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The Observational Foundations of Babylonian Astronomy

1. Introduction

Of the excavated Mesopotamian astronomical cuneiform tablets, a small fraction utilized advanced mathematical algorithms to calculate astronomical quantities. This corpus comprises about 450 tablets and fragments, which were found in the ancient towns of Babylon and Uruk. Scientific excavation produced the material from Uruk, while the Babylonian tablets were obtained through the antiquities-market. The texts, which were written during the period 450–50 BC, are classified as the corpus of Babylonian mathematical astronomy. This corpus consists of two different types of texts: tabular texts and procedure-texts.

Tabular texts contain computed astronomical quantities arranged in rows and columns, while procedure-texts instruct how to calculate those astronomical quantities for the Moon, the Sun, and planets. This chapter discusses the empirical foundation of this corpus. In order to do this, however, we must take into consideration older astronomical texts, since these texts may give hints as to which observations and astronomical conceptions the Babylonian astronomers had at their disposal, which will in turn shed light on how the mathematical algorithms were likely to have been developed. The earliest known cuneiform texts concerned with astronomy date back to the beginning of the second millennium BC.

Since 1955, when Neugebauer's *Astronomical Cuneiform Texts* was published, we have known fairly well what the Babylonian astronomers calculated and how. They utilized elegant numerical functions representing different astronomical quantities, such as the synodic arc or angular displacement of the Sun, the Moon, or a planet from one synodic event to the next or the changing length of daytime or of nighttime throughout the year.

Note the way in which Babylonian astronomy differs from ours: we calculate the positions of celestial bodies as continuous functions of time, whereas the Babylonians concentrated on isolated events (synodic phases), for which they calculated the positions in the zodiacal circle and the times they were expected to occur [see ch. 4.1 §3, p. 59]. In the case of the Moon, the days around the New and Full Moons were of special interest. For these phases, not only the day of the event but also the time between the rising and setting of the Sun and Moon were observed, predicted, and calculated. These time-intervals, although rather complicated from a theoretical point of view, are visually obvious and thus easy to observe.

The procedure-texts give quite abbreviated instructions on how to calculate the tabular texts. They do not explain or indicate how the algorithms were developed or give any information regarding their theoretical or empirical foundation. But the two types of texts, the tabular and the procedural, helped in the original modern decipherment of the cuneiform astronomical texts. Our knowledge of Babylonian mathematical astronomy has been extended by numerous papers published after 1955 and lately deepened by a monograph by Ossendrijver [2012], which provides new editions of the procedure-texts and a semantic, mathematical, and astronomical analysis.[1] Nevertheless, we know relatively little about how this elegant and efficient mathematical astronomy was developed; nor do we know in detail what sorts of observations provided its foundation.

2. Babylonian astronomical observations

The earliest hints about astronomical observations are found in early omen-texts, which seem to connect observed lunar eclipses with the death of kings perhaps in the the Ur III dynasty (*ca* 2100 BC); another omen-text gives the dates of the first and last visibility of Venus observed in the years of the Old-Babylonian king Ammiṣaduqa. Later, probably starting around 747 BC, the Babylonians observed the sky regularly over the course of many hundreds of years, recording this "regular watching" on cuneiform tablets. All such tablets known to us have been published by Sachs and Hunger in their *Astronomical Diaries* [1988–1996]. The earliest surviving Diary dates from 652 BC; the latest, from 61 BC. Hunger estimates that only 5% of the written Diaries have been found. Other collections of observed and predicted quantities are published in Hunger 2001–2012.

Along with weather, prices, river levels, and historical events, a typical Diary contains information on the Moon, planets, solstices and equinoxes, Sirius-phenomena, meteors, and comets. With these ancient records of observations, we know what Babylonian astronomers observed[2] and we are able to check the accuracy of their observations. The movements of the Moon and planets were traced by means of their passings by some

[1] For an excellent introduction to Babylonian astronomy, see Ossendrijver 2012, 1–53.

[2] For further details, see the introduction in Sachs and Hunger 1988–1996, 1.11–27. See also Hunger 2001–2012.

special so-called Normal Stars [see Glossary, p. 574] for the purpose of establishing the positions of their synodic phenomena. The following planetary phases were observed:

- their first and last visibilities,
- stationary points, and the
- acronychal risings[3] of the outer planets.

Only by means of solar and lunar eclipses, which were recorded carefully, were the conjunctions and oppositions of the Sun and Moon observable. Since, normally, such syzygies cannot be observed directly, the Babylonians observed the risings and settings of the Sun and Moon in the days around New and Full Moon.

Six time-intervals, called the Lunar Six, were measured and recorded together with the day on which they were observed:

NA_1 = time between sunset and the first visible setting of the new crescent.

At Full Moon the Babylonians regularly measured the following time-intervals, known as the Lunar Four:

ŠÚ = time from moonset to sunrise, measured at last moonset before sunrise.
NA = time from sunrise to moonset, measured at first moonset after sunrise.
ME = time from moonrise to sunset, measured at last moonset before sunset.
GE_6 = time from sunset to moonrise, measured at first moonrise after sunset.

Toward the end of a month, they also recorded:

KUR = time from last visible moonrise before conjunction to sunrise.

Note that it is exactly those synodic phenomena of planets and of the Moon observed and recorded in the Diaries that were calculated in the tabular texts. Before the methods evident in the texts collected in Neugebauer 1955 were developed, the Babylonians had found an easy and elegant method for their prediction by means of earlier observations.

There have been several attempts to reconstruct the numerical functions of the tabular texts by means of observations, but we are still far from a comprehensive reconstruction. We must keep in mind that a reconstruction only shows that we have been able to derive the astronomical parameters of a numerical function from Babylonian observations. It does not prove that this was how the Babylonians did it; there may be other ways to construct the same algorithms from observations. Thus, each reconstruction should be confirmed by other means. Strictly speaking, we can only determine the empirical basis of the numerical systems when we know how they were constructed in the first place. Here the non-mathematical astronomical tables that show us earlier stages of the developing astronomy and reveal astronomical concepts and methods for predictions are of great help.

3 The opposition of a planet cannot be observed directly, so the Babylonians observed its rising at sunset, i.e., its acronychal rising, instead. See ch. 4.1 §3.1, p. 60.

3. Preconditions for practicing mathematical astronomy

The Babylonian scribes were aided in their development of astronomy by a well-functioning writing system and their mathematical training in a sexagesimal number system [see ch. 3.1 §7, p. 47] in which the standard calculations were easily carried out. In addition, the Babylonian calendar and the division of the zodiacal circle into segments of 30° or signs offered a suitable frame for recording times and positions of astronomical events. The empirical basis of this frame is quite well known.

The Babylonian calendar itself is very old: it is attested in texts dating from 2600 BC. It was determined by lunar phases and by the solar year: the day began at sunset; the first day of a new month began on the evening when the new crescent became visible for the first time after conjunction. This event took place either 29 or 30 days after the previous first visibility. However, the sequence of 29- and 30-day months is highly irregular and, hence, troublesome to handle. The normal year had 12 months but 12 synodic months are some 10.88 days shorter than the solar year. Therefore, on average, an intercalated 13th month was necessary roughly every three years in order to keep the months in tune with the seasons.

This calendar evidently was formed empirically by observations of sunset, the new crescent, and the positions of the Babylonian months within the solar year, whereby a shift of the lunar months with respect to the seasons would show the need for intercalating a 13th month. Period-relations between lunar months and solar years helped to organize the intercalations. John Britton [2007a, 119–130] analyzed the patterns of the intercalations in early texts and of the solar longitude at the beginnings of Babylonian years. He shows that such period-relations improved from 750 to 484 BC, after which time intercalations were systematically governed by the 19-year cycle in which 19 years = 235 synodic months. (The simplest way to discover this cycle would be to notice that eclipses separated by 235 months recur at the same place in the sky.)

Before the intercalations were regulated, the need for an extra month could be indicated by the date of the heliacal rising of a bright star like Sothis (Sirius). Other indicators were the points at the horizon where the Sun rises or sets. Since early times, the Babylonians knew that the rising point of the Sun changes throughout the year; they regulated their calendar such that the day at which the Sun rose straight east (spring equinox) took place in month XII or month I. Such observations may also have been used to find and improve the intercalation cycles. The problem of intercalation was settled in the early fifth century BC before the ephemerides in Neugebauer 1955 had been developed; the trouble with the variable length of lunar months remained but eventually it was elegantly solved.

The Babylonians had, from the earliest bureaucratic texts (3000 BC) onward, an easy way to cope with the irregular length of the month: each month was reckoned as 30 days independently of its actual length. This "schematic year" with 12 months of 30 days was a practical approximation to the irregular Babylonian lunisolar year. In the later mathematical astronomical texts, we find traces of this schematic year in the useful unit *tithi* (= 1⁄30 synodic month) [see Glossary, p. 576]. The schematic year was used in early astronomical schemes (from around 1100 BC) to record how the rising point of the Sun, the length of daytime, and the time of the Moon's first visibility changed throughout the year.

In this period, the movements of the Sun, the Moon, and the planets were traced along some 17 constellations called the "path of the Moon". Later, around 450 BC, that path was divided into 12 sections of 30°. I presume that this division was conceived in analogy to the schematic year, the reason being that several crude astronomical schemes identify the schematic year with the 12 zodiacal signs of 30°. This corresponds to the fact that the Sun travels roughly 1° per day and 30° per month. Huber 1958 points to the fact that some bright stars are situated at the beginnings of zodiacal signs, implying that these were the stars used to define the boundaries of the signs.

This abstraction of the Moon's path facilitated the computations in the tabular texts. But the positions of the Moon and planets on the zodiacal circle or ecliptic cannot be observed directly. In the Diaries, the movements of the Moon and planets were surveyed by means of their passings by Normal Stars. A newly edited text gives the zodiacal positions of the Normal Stars [Roughton, Steele, and Walker 2004]. In this way, observed positions could easily be converted to positions in the zodiacal circle. The Diaries also note how much above or below the Normal Stars the passing took place. Such observations were also used to survey movement in latitude—or, more precisely, to find the positions of the Moon or the planets within the "zodiacal band". Steele 2007b shows that the zodiacal band was identified by Normal Stars. For each of these special stars, the Babylonian astronomers knew the interval of 6 cubits or 12° (= 6 × 2°) within which the variation in lunar latitude took place and they knew the smaller central path within which eclipses took place.

To summarize, the days of the synodic phenomena of the Moon and planets were observed directly, while their positions within the zodiacal band (longitude and latitude) were found by converting their observed positions relative to Normal Stars. Evidently, such observations constituted the empirical basis of all functions capturing movement in longitude and latitude. Note that the Babylonian conception of the zodiacal circle or ecliptic differs from ours: for them, its starting-point is fixed with respect to the stars; whereas, for us, its starting-point is the vernal or spring equinoctial point, which moves slowly backward (to the west) along the zodiacal circle [see ch. 1 §7, p. 20].

4. Methods of astronomical prediction

Eclipses played an important role in Mesopotamia. They occur when the Moon in its full or new phase passes the middle of the zodiacal band. This happens at intervals of six or sometimes five months. Eclipses were predicted by means of the Saros, an eclipse-cycle of 223 months, which equaled 18 years plus 11 days. The Babylonians named the cycle "18" and identified 38 months within each cycle in which eclipses could possibly occur [Steele

2000b]. We call such “dangerous” months eclipse-possibilities (EPs). Since EPs repeat after 1 Saros, they can be arranged in a matrix-like scheme of columns and lines by which all future EPs may be predicted.

Each column covered 1 Saros, listing its 38 EPs. The next column listed the EPs of the next Saros, and so did each line of the scheme, giving a series of EPs situated 1 Saros apart. The largest eclipse scheme comprised 24 cycles, covering the time from 747 BC to 333 BC. We do not know exactly when such Saros-schemes for predicting eclipses were developed—probably by the seventh century BC. From then on, the months, but not the times, of future EPs were known beforehand. When the time of an eclipse also was known, the times of the EP expected 1 Saros later could be predicted [Brack-Bernsen and Steele 2005]. The basis for such predictions would be the observed eclipses recorded in the Diaries. Here the years, months, days, times, and other details of eclipses are recorded; however, the measured times were only accurate when the eclipses took place near sunset or sunrise.

The empirical basis of Babylonian mathematical astronomy is certainly to be found in the Diaries. But we have the problem that only about 5% of all such texts are at our disposal, so it is on a thin basis that we evaluate the quantity and quality of observed events. In addition, the Diaries do not tell us which kind of practical experience and knowledge the Babylonian astronomers possessed. In planetary texts, one often finds notes such as this:

> Month VIII, the 25th, Jupiter’s first appearance in Scorpius, 3½ cubits behind alpha Scorpii; rising of Jupiter to sunrise: 11°; (ideal) first appearance on the 24th. [Hunger 2001–2012, 6.263]

The interval of 11° of time between the risings may have played a role for finding the ideal date but we do not know exactly how. Often the Diaries give days and times between the risings of the Sun and a planet or the Moon in addition to comments like “clouded over”, “I did not watch,” and so on.

In the case of the Moon, we know how such missing data were reconstructed by means of the Goal-Year method. In the case of the planets we do not know the details.

Goal-Year Texts are collections of earlier observations, presumably excerpts from the Diaries, that were used for the prediction of astronomical events in a particular year, Y, the “Goal-Year”. The Goal-Year table for year Y would collect the synodic phenomena of Jupiter from year $Y-71$ and Jupiter’s passings by the Normal Stars from year $Y-83$. Evidently, the Babylonians knew the 71- and 83-year periods of Jupiter, and they probably had ways to adjust the days of the phenomena to year Y.

In the case of the Moon, the Goal-Year tables collected lunar data from year $Y-18$, that is, the Lunar Sixes and eclipses of that year. Lunar Six data were skillfully used to predict their expected values for year Y by means of the Goal-Year method [Brack-Bernsen 1997, 1999; Brack-Bernsen and Hunger 2002]. This method is easy and very accurate—and it was known at least since the sixth century BC. The times $\check{S}\acute{U}_i$ and NA_i for a month i could be found by easy calculations from the data $\check{S}\acute{U}_{i-223}$ and NA_{i-223} of month $i-223$, that is, 1 Saros earlier, measured on two consecutive mornings:

$$\check{S}\acute{U}_i = \check{S}\acute{U}_{i-223} + \frac{(\check{S}\acute{U} + NA)_{i-223}}{3}$$

$$NA_i = NA_{i-223} - \frac{(\check{S}\acute{U} + NA)_{i-223}}{3}$$

These rules for prediction are based on elegant combinations of the following empirical considerations:

(1) After 1 Saros, the time of opposition with respect to sunrise is shifted by ⅓ of a day.

(2) The sum ŠÚ + NA is connected to how fast the Full Moon moves with respect to the Sun. It is the setting time of the Moon's movement relative to the Sun on the day of the Full Moon. Analogously, ME + GE_6 is its rising-time.

(3) The sums ŠÚ + NA and ME + GE_6 repeat after 1 Saros. When the difference for finding NA (or other Lunar Six) became negative, the calculations were revised and the day of the phenomenon changed.

The Goal-Year method thus provided a means to determine the length of the month in advance [Brack-Bernsen and Hunger 2008; Brack-Bernsen 2011].

5. The tabular texts: the planets

The sequence of synodic phenomena for the inner planets is different from that of the outer planets but the numerical methods are similar. I shall, therefore, present one planet, Jupiter, as an example. The five characteristic synodic phenomena of an outer planet are:

- Ω disappearance,
- Γ appearance or heliacal rising,
- Φ first stationary station,
- Θ opposition, and
- Ψ second station [see ch. 4.1 §3, p. 59].

A complete tabular text gives in separate columns the times and zodiacal positions of these five synodic phenomena for consecutive synodic periods 1, 2, 3, and so on [Table 1, p. 157]. In most cases, the column tabulating position (in longitude) was calculated separately[4] by means of the synodic arc $\Delta\lambda$, that is, the angular distance between successive events of the same sort: $\lambda_\Gamma(i+1) = \lambda_\Gamma(i) + \Delta\lambda_i$, whereby the Babylonians utilized two main types of numerical functions to determine the synodic arc $\Delta\lambda$:

4 Sometimes, the data of one phase were used to find those of the next by means of some pushes [see ch. 4.5 §4, p. 119]. A decent value for the angular distance between the phases can be found by observation.

	$t\,\Gamma$	$\lambda\,\Gamma$	$t\,\Phi$	$\lambda\,\Phi$	$t\,\Theta$	$\lambda\,\Theta$	$t\,\Psi$	$\lambda\,\Psi$	$t\,\Omega$	$\lambda\,\Omega$
1	.	.	.	.	.	.	.	.	.	.
2	.	.	.	.	.	.	.	.	.	.
3	.	.	.	.	.	.	.	.	.	.

Table 1. The structure of a typical planetary ephemeris

Columns 1 and 2 tabulate times and positions (longitudes) of the planet's consecutive appearances or heliacal risings. Columns 3 and 4 tabulate times and positions (longitudes) of its first stations, and so on. Each line follows the planet through one synodic period, with the next period given in the line below.

System A
: calculates $\Delta\lambda$ from a step-function of its position in the zodiacal circle. The synodic arc $\Delta\lambda$ of Jupiter is, for instance, approximated to be 36° on one part of the zodiacal circle and 30° on another.[5] When a step of the functions is passed, the value of $\Delta\lambda$ is found by linear interpolation.

System B
: gives the synodic arc through a linear zigzag-function of the number of the phenomenon: $\Delta\lambda$ varies linearly between a maximum M and a minimum m with the amount of $\pm d$ from one line to the next.

For the corresponding times of the events, a similar calculation was used:

$$t_\Gamma(i+1) = t_\Gamma(i) + \Delta t_i.$$

In most cases, Δt_i was derived from $\Delta\lambda_i$ by the following (surprising) connection, which was used for all planets:

$$\Delta\lambda + C = \Delta t,$$

where C is a constant with λ measured in degrees and the time, in *tithis* [see ch. 4.5 §2, p. 113]. Van der Waerden called this feature the "Sonnenabstandsprinzip" ("The Solar-Distance Principle"). It allows for the fact that the Sun travels 1° per day and that 1 day ≈ 1 *tithi*.

Such a connection may have been found by observation, for example, of acronychal risings of Jupiter or Saturn and then transferred to the other synodic phases of the planet. It might even have been extended to other planets, once the constant was adjusted. I do not, however, see the principle as a basic concept according to which the functions were constructed by means of dynamic models of solar and planetary movements. Note that the conversion $\Delta t_i = \Delta\lambda_i + C$ is far more suitable for acronychal rising than for heliacal rising and that the same system was used for all phenomena of Jupiter.

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5 Evidently, the Babylonians had noticed that the angular distance between successive Γ-phenomena was smaller on one part of the zodiacal circle and larger on the other part, and they had developed numerical methods to describe this variation.

The numerical functions in both Systems A and B were constructed such that they satisfy a period-relation, tying them to actually occurring phenomena quite well, even over a number of centuries. A number Π was determined such that the planet after Π synodic periods would return to the same position in the zodiacal circle. Such a period corresponds to an integer number Z of sidereal revolutions of the phenomenon in question (i.e., to the number of times that the phenomenon returns to the same star) and to a whole number Y of revolutions of the Sun (i.e., to the number of times that the Sun returns to the same point on the zodiacal circle as calculated in the Babylonian scheme at use) and, therefore, also to Y years. Most of the Jupiter texts are based on the following period-relation:

391 synodic occurrences = 36 sidereal rotations = 427 years.

The exact period-relations were not the result of observations over hundreds of years but were—no doubt—constructed from shorter periods by means of corrections to these periods. From Goal-Year tables we know approximate periods for all planets and we presume that the Babylonians used Goal-Year data from earlier periods in order to fill in non-observable data in the Diaries. This means that, in the case of Jupiter, the scribes compared observations some 71 and 83 years apart and that they knew how to adjust the data from the Goal-Year tablets to the actual year, presumably by some knowledge of the shift in date and location after 71 and 83 years.

Precisely such empirical data can be used to find the period-relations on which the numerical functions of Systems A and B are based. Once the period-relation was determined, other parameters could be fixed. System B is determined by the period-relation and one additional parameter, for example, the maximum M of the linear zigzag-function. System A step-functions are more flexible in that they are determined by more than one parameter besides the period-relation. There is some freedom in choosing the number and length of the arcs (into which the zodiacal circle was subdivided) together with the respective synodic arcs of the planets.

The constant C, which enabled converting synodic arc into synodic time and *vice versa* using $\Delta\lambda + C = \Delta t$, was determined such that the sum of the synodic arcs over Π periods equals $Z \times 360°$ and that the sum over the synodic times over Π periods equals Y years. Given the numbers Π, Z, and Y and the length of the year measured in *tithis*, one can find the constant C for each planet as the difference between the mean values of Δt and $\Delta\lambda$. Lunar tables of System A contain implicitly two different values for the year: the column recording the dates, where intercalations are regulated in the 19-year cycle, have a year-length of 6,11;3 *tithis*, while the velocity function of the Sun[6] in the same system gives the length of the year to be 6,11;4 *tithis*. Barring one procedure-text [Neugebauer 1955, no. 813 (pp. 286, 412)], the Babylonians utilized the latter value to determine C for the planets.

6 The velocity of a celestial body is the daily displacement in its position on the zodiacal circle during a given time-interval; it is typically assessed in degrees per day.

There have been several attempts to construct the numerical functions in the ephemerides from observations. Asger Aaboe [1958] compared positions and times of planetary events (calculated in the Babylonian tabular texts) with modern astronomical calculations. In some cases, the positions of the events showed a good fit but not the times; and sometimes *vice versa*. Therefore, Aaboe proposed that some might have been constructed from the positions of consecutive phenomena, others by the times of their occurrences. For System A for Mars, he found a nearly perfect agreement with synodic arcs derived from the longitudes of opposition.

Later [1965], Aaboe showed that the Babylonian step-function for calculating consecutive positions of one synodic phenomenon results in an uneven distribution of the events in the zodiacal circle. This corresponds to reality. Aaboe thus proposed that a simple counting of the numbers of events taking place within different zones of the zodiacal circle could lead to the Babylonian step-function. He also showed that, next to the desire to reproduce a planet's behavior, purely arithmetic considerations and demands of the models co-determined the choice of numbers in the period-relation.

The texts in Neugebauer 1955 exhibit a variety of numerical methods for determining the synodic arc (procedure-texts for Jupiter bear witness to eight variations of Systems A and B), showing how the scribes tried different way of finding good "fits" to observational experience through their elaboration of the methods of Systems A and B and the choice of numbers. These arithmetic schemes deliver a phenomenological numerical description of periodic phenomena. For Jupiter, for instance, the same function for the synodic arc could be used, and was in fact used, for all characteristic phenomena, providing a good fit.

This was not the case for Mercury, for which the different phenomena were fitted individually by different functions for $\Delta\lambda$ or Δt. The ephemerides in Neugebauer 1955 are not the result of one man's work: an entire group of scribes was involved that no doubt utilized different types of observations for their fits. What they shared were the observations, the arithmetic methods, and a number of conceptions based in practical knowledge, for instance, on how to predict by means of the data collected in the Goal-Year tablets.

Noel Swerdlow's *The Babylonian Theory of the Planets* [1998] gives a solid analysis of all systems of planetary computation and of the planetary data as recorded in the Diaries. Since the dates of the phenomena were given with much more precision than the positions in the zodiac,[7] Swerdlow investigated whether the dates might have been the empirical basis of the numerical functions and he presented a detailed reconstruction of all schemes based on observed dates. For the planetary phases, he determined "observed" synodic times in two ways:

(1) from recordings in the Diaries and

[7] The month, day, and zodiacal sign was noted for ΓΦΨΩ but not the position within the sign. Only the date is found with ϑ.

(2) through modern computation of first or last visibility.

He then compared these data with the synodic times of the Babylonian numerical functions, exhibiting substantial deviations. It was consequently not evident how the parameters of the models were determined from "observed" values. Further know-how was needed. Therefore, Swerdlow presented a reconstruction of all the planetary systems based on observations of the times of the phenomena, starting with "well-chosen" values of Δt. In a later paper [1999a], Swerdlow showed that computer-simulated dates of acronychal rising fit the numerical functions much better than dates of heliacal rising, which he had used in the first reconstruction. He thus proposed that the functions for the outer planets were constructed from the dates at which the acronychal rising of the planets had been observed.

Since the tabular texts establish a fixed connection between synodic arc and synodic time ($\Delta\lambda + C = \Delta t$), it is possible that the synodic times were calculated first and the synodic arcs were found by means of C or *vice versa*. The Babylonians utilized the connection in both directions. I believe that the Babylonians first developed their systems from the positions of Jupiter and Saturn, as Aaboe first proposed. Later, they may have applied the methods to data of observed times and may thus have been able to construct the numerical functions, for example, of the irregular planet Mercury, as reconstructed by Swerdlow. In either case, more precise data or some extra know-how is needed than what we find in the Diaries.

It is, however, not too serious that only the zodiacal sign was given for ΓΦΨΩ. The passings by Normal Stars were also recorded, so it is quite possible that the Babylonians could find the positions of the phases more precisely, for example, by estimating the planetary movement in its different phases. They were excellent observers and much more experienced than what we find in the Diaries. On the basis of an early observation text for Mars, Britton [2004, 33–55] showed how ongoing observations of planetary phenomena were improved and systematized during the time from 668 BC to around 600 BC, when planetary observations included nearly all elements reflected in the later observation texts, showing that a motion of ¼° was perceptible to sixth-century observers.

6. The lunar systems

The lunar ephemerides are the most remarkable achievement of Babylonian astronomy. They skillfully combine the effects of all variables that determine the synodic lunar phenomena and succeed in calculating the Lunar Six time-intervals near New or Full Moon and eclipses. A typical table will have 12 or 13 lines, one for each lunation within a year, and up to 13 or 19 columns recording the numerical functions necessary for the calculation of, for example, the day when the new crescent was visible for the first time together with NA_1, the time from sunset to its setting. NA_1 is a quite complicated quantity. It depends on the position of the Moon in the zodiacal circle, its latitude, the time from conjunction to the sunset in question, the length of daytime, and the lunar velocity. Each of these variables was taken into account in separate columns and then combined elegantly to find NA_1. Neugebauer identified the columns by the following letters:

T, Φ, [A], B, C, [D], E, Ψ, F, G, J, C′, K, [L], M, [N, O, Q, R], P.

The tabular lunar texts focused on New or Full Moon. Correspondingly, the astronomical quantities in the different columns are calculated for the moment of New and Full Moon.

The lunar texts are grouped into Systems A and B. The columns in square brackets are only present in System B texts, while columns Φ and C′ are specific to System A. Column T records the year and month of the lunation in question, the intercalations being regulated by means of a 19-year cycle, where 19 years = 235 synodic months. This means that Column T implicitly has the year-length of 12;22,6,19 synodic months, an excellent approximation to the tropical year, the period of the Sun's return to the vernal equinoctial point. However, the Babylonians did not distinguish between the tropical and the sidereal year. Column B was based on another year-length.

Column B calculates the position of the Moon $\lambda_{☾}$ at conjunction or opposition. In System A this calculation is made by means of a step-function, while System B uses a linear zigzag-function. At conjunctions, the position of the Sun ($\lambda_{\odot}$) equals $\lambda_{☾}$; at opposition, it equals $\lambda_{☾} + 180°$. Surprisingly, the synodic arc $\Delta\lambda$ depends heavily on the solar velocity, while the lunar contribution is negligible. Therefore, observations of positions of new crescents or Full Moons may have served as bases for the construction of Column B [Bernsen 1969]. Such data could have been combined with the empirical knowledge of the lunar movement in elongation and with the exact position of the Moon observed at eclipses. Function B of System A has the period P = 12;22,8 months, reckoning the excess of the year over 12 months as 0;22,8 synodic months = 11;4 *tithis*.

This value is used heavily in both Systems A and B in spite of the fact that the period of B in System B is 12;22,13,20 months. That different values of the length of the year were used within each system indicates that the numerical functions were numerical approximations to astronomical phenomena and not derived theoretically from one basic model. A newly published procedure-text [Britton, Horowitz, and Steele 2007] shows how the Babylonians in the late second century BC seemingly aimed at minimizing such small inconsistencies within System B. In comparison to the values known from tabular texts, some more precise parameters for the daily movement of the Sun and Moon are recorded here.

Column Φ belongs to System A and is connected to the lunar velocity. Column C gives the length of daytime at the beginning or middle of the month, respectively. Column E approximates the lunar latitude and F its momentary velocity, i.e., the velocity of the Moon measured at New or Full Moon. The length L of the synodic month is approximated by 29 days + G + J, where G is the contribution due to the varying lunar velocity, while J gives the solar contribution. K gives the time of conjunction (or opposition) with respect to sunset, and finally Column P contains the calculated value of NA_1 and the day at which it occurs.

Column C approximates the actual length of daytime with a "sine-like" curve. The lengths of daytime and nighttime equal the rising and setting time of the arc on the zodiacal circle between $\lambda_{\odot}°$ and $\lambda_{\odot}$+180°; therefore, they are connected to rising-times of arcs on the zodiacal circle. By means of Babylonian methods, these can be found easily from the partial sums of the Lunar Four, ME + GE_6 or ŠÚ + NA, which were observed and collected regularly.

7. Some complications

In System A, all functions accounting for the lunar velocity were derived from the numerical function recorded in the second Column Φ. Also of interest is a series of variable time-intervals that were also recorded in System A: time-intervals at 1, 6, and 12 synodic months. These time-intervals depend on the velocities of the Sun and Moon, and the Babylonians calculated them, correctly, as a sum of two terms: one depending only on the solar velocity and the other only on the lunar anomaly.[8] The solar component was deduced from Column B, while the lunar component was derived from Column Φ.

The burning questions are how the Babylonians were able to separate lunar and solar anomaly on the basis of their observed data and how Φ was derived from observations. Column Φ can be interpreted as the lunar contribution to the length of the Saros, a cycle of 223 lunar months. I have called its duration $\Delta^{223}t$, indicating that it is the difference in time between the beginnings of 2 months, situated 223 months apart. In the case of $\Delta^{223}t$, it is the solar contribution that dominates. Observed values of $\Delta^{223}t$ vary with the period of the year and not with the period of the lunar velocity, but no Babylonian estimate for the solar contribution had yet been found. Without that, Φ cannot be derived from observations of, for example, eclipses situated some *n* Saroses apart [Brack-Bernsen 1980].

Some other lunar observations must have been used: this is why, in my own research, I have focused on the Lunar Six time-intervals, because they incorporate the lunar velocity and investigated whether it is possible to find the period of Φ from the Lunar Six. It turns out that the sum of the Lunar Four, Σ = ŠÚ + NA + ME + GE_6, indeed, oscillates with the period P_{Φ} of Φ. Each of the partial sums, ŠÚ + NA or ME + GE_6, oscillates in tune with the year. The reason is that they measure the setting and rising-times, respectively, of the Moon's movement relative to the Sun within one day. Such rising- and setting-imes depend heavily on the momentary angle of the zodiacal circle at the eastern and western horizons, respectively.

This dependence is reduced effectively by adding the two, so that their sum Σ oscillates in tune with the lunar velocity and, hence, with Φ. The function Σ is quite noisy (i.e., it exhibits an irregular pattern) but it repeats nicely after 1 Saros; and the linear zigzag-function $\hat{Z}$,

[8] The amplitudes of the two variables, that is, the differences between their highest and lowest values, were well determined in all cases. See Brack-Bernsen 1980. A modern mathematical analysis of interference patterns resulting from the superposition of two periodic functions with slightly different periods, here $P_{\odot}$ and $P_{☾}$, shows how to separate the two dependences graphically.

which approximates it, has the same period, amplitude, and phase as Φ. This has led to the hypothesis that the numerical function Φ was derived empirically from the sum Σ of the Lunar Four. In the meantime, a function giving the solar contribution to the Saros has been found [Brack-Bernsen and Hunger 2002, 80–85; Brack-Bernsen and Steele 2005]. Thus, on this basis, the former identification of Φ as the lunar contribution to $\Delta^{223}t$ can now be accepted. The question still is, however, how Column Φ was constructed and how it still holds that the period P_Φ, which equals the period of the lunar velocity, was derived from horizon observations of the Lunar Four.

Britton worked intensively with the astronomy of the texts in Neugebauer 1955 and presented his reconstruction of the lunar systems in two major papers [2007b, 2009]. His starting-point was two ancient cycles, the Saros (= 223 months) and the 19-year calendar cycle (= 235 months). He suggested that System A was the work of one very clever author who, in addition to the ancient cycles, assembled the following empirical elements necessary for the construction of System A functions:

- an accurate anomalistic period-relation,
- estimates of the extremes and amplitudes of key eclipse-intervals, and
- an improved estimate of the length of the mean synodic month.

The next and crucial theoretical step consisted in building a mathematical model of the amplitude of 235 months in units equal to the change over 223 months due to lunar anomaly in the length of 223 months [Britton 2007b, 86].

In order to derive the different values for the length of the year, implicitly given by period-relations of Systems A and B, Britton postulated that the Babylonians used the positions of the Moon with respect to Normal Stars observed at eclipses. Well-chosen eclipses that had taken place 334, 335, and 804 months apart could lead to the estimate that after 19 years, the position of the Moon with respect to the Normal Stars would be shifted by 0;15° = ¼. This insight implies that:

(1) the variation of 235 months was due solely to lunar anomaly (uniquely among intervals bounded by eclipses), and
(2) the sidereal year was of 12;22,8 months, the period of Column B in System A.

In order to reconstruct the exact parameters of those functions accounting for the variable lunar velocity, Britton started with simple period-relations between synodic months m and anomalistic months m_a: $14m = 15m_a$ or $223m = 239m_a$. The Babylonians may have recognized that these were only approximate and so tried to find more accurate relations of the form $14k - 1m = 15k - 1m_a$. In order to determine k, Britton analyzed Lunar Four data and utilized the mean values of their sum Σ taken over 7 months and got the estimate: $17.85 < k < 18.25$. The value of k determined as 17;55,12 leads to the period-relation of Column Φ in System A, while $k = 18$ gives the period-relation of F in System B.

In his reconstruction, Britton then determined extremes and amplitudes of the following time-intervals: 6, 12, 223, and 235 synodic months, which all occur as intervals between lunar eclipses. To this end, Britton utilized computer-simulated observations of times and positions of the eclipses arranged according to the Saros-scheme mentioned above. In Britton 2009, the reconstruction of how the effects of lunar and solar anomaly were separated was based on one crucial insight: that the variation of the 235-month eclipse-interval is solely due to lunar anomaly. Its amplitude can be found from Britton's Saros-scheme, whereby the amplitudes of 223 and 12 months could be deduced by an elegant mathematical model.

Britton was right in his insistence that Column Φ is closely connected to lunar eclipses, and his reconstruction is one possibility. I would prefer a reconstruction that grows directly out of the Goal-Year method and proceeds without our heavy tool of algebraic notation.

That the numerical functions giving the length $\Delta^{235}t$ of 235 months show nice symmetries may simply be the result of the way they can be constructed from two versions R and S of Φ. We still do not know which kind of concept or what consideration determined the use of versions R and S of Φ, shifted by 1, 6, or 12 months, respectively, in order to find the lunar component of $\Delta^{1}t$, $\Delta^{6}t$, and $\Delta^{12}t$. Britton's reconstruction is based heavily on the structure of the Saros-scheme into which quite accurate times of the EPs are added.

Two questions arise: did the Babylonians have access to such a complete list of timed eclipses, and if so how did they know which special eclipses, situated far apart, they should pick in order to determine their parameters? Ptolemy approached the problem in that way but it is not evident that the Babylonians did. We still do not know with any certainty how all the columns associated with lunar velocity were developed. A method more closely related to the Goal-Year method, which is applicable to every month, seems to be preferable from a historical standpoint. This method was used by the Babylonians and it automatically gave them a means of comparing Lunar Six values of each syzygy with that of 1 Saros earlier.

8. Conclusion

Some years ago, Otto Neugebauer wrote:

> For the cuneiform ephemerides we can penetrate the astronomical significance of the individual steps, as one may expect with any sufficiently complex mathematical structure. But we have no concept of the arguments, mathematical as well as astronomical, which guided the inventors of these procedures. [Neugebauer 1975, 348]

Much has been achieved since that time. Aaboe and Swerdlow have shown how the planetary schemes can be reconstructed from Babylonian observations. Hunger has in six volumes edited the astronomical Diaries and related texts, so that we now have all the Babylonian observations which have survived on cuneiform tablets at our disposal. The edition by Roughton, Walker, and Steele of a text on Normal Stars has given us insight into how observed positions of the Moon and the planets with respect to Normal Stars could be transferred into positions in the zodiacal circle. This means that Babylonians had many more accurate planetary positions at their disposal than just the location given by zodiacal sign in the Diaries. We understand now how the movement of the Moon through the zodiacal band was surveyed by means of Normal Stars, and Ossendrijver's edition of procedure-texts includes many insights. There has also been progress in the reconstruction of Babylonian lunar theory. Britton has shown how one could construct many period-relations and numerical functions using Babylonian lunar data. His analyses have brought new insight as well. Still, I remain convinced that we have not yet found the final solution. The insight that observed eclipses could not be used for the construction of column Φ has led me to concentrate on the only other Babylonian observations which contain information on lunar velocity, the Lunar Six time-intervals, which until then had not been investigated. This has proved very fruitful. All reconstructions of column Φ and F are in a crucial step based on the sum Σ. But more important is that the research on Lunar Six has led to the discovery of the Goal-Year method and the edition of the early procedure-text TU 11 containing many rules for prediction. These give us insight into the methods, empirical knowledge, and astronomical concepts that guided the Babylonian scribes in developing their astronomy.