



# Project risk neutrality in the context of asymmetric information<sup>☆</sup>

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## ABSTRACT

Using the modeling framework of [Stiglitz and Weiss \(1981\)](#), we show that – perhaps surprisingly – there is no influence of average project risk on the capital market equilibrium. The savings interest rate fully determines the amount of credit rationing and the nature of an equilibrium (adverse selection, two-prices etc.). This rate is, in turn, fully determined by the relative probabilities of success of firms' projects (and, thus, repayment of their debt). Hence, making capital markets overall "less risky", which may for example be the case when financial markets become greener, does not alleviate concerns of asymmetric information. The result holds both for cases of hidden information and for those of hidden actions.

## 1. Introduction

Recent literature suggests that banks financing green firms and green projects, respectively, face a lower risk of default (see, for example, [Cui et al., 2018](#) or [An & Pivo, 2020](#)). A reasonable rationale for the observation that green projects seem to be especially safe ones is posited by the *carbon risk premium hypothesis*: climate-damaging firms are subject to additional bankruptcy risk because the price of carbon may be further increased in the future ([Bolton & Kacperczyk, 2021](#)). From a financial economics viewpoint, we should consequently expect lenders to charge lower interest payments on this kind of debt. This intuition is confirmed by [Jung et al. \(2018\)](#) who at the 10%-level of significance find that a one standard deviation increase in carbon emissions of a firm increases their cost of debt by 38 basis points (p. 1163). At first sight, the above observations seem to prompt that, once the transition from an environmentally deficient to a green economy is completed, the way for a debt market with few to no problems of intermediation resulting in credit rationing and other inefficiencies is paved. We discourage this hypothesis by inspection of the famous Stiglitz–Weiss (SW) model ([Stiglitz & Weiss, 1981](#)).

Despite the fact that much work has focused on, expanded, and sometimes even corrected said model, there is only little discussion of the role of overall project riskiness yet. However, its inspection yields quite a grand result which roughly goes like the following. Start from any capital market equilibrium in the SW model. Now apply an equi-proportionate mean-preserving change to the success probabilities of all projects available for conduction by firms. Then the result is always an analogous equilibrium with identical savings interest rates and the same amount of capital allotted to the same types of firms. Thus, the absolute level of average project risk is neutral.

To the best of our knowledge, the only similar contribution on the role of probabilities in this model appears to stem from [Stiglitz and Weiss \(1992\)](#) themselves. Their analysis on success probabilities in a very related model setup, however, produces results that are qualitatively different from ours due to a difference in assumptions. We shall address this matter in more detail later and aim

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to induce an open discussion on the subject.

Before proceeding to our hypothesis, noteworthy contributions on the Stiglitz–Weiss model are reviewed in Section 2 and the model is briefly set up in Section 3. We begin the actual model analysis in Section 4 by analyzing the benchmark case without asymmetric information. The latter is introduced in Section 5 by assuming different project endowments of firms which are not distinguishable to outsiders. This corresponds to the case of hidden information. Hidden action with non-enforceable project choice of corporates, on the other hand, is analyzed in Section 6. Each of the three last-mentioned Sections is accompanied by a numerical example for illustrative purposes. We briefly discuss neutrality of the method of financing by inspecting trade in shares rather than fixed-interest loans in Section 7. Section 8 concludes the paper.

## 2. Literature overview

As a look into the References Section reveals, much related work originates from Stiglitz and Weiss themselves. A somewhat definitive paper among these is [Stiglitz and Weiss \(1992\)](#). There, the model is set up in a slightly different, yet largely parallel way. It aims to consider the credit market under the lens of a macroeconomic model where the project owners are specified in more detail. They have rich or poor endowments, decreasing absolute risk aversion and the choice between safe and risky projects. As a result, both adverse selection and moral hazard are integrated into the model: The rich switch to the riskier project already at lower interest rates than the poor while the latter may be rationed more in equilibrium. Banks offer contracts as combinations of interest rate and collateral. Various types of equilibria can arise (pure rationing, two-prices etc.).

[Suarez and Sussman \(1996\)](#) integrate the SW-model into a dynamic setting, namely one of overlapping generations (OLG). There, firms produce output deterministically in one period, but stochastically in the subsequent one. Consequently, an alternating price structure – which can be an endogenous result of that model – of low and high prices (induced by booms and busts, respectively) leads to intergenerational disparities. As is standard in OLG-models, a redistribution scheme can be found that enhances welfare.

A contribution by [Coco \(1999\)](#) employs risk aversion of firms in the model as well. Accordingly, project owners are considered as “entrepreneurs” (individuals with risk preferences) rather than faceless conglomerates there. He identifies risk aversion itself as a possible driver of credit rationing as the risk averse owners of relatively safe projects are the first victims of adverse selection. Their reluctance to post collateral (which is also endogenous in [Coco’s 1999](#) version of the model) exacerbates this problem.

[Coco \(1997\)](#) and [Arnold and Riley \(2009\)](#) show independently of each other that the return on lending achieves its unique global maximum at the highest possible credit interest rate which still produces positive capital demand. At that rate, the riskiest firms are just indifferent between applying for a loan to conduct their project and staying inactive. The assumption that firms still apply when indifferent is common and posits a version of epsilon altruism, that is, the desire to make the other market participant better-off when not harming oneself (see also [Hillier, 1997](#), p. 38). Thus, all the (expected) project return goes to lenders which can then, in turn, pay savers the entire (expected) project return as the savings interest rate. Since all projects are expectationally equivalent, this is impossible with lower credit rates at which the riskiest corporates make positive profits.

[Arnold et al. \(2014\)](#) analyze non-diversifiable risk and risk aversion of savers. Employing the former, individuals cannot be guaranteed a definite rate of return on their deposits. Rather, different states of nature have to be distinguished. Consumers dislike the resulting uncertainty and, thus, tend to save less, which gives room to additional credit rationing in equilibrium.

[Su and Zhang \(2017\)](#) again endogenize collateral in addition to interest rates of a contract, just as [Stiglitz and Weiss \(1992\)](#) and [Coco \(1999\)](#). In their work, just like in the first-mentioned, it serves the purpose of allowing for the co-existence of both adverse selection and moral hazard. They find that the two phenomena can co-exist when there is second-order stochastic dominance of some projects against others. The intuition is that for a given (relatively high) rate of repayment, potential borrowers that can choose from a “good” (not too risky) pool of projects may not demand capital at all while those with the ability to choose from some other “bad” (more risky) pool will pick a very risky alternative among these (to ensure some positive net profit in case of a success, say).

## 3. Model

The notation mostly corresponds to the one used in [Arnold et al. \(2014\)](#), stripped of the time dimension. It is also the one used in Chapter VII of [Arnold’s \(2020\)](#) textbook, which the following exposition follows closely.

On the capital demand side, we have  $N_j$  firms of each risk class  $j$ . More precisely, each risk class encompasses a continuum of length  $N_j$  of corporates. This continuum assumption ensures that capital lenders can take expected values as definitive payoff due to perfect diversification (given uncorrelated failure events) as the strong law of large numbers applies. Hence, financial intermediaries do not bear any risk, so we do not have to specify their risk preferences.

Each project is all-or-nothing in the sense that it has a probability of success  $p_j$ , in which case it delivers a return  $R_j$ , while with the complementary probability  $1 - p_j$ , its payoff is zero. The risk classes are ordered by decreasing success probabilities, i.e.  $0 < p_J < p_{J-1} < \dots < p_1 < 1$ . However, no project is unambiguously superior to another as they are all expectationally equivalent:  $p_j R_j = E(R)$ ,  $j = 1, \dots, J$ . In other words, projects are mean-preserving spreads from each other. It follows that  $R_1 < R_2 < \dots < R_J$ .

Of course, the projects yielding higher returns in case of a success have a greater variance than those promising lower returns. Thus, we make a stark assumption contradicting a large body of the financial markets literature following [Markowitz \(1952\)](#) here by saying that they are expectationally equivalent, namely that risk is not rewarded. In other words, there is “a linear relationship between risk and return” ([Basu, 1992](#), p. 6), but none between risk and expected return. For now, this serves the mere purpose of ensuring an adequate form of capital demand, such that equilibria under asymmetric information can be characterized by adverse selection. If we were to make riskier projects more promising, the term **adverse** selection may become inadequate as both market

sides could then prefer the conduct of a riskier over a safer project, a point already noted by Riley (1987). Making safer projects better in expected values, on the other hand, is to some degree allowed, as long as second-order stochastic dominance of safer over riskier projects is warranted (see Su & Zhang, 2017, p. 1065). The assumption of equal expected returns is immaterial for the analysis of what happens when riskiness of the overall market changes. Indeed, the SW-model “is not based on an investigation of factors that determine the risk of projects” (Basu, 1992, p. 4).

In order to finance their corresponding project, firms need up-front investment capital  $B$  the only potential source of which is a loan from a bank. Projects are assumed to be socially desirable in the sense that  $E(R) > B$ .<sup>1</sup> In exchange for the borrowed amount, firms pledge collateral  $C$  to the bank and promise to repay their debt including interest rate  $r$ . We adopt the assumption of exogenous collateral in order to simplify the analysis. Wette (1983) shows that, even with risk neutral rather than risk averse borrowers, adverse selection problems are exacerbated via additional pledged collateral.<sup>2</sup> That state of affairs serves to diminish expected lender profits. As this leaves credit interest rates as the more attractive rationing device, modeling the former as the single latter suffices. If corporates fail to repay, the collateral is withheld by the bank. Of course, firms could also choose to self-finance in order to avoid paying interest. This case is, however, not of huge interest as it would limit focus on large firms (those capable of self-financing) and essentially eliminate the role of risk in the model. To close this channel, we assume  $C < B$ . To sum up, firm profits follow

$$E(\pi_j^F) = p_j[R_j - (1 + r)B] + (1 - p_j)(-C). \quad (1)$$

On the financial intermediary side, there are banks charging a fixed rate of interest  $r$  on the loans they give out. To begin with, assume that these credit interest rates can be tailored to each risk class of firms at first (in Section 4). Afterward, we disable financial intermediaries from observing the project a contracting firm has access to. Hence, the interest charged on loans can no longer be firm-specific. This is where asymmetric information finds its way into the model. If debt cannot be served, lenders<sup>3</sup> can claim the pledged collateral. Thus, their expected repayment payoff from financing a type- $j$  firm takes the form

$$E(\pi_j^L) = p_j(1 + r)B + (1 - p_j)C. \quad (2)$$

Finally, there is also a capital supply side of the model. Individuals provide savings to the financial intermediaries in exchange for some savings interest rate  $i$ . Their capital supply function can take any arbitrary shape as long as it is increasing in  $i$ :

$$S(i), \quad \frac{dS(i)}{di} > 0 \quad (3)$$

Banks cannot choose this rate haphazardly. Rather, they are driven down to zero profits by Bertrand competition (cf. Coco, 1999, p. 562), which is the standard result of price competition. In a more general setting of repeated capital allocation, Stiglitz and Weiss (1983) argue that banks indeed cannot make positive profits when forced to attract both borrowers and depositors (cf. pp. 919–920). Arnold (2012) shows that this is the only Nash equilibrium of mutually interdependent bank behavior (cf. p. 222). Our analysis of variations in success probabilities will take a specific form for the main body of this paper: They are assumed to change by 100x%. More precisely, we resort to the special case of

$$p'_j = (1 + x)p_j, \quad j = 1, \dots, J.$$

While this may look like a stark restriction at first, it clearly disentangles the hypothesis intended to be proven from another effect, namely risk dispersion. We inspect only proportionate changes in all risk classes because our goal is to analyze effects of the overall level of project riskiness. Further, note that expected revenues remain fixed at  $E(R)$ :

$$R'_j = \frac{1}{1 + x} R_j, \quad j = 1, \dots, J.$$

#### 4. Complete markets

The analysis of no information asymmetries at all is meant to serve as a benchmark and justify the further analysis. The equilibrium with complete markets turns out to be fairly simple.

##### 4.1. Equilibrium with complete markets

As lenders have to make zero profits overall and borrowers are distinguishable, lender profits have to equal interest services to savers for any project financed. They choose  $r$  firm-specifically such that, with each class  $j$ , they have expected (and, by continuity, definitive) profits of

$$E(\pi_j^L) = p_j(1 + r)B + (1 - p_j)C = (1 + i)B. \quad (4)$$

Furthermore, it is straightforward that the profits of borrowers and lenders have to add up to the total project return,<sup>4</sup> i.e.

$$E(\pi_j^F) + E(\pi_j^L) = E(R). \quad (5)$$

<sup>1</sup> Note that this additionally implies  $R_1 > B$  which is equivalent to some capital demand by every firm in the plausible interval of credit rates  $r \geq 0$ .

<sup>2</sup> Stiglitz and Weiss themselves acknowledge this contribution to their model (cf. Stiglitz & Weiss, 1983, p. 914 and Stiglitz & Weiss, 1992, p. 694).

<sup>3</sup> We use the superscript  $L$  for “lenders” to refer to banks in order to avoid confusion as  $B$  could mean “borrower” – a firm – as well. Additionally, the symbol is already in use for the borrowed amount of capital.

<sup>4</sup> One can also obtain this result from adding up (1) and (2).

Together, (4) and (5) imply

$$E(\pi_j^F) = E(R) - (1+i)B. \quad (6)$$

Hence, neither firm profits nor bank profits depend directly on  $r$ . The latter is set in such a way that the differences in  $p_j$  among risk classes are completely irrelevant. More precisely, manipulation of (4) yields

$$r = \frac{1+i}{p_j} - \frac{1-p_j}{p_j} \frac{C}{B} - 1 \quad (7)$$

which is uniquely determined by  $i$  for each firm. Furthermore, it reveals that firms conducting safer projects pay lower interest rates as the derivative

$$\frac{\partial r}{\partial p_j} = -\frac{1+i-C/B+p_j^2}{p_j^2}$$

is negative due to  $C < B$ .

Determining the capital market equilibrium requires equating capital supply and demand. Capital demand is simply the required capital of all firms taken together as long as they are not drained the entire expected project return:

$$I(i) = \begin{cases} \sum_{j=1}^J N_j B, & \text{for } i \leq \frac{E(R)}{B} - 1 \\ 0, & \text{else.} \end{cases} \quad (8)$$

Equating this with  $S(i)$  yields the equilibrium savings rate  $i$  and, thus, (implicitly) a value of  $r$  for each firm.

To return to the hypothesis of this paper, what is the effect of changes in  $p_j$  on the capital market equilibrium? For  $E(R)$  constant, which we will assume throughout, the answer is plainly: nothing.  $i$  is determined independently of  $r$  at first. Here is where the type of equilibrium is fixed: Either there is full allotment of capital or, if savers demand to high saving rates, the entire project return goes to lenders (who pass it through to savers) and credit is rationed by  $I((E(R)/B) - 1) - S((E(R)/B) - 1)$ . As this expression only depends on the  $p_j$ 's via  $E(R)$ , it is constant for mean-preserving changes in project risks. In fact, changes in  $p_j$  may so far even be arbitrary.

**Proposition 1.** *Under complete markets, arbitrary mean-preserving changes in the success probabilities  $p_j$  are neutral.*

Thus, it appears that there is room for a neutrality result in versions of the model with asymmetric information as well.

#### 4.2. Complete markets equilibrium: Example

The numerical example in this and the following sections will restrict itself to two risk classes. While this may seem like an oversimplification at first, [Stiglitz and Weiss \(1987\)](#) argue that the number of borrower types is often irrelevant for the amount of rationing in equilibrium. We thus content ourselves with the minimum number of types necessary to obtain equilibria with adverse selection or moral hazard, namely two, for expositional ease.

Let the two firm or project types be characterized by  $p_1 = 0.8$ ,  $R_1 = 125$ ,  $p_2 = 0.5$  and  $R_2 = 200$ . There are  $N_1 = N_2 = 250$  firms of each type. Hence, the expected payoff is  $E(R) = 100$ . Firms need to finance a capital input of  $B = 80$  for which they can pledge collateral of  $C = 50$ . Capital demand is  $S(i) = 200000i$ .

The entire project return of  $(E(R)/B) - 1$  here takes on the value 0.25. By (8), we have capital demand of  $(250+250) \times 80 = 40000$  up to that value. Equilibrating demand and supply yields an interior equilibrium where  $i^* = 0.2$ . By (7) we know that type-1 firms pay an interest of

$$r = \frac{1+0.2}{0.8} - \frac{1-0.8}{0.8} \times \frac{50}{80} - 1 = \frac{11}{32}$$

on their loans, while type 2 has to provide more:

$$r = \frac{1+0.2}{0.5} - \frac{1-0.5}{0.5} \times \frac{50}{80} - 1 = \frac{31}{40}.$$

A visualization of this equilibrium is viewable in [Fig. 1](#).

Turning to changes in the  $p_j$ 's, we set  $x = 0.1$  such that  $p'_1 = 0.88$ ,  $R'_1 = 1250/11$ ,  $p'_2 = 0.55$  and  $R'_2 = 2000/11$ . Note that  $E(R') = 100$  still.

All of the steps up to and including the equilibration of demand and supply follow the deduction above verbatim. Thus, [Fig. 1](#) posits a valid representation of this altered version of the economy as well. We only obtain different implied interest rates on loans, namely

$$r' = \frac{1+0.2}{0.88} - \frac{1-0.88}{0.88} \times \frac{50}{80} - 1 = \frac{49}{176}$$

for firms of the first type and

$$r' = \frac{1+0.2}{0.55} - \frac{1-0.55}{0.55} \times \frac{50}{80} - 1 = \frac{59}{88}$$

for those of the second type.

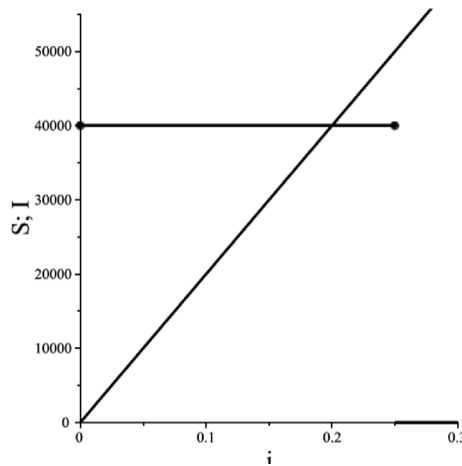


Fig. 1. Equilibrium with complete markets.

## 5. Hidden information

In this Section, borrowers are at an informational advantage: they know something the lenders do not, namely their own risk class. The odd result that this can be disadvantageous for some of the borrowers is at the core of the original [Stiglitz and Weiss \(1981\)](#) paper. The general idea dates back at least to [Akerlof \(1970\)](#).

### 5.1. Equilibrium with adverse selection

We proceed with a version of the model where there are still  $J$  types of borrowers. Those are, however, not distinguishable from the lenders' point of view. This means that there can only be one single interest rate  $r$  charged by banks on all firms' loans. Hence, while firm profits are still given by (1), Eq. (2) now only gives lender profits from one single debt contract with a type- $j$  borrower. Therefore,  $p_j$  has to be replaced by an average success probability. This depends, of course, on the mix of borrowers applying for a loan. We can derive the condition for every firm type  $j$  to demand capital from

$$E(\pi_j^F) \geq 0$$

as 0 is the outside option of each firm. With (1), we obtain

$$r \leq \frac{E(R) - (1 - p_j)C}{p_j B} - 1 \equiv r_j. \quad (9)$$

This threshold clearly depends negatively on  $p_j$ <sup>5</sup>:

$$\frac{\partial r_j}{\partial p_j} = -\frac{1}{B p_j^2} (E(R) - S) < 0. \quad (10)$$

Hence, it is the owners of safe projects that can tolerate only lower interest rates before leaving the capital market. This is precisely the core of the adverse selection problem. It reveals that expected success probabilities  $E(p|r \leq r_j)$  experience non-monotonous jumps downwards at each  $r_j$ . Therefore, lender profits

$$E(\pi^L|r \leq r_j) = E(p|r \leq r_j)(1 + r)B + (1 - E(p|r \leq r_j))C \quad (11)$$

are characterized by upward sloping segments in their dependence on  $r$  with discontinuous negative jumps at the critical  $r_j$ 's. As both [Coco \(1997\)](#) and [Arnold and Riley \(2009\)](#) show, this function reaches its global maximum at  $r_j$ . Thus, so does the percentage return

$$i(r) = \frac{E(\pi^L|r \leq r_j)}{B} - 1$$

which is paid to savers as deposit interest rate. Its maximum value is, of course, total project return<sup>6</sup>

$$i(r_j) = \frac{E(R)}{B} - 1.$$

<sup>5</sup> Remember  $E(R) > B > C$  and  $p_j \in (0; 1)$ ,  $j = 1, \dots, J$ .

<sup>6</sup> To see this, either plug  $r_j$  into (11) or simply remember  $E(R) = E(\pi_j^L) + E(\pi_j^F)$ .

What follows now is the actual analysis of the influence of project risk on profits for both sides of the market and, thus, the capital market equilibrium. Along the same line of thought that determined  $i(r_j)$ , we know that each project  $j$  yields zero profit to its owner at the corresponding  $r_j$ : Denoting lender profits from financing this project as in (2), the former are thus given by  $E(\pi_j^L | r = r_j) = E(R)$ . Risk classes  $1, \dots, j-1$  at this point have already left the market. Riskier projects' owners  $j' \in \{j+1, \dots, J\}$  generate expected profits

$$E(\pi_{j'}^F | r = r_j) = p_{j'}(R_{j'} - (1 + r_j)B) + (1 - p_{j'})(-C).$$

Plugging in (9), we obtain

$$E(\pi_{j'}^F | r = r_j) = \left(1 - \frac{p_{j'}}{p_j}\right)(E(R) - C). \quad (12)$$

We arrive at the conclusion that borrower profits at critical interest rates do not depend on their individual success probability per se, but only on the latter in relation to that of the last just-applying project. Hence, varying the success probabilities  $p_j$  of all firms proportionately is sufficient to obtain identical profits for them. By (5), the same automatically also holds for bank profits from financing that very firm. They are given as

$$E(\pi_{j'}^L | r = r_j) = \frac{p_{j'}}{p_j}E(R) + \left(1 - \frac{p_{j'}}{p_j}\right)C.$$

The percentage return on the loan to a firm of type  $j'$  is then

$$i_{j'} = \frac{E(\pi_{j'}^L | r = r_j)}{B} - 1 \quad (13)$$

and, trivially,  $i_j = E(R)/B - 1$ . Now  $i(r_j)$  can be computed simply as an average value:

$$i(r_j) = \frac{\sum_{k=j}^J N_k i_k}{\sum_{k=j}^J N_k}. \quad (14)$$

If we switch to the  $p_j'$ -regime, the critical interest rates are now given by

$$r_j' = \frac{E(R) - (1 - (1+x)p_j)C}{(1+x)p_j B} - 1.$$

They clearly differ from (9) due to the changes in the  $p_j$  which are unambiguously inversely related to  $r_j$ , see (10). At this new critical interest rate, firm  $j$  obviously makes zero profits again. For all other corporates (those in  $j'$ ), we get

$$E(\pi_{j'}^F | r = r_j') = (1+x)p_{j'}(R_{j'} - (1+r_j')B) + (1 - (1+x)p_{j'})(-C)$$

which collapses to the same expression as (12). By the same argument as before, we obtain bank profits identical to their previous level,  $E(\pi_{j'}^L | r = r_j') = E(\pi_{j'}^L | r = r_j)$  and, hence, the same relative return  $i_{j'}$ . It follows that, albeit the credit interest rates  $r_j$  where the jumps happen change, the pooled return  $i(r_j)$  available to banks as a savings interest rate they can offer individuals remains the same at all those critical credit interest rates. This insight proves to be an important corollary in our venture to show that the wealth distribution between borrowers and lenders along with the amount of rationing is unaltered not only at those critical rates, but at any equilibrium.

Before analyzing the equilibrium, we first have to close the model by specifying its demand side. Firms will obviously wish to conduct their projects as long as those are lucrative in expectations. Non-negative firm profits  $E(\pi_j^F) \geq 0$  range until critical interest rates  $r_j$ . Hence, capital demand is given by

$$I(r) = \begin{cases} \sum_{k=1}^J N_k B, & \text{for } r \leq r_1 \\ \sum_{k=j}^J N_k B, & \text{for } r_{j-1} < r \leq r_j, \quad j = 2, \dots, J \\ 0 & \text{for } r > r_J. \end{cases}$$

Consider a market for credit where there is adverse selection such that all projects with returns strictly below  $R_j$  do not apply for a loan. Firms  $j$  through  $J$  obtain capital at an interest rate  $r^* \in (r_{j-1}; r_j]$  where demand and supply intersect. Within that interval, any lower interest rate would yield a lower return and excess demand  $S(i(r)) < I(r)$  while higher rates are associated with excess supply  $S(i(r)) > I(r)$ . Although a situation of excess supply implies higher returns  $i(r)$  by  $dS(i)/di > 0$ , there is no means available to banks that actually generates more of this return, i.e., no residual demand. Suppose further that  $i(r) < i(r^*)$  for all  $r < r^*$  such that higher returns are in fact not possible through lower debt interest rates. This ensures that  $r^*$  is in fact a unique-price-equilibrium. The converse case of equilibria with multiple prices is delegated to one of the examples found in the following Subsection.

With probabilities  $p_j'$ , we obtain  $S(i(r_j)) = S(i(r_j'))$  as well as  $I(r_j) = I(r_j')$  for any critical interest rate  $r_j$ . Therefore, we know specifically that there is still excess demand when firm  $j-1$  is on the verge of exiting the market due to  $S(i(r_{j-1})) < \lim_{\varepsilon \rightarrow 0+} I(r_{j-1} + \varepsilon) \Rightarrow S(i(r_{j-1}')) < \lim_{\varepsilon \rightarrow 0+} I'(r_j' + \varepsilon)$ .<sup>7</sup> Similarly, before firm  $j$  exits the market, there is an intersection of demand and supply:  $S(i(r_j)) > I(r_j) \Rightarrow S(i(r_j')) > I(r_j')$ . Finally,  $i$  is fixed at that value which obtains  $S(i) = \sum_{k=j}^J N_k B$ . This concludes the proof for neutrality of success probabilities in the case of adverse selection.

<sup>7</sup>  $S(i(r_{j-1})) < I(r_{j-1})$  would suffice for no unique equilibrium to the left of that under consideration, but would still allow for a two-prices-equilibrium to happen.

**Proposition 2.** *Equi-proportionate changes of the success probabilities  $p_j$  are neutral if there is hidden information.*

The above neutrality result is in stark contrast to the findings of [Stiglitz and Weiss \(1992\)](#). This is due to the fact that, there, the assumption of a constant expected return is dropped, which makes sense for their interpretation of changes in  $p_j$  that refers to business cycles. We, on the other hand, analyze changes in the average project risk, leaving their quality as measured by expected return constant. Our results have in common that it is only relative probabilities that matter. The effect of changes in expected return are addressed in Subsection 5.3.

## 5.2. Adverse selection and two-price-equilibria: Examples

We stick with to example from Section 4.2. Using (9), we can calculate interest rates when capital demand of each type drops to zero. They take on values of

$$r_1 = \frac{100 - 0.2 \times 50}{0.8 \times 80} - 1 = \frac{13}{32}$$

and

$$r_2 = \frac{100 - 0.5 \times 50}{0.5 \times 80} - 1 = \frac{7}{8}.$$

They define when firms leave the market and, thereby, the structure of capital demand. In order to obtain a regular (unique-price) adverse selection equilibrium, we change firm numbers to  $N_1 = 150$  and  $N_2 = 350$ . The former case is treated below as an illustration of the two-price-equilibrium (see also ([Arnold et al., 2014](#))). As critical loan interest rates change in the same way when altering  $p_j$ 's, i.e., turning to the safer variant of this market (with  $p'_j = 1.1p_j$ ,  $j = 1, 2$ ), critical interest rates become

$$r'_1 = \frac{100 - 0.12 \times 50}{0.88 \times 80} - 1 = \frac{59}{176}$$

and

$$r'_2 = \frac{100 - 0.45 \times 50}{0.55 \times 80} - 1 = \frac{67}{88}$$

independently of the  $N_j$ 's.

### 5.2.1. Adverse selection: Example

With the above numbers (lengths of the continua) of firms,  $N_1 = 150$  and  $N_2 = 350$ , capital demand is

$$I(r) = \begin{cases} (150 + 350) \times 80 = 40\,000, & \text{for } r \leq \frac{13}{32} \\ 350 \times 80 = 28\,000, & \text{for } \frac{13}{32} < r \leq \frac{7}{8} \\ 0, & \text{for } r > \frac{7}{8}. \end{cases}$$

Using lender profits from (11), we can calculate generated returns to the bank depending only on  $r$ . They are

$$i(r) = \begin{cases} 0.59r - \frac{123}{800}, & \text{for } r \leq \frac{13}{32} \\ 0.5r - 0.1875, & \text{for } \frac{13}{32} < r \leq \frac{7}{8}. \end{cases}$$

Inserting the critical rates, we have  $i(r_1) = 11/128$  and  $i(r_2) = 0.25$ . The corresponding levels of capital supply are  $S(i(r_1)) = 17\,187.5$  and  $S(i(r_2)) = 50\,000$ . The equilibration of demand and supply can clearly only happen in the range  $(r_1; r_2]$  here as  $S(i(r_1)) < I(r_1)$  but  $S(i(r_2)) \geq I(r_2)$ . Indeed, we obtain a (single-price) equilibrium with

$$28\,000 = 200\,000 \times (0.5r - 0.1875) \Leftrightarrow r^* = 0.655.$$

The equilibrium is therefore characterized by adverse selection: only the owners of riskier projects (type 2) obtain capital at a relatively high loan interest rate. A graphic illustration is provided in [Fig. 2](#).

If we increase success probabilities in a mean-preserving way by 10% here, we get returns of

$$i'(r) = \begin{cases} 0.649r - \frac{1053}{8000}, & \text{for } r \leq \frac{59}{176} \\ 0.55r - \frac{27}{160}, & \text{for } \frac{59}{176} < r \leq \frac{67}{88}. \end{cases}$$

Returns obtain identical maximum values of  $i'(r'_1) = 11/128 = i(r_1)$  and  $i'(r'_2) = 0.25 = i(r_2)$  as before. Hence, at  $r'_1$ , capital supply falls short of capital demand by the same amount as before at  $r_1$ . The equilibrium is located in the second segment of capital demand where the equilibrium of the latter with supply accounts to

$$28\,000 = 200\,000 \times \left(0.55r - \frac{27}{160}\right) \Leftrightarrow r^{*'} = \frac{247}{440}.$$

This equilibrium suffers from the same imperfect information frictions since the same types of borrowers as in the benchmark, namely those with higher success probabilities, are discouraged from applying for a loan due to too high repayment obligations – there is still adverse selection.

We illustrate this new equilibrium by the green curves in [Fig. 3](#) where we simply add the new lines for capital demand and supply to [Fig. 2](#). The fact that it looks like a compression of the (black) reference case is no mere coincidence: Graphically, when changing all  $p_j$  simultaneously by factor  $(1 + x)$ , we always stretch ( $x < 0$ ) or clinch ( $x > 0$ ) the entire diagram.



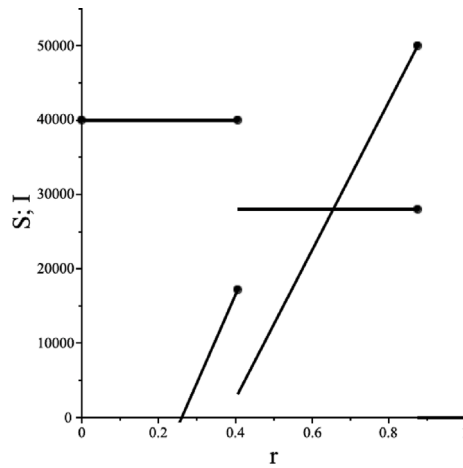


Fig. 2. Equilibrium with adverse selection.

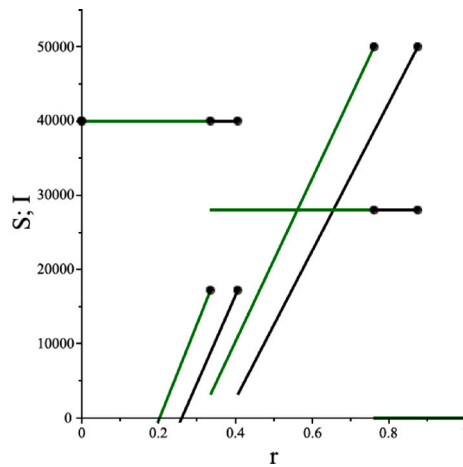


Fig. 3. Equilibria with adverse selection under different probability regimes.

### 5.2.2. Two-price-equilibrium: Example

Returning to  $N_1 = N_2 = 250$ , we obtain capital demand

$$I(r) = \begin{cases} (250 + 250) \times 80 = 40\,000, & \text{for } r \leq \frac{13}{32} \\ 250 \times 80 = 20\,000, & \text{for } \frac{13}{32} < r \leq 0.875 \\ 0, & \text{for } r > 0.875. \end{cases}$$

As in the previous example, we use lender profits from (11) to calculate banks' returns as a function of  $r$  exclusively:

$$i(r) = \begin{cases} 0.65r - \frac{21}{160}, & \text{for } r \leq \frac{13}{32} \\ 0.5r - 0.1875, & \text{for } \frac{13}{32} < r \leq 0.875. \end{cases}$$

Consequently, the critical rates give rise to maximum possible returns for given amounts of selection of  $i(r_1) = 17/128$  and  $i(r_2) = 0.25$ , which can pool capital supply of  $S(i(r_1)) = 26\,562.5$  and  $S(i(r_2)) = 50\,000$ , respectively.

The naïve approach to equate demand and supply in their second segments does not yield an equilibrium here. If banks lent capital only to firms of type 2 at the rate  $r^* = 0.575$  (at which demand and supply intersect), profits could be raised by charging  $r_1 = 13/32$  from project-1-owners that have not applied for a loan so far. Hence, lenders will charge  $r_1$  right away. In this process, credit is rationed to an extent such that residual demand can be exactly satisfied by pooling savings at a higher interest rate such that the same rate of return is secured. A lower rate would lead banks to skip this second round, while a higher one is not achievable due to competitive pressure. Both firm types are rationed in proportion to their mass. The commensurate distribution of rationing happens with certainty as there is a continuum of firms of each type. Afterward, some firms of the second type (those who turned out unlucky in the first round) would still demand capital even at a higher rate. Banks can satisfy this residual demand by equating it to residual supply as long as the low-interest round indeed happened first. Hence, by charging  $\tilde{r}$  such that  $i(\tilde{r}) = i(r_1)$ , no residual



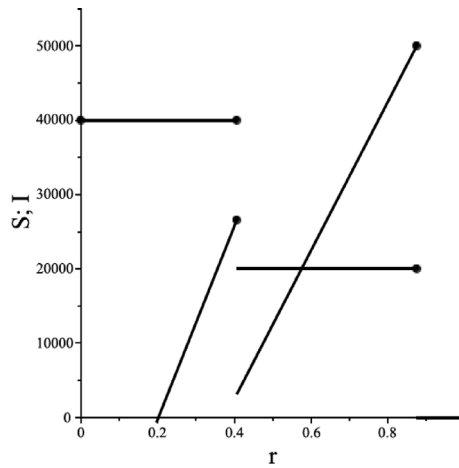


Fig. 4. Equilibrium with two prices.

demand is left and returns are not diminished. This is precisely the case when

$$\frac{17}{128} = 0.5r - 0.1875 \Leftrightarrow \tilde{r} = \frac{41}{64}.$$

In this equilibrium, all firms of type 2 obtain capital. Each firm of type 1 is only provided a loan with probability  $\tilde{S}/(N_1 + N_2)B$ , where  $\tilde{S}$  denotes allocated capital from the first round. Equilibrating residual demand and residual supply delivers  $\tilde{S} = 13\,125$ , which is equivalent to a fraction of  $43/64$  type-1-firms, a total of roughly 168, being rationed. We illustrate the equilibrium graphically in Fig. 4.

After the increase in the  $p_j$ 's, capital demand jumps at lower values of  $r$  and is otherwise unaltered:

$$I'(r) = \begin{cases} 40\,000, & \text{for } r \leq \frac{59}{176} \\ 20\,000, & \text{for } \frac{59}{176} < r \leq \frac{67}{88} \\ 0, & \text{for } r > \frac{67}{88}. \end{cases}$$

Dividing lender profits from (11) by  $B$  and subtracting 1 gives returns as

$$i'(r) = \begin{cases} 0.715r - \frac{171}{1600}, & \text{for } r \leq \frac{59}{176} \\ 0.55r - \frac{27}{160}, & \text{for } \frac{59}{176} < r \leq \frac{67}{88}. \end{cases}$$

Obtainable returns and corresponding levels of capital remain the same as before the change in success probabilities:  $i'(r'_1) = 17/128 = i(r_1)$ ,  $i'(r'_2) = 0.25 = i(r_2)$ ,  $S(i'(r'_1)) = 26\,562.5 = S(i(r_1))$  and  $S(i'(r'_2)) = 50\,000 = S(i(r_2))$ . As capital supply again enters the saltus of capital demand at  $r'_1 = 59/176$ , we still obtain an equilibrium with two rounds of capital allotment. At  $r'_1$ , equal portions of firm types receive capital. Banks use that interest rate to gather proceeds of  $\tilde{S} = 13\,125$ . The remaining type-2 firms apply again at

$$\frac{17}{128} = 0.55r - \frac{27}{160} \Leftrightarrow \tilde{r}' = \frac{193}{352}$$

for which they all receive capital. It is still 168 of the type-1-borrowers that do not get their desired capital. Hence, the equilibria under both probability regimes are equivalent in real terms.

To see that credit rationing is in fact of identical magnitude in both cases, one can also consider Fig. 5. There, the vertical distance between capital demand and supply at  $r_1$  and  $r'_1$ , respectively, is the same.

### 5.3. Generalizations

Two assumptions appear to be crucial to our results thus far: independence of expected project return and the corresponding project's risk level as well as equi-proportionality of success probability changes. We devote this Subsection to showing a strong case for relevance of the former and potential irrelevance of the latter assumption, thus disentangling the channels of risk and return.

#### 5.3.1. Lower returns in the green loan market

Even if one is willing to uphold the assumption that risk is not directly rewarded, one should include the possibility that the green loan market is earmarked by reduced returns. Theoretical backgrounds of this consideration encompass investor tastes, that is, willingness to forego returns, or simply the necessity of up-front investments. The evaluation of fixed-interest green financial products may thus reveal a "greenium", i.e., lower returns compared to similar non-green counterparts. There is a rather mixed literature on whether or not this is the case. Cheong and Choi (2020) provide an overview, suggesting that a positive greenium is

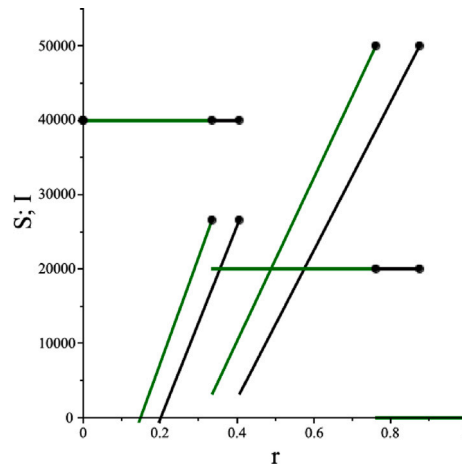


Fig. 5. Equilibria with two prices under different probability regimes.

the greater consensus. A seminal contribution is the one by [Baker et al. \(2022\)](#). Studies documenting an absent greenium often rely on comparisons of different financial products of the same entity, promising to use the respective proceeds for different projects (see, for example, [Larcker & Watts, 2020](#)). Since we restrict ourselves to firms conducting only a single project, these arguments do not apply here. Furthermore, doubts about the influence of differently framed financing tools on firms' business plans are brought forth by [Krahen et al. \(2023\)](#).

Contrary to what has been assumed so far, let project returns in case of a success decrease to

$$R'_j = \frac{1}{1+y} R_j,$$

where  $y > x$ , when switching to the low-risk-regime. We obtain

$$E(R') = \frac{1+x}{1+y} E(R).$$

Consequently,  $E(R') < E(R)$ . The intuition is that, in order to make production processes and the financial market turn greener, investments into cleaner technology are necessary that reduce firm profits in case of a success. This story aligns with the one behind the classical model of [Heinkel et al. \(2001\)](#). Furthermore, it incorporates the traditional risk-return relation into our model on an “across-regimes” basis. The absence of such a relation “within regimes” has already been argued to be an innocuous assumption serving merely expositional purposes in Section 3.

As project returns go down in expectation, so do tolerated credit rates as indicated by (9). They are now

$$r'_j = \frac{E(R') - (1 - p'_j)C}{p'_j B} - 1.$$

Note that the owners of riskier projects are still those who tolerate higher credit rates as the derivation of (10) remains valid even when replacing  $E(R)$  with  $E(R')$ . Evidently, plugging the critical rate into firm profits for any firm  $j'$  still applying for a loan,

$$E(\pi_{j'}^{F'} | r = r'_j) = (1+x)p_{j'} \left[ \frac{1}{1+y} R_{j'} - (1+r'_j)B \right] + [1 - (1+x)p_{j'}](-C),$$

reveals that firm profits fall from (12) to

$$E(\pi_{j'}^{F'} | r = r'_j) = \frac{1+x}{1+y} \left( 1 - \frac{p_{j'}}{p_j} \right) (E(R) - C).$$

Unlike in the baseline model, decreasing firm profits do not imply increasing lender profits here because the allocable return has also decreased. Indeed, banks attain only

$$E(\pi_{j'}^{L'} | r = r'_j) = \frac{1+x}{1+y} \frac{p_{j'}}{p_j} E(R) + \left( 1 - \frac{p_{j'}}{p_j} \right) C,$$

that is, less than before the decrease in market risk.

The above analysis shows that each firm type will stop demanding capital at lower interest rates compared to when expected returns stayed constant. At those rates, banks generate reduced profits as well. This, in turn, diminishes the savings interest rate they can pay to their customers. So capital supply changes in a fundamental way, implying a variation in the equilibrium structure, save for special cases. One of these special cases is pursued in the next Subsubsection.

We can conclude that there is a channel through which greener finance can influence the capital market equilibrium. This channel operates entirely via project returns, not project risk. Moreover, the consequences of lower-return projects is destined to be negative, that is, entail more rationing or fewer project types being financed, given the existence of a greenium.

### 5.3.2. Special case of identical adverse selection

Consider an economy with  $J = 2$  risk classes. Let there be adverse selection such that the equilibrium is as in Fig. 2. From there, let  $R_j$  decrease by an arbitrary factor, affecting  $p_j$  only to an extent such that the result is still  $E(R') < E(R)$ . Of course, someone has to lose from the decrease in expected returns, implying some change in the structure of the equilibrium. However, the amount of firms financed in equilibrium may remain identical. For this to be the case, two conditions must be met.

First, we must assume that  $i'(r'_1)$  does not exceed  $i(r_1)$  by too much, such as not to trigger a two-prices-equilibrium. Capital supply did not enter the saltus in capital demand, that is,  $S(i(r_1)) < \lim_{\epsilon \rightarrow 0+} I(r_1 + \epsilon)$ , and must continue not to do so,  $S(i'(r'_1)) < \lim_{\epsilon \rightarrow 0+} I'(r'_1 + \epsilon)$ . Lenders obtain the entire project return from firms of type 1 at  $r'_1$ , but this is now less than before due to  $E(R') < E(R)$ . Their profits from financing type 2,

$$E(\pi_2^L | r = r'_1) = \frac{p'_2}{p'_1} E(R') + \left(1 - \frac{p'_2}{p'_1}\right) C,$$

can only overcompensate this if the corresponding probability of success increases accordingly. Since making projects more similar decreases the severity of asymmetric information, a change in equilibrium structure achieved that way must be attributed to the reduction in information asymmetries, not changes in project risk per se. Hence, we consider the assumption  $S(i'(r'_1)) < \lim_{\epsilon \rightarrow 0+} I'(r'_1 + \epsilon)$  to be innocuous.

Second, as all type-2-firms were financed before raising the  $p_j$ 's,  $S(i(r_2)) \geq I(r_2)$ , there must not be rationing among these corporates with their projects becoming safer as well,  $S(i'(r'_2)) \geq I'(r'_2)$ . This simply means that projects must not become too unprofitable, ensuring that capital supply from the entire project return is sufficient to finance all risky firms:

$$S\left(\frac{E(R')}{B} - 1\right) \geq I'(r'_2)$$

Due to  $E(R') < E(R)$  and  $I(r_2) = I'(r'_2)$ , this condition is stricter than what the previous equilibrium warrants and may thus be violated. If it is, some of the applying firms are rationed. If not, the equilibrium features all projects of type 2 being financed and an unchanged savings interest rate  $i$  as before. Firm profits, however, decrease as they have a prospect of lower expected returns while being forced to finance the same savings rate to pool enough capital supply.

### 5.3.3. Disproportionate probability changes

The assumption followed in the main body so far that probability changes are always equi-proportionate with factor  $1 + x$  may turn out immaterial in some applications: The neutrality result can hold even if only one success probability changes. That is, there exist cases in which even disproportionate probability changes leave both the kind of equilibrium (including the amount of rationing) and the savings interest rate unaltered. For ease of exposition, we again restrict ourselves to  $J = 2$  risk classes and return to  $E(R) = E(R')$ .

Suppose the capital market equilibrium is of the same qualitative structure as seen in Fig. 2: There is adverse selection such that  $j = 1$ -firms do not apply for a loan. Any change in  $p_2$  modifies the shape of capital supply in both segments of the graph. The reason for this is that the return function changes in response to the average success probability being altered independently of whether both firms apply or only type 2. However, capital supply in the second segment remains more than sufficient to cover demand as the return still grows up to its global maximum of  $E(R)/B - 1$ . As the return function experiences a downward jump at  $r_1$ , it is sufficient to assume  $S(i(r_1)) < \lim_{\epsilon \rightarrow 0+} I(r_1 + \epsilon)$  in order to uphold the unique intersection of demand and supply in the second segment (preempting a two-prices-equilibrium at the same time). Both the savings interest rate and capital allotment are then unvaried. So we simply have to prove that there is a plausible interval of  $p'_2$ 's for which  $S(i(r_1)) < \lim_{\epsilon \rightarrow 0+} I(r_1 + \epsilon)$  continues to hold.

The average success probability of a financed project in the two-type-case before adverse selection comes into play is

$$E(p | r \leq r_1) = \frac{N_1 p_1 + N_2 p_2}{N_1 + N_2}. \quad (15)$$

That probability plays a part for expected bank profits at  $r_1$ :

$$E(\pi^L | r = r_1) = E(p | r \leq r_1) [(1 + r_1)B - C] + C. \quad (16)$$

Those are necessary to determine the return generated by lenders which they pass on as the savings interest rate. More precisely,  $S(i(r_1)) < \lim_{\epsilon \rightarrow 0+} I(r_1 + \epsilon)$  can be expressed as

$$S\left(\frac{E(\pi^L | r = r_1)}{B} - 1\right) < N_2 B.$$

Inserting (15) as well as (16) and rearranging terms yields

$$p_2 < \frac{(N_1 + N_2) \{ [1 + S^{-1}(N_2 B)] B - C \} + C}{N_2(1 + r_1)B} - \frac{N_1}{N_2} p_1. \quad (17)$$

Inequality (17) reveals that there are most definitely values in the neighborhood of the initial  $p_2$  which obtain the neutrality result. Firstly, we can reduce  $p_2$  and make it arbitrarily close to zero as only a too high  $p'_2$  would be problematic.<sup>8</sup> Secondly, as the initial

<sup>8</sup> We must, however, exclude  $p_2 = 0$  because this would necessitate  $R_2 \rightarrow +\infty$  for constant  $E(R) > 0$ .

$p_2$  must have fulfilled (17), there must also exist some larger values such that it still holds ( $p'_2 = p_2 + \varepsilon$  where  $\varepsilon$  is sufficiently close to zero). To sum up, we can conclude that, denoting the RHS of (17) as  $p_2^*$ , any  $p'_2 \in (0, p_2^*) \neq \emptyset$  for given  $p_1$  obtains a neutrality result in the sense discussed above. Hence, it becomes evident that our result also extends to disproportionate probability changes.

By a similar logic, one can also obtain that, for constant  $p_2$ , we can make  $p_1$  arbitrarily close to unity or reduce it to a value just above some  $p_1^*$  without altering the adverse-selection-structure of the benchmark equilibrium (including the interest rate on savings and capital allotment). This serves to strengthen the generality of our neutrality result. Firm profits, however, will again differ.

**Proposition 3.** *Lower yields from projects becoming safer overall as well as disproportionate probability changes may leave the number of non-applying firms under adverse selection and, hence, the equilibrium savings rate constant, but will alter profits.*

## 6. Hidden actions

Equilibria characterized by moral hazard require a different kind of information imperfection than the one employed so far: the market side at an informational advantage must be able to make some hidden choice. To incorporate this, the model will be modified in a straightforward way.

### 6.1. Equilibrium with moral hazard

To depict hidden action, we no longer consider firms to be endowed with one single project. Rather, each of the  $N$  firms is capable of conducting all  $J$  projects. While it would be in the lenders' interest to secure the implementation of project 1, they cannot force borrowers to do so directly. The project choice depends on the charged loan interest rate  $r$ . With firm profits conditional on the choice of project  $j$  given by (1), we can immediately conclude that each firm would always choose the riskiest project as  $E(\pi_j^F) \geq E(\pi_{j+1}^F)$ ,  $j = 1, \dots, J-1$  necessitates

$$r \leq \frac{C}{B} - 1 < 0.$$

In words, incentivizing the conduction of safe projects requires banks to actually pay corporates for doing so.<sup>9</sup> As then  $i < 0$ , no capital supply can be attracted. The only possible equilibrium entails project choice  $J$  by all firms.

To obtain equilibria characterized by non-trivial moral hazard (like the one described above), we alter project returns by  $\Delta R_1 > \Delta R_2 > \dots > \Delta R_J$  the sign of which is irrelevant. On the outset, the only restriction we have to impose on the  $\Delta R_j$ 's is that they are not too high such that the resulting  $R_j$  still obey  $R_1 < R_2 < \dots < R_J$ .<sup>10</sup> We denote expected returns by  $E(R_j)$  and define  $\alpha_j > 1$  as a mark-up factor which states by how much project  $j$  is, on average, better than  $j+1$ :

$$E(R_j) = \alpha_j E(R_{j+1}), \quad j = 1, \dots, J-1. \quad (18)$$

By (1) and (18), we know that a safer project is preferred to the next-riskier one by borrowers as long as

$$r \leq \frac{(\alpha_j - 1)E(R_{j+1}) + (p_j - p_{j+1})C}{(p_j - p_{j+1})B} - 1 \equiv r_j (> 0), \quad j = 1, \dots, J-1. \quad (19)$$

The fact that corporates choose the safer alternative when indifferent posits another form of epsilon altruism (see also Hillier, 1997, p. 38). Firms stop demanding capital if not even the riskiest project is worth financing anymore due to a too high interest burden.  $E(\pi_J^F) \geq 0$  requires

$$r \leq \frac{E(R_J) - (1 - p_J)C}{p_J B} - 1 \equiv r_J. \quad (20)$$

To ensure that both moral hazard per se as well as the switch-inducing rates in (19) and (20) are meaningful for our model, we have to assume that those rates are increasingly ordered. That is, we need  $r_j < r_{j+1}$ ,  $j = 1, \dots, J-1$ . This condition yields a threshold for each  $\alpha_j$  but the last one:

$$\alpha_j < \frac{p_j - p_{j+1}}{p_{j+1} - p_{j+2}} \left( 1 - \frac{1}{\alpha_{j+1}} \right) + 1, \quad j = 1, \dots, J-2. \quad (21)$$

As the above threshold is always greater than one (as long as  $\alpha_{j+1} > 1$ , which has to be true), it can never be logically inconsistent. Further, it depends positively on  $\alpha_{j+1}$ , which tells us that, in comparative terms, avoiding risk has to become sufficiently less attractive the fewer risk is already taken in order to be worth entertaining more of it as the burden of repayment grows. In other words, each additional risk taken has to be disproportionately more promising in expectations than the previous risk increase. Whenever  $r_{J-1} < r_J$  additionally holds,  $r_J$  constitutes the unique global maximum of the return function again. This is the case whenever the gain in surplus from avoiding the worst type of risk is not too high:

$$\alpha_{J-1} < \frac{p_{J-1}}{p_J} - \frac{p_{J-1} - p_J}{p_J} \frac{C}{E(R_J)}. \quad (22)$$

<sup>9</sup> It is worth noting that firms will in fact have no incentive to switch back to the riskiest project. This is due to the fact that a riskier project choice would amplify the loss in case of a failure as the cheap repayment is then more likely to be missed out on.

<sup>10</sup> We abstain from introducing a new notation for payoffs here in order to warrant readability.

If we were to restrict ourselves to  $J = 2$  risk classes, (22) alone would suffice. Neglecting collateral would obviate the above derivations of (21) and (22) because then, no matter what, more risk eventually becomes attractive as  $r$  rises: there is never anything to loose from conducting a project but always some chance of positive profits whenever  $R_j > (1 + r)B$ , which always holds for  $j + k, k \geq 1$  if it holds for given  $j$ , i.e., if even more risk is being taken, but not necessarily the other way round (see also Arnold, 2020, pp. 246–248).

We can now determine firms' actions depending on  $r$ . They choose project 1 for  $r < r_1$ , switch to project 2 there, which they conduct up until  $r_2(> r_1)$  where they switch to project 3, and so on until they quit demanding capital at  $r_J$ . Note that every firm acts identical as they all choose from the same pool of projects. Formally, capital demand satisfies

$$I(r) = \begin{cases} NB, & \text{for } r \leq r_J \\ 0, & \text{for } r > r_J. \end{cases}$$

Bank profits are given by (2) where  $p_j$  is determined uniquely as the one for which  $r_{j-1} < r \leq r_j$  holds. Thus, returns are

$$i(r) = \begin{cases} p_1(1+r) + (1-p_1)\frac{C}{B} - 1, & \text{for } r \leq r_1 \\ p_j(1+r) + (1-p_j)\frac{C}{B} - 1, & \text{for } r_{j-1} < r \leq r_j \text{ } j = 2, \dots, J. \end{cases} \quad (23)$$

Inserting critical interest rates (those where switches happen) into (23), we obtain

$$i(r_j) = \frac{C}{B} + \frac{p_j}{p_j - p_{j+1}} \frac{(\alpha_j - 1)E(R_{j+1})}{B} - 1, \quad j = 1, \dots, J-1$$

and, of course, the entire project return at  $r_J$ :

$$i(r_J) = \frac{E(R_J)}{B} - 1.$$

Now, we turn to the influence of  $p_j$  again. Altering them all simultaneously by  $(1+x)$  and  $R_j$  by  $1/(1+x)$  just as in the main part of Section 5, we obtain  $E(R'_j) = E(R_j)$ ,  $j = 1, \dots, J$ . The switch-inducing loan rates from (19) become

$$r'_j = \frac{(1+x)(p_j - p_{j+1})C + (\alpha_j - 1)E(R_{j+1})}{(1+x)(p_j - p_{j+1})B} - 1, \quad j = 1, \dots, J-1.$$

Similarly, we get

$$r'_J = \frac{E(R_J) - (1+x)(1-p_J)C}{(1+x)p_J B} - 1$$

from (20). Inserting both into (23) immediately yields

$$i'(r'_j) = i(r_j), \quad j = 1, \dots, J.$$

Hence, whenever the loan interest rate is reached at which corporates alter their choice of project conduction, the return thus generated is already fixed. Due to constant capital demand in the range of acceptable loan rates it is therefore certain that, if credit rationing arises, its extent is numerically identical under both distributions of risk. By the same logic as in the previous sections, the equilibrating  $i$  determines the conducted project uniquely.

**Proposition 4.** *Equi-proportionate changes of the success probabilities  $p_j$  are neutral if there is hidden project choice.*

## 6.2. Moral hazard equilibrium: Example

In order to make our above example fit into the variant of the model where moral hazard plays a role, we need  $E(R_1) > E(R_2)$ . Therefore, we now use  $R_2 = 170$  such that  $E(R_1) = 100 > 85 = E(R_2)$ . By (18), this gives  $\alpha_1 = 20/17$ . This, in turn, allows us to calculate the switching rate beyond which project 2 is financed via (19) as

$$r_1 = \frac{(0.8 - 0.5) \times 50 + (\frac{20}{17} - 1) \times 85}{(0.8 - 0.5) \times 80} - 1 = 0.25.$$

Firms stop demanding capital if credit interest rates go beyond

$$r_2 = \frac{85 - 0.5 \times 50}{0.5 \times 80} - 1 = 0.5$$

by (20). Until then, capital demand is constant at  $NB = 40\,000$ . The return from financing follows (23), i.e.,

$$i(r) = \begin{cases} 0.8(1+r) + 0.2\frac{50}{80} - 1, & \text{for } r \leq 0.25 \\ 0.5(1+r) + 0.5\frac{50}{80} - 1, & \text{for } 0.25 < r \leq 0.5, \end{cases}$$

which gives rise to  $i(r_1) = 0.125$  and  $i(r_2) = 0.0625$ . Consequently, capital supply is equal to  $S(i(r_1)) = 25\,000$  and  $S(i(r_2)) = 12\,500$ . From this we know that there can be no equilibrium without credit rationing. Rather, in equilibrium, banks generate the maximum possible return of 12.5%, project 1 is conducted and credit is rationed. A continuum of length  $15\,000/80 = 187.5$  of firms is denied a loan. We depict this equilibrium graphically in Fig. 6 using the usual formatting. Interestingly, full allotment would be part of an equilibrium if project 2 did not exist or could be ruled out contractually: Firms would then tolerate loan rates up to 0.40625 such

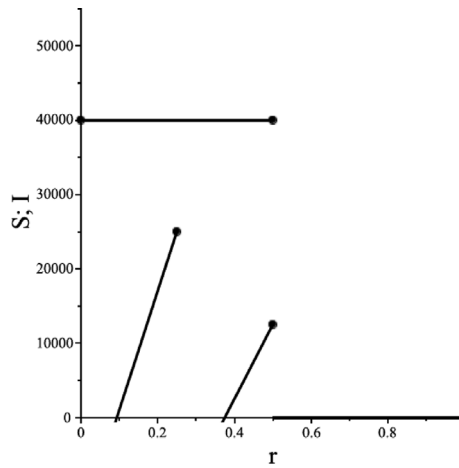


Fig. 6. Equilibrium with moral hazard.

that supply and demand can be equilibrated at  $r = 11/32$ . Their profits in this case would amount to 4, while with rationing they are zero independently of whether they obtained capital or not. This reflects the fact that their mere ability to act adversely leads to losses for the proprietors of hidden actions (see also Hillier, 1997, p. 42).

Now, let  $p_1$  and  $p_2$  rise by 10% again (with  $R'_1 = 2000/11$  as before and  $R'_2 = 1700/11$ ). The new critical interest rates are

$$r'_1 = \frac{(0.88 - 0.55) \times 50 + (\frac{20}{17} - 1) \times 85}{(0.88 - 0.55) \times 80} - 1 = \frac{17}{88}$$

and

$$r'_2 = \frac{85 - 0.45 \times 50}{0.55 \times 80} - 1 = \frac{37}{88}.$$

Consequently, returns are

$$i(r) = \begin{cases} 0.88(1+r) + 0.12\frac{50}{80} - 1, & \text{for } r \leq \frac{17}{88} \\ 0.55(1+r) + 0.45\frac{50}{80} - 1, & \text{for } \frac{17}{88} < r \leq \frac{37}{88} \end{cases}$$

and achieve local maxima at  $i'(r'_1) = 0.125 = i(r_1)$  and  $i'(r'_2) = 0.0625 = i(r_2)$ . Of course, capital supply at those rates equals its previous levels. We therefore obtain an equilibrium with  $r^{*'} = 17/88$ ,  $i^{*'} = 0.125$  and credit rationing of 15 000 units of capital (187.5 firms) where all firms decide on project 1. The new equilibrium is depicted in Fig. 7 in green as a compression of the former one (in black).

## 7. Financing projects via shares

Financing via shares may bring about an easy solution to the informational inefficiencies arising in the SW model. The result that underinvestment can be mitigated this way dates back to de Meza and Webb (1987). For either variant of information imperfection, we can propose financing via shares as an alternative source of capital. The results will, however, turn out to be simple to the point of triviality. For a given, common level of collateral, adverse selection and moral hazard can be brought about in the following way. If firms pledge some fraction  $s$  of their net worth in exchange for the provision of capital  $B$ , their changes in profits starting from no involvement of the lender<sup>11</sup> follow

$$E(\pi_j^F) = (1-s)(E(R_j) + C) - C.$$

Applying this to the hidden information case reveals that the critical proportion any firm is willing to give up has to satisfy  $E(\pi_j^F) \geq 0$ , hence

$$s \leq \frac{E(R_j)}{E(R_j) + C} \equiv s_j, \quad j = 1, \dots, J.$$

With equal  $E(R)$  as in Section 5, firms do not differ at all. With higher  $E(R_j)$  for safer  $j$ , the riskier firms are actually those that are rationed first. This essentially posits a case of advantageous selection, a term coined by de Meza and Webb (1999) (although, there, the context is overinvestment). It can be turned back into adverse selection by assuming  $E(R_1) < E(R_2) < \dots < E(R_J)$  instead, which is consistent with traditional financial market theory.

<sup>11</sup> It is perhaps more appropriate to speak of a shareholder rather than a bank here.

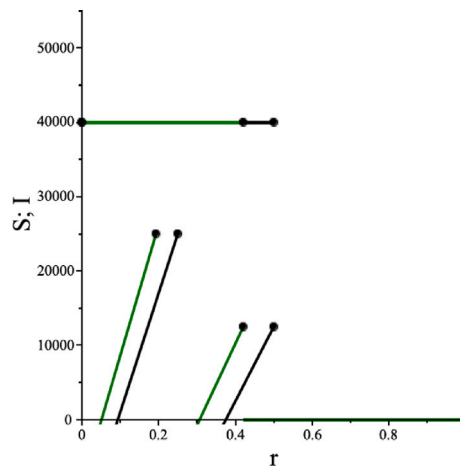


Fig. 7. Equilibria with moral hazard under different probability regimes.

For the case of hidden actions, firms will always choose the project yielding the highest expected return. Depending on the ranking of the  $E(R_j)$ 's, this will result in moral hazard or moral harmony (or simply total indifference):

$$E(\pi_j^F) \geq E(\pi_{j+1}^F) \Leftrightarrow E(R_j) \geq E(R_{j+1}), \quad j = 1, \dots, J-1.$$

Either viewpoint can be used to yield an equilibrium with moral hazard or adverse selection. However, it is clear that the  $p_j$ 's do not have any influence on it whatsoever because they only affect tolerated share issues via  $E(R_j)$  which is assumed to be constant.

Differences in firm 'quality' that can yield a rationing-type equilibrium are obtained once the collateral is allowed to vary across corporates. Therefore, one would have to model risky firms as those who are able to pledge lower collateral.<sup>12</sup> If one then even allows  $C_j$  to vary when switching from the  $p_j$ - to the  $p'_j$ -regime (assuming a positive association of both variables), the neutrality result could fail to hold and additional safety would even exacerbate the informational problems of the market.

## 8. Conclusion

Using the model by [Stiglitz and Weiss \(1981\)](#), we investigate the role of average project risk for market equilibria. Expected project returns are left constant. In this way, we isolate the effect of success probabilities per se from others resulting from growth or business cycles. The ongoing transition to more green markets may be considered a current example.

Looking at complete markets as a benchmark, we obtain a neutrality result: the amount of rationing and the savings interest rate obtained in a capital market equilibrium is unchanged. This motivates a further look into cases of asymmetric information.

For a market of hidden information, we find that problems of unidentifiability of firms by banks cannot be mitigated by a safer pool of borrowers. Rather, credit interest rates charged per risk class simply fall (or rise in case of a riskier pool) in such a way that savings interest rates remain at their pre-risk change level. This leaves the volume of investments financed in equilibrium constant. The latter may decrease if higher success probabilities are accompanied by reduced expected revenues.

A similar result holds if firms are characterized by their ability to take hidden actions: again, the only change happening after an alteration of success probabilities is in the interest rate on loans. Firms finance the same projects as before and credit rationing remains at a numerically identical level.

Considering different ways of financing such as a stock market can also maintain the neutrality result. However, there are variants of the model where this is not the case, for example when project risk influences pledged collateral. We consider the influence of both collateral choice and financing method to be interesting avenues of further research on the topic of project risk neutrality.

## Declaration of competing interest

None.

<sup>12</sup> If risky firms were the ones able to pledge a higher value of  $C$ , the term "risky" would simply become inadequate. Additionally, they suffer more from the danger of losing it due to their low  $p_j$ .



## References

- Akerlof, George A. (1970). The market for 'lemons': Quality uncertainty and the market mechanism. *Quarterly Journal of Economics*, 84(3), 488–500.
- An, Xudong, & Pivo, Gary (2020). Green buildings in commercial mortgage-backed securities: The effects of LEED and energy star certification on default risk and loan terms. *Real Estate Economics*, 48(1), 7–42. <http://dx.doi.org/10.1111/1540-6229.12228>.
- Arnold, Lutz G. (2012). A game-theoretic foundation for competitive equilibria in the Stiglitz–Weiss model. *German Economic Review*, 13(2), 211–227.
- Arnold, Lutz G. (2020). *Makroökonomik*, 6. Auflage, Tübingen: Mohr Siebeck.
- Arnold, Lutz G., Reeder, Johannes, & Trepl, Stefanie (2014). Single-name credit risk, portfolio risk and credit rationing. *Economica*, 81, 311–328. <http://dx.doi.org/10.1111/ecca.12075>.
- Arnold, Lutz G., & Riley, John G. (2009). On the possibility of credit rationing in the Stiglitz–Weiss model. *The American Economic Review*, 99(5), 2012–2021.
- Baker, Malcolm, Bergstresser, Daniel, Serafeim, George, & Wurgler, Jeffrey (2022). The pricing and ownership of US green bonds. *Annual Review of Financial Economics*, 14, 10.1–10.23.
- Basu, Santanu (1992). *Asymmetric information, credit rationing and the Stiglitz and Weiss model: Research Paper No. 360*, School of Economic and Financial Studies.
- Bolton, Patrick, & Kacperczyk, Marcin (2021). Do investors care about carbon risk? *Journal of Financial Economics*, 142, 517–549.
- Cheong, Chiyoung, & Choi, Jaewon (2020). Green bonds: a survey. *Journal of Derivatives and Quantitative Studies*, 28(4), 175–189. <http://dx.doi.org/10.1108/JDQS-09-2020-0024>.
- Coco, Giuseppe (1997). *Credit rationing and the welfare gain from usury laws: Discussion Paper 97/15*, University of Exeter.
- Coco, Giuseppe (1999). Collateral, heterogeneity in risk attitude and the credit market equilibrium. *European Economic Review*, 43, 559–574.
- Cui, Yujun, Geobey, Sean, Weber, Olaf, & Lin, Haiying (2018). The impact of green lending on credit risk in China. *Sustainability*, 10, 2008. <http://dx.doi.org/10.3390/su10062008>.
- de Meza, David, & Webb, David C. (1987). Too much investment: A problem of asymmetric information. *Quarterly Journal of Economics*, 102(2), 281–292.
- de Meza, David, & Webb, David C. (1999). Wealth, enterprise and credit policy. *The Economic Journal*, 109(April), 153–163.
- Heinkel, Robert, Kraus, Alan, & Zechner, Josef (2001). The effect of green investment on corporate behavior. *The Journal of Financial and Quantitative Analysis*, 36(4), 431–449.
- Hillier, Brian (1997). *The economics of asymmetric information*. London: Palgrave.
- Jung, Juhyun, Herbohn, Kathleen, & Clarkson, Peter (2018). Carbon risk, carbon risk awareness and the cost of debt financing. *Journal of Business Ethics*, 150, 1151–1171.
- Krahnert, Jan, Rocholl, Jörg, & Thum, Marcel (2023). A primer on green finance: From wishful thinking to marginal impact. *Review of Economics*, 74(1), 1–19.
- Larcker, David F., & Watts, Edward M. (2020). Where's the greenium? *Journal of Accounting and Economics*, 69, Article 101312.
- Markowitz, Harry M. (1952). Portfolio selection. *The Journal of Finance*, 7, 77–91.
- Riley, John G. (1987). Credit rationing: A further remark. *The American Economic Review*, 77(1), 224–227.
- Stiglitz, Joseph E., & Weiss, Andrew (1981). Credit rationing in markets with imperfect information. *The American Economic Review*, 71(3), 393–410.
- Stiglitz, Joseph E., & Weiss, Andrew (1983). Incentive effects of terminations: Applications to the credit and labor markets. *The American Economic Review*, 73(5), 912–927.
- Stiglitz, Joseph E., & Weiss, Andrew (1987). Credit rationing: Reply. *The American Economic Review*, 77(1), 228–231.
- Stiglitz, Joseph E., & Weiss, Andrew (1992). Asymmetric information in credit markets and its implications for macro-economics. *Oxford Economic Papers*, 44, 694–724.
- Su, Xunhua, & Zhang, Li (2017). A reexamination of credit rationing in the Stiglitz and Weiss model. *Journal of Money, Credit and Banking*, 49(5), 1059–1072.
- Suarez, Javier, & Sussman, Oren (1996). Endogenous cycles in a Stiglitz–Weiss economy, *Seminários de Pesquisa Econômica II* (2ª parte).
- Wette, Hildegard C. (1983). Collateral in credit rationing in markets with imperfect information: Note. *The American Economic Review*, 73(3), 442–445.