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Erratum: Unorientable topological gravity and orthogonal random matrix universality

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Abstract: Reevaluating our published work, we found eq. (4.3) in the paper to contain an error that propagates into the following discussion. In this erratum we fix this error and adjust the discussion that follows. This does not change our main observation, i.e. the agreement up to order τ^4 between the canonical spectral form factor of unorientable topological gravity and the prediction of universal random matrix theory. Moreover, the corrected form of eq. (4.3) makes the extension to higher orders more transparent.

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In going from eq. (4.2) to (4.3) of the paper, we mistakenly added a factor of β to the term

$$\frac{\tau^6\,(2\pi\beta)^{3/2}}{30\pi^2}\left(\frac{t}{\beta}\right)^{3/2} = \mathcal{O}\big(\beta^0\big), \tag{1}$$

which appears in eq. (4.3) as

$$\frac{\beta\tau^5}{\pi}\frac{t\sqrt{2\pi}}{15}(t\tau^2)^{1/2} = \mathcal{O}(\beta). \tag{2}$$

We intentionally neglected terms of order $\mathcal{O}(\tau^8)$ in eq. (4.2) and terms of order $\mathcal{O}(\tau^7)$ except one in eq. (4.3); this was because we were focusing only on the τ^3 coefficient. However, upon reevaluating we believe all terms with like powers of β should be grouped together, and we believe there to be an infinity of such terms. We would like to rewrite eq. (4.3) in a way that demonstrates this. To increase clarity, we keep all terms we have computed in eqs. (2.63)–(2.70), i.e. up to $g = 7/2$ in the loop equations, which changes eq. (4.3) to the following:

$$\begin{aligned}
e^{-S_0}\kappa_\beta^{\mathrm{WP}}(\tau) &= \frac{\tau}{2\pi\beta} - \frac{\tau^2}{\sqrt{2\pi\beta}} + \frac{8\sqrt{2\pi\beta}}{3\pi}\tau^4 - \frac{64(2\pi\beta)^{\frac{3}{2}}}{15\pi^2}\tau^6 + \frac{512(2\pi\beta)^{5/2}}{105\pi^3}\tau^8 + \dots \\
&+ \frac{\tau^3}{\pi}\left[\frac{-10}{3} + \log\left(\frac{2t}{\beta}\right) - \frac{\sqrt{2\pi}}{3}\left(t\tau^2\right)^{1/2} + \frac{\sqrt{2\pi}}{15}\left(t\tau^2\right)^{3/2} - \frac{2\left(t\tau^2\right)^2}{45} + \frac{\sqrt{2\pi}}{210}(\tau^2 t)^{5/2} + \dots\right] \\
&+ \frac{\tau^5\beta}{\pi}\left[\frac{163}{15} - 4\log\left(\frac{2t}{\beta}\right) + \frac{17\sqrt{2\pi}}{12}\left(t\tau^2\right)^{1/2} - \frac{3\sqrt{2\pi}}{20}\left(t\tau^2\right)^{3/2} + \dots\right] \\
&+ \frac{\tau^7\beta^2}{\pi}\left[\frac{-1658}{63} + \frac{32}{3}\log\left(\frac{2t}{\beta}\right) - \frac{881\sqrt{2\pi}}{240}(\tau^2 t)^{1/2} + \dots\right] \\
&+ \dots
\end{aligned} \tag{4.3$'$}$$

This differs from the paper because we have fixed the aforementioned mistake (the fourth term of the second line, put here in red) and kept higher order terms that we initially left out in order to focus on the τ^3 term. Furthermore, we have added dots to make precise where additional terms would appear upon computing volumes of higher genus. Specifically, in the first line the dots indicate terms of even power in τ and half integer power in β that appear as the t-independent terms from half-integer genus contributions. As stated in the paper, all of these terms agree with universal RMT, i.e. the expansion of eq. (3.22). In the subsequent lines, we group together terms having the same dependence on β. This is the same logic applied in the original equations, except now we include all the available higher order terms. The dots in the groupings of these terms mean t-dependent terms of the same β-dependence from higher genus. Here, the type of additional terms is unambiguously two-fold. First, there are terms depending on $\sqrt{t}$, which come from half-integer genus contributions. Secondly, there are terms that do not depend on $\sqrt{t}$ but on t, for which there is only one example available at present, given by the fifth term, marked in blue, in the second line of eq. (4.3′), arising from the $g = 3$ contribution. We believe both these types of terms will continue for higher genus, thus the dots. The dots in the last line of eq. (4.3′) indicate terms of higher order in β and τ that definitely arise at higher genus, again with the sort of t-dependent terms discussed.

The identity in eq. (4.6) of the paper does not need to be altered, and is still sufficient for demonstrating equivalence for the τ^3 and τ^4 coefficient. This remains true because the

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mistaken term, and the additional terms we chose to include here, in brackets in the second line of eq. (4.3′), are all higher order in τ. However, in the paper we incorrectly state that the $\mathcal{O}(\tau^9)$ terms in eq. (4.6), originating from eq. (4.4), would contribute to higher order odd powers of τ as written in eq. (4.3) of the paper. In light of the β-dependence in eq. (4.3′), this statement should be altered to saying that it would contribute to canceling the remaining β-independent terms in the second line of eq. (4.3′). Based on eqs. (4.4)–(4.6), we see that the $\sqrt{t}$-type terms (and the logarithmic terms) come from the asymptotic expansion of the hypergeometric functions, while the t-type terms come from the convergent expansion of the functions. The identity eq. (4.6) cancels the first convergent term, i.e. the blue term in eq. (4.3′), using the $\log(2t/\beta)$ term and the first $\sqrt{t}$-type term in favor of the τ^3 RMT result. The next convergent term would appear at either $\mathcal{O}(\tau^9)$ or $\mathcal{O}(\tau^{11})$ and cannot be known for certain without higher genus contributions, but we expect it to cancel against the next $\sqrt{t}$-type terms using analogous identities to eq. (4.6). The new hypergeometric functions responsible for these identities would have to have a convergent series that begins at $\propto \tau^3(t\tau^2)^3$ and an asymptotic series that begins at $\propto \tau^3(t\tau^2)^{\frac{3}{2}}$, which certainly exists.[1]

After discussing the coefficient of τ^3, we speculated on how the coefficient of τ^5 could be made to agree in an analogous manner. The error we made in equation (4.3) was propagated into equations (4.7) and (4.8), which should instead read:

$$\frac{\tau^5\beta}{\pi}\left[\frac{163}{15}-4\log\left(\frac{2t}{\beta}\right)+\frac{17\sqrt{2\pi}}{12}\left(t\tau^2\right)^{1/2}-\frac{3\sqrt{2\pi}}{20}\left(t\tau^2\right)^{3/2}+\dots\right] \tag{4.7′}$$

$$\frac{\tau^5\beta}{\pi}\left[\frac{163}{15}-4\log\left(\frac{2t}{\beta}\right)+\frac{17\sqrt{2\pi}}{12}\left(t\tau^2\right)^{1/2}-\frac{3\sqrt{2\pi}}{20}\left(t\tau^2\right)^{3/2}+a(t\tau^2)^2+\dots\right]. \tag{4.8′}$$

Nevertheless, this does not alter the main claim we were making, that an analogous identity to (4.6) can be used to show agreement at the level of τ^5, and that this would require the $g=4$ volume, i.e. to determine the value of a in (4.8′). These corrections actually simplify eq. (4.9) in the original text, as C can be set to zero. As a final point, comparing the τ^5 bracket in eqs. (4.3) and (4.7) to eq. (4.3′), the $\mathcal{O}(\tau^4)$ in the brackets should have been $\mathcal{O}(\tau^3)$.

To conclude, we believe the brackets associated with odd powers of τ form an infinite series that, in the limit of $t\to\infty$, goes to the universal RMT result, i.e. (3.23) in the paper. The identity proven in the paper, eq. (4.6), gives a mechanism for how this occurs, and demonstrates agreement of the τ^3 and τ^4 coefficient. To extend the program, i.e. cancel more t-dependent terms and extend agreement to higher order powers of τ, we would need contributions from higher genus, which was the claim in the paper.

Data Availability Statement. This article has no associated data or the data will not be deposited.

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[1] E.g. $\tau^3(t\tau^2)^3{}_1F_1\left(3/2,5/2;-\frac{t\tau^2}{2}\right)$.

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