Beyond the ensemble paradigm in low-dimensional quantum gravity: Schwarzian density, quantum chaos and wormhole contributions

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Based on periodic orbit theory we address the individual-system versus ensemble interpretation of quantum gravity from a quantum chaos perspective. To this end we show that the spectrum of geodesic motion on high-dimensional hyperbolic manifolds, described by the Selberg trace formula, displays a Schwarzian ($\sinh 2\pi\sqrt{E}$) mean level density. Due to its chaotic classical limit, this quantum system also shows all universal signatures of quantum chaos. These two properties imply a possible duality to Jackiw-Teitelboim-type quantum gravity at the level of a *single system* instead of an *ensemble of systems* like matrix theories and SYK models. Beyond the universal regime we show how the full wormhole geometry on the gravity side emerges from the discreteness of the set of periodic orbits. Thereby, we take initial steps towards a duality between gravitational and mesoscopic chaotic quantum systems through the topological, respectively, periodic orbit expansions of their correlators.

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Low-dimensional gravity has been a fruitful area for studying holography in recent years, with much interest arising from the paradigm that quantum gravitational degrees of freedom display characteristic signatures of quantum chaos in the presence of a horizon [1]. A prime example of this is Jackiw-Teitelboim (JT) gravity [2, 3], whose quantization has been shown to be exactly dual to a matrix ensemble with a universal limit, given by Random Matrix Theory (RMT) [4]. Originally devised to consider specific signatures of chaos for the unitary case of broken time-reversal symmetry (see [5] for a recent review), further work clarified how to extend the duality to other universality classes [6–10] and even used it to explore properties of hyperbolic manifolds [4, 11, 12].

JT gravity displays a characteristic density of states $\sim \sinh\left(2\pi\sqrt{E}\right)$, a hallmark of systems with a black hole dual [13]. It arises from the Schwarzian action describing the leading order (disk) contribution to the 1-point spectral function. Given the expectation of gravitational systems as being maximally chaotic [14, 15], a possible duality requires the presence of general universal signatures of late-time quantum chaos, but a complete duality occurs only when the correlation functions of both theories are identical even at the model-specific level. Summarizing the state of the art, both the celebrated matrix ensemble of [4] and the "universe approach" of [16, 17] reach full duality by using the Schwarzian density as an external input, while the SYK [18, 19] and related models [1, 20, 21] do display the Schwarzian action but only match the gravity side in the universal regime [22].

The study of JT gravity has allowed to shed light on how to deal with higher-than-disk topologies in the gravitational path integral, but at the same time it raised the question of how to understand such topologies from the holographic perspective. In particular, wormhole geometries correspond to non-factorizing correlation functions in the dual theory. In JT gravity, the emergence of non-vanishing connected 2-point correlators can be explained by an ensemble interpretation of the dual theory [23–26], but in higher dimensions this interpretation is not available and the so-called factorization problem remains [27].

Although the ensemble interpretation is supported by the existence of matrix and SYK-related dual theories, it apparently contradicts the holographic paradigm embodied by the AdS/CFT correspondence, where the duality relates two *individual* systems, see e.g. [28–30]. The study of JT gravity from the individual-system perspective is therefore an active field, with ideas being pursued in three-dimensional gravity [31], constrained matrix models [25], non-perturbative corrections [32] and the effective description as a "universe" field theory [16, 17].

Here we appeal to the success of periodic-orbit theory [33, 34] in explaining both universal [35–37] and non-universal [38, 39] features of quantum chaotic systems, see [40] for a recent review. Using this semiclassical approach, we explicitly construct an *individual* chaotic quantum system, i.e., high-dimensional hyperbolic dynamics [41], that possesses the same spectral (Schwarzian) density as JT gravity. As for any quantum chaotic model, the standard semiclassical smoothing defined by averaging over small parameter windows [26, 37] reproduces the *universal*, RMT, limit of the leading term in the JT 2-point correlation function [1, 4], the so-called ramp contribution, in systems with both broken and preserved time-reversal invariance [35]. Remarkably, discreteness of the set of periodic orbits allows us to recover the full gravitational result as given by the double-trumpet topology [4], in the whole regime of parameters including universal and JT-specific properties. This equivalence appears when a statistical description of the set of periodic orbits is induced by a coarse graining of the classical phase space at a finite action scale.

We begin with introducing JT gravity as a two-dimensional theory of dilaton gravity defined by the action

$$S_{JT}[g,\phi] = -\frac{S_0}{2\pi}\left[\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}R + \int_{\partial\mathcal{M}}\sqrt{h}K\right] - \left[\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}\phi(R+2) + \int_{\partial\mathcal{M}}\sqrt{h}\phi(K-1)\right], \quad (1)$$

where S_0 is a large parameter setting the entropy scale and suppressing topology change, $\mathcal{M}$ is a two-dimensional Riemannian manifold, $\partial\mathcal{M}$ its boundary, g and R the metric determinant and Ricci scalar on $\mathcal{M}$, h and K the induced metric and curvature on $\partial\mathcal{M}$ and ϕ a complex scalar field called the dilaton. Equation (1) arises universally as the near-horizon geometry of higher-dimensional near-extremal black holes, see e.g. [42].

In its quantized form, the theory is defined through the n-boundary connected partition functions

$$\langle Z(\beta_1)\dots Z(\beta_n)\rangle^{(c)} \simeq \sum_{g=0}^{\infty} e^{(2-2g-n)S_0} Z_{g,n}(\beta_1,\dots,\beta_n), \tag{2}$$

expressed as a topological (genus) expansion. The right-hand side in Eq. (2) is computed by path integrating over the action (1), whose topological term (the first bracket) induces the genus decomposition through the Gauß-Bonnet theorem. The left-hand side sets the boundary conditions, namely that one should integrate over connected manifolds with n asymptotic AdS boundaries of renormalized lengths $\beta_1,\dots,\beta_n$ (see [43] for an overview). One can compute the functions $Z_{g,n}$ exactly for all orders n and genus g [4], but in this Letter, we will focus on the disk and double trumpet geometries, $g=0$ and $n=1,2$, respectively. These two correlators depend directly on integrating the boundary reparametrization mode, $w(u)$, against the emergent Schwarzian action [44],

$$S[w,\phi] = -\frac{S_0}{2\pi}\int du\phi_r(u)\mathrm{Sch}(w,u)\,. \tag{3}$$

It is the simplest action invariant under $\mathrm{SL}(2,\mathbb{R})$, also describing the low energy regime of the SYK model [18, 45]. Integrating out the Schwarzian action leads, firstly, to the well-known disk partition function

$$Z_{0,1}(\beta) = Z^{\mathrm{d(isk)}}_{\mathrm{Sch}}(\beta) = \int_0^\infty dE e^{-\beta E}\rho_0(E), \tag{4}$$

depending on the characteristic density of states

$$\rho_0(E) = \frac{1}{4\pi^2}\sinh\left(2\pi\sqrt{E}\right). \tag{5}$$

Secondly, it yields the wormhole (double-trumpet) contribution to the 2-point function,

$$Z_{0,2}(\beta_1,\beta_2) = \int_0^\infty b\,db\; Z^t_{\mathrm{Sch}}(\beta_1,b)Z^t_{\mathrm{Sch}}(\beta_2,b) \tag{6}$$

in terms of the characteristic trumpet partition function

$$Z^{\mathrm{t(rumpet)}}_{\mathrm{Sch}}(\beta,b) = \frac{1}{2\sqrt{\pi}\beta^{1/2}}e^{-b^2/4\beta}. \tag{7}$$

We claim that these central quantities, $Z_{0,1}$ and $Z_{0,2}$, can be exactly reproduced by an individual quantum chaotic, non-gravitational, dynamical system. To show this, we consider a free particle of mass $M=1/2$ moving on an f-dimensional Riemannian manifold $\mathcal{K}$ of constant negative curvature $R=-2$ (or correspondingly, with curvature radius L set to unity). In the regime of interest here, $f\geq 3$, this system is fully chaotic and its quantization relies on the Gutzwiller trace formula [33], which provides an approximation for e.g. the *quantum* partition function using properties of the system's *classical* periodic orbits. In the case at hand, i.e. free motion on a hyperbolic f-manifold $\mathcal{K}$, Gutzwiller [41] found that the semiclassical approximation is identical [46] to the Selberg trace formula (STF) relating the spectrum of the Laplacian Δ on $\mathcal{K}$ to the length spectrum of the set Γ of closed primitive geodesics γ on $\mathcal{K}$.

Using dimensionless variables, the STF is given by [47]

$$\sum_{n=0}^{\infty} h(p_n) = \mathcal{V}\int_0^\infty h(p)\Phi_f(p)dp + \sum_{\gamma\in\Gamma}\sum_{m=1}^{\infty}\frac{\chi_\gamma^m l_\gamma g(ml_\gamma)}{2S_f(m,l_\gamma)}, \tag{8}$$

where we parameterized the eigenvalues of Δ as $\lambda_n = (p_n^2{+}(f{-}1)/4)$. The even function $h(p)$ satisfies certain technical conditions on its large-p behavior and analytic structure. $\mathcal{V}$ is the volume of $\mathcal{K}$ and $\Phi_f(p)$ is the *Plancherel measure* of $\mathcal{K}$, a quantity originating in the group theoretical underpinnings of the STF. In the present context, importantly, it plays the role of the mean spectral density or Weyl term. The first sum ranges over primitive closed geodesics of $\mathcal{K}$ (i.e. the periodic orbits of the dynamical system with $\hat{H}=-\Delta$), the second accounts for their repetitions. Lastly, l_γ is the length of the γ-th orbit (in units of $2L$), l_γ/S_f its stability amplitude, χ_γ are phases arising from possible fluxes breaking time-reversal-invariance and $g(l)$ is the Fourier transform of $h(p)$.

Interestingly, the Plancherel measure appears as the spectral density in the exact Weyl term via the orthonormalization condition of radial eigenfunctions of Δ on $\mathcal{K}$, $(\phi_p,\phi_{p'}) = \delta(p-p')/\Phi_f(p)$ [48]. This is exactly the way how the famous $\sinh(\pi p)$ density (5) enters in JT gravity when understood as a so-called BF gauge theory [49].

The known result [50] for $\Phi_f(p)$, quoted in Eq. (33), can be written for odd f as [51]

$$\Phi_f(p) = \frac{\frac{f-1}{2}!\left(\frac{f-3}{2}!\right)^2}{\pi^{\frac{f+3}{2}}(f-1)!}(\pi p)\sinh_f(\pi p), \tag{9}$$

with the truncated-product approximant

$$\sinh_f(\pi p) \equiv \pi p\prod_{k=1}^{\frac{f-3}{2}}\left(1+\frac{p^2}{k^2}\right). \tag{10}$$

In a usual semiclassical treatment, the Weyl term would be estimated by a Thomas-Fermi approximation $\rho_f^{\mathrm{TF}}(p^2)$, i.e. an integral over an energy shell in phase space [52, 53]. This agrees with the exact density $\Phi_f(p)$ at large energies,

$$\mathcal{V}\Phi_f(p) = \rho_f^{\mathrm{TF}}(p^2)\left(1+\mathcal{O}(1/p)\right). \tag{11}$$

The Plancherel measure $\Phi_f(p)$ can, indeed, be understood to contain *all quantum corrections* to $\rho_f^{\mathrm{TF}}(p^2)$ [46]. This exactness allows us to use the limit

$$\lim_{f\to\infty} \sinh_f(\pi p) = \sinh(\pi p)\,. \tag{12}$$

Absorbing the combinatorial factors in Eq. (9) into the appropriately chosen volume $\mathcal{V}$, the spectral density appearing in Eq. (8) finally reads $\mathcal{V}\sinh(\pi p)$.

To proceed further, we need to specify the spectral function $h(p)$. In the JT gravity matrix model [4], the operator inserted on the matrix side to compute the JT gravity path integral with standard boundary conditions is the trace of the heat kernel. In our approach, the appropriate choice turns out to be essentially the same, namely $h(p) = e^{-i\tau p^2/4}$. When summed over the spectrum of the Hamiltonian, this choice yields the trace of the (analytically continued) heat kernel up to an interesting scaling of the complex time $\tau = t - i\beta, \beta > 0$ by a factor of four [54]. Plugging this into Eq. (8) yields for the Weyl term

$$\begin{aligned} &\mathcal{V}\int_0^\infty \frac{p\,dp}{2\pi^2} e^{-i\tau p^2/4}\sinh \pi p \\ &= \mathcal{V}\int_0^\infty e^{-i\tau E}\frac{1}{4\pi^2}\sinh\Big(2\pi\sqrt{E}\Big)dE\,. \end{aligned} \tag{13}$$

It is exactly the JT gravity disk partition function including, crucially, the sinh-type spectral density, Eq. (5). It is interesting to note that, up to factors absorbed into the choice of $\mathcal{V}$, $\Phi_{f=3}(p) \propto p^2$ exactly reproduces the spectral density of the Airy model [55]. The Weyl term of the STF therefore interpolates between the Airy and JT spectral density for $f = 3$ and ∞, in a way akin, but not identical to the $(2, q)$ minimal string for $q = 1, \infty$ [4, 56].

Having identified the gravitational correlator $Z_{0,1}$ with the Weyl contribution of an indivudual dynamical system via the periodic orbit approach, we now consider the leading JT wormhole partition function. Emulating the semiclassical 1-point calculation, we propose to compute the leading-order contribution to the 2-point correlator of partition functions representing them as

$$Z(\tau) = \mathrm{tr}\Big(e^{-i\tau\hat{H}/4}\Big) = Z_{\mathrm{Weyl}}(\tau) + Z_{\mathrm{PO}}(\tau)\,, \tag{14}$$

with the periodic-orbit contribution

$$Z_{\mathrm{PO}}(\tau) = \sum_{\gamma\in\Gamma}\sum_{m=1}^{\infty}\frac{\chi_\gamma^m l_\gamma}{S_f(m,l_\gamma)}\frac{e^{-\frac{m^2 l_\gamma^2}{2i\tau}}}{\sqrt{2\pi i\tau}} \tag{15}$$

expressed through the STF (8). Note the remarkable appearance of $Z^{\mathrm{t}}_{\mathrm{Sch}}(i\tau, l_\gamma)$, Eq. (7), with its exponent now admitting an interpretation as the classical action of the periodic orbit γ in units of $\hbar$.

Assuming $|t| \gg \beta$ these exponents give rise to highly oscillatory contributions to $Z_{\mathrm{PO}}(\tau)$. Usually, correlation functions involving periodic-orbit sums are semiclassically evaluated through a *smoothing* procedure that acts on *oscillatory* expressions as Eq. (15). Thereby quantum interference between amplitudes associated with so-called correlated periodic orbits, surviving this smoothing, is highlighted [36, 40, 57]. Note that this average, furtheron denoted as $\langle\cdot\rangle_{\mathrm{sc}}$, operates on the level of a *single* system, without the necessity to introduce disorder averages or some other type of ensemble.

Accordingly we define semiclassical connected correlators of partition functions $Z(\tau)$, Eq. (14), through the smoothing of (products of) their periodic-orbit terms (15) (for $|t| \gg \beta$) and identify contributions to the single and double sums that do not vanish under $\langle\cdot\rangle_{\mathrm{sc}}$. For the 1-point function this simply means

$$\langle Z(\tau)\rangle_{\mathrm{sc}} = Z_{\mathrm{Weyl}}(\tau) = Z_{0,1}(i\tau)\,, \tag{16}$$

since Z_{Weyl} is by definition smooth and we have shown $Z_{\mathrm{Weyl}} = Z^{\mathrm{d}}_{\mathrm{Sch}}$. However, the semiclassical (connected) 2-point function,

$$\mathcal{Z}^{\mathrm{c}}_{\mathrm{sc}}(\tau_1,\tau_2) = \langle Z_{\mathrm{PO}}(\tau_1)Z_{\mathrm{PO}}(\tau_2)\rangle_{\mathrm{sc}} \tag{17}$$

critically depends on the relative sign of the time variables t_1, t_2, which controls the existence of slow oscillatory contributions arising from quantum interference. Inspection of the products involved then shows

$$\mathcal{Z}^{\mathrm{c}}_{\mathrm{sc}}(\tau_1,\tau_2) = \begin{cases} 0 & \text{for } t_1t_2 > 0 \\ \mathcal{Z}^{\mathrm{corr}}_{\mathrm{sc}}(\tau_1,\tau_2) & \text{for } t_1 \simeq -t_2 \end{cases} \tag{18}$$

where $\mathcal{Z}^{\mathrm{corr}}_{\mathrm{sc}}$ comprises pairs of *correlated* orbits with systematically small action differences [35, 36]. A well established result of the semiclassical analysis, Berry's diagonal approximation [35], is that for times shorter than a scale set by ρ_0 the double sum is dominated by its diagonal part, i.e., $\mathcal{Z}^{\mathrm{corr}}_{\mathrm{sc}} \simeq \mathcal{Z}^{\mathrm{dg}}_{\mathrm{sc}}$, where

$$\mathcal{Z}^{\mathrm{dg}}_{\mathrm{sc}}(\tau_1,\tau_2) = \sum_{\gamma}\sum_{m=1}^{\infty}\frac{\kappa l_\gamma^2}{S_f^2(m,l_\gamma)}\frac{e^{-\frac{m^2l_\gamma^2}{2i\tau_1}}}{\sqrt{2\pi i\tau_1}}\frac{e^{-\frac{m^2l_\gamma^2}{2i\tau_2}}}{\sqrt{2\pi i\tau_2}}\,. \tag{19}$$

This is corrected by off-diagonal terms [36, 37] representing quantum interference effects. The presence (absence) of time-reversal-symmetry, encoded in $\kappa = 2(1)$ [58], is responsible for the first (weak localization) correction.

As long as $i\tau_1 \simeq (i\tau_2)^*$, the function $\mathcal{Z}^{\mathrm{dg}}_{\mathrm{sc}}(\tau_1,\tau_2)$ is by definition smooth. Thus, the sum can be written as an integral by introducing the weak limit $\bar{\eta}(l)$ of the density

$$\eta(l) := \sum_{\gamma}\delta(l - l_\gamma) \overset{\mathrm{weak}}{=} \bar{\eta}(l) = \frac{\sinh\left[(f-1)l/2\right]}{l/2} \tag{20}$$

of closed geodesic lengths. The last equality states the exponential proliferation of closed geodesics [59, 60], valid for $l > l_0$ where l_0 is set by the shortest one. Using the explicit form $S_f(m,l) = \sinh^{m(f-1)}(l/4)$[47, 48] for the

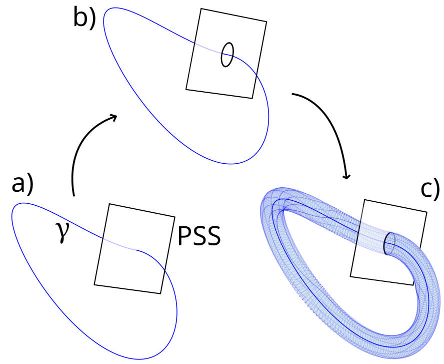

FIG. 1. a) Every periodic orbit γ in phase space is determined by the points where it pierces the Poincare Surface of Section (PSS). b) When a finite resolution on the PSS is introduced, any property of the periodic orbits, like the joint distribution of their lengths, (c) cannot be determined with infinite precision.

stabilities in Eq. (15), and known results in hyperbolic geometry [61] implying $l_0 \gg 1/\mathcal{V}$, within the regime of integration, the decaying squared stability factors exactly cancel the exponential proliferation, but only for $m = 1$. Terms with $m \geq 2$ are also suppressed since the number of primitive geodesics grows exponentially faster than the number of repetitions [59].

We now quantify the range of $\tau_{1,2} = t_{1,2} - i\beta_{1,2}$ where Eq. (19) is valid by defining $t_1 = t - \epsilon, t_2 = -t - \epsilon$. Using semiclassical techniques [35], we expand the actions in Eq. (19) to first order in $\epsilon/t, \beta_{1,2}/t$, evaluate all prefactors at zeroth order and extend the integration range, to get

$$\mathcal{Z}^{\rm dg}_{\rm sc}(t-\epsilon-i\beta_1,-t-\epsilon-i\beta_2) = \kappa\int_0^\infty \frac{l dl {\rm e}^{-\frac{l^2}{2t^2}(\bar\beta+i\epsilon)}}{2\pi|t|} + \mathcal{O}\left(\frac{\beta_{1,2}}{t},\frac{\epsilon}{t}\right) \simeq \frac{\kappa|t|}{2\pi}\frac{1}{\bar\beta+i\epsilon} \tag{21}$$

with $\bar\beta = (\beta_1+\beta_2)/2$. This result contains in particular the so-called ramp [4], $Z_{0,2}(i\tau, -i\tau^*) = \kappa|t|/2\pi\beta$, valid for $|t| \gg \beta$ (and reported for $f = 2, \kappa = 1$ in [62]).

To summarize, the diagonal approximation yields the gravitational result for $Z_{0,2}$ for $t_1 \simeq -t_2$ and $|t_{1,2}| \gg \beta_{1,2}$ while it fails in any other regime, see Eq. (18). A possible duality however demands the gravitational result for the wormhole to be given by the double trumpet, Eq. (6), and to hold for *all* values of the boundary lengths $\tau_{1,2}$. [63]

Aiming to fully reconcile the semiclassical and gravitational results for $Z_{0,2}$ we reconsider the notion of smoothing average again, now from the general perspective of the loss of information that takes place when neglecting highly oscillatory contributions in sums over amplitudes. To go beyond this picture, but still in a way that we define the correlation functions for a single individual system, we follow standard methods of statistical physics [64] and introduce a *coarse grain* description. This coarse graining is implemented by introducing a finite action scale, blurring structures in phase space, with the consequent loss of information. The corresponding probabilistic description is then a consequence of our ignorance, rather than the existence of a physical ensemble such as in disordered systems. Our ideas get support from two recent proposals, in [65] based on stochastic methods and in [16, 17] where JT correlators are derived within the "universe" (Kodaira-Spencer) field theory approach as expectation values of operators insensitive to certain microscopic degrees of freedom.

Considering for simplicity the orthogonal ($\kappa = 2$) case of $\chi_\gamma = 1$ in Eq. (8), we implement the coarse graining as follows. To specify a periodic orbit in phase space it is enough to know the points where it pierces any manifold transversal to the classical flow [66], so-called Poincaré surface of section. As illustrated in Fig. 1 a finite resolution on this manifold introduces a degree of ignorance about the orbits. In particular the exact, microscopic (mc) distribution $P^{\rm mc}_{\mathbf{l}(\mathbf{\Gamma})}$ of lengths $\mathbf{l} = (l^{(1)}, l^{(2)}, \ldots)$,

$$P^{\rm mc}_{\mathbf{l}(\Gamma)}(\mathbf{l}) = \prod_\gamma \delta(l^{(\gamma)} - l_\gamma), \text{ with } \mathbf{l}(\Gamma) = (l_1, l_2 \ldots)\,, \tag{22}$$

will no longer be sharp. Its coarse-grained (cg) version $P^{(\rm cg)}$ provides instead a statistical description of the length set, including the systematic correlations rigorously shown to exist [67, 68] and to be responsible for the emergence of universal RMT spectral fluctuations [37]. As any object built upon the length spectrum, the length density $\eta(l)$ is no longer sharp in the l_γ's but described by its statistical fluctuations under the coarse-grained distribution. In particular, its first two moments read

$$\begin{aligned}\langle\eta(l)\rangle &= \sum_\gamma \int d\mathbf{l}' P^{\rm cg}(\mathbf{l})\delta(l-l_\gamma)\,,\\ \langle\eta(l)\eta(l')\rangle &= \sum_{\gamma,\delta}\int d\mathbf{l}P^{\rm cg}(\mathbf{l})\delta(l-l^{(\gamma)})\delta(l'-l^{(\delta)})\,.\end{aligned} \tag{23}$$

By means of the corresponding (c)onnected correlator

$$\langle\eta(l)\eta(l')\rangle^{\rm c} = \langle\eta(l)\eta(l')\rangle - \langle\eta(l)\rangle\langle\eta(l')\rangle \tag{24}$$

the connected 2-point correlation function of periodic orbit sums (15) is written, in view of Eq. (7), as

$$\begin{aligned}\langle Z_{\rm PO}(\tau_1)Z_{\rm PO}(\tau_2)\rangle^{\rm c} \;=\; &\int_0^\infty \frac{l dl}{S_f(1,l)}\frac{l' dl'}{S_f(1,l')}\langle\eta(l)\eta(l')\rangle^{\rm c}\\ &\times Z^{\rm t}_{\rm Sch}(i\tau_1,l)Z^{\rm t}_{\rm Sch}(i\tau_2,l')\,.\end{aligned} \tag{25}$$

Here, consistently with the derivation of Eq. (21), terms with $m \geq 2$ have been neglected.

After having specified the way statistical properties of the set of lengths determine the semiclassical correlation functions we ask for the properties of $P^{\rm cg}(\mathbf{l})$. Following a standard approach of periodic orbit theory, we start by considering the lengths as uncorrelated random variables [69]. The self-correlations, however, have a distinct role when (as in our case) the probability distribution is defined over a discrete space. For uncorrelated variables, the definitions (23), where the diagonal term $\gamma = \delta$ produces a singular contribution, imply the existence of contact terms in correlators of the form (24) [70]. Invoking the fundamental property of chaotic systems that the set of periodic orbits is *discrete* [66] and the straightforward condition $\langle \eta(l) \rangle = \bar{\eta}(l)$, we get

$$\langle \eta(l)\eta(l') \rangle^{\mathrm{c}}_{\mathrm{un}} = \kappa \bar{\eta}(l)\delta(l-l') \tag{26}$$

for uncorrelated lengths, where κ is reintroduced following the analysis of [58]. Substitution in Eq. (25) yields

$$\begin{aligned} &\langle Z_{\mathrm{PO}}(\tau_1) Z_{\mathrm{PO}}(\tau_2) \rangle^{\mathrm{c}}_{\mathrm{un}} \\ &= \kappa \int_0^\infty \frac{\bar{\eta}(l) l^2 dl}{S_f^2(1,l)} Z^{\mathrm{t}}_{\mathrm{Sch}}(i\tau_1, l) Z^{\mathrm{t}}_{\mathrm{Sch}}(i\tau_2, l)\,. \end{aligned} \tag{27}$$

Finally, using the same considerations about $\bar{\eta}(l)/S_f^2(1,l)$ that lead to Eq. (21), we find, in accordance with Eq. (6),

$$\langle Z_{\mathrm{PO}}(\tau_1) Z_{\mathrm{PO}}(\tau_2) \rangle^{\mathrm{c}}_{\mathrm{un}} = \kappa Z_{0,2}(i\tau_1, i\tau_2)\,. \tag{28}$$

This result holds for *all complex values of* $\tau_{1,2}$, in particular for the universal regime $t_1 \simeq -t_2, |t_{1,2}| \gg \beta_{1,2}$, where it coincides with the diagonal approximation, Eq. (21). Moreover, identifying the periodic orbit length l with the length of the internal gluing geodesics b appearing in the computation of JT gravity partition functions, Eq. (6), Eq. (28) gives the exact result for the JT "double trumpet" or wormhole partition function. This agreement extends well beyond the ramp, Eq. (21) as it holds at the more fundamental level of its full integral form in Eq. (6). In this way the Weil-Petersson measure ldl, the Schwarzian trumpets of [4] and the κ index reflecting the orientability of the manifolds in the gravitational genus expansion, all admit a periodic orbit interpretation.

To conclude, based on the Selberg trace formula we studied spectral correlations of quantized chaotic motion in high-dimensional manifolds of constant negative curvature and showed that it represents a possible dual to JT quantum gravity. We support this claim by deriving exact results for the leading-order terms of two JT correlation functions. First, we show the strict equivalence of the two theories at the level of the 1-point correlator where the Weyl term representing the smooth spectral density of the hyperbolic dynamical system exactly coincides with the gravitational result, as given by the Schwarzian action on a disk topology. Second, while under the usual semiclassical smoothing, the 2-point correlator of the hyperbolic system reproduces the JT result for $Z_{0,2}(i\tau_1, i\tau_2)$ only in the random matrix theory regime $\mathrm{Im}\,\tau_1 \sim \mathrm{Im}\,\tau_2 \gg \mathrm{Re}\,\tau_{1,2}$ (the so-called ramp contribution), the combination of phase space coarse graining and discreteness of the underlying periodic orbit set extends this result to the full wormhole geometry and all complex times. In this way, our approach provides a periodic orbit interpretation of distinct features on the gravitational side. Together, these results imply a possible duality at the level of an individual quantum system, as opposed to the ensemble paradigm (e.g. [23, 30] and references therein) in low-dimensional quantum gravity, with the large dimensionality of the hyperbolic system hinting towards chaotic many-body dynamics.

In order to fully establish the duality, it is desirable to show whether the geodesic motion in high dimensional manifolds can be described by the Schwarzian action or to identify the telltale $\mathrm{SL}(2,\mathbb{R})$ symmetry, both characteristic properties of JT gravity also present in for example, the SYK model. This, and the possible existence of a genus-like expansion on the semiclassical side emerging from the interplay between the encounter mechanism [8, 36, 37] and the finite resolution introduced here, are subject of present scrutiny.

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End Matter

We now elaborate on how the Plancherel measure appears as a deep geometrical structure in both JT gravity and the Selberg trace formula.

In order to argue that the Plancherel measure in the STF has the same origin as the sinh spectral density in JT gravity, we need to recall some important steps in the derivation of the STF [47]. The manifold $\mathcal{K}$ on which the system considered in the main text is defined can be understood as a quotient manifold $\mathcal{K} = \mathbb{H}^f/\Gamma$, where $\mathbb{H}^f$ is the hyperbolic space in f dimensions and Γ some discrete hyperbolic subgroup of the group of orientation-preserving isometries on $\mathbb{H}^f$. For $f = 2, 3$, Γ would be a Fuchsian or Kleinian group respectively, and more generally, is isomorphic to the fundamental group $\pi_1(\mathcal{K})$, i.e. the set of closed geodesics on $\mathcal{K}$. Note that in the simplified notation of the main text, Γ refers only to the simple closed geodesics, or equivalently, the primitive elements of the fundamental group of $\mathcal{K}$, usually denoted in the literature by Γ^*. We consider the Laplacian Δ on $\mathbb{H}^f$ and define the notion of point-pair invariants $k(x, y) = k(d(x, y))$, i.e. smooth, even functions of compact support, which depend only on the distance between two points $(x, y) \in \mathbb{H}^f \times \mathbb{H}^f$. Suppose now we have some radial eigenfunction of the Laplacian, *viz.* a function ϕ satisfying $\Delta\phi = \lambda\phi$ for some λ and radial about a point p. Then for such a ϕ, it is not difficult to show that

$$\int_{\mathbb{H}^f} k(x, y)\phi(y)dy = h(\lambda)\phi(x), \tag{29}$$

with a function $h(\lambda)$ that does not depend on ϕ. The left-hand side of (29) can be expressed as an integral over the fundamental domain F of Γ, since $\Gamma F = \mathbb{H}^f$:

$$\int_{\mathbb{H}^f} k(x, y)\phi(y)dy = \int_F \left(\sum_{\gamma\in\Gamma} k(x, \gamma y)\right)\phi(y)dy, \tag{30}$$

and combining Eqs. (29) and (30) for a complete set of eigenfunctions $\phi_0, \phi_1, \phi_2, \ldots$, we recover Selberg's pretrace formula,

$$\sum_\gamma k(x, \gamma y) = \sum_n h(\lambda_n)\phi_n(x)\overline{\phi}_n(y). \tag{31}$$

The argument is perhaps a bit more subtle than a first glance would suggest. One needs to recognize that the LHS of (31) generates a Hilbert-Schmidt operator whose action is given by the RHS of (30). According to (29) then, the ϕ_n are eigenfunctions of this operator, and hence by Hilbert-Schmidt theory, the kernel of the operator admits an expansion as the RHS of (31). Taking the trace in Eq. (31) gives the STF, and the term on the LHS coming from the conjugacy class of the identity $\gamma = \{\mathbb{1}\}$ takes the form of the Weyl term

$$\mathcal{V}\int_0^\infty h(\lambda)\Phi_f(\lambda)d\lambda, \tag{32}$$

with the Plancherel measure, explicitely given by

$$\Phi_f(p) = \frac{f}{(4\pi)^{f/2}\Gamma(\frac{f+2}{2})}\frac{|\Gamma(ip + (f-1)/2|^2}{|\Gamma(ip)|^2}, \tag{33}$$

coming in as the density of the radial eigenfunctions traced out on the RHS of (31),

$$\int_{\mathbb{H}^f} \phi_p(x)\overline{\phi}_{p'}(x) = \frac{1}{\Phi_f(p)}\delta(p - p'), \tag{34}$$

where now p is a variable parameterizing the continuous part of the spectrum of Δ and $\Delta\phi_p = \lambda(p)\phi_p$ [48].

To see that the origin of the spectral density of JT gravity can be understood in the same way, we need to recall its formulation as a $\mathfrak{sl}(2, \mathbb{R})$ BF gauge theory. This theory is defined by the action

$$S_{BF} = -i\int \mathrm{tr}(BF), \tag{35}$$

where B is a scalar field and F the field strength computed from a gauge connection A. The integration is to be performed over the (as yet unspecified) gauge group manifold. This action can be straightforwardly mapped to the JT gravity bulk dilaton action, while the action of the boundary mode depends on the global structure of the gauge group, rather than just the gauge algebra. Taking the gauge group to be the universal cover $\widetilde{SL}(2,\mathbb{R})$ of groups with the Lie algebra $\mathfrak{sl}(2,\mathbb{R})$, the disk partition function can generically be expressed as an integral over the irreducible representations (irreps) of $\widetilde{SL}(2,\mathbb{R})$ [49],

$$Z_{\text{disk}} \propto \int dR\Phi(R)e^{-\frac{\beta}{2C}\left[C_2(R)-\frac{1}{4}\right]}, \tag{36}$$

with the quadratic Casimir $C_2(R)$ and the Plancherel measure

$$\Phi(R)dR = \frac{(2\pi)^{-2}s\sinh(2\pi s)}{\cosh(2\pi s)+\cos(2\pi\mu)}d\mu ds. \tag{37}$$

Analytically continuing $\mu \to i\infty$ yields the sinh spectral density, and hence the disk partition function of JT gravity. The Plancherel measure comes in, once again, as the density of the basis functions of the "irrep space", namely the irreps U_R themselves:

$$(U_R, U_{R'}) = \frac{1}{\Phi(R)}\delta(R,R'), \tag{38}$$

where $\delta(R,R')$ is the product of a Dirac (Kronecker) δ for every continuous (discrete) representation label in R. The correspondence between Eqs. (34) and (38), and hence of the Weyl term in the STF and the disk partition function of JT gravity, including the functional form of the Plancherel measure suggests a tantalizing unexplored connection between group quotients of high-dimensional hyperbolic spaces $\mathbb{H}^f/\Gamma$ and groups like the gauge group of JT gravity, which is technically a purely hyperbolic sector of a central extension of $\widetilde{SL}(2,\mathbb{R})$.