Progressive Pensions
and Labor Supply Incentives

Dissertation zur Erlangung des Grades eines
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Chapter 1

Introduction

Several Western economies operate pay-as-you-go pension systems, in which today's working generation finances the pensions of today's retired generation. Demographic change is putting significant pressure on the financial stability of these systems. Providing sufficient resources for the elderly becomes quite expensive in aging societies. Many countries have adopted reforms to mitigate the costs. These include raising the normal retirement age, linking pension payments to changes in life expectancy, and encouraging the use of private pension savings. This may still not be enough to maintain current living standards for the elderly, which is particularly problematic for households with low incomes and low levels of private savings. These tend to be people with little education, single mothers, and single women in general, people with a migration background, and the long-term unemployed, see Haan et al. (2017) and OECD (2017).

Most public pension systems in OECD countries provide some minimum retirement income independent of past earnings as a safety net for poor pensioners (OECD (2021a))[1]. For example, the Netherlands has a flat-rate basic pension that is related to the minimum wage. In 2020, the monthly pension payments valued 1,300 euro for a single household and 1,770 euro for a couple household.[2] In Switzerland, pension benefits are generally based on lifetime earnings and the number of years of contributions, but are also subject to a cap and a floor. Pension benefits value at least 16 percent of gross average earnings (2020: CHF 1,180) for individuals with a full contribution history, see OECD (2021f). The Czech Republic provides a basic pension equal to 10 percent of the legislated average wage (2020: CZK 3490) and an earnings-related benefit on top, see OECD (2021b). Only four OECD countries have

[1] These payments are often subject to further eligibility requirements regarding years of residence or years of pension contributions in that country.

[2] The basic benefit are 2 percent of the full value for each year a worker lives or works in the Netherlands, see OECD (2021d).

no basic or minimum pension, namely Australia, Finland, the US and, Germany, see OECD (2021a).[3] However, for example, the US supports the income poor in old age through a progressive component in the pension formula. Progressive pension systems are redistributive and provide disproportionately high pensions to those with low earnings records.

The German public pension system is largely proportional in its status quo. This means that pension payments in old age are proportional to lifetime earnings. The principle of proportionality, also known as the principle of equivalence, has a historical basis and is one of the core pillars of the German public pension system. It was introduced with the pension reform of 1957 and is intended to incentivize pension contributions and thus labor supply, see Rossbach (2022). Proportionality in the pension system is often perceived as fair. However, it ignores that life expectancy is a major determinant of the internal rate of return an individual obtains from her pension contributions. Life expectancy varies largely across income groups. Haan et al. (2017) report a life expectancy gap of 7 years between the 10 percent earnings riches and the 10 percent earnings poorest German men. This reflects into the pension system. Haan et al. (2020) document that the German pension system is actually regressive when accounting for heterogeneous life expectancy. A couple of studies for the US find that the progressivity of US Social Security is undone by the differences in life expectancy across income groups, see for example Goda et al. (2011). Although this makes a strong case for a redistributive component in the pension system, a progressive pension reform is politically difficult to implement. The history of German pension reforms provides some examples of deviations from the equivalence principle, but politicians have not yet dared to reform the pension system in a fundamental way to systematically redistribute income in old age. The recent introduction of the Grundrente in 2021 might be a first step in that direction. Since the Grundrente is tax-financed, it does not generate redistribution within the pension system. It is basically a transfer payment to pensioners with a long record of pension contributions. Eligible pensions are topped-up to a maximum value of 80% of the average pension benefit. Still, the Grundrente is largely criticized. The eligibility requirements are extremely complex and poorly communicated. The literature questions whether the Grundrente is well-targeted. Börsch-Supan and Goll (2021) estimate that 76 percent of the individuals who are poor in old age are not meeting the eligibility requirements, while 20 percent of beneficiaries are wealthy. Ragnitz (2020) finds that the Grundrente top-up is insufficient to significantly increase the pension payments of most poor individuals. The income of many eligible pensioners is still below the subsistence level.

[3]Still, these four countries provide targeted benefits that are means-tested.

Besides the discussion about the German Grundrente, there is a large literature on redistribution through the pension system in general, which is summarized in Chapter 2. Overall, the literature argues that higher pension progressivity leads to more redistribution and insurance provision on the one hand, but increases implicit taxes and therefore distorts labor supply choices on the other, see Fehr and Habermann (2008) Fehr (2000), and Nishiyama and Smetters (2008). That equity-efficiency trade-off is also discussed in the labor taxation literature. A seminal study by Emmanuel Saez in 2002 analyzes optimal income transfer programs for low earners. He shows that, when labor supply responses are concentrated along the extensive margin, an optimal labor tax policy explicitly subsidizes employment. This is empirically often the case for low earners, see for example Meyer (2002). I apply the mechanism identified by Saez (2002) to the pension system.

The general research question of this dissertation is: To what extent can a progressive pension system that subsidizes employment of low earners mitigate the economic costs of income redistribution in old age, and what is a feasible way to implement it? I use a structural model with heterogeneous agents to quantify the effects of different pension reform scenarios on labor supply, welfare, and inequality for different social groups. The model is calibrated to the German economy.

In particular, I propose a pension formula, that links pension payments to both individual earnings and an individual's employment status. I first show in an analytical model that it is possible to mitigate the economic costs of old age income redistribution by incentivizing employment participation of the beneficiary group. Next, I evaluate the effects of such a pension system in a quantitative stochastic overlapping generations model with heterogeneous agents. Households face an explicit labor force participation decision and can on top choose their working hours. They are subject to persistent shocks to labor productivity and longevity risk as well as shocks to individual life expectancy. They can partially self-insure through saving in a riskless asset. A government collects progressive taxes on labor earnings and taxes on consumption to finance government expenditures. Moreover, it operates a pay-as-you-go pension system that is financed by payroll taxes. The model is calibrated to the German economy in 2017. The starting point of the analysis is a situation with a proportional pension system, in which old age pension benefits are directly proportional to lifetime earnings. In the first reform scenario, I increase the progressivity of the pension system by allowing for a disproportional high accumulation of pension claims for earnings-poor working households, which comes at the expense of a cut in pension claims for high-earning individuals. The progressive pension component is directly linked to the individual employment decision and hence, households acquire pension claims for every year they were employed, irrespective of how much they earned. I implicitly assume that the public pension insurer is perfectly informed about an individual's employment status. I find long-run welfare gains of 0.31 percent and an increase in the employment rate of 1.3 percentage points. The aggregate efficiency effect is positive and amounts to a permanent rise in consumption of 0.73 percent. Still, aggregate labor supply and thus output decline by more than one percent in the long run.

However, in reality, the government may have problems in observing employment decisions. Individuals might only have a fictitious working contract or work minimal hours to become eligible to the pension subsidy. Therefore, direct employment subsidies are generally not feasible. Saez (2002) shows for the case of the tax system that a second-best policy looks quite similar to the Earnings Income Tax Credit (EITC) in the US. This is an income transfer program for low-earnings individuals. I appreciate that fact and study an EITC-style progressive pension system that solely relies on individual earnings as a measure to calculate pension benefits. The pension formula has a phase-in and a phase-out region similar to the EITC.[4] Households with earnings less than a certain threshold accumulate disproportional high pension claims for each year they are in employment. Consequently, the system sets incentives for both labor force participation and higher labor hours at the lower end of the earnings distribution. I show in a simulated model that the second-best policy is less efficient at stimulating labor supply, but can still preserve a large fraction of the positive employment, welfare, and efficiency effects.

The quantitative model just described makes a number of simplifying assumptions in order to reduce the state space and thus make the simulation of a transition path feasible. However, it comes at the cost of reduced realism. In particular, the model is silent on the effects for the largest group at risk of poverty in old age, women. I, therefore, provide an extension to the model, which is much more complete in modeling demographics. It accommodates single and couple households, male and female individuals, and fertility shocks to women. Moreover, I question the implicit assumption from the previous model that old age earnings are redistributed based on information on annual earnings. In fact, in some Western countries, such as Portugal[5], the Czech Republic[6] and the US[7], support low-income earners in old age through a progressive pension system. While the pension formulas differ, they

[4]See Chapter 2 for a description of the EITC scheme.

[5]Pension benefits are proportional to reference earnings. The earnings measure was the best 10 of the final 15 years, but that was extended to lifetime average earnings from 2017. For people with more than 40 contribution years, only the best 40 count, see to OECD (2021e).

[6]According to OECD (2021b), the earnings measure for the earnings-related pension averages across all years since 1986, but will gradually reach lifetime average. See also for a formal definition OECD (2020).

[7]See Section 3.3 for a description of US Social Security.

all redistribute based on information on aggregate lifetime earnings. The pension system is a tax instrument that facilitates the redistribution of lifetime earnings. But is it also favorable? Theory predicts substantial welfare gains from history-dependent income taxation, see for example Kapička (2022). I compare the case of redistributing old age income (1) given information on annual earnings during working life to (2) given information on lifetime earnings at retirement entry. I use exactly the same redistribution scheme in both cases. In particular, I again use a functional form similar to the EITC to exploit positive employment incentives for low-earning individuals.

I first show in a stylized analytical model that the EITC employment incentive applies to a broader population group if it is based on annual earnings. The results of the simulation model are in line with the theory. Old-age income redistribution based on annual earnings dominates the lifetime earnings-based redistribution. The labor supply incentive of the EITC-style redistribution scheme is substantial if applied to annual earnings. It incentivizes low earners to participate in the labor market every year. If an individual is not employed, the pension credit subsidy for that year is lost forever. Lifetime earnings-based redistribution does not exhibit such a strong extensive margin incentive. Moreover, individuals are tempted not to work in years with adverse productivity shocks due to increased insurance. Encouraging steady employment during working life is an important tool to reduce old-age poverty risk. Haan et al. (2017) document that individuals with long employment histories exhibit a very low risk of old-age poverty. Intuitively, redistribution should be more targeted if based on lifetime information, as short-term earnings fluctuations are balanced over time. However, our simulations show that the impact of increased labor supply of the earnings poor is outweighing the gains of more targeted redistribution. I find positive employment effects of 0.73 percentage points and long-run welfare gains of 0.52 percent with an annual earnings-based system. Lifetime accounting shows adverse effects on employment rates and very little welfare gains.

The dissertation is structured as follows. The next chapter provides an overview of the literature that is closely related to the dissertation. Chapter 3 describes the status quo of the German pension system in light of demographic change, and Chapter 4 discusses facts about lifetime inequality, that affect old age income inequality. In Chapter 5, I show that a well designed pension system can mitigate the economic costs of increased old-age income redistribution by incentivizing employment. In Chapter 6, I explore a novel channel in the design of progressive pensions, the redistribution base. I show that a progressive pension system that redistributes based on annual earnings dominates a pension system that redistributes based on lifetime earnings. The underlying model is calibrated to the German economy and can be used to explore many other social policy reform ideas. Finally, Chapter 7 provides summary remarks. The appendices provide further mathematical derivations as well as supplement tables and figures.

Chapter 2

Literature Review

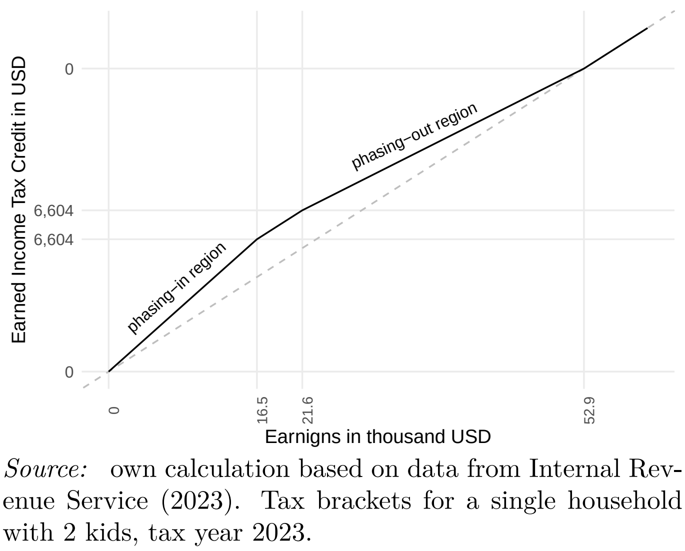
Methodologically, the dissertation is related to the broad literature that uses quantitative general equilibrium models with heterogeneous agents to analyze the incentive effects and welfare implications of redistributive fiscal policy. Popular themes of papers in this field include the optimal progressivity of the income tax code or the optimal taxation of capital income, see for example Domeij and Heathcote (2004), Conesa and Krueger (2006), Conesa et al. (2009), and Kindermann and Krueger (2022). In particular, I contribute to the following strands of literature.

Quantitative literature on progressive pension reforms I add to the large literature on labor supply effects and welfare consequences of progressive pension systems in heterogeneous agent life-cycle models. Huggett and Ventura (1999) were among the first to evaluate the welfare implications of redistributive social security in a quantitative macroeconomic model. They explore the case of replacing US Social Security with a two-tier system. The first tier is a retirement annuity which is proportional to pension contributions, the second tier provides a minimum pension for low earners. The main finding is that the two-tier system is hard to justify as it redistributes away from median-productive individuals, who constitute the majority. Nishiyama and Smetters (2008) find that the largest insurance gains in US Social Security are realized by a relatively long (35 years) averaging period. The optimal earnings-benefit link is almost proportional, as a progressive scheme would lead to sizable labor supply distortions while realizing only little further insurance gains. Newer studies such as Jones and Li (2023) are much more complete in modeling demographics (for instance education and health) as well as a government sector that provides further social insurance during working life. They show that welfare is maximized with flat baseline pension benefits and reduced claiming adjustment for both retiring early and late. O'Dea (2018) shows that substantial welfare benefits can be generated by strengthening a countrys means-tested old-age income floor at the cost of reducing pension payments related to the individual earnings history. Nam (2022) points to the fact that progressive pensions are an adequate measure for counteracting heterogeneity in job stability over the life cycle. The optimal degree of progressivity in the PIA function of US Social Security can lead to welfare gains of 0.52 percent of lifetime consumption relative to the current system. Golosov et al. (2013) determine the optimal structure of the PIA function for the year 2000. They use a model with an intensive- and extensive labor supply decision and find aggregate welfare gains equivalent to 2.72 percent. They choose a replacement rate of 100 percent for the first region and set the first bend point to 1,621 dollars of Average Indexed Monthly Earnings (AMIE). Afterward the optimal replacement rate is zero, which means that pension benefits are bounded at 1,621 dollars. Fehr and Habermann (2008) study the welfare effects of introducing a more progressive pension system with basic allowances for contributions and a flat-rate benefit in the German system. They find that positive liquidity and income insurance effects outweigh negative distortions in labor supply. This enhances overall efficiency. Another study on Germany is by Fehr et al. (2013). They introduce progressivity into a proportional pension system and measure transitional effects. A pension formula that puts 30% weight on a flat-rate component and the rest on an earnings-related component maximizes aggregate economic efficiency.

Empirical literature on pension reforms When conducting simulations with structural models, a fundamental question is how individuals in the real world respond to the reform. The empirical literature shows that labor supply reactions to pension reforms are present and sizable. A recent study by French et al. (2022) exploits a 1999 pension reform in Poland that affected men born after 1948. Individuals in that study are aged 51-54. The reform hits them 15-11 years before retirement entry. Poland switched from a Defined Benefit to a Notional Defined Contribution scheme, which tightened the contribution-benefit link in social security. They estimated an forward-looking employment elasticity of 0.44 with respect to the return to work. This study demonstrates that pension reforms have an impact on individuals' decisions regarding their labor supply many years prior to retirement. There are many studies that observe individuals close to retirement entry. Gelber et al. (2016) exploit exogenous variation due to the US Social Security Notch in 1977 to estimate the effect of pension income on earnings. A one-dollar increase in pension benefits causes earnings in the elderly years to fall by 46-61 percent due to a positive income effect. Manoli and Weber (2016) use data from Austria to estimate the effect of retirement benefits on the extensive margin labor supply decision at retirement entry. Overall, the empirical literature provides evidence that individuals adjust their labor supply in response to pension reforms both shortly before and years leading up to retirement entry.

Extensive margin labor supply Studies like Sánchez-Martín (2010) or Wallenius (2013) point to the fact that social security reforms can also have an impact on extensive margin labor supply choices. Yet, they only look at a household's optimal decision to retire. The dissertation more generally connects to the literature on extensive margin labor supply responses and the role of the fiscal tax and redistribution system. Saez (2002) was among the first to show that, when labor supply responses are concentrated along the extensive margin, an optimal labor tax policy explicitly subsidizes employment. As direct employment subsidies are generally not feasible,[1] a second-best policy looks quite similar as the EITC in the US. The EITC is a major income transfer program in the US that aims to encourage low-income individuals to work by partially matching earnings with a refundable tax credit. It was introduced in 1975 and has been extended several times. The size of the credit depends on the marriage status and the number of children in the family. The main beneficiaries are single-parent families. Regardless of the composition of the household, the subsidy starts at the first dollar earned and hence targets all low earners. Figure 2.1 shows the transfer scheme for a single parent with two children in 2023. In the phasing-in region, every earned dollar is matched with 40 cents by the government. The credit is maximal for annual earnings between 16,510 and 21,560 dollars and amounts to 6,604 dollars. It phases out afterward and is zero for earnings above 52,918 dollars. The dashed reference line shows the situation without the EITC.

Figure 2.1: Earned Income Tax Credit (EITC) 2023



Source: own calculation based on data from Internal Revenue Service (2023). Tax brackets for a single household with 2 kids, tax year 2023.

A series of empirical studies have quantified the EITC's impact on labor supply. There is a large consensus on the positive employment effects for low-earners. Most

[1]Households might only have a fictitious working contract or work minimal hours.

of the studies use data from the Current Population Survey (CPS) and focus on single mothers, as the EITC reforms in the 1990s have created strong work intensives for that group. For instance, Eissa and Liebman (1996) find an increase in labor force participation of single mothers by up to 2.8 percentage points, while intensive margin labor supply is not distorted for those who are already in the workforce. Meyer and Rosenbaum (2001) show that the EITC and other tax changes account for over 60 percent of the 1984-1996 increase in employment of single mothers compared to childless single women. Hotz et al. (2001) exploit regional variation from a natural experiment in California to identify the effects of an EITC expansion on the local labor market. They present evidence that the EITC sets strong participation incentives for low-skilled workers. However, a recent study by Kleven (2021) questions the previous findings. He postulates that the increase in participation rates in the 1990s was mainly driven by confounders. First, the state welfare reforms between 1992-96 and the federal welfare reform act of 1996 introduced time limits and work requirements for welfare payments. Second, the booming macroeconomy, and finally, a change in social norms.

The dissertation adds to the quantitative EITC literature as it demonstrates that the EITC mechanism can be applied to the pension system as well. It is most effective when redistributing old-age income based on information on annual earnings during one's working life, as it encourages a steady employment history.

Lifetime earnings redistribution This dissertation also adds to the literature on lifetime earnings taxation. In the US, income redistribution is organized by two different tax instruments, namely Social Security and the income tax system. US Social Security redistributes based on the AIME, which is a measure that approximates lifetime earnings, see Section 3.3. The tax system, which includes income transfer systems such as the EITC, is based on annual earnings.[2] This division might be justified by the nature of the tax and pension system. Pension payments are activated when working life ends and can be based on information on the full earnings history. Income taxes are due annually. However, there are good reasons to consider lifetime income taxation. Haan et al. (2019) find that taxes would provide more insurance if based on lifetime instead of annual earnings. Vickrey (1939) and Vickrey (1947) propose a cumulative averaging system to realize lifetime income taxation. Vickrey suggests annual tax payments, where the tax base is not only income from the actual

[2]According to Liebman (2002a), the US tax system allowed for tax averaging in the years 1964 to 1986. Taxpayers who experienced a large increase in income could allocate some of their income to the previous four years. The New Income Tax Act of 1971 introduced tax averaging in Canada. It was abolished in 1988, see Davies (1975) and Sato (2021). Tax averaging clearly puts an immense administrative burden on the tax authorities. However, advances in digitization and the management of big data should be able to solve this problem.

calendar year but accumulated income of the averaging period. The current year is treated as the end of an averaging period. The overall tax burdens are calculated based on all earnings in the averaging period. The amount due in the current calendar year is the difference between the overall tax burdens and the sum of the past tax payments. Payments are hence lower in years with relatively little earnings. In the extreme case, the averaging period can be a lifetime. Vickrey's scheme aims to realize horizontal equity[3] and has been recognized by the literature of capital income taxation, see for instance Auerbach (1991). The literature on extending the tax base of labor income to longer periods is relatively small. Liebman (2002b) uses a simple model to estimate efficiency gains from lifetime taxation. A switch from annual to lifetime taxation leads to an increase in earnings and tax revenues of 1 percent. This leads to a 7 percent reduction in total deadweight loss from all tax instruments. Age-dependent taxation is a related idea that is more prominent in the literature. A couple of recent studies find substantial welfare gains from age-dependent income taxation such as Kapička (2022) and Golosov et al. (2016). This dissertation adds to the literature on lifetime earnings taxation. I analyze labor supply incentives of progressive pension systems that redistribute based on annual earnings vs. progressive pension systems that redistribute based on lifetime earnings. I use a quantitative model with detailed demographics to simulate the effect of both reforms. I show that redistribution based on annual earnings sets strong incentives to participate in the workforce every year, while redistribution based on lifetime earnings provides more insurance and causes larger economic costs.

Very heterogeneous agent models The dissertation relates to a recent literature that uses large-scale quantitative simulation models with very detailed heterogeneity on the household level. These studies analyze the impact of public policies on individuals of different gender or family type. A recent paper by Guner et al. (2021) explores the replacement of means-tested transfers by simpler policies such as Universal Basic Income or Negative Income Taxes. As in my model, individuals make intensive- and extensive labor choices, face fertility and earnings risk, and differ in education, gender, and household type. Kurnaz (2021) studies optimal taxation of families in a similar framework. Kaygusuz (2015) conducts four reform exercises in a model with single and couple households that accounts for detailed assortative mating behavior. In particular, the paper quantifies labor supply effects when eliminating redistributive features[4] in US Social Security. Dropping spousal and survivor's benefits has the largest consequences. Eliminating only the progressive benefit formula increases intensive margin labor supply (0.2% for men, 0.3%

[3] Individuals with the same average income over time should pay the same average tax rate.

[4] Spousal and survivor's benefits, a concave benefit-contribution link, and a contribution cap for high earner

for women), but distorts employment rates of married women (-0.7%). This is, to some extent, a vice-versa experiment to my reform in Chapter 6 and in line with my findings. However, Kaygusuz (2015) abstracts from any sort of earnings risk and can hence not quantify insurance effects. This dissertation provides a large-scale simulation model that is calibrated to the German economy. The model is flexible enough to evaluate many other social policy reform ideas in future research.

Chapter 3

The Status Quo of the German Pension System

In order to justify the need for a reform of the German pension system, this chapter describes its status quo. I provide a brief overview of how pensions are currently calculated and describe the financial situation of the system in light of demographic change. As the old-age poverty risk rate is rising, a progressive pension reform could help to mitigate the impact of demographic change on the poor in old age. An example of a progressive pension system is US Social Security, which is briefly described. Finally, I show elements of the German pension system that are redistributive today or used to in the past.

3.1 The German pension system

The German pension system is based on a three-pillar model consisting of the public pension insurance (Gesetzliche Rentenversicherung), the occupational pension scheme (Betriebliche Altervorsorge), and private pension plans (Private Rentenversicherung). This dissertation discusses only the public pension system.

All employees whose income is subject to social security contributions contribute to the public pension scheme. Individuals who are self-employed or not working can enroll voluntarily. The system is based on a pay-as-you-go model, whereby current contributions from workers are used to fund the benefits of current pensioners. The 2023 contribution rate is 18.6 percent of gross earnings and contributions are evenly split between employers and employees. As a reward for their pension contributions, individuals are credited pension entitlements, known as earnings points, each year of their working lives. The normal retirement age is 66 in 2023, and it is gradually set to increase to 67 by 2031. However, in reality, many workers tend to retire earlier than the official retirement age, either by making use of early-retirement programs for long-term insured or by applying for disability pensions. For instance, in 2021, the actual retirement age was 64.1 for old-age pensioners and 53.6 for disability pensioners, see Deutsche Rentenversicherung Bund (2022b). As of December 2020, the public pension scheme counted 82.5 million members. 39 million of them were actively insured (they are compulsory or voluntary contributing to the pension system), 17.7 million were passively insured (they are currently not contributing to the system, but used to) and 25.8 million received pension benefits, see Deutsche Rentenversicherung Bund (2022b). These figures show that most Germans participate in the public pension system at some point in their lives, and many depend on pension benefits in old age. Ensuring the stability of the system is an ongoing social policy task.

3.1.1 Calculation of pensions

Individuals are credited with pension entitlements in proportion to their pension contributions during their working lives. Once retired, the monthly pension is recalculated each year based on the so-called current pension value. This section describes the pension calculation scheme and largely follows a study by the scientific board of the Federal Ministry for Economic Affairs (Wissenschaftlicher Beirat) in Bundesministerium für Wirtschaft und Energie (BMWi) (2021).

Accumulation of pension entitlements Any earnings y_j below the contribution cap $y_{max,j}$ are subject to pension contributions at rate τ_p. Individuals obtain pension entitlements in proportion to their contribution, which are recorded as earnings points ep_j.[1] The final earnings points value at retirement entry in period j_r is denoted ep_{j_r}. The accumulation formula reads

$$ep_{j_r} = \sum_{j=1}^{j_r-1} ep_j + \frac{\min(y_j, y_{max,j})}{\bar{y}_j},$$

where $\bar{y}_j$ is a reference salary, which corresponds to the average earnings of the actively insured population in the previous calendar year.[2] In addition to obtaining earning points as a reward for pension contributions, people can also be credited earnings points through mandatory insurance. This applies for example to mothers of young children and short-term unemployed.

[1]The contribution cap is adjusted annually for the development of wages. In 2022 $y_{max} = 84,600$, the reference salary $\bar{y} = 38,901$ and ep_{max} is hence 2.14, see Deutsche Rentenversicherung Bund (2022a).

[2]Earnings points are actually recorded on a monthly basis, for simplification I abstract from that in this dissertation and assume annual records.

The current pension value The current pension value is denoted by pv and defines the monetary value of one earnings point. In 2022, one earnings point was worth 36.02 euros in monthly pension payments, see Deutsche Rentenversicherung Bund (2022a). The pension value is calculated annually according to the following formula

$$pv = pv^{-} \times wf \times cf \times sf.$$

In particular, the pension value of the previous year pv^{-} is multiplied by the wage factor wf, the contribution factor cf, and the sustainability factor sf. The current pension value pv hence adjusts to the development of

- gross wages w through the wage factor $wf = \frac{w}{w^{-}}$,
- the contribution rate τ_p through the contribution factor $cf = \frac{\tau_p}{\tau_p^{-}}$ and
- the ratio of pensioners to actively insured PR through the sustainability factor $sf = 1 + \left(1 - \frac{PR}{PR^{-}}\right) \times 0.25$.

The pension value increases with growing wages, but its development is dampened by the contribution factor and by the sustainability factor in order to reduce the burden on the working generation. The sustainability factor was 1.0076 in 2022, see Deutsche Rentenversicherung Bund (2022b), but will fall well below one with the retirement of the Baby Boomer. This should reduce the pension level markedly. However, two laws are currently interrupting this mechanism. The first is the pension guarantee, which was enacted in 2009 and prohibits pensions from falling. This guarantee includes the so-called catch-up factor, which halves future increases in the pension value until it is restored to its previous level.[3] The second law is a pension reform from 2019, the so-called Doppelte Haltelinie. It sets a lower bound for the pre-tax security level and hence for the current pension value pv, as outlined in Section 3.1.3.

The pension formula Individual monthly pensions p are calculated by

$$p = ep \times ef \times ptf \times pv,$$

where ef is the entry factor and ptf is the pension type factor. The entry factor is 1 for individuals who claim retirement benefits at the normal retirement age. Otherwise, the entry factor is reduced by 0.03 for each month of early retirement and increased by 0.05 for each month of delayed retirement. The pension type factor is 1 for old age pensions and lower for disability and widow pensions. For example,

[3] The catch-up factor - Nachholfaktor - is suspended since the pension reform 2019.

a person retiring at normal retirement age with 45 earnings points will receive a monthly pension of 1,620.90 euros in 2022.

Lifetime earnings are the main determinant of the relative income position in old age. However, the overall generosity of the system is determined by the pension value, which is set by policymakers. It is supposed to adapt to the development of the economy through the wage factor as well as to the demographic development through the sustainability factor. Germany is facing a major change in the age structure of its population and it is very questionable whether the pension system can cope with it. The current system differentiates only along one dimension, the old versus the young generation. Thus, the future burden is borne by the working generation through higher pension contributions, or by the old generation through lower pension benefits, or a combination of both. Other concepts, such as increasing the burden on the rich (young, old, or both), do not fit.

3.1.2 Demographic trends

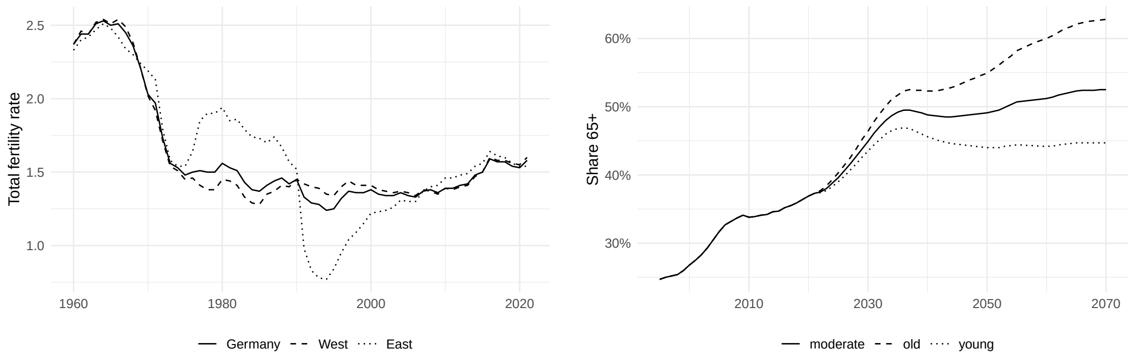
This section presents some statistics on the demographic development in Germany. Life expectancy is rising dramatically. According to the Statistisches Bundesamt (Destatis), a 65-year-old man (woman) had a remaining life expectancy of 14.3 (18.2) years in 1993 and 17.8 (21.1) years in 2019. This is an increase of 3.5 (2.9) years within one generation. The Germans not only live longer, they also have fewer children. The total fertility rate fell from 2.5 in 1965 to 1.4 children per woman in 1985 and has been stagnant for some time.[4] Although the fertility rate has slightly increased and is now above 1.5, it is still far from the level of 2.1 children per woman needed to replace the parental generation. The left panel of Figure 3.1 shows the development of the total fertility rate for Germany as a whole, as well as for West- and East Germany separately. The trends in birth development exhibit quite some differences between West and East Germany. After the baby boom ended in the late 1960s, fertility in West Germany remained at a constant low level. In East Germany, family policy measures such as birth allowances and marriage loans led to an increase in fertility since 1972. After reunification, there was another sharp decline in birth rates. Since then, the total fertility rate has recovered and has even been slightly higher than in West Germany in recent years, see Bundesinstitut für Bevölkerungsforschung (2022).

The combination of both trends leads to a substantial increase in the old age de-

[4]The total fertility rate describes the relative frequency of births among women during a specific period. It represents the average number of children that a woman would have throughout her lifetime, assuming the fertility conditions of the given year were to apply to her from age 15 to age 49. This number of children per woman is hypothetical, as it reflects the fertility rate of a modeled generation of women rather than a specific one.

pendency ratio (OAD ratio). The OAD ratio measures the size of the working-age population (ages 20-64) relative to the size of the old-age population (ages 65+). The right panel of Figure 3.1 shows the OAD ratio for Germany in the years 1995-2021 as well as projections until 2070 as reported by the Statistisches Bundesamt (Destatis). In 1995, there were 25 individuals aged 65+ for every 100 working-age individuals. Until 2021, this ratio has increased to 37:100. The projections are based on different assumptions regarding the birth rate, life expectancy, and migration. The figure shows the scenarios for a moderately aging population, a relatively young, and a relatively old population. In all scenarios, the OAP ratio will substantially increase until 2035 due to the aging of the Baby Boomer generation and will remain well above 40% afterward. There is no sign that Germany will return to the age structure of the last century even when the boomer passes away. This would require a significantly higher birth rate in the long term. The age structure is changing significantly and this will soon be reflected in the financial situation of the pension system.

Figure 3.1: Old age dependency ratio



Source total fertility rate: own calculation based on data from the Bundesinstitut für Bevölkerungsforschung (2022).
Source OAD rate: own calculation based on data from the Statistisches Bundesamt (Destatis). Projection variants G2L2W2 (moderate), G1L3W1 (old) and G1L3W1 (young).

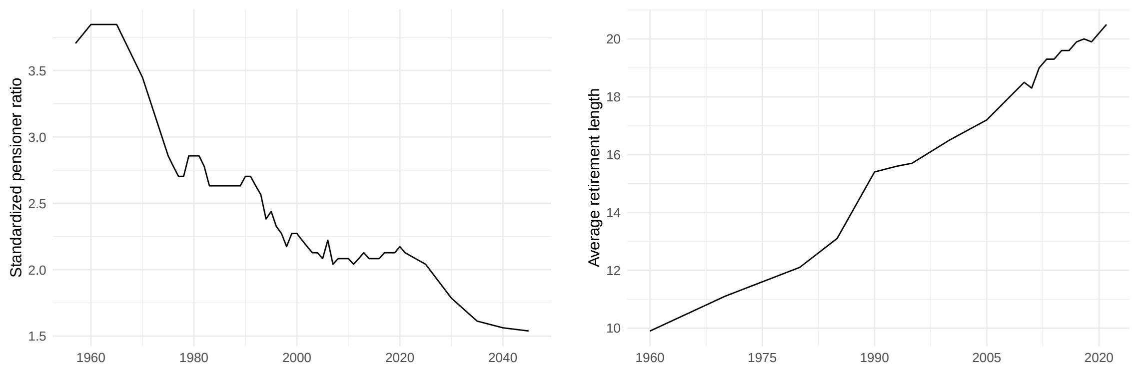
Demographic change has more dimensions than the aging of the population. For example, family structures are also changing. Marriage rates have been falling for decades. According to the Statistisches Bundesamt (Destatis), in 1965 8.2 marriages were formed per 1000 individuals. This value has decreased to 4.3 in 2021 and therefore almost halved. Moreover, marriages are less stable. While only 20 percent of all marriages formed in 1965 were divorced within 25 years (until 1990), 39 percent of the marriages formed in 1995 have been divorced 25 years later, see Bundesinstitut für Bevölkerungsforschung (2020). These developments suggest that the family, as an institution, is becoming a less reliable source of individual old-age security. This factor must be taken into account when considering pension reforms, for example by providing labor supply incentives for the second earner of a family.

3.1.3 The financial situation

This section analyzes the financial situation of the pension system today and provides projections for the near future in light of demographic change. The central trade-off in a pay-as-you-go pension system is the balance between the financial burden on the younger generation and the provision of adequate pension benefits for the old. The burdens to the young are usually measured through the contribution rate τ_p, which is currently 18.6% of gross earnings. The generosity of the pension system for the elderly is measured by the pre-tax security level.[5] It is calculated as the standard pension[6] divided by the average earnings (both after taxes and social security contribution) and is currently at 48.1%.

A report by the Deutsche Rentenversicherung Bund (2022c) shows that the effects of demographic change are already visible in the pension system today. For example, the standardized pension ratio (vereinfachter Rentnerquotient), which measures the ratio of contributors to pensioners, was 3.8 in 1960 and declined to 2.3 in 2000 and to 2.1 in 2021.[7] Projections suggest that it will drop further to 1.5 in 2045. Hence, only 1.5 standard workers will finance the pension of one standard pensioner. The average retirement duration was 9.9 years in 1960 and has increased to 20.5 years in 2021. Figure 3.2 shows the development of the standardized pensioner ratio and the average retirement duration over time.

Figure 3.2: Aging population and the pension system



Source: own calculation based on data and projections from Deutsche Rentenversicherung Bund (2022c).

Table 3.1 provides details on the current financial situation of the pension system. Pension contributions make up 77 percent of the funds, with the remainder being

[5]The pre-tax security level is a highly standardized and stylized measure. The OECD (2021c) estimates a gross replacement rate of 45.1 percent and a net replacement rate of 52.9 percent for mean earners in 2020. This means net pension payments value on average 52.9 percent of previous net earnings.

[6]The pension benefits of an individual who worked for 45 years and earned every year the average income.

[7]This is $\frac{\sum \text{pension payments}}{\text{standard pension}} \times \frac{\sum \text{pension contribuitions}}{\text{standard contribution}}$.

transfer payments from the federal government.[8] While the pension system generated a surplus of 2.1 billion euros in 2022, it will likely run on a deficit in the near future. For instance, the pension report by the Bundesministerium Arbeit und Soziales (2022b) predicts a deficit of 1.125 billion euros in 2023, which will increase to 16.725 billion euros in 2026. Federal transfers, valuing 81 billion in 2022 (22.8% of the overall expenditures), will increase to 140.1 billion in 2036 (23.3%). This suggests that the burdens to the taxpayer will increase substantially in the next decade due to both a higher contribution rate and growing federal transfers. Still, the security level for the pensioners will shrink.

Table 3.1: Revenues and expenditures GRV in 2022

	2022
Revenues	356,777
contributions	274,520
federal subsidies	326,399
Expenditures	354,661
pension payments	308,382
miners pensions	23,777
rehabilitation measures	6,700
administration costs	4,600
Surplus	2,116

Source: Deutsche Rentenversicherung Bund (2022a), in million euros.

To counteract the deficit, pension parameters will need to be adjusted with the retirement of the Baby Boomer cohort. According to projections by the Bundesministerium Arbeit und Soziales (2022b), the contribution rate will remain at 18.6 percent for three more years and will increase slowly from 2027 onward. At the same time, the pre-tax security level will decline to 44.9 percent in 2036. The study by the scientific board of the Federal Ministry for Economic Affairs (Wissenschaftlicher Beirat) Bundesministerium für Wirtschaft und Energie (BMWi) (2021) provides projections until 2060. In a scenario with no fundamental reforms, they see the long-term values for the contribution rate and the pre-tax security level at 24.5 and 42 percent, respectively. In other words, there will be an increase in the burden on young people and a decrease in pension benefits.

Of course, none of this comes as a surprise and has been seen for a long time. Since the 2000s, a couple of reforms were passed to adjust the pension system to

[8]These subsidies are used to cover non-insurance services (versicherungsfremde Leistungen). These are expenditures that are not directly covered by contributions such as pension entitlements for short-term unemployed or child-raising mothers. Moreover, the subsidies help to ensure financial stability and the pension system to cover potential deficits, see Bundesministerium Arbeit und Soziales (2022a).

demographic development. Earlier reforms were primarily aimed at protecting the working generation from excessive burdens imposed by the pension system. This includes the introduction of the sustainability factor in 2004 and the increase of the retirement age in 2007. However, as the reforms started taking their first effects, the debate on old-age poverty was becoming more pronounced. Although the demographic situation was as tense as before, the pension policy became more generous from 2014 onward. Early retirement options for the long-term insured were introduced, and the pension adjustment rules were limited until 2025 by the Doppelte Haltelinie, which sets a cap on the contribution rate of 20 percent and limits the pre-tax security level to a minimum of 48 percent. Finally, in 2022 the Grundrente was introduced. This is a tax-financed transfer payment for poor pensioners with a long record of pension contributions.

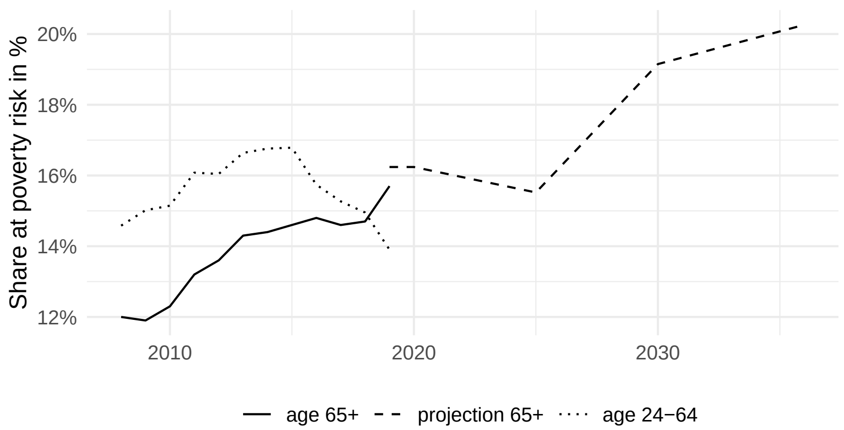
3.2 Old-age poverty

None of the latest pension reforms will be able to contain increasing poverty among the elderly in the long term. Per capita pension benefits will be significantly lower for future generations, which leads to a linear decline in pension payments for all population groups in a proportional pension system. Still, this does not necessarily mean that all social groups will be equally affected. Low-income pensioners are likely to suffer the most, as they have a high consumption ratio and tend to have little private retirement savings. Poverty among the elderly will likely increase.

Poverty can be measured by the poverty risk rate, which is defined as the percentage of households whose disposable income falls below 60 percent of the national median income. According to the latest data from the Statistisches Bundesamt (Destatis), the poverty risk rate was 14.8 percent in 2019, which corresponds to 12 million people in Germany. While the working generation used to face a higher risk of poverty compared to the older generation, this is now changing. Since 2018, the poverty risk rate of generation 65+ has been higher. Figure 3.3 shows that development graphically. A study by Haan et al. (2017) estimates that the old-age poverty risk rate will reach 20 percent in 2036. Several factors have contributed to that trend. These include previous pension reforms, in particular the introduction of the sustainability factor, lower pension entitlements due to a growing low-income sector, and high unemployment rates in the 1990s and early 2000s.

The amount of earnings points at retirement entry e_{j_r} is an important indicator of poverty risk in old age. Figure 3.4 shows the distribution of earnings points e_{j_r} for individuals with a record of at least 20 pension-relevant years in the 2017

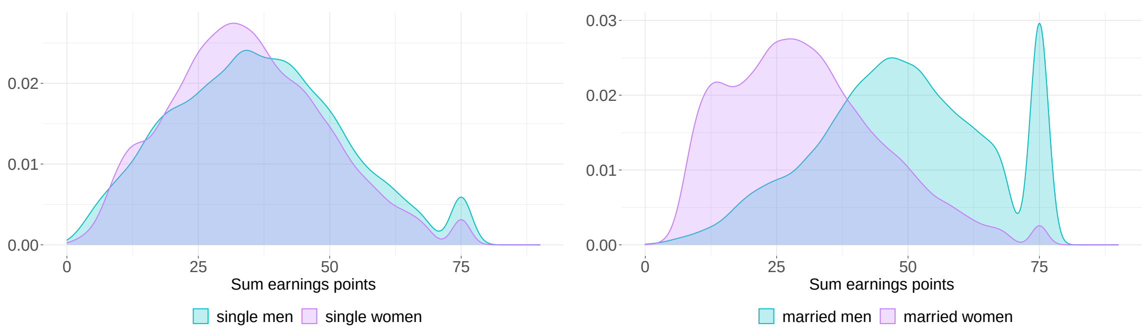
Figure 3.3: Poverty risk in Germany



Source: own calculation based on data from the Statistisches Bundesamt (Destatis), projections from Haan et al. (2017).

retirement cohort.[9] Single women accumulated on average 34.5 earnings points (mothers: 33.3, childless: 37.7), which is only slightly below the average for single men of 36.4 earnings points. Married women accumulated on average 30.6 earnings points (mothers: 29.5, childless: 39.6) and married men 49.0.[10] Although married mothers receive on average lower pension payments than single mothers, they are usually better of in old age as they can rely on their husbands' pension benefits. Note, the 25th percentile of single women is 24 earnings points. This means that 25 percent of single women in the sample live on pension payments of less than 864 euros per month.

Figure 3.4: Kernel density: earnings points



Source: own calculation based on data from FDZ-RV – SUFVVL2017. 75 represents the mean value of those with more than 70 earnings points.

Since pension payments are proportional to lifetime earnings, poverty is transferred

[9] This refers to the actual number of earnings points a person accumulates upon retirement. This includes earnings points due to own contributions, compulsory insurance such as for raising children, as well as earnings points transfers resulting from a divorce.

[10] The mode of the distribution of married men is 75. This is not the actual mode, but the mean value for pension entitlements beyond the censoring threshold of 70.

to retirement age and even amplified. To increase retirement income, poor pensioners have to apply for the means-tested minimum income program for the elderly (Grundsicherung im Alter), which is recommended for (single) pensioners with a monthly income of less than 973 euros, see Deutsche Rentenversicherung Bund (2023a). The Statistisches Bundesamt (Destatis) reports that in September 2022, 650.000 pensioners received benefits from the minimum income program, and the federal expenditures for the minimum income transfer program amounted to 8.1 billion euros in 2021, see Bundesministerium für Arbeit und Soziales (2021). However, many more might be eligible. A study by Buslei et al. (2019) estimates that the take-up rate is only 40 percent.[11] In particular pensioners older than 77 years and property owners are often not applying. This implies that almost one million pensioners are currently living below the subsistence level. Low pension benefits are problematic, in particular as welfare programs are not fully utilized. The figures show that old-age poverty is already problematic today and will be worse in the near future. Many pensioners live on very low pension payments today although they contributed to the system for many years. A decline in the pension level will push many additional pensioners below the subsistence level.

3.3 US Social Security: a progressive pension system

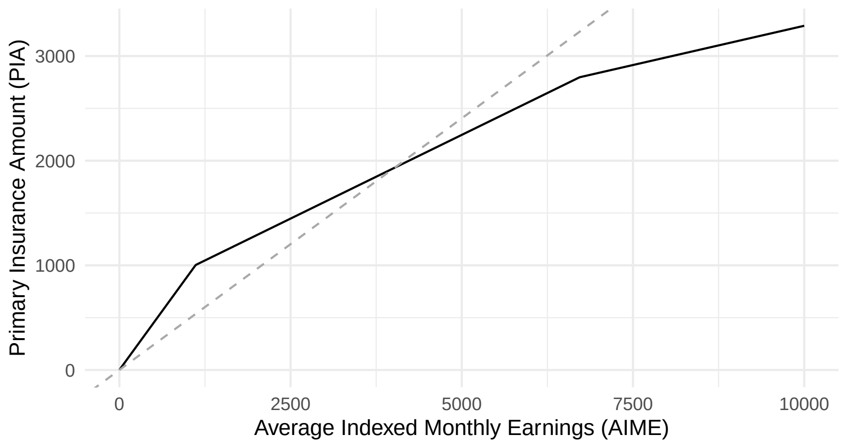
One option to mitigate the effects of demographic change for the poor is to reform the distribution scheme of the pension system. So far, the system redistributes only between generations, with the contributions of the young being used to finance the pensions of the old (intergenerational redistribution). This could be extended by a redistributive component within the pensioner generation (intragenerational redistribution). Some Western countries, such as Portugal, the Czech Republic, and the US, support low-income earners in old age through a progressive pension system, see OECD (2021a). This section briefly describes the US system as a comparative case.

Similar to the German system, pension payments in the US are based on an individual's previous earnings. The first step in calculating the monthly pension benefits is to determine the Average Indexed Monthly Earnings (AIME). To do this, an individual's earnings history is adjusted for inflation and then the average of the highest 35 years of earnings is computed. This average is then divided by 12 to arrive at the AIME, see Social Security Administration (2023). The AIME is used to determine the individual's primary insurance amount (PIA). The PIA is calculated using a progressive formula that includes two bend points, which depend on the year in

[11] The households that do not claim their entitlement could increase their income by almost 30 percent on average.

which a worker becomes eligible for retirement benefits.[12] The pension formula for the 2023 cohort is presented in Figure 3.5 (solid line). It consists of three parts: the first 1,115 dollars of the AIME value is multiplied by 90%, the next 5,606 dollars of the AIME value is multiplied by 32%, and any AIME value over 6,721 dollars is multiplied by 15%. This sums up the PIA, i.e. the monthly pension benefit.

Figure 3.5: US Social Security PIA formula



Source: own calculation based on data from SSA, Social Security Administration (2023).

The dashed line in Figure 3.5 plots a proportional system with a replacement rate of 48.1 percent. This is the pre-tax security level of the German pension insurance in 2023. US Social Security provides substantially more redistribution. Income-poor individuals receive disproportional high benefits, which are financed by a lower pension level for the rich. This example raises the question if the German system should also accommodate a progressive element in order to support the poorest pensioners.

3.4 Progressivity in the German pension system

One of the main concerns about the introduction of progressivity in the German pension system is the violation of the so-called equivalence principle. This is a key element of the pension system since the so-called Great Pension Reform 1957, see Rossbach (2022). Pension payments are supposed to be proportional to pension contributions in order to achieve a sense of fairness. However, that idea was introduced in 1957. At that time, the population structure was fundamentally different from today. The average length of retirement was about 10 years and the population was growing. Given the demographic developments of the last fifty years and the resulting new requirements, it is legitimate to question the idea of the equivalence

[12]The worker turns 62 or becomes disabled before age 62, or dies before attaining age 62.

principle. Moreover, the history of the German public pension insurance provides a number of reforms that deviate from the principle of proportionality.

Reform 1972: Pension by minimum earnings (Rente nach Mindesteinkommen, see Steffen (2011))
Any earnings points earned prior to 1973 that were less than 0.75 earnings points in a given year were upgraded to a value of 0.75.
Eligibility requirements:

- at least 25 years of mandatory contributions upon retirement[13]
- accumulated earnings points until 1973 valued on average less than 0.75 per year.

The reform was justified by a significant gender wage gap and large regional wage differences in the post-war period, which should not affect future pension payments. This reform was particularly generous for long-term part-time workers.

Reform 1992: Pension by minimum earnings points (Rente nach Mindestentgeltpunkten, see Steffen (2011))
Earnings points earned prior to 1992 that value less than 0.0625 in a month (this is 75 percent of the average salary) were upgraded by factor 1.5 to a maximum of 0.0625.[14] Individuals with

- at least 35 years of pension-relevant employment (including times before and after 1992)[15]
- and an average earnings points value of 0.75 per year at retirement entry

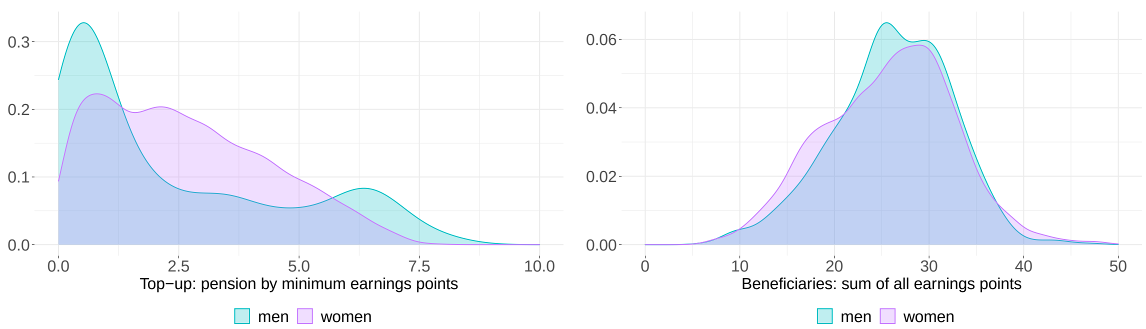
were eligible. I use data on the 2004 retirement cohort from the scientific usefile *Vollendete Versichertenleben* (FDZ-RV (2017a)) provided by the Deutsche Rentenversicherung to analyze the effect of the 1992 reform. Most of the individuals in the sample were born between 1940 and 1945, so they have been in the workforce from about 1960 to 2004 and potentially benefited from the reform. Among the individuals with at least 35 pension-relevant years, 4.8 percent of the men and 40.8 percent of the women received a pension top-up. Male beneficiaries got on average 2.3 and female beneficiaries 2.7 additional earnings points. This corresponds to an increase in monthly pension benefits of 83.0 and 97.4 euros in 2022.

The left panel of Figure 3.6 shows that the distribution of the top-up differs between men and women. The male distribution has two modes, one a the very left end of the

[13]In particular years with so-called Pfichtbeitragszeiten, Ersatzzeiten, and Zurrechnungszeiten.
[14]As the top-up is proportional, part-time workers benefit less than in the 1973 reform.
[15]Years with so-called Pfichtbeitragszeiten, Ersatzzeiten, Zurrechnungszeiten, Anrechnungszeiten, Zeiten freiwilliger Beträge, Berücksichtigungszeiten für Kindererziehung.

distribution and one at a relatively high value of 6.5 earnings points. The female distribution is triangle-shaped. This indicates that the underlying labor supply pattern differs across genders. However, the final distribution of earnings points at retirement entry is fairly similar between men and women, as shown in the right panel of Figure 3.6. Male beneficiaries enter retirement on average with 26.2 earnings points and female beneficiaries with 25.8 earnings points. Nevertheless, women are clearly the main beneficiary group. About 40 percent of the women with a long employment history have benefited. Their pensions increase on average by more than 10 percent.

Figure 3.6: Kernel density: earnings points top-up



Source: own calculation based on data from FDZ-RV – SUFVVL2004-2017.

Reform 2019: Reduced pension contributions for midi-jobs (Beschäftigung im Übergangsbereich, see Deutsche Rentenversicherung Bund (2023c))

Individuals earning between 520 and 2,000 euros per month (so-called midi-jobbers) are subject to a reduced pension contribution rate while they acquire the full amount of earnings points. According to Buslei et al. (2023), as of 2023 about 6.2 million people (i.e. 18 percent of all regular workers) fall in this earnings range. 74.9 percent of the midi-job worker are female and 68.9 percent work part-time.

Pension contributions are usually split equally between the employer and the employee, but the rule is different for midi-job workers. The left panel of Figure 3.7 shows the composition of the pension contributions for this group. The light green area is the share of the contributions covered by the employer and the dark green area is the share covered by the worker. The red area is the share that is not covered at all by contributions. In fact, the uncovered earnings points top-up is quite small. For instance, an individual with monthly earnings of 1,000 euros[16] acquires 0.2781 earnings points per year and the actual contributions would value 0.2481

[16] Avg. monthly earnings 2023: $\bar{y} = 3,595.17$, $y = 1,000$, earnings points $ep = \frac{y}{\bar{y}} = 0.2781$
Contribution base midi-job: $1.1081459459 \times y - 216.2918918918 = 891.8541$
Earnings points due to contributions: $ep_c = \frac{891.8541}{\bar{y}} = 0.2481$

earnings points. The annual top-up is 0.03 earnings points. If that individual works for 45 years, she accumulates 1.35 uncovered earnings points at retirement entry. This corresponds to a monthly pensions increase of $1.35 \times 36.02 = 48.63$ euros as of 2022. The main benefit for the worker is the disproportional high contribution

Figure 3.7: Pension contributions in midi-jobs

Source: own calculation based on information from Deutsche Rentenversicherung Bund (2023c).

share of the employer. The right panel of Figure 3.7 shows that roughly 70 percent of the monthly contribution waiver is covered by the employer, the remainder is uncovered. If the contributions of an individual who earns 1,000 euros per month were, as usually, 1:1 matched by the employer, the contributions would only cover 0.1804 earnings points per year.[17] The actual earnings points top-up for the worker is hence 0.0977 (uncovered: 0.0301, employer: 0.0676). This increases the worker's pensions by 158.40 euros per month after 45 working years.[18]

The midi-job rule eases the burden on employees during working life, as pension contributions are reduced. For example, an employee with a monthly salary of 1,000 euros saves 32.68 euros in contributions per month. In fact, the midi-job rule does not increase the pensions of the poor, it just lowers contributions during working life. Moreover, it subsidizes part-time work and is hence not an appropriate measure to reduce old-age poverty risk.

Reform 2021: Grundrente (Grundrente, see DRV Bund 2023b)

In January 2021, Germany introduced the Grundrente. This program is designed to increase the pension payments of individuals who

- have contributed to the pension system for many years,
- are not eligible for the minimum income program for the elderly,
- but are still at risk of old-age poverty.

[17] Contribution base worker c_w: $1.3513513513 \times y - 702.7027027027 = 648.6486$
Earnings points: $ep = \frac{648.6486}{\bar{y}} = 0.1804$

[18] All calculations are based on the year 2023, assuming that nothing changes in the future.

The aim of the Grundrente is to address this gap and provide additional support to those with low pensions. The Grundrente is tax-financed and a top-up in addition to the proportional pension payments. About 1.1 million pensioners received the Grundrente in January 2023 and it is valued on average 86 euros per capita and month, see Tagesschau (2023). A major criticism is the large administrative costs, which amounted to 380 million euros in 2021, see Bockenheimer (2021). This is one-third of the aggregate Grundrente benefits. A study by Börsch-Supan and Goll (2021) estimate that 81 percent of the beneficiaries are women.

To explain the Grundrente benefit formula, it is easiest to assume that the pension insurance runs two different pension accounts for each individual. One is a record for the proportional pension payments and the other one is a record for the Grundrente. Eligibility requirements differ largely. While the calculation of proportional pension payment is straightforward, the Grundrente is not. It depends on two sorts of assessment times as well as the distribution of earnings points over the years.

Assessment times The first measure is the number of years n_{SS} with relevant contributions to the pension system. These are contributions due to employment, caregiving, child-rearing, and sickness (Grundrentenzeiten). Periods of mini-job work without personal contributions, voluntary contributions, or unemployment do not count as years n_{SS}. The second measure is the number of years n_{GR} ($n_{GR} \in n_{SS}$) with earnings $\frac{y_j}{\bar{y}} \geq 0.3$, the so-called Grundrentenbewertungszeiten. This corresponds to gross earnings of more than 1,079 euros per month in 2023.

Annual earnings point accumulation The dashed line in Figure 3.8 (left) shows the accumulation formula for earnings points. Earnings points, which are proportional to earnings, are only credited in years that count as Grundrentenbewertungszeiten n_{GR}. At retirement entry, the average earnings points ep_{avg} are calculated as

$$ep_{avg} = \frac{\sum_{j=1}^{n_{GR}} ep_j}{n_{GR}}.$$

Note, ep_{avg} is either zero or greater than or equal to 0.3 by construction and differs from the average of the earnings points in the account for the proportional pension.

Top-up cap Given the years of relevant contributions n_{SS}, the value ep_{max} is computed as

$$ep_{max} = \begin{cases} 0.8 & \text{if } n_{SS} \geq 35 \\ 0.6 & \text{if } n_{SS} = 34 \\ 0.4 & \text{if } n_{SS} = 33 \\ 0.0 & \text{if } n_{SS} < 33. \end{cases}$$

The Grundrente formula At retirement entry, the final earnings points ep_{GR} are computed according to

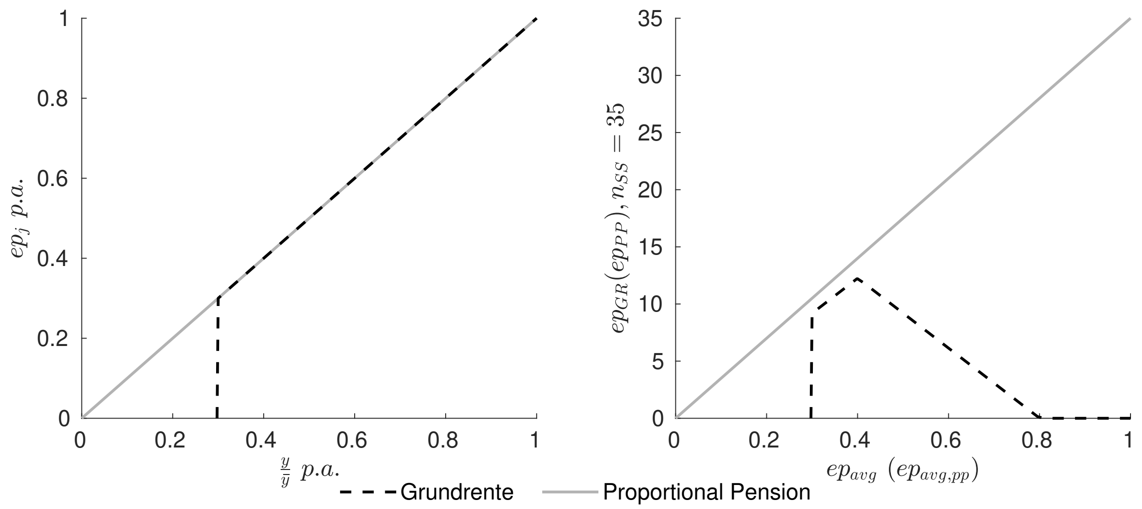
$$ep_{GR} = \max\left[\min\left[2 \times ep_{avg}, ep_{max}\right] - ep_{avg}, 0\right] \times 0.875 \times \min\left[n_{GR}, 35\right]. \tag{3.1}$$

To compute the final earnings points ep_{GR}, the average annual earnings points ep_{avg} are doubled but only up to the maximum value ep_{max}, which is defined by the number of years with relevant contributions n_{SS}. Next, the average earnings points ep_{avg} are deducted and the remainder is multiplied by 0.875. This value is then multiplied by the number of years with pension contributions n_{SS}, but with a maximum of 35 years. The result is the final earnigns points value ep_{GR} in the Grundrente pension account. The right panel of Figure 3.8 shows the average annual earnings points ep_{avg} on the horizontal axis and the corresponding final earnings points ep_{GR} on the vertical axis for an individual with ($n_{SS} = n_{GR} = 35$) graphically. Individuals with average earnings points $ep_{avg} = 0.4$ archive with $ep_{GR} = 12.25$ the highest final Grundrente earnings point value in their accounts. Individuals with $ep_{avg} > 0.8$ end up with zero final Grundrente earnings points.

Grundrente pension payments As in the proportional system, earnings points ep_{GR} are multiplied by the current pension value to compute monthly pension payments. A pensioner with $ep_{avg} = 0.4$ and $n_{SS} = 35$ receives in 2023 a monthly Grundrente payment of $0.4 \times 0.857 \times 35 \times 36.02 = 441.25$ euros. This is a top-up to her proportional pension of $0.4 \times 35 \times 36.02 = 504.28$ euros and hence substantial. However, the Grundrente is subject to an income test.

Income test The pension payments from the Grundrente are subject to an income test at the household level. In 2023, singles with a monthly income of max. 1,317 euros and married couples with a monthly income of at most 2,055 euros receive the full amount of Grundrente as outlined in the previous paragraph. 60 percent of any

Figure 3.8: Grundrente earnings points



income between 1,318 and 1,685 euros (couples: 2,056 and 2,423 euros) is deducted from the Grundrente. Any incomes above 1,686 (2,424) euros are fully deducted, see Deutsche Rentenversicherung Bund (2023b). The income test ensures that the Grundrente is limited to low-income households. However, there is no means test with respect to wealth.

This section provides a number of examples that are clearly not in line with the equivalence principle. That list could be extended to include the provision of non-insurance benefits, survivors' benefits, and the fact that different life expectancies are not taken into account. The mantra of the equivalence principle should not hinder future progressive pension reform, because it did not in past reforms either.

Chapter 4

Facts about Lifetime Inequality

This chapter documents salient facts on different dimensions of inequality and risk that households face over their life cycle. I restrict my attention to mechanisms that have a first-order impact on old age income and that could shape the need for redistribution through the pension system. These facts will guide me in constructing my simulation model in Chapter 5. Specifically, I first examine the statistical properties of the household's labor earnings process using administrative data. In the next step, I discuss the relationship between lifetime income and individual life expectancy. The chapter is largely based on a revised version of the working paper *Progressive Pensions as an Incentive for Labor Force Participation* by Kindermann and Püschel (2021).

4.1 Inequality in labor earnings

Providing a proper model for the household's life cycle labor earnings process is crucial if one wants to assess the benefits of fiscal redistribution and insurance. To this end, I use administrative data from the German public pension insurance system to estimate life cycle labor earnings profiles and earnings risk. I use data from Germany in this discussion, as the German public pension insurance system (Deutsche Rentenversicherung) offers an administrative dataset with detailed information on the earnings histories of a subsample of all insured households. What I find in this data is consistent with recent research from other countries, especially the US. In particular, I will argue that a simple log-normal AR(1) process is not a good description of the dynamics of individual labor earnings, a fact also supported by the work of Guvenen et al. (2021), Busch and Ludwig (2020) and de Nardi et al. (2019). In addition, I document a strong positive correlation between individual lifetime income and life expectancy. Meara et al. (2008), Mackenbach et al. (2015), de Gelder et al. (2017), Waldron (2007) and Cristia (2011) find a similar relationship for various other countries.

4.1.1 Data selection

My dataset, the scientific use file of the Versichertenkontenstichprobe 2017 (FDZ-RV – SUFVSKT2002-2017), contains information from the insurance accounts of 69,520 individuals actively insured under the public mandatory German pension scheme.[1] The data set consists of two parts: One provides demographic characteristics such as age, gender and education for the year 2017. The other one records the entire history of an individual's accumulated pension claims and employment status on a monthly basis together with an indicator of the source these claims were accumulated from (like labor earnings, unemployment, child care, etc.). The sample covers worker who were born between 1950 and 1987 and who were not permanently retired in 2017. A historical record starts in the year an individual turns 14 and ends when she turns 65. Hence, the maximum length of an employment history is 624 month. Overall, the data set includes more than 28 million worker-month observations for the years 1964 to 2017. As the sample ends in Dezember 2017, individuals who were born in 1953 or later have shorter histories (e.g. 612 month for the 1953 cohort). Those who have never been employed are not represented, as they never were registered with the insurance. Although the monthly records start in 1964, I only consider observations for the years 2000 to 2016. This has certain advantages: First, my estimates are based on recent data; second, I avoid structural breaks arising from German reunification and policy-changes in the 1990s and third, different age cohorts are represented in the sample at similar shares in each year (early sample years cover only young individuals). The data-selection process is summarized in Table 4.1.

I restrict the sample such that it targets workers who are attached to the labor market. I therefore limit my attention to men aged between 25 and 60 who are likely to already have finished education and military service and are not in the process of retiring. Individuals who received pensions such as disability pensions or early-retirement pensions are dropped. I divide the sample into two educational groups[2] in accordance with the scheme to the International Standard Classification of Education of the UNESCO (ISCED 2011). An individual is defined as college-educated[3] if she is classified ISCED 6 (Bachelors or equivalent level) or above, excluding ISCED 65 (trade and technical schools, including master craftsman training). She is high

[1]The German pension scheme covered a total of 38 million actively insured individuals in 2017.

[2]The data set provides the variable `TTSC3_KLDB2010` which indicates an individual's highest degree in 2017 according to the classification of education scheme of the Federal Statistical Office of Germany (Klassifikation der Berufe 2010 - KldB 2010).

[3]Corresponds to KldB 2010 4-6.

Table 4.1: Data selection

	Individuals	Observations
Initial data set (1975 - 2017)	69,520	28,166,952
Initial data set (2000 - 2016)	69,520	14,139,972
- Women	−36,634	−7,451,736
- Ages < 25		−1,014,120
- Ages > 60		−152,976
	32,886	5,521,140
- Ind. that receive pensions	−3,606	−605,208
	29,280	4,915,932
- Ind. with unknown education	−13,677	−2,346,840
	15,603	2,569,092
Annualized data (2000 - 2016)	15,603	214,091
No contributory earnings in 2000 - 2016	−361	−6,137
No contributory earnings in entire year		−18,770
Final data set	15,242	189,184
High school education	11,821	149,929
College education	3,421	39,255
Observations on regular workers		181,469
Observations on low earners		7,715

Source: own calculation, based on data from FDZ-RV – SUFVSKT2002-2017.

school-educated[4] if she is classified ISCED 5 and below or ISCED 65. I drop individuals with unknown education status.

For estimating earnings profiles I use all pension claims that stem from (1) regular-employment, (2) mini-jobs or (3) unemployment benefits (short-term, max. 12 month)[5]. Since individuals are productive when searching for a new job, I consider short-term unemployment as an employment type. Table 4.2 shows the distribution of employment states across monthly observations. About 13 percent of all observations are on months with no contributory earnings. Such observations emerge when individuals become self-employed or civil servants, when they take care leave, face a longer spell of unemployment or just decide to drop out of the workforce. I code non-contributory months as periods of zero earnings.

To make the data comparable with my simulation model, I change the time-dimension of the panel from monthly to annual, by computing the sum of acquired pensions

[4]Corresponds to KldB 2010 1-3. High school education means that somebody holds no college degree. Still, that individual has very likely vocational training.

[5]According to the variable `SES`.

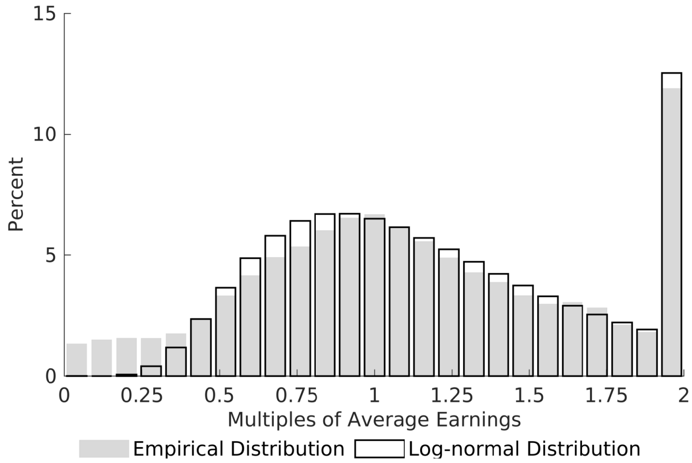
Table 4.2: Distribution of employment states (across monthly observations)

Employment Status	Observations	Percent
Regular employment	2,139,302	83.27
Mini-job	44,113	1.72
Unemployment (short-term)	55,138	2.15
No contributory earnings	330,539	12.86
Total	2,569,092	100.00

Source: own calculation, based on data from FDZ-RV – SUFVSKT2002-2017.

claims for each calendar year. Finally, I exclude observations with no contributory earnings in an entire calendar year, see Table 4.1. My final data set is an unbalanced annual panel for the years 2000 to 2016 with 15,242 individuals – of which 22.4 percent are college-educated – and a total of 189,184 observations.

Figure 4.1: Histogram of earnings points



Source: own calculation, based on data from FDZ-RV – SUFVSKT2002-2017.

Figure 4.1 shows a histogram of raw individual annual earnings (gray bars) expressed as multiples of average labor earnings of the total population. The figure reveals two salient features of the data: First, the data are top-coded at about two times average earnings. This is owing to the presence of a contribution ceiling in the German pension system. Second and more importantly, there is a substantial mass at values below 0.25, which is atypical under the usual assumption of log-normally distributed earnings. To strengthen this point, the framed bars in Figure 4.1 show the histogram of a log-normal distribution that provides the best fit to my data. Under log-normality, the share of households at the lower end of the earnings distribution is almost zero. My sample hence looks stratified and using the assumption of a common log-normal distribution to describe individual earnings seems invalid.

In order to take account of the substantial mass of individuals at the lower end of the earnings distribution, I split the sample into two sub-samples. The first one contains individuals with normal labor earnings and the second one those with extraordinarily low earnings. An individual i with education s and age j is defined as a low earner in year t if she acquires pension claims y^p_{isjt} that are less than those of somebody who is working full-time for six month at minimum wage. With 250 annual working days, 8 hours of work per day, a minimum wage of 8.50 euros and an average income of 36,187 euros in 2016, the threshold below which an individual counts as low earner is

$$\frac{125 \times 8 \times 8.5}{36,187} = 0.23. \tag{4.1}$$

Within the sample, 95.9% of observations are regular earnings and 4.1% are low earnings. I use observations from regular workers to estimate earnings profiles as shown in the left panel of Figure 4.2. All those with earnings below the threshold are low earnings individuals. Low earnings individuals can be thought of as having some months of temporary unemployment or non-employment throughout a year or as being marginally employed (i.e. having a so-called mini-job).

4.1.2 Earnings measurement

Earnings y_{isjt} of an individual i of education s and age j at time t are subject to social security contribution. There is a contribution threshold $y_{max,t}$ and any earnings beyond that value are non-contributory. Contributory earnings hence amount to $\min\left(y_{isjt},\ y_{max,t}\right)$. They are converted into pension claims y^p_{isjt} by diving them through average earnings $\bar{y}_t$. I account for the fact that pension claims from so-called mini-and midi jobs are subject to a reduced pension contribution rate.[6] Both, the contribution threshold $y_{max,t}$ and average earnings $\bar{y}_t$ are adjusted annually to account for wage growth. The contribution threshold $y_{max,t}$ currently amounts to about twice the average earnings $\bar{y}_t$.[7]

For my analysis, it is most convenient to use pension claims y^p_{isjt} as an earnings measure, as they are stationary over time. In particular, I define

$$y^p_{isjt} = \frac{\min\left(y_{isjt},\ y_{max,t}\right)}{\bar{y}_t}. \tag{4.2}$$

Obviously, the data are right-censored at $y_{max,t}$, see also Figure 4.1.

[6] In a mini-job, an individual can earn a maximum of EUR 450. Midi-jobs cover earnings from 451 to 850 euros in 2016.

[7] See Section 11 in Deutsche Rentenversicherung Bund (2020) for a full history of reference values.

4.1.3 The dynamics of normal earnings

I describe the earnings dynamics of the normal earner sample by a standard AR(1) process in logs. I therefore split the normal labor earnings sample according to an individuals' education level $s \in \{0, 1\}$. $s = 0$ summarizes all individuals with high school education, while $s = 1$ indicates the college-educated workforce. For each education group, I derive a deterministic life cycle labor earnings profile as well as an AR(1) process for residual log-labor earnings. More specifically, I estimate the statistical model

$$\log\left(y_{isjt}\right) = \kappa_{t,s} + \theta_{j,s} + \eta_{isjt} \quad \text{with} \quad \eta_{isjt} = \rho_s \eta_{isj-1,t-1} + \varepsilon_{isjt}, \tag{4.3}$$

for labor earnings y_{isjt} of an individual i with education s at age j in year t. $\kappa_{t,s}$ is a year fixed effect that controls for earnings changes along the business cycle. $\theta_{j,s}$ is an age fixed effect that informs about the age-earnings relationship. The noise term ε_{isjt} is assumed to follow a normal distribution with mean 0 and variance $\sigma^2_{\varepsilon,s}$. Furthermore, I let the stochastic process start from its long-run variance σ^2_s. This means that

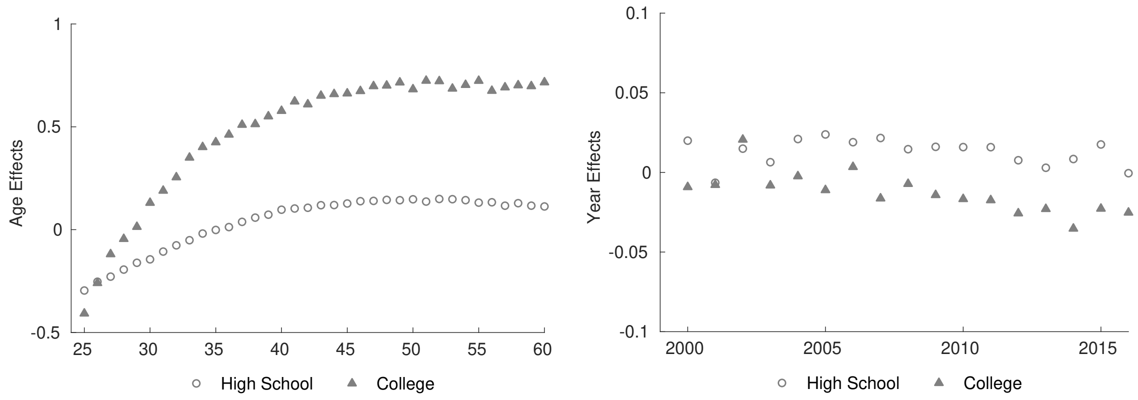
$$\varepsilon_{isjt} \sim N(0, \sigma^2_{\varepsilon,s}) \quad \text{and} \quad \eta_{is20t} \sim N\left(0, \sigma^2_s\right) \text{ with } \sigma^2_s = \frac{\sigma^2_{\varepsilon,s}}{1 - \rho^2_s}.$$

I use a generalized method of moments estimator to determine the parameters of this model. I thereby control for the fact that the data are top-coded at the threshold $y_{max,t}$ and that I truncated them at the low earner threshold $y_{min} = 0.23$. See Appendix A for estimation details.

Lifetime earnings profiles The results of this estimation process are quite standard in the sense that the estimates exhibit typical life cycle labor earnings profiles, a significant college wage premium as well as a high auto-correlation of earnings.

The left panel of Figure 4.2 visualizes the point estimates of the age fixed effects by education level. Up to the age of 45, earnings steeply increase for both education groups, especially so for the college-educated. Afterwards, they stagnate or decline slightly for the rest of an individual's working life. This shape of life cycle earnings is quite common in the empirical literature and has been found for other countries as well, see for example Heckman et al. (1998) or Casanova (2013). The college-wage premium implied by these profiles is equal to 60 percent, which is in line with empirical findings (OECD, 2016). The right panel of the figure shows the year fixed effects. These are generally small relative to the age effects and exhibit some cyclical dynamics.

Figure 4.2: Age fixed-effects and year fixed-effects



Source: own estimation, based on data from FDZ-RV – SUFVSKT2002-2017.

Table 4.3 summarizes the estimation results for the residual earnings process.

Table 4.3: Estimates of residual log-earnings process

	High School $s = 0$	College $s = 1$
Autocorrelation $\hat{\rho}_s$	0.9881	0.9900
Innovation Variance $\hat{\sigma}^2_{\varepsilon,s}$	0.0042	0.0040
Unconditional Variance $\frac{\hat{\sigma}^2_{\varepsilon,s}}{1-\hat{\rho}^2_s}$	0.1787	0.2016

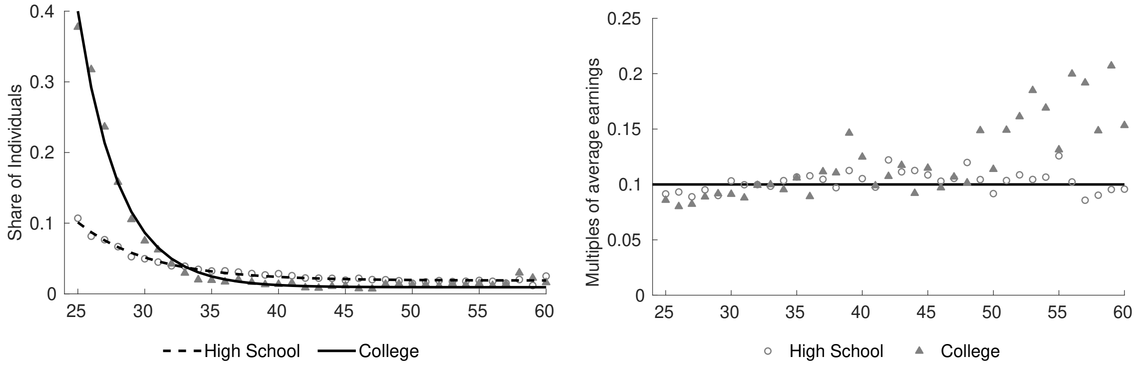
Source: own estimation, based on data from FDZ-RV – SUFVSKT2002-2017.

The parameter estimates are again fairly standard. Both high school and college-educated workers exhibit a high persistence in labor earnings with an unconditional earnings variance of around 15 to 20 percent. This is in line with what has been found in Bayer and Juessen (2012), for example.

4.1.4 The low earnings group

As discussed before, a simple log-normal distribution is not enough to capture the bimodal distribution of the earnings data. In a second step, I therefore examine the statistical properties of the low labor earnings sample. The left hand side of Figure 4.3 shows – for each age between 25 and 60 – the fraction of individuals in an age cohort that is a member of the low earnings group (circles for high school and triangles for college-educated workers). This fraction declines over time, which indicates that individuals transition between the states of low and normal labor earnings while moving through their life cycle. College-educated workers predominantly experience low labor earnings early in their career, for example when doing internships or while working in addition to studying in college. At later ages, the share of individuals

Figure 4.3: Life cycle dynamics of low labor earnings



Source: own estimation, based on data from FDZ-RV – SUFVSKT2002-2017.

in the low earnings region converges to almost zero. For high school workers, on the other hand, experiencing a low earnings episode is a phenomenon that is more equally distributed across ages. Labor earnings of individuals in the low earnings group are by and large independent of age and education.[8] The right panel of Figure 4.3 shows the age-education-earnings relationship of the low earnings segment of the population. For all ages and education types, average earnings of the low earnings group is approximately equal to 10 percent of average labor earnings. With average earnings amounting to roughly 37,000 euros a year, the typical low earnings individual makes 3,700 euros a year, or 308 euros a month.

I interpret the findings in Figure 4.3 in the following way: Following empirical evidence from the labor literature that starts with Hall (1982), I assume that individuals face different degrees of career stability. While some exhibit stable career paths, others frequently transition into and out of employment.[9] I use these findings to calibrate my model in Chapter 5. In particular, I model career stability as a one-time discrete shock $m \in \{0, 1\}$ that a fraction ϕ_m of the population in each education groups draws at the beginning of working life. While individuals with $m = 0$ face a stable career path and consequently never experience a low earnings episode, those with $m = 1$ may transition into and out of low earnings throughout their entire working life.

4.1.5 The transition process for low earnings episodes

I model the transition into and out of low earnings as a first-order discrete Markov process with a transition matrix as shown in equation (4.4). In particular, I assume

[8]Partly this may be owing to my choice of the earnings threshold that separates normal and low earners, which is independent of age and education as well.

[9]See Kuhn and Ploj (2020) for a recent investigation of the importance of career stability for heterogeneity in household wealth.

that households with unstable careers ($m = 1$) face the education-specific transition matrix

$$\Pi^s_{low} = \begin{bmatrix} 1 - \pi^s_{low,0} & \pi^s_{low,0} \\ 1 - \pi^s_{low,1} & \pi^s_{low,1} \end{bmatrix}. \tag{4.4}$$

The probability $\pi^s_{low,0}$ indicates the likelihood of a normal earner to transition into the low earnings state in the next period, while $\pi^s_{low,1}$ is the probability to remain in the low earnings state. I assume that at age 25, a fraction

$$\Omega_{25,s} = \omega^s_{low}$$

of all individuals with an unstable career path ($m = 1$) start out in the low earnings state. Over time, the share of low earnings individuals evolves according to

$$\Omega_{j+1,s} = \Omega_{j,s} \times \pi^s_{low,1} + (1 - \Omega_{j,s}) \times \pi^s_{low,0}.$$

Knowing that only a share ϕ_m of the population of education level s is exposed to low earnings shocks at all, I can calculate the fraction of individuals in each education-age bin that currently experiences a low earnings episode as

$$\Phi_{j,s} = \phi_m \times \Omega_{j,s}.$$

I use the empirical counterparts to these shares $\hat{\Phi}_{j,s}$ shown in the left panel of Figure 4.3 to estimate the six free parameters ω^s_{low}, $\pi^s_{low,0}$ and $\pi^s_{low,1}$ for $s \in \{0, 1\}$ of this statistical model. My choices of parameter minimizes a simple residual sum of squares between the empirical and the model based moments $\Phi_{j,s}$. Table 4.4 summarizes the point estimates that provide the best fit to the data in a least squares sense. The solid and dashed lines in the left panel of Figure 4.3 indicate the model's predicted share of households in the low earnings group. As noted above,

Table 4.4: Estimates of low-earnings transition process

	High School $s = 0$	College $s = 1$
Productivity level $\exp(\eta_0)$	0.0675	0.0675
Initial share of low income earners ω^s_{low}	0.2022	0.8005
Probability to transition to low earnings $\pi^s_{low,0}$	0.0064	0.0052
Probability to stay low income earner $\pi^s_{low,1}$	0.8374	0.7282

Source: own estimation, based on data from FDZ-RV – SUFVSKT2002-2017.

college-educated workers experience low earnings episodes predominantly early in their life, while for high school workers the risk of drawing a low income shock is more equally distributed over the life cycle. This is reflected in the estimates of ω^s_{low}, i.e. the share of low earners at age 25. Throughout her working life, the chance for a regular worker to transition into a low earnings episode is very small (less than 1 percent for both education groups). Being in the low income state however has quite some persistence. With a persistence of 0.84 and 0.73, the average duration of a low earnings episode is 6.15 years for high school workers and 3.68 years for the college-educated, respectively.

Summing up, the investigation of the labor earnings process of individuals in my administrative dataset has shown that a simple log-normal AR(1) process is not rich enough to describe the earnings dynamics of households. While it might be a fair description of what happens in "normal" times, individuals can also experience very low earnings episodes. I provid a statistical model that can fit the data on low earners by age and education. Note that the recent literature on fiscal redistribution has highlighted the importance of generating a realistic earnings distribution, see for example Castaneda et al. (2003) or Kindermann and Krueger (2022), which can not simply be captured by a single AR(1) labor productivity component. While the aforementioned papers concentrate on income at the top end of the distribution, I use a similar methodology to more realistically characterize households at the bottom, who might be more loosely attached to the labor force and therefore responsive to employment incentives.

4.2 Inequality in life expectancy

From the perspective of the pension system, inequality in earnings and earnings risk is not the only factor that can justify redistributive elements. While individual life expectancy has increased substantially for younger cohorts, a recent literature also documents that the increase in life expectancy is not equally distributed within cohorts. Meara et al. (2008) show that the decline in mortality rates at older ages in the US in between 1980 and 2000 can almost exclusively be attributed to a rising life expectancy of highly educated individuals. For the lower skilled, life expectancy has stagnated in the same time period, leading to a 30 percent increase in the longevity-education gap. Mackenbach et al. (2015) and de Gelder et al. (2017) find similar dynamics in individual life expectancy for selected European countries. Yet, it is not only education that correlates with life expectancy. Waldron (2007) uses data from the US social security system to calculate life expectancy at age 65 for the cohorts born in 1912 and in 1941. While for the lowest income group life expectancy of the 1941 cohort is only about half a year greater than that of the 1912 cohort, this difference amounts to 5.6 years for the highest income group. Cristia (2011) supports these findings.

Life expectancy is a major determinant of the internal rate of return an individual obtains from a public pension system, as those systems pay out annuity streams of income. The amount of payment an individual gets typically is related to the average life expectancy of all pensioner. Hence, an individual with an unusually low life expectancy makes a low return on pension contributions, and vice versa for those with a high individual life expectancy. Liebman (2002a), Goda et al. (2011) and Bosworth et al. (2016) calculate the internal rates of return for individuals of different income groups in the US paying particular attention to the group specific life expectancy. All these studies find that the progressivity of the US system – that leads low income earners to get a higher replacement rate than high income individuals – is undone by the differences in life expectancy across income groups. In some cases, the internal rate of return is even lower for low income earners than for higher income groups.

In Germany, the relation between education or income and longevity is comparable to the international evidence. Luy et al. (2015) find individuals with college education to live on average 2.5 years longer than those with lower education levels. Haan et al. (2020) report a life expectancy gap of around 7 years between individuals in the top and the bottom lifetime labor earnings decile using administrative data from the German pension insurance system.

As the German pension system is fully earnings related, the differences in life expectancy along the income distribution lead the internal rate of return to be particularly low for low income individuals. Taking this into account, the German statutory pension system is in fact regressive, redistributing income from the income poor to the rich through the life expectancy channel. Consequently, Breyer and Hupfeld (2010) argue in favor of a more progressive pension formula that explicitly takes the earnings-longevity relationship into account and guarantees a constant internal rate of return along the income distribution.

Overall, this section has shown that there can be multiple reasons for having a progressive pension system. On the one hand, individuals are exposed to a significant amount of earnings risk, much richer than the typical AR(1) process for log-labor earnings would predict. Most importantly, individuals face a serious portion of low income episodes, which not only lowers their lifetime earnings, but also makes them marginally attached to the labor force. On the other hand, differences in life expectancy along the education and income distribution alter the implicit rate of return an individual can expect from its pension contributions. Whether the potential benefits of redistribution, insurance and of equalizing individuals' rates of return outweigh the labor supply distortions inherent in any progressive pension system is an open question which I now address using a quantitative model.

Chapter 5

Progressive Pensions as an Incentive for Labor Force Participation

The previous chapters have shown that a reform of the German pension system is necessary to cope with demographic change. The introduction of a progressive component is one way of supporting the poor in old age. However, we know from the literature that income redistribution comes with distortions to labor supply. This chapter shows, that a well-designed progressive pension formula can limit economic costs in the form of labor supply distortions on workers, while still providing adequate benefits to poor pensioners. I build on a literature starting with Saez (2002) that analyzes optimal income transfer programs for low-income workers. When labor supply responses are concentrated along the extensive margin, as it is empirically the case for low-earners (see e.g. Meyer (2002)), an optimal labor tax policy explicitly subsidizes employment. Ideally, a public pension system therefore links pension payments to both individual earnings and an individual's employment status. While pension insurers typically have detailed records of an individual's employment history available, one might nevertheless fear that substantial employment subsidies may cause households to extensively engage in minimum-hours contracts or even in fictitious contracts to just become eligible for social security.[1] A second-best policy hence looks quite similar to the Earned Income Tax Credit (EITC) in the US. An EITC-style policy links transfer payments solely to individual earnings, which are observable by the government. Earnings below a threshold are partially matched by the government, which increases the return-to-work and pulls low-income workers into employment. I first show that mechanism in a stylized model. Next, I quan-

[1] In the end, the government still is bound by not being able to observe individual productivity.

tify the importance of labor supply distortions, redistribution, and insurance for aggregate efficiency and long-run welfare in a quantitative simulation model.

The quantitative simulation model is calibrated to the German economy with a proportional pension system. I first quantify the effects of an purely employment-based progressive pension reforms on individual labor force participation and labor hours. The positive employment effects can be sizable and are concentrated among workers with adverse productivity shocks. In the long run, the overall employment rate increases by 1.3 percentage points. Most of the employment gains stem from high school-educated workers, but college-educated workers react positively, too. As an example, the introduction of an employment-linked progressive pension system leads the least productive 35-year-old high school-educated worker to increase their employment by 14 percentage points. An earnings-based progressive pension system, the second-best policy, is somewhat less efficient in stimulating employment, as it can not directly tackle the individual employment decision. Nevertheless, the projected employment gains are still substantial, in the order to 1.1 percentage points for the working population at large. Along the intensive margin, the labor supply decision of households is mostly distorted downwards, leading to an overall decline of about 0.9 hours per week. Intensive margin distortions also affect high productive workers, but positive employment effect are concentrated among low productive individuals. In total, this means that aggregate labor input declines by roughly 1 percent.

Both reforms I consider substantially reduce old-age income inequality and provide insurance against labor productivity shocks. By stimulating employment at the lower end of the productivity distribution, they also alter the risk-properties of labor earnings during working life. The reduced need for self-insurance leads to a decline in aggregate savings along a transition path. The reforms also induce a drop in aggregate consumption. In the initial periods of the transition path, consumption falls by about 0.8 percent. As private assets shrink along the way to the new long-run equilibrium, the consumption decline becomes more pronounced.

Finally, I evaluate the welfare and efficiency effects of progressive pension reforms. My preferred measure of household welfare is ex-ante expected lifetime utility. I calculate the consumption equivalent variation that each cohort affected by a pension reform[2] experiences. As the welfare effects of pension reforms can vary a lot across different generations, I also derive an aggregate measure of the economic efficiency effect that takes into account the welfare changes of all affected cohorts. The introduction of progressive pensions increase the welfare of almost all cohorts, except for the already retired at the time of the reform. The latter experience a small

[2]This is the initial cross-section of households at the time of reform as well as all new-born generations along the transition path

welfare loss from a rise in the consumption tax rate. The aggregate efficiency effect of introducing an employment-linked progressive pension is positive. It amounts to a permanent rise in consumption of 0.73 percent. EIPC systems are less effective tools, as they can not directly condition on the individual employment decision. Yet, they can still recover around 90 percent of the efficiency gains of an employment-linked progressive pension. Positive welfare effects predominantly stem from high school workers. College graduates on average experience welfare losses. The chapter is largely based on a revised version of the working paper *Progressive Pensions as an Incentive for Labor Force Participation* by Kindermann and Püschel (2021).

5.1 The analytical model: a two-period framework

Before setting out the large-scale simulation model, I want to build some intuition for the main mechanisms at work using a much simpler and stylized framework. The starting point of my analysis is a situation with a proportional pension system, in which old-age pension benefits are directly proportional to lifetime earnings. In the reform scenarios, I increase the progressivity of the pension system by allowing for a disproportionately high accumulation of pension claims for earnings-poor working households, which comes at the expense of a cut in pension claims for high-earnings individuals. I do so in two steps. First, I introduce a progressive pension component that is directly linked to the individual employment decision. Through this component, households acquire pension claims for every year they were employed, irrespective of how much they earned. I use this employment-linked progressive pension system (ELS) as a benchmark case, with the implicit assumption being that the public pension insurer is perfectly informed about an individual's employment status. In a second step, I appreciate the fact that the government may have problems in observing employment and study an EITC-like progressive pension which I call the Earned Income Pension Credit (EIPC). This system solely relies on individual earnings as a measure to calculate pension benefits. The pension formula has a phase-in and a phase-out region. Households with earnings less than a certain threshold accumulate disproportional high pension claims for each year they are in employment. Consequently, the system sets incentives for both labor force participation and higher labor hours at the lower end of the earnings distribution.

Households in this framework live for two periods $j = 1, 2$. At each date t, a new generation of mass N_t is born. At the moment they enter the economy, households draw two different shocks: (i) a labor productivity z according to the cumulative distribution function $\Phi_z(\cdot)$ and (ii) a utility cost of employment ξ according to the cumulative distribution function $\Phi_\xi(\cdot)$. I assume both shocks to be independent and identically distributed across households. The interest rate r as well as the wage rate w for effective labor are exogenous. I consider steady state allocations only.[3]

5.1.1 The household decision problem

Households can supply labor only in the first period of life, in the second period they are retired. The labor supply decision consists of two stages: an extensive and an intensive one. Households first have to decide whether to work or not. I denote the choice to be non-employed or employed by $e \in \{0, 1\}$. Once they joined the labor force, agents choose their optimal number of labor hours ℓ. Individuals derive utility from consumption c_j in each period and suffer disutility from working. For analytical tractability, I assume that preferences are quasi-linear in consumption and that the time discount rate equals the interest rate r. More specifically, I let preferences be represented by the utility function

$$U(c_1, c_2, \ell, e) = c_1 + \frac{c_2}{1+r} - \frac{\ell^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \xi e. \tag{5.1}$$

Consistent with household choices, disutility from labor is due to an intensive and an extensive margin component. The former is mainly governed by the Frisch elasticity of labor supply χ. The latter kicks in through utility costs of employment ξ, which emerge whenever an individual is employed ($e = 1$).

Households maximize utility in (5.1) subject to the present value budget constraint

$$c_1 + \frac{c_2}{1+r} = (1-\tau_p)wze\ell + \frac{p}{1+r} + R, \tag{5.2}$$

where R denotes some unearned income. Households pay contributions to the pension system in the form of a payroll tax τ_p on their total labor earnings $wze\ell$. As a reward, they receive a pension payment p when retired. Note that households only have the capacity to earn income by providing hours ℓ if they formally joined the labor force ($e = 1$).

Following the previous discussion, I analyze two progressive pension systems. The first system creates redistribution based on an indidividual's employment decision e. I refer to this system as the Employment-Linked System (ELS). The second system is closely related to the Earned Income Tax Credit in that it wants to set employment incentives. In contrast to the ELS, however, it is entirely based on individual earnings y. I refer to this system as the Earned Income Pension Credit (EIPC).

[3] I hence drop the time index t wherever possible.

5.1.2 The employment-linked system

Pension payments p are due to two components: First, the household's employment status in period 1 and second, her individual labor earnings. Specifically, I let

$$p = \kappa \times \Big[\lambda \bar{y} e + (1-\lambda) wze\ell\Big]. \tag{5.3}$$

κ denotes the replacement rate of the pension system and $\bar{y}$ average labor earnings of the employed. When being employed, households receive a fixed pension reward for employment, which is indexed to average earnings and independent of the households own income position, plus an earnings-tied pension. The factor λ indicates the strength of the employment component relative to the earnings-related component. Since the size of the employment component is independent of individual income, λ is also a measure for the progressivity of the pension system. If $\lambda = 0$, pension claims are purely earnings related. I call such a system a proportional pension. If $\lambda > 0$, I call the pension system progressive. Note, however, that redistribution within the pension system is limited to the employed, since households do not acquire any pension claims when they are not in employment in the first period ($e = 0$). I therefore also call the system employment-related. The left panel of Figure 5.1 depicts this system graphically. On the horizontal axis, the figures shows the earnings of an individual relative to the average earnings of the population $\frac{y}{\bar{y}}$. The vertical axis indicates a worker's pension benefit normalized by the pension replacement rate $\frac{p}{\kappa}$. The dashed line indicates a proportional pension system, while the solid line illustrates the ELS with a value of $\lambda = 0.5$.

Implicit taxes and participation subsidies In the following, I deliberately assume that the population growth rate of the economy, which defines the implicit return on pension contributions, is equal to the interest rate on financial investments, i.e. $r = n$. In the context of my model, this means that $\tau_p = \frac{\kappa}{(1+r)}$.[4] Combining the household budget constraint (5.2) with the pension formula (5.3) as well as the return assumption on pension payments, I can write the budget constraint as

$$c_1 + \frac{c_2}{1+r} = \Big[1 - \underbrace{\lambda \tau_p}_{=:\tau_p^{\text{imp}}}\Big] wze\ell + \underbrace{\lambda \tau_p \bar{y}}_{=:\tau_p^{\text{sub}}} e + R. \tag{5.4}$$

The pension system influences the household budget constraint in two ways. On the one hand, it imposes an implicit tax on intensive labor supply $\tau_p^{\text{imp}} = \lambda \tau_p$. This implicit tax is equal to zero when the pension system is fully earnings related ($\lambda = 0$). In this case, any additional euro a household contributes to the system

[4]In Appendix B, I show that my results also hold in a more general framework where $r \neq n$. The intuition is exactly the same, with the only difference that formulas get more complicated.

pays the same return as a financial investment. Hence, contributing to the system is as valuable to the household as not contributing and saving the money in a private financial account. Yet, when I weaken the link between pension contributions and pension payments by setting $\lambda > 0$, the implicit tax rate of the pension system increases. In the extreme case where $\lambda = 1$, an increase in intensive margin labor supply has no effect on the size of the pension a household receives. Consequently, $\tau_p^{\text{imp}} = \tau_p$, meaning that all of the pension contribution is perceived as a tax.

On the other hand, the pension system comes with a subsidy to employment $\tau_p^{\text{sub}} = \lambda \tau_p \bar{y}$. This subsidy emerges when the pension system pays benefits that are independent of individual income, but are linked only to the employment status of a household. A larger λ implies a greater importance of the employment component, and therefore leads to a higher employment subsidy. Summing up, a higher pension progressivity λ has two opposing effects: it distorts labor supply on the intensive margin by imposing a higher implicit tax rate on households, but it encourages employment by providing a greater participation subsidy.

Intensive and extensive margin choices I now take a deeper look at the household's labor supply problem and determine the incentive effects of an increase in pension progressivity more formally.[5] I start with the intensive margin labor supply decision of an employed household with productivity z. Maximizing utility in (5.1) subject to the household budget constraint (5.4) yields

$$\ell(z|e = 1) = \left[(1 - \tau_p^{\text{imp}})wz\right]^{\chi}.$$

In the absence of income effects, the intensive margin labor supply choice is immediately determined by individual productivity z as well as the implicit tax rate τ_p^{imp} of the pension system.

To make an employment choice at the extensive margin, the household has to compare her utility from working to the utility from not working. This utility difference is

$$U(e = 1) - U(e = 0) = \frac{\left[(1 - \tau_p^{\text{imp}})wz\right]^{1+\chi}}{1 + \chi} + \tau_p^{\text{sub}} - \xi.$$

Consequently, given the distribution of the utility costs of employment ξ, the prob-

[5] All formal derivations can be found in Appendix B.

ability that an individual with labor productivity z chooses to be employed is

$$P(e = 1|z) = \Phi_\xi \left(\frac{\left[(1 - \tau_p^{\text{imp}})wz\right]^{1+\chi}}{1 + \chi} + \tau_p^{\text{sub}} \right).$$

The term in parentheses denotes the utility gain from working and marks the indifference point of households. Any individual with ξ below this utility gain chooses to be employed, anyone with ξ larger than the respective utility gain chooses to not be employed. Total labor supply of all households with labor productivity z consequently is

$$h(z) = P(e = 1|z) \times \ell(z|e = 1).$$

The incentive effects of progressive pensions Equipped with the solution to the household's labor supply choice problem, I can study how a change in pension progressivity λ impacts on the intensive and the extensive labor supply decision of a household. Taking the derivative with respect to λ, I immediately obtain

$$\frac{\partial \ell(z|e = 1)}{\partial \lambda} = -\tau_p \times \chi \times \frac{\ell(z|e = 1)}{1 - \tau_p^{\text{imp}}} < 0. \tag{5.5}$$

As already argued above, an increase in pension progressivity leads to an increase in the implicit tax rate and therefore directly distorts labor supply on the intensive margin. The extent of this distortion is due to two factors: first, the size of the pension system as indicated by its contribution rate τ_p; second, the elasticity χ that governs the reaction to changes in the price of labor.

Regarding the employment decision, I find that

$$\frac{\partial P(e = 1|z)}{\partial \lambda} = \tau_p \times \phi_\xi(\cdot) \times \left[\bar{y} - wz\ell(z|e = 1)\right]. \tag{5.6}$$

This derivative again depends on the size of the pension system τ_p and on the extent to which individuals react to changes in participation incentives. The latter is determined by the density $\phi_\xi(\cdot)$ of households located exactly at the indifference point between employment and non-employment. Most importantly, however, the degree to which pension progressivity incentivizes labor force participation depends on the relative labor market position of a household. All households who would earn an income below-average labor earnings $\bar{y}$ are encouraged to be employed, while households earning more than $\bar{y}$ are discouraged. This is owing to the progressive nature of the employment component that pays a relatively high subsidy to the earnings poor, but a relatively low subsidy to the earnings rich.

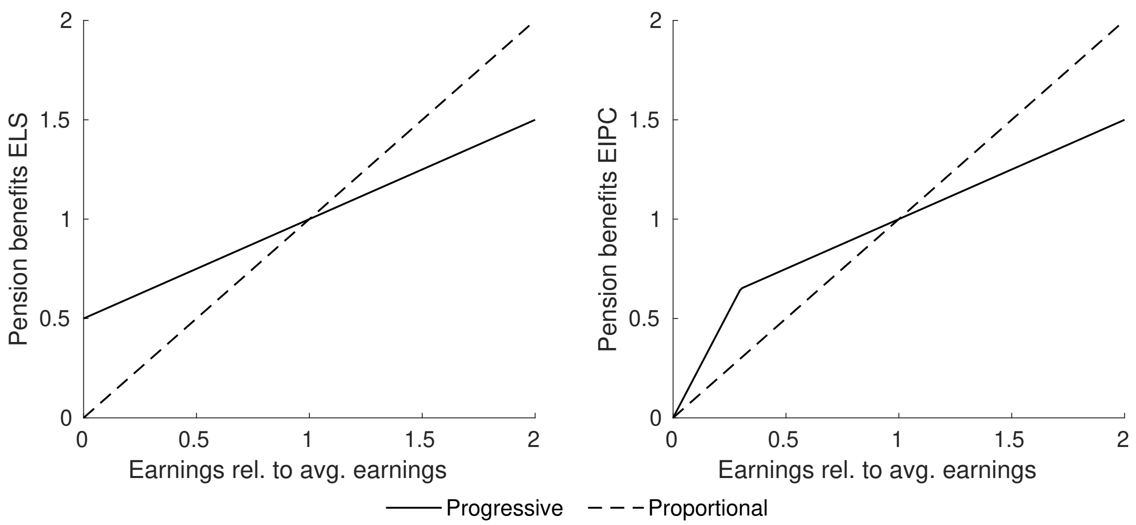
5.1.3 The Earned Income Pension Credit

The EIPC can only grant pension payments on the basis of individual earnings y. To make the system consistent with the ELS, I propose the functional form

$$p = \kappa \times \begin{cases} \lambda \frac{y}{b} + (1-\lambda)y & \text{if } y < b\bar{y} \text{ and} \\ \lambda \bar{y} + (1-\lambda)y & \text{otherwise,} \end{cases} \tag{5.7}$$

where $b \in (0,1)$. The right panel of Figure 5.1 indicates the shape of this pension formula. The dashed line again indicates the proportional pension system, while the solid line illustrates the EIPC pension formula proposed in (5.7) with values $\lambda = 0.5$ and $b = 0.3$.

Figure 5.1: Progressive pension formulas



The phase-in and phase-out structure in the spirit of the Earned Income Tax Credit becomes immediately apparent from the figure. The proposed pension formula defines a threshold level b as a fraction of average earnings $\bar{y}$ at which a worker can enjoy the maximum employment subsidy $\tau_p^{\text{sub}} = \tau_p \lambda \bar{y}$, see (5.4). The incentive effects of this system are in fact identical to those of the ELS for all workers with earnings greater than the threshold level $b\bar{y}$. For workers with labor earnings less than the threshold, I can write the budget constraint as

$$c_1 + \frac{c_2}{1+r} = \Big[1 - \underbrace{\lambda \tau_p}_{=:\tau_p^{\text{imp}}}\Big] y + \underbrace{\lambda \tau_p \frac{y}{b}}_{=:\tau_p^{\text{sub}}}. \tag{5.8}$$

The size of the employment subsidy is now conditional on a worker's labor earnings y. The more earnings an individual can generate from working, the higher the implicit employment subsidy will be, and the more likely that individual is to participate in the labor market. Additionally, low-earning individuals have a strong incentive to increase their intensive margin labor supply up to the bend point b, as their pension benefits increase disproportionately with any additional euro earned. It should be noted that in this case, I cannot strictly separate the employment subsidy from the implicit tax rate. Note further that as b approaches zero, the EIPC looks more and more like the ELS, with all the benefits and problems.

Summing up, the progressive pension systems affects aggregate labor in two ways. While distorting labor supply along the intensive margin, they also provides incentives for employment along the extensive margin, especially for the earnings poor. The effect on total labor earnings is therefore ambiguous and depends on the exact choices of the intensive margin labor supply elasticity, the distribution of participation costs and the distribution of labor earnings in the population. What is more, a progressive pension system not only influences households' labor supply decisions. It also alters the distribution of household income at old age by redistributing between households with different life-time incomes and life-expectancies and by providing insurance against productivity fluctuations over the life cycle.

5.2 The quantitative simulation model

My full quantitative simulation model is based on the previous theoretical considerations and informed by the empirical facts regarding lifetime inequality as discussed in Chapter 4. In particular, I employ a general equilibrium overlapping generations model with survival risk in the spirit of Auerbach and Kotlikoff (1987). Households draw persistent shocks to their labor productivity, like in Conesa et al. (2009), and have to decide about whether to be employed, how many hours to supply and about how much to consume and save. In addition, individuals face shocks to their life expectancy. The government operates a (potentially progressive) pay-as-you-go pension system financed by payroll taxes and collects resources through a consumption tax and a progressive tax on labor earnings in order to finance general government expenditure. I consider an open economy framework, so that the prices for capital and labor are fixed, but government parameters adjust in order to keep the fiscal tax and transfer systems balanced. My simulations start from a long-run equilibrium calibrated to the German economy. Any reform to the pension system puts the economy on a transition path to a new steady state. I calculate this entire transition path and measure the welfare effects on different cohorts and households along the transition. The computational details are provided in Appendix C.1.2.

5.2.1 Demographics

The economy is populated by overlapping generations of heterogeneous individuals.[6] At each point in time t, a new generation of size N_t is born. I assume that the population grows at a constant rate n. Households start their economic life at age $j = 20$ and live up to a maximum of J years, after which they die with certainty. They can supply labor to the market until they reach the mandatory retirement age j_r. Throughout their entire life, individuals are subject to idiosyncratic survival risk. Specifically, I denote by $\psi_{j,h}$ the conditional probability of an agent to survive from period $j-1$ to period j, with $\psi_{20,h} = 1$ and $\psi_{J+1,h} = 0$. Survival probabilities, and hence life expectancy, depend on the individual health status h, discussed in more detail below.

As population grows with a constant rate n, a long-run equilibrium in this economy is characterized by all aggregate variables growing at this very same rate. To make aggregates stationary again, I express all variables in per capita terms of the youngest generation at a certain date t. I denote by m_j the time-invariant relative size of a cohort aged j at any point in time.

5.2.2 Technology

A continuum of identical firms produce a single good Y_t under perfect competition. They hire both capital K_t at price r_t and labor L_t at price w_t on competitive spot markets. Firms operate a constant returns to scale technology

$$Y_t = \Omega K_t^{\alpha} L_t^{1-\alpha}. \tag{5.9}$$

Ω denotes the aggregate level of productivity, whereas α is the elasticity of output with respect to capital. In the process of production, a fraction δ of the capital stock depreciates. Given the assumptions about competition and technology, I can safely assume the existence of a representative firm that takes prices as given and operates the aggregate technology in (5.9). In addition to employing factor inputs, the firm has to invest I_t into its capital stock. The law of motion for the capital stock reads

$$(1+n)K_{t+1} = (1-\delta)K_t + I_t.$$

5.2.3 Preferences and endowments

Preferences Households have preferences over stochastic streams of consumption $c_{j,t} \geq 0$, labor supply $\ell_{j,t} \geq 0$ and employment $e_{j,t} \in \{0,1\}$. They maximize a

[6] I use the terms individual, household and agent synonymously.

discounted, generalized recursive, expected utility function

$$U_{j,t} = u(c_{j,t}, \ell_{j,t}, e_{j,t}) - \beta\psi_{j+1,h}E_t\left[(-U_{j+1,t+1})^{1+\gamma}\right]^{\frac{1}{1+\gamma}}.$$

My preference formulation follows Swanson (2018) and is a generalization of Epstein and Zin (1989) that allows to separate intertemporal substitution from risk aversion. Individuals form expectations with respect to future labor productivity and health and incur a utility loss from being employed. They discount the future with the constant time discount factor β as well as their individual survival rate. For the sake of notational ease, I deliberately drop the time index t on all household level variables.

Labor productivity Households are ex-ante homogeneous, but differ ex-post in their labor productivity $z(j, s, \eta)$. At the beginning of life, they draw one of two education levels: high school education ($s = 0$) or college education ($s = 1$); the probability to draw $s = 1$ is ϕ_s. All individuals of education s share a common deterministic age-specific labor productivity profile $\theta_{j,s}$.

Throughout their working life, households' labor productivity is due to idiosyncratic shocks η. For individuals with normal labor earnings, I assume that their productivity follows a standard, education-specific AR(1) process in logs

$$\eta^+ = \rho_s\eta + \varepsilon^+ \quad \text{with} \quad \varepsilon^+ \sim N(0, \sigma^2_{\varepsilon,s}), \tag{5.10}$$

where innovations ε^+ are iid across households.

The evidence provided in Section 4 has shown that a simple AR(1) process is not enough to describe the earnings distribution of households. To cope with the fact that a significant part of workers experiences low-earnings episodes I proceed as follows: I assume that, knowing their education level, households divide into two groups $m \in \{0, 1\}$. m is a permanent state that indicates whether an individual faces a stable career path ($m = 0$) or an unstable career path ($m = 1$). The probability to draw the state $m = 1$ is denoted by ϕ_m. The labor productivity dynamics of workers with stable careers is described solely by the AR(1) process shown above. On top, agents with an unstable career can be hit by an additional persistent (but not permanent) low productivity shock, regardless of their current productivity. When exiting the low productivity state, agents revert to normal AR(1) productivity. I provide details on the exact parameterization of low productivity shocks in the calibration section.[7]

[7]This approach is consistent with empirical evidence from the labor literature that starts with Hall (1982). More recently, Kuhn and Ploj (2020) investigate the importance of career instability

I denote by $\pi_\eta(\eta^+|\eta, j, s, m)$ the probability distribution of next-period's productivity η^+, conditional on current labor productivity η, age j, education s and career stability m. Finally, the wage an individual faces equals the product of the wage rate per efficiency unit of labor and her individual labor productivity $w_t \times z(j, s, \eta)$.

Budget constraint Markets are incomplete. Like in Bewley (1986), Imrohoruglu (1989), Huggett (1993), and Aiyagari (1994), households can only self-insure against fluctuations in individual labor productivity by saving in a risk-free asset a with return r_t. Savings are subject to a tight borrowing constraint, so that household wealth needs to satisfy $a \geq 0$. Households' resources are composed of their current wealth (including returns), their income from working $y = w_t z(j, s, \eta)e\ell$, intergenerational transfers b,[8] as well as pension payments p. They use these resources to finance consumption expenditure $(1 + \tau_{c,t})c$ (including consumption taxes) and savings into the next period a^+, contributions to social security $T_{p,t}(y)$ as well as progressive income taxes $T_t\Big(y - T_{p,t}(y) + p\Big)$. Households can deduct social security contributions from gross income for the purpose of taxation. In turn, all pension benefits are liable for taxation.

Individual life expectancy A household's savings behavior is shaped by the interest rate, the discount factor, productivity risk and individual life expectancy. As for the latter, I assume that individual survival probabilities are defined by some health state h. Each health level is associated with a set of age specific survival probabilities $\psi_{j,h}$ that lead to a certain life expectancy. An agent's health status can change over the life cycle according to the probability distribution $\pi_h(h^+|h, j, s, \eta)$. Future health h^+ hence is conditional on current health, age, education and individual labor productivity.

Dynamic optimization problem The current state of a household is described by a vector $\mathbf{x} = (j, s, m, \eta, h, a, ep)$ that summarizes the household's age j, education s, career stability m, her current labor productivity shock η, health h, her wealth position a as well as the amount of already accumulated pension claims ep. The

for heterogeneity in household wealth. Nam (2022) analyzes the consequences of career instability for the optimal progressivity of the pension system. The approach also follows Castaneda et al. (2003) or Kindermann and Krueger (2022), who augment standard AR(1) processes for labor productivity with additional shocks to paint a realistic picture or top 1% earnings and wealth heterogeneity in the US.

[8] Intergenerational transfers consist only of accidental bequests that households might leave if they die before the terminal age J. I assume that the total of those accidental bequests is distributed lump-sum to all working-age households.

dynamic optimization problem of an individual then reads

$$v(\mathbf{x}) = \max_{c,\ell,e,a^+,ep^+} \ u(c,\ell,e) - \beta\psi_{j+1,h} E\left[\Big[-v_{t+1}(\mathbf{x}^+)\Big]^{1+\gamma} \ \middle| \ \mathbf{x}\right]^{\frac{1}{1+\gamma}}, \tag{5.11}$$

with $\mathbf{x}^+ = (j+1, s, m, \eta^+, h^+, a^+, ep^+)$. Households maximize (5.11) subject to the borrowing constraint $a^+ \geq 0$, the budget constraint

$$(1+\tau_{c,t})c + a^+ + T_{p,t}(y) + T_t\Big(y - T_{p,t}(y) + p\Big) = (1+r_t)a + y + p + b$$
$$\text{with } \ y = w_t z(j,s,\eta)e\ell,$$

the accumulation equation for pension claims ep^+ discussed in Section 5.2.4 as well as the laws of motion for labor productivity π_η and health π_h. The result of this dynamic program are policy functions c, ℓ, e, a^+, and ep^+ that all depend on the household's current state $\mathbf{x}$. I derive the first-order conditions in Appendix C.1.

5.2.4 The pension system

The pension system has a contribution ceiling equal to two times average labor earnings of the employed. I therefore define pension-relevant earnings y^p as

$$y^p = \min\Big(wz(j,s,\eta)e\ell, \ 2\bar{y}_t\Big).$$

Households pay payroll taxes at rate τ_p on relevant earnings. In reward for their contributions, they earn pension claims ep. I can write

$$T_{p,t}(y) = \tau_{p,t} \times y^p \quad \text{and} \quad ep^+ = ep + f_t(y^p), \tag{5.12}$$

where the function f_t determines the relationship between relevant labor earnings and pension claims. In the initial equilibrium denoted by $t = 0$, I assume that the pensions system is purely proportional (as it is in Germany) and therefore set $f_0(y) = y$.

Finally, individual pension benefits $p(ep)$ are calculated from the life-time average of earned pension claims as

$$p(ep) = \kappa_t \times \frac{ep}{j_r - 20},$$

where κ_t is the replacement rate.

The pension system operates on a pay-as-you-go basis. In the initial equilibrium, total pension contributions hence need to be equal to the total amount of pension payments. Letting Φ_t denote the cross-sectional measure of households over the state space,[9] I require

$$\tau_{p,0} \times \underbrace{\int y^p \, d\Phi_0}_{\text{contribution base}} = \underbrace{\int p(ep) \times \mathbb{1}_{j \geq j_r} \, d\Phi_0}_{\text{total pension claims}} . \tag{5.13}$$

I will depart from the notion of period-by-period budget balance along the transition path in order to smooth the costs and benefits of pension reforms over multiple generations. I provide more details in Section 5.4.2.

5.2.5 The tax system and government expenditure

The government collects proportional taxes on consumption expenditure and progressive taxes on labor earnings net of social security contributions as well as pension payments. In addition, it can issue debt B_t. Fiscal revenue is used to finance (wasteful) government spending as well as debt services. The government budget constraint reads

$$\tau_{c,t} \times C_t + \int T_t\Big(y - T_p(y_p) + p\Big) \, d\Phi_t + (1+n)B_{t+1} = G_t + (1+r_t)B_t$$
$$\text{with } y = w_t z(j,s,\eta)e\ell. \tag{5.14}$$

C_t denotes aggregate consumption and T_t the progressive income tax schedule. I assume that government consumption is fixed per capita. Consequently, I adjust the tax system to keep the fiscal system in balance.

5.2.6 Capital markets, trade and equilibrium

I model a small open economy that freely trades capital and goods on competitive international markets. All private savings that are not absorbed by the domestic production sector or the government are invested abroad at the international interest rate $\bar{r}$. The capital market equilibrium reads

$$K_t + B_t + Q_t = A_t,$$

where A_t are aggregate private savings and Q_t is the country's net foreign asset position. As the economy grows at rate n, the net foreign asset position increases over time such that the capital account is $Q_t - (1+n)Q_{t+1}$. Net income from abroad, on the other hand, amounts to $\bar{r}Q_t$. According to the balance of payments identity,

[9] Φ_t is a measure and indicates the mass of households on each subset of the state space. I require that for each age j, Φ_t sums up to the total mass of households in a cohort m_j. A detailed analytical description of Φ_t can be found in Appendix C.1.1.

I therefore have a trade balance of

$$TB_t = (1+n)Q_{t+1} - (1+\bar{r})Q_t. \tag{5.15}$$

The economy's interest rate is then equal to the world-wide interest rate $r_t = \bar{r}$.

I assume that the government collects all accidental bequests and redistributes them in a lump-sum way among the surviving working-age population. Consequently,

$$b_{j,t} = \frac{\int \frac{1-\psi_{j,h}}{\psi_{j,h}} \times (1+r_t)a \; d\Phi_t}{\int \mathbb{1}_{j<j_r} \; d\Phi_t} \quad \text{if } j < j_r. \tag{5.16}$$

5.2.7 Recursive competitive equilibrium

Given an international interest rate $\bar{r}$, government expenditures G, a consumption tax rate τ_c, a progressive tax system $T(\cdot)$ as well as a characterization of the pension system $\{\tau_p, \kappa\}$, a stationary recursive equilibrium with population growth n is a collection of value and policy functions $\{v, c, \ell, e, a^+, ep^+\}$ for the household, optimal production inputs $\{K, L\}$, accidental bequests $\{b_j\}_{j=20}^{J}$, a net foreign asset position and a trade balance $\{Q, TB\}$ as well as factor prices $\{r, w\}$ that satisfy

1. **Household optimization** Given prices and characteristics of the tax and pension system, the value function v satisfies the Bellman equation (5.11) together with the budget constraint, the accumulation equation for pension claims, the borrowing constraint and the laws of motion for productivity risk and health. c, ℓ, e, a^+, and ep^+ are the associated policy functions.

2. **Firm optimization** Given the international interest rate $\bar{r}$ as well as the wage rate w, firms employ capital and labor according to the demand functions

$$\bar{r} = \Omega\alpha \left(\frac{L}{K}\right)^{1-\alpha} - \delta \;\text{ and }\; w = \Omega(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

3. **Government constraints** The budget constraints of the pension system (5.13) and the tax system (5.14) hold, and accidental bequests are calculated from (5.16).

4. **Market clearing:**

   (a) The labor market clears:

$$L = \int z(j, s, m, \eta)e(\mathbf{x})l(\mathbf{x}) \; d\Phi$$

(b) The capital market clears:

$$K + Q = \int a \; d\Phi$$

(c) The balance of payments identity is satisfied:

$$TB = (n - \bar{r})Q$$

(d) The goods market clears:

$$Y = \int c(\mathbf{x}) \; d\Phi + (n + \delta)K + G + TB.$$

5. **Consistency of probability measure** Φ The invariant probability measure is consistent with the population structure of the economy, with the exogenous processes of labor productivity η and health h, and the household policy functions a^+ and ep^+. A formal definition is provided in Appendix C.1.1.

5.3 Calibration

This section discusses my choices of functional forms and parameters. I pay particular attention to the labor supply decision of households along the extensive and the intensive margin. I calibrate my model to the German economy in the year 2017. This is four years prior to the introduction of the Grundrente. Germany features a proportional pension system in line with the one described in Chapter 3. Germany therefore serves as a good benchmark for reforms that aim at introducing progressivity into the pension formula.

5.3.1 Demographics

I assume a population growth rate of $n = 0.0$, which is a compromise between the average growth rate of 0.4% reported in the period 2012 to 2017 for the German population at large, and the fact that most of the German population growth came from refugee migration, see Statistisches Bundesamt (Destatis).[10] I let households start their economic life at the age of 20 and allow for a maximum life span of 99 years. Mandatory retirement is at the age of 64, which equals the current average retirement age of the German regular retirement population, see Deutsche Rentenversicherung Bund (2019).

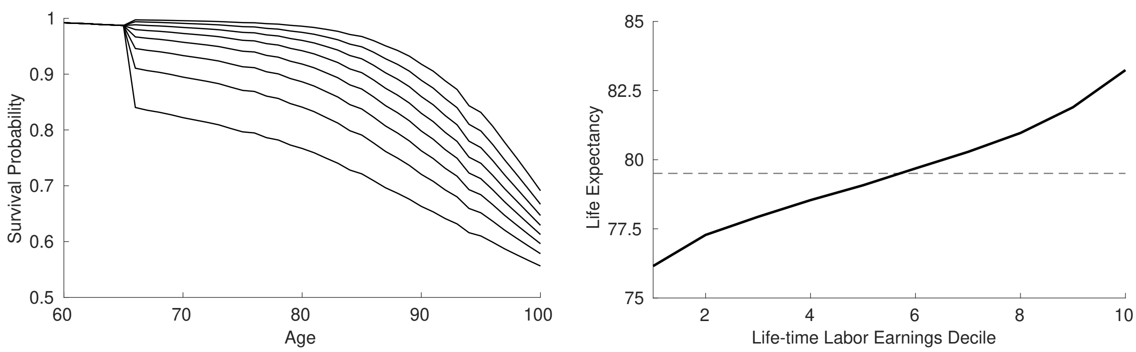
[10] In fact, the growth rate of the native population was -0.2% in the same time period.

With regards to life expectancy, I extract the 2017 annual life tables for men from the Human Mortality Database (2020) to calculate average survival probabilities $\bar{\psi}_j$ of the overall population. This is the average probability of an individual of age j to survive to age $j+1$. During working life ($j < j_r$) I set the individual survival probabilities $\psi_{j,h}$ equal to $\bar{\psi}_j$. When entering retirement, each individual draws one out of eight different health shocks $h \in \{0, \ldots, 7\}$ according to a probability distribution $P(h|s, \eta)$. A health shock is associated with a set of survival probabilities $\psi_{j,h}$ that I calculate from a logistic model, where

$$\psi_{j,h} = \frac{1}{1 + \exp(-\iota_h \times \bar{x}_j)} \quad \text{with} \quad \bar{x}_j = \log\left(\frac{1}{\bar{\psi}_j} - 1\right). \tag{5.17}$$

I choose the multipliers ι_h such that (i) life expectancy at the lowest health shock $h = 0$ is ten years below-average, (ii) life expectancy at the highest health shock $h = 7$ is ten years above-average and (iii) life expectancy evolves linearly with health shocks h.[11] The left panel of Figure 5.2 shows the resulting survival probability profiles.

Figure 5.2: Survival probabilities and life expectancy



The probabilities $P(h|s, \eta)$ to draw a certain health shock upon entering retirement depend on the individual's education s and on the labor productivity shock η at the date directly prior to retirement. This modeling choice is grounded on two pieces of empirical evidence: First, Luy et al. (2015) find that in Germany individuals with college education live on average 2.5 years longer than those with lower education levels. Second, Haan et al. (2020) report a life expectancy gap of around 7 years between individuals in the top and the bottom life-time labor earnings decile.

To incorporate these empirical facts, I assume $P(h|s, \eta)$ to be the probability mass function of a binomial distribution with success probabilities $p_{s,\eta}$ depending on ed-

[11]Note that for $\iota_h = 1$, I recover the average survival probability $\psi_{j,h} = \bar{\psi}_j$.

ucation and labor productivity. In particular, I let

$$p_{s,\eta} = \Phi\left(\iota_0 + \iota_1 \times \mathbb{1}_{s=\text{college}} + \iota_2 \times \eta\right), \tag{5.18}$$

where Φ is the probability distribution function of the standard normal distribution and $\mathbb{1}_{s=\text{college}}$ is an indicator function that takes a value of one for households with college education. I set the parameters $\iota_1 = 0.32$ and $\iota_2 = 0.64$ to target the reported life expectancy gaps by education level and life-time labor earnings. Finally, I choose $\iota_0 = -0.06$ such that the average life expectancy of the total population amounts to 79.5 years, the value I obtain from the Human Mortality Database (2020) life tables. The right panel of Figure 5.2 shows the relation between lifetime labor earnings and life expectancy. While individuals in the bottom decile expect their life to be about four years shorter than that of the population average, the average life of a top decile earner is three years longer.

To incorporate these probabilities into the model, I let the transition probability

$$\pi_h(h^+|h, j, s, \eta) = \begin{cases} P(h|s,\eta) & \text{if } j = j_r - 1 \text{ and} \\ \mathbf{I} & \text{otherwise,} \end{cases}$$

with $\mathbf{I}$ being the identity matrix. Hence, the model features one single health shock that individuals are exposed to right before entering retirement. After the individual health status is revealed, households retain their health level for the rest of their life. While agents share a common set of survival probabilities during their entire working life, they still form expectations with respect to their survival chances at retirement. Hence, the need for old-age savings differs across individuals of different education levels and labor productivity.

5.3.2 Technology

On the technology side I choose a depreciation rate of $\delta = 0.07$, leading to a realistic investment to output ratio of 21 percent. I set the capital share in production at $\alpha = 0.3$ and normalize the technology level Ω such that the wage rate per efficiency unit of labor w_t is equal to 1. Finally, I assume an international interest rate of $\bar{r} = 0.03$, which constitutes as mix between the (in 2017) very low interest rates on deposits and long-run investment opportunities that offer higher returns.

5.3.3 Preferences and endowments

Preferences I let the period utility function be

$$u(c, \ell, e) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu_s \frac{\ell^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \xi_s e.$$

I choose an intertemporal elasticity of substitution σ of 0.8. The choice of σ has important implications for the size of the income effect of wage changes on labor supply and therefore for the life-cycle profiles of participation and labor hours, see Section 5.4.1. My choice of σ ensures that my model is able to match empirical life-cycle profiles. My preferred value for the Frisch elasticity is $\chi = 0.4$, which is a medium range value, see for example Keane (2011). I choose the education-specific level parameters of intensive labor supply $\nu_1 = 46.55$ and $\nu_2 = 32.80$ so as to target a 38.1 hour and a 40.0 hour work week for high school and college-educated employed workers, respectively. According to Swanson (2018), the relative risk aversion with respect to fluctuations in individual consumption in my utility formulation is approximately equal to

$$R_c \approx \frac{1}{\sigma + \chi} + \frac{\gamma(1-\sigma)}{\sigma + \frac{1-\sigma}{1+\frac{1}{\chi}}}.$$

I set $\gamma = 9.286$ so that relative risk aversion is equal to 3.[12] Finally, I set the time discount factor to $\beta = 0.9835$ so that all capital and public debt is entirely absorbed by private savings in the initial equilibrium, and net foreign assets as well as the trade balance are zero.

The micro Frisch elasticity χ only is an intensive margin elasticity and does not incorporate extensive margin choices. The macro labor supply elasticity, which incorporates both intensive and extensive margin choices, is typically larger, see the discussion in Keane and Rogerson (2012) or Peterman (2016). The extensive margin labor supply reaction to a change in wages is to a large degree determined by the probability density of the utility costs of employment ξ. My calibration strategy for the distribution of participation costs ξ is the following: I assume that ξ is iid across households and independent of the household's labor productivity $z(j, s, \eta)$. I let ξ follow a log-normal distribution with education-specific mean $\mu_{\xi,s}$ and a common variance σ_ξ^2. The means are set so as to target employment-to-population ratios for the 25 to 54 year old by education level. The variance is chosen to target evidence on participation elasticities in Bartels and Pestel (2016), see Appendix C.2 for further details.

[12]Note that in the absence of additional curvature, consumption risk aversion would only be 0.83.

Labor productivity In Section 4.1, I already sketched the dynamics of labor earnings using administrative data on the German working population. However, in my quantitative model I need to parameterize labor productivity, which differs from labor earnings when individual labor hours vary across ages and states.

I parameterize the age-productivity relationship using the functional form

$$\theta_{j,s} = b_{0,s} + b_{1,s}\frac{\min(j, j_{M,s})}{10} + b_{2,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^2 + b_{3,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^3. \quad (5.19)$$

This functional form is flexible enough to capture both a hump-shaped ($j_{M,s} = \infty$) and a stagnating ($j_{M,s} < j_r$) life-cycle labor productivity profile. Note that in the case of a stagnating profile, labor productivity is constant from age $j_{M,s}$ onward. I let labor productivity risk of workers be guided by a standard first-order autoregressive process with parameters ρ_s and $\sigma^2_{\varepsilon,s}$ as in (5.10). In addition, I assume that workers with an unstable career path ($m = 1$) are exposed to an additional first-order Markov process of the form

$$\Pi^s_{low} = \begin{bmatrix} 1 - \pi^s_{low,0} & \pi^s_{low,0} \\ 1 - \pi^s_{low,1} & \pi^s_{low,1} \end{bmatrix} \quad \text{with initial distribution} \quad \begin{bmatrix} \omega^s_{low} \\ 1 - \omega^s_{low} \end{bmatrix}. \quad (5.20)$$

This process governs the transition into and out of the low-earnings state, in which individuals face a labor log-productivity of η_0. $\pi^s_{low,0}$ consequently denotes the probability to receive a low-earnings shock, while $\pi^s_{low,1}$ is an indicator of the persistence of the low-earnings state.

To provide a suitable calibration for the labor productivity process, I first set the share of college-educated workers to $\phi_s = 0.2373$ in accordance with the data and assume that the fraction of worker that are subject to low-earnings shocks of each education group is $\phi_m = 0.5$. I then estimate a subset of parameters directly from the earnings data, see Tables 4.3 and 4.4 in Section 4.1.3 for details. This includes the autocorrelation ρ_s of normal labor productivity risk, the initial distribution ω^s_{low}, and the probabilities $\pi^s_{low,0}$ and $\pi^s_{low,1}$ of the low labor productivity shock process.

This leaves a total of 13 parameters that need to be calibrated:

1. the 10 parameters $b_{i,s}$ and $j_{M,s}$ of the polynomials in (5.19) for high school and college-educated workers;
2. the innovation variances $\sigma^2_{\varepsilon,s}$ of the normal labor productivity processes for each education level;
3. the labor productivity η_0 of low productivity workers.

I calibrate these parameters within my simulation model such that the model-implied statistics for labor earnings match their empirical counterparts. In particular, I target the following statistics:

1. the results of an age fixed-effects regression for labor earnings, see Figure 5.3 for a comparison between empirical and model implied life-cycle earnings;
2. the variance of normal labor earnings in Table 4.3 in Section 4.1.3;
3. average labor earnings of low productivity individuals as shown in the right panel of Figure 4.3.

Figure 5.3: Empirical and model-implied average life-cycle earnings profiles

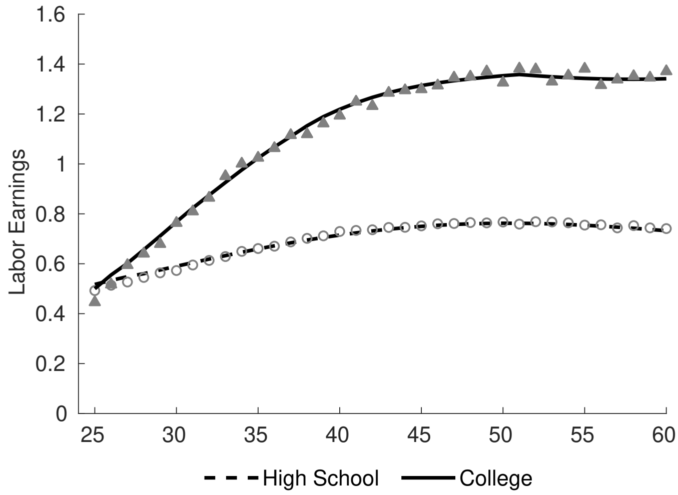


Table 5.1 summarizes the parameters of labor productivity profiles and risk. More details on the calibration process as well as the formulation of the productivity process in model terms can be found in Section 4.1 and in Appendix C.2.

5.3.4 Government policies

I set the pension contribution rate at $\tau_p = 0.187$, the contribution rate of the German pension system in 2017. In equilibrium, my choice of τ_p results in a value of $\kappa = 0.455$, the gross replacement rate of the system, which is close to the gross standard replacement rate of 48.3 percent in Germany in 2017, see Deutsche Rentenversicherung Bund (2020). In my initial economy, I fix government consumption at 19 percent of GDP. I employ the statutory German income tax code for the year 2017 to labor earnings and pension income. Individuals with earnings less than 0.24 times the average earnings are exempt from taxes. For earnings between 0.24 and 1.46 times the average, the marginal tax rate increases from 14 to 42 percent. For earnings exceeding 6.93 times the average, the top marginal tax rate of 45 percent is applied. Figure 5.4 shows the tax code $T_{I,t}$ in the case of individual taxation. I ac-

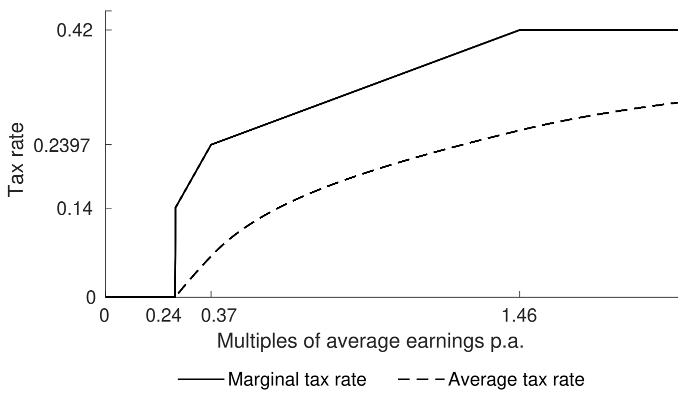
Table 5.1: Parameter values of labor productivity profiles and risk

	High School $s=0$	College $s=1$
Normal labor productivity		
Intercept $b_{0,s}$	-2.0732	6.4829
Linear age term $b_{1,s}$	0.6238	3.6932
Quadratic age term $b_{2,s}$	-0.0595	-0.7130
Cubic age term $b_{3,s}$	0.0000	0.0467
Stagnation threshold $j_{M,s}$	∞	51
Autocorrelation ρ_s	0.9881	0.9900
Innovation Variance $\sigma^2_{\varepsilon,s}$	0.0045	0.0042
Low labor productivity		
Productivity level $\exp(\eta_0)$	0.0675	0.0493
Initial share of low productivity earners ω^s_{low}	0.2022	0.8005
Probability to transition to low productivity $\pi^s_{low,0}$	0.0064	0.0052
Probability to stay low productivity earner $\pi^s_{low,1}$	0.8374	0.7324

count for the fact that about two-thirds of working-age German households consist of couples, as reported by the RDC of the FSO (2017). They enjoy a tax advantage in the form of income splitting. Hence, I set the splitting factor to 1.65. This results in

$$T_t\Big(y - T_{p,t}(y) + p\Big) = 1.65 \times T_{I,t}\left(\frac{y - T_{p,t}(y) + p}{1.65}\right).$$

Figure 5.4: Marginal and average tax rates for labor earnings and pension income



Finally, I set the consumption tax rate at $\tau_c = 0.207$ to balance the fiscal budget. Table 5.2 summarizes the parameters of my model.

Table 5.2: Summary of model parameters

Exogenous parameters	Value	Endogenous Parameter	Value
Share college-educated ϕ_{Col}	0.237	Depreciation rate δ	0.070
Share unstable careers ϕ_m	0.500	Technology level Ω	0.923
Population growth rate n	0.000	Disutility of labor hours ν_{HS}	46.55
Retirement age	64	Disutility of labor hours ν_{Col}	32.80
Pension contribution rate τ_p	0.187	Mean disutility empl. $\mu_{\xi,HS}$	1.013
International interest rate $\bar{r}$	0.030	Mean disutility empl. $\mu_{\xi,Col}$	0.590
Capital share in production α	0.300	Var. disutility empl. σ_ξ^2	0.138
Intert. elasticity of substitution σ	0.800	Discount factor β	0.984
Frisch elasticity of labor supply χ	0.400	Consumption tax rate τ_c	0.207
Expected utility curvature γ	9.286	Replacement rate κ	0.455

5.4 Simulation results

In this section, I present simulation results from my quantitative model. I start by showing the central features of my initial equilibrium economy. I then turn to counterfactual policy simulations, in which I introduce progressive components into the pension formula.

5.4.1 The initial equilibrium

Table 5.3 summarizes central macroeconomic aggregates of my initial equilibrium economy with a proportional pension system as outlined in Section 5.2.4 and compares it to data from the German economy in 2017. I calibrated the discount factor such that private savings cover total demand by firms and the government. In reality, private savings are somewhat higher than capital plus public debt. However, a substantial part of these assets come from the top 1 percent wealth holders, a particular group that I do not include in my model. As a result, the German economy holds net foreign assets worth about 45 percent of GDP.

On the goods market, government consumption and investment almost perfectly match their empirical counterparts. The trade balance in my model is zero, like the net foreign asset position, which implies private consumption to be higher than in the data.[13] The average work week of prime aged workers is equal to 38.2 hours for high school and 40.1 hours for college-educated workers, just like in data from the RDC of the FSO 2017. The employment-to-population ratio is at 84.4 and 95.1 percent, respectively.

[13]Note that Germany has both a positive trade balance and a positive net foreign asset position. In a long-run equilibrium, this is impossible to achieve without a permanently positive balance of payments. Hence, I decided to strike a balance by having both the net foreign asset position and the trade balance equal to zero.

Table 5.3: Macroeconomic aggregates

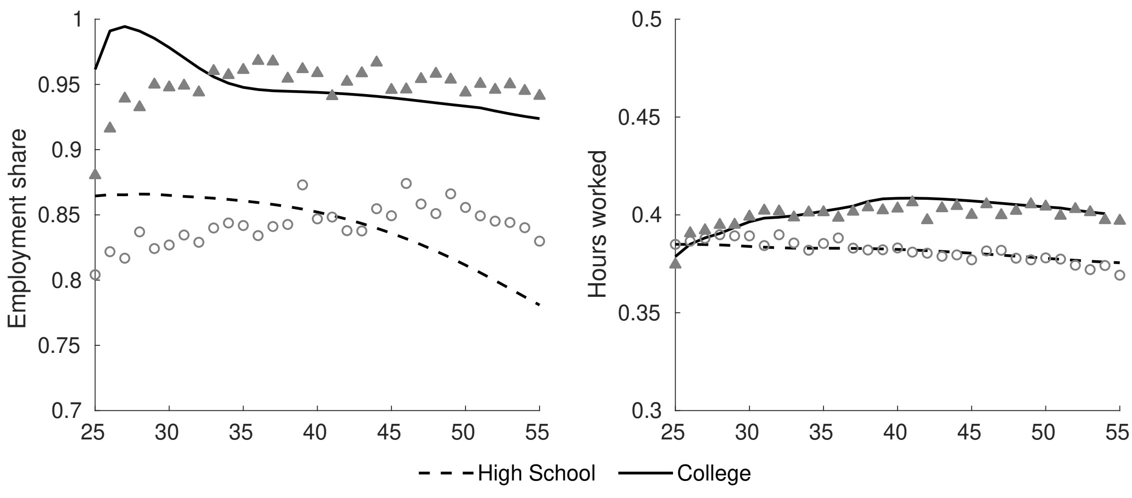
Variable	Value (HS/Col)	Data 2017
Private Assets	360.00	433.09
Capital Stock	300.00	305.24
Public Debt	60.00	64.60
Net Foreign Assets	0.00	44.25
Private Consumption	60.00	52.11
Government Consumption	19.00	19.84
Investment	21.00	20.96
Trade Balance	0.00	7.09
Labor Tax Revenue	8.38	8.35
Consumption Tax Revenue	12.42	8.74
Average Work Week of Employed 25-54 (in hrs)	38.2/40.1	38.1/40.0
Employment-to-Population Ratio 25-54 (in %)	84.4/95.1	84.4/95.1

Variables in percent of GDP if not indicated otherwise.

Data sources: PA: Alvaredo et al. (2022), CS: Statistisches Bundesamt (Destatis) (2023), PD, NFA: Deutsche Bundesbank (2022), PC, GC, I, TB: Statistisches Bundesamt (Destatis) (2023), LTR, CTR, AWW, EtP: RDC of the FSO (2017).

The left panel of Figure 5.5 compares the labor force participation profiles of high school (dashed/circles) and college (solid/triangles) workers with their empirical counterparts derived from the RDC of the FSO 2017. The right panel shows life-cycle labor hours by education level. Overall, my model fits the data decently.

Figure 5.5: Labor force participation and hours over the life cycle



Data source empirical profiles: own estimation, based on data from the RDC of the FSO (2017).

Yet, as households start their life with zero assets, the employment share is somewhat too high early in life. As households become older and have accumulated some wealth, they successively withdraw from the labor force. Note that the life-cycle labor productivity profile of high school workers is much flatter than that of college graduates, see Figure 5.3. As a result, labor force participation of the former drops faster than that of the latter. The model-implied labor hours profiles, on the other hand, match the data almost perfectly.

5.4.2 The thought experiment

I present results from counterfactual policy analyses arising from the introduction of either an employment-linked progressive pension system (ELS) or an Earned Income Pension Credit (EIPC).[14] In both reform exercises, households pay payroll taxes at rate τ_p on pension relevant earnings y^p.

The employment-linked progressive pension system The first pension system I propose is the ELS, which is closely related to the pension system discussed in Section 5.1.2. Compared to the benchmark model, I modify the function $f(y^p)$ in the accumulation formula for pension claims by adding an employment component. Pension claims ep^+ then evolve according to

$$ep^+ = ep + \Big[\lambda \bar{y} e + (1-\lambda) y^p\Big].$$

For each year in which they are employed ($e = 1$), households receive pension claims of size $\lambda \bar{y}$ through the employment component, which is explicitely indexed to average labor earnings and not to individual income. In addition to employment, households are rewarded for higher contributions to the system through the earnings component, which is scaled with $1-\lambda$. The factor λ governs the weight on the two different components and defines the degree of progressivity of the pension system. A more progressive system incentivizes employment especially for the income poor population. It does so, however, at the expense of the earnings component.[15] Note that the benchmark system can be restored by simply setting $\lambda = 0.0$.

Increasing λ encourages employment, but it distorts intensive margin labor supply ℓ. This can readily be seen from the first-order condition for labor supply, which

[14]See Section 5.1.2 and 5.1.3 for a detailed discussion of the reforms and the corresponding labor supply incentives.

[15]I assume that λ only changes the weight of the employment and the earnings component, and do not allow the pension system to increase or decrease in its overall size.

reads for $y \leq 2\bar{y}$

$$\nu \ell^{\frac{1}{\chi}} = \Bigg[(1-\tau_p)\Big(1 - T'(y - T_p(y^p))\Big)\frac{c^{-\frac{1}{\sigma}}}{1+\tau_c} + (1-\lambda)\beta\psi_{j+1,h} E\left[v_{ep}(\mathbf{x}^+) \;\Big|\; j,s,m,\eta,h\right]\Bigg] wz(j,s,\eta)e.$$

The marginal disutility of providing an additional hour of work has to equal the marginal benefits. Providing an additional hour of work increases gross income by an amount $wz(j,s,\eta)$. This has an instantaneous benefit, as it allows the household to increase consumption, yet, only after paying contributions to the pension system and taxes. In addition, earning more has an impact on future pension income. The term $E\Big[v_{ep}(\mathbf{x}^+) \,\Big| j,s,\eta,h\Big]$ measures the utility value of accumulating additional pension claims. When λ increases and the pension system becomes more progressive, the link between earning more income and accumulating more pension claims is weakened and the return to providing additional working hours declines.

The Earned Income Pension Credit The second pension system I propose is the EIPC, which is closely related to the pension system discussed in Section 5.1.3. Compared to the benchmark model, I modify the function $f(y^p)$ in the accumulation formula such that it has two regions and mimics the EITC structure. Pension claims ep^+ then evolve according to

$$ep^+ = ep + \begin{cases} \Big(\frac{\lambda}{b} + 1 - \lambda\Big)y^p & \text{if } y^p < b\bar{y} \text{ and} \\ \lambda\bar{y} + (1-\lambda)y^p & \text{otherwise.} \end{cases}$$

Note, the accumulation equation does not depend on the employment decision e anymore. Individuals with earnings y^p above the bendpoint $b\bar{y}$ experience the same intensive margin labor supply incentives as workers in an economy with the ELS system. For individuals with labor earnings less than the threshold, the size of the employment subsidy is now conditional on earnings y^p. The more earnings an individual can generate from working, the higher the implicit employment subsidy will be, and the more likely that individual is to participate in the labor market.

The first-order condition for intensive margin labor supply for workers with earnings

$y^p < b\bar{y}$ now reads

$$\nu\ell^{\frac{1}{\chi}} = \Bigg[(1-\tau_p)\Big(1-T'(y-T_p(y^p))\Big)\frac{c^{-\frac{1}{\sigma}}}{1+\tau_c} + \Big(1-\lambda+\frac{\lambda}{b}\Big)\beta\psi_{j+1,h}E\left[v_{ep}(\mathbf{x}^+)\ \Big|\ j,s,m,\eta,h\right]\Bigg]wz(j,s,\eta).$$

The factor $\left(1-\lambda+\frac{\lambda}{b}\right)$ is always greater than one, as $b \in (0,1)$. This means that future pension income grows disproportionately with current earnings y^p. Low-earning individuals have hence a strong incentive to increase their intensive margin labor supply up to the bend point b.

Finally, both in the ELS and in the EIPC system individual pension benefits $p(ep)$ are calculated from the life-time average of earned pension claims ep_{j_r} as

$$p(ep) = \kappa_t \times \frac{ep_{j_r}}{j_r - 20},$$

where κ_t is the replacement rate. As in the benchmark, the pension system operates on a pay-as-you-go basis. In the initial equilibrium, total pension contributions hence need to be equal to the total amount of pension payments. Hence, equation 5.13 must be satisfied.

For my analysis, I selected a medium-range progressivity parameter of $\lambda = 0.5$. This means that 50% of pension payments are proportional to earnings, while the other 50% are subject to redistribution. I conducted simulations for the EIPC with bend points $b \in (0.2, 0.4)$ to investigate how much of the efficiency gains inherent in a genuinely employment-linked pension system can be recovered.

To ensure comparability between simulations, I use the same set of structural parameters, but fix per-capita government consumption over time. The contribution rate of the pension system remains at the initial equilibrium level in order to ensure that my reform does not change the system's size. I calculate full transition paths. Starting from an initial long-run equilibrium (indicated by $t = 0$), I assume that the economy is surprised by the reform of the pension formula and therefore enters a transition path at date $t = 1$. It then converges towards a new long-run equilibrium.

I allow the government to smooth the benefits and costs of the pension reform over time. To this end, I let the consumption tax rate balance the intertemporal budget of the government. The balancing consumption tax rate τ_c can be calculated from

$$\tau_c \cdot \sum_{t=1}^{\infty} R_t C_t + \sum_{t=1}^{\infty} R_t \int T_t\big(\cdot\big)\ d\Phi_t = \sum_{t=1}^{\infty} R_t G_t \quad \text{with} \quad R_t = \left[\frac{1+n}{1+\bar{r}}\right]^t.$$

I choose the same approach to calculate a replacement rate κ that balances the intertemporal budget of the pension system. All instantaneous budget imbalances are financed by issuing or repaying public debt.

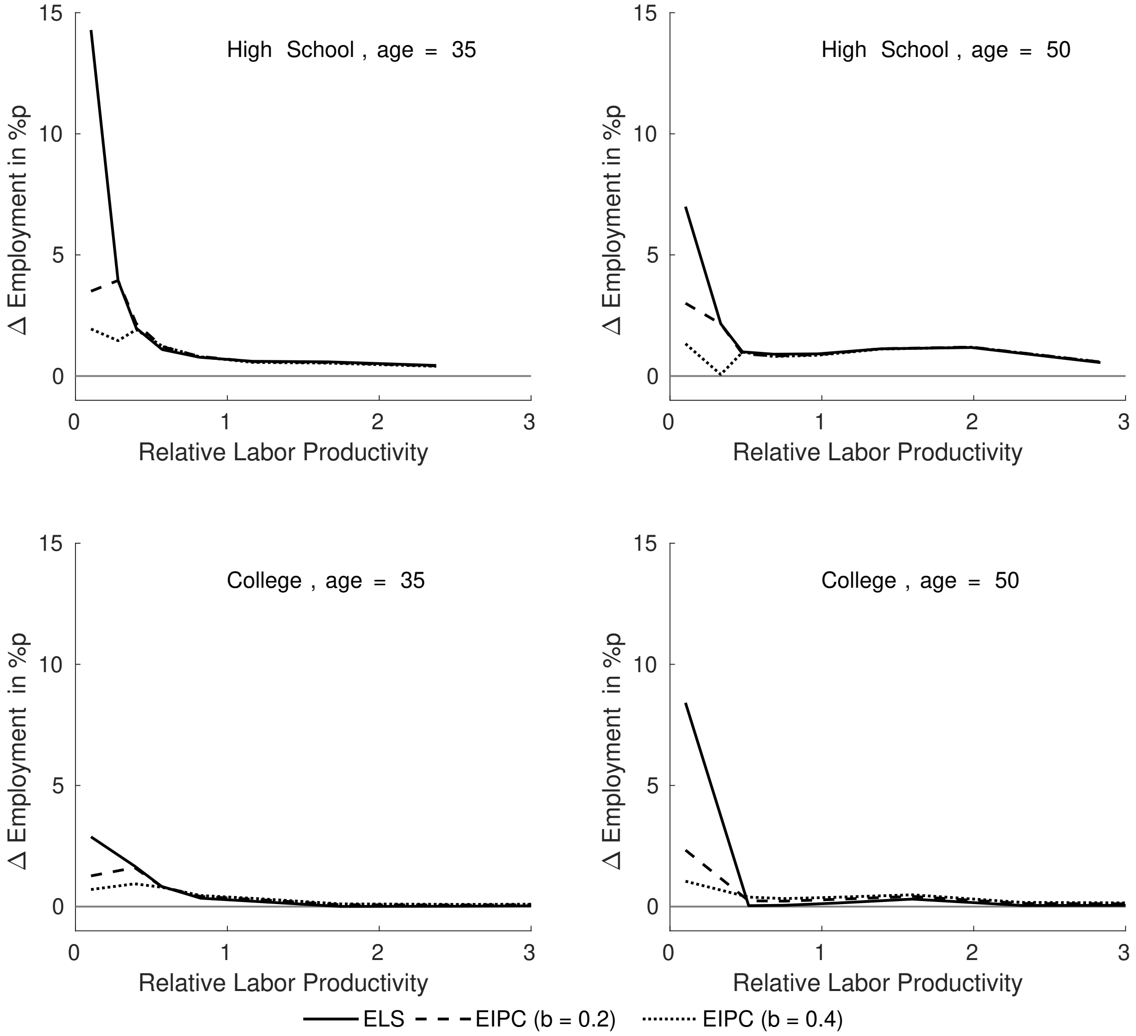
5.4.3 Labor supply effects of pension progressivity

Before studying full transitional dynamics, I illustrate some of the long-run effects of progressive pensions that are important for understanding welfare and efficiency effects. Figure 5.6 shows the long-run employment effects induced by the progressive pension reforms. The horizontal axis denotes an agent's labor productivity relative to the average labor productivity of the working-age population. On the vertical axis, I plot the change in employment between the initial proportional system and the new progressive pension system in percentage points. The first row shows the employment effects for 35- and 50-year old high school workers, the second row shows the results for the college-educated workforce. Employment changes are evaluated at the average distribution of wealth and pension claims of an agent in a respective age and education bin.

I first focus on the solid lines, which represent the employment effects that come with the introduction of an ELS. Regardless of age, all high school-educated households experience an increase in labor force participation. The effects are most pronounced for the productivity poor, as they experience the highest implicit employment subsidy, see discussion in Section 5.1.2. A rising labor force participation of households with high productivity, on the other hand, is the result of a negative income effect stemming from increased pension progressivity. At young ages, where individuals do not have a lot of wealth, the employment effect is quite high for the productivity poor. For example, the employment rate increases by 14 percentage points among the productivity poorest 35 year old high school worker. The effect fades out somewhat for older workers, as individual wealth increases. Yet, employment of the productivity-poorest 50-year-old high school-educated worker still increases by a remarkable 7.0 percentage points. The participation rates among college-educated worker increase as well, but the effect is much smaller.

The dashed and the dotted lines indicate the employment effects of an EIPC with bend points $b = 0.2$ and $b = 0.4$, respectively. As outlined in Section 5.1.3, under such purely earnings-related pension systems, the employment subsidy increases with earnings for the productivity poorer. It is hence not surprising that these systems are less effecting in stimulating employment at the lower end of the earnings distribution. Still employment effect equals to about one third of the truly employment linked system for $b = 0.2$ among the high school-educated worker. For higher productivity workers, however, the employment incentive effects are almost identical

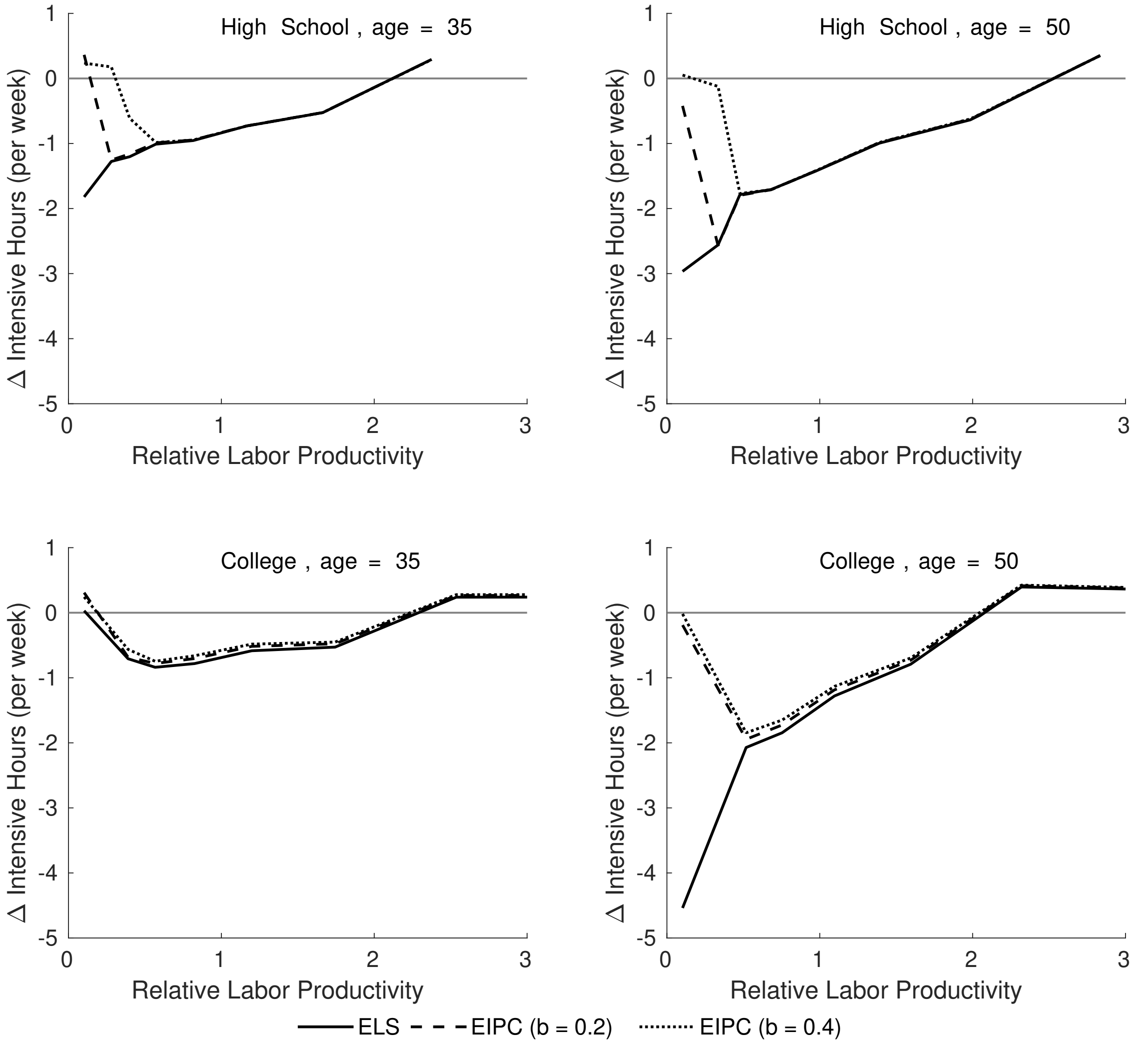
Figure 5.6: Employment changes and labor productivity



between the three systems.

Figure 5.7 shows the intensive margin labor supply response to increased pension progressivity. The structure of this figure is the same as the previous one, though on the vertical axis I show the change in intensive margin labor hours of employed individuals. For the ELS, the picture is almost inverse to the previous one. Weakening the link between accumulated pension claims and individual earnings leads to a higher implicit tax rate of the pension system, see discussion in Section 5.1.2. Hence, increased pension progressivity comes with negative labor supply incentives at the intensive margin, and especially so for earning poorer households. The negative incentive effect, however, only kicks in for individuals with labor earnings below the contribution ceiling of $2\bar{y}$. Once a household's income is greater than this ceiling – which happens if labor productivity is large – any additional euro of income earned is not subject to the payroll tax anymore. It is consequently not surprising that the negative intensive labor supply effect fades out with productivity. For the richest

Figure 5.7: Intensive margin labor supply changes and labor productivity



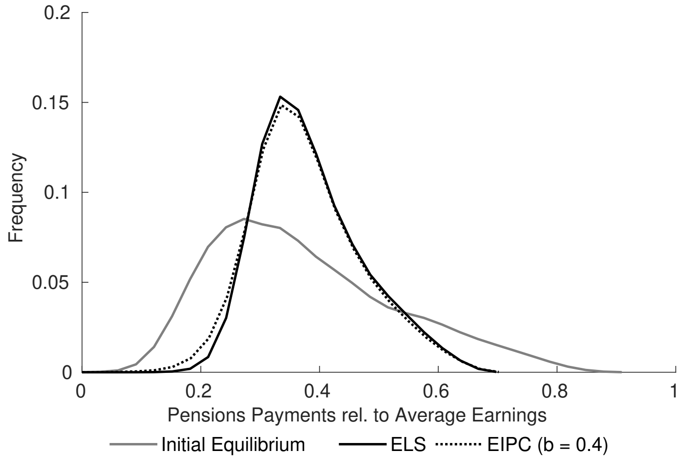
households, there is even a slight increase in hours, which stems from the negative income effect of higher pension progressivity.

The picture is again different for the EIPC. As noted in Section 5.1.3, the EIPC is less effective at stimulating employment at the lower end of the productivity distribution, since the implicit employment subsidy increases with earnings up to the bend point b. However, this increasing subsidy does stimulate intensive margin labor supply for low-earning individuals. This can be observed directly in Figure 5.7.

5.4.4 Progressivity and the distribution of pension claims

Increased pension progressivity not only comes with labor supply effects, it also alters the distribution of pension claims a household accumulates over her working life. Figure 5.8 shows the distribution of pension payments relative to average labor earnings at the retirement age j_r under different pension systems.

Figure 5.8: The distribution of pension benefits



The gray line displays the distribution of pension payments in the initial equilibrium. As pension claims are perfectly earnings related, this distribution is closely linked to the lifetime earnings distribution of households. Recall that the replacement rate is $\kappa = 0.455$ in the initial equilibrium. However, the mode of the pension payment distribution is somewhat lower at around 0.25. This is owing to potential interruptions in the individual's employment history and the fact that the accumulation of pension claims is capped at twice the average earnings.

The distribution of pension claims is much more concentrated with an ELS, as shown by the black line in Figure 5.8. In fact, the mass of individuals with a pension of less than 20 percent of average earnings shrinks to almost zero. The dotted line finally indicates the distribution of pension payments under an EIPC with bend point $b = 0.4$.[16] The system is slightly less efficient in mitigating inequality in pension payments compared to the ELS, but the differences are small.

5.4.5 A macroeconomic evaluation

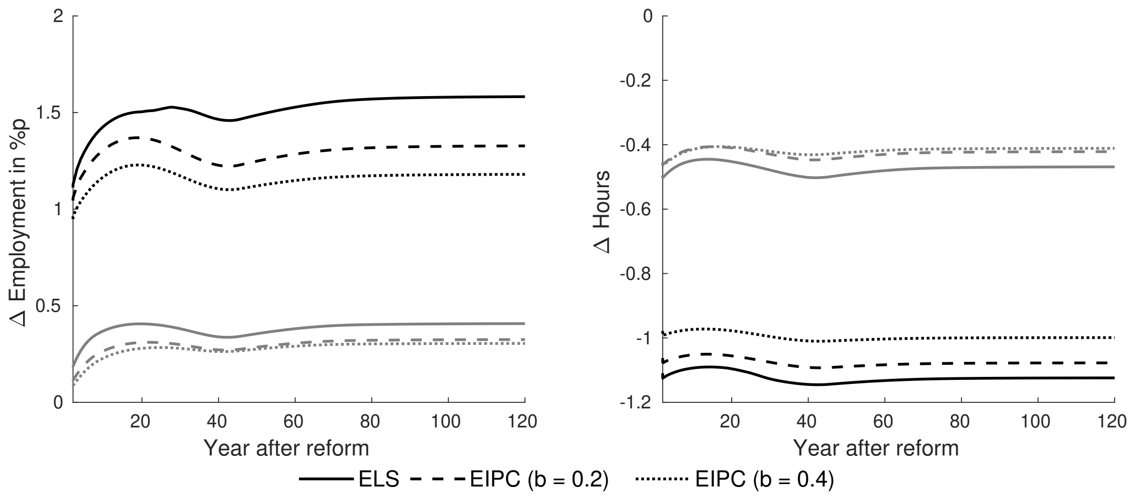
In this section, I evaluate the macroeconomic consequences of progressive pension reforms along a full transition path. Recall that I indicate the initial long-run equilibrium by $t = 0$. The pension reform comes at a surprise in period $t = 1$ and induces a transition path to a new long-run equilibrium. Computational details on the transition path are provided in Appendix C.1.2.

Figure 5.9 shows the employment and intensive labor supply effects for high school (black) and college (gray) educated workers. Overall, the effects are quite evenly

[16] The distribution of a pension system with bend point $b = 0.2$ would obviously lie in the middle. But since the differences are small anyway, I don't show the results for such a system in this graph.

distributed over time. Employment of college-educated workers rises by about 0.25 percentage points on average. The employment effect is much larger for high school workers, with a peak effect of 1.5 percentage points. The adverse impact on working hours can be seen in the right panel of Figure 5.9. The pattern is quite similar, with a smaller decline in hours for those with a college degree and larger effects for high school workers. Additionally, the impact on both extensive and intensive margin labor supply is most pronounced under the ELS. When looking at the EIPC, I find that a higher bend point reduces both the overall positive employment effects as well as the negative intensive margin distortions.

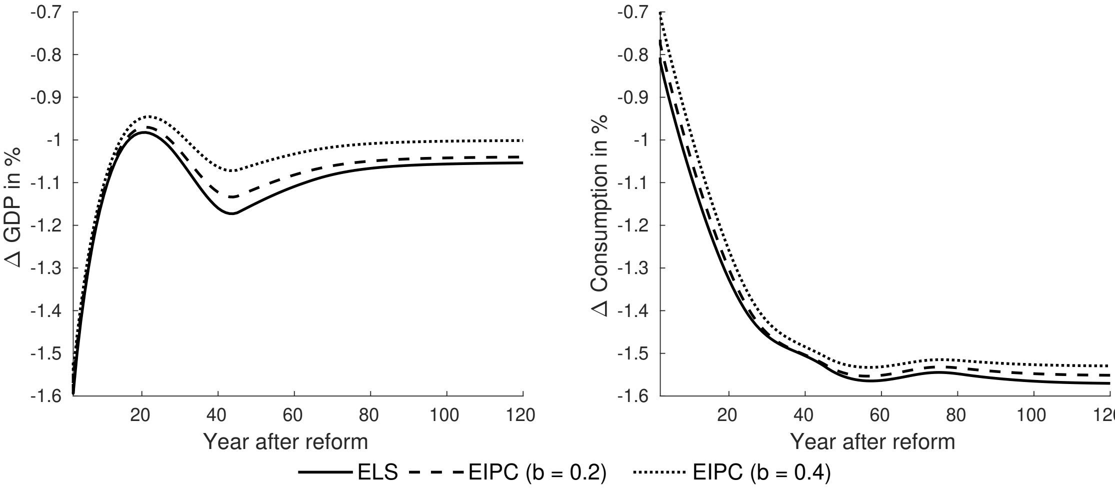
Figure 5.9: Aggregate labor supply effects



Combining the extensive and intensive margin labor supply responses, I find that overall labor input declines. This can be seen from the left panel of Figure 5.10, which shows the evolution of GDP over time. Recall that I consider a small open economy setting. Hence, aggregate capital input, labor input and GDP all move synchronously. The overall decline in labor input is not surprising in light of the fact that the employment effect is most pronounced for low-productivity workers, but the negative intensive margin distortions are distributed more evenly across productivity types. The drop in labor hours and therefore GDP is the strongest in the period directly after the reform ($t = 1$). It is mitigated somewhat by a decline in aggregate savings over time, see below.

The right panel of Figure 5.10 shows the consequences of the pension reforms for aggregate consumption. Not surprisingly, the decline in GDP causes aggregate consumption to drop immediately as I introduce progressive pensions into the economy. In the short-run, individuals can still live on their private assets, which damps the immediate consumption response. As private assets melt down over time, however,

Figure 5.10: GDP and aggregate consumption



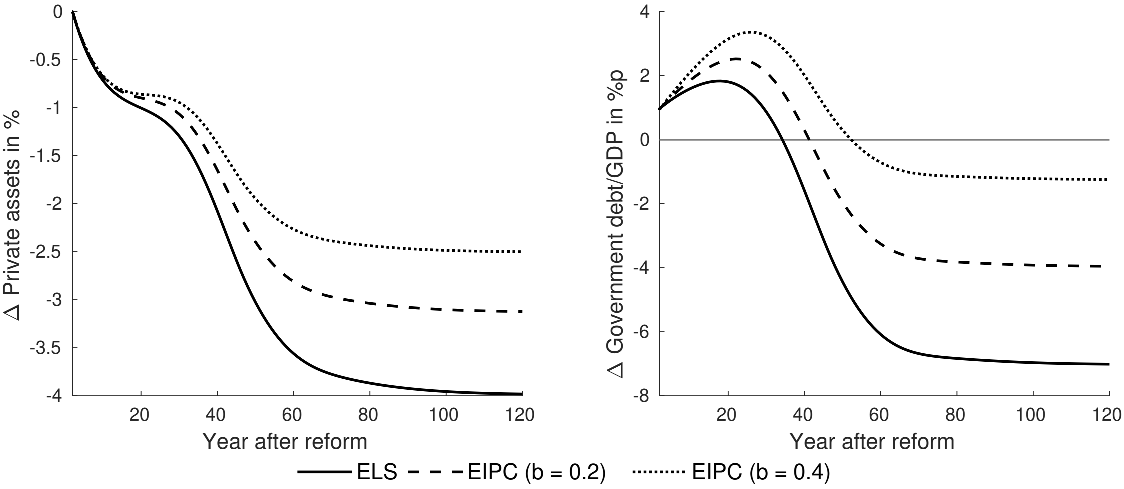
the consumption response becomes more pronounced. In the long-run, aggregate consumption declines by approximately 1.6 percentage points.

The left panel of Figure 5.11 illustrates the gradual meltdown of private assets along the transition. As opposed to the previous graphs, there is a substantial difference in the long-run effects of different pension system designs. The decline in private assets is most pronounced for the ELS, while it is much more moderate for an EIPC with bend point $b = 0.4$. The differences can be explained by the risk properties inherent in the different systems. The introduction of an ELS has the largest employment effect for productivity-poor individuals. As labor income in the poorest productivity states rises, the need for precautionary savings to insure a short-fall in labor earnings upon adverse productivity shocks declines. This impacts on private asset accumulation. The EIPC system with bend point $b = 0.4$ is much less successful in stimulating employment at the bottom end of the productivity distribution. Hence, households rely more on precautionary savings which damps the asset meltdown over time.

Finally, the right panel of Figure 5.11 shows the evolution of public debt over time. In the short-run, a strong decline in labor hours causes both a short-fall in tax revenue and pension contributions. To cope with the resulting budget imbalance, the government has to issue additional debt. As labor supply stabilizes in the medium- and long-run, however, the government is even able to reduce its debt level by about 7 percentage points. This comes with a relief for future generations.

Summing up, the simulation results indicate that the macroeconomic consequences of the proposed pension reforms are generally negative. The stimulation of labor force participation at the lower end of the productivity distribution somewhat mit-

Figure 5.11: Capital and public debt



igates the burden from larger labor supply distortions at the intensive margin. Yet, GDP and aggregate consumption still decline by 1 and 1.6 percent, respectively. However, I also see that the introduction of progressive pensions alters the risk properties of labor earnings risk, which affects aggregate savings. It also mitigates differences in pension income and therefore reduces consumption inequality especially at old age. To jointly evaluate the negative level and positive distributional consequences, I next take a look at aggregate welfare and economic efficiency.

5.4.6 Welfare analysis

This section evaluates the welfare and efficiency effects of progressive pensions. My preferred measure of household welfare is ex-ante expected life-time utility EV_t before any information about the household's education level or labor productivity has been revealed. I calculate ex-ante utility for any generation that is affected by a progressive pension reform, i.e., the initial cross-section of households at the reform date $t = 1$ as well as all new-born generations along the transition path. I distinguish affected generations by their birth date $t \in \{-J+1, \dots, 0, 1, \dots, \infty\}$. I compare the utility measures of these generations to the utility level $\overline{EV}$ of a generation that was born and has lived entirely through the initial equilibrium with a proportional pension system. To give the welfare numbers a meaningful interpretation, I calculate the corresponding consumption equivalent variation CEV_t. The consumption equivalent variation indicates by how many percent I would have to increase or decrease the consumption level of households at each age and each potential state in the initial equilibrium in order to make them as well off as in a reform scenario with progressive pensions. A positive value for CEV_t indicates that a progressive pension system increases welfare of a particular cohort t and that households of this cohort would be willing to pay a positive amount of resources in order to live in a world with progressive pensions. Details on the welfare measure are provides in Appendix C.1.5.

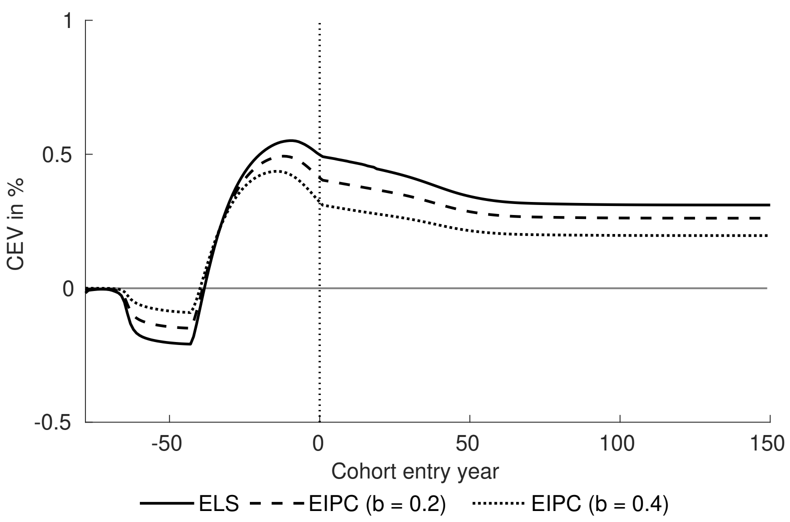
The welfare effects of pension reforms can vary a lot across different cohorts because of intergenerational redistribution. To derive a meaningful measure of the economic efficiency effect of pension reforms, I have to find a way to aggregate different welfare changes across cohorts to one aggregate efficiency measure. My method follows Fehr and Kindermann (2015) and Kindermann and Krueger (2022). I calculate the monetary transfer Ψ_t that each affected generation would have to pay in order to be indifferent between living along the reform path and in the initial equilibrium. I then derive the present value of all of these transfers, which provides a wealth-based measure W of economic efficiency. To turn this into a consumption based measure C, I convert the wealth-based measure into an annuity that pays out a constant stream along the transition path and in the new long-run equilibrium:

$$W = \sum_{t=-J+1}^{\infty} \left[\frac{1+n}{1+\bar{r}}\right]^t \Psi_t \quad \text{ăand} \quad C = W \times \left[\sum_{t=1}^{\infty} \left[\frac{1+n}{1+\bar{r}}\right]^t\right]^{-1}.$$

I express the resulting time-invariant welfare gain C in percent of the initial equilibrium consumption level C_0.

Figure 5.12 shows the welfare effects of the three reform scenarios. The vertical dotted line separates cohorts that were already alive in the initial equilibrium and that were surprised by the reform at some date in their life cycle from those cohorts born along the transition path $t > 0$. All three reforms generally come with positive

Figure 5.12: Welfare effects along the transition path



welfare effects for current working cohorts as well as all newborns. Only generations that were already retired at the time of the pension reform loose slightly from an increase in consumption taxes. Young workers as well as newborns at the reform date experience the highest welfare gains, in the order of 0.5 percent of lifetime consumption. As the transition proceeds and private assets melt down, welfare again declines, but remain positive even in the long-run.

Welfare effects differ across reform scenarios. The introduction of an ELS results in the highest welfare gains for current workers and future generations, but current retirees experience the highest welfare losses. The effects are more moderate for an EIPC system with bend points $b = 0.2$ and $b = 0.4$. There are two possible explanations for this. First, as shown in Figure 5.11, the asset melt-down is least pronounced for $b = 0.4$, which suggests a lower degree of intergenerational redistribution under this scenario. Second, a higher value for the bend-point b reduces the effectiveness of the progressive pension system in stimulating employment and leads to fewer risk-sharing opportunities, resulting in lower economic efficiency.

To disentangle intergenerational redistribution from economic efficiency, the first row of Table 5.4 shows the aggregate efficiency effects of the different progressive pension systems. The ELS turns out to be the most efficient system to implement. It generates a permanent increase in welfare worth 0.73 percent of aggregate consumption. This is not surprising in light of the fact that this system operates under the assumption that the government can condition pension payments on the individual employment decision. If the government is bound by informational constraints and can only condition pension payments on income, aggregate efficiency has to deteriorate. However, even under the "second-best" policies with bend points $b = 0.2$ and $b = 0.4$, the government can still recover 90% and 78% of the original efficiency effect, respectively.

The second panel of Table 5.4 shows the welfare consequences for the four different permanent types $(s, m) \in \{0, 1\} \times \{0, 1\}$. The major beneficiaries of progressive pensions are high school workers, as they tend to be the recipients of employment subsidies. The college-educated lose because of higher labor supply distortions and a reduction in their pension benefits. Within the group of high school workers, it is those with a stable career path that experience the highest welfare gains. The welfare gains of workers with an unstable career path are only half the size. The reason is that workers with a stable career tend to have a higher labor market attachment. As such, they can enjoy the full benefits of redistribution through the progressive pension without incurring any major extra cost. Those with an unstable career are motivated to stay attached to the labor force even when they experience a low labor productivity shock. They do so to enjoy the employment subsidy embedded in the progressive pension. However, labor market participation comes at a higher utility

Table 5.4: Welfare effects of increased pension progressivity

		EIPC	
Variable	ELS	b = 0.2	b = 0.4
Change in aggregate efficiency	0.73	0.66	0.57
Change in ex-ante long-run welfare	0.31	0.26	0.20
Long-run Welfare by Permanent Types			
– for high school with unstable career	0.31	0.22	0.14
– for high school with stable career	0.52	0.57	0.52
– for college with unstable career	−0.14	−0.22	−0.26
– for college with stable career	−0.19	−0.14	−0.18
Long-run Welfare Decomposition			
– average utility of consumption	−0.26	−0.26	−0.25
– average disutility of labor	−0.15	−0.09	−0.07
– risk sharing possibilities	0.71	0.61	0.51

Table reports CEV over initial equilibrium in percent.

participation cost, which damps the welfare benefits of such households. This effect becomes even more pronounced under an earnings-based system with bend point $b = 0.2$ or $b = 0.4$ that reduces the employment subsidy for households with very low labor earnings.

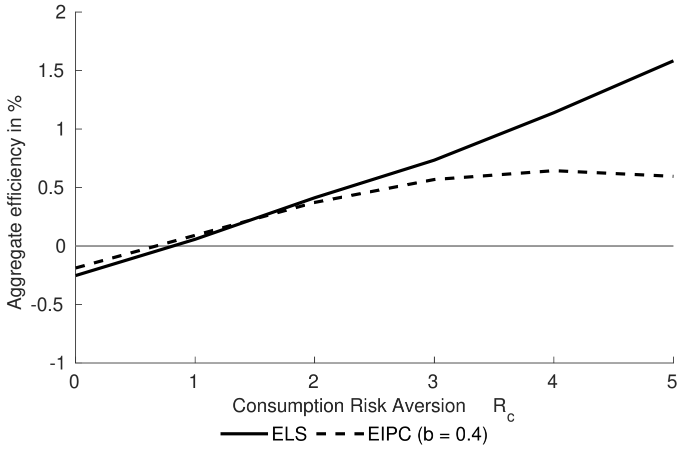
Household welfare gains can stem from (i) increases in average life-cycle consumption, (ii) a decline in the average disutility of labor, or (iii) increased risk-sharing possibilities that lead to a decline in the variance of consumption and/or labor hours. In the last panel of Table 5.4, I decompose the long-run welfare gain into effects coming from exactly these three components. The welfare gains of progressive pensions are entirely due to improved risk-sharing possibilities. As I already discussed before, average life-cycle consumption declines, and the utility costs of labor force participation increase. Both effects reduce long-run welfare. However, a declining variance of consumption and labor hours within age groups overcompensates these negative effects and leads to an overall welfare increase.

5.4.7 Sensitivity analysis

This section provides sensitivity checks with respect to two central elements of my quantitative model: individual risk aversion and the structure of the labor market.

Risk aversion Figure 5.13 shows how the aggregate efficiency effects of progressive pensions depend on household risk aversion. In my preferred calibration, I use a consumption risk aversion of $R_c = 3$. A higher risk aversion leads to additional welfare gains from increased social insurance. The effect is quite strong for the ELS,

Figure 5.13: Sensitivity analysis



as this system comes with the best risk-sharing possibilities for households. With a risk aversion of $R_c = 5$, aggregate efficiency increases to a remarkable 1.6 percent. An EIPC system with bend point $b = 0.4$ is much less successful in insuring labor productivity risk, especially for households with very low productivity shocks. This is directly reflected in the aggregate efficiency numbers.

In addition to showing that a higher risk aversion raises the size of efficiency gains from redistribution and social insurance, there is another point of interest in Figure 5.13. When choosing a value of $\gamma = -3.57$, individual risk aversion drops to zero. In this case, the gains from redistribution are absent and the efficiency effects from progressive pensions emerge solely from labor supply distortions. Aggregate labor supply distortions are most pronounced for the ELS. While this system sets the highest employment incentives, it also comes with a positive implicit tax rate for all working individuals. An earnings-based system with $b = 0.4$, on the other hand, sets additional positive labor supply incentives at the intensive margin for the productivity poor, which limits aggregate efficiency losses.

Structure of the labor market Table 5.5 displays the aggregate efficiency consequences of introducing an earnings-based progressive pension with bend point $b = 0.4$ for different assumptions about the structure of the labor market. In my benchmark scenario, I assumed that 50 percent of the population is exposed to low productivity shocks, while the other half faces stable career paths. To check the importance of this assumption, I let the whole population be exposed to low productivity shocks $\phi_m = 1$ and recalculate the respective shock process to guarantee consistency with the data. As the results in Table 5.4 reveal, the consequences for both aggregate efficiency and long-run welfare are only minor.

The last row of the table reports the results from simulations in which I assume away

Table 5.5: Sensitivity analysis

	EIPC (b = 0.4)	
	agg. efficiency	long-run welfare
Benchmark simulation	0.57	0.20
Career Stability: $\phi_m = 1$	0.49	0.18
No extensive margin costs ($\xi = 0$)	0.44	0.17

Table reports CEV over initial equilibrium in percent.

participation costs. Without participation costs, households are always employed, regardless of their productivity shock. Hence, setting extensive margin employment effects can not improve economic efficiency by definition. Consequently, the aggregate efficiency effect of introducing progressive pensions shrinks by about 25 percent as compared to my benchmark scenario.

5.5 Conclusion

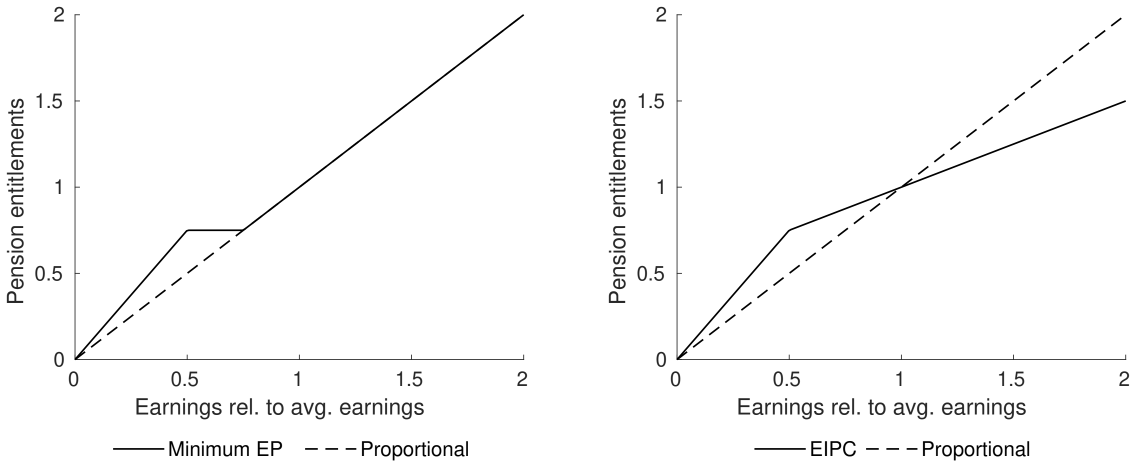
This chapter studies the introduction of a progressive component in a proportional pension system. I show in an analytical model that linking pension payments to the employment decision of households can mitigate overall labor supply distortions. I quantify the effects of progressive pension systems in a stochastic overlapping generations model with labor supply responses at the intensive and at extensive margin. The focus of my analysis is extensive margin labor supply reactions to progressive pension reforms. A pension system with an employment-linked component increases labor force participation and hence mitigates negative labor supply effects at the intensive margin. Aggregating the resulting welfare effects along the transitional path and in a new long-run equilibrium shows that such a reform is efficiency improving. I address potential feasibility concerns and propose a second reform, the Earned Income Pension Credit, which redistributes pension claims solely based on earnings. My simulation results indicate that a substantial share of the efficiency gains from the employment-linked system can be restored with the EIPC.

The proposed EIPC pension formula is related to the 1992 pension reform (Rente nach Mindestentgeltpunkten) as outlined in Section 3.4. In that reform, annual earnings points credited prior to 1992 that value less than 0.75 were upgraded by factor 1.5 to a maximum of 0.75.[17] Individuals with at least 35 years of pension-relevant employment (including times before and after 1992) and an average earnings points value of 0.75 per year at retirement entry were eligible for the top-up. Figure 5.14 compares the two reforms (EIPC with $b = 0.5,\ \lambda = 0.5$). Obviously, the 1992

[17] The adjustment actually happened on a monthly basis, see Section 3.4.

reform did not cut the pensions of the rich, but apart from that the functional form exhibits some similarities with the EIPC. Still, that reform can hardly be compared to the EIPC exercise conducted in that chapter, as it was imposed ex-post and only applied to pension entitlements that have already been credited. The effects on labor supply decisions are hence minor. Only the participation eligibility criteria of 35 years with contributions might have pulled some workers into employment after the reform was communicated.

Figure 5.14: Pension by minimum earnings points and the EIPC



However, I still learn from the data on the 1992 reform that the main beneficiary group of the reform was women.[18] In particular, 40.8 percent of the women and only 4.8 percent of the men in the sample received a pension top-up. Unfortunately, the model presented in this chapter is silent about the effects on women. Women face considerable income risk over the life cycle and exhibit higher elasticities at both margins of response. Hence, I expect that a large fraction of women would react to the proposed pension reforms. My welfare results can thus be regarded as a conservative estimate. I address that issue in Chapter 6 of the dissertation, which provides an extension of the model in an economy with single and couple households. Moreover, the following chapter discusses the role of the earnings base, namely annual earnings or lifetime earnings, to which the progressive pension formula is applied.

[18]I use data from FDZ-RV – SUFVVL2004-2017, see Section 3.4.

Chapter 6

Lifetime Earnings Redistribution - Favorable or Costly?

In some Western countries, such as Portugal[1], the Czech Republic[2] and the US[3], low-income individuals are supported through a progressive pension system in old age. While the pension formulas differ, they all redistribute based on information on aggregate lifetime earnings.[4] For instance, US Social Security calculates an index of average earnings for each retiree and then applies different replacement rates. A disproportional high replacement rate at the bottom of the income distribution therefore generates substantial pension progressivity, see Section 3.3. The pension system is a tax instrument that facilitates the redistribution of lifetime earnings. But is it also favorable? Theory predicts substantial welfare gains from history-dependent income taxation, see for example Kapička (2022). This chapter of the dissertation studies the gains and costs of lifetime earnings redistribution through the pension system. I compare the case of redistributing old age income (1) given information on lifetime earnings at retirement entry to (2) given information on annual earnings during working life. I use the same redistribution scheme in both experiments.

As in the previous chapter, I apply a redistribution scheme that is closely related to

[1]Pension benefits are proportional to reference earnings. The earnings measure was the best 10 of the final 15 years, but that was extended to lifetime average earnings from 2017. For people with more than 40 contribution years, only the best 40 count, see to OECD (2021e).

[2]According to OECD (2021b), the earnings measure for the earnings-related pension averages across all years since 1986, but will gradually reach lifetime average. See also for a formal definition OECD (2020).

[3]See Section 3.3. for a description of US Social Security.

[4]Old age income redistribution based on annual information is only rarely used in practice. One example where annual extensive margin decisions are relevant is the German Grundrente. To qualify for the transfer, individuals need to have contributed for a minimum of 33 years towards the statutory pension insurance.

the Earned Income Tax Credit (EITC) in the US. There are two obvious ways to introduce EITC-style redistribution into the pension system. First, through the accumulation formula of pension entitlements and second, through the pension formula. The accumulation formula credits pension entitlements based on labor earnings in every year during working life. The pension formula converts pension entitlements into pension payments at retirement entry. Hence, redistribution is based on annual earnings if the transfer scheme is applied to the accumulation formula and it is based on lifetime earnings if applied to the pension formula. Does it make a difference?

I am mostly interested in changes in female labor supply, old-age income inequality, and long-run welfare. My simulation results suggest, that redistribution based on annual earnings dominates the lifetime earnings-based system. The labor supply incentive of the EITC-style redistribution scheme is substantial if applied to annual earnings. It incentivizes low earners to participate in the labor market every year. If an individual is not employed, the pension credit subsidy for that year is lost forever. Lifetime earnings-based redistribution does not exhibit such a strong extensive margin incentive. Moreover, individuals are tempted not to work in years with adverse productivity shocks due to increased insurance. Encouraging steady employment during working life is an important tool to reduce old-age poverty risk.[5] Intuitively, redistribution should be more targeted if based on lifetime information, as short-term earnings fluctuations are balanced over time. However, the simulations show that the impact of increased labor supply of the earnings poor is outweighing the gains of more targeted redistribution.

I quantify the individual and macroeconomic consequences of these pension reforms using a simulated overlapping generations model. When entering the economy, individuals draw a specific gender and education level at random. They are then potentially matched to a partner of the opposite gender to form a marriage. Martial status is invariant over the entire life, meaning that marriages don't get divorced and individuals who don't get matched are singles forever. I allow for assortative mating with respect to education. Throughout their life, individuals make labor supply decisions at the extensive and the intensive margin. In particular, women can decide whether to work full time, part time, in a mini-job or not at all. Men can only work full time or not at all.[6] Children arrive according to a stochastic process. The presence of children induces both time and monetary costs to the family. As a result, mothers have to cut back on labor hours which, in case they are single,

[5] Haan et al. (2017) document that individuals with long employment histories exhibit a very low risk of old-age poverty

[6] Only few men work part time in reality and I neglect that option for simplicity. I am more specific in modeling female labor supply, as women bear a larger old-age poverty risk and show more variation in labor supply.

puts them at a substantial risk of poverty. This risk may spill over to retirement in the form of reduced pension benefits. By accounting for many demographic details, I can study the behavior of various social groups over their life cycle. I can identify old-age poverty risk groups and study group-specific responses to the pension reforms. I calibrate the model to the German economy, which features only little redistribution within the pension system. Starting from this benchmark, I quantify the long-run effects on individual labor supply decisions, old-age poverty, the macroeconomy and welfare of both reforms. My simulation results show that annual earnings-based redistribution provides positive labor supply incentives for low earner in every working year. The activation of a broader workforce, especially married women, mitigates overall negative macroeconomic consequences and long-run welfare increases by 0.52 percent. Lifetime earnings-based redistribution is less successful in stimulating labor supply. The macroeconomic consequences of increased redistribution are more severe and the long-run welfare effects are slightly negative.

The chapter is largely based on a revised version of the working paper *An Earned Income Pension Credit as a means to reduce old-age poverty risk* by Kindermann and Püschel (2022).[7]

6.1 Building intuition: a three-period framework

Before setting out my large-scale simulation model, I want to build some intuition for the main mechanisms at work using a much simpler and stylized framework. Households in this framework live for three periods $j = 1, 2, 3$. They supply labor and contribute to the pension system in the first two periods of life, in the final period they are retired. The wage rate w_j for effective labor is exogenous. Individuals derive utility from consumption c_j in each period and suffer disutility from working ℓ_j. For analytical tractability, I assume that preferences are quasi-linear in consumption and that the time discount rate equals the interest rate $r = 1$.[8] Disutility from labor is governed by the Frisch elasticity of labor supply χ. More specifically, I let preferences be represented by the utility function

$$U(c_1, c_2, c_3, \ell_1, \ell_2) = c_1 + c_2 + c_3 - \frac{\ell_1^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \frac{\ell_2^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}}. \tag{6.1}$$

Households maximize utility in (6.1) subject to the present value budget constraint

$$c_1 + c_2 + c_3 = (1 - \tau_p)(w_1\ell_1 + w_2\ell_2) + p(e_{j_r}). \tag{6.2}$$

[7] A newer version of the paper circulates with the title *Lifetime Earnings Redistribution - Favorable or Costly?*

[8] I relax all of these assumptions later on in my quantitative model in Section 6.2.

The mechanics of a pay-as-you-go (PAYG) pension system are quite simple. The system collects contributions in the form of a payroll tax τ_p on earnings $w_j \ell_j$ from current workers. In reward for their annual contributions, workers are credited pension entitlements e_j.[9] Finally, the sum of all contributions is redistributed as pensions p to all retirees in relation to their pension entitlements.

6.1.1 The proportional pension system

The proportional pension system is the starting point of the analysis. Here, both annual pension entitlements and pension payments are proportional to a worker's earnings or earnings history. It is characterized by the accumulation formula for pension entitlements

$$e_{j_r} = \underbrace{\frac{w_1 \ell_1}{2}}_{e_1} + \underbrace{\frac{w_2 \ell_2}{2}}_{e_2}. \tag{6.3}$$

and the pension formula

$$p = \kappa \times e_{j_r}. \tag{6.4}$$

During working life, the household accumulates annual pension entitlements e_j in direct proportion to her contributions and hence earnings $w_j \ell_j$. At retirement entry, the pension formula converts the sum of annual pension entitlements e_{j_r} into pension payments by multiplying it with a scale factor κ.[10] I now introduce a progressive component into the pension system. First, I analyze the case of a progressive pension formula and second, the case of a progressive pension entitlement formula.[11]

6.1.2 Redistribution based on lifetime earnings: the LEAP system

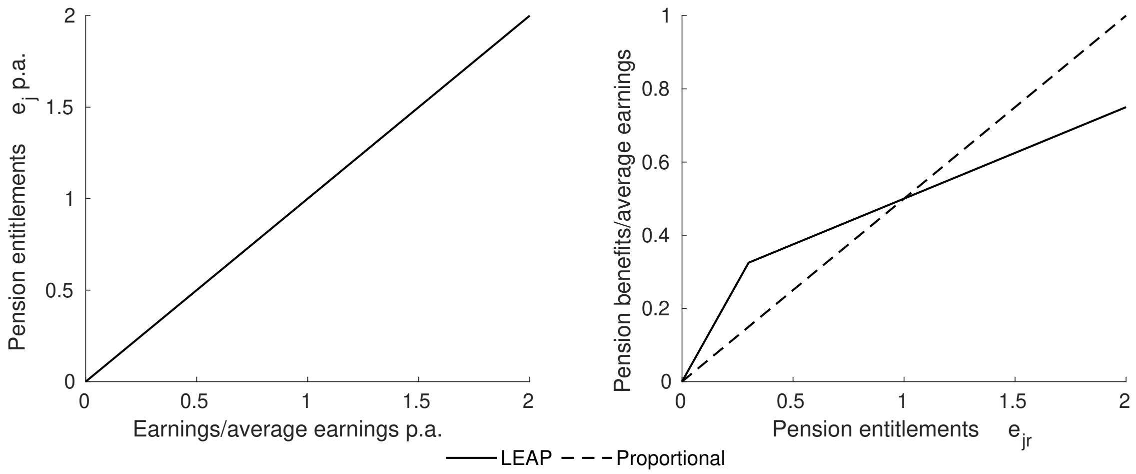
In order to provide some intuition on my first redistribution scheme, let me start with examining the system graphically. The left panel of Figure 6.1 shows the formula for annual pension entitlements e_j. It is similar to the one in the proportional

[9] Pension entitlements are usually some index of the worker's earnings or contribution history.

[10] Note that in practice many pension systems feature a contribution and/or accumulation ceiling. For now, I want to keep the discussion as simple as possible. I will, however, include such a ceiling in my quantitative model.

[11] Strictly speaking, the government could also introduce progressivity into the contribution formula. There are, however, several reasons why this hardly makes sense. First, there is a one-to-one mapping between progressive pension contributions and a progressive entitlement formula, as the only thing that matters for the individual is the relationship between contributions and entitlements. Second, when the aim of the government is to free up resources or redistribute during working years it should resort to the income tax and not the pension system.

Figure 6.1: The LEAP-system



pension system, where annual entitlements e_j are proportional to annual earnings $w_j\ell_j$. Hence, final pension entitlements e_{j_r} are simply average lifetime earnings

$$e_{j_r} = \underbrace{\frac{w_1\ell_1}{2}}_{e_1} + \underbrace{\frac{w_2\ell_2}{2}}_{e_2}. \tag{6.5}$$

The right panel illustrates the pension formula, which is clearly progressive. Compared to the proportional pension system, individuals with below-average lifetime earnings receive a top-up to their pension payments while the benefits of the above-average earners are cut. The average earner (an individual that earns on average $\bar{y}$) sits at $e_{j_r} = 1$ and is indifferent between both systems. Moreover, the pension formula includes a phased-in and phased-out structure with two different pension replacement rates, resembling the structure of the EITC. In particular, the pension formula reads

$$p(e_{j_r}) = \kappa \times \begin{cases} \left[\frac{\lambda}{b} + (1-\lambda)\right]e_{j_r} & \text{if } e_{j_r} < b\bar{y} \text{ and} \\ \lambda\bar{y} + (1-\lambda)e_{j_r} & \text{otherwise.} \end{cases} \tag{6.6}$$

The two regions are separated by a threshold which is defined as the multiple $b \in (0,1)$ of average earnings $\bar{y}$. In the phase-in region, pension payments increase disproportionately strongly with average lifetime earnings. Pensioners with average lifetime earnings equal to $b\bar{y}$ enjoy the highest pension top-up. In the phase-out region, pension payments increase at rate $(1-\lambda)$. λ is hence the progressivity factor of the pension system. In the extreme case of $\lambda = 1$, the pension formula would be flat beyond the bendpoint, and pension payments would not increase with additional earnings. Since I feed average lifetime earnings e_{j_r} into the pension formula, redistribution is based on lifetime earnings information. I denote the pension system as Lifetime Earnings Progressive (LEAP) system.

Deriving the intensive margin labor supply choice ℓ_j reviles the effect of the pension system on labor supply.[12]

$$\ell_j = \begin{cases} \Big[w_j \times \Big(1 - \underbrace{\tau_p \lambda}_{\tau_p^{imp}} + \underbrace{\tau_p \frac{\lambda}{b}}_{\tau_p^{sub}}\Big)\Big]^{\chi} & \text{if } \frac{w_1\ell_1 + w_2\ell_2}{2} < b\bar{y} \text{ and} \\ \Big[w_j \times \Big(1 - \underbrace{\tau_p \lambda}_{\tau_p^{imp}}\Big)\Big]^{\chi} & \text{otherwise.} \end{cases} \tag{6.7}$$

The two regions of the pension formula are mirrored in the policy function for labor supply. Individuals in the first region experience an earnings dependent subsidy to working $\tau_p^{sub} = \tau_p \frac{\lambda}{b}$. Since the amount of average lifetime earnings determines the region, only individuals with permanent low earnings can benefit from the subsidy. Individuals with temporarily low earnings are in the second region and do not experience the incentive effect. Hence, labor supply in one period is determined by the current wage as well as earnings in the other period. The distribution of earnings across periods is irrelevant.

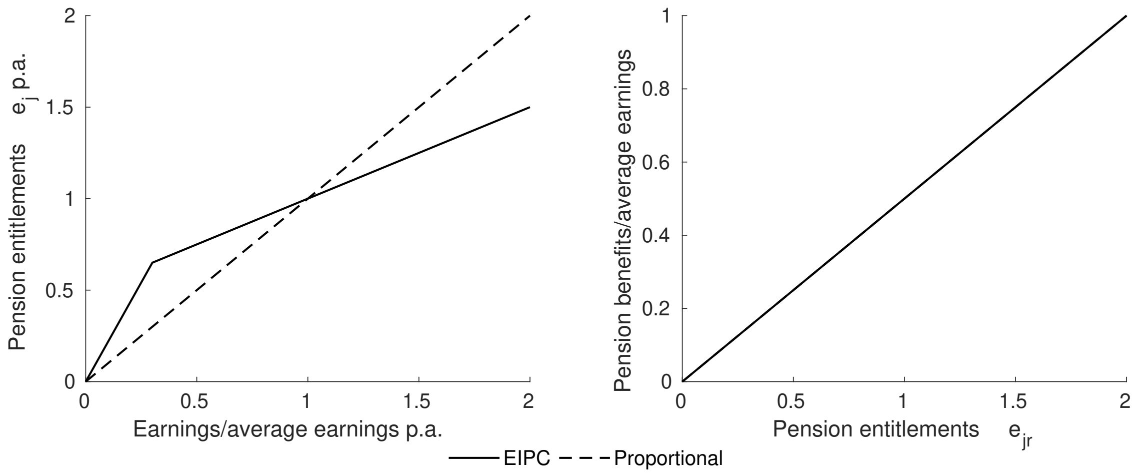
There are also costs. First, there is a negative substitution effect. Households perceive a fraction λ of their pension contributions τ_p as an implicit tax $\tau_p^{imp} = \tau_p \lambda$. This distorts intensive margin labor supply in both regions. Second, there are efficiency costs of increased insurance. Individuals may be tempted to reduce their labor hours or to not work at all during periods with adverse productivity shocks in order to limit their lifetime earnings. This raises the value of accumulated pension entitlements, as the corresponding pension benefits are higher.

6.1.3 Redistribution based on annual earnings: the EIPC system

Let's now begin by analyzing Figure 6.2, which graphically represents the alternative pension system that redistributes based on information on annual earnings. The structure of the graph is the same as in the previous case. The functional form of the progressive function is exactly the same as in the LEAP system, but it is now applied to the accumulation formula. The pension formula, illustrated in the right

[12] I deliberately assume that $\frac{\kappa}{2\tau_p} = 1$.

Figure 6.2: The EIPC-system



panel, is proportional to final pension entitlements and simply reads

$$p(e_{j_r}) = \kappa \times e_{jr}. \tag{6.8}$$

To gain more understanding of the differences between the two systems, one has to take a closer look at the accumulation formula. Compared to the proportional system, individuals with below-average annual earnings receive a top-up to their annual pension entitlements e_j while the above-average earner is credited less (see left panel). Hence, old-age income redistribution is based on information on annual earnings during working life. Period pension entitlements e_j are computed as

$$e_j = \frac{1}{2} \times \begin{cases} \left[\frac{\lambda}{b} + (1-\lambda)\right] w_j \ell_j & \text{if } w_j \ell_j < b\bar{y} \text{ and} \\ \lambda \bar{y} + (1-\lambda) w_j \ell_j & \text{otherwise,} \end{cases} \tag{6.9}$$

and final pension entitlements are again $e_{j_r} = e_1 + e_2$. Whether an individual sits in either the phase-in or the phase-out region is now reassessed in every period of working life. This also affects the labor supply decision of households as shown in Equation (6.10).

$$\ell_j = \begin{cases} \left[w_j \times \left(1 - \underbrace{\tau_p \lambda}_{\tau_p^{imp}} + \underbrace{\tau_p \frac{\lambda}{b}}_{\tau_p^{sub}}\right)\right]^{\chi} & \text{if } w_j \ell_j < b\bar{y} \text{ and} \\ \left[w_j \times \left(1 - \underbrace{\tau_p \lambda}_{\tau_p^{imp}}\right)\right]^{\chi} & \text{otherwise.} \end{cases} \tag{6.10}$$

The policy function ℓ_j exhibits again two regions, where the first one provides a subsidy $\tau_p^{sub} = \tau_p \frac{\lambda}{b}$. However, the subsidy is now also available for workers with temporary low earnings. This provides an incentive to increase the labor supply for all individuals in periods with low earnings. The labor supply decision in period j is only determined by the current wage w_j and does not directly depend on earnings in other periods.

Moreover, the system sets a positive extensive margin labor supply incentive, as only working individuals benefit from the top-up. Hence, the insurance effect is also limited to the employed, which caps the efficiency costs of increased insurance compared to the LEAP system. The distortion due to the implicit tax $\tau_p^{imp} = \tau_p \lambda$ is similar to the LEAP system.

I denote this pension system as Earned Income Pension Credit (EIPC) because it is very closely related to the EITC in the US. The functional form (phase-in and phase-out structure), the assessment base (annual earnings), and the labor supply incentives (intensive + extensive margin) are similar. The main difference is that the benefits are distributed during retirement rather than working life.

6.1.4 Annual earnings vs. lifetime earnings redistribution

The discussion boils down to one question: Should old-age income redistribution f be based on the sum of annual earnings $p = f(\sum_{j=1}^{j_r-1} y_j)$ or on annual earnings $p = \sum_{j=1}^{j_r-1} f(y_j)$? Economic theory suggests, that annual accounting leads to more favorable labor market outcomes. While the incentive effects of both progressive systems are similar for individuals with permanently low earnings, they differ for individuals with temporarily low earnings. In the LEAP system, a worker with a one-time adverse productivity shock is treated as in any other period. The pension system imposes a negative labor supply incentive through the implicit tax τ_p^{imp} and increased insurance may lead individuals to work less or not at all. The EIPC system provides a subsidy to labor supply $\tau_p^{sub} > \tau_p^{imp}$ for periods of low earnings. To become eligible for the top-up, an individual must work during times of low earnings. Hence, the costs of increased insurance are limited. However, lifetime accounting should also provide benefits as it allows for more targeted redistribution. Short-term fluctuations in earnings are balanced over time and redistribution is based on more information. To elaborate on the size of labor supply effects and the benefits of more targeted redistribution, I now investigate the effects of both pension systems in a quantitative model.

6.2 The quantitative simulation model

My full quantitative simulation model is based on previous theoretical considerations and informed by empirical facts. I employ a general equilibrium overlapping generations model with population growth and survival risk in the spirit of Auerbach and Kotlikoff (1987). The model features single and couple households. Each individual draws a persistent shock to gender, marital status and, labor productivity at the beginning of life. Marriages are stable, and couples retire and die jointly. Households decide about labor supply at the intensive and extensive margin as well as about consumption and savings. Couples make joint decisions. Over time, individuals are subject to transitory income shocks, labor flexibility shocks, and fertility shocks. Parents bear both time and monetary costs when raising kids. In the benchmark model, the government operates a proportional pay-as-you-go pension system financed by payroll taxes. Further, the government collects resources through the progressive taxation of labor earnings and a proportional consumption tax to cover general government expenditures and transfer payments to families with kids. I consider an open economy framework, so that the prices for capital and labor are fixed, but government parameters adjust in order to keep the fiscal tax and transfer systems balanced. Since I only consider long-run equilibria, I omit the time index t in the following wherever possible. The benchmark economy is calibrated to the German economy. The computational details are provided in Appendix D.3.

6.2.1 Demographics

The economy is populated by overlapping generations of heterogeneous individuals. At each point in time t, a new generation of size N_t is born. Individuals are either male or female $g \in \{m, f\}$, high-school or college-educated $s \in \{0, 1\}$ and live in a single or a couple household $i \in \{s, c\}$. These states do not change over the life cycle. Couples are born and die jointly, marriages don't get divorced and individuals who don't get matched are singles forever. I assume that the population grows at a constant rate n. Households start their economic life at age $j = 20$ and live up to a maximum of J years. Whether a household is still alive in the next period is uncertain and depends on the age and gender-specific survival probability $\psi^i_{j,g}$. This is the conditional probability of a household to survive from period $j - 1$ to period j, with $\psi^i_{20,g} = 1$ and $\psi^i_{J+1,g} = 0$. A fraction of women give birth to two kids k during working life. Fertility is modeled as a stochastic process where both the timing and the outcome of the shock is uncertain. The kids never enter the economy as productive agents. Individuals can supply labor to the market until they reach the mandatory retirement age j_r. As the population grows with a constant rate n, a long-run equilibrium in this economy is characterized by all aggregate variables growing at this very same rate. To make aggregates stationary again, I express all variables in per capita terms of the youngest generation at a certain date t. I denote by m_j the time-invariant relative size of a cohort aged j at any point in time.

6.2.2 Technology

A continuum of identical firms produces a single good Y under perfect competition. They hire both capital K at price r and labor L at price w on competitive spot markets. Firms operate a constant returns to scale technology

$$Y = \Omega K^{\alpha} L^{1-\alpha}. \tag{6.11}$$

Ω denotes the aggregate level of productivity, whereas α is the elasticity of output with respect to capital. In the process of production, a fraction δ of the capital stock depreciates. Given the assumptions about competition and technology, I can safely assume the existence of a representative firm that takes prices as given and operates the aggregate technology in (6.11). In addition to employing factor inputs, the firm has to invest I_t into its capital stock. The law of motion for the capital stock reads

$$(1+n)K_{t+1} = (1-\delta)K_t + I_t.$$

6.2.3 Preferences and endowments

Preferences Individuals have preferences over stochastic streams of consumption $c_j \geq 0$ and labor supply $\ell_j \geq 0$. Single households maximize the discounted expected utility

$$U_0^s = E_0 \left[\sum_{j=20}^{J} \psi_{j+1,g}^s \beta^{j-20} u(c_j, \ell_j) \right],$$

and couple households maximize the discounted expected utility

$$U_0^c = E_0 \left[\sum_{j=20}^{J} \psi_{j+1}^c \beta^{j-20} \Big(u(c_{j,m}, \ell_{j,m}) + u(c_{j,f}, \ell_{j,f}) \Big) \right],$$

where c_m and c_f denote consumption of the husband and the wife, respectively. Expectations are formed with respect to future labor productivity, the future labor choice set, and fertility. They discount the future with the constant time discount factor β as well as their individual survival rate.

Labor productivity Households are ex-ante homogeneous, but differ ex-post in their labor productivity. At the beginning of life, individuals draw one of two education levels. They are either high school or college-educated $s \in \{0, 1\}$ for the rest of their life. All individuals of education s share a common deterministic age-specific labor productivity profile $\theta_{j,s}$. Throughout working life, they are subject to idiosyncratic productivity shocks η, which follow a standard AR(1) process in logs

$$\eta^+ = \rho_s \eta + \varepsilon^+ \quad \text{with} \quad \varepsilon^+ \sim N(0, \sigma_{\varepsilon^2,s}), \tag{6.12}$$

where innovations ε^+ are iid across and within households. $\pi_\eta(\eta^+|\eta, s)$ denotes the probability distribution of next-period's productivity η^+, conditional on current labor productivity η, and education s. The general productivity level is denoted as $z(j, s, \eta)$ and incorporates the deterministic age-specific productivity profile $\theta_{j,s}$ as well as transitory shocks η. Part-time and female workers are less productive and the arrival of kids limits a woman's productivity further. Finally, individual wages $w(j, s, \eta, g, k, \ell)$ equal the product of the wage rate per efficiency unit of labor w, the general productivity level $z(j, s, \eta)$, the part-time gap $w_p(\ell)$ and the gender gap $w_g(j, g, k)$.

Families and fertility Women are subject to a fertility shock k. They give birth to two kids, that arrive according to the probability distribution $\pi_k(k^+|k, j, i, 1)$. The probability to give birth depends on age j as well as the marital status i. Kids live on average for 18 years in the household. Parents suffer age-dependent disutility from raising children. Children are hungry and families[13] benefit from economies of scale in consumption. Consumption expenditure $\hat{c}$ is given by:

$$\hat{c} = c \times \upsilon(j, k, i), \tag{6.13}$$

where $\upsilon(j, k, i)$ is a scale factor that depends on the age and the composition of the household. I apply the new OECD equivalence scale.[14]

Notation: I denote by c the aggregate household consumption $c_m + c_f$, thereby avoiding the individual subscripts wherever possible. I use the same notation for any other household-level variable, like earnings y, consumption expenditures $\hat{c}$, mini-job earnings y_{mini}, pensions p and, bequests b.

[13] I refer to a household with more than one member to families. Families can take the form of single mothers, couples, and couples with children.

[14] Each member of the household is given an equivalence value: 1.0 to the first adult, 0.5 to the second, and 0.3 to each child. I don't distinguish between young and old children.

Budget constraint Markets are incomplete. Like in Aiyagari (1994) households can only self-insure against fluctuations in individual labor productivity by saving in a risk-free asset a with return r. They cannot borrow, such that assets must satisfy $a \geq 0$. Households' resources are composed of their current wealth a (including returns), their income from working in a regular job y or in a mini-job y_{mini}, government transfers t_{cb} and t_{cs}, intergenerational transfers b,[15] and pension payments p. They use these resources to finance consumption expenditure $(1+\tau_c)\times\hat{c}$, savings into the next period a^+, contributions to social security $T_p(y^p)$ on pension relevant earnings y^p, as well as progressive income taxes $T(y,p)$. Households can deduct social security contributions from gross income for the purpose of taxation. In turn, all pension benefits are liable for taxation.

Dynamic optimization problem - singles: The current state of a single household is described by a vector $\mathbf{x_s} = (j, g, s, \eta, k, h, \xi, a, e)$ that summarizes the household's age j, gender g, education s, her current labor productivity shock η, the presence and age of kids k, her labor choice set h, her employment costs ξ, her wealth position a as well as pension entitlements e. The dynamic optimization problem of a single reads

$$v(\mathbf{x_s}) = \max_{c,\ell\leq h,a^+,e^+} u(c,\ell) + \beta\psi^s_{j+1,g} E\Big[v(\mathbf{x_s^+}) \Bigm| \mathbf{x_s}\Big] \tag{6.14}$$

with $\mathbf{x_s^+} = (j+1, g, s, \eta^+, k^+, h^+, \xi^+, a^+, e^+)$. Households maximize (6.14) subject to the borrowing constraint $a^+ \geq 0$, the budget constraint

$$(1+\tau_c)\hat{c}+a^+ +T_p(y^p)+T\Big(y-T_p(\min(y,2\bar{y}))+p\Big) = (1+r)a+y+y_{mini}+t_{cb}+t_{cs}+b,$$
$$\text{with } y = w(j,s,\eta,g,k,\ell)\times\ell,$$

the accumulation equation for pension entitlements e^+ as well as the laws of motion for labor productivity η, fertility k and the labor choice set h. The result of this dynamic program are policy functions c, ℓ, and a^+ that all depend on the household's current state $\mathbf{x_s}$.

Dynamic optimization problem - couples: The current state of a couple household is described by a vector $\mathbf{x_c} = (j, s_m, s_f, \eta_m, \eta_f, k, h, \xi, a, e_m, e_f)$. It summarizes the joint household states age j, the presence and age of kids k, the labor choice set h, the employment costs ξ, and wealth a. The individual-specific states are education s_m, s_f, labor productivity shocks η_m, η_f, and pension entitlements e_m, e_f

[15] Intergenerational transfers consist only of accidental bequests that households might leave if they die before the terminal age J. I assume that the total of those accidental bequests is distributed lump-sum to all working-age households.

for the husband and the wife, respectively. The dynamic optimization problem of a couple reads

$$v(\mathbf{x_c}) = \max_{\substack{c_m, c_f, \ell_m, \ell_f \le h, \\ a^+, e_m^+, e_f^+}} \Big[u(c_m, \ell_m) + u(c_f, \ell_f)\Big] + \beta \psi_{j+1}^c E\Big[v(\mathbf{x_c^+}) \;\Big|\; \mathbf{x_c}\Big] \tag{6.15}$$

with $\mathbf{x_c^+} = (j+1, s_m, s_f, \eta_m^+, \eta_f^+, k^+, h^+, \xi^+, a^+, e_m^+, e_f^+)$. Couples maximize (6.15) subject to the borrowing constraint $a^+ \geq 0$ and the budget constraint

$$\begin{aligned}(1+\tau_c)\hat{c} + a^+ + T_p(y^p) + 2T\Big[0.5\Big(y - T_p(\min(y, 2\bar{y})) + p\Big)\Big]\\ = (1+r)a + y + y_{mini} + p + t_{cb} + b,\end{aligned}$$

the accumulation equation for pension entitlements e^+ as well as the laws of motion for labor productivity η, fertility k and the labor choice set h. The result of this dynamic program are policy functions $c_m, c_f, \ell_m, \ell_f, e_m^+, e_f^+$ and a^+ that all depend on the household's current state $\mathbf{x_c}$.

Labor supply The time endowment per period is 1 and can be used for labor and leisure. A woman can choose to work full-time, part-time in a mini-job, or not at all. Men can only work full-time or not at all. Couples decide about each partner's optimal labor supply jointly to maximize household utility. I provide an analytical solution to the household problem in Appendix D.1.

6.2.4 The pension system

The pension system collects payroll taxes at rate τ_p on pension-relevant earnings y^p. These are any earnings y below the contribution ceiling equal to two times average labor earnings $\bar{y}$ of the full-time and part-time worker, as well as a fraction $\frac{\tau_p^{mini}}{\tau_p}$ of mini-job earnings y_{mini}. y^p are defined as

$$y^p = \min\left[y + \frac{\tau_p^{mini}}{\tau_p} y_{mini},\ 2\bar{y}\right].$$

Pension contributions $T_p(y^p)$ are given by

$$T_p(y^p) = \tau_p \times y^p.$$

In reward for contributing to the system, individuals increase pension entitlements e according to

$$e^+ = e + f(y^p). \tag{6.16}$$

Mothers get an annual pension entitlement credit of e_k on their pension account as long as young kids ($k = 1$) are in the house.[16] This compensates mothers for their reduced pension contributions while raising their kids. Finally, individual pension benefits p are calculated from accumulated pension entitlements at retirement entry e_{j_r} as

$$p(e_{j_r}) = \bar{y} \times \kappa \times \frac{e_{j_r}}{j_r - 19},$$

where κ is the replacement rate.

The pension system runs on a pay-as-you-go basis. In equilibrium, total pension contributions need to be equal to the total amount of pension payments. Letting Φ^s and Φ^c denote the cross-sectional measure of households over the state space,[17] I require

$$\underbrace{\int p(e_{j_r}) \times \mathbb{1}_{j \geq j_r} \; d\Phi^s + \int (p(e^m_{j_r}) + p(e^f_{j_r})) \times \mathbb{1}_{j \geq j_r} \; d\Phi^c}_{\text{total pension claims}} = \underbrace{\tau_p \times \Big(\int y^p \; d\Phi^s + \int y^p \; d\Phi^c \Big)}_{\text{total contributions}}. \tag{6.17}$$

6.2.5 The tax system and government expenditure

The government collects proportional taxes on consumption expenditure and progressive taxes on labor earnings y net of social security contributions as well as pension payments p. Earnings from mini-jobs y_{mini} are tax-free. Tax revenue is used to finance (wasteful) government spending G, child benefits T_{cb} to families with kids, and additional child support transfers T_{cs} to single mothers.

As I abstract from any government debt, the tax system is balanced whenever

$$\tau_c \times \hat{C} + \int T\Big(y - T_p(\min(y, 2\bar{y})) + p\Big) \; d\Phi^s + \int 2T\Big(\frac{y - T_p(\min(y, 2\bar{y})) + p}{2}\Big) \; d\Phi^c = G + T_{cb} + T_{cs} \tag{6.18}$$

holds. $\hat{C}$ denotes aggregate consumption expenditures and T the progressive income tax schedule for single and couple households. Φ_s and Φ_c are the cross-sectional measures of single and couples households over the state space. Government consumption is fixed per capita. I adjust the consumption tax rate τ_c to keep the fiscal

[16] Equation 6.16 changes to $e^+ = e + \min\left[y^p + e_k, \; 2\bar{y}\right]$ for women with $k = 1$.

[17] Φ^s and Φ^c are cress-sectional measures indicate the mass of single and couple households on each subset of the state space. I require that for each age j, $\Phi^s + \Phi^c$ sums up to the total mass of households in a cohort m_j.

system in balance.

6.2.6 Capital markets, trade and equilibrium

I model a small open economy that freely trades capital and goods on competitive international markets. All private savings that are not employed by the domestic production sector are invested abroad at the international interest rate $\bar{r}$. The capital market equilibrium reads

$$K + Q = A,$$

where A is aggregate private savings and Q is the country's net foreign asset position. As the economy grows at rate n, the net foreign asset position increases over time such that the capital account is $-nQ_{t+1}$. Net income from abroad amounts to $\bar{r}Q_t$. According to the balance of payments identity, I, therefore, have a trade balance of

$$TB = (n - \bar{r})Q. \tag{6.19}$$

I assume that the government collects all accidental bequests b and redistributes them in a lump-sum way among the surviving working-age population. Consequently,

$$b_{j,t} = \frac{\int \frac{1-\psi^s_{j,g}}{\psi^s_{j,g}} \times (1+r)a \; d\Phi^s_t + \int \frac{1-\psi^c_j}{\psi^c_j} \times (1+r)a \; d\Phi^c_t}{\int \mathbb{1}_{j<j_r} \; d\Phi^s_t + \int \mathbb{1}_{j<j_r} \; d\Phi^c_t} \quad \text{if } j < j_r. \tag{6.20}$$

Given an international interest rate and the exogenous fiscal policy parameters, a recursive competitive equilibrium of this model is a set of household policy functions, a measure of households, optimal production inputs, factor prices, accidental bequests, a net foreign asset position, and a trade balance that are consistent with individual optimization and market clearance.

6.3 Stationary recursive competitive equilibrium

Given an international interest rate $\bar{r}$, government expenditures G, a consumption tax rate τ_c, a progressive tax system $T(\cdot)$ as well as a characterization of the pension system $\{\tau_p, \tau_p^{mini}, \kappa\}$, a stationary recursive equilibrium with population growth n is a collection of value and policy functions $\{v, c, \ell, a^+, ep^+\}$ for a single and $\{v, c_m, c_f, \ell_m, \ell_f, a^+, e^+_m, e^+_f\}$ for a couple household, optimal production inputs

$\{K, L\}$, accidental bequests $\{b_j\}_{j=20}^{J}$, a net foreign asset position and a trade balance $\{Q, TB\}$ as well as factor prices $\{r, w\}$ that satisfy

1. **Household Optimization** Given prices and characteristics of the tax and pension system, the value function v satisfies the Bellman equation (6.14) for singles and (6.15) for couples together with the budget constraint, the accumulation equation for pension claims, the borrowing constraint and the laws of motion for productivity risk, the career choice set and fertility. c, ℓ, a^+ and e^+ are the associated policy functions for single households and $c_m, c_f, \ell_m, \ell_f, a^+$, e_m^+, e_f^+ for couple households.

2. (Firm Optimization) Given the international interest rate $\bar{r}$ as well as the wage rate w, firms employ capital and labor according to the demand functions

$$\bar{r} = \Omega\alpha \left(\frac{L}{K}\right)^{1-\alpha} - \delta \text{ and } w = \Omega(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}.$$

3. **Government Constraints** The budget constraints of the pension system (6.17) and the tax system (6.18) hold, and accidental bequests are calculated from (6.20).

4. **Market Clearing**

   (a) The labor market clears:

$$L = \int w(j, s, \eta, g, k, \ell)\ell(\mathbf{x}_s)\ d\Phi^s + \\ \int w(j, s_m, \eta_m, 0, k, \ell_m)\ell_m(\mathbf{x}_c) + w(j, s_f, \eta_f, 1, k, \ell_f)\ell_f(\mathbf{x}_c)\ d\Phi^c$$

   (b) The capital market clears:

$$K + Q = \int a\ d\Phi^s + \int a\ d\Phi^c$$

   (c) The balance of payments identity is satisfied:

$$TB = (n - \bar{r})Q$$

   (d) The goods market clears:

$$Y = \int c(\mathbf{x}_s) \times \upsilon_{j,k,s}\ d\Phi^s + \int \Big(c_m(\mathbf{x}_c) + c_f(\mathbf{x}_c)\Big) \times \upsilon_{j,k,c}\ d\Phi^c \\ + (n+\delta)K + G + TB.$$

5. **Consistency of Probability Measure** Φ The invariant probability measure is consistent with the population structure of the economy, with the exogenous processes of labor productivity η, the labor choice set h and fertility k, as well as the household policy functions a^+, e^+ and a^+, e_m^+, e_f^+, for singles and couples, respectively. A formal definition is provided in Appendix D.2.

6.4 Calibration

This section discusses the choices of functional forms and parameters in detail. I pay particular attention to matching empirical evidence on labor supply for different household types. I calibrate the model to the German economy, which currently features a proportional pension system. Germany, therefore, serves as a good benchmark for reforms that aim at introducing progressivity into the pension formula.[18] The average regular earnings amounted 37,000 euros in the base year 2017 in Germany.[19] This value serves as the empirical counterpart to the model implied $\bar{y}$ in the calibration process. Note, the Grundrente was introduced in 2021 and is hence neglected in my model.

6.4.1 Demographics

I assume a population growth rate of $n = 0.0$, which is a compromise between the average growth rate of 0.4% reported in the period 2012 to 2017 for the German population at large, and the fact that most of the German population growth came from refugee migration, see Statistisches Bundesamt (Destatis).[20] I let households start their economic life at the age of 20 and allow for a maximum life span of 99 years. Mandatory retirement age j_r is at 64, which equals the current average retirement age of the German regular retirement population, see Deutsche Rentenversicherung Bund (2019).

Life expectancy I extract the 2017 annual life tables for men and women from the Human Mortality Database (2020) to calculate average survival probabilities $\psi_{j,g}$. The joint survival probabilities ψ_j^c of couples are the average of the male and the female survival probabilities, i.e. $\psi_j^c = (\psi_{j,m}^s + \psi_{j,f}^s) \times 0.5$. Single men die on average at age 79.5, single women at age 84.1, and couples at age 81.7. The left panel of Figure 6.3 shows the respective survival probability profiles.

[18] US Social Security would not make a good benchmark case. It redistributes strongly to single-earner married households due to spousal and survivor benefits. The German system has survivor benefits as well, but no spousal transfers.

[19] This is the average of contributory earnings in Germany, see Deutsche Rentenversicherung Bund (2020)

[20] In fact, the growth rate of the native population was -0.2% in the same time period.

Gender, education and marriage I use 2017 data from the German microcensus, the RDC of the FSO 2017, (age cohorts 35-49) to estimate the following demographic parameters. The proportion of men is 50.78%. 33.02% of the men have a college education ($s = 1$), and 67.70% of the men live in a couple household.[21] 27.70% of women have a college education. I also examine patterns of assortative mating and find that 85.69% of high school-educated men are married to a high school-educated woman. 54.81% of college-educated men are married to a college-educated woman. Informed by these empirical facts, I set the demographic parameters of the model as follows:

A household is with probability $\phi_c = 0.6770$ a couple household. The education level of the two partners in a couple household (s_m, s_f) are assigned according to the joint probability distribution

$$\phi_s^c(s_m, s_f) = \begin{bmatrix} \phi_s^c(0,0) = 0.5740 & \phi_s^c(0,1) = 0.0958 \\ \phi_s^c(1,0) = 0.1492 & \phi_s^c(1,1) = 0.1810 \end{bmatrix}, \tag{6.21}$$

where $\phi_s^c(0,0)$ indicates that both partners are high school-educated and $\phi_s^c(1,1)$ that both are college-educated. Single households are with probability $\phi_m = 0.5078$ male and with probability $\phi_f = 0.4922$ female. A male single household is with probability $\phi_s^m = 0.3306$ and a female single household is with probability $\phi_s^f = 0.2776$ college-educated. This calibration strategy provides a model consistent measure of households, as outlined in Appendix D.3, and incorporates the empirical targets regarding household formation, gender, and education distribution at the same time.

Children Motherhood is denoted by the state k, which can take values $k \in (0, 1, 2, 3)$. I model the transition of k as a first-order Markov process with the probability distribution $\pi_k(k^+|k, j, i, g)$. The transition matrix for women reads

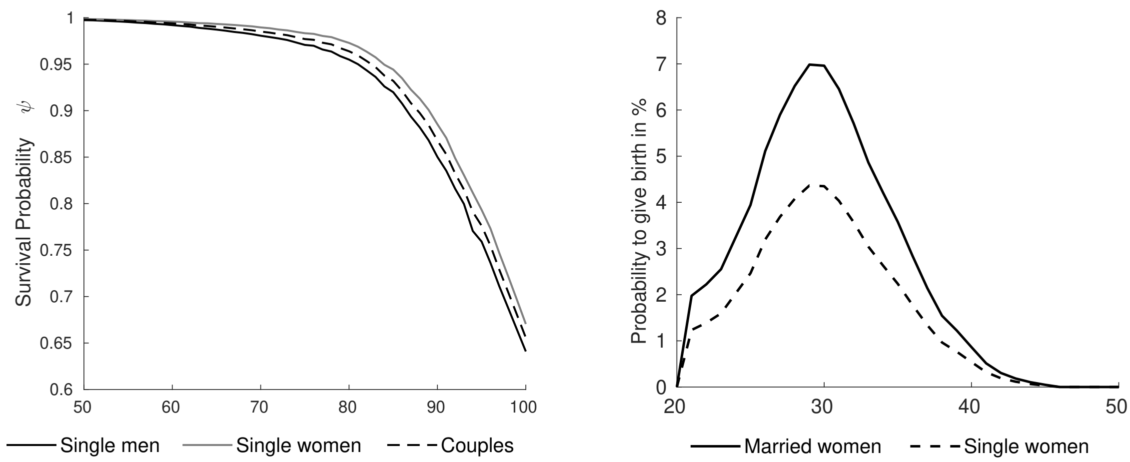
$$\pi_k(k^+|k, j_{<j_r}, i, 1) = \begin{bmatrix} \pi_{0,0}^{j,i} = 1 - \pi_{0,1}^{j,i} & \pi_{0,1}^{j,i} = \phi_k^{j,i} & \pi_{0,2} = 0 & \pi_{0,3} = 0 \\ \pi_{1,0} = 0 & \pi_{1,1} = 1 - \pi_{1,2} & \pi_{1,2} = \frac{1}{6} & \pi_{1,3} = 0 \\ \pi_{2,0} = 0 & \pi_{2,1} = 0 & \pi_{2,2} = 1 - \pi_{2,3} & \pi_{2,3} = \frac{1}{12} \\ \pi_{3,0} = 0 & \pi_{3,1} = 0 & \pi_{3,2} = 0 & \pi_{3,3} = 1 \end{bmatrix}. \tag{6.22}$$

Women can give birth until age 45 ($\phi_k^{j>45,i} = 0$) and kids leave the house at age j_r with certainty ($\pi_{1,3}^{j_r,i} = 1$, $\pi_{2,3}^{j_r,i} = 1$). Men are never hit by a fertility shock and are hence always in state $k = 0$.

[21] This includes individuals who live in a couple household but are not formally married

Women enter the economy childless which is denoted by $k = 0$. $\pi_{0,1}^{j,i} = \phi_k^{j,i}$ is the probability for a woman of age j and marital status i to give birth and hence transition from $k = 0$ to $k = 1$. The state $k = 1$ indicates that two young children are in the household, which imposes high time costs on the woman and considerable monetary costs on the household. Women transition with probability $\pi_{1,2} = \frac{1}{6}$ to state $k = 2$, which indicates that the children have grown older and are less time demanding. Women transition with probability $\pi_{2,3} = \frac{1}{12}$ to state $k = 3$, where the kids have left the family and do not impose any costs anymore. Women remain on average for 6 years in the state $k = 1$ and for 12 years in the state $k = 2$. Consequently, children leave the house on average at the age 18. Over time, 80.02%

Figure 6.3: Motherhood and survival



of the married women and 47.53% of the single women give birth, which is similar to the data as reported by the Statistisches Bundesamt (Destatis). This results in a model birthrate of 1.40, which is slightly below the true 2017 value of 1.57. I use data from Eurostat (2023) on the mother's age at the birth of her first child to calibrate the probability of giving birth over the life cycle. The right panel of Figure 6.3 shows the probabilities $\phi_k^{j,i}$ graphically.

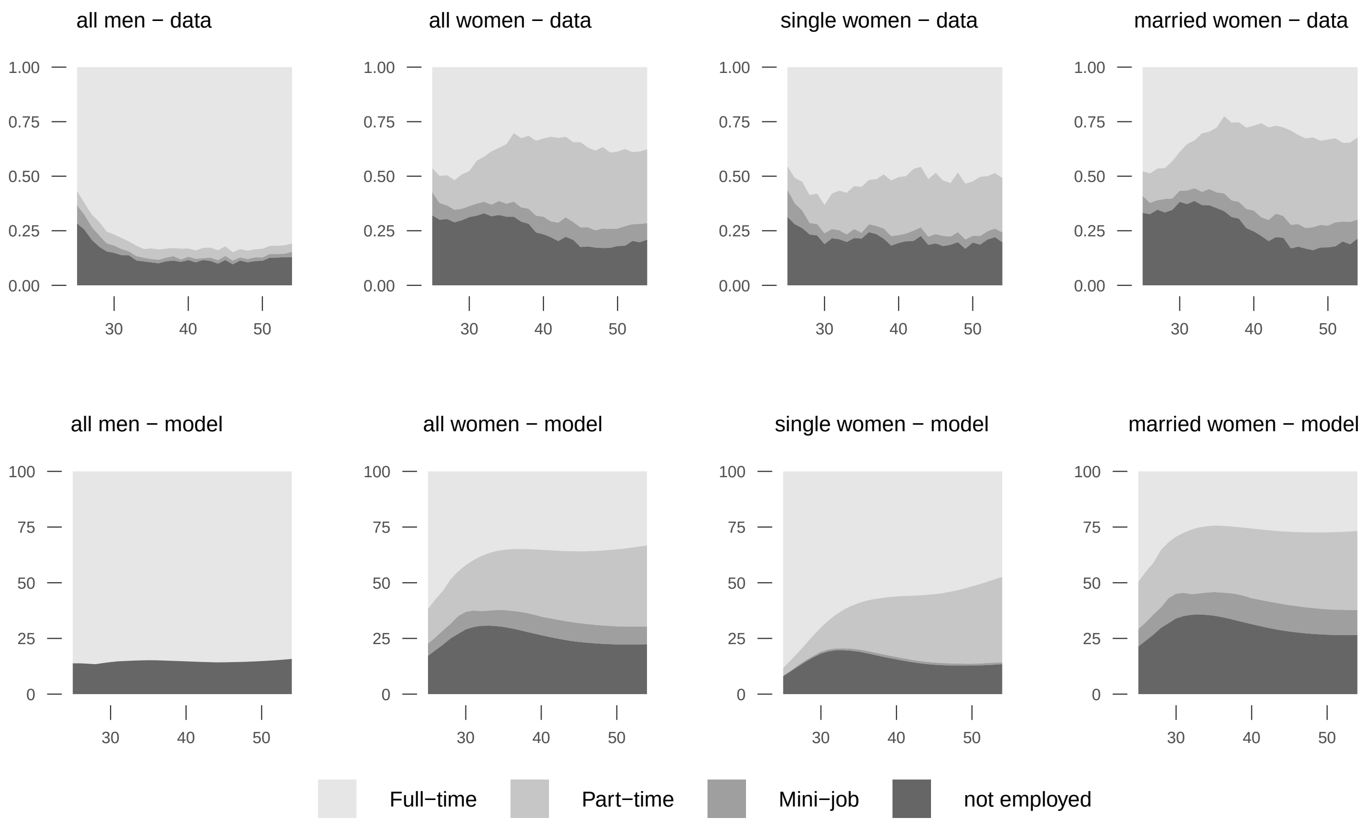
6.4.2 Technology

I choose a depreciation rate of $\delta = 0.07$, leading to a realistic investment to output ratio of 21%. The capital share in production is $\alpha = 0.3$ and I normalize the technology level Ω such that the wage rate per efficiency unit of male labor w is equal to 1. I assume an international interest rate of $\bar{r} = 0.03$.

6.4.3 Preferences and endowments

Providing a model that replicates empirical labor supply patterns over the life cycle is a crucial starting point to assess pension reforms. My benchmark is the labor supply data from the German microcensus, the RDC of the FSO (2017), for prime-age worker as displayed in the first row of Figure 6.4.[22] The figure reveals some features of the data: First, women show much more variation in labor supply over the life cycle than men. While a sizable fraction of women work in a mini-job or part-time, the mass of men works either full-time or not at all. Second, women reduce labor hours in their 30s to raise kids. Most mothers continue to work part-time even as their children grow older. Finally, couple women are much more likely to work part-time than single women. Only about 25% of married women work full-time from their 30s onward. I calibrate the model to match these findings. Table 6.2 provides an overview of all exogenous model parameters, and Table 6.3 on all endogenous set parameters.

Figure 6.4: Empirical vs. simulated labor supply



Source empirical profiles: own estimation, based on data from the RDC of the FSO (2017).

[22]The entire analysis on labor supply is based on results of prime-age worker, i.e. ages 25-54

Preferences I let the period utility function be

$$u(c_j, \ell_j),$$

and

$$u(c_{j,m}, \ell_{j,m}) + u(c_{j,f}, \ell_{j,f}),$$

for singles and couples, respectively. The functional form is given by

$$u(c_j, \ell_j) = \frac{c_j^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu_g \frac{(\lambda^i_{\zeta_{g,k}} \zeta_k + \ell_j)^{1+\frac{1}{\chi_g}}}{1+\frac{1}{\chi_g}} - \xi \times \mathbb{1}_{\ell_j > 0}. \tag{6.23}$$

I choose an intertemporal elasticity of substitution σ of 0.67.[23] The choice of σ has important implications for the size of the income effect of wage changes on labor supply. Heathcote et al. (2014) estimate a similar value for this parameter in a life cycle model using cross-sectional data on earnings and consumption from PSID and CEX. I set the time discount factor to $\beta = 0.98$, a standard value in the literature. My preferred value for the Frisch elasticity is $\chi_m = 0.4$ for men and $\chi_w = 0.875$ for women. This is in line with the empirical literature that estimates a much higher labor supply elasticity for women than for men, see for example Keane (2011). ξ describes the utility costs of employment. I assume that ξ is iid across households, drawn at the household level and independent of the individual labor productivity.

Labor supply ℓ is modeled as a discrete choice. Women can work full-time, part-time, in a mini-job or not at all. According to data from the RDC of the FSO (2017), full-time employees work an average of 40.4 hours per week, whereas part-time employees work 20.8 hours per week. I calibrate $\ell = 0.4$ and $\ell = 0.2$ for full-time and part-time work, respectively.[24] Labor hours of mini-job worker are set to $\ell = 0.1$, which corresponds to a 10-hours work week. Men decide only on their extensive labor supply, working full-time or not at all. This modeling choice is based on empirical evidence, which shows that only very few employed men work part-time.

Women bear the lion's share of the time costs ζ_k to raise children. I calibrate ζ_k to meet empirical targets with respect to female labor supply over the life cycle. At state $k = 1$, kids are young and impose time costs of $\zeta_1 = 0.6$ on the family, while

[23] In a model with inelastic labor supply, the implied risk aversion would then be equal to 1.5.

[24] An individual can work up to 100 hours per week, which would correspond to $\ell = 1$. 68 hours are required for sleeping, eating, and personal hygiene.

older kids, $k = 2$, impose $\zeta_2 = 0.15$. As kids grow up ($k = 3$), the time costs shrink to zero, i.e. $\zeta_3 = \zeta_0 = 0$. Single mothers bear these costs alone ($\lambda^s_{\zeta_{f,k}} = 1$), while couples can share them. A couple mum takes the share $\lambda^c_{\zeta_{f,1}} = 0.9$ for young kids and $\lambda^c_{\zeta_{f,2}} = 0.75$ afterwards. A couple dad covers $\lambda^c_{\zeta_{m,k}} = 1 - \lambda^c_{\zeta_{f,k}}$.

Labor productivity I estimate earnings profiles which are based on the same data as in the previous chapter (FDZ-RV – SUFVSKT2002-2017), but I proceed differently in the sample selection process.[25] First, since I don't include low-productivity shocks as outlined in Section 5.3.3 in this model, I keep low-earning worker in the sample. Second, I drop the mini-job worker from the sample. The reason is that mini-job workers in the model a paid a flat salary and hence I am only interested in the productivity profiles of regular workers.[26] Appendix D.4.1 provides more details on the sample selection process. I estimate earnings profile for both high school and college-educated worker, following the same estimation procedure as outlined in Section 4.1.3.[27]

In particular, I describe the earnings dynamics by a standard AR(1) process in logs. I split the sample according to an individual's education level and derive a deterministic life cycle labor earnings profile as well as an AR(1) process for residual log-labor earnings for each education group. More specifically, I estimate the statistical model

$$\log(y_{isjt}) = \kappa_{t,s} + \theta_{j,s} + \eta_{isjt} \quad \text{with} \quad \eta_{isjt} = \rho_s \eta_{isj-1,t-1} + \varepsilon_{isjt}, \tag{6.24}$$

for labor earnings y_{isjt} of an individual i with education s at age j in year t. $\kappa_{t,s}$ is a year-fixed effect that controls for earnings changes along the business cycle. $\theta_{j,s}$ is an age-fixed effect that informs about the age-earnings relationship. The noise term ε_{isjt} is assumed to follow a normal distribution with mean 0 and variance $\sigma^2_{\varepsilon,s}$. Furthermore, I let the stochastic process start from its long-run variance σ^2_s. This means that

$$\varepsilon_{isjt} \sim N(0, \sigma^2_{\varepsilon,s}) \quad \text{and} \quad \eta_{is20t} \sim N\left(0, \sigma^2_s\right) \text{ with } \sigma^2_s = \frac{\sigma^2_{\varepsilon,s}}{1-\rho^2_s}.$$

I use a generalized method of moments estimator to determine the parameters of this model. I thereby control for the fact that the data are top-coded at the threshold

[25] In particular, I use the 2017 wave of the Scientific Use File of the Versichertenkontenstichprobe (FDZ-RV – SUFVSKT2002-2017), which contains information from the insurance accounts of 69,520 insured individuals. This is about 0.18% of the active insured population.

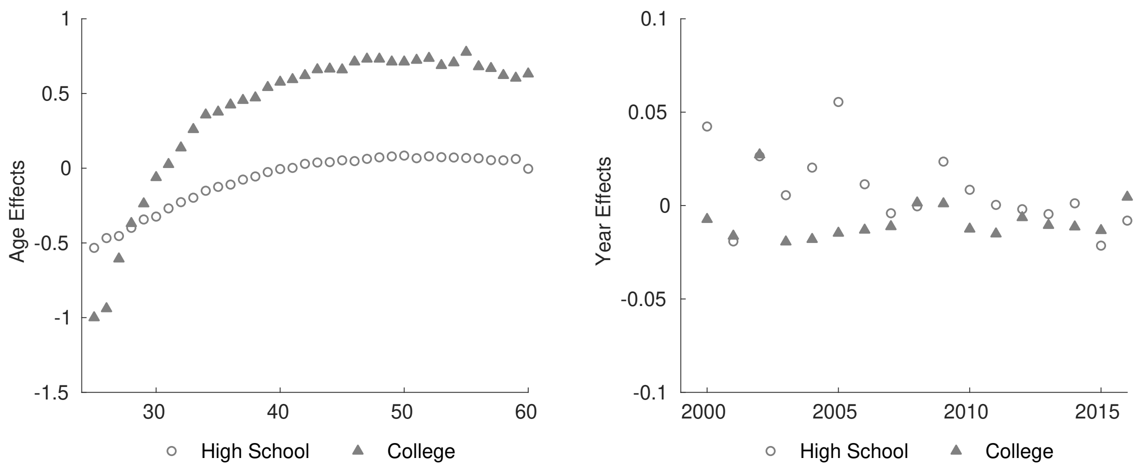
[26] Only the least productive mini-job worker a paid by their productivity, as outlined later in this section.

[27] Since the data is not subject to truncation at the left end, I set the threshold y_{min} to a very small positive value.

$y_{max,t}$.

The left panel of Figure 6.5 shows the resulting labor earnings profiles. The earnings profiles exhibit a significant college wage premium as well as a high auto-correlation of earnings. Up to age 45, earnings steeply increase for both education groups, especially so for the college-educated. Afterward, they stagnate or decline slightly for the rest of an individual's working life. This shape of life cycle earnings is quite common in the empirical literature and has been found for other countries as well, see for example Heckman et al. (1998) or Casanova (2013). The college-wage premium implied by these profiles is equal to 60 percent, which is in line with empirical findings (OECD, 2016). The right panel of the figure shows the year fixed effects. These are generally small relative to the age effects and exhibit some cyclical dynamics. The autocorrelation parameter ρ_s of the residual earnings process is 0.93 and 0.99, the innovation Variance $\sigma^2_{\varepsilon,s}$ is 0.0372 and 0.0059 for high school and college-educated worker, respectively. Finally, the unconditional variance of the process $\frac{\hat{\sigma}^2_{\varepsilon,s}}{1-\hat{\rho}^2_s}$ is 0.2756 and 0.2983. The estimated process exhibits considerably more variation than the one estimated in Chapter 4. This is because I keep most of the low-earnings individuals in the sample, who might be only loosely attached to the labor market.

Figure 6.5: Age fixed-effects and year fixed-effects



Source: own estimation, based on data from FDZ-RV – SUFVSKT2002-2017.

I use the estimated earnings profile to calibrate labor productivity in the quantitative model. I parameterize the deterministic age-productivity relationship $\theta_{j,s}$ using the same functional form as in Chapter 4. In particular, I let

$$\theta_{j,s} = b_{0,s} + b_{1,s}\frac{\min(j, j_{M,s})}{10} + b_{2,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^2 + b_{3,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^3, \quad (6.25)$$

which is flexible enough to capture both a hump-shaped ($j_{M,s} = \infty$) and a stagnating ($j_{M,s} < j_r$) life cycle labor productivity profile. Note that in the case of a stagnating profile, labor productivity is constant from age $j_{M,s}$ onward.

I let the labor productivity risk of workers be guided by a standard first-order autoregressive process as outlined in Equation (6.12). The parameters ρ_s and $\sigma^2_{\varepsilon,s}$ are directly estimated from the earnings data. I discretize the AR(1) process by a three-state Markov chain using a Rouwenhorst method. In doing so, I obtain a set of three productivity realizations $\{\eta_{1,s}, \eta_{2,s}, \eta_{3,s}\}$, the initial distribution over theses realizations at age 20

$$\pi_{\eta,20}(\eta\,|s) = \left[\phi^s_\eta(1), \quad \phi^s_\eta(2), \quad \phi^s_\eta(3),\right]. \tag{6.26}$$

as well as a transition matrix $\pi_\eta(\eta^+|\eta, s)$ that governs the transition between the states.

This leaves 10 parameters $b_{i,s}$ and $j_{M,s}$ of the polynomials in (6.25) for high school and college-educated workers that need to be calibrated. I calibrate these parameters such that the model-implied earnings profiles match their empirical counterparts as shown in the left panel of Figure 6.6. Combining the deterministic productivity profile $\theta_{j,s}$, and the transitory state η, provides the general productivity $z(j, s, \eta)$ for each age, education and productivity level. Table 6.1 summarizes the productivity parameters and the productivity risk process.

Table 6.1: Parameter values of labor productivity profiles and risk

	High School $s = 0$	College $s = 1$
Intercept $b_{0,s}$	-2.0732	-17.2099
Linear age term $b_{1,s}$	0.7833	11.8163
Quadratic age term $b_{2,s}$	-0.0572	-2.6345
Cubic age term $b_{3,s}$	-0.0026	0.1984
Stagnation threshold $j_{M,s}$	∞	44.26
Autocorrelation ρ_s	0.9300	0.9900
Innovation Variance $\sigma^2_{\varepsilon,s}$	0.0372	0.0059

Source: own estimation, based on data from FDZ-RV – SUFVSKT2002-2017.

Wages I use the same productivity profiles $z(j, s, \eta)$ for men and for women.[28] However, I account for the fact that women are often less productive to firms and impose a gender gap w_g of (1 - 0.0865) on all female employees. This gap is steadily increasing for mothers. I have calibrated w_g according to estimates by Schrenker and

[28] Women exhibit much more variation in weekly hours than men. Without information on hours worked I can not extract information on productivity from earnings data.

Zucco (2020) on the empirical gender wage gap in Germany, see Figure 6.3 (right). I assume that part-time workers provide 97.5 percent of the productivity of full-time workers. This is mirrored in the wage gap for part-time work $w_p = (1 - 0.025)$. I have calibrated that value in order to meet empirical targets of the fraction of part-time working women. Individual wages $w(j, s, \eta, g, k, \ell)$ equal the product of the wage rate per efficiency unit of labor w, the general productivity level $z(j, s, \eta)$, the part-time gap $w_p(\ell)$ and the gender gap $w_g(j, g, k)$. Hence,

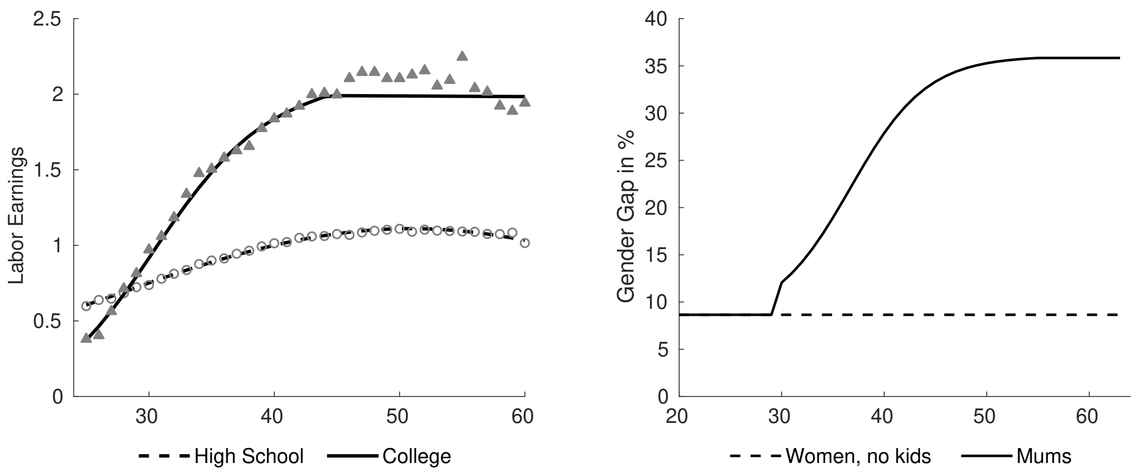
$$w(j, s, \eta, g, k, \ell) = w \times z(j, s, \eta) \times w_p(\ell) \times w_g(j, g, k).$$

The left panel of Figure 6.6 shows that the model-implied earnings profile of regular workers matches its empirical counterpart very well.

Mini-jobs Mini-job workers in Germany can earn at most 5,400 euros per year, which is equal to $0.1459 \times \bar{y}$.[29] I assume that mini-job workers in the model are paid a flat salary of that value. In case their individual labor productivity values less than that, they are paid according to their productivity if working $\ell = 0.1$. Hence, I set mini-job earnings y_{mini} to

$$y_{mini} = \min\left[0.1459 \times \bar{y}, 0.1 \times w(j, s, \eta, g, k, 0.1)\right].$$

Figure 6.6: Empirical and model-implied average life-cycle earnings profiles



Source empirical profiles: own estimation, based on data from FDZ-RV – SUFVSKT2002-2017 and Schrenker and Zucco (2020).

[29]This corresponds to a wage of 10.50 euros per hour, which is above the 2017 German minimum wage of 8.84 euros.

Labor supply The level parameter ν guides the intensive margin labor supply choices of the households. I set $\nu_m = 75$ and $\nu_f = 15.25$ to target empirical labor supply rates in mini-jobs, part-time and full-time work as given in Table 6.4. Participation rates are mainly governed by the probability density of the utility costs of employment ξ.[30] My calibration strategy for the distribution of participation costs ξ is the following: I assume that ξ is iid across households, drawn at the household level and independent of the individual labor productivity. I let ξ follow a log-normal distribution with mean μ_ξ and variance σ_ξ^2. The mean $\mu_\xi = 0.52$ and the variance $\sigma_\xi^2 = 2.75$ are chosen to generate a male participation rate of 86% and a female participation rate of 75%.

In order to replicate the empirical finding of long-lasting part-time contracts among women, I constrain the labor choice set ℓ. Whether a woman can work full-time or not depends on the status of the labor choice set $h \in \{0.2, 0.4\}$. h is initially 0.4 which provides the full set of labor choices. The probability to switch to $h = 0.2$ (no full-time option), is positive whenever a woman was not working full-time in the previous period. I model the transition of h as a first-order discrete Markov process with the probability distribution $\pi_h(h^+|h, g, \ell)$. In particular, I assume that women face the following transition matrix

$$\Pi_h^{\ell<0.4}(h|h^+, 1, \ell < 0.4) = \begin{bmatrix} 1 - \pi_{0,1}^{\ell<0.4} & \pi_{0,1}^{\ell<0.4} \\ 1 - \pi_{1,1}^{\ell<0.4} & \pi_{1,1}^{\ell<0.4} \end{bmatrix}. \tag{6.27}$$

A woman who is not working full-time transitions with probability $\pi_{0,1}^{\ell<0.4} = 0.90$ from $h = 0.4$ to $h = 0.2$ in the next period. Once she switched to $h = 0.2$, she is on average for 10 years excluded from full-time work. This is guided by the probability $\pi_{1,1}^{\ell<0.4} = 0.90$ to remain in $h = 0.2$. Women who work full-time can not transition into $h = 0.2$ and consequently $\pi_{0,1}^{\ell=0.4} = 0$.

6.4.4 Government policies

The pension budget I fix the pension contribution rate at $\tau_p = 0.187$, the 2017 statutory rate of the German pension system. Earnings from mini-jobs are subject to a reduced contribution rate (and hence reduced pension entitlements) of $\tau_p^{mini} = 0.150$, see Deutsche Rentenversicherung Bund (2023c). This results in a gross pension replacement of $\kappa = 0.41$, which is similar to the German gross replacement rate for the mean earner as reported by OECD (2021c).

[30] If a large fraction of households is located directly at the threshold between not working and working, an increase in the return to work causes a greater fraction of households to switch from non-employment to employment.

Mother benefits Mothers are compensated for foregone pension contributions while raising children. For each child, they are credited pension entitlements equivalent to that of an average earner for three years. In order to account for this in the model, I set the mother pension benefits to $e_k = 1$ whenever a woman is in state $k = 1$ and $e_k = 0$ otherwise. As a mother remains on average for six years in $k = 1$, her pension account increases by $\frac{6}{j_r - 1}$. This is similar to the credit a German mother with two children gets.

The tax budget The government raises taxes on consumption and labor earnings to cover government expenditures. I employ the 2017 statutory German progressive income tax code as depicted in Figure 6.7. Households can deduct social security contributions from gross income for the purpose of taxation. In turn, all pension benefits are liable for taxation. The tax function reads

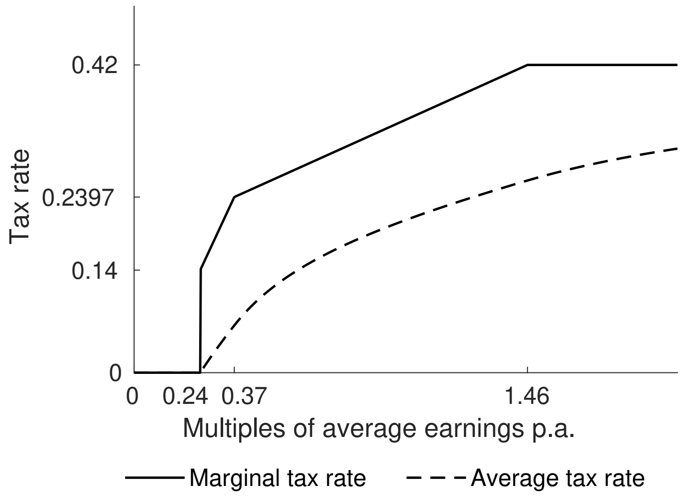
$$T = T\Big(y - T_p(\min(y, 2\bar{y})) + p\Big).$$

Couples enjoy a tax advantage in the form of income splitting. I set the proportional consumption tax rate at $\tau_c = 0.17$ to balance the fiscal budget. This is in line with the German VAT tax. Although consumption goods are generally taxed at a rate of 19%, many goods (such as food) are taxed at a lower rate. A consumption tax rate of 17 percent is thus a good compromise. I fix (wasteful) government consumption at 19% of GDP in the benchmark economy. Moreover, the government provides two transfer programs to support families with kids in the household. In 2017 parents received a child benefit of 192 euros per child and month. Hence, in the model, a family with two kids receives $t_{cb} = \frac{2\times192\times12}{37{,}000} = 0.1245\bar{y}$. Moreover, tax-financed child support t_{cs} for single mothers replaces real-world alimony payments. Setting $t_{cs} = 3 \times 0.1245 \times \bar{y}$, I calibrate monthly child support payments to 576 euros per child. In Germany, alimony payments depend on individual income and the age of the child. The so-called Düsseldorfer Table (2017) specifies the exact amount. For monthly net earnings between 1,500 – 5,100 euros, monthly payments amounted 360 – 736 euros in 2017.

6.5 Simulation results

In this section, I present simulation results from my quantitative model. I start by showing the central features of the economy in the initial equilibrium. I then turn to counterfactual policy simulations, in which I introduce progressive components into the pension formula.

Figure 6.7: Labor tax schedule



Data source: own calculation, according to §32a Einkommensteuertarif 2017. The top marginal tax rate of 45 percent applies to earnings beyond $6.93 \times \bar{y}$.

6.5.1 The benchmark economy

Figure 6.4 compares the model-implied labor choices over ages to their empirical counterpart. In earlier years, model individuals are liquidity constrained, which forces them into employment. Hence, the share of full-time workers in the model is somewhat too high early in life. In the data, young individuals work less as they spend more time acquiring education and might have savings and support from their parents. From age 30 onward, the model implied labor supply profiles are largely in line with the data. Most importantly, the model can replicate labor supply patterns over the life cycle for both single and married women. Table 6.4 shows the aggregate labor supply pattern for prime-age worker which also fit the data very well. This is a good starting point to derive meaningful results from the policy experiments.

Table 6.5 summarizes central macroeconomic aggregates of the benchmark economy and compares them to the data on the German economy in 2017. In reality, private savings are somewhat higher than capital plus public debt. However, a substantial part of these assets come from the top 1 percent of wealth holders, a particular group that I do not include in my model. As a result, the German economy holds net foreign assets worth about 45 percent of GDP. In the model, I abstract from government debt and calibrate the discount factor such that private savings almost cover the total capital demand by firms. As the model neglects various tax sources, consumption and labor tax revenues are somewhat higher than in the data.

Table 6.2: Summary of exogenous model parameters

	Value	Source
Demographics		
Share couples ϕ_c	0.6770	RDC of the FSO (2017)
Share women ϕ_f	0.492	RDC of the FSO (2017)
Share college-educated men ϕ_s^m	0.331	RDC of the FSO (2017)
Share college-educated women ϕ_s^f	0.278	RDC of the FSO (2017)
Assortative mating:		RDC of the FSO (2017)
HS – HS $\phi_s^c(0,0)$	0.5740	RDC of the FSO (2017)
Col – HS $\phi_s^c(1,0)$	0.1492	RDC of the FSO (2017)
HS – Col $\phi_s^c(0,1)$	0.0958	RDC of the FSO (2017)
Col – Col $\phi_s^c(1,1)$	0.1810	RDC of the FSO (2017)
Fertility women:		RDC of the FSO (2017)
$\sum_{j=21}^{45} \phi_k^{j,s}$	0.475	RDC of the FSO (2017)
$\sum_{j=21}^{45} \phi_k^{j,c}$	0.800	RDC of the FSO (2017)
Max. age J	99	
Population growth rate n	0.000	Statistisches Bundesamt (Destatis)
Retirement age j_r	64	DRV Bund (2019)
Others		
Child benefits t_{cb}	$0.1245\bar{y}$	Statistisches Bundesamt (Destatis)
Child benefits t_{cs}	$3 \times t_{cb}$	Düsseldorfer Tabelle (2017)
Mother benefits e_k	1	Statistisches Bundesamt (Destatis)
Pension contribution rate τ_p	0.187	DRV Bund 2022c
Pension contribution rate τ_p^{mini}	0.150	DRV Bund 2023c
International interest rate $\bar{r}$	0.030	
Capital share in production α	0.300	
Intert. elast. of substitution σ	0.670	Heathcote et al. (2014)
Frisch elast. of labor supply χ_m	0.400	Keane (2011)
Frisch elast. of labor supply χ_f	0.875	Keane (2011)
Gender gap w_g	Figure 6.6	Schrenker and Zucco (2020)
Survival probabilities $\bar{\psi}_{j,g}$	Figure 6.3	HMD (2020)

6.5.2 Pension reforms

I present long-run effects resulting from the proposed pension reforms. The benchmark case is always the proportional pension system. In the first exercise, the economy switches to the LEAP system, which redistributes based on lifetime earnings. The second exercise analyzes a reform to the EIPC system, that redistributes based on annual earnings. I selected a medium-range progressivity parameter of $\lambda = 0.5$ and a bend point $b = 0.3$ in both reform scenarios. To ensure comparability between simulations, I use the same set of structural parameters but fix per-capita government consumption over time. I assume that the contribution rate of the pension system remains at the initial equilibrium level. In doing so, I ensure that the size of the pension system relative to total labor hours is constant for all reforms. I

Table 6.3: Summary of endogenous model parameters

	Value	Target
Depreciation rate δ	0.07	Investmetn/Output: 21.0
Technology level Ω	0.92	Wage per efficiency unit $w = 1$
Discount factor β	0.98	Closed economy: NFA = 0.00
Consumption tax rate τ_c	0.17	Government budget balance
Replacement rate κ	0.41	Pension budget balance
To target labor supply		Table 6.4 & Figure 6.4
Disutility labor ν_m	75.00	
Disutility labor ν_f	15.25	
Disutility empl. mean μ_ξ	0.52	
Disutility empl. var σ^2_ξ	2.75	
Time costs kids ζ_1	0.60	
Time costs kids ζ_2	0.15	
Mum share $\lambda^c_{\zeta_{f,1}}$	0.85	
Mum share $\lambda^c_{\zeta_{f,2}}$	0.75	
Part-time gap w_p	0.975	

Table 6.4: Benchmark: labor supply

	Women				**Men**	
	not empl	mini-job	PT	FT	not empl	FT
Data	25.11	7.50	29.20	38.19	14.22	85.78
Model	25.26	7.75	28.28	38.71	14.63	85.37

Values in %p.

Table 6.5: Macroeconomic aggregates

Variable	Value (HS/Col)	Data 2017
Private Assets	282.50	433.09
Capital Stock	300.00	305.24
Public Debt	0.00	64.60
Net Foreign Assets	−17.50	44.25
Private Consumption	59.56	52.11
Government Consumption	19.00	19.84
Investment	21.00	20.96
Trade Balance	0.44	7.09
Labor Tax Revenue	10.83	8.35
Consumption Tax Revenue	10.14	8.74

Variables in percent of GDP if not indicated otherwise.

Data sources: PA: Alvaredo et al. (2022), CS: Statistisches Bundesamt (Destatis) (2023), PD, NFA: Deutsche Bundesbank (2022), PC, GC, I, TB: Statistisches Bundesamt (Destatis) (2023), LTR, CTR, AWW, EtP: RDC of the FSO (2017).

use the replacement rate κ to balance the pension budget.[31]

[31]Note that alternatively, I could fix the total expenditure of the pension system at the initial equilibrium level. This is, however, counterfactual to the nature of a pay-as-you-go system. With

The LEAP-system follows the description in Section 6.1.2. The accumulation formula for pension entitlements is proportional to pension contributions, with

$$e^{+}(e, y^{p}) = e + \frac{y^{p}}{\bar{y}}.$$

The pension formula is progressive. It has two regions and reads

$$p(e_{j_r}) = \begin{cases} \left[\frac{\lambda}{b} + (1-\lambda)\right] \times \frac{e_{j_r}}{j_r - 20} \times \kappa \times \bar{y} & \text{if } \frac{e_{j_r}}{j_r - 20} < b \\ \left[\lambda + (1-\lambda) \times \frac{e_{j_r}}{j_r - 20}\right] \times \kappa \times \bar{y} & \text{else.} \end{cases}$$

Since e_{j_r} is proportional to lifetime earnings and enters the progressive formula, the LEAP system redistributes based on lifetime earnings.

The EIPC-system follows the description in Section 6.1.3. The accumulation formula for pension claims has two regions and reads

$$e^{+}(e, y^{p}) = \begin{cases} e + \left[\frac{\lambda}{b} + (1-\lambda)\right] \times \frac{y^{p}}{\bar{y}} & \text{if } \frac{y^{p}}{\bar{y}} < b \\ e + \left[\lambda + (1-\lambda) \times \frac{y^{p}}{\bar{y}}\right] & \text{else,} \end{cases}$$

while the pension formula is proportional and is given by

$$p(e_{j_r}) = \bar{y} \times \kappa \times \frac{e_{j_r}}{j_r - 20}.$$

The EIPC system redistributes based on annual earnings, as annual earnings y^{p} are fed into the progressive formula.

6.5.3 Labor supply effects

The pension reforms affect labor supply both in a positive and in a disruptive way, see discussion in Section 6.1. This section evaluates the strength of both forces in a quantitative model for different population groups. Table 6.6 summarizes the aggregate labor supply effects for men and women. Figure D.2 in the appendix shows the benchmark employment rates over the life cycle for comparison.

Women The effects are perfectly in line with the analytical model. On the one hand, I clearly see the positive intensive margin effect for individuals with low labor earnings. The subsidy τ_{p}^{sub} pulls mini-job workers into part-time jobs, increasing

fixed total expenditure, an increase in labor force participation or labor hours would lead to a decline in per capita pension payments and therefore lead to a cut in pension benefits which would counteract the positive effects of my pension reforms.

Table 6.6: Reform: labor supply responses

	Women				**Men**	
	not empl	mini-job	PT	FT	not empl	FT
Benchmark	25.26	7.75	28.28	38.71	14.63	85.37
LEAP	0.32	-0.52	2.53	-2.33	0.61	-0.61
EIPC	-0.65	-2.24	5.70	-2.82	-0.58	0.58

Values in %p.

both their current earnings and their future pension payments. The fraction of mini-job workers drops by 0.5 and 2.25 percentage points after the LEAP and the EIPC reform, respectively. This reduces the share of women in mini-jobs by a remarkable 30 percent in the case of the EIPC reform. The effect of the EIPC reform is so much stronger because the subsidy τ_p^{sub} also affects individuals with temporarily low earnings. In the LEAP system only individuals with permanent low earnings benefit. On the other hand, I also observe the distorting effect of the implicit tax τ_p^{sub}. As the benefit-contribution link is weakened, workers reduce labor hours. Hence, the share of full-time working women drops by 2.3 and 2.8 percentage points in the LEAP and the EIPC case, respectively.

Both reforms set extensive margin labor supply incentives through the pension top-up for individuals with below-average lifetime earnings. The LEAP system provides considerable insurance against temporarily adverse earnings shocks. Periods of low earnings basically increase the value of accumulated pension entitlements as lifetime earnings shrink. This increases the return to pension contributions whenever $e_{j_r} > b\bar{y}$ holds. Individuals can fully exploit that insurance effect if they don't work at all during periods of low earnings. As a result, the fraction of not employed women increases by 0.3 percentage points. The EIPC system shows an opposite effect. To obtain the pension top-up, individuals need to be employed during periods of low earnings. Consequently, female labor force participation increases by 0.65 percentage points in the EIPC economy.

Figure 6.8 provides a deeper insight on the extensive margin effect over the life cycle. Let's start with analyzing the LEAP system (dashed lines). The employment rate of single women is declining, especially in later years. The more wealth individuals accumulate, the more the importance of immediate income declines and they can afford not to work in times of adverse productivity shocks. Increased insurance through the LEAP pension system amplifies this behavior and employment rates drop by roughly 1.5 percentage points at age 60. The effect is different for married women. Here, employment rates are slightly increasing. This is because many of them retire with average lifetime earnings below the bend point $e_{j_r} < b\bar{y}$ and thus experience a positive incentive effect. The value of their pension entitlements increases with additional earnings. In particular, about 28 percent of the married mothers with high school education (22 percent with college) retire with pension entitlements that value less than $b\bar{y}$. The fraction among married women without kids is above 20 percent. This compares to 14 percent of the single mothers with high school education (7 percent with college) and roughly 1 percent of the childless single women. Table D.3 in the appendix shows these numbers for all population groups.

The EIPC system sets strong positive employment incentives for married women. They increase labor force participation at all ages and the effect is particularly strong (+3 percentage points) at the end of working life. They are usually the second earner in the family and benefit the most from reforms that subsidize part-time workers and low earners. Single women are more likely to reduce labor force participation at younger ages, but that effect is almost compensated by higher employment rates in later years. Overall, the picture suggests that high school-educated individuals react stronger (more positive) to the EIPC reform than their college-educated counterparts. The negative effects of the LEAP reform are also more severe for that group.

Men The LEAP system distorts employment rates because of the increased insurance. Single men without a college education are tempted to withdraw early from the workforce.

The EIPC system sets overall positive employment incentives for male workers. To understand these effects it is helpful to consider the employment rates in the benchmark scenario as shown in Figure D.2 in the appendix. Only a few single men are not employed in the initial equilibrium and there is quantitatively not much potential for improvement. Therefore, the effects are almost zero for this group. There are families, in which the husband is the second earner. Here, a positive employment effect can be observed in all age groups, which is strongest for men with low education.

While both progressive reforms come with considerable distortions to full-time work, the EIPC system is more successful in pulling marginal and non-employed individuals into the workforce. Nevertheless, total labor supply falls by 0.37 percent under the EIPC reform, compared with a 1.43 percent decline under the LEAP reform. The EIPC reform, and thus annual earnings-based redistribution, dominates lifetime earnings-based redistribution in terms of labor market outcomes.

Figure 6.8: Changes in employment rates women

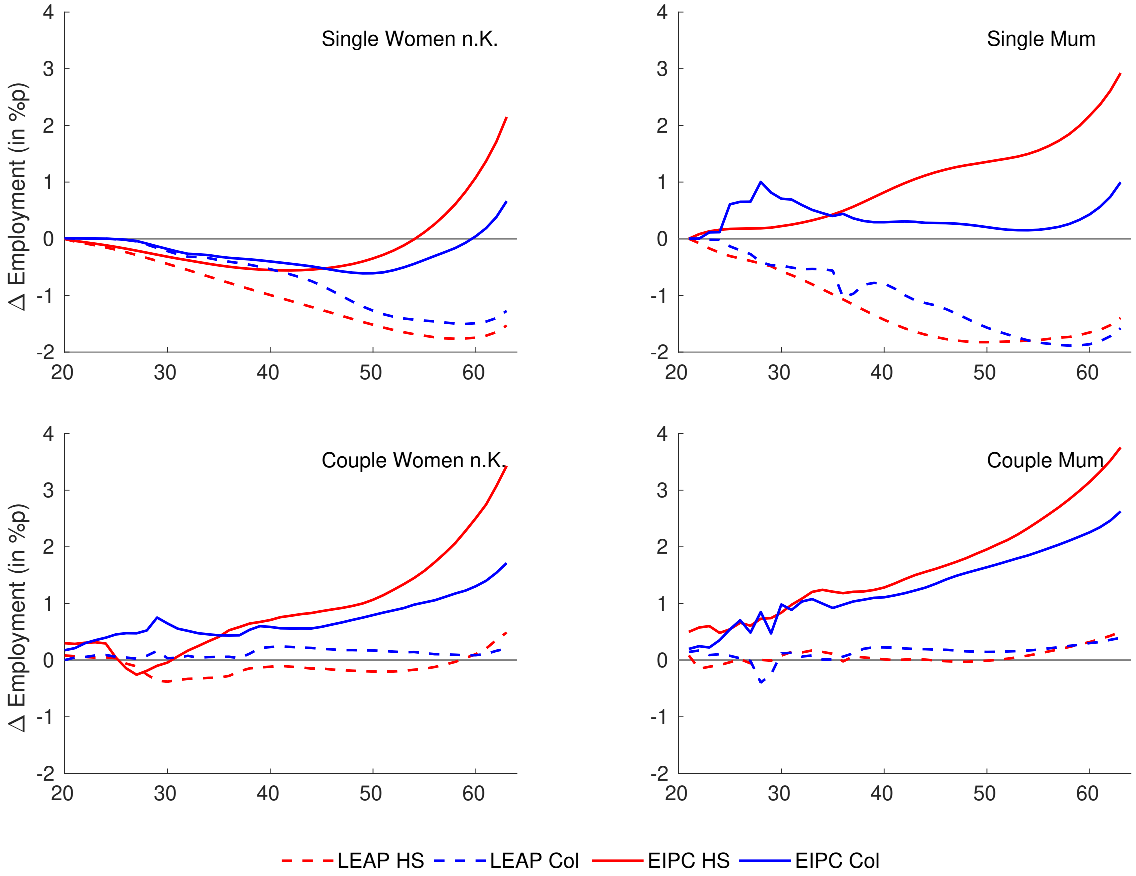
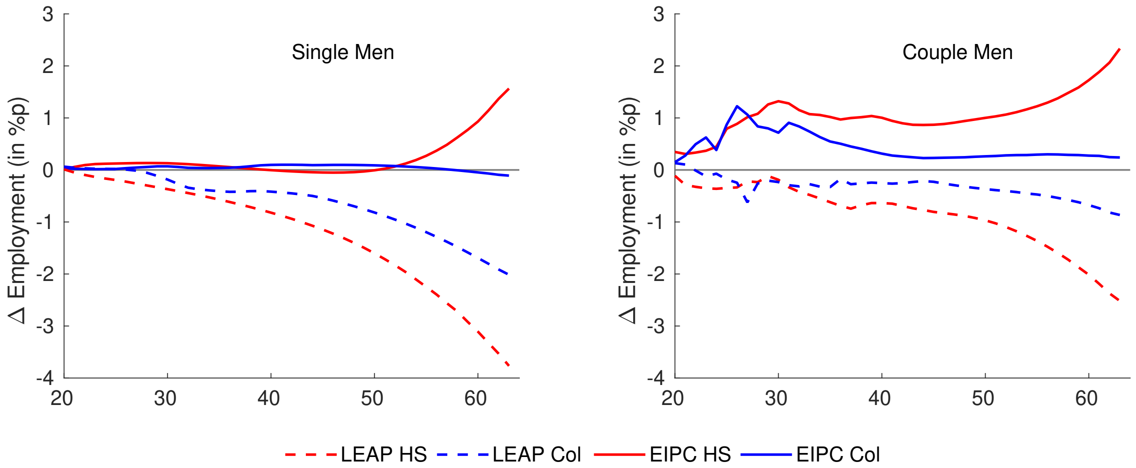


Figure 6.9: Changes in employment rates men



6.5.4 A macroeconomic evaluation

Table 6.7 shows the macroeconomic consequences of the proposed pension reforms. The simulation results are generally negative. Since aggregate labor supply shrinks, aggregate capital and output drop at the same rate. As the reformed pension systems provide more insurance against earnings risks, aggregate savings decline as well. To balance the government budget, the consumption tax rate adjusts after the decline in the labor tax revenue. However, the EIPC mechanism stimulates labor supply at the lower end of the productivity distribution and mitigates the burden of increased redistribution. The decline in labor supply, and hence output, amount to only one-third compared to the LEAP reform. Nevertheless, total consumption is down by 0.6 percent.

6.5.5 The effect on inequality

In this section, I discuss the effects of the two progressive reforms on inequality. I measure inequality in terms of pension payments, income, and consumption inequality across different population groups as well as the variance of consumption and wealth.

Old-age pensions Progressive pension reforms aim to change the distribution of pension benefits. Figure 6.10 shows the distribution of pension payments at retirement entry relative to average labor earnings $\bar{y}$. The dotted lines display the benchmark case. The effect of the reforms is fairly similar for all groups. Both the EIPC (solid line) and the LEAP (dashed line) reform reduce income inequality in old age. The distributions in the reform scenarios are much narrower and there is less weight on the right tails. Comparing the two reforms, the EIPC distribution is shifted more to the right than the LEAP distribution. Fewer households live on very little pension benefits. The positive extensive margin incentive of the EIPC system is clearly reflected in pension payments. For instance, the fraction of married women without any pension payments is roughly half. In the case of the LEAP system, the proportion of married men without any pension benefits actually increases.

Income inequality Table 6.8 and 6.9 describe the relative income and consumption position of various population groups during working life and in retirement. The measures are calculated at the household level and are not adjusted for household size and composition. The measure y_{net} denotes after-tax earnings plus transfer payments from the government, p_{net} denotes after-tax pension payments and c_y and c_p denote consumption during working life and in retirement, respectively. The corresponding population averages are $\bar{y}_{net}$, $\bar{p}_{net}$, $\bar{c}_y$ and $\bar{c}_p$. Let's start with the analysis of column PP in Table 6.8, which describes the situation of single households in an economy with a proportional pension system.

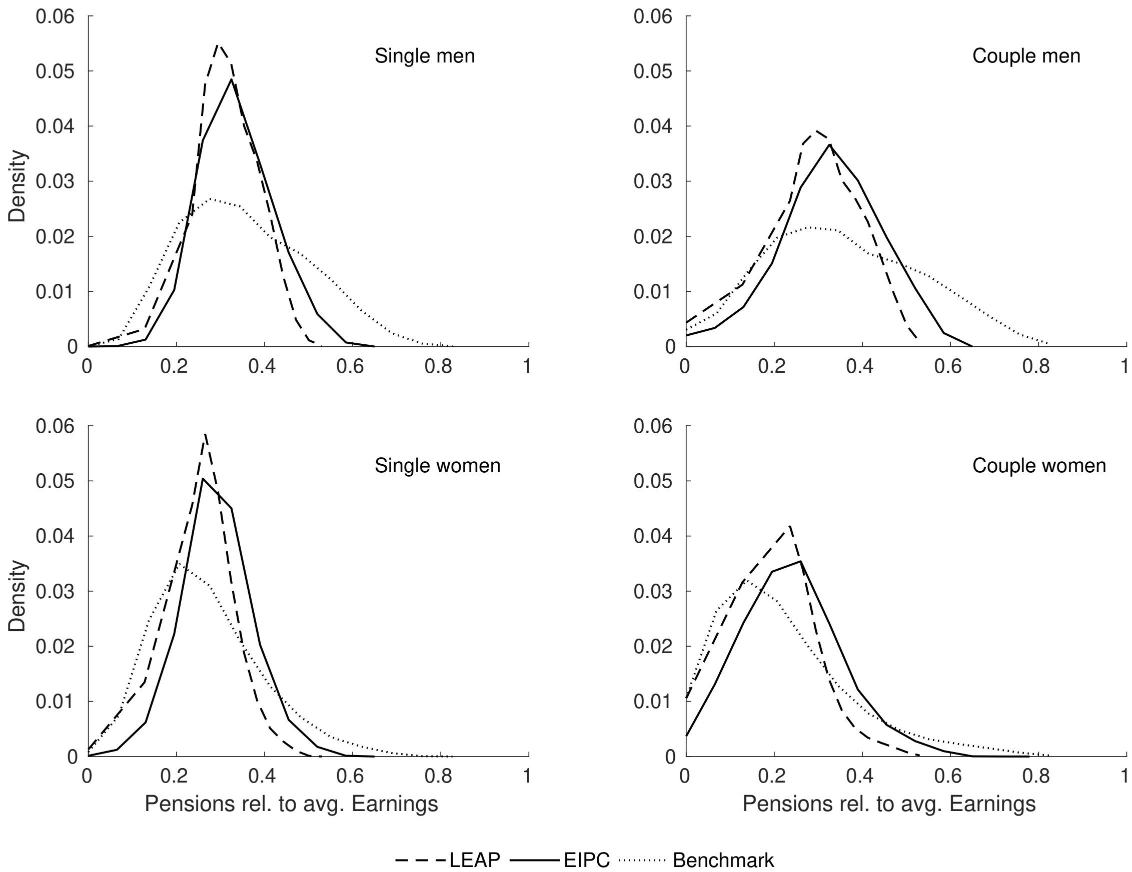
The income gap between single women with and without kids is relatively small

Table 6.7: Reform: macroeconomic aggregates

Variable	LEAP	EIPC
Private Savings	−1.27	−1.34
Capital Stock	−1.43	−0.37
Net Foreign Assets	−4.08	15.34
Private Consumption	−1.86	−0.63
Government Consumption	0.00	0.00
Investment	−1.43	−0.37
Trade Balance	−4.66	18.48
Consumption Tax Rate in%p.	0.94	0.40
Consumption Tax Revenue	3.53	1.70
Labor Tax Revenue	−3.31	−1.59

Variables in percent of GDP if not indicated otherwise.

Figure 6.10: The distribution of pension benefits



during working life. High school-educated mothers earn 82 percent of the mean income, while the childless earn 89 percent. This indicates that the tax-financed transfer payments compensate mothers well for missed labor earnings. However, there remains a significant consumption gap due to the monetary costs of kids. As a result, high school-educated single mothers consume only 61 percent of the average, while this value is 78 percent for corresponding childless women. The income situation changes considerably in old age. While childless women are able to maintain their relative income position, single mothers decline. The mother's benefits e_k are not sufficient to compensate for reduced pension contributions. Nevertheless, single mothers are still better off in terms of their relative consumption position than during their working lives. This is because the children have left the family and no longer impose monetary costs. This effect applies to all households with kids. Income-rich singles, best represented by college-educated men, descend in the income distribution when they retire. This is due to the pension assessment ceiling, which limits pension payments even in a proportional pension system. Still, they are able to maintain their consumption level due to large private old-age savings.

The situation of couple households is presented in Table 6.9. They can handle a low income better than singles. For instance, a low-educated couple without kids earns 1.79 times the average income but consumes 2.08 times the average consumption. A low-educated single man earns the mean income but consumes less than 90 percent of the average. The reason is that couples benefit from economies of scale and that the second earner reduces the need for precautionary savings. Moreover, kids have a less disruptive effect on the pension income of couple households, than on the pension income of single households. The relative income position of couples with children actually increases in retirement.

Let's now evaluate the effects on inequality of the different pension systems. The proportional pension system caps the pensions of the rich slightly through the contribution ceiling, but it lacks a mechanism that increases the pensions of the poor. Thus, income inequality is largely transferred to old age. Both progressive reform scenarios reduce old-age income inequality considerably. For example, single mothers with a high school education achieve a retirement income position of 0.85 under EIPC reform compared to 0.69 with the proportional pension system. 49 percent of her working-age income is replaced by old-age pensions, up from 39 percent. At the same time, the pensions of the rich are cut. The college-educated childless couple descends in the income position from 2.34 in the benchmark to a value of 2.09 after the EIPC reform. Only 38 percent of working-age income is replaced in retirement, compared with 43 percent in the proportional system.

Although the proposed reforms alter the old-age income distribution substantially, it hardly affects consumption inequality. Households smooth their consumption in all scenarios through private savings. I further see little effect in the distribution of working-age income. The positive and negative labor supply effects are hidden in aggregate numbers. Overall, the differences between the EIPC system and the

LEAP system are minor with respect to inequality. The analysis suggests that the LEAP system is slightly more successful in supporting the poor. For example, low-educated single mothers achieve a slightly higher old-age income position and a higher pension replacement rate under the LEAP reform.

Table 6.8: Income inequality: singles

	Women, no kids						**Women with kids**					
	HS			College			HS			College		
	PP	LEAP	EIPC	PP	LEAP	EIPC	PP	LEAP	EIPC	PP	LEAP	EIPC
Income inequality												
$y_{net}/\bar{y}_{net}$	0.89	0.87	0.87	1.25	1.24	1.23	0.82	0.82	0.82	1.09	1.08	1.08
$p_{net}/\bar{p}_{net}$	0.93	0.99	1.02	1.24	1.13	1.15	0.69	0.86	0.85	0.93	0.98	0.98
p_{net}/y_{net}	0.48	0.54	0.55	0.46	0.43	0.44	0.39	0.50	0.49	0.40	0.43	0.43
Consumption inequality												
$c_y/\bar{c_y}$	0.78	0.78	0.78	1.04	1.02	1.02	0.61	0.62	0.62	0.76	0.76	0.76
$c_p/\bar{c_p}$	0.78	0.77	0.78	1.11	1.09	1.09	0.69	0.70	0.70	0.87	0.87	0.87
c_p/c_y	1.14	1.14	1.14	1.22	1.21	1.21	1.29	1.29	1.29	1.31	1.30	1.30

	Men					
	HS			College		
	PP	LEAP	EIPC	PP	LEAP	EIPC
Income inequality						
$y_{net}/\bar{y}_{net}$	1.01	1.01	1.02	1.47	1.48	1.48
$p_{net}/\bar{p}_{net}$	1.07	1.06	1.09	1.38	1.20	1.20
p_{net}/y_{net}	0.49	0.50	0.50	0.44	0.38	0.38
Consumption inequality						
$c_y/\bar{c}_y$	0.89	0.89	0.89	1.23	1.21	1.22
$c_p/\bar{c}_p$	0.83	0.83	0.84	1.24	1.22	1.22
c_p/c_y	1.08	1.07	1.08	1.15	1.15	1.15

$y_{net}/\bar{y}_{net}$ denotes the average income position of a household. For $y_{net}/\bar{y}_{net} = 1$, the household earns the average income.
p_{net}/y_{net} is the pension replacement rate, c_p/c_y is the consumption replacement rate. $c_p/c_y > 1$ indicates that the old-age consumption is larger than consumption during working life.

Consumption variance The left panel of Figure 6.11 shows the consumption variance over the life cycle, which decreases markedly for all cohorts after the reforms. The reason for this is two-folded. First, earnings-rich households, who are the main payer of the reform, save more during working life (and hence consume less) in order to compensate for reduced pension payments in old age. Second, lower income inequality in pension payments reduces the variance of consumption in retirement. This effect is especially large for the very old cohorts. With dropping survival probabilities, households turn into hand-to-mouth consumers and live solely

Table 6.9: Income inequality: couples

No kids	M = HS F = HS	M = Col F = HS	M = HS F = Col	M = Col F = Col
	PP LEAP EIPC	PP LEAP EIPC	PP LEAP EIPC	PP LEAP EIPC
Income inequality				
$y_{net}/\bar{y}_{net}$	1.79 1.79 1.79	2.27 2.28 2.27	2.19 2.20 2.19	2.57 2.58 2.57
$p_{net}/\bar{p}_{net}$	1.86 1.93 1.92	2.12 2.00 1.98	2.12 2.02 1.98	2.34 2.11 2.09
p_{net}/y_{net}	0.48 0.51 0.51	0.44 0.41 0.41	0.45 0.43 0.42	0.43 0.39 0.38
Consumption inequality				
$c_y/\bar{c}_y$	2.08 2.08 2.09	2.54 2.54 2.53	2.46 2.45 2.44	2.83 2.81 2.80
$c_p/\bar{c}_p$	1.95 1.96 1.96	2.49 2.49 2.48	2.43 2.42 2.41	2.90 2.87 2.86
c_p/c_y	1.07 1.07 1.07	1.12 1.12 1.12	1.13 1.13 1.13	1.17 1.17 1.17

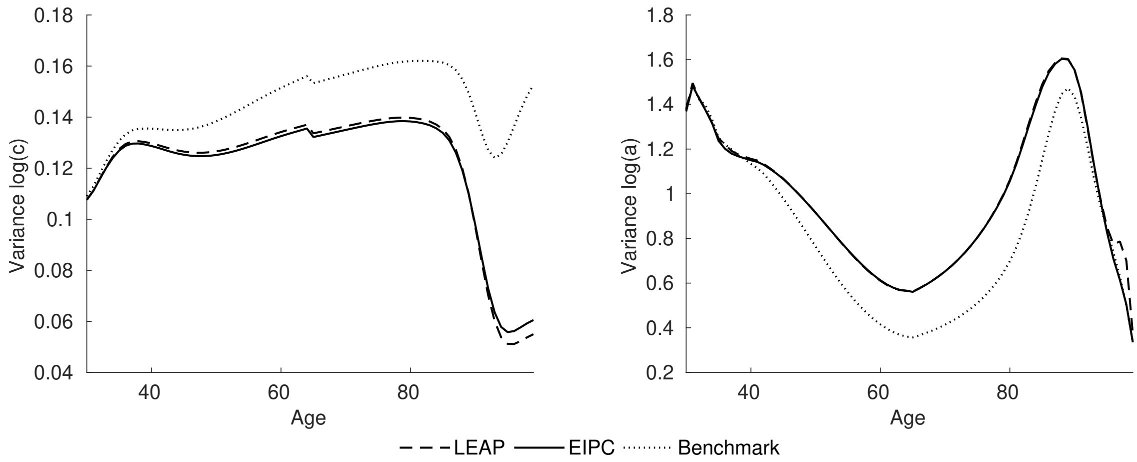
With kids	M = HS F = HS	M = Col F = HS	M = HS F = Col	M = Col F = Col
	PP LEAP EIPC	PP LEAP EIPC	PP LEAP EIPC	PP LEAP EIPC
Income inequality				
$y_{net}/\bar{y}_{net}$	1.57 1.57 1.58	2.12 2.14 2.14	1.88 1.88 1.87	2.32 2.33 2.33
$p_{net}/\bar{p}_{net}$	1.77 1.89 1.88	2.08 1.99 1.95	2.02 2.00 1.98	2.26 2.09 2.07
p_{net}/y_{net}	0.52 0.57 0.56	0.46 0.44 0.43	0.50 0.50 0.50	0.45 0.42 0.42
Consumption inequality				
$c_y/\bar{c}_y$	1.62 1.63 1.63	2.08 2.08 2.07	1.85 1.84 1.84	2.24 2.23 2.23
$c_p/\bar{c}_p$	1.73 1.75 1.75	2.31 2.31 2.30	2.03 2.03 2.03	2.56 2.54 2.53
c_p/c_y	1.22 1.23 1.23	1.27 1.27 1.27	1.26 1.26 1.26	1.30 1.30 1.30

on their pension payments. Hence, income inequality maps directly into consumption inequality.

A comparison of the two reforms shows that the impact is roughly the same, but still qualitatively different. The EIPC reform is more successful in reducing consumption inequality during working life, while the LEAP system performs better in very old age. This is because of the positive employment incentive of the EIPC system. Households with adverse productivity shocks are pulled into the workforce in every period of working life. This reduces inequality in earnings and hence inequality in consumption. The situation flips for the very old cohorts. The LEAP system is more successful in reducing consumption inequality among hand-to-mouth consumers. This indicates that lifetime earnings-based redistribution is more targeted in the end.

Asset variance The right panel of Figure 6.11 shows the asset variance over the life cycle. The wealth gap is substantially widening as an unwanted side effect of the reform. The reason is that poor households will save less because of increased insurance through the pension system. The rich will save more to compensate for lower pension benefits in old age. The gap starts to increase from age 40 onward and remains large until the early 80s.

Figure 6.11: Variance of consumption log(c) and wealth log(a)



6.5.6 Welfare analysis

To jointly evaluate the negative macroeconomic and positive distributional consequences, I take a look at long-run welfare. To this end, I calculate the ex-ante expected lifetime utility EV before any information about the household's education level or labor productivity has been revealed. I then compare two steady state allocations: the benchmark scenario with a proportional pension system and utility level EV_0, and each reformed system with an associated utility level EV_∞. To give the welfare numbers a meaningful interpretation, I calculate the consumption equivalent variation CEV between the two utility levels. The CEV indicates by how many percent I would have to increase or decrease the consumption level of households at each age and each potential state in the benchmark equilibrium in order to make them as well off as in a reform scenario with progressive pensions. A negative value for CEV indicates that a reform of the pension system deteriorates long-run welfare and that households would be willing to pay a positive amount of resources in order to stay in the benchmark equilibrium.

The first row of Table 6.10 shows aggregate ex-ante welfare effects and the subsequent rows show group-specific effects. The LEAP reform is almost neutral with respect to aggregate welfare and single women are the only beneficiaries. Married women are hardly affected and both married and single men lose. The EIPC system provides aggregate welfare gains, CEV increases by more than 0.5 percent. The main beneficiaries are single women and the group of married men is the only team that is losing.

Overall, the results indicate that single women are the main beneficiary group of the progressive pension reforms. They have often severely interrupted employment histories due to the arrival of children and can not provide enough private savings for a decent life in old age. They would benefit greatly from both reform ideas due to increased insurance and redistribution effects. Still, the EIPC reform is clearly superior. It is very successful in incentivizing the labor supply of the earnings poor. Thereby, it is not only redistributes old-age income, but also increases the earnings of the poor during working life. Men have the highest incomes and are therefore the main payers of the reform. This is reflected in the welfare results, which are mostly negative.

Table 6.10: Welfare effects

Variable	LEAP	EIPC
Change in ex-ante long-run welfare	**-0.022**	**0.524**
– for Single Men	−0.486	0.147
– for Single Women	0.787	1.415
– for Married Men	−0.316	−0.126
– for Married Women	0.087	0.900

Welfare effects are reported as CEV over initial equilibrium in percent.

6.6 Conclusion

So far, the literature on pension design has considered various pension formulas and degrees of progressivity. I am exploring a novel channel: the redistribution base. My simulations show that this has a major effect on labor supply responses and hence the aggregate economy. Starting from a purely proportional pension system, I conduct two reform exercises. In reform one, I introduce progressivity into the pension formula. In reform two, I introduce progressivity into the accumulation formula. I apply exactly the same transfer scheme in both cases. In the first case, transfers are based on information on lifetime earnings, while in the second case, transfers are based on information on annual earnings during working life. My analysis shows that the behavioral responses differ substantially.

I find that annual earnings-based redistribution provides large positive welfare effects. It leads to considerable old-age income redistribution while the economic costs are limited. Lifetime earnings-based redistribution generates more severe macroeconomic distortions that soak up the positive effects of reduced income inequality in old age. This mechanism should not be ignored in future reform considerations in the debate on progressive pensions.

Still, some open questions remain. First, the welfare numbers are silent about the source of the effects. Both reforms change the insurance possibilities and redistribute resources across households. Moreover, it is not clear whether the welfare gains stem from a better allocation of consumption across types or from an increase in the time allocated to leisure. Conesa et al. (2009) provide a decomposition of welfare effects in these categories, which could be adopted. However, this approach is not straightforward to apply in my model due to the presence of couple households. Second, I only analyze long-run effects and hence the periods of the transition path are completely ignored. I can not say whether the welfare gains of the EIPC reform are only due to intergenerational redistribution, or if the reform has really improved the allocation of resources. However, the EIPC experiment is closely related to the EIPC reform outlined in Chapter 5, which provides efficiency gains due to improved risk-sharing possibilities.

Chapter 7

Conclusion

Given the demographic trends and the government's inaction over the past 15 years, fundamental pension reform must be a top priority on the political agenda for the 2020s. One possibility to counteract an increase in old age poverty rates in the near future is the introduction of a progressive component into the pension system. I argue that it is more important to prevent a decline in pension levels for the poor than to cling to the outdated concept of the equivalence principle. So far, the courage for comprehensive reform to move officially away from that principle has been lacking. The introduction of the Grundrente in 2021 should not be seen as a progressive pension reform in the sense proposed in this dissertation. The Grundrente does not lead to more redistribution within the pension system. It simply provides a tax-financed transfer to pensioners with a long contribution history. The literature shows that the Grundrente will do very little to reduce the risk of poverty in old age in the future since most at-risk groups do not meet the eligibility requirements.

This dissertation explores several approaches to introduce progressivity into the German pension system. One option that has proven to be superior to others is the EITC mechanism applied to annual earnings. This system is feasible and provides sizable welfare gains at the same time. Adverse economic consequences are limited both in the long run and along the transition. I provide simulation results on the effects of such a pension reform for various social groups. I find that single women, the largest old age poverty risk group, would benefit most from such a reform. I put a particular focus on the labor supply effects. I show that a well-designed reform has the potential to limit labor supply distortions and even increase the overall employment rate. Still, the proposed mechanism can not fully neutralize the distortion of the implicit tax on labor earnings due to redistribution. A major strength of the EIPC system is its simplicity. The mechanism is clean, easy to communicate, and easy to adapt to. Debets et al. (2022) and others provide evidence that pension knowledge has a positive causal effect on active pension decisions. Policymakers should exploit this channel in future reforms and make reforms that people understand.

One important question that is not addressed in this dissertation is the interaction of an EIPC pension system with an EITC tax system. The literature has shown that the EITC mechanism in the tax system is successfully incentivizing employment among the poor in the US. The simulation models in Chapter 5 and 6 are calibrated to the German economy with the German income tax code. Likely, a large fraction of the observed employment effect would also be absorbed by an EITC tax system. To isolate the effect of the EIPC pension reform, the model should instead be calibrated to the US economy with the EITC tax code. It would be even more interesting to model a unified tax and transfer system, in which higher pension progressivity is offset by a decrease in the progressivity in the tax system.

Although I argue that the introduction of the Grundrente should not be misunderstood as a progressive pension reform, an analysis of its effects on labor supply and welfare in a structural model would be an interesting contribution to the literature. In particular, the requirement to contribute to the pension system for at least 33 years should provide a significant extensive margin incentive to work. The framework presented in Chapter 6 is a good starting point to implement the Grundrente mechanism and address that question. This would facilitate quantifying the welfare effects of the reform for different social groups and identifying possible weaknesses and improvements of the Grundrente mechanism. However, one feature of the Grundrente that is difficult to capture in a structural model with rational agents is the extremely complicated eligibility requirements. Agents in the model will react perfectly well to the mechanism, but it is very unlikely that individuals in the real world do as well.

Appendix A

Appendix Inequality

A.1 Censoring threshold

The starting point is the data set of regular workers with 189,184 observations as summarized in Table 4.1. While I fixed the bottom threshold that marks the difference between a regular worker and a low earner at a constant value of 0.23, see equation (4.1), identifying the top censoring threshold is not as straightforward. Although the German public pension insurance provides an official contribution ceiling $\tilde{y}_{max,t}$ for contributory earnings in every year, see Deutsche Rentenversicherung Bund (2020), I cannot take this value directly. The reason is that the ceiling is applied on a monthly basis while I am working with annual data. Hence, my observations could be subject to censoring, although the observed annual earnings y^p_{isjt} are below the official cut-off value. This is the case if the contribution threshold is reached in some months of the year, but not in others (for instance because of salary changes). In addition, I observe a few outliers where annual pension claims y^p_{isjt} are beyond the corresponding official threshold, which might be due to value adjustments.

To overcome these problems, I use the following strategy to identify a threshold $y_{max,t}$ for every year that captures most observations that have been top-coded at least in one month:

1. First, I find the value of pension claims $mode_{y,t}$ at the upper end of the distribution where most of the observations pile up and compare it to the official threshold $\frac{\tilde{y}_{max,t}}{\bar{y}_t}$. $mode_{y,t}$ typically is in the order of 0.0002 smaller than $\frac{\tilde{y}_{max,t}}{\bar{y}_t}$, which corresponds to about 7 euros in 2016 compared to an average income of 36,000 euros.

2. Next, I define the censoring threshold as

$$\frac{y_{max,t}}{\bar{y}_t} = mode_{y,t} - 0.0003.$$

   This guarantees that (i) $y_{max,t}$ is always smaller than $\tilde{y}_{max,t}$ and (ii) as little information as possible is cut off.

3. Next, I identify outliers as observations with

$$y^p_{isjt} > 1.05 \times \frac{y_{max,t}}{\bar{y}_t},$$

   that is those that exceed the contribution ceiling by more than 5 percent. These outliers are treated as observations with no contributory earnings and therefore deleted from the data set (285 observations).

4. Finally, I recalculate pension claims for all individuals that exceed the contribution ceiling by less than the outlier threshold. Specifically, I set

$$y^p_{isjt} = \frac{y_{max,t}}{\bar{y}_t} \text{ for all } i \text{ with } y^p_{isjt} > \frac{y_{max,t}}{\bar{y}_t}.$$

   This modifies 16,597 observations.

After these steps, the data is subject to a sharp annual censoring threshold $y_{max,t}$. Table A.1 shows the exact values of $\tilde{y}_{max,t}$, $y_{max,t}$, and the share of observation at both thresholds for each year. About 7 to 12 percent of the annual observations are on the threshold value $y_{max,t}$.

A.2 Statistical model and moments

I use a generalized method of moments estimator to determine the parameters of this model. I thereby control for the fact that the data are top-coded at the threshold $y_{max,t}$ and that I truncated them at the low earner threshold $y_{min} = 0.23$. Using

$$x_{sjt} = \frac{\log(y_{min}) - \kappa_{t,s} - \theta_{j,s}}{\sigma_s} \quad \text{and} \quad z_{sjt} = \frac{\log(y_{max,t}) - \kappa_{t,s} - \theta_{j,s}}{\sigma_s}$$

as notation for the standardized truncation and censoring thresholds, the age-, education-, and year-specific mean of the left-truncated and right-censored distribution of earnings is

$$\begin{aligned} E_{sjt} &= E\Big[\log(y_{isjt})\Big|\, y_{min} \le y_{isjt} \le y_{max,t}\Big] = \\ &= [1 - P_{sjt}] \times \left[\kappa_{t,s} + \theta_{j,s} + \sigma_s \frac{\phi(x_{sjt}) - \phi(z_{sjt})}{\Phi(z_{sjt}) - \Phi(x_{sjt})}\right] + P_{sjt} \times \log(y_{max,t}) \end{aligned}$$

Table A.1: Identification of $y^*_{max,t}$

Year t	$\tilde{y}_{max,t}$	% at $\tilde{y}_{max,t}$	$y_{max,t}$	% at $y_{max,t}$	Observations n
2000	1.9021	0.9140	1.9017	9.0382	6,893
2001	1.8908	8.4678	1.8905	9.5849	7,251
2002	1.8864	1.2084	1.8858	10.0832	7,696
2003	2.1149	0.2959	2.1143	7.2115	8,112
2004	2.1266	0.6251	2.1261	7.6197	8,478
2005	2.1368	7.4983	2.1365	7.6889	8,922
2006	2.1360	7.3366	2.1358	7.4732	9,514
2007	2.1034	0.9538	2.1029	8.5742	10,170
2008	2.0767	1.0249	2.0763	9.1874	10,830
2009	2.1242	0.4134	2.1239	8.4528	11,369
2010	2.1192	8.6243	2.1191	8.6578	11,943
2011	2.0561	0.6724	2.0556	9.6590	12,641
2012	2.0362	9.4922	2.0361	9.6429	13,274
2013	2.0678	9.6261	2.0675	10.0647	13,453
2014	2.0687	0.7156	2.0683	10.2464	13,556
2015	2.0530	10.6598	2.0528	10.7109	13,687
2016	2.0560	0.7675	2.0553	11.6082	13,680
					181,469

[*] Values for $\tilde{y}_{max,t}$ and $y_{max,t}$ are expressed relative to average earnings $\bar{y}_t$.

with

$$P_{sjt} = P\Big(\{y_{isjt} = y_{max,t}\}\Big) = \frac{1 - \Phi(z_{sjt})}{1 - \Phi(x_{sjt})}.$$

When calculating the variance, I exclude the censored data, i.e. all observations with $y_{isjt} = y_{max,t}$. The variance of the double-truncated distribution of earnings then reads

$$\begin{aligned}\text{Var}_{sjt} &= \text{Var}\Big[\log(y_{isjt})\Big|\, y_{min} \leq y_{isjt} < y_{max,t}\Big] = \\ &= \sigma_s^2 \times \left[1 + \frac{x_{sjt}\phi(x_{sjt}) - z_{sjt}\phi(z_{sjt})}{\Phi(z_{sjt}) - \Phi(x_{sjt})} - \left(\frac{\phi(x_{sjt}) - \phi(z_{sjt})}{\Phi(z_{sjt}) - \Phi(x_{sjt})}\right)^2\right].\end{aligned}$$

Following Manjunath and Wilhelm (2012), I derive the intertemporal covariance of the double-truncated distribution of earnings as

$$
\begin{aligned}
\mathrm{Cov}_{sjt} &= \mathrm{Cov}\Big[\log(y_{isjt}), \log(y_{isj+1,t+1}) \\
&\qquad\qquad \Big| \; y_{min} \leq y_{isjt} < y_{max,t} \wedge y_{min,t+1} \leq y_{isj+1,t+1} < y_{max,t+1}\Big] \\
&= \rho\sigma_s^2 \Bigg\{ 1+ \\
&\quad + M x_{sjt}\phi(x_{sjt}) \left[\Phi\left(\frac{z_{sj+1,t+1} - \rho x_{sjt}}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{x_{sj+1,t+1} - \rho x_{sjt}}{\sqrt{1-\rho^2}}\right)\right] \\
&\quad - M z_{sjt}\phi(x_{sjt}) \left[\Phi\left(\frac{z_{sj+1,t+1} - \rho z_{sjt}}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{x_{sj+1,t+1} - \rho z_{sjt}}{\sqrt{1-\rho^2}}\right)\right] \\
&\quad + M x_{sj+1,t+1}\phi(x_{sj+1,t+1}) \left[\Phi\left(\frac{z_{sjt} - \rho x_{sj+1,t+1}}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{x_{sjt} - \rho x_{sj+1,t+1}}{\sqrt{1-\rho^2}}\right)\right] \\
&\quad - M z_{sj+1,t+1}\phi(x_{sj+1,t+1}) \left[\Phi\left(\frac{z_{sjt} - \rho z_{sj+1,t+1}}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{x_{sjt} - \rho z_{sj+1,t+1}}{\sqrt{1-\rho^2}}\right)\right] \\
&\quad + M \frac{\sigma_\varepsilon^2}{\rho} \left[\phi_{0,\Sigma}\begin{pmatrix} x_{sjt} \\ x_{sj+1,t+1} \end{pmatrix} - \phi_{0,\Sigma}\begin{pmatrix} x_{sjt} \\ z_{sj+1,t+1} \end{pmatrix}\right] \\
&\quad - M \frac{\sigma_\varepsilon^2}{\rho} \left[\phi_{0,\Sigma}\begin{pmatrix} z_{sjt} \\ x_{sj+1,t+1} \end{pmatrix} - \phi_{0,\Sigma}\begin{pmatrix} z_{sjt} \\ z_{sj+1,t+1} \end{pmatrix}\right]\Bigg\} \\
&\quad - \sigma_s^2 \left[\frac{\phi(x_{sjt}) - \phi(z_{sjt})}{\Phi(z_{sj+1,t+1}) - \Phi(x_{sj+1,t+1})}\right]\left[\frac{\phi(x_{sj+1,t+1}) - \phi(z_{sj+1,t+1})}{\Phi(z_{sj+1,t+1}) - \Phi(x_{sj+1,t+1})}\right],
\end{aligned}
$$

where

$$
M = \left[\Phi_{0,\Sigma}\begin{pmatrix} z_{sjt} \\ z_{sj+1,t+1} \end{pmatrix} - \Phi_{0,\Sigma}\begin{pmatrix} x_{sjt} \\ x_{sj+1,t+1} \end{pmatrix}\right]^{-1} \quad \text{and} \quad \Sigma = \begin{bmatrix} 1 & \rho^2 \\ \rho^2 & 1 \end{bmatrix}.
$$

Moment conditions and estimation To estimate the statistical model in (4.3) with my data, I have to determine a total of 110 parameters:

1. 34 year fixed effects $\kappa_{t,s}$ for the years 2000 to 2016 and the education levels $s \in \{0, 1\}$;
2. 72 age fixed effects $\theta_{j,s}$ for the ages 25 to 60 for each education level s;
3. the two unconditional variances σ_s^2;
4. the two autocorrelation parameters ρ_s.

In order to estimate these parameters, I use the labor earnings data y_{isjt}^p to calculate the empirical moments that correspond to the means E_{sjt}, censoring shares P_{sjt}, variances Var_{sjt} and covariances Cov_{sjt} discussed above for each education level s, age j and year t. I exclude moments when the number of individuals in the corresponding education-age-year bin is smaller than 30, or when the empirical standard error of the moment is equal to zero. This gives me the following moments:

- **sample means:** I estimate 974 means $\hat{\mu}_{sjt}$ of $\log(y^p_{isjt})$ including the censored observations $y_{isjt} = y_{max,t}$ and the corresponding standard errors $\frac{\hat{\sigma}_{sjt}}{\sqrt{n_{sjt}}}$;
- **share of observations at threshold $y_{max,t}$:** I compute 930 shares $\hat{s}hr_{sjt}$ of the observations that sit exactly on the threshold $y_{max,t}$ and the corresponding standard errors $\sqrt{\frac{shr_{sjt}(1-shr_{sjt})}{n_{sjt}}}$;
- **sample variances:** I estimate 943 variances $\hat{\sigma}^2_{sjt}$ of $\log(y^p_{isjt})$ excluding the censored observations as well as the corresponding standard errors of the variance $\hat{\sigma}^2_{sjt}\frac{\sqrt{2}}{n_{sjt}-1}$;
- **sample covariances:** I compute 877 covariances $\hat{\sigma}_{sjt,t+1}$ of $\log(y_{isjt})$ excluding the censored observations as well as the corresponding standard errors of the covariance $\sqrt{\frac{(\hat{\sigma}_{sjt,t+1})^2+\hat{\sigma}^2_{sjt}\hat{\sigma}^2_{sj+1,t+1}}{n_{sjt}-1}}$.

I use these 3724 empirical moments to calculate a residual sum of squares measure. I use a diagonal weighting matrix that has the inverse of the squared standard errors of the empirical moments on the diagonal. To minimize the residual sum of squares and account for multiple local minima, I use the method of simulated annealing, see Du and Swamy (2016). I estimate parameters separately for each education level s.

Appendix B

Analytical Model

In this Appendix, I present a (partial) equilibrium version of the simple model discussed in Section 5.1.2. Households in this framework live for two periods $j = 1, 2$. At each date t, a new generation of mass N_t is born. At the moment they enter the economy, households draw two different shocks: (i) a labor productivity z according to the cumulative distribution function $\Phi_z(\cdot)$ and (ii) a utility cost of employment ξ according to the cumulative distribution function $\Phi_\xi(\cdot)$. I assume both shocks to be independent and identically distributed across households. The interest rate r as well as the wage rate w for effective labor are exogenous. I consider steady state allocations only.[1]

B.1 The household decision problem

As in Section 5.1.2 households maximize utility

$$U(c_1, c_2, \ell, e) = c_1 + \frac{c_2}{1+r} - \frac{\ell^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \xi e. \tag{B.1}$$

subject to the budget constraint

$$c_1 + \frac{c_2}{1+r} = (1-\tau_p)wze\ell + \frac{p}{1+r}. \tag{B.2}$$

The government operates an employment-linked pension system (ELS), such that

$$p = \kappa \times \Big[\lambda \bar{y} e + (1-\lambda)wze\ell\Big]. \tag{B.3}$$

[1] I hence drop the time index t wherever possible.

Plugging the pension formula into the household's budget constraint, I can write

$$
\begin{aligned}
c_1 + \frac{c_2}{1+r} &= (1-\tau_p)wze\ell + \frac{\kappa \times \left[\lambda \bar{y} e + (1-\lambda) wze\ell\right]}{1+r} \\
&= \left[1 - \tau_p + \frac{\kappa}{1+r} \times (1-\lambda)\right] wze\ell + \frac{\kappa}{1+r} \times \lambda \bar{y} e.
\end{aligned}
$$

B.2 The equilibrium pension system

For an equilibrium in this economy to exist, I require $r, n \geq -1$, which is not restrictive. Recall that labor productivity z is distributed in this economy according to the distribution function Φ_z. Further, denote by $e(z)$ and $\ell(z)$ the optimal household choices as functions of labor productivity, which I discuss in more detail below. Average labor earnings of the employed then are given by

$$
\bar{y} = \frac{\int wze(z)\ell(z)\ \Phi_z(dz)}{\int e(z)\ \Phi_z(dz)}.
$$

The pension system collects pension contributions $\tau_p wze(z)\ell(z)$ from each employed households and pays pensions according to the pension formula discussed above. Letting population growth be constant over time and let n denote the population growth rate. In a balanced-budget pay-as-you-go pension system the sum of pension contributions needs to be equal to the sum of pension payments, i.e.

$$
\int \tau_p wze(z)\ell(z)\ \Phi_z(dz) = \frac{\int \kappa \times [\lambda \bar{y} e + (1-\lambda) wze\ell]\ \Phi_z(dz)}{1+n}.
$$

Dividing this equation by the measure of employed households, I immediately obtain

$$
\tau_p \times \bar{y} = \frac{\kappa}{1+n} \times \left[\lambda \bar{y} + (1-\lambda)\bar{y}\right].
$$

The equilibrium replacement rate of the pension system hence is

$$
\kappa = (1+n)\tau_p. \tag{B.4}
$$

B.3 Implicit taxes and employment subsidies

Let's denote by $\varrho = \frac{1+n}{1+r}$ the ratio between population growth and the economy's interest rate. ϱ is an indicator for the rate-of-return difference between the pension system and the capital market. The smaller is ϱ, the higher is the return to financial investments relative to investments into public pensions. In the benchmark case in Section 5.1.2, I assumes that $r = n$ and therefore $\varrho = 1$. However, I now want to prove our results more generally.

Using the relationship in (B.4), the household budget constraint becomes

$$c_1 + \frac{c_2}{1+r} = \Big[1 - \underbrace{\Big(1 - \varrho(1-\lambda)\Big)\tau_p}_{=:\tau_p^{\text{imp}}}\Big] wze\ell + \underbrace{\lambda\varrho\tau_p\bar{y}}_{=:\tau_p^{\text{sub}}} e. \tag{B.5}$$

τ_p^{imp} is the implicit tax rate. Note that I have

$$\tau_p^{\text{imp}} \geq 0 \quad \text{whenever} \quad n \leq r + \frac{\lambda}{1-\lambda}(1+r).$$

In a proportional pension system with $\lambda = 0$, the implicit tax rate on labor earnings is hence non-negative if $n \leq r$, and it is zero in case of $n = r$. In a dynamically efficient economy ($n \leq r$), the implicit tax rate is always positive for any $\lambda > 0$. τ_p^{sub} is an employment subsidy. This subsidy is positive whenever $\lambda > 0$.

B.4 Optimal choices

Using the budget constraint in (B.5), the household optimization problem becomes

$$\begin{aligned} \max_{c_1, c_2, \ell, e} \; u(c_1, c_2, \ell, e) &= c_1 + \frac{c_2}{1+r} - \frac{\ell^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \xi e \\ \text{s.t. } c_1 + \frac{c_2}{1+r} &= \Big[1 - \tau_p^{\text{imp}}\Big] wze\ell + \tau_p^{\text{sub}} e. \end{aligned}$$

The first-order condition with respect to intensive margin labor supply is

$$\begin{aligned} -\ell(z|e=1)^{\frac{1}{\chi}} + \Big[(1-\tau_p^{\text{imp}})wz\Big] &= 0 \\ \Leftrightarrow \qquad \ell(z|e=1) &= \Big[(1-\tau_p^{\text{imp}})wz\Big]^{\chi}. \end{aligned} \tag{B.6}$$

Plugging $\ell(z|e=1)$ into the household utility function, I immediately obtain

$$\begin{aligned} U(z|e=1) &= [1-\tau_p^{\text{imp}}]wz[(1-\tau_p^{\text{imp}})wz]^{\chi} + \tau_p^{\text{sub}} - \frac{[(1-\tau_p^{\text{imp}})wz]^{1+\chi}}{1+\frac{1}{\chi}} - \xi \\ &= \frac{\Big[(1-\tau_p^{\text{imp}})wz\Big]^{1+\chi}}{1+\chi} + \tau_p^{\text{sub}} - \xi. \end{aligned}$$

As $\ell(z|e=0) = 0$, I have $U(z|e=0) = 0$ and hence the utility difference between being employed and not is

$$U(z|e=1) - U(z|e=0) = \frac{\Big[(1-\tau_p^{\text{imp}})wz\Big]^{1+\chi}}{1+\chi} + \tau_p^{\text{sub}} - \xi.$$

Given the distribution Φ_ξ of the utility costs of employment, the probability that an individual with labor productivity z is employed is given by

$$\begin{aligned} P(e=1|z) &= P\left(\left\{U(z|e=1) - U(z|e=0) \geq 0\right\}\right) \\ &= \Phi_\xi \left(\frac{\left[(1-\tau_p^{\text{imp}})wz\right]^{1+\chi}}{1+\chi} + \tau_p^{\text{sub}} \right). \end{aligned} \tag{B.7}$$

B.5 Incentive effects of progressive pensions

To study the incentive effects of employment-linked progressive pensions on labor supply, I take the derivative of a household's employment decision with respect to λ. For the intensive hours choice in (B.6) this derivative is

$$\frac{\partial \ell(z|e=1)}{\partial \lambda} = -\tau_p \times \varrho \times \chi \times \frac{\ell(z|e=1)}{1-\tau_p^{\text{imp}}} < 0.$$

The probability of being employed in (B.7) changes with λ according to

$$\begin{aligned} \frac{\partial P(e=1|z)}{\partial \lambda} &= \phi_\xi(\cdot) \cdot \left[\left[(1-\tau_p^{\text{imp}})wz\right]^{\chi} (-wz) \cdot \frac{\partial \tau_p^{\text{imp}}}{\partial \lambda} + \frac{\partial \tau_p^{\text{sub}}}{\partial \lambda} \right] \\ &= \phi_\xi(\cdot) \cdot \left[-wz\ell(z|e=1) \cdot \frac{\partial \tau_p^{\text{imp}}}{\partial \lambda} + \frac{\partial \tau_p^{\text{sub}}}{\partial \lambda} \right] \end{aligned}$$

With $\frac{\partial \tau_p^{\text{imp}}}{\partial \lambda} = \varrho \tau_p$ and $\frac{\partial \tau_p^{\text{sub}}}{\partial \lambda} = \varrho \tau_p \bar{y}$, I get

$$\frac{\partial P(e=1|z)}{\partial \lambda} = \tau_p \times \varrho \times \phi_\xi(\cdot) \times \left[\bar{y} - wz\ell(z|e=1)\right],$$

where the sign of the effect depends on the relative income position of the household. It is positive for all individuals with earnings less than the average earnings of the workforce, and negative otherwise.

Appendix C

Progressive Pensions as an Incentive for Labor Force Participation

C.1 First-order conditions for the ELS

In the following, I describe the first-order conditions of the household problem under an employment-linked pension system.

The dynamic household optimization problem reads

$$v(\mathbf{x}) = \max_{c,\ell,e,a^+,ep^+} \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu \frac{\ell^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \xi e + \frac{\beta \psi_{j+1,h}}{1-\frac{1}{\sigma}} E\left[\left[\left(1-\frac{1}{\sigma}\right) v(\mathbf{x}^+)\right]^{1+\gamma} \,\middle|\, j,s,m,\eta,h\right]^{\frac{1}{1+\gamma}}$$

with $\mathbf{x} = (j, s, m, \eta, h, a, ep)$ and $\mathbf{x}^+ = (j+1, s, m, \eta^+, h^+, a^+, ep^+)$. Households maximize their utility with respect to the budget constraint

$$(1+\tau_c)c + a^+ + T_p(y) + T\Big(y - T_p(y) + p\Big) = (1+r)a + y + p + b \quad \text{with } y = wz(j,s,m,\eta)e\ell$$

and the accumulation equation for pension claims

$$ep^+ = ep + \Big[\lambda \bar{y} e + (1-\lambda)\min\Big(wz(j,s,m,\eta)e\ell,\ 2\bar{y}\Big)\Big].$$

In the following, I assume that $y < 2\bar{y}$, meaning that the household is below the contribution ceiling of the pension system. Let us denote by μ_1 and μ_2 the multipliers on the budget constraint and the pension accumulation equation in the Lagrangian $\mathcal{L}$, respectively. The first-order conditions of the household then read

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c} &= c^{-\frac{1}{\sigma}} - \mu_1(1+\tau_c) = 0 \\
\frac{\partial \mathcal{L}}{\partial \ell} &= -\nu \ell^{\frac{1}{\chi}} + \Big[(1-\tau_p)\big(1-T'(y_{tax})\big)\mu_1 + (1-\lambda)\mu_2\Big]\, wz(j,s,m,\eta)e = 0 \\
\frac{\partial \mathcal{L}}{\partial a^+} &= -\mu_1 + \beta\psi_{j+1,h} E\left[M(\mathbf{x}^+)V_a(\mathbf{x}^+) \,\Big|\, j,s,m,\eta,h\right] = 0 \\
\frac{\partial \mathcal{L}}{\partial ep^+} &= -\mu_2 + \beta\psi_{j+1,h} E\left[M(\mathbf{x}^+)V_{ep}(\mathbf{x}^+) \,\Big|\, j,s,m,\eta,h\right] = 0
\end{aligned}$$

where $y_{tax} = y - T_p(y) + p$ and

$$M(\mathbf{x}^+) = E\left[\left[\left(1-\frac{1}{\sigma}\right)v(\mathbf{x}^+)\right]^{1+\gamma} \,\Bigg|\, j,s,m,\eta,h\right]^{\frac{-\gamma}{1+\gamma}} \times \left[\left(1-\frac{1}{\sigma}\right)v(\mathbf{x}^+)\right]^{\gamma}. \quad \text{(C.1)}$$

Note that the state-specific discount factor $M(\mathbf{x}^+)$ determines the weight a household attaches to different future events. In the case of standard CRRA preferences, i.e. when $\gamma = 0$, I have $M(\mathbf{x}^+) = 1$ and risk aversion solely emerges from the curvature of the household's utility functions. In case of $\gamma > 0$, the household attaches a higher weight to negative future events and therefore risk aversion increases.

Using the envelope theorem, I immediately obtain

$$\begin{aligned}
V_a(\mathbf{x}) &= (1+r)\mu_1 \quad \text{and} \\
V_{ep}(\mathbf{x}) &= \begin{cases} \mu_2 & \text{if } j < j_R \text{ and} \\ (1-T'(y_{tax}))\frac{\kappa}{j_R-20}\mu_1 + \mu_2 & \text{otherwise.} \end{cases}
\end{aligned}$$

Under the assumption of a time-invariant consumption tax rate, the Euler equation then reads

$$c(\mathbf{x})^{-\frac{1}{\sigma}} = (1+r)\beta\psi_{j+1,h} E\left[M(\mathbf{x}^+)c(\mathbf{x}^+)^{-\frac{1}{\sigma}} \,\Big|\, j,s,m,\eta,h\right]. \quad \text{(C.2)}$$

The first order condition for labor supply is

$$\begin{aligned}
\nu\ell(\mathbf{x})^{\frac{1}{\chi}} = &\Bigg[(1-\tau_p)\big(1-T'(y_{tax})\big)\frac{c(\mathbf{x})^{-\frac{1}{\sigma}}}{1+\tau_c} \\
&+ (1-\lambda)\beta\psi_{j+1,h} E\left[M(\mathbf{x}^+)V_{ep}(\mathbf{x}^+) \,\Big|\, j,s,m,\eta,h\right]\Bigg] wz(j,s,m,\eta)e(\mathbf{x}).
\end{aligned}$$

C.1.1 The measure of households

First, I construct the measure of households at age 20 across the characteristics (s, m, η, h, a, ep). Households draw one of two possible education levels $s \in \{0, 1\}$, where $s = 1$ occurs with probability ϕ_s. They are also assigned a career-path characteristic $m \in \{0, 1\}$, where $m = 1$ occurs with probability ϕ_m. Conditional on their career path m, households draw an initial labor productivity η at age 20 from the distribution $\pi_{\eta,20}(\eta \,|\, s, m)$, see equation (C.6). Finally, households enter the economy with average health $\bar{h}$, zero assets and zero pension claims. Thus,

$$\begin{aligned}
&\Phi(\{20\}, \{s\}, \{m\}, \{\eta\}, \{\bar{h}\}, \{0\}, \{0\}) = \\
&\qquad = \Big[s\phi_s + (1-s)(1-\phi_s)\Big] \times \Big[m\phi_m + (1-m)(1-\phi_m)\Big] \times \pi_{\eta,20}(\eta \,|\, s, m)
\end{aligned}$$

and zero otherwise.

I can then construct the probability measure for all ages $j > 1$. For all Borel sets of assets $\mathcal{A}$ and pension claims $\mathcal{EP}$ I have

$$\begin{aligned}
&\Phi(\{j+1\}, \{s\}, \{m\}, \{\eta^+\}, \{h^+\}, \mathcal{EP}, \mathcal{A}) = \\
&= \frac{\psi_{j+1,h} \times \pi_\eta(\eta^+ \mid \eta, j, s, m) \times \pi_h(h^+ \mid h, j, s, \eta)}{1+n} \\
&\quad \times \int \mathbb{1}_{\{a^+(j,s,m,\eta,h,a,ep) \in \mathcal{A}\}} \times \mathbb{1}_{\{ep^+(j,s,m,\eta,h,a,ep) \in \mathcal{EP}\}} \, \Phi(\{j\}, \{s\}, \{m\}, \{\eta\}, \{\bar{h}\}, dep, da)
\end{aligned}$$

where the integral is the measure of assets a and pension claims ep today such that, for fixed (j, s, m, η, h), the optimal choice today of assets for tomorrow $a^+(j, s, m, \eta, h, a, ep)$ lies in $\mathcal{A}$ and the optimal choice today of pension claims for tomorrow $ep^+(j, s, m, \eta, h, a)$ lies in $\mathcal{EP}$.

C.1.2 Computational algorithm

Following Kindermann and Krueger (2022), I solve the model in three steps. I apply the method of endogenous grid points to solve the household problem. I can than compute policy functions $c(x), \ell(x)$ and $a^+(x)$, as well as the value function $v(x)$. Second, I determine equilibrium quantities and prices following closely the Gauss-Seidel-Quasi-Newton procedure proposed in Ludwig (2007). Finally, I calculate compensating transfers using a standard rootfinding method.

Computation of policy and value functions

I use the method of endogenous gridpoints to compute the policy and value functions. The state space of the quantitative model is $\mathbf{x} = (j, s, m, \eta, h, a, ep)$. To solve the model on a computer, I start with discretizing the continuous elements a, ep and η. I use routines provided by the toolbox that accompanies Fehr and Kindermann (2018).

- I specify the asset grid $\hat{\mathcal{A}} = \{\hat{a}_0, \ldots, \hat{a}_{100}\}$ as nodes with growing distance on the interval $[\bar{a}_l, \bar{a}_u]$. In particular, I let

$$\hat{a}_i = \bar{a}_l + \frac{\bar{a}_u - \bar{a}_l}{(1+g_a)^{100} - 1} \times [(1+g_a)^i - 1] \text{ for } i = 0, 1, \ldots, 100.$$

  The lower limit of the asset grid is $\bar{a}_l = 0$, the upper limit of the asset grid is $\bar{a}_u = 70$, the growth rate of gridpoints $g_a = 0.08$.
- I specify the earnings points grid $\widehat{\mathcal{EP}} = \{\hat{ep}_0, \ldots, \hat{ep}_{30}\}$ as a grid with $\bar{ep}_l = 0$, $\bar{ep}_u = 2$ and equally spaced nodes.
- I approximate the stochastic process of the AR(1) labor productivity process of normal labor earnings by a Markov chain. I use the Rouwenhorst method to discretize the stochastic process of the innovations[1] $\hat{\mathcal{E}} = \{\hat{\eta}_{1,s}, \ldots, \hat{\eta}_{7,s}\}$ and to determine a transition matrix

$$\pi_\eta(\eta^+|\eta, j, s) = \begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{17} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{27} \\ \vdots & \vdots & \ddots & \cdots \\ \pi_{71} & \pi_{72} & \cdots & \pi_{77} \end{bmatrix} \tag{C.3}$$

  for each states s and age j.
- In order to account for the low productivity shocks, I extend the stochastic process $\hat{\mathcal{E}}$ to $\hat{\mathcal{E}} = \{\hat{\eta}_{0,s}, \hat{\eta}_{1,s}, \ldots, \hat{\eta}_{7,s}\}$ and augment the 7×7 Markov transition

[1] Where ρ_s and $\sigma^2_{\varepsilon,s}$ are as specified in Table 5.1 and $\mu = 0$.

matrix as outlined in Appendix C.2.2.

- I determine the health shocks $h \in \{0, \ldots, 7\}$ and the transitions matrix $\pi_h(h^+|h, j, s, \eta)$ as outlined in Section 5.3.1.

The policy and value functions can now be solved via backward induction. In the last possible age J, the household will not work[2] and not save, but will consume all remaining resources. This determines the policy functions

$$\begin{aligned} c(J, s, m, \hat{\eta}_g, h, \hat{a}_i, \hat{ep}_k) &= \frac{(1+r) \times \hat{a}^i + p - T(p) + b}{1+\tau_c}, \\ l(J, s, m, \hat{\eta}_g, h, \hat{a}_i, \hat{ep}_k) &= 0, \\ a^+(J, s, m, \hat{\eta}_g, h, \hat{a}_i, \hat{ep}_k) &= 0 \end{aligned}$$

and the value function

$$v(J, s, m, \hat{\eta}_g, h, \hat{a}_i, \hat{ep}_k) = \frac{\left[c(J, s, m, \hat{\eta}_g, h, \hat{a}_i, \hat{ep}_k)\right]^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

for all $i = 0, \ldots, 100,\ k = 0, \ldots, 30,\ g = 0, \ldots, 7$.
With the final period policy functions and value function at hand, I can iterate backwards over ages to determine the full history of household decisions. I describe the procedure for working-age households. Assume the problem is solved for age $j+1$, then the problem for an individual at state $\mathbf{x} = (j, s, m, \eta, h, a, ep)$ reads

$$\begin{aligned} v(\mathbf{x}) = \max_{c,\ell,e,a^+,ep^+} &\ \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu \frac{\ell^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \xi e \\ &- \beta\psi_{j+1,h} \left[\sum_{\eta^+} \pi_\eta(\eta^+|\eta) \sum_{h^+} \pi_h(h^+|h) \cdot [-v(\mathbf{x}^+)]^{1+\gamma}\right]^{\frac{1}{1+\gamma}} \end{aligned} \tag{C.4}$$

with $\mathbf{x} = (j, s, m, \eta, h, a, ep)$ and $\mathbf{x}^+ = (j+1, s, m, \eta^+, h^+, a^+, ep^+)$. Households maximize their utility with respect to the budget constraint

$$(1+\tau_c)c + a^+ + T_p(y) + T\Big(y - T_p(y) + p\Big) = (1+r)a + y + p + b \qquad \text{with } y = wz(j, s, m, \eta)e\ell,$$

the accumulation equation for pension claims

$$ep^+ = ep + \Big[\lambda\bar{y}e + (1-\lambda)\min\Big(wz(j, s, m, \eta)e\ell,\ 2\bar{y}\Big)\Big].$$

and the positive asset restriction $a^+ \geq 0$ and the time restriction $0 \leq \ell \leq 1$. The

[2]Remember, the compulsory retirement age is j_r.

first order conditions are outlined in Appendix C.1.

I now apply the method of endogenous gridpoints. I first define an exogenous grid on the state space $\{\hat{a}_v\}_{v=0}^{100}$, which denotes the remainder of assets in the next period, i.e. $a^+ = \hat{a}_v$. For each state $\tilde{\mathbf{x}} = (j, s, m, \eta, h, a^+, ep)$, I

- search for the optimal $\ell(\tilde{\mathbf{x}})$ according to the first order condition (C.2) using a quasi-Newton rootfinding method
  1. given $\ell(\tilde{\mathbf{x}})$[3] I determine

$$ep^+ = \frac{(j-1)ep}{j} + \frac{\lambda\bar{y}e + (1-\lambda)\min\Big(wz(j,s,m,\eta)e\ell,\ 2\bar{y}\Big)}{j}$$

  2. given a^+ and ep^+ I determine $c(\tilde{\mathbf{x}})$ from the Euler Equation (C.2)
  3. with $l(\tilde{\mathbf{x}})$ and $c(\tilde{\mathbf{x}})$, I use the budget constraint (C.5) to get $a(\tilde{\mathbf{x}})$
- once $l(\tilde{\mathbf{x}})$, $c(\tilde{\mathbf{x}})$ and $a(\tilde{\mathbf{x}})$ are solved, I can interpolate along a to obtain the policy functions $l(\mathbf{x})$, $c(\mathbf{x})$ and $a^+(\mathbf{x})$ as well as the value function

$$v(\mathbf{x}) = \frac{c(\mathbf{x})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu\frac{\ell(\mathbf{x})^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \xi e(\mathbf{x}) - \beta\psi_{j+1,h}\left[\sum_{\eta^+}\pi_\eta(\eta^+|\eta)\sum_{h^+}\pi_h(h^+|h)\cdot[-v(\mathbf{x}^+)]^{1+\gamma}\right]^{\frac{1}{1+\gamma}}, \quad \text{(C.5)}$$

  for each todays asset value $\hat{a}_i$, $i = 0, \ldots, 100$ and earnings points amount $\hat{ep}_k, k = 0, \ldots, 30$ by piecewise linear interpolation[4]

In case the asset restriction $a^+ \geq 0$ is binding, I extend the interpolation data by another point of value 0 on the left and determine the policy and value functions at this point. I assume the household consumes all available resources and has no savings left over for tomorrow.

C.1.3 The initial equilibrium of the macroeconomy

I model a small open economy, hence prices r and w are fixed. In order to determine aggregate quantities and policy parameters in the initial equilibrium ($t = 0$) I need to determine the following four variables numerically:

[3] I guess $\ell = \ell^+$ in the first iteration

[4] Note, I interpolate $\left[(1-\frac{1}{\sigma})v(x)\right]^{\frac{1}{1-\frac{1}{\sigma}}}$ rather than $v(x)$ directly and than transform it back to the original shape. This leads to more accurate results for discretized functions with high curvature.

- the government budget balancing consumption tax rate τ_c as outlined in Equation (5.14)
- the pension replacement rate κ that balances pension contributions and pension payments as outlined in equation (5.13)
- average earnings $\bar{y}_t$[5]
- aggregate bequests $\bar{B}$, which immediately allows us to compute cohort bequests $\{b_j\}_{j=20}^{J}$ (equally shared between all working-age individuals).

Once a guess of these four variables is available, I can use the following algorithm to compute the remainder individual and aggregate variables of the economy:

1. I solve the household optimization problem using the guesses for $\tau_c, \kappa, \bar{y}, \bar{B}$ and determine the measure of households.
2. I compute aggregate quantities $\{L, K, TB, Y, C, G, I, \Omega, B\}$ from individual decisions and the measure of household and determine the gap $D = Y - C - I - G$ between demand and supply.

I determine the four central parameters $(\tau_c, \kappa, \bar{y}_t, \bar{B})$ by means of a quasi-Newton rootfinding method. The method receives an initial guess of these variables and updates them in each iteration step using the Jacobian of the determining equation system. The iteration process stops when the government and the pension budget are in equilibrium and the model implied average earning and aggregate bequest equal the guess provided by the method. After the iteration procedure has finished, I extract the Jacobian which is essential for running a Gauss-Seidel-Quasi-Newton method as proposed in Ludwig (2007) to compute the transitional dynamics.

C.1.4 The transition path of the macroeconemy

To quantify the intergenerational effects of the pension reforms, I simulate the economy along the transition path. I distinguish between different simulation periods $t = 0, 1, \ldots, T$, where period $t = 0$ is the initial equilibrium as determined before. In period $t = 1$, the pension reform is introduced such that households adopt their decisions and the macroeconomy adjusts. The economy slowly converges to a new long-run equilibrium, which is reached after $T = 300$ periods in the numerical model.

While most variables and parameters are time dependent, government consumption G is fixed. The measure for average earnings $\bar{y}_t$ adjusts along the transition. However, I use the value from the initial equilibrium $\bar{y}_0$ as reference value for computing

[5] This is an important parameter, as it determines the pension contribution cap, pension payments and earnings of the low-earning group.

policy parameters such as pension payments. As a result, I only have to determine $\tau_{c,t}, \kappa_t$ and aggregate bequests $\bar{B}_t$ in each period $t = 1, 2, \ldots, T$ of the transition.

I use the Gauss-Seidel-Quasi-Newton procedure proposed by Ludwig (2007) to solve for our variables of interest. This procedure works like a standard rootfinding method with the difference that the Jacobian is not computed numercially but initialized using the initial equilibrium Jacobian. To speed up the computational process, I use openMP to parallelize the computation of household decisions and invariant household measure across different cohorts.

C.1.5 Welfare calculations

The welfare calculation follows Fehr and Kindermann (2015) and Kindermann and Krueger (2022).

Consumption equivalent variation

I calculate utility of each household recursively according to

$$v_t(\mathbf{x}) = u\Big(c_t(\mathbf{x}), \ell_t(\mathbf{x}), e_t(\mathbf{x})\Big) - \beta\psi_{j+1,h} E_t \left[\Big(-v_{t+1}(\mathbf{x}^+)\Big)^{1+\gamma}\right]^{\frac{1}{1+\gamma}}.$$

In addition, I can calculate the discounted marginal utility of consumption as

$$PVC_t(\mathbf{x}) = u_c\Big(c_t(\mathbf{x}), \ell_t(\mathbf{x}), e_t(\mathbf{x})\Big) - \beta\psi_{j+1,h} E_t \left[M(\mathbf{x}^+) PVC_{t+1}(\mathbf{x}^+)\right],$$

with the stochastic discount factor defined as in (C.1).

Ex-ante expected utility of a cohort born at some date t then is given by

$$EV_t = -E_0 \left[(-v_t(x))^{1+\gamma}\right]^{\frac{1}{1+\gamma}}.$$

where $\mathbf{x} = \{1, s, m, \eta, \bar{h}, 0, 0\}$ and E_0 uses the invariant distribution of this cohort at age $j = 1$. Similarly, I can calculate the ex-ante discounted marginal utility of consumption as

$$PVC_t = E_0 \left[M(\mathbf{x}) PVC_t(x)\right].$$

Let us denote by $\overline{EV}$ and $\overline{PVC}$ the ex-ante welfare measure and the discounted marginal utility of consumption of a cohort that was born and has lived entirely in the initial equilibrium with a proportional pension system. I compute the consumption equivalent variation welfare measure between this initial cohort and any other cohort that was born at some date t and has experienced the pension reform as

$$CEV_t = \frac{EV_t - \overline{EV}}{\overline{PVC}}.$$

Efficiency measure

To derive our measure of aggregate efficiency, I numerically compute the transfer payment Ψ_t that I have to give to each cohort affected by the pension reform so as to make this cohort as well of as the initial equilibrium cohort. Technically, I use a quasi-Newton method and determine the payment Ψ_t such that

$$EV_t(\Psi_t) = \overline{EV}.$$

The negative of Ψ_t is a monetary measure of the welfare increase the cohort t experiences from the pension reform.

I then derive the present value of all transfers, which gives us a wealth-based measure W of the economic efficiency effect

$$W = \sum_{t=-J+1}^{\infty} \left[\frac{1+n}{1+\bar{r}}\right]^t \Psi_t.$$

I convert the wealth-based measure into an annuity that pays out a constant stream along the transition path and in the new long-run equilibrium:

$$C = W \times \left[\sum_{t=1}^{\infty} \left[\frac{1+n}{1+\bar{r}}\right]^t\right]^{-1}.$$

Our final measure of economic efficiency relates this annuity stream to the initial equilibrium aggregate consumption level, i.e., I compute

$$\varphi = -\frac{C}{C_0} \cdot 100.$$

Note that I have to use $-C$ in this computation, as I want our measure to be positive when the economy experiences aggregate efficiency gains.

C.2 Further information on the calibration process

C.2.1 Estimating model-implied participation elasticities

For estimating participation elasticities I follow the evidence from Table 2(2) in Bartels and Pestel (2016). They empirically test to what extent a lower participation tax rate PTR is associated with an increased probability of taking up work. They define a household's participation tax rate as

$$PTR_{ih} = \frac{T(y_h^E) - T(y_h^U)}{y_i^{E,w}},$$

where y_h^E is gross household income (i.e. the sum of labor earnings, asset income and transfers of all household members), $T(y_h^E)$ is a household's net taxes and $y_i^{E,w}$ are labor earnings of individual i when being employed E. $T(y_h^U)$ denotes a household's net taxes if individual i is unemployed U. The binary outcome variable *switch* takes a value of one if individual i switches from non-participation in period $t-1$ to participation in period t. Bartels and Pestel (2016) estimate the effect of changes in the short-term participation tax rate Δ_{PTR} on male labor force participation in Germany, evaluated at 40 h, using the following statistical model:

$$\begin{aligned} switch = b_1\Delta_{PTR} + b_2 Age_{35-44} + b_3 Age_{45-54} + b_4\Delta_{U-rate} + b_5 East \\ + b_6\, Year_{FE} + b_7 HH_{FE} + b_8 Skill_{FE} + \epsilon. \end{aligned}$$

b_1 is the coefficient of interest, which takes a value of -0.106 and is significant at the 1% level. The impact of changes in the the short-term participation tax rate on the probability to take up work is substantial. Reducing the participation tax rate by 10 percentage points increases the probability of taking up work by 1.06 percentage points. Coefficients on age-group dummies, changes in the unemployment rate and on whether a household is located in East Germany b_2, b_3, b_4 and b_5 are all insignificant.

I adopt this method to estimate the participation elasticity implied by my model using simulated data. I restrict the simulated data such that it corresponds to the data selection of Bartels and Pestel (2016). I meet most of the specifications by construction as, for instance, self-employed, civil servants and disabled individuals are not represented in my model anyway. I limit the analysis to individuals of ages 25 to 54.

The measure for PTR is constructed as follows: I estimate participation taxes in the benchmark equilibrium of my model that most closely resembles the German economy. For each potential household characterized by the state vector

$\mathbf{x} = (j, s, m, \eta, h, a, ep)$ with $j \in \{25, \ldots, 54\}$, I compute the initial share of employed individuals $e(\mathbf{x})$, the initial taxable income

$$y_{tax}(\mathbf{x}) = y(\mathbf{x}) - \tau_p \min\Big(y(\mathbf{x}), 2\bar{y}\Big) \quad \text{with} \quad y(\mathbf{x}) = wz(j, s, m, \eta)\ell(\mathbf{x})$$

and the initial participation tax rate as

$$PTR(\mathbf{x}) = \frac{T_p\Big(y(\mathbf{x})\Big) + T\Big(y_{tax}(\mathbf{x})\Big)}{y(\mathbf{x})}.$$

Next, I reduce the contribution rate to the pension system τ_p by 10 percentage points without recalculating equilbrium prices. Under this new contribution rate, I compute a new share of employed households $e_{new}(\mathbf{x})$ and a new participation tax rate $PTR_{new}(\mathbf{x})$.

Under the benchmark equilibrium, a fraction $1 - e(\mathbf{x})$ of households was not in employment. Under the system with a lower pension contribution rate, the fraction of non-employed changed to $1 - e_{new}(\mathbf{x})$. I split the sample of $1 - e(\mathbf{x})$ non-employed individuals into those $e_{new}(\mathbf{x}) - e(\mathbf{x})$ that switched from non-employment to employment and assign to them a value of 1 for the variable *switch*. For the other $1 - e_{new}(\mathbf{x})$ that remained in non-employment, *switch* takes a value of 0. The change in the participation tax rate of these individuals is equal to

$$\Delta_{PTR} = PTR_{new}(\mathbf{x}) - PTR(\mathbf{x}).$$

To account for the distribution of households over the state-space, I create a weighted data set using the distribution $\Phi(\cdot)$ as individual weights. In addition, I collect households' age and education level.

Employing this simulated data and the empirical evidence of Bartels and Pestel (2016), I use the method of indirect inference to calibrate the variance σ_ξ^2 of participation costs ξ. In particular, I run the following regression on my simulated data

$$switch = b_0 + b_1\Delta_{PTR} + b_2 Age_{35-44} + b_3 Age_{45-54} + b_8 College + \epsilon$$

and target a participation elasticity b_1 of -0.106. Stetting σ_ξ^2 to 0.138 delivers exactly this value. This means that the probability of switching from non-employment to employment after reducing the pension contribution rate τ_p by 10 percentage points (from 0.1870 to 0.0870) increases by 1 percentage point. This change is substantial given a benchmark participation rate of 87% for the age group 24-54. Unlike in Bartels and Pestel (2016), coefficients on the age and college dummies are significant. However, this is not surprising given that the simulated data set features more than 1.6 million observations. Table C.1 provides details on the estimation results from my simulated data.

Table C.1: Effect of Δ_{PTR} on the probability of taking up work

Switch (U → E)	
Δ_{PTR}	−0.099 (0.0175)
Age_{35-44}	−0.0055 (0.0005)
Age_{45-54}	0.0223 (0.0005)
$College$	−0.0023 (0.0007)

Observations: 1,639,696, standard errors in parenthesis.

C.2.2 Parameterizing labor productivity

This section provides further details on the calibration of labor productivity profiles and productivity risk as outlined in Section 5.3.3.

Normal labor productivity I first concentrate on normal labor productivity, meaning the labor productivity process of individuals with permanent state $m = 0$. Labor earnings and labor productivity are not identical when individual labor hours vary across ages and states, as they do in my quantitative model. Hence, I can not simply take the labor earnings estimates one for one. Instead, to calibrate the process of normal labor productivity, I proceed as follows: I assume the average labor productivity profile to evolve according to

$$\theta_{j,s} = b_{0,s} + b_{1,s}\frac{\min(j, j_{M,s})}{10} + b_{2,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^2 + b_{3,s}\left[\frac{\min(j, j_{M,s})}{10}\right]^3.$$

This functional form is flexible enough to capture both a hump-shaped ($j_{M,s} = \infty$) and a stagnating ($j_{M,s} < j_R$) life-cycle labor productivity profile. Note that in the case of a stagnating profile, labor productivity is constant from age $j_{M,s}$ onward. I calibrate the coefficients of this polynomial such that my model implied average labor earnings profile for each education type matches its empirical counterpart.

Figure 5.3 compares the empirical and model implied average earnings profiles.[6] The top panel of Table 5.1 in the main text shows the calibrated values for the polynomial coefficients $b_{i,s}$ and the stagnation thresholds $j_{M,s}$.

Next, I model residual labor productivity as an AR(1) process. In particular, I discretize the AR(1) process by a seven state Markov chain using a Rouwenhorst method, see Kopecky and Suen (2010). As autocorrelation parameter ρ_s I directly use the estimates from Table 4.3. I then calibrate the innovation variance $\sigma^2_{\varepsilon,s}$ such that the model implied variance of residual labor earnings equals its empirical counterpart, see Table 4.3. In doing so, I obtain a set of seven productivity realizations $\{\eta_{1,s}, \dots, \eta_{7,s}\}$ as well as a transition matrix π^s that governs the transition between these seven normal productivity states.

Low labor productivity shocks The shock process for low labor productivity shocks follows the structure discussed in Section 4.1.4. In particular, I assume that at the beginning of life ($j = 1$) a fraction ω^s_{low} of households with permanent state $m = 1$ starts in the low productivity state. The share $1 - \omega^s_{low}$ has normal labor productivity. Individuals transition between the state of normal productivity and a low productivity shock according to the transition matrix specified in (5.20). I take the estimates of the initial share of households as well as the transition matrix directly from my empirical findings as summarized in Table 4.4. When individuals draw the low labor productivity shock, they get assigned a labor productivity level of $\exp(\eta_0) = 0.17$. This productivity level ensures that the average earnings of low productivity workers are equal to 10 percent of the average labor earnings of the total population, see the right panel of Figure 4.3.

Bringing the two processes together At the beginning of life, a fraction ϕ^s_m of households of education level s draws a permanent shock $m = 1$. These households face a labor productivity process that combines normal labor productivity with low productivity shocks. Households with $m = 0$, on the other hand, only experience a normal labor productivity process. I set the transition matrix between potential labor productivity states $\{\eta_0, \eta_{1,s}, \dots, \eta_{7,s}\}$ to

[6]Note that, owing to the log-normal nature of labor productivity shocks, the model-implied average life-cycle wage profile is equal to

$$\exp\left(\theta_{j,s} + \frac{\sigma_s^2}{2}\right).$$

$$\pi_\eta(\eta^+|\eta, j, s, m) = \begin{bmatrix} m\pi^s_{low,1} & (1-m\pi^s_{low,1})\phi^s_\eta(1) & \dots & (1-m\pi^s_{low,1})\phi^s_\eta(7) \\ m\pi_{low,0} & (1-m\pi^s_{low,0})\pi^s_{11} & \dots & (1-m\pi^s_{low,0})\pi^s_{17} \\ m\pi_{low,0} & (1-m\pi^s_{low,0})\pi^s_{21} & \dots & (1-m\pi^s_{low,0})\pi^s_{27} \\ \vdots & \vdots & \dots & \vdots \\ m\pi_{low,0} & (1-m\pi^s_{low,0})\pi^s_{71} & \dots & (1-m\pi^s_{low,0})\pi^s_{77} \end{bmatrix}.$$

Hence, when being in the normal productivity state, households transition into the low productivity state η_0 with a constant probability $m\pi_{low,0}$, meaning 0 when $m = 0$ and $\pi_{low,0}$ when $m = 1$. Once they are facing low productivity, they stay in the low productivity state with probability $m\pi^s_{low,1}$. If they revert to normal productivity, they draw a regular productivity shock from the unconditional distribution $\phi^s_\eta(i)$.

At the beginning of life, individuals are distributed over the potential productivity levels $\{\eta_0, \eta_{1,s}, \dots, \eta_{7,s}\}$ according to the distribution

$$\pi_{\eta,20}(\eta \,|\, m, s) = \begin{bmatrix} m\omega^s_{low} & (1-m\omega^s_{low})\phi^s_\eta(1) & \dots & (1-m\omega^s_{low})\phi^s_\eta(7) \end{bmatrix}. \tag{C.6}$$

Hence, those individuals who do not experience low productivity from the outset of their life draw an initial labor productivity from the unconditional distribution of the normal productivity process. Finally, individual labor productivity is given by

$$z(j, s, m, \eta_{i,s}) = \begin{cases} \exp(\theta_{j,s} + \eta_{i,s}) & \text{if } i > 0 \text{ and} \\ \exp(\eta_0) & \text{otherwise.} \end{cases}$$

Agents with a low productivity shock consequently have a productivity level that is independent of age.

Appendix D

Lifetime Earnings Redistribution - Favorable or Costly?

D.1 The household problem

D.1.1 First-order conditions for the single household

In the following, I describe the solution of the single's household problem in an economy with a proportional pension system. Since labor supply constitutes a discrete choice I can not formulate a first-order condition for labor supply. Instead I follow Fehr and Kindermann (2018) and solve the problem in two steps. First, I assume the household had already made a labor supply choice $\ell \leq h$. Conditional on this labor supply decision, I determine the optimal consumption-saving decision by solving the conditional optimization problem $\tilde{v}(\mathbf{x_s}, \ell)$.

Consumption – savings choice: The dynamic household optimization problem reads

$$\tilde{v}(\mathbf{x_s}, \ell) = \max_{c,a^+} \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu \frac{\left(\zeta_k + \ell\right)^{1+\frac{1}{\chi_g}}}{1+\frac{1}{\chi_g}} - \xi \times \mathbb{1}_{\ell>0} + \beta \psi^s_{j+1} E\left[v(\mathbf{x_s^+}) \middle| j, s, \eta, g, h, k, \ell \right]$$

with $\mathbf{x_s} = (j, g, s, \eta, h, k, \xi, a, e)$ and $\mathbf{x_s^+} = (j+1, g, s, \eta^+, h^+, k^+, \xi^+, a^+, e^+)$. Households maximize their conditional utility with respect to the borrowing constraint

$a^+ \geq 0$, and the budget constraint

$$c \times (1+\tau_c)\upsilon_{j,k,i} + a^+ + T_p(\min(y, 2\bar{y})) + T\Big(y - T_p(y^p) + p\Big)$$
$$= (1+r)a + y + y_{mini} + h + t_{cb} + t_{cs} + b. \quad \text{(D.1)}$$

The first-order conditions of the household then read

$$\frac{\partial \mathcal{L}}{\partial c} = c^{-\frac{1}{\sigma}} - \mu(1+\tau_c)\upsilon_{j,k,s} = 0$$
$$\frac{\partial \mathcal{L}}{\partial a^+} = -\mu + \beta\psi_{j+1} E\left[v_a(\mathbf{x}_\mathbf{s}^+) \;\Big|\; j, s, \eta, g, h, k, \ell\right] = 0,$$

where μ is the multiplier on the budget constraint in the Lagrangian $\mathcal{L}$. Using the envelope theorem, I immediately obtain

$$v_a(\mathbf{x}_\mathbf{s}^+) = (1+r)\mu^+.$$

The Euler equation then reads

$$\frac{c(\mathbf{x}_\mathbf{s}, \ell)^{-\frac{1}{\sigma}}}{\upsilon_{j,k,s} \times (1+\tau_c)} = (1+r)\beta\psi_{j+1,g} E\left[\frac{c(\mathbf{x}_\mathbf{s}^+, \ell^+)^{-\frac{1}{\sigma}}}{\upsilon_{j+1,k^+,s} \times (1+\tau_c^+)} \;\Bigg|\; j, s, \eta, g, h, k, \ell\right]. \quad \text{(D.2)}$$

The Euler equation (D.2) and the budget constraint (D.1) define the optimal level of consumption $c(\mathbf{x}_\mathbf{s}, \ell)$, savings $a^+(\mathbf{x}_\mathbf{s}, \ell)$ and the utility value $\tilde{v}(\mathbf{x}_\mathbf{s}, \ell)$ conditional on a certain labor supply decision $\ell \in (0, 0.4)$ for male and $\ell \in (0, 0.1, 0.2, 0.4)$ for female households.[1]

Furthermore, the law of motion of pension entitlements

$$e^+(\mathbf{x}_\mathbf{s}, \ell) = e + \min\left[y(\mathbf{x}_\mathbf{s}, \ell) + \frac{\tau_p^{mini}}{\tau_p} y_{mini}(\mathbf{x}_\mathbf{s}, \ell),\ 2\bar{y}\right] \quad \text{(D.3)}$$

determines the conditional pension entitlements $e^+(\mathbf{x}_\mathbf{s}, \ell)$.

Labour supply decision: Given the utility $\tilde{v}(\mathbf{x}_\mathbf{s}, \ell)$ for every possible $\ell \leq h$ the utility maximizing labor supply decision is

$$\ell(\mathbf{x}_\mathbf{s}) = \arg\max \tilde{v}(\mathbf{x}_\mathbf{s}, \ell).$$

[1] The female household might be constraint in the labor choice set, $\ell \leq h$ must hold.

D.1.2 First-order conditions for the couple household

The solution strategy to the couple's household problem is similar. I first solve the consumption-saving problem, conditional on the labor supply choices $\ell_m \in (0, 0.4)$ and $\ell_f \in (0, 0.1, 0.2, 0.4)$. I compute for every possible $(l_m, l_f \leq h)$ combination

$$
\begin{aligned}
\tilde{v}(\mathbf{x_c}, \ell_m, \ell_f) = \max_{\substack{c_m, c_f, \\ a^+, e_m^+, e_f^+}} & \left[\frac{c_m^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu_m \frac{(\lambda^c_{\zeta_{m,k}} \zeta_k + \ell_m)^{1+\frac{1}{\chi_m}}}{1+\frac{1}{\chi_m}} - \xi \times \mathbb{1}_{\ell_m > 0} \right. \\
& \left. + \frac{c_f^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu_f \frac{(\lambda^c_{\zeta_{f,k}} \zeta_k + \ell_f)^{1+\frac{1}{\chi_f}}}{1+\frac{1}{\chi_f}} - \xi \times \mathbb{1}_{\ell_f > 0} \right] \\
& + \beta \psi^c_{j+1} E \Big[v(\mathbf{x_c^+}) \,\Big|\, j, s_m, s_f, \eta_m, \eta_f, h, k, \ell_f \Big]
\end{aligned}
$$

with $\mathbf{x_c} = (j, s_m, s_f, \eta_m, \eta_f, k, h, \xi, a, e_m, e_f)$ and $\mathbf{x_c^+} = (j+1, s_m, s_f, \eta_m^+, \eta_f^+, k^+, h^+, \xi^+, a^+, e_m^+, e_f^+)$. Couples maximize their conditional utility with respect to the borrowing constraint $a^+ \geq 0$, and the budget constraint

$$
\begin{aligned}
a^+ + (c_m + c_f)(1+\tau_c)\upsilon_{j,k,c} + T_p(y^p) + 2T\Big[0.5\big(y - T_p(\min(y, 2\bar{y})) + p\big)\Big] \\
= (1+r)a + y + y_{mini} + p + t_{cb} + b. \quad \text{(D.4)}
\end{aligned}
$$

The first-order conditions read

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_g} &= c_g^{-\frac{1}{\sigma}} - \mu(1+\tau_c)\upsilon_{j,k,c} = 0 \\
\frac{\partial \mathcal{L}}{\partial a^+} &= -\mu + \beta \psi_{j+1} E\left[v_a(\mathbf{x_c^+}) \,\Big|\, j, s_m, s_f, \eta_m, \eta_f, h, k, \ell_f \right] = 0,
\end{aligned}
$$

where μ is the multiplier on the budget constraint in the Lagrangian $\mathcal{L}$. It immediately follows $c_m = c_f$. Using the envelope theorem, I obtain

$$
v_a(\mathbf{x_c^+}) = (1+r)\mu^+.
$$

The Euler equation then reads

$$
\frac{c_g(\mathbf{x_c}, \ell_f, \ell_m)^{-\frac{1}{\sigma}}}{\upsilon_{j,k,c}(1+\tau_c)} = (1+r)\beta\psi^c_{j+1} E\left[\frac{c_g^{-\frac{1}{\sigma}}(\mathbf{x_c^+}, \ell_f^+, \ell_m^+)}{\upsilon_{j+,k^+,c}(1+\tau_c^+)} \Bigg| \, j, s_m, s_f, \eta_m, \eta_f, h, k, \ell_f \right]. \quad \text{(D.5)}
$$

The Euler equation (D.5) and the budget constraint (D.4) define the optimal level of consumption $c_g(\mathbf{x_c}, \ell_m, \ell_f)$, savings $a^+(\mathbf{x_c}, \ell_m, \ell_f)$ and the utility value $\tilde{v}(\mathbf{x_c}, \ell_m, \ell_f)$ conditional on a certain labor supply decision (ℓ_m, ℓ_f).

Furthermore, the law of motion of pension entitlements

$$e_g^+(\mathbf{x_c}, \ell_m, \ell_f) = e_g + \min\left[y^g(\mathbf{x_c}, \ell_m, \ell_f) + \frac{\tau_p^{mini}}{\tau_p} y_{mini}^g(\mathbf{x_c}, \ell_m, \ell_f),\ 2\bar{y}\right] \tag{D.6}$$

determines the conditional pension entitlements for each partner $e_g^+(\mathbf{x_c}, \ell_m, \ell_f)$.

Labour supply decision: Given the conditional utility $\tilde{v}(\mathbf{x_c}, \ell_m, \ell_f)$ for every possible $(\ell_m, \ell_f \leq h)$ combination, the household chooses the utility maximizing labor supply combination

$$\Big(\ell_m(\mathbf{x_c}), \ell_f(\mathbf{x_c})\Big) = \arg\max \tilde{v}(\mathbf{x_c}, \ell_m, \ell_f).$$

D.2 The measure of households

The population consists of couple and single households which operate on different state spaces. At age 20, the mass of couple households Φ^c and the mass of single households Φ^s sum to one.

Couple households At age 20, couple households draws one of four possible education level (s_m, s_f) from the joint distribution $\phi_s^c(s_m, s_f)$. Conditional on the their education level, each partner draws an initial labor productivity η from the distribution $\pi_{\eta,20}$, see equation (6.26). The household enters the economy without kids $k = 0$, the full labor choice set $h = 0.4$, zero assets $a = 0$ and zero pension claims $e_m = e_f = 0$. The realization of ξ follows a log-normal distribution with mean μ_ξ and variance σ_ξ^2. Thus, the measure of couple households with characteristics $\mathbf{x_c} = (s_m, s_f, \eta_m, \eta_f, h, k, \xi, a, e_m, e_f)$ is constructed as

$$\Phi^c(\{20\}, \{s_m\}, \{s_f\}, \{\eta_m\}, \{\eta_f\}, \{0\}, \{0.4\}, \{\xi\}, \{0\}, \{0\}, \{0\}) = \\ \phi_i \times \phi_s^c(s_m, s_f) \times \pi_{\eta_m,20}(\eta_m \mid s_m) \times \pi_{\eta_f,20}(\eta_f \mid s_f),$$

and zero otherwise.

I can then construct the probability measure for all ages $j > 1$. For all Borel sets of assets $\mathcal{A}$ and pension claims of the husband $\mathcal{EP}_m$ and the wife $\mathcal{EP}_f$ I have

$$\Phi^c(\{j+1\}, \{s_m\}, \{s_f\}, \{\eta_m^+\}, \{\eta_f^+\}, \{h^+\}, \{k^+\}, \{\xi^+\}, \mathcal{A}, \mathcal{EP}_m, \mathcal{EP}_f) = \\ = \frac{\psi_{j+1}^c \times \pi_\eta(\eta_m^+ \mid \eta_m, s) \times \pi_\eta(\eta_f^+ \mid \eta_f, s) \times \pi_h(h^+ \mid h, \ell_f, 1) \times \pi_k(k^+ \mid k, j, 1) \times \pi_\xi(\xi^+)}{1+n} \\ \times \int \mathbb{1}_{\{a^+(\mathbf{x_c}) \in \mathcal{A}\}} \times \mathbb{1}_{\{e_m^+(\mathbf{x_c}) \in \mathcal{EP}_m\}} \times \mathbb{1}_{\{e_f^+(\mathbf{x_c}) \in \mathcal{EP}_f\}} \\ \Phi^c(\{j\}, \{s_m\}, \{s_f\}, \{\eta_m\}, \{\eta_f\}, \{k\}, \{h\}, \{\xi\}, da, de_m, de_f),$$

where the integral is the measure of assets a and pension claims e_m and e_f today, such that for fixed $(j, s_m, s_f, \eta_m, \eta_f, h, k, \ell_f)$, the optimal choice today of assets for tomorrow $a^+(\mathbf{x_c})$ lies in $\mathcal{A}$ and the optimal choice today of pension claims for tomorrow $e_m^+(\mathbf{x_c})$ and $e_f^+(\mathbf{x_c})$ lie in $\mathcal{EP}_m$ and $\mathcal{EP}_f$, respectively.

Single households Next, I construct the measure of single households across the characteristics $\mathbf{x_s} = (g, s, \eta, h, k, \xi, a, e)$. At age 20, households draw a gender $g \in \{0, 1\}$ and a education level $s \in \{0, 1\}$, where $g = 1$ occurs with probability ϕ_g and $s = 1$ with probability ϕ_s^g. Conditional on the education level, households draw an initial labor productivity η from the distribution $\pi_{\eta,20}$, see equation (6.26). Households enter the economy without kids, the full labor choice set $h = 0.4$, zero assets and zero pension claims. Thus,

$$\Phi^s(\{20\},\{g\},\{s\},\{\eta\},\{0\},\{0.4\},\{\xi\},\{0\},\{0\}) = \phi_g(g) \times \phi_s^g(s_g) \times \pi_{\eta_g,20}(\eta_g \mid s_g),$$

and zero otherwise.

I can then construct the probability measure for all ages $j > 1$. For all Borel sets of assets $\mathcal{A}$ and pension claims $\mathcal{EP}$ I have

$$\begin{aligned}
&\Phi(\{j+1\},\{g\},\{s\},\{\eta^+\},\{h^+\},\{k^+\},\{\xi^+\},\mathcal{EP},\mathcal{A}) = \\
&= \frac{\psi_{j+1,g}^s \times \pi_\eta(\eta^+ \mid \eta, s) \times \pi_h(h^+ \mid h, \ell, g) \times \pi_k(k^+ \mid k, j, g) \times \pi_\xi(\xi^+)}{1+n} \\
&\quad \times \int \mathbb{1}_{\{a^+(\mathbf{x_s}) \in \mathcal{A}\}} \times \mathbb{1}_{\{e^+(\mathbf{x_s}) \in \mathcal{EP}\}} \, \Phi(\{j\},\{s\},\{\eta\},\{h\},\{k\},\{\xi\}, da, de)
\end{aligned}$$

where the integral is the measure of assets a and pension claims e today such that, for fixed $(j, g, s, \eta, h, k, \xi)$, the optimal choice today of assets for tomorrow $a^+(\mathbf{x_s})$ lies in $\mathcal{A}$ and the optimal choice today of pension claims for tomorrow $e^+(\mathbf{x_s})$ lies in $\mathcal{EP}$.

D.3 Computational algorithm

The solution method is very similar to the one outlined for the model in Chapter 5. Following Kindermann and Krueger (2022), I solve the model in three steps. I apply the method of endogenous grid points to solve the household problem. I can than compute policy functions $c(\mathbf{x_s}), \ell(\mathbf{x_s}), a^+(\mathbf{x_s})$ for single households and $c(\mathbf{x_c}), \ell_m(\mathbf{x_c}),\ \ell_f(\mathbf{x_c}),\ a^+(\mathbf{x_c})$ for couple households as well as the value functions $v(\mathbf{x_s})$ and $v(\mathbf{x_c})$. Second, I determine equilibrium quantities and prices using a Quasi-Newton procedure.

Computation of policy and value functions

This section presents the method for computing policy and value functions of single households using the method of endogenous gridpoints. The solution method for couple households is similar. The state space of the quantitative model is $\mathbf{x_s} = (j, g, s, \eta, h, k, \xi, a, e)$. To solve the model on a computer, I start with discretizing the continuous elements a, e, η, h and k. I use routines provided by the toolbox that accompanies Fehr and Kindermann (2018).

- I specify the asset grid $\hat{\mathcal{A}} = \{\hat{a}_0, \ldots, \hat{a}_{40}\}$ as nodes with growing distance on the interval $[\bar{a}_l, \bar{a}_u]$. In particular, I let

$$\hat{a}_i = \bar{a}_l + \frac{\bar{a}_u - \bar{a}_l}{(1+g_a)^{40} - 1} \times [(1+g_a)^i - 1] \text{ for } i = 0, 1, \ldots, 40.$$

  The lower limit of the asset grid is $\bar{a}_l = 0$, the upper limit of the asset grid is $\bar{a}_u = 50$, the growth rate of gridpoints is $g_a = 0.14$.
- I specify the earnings points grid $\widehat{\mathcal{EP}} = \{\hat{ep}_0, \ldots, \hat{ep}_{12}\}$ as a grid with $\bar{ep}_l = 0$, $\bar{ep}_u = 2$ and equally spaced nodes.
- I approximate the stochastic process of the AR(1) labor productivity process of normal labor earnings by a Markov chain. I use the Rouenhorst method to discretize the stochastic process of the innovations[2] $\hat{\mathcal{E}} = \{\hat{\eta}_1, \ldots, \hat{\eta}_3\}$ and to determine a transition matrix

$$\pi_\eta(\eta^+|\eta, s) = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{17} \\ \pi_{21} & \pi_{22} & \pi_{27} \\ \pi_{31} & \pi_{32} & \pi_{37} \end{bmatrix}. \tag{D.7}$$

- I determine the labor choice shocks $h \in \{0, 1\}$ and the transitions matrix

[2] Where ρ_s and $\sigma^2_{\varepsilon,s}$ are as specified in Table 6.1 and $\mu = 0$.

$\pi_{h^+}(h^+|h,g,\ell)$ as outlined in (6.27).

- I determine the fertility shocks $k \in \{0,3\}$ and the transitions matrix $\pi_{k^+}(k^+|k,j,i,g)$ as outlined in (6.22).

The policy and value functions can now be solved via backward induction. In the last possible age J, the household will not work[3] and not save, but will consume all remaining resources. This determines the policy functions

$$\begin{aligned} c(J,s,\hat{\eta}_g,\hat{h}_m,\hat{k}_n,\xi,\hat{a}_i,\hat{e}_k) &= \frac{(1+r)\times \hat{a}^i + p - T(p) + b}{\upsilon_{j,k,s}(1+\tau_c)}, \\ l(J,s,\hat{\eta}_g,\hat{h}_m,\hat{k}_n,\xi,\hat{a}_i,\hat{e}_k) &= 0, \\ a^+(J,s,\hat{\eta}_g,\hat{h}_m,\hat{k}_n,\xi,\hat{a}_i,\hat{e}_k) &= 0 \end{aligned}$$

and the value function

$$v(J,s,\hat{\eta}_g,\hat{h}_m,\hat{k}_n,\xi,\hat{a}_i,\hat{e}_k) = c(J,s,\hat{\eta}_g,\hat{h}_m,\hat{k}_n,\xi,\hat{a}_i,\hat{e}_k)$$

for all $i = 0,\dots,100,\ k = 0,\dots,30,\ g = 0,\dots,3,\ m = 0,1,\ n = 0,\dots,3.$
With the final period policy functions and value function at hand, I can iterate backwards over ages to determine the full history of household decisions. I describe the procedure for working-age households. Assume the problem is solved for age $j+1$, then the problem for an individual reads

$$\begin{aligned} v(\mathbf{x}_s) = \max_{c,\ell,a^+,e^+} \ & \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu\frac{(\zeta_k+\ell)^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \xi \times \mathbb{1}_{\ell>0} \\ & + \beta\psi^s_{j+1}\left[\sum_{\eta^+}\pi_\eta(\eta^+|\eta)\sum_{h^+}\pi_h(h^+|h)\sum_{k^+}\pi_k(k^+|k)\cdot E_\xi\Big[v(\mathbf{x}_\mathbf{s}^+)\Big]\right] \end{aligned} \tag{D.8}$$

with $\mathbf{x}_s = (j,s,\eta,h,k,\xi,a,e)$ and $\mathbf{x}_\mathbf{s}^+ = (j+1,s,\eta^+,h^+,k^+,\xi^+,a^+,e^+)$. Households maximize their utility with respect to the budget constraint

$$\begin{aligned} \upsilon_{j,k,s}(1+\tau_c)c + a^+ + T_p(y^p) + T\Big(y - T_p(\min(y,2\bar{y})) + p\Big) = \\ (1+r)a + y + y_{mini} + t_{cb} + t_{cs} + p + b, \end{aligned} \tag{D.9}$$

the accumulation equation for pension claims

$$e^+ = e + \min\Big(y + y_{mini},\ 2\bar{y}\Big).$$

and the positive asset restriction $a^+ \geq 0$. The first order conditions are outlined in Appendix D.1.1.

[3] Remember, the compulsory retirement age is j_r.

I now apply the method of endogenous gridpoints. I first define an exogenous grid on the state space $\{\hat{a}_v\}_{v=0}^{40}$, which denotes the remainder of assets in the next period, i.e. $a^+ = \hat{a}_v$. For each state $\tilde{\mathbf{x}}_\mathbf{s} = (j, s, \eta, h, k, \xi, a^+, e)$ and possible labor choice $\ell \in (0, 0.1, 0.2, 0.4)$

1. I determine

$$e^+ = \frac{(j-1)e}{j} + \frac{\min\Big(y + y_{mini},\ 2\bar{y}\Big)}{j}$$

2. given a^+ and e^+ I determine $c(\tilde{\mathbf{x}}_\mathbf{s})$ from the Euler Equation (D.2)
3. with $l(\tilde{\mathbf{x}}_\mathbf{s})$ and $c(\tilde{\mathbf{x}}_\mathbf{s})$, I use the budget constraint (D.9) to get $a(\tilde{\mathbf{x}}_\mathbf{s})$

once $l(\tilde{\mathbf{x}}_\mathbf{s})$, $c(\tilde{\mathbf{x}}_\mathbf{s})$ and $a(\tilde{\mathbf{x}}_\mathbf{s})$ are solved, I can interpolate along a to obtain the policy functions $l(\mathbf{x}_\mathbf{s})$, $c(\mathbf{x}_\mathbf{s})$ and $a^+(\mathbf{x}_\mathbf{s})$ as well as the value function

$$\begin{aligned} v(\mathbf{x}_\mathbf{s}) = & \frac{c(\mathbf{x}_\mathbf{s})^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu \frac{(\zeta_k + \ell(\mathbf{x}_\mathbf{s}))^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}} - \xi \times \mathbb{1}_{\ell>0} \\ & + \beta\psi_{j+1}^s \left[\sum_{\eta^+} \pi_\eta(\eta^+|\eta) \sum_{h^+} \pi_h(h^+|h) \sum_{k^+} \pi_k(k^+|k) \cdot E_\xi\Big[v(\mathbf{x}_\mathbf{s}^+)\Big]\right] \end{aligned} \tag{D.10}$$

for each todays asset value $\hat{a}_i,\ i = 0, \ldots, 40$ and earnings points amount $\hat{e}_k, k = 0, \ldots, 12$ by piecewise linear interpolation.[4]

In case the asset restriction $a^+ \geq 0$ is binding, I extend the interpolation data by another point of value 0 on the left and determine the policy and value functions at this point. I assume the household consumes all available resources and has no savings left over for tomorrow. The solution method for couple households is similar.

The initial equilibrium of the macroeconomy

I model a small open economy, hence prices r and w are fixed. In order to determine aggregate quantities and policy parameters in the initial equilibrium ($t = 0$) I need to determine the following four variables numerically:

- the government budget balancing consumption tax rate τ_c as outlined in Equation (6.18)

[4]Note, I interpolate $\Big[(1-\frac{1}{\sigma})v(x_s)\Big]^{\frac{1}{1-\frac{1}{\sigma}}}$ rather than $v(x_s)$ directly and than transform it back to the original shape. This leads to more accurate results for discretized functions with high curvature.

- the pension replacement rate κ that balances pension contributions and pension payments as outlined in equation (6.17)
- average earnings $\bar{y}_t$
- and aggregate bequests $\bar{B}$, which immediately allows us to compute cohort bequests $\{b_j\}_{j=20}^{J}$ (equally shared between all working-age individuals).

Once a guess of these four variables is available, I can use the following algorithm to compute the remainder individual and aggregate variables of the economy:

1. I solve the household optimization problem using the guesses for $\tau_c, \kappa, \bar{y}, \bar{B}$ and determine the measure of households.
2. I compute aggregate quantities $\{L, K, TB, Y, C, G, I, \Omega, B\}$ from individual decisions and the measure of household and determine the gap $D = Y - C - I - G$ between demand and supply.

I determine the three central parameters $(\tau_c, \kappa, \bar{B})$ by means of a quasi-Newton rootfinding method and search manually for $\bar{y}_t$. The method receives an initial guess of these variables and updates them in each iteration step using the Jacobian of the determining equation system. The iteration process stops when the government and the pension budget are in equilibrium and the model implied average earnings and aggregate bequests equal the guess provided by the method.

D.4 Datawork

The productivity profiles in this paper are based on administrative data from the German Pension Insurance. In particular I use the 2017 wave of the scientific usefile of the Versichertenkontenstichprobe (FDZ-RV – SUFVSKT2002-2017) that contains monthly earnings data of 69,520 insured individuals. This is about 0.18% of the actively insured population.[5] I restrict our attention to the male sample population aged betIen 25 and 60 of which I have information on the education level. My measure of monthly labor earnings comprises income from regular work and short-term unemployment (up to one year). I count all other source of pension accumulation (like times of care for children, sickness or mini-job employment) as zero earnings months. I sum up monthly earnings observations to construct an annual earnings measure for each individual. This appendix explains the data selection and estimation process in detail.

D.4.1 The administrative dataset

I use the same dataset and a similar estimation strategy as in Chapter 4. However, the approach differs with respect to low earnings individuals. While I define a low-earnings group (observations on annual earnings ($y < 0.23\bar{y}$)) in Chapter 4 that is excluded from the sample, I instead simply exclude mini-job worker from the sample for this chapter. The reason is that mini-job worker in the model a paid a flat salary and hence I am only interested in productivity profiles of regular worker.

Earnings measurement Earnings y_{isjt} of an individual i of education s and age j at time t are subject to social security contribution. There is a contribution threshold $y_{max,t}$ and any earnings beyond that value are non-contributory. Contributory earnings hence amount to $\min\left(y_{isjt},\ y_{max,t}\right)$. They are converted into pension claims y^p_{isjt} by diving them through average earnings $\bar{y}_t$. I account for the fact that pension claims from so-called midi jobs[6] are subject to a reduced pension contribution rate. Both, the contribution threshold $y_{max,t}$ and average earnings $\bar{y}_t$ are adjusted annually to account for wage growth. The contribution threshold $y_{max,t}$ currently amounts to about twice the average earnings $\bar{y}_t$.[7]

It is most convenient to use pension claims y^p_{isjt} as an earnings measure, as they are

[5] The German pension scheme covered of 38 million actively insured individuals in 2017.

[6] In a mini-job, an individual can earn a maximum of EUR 450. Midi-jobs cover earnings from 451 to 850 Euros.

[7] See Section 11 in Deutsche Rentenversicherung Bund (2020) for a full history of reference values.

stationary over time. In particular, I define

$$y^p_{isjt} = \frac{\min\left(y_{isjt},\ y_{max,t}\right)}{\bar{y}_t}. \tag{D.11}$$

Data selection Although the monthly records start in 1964, I only consider observations for the years 2000 to 2016. This has certain advantages: First, our estimates are based on recent data; second, I avoid structural breaks arising from German reunification and policy-changes in the 1990s and third, different age cohorts are represented in the sample at similar shares in each year (early sample years cover only young individuals). The data-selection process is summarized in Table D.1.

Table D.1: Data selection

	Individuals	Observations
Initial data set (1975 - 2017)	69,520	28,166,952
Initial data set (2000 - 2016)	69,520	14,139,972
- Women	−36,634	−7,451,736
- Ages < 25		−1,014,120
- Ages > 60		−152,976
	32,886	5,521,140
- Ind. that receive pensions	−3,606	−605,208
	29,280	4,915,932
- Ind. with unknown education	−13,677	−2,346,840
	15,603	2,569,092
Annualized data (2000 - 2016)	15,603	214,091
No contributory earnings	−391	−25,197
Final data set	**15,212**	**188,894**
Non-college education	11,800	149,757
College education	3,412	39,137

Source: own calculation, based on data from FDZ-RV – SUFVSKT2002-2017.

I restrict the sample such that it targets workers who are attached to the labor market. I therefore limit our attention to men aged between 25 and 60 who are likely to already have finished education and military service and are not in the process of retiring. I drop all individuals who already received pensions such as disability pensions or early-retirement pensions.

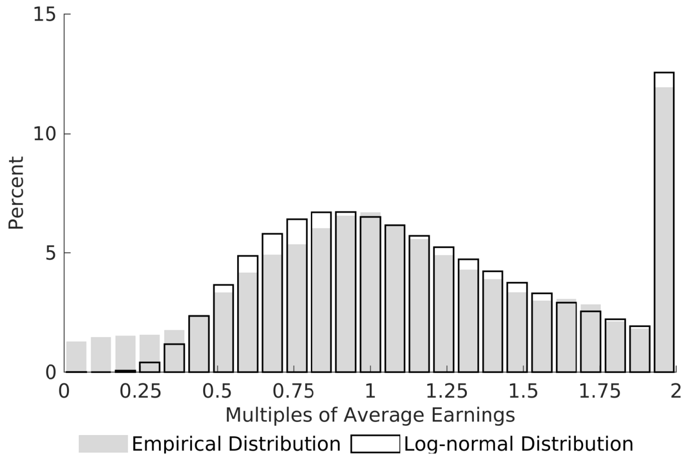
I divide the sample into two educational groups. I adapt the scheme to the International Standard Classification of Education of the UNESCO (ISCED 2011) to allow for international comparison. A person is defined to be college-educated[8] if

[8]corresponds to KldB 2010 4-6

she is classified ISCED 6 (Bachelors or equivalent level) or above, excluding ISCED 65 (trade and technical schools, including master craftsman training). She is high school-educated[9] if she is classified ISCED 5 and below or ISCED 65. I drop those with unknown education status.

For estimating earnings profiles I use all pension claims y^p_{isjt} that stem from regular-employment or unemployment benefits (short-term, max. 12 month) according to the variable `SES`. Since individuals are productive when searching for a new job, I consider short-term unemployment as an employment type. Figure D.1 shows the distribution of earnings. Obviously, the data are right-censored.

Figure D.1: Histogram of earnings points



Source: own calculation, based on data from FDZ-RV – SUFVSKT2002-2017.

Table D.2 shows the distribution of employment states across monthly observations. About 14.6 percent of all observations are on months with no contributory earnings. Such observations emerge when individuals become self-employed or civil servants, when they take care leave, face a longer spell of unemployment, work in a mini-job(1.7 percent of the total observations) or just decide to drop out of the workforce. I code non-contributory months as periods of zero earnings.

To make the data comparable with our simulation model, I have to change the time-dimension of the panel from monthly to annual. I do so by computing the sum of acquired pensions claims for each calendar year. Finally, I drop all sample individuals who had no contributory earnings at all in the period from 2000 to 2016. I exclude observations with no contributory earnings in an entire calendar year, see Table D.1. The final data set is an unbalanced annual panel for the years 2000 to 2016 with 15,212 individuals – of which 22.4 percent are college-educated – and a total of 188,894 observations.

[9] corresponds to KldB 2010 1-3

Table D.2: Distribution of employment states (across monthly observations)

Employment Status	Observations	Percent
Regular employment	2,139,302	83.27
Unemployment (short-term)	55,138	2.15
No contributory earnings (incl. mini-jobs)	374,652	14.58
Total	2,569,092	100.00

Source: own calculation, based on data from FDZ-RV – SUFVSKT2002-2017.

D.4.2 Earnings estimates

From here onward, I follow the same estimation strategy es outlined in Appendix A. I first identify the censoring threshold, which is similar in both Chapters. I set the low earner threshold y_{min} as used in Section A.2 to a very small value, to neutralize the truncation threshold from below and use the generalized method of moments estimator to determine the parameters of the statistical model as outlined in Equation (6.24). The estimation results are summarized in Table 6.1 and graphically represented in Figure 6.5.

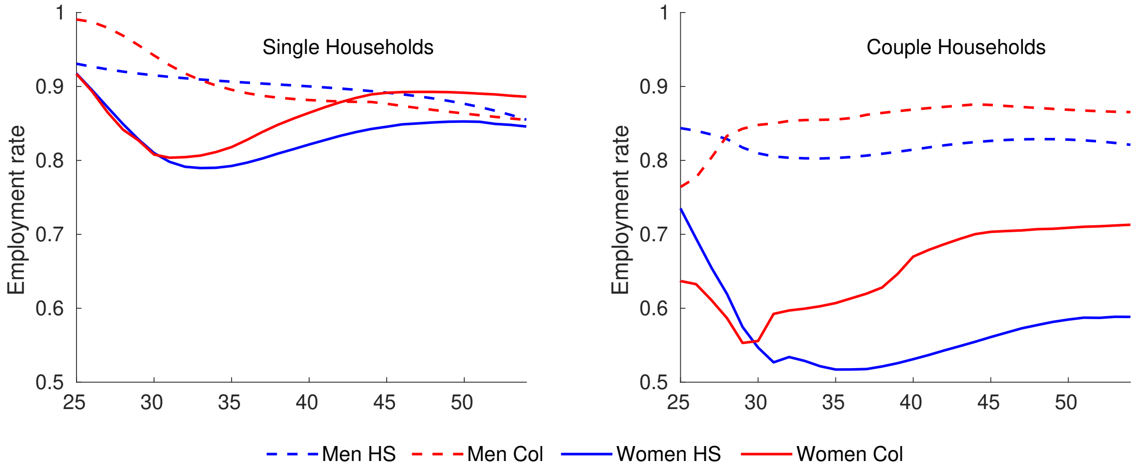
D.5 Further simulation results

Table D.3: Individuals who retire with $e_{j_r} < b\bar{y}$ in the benchmark economy

	No kids		**Kids**	
	High School	College	High School	College
Singles				
women	2.00	0.92	13.95	6.98
men	1.14	0.63	-	-
Couples				
women	23.67	17.32	28.54	21.97
men	8.43	6.94	5.20	3.85
Married women				
husband HS	20.40	9.97	26.21	13.76
husband Col	36.28	21.21	37.50	26.32
Married men				
wife HS	6.56	2.45	3.84	1.31
wife Col	19.64	10.65	13.34	5.95

Values in percent of group total.

Figure D.2: Labor supply by education and gender



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