14-Month Intervals of Lunar Velocity and Column Φ in Babylonian Astronomy: Atypical Text C*

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Abstract

At the heart of the Babylonian lunar theory known as System A is Column Φ, a function that represents the length of the Saros assuming that the solar velocity is at its maximum, and from which all functions that reflect the moon's anomalistic motion are derived. In this paper we discuss the Q-polygon for analysing linear zig-zag functions and apply it to Column Φ. We show that using multiples of the smallest difference between Φ values provides a convenient way of calculating the change in Φ over long periods. We use this approach to analyse the Φ and related numbers on the so-called Atypical Text C. We draw attention to our new reading of some of the Φ-values and related numbers. These new readings show that the Φ-values are closely connected to the other, related, numbers in the text, and they make it obvious that there is a connection between the Φ-numbers, the 14-month anomalistic period and the truncated version of Φ, which was used for finding Λ and G from Φ.

1 Introduction

This paper is part of the talk given by Brack-Bernsen at the symposium in honour of Hermann Hunger in Vienna July 2007. In the first part of the talk the results of the collaboration with Hermann Hunger were presented: We had aimed at understanding some difficult astronomical texts from periods during which the formation of the astronomical theories took place. And we have succeeded in understanding several cuneiform texts and reconstructing many predicting rules for finding, among others, lunar phases (the Lunar Six), duration of the lunar month and the times of lunar eclipses. Such early predicting rules are very important for a deeper understanding of the Babylonian Astronomy—they help us to know which astronomical concepts the Babylonians utilized, how they thought and argued: in short, how their early astronomical theories worked. Such knowledge is indispensable for the far goal of giving a credible reconstruction of how the Babylonian mathe-

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matical astronomy was developed and how its parameters were determined. The decoded (or reconstructed) predicting rules have already been published (see Brack-Bernsen and Hunger 2002, 2006, 2007, and 2008). Therefore at this place we only reproduce the last part of the talk, which was an account of work in progress on the so-called atypical astronomical text C. This investigation, undertaken in collaboration with John Steele, has now been finished. And it fits well into the theme of this symposium: Assyriology and empiricism. Text C has some numbers (given in the Babylonian sexagesimal system) which clearly belong to
2 column Φ, the first calculated number column in a lunar ephemeris of system A. Text C also has some coefficients, which are special numbers used for specific calculations (with Φ-numbers). Column Φ has caused the researchers many problems. It has two functions: it is the basic number function for grasping the variable lunar velocity—and it gives the lunar contribution to the duration of one Saros = 223 lunar months, an important eclipse period which was known and utilized early by the Babylonian astronomers. Φ gives a very good approximation to the period of the varying lunar velocity, and not only the Babylonian numerical function for the lunar velocity is derived from it but also the lunar contribution to the duration of one and 12 synodic months, respectively. (A synodic month is the period between two consecutive full or new moons.)

The question is how Φ was constructed. And in this connection, text C is very important, since its Φ numbers with the connected coefficients give us hints to how the Babylonians calculated and worked with Φ numbers. Text C has been published earlier by Neugebauer and Sachs (1967), but we have some new and better readings and a deeper understanding of the numbers.

There have been several proposals for a reconstruction of Φ. It is generally agreed on that the period of Φ (= the period of the varying lunar velocity) was found from the sum Σ of the Lunar Four. The Lunar Four are time differences between risings and settings of the sun and the full moon, measured in the days around opposition. These time differences were observed regularly since 700 BC, and since the sixth century, the Babylonian astronomers knew how to predict them by means of the so called Goal-Year method (see Brack-Bernsen and Hunger 2002). This method used sums of the Lunar Four in a way showing that the Babylonians knew that the sums reflected the movement of the moon relatively to the sun. Therefore we understand that and how the Babylonians empirically could determine the period of the lunar velocity by means of the Lunar Four time intervals. The concepts underlying the Goal-Year method show us that the Babylonians knew what they were doing by adding the Lunar Four.

There have been several proposals for how the amplitude and phase of Φ was found: Brack-Bernsen (1991) proposed it to be a pure empirical fit to Σ, but now we know that the Babylonians also had a good approximation for the dominating solar contribution to the Saros (Brack-Bernsen and Hunger 2002, and Brack-Bernsen and Steele 2005). Therefore we now see the possibility that time differences between lunar eclipses were used for the construction of Φ. Text C also gives us the impression that lunar eclipses are connected to the Φ numbers written in the text. Teije de Jong (private communication) has proposed that Φ was constructed by means of an early and rather imprecise period relation: 27 solar years = 334 synodic months, while John Britton (preprint 2008) has presented a (rather modern) reconstruction of Φ based among others on the shortest and longest duration of the 235 months period (= 1 Saros + 12 months) and on the shortest 6 months intervals between lunar eclipses. Cuneiform texts show that the Babylonians were concerned with all these periods. But there is still a controversy about Φ and many open questions remain. Thus, text C is a welcome source of information on Φ. Its Φ numbers give us hints to which calculations were performed and they seem also to refer to the schemes for finding the duration of 1 month and 12 months from Φ (reconstructed by Asger Aaboe 1971).

Φ is a numerical linear zigzag function. Olaf Schmidt has introduced a mathematical tool, the Q-polygon, which is very useful when working with linear zigzag functions. It helps us in understanding the derivation of the 1 or 12 months' duration from Φ and in analyzing the numbers in text C. The mathematical analysis with technical details is given in section 2 of this paper, while text C is presented and discussed in section 3.

2 Column Φ, δ and the Q-polygon

The numbers given in column Φ of a system A lunar ephemeris are discrete but are all situated on the branches of a linear zig-zag function. It is well known how the numbers were calculated: from the number ϕ_i in line i (lunation i) to the next, ϕ_{i+1}, add or subtract d_Φ, depending on whether ϕ_i is situated on the ascending or descending branch: $\phi_{i+1} = \phi_i \pm \mathrm{d}_\Phi$

The numbers around the maximum M_Φ obey the equation:

$$2\mathrm{M}_\Phi - \mathrm{d}_\Phi = \phi_i + \phi_{i+1}$$

the rule applies when ϕ_i is situated on the ascending branch and $\phi_i + \mathrm{d}_\Phi > \mathrm{M}_\Phi$ while the numbers around a minimum m_Φ obey the equation:

$$2\mathrm{m}_\Phi + \mathrm{d}_\Phi = \phi_i + \phi_{i+1}$$

the rule applies when ϕ_i is situated on the descending branch and $\phi_i - \mathrm{d}_\Phi > \mathrm{m}_\Phi$

Therefore, knowing all the parameters of column Φ, it is in principle possible to calculate by hand the ϕ values of all coming lunations—e.g. ϕ_{i+m} for the lunation (i + m) situated m months later than lunation(i).

In his lectures on Babylonian Astronomy given at the University of Copenhagen, Olaf Schmidt presented the Q-Polygon, which can be used to facilitate such calculations. We shall here give an overview of his introduction of the Q-Polygon and then utilize it in connection with column Φ.

4 | Below we repeat the parameters of column Φ

Maximum: $\mathrm{M}_\Phi = 2{,}17;04{,}48{,}53{,}20$

$$\Delta_\Phi = \mathrm{M}_\Phi - \mathrm{m}_\Phi = 19;16{,}51{,}06{,}40$$

Minimum: $\mathrm{m}_\Phi = 1{,}57;47{,}57{,}46{,}40$

and the monthly difference $\mathrm{d}_\Phi = 2;45{,}55{,}33{,}20$

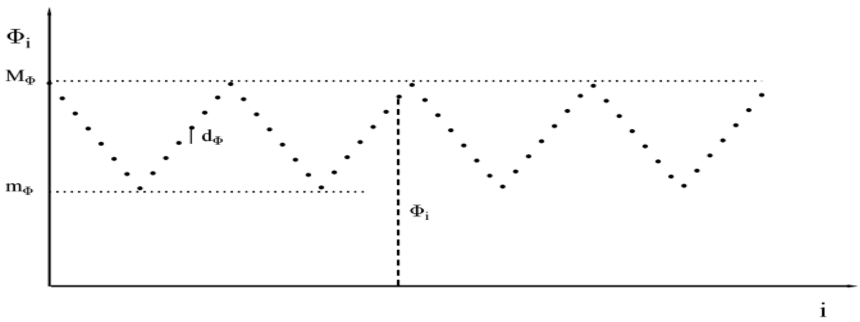
all numbers given in units of uš (= time-degrees).
Hence, the Period of Φ

$$\mathrm{P}_\Phi = \frac{2\Delta_\Phi}{d_\Phi} = \frac{38;33{,}42{,}13{,}20}{2;45{,}55{,}33{,}20} = \frac{1{,}44{,}7}{7{,}28}$$

or written in our decimal system

$$\mathrm{P}_\Phi = \frac{6247}{448} = \frac{\Pi}{Z}$$

This means that there are $\Pi_\Phi = 1{,}44{,}7$ (or 6247) different Φ_1 (and Φ_2) numbers, before the numbers repeat, having fulfilled $Z_\Phi = 7{,}28$ (or 448) Periods, P_Φ. There are two disjunct series of Φ-numbers: those who are used by new moon are normally called Φ_1-numbers, while we call those used by full moon Φ_2-numbers.



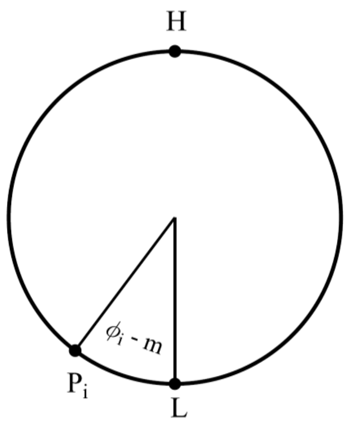
Φ_1, the function used by new moons, can assume the minimum m but not M, whereas Φ_2, used in full moon ephemerides, assumes the maximum M but not m. The Φ_1 and Φ_2 numbers are all different.

Imagine all consecutive the Φ_2 numbers (ϕ_0, ϕ_1, ϕ_2, ϕ_3, ...) mapped in a coordinate system, starting at the maximum M, then the discrete ϕ-points will be placed equidistantly on a linear zig-zag function, returning back to M after 6247 points and exactly 448 zig-zags.

Let this zig-zag function with all its ϕ-points be mapped on the periphery of a circle with circumference equal to 2Δ in the following way:

(Heuristically one can describe the process as follows: push all the 448 zig-zags with their ϕ-points together into the first zig-zag starting at m. We will then end up with one zig-zag starting at m going up to M and ending back at m. All 6247 ϕ-points will be situated on this single zig-zag. Take now the ends, i.e., the two minima m and bend them together forming a circle instead of a zig-zag, so that the straight zig-zag lines will be bend to half circles.)

The minimum m shall be depicted into the lowest point L of the circle and the maximum M shall be depicted on the highest point H of the circle. A ϕ_i situated on the ascending branch is depicted on the point P_i of the left half circle so determined that P_i has the same distance to L as ϕ_i has to m, i.e. the circle-arc (L, P_i) be shall equal to $\phi_i - \text{m}$:



$$\phi_i - \text{m} = \widehat{LP_i}$$

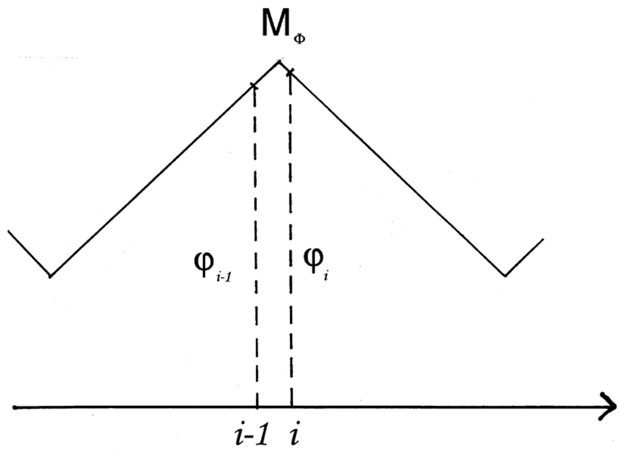
and similarly, a ϕ_j on the descending branch is depicted on the point P_j on the right half of the circle which obeys:

$$\phi_j - \text{m} = \widehat{P_j L}$$

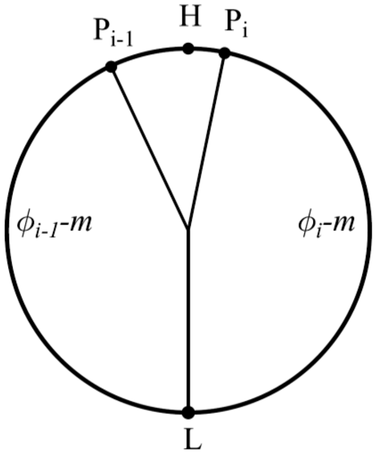
In this way all (ϕ-numbers or) all points ϕ_i on the zig-zag function are depicted on points P_i on the circle:

$$\begin{array}{cccccc} \phi_1, & \phi_2, & \phi_3,\ldots & \phi_i,\ldots & \phi_\Pi \\ \downarrow & \downarrow & \downarrow \ldots & \downarrow \ldots & \downarrow \\ P_1, & P_2, & P_3,\ldots & P_i, \ldots, & P_\Pi, \end{array}$$

We shall now show that this mapping also fits around the highest point, H, of the circle. Do we really get the correct distance, d, between two points on each side of a maximum?



Let ϕ_{i-1} and ϕ_i (two arbitrary consecutive ϕ-numbers be situated before and after the maximum M. For their mapping-points P_{i-1} and P_i on the circle we have:



$$\widehat{LP_{i-1}} = \phi_{i-1} - m$$

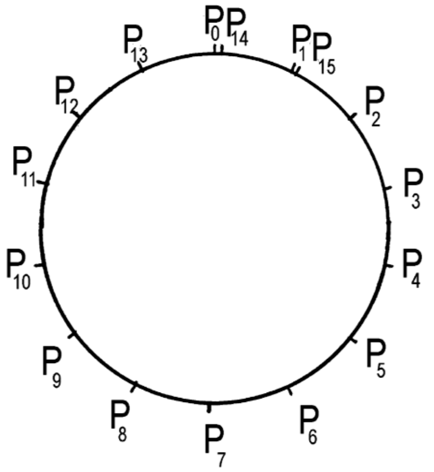
$$\widehat{P_i L} = \phi_i - m$$

We can now calculate

$$\begin{aligned} &\text{the circumference of the circle} - \widehat{P_{i-1}P_i} &&= \widehat{LP_{i-1}} + \widehat{P_iL} = \\ &2\Delta \qquad\qquad\qquad\qquad - \widehat{P_{i-1}P_i} &&= \phi_{i-1} + \phi_i - 2\mathrm{m} \\ & &&= 2\,\mathrm{M} - \mathrm{d} - 2\,\mathrm{m} \\ & &&= 2\,\Delta - \mathrm{d} \\ &\text{Hence, we have:} \qquad\qquad \widehat{P_{i-1}P_i} &&= \mathrm{d}, \end{aligned}$$

and we have demonstrated, that the mapping also fits at the top of the circle, around the maximum H. In other words for all *i* we have: $\widehat{P_{i-1}P_i} = \mathrm{d}$.

Since there are 6247 P-points on the circle, evidently, there will be many points between two points with consecutive numbers *i*.



We shall therefore give the P-points new names Q_1, Q_2, Q_3, ..., Q_Π so that Q_2 is the first P-point we meet, when we from Q_1 go clockwise around the circle, Q_3 is the next P-point, and so on. These Q-points are spread equidistantly on the periphery of the circle:

We shall now show, that $\widehat{Q_{i-1}Q_i} = \widehat{Q_iQ_{i+1}}$ for all $i = 2, 3, \dots, \Pi$-1

Proof:

Either the distances between consecutive Q-points are all equal or there exists one Q-point which is closest to the next:

∃r so that $\widehat{Q_rQ_{r+1}} \leq \widehat{Q_iQ_{i+1}}$ for all i.

The proof is finished when we have demonstrated, that for all i:

$\widehat{Q_rQ_{r+1}} = \widehat{Q_iQ_{i+1}}$

Let P_s be the P-point equal to Q_r: $P_s = Q_r$, and let P_t be the P-point $= Q_{r+1}$. If s ≺ t, we come from P_s to P_t by moving t – s = m steps of length d forward. (If s ≻ t, we come from P_s to P_t by moving s – t steps of length d backward, i.e., by moving t – s = m steps of length d forward.)

If we now start by P_t and again move m d-steps forward, we will come to point $P_{s+2\mathrm{m}}$ for which we know that

$\widehat{P_{s+m}P_{s+2m}} = \widehat{P_sP_{s+m}}$ = the smallest possible distance between Points.

Now $P_{s+2\mathrm{m}}$ equals some Q-point, and it must be Q_{r+2} the one closest to Q_{r+1}. Therefore we know that:

$\widehat{Q_rQ_{r+1}} = \widehat{Q_{r+1}Q_{r+2}} = \widehat{Q_{r+2}Q_{r+3}} = \dots = \delta$

We have demonstrated that the Q-points are placed equidistantly, so we can find δ, the difference between two consecutive Q-points (Olaf Schmidt called this smallest difference γ. Since then J. Britton (1999) has also introduced this quantity, calling it δ. We shall here follow his notation.):

$$\widehat{Q_r Q_{r+1}} = \delta = \frac{2\Delta_\Phi}{\Pi_\Phi} = \frac{38;33,42,13,20}{1,44,7,} = 0;0,22,13,20$$

In his lectures, Olaf Schmidt used such a mapping of Column A in system B for calculating (sums of A values) how far the sun has gone during 12, 13, and Π_A
8 months. Since the linear zig-zag function A(i) is a function of the time (line number) and not (directly) of the position in the ecliptic, it could be shown, that the sun does not always move the same distance during a year—of the period P_A—How far it moves according to the model, depends on the starting position of the A-value within the Q-polygon. Therefore, according to our criteria, such a procedure can not correspond to a "velocity function." Column B, giving the position of the moon at conjunction or opposition, respectively, is calculated from column A. $\text{Position}_{i+1} = \text{Position}_i + A_i$. In other words column A is the difference column of column B.

Contrarily to the step function behind column B in system A, which really can be interpreted as a "velocity function" of new moon or full moon, column A in System B does not function alike—and this fact makes it awkward to work with. (See also Schmidt 1987.)

We shall here utilize the Q-polygon differently: to analyse the function Φ(i). Let us again look at the parameters of column Φ:

$$\delta = \frac{2\Delta_\Phi}{\Pi_\Phi} = \frac{d_\Phi}{Z_\Phi}$$

Therefore, d_Φ, the monthly difference between Φ values, equals $\delta \times Z_\Phi$: $d_\Phi = 448 \times \delta$. This means that we come from one P-point to the next by moving 448 δ-steps = 448 Q-points forward on the circle.

This knowledge is quite practical when one wants to calculate by hand Φ numbers m lunations apart. Starting with an arbitrary ϕ_i we can easily calculate ϕ_{i+m}, the value of column Φ m months later, if ϕ_i and ϕ_{i+m} are situated on the same branch of Φ:

$$\phi_{i+m} = \phi_i + \mathrm{m} \times \mathrm{d}_\Phi = \phi_i + \mathrm{m} \times 448\delta \text{ (modulo } 2\Delta)$$

If ϕ_i and ϕ_{i+m} are situated on different branches, then ϕ_{i+m} found above will be larger than M. The difference ϕ_{i+m} – M, used appropriately will then give the correct ϕ_{i+m}.

The calculations are more easily done by decimal numbers, when we forget that all numbers are given in units of uš, so that we multiply them all by 60^4 and treat them as integers:

$\delta = 22{,}13{,}20 = 80000$
$d_\Phi = 2{,}45{,}55{,}33{,}20 = 448 \times 80000$
$M_\Phi = 2{,}17{,}4{,}48{,}53{,}20 = 1776560000 = 22207 \times 80000$
$m_\Phi = 1{,}57{,}47{,}57{,}46{,}40 = 1526680000 = 19083.5 \times 80000$

The function Φ_2 assumes M but not m, while the function Φ_1 can assume the value m but not M.

From this we see, that the Q_1-polygon is shifted by $1/2\ \delta$ in comparison to the Q_2-polygon. This makes sense, since new moons take place half way between full moons. The smallest distance between Q_1- and Q_2-points being $1/2\ \delta\ =\ 0;0,11,6,40$

We can now use the Q-technique to calculate the shift in Φ- value after 12 and 14 Months and after 223 months = 1 Saros, knowing that $d_\Phi = \Delta^1\Phi = 448\ \delta$ (See also Britton 1999, 203):

$$\Delta^{12}\Phi = 12 \times 448\ \delta = 5376\ \delta = -871\ \delta\ (\text{modulo } 6247\delta)$$
$$\Delta^{14}\Phi = 14 \times 448\ \delta = 6272\ \delta = 25\ \delta\ (\text{modulo } 6247\delta)$$
$$\Delta^{223}\Phi = 223 \times 448\ \delta = 16\ \times 6247\delta - 48\delta = -48\delta\ (\text{modulo } 6247\delta)$$

Function Φ is, correctly, constructed in such a way, that lunations situated 14 or 223 synodic months apart will have almost the same Φ-value and by this also almost the same lunar velocity.

The shift after $2\ \times\ 14\ +\ 223\ =\ 251$ synodic months will be very little:

$$\Delta^{251}\Phi = 2 \times 25\ \delta - 48\delta = 2\delta\ (\text{modulo } 6247\delta)$$

3 Evidence for the Babylonian Use of *δ*: Atypical Text C

BM 36301 (= 80-6-17, 27) was published by Neugebauer and Sachs (1967) as Atypical Text C. BM 36301 is a well preserved tablet missing only a small part at the bottom left corner, and an even smaller piece at the top right. The obverse is divided into three columns. Column I continues around the lower edge and for a further two lines on the reverse which is otherwise uninscribed. It contains dates (years and months but no day numbers) and longitudes of first and last visibilities and stationary points of Mars calculated using a variant to the common System A scheme. Column II contains dates (years, months and day numbers) of Venus phenomena. The first five lines contain the word *ḫi-pi* "broken," indicating that the tablet is a copy and that the original tablet was broken here. The bottom of column II, separated by a dividing line from the Venus data and extending into column III contains values Φ_2 followed by an enigmatic collection of month names, numbers and the king's name Kandalanu. Φ_2 numbers continue for five lines at the top of column III, then, following a horizontal dividing line other numbers related to Φ are given. The first of these numbers is followed by the term igi-gub-ú-meš.

Neugebauer and Sachs investigated the whole tablet. They showed that the Mars data was probably calculated for either the reign of Artaxerxes III (middle of the fourth century BC) or the beginning of the Seleucid Era 47 years later, and
10
the Venus data may date to Artaxerxes I, although this is far from certain. They also noted that the Φ_2 values date from the beginning of the fourth century BC if they are assumed to be on the ascending branch. In the following we shall be concerned only with the lunar part of the text.

We give in figure 1 a revised transcription of the lunar sections of Text C, incorporating collations by H. Hunger and J. Steele. We have refrained from making any restorations to missing text.

I	II	III
		1,59,5⸢4⸣[+x? …]
		2,10,33[…]
		2,5,18,31,[…]40
		2,5,9,15,33[…]
		2,10,52,35,31[+x …] x x x
		2,46,40 igi-gub-ú-meš
		2,:,22,13,20
	(Venus	2,28,8,53,20
(Mars Section)		2,18,53,20
	Section)	2,18,53,20
		2.18,8,53,20
		48,8,53,20
		1,19,37,46,40
		3,15,33,20
		4,56,17,46,40
		⸢5⸣,14,48,53,20
		1,51,6,40
		4,56,17,46,40
		3,9,37,46,40
	2,3,27,24,20,40 36 kan-da-la-nu	
	2,6,41,51,6,40 sig 1,40	
	2,8,51,28,53,20 gan	
	2,1,17,46,40 sig 2	
	2,13,20 apin 28	

Figure 1: The revised transcription of the lunar sections of Text C

Critical Apparatus:

Obv. III 1: The final digit is damaged and could be 4, 5, 6, 7 or 8.

Obv. III 2: Neugebauer and Sachs read 20 after 33, but neither Hunger or Steele could see it.

Obv. III 3: Hunger writes: "After 31, traces of two verticals as lower part of 5 or 8; upper part gone so it could be 6; but then much space (broken) until 40."

Obv. III 5: Of the 33, only 31 + × remains. At the end of the line, extending around the edge, are three or four unreadable signs.

Obv. III 7: There is a clear separation mark between the 2 and the 22, indicating that we must read this number as 2,0,22,13,20. Neugebauer and Sachs noted this separation mark in the transcription but ignored it in their analysis of the numbers.

Obv. III 12: Neugebauer and Sachs read 1,48,8,53,20 but there is no 1 at the beginning of the number.

Obv. III 16: Neugebauer and Sachs read 6,14,48,53,20 but both Hunger and Steele read the initial number as either 5 or 6.

Obv. III 17: Hunger writes concerning the 20: "last wedge double."

3.1 The igi-gub-ú-meš numbers

For reasons that will become apparent, we begin our discussion with the numbers given in column III 6-19. The text calls the numbers igi-gub-ú-meš, customarily translated as "coefficient." We know the term igi-gub or igi-gub-ú from the so-called "coefficient lists" of the Old Babylonian period (Robson 1999 pp. 193–207), collections in long lists of standard numbers used for calculating (e.g. the coefficient 5 was used for calculating the area of a circle starting with the square of its diameter), and also from Late Babylonian astronomical procedure texts (Neugebauer 1955, p. 476), where it also seems to refer to numbers used in calculations.

Neugebauer and Sachs divided the numbers into two groups (which the text does not do) and identified them as multiples of 11,6,40 and of 17,46,40 respectively. In doing so, however, they had to correct some numbers. After collating the text we can identify all the numbers (without any correction) as multiples of $\delta = 22{,}13{,}20$.

In table 1 below, we have identified all the igi-gub-ú-meš numbers as multiples of δ. This can not be an accident. The numbers called igi-gub-ú-meš in Text C are a collection of numbers of the form $\mathrm{n} \times \delta$.

Not only are the igi-gub-ú-meš numbers all multiples of δ, most are also multiples of 25δ. We recall that 25δ is the change in Φ over 14 months. This is significant because 14 months is the well-known shortest approximate period of return in lunar anomaly. The period relation is:

14 synodic months = 15 anomalistic months.

Other anomalistic period relations are connected to multiples of the 14 month period:

41 months = 3 × 14 – 1 months
223 months = 16 × 14 – 1 months

251 months = 18 × 14 – 1 months

N	value		d×δ		s×δ	(ev.)	p×25δ	p×48δ
N_1	2,46,40	=	450 δ	=	7,30 δ	=	18 × 25δ	
N_2	2, 0,22,13,20	=	325 δ	=	5,25 δ	=	13 ×25δ	
N_3	2,28, 8,53,20	=	400 δ	=	6,40 δ	=	16×25δ	
N_4	2,18,53,20	=	375 δ	=	6,15 δ	=	15×25 δ	
N_5	2,18,53,20	=	375 δ	=	6,15 δ	=	15×25 δ	
N_6	2,18, 8,53,20	=	373 δ	=	6,13 δ			
N_7	0,48, 8,53,20	=	130 δ	=	2,10 δ			
N_8	1,19,37,46,40	=	215 δ	=	3,35 δ			
N_9	3,15,33,20	=	528 δ	=	8,48 δ	=		11×48δ
N_{10}	4,56,17,46,40	=	800 δ	=	13,20 δ	=	32×25δ	
N_{11}	5,14,48,53,20	=	850 δ	=	14,10 δ	=	34×25δ	
N_{12}	1,51, 6,40	=	300 δ	=	5, 0 δ	=	12×25δ	
N_{13}	4,56,17,46,40	=	800 δ	=	13,20 δ	=	32×25δ	
N_{14}	3, 9,37,46,40	=	512 δ	=	8,32 δ			

Table 1: The numbers called igi-gub-ú-meš in Text C

The number N_3 is of special interest in this context. $N_3 = 2{,}28, 8{,}53{,}20 = 400\delta$ $(= 16 \times 25\delta = 448\delta - 48\delta)$ can be seen as the change in Φ after 16 × 14 months = 224 Months, which is obviously the change after one month plus a Saros. Similarly, the difference between N_5 and N_6 equals 2δ, which is the difference between Φ-values situated 251 months apart, and $N_6 = 373\delta$ is the difference between Φ-values situated 41 months apart.

Clearly the igi-gub-ú-meš numbers on Text C provide evidence of an interest in calculating with column Φ and that the period of 14 months and its multiples played a major role. Furthermore, it seems likely that the minimal difference between Φ values, δ might have been used in these calculations. To us it seems as if the numbers could have been used for long term calculation of Φ values in a way very similar to calculations undertaken by L. Brack-Bernsen by hand before we had a computer program which made it possible to calculate all values of Φ automatically.

3.2 The Φ_2 numbers

The ten Φ values found at the bottom of column II and the top of column III are all Φ_2 numbers, that is Φ values associated with full moons. We label those at the bottom of column II a-e and those at the top of column III f-j.

Neugebauer and Sachs noted that the difference between most of the Φ_2 values equals a multiple of 25δ. We point at the additional fact that for most of the Φ_2 numbers the distance to the extreme values M' and m' of the truncated Φ function are also multiples of 25δ.

$$\text{M}' = 2{,}13;20,0,0,0 = 21600\delta \quad \text{and} \quad \text{m}' = 1{,}58;31,6{,}40,0 = 19200\delta$$
$$\text{M}' - \text{m}' = 50 \times 48\delta = 96 \times 25\delta$$

Only two Φ-numbers, j and f, do not have this quality. We propose that there is an scribal error in j: if instead of 2,10,**5**2,35,33,20 we read j = 2,10,**4**2,35,33,20, then this is again a Φ-value which now also has the distance of n times 25δ to M', m', and to the other eight Φ-numbers. By this correction j will have the same property as all the other 8 Φ-numbers have. We see this as a strong indication that j contains a scribal error, which we now have corrected. Of f only 1,59,50[+ x...], was visible and Neugebauer and Sachs reconstructed it as 1,59,55,[11, 6,40], with a notation that the last 5 in 55 was not clearly readable. We propose another reading: f must be a Φ_2 number; it can, however, also be reconstructed such that it, similar to the other eight Φ_2-numbers on text C, is situated within in the pattern of n × 25δ values. We reconstruct f as 1,59,5[4,26,40, 0].

Label of Φ_2 numbers	new reading of the Φ_2 numbers	distance to M'	distance to m'
a	2, 3,27,24,26,40	$64\times25\delta$	$32\times25\delta$
b	2, 6,41,51, 6,40	$43\times25\delta$	$53\times25\delta$
c	2, 8,51,28,53,20	$29\times25\delta$	$67\times25\delta$
d	2, 1,17,46,40, 0	$78\times25\delta$	$18\times25\delta$
e	2,13,20, 0, 0, 0	$0\times25\delta$	$96\times25\delta$
f	1,59,5**4,26,40, 0**	$87\times25\delta$	$9\times25\delta$
g	2,10,33,20, 0, 0	$18\times25\delta$	$78\times25\delta$
h	2, 5,18,31, 6,40	$52\times25\delta$	$44\times25\delta$
i	2, 5, 9,15,33,20	$53\times25\delta$	$43\times25\delta$
j	2,10,**4**2,35,33,20	$17\times25\delta$	$79\times25\delta$

Table 2: Our reading of the Φ numbers on Text C, their distance to M' and m'.

With this reading of the Φ_2 numbers we have achieved that the distances between the corresponding dates (contrarily to Neugebauer and Sachs' reading used in their Table 15) all are multiples of 14 Months. This clearly shows that the Φ_2 numbers on Text C were chosen because of their relation with the truncated maximum and minimum of Φ, pointing to the importance of these truncated maximum and minimums.

Because of the long number period of Φ_2, over 505 years, it is possible to find unique dates for the Φ_2 values. Neugebauer and Sachs found dates around the beginning of the fourth century BC for the Φ_2 values if they are assumed to be on the ascending branch of Φ_2. If they are assumed to be on the descending branch, we obtain two further possible dates (presented in table 3), one in the seventh/sixth century BC and the other in the second/first century BC. Table 4 below lists the two most plausible datings. These datings take into account our new reconstructions of j and f.

Label	Φ_2 numbers	early GN	early Date	late GN	late Date
a	2, 3,27,24,26,40	5385	-565 Jun 12	11632	-60 Jul 8
b	2, 6,41,51, 6,40	5091	-589 Sep 4	11388	-84 Sep 30
c	2, 8,51,28,53,20	4895	-605 Oct 30	11142	-100 Nov 25
d	2, 1,17,46,40, 0	5581	-549 Apr 17	11828	-44 May 13
e	2,13,20, 0, 0, 0	4489	-637 Jan 2	10736	-132 Jan 29
f	1,59,5[4,21,40, 0]	5707	-539 Jun 24	11954	-34Jul 21
g	2,10,33,[20, 0, 0]	4741	-617 May 19	10988	-112 Jun 13
h	2, 5,18,31,[6],40	5217	-579 Nov 11	11464	-74 Dec 8
i	2, 5, 9,15,33,[20]	5231	-578 Dec 30	11478	-72 Jan 25
j	2,10,42,35,3[3,20]	4727	-618 Mar 31	10974	-113 Apr 27

Table 3: Dates when Φ_2 numbers are assumed on descending branches

Of the three possible dates for the Φ_2 values, the late dating for the descending branch in the second/first century BC is almost certainly too late. The early date for the descending branch at first sight seems too early, but it is worth remarking that the date of one of the Φ_2 values, -637 Jan 2, is within the reign of Kandalanu and that his name is given after the first Φ_2 number at the bottom of column II. In support of fifth/fourth century BC dates if we assume the ascending branch we note that the Mars and Venus data on the tablet probably correspond to the fourth century BC. However, since the present section of the text seems to be concerned with calculating Φ over long periods, we cannot rule out any of the datings. In the following analysis we will only be concerned with the differences in months between the Φ_2 values which remain the same in all three datings.

Label	Φ_2 numbers	ascending GN	branch Date	descending GN	branch Date
a	2, 3,27,24,26,40	7411	-401 Apr 2	5385	-565 Jun 12
b	2, 6,41,51, 6,40	7705	-378 Dec 24	5091	-589 Sep 4
c	2, 8,51,28,53,20	7901	-362 Nov 13	4895	-605 Oct 30
d	2, 1,17,46,40, 0	7215	-417 May 28	5581	-549 Apr 17
e	2,13,20, 0, 0, 0	8307	-329 Sep 10	4489	-637 Jan 2
f	1,59,5[4,21,40, 0]	7089	-427 Mar 20	5707	-539 Jun 24
g	2,10,33,[20, 0, 0]	8055	-349 Apr 27	4741	-617 May 19
h	2, 5,18,31,[6],40	7579	-388 Oct 31	5217	-579 Nov 11
i	2, 5, 9,15,33,[20]	7565	-389 Sep 14	5231	-578 Dec 30
j	2,10,42,35,3[3,20]	8069	-348 Jun 13	4727	-618 Mar 31

Table 4: Plausible Dates of Φ_2 numbers (with Goldstine Numbers)

We know that Column Φ contains information on the lunar velocity, and therefore it is worth repeating Neugebauer and Sachs' notice, that the dates are almost all situated an entire number times 14 months apart. The interval of 14 synodic months is a near approximation to the period of 13;56,39,6,25,... synodic months of column Φ. Neugebauer and Sachs write: "It seems likely that the significance of these multiples of 14 lines (or months) is connected to the fact, that 14 lines is a convenient control unit, following one traversal of the zig-zag, with the difference D = 0,0,9,15,33,20" ($25\delta = 0;\ 0,9,15,33,20$).

In the table 5 below, we give the line difference (= difference in lunation number) between the Φ_2 values of text C as we read them. In the first column we give the difference between the Φ lunation, when their lunation numbers (= GN-number) have been rearranged in chronological order, as Neugebauer and Sachs did in their scheme 15, and in the second column we present the differences between the dates, when given according to the order in which the Φ_2-dates are found on the tablet:

in chronological order:			as occurring in the text		
238	=	17 × 14	294	=	21 × 14
14	=	1 × 14	196	=	14 × 14
154	=	11 × 14	-686	=	-49 × 14
196	=	14 × 14	1092	=	78 × 14
126	=	9 × 14	-1218	=	-87 × 14
14	=	1 × 14	966	=	69 × 14
154	=	11 × 14	-476	=	-34 × 14
196	=	14 × 14	-14	=	-1 × 14
126	=	9 × 14	404	=	36 × 14

Table 5: Line-differences between the Φ_2 values on Text C

When we arrange the Φ_2 values in chronological order we see that there is a clear pattern in their distribution. All the $\Phi_2$values correspond to dates that are separated by a multiple of 14 months. Furthermore, the intervals between the Φ numbers display a repeating pattern of 1 – 11 – 14 – 9 times 14 months (see Figure 2).

Although we now understand that the Φ_2 values recorded on Text C were chosen because they correspond to dates that were separated by 14 months, we cannot explain the ordering of the Φ_2 values on the text. They are arranged neither in chronological order, nor in order of their size. There might, however, be a connection between the Coefficients, N_i, and the Φ_2 values: there are $N_1 = 18 \times 14$ months between date g and date e, $N_{12} = 12 \times 14$ months between day c and j, and $N_{11} = 34 \times 14$ is the number of months between date f and i.

We do not know how and why the 10 Φ-numbers (a, b, ..., j) were selected. But they are surely somehow connected to the truncations at M' and m', and they are situated on some of the points intersecting the line between M' and m' in 95 equal distances of 25δ. From many ACT texts we know that the Babylonians used another series of points (situated equidistantly between M' and m' at the distance of 48δ) in order to calculate the lunar contribution, G and Λ, to the duration of 1 or 12 months (See Aaboe 1971 and Neugebauer 1975).

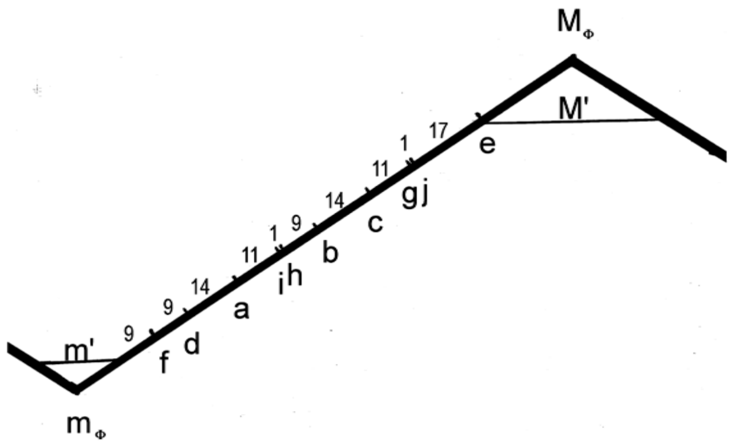


Figure 2: Positions of the Φ_2-Dates on the ascending branch. The numbers indicate the multiples of 14 months intervals between the dates

In a coming paper, the useful tool of the Q-polygon (presented in this paper) shall be utilized to illustrate how column Φ was used to generate the other columns (Λ and G) depending on the lunar velocity. And it shall be investigated, what a similar numerical method using Φ-values at the distance of 25 δ instead of 48 δ could deliver (Brack-Bernsen: Φ, Λ, and BM 77224+, a table with corresponding Φ- and Λ-values).

3.3 Kandalanu and the month names and numbers following the Φ_2 values at the bottom of column II

Following the five Φ_2 values at the bottom of column II and extending across column III we find the name of the seventh century BC king Kandalanu, a series of month names, and various numbers:

We do not know the significance of any of this text. The series of month names does not correspond to the months of the Φ_2 values. One of the Φ_2 values by the earliest dating, value e, corresponds to a date during the reign of Kandalanu, but we do not know whether this is merely coincidence, especially as we cannot be certain that the early dating of the Φ_2 values is the correct one. As noted by Neugebauer and Sachs, the pattern of the month names is suggestive of the pattern of successive eclipse possibilities. The 36 before the name Kandalanu could refer to 36 years, twice the Saros cycle, but this is no more than a guess. Similarly, the numbers 1,40 and 2(,0) could be the same type of numbers that we find on LBAT 1413 and LBAT 1414 that refer to the length of the Saros (Brown 2000, p. 205; Steele 2002, Brack-Bernsen and Steele 2005), but again this is only a guess. What follows is therefore quite speculative.

Label	Φ_2 value	following text
a	2,3,27,24,26,40	36 kan-da-la-nu
b	2,6,41,51,6,40	sig 1,40
c	2,8,51,28,53,20	gan
d	2,1,17,46,40	sig 2
e	2,13,20	apin 28

There may be a connection between the Φ_2-values and the Saros Cycle Scheme for predicting lunar eclipses (For details of the Saros Cycle Scheme, see Steele (2000)): If we assume the Φ-points to be on the ascending branch, then the dates corresponding to e (=M'), h, d, and also m' are all dates of lunar eclipse possibilities occurring in the Saros Cycle Scheme. However, none of them are dates of lunar eclipses which are visible from Mesopotamia.

According to the "old dating" where we locate the Φ_2 points on the descending branch, three of the Φ_2-values (b, f, and j) correspond to lunar eclipses. (Whereas neither M' nor m' refer to days of lunar eclipse possibilities.)

j corresponds to -618 Mar. 31, where a lunar eclipse of magnitude 0.72 occurred. The eclipse started around an hour before moonrise, so that the moon rose eclipsed. The same is true for f, corresponding to -539 June 24, which again is the date of a total lunar eclipse starting over three hours before moonrise and ending 0.43 hours = 6;27 uš after moonrise. (The numerical details of the lunar eclipses are taken from Huber and de Meis, 2004, pp. 184–187.)

The date -589 Sep. 4, corresponding to b, is not the date of an observable eclipse, but only of an eclipse-possibility. However, the eclipse, taking place two Saros earlier on date -625 Aug. 13, is visible from Babylon. This is the eclipse of Kandalanu Year 22 month VI. Could this eclipse possibly have something to do with our eclipse date b?

18 | In this regard it may be worth noting that if the remark “36 Kan-da-la-nu,” which follows number a, is shifted 1 line downwards, we could read it as comment to b = 2, 6,41, 6,40, and then it would make sense: 36 [Years] are often in cuneiform texts used for 2 Saros, and 2 Saros before the EP of -589 Sep. 4, we get to a lunar eclipse where the moon rose eclipsed within the time of Kandalanu (that of -625 Aug. 13). Around the time of this eclipse we have a very nice fit between the curves Σ and Φ. Σ is the sum of the Lunar Four, which constitute the observational basis of the period relation underlying Φ. In Figure 3 we have an instance where Σ and Φ appear nicely in phase. This is not an unique event. The period relation behind Φ is a very nice approximation to the period of the lunar velocity. Therefore there will be many such instances of nice agreement in phase between Σ and Φ during the time from 650 BC to 200 BC.

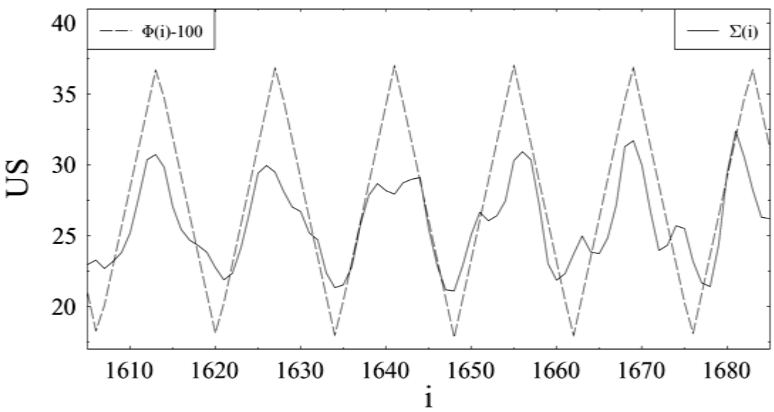


Figure 3: A graphical comparison between Σ and Φ. Around lunation i = 1645 there is a optimal fit between the curves Σ and Φ – 100°. Lunation 1645 is the full moon of -625 Aug. 13, the date 2 Saros before date b = -589 Sep. 4.

We mention this as a possibility, that the text may contain some scattered notes giving hints to how Φ was constructed or how it was used for long term calculations, using: some special Φ_2 values—coefficients connected to 14 months, which is a good approximation to the period $P_{☾} = P_{\Phi}$ of the lunar velocity, and finally—36 Kandalanu followed by month names written in the pattern of consecutive eclipse possibilities. We remind the reader, however, that the evidence for the interpretation of this part of Atypical Text C is very limited, and we do not believe that any firm conclusions can be drawn from it.

An additional reason for bringing Figure 3 is to point at another possible consequence of the numbers in Text C. Clearly the 14-month anomalistic period played a major role, and it was closely connected to Φ. This connection may have been utilized in different ways. In Figure 3 we have the "lucky" case of a lunar eclipse which took place at a time when Σ was nice and linear. But Figure 3 also illustrates how the 14 month period eventually could have been used in a different way: for replacing "shaky parts" of Σ with straight parts, just by going a few times 14 months forward or backward. Whenever Σ has been recorded over a longer period, it is possible to find linear sections and e.g. to connect them to eclipses which can be timed quite exactly—as for instance eclipses where the moon rises or sets eclipsed (which was the case in the Figure 3). Or expressed the other way around: if one has an observed event which takes place at a time where Σ is shaky, then one can exchange the shaky part by a nicer, linear, part to be found some 14 months earlier or later. Such straight-lined parts of Σ may have been used to find the period of Φ. We have no idea if the Babylonians worked like this; but still we think it is worth mentioning it as a possibility. To sum up: we give three different possibilities for how the interval of 14 months may have been utilized:

1. In order to exchange "shaky" parts of Σ by "usable," linear parts,
2. for something alike the scheme calculating Λ or G from Φ; but now with steps of 25δ instead of 48δ-steps, or
3. for long term calculations of Φ-values.

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