested in the Uruk IV – III Periods (3100 BC – 2900 BC), i.e., in periods much earlier than any secure attestation of the Sumerian Language (see Englund (1988), p. 122). Powell (1972) sees the situation differently. He has argued for the fact that within the Sumerian language number words have a sexagesimal structure and so concludes that the sexagesimal notations of the period must have originated in a spoken Sumerian.
6. See Englund (1988), p. 123, footnote 2.
7. See Englund (1988), p. 123.
8. Englund (1988), p. 123, footnote 3.
9. Englund (1988), pp. 143–147.
10. See Nissen et al. (1990).
11. See Englund (1988), p. 133, footnote 10.
12. See Beaulieu (1993) and Brown (2000), pp. 146–153.
13. See Brack-Bernsen and Hunger (2002).
14. Neugebauer (1955).
15. von Weiher (1988), pp. 198–200.
16. For futher details, see Brack-Bernsen and Steele (2004) and Steele (2006).
17. Another example can be mentioned here: Two sections of the Atypical Astronomical Cuneiform Text E

published by Neugebauer and Sachs (1967) are concerned with lunar latitude. A new analysis of text E has shown how the "ideal year" was utilized for the construction of a zigzag function for the lunar latitude; see Brack-Bernsen and Hunger (2007).
18. See de Meyer, 1982, pp. 274–275. I thank H. Hunger for this reference.
19. See Sjöberg (1975), p. 145, who gives a German translation.
20. See also Friberg (1987–90), p. 578.
21. See also Englund (1988), p. 177 where, commenting of Ur III workload texts, he surmises that a variant of the Ur III workload system may have been in use already in presargonic Lagash.
22. Hunger and Pingree (1988), pp. 163–164.
23. See Brack-Bernsen (2005), pp. 7–13.

References

Beaulieu, P.-A., 1993, "The Impact of Month-lengths on the Neo-Babylonian Cultic Calendar", *Zeitschrift für Assyriologie* 83, 66–87.

Brack-Bernsen, L., 2005, "The 'days in excess' from MUL.APIN. On the 'First Intercalation' and 'Waterclock' Schemes from MUL.APIN", *Centaurus* 47, 1–29.

——, and Hunger, H., 2002, "TU 11, A Collection of Rules for the Prediction of Lunar Phases and of Month Lengths", *SCIAMVS* 3, 3–90.

——, and Hunger, H., 2007, "On the Atypical Astronomical Cuneiform Text E, A Meanvalue Scheme for Predicting Lunar Latitude", *Archiv für Orientforschung*, in press.

——, and Steele, J. M., 2004, "Babylonian Mathemagics: Two Mathematical Astronomical-Astrological Texts", in C. Burnett, J.P. Hogendijk, K. Plofker and M. Yano (eds.), *Studies in the History of the Exact Sciences in Honour of David Pingree* (Leiden, Boston: Brill), 95–125.

Brown, D, 2000, *Mesopotamian Planetary Astronomy-Astrology* (Groningen: Styx).

Damerow, P. and Englund, R. K., 1987, "Die Zahlenzeichensysteme der archaischen Texte aus Uruk", in *Archaische Texte aus Uruk* 2 (Berlin: Gbr. Mann Verlag) 117–166.

Englund R. K., 1988, "Administrative Timekeeping in Ancient Mesopotamia", *Journal of the Economic and Social History of the Orient* 31, 121–185.

——, 1998, "Texts from the late Uruk Period", in *Mesopotamien, Späturuk-Zeit und Frühdynastische Zeit*, Orbis Biblicus et Orientalis 160/1, 15–233.

——, 2001, "Grain Accounting Practices in Archaic Mesopotamia", in J. Høyrup and P. Damerow (eds.), *Changing Views on Ancient Near Eastern Mathematics* (Berlin: Dietrich Reimer Verlag), 1–35

Friberg, J., 1978, *The Early Roots of Babylonian Mathematics* I (Göteborg).

——, Hunger, H. and al-Rawi, F. N. H., 1987–90, "Seed and Reeds, A Metro-Mathematical Topic Text from Late Babylonian Uruk", *Baghdader Mitteilungen* 21, 483–557.

Green, M. W., Nissen, H. J., Damerow, P. and Englund, R. K., 1987, *Zeichenliste der Archaischen Texte aus Uruk. Archaische Texte aus Uruk* 2 (Berlin: Gebr. Mann Verlag).

Hunger, H. and Pingree, D., 1989, *MULAPIN: An Astronomical Compendium in Cuneiform*, Archiv für Orientforschung Beiheft 24 (Horn: Verlag F. Berger).

Lansberger, B., 1949, "Jahreszeiten im Sumerisch-Akkadischen", *Journal of Near Eastern Studies* 8, 248–297.

de Meyer, L., 1982, "Deux prières *ikribu* du temps d'Ammisaduqa", in G. van Driel *et al.* (ed.), *Zikir shumim: Assyriological Studies presented to F. R. Kraus* (Leiden: Brill), 274–275.

Neugebauer, O., 1955, *Astronomical Cuneiform Texts* (London: Lund Humphries).

——, and Sachs, A., 1967, "Some Atypical Astronomical Cuneiform Texts I", *Journal of Cuneiform Studies* 21, 183–218.

Nissen, H. J., Damerow, P. and Englund, R. K., 1990, *Frühe Schrift und Techniken der Wirtschaftsverwaltung im alten Vorderen Orient* (Berlin: Verlag Franzbecker).

Powell, M. A., 1972, "Sumerian Area Measures and the Alleged Decimal Substratum", *Zeitschrift für Assyriologie* 62, 165–221.

Robson, E., 1999, *Mesopotamian Mathematics, 2100–1600 BC, Technical Constants in Bureaucracy and Education*, Oxford Editions of Cuneiform Texts 14 (Oxford: Clarendon Press).

Sallaberger, W., 1993, *Der kultische Kalender der Ur III–Zeit* (Berlin, New York: Walter de Gruyter)

Schmandt-Besserat, D., 1977, “An Archaic Recording System and the Origin of Writing”, *Syro-Mesopotamian Studies* 1, 31–70.

Sjöberg, Å. W., 1975, “Der Examenstext A”, *Zeitschrift für Assyriologie* 64, 137–176.

Steele, J. M., 2006, “Greek Influence on Babylonian Astronomy?”, *Mediterranean Archaeology and Archaeometry* 6/3, 149–156.

Tanret, M., 2004, “What a Difference a Day Made...: On Old Babylonian Month Lengths”, *Journal of Cuneiform Studies* 56, 5–12.

Vajman, A. A., 1989, “Beiträge zur Entzifferung der Archaischen Schriften Vorderasiens. Erster Teil”, *Baghdader Mitteilungen* 20, 91–138.

——, 1990, “Beiträge zur Entzifferung der Archaischen Schriften Vorderasiens. Zweiter Teil”, *Baghdader Mitteilungen* 21, 91–124.

von Weiher, E., 1988, *Spätbabylonische Texte aus Uruk* 3 (Berlin: Gebr. Mann Verlag).