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Green central banking and game theory: The *Chicken Game*-approach[☆]

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ABSTRACT

This paper investigates the determinants of the probability that a central bank chooses to make its financial sector green. We derive a mixed-strategy Nash equilibrium from a strategic setting of two monetary authorities choosing simultaneously between the alternatives of greening and conducting business as usual. Using a very general setup, we obtain a model that nests most of the usual 2 × 2-situations in game theory. "Green" avoids a country's contribution to an externality experienced by both, but also encompasses a sacrifice of slowing down economic performance. The probability of greening is found to decrease whenever "greening" means a larger sacrifice for the other country, while it increases with the size of both countries, the rate of internalization applied to the externality as well as the severity of this externality. Unlike the typical (pure) free-riding approach to international coordination on environmental issues, we find some willingness of countries to sacrifice wealth for the sake of avoiding a worst case. In a repeated setting, co-operative solutions can be established. The influence of discounting on the stability of these solutions is ambiguous.

1. Introduction

The contribution of our paper to the literature is two-fold. First, we provide some mathematical analysis to the (so far) mainly verbal theory of green central banking. Our goal is to thereby draw attention to the importance of green finance, i.e., the integration of non-financial goals into the financial sector. Second, we provide a (fully-discussed) analytical example of a *Chicken Game* in the context of climate change policies, including repeated games. Most of the analysis carries over to other kinds of climate policies where international coordination is relevant by mere reinterpretation.

1.1. Green central banking

Following the European Investment Bank's first issuance of a green bond in 2007 (cf. Banga, 2019, p. 18), a sizeable strand of literature dedicated to the analysis of both drivers and effects of green loans, green bonds and similar financial products has emerged. The empirical analyses among these lines have mostly documented a shift in demand from regular towards green bonds as can be seen in, for example, Hu et al. (2020). While we acknowledge that this leads to a certain degree of greening in the financial system, there is

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still room for further greenness. Researchers such as Dikau and Volz (2018) argue that central banks (henceforth, CBs) are both able and obliged to play a role in this context. A few years ago, various CBs around the world, such as the Banco Central do Brasil and the Bangladesh Bank, already began making efforts to do their part in the greening of the financial system in their respective countries (cf. Dikau and Volz, 2018, p. 7). Even the European Central Bank has recently expressed interest in helping to mitigate climate change and has begun to take action (European Central Bank, 2022).

Means to achieve this include, among other things, reductions in reserve requirements and risk-management standards based on environmental and social criteria (cf. Dikau and Volz, 2018, pp. 6-12). Caveats on a potential overstretching of a CB's mandates and involved trade-offs such as a higher tolerated rate of inflation are propounded by, e.g., Simandan and Paun (2021). The rationale behind this is simple: As was frequently noted (for example, by Simandan and Paun, 2021, p. 4), the number of policy instruments available linearly pins down the number of achievable goals as stated by the Tinbergen rule. As a result, other monetary authorities, such as the Federal Reserve System, remain somewhat distant from this issue.

It remains an open discussion whether environmental responsibility is less of an additional goal rather than one CBs must incorporate in order to fulfill their existing mandate(s). While a somewhat sound monetary policy in environmental terms may even be required to achieve a CB's top-tier goals to guarantee the stability of, primarily, prices, but also the financial system as a whole (as is argued by, e.g., Dikau and Volz, 2018, p. 2), we do not follow this line of reasoning. Rather, we focus on a trade-off between contributing to the economy's sustainability but also diminishing its performance. Moreover, the intention of this paper is neither to promote nor to dismiss the idea of green central banking per se but simply to provide some formal analysis on potential reasons why monetary authorities may choose to pursue it. The fact that (the intensity of) this pursuit varies widely across countries can be seen from the comprehensive table assembled by Eames and Barmes (2022, p. 8) where a fat zero flaunts for the number of green monetary policy actions taken by many G20-countries. Of course, one might object that those countries may rely on other policy measures steered at achieving climate friendliness instead. However, this argument loses its bite when we remind ourselves that central banks are (by and large from a legal perspective) institutions independent from national governments.

Our goal is to contribute to the literature on green central banking by building a theoretical model of a CB's strategic decision-making on whether to follow a green mandate or not.[1,2] For most of this paper, we consider payoffs in present value terms in order to allow for a simple game-theoretic simultaneous-move game. From this setup, we can determine the Nash equilibria of this game (which depend on the precise parametrization) and, based on these, identify the probability of a greening to take place for the case of multiple equilibria. Thereby, we hope to enhance understanding on why some CBs choose to conduct a green policy while others don't.

1.2. Climate change game theory

Frequently, the insufficient greenness of national policies is modelled as a result of a lack of international coordination. Essentially, this state of affairs is considered as the tragedy of the commons: when an action grants private utility that exceeds an associated commonly shared cost, individual actors are prone to exhaust those private benefits, ultimately leading to a collectively undesirable result (cf. Ostrom, 2010). It is typically modelled as a *Prisoners' Dilemma* equilibrium, which in most cases is the first equilibrium of static game theory to be analyzed in any seminal textbook (cf. Gibbons, 2011, pp. 2–3). There, private benefits are modelled in such a way that they crowd out public good provision in equilibrium, leading to an inefficient (i.e., a not welfare-maximizing) solution. The same holds true for avoidance of public bads. The decision to sacrifice something individually for the sake of contributing to a global good is not carried out due to free-riding incentives, as argued by, e.g., Wood (2011). Arguably, the presence of more than one other country facing a similar choice exacerbates this mechanism. Alternatively, one can interpret the players not as individual countries (or rather, their policy makers) but as groups of them. Forgó et al. (2005) consider sensible groups such as the European Union (which gains additional plausibility in our setting of CBs due to their common monetary authority, the ECB), US + Canada + Japan, or the Former Soviet Union. Pittel and Rübbelke (2008) consider a more holistic approach and simply subdivide the world into developing and industrialized countries. The question of the correct partitioning remains open and an issue of international policy coalitions.

Aside from literature on the previously mentioned static games, there is also some work on dynamic games with a clear timing structure and extensive form representations. A good example is Forgó et al. (2005). The corresponding strand of the literature is dedicated to the analysis of *negotiations* about climate policies – an area we do not touch on here.

The model we provide serves as a rationale for the fact that green policies are not fully dismissed by any country regardless of the above logic: In order to avoid a worst case, individual actors may volunteer to sacrifice themselves to be free-ridden on. The implied type of (Nash) equilibrium is that of a *Chicken Game* whose applicability to climate issues is assessed by DeCanio and Fremstad (2013). Robinson and Goforth (2005) consider all possible variations of (ordinally ranked) 2×2 games (cf. pp. 19–20). Our game is, in essence, a version of their g_{122}, as was already noted by DeCanio and Fremstad (2013, p. 182). The same authors identify four other (climate relevant) *Chicken*-type 2×2 games (cf. pp. 182–183). Each of those differs mainly from the version we discuss here in that they obtain unique equilibria according to the Nash principle with a clear free-rider (while maxi-min leads to the greenest allocation).

[1] In principle, the analysis conducted here is compatible with various different interpretations. For example, one could also interpret the agents as the governments of the respective countries that are faced with the possibility of implementing some kind of green political strategy.

[2] We do not mean to imply here that no other notable mathematical contributions exist. See, for example, Giovanardi et al. (2023) on the ineffectiveness of green central banking in the presence of an optimum carbon tax.

In a setting similar to ours, Pittel and Rübbelke (2008) analyze the importance of ancillary (or secondary) benefits from climate protection. Those are characterised by avoidance of a more local kind of externality and include examples such as noise from traffic. They may serve as an additional rationale for the different rates of internalization δ_i which capture an (inverse) tolerance for negative externalities[3] and will be introduced later. It could also be said that they incorporate the fact that some countries tend to be more far-sighted and, thus, environmentally responsible than others despite the existence of some objectively verifiable level of the externality.

2. Model setup

We consider two economies $i = 1, 2$ in each of which a CB can choose its action among two policies: Aiding the greening of the financial system ("green") or conducting business as usual following a narrow mandate of price stability ("brown"). If they choose "green", the overall economic performance of their economy will be hampered such that it falls from R_i^b to $R_i^g < R_i^b$.[4] Generally speaking, we thus incorporate a result from the Stern Review that sacrificing "around 1 % of global GDP each year [can avoid the worst impacts of climate change]" (Stern, 2006, p. vi) without predetermining the exact size of the sacrifice to be undertaken by each actor. Any potential harm from not doing so is assumed to be captured by the (global) externality $f(E)$, where E denotes, say, greenhouse gas (GHG) emissions. We evade the (stark) assumption of non-locality implied therefrom by adding flexibility via an internalization parameter δ_i, $i = 1, 2$, which can vary across countries and will be introduced in Section 3. One can also interpret the decrease in output as the result of an increasing number of "green loans" (cf. Giraudet et al., 2020). Because these are typically associated with a lower rate of return due to a "greenium", as documented in a large body of empirical observations by, e.g., Fatica et al. (2021), the payoffs of projects financed in this manner should overall be (safer, but) lower.[5, 6] In exchange for this, they know that a contribution to the externality $f(E)$ that materializes itself (equally bad for both countries, but possibly internalized in different intensities) will be circumvented. To depict this, we define $E = \omega_1 e_1^{s_1} + \omega_2 e_2^{s_2}$ where $s_i \in \{b, g\}$, $i = 1, 2$ defines the policy choice (between becoming g reen and staying b rown) and

$$e_i^{s_i} = \begin{cases} 0 & \text{if } s_i = g \\ e & \text{if } s_i = b \end{cases}, \quad i = 1, 2.$$

The parameters $\omega_i > 0$, $i = 1, 2$ allow for different sizes of both economies.[7] In order to depict convex worsening of the externality, we express it as the quadratic environmental damage it inflicts, i.e., by the convex function $f(E) = E^2$.

3. Utility representation

In order to allow for some reluctance on the part of CBs towards introducing the green mandate, we assume a (potentially country-specific) rate of internalization $0 < \delta_i < 1$ which they use as a weight placed on the global externality $f(E)$.[8] Thus, a different valuation (or emphasis) of their various goals is explicitly allowed. Upon comparing present value terms, we allow for a rather broad interpretation of the $R_i^{s_i}$ as already representing an adequately discounted output measure. Hence, we obtain the following utility function characterizing each central bank:

$$U_i^{s_i} = R_i^{s_i} - \delta_i f(E), \; s_i \in \{b, g\}, \; i = 1, 2 \tag{1}$$

where s_i again represents the policy chosen by i. Of course, both s_1 and s_2 enter utility of each i through E – the environment is a (global) public good. Modelling the choices of both CBs as a one-shot[9] simultaneous-move game, we obtain the game matrix depicted

[3] Note, however, that these rates cannot fully substitute the concept of a local element of externalities; the externality in our model is a global public bad. That is, if our model were about cars, it would encompass their carbon dioxide emissions, but not the noise they make. The former may, however, hurt different individuals in a different way.

[4] One may also consider those output measures as including, in some form, the price stability of a certain country. For example, as argued by Simandan and Paun (2021, p. 12), unduely high expectations of the people about a CB's potential effect on environmental issues could lead to a "loss of reputation [that] may render it unable to counter inflation via little credibility over inflationary expectations."

[5] A comprehensive overview over different empirical studies documenting a positive premium is given by Cheong and Choi (2020). According to those same authors, findings documenting the absence or negativity of the greenium seem to be considerably fewer in number and subject to drawbacks. The conjectured property of safer returns can be justified on the grounds of a hedge against potential emission pricing in the future (see Bolton and Kacperczyk, 2021).

[6] One may object that the financial markets' perspective used as an argument here is inadequate as asset prices may reflect investor tastes and thus generally only report intrinsic value imperfectly (see Fama and French, 2007). Arguing from the prespective of true economic performance instead, it seems obvious that cleaner production always premises some form of costly innovation, a state of affairs already incorporated in the work of Heinkel et al. (2001).

[7] An alternative interpretation of ω_i could be a country's energy intensity: A very energy-intensive economy can be expected to have a massive role to play in the size of the worldwide externality given by total greenhouse gas emissions when choosing between building coal generators and solar panels.

[8] We could, instead, consider δ_i to be a standard discounting factor, enabling different timing of output and the externality to take place. A combination of both motives would be possible as well. The difference in its interpretation is straightforward. We stick with the logic of internalization because of the aforementioned flexibility it grants and because genuine discounting is applied in the repeated setting from Section 7.

[9] Thus far, the opportunity to save the environment is "now or never". We soften this assumption in our dynamic analysis in Section 7.

Table 1
Static game-matrix.

		CB2 green	brown
CB1	green	$R_1^g; R_2^g$	$R_1^g - \delta_1\omega_2^2e^2; R_2^b - \delta_2\omega_2^2e^2$
	brown	$R_1^b - \delta_1\omega_1^2e^2; R_2^g - \delta_2\omega_1^2e^2$	$R_1^b - \delta_1(\omega_1+\omega_2)^2e^2; R_2^b - \delta_2(\omega_1+\omega_2)^2e^2$

in Table 1 by applying the two possible choices of s_i by each CB and corresponding materialised values of E to equation (1). As is standard in game theory, the resulting game structure is assumed to be common knowledge. In our depiction, CB1 is assumed to be the row player while CB2 represents the column player.

It is rather obvious that for a sufficiently high level of the externality or, similarly, its rate of internalization by both monetary authorities as well as a sufficiently low difference in economic performances under the different regimes $R_i^b - R_i^g$, $i = 1, 2$, acting according to a green mandate will be a strictly dominant strategy and vice versa for the business-as-usual-case. Both cases can be found below. Later, we will, on the contrary, be looking for intermediate parameter constellations so that there is a mixed-strategy Nash equilibrium (alongside two pure-strategy equilibria),[10] which will allow us to identify the various determinants of the probability of a CB going green in a strategic setup. We acknowledge critique on considering mixed strategies in geopolitical settings brought forth by authors such as DeCanio and Fremstad (2013, p. 179). However, we do not agree that considerations of this kind are meaningless. Obviously, no government or CB would toss a coin in order to determine its political course of action. But neither is the decision to "go green" made once and for all with no possibility of corrective measures. Furthermore, there is no clear rationale for one equilibrium to be reached rather than another in the presence of multiple equilibria, meaning that the mixing itself may be considered as a means to achieve coordination (albeit an imperfect one). The discussion in Section 7 sheds light on this matter. Another argument brought forth by Pittel and Rübbelke (2008) is the relevance of participation probabilities when ratification processes of international agreements extend over longer periods of time.

What has not been addressed yet is the length of the time period under consideration, i.e., for how long ahead the decision in favor of a certain policy choice is made. If we stick with the one-shot game, the term period of a CB's president may be adequate, given that they are not interested in the course of the economy after their incumbency. A repeated version may use the same period length or an even shorter one as long as this would not make policy choices too frequently changeable, potentially leading to unprecedented (and, thus, uncaptured by the model) instability of prices or the economic performance.

4. Unique equilibria

In order to provide a strategic solution to our game, two main principles are available. We employ the concept of Nash equilibrium where chosen policies need to be mutual best responses. A maxi-min strategy would only be adequate for especially risk averse[11] actors (cf. DeCanio and Fremstad, 2013, p. 183). The latter should not be expected when considering policy authorities, hence we limit attention to the Nash-concept in order not to miss any potential equilibria. To provide a rigorous discussion of all contingencies, we begin our equilibrium analysis with unique Nash equilibria and the conditions under which they arise before proceeding to our core idea of multiple equilibria and mixed strategies. Further, note that we do not consider knife-edge cases with equal payoffs from different strategies due to their lack of materiality. Each theoretical discussion is followed by a numerical example for illustrative purposes.

4.1. Prisoners' Dilemma

For the sake of completeness, we show that our model representation nests the famous *Prisoners' Dilemma* (PD). This occurs whenever (brown, brown) is the unique Nash equilibrium in strictly dominant strategies, which is, in turn, the case when CB i prefers to play "brown" in reaction to both "green" and "brown" as the policy choice of $-i$, $i = 1, 2$. "Green" is always accompanied by "brown" iff

$$R_i^b - \delta_i\omega_i^2e^2 > R_i^g$$

or, equivalently,

$$R_i^b - R_i^g > \delta_i\omega_i^2e^2,$$

i.e., if the benefit in economic performance (or financial stability) outweighs the single-handedly created damage caused by, say, GHG emission. The condition for "brown" to be a best response to "brown", too, is

$$R_i^b - \delta_i(\omega_1+\omega_2)^2e^2 > R_i^g - \delta_i\omega_{-i}^2e^2.$$

[10] We recognize some similarity to Obstfeld's (1996) model on currency crises. In our setup, however, the equilibria for an intermediate parametrization are based on coordination to contrariant rather than complementary policies.

[11] In the context of variable-sum games such as the one under consideration here, the assumed risk aversion would in fact have to tend towards infinity.

Bringing this into the form

$$R_i^b - R_i^g > \delta_i(\omega_i^2 + 2\omega_1\omega_2)e^2 \quad (2)$$

reveals another, stricter, precondition. Assuming the latter hence makes the former unnecessary. The interpretation of (2) is that the financial gain from staying brown has to outweigh not just the cost of one's own environmental damage caused by doing so, but also the full part that is due to global climate neglection, i.e., everything but what is unambiguously "the other one's fault".

If the above scheme really is meant to depict a *Prisoners' Dilemma* (or, equivalently, the tragedy of the commons), we need to further impose that

$$R_i^g > R_i^b - \delta_i(\omega_1 + \omega_2)^2e^2, \ i = 1, 2,$$

i.e., that (green, green) would be socially desirable despite the fact that it is unachievable by rational decision making. For this to be the case, the benefit from staying brown individually must not reach or exceed total environmental damage:

$$R_i^b - R_i^g < \delta_i(\omega_1 + \omega_2)^2e^2. \quad (3)$$

The results are summarized under the following proposition.[12]

Proposition 1. Under (2) and (3) for $i = 1, 2$, the model obtains a classical *Prisoners' Dilemma*-type of equilibrium.

Example 1. Consider two economies with $\delta_1 = \delta_2 = 0.9$, $\omega_1 = \omega_2 = 0.5$, and $e = 2$. We will re-use this parametrization of internalization rates, sizes, and emissions in all subsequent examples and vary only economic performance under the different regimes to illustrate the various cases that can arise. Here, we choose $R_1^b = R_2^b = 6$ and $R_1^g = R_2^g = 3$ such that (2) and (3) hold: $R_i^b - R_i^g = 3$ lies between $\delta_i(\omega_i^2 + 2\omega_1\omega_2)e^2 = 2.7$ and $\delta_i(\omega_1 + \omega_2)^2e^2 = 3.6$ for both $i = 1, 2$. The corresponding game is depicted in Table 2 and has the unique and inefficient equilibrium (brown, brown).

We can also visualize the game structure resulting from various parameter constellations graphically and do so by means of Fig. 1. It is limited in both directions by $\delta_i(\omega_1 + \omega_2)^2e^2 = 3.6$, the threshold of economic performance losses when going green beyond which doing so would not be socially desirable (cf. (3)). As the Figure shows, the *Prisoners' Dilemma* (on the far north-east of the Figure) is a rather extreme case of high losses and requires both players' situations to be very similar.

4.2. Harmony Reigns

Uncovering the preconditions for the converse of what the previous Subsection has established, namely a unique (green, green) Nash equilibrium (*Harmony Reigns*, HR), is straightforward. Monetary authorities answer to "green" with "green" under the exact opposite as when they would choose "brown", i.e., as long as

$$R_i^b - R_i^g < \delta_i\omega_i^2e^2. \quad (4)$$

Furthermore, "brown" induces "green" if

$$R_i^b - R_i^g < \delta_i(\omega_i^2 + 2\omega_1\omega_2)e^2,$$

which poses a weaker condition. Hence we can conclude[13]:

Proposition 2. Under (4) for $i = 1, 2$, *Harmony Reigns*.

While we are forced to admit that this result is mathematically possible, we consider it unduely optimistic when looking at the real world situation: As mentioned in the introduction, not every CB follows a green mandate.

Example 2. Consider two economies with δ_i, ω_i and e as in Example 1. Further, let $R_1^b = R_2^b = 1$ and $R_1^g = R_2^g = 0.5$. These output levels ensure that (4) holds as then $R_i^b - R_i^g = 0.5 < 0.9 = \delta_i\omega_i^2e^2$, $i = 1, 2$. The resulting game is depicted in Table 3 and yields an equilibrium where *Harmony Reigns*.

The corresponding place in Figure 1 is to the south-west where economic losses are very low for both CBs.

4.3. Environmental Responsibility Assignment

The final possibility of unique equilibria is given by an asymmetric equilibrium, which is located either in the north-east or the south-west L-shaped corner of Table 1. We shall consider them both jointly as they are, in fact, symmetric. In such constellations, one country is unambiguously assigned the role of the responsible actor: For a given parametrization, the equilibrium really could not be the other way round in terms of policy choices. Hence, we refer to these as cases of *Environmental Responsibility Assignment* (ERA).

[12] Axelrod and Keohane (1985) argue that, additionally, the payoff from cooperation also has to exceed that from an even chance of being exploiter or exploited (cf. p. 229). In our setup, this would require $R_i^b - R_i^g < \delta_i(\omega_1^2 + \omega_2^2)e^2$ – an inequality that always contradicts (2) for at least one i. As that assumption is immaterial for the resulting game structure, we consider it to be negligible.

[13] Of course, for this outcome to indeed be *harmonic*, we still need desirability of avoiding the worst case. But the latter, given by (3), is already implied by (4).

Table 2
Bi-matrix with parameters yielding a *Prisoners' Dilemma*-equilibrium.

		CB2	
		green	brown
CB1	green	3; 3	2.1; 5.1
	brown	5.1; 2.1	2.4; 2.4

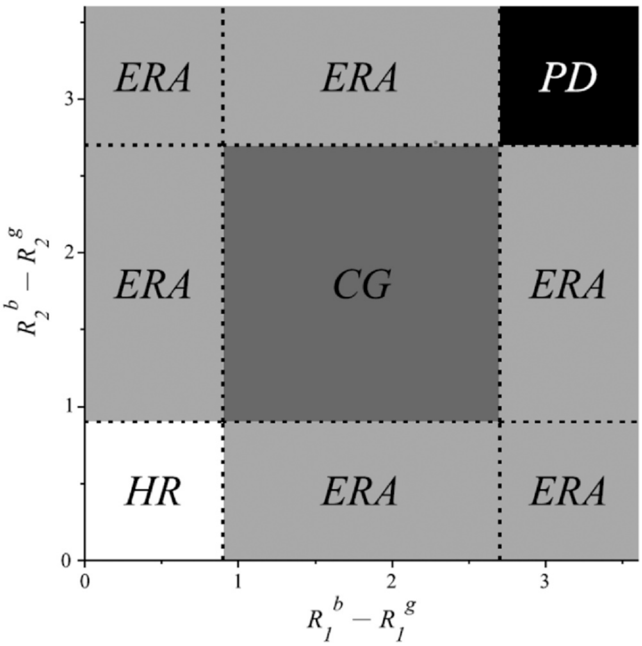

Fig. 1. Various types of games arising for different parametrizations.

Table 3
Bi-matrix with parameters yielding a *Harmony Reigns*-equilibrium.

		CB2	
		green	brown
CB1	green	0.5; 0.5	− 0.4; 0.1
	brown	0.1; − 0.4	− 2.6; − 2.6

Off-diagonal Nash equilibria are achieved once we assume that for at least one CB, either policy constitutes a strictly dominant strategy. In the simplest cases, both do. For this variant of the model, consider i with strictly dominant strategy "brown", such that (2) holds for them while the other, $-i$, always wishes to play "green":

$$R_{-i}^b - R_{-i}^g < \delta_{-i}\omega_{-i}^2 e^2. \tag{5}$$

Without further restrictions, there is no sensible manipulation available to obtain readily interpretable results here. But if we assume equal economic losses to both countries when going green, we can merge both conditions to yield

$$\frac{\delta_i}{\delta_{-i}} < \frac{\omega_{-i}^2}{\omega_i^2 + 2\omega_1\omega_2}.$$

Hence, we know that the equilibrium-"green" country is both large ($\omega_{-i} \gg \omega_i$) and its monetary authority internalizes much of the externality ($\delta_{-i} \gg \delta_i$).[14] The story behind this is that $-i$ is the big, responsible part that is willing to bear the costs of fighting climate change on its own, while the small, less responsible i can partially free-ride (without actually reaching an undesirable outcome as was the case with the *Prisoners' Dilemma*-parametrization).

The above focus on equilibria in strictly dominant strategies is too narrow a perspective to cover all possible parameter constellations and, thus, capture all potential pure-strategy equilibria. We can easily relax this to one strictly dominant strategy and an optimally asymmetric reaction. If we still let i strictly prefer to play "brown" no matter what, all we need to know about $-i$ is when

[14] In fact, equal sizes $\omega_i = \omega_{-i}$ necessitate a difference in internalization rates of more than factor 2: $\delta_i < 0.5\delta_{-i}$.

they react to this with “green”. We know from before that this is the case exactly if

$$R^b_{-i} - R^g_{-i} < \delta_{-i}(\omega^2_{-i} + 2\omega_1\omega_2)e^2, \tag{6}$$

which is a weaker condition than (5). The rationale for this constellation is simple: The commitment of *i* to conduct no “green” policy is most credible if they are observed to be subject to considerable economic losses following it. In its desire to avoid the worst case, $-i$ chips in.

If, on the other hand, we take (5) as given, the question of when *i* plays “brown” is reduced to when “brown” is a best response to “green”. This is the case as long as

$$R^b_i - R^g_i > \delta_i\omega_i^2 e^2, \tag{7}$$

which, again, turns out to be a weaker condition than (2). So there is a third reasonable story that can be told under the umbrella of *Environmental Responsibility Assignment*: Given that one country is particularly fond of playing “green”, the other one can kick back and let the first-mentioned handle saving the environment.

Proposition 3. There is a unique equilibrium in asymmetric strategies whenever

i. either (2) and (6)
ii. or (5) and (7)

hold simultaneously.

Example 3. Consider the same economies as in the previous examples, but now let $i = 1$ have $R^b_1 = 1$ and $R^g_1 = 0.5$ as in Example 2 while $i = 2$ obtains $R^b_2 = 6$ and $R^g_2 = 3$ as in the first Example.[15] Then the game is as in Table 4 and assigns environmental responsibility to the CB of country 1.

The various types of *Environmental Responsibility Assignment* can also be found in Figure 1. That of Example 3 is located to the far north-west. Consider as an additional illustration the eastern rectangle. There, country 1’s CB is faced with very high economic damages when avoiding externalities. It will therefore always go with the “brown” policy. CB2, on the other hand, is faced with intermediate losses and is accordingly prepared to take action if it has to, which is certain to be the case. Hence, CB2 is assigned environmental responsibility.

5. Multiple equilibria

Before moving on to the consideration of multiple equilibria and mixed strategies, a few words of justification are in order. As hinted in Section 3, our objective to determine the probabilities assigned to the different policies in a mixed-strategy equilibrium has not been set because one should truly imagine CBs operating some randomization device to determine the course of their policy. Rather, in a world where the cost of climate change mitigation will eventually have to be borne by someone if a global catastrophe is to be avoided, it is unclear who will have to bear that burden (unless we happen to be in the case of Subsection 4.3). Our argument is thus that monetary authorities should be interested in the probability that it really has to be them who is making that sacrifice.[16] Estimates of these probabilities might come from observations of previous actions of the other CB when viewing the game as embedded in a repeated setting (see Section 7) given that we are no longer in period $t = 0$. For now, we restrict ourselves to what those probabilities have to be for one player to support mixing of the other and vice versa, which is the theoretical precondition for existence of mixed-strategy equilibria (see Section 1.3 of Gibbons, 2011).

5.1. Complementary equilibrium

In order to determine a Nash equilibrium where players mix their strategies, we need the game represented in Table 1 to obtain no (weakly) dominant strategy such that there are two pure-strategy equilibria.[17] In other words, we want to create an issue of coordination. This can be achieved in one of two ways: Firstly, players might wish to perform actions complementary to their opponent’s, i.e., respond with “green” to “green” and with “brown” to “brown”. While this constellation would match the somewhat standard problem of coordination implied by a *Stag Hunt* [18] -type of structure, we do not obtain such a result here. On the contrary,

[15] That is, we limit attention to a demonstration of strictly dominant strategies.

[16] So what player *i* should really be interested in is the probability of “brown” by $-i$, or in subsequent notation, $p^{b*}_{-i} = 1 - p^{g*}_{-i}$. However, the latter can only be supported as long as *i* randomizes with p^{g*}_i (determined below).

[17] As is noted in any seminal textbook dealing with game theory, any two-player-two-strategy-game obtaining two pure-strategy Nash equilibria automatically also has a mixed-strategy Nash equilibrium (see, for example, Gibbons, 2011, p. 43).

[18] DeCanio and Fremstad (2013, p. 181) speak of *Coordination games* as a more open term within the context of climate policy. This addresses not only *Stag Hunts*, but also *Battles of the Sexes* (cf. McAdams, 2009, p. 222). A distinguishing feature of *Battle of the Sexes* is the non-unanimous preference ordering for the two equilibria among both players: Each prefers a different one than does the other. In a *Stag Hunt*, on the other hand, there is a unanimously preferred coordination goal (but fear of missing out). Due to, for instance, the individuality of internalization rates δ_i, we cannot argue for any expression to be more appropriate than the other inside our framework. Further, note that *Chicken* is also a game of coordination. There, however, the aim is to coordinate towards different strategies.

Table 4
Bi-matrix with parameters yielding an *Environmental Responsibility Assignment*-equilibrium in strictly dominant strategies.

		CB2	
		green	brown
CB1	green	0.5; 3	− 0.4; 5.1
	brown	0.1; 2.1	− 2.6; 2.4

we begin by showing that this version of multiple equilibria cannot occur in our model. To see this, note that for each CB $i = 1, 2$ we must have

$$R_i^g > R_i^b - \delta_i \omega_i^2 e^2$$

in order to obtain "green" as a best response to "green", which can be rearranged to yield

$$R_i^b - R_i^g < \delta_i \omega_i^2 e^2.$$

For the south-east corner to be achievable as well, we further need

$$R_i^b - \delta_i (\omega_1 + \omega_2)^2 e^2 > R_i^g - \delta_i \omega_{-i}^2 e^2$$

or, equivalently,

$$R_i^b - R_i^g > \delta_i [(\omega_1 + \omega_2)^2 - \omega_{-i}^2] e^2.$$

Those two conditions[19] both have $R_i^b - R_i^g$ on one side of the inequality sign (pointing into different directions) and can thus be combined to yield the necessary condition

$$\delta_i \omega_i^2 e^2 > R_i^b - R_i^g > \delta_i [(\omega_1 + \omega_2)^2 - \omega_{-i}^2] e^2.$$

Straightforward manipulation of the inequality

$$\delta_i \omega_i^2 e^2 > \delta_i [(\omega_1 + \omega_2)^2 - \omega_{-i}^2] e^2$$

obtained by considering only the far-ends and using the first binomial formula yields

$$\omega_1^2 + \omega_2^2 > \omega_1^2 + \omega_2^2 + 2\omega_1 \omega_2,$$

or simply $\omega_1 \omega_2 < 0$, which can never be true as long as both weights have the same sign, i.e., $\omega_i \lessgtr 0$ for both $i = 1, 2$ simultaneously. Since we have (reasonably) assumed $\omega_i > 0$, $i = 1, 2$ (recall that ω_i defines the size of economy i), we can conclude that there is no mixed-strategy equilibrium resulting from the attempt to achieve complementary strategies:

Proposition 4. There can never be an equilibrium where each player wants to choose the same strategy as the other unconditional on what that other player does, i.e., there is no *Stag Hunt*- or *Battle of the Sexes*-type of equilibrium.

With this result, we can now restrict attention to a mixed-strategy Nash equilibrium containing opposite strategies.

5.2. Asymmetric equilibrium

We now investigate the opposite and, in our opinion, more intriguing case where each of the monetary authorities wishes to do the exact opposite of what the other country's CB does. Note that this is still a game of coordination as multiple equilibria exist. One can interpret the desire to act disparately as a case of partial free-riding: As long as someone else does their part in avoiding some of the externality, the own part that could be fulfilled is simply considered as less important. The resulting payoffs are in essence a version of the standard *Chicken game* (CG) (cf. McAdams, 2009, p. 223). Madani (2013, p. 71, italics added) also claims that "no matter what the current climate game structure is […], *Chicken* may be the future of the international climate game, should all parties keep defecting". We would argue that this future has arrived: The refusal of major countries' monetary authorities, such as the United States' Fed as mentioned in the introduction, to partake in green central banking as, in turn, practiced by the ECB, may serve as an indicator that this kind of situation has indeed been reached.

The necessary and sufficient condition for "brown" to be a best response to "green" for CB $i = 1, 2$ is

$$R_i^b - \delta_i \omega_i^2 e^2 > R_i^g.$$

Slight modification of this inequality obtains

$$R_i^b - R_i^g > \delta_i \omega_i^2 e^2. \tag{8}$$

"Green" being a best response to the other CB playing "brown", on the other hand, necessitates

[19] To be precise, the two inequalities are jointly sufficient: both being satisfied simultaneously presents a necessary and sufficient condition for the discussed multiple equilibria to exist, while each inequality on its own posits a necessary condition.

$$R_i^g - \delta_i \omega_{-i}^2 e^2 > R_i^b - \delta_i(\omega_1 + \omega_2)^2 e^2$$

or, equivalently,

$$R_i^b - R_i^g < \delta_i(\omega_i^2 + 2\omega_1\omega_2)e^2, \tag{9}$$

in an analogous fashion to that used in the previous case. Again, the implied inequality chain

$$\delta_i(\omega_i^2 + 2\omega_1\omega_2)e^2 > R_i^b - R_i^g > \delta_i \omega_i^2 e^2,$$

is condensed into a necessary condition as given by its far-ends:

$$\delta_i(\omega_i^2 + 2\omega_1\omega_2)e^2 > \delta_i \omega_i^2 e^2.$$

As this now implies the exact opposite of the condition derived in Subsection 5.1, i.e., $\omega_1\omega_2 > 0$, we can conclude that the necessary condition for a mixed-strategy Nash equilibrium of the asymmetric strategy-type is always true. Whether or not both inequalities separately hold true as well, in other words whether the (necessary and) sufficient condition is also fulfilled, fully depends on the parametrization of the model.[20] Note further that (green, green) is always preferred to (brown, brown) as (9) can be reformulated as

$$R_i^g > R_i^b - \delta_i(\omega_i^2 + 2\omega_1\omega_2)e^2 > R_i^b - \delta_i(\omega_1 + \omega_2)^2 e^2.$$

Given that coordination fails, both agree that it should fail in favor of climate protection, whereas the "myopic pursuit of self-interest can be disastrous" (Axelrod and Keohane, 1985, p. 231). For now, we can conclude:

Proposition 5. The game from Table 1 can obtain a situation of multiple equilibria with different strategies as coordination goals, i.e., a *Chicken Game*-structure.

Example 4. With the values of the structural parameters as known by now, let $R_1^b = R_2^b = 4$ and $R_1^g = R_2^g = 2$ such that $R_i^b - R_i^g = 2$ lies between $\delta_i \omega_i^2 e^2 = 0.9$ and $\delta_i(\omega_i^2 + 2\omega_1\omega_2)e^2 = 2.7$ as necessitated by (8) and (9). Then the game is as in Table 5 and has two equilibria where strategies are opposite.

The *Chicken Game* posits the final missing tile from Figure 1: the big center square. While it still requires a certain degree of symmetry among actors, it extends considerably beyond the main diagonal.

To determine the probability of each country's CB choosing "green" in the resulting mixed equilibrium, we analyze the situation of an arbitrary monetary authority, CB $-i$ say. It will choose to green its domestic financial system if its expeted utility then (weakly) exceeds that of not doing so. Given that CB i (located in the other country) randomizes strategies with probabilities p_i^g for "green" and $(1 - p_i^g)$ for "brown", we consequently need

$$p_i^g R_{-i}^g + (1 - p_i^g)[R_{-i}^g - \delta_{-i}\omega_i^2 e^2] \geq p_i^g[R_{-i}^b - \delta_{-i}\omega_{-i}^2 e^2] + (1 - p_i^g)[R_{-i}^b - \delta_{-i}(\omega_1 + \omega_2)^2 e^2]$$

to hold. The nested case of equality tells us exactly when $-i$ is made indifferent between its pure strategies by i's mixing such that it is prepared to perform an arbitrary mixing itself. Solving for p_i^g, we obtain

$$p_i^g \leq 1 + 0.5\frac{\omega_{-i}}{\omega_i} - \frac{R_{-i}^b - R_{-i}^g}{2\delta_{-i}\omega_1\omega_2 e^2}. \tag{10}$$

As becomes visible from inequality (10), any CB switches from playing "brown" to playing "green" once the probability of the other one playing "green" falls below a certain threshold:

Proposition 6. There is a mixed-strategy Nash equilibrium where each CB i chooses "green" with probability $p_i^{g*} = 1 + 0.5\omega_{-i}/\omega_i - (R_{-i}^b - R_{-i}^g)/(2\delta_{-i}\omega_1\omega_2 e^2)$.

This probability acts as a threshold that determines beyond what kind of mixing i induces $-i$ to change their policy choice. At that level, indifference of $-i$ follows. The latter is thus prepared to perform an arbitrary strategy mix itself. Hence, if the converse is also true, we have found mutual best-responses, i.e., a Nash equilibrium. For each country, the probability in this mixed-strategy equilibrium is determined by the sizes of both economies, the extent of the externality, its rate of internalization (or, equivalently for now, time preferences) and the difference in economic performance under both regimes. The last two parts are, respectively, only relevant for the other CB (i.e., the one observing the mixing). We will refer to this threshold (i.e., to (10) with equality) as p_i^{g*} in what follows because it represents the chosen mixing probability in the mixed-strategy Nash equilibrium.

Example 5. The game from Example 4 has a mixed-strategy Nash equilibrium with $p_1^g = p_2^g \approx 0.3889$. There, coordination to (brown, green) or (green, brown) is achieved with a probability of roughly 0.2377 each. Miscoordination to (green, green) happens with a chance of approximately 0.1512, while the worst case happens in about 0.3735, i.e., the largest fraction of all cases.

Figure 2 plots the attainable utility levels to both CBs. Every combination of utility levels inside the tetragon is theoretically attainable by adequate mixing. The corners represent pure strategy pairings, while the lines connecting them show one-sided and the space inside two-sided mixing of strategies. Specifically, the mixed-strategy Nash equilibrium is marked at (1.45, 1.45), the values

[20] We need the constellations obtaining dominant strategies verbalized in Section 3 to be violated, i.e., as argued above, an intermediate parametrization. The resulting maxi-min equilibrium (assuming that (brown, brown) is indeed the worst case) of such a game was already determined by DeCanio and Fremstad (2013, cf. p. 182) to be (green, green).

Table 5
Bi-matrix with parameters yielding *Chicken Game*-equilibria.

		CB2	
		green	brown
CB1	green	2; 2	1.1; 3.1
	brown	3.1; 1.1	0.4; 0.4

obtained with Examples 4 and 5. Further, note that all allocations on the north-eastern border of the tetragon, i.e., the kinked line connecting (1.1, 3.1) with (3.1, 1.1) via (2, 2), are Pareto-optimal for the stage game with no instruments of communication: increasing U_1 is only possible when decreasing U_2 and vice versa. What this implies is that any policy combination with no risk of ending up with (brown, brown) is, in a Pareto-sense, efficient.

One can also visualize the mixed-strategy equilibrium along with the two pure-strategy equilibria using best-response functions. They are given by

$$p_i^g(p_{-i}^g) = \begin{cases} 1 & \text{if } p_{-i}^g < p_{-i}^{g*} \\ \in [0; 1] & \text{if } p_{-i}^g = p_{-i}^{g*} \\ 0 & \text{if } p_{-i}^g > p_{-i}^{g*} \end{cases}$$

and are depicted in Figure 3 using the numbers from Example 5. There, the dashed line corresponds to CB1 and the dotted line to CB2. The three equilibria (i.e., all strategy combinations involving mutual best-responses) are marked with fat dots.

6. Interpreting the greening-probability

In order to provide some intuition for the size of the probabilities identified above, we conduct two methods of analysis. First, in a similar vein as Pittel and Rübbelke (2008), we ascertain under which parameter constellations one player has a higher greening-probability than the other. Second, we calculate the precise influence of all relevant parameters, that is, the various (partial) derivatives of p_i^{g*}, for a single CB with respect to those variables. Note that the second type of analysis may be regarded as a generalization of the first, ultimately allowing the same conclusions to be reached.

6.1. Comparing the probabilities of both countries

The CB of country i is more likely to go green than that of $-i$ whenever

$$1 + 0.5\frac{\omega_{-i}}{\omega_i} - \frac{R_{-i}^b - R_{-i}^g}{2\delta_{-i}\omega_1\omega_2 e^2} > 1 + 0.5\frac{\omega_i}{\omega_{-i}} - \frac{R_i^b - R_i^g}{2\delta_i\omega_1\omega_2 e^2}. \tag{11}$$

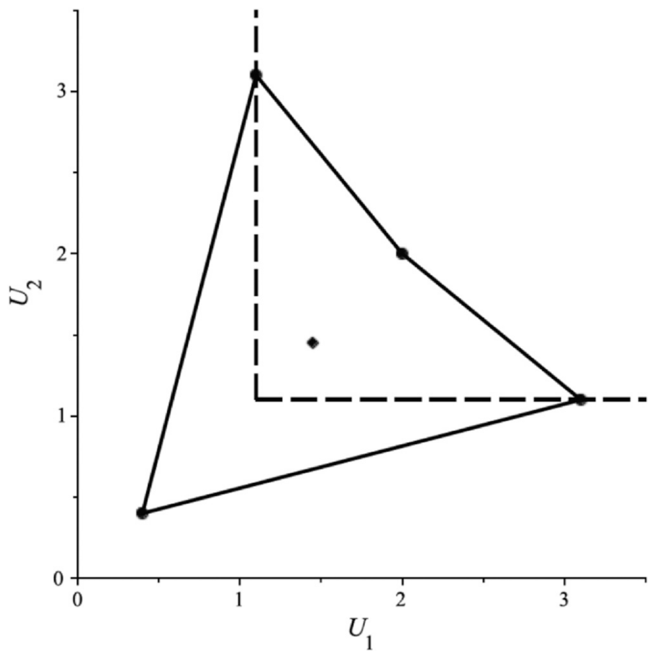

Fig. 2. Achievable points in utility space for a *Chicken Game*.

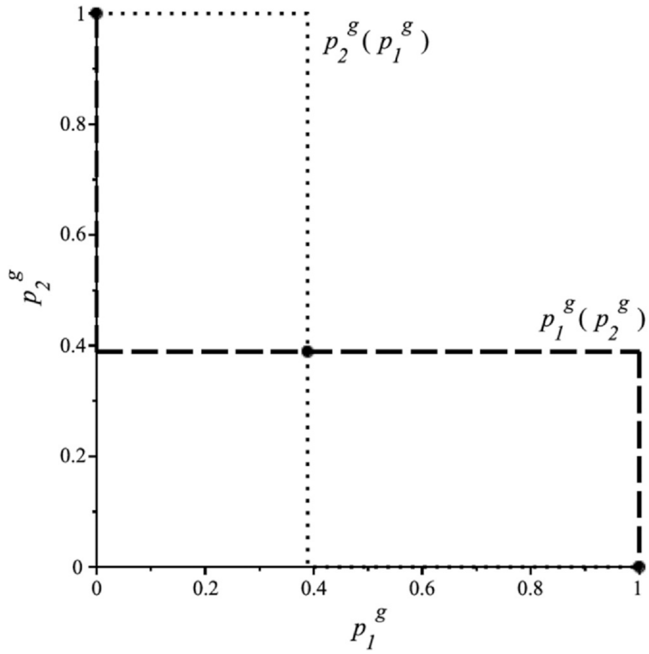

Fig. 3. Best-response functions in a *Chicken Game*.

There is little insight to be gained from this inequality without imposing further restrictions. However, it seems interesting to compare two countries that are equal in every respect but one. We begin by considering two countries that differ only in size, i.e., we impose $\omega_i \neq \omega_{-i}$. At the same time, the economic losses due to greening are the same, $R_{-i}^b - R_{-i}^g = R_i^b - R_i^g$, and the same holds true for internalization rates, $\delta_{-i} = \delta_i$. Thus, (11) simplifies to

$$\omega_{-i}^2 > \omega_i^2.$$

Due to our assumption of $\omega_i, \omega_{-i} > 0$ we can omit the squares and conclude that i is the country whose CB is more likely to green its financial system whenever i is also the smaller country. Thus, a larger country can more easily exploit a smaller country's readiness to sacrifice itself for the sake of preventing a worst case.

Next, we let countries be of the same size ($\omega_i = \omega_{-i}$) and continue to assume $R_{-i}^b - R_{-i}^g = R_i^b - R_i^g$. We do, however, allow for different rates of internalization $\delta_i \neq \delta_{-i}$. This turns (11) into

$$\delta_{-i} > \delta_i,$$

meaning that, surprisingly, the CB that cares less about the externality is more prone to choose "green". The intuition is that, in an equilibrium with mixed strategies, the less responsible player, i say, wants to make the other one, $-i$, mix rather than just play "green" as would be more in their nature due to them being the more responsible actor. This, in turn, necessitates a higher chance of greening by i such that the inadvertent coordination to (green, green)[21] is worth being prevented by $-i$.

One last example arises when assuming same sizes, $\omega_i = \omega_{-i}$, and rates of internalization, $\delta_i = \delta_{-i}$, but different economic losses from greening, i.e., $R_{-i}^b - R_{-i}^g \neq R_i^b - R_i^g$. With these assumptions, (11) becomes

$$R_i^b - R_i^g > R_{-i}^b - R_{-i}^g.$$

So the country that has more to lose from going green has a CB that is more likely to do the latter. Again, at first glance, this may seem contradictory. The logic is once more that of incentives to mix: To keep the party with little to lose mixing instead of greening straight away, the chance of the other one playing "green" needs to be sufficiently high in order to make a possible miscoordination worth preventing.

6.2. *Determinants of a single CB's greening-probability*

To be concise in interpreting the critical probability derived in Subsection 5.2, we formally ascertain its derivatives with respect to all relevant parameters. Note that in the following comparative statics analysis, we restrict attention to "small" changes in

[21] We have not yet said anything about welfare or how desirable any allocation is compared to the others from a (macro)economic point of view. Indeed, it would seem natural to assume (green, green) to be socially desirable and the only obstacle in the course of its implementability to be that internalization rates lie below one. We do not further elaborate on this point but instead simply note that that allocation is not a static Nash equilibrium. This makes achieving the concerned strategy profile undesirable from the individual viewpoints of both actors.

parameters. That is to say, we always take for granted that (8) and (9) continue to hold after the modifications. This simply ensures the further existence of (two pure-strategy, and thus) a mixed-strategy equilibrium. We start by noting that the difference in the other economy's performance under a green versus a business-as-usual regime exerts influence according to

$$\frac{\partial p_i^{g*}}{\partial (R_{-i}^b - R_{-i}^g)} = -\frac{1}{2\delta_{-i}\omega_1\omega_2 e^2}.$$

Due to the (obviously) negative sign of this derivative, the threshold probability p_i^{g*} is lower for higher differences in economic performance of the other player's country. The intuition behind it is the following: Observing that the other economy will incur heavy losses when greening its financial sector, inducing it to still play "green" requires a rather high probability of the other one doing the opposite and, thus, a lower p_i^{g*} in equilibrium.

Another important driver of this probability is the size of each economy. It seems rather intuitive to assume that incentives for free-riding persist foremost among smaller countries.[22] Starting with the influence of economy i (i.e., the same one), we obtain

$$\frac{\partial p_i^{g*}}{\partial \omega_i} = \frac{1}{\omega_i^2}\left(\frac{R_{-i}^b - R_{-i}^g}{2\delta_{-i}\omega_{-i}e^2} - 0.5\omega_{-i}\right).$$

We wish to show that larger countries' monetary authorities will be more likely to take responsibility and choose "green" with a higher probability in equilibrium. Using (8), we can conclude that the above derivative obeys

$$\frac{\partial p_i^{g*}}{\partial \omega_i} > \frac{1}{\omega_i^2}\left(\frac{\delta_{-i}\omega_{-i}^2 e^2}{2\delta_{-i}\omega_{-i}e^2} - 0.5\omega_{-i}\right).$$

Since the right-hand side shortens to 0, the derivative always shows the intended positive sign, which goes to show that a sufficient size for the desired positive influence of size itself can be taken for granted under no further restrictions. Larger countries (or their CBs, respectively) are therefore increasingly prepared to take action against climate change in our model.

As for the size of the country where CB $-i$ is located, we can simlarly determine its effect to be

$$\frac{\partial p_i^{g*}}{\partial \omega_{-i}} = \frac{1}{2\omega_i} + \frac{R_{-i}^b - R_{-i}^g}{2\delta_{-i}\omega_i\omega_{-i}^2},$$

which is unambiguously positive. The rationale for this state of affairs is simple: If $-i$ is a large economy, the threshold probability p_i^{g*} of i choosing "green" below which "green" is a best response increases. That is, the monetary authority of i has stronger incentives to green its financial sector, as it recognizes its own potential harm from not doing so. More specifically, i acknowledges how bad additional externalities actually are (due to the convex shape of the damage function) given that $-i$ chooses "brown" – which may happen in a mixed equilibrium. Probability mass is shifted towards greening via this channel because both actors suffer from the larger accruing externality in a similar way.

Before advancing towards the impact of the internalization rate δ_{-i}, we note that for the equilibrium probability of choosing "green" of either CB, its own such value, δ_i, will be irrelevant:

$$\frac{\partial p_i^{g*}}{\partial \delta_i} = 0.$$

That is, any part in this equilibrium probability enacted by one of the CBs' rates of internalization will emanate from a strategic consideration of the other one's parameter. The impact of δ_{-i} is given by

$$\frac{\partial p_i^{g*}}{\partial \delta_{-i}} = \frac{R_{-i}^b - R_{-i}^g}{2e^2\delta_{-i}^2\omega_1\omega_2}.$$

Thus, a high rate of internalization of the, respectively, other CB will lead to a high probability of choosing "green". So, despite the pessimist introduction toward this parameter's role, a shift in it will still lead to a stronger concentration of the probability mass on greening. The intuition is that, in order to keep a very socially conscious CB from playing "green", it needs to consider the event of a mistaken coordination towards (green, green) sufficiently likely. Therefore, raising awareness of the severity of externalities may indeed contribute to more responsible finance (via more likely green CB policies) taking place.

Last but not least, it is time to examine the influence of the externality level e itself. Computing the derivative of p_i^{g*} with respect to e yields

$$\frac{\partial p_i^{g*}}{\partial e} = \frac{R_{-i}^b - R_{-i}^g}{\delta_{-i}\omega_1\omega_2 e^3},$$

from which we can, again, conclude an unambiguously positive sign. Thus, any CB $-i$ chooses "green" as long as the probability of CB i doing the same does not exceed a now higher threshold. So with a higher level of the externality, the probability of avoiding it

[22] In reality, this is not necessarily the case: As Cornell (2010) argues, many least developed countries show considerable interest and willingness to act in an environmentally responsible manner (especially when financial matters are concerned).

increases simultaneously for both CBs: “Green” is then more easily incentivized as an accidental coordination towards the allocation resulting from (green, green) is not considered as too bad compared to the losses when ending up at (brown, brown).

7. Repeated *Chicken Games*

The concepts of Nash equilibria and mutual best responses, including the mixing of strategies, all assumed a one-shot game with terminal, unchangeable decisions. In reality, monetary authorities can make new choices about their degree of greenness on a day-to-day- (or at least meeting-to-meeting-) basis. Therefore, a dynamic (repeated) version of the model may be of particular interest. Notation is mostly borrowed from Mailath and Samuelson (2006).

The model as it stands can readily be applied to such a dynamic setting if one interprets the economic variables of interest as sufficiently short-term ones (e.g. let $R_i^{s_i}$ be i's economic or financial performance for the next year instead of a present value over some longer horizon). We simply need to make the payoffs of various periods comparable using a standard geometric discount factor $0 < \beta_i < 1, i = 1, 2$. Furthermore, the term “strategy” (as opposed to the stage-game “policies”) must always refer to a complete plan of play for the infinite horizon, potentially conditioned on previous actions of the other player. As the action of one CB always has a direct and measurable effect on the payoff of the other via the externality, it seems natural to assume perfect monitoring (see also Chapter 2 in Mailath and Samuelson, 2006). So strategies σ_i are functions of the repeated game's complete histories. The latter collect as a time-ordered a^{2t}-vector at each $t > 0$ all previously observed actions $a = \{b, g\}$ and are denoted as $h^t = ((s_{i,0}, s_{-i,0}), \ldots, (s_{i,t-1}, s_{-i,t-1}))$, where $s_{i,t} \in a$ represents the policy realized by CB i in a certain time period (regardless of the mixing this action may result from). All potential paths of such histories up to time t are stored in the corresponding set H^t. Of course, $H^0 = \{\varnothing\}$. Those strategies map into a stage-game policy choice any observable history at any point in time, $H = \bigcup_{t=0}^{\infty} H^t$. As p_i^g includes all mixed and pure stage-game strategies, we can write

$$\sigma_i: H \to p_i^g.$$

Our analysis is restricted to repeated *Chicken Games* as the latter constitute the focus of this paper. Hence, we always take (8) and (9) as given for the remainder of this Section. Such repeated *Chicken Games* involve a particularly tricky incentive scheme: “[E]ach player has a strong incentive to avoid cooperation in the short run in order to develop a reputation for firmness in the long run” (Axelrod and Keohane, 1985, p. 243). Indeed, whenever one of the equilibria (green, brown) or (brown, green) is reached, we should not expect recoordination towards the other equilibrium. The only plausible exception is posed by (a credible commitment to) *Tit-for-Tat* by both sides.

We focus on two specific out of a virtually infinite number of possible strategy combinations. The analysis follows the impetus to treat countries (ex ante) as identical in the sense that their strategies for the dynamic game do not prescribe different reactions to the same kind of situation. The first of those strategies (Subsection 7.2) assesses a *Folk*-statement on conditions under which mutual cooperation, i.e., (green, green), can be upheld forever despite not constituting a static Nash solution if that cooperation is threatened to be lost after a single defection by one of the two involved parties. The second one (Subsection 7.3) deals with the question of dynamic stability of a static Nash solution, that is, when the “green”-playing CB will not wish to reverse the situation in favor of becoming the “brown”-player instead, incurring losses for both in order to signal unwillingness to uphold the prior arrangement.

When considering the way we move through a repeated game, the question of an adequate starting point arises. For the *Folk*-analysis, it trivially has to be (green, green). Whether or not this is the result of a previous mixing of strategies is irrelevant for the stability analysis conducted below. But for our other application, things are less clear. We would argue that no arbitrary starting point is justifiable. This is where (10) plays a role again: Given the game structure, the only way to pick a strategy non-arbitrarily, based on non-arbitrary beliefs, is the precise mixing implied therein.[23] Starting from the ensuing (randomly) achieved allocation, we can check what kind of long-run allocations are stable, i.e., supportable given time preferences β_i. While a missed coordination would probably lead to some phasing-in, either static Nash equilibrium is achieved after a finite number of retrials. The analysis of Subsection 7.3 below explores conditions under which that allocation can be upheld despite reversion incentives.

7.1. Repeated mixing

Before moving on to analyze dynamic stability of strategy pairs, it seems adequate to further justify our starting point. If the choice of p_i^{g*} forever were inferior to some other strategy choice given the repeated setting, assuming that players start with it would lead to misconclusions. However, revisitting Figure 2, one can easily verify that mixing forever on both sides indeed posits a scenario of mutual best-responses.

Following the arguments of Gossner and Tomala (2020, cf. p. 144), any *individually rational* strategy can be an equilibrium of the repeated game, provided that players are sufficiently patient. This means that, in the long run, if one player chooses policies such to minimize the other one's payoff, the latter must respond optimally and maximize her own payoff from this scenario. In our setting, this minmax-payoff for player i results from “green” by i and “brown” by $-i$. That is, in Figure 2, any utility combination resulting from mixes on and beyond (i.e., north-east of) the dashed lines can be attained given adequate time preferences. For the underlying

[23] It appears that, somewhere along the younger path of history, CBs around the world have realized either the fact that they are playing such a game per se or that they have transitioned from a *Prisoners' Dilemma* to a *Chicken Game* (perhaps because their δ_i have risen), while some may not have reached this point yet. So the question about an adequate starting point is indeed a current geopolitical issue and not just a theoretical concern.

example, the Figure shows that infinite repetition of the mixed-strategy Nash equilibrium is indeed a valid strategy for the repeated game despite not being Pareto-efficient.

For the general case, the Figure is suggestive, but does not provide a sound proof of the fact that the utility levels obtained from mixing lie above the minmax-payoff. However, it is easy to verify that this always has to be the case. To do so, note that, provided the other CB mixes with their p_{-i}^{g*}, any policy choice in the stage game is equivalent for i: mutual best-responses require them to play p_i^{g*}, but unilaterally, this is just as good as any mixing or the pure strategies. In particular, if they were to just play "green", their expected utility would also be the same. This utility, in turn, lies somewhere between R_i^g and $R_i^g - \delta_i\omega_{-i}^2 e^2$, implying that it is (below the former and) above the latter. As that one is just the minmax-payoff, the utility from mixing must exceed it.

7.2. Sustained cooperation in harmonic green

Suppose the two countries end up in a (green, green)-situation as an outcome of their mixed strategies. As seen in Subsection 5.2, this is not a static Nash solution with the assumed parametrization. It may, however, be worthwhile to keep this solution forever if the planning horizon is infinite.[24] To sustain this allocation, we need a (credible) *Grim-Trigger-Strategy* that both players are committed to. That is, both threaten each other with eternally playing "brown" starting after they observe the other one playing "brown" for the first time.[25] Other than that, both always play "green". Formally, for all $t = 0, \ldots, \infty$,

$$\sigma_{i,t} = \begin{cases} \text{brown}, & \text{if } \exists\, \tau\text{: } h^\tau = ((g,g)^{\tau-1}, (g,b)) \\ \text{green}, & \text{else} \end{cases}$$

i's payoff from sticking with "green" is therefore

$$\sum_{t=0}^{\infty} \beta_i^t R_i^g. \tag{12}$$

We refrain from normalizing via multiplying infinite sums with $(1-\beta_i)$ (cf. Mailath and Samuelson, 2006, p. 3) as the latter would cancel out in each of the subsequent payoff comparisons. Now consider a one-shot deviation: If i decides to defect right away, its payoff consists of the individually best outcome from (brown, green) now and the comparatively mediocre (green, brown) in all subsequent periods.[26] The intuition is that i's best response to $-i$'s *Grim-Trigger* is to go with "green" forever. Note that both continue to follow $\sigma_{i,t}$ from above. Assuming a continued (brown, brown)-situation, on the other hand, does not seem reasonable as there would always be an incentive to deviate. Furthermore, the implied stability condition (see below) would then be laxer, so we decide to be conservative. The overall payoff is therefore given by

$$R_i^b - \delta_i\omega_i e^2 + \sum_{t=1}^{\infty} \beta_i^t (R_i^g - \delta_i\omega_{-i}^2 e^2). \tag{13}$$

Cooperation, i.e., (green, green), can be upheld as long as i prefers to do so. Hence, (12) must exceed (13). Using the solution formula for the geometric row, we can collapse this into the stability condition

$$R_i^b - R_i^g < \delta_i e^2 \left(\omega_i^2 + \beta_i \frac{\omega_{-i}^2}{1-\beta_i} \right). \tag{14}$$

As (14) contradicts neither (8) nor (9) necessarily, we can conclude that parameter constellations exist such that international cooperation in the form of common greenness can be sustained.

Proposition 7. (Green, green) can be a stable long-run equilibrium despite not being an equilibrium of the stage-*Chicken Game* if future valuation β_i is sufficiently high (close to one) on both sides.

Essentially, Proposition 7 constitutes an application of the *Folk-theorem*.

Having shown that upholding cooperation can be a rational choice of both players, the question remains whether the associated repeated policy pair is in fact desirable from a social point of view, that is, Pareto-efficient. Graphically speaking, the answer depends on the shape of Figure 2 and is in the affirmative if the latter takes on a kite-shape. If, however, the north-eastern kink turns out to be bent towards the origin, as in Example 4, a long-run average of payoffs above that from (green, green) forever is possible for both players. Hence, they can Pareto-improve by randomly assigning a "green"-leader and a "green"-follower with periodically alternating policies, most intuitively assigning a 50 %-chance of obtaining each role to both actors.[27] We assume that each CB obtains the role of "green"-leader, associated with a total payoff of

$$\sum_{t=1}^{\infty} \beta_i^{2t-2} (R_i^g - \delta_i\omega_{-i}^2 e^2) + \sum_{t=1}^{\infty} \beta_i^{2t-1} (R_i^b - \delta_i\omega_i^2 e^2), \tag{15}$$

[24] This is essentially the seminal argument used in infinitely repeated *Prisoners' Dilemma*ta.

[25] We allow the defecting i to still always react optimally in order to be conservative with the subsequent stability conditions.

[26] Note that stage-game actions are denoted as $(s_{i,t}, s_{-i,t})$ rather than $(s_{1,t}, s_{2,t})$ for the sake of notational ease.

[27] The story behind such a scenario is that both players meet, flip a coin and enforceably agree on who does which policy first given each outcome.

with probability 0.5. The same holds true for the role of the "green"-follower with payoff

$$\sum_{t=1}^{\infty} \beta_i^{2t-2}(R_i^b - \delta_i \omega_i^2 e^2) + \sum_{t=1}^{\infty} \beta_i^{2t-1}(R_i^g - \delta_i \omega_{-i}^2 e^2). \tag{16}$$

The average of (15) and (16) exceeds (12) under the condition

$$R_i^b - R_i^g > \delta_i(\omega_i^2 + \omega_{-i}^2)e^2. \tag{17}$$

Note that (17) was true in Example 4 but need not be true for a *Chicken Game*-structure generally.

The above result merely demonstrates a possible Pareto-improvement in an ex ante-sense. That is, after one CB has been assigned the role of "green"-leader, it may rationally wish to bail out of the arrangement. For ex post Pareto-efficiency, this must not be the case, i.e., we need (16) to exceed (12). As can be seen from the resulting inequality

$$R_i^b - R_i^g > \frac{\delta_i(\beta_i \omega_i^2 + \omega_{-i}^2)}{\beta_i}, \tag{18}$$

ex post Pareto-efficiency is always given for perfect patience ($\beta_i \to 1$)[28] and never for perfect myopia ($\beta_i = 0$). One can also condense (18) to yield a critical level of patience: ex post Pareto-efficiency requires

$$\beta_i > \frac{\delta_i \omega_{-i}^2 e^2}{R_i^b - R_i^g - \delta_i \omega_i^2 e^2},$$

which is more easily fulfilled if economic losses from greening are high and if potential environmental damage is low. A plausible rationale for the latter is that (green, green) becomes less attractive as the possible impact of climate-friendly policy grows smaller.

7.3. Reversing responsibility

The starting point for this Subsection is achieved coordination towards (green, brown) or (brown, green). We assume that it is the result of (possibly, repeated) mixing. Two scenarios about how to go on from there seem plausible:

- The strategies observed are repeated *ad infinitum*.
- Mutual *Tit-for-Tat* creates a periodic alternation of both static equilibria.

We focus on the first-mentioned case. The reason for this is that a stability condition for the "green"-locked player also implies stability for both the "green"-leader and -follower in the alternating case. Hence, proving the possibility of the former type of long-run equilibrium also proves the latter.

The underlying strategy at any point in time $t = 0, \ldots, \infty$ is for both players

$$\sigma_{i,t} = \begin{cases} \text{green}, & \text{if}[s_{i,t-1} = g \wedge s_{-i,t-1} = b] \vee \\ & [s_{i,t-2} = s_{i,t-1} = s_{-i,t-1} = b \wedge s_{-i,t-2} = g] \\ \text{brown}, & \text{if}[s_{i,t-1} = b \wedge s_{-i,t-1} = g] \quad \vee \\ & \exists\, \tau \in (0, t): [s_{i,\tau-1} = g \wedge s_{-i,\tau-1} = s_{i,\tau} = s_{-i,\tau} = b] \\ p_i^{g*}, & \text{else} \end{cases},$$

where the limits on τ ensure that we in fact consider only (past, hence) observable points in time (the policy actions of which can be found in h^t). Both CBs keep mixing stage-game policies until coordination is achieved. They continue to play that Nash solution once it has been reached. To make the incentive to deviate as strong as possible (taking on a conservative stance again), we assume that after one switch to the "brown" policy by i, resulting in a worst case, $-i$ "gets the message" that a new responsible player is required and takes on this role instead. So each is willing to stick with the "brown" action forever if that is what it takes to reverse the Nash equilibrium from (green, brown) to (brown, green) after a single defection on their own account.

If player i is trapped in the position of being the environmental savior, the discounted value of his perpetual payoff stream is[29]

$$\sum_{t=0}^{\infty} \beta_i^t (R_i^g - \delta_i \omega_{-i}^2 e^2). \tag{19}$$

On the other hand, i values its defective one-shot deviation strategy with

$$R_i^b - \delta_i(\omega_1 + \omega_2)^2 e^2 + \sum_{t=1}^{\infty} \beta_i^t (R_i^b - \delta_i \omega_i^2 e^2). \tag{20}$$

The geometric row reveals stability of the perpetual (green, brown)-equilibrium, i.e., (19) > (20), as long as

[28] The derivation of (18) requires $\beta_i \neq 1$.

[29] The following deduction posits a slight abuse of notation as we implicitly reset t to zero (from some positive integer) at the time of inspection.

$$R_i^b - R_i^g < \delta_i e^2[\omega_i^2 - \beta_i \omega_{-i}^2 + 2(1-\beta_i)\omega_1\omega_2]. \tag{21}$$

Obviously, stability is more easily achieved with higher rates of internalization δ_i and stronger environmental damages e again. Contrary to what has been established from (14), however, patience (high β_i) now deteriorates stability. The intuition here is that the reputation of being a firmly "brown" actor eventually pays off due to the avoidance of economic sacrifices despite a rather clean environment. As (21) does not automatically contradict (8) or (9), we can conclude:

Proposition 8. Infinite repetition of each of the static *Chicken*-equilibrium strategies is an equilibrium of the repeated game for a sufficiently low level of patience (β_i sufficiently close to 0) on the part of the "green"-locked CB.

The alternating structure from mutual *Tit-for-Tat* is probably not realistic. National institutions are mostly associated with some stability of their political practice. Additionally, the caveats on our model not aptly capturing consequences of such a jumpy policy noted at the end of Section 3 apply. Formally emplyoing the strategy choice

$$\sigma_{i,t} = \begin{cases} \text{green}, & \text{if}[s_{i,t-1} = b \wedge s_{-i,t-1} = g] \quad \vee \\ & \exists\, \tau \in (0,t)\colon [s_{i,\tau-1} = g \wedge s_{-i,\tau-1} = s_{i,\tau} = s_{-i,\tau} = b] \\ \text{brown}, & \text{if}[s_{i,t-1} = g \wedge s_{-i,t-1} = b] \quad \vee \\ & \exists\, \tau \in (0,t)\colon [s_{i,\tau-1} = s_{i,\tau} = s_{-i,\tau} = b \wedge s_{-i,\tau-1} = g] \\ p_i^{g*}, & \text{else} \end{cases}, \quad t = 0, \ldots, \infty,$$

we could still check under which condition the "green"-leader's payoff[30] (15) exceeds (20). It is easy to see that the resulting condition is laxer than (21), as some terms on the LHS (to the left of the " $>$ "-sign) are now even greater than those from (19). Note that in our analysis of stability of the "locked" equilibrium, we have only looked at the stability condition of a single CB (the one free-ridden on) because, there, for the other one that point was trivial. To validate the stability argument with alternation, it is sufficient to assume that (21) holds for both $i = 1, 2$. Hence, if (green, brown) and, mirror-invertedly, (brown, green) are stable, alternation is most definitely stable as well.

8. Conclusion and outlook

Starting from a strategic setup of two national central banks, we solve for a Nash equilibrium in mixed strategies where each monetary authority will try to free-ride on the other but not at the risk of a failed coordination towards the unfavorable allocation of no-one going green. The converse case of a complementary-strategy equilibrium does not exist in our model, whereas it can also be used to model several classical single-equilibrium cases. From the case of asymmetric coordination goals, we gain helpful insights. We see that strategic considerations taking into account another country's fundamentals such as its size or environmental responsibility matter for a CB's likelihood of going green. To sum up, the probability of greening is found to increase whenever the counterparty has only little economic losses to expect from following a green mandate itself, whenever it is large and if it internalizes much of the externality. A positive influence also stems from a country's own size as well as the severity of the externality itself. In a repeated setting, the role of discounting for stability is ambiguous: while the usual Folk-theorem result necessitates high patience, a sufficing degree of impatience is crucial if the party playing "green" in the stage-game Nash equilibrium is to keep choosing this policy repeatedly. Overall, a strong awareness of high potential damages fosters cooperation.

A potentially interesting extension to our model would be to make the game truly dynamic (as opposed to simply repeated) by making externalities persistent over time. As the Madani (2013)-quote at the beginning of 5.2 hinted, the structure of our world's climate game could be changing over time. A rigorous modelling of this state of affairs, however, requires an even more elaborate framework since the involved accumulation of externalities most likely brings about responses from other economic actors not modelled here, such as the society and national policymakers. An extension of this kind is therefore beyond the scope of this paper. Nonetheless, we consider it to be an interesting venue for future research.

CRediT authorship contribution statement

Fabian Alex: Writing – review & editing, Writing – original draft, Visualization, Methodology, Formal analysis, Conceptualization.

Data Availability

No data were used in the process of writing this article.

Declaration of Competing Interest

None.

[30] Of course, the "green"-follower in its position as player starting with "brown" has no means available to convey his potential intention to defect at $t = 0$.

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