Interplay between quantum dissipation and an in-plane magnetic field in the spin ratchet effect

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We investigate the existence of the pure spin ratchet effect in a dissipative quasi-one-dimensional system with Rashba spin-orbit interaction. The system is additionally placed into a transverse uniform stationary in-plane magnetic field. It is shown that the effect exists at low temperatures and pure spin currents can be generated by applying an unbiased ac driving to the system. An *analytical* expression for the ratchet spin current is derived. From this expression it follows that the spin ratchet effect appears as a result of the simultaneous presence of the spin-orbit interaction, coupling between the orbital degrees of freedom and spatial asymmetry. In this paper we consider the case of a broken spatial symmetry by virtue of asymmetric periodic potentials. It turns out that an external magnetic field does not have any impact on the existence of the spin ratchet effect, but it influences its efficiency enhancing or reducing the magnitude of the spin current.

I. INTRODUCTION

It is well known that a directed stationary flow of particles in a system can be created by unbiased external forces. In general this possibility arises when the system is not invariant under reflections in real space. This fact is mainly independent of the mechanics which underpins the particle motion, classical or quantum. However, the microscopic origin of this effect, conventionally called the ratchet effect, is different in the classical and quantum case. One principle source of that difference is quantum mechanical tunnelling which does not have analogs in the classical mechanics. Correspondingly, one usually distinguishes between classical and quantum ratchet effects. In this paper we concentrate on the latter one in a dissipative system. Such dissipative ratchet systems act as Brownian motors[1,2] turning Brownian into directed motion. The existence of the ratchet effect in a quantum dissipative one-dimensional (1D) system which lacks the spatial symmetry has been first theoretically predicted in Ref. 3. Later, within a tight-binding model where the lowest bands are narrow, it has been disclosed that a ratchet state of the particle transport can only be achieved when at least the two lowest Bloch bands contribute to transport[4]. To obtain the ratchet effect in systems with weak periodic potentials at least two harmonics of the potential should enter the dynamical equations[5]. Rectification can also take place in a single-band tight-binding model where the spatial asymmetry is concealed from the electron dynamics. One way to achieve this is to use unbiased external forces with harmonic mixing[6].

Coherent charge ratchets based on molecular wires with an asymmetric level structure of the orbital energies were proposed in Ref. 7. In this case weak dissipation originates from a weak coupling between the wire edges and leads which represent fermionic reservoirs. In contrast to the systems described above in this system there is no dissipation in the wire. The ratchet effect is a result of the dissipative coupling of the wire to fermionic baths.

In a different branch of condensed matter a new research field has emerged during the last decade, namely spintronics, where one tries to make use of the spin degree of freedom of a particle instead of only the charge one. One essential difference between spin and charge is that a particle can have more than one spin state while it has only one charge state. In the context of transport it is important that the spin state of a particle can strongly depend on the transport conditions, in particular on the transport direction, as it happens for example in systems with spin-orbit interaction. This fact has founded a new arena for different spin devices used to store, transform and transfer miscellaneous information. The possibility to transfer the spin separately from charge plays an important role. This can be implemented by so-called pure spin currents, that is spin currents which are not accompanied by charge currents. Thus the generation of such currents has been extensively discussed. Among different mechanisms of spin-orbit interaction Rashba spin-orbit interaction (RSOI)[8] plays a distinguished role because it provides an opportunity to control the spin-orbit coupling strength by an external electric field. The change in the band structure spawned by the spin-orbit interaction leads to one of the most remarkable effects in spintronics, the intrinsic spin-Hall effect, first predicted by Murakami *et al.*[9] for hole-doped semiconductors with the spin-orbit interaction of the effective Luttinger model for holes and later by Sinova *et al.*[10] in a high-mobility two-dimensional electron gas (2DEG) with RSOI. The spin current which results from the intrinsic spin-Hall effect is pure and its experimental detection was discussed, *e.g.*, by Wunderlich *et al.*[11]. Another kind of spin-Hall effect, the extrinsic spin-Hall effect, is a result of the spin-orbit interaction as well. The spin currents related to the extrinsic spin-Hall effect are also pure. Such pure spin currents were experimentally detected through optical measuring of electron spin accumulation at the edges of the samples[12] and through the reciprocal spin-Hall effect[13,14] in Ref. 15. Another approach to create pure spin currents is to use polarized light. For example in noncentrosymmetric semiconductors one-photon absorption of linearly polarized light induces pure spin currents[16]. The pure spin current response to linearly and circularly polarized light irradiation, exciting electrons from valence bands into the conduction bands, was studied by Li *et al.*[17] and by Zhou *et al.*[18] for 2DEGs with RSOI. The pure spin current generation using the spin ratchet

effect in coherent and dissipative systems with RSOI was investigated in Refs. 19 and 20, respectively. The spin ratchet effect in the presence of a non-uniform static magnetic field without spin-orbit interaction, the Zeeman ratchet effect, was studied in Ref. 21 for coherent quantum wires formed in a 2DEG. However, the spin ratchet effect in a dissipative system in an external magnetic field has not been considered up to now.

In this paper we extend the results of Ref. 20 to include a transverse in-plane uniform stationary magnetic field. Specifically, we consider non-interacting electrons in a quantum wire formed by a harmonic transverse confinement in a 2DEG with RSOI. The electrons are also subject to a 1D periodic potential along the wire direction and the in-plane magnetic field perpendicular to the wire. An orbital coupling between this originally isolated system and an external environment causes dissipative processes affecting indirectly the spin dynamics through RSOI.

An external ac driving originates in our work from an applied ac electric field. We show that for such a driving the net stationary charge current is strongly suppressed if the transport is governed only by electrons of the Bloch sub-bands related to the same Bloch band which would result from the corresponding truly 1D problem without RSOI. However, at the same time and under the same conditions a net stationary spin current turns out to be activated in a spatially asymmetric situation and for finite values of the spin-orbit coupling strength and the coupling strength between the orbital degrees of freedom. The magnetic field does not destroy this picture, but it can partly reduce or on the contrary enhance the ratchet effect.

The paper is organized as follows. In Section II we describe a model within which a ratchet like behavior of the spin transport can be achieved and present a master equation in terms of populations and transition rates between the basis states used to calculate the charge and spin currents. These basis states are then thoroughly discussed in Section III. A tight-binding model is formulated in Section IV. In Section V we present the transition rates and their properties. Finally, in Section VI we derive *analytical* expressions for the charge and spin currents and explore the spin ratchet effect in the system.

II. FORMULATION OF THE PROBLEM

The full Hamiltonian of our problem is

$$\hat{H}_{\text{full}}(t) = \hat{H} + \hat{H}_{\text{ext}}(t) + \hat{H}_{\text{bath}}, \tag{1}$$

where $\hat{H}$ is the Hamiltonian of the isolated periodic system, $\hat{H}_{\text{ext}}(t)$ describes an external driving and $\hat{H}_{\text{bath}}$ represents the term responsible for dissipative processes.

The isolated quasi-1D periodic system is formed in a 2DEG ($x-z$ plane) with RSOI using a periodic potential along the x-axis and a harmonic confinement along the z-axis. The whole system is in a uniform stationary magnetic field along the z-axis:

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m} + \frac{m\omega_0^2 \hat{z}^2}{2} - \frac{\hbar^2 k_{\text{so}}}{m}\left(\hat{\sigma}_x \hat{k}_z - \hat{\sigma}_z \hat{k}_x\right) + $$
$$+ U(\hat{x})\left(1 + \gamma\frac{\hat{z}^2}{L^2}\right) - g\mu_{\text{B}}\hat{\sigma}_z H_0, \tag{2}$$

where H_0 is the z-component of the magnetic field $\mathbf{H}_0 = (0, 0, H_0)$, and we have used the gauge in which the components of the vector potential are $A_x = -H_0 y$, $A_y = A_z = 0$ (Landau gauge). Additionally, we have taken into account the fact that in a 2DEG $y = 0$. In Eq. (2) the operator $\hat{\mathbf{k}}$ is related to the momentum operator $\hat{\mathbf{p}}$ as $\hat{\mathbf{p}} = \hbar\hat{\mathbf{k}}$, ω_0 is the harmonic confinement strength, k_{so} the spin-orbit interaction strength, γ the strength of the coupling between the orbital degrees of freedom x and z, g the electron spin g-factor, μ_{B} the Bohr magneton, and $U(\hat{x})$ denotes the periodic potential with period L,

$$U(x + L) = U(x). \tag{3}$$

In the following we assume that the periodic structure is subject to an external homogeneous time-dependent electric field. Only the x-component of the electric field vector is nonzero, that is the electric field is parallel or anti-parallel to the x-axis. Experimentally this can be implemented using for example linearly polarized light. The external force thus couples only to the x-component of the electron coordinate operator:

$$\hat{H}_{\text{ext}} = -F(t)\hat{x}, \tag{4}$$

where the force $F(t)$ is unbiased. In this work we use the time-dependence

$$F(t) = F\cos(\Omega(t - t_0)). \tag{5}$$

The system is also coupled to an external bath. In the present work we assume the transverse confinement to be strong enough so that the probabilities of direct bath-excited transitions between the transverse modes are negligibly small. In other words, the wire is truly 1D from the point of view of the bath which directly changes only the dynamics along the wire. Thus in our model the external environment couples to the electronic degrees of freedom only through $\hat{x}$. The bath itself as well as its interaction with the quantum wire are described within the Caldeira-Leggett model[22,23],

$$\hat{H}_{\text{bath}} = \frac{1}{2}\sum_{\alpha=1}^{N_O}\left[\frac{\hat{p}_\alpha^2}{m_\alpha} + m_\alpha\omega_\alpha^2\left(\hat{x}_\alpha - \frac{c_\alpha}{m_\alpha\omega_\alpha^2}\hat{x}\right)^2\right]. \tag{6}$$

The bath is fully characterized by its spectral density defined as

$$J(\omega) \equiv \frac{\pi}{2}\sum_{\alpha=1}^{N_O}\frac{c_\alpha^2}{m_\alpha\omega_\alpha}\delta(\omega - \omega_\alpha). \tag{7}$$

It is important to emphasize that, due to the spin-orbit interaction and orbit-orbit coupling, the direct dissipative interaction

between the longitudinal dynamics in the wire and the external environment has an indirect impact on the transition rates between different transverse modes. The transverse dynamics in the wire indirectly feels the presence of the external bath through the spin-orbit interaction and orbit-orbit coupling.

The dynamical quantities of interest are the charge and spin currents. Specifically, the longitudinal charge current $J_C(t)$ is given as a statistical average of the longitudinal charge current operator $\hat{J}_C(t)$, *i.e.* the product of the velocity operator $\hat{v}(t)$ and the elementary electronic charge $-e$,

$$\hat{J}_C(t) = -e\hat{v}(t), \qquad (8)$$

$$J_C(t) = -e\frac{d}{dt}\text{Tr}[\hat{x}\hat{\rho}(t)], \qquad (9)$$

where $\hat{\rho}(t) = \text{Tr}_{\text{bath}}\hat{W}(t)$ is the reduced statistical operator of the system, that is the full one $\hat{W}(t)$ with the bath degrees of freedom traced out.

For the longitudinal spin current operator we use the definition suggested by Shi *et al.*[24],

$$\hat{J}_S^i(t) = \frac{d}{dt}\left(\hat{\sigma}_i\hat{x}\right), \qquad (10)$$

which was further developed and applied to a two-dimensional hole gas by Zhang *et al.*[25] The advantage of this definition over the conventional one ($\hat{J}_S^i = \hat{\sigma}_i\hat{v}$) is that using the corresponding spin current,

$$J_S^i(t) = \frac{d}{dt}\text{Tr}\left(\hat{\sigma}_i\hat{x}\hat{\rho}(t)\right), \qquad (11)$$

the continuity equation for the spin density can often be written without a source term, which means that the spin current defined in this way is conserved. This conserved spin current can be uniquely related to the spin accumulation at a sample boundary. The out-of-plane polarized spin accumulation can experimentally be measured with Kerr rotation microscopy[26] or the Faraday rotation technique[27]. The in-plane spin polarization is not directly measured by Kerr rotation microscopy, but it can still be scanned by a magneto-optic Kerr microscope using, *e.g.*, the cleaved edge technology as discussed by Kotissek *et al.*[28] Even when the continuity equation contains a source term, there is still one advantage of the spin current operator definition (10). This definition leads to a very reasonable physical result: the corresponding spin current in (11) vanishes in insulators. In Section VI we will return to this point and analytically prove that when the periodic potential gets stronger and as a result the energy bands get narrower, that is when the system turns into an insulator, the spin current given by Eq. (11) goes to zero. Below we will calculate only the spin current polarized along the z-axis and denote this current as J_S, *i.e.*, $J_S(t) \equiv J_S^z(t)$. The components of the spin current polarized along the x and y axes are zero as shown in Appendix B.

It is convenient to calculate the traces in (9) and (11) using the basis which diagonalizes both $\hat{x}$ and $\hat{\sigma}_z$, because this requires to determine only the diagonal elements of the reduced density matrix. In a quasi-1D periodic system with

RSOI the energy spectrum can be related to the one of the corresponding truly 1D problem without RSOI[29]. This links the Bloch bands of that truly 1D problem to the Bloch sub-bands of the quasi-1D problem. The general structure of the results obtained in Ref. 29 is retained in the presence of the orbit-orbit coupling and a uniform stationary magnetic field along the z-axis. A slight change of the theory is given in Appendix C. We shall consider electron transport under such conditions when only a finite number of the Bloch sub-bands is involved. The basis which diagonalizes $\hat{x}$ and $\hat{\sigma}_z$ becomes in this case discrete. The total number of the Bloch sub-bands is equal to the product of the number, N_B, of the Bloch bands from the corresponding truly 1D problem without magnetic field and without spin-orbit coupling, the number, N_t, of the transverse modes and the number of the spin states. In this work we shall use the model with $N_B = 1$, $N_t = 2$. Since there are only two spin states, the total number of the Bloch sub-bands in our problem is equal to four. The representation in terms of the eigen-states of the coordinate operator for a model with discrete x-values is called discrete variable representation (DVR)[30]. Let us call σ-DVR the representation in which both the coordinate and spin operators are diagonal. Denoting the σ-DVR basis states as $\{|\alpha\rangle\}$ and eigen-values of $\hat{x}$ and $\hat{\sigma}_z$ in a state $|\alpha\rangle$ through x_α and σ_α, respectively, the charge and spin currents (9) and (11) are rewritten as

$$J_C(t) = -e\sum_\alpha x_\alpha \frac{d}{dt}P_\alpha(t),$$
$$J_S(t) = \sum_\alpha \sigma_\alpha x_\alpha \frac{d}{dt}P_\alpha(t), \qquad (12)$$

where $P_\alpha(t) \equiv \langle\alpha|\hat{\rho}(t)|\alpha\rangle$ is the population of the σ-DVR state $|\alpha\rangle$ at time t.

We are interested in the long-time limit of the currents $\bar{J}_C(t)$ and $\bar{J}_S(t)$ averaged over the driving period $T = 2\pi/\Omega$ with the time average of a time dependent function $f(t)$ defined as $\bar{f}(t) \equiv (1/T)\int_t^{t+T}dt'f(t')$. From (12) it follows

$$\bar{J}_C(t) = -e\sum_\alpha x_\alpha \frac{d}{dt}\bar{P}_\alpha(t),$$
$$\bar{J}_S(t) = \sum_\alpha \sigma_\alpha x_\alpha \frac{d}{dt}\bar{P}_\alpha(t). \qquad (13)$$

The advantage of working in the σ-DVR basis is that real-time path integral techniques can be used to exactly trace out the bath degrees of freedom[31,32]. Moreover, at driving frequencies larger than the ones characterizing the internal dynamics of the quasi-1D system coupled to the bath, the averaged populations $\bar{P}_\alpha(t)$ can be found from the master equation,

$$\frac{d}{dt}\bar{P}_\alpha(t) = \sum_{\substack{\beta \\ (\beta\neq\alpha)}}\bar{\Gamma}_{\alpha\beta}\bar{P}_\beta(t) - \sum_{\substack{\beta \\ (\beta\neq\alpha)}}\bar{\Gamma}_{\beta\alpha}\bar{P}_\alpha(t), \qquad (14)$$

valid at long times. In Eq. (14) $\bar{\Gamma}_{\alpha\beta}$ is an averaged transition rate from the state $|\beta\rangle$ to the state $|\alpha\rangle$. In order to obtain

concrete expressions for the averaged currents the σ-DVR basis must be found explicitly. This is the subject of the next section.

III. DIAGONALIZATION OF $\hat{\sigma}_z$ AND $\hat{x}$: THE σ-DVR BASIS

The eigen-states of the $\hat{\sigma}_z$ operator were found in Ref. 29 (see Eq. (12) therein) for a model without coupling between the orbital degrees of freedom and magnetic field. The changes necessary to include those two effects are discussed in Appendix C. The eigen-value equation for the $\hat{\sigma}_z$ operator is

$$\hat{\sigma}_z|l, k_{\mathrm{B}}, j, \sigma\rangle_{\gamma,j} = \sigma|l, k_{\mathrm{B}}, j, \sigma\rangle_{\gamma,j}. \tag{15}$$

In Eq. (15) l, k_{B}, j, σ stand for the Bloch band index, quasi-momentum, transverse mode index and z-projection of the spin, respectively. Since in the presence of the orbit-orbit coupling the periodic potential $U_{\gamma,j}(x)$ (see Appendix C) depends on γ and j, we have labeled the ket-symbol with the subscript γ, j. In the ensuing analysis we follow the same rule and label all the bra- and ket-symbols with the subscript γ, j, that is $_{\gamma,j}\langle\cdots|$ and $|\cdots\rangle_{\gamma,j}$.

It is convenient to start the diagonalization of the coordinate operator writing its matrix in the $\{|l, k_{\mathrm{B}}, j, \sigma\rangle_{\gamma,j}\}$ representation:

$$_{\gamma,j'}\langle l', k'_{\mathrm{B}}, j', \sigma'|\hat{x}|l, k_{\mathrm{B}}, j, \sigma\rangle_{\gamma,j} = \\ = \delta_{j',j}\delta_{\sigma',\sigma} \, _{\gamma,j}\langle l', k'_{\mathrm{B}} + \sigma k_{\mathrm{so}}|\hat{x}|l, k_{\mathrm{B}} + \sigma k_{\mathrm{so}}\rangle_{\gamma,j}. \tag{16}$$

The diagonal blocks,

$$_{\gamma,j}\langle l', k'_{\mathrm{B}}, j, \sigma = 1|\hat{x}|l, k_{\mathrm{B}}, j, \sigma = 1\rangle_{\gamma,j} = \\ = \, _{\gamma,j}\langle l', k'_{\mathrm{B}} + k_{\mathrm{so}}|\hat{x}|l, k_{\mathrm{B}} + k_{\mathrm{so}}\rangle_{\gamma,j}, \quad \forall j, \\ _{\gamma,j}\langle l', k'_{\mathrm{B}}, j, \sigma = -1|\hat{x}|l, k_{\mathrm{B}}, j, \sigma = -1\rangle_{\gamma,j} = \\ = \, _{\gamma,j}\langle l', k'_{\mathrm{B}} - k_{\mathrm{so}}|\hat{x}|l, k_{\mathrm{B}} - k_{\mathrm{so}}\rangle_{\gamma,j}, \quad \forall j, \tag{17}$$

are unitary equivalent for a given value of the index j and thus the eigen-values of $\hat{x}$ do not depend on σ.

As it is shown in Appendix A, the eigen-values of the matrix $_{\gamma,j}\langle l', k'_{\mathrm{B}}|\hat{x}|l, k_{\mathrm{B}}\rangle_{\gamma,j}$ are

$$x_{\gamma;\zeta,m,j} = mL + d_{\gamma;\zeta,j}, \tag{18}$$

where $m = 0, \pm 1, \pm 2 \ldots$, $\zeta = 1, 2, \ldots, N_{\mathrm{B}}$ and the eigen-values $d_{\gamma;\zeta,j}$ are distributed within one elementary cell. If, for example, the system is divided into the elementary cells in such a way that the origin of coordinates is at the center of an elementary cell, then $-L/2 < d_{\gamma;\zeta,j} \leqslant L/2$. In Eq. (18) we have taken into account that the periodic potential $U_{\gamma,j}(x)$, introduced in Appendix C, depends on γ and j, and thus the eigen-values distributed within one elementary cell also acquire a dependence on γ and j.

From (16) and (18) it follows that one can label the eigen-states of $\hat{x}$ with the quantum numbers ζ, m, j, σ, that is

as $|\zeta, m, j, \sigma\rangle_{\gamma,j}$, and in the $\{|l, k_{\mathrm{B}}, j, \sigma\rangle_{\gamma,j}\}$ representation these eigen-states have the form:

$$_{\gamma,j'}\langle l, k_{\mathrm{B}}, j', \sigma'|\zeta, m, j, \sigma\rangle_{\gamma,j} = \\ = \delta_{j',j}\delta_{\sigma',\sigma} \, _{\gamma,j}\langle l, k_{\mathrm{B}}, j, \sigma|\zeta, m, j, \sigma\rangle_{\gamma,j}. \tag{19}$$

The corresponding eigen-values are $x_{\gamma;\zeta,m,j,\sigma} = x_{\gamma;\zeta,m,j}$. From the eigen-value equation

$$\hat{x}|\zeta, m, j, \sigma\rangle_{\gamma,j} = x_{\gamma;\zeta,m,j}|\zeta, m, j, \sigma\rangle_{\gamma,j} \tag{20}$$

written in the $\{|l, k_{\mathrm{B}}, j, \sigma\rangle_{\gamma,j}\}$ representation through the use of (16),

$$\sum_{l', k'_{\mathrm{B}}} \, _{\gamma,j}\langle l, k_{\mathrm{B}} + \sigma k_{\mathrm{so}}|\hat{x}|l', k'_{\mathrm{B}} + \sigma k_{\mathrm{so}}\rangle_{\gamma,j} \times \\ \times \, _{\gamma,j}\langle l', k'_{\mathrm{B}}, j, \sigma|\zeta, m, j, \sigma\rangle_{\gamma,j} = \\ = x_{\gamma;\zeta,m,j} \, _{\gamma,j}\langle l, k_{\mathrm{B}}, j, \sigma|\zeta, m, j, \sigma\rangle_{\gamma,j}, \tag{21}$$

it follows that

$$_{\gamma,j}\langle l, k_{\mathrm{B}}, j, \sigma = 1|\zeta, m, j, \sigma = 1\rangle_{\gamma,j} = \\ = \, _{\gamma,j}\langle l, k_{\mathrm{B}} + k_{\mathrm{so}}|\zeta, m\rangle_{\gamma,j}, \\ _{\gamma,j}\langle l, k_{\mathrm{B}}, j, \sigma = -1|\zeta, m, j, \sigma = -1\rangle_{\gamma,j} = \\ = \, _{\gamma,j}\langle l, k_{\mathrm{B}} - k_{\mathrm{so}}|\zeta, m\rangle_{\gamma,j}. \tag{22}$$

Since $|\zeta, m, j, \sigma\rangle_{\gamma,j}$ is also the eigen-state of $\hat{\sigma}_z$ corresponding to the eigen-value $\sigma_{\zeta,m,j,\sigma} = \sigma$, we infer that the σ-DVR basis states $|\alpha\rangle$ from the previous section are just the $|\zeta, m, j, \sigma\rangle_{\gamma,j}$ states, that is $\{|\alpha\rangle\} \equiv \{|\zeta, m, j, \sigma\rangle_{\gamma,j}\}$.

IV. σ-DVR REPRESENTATION AND ITS TIGHT-BINDING MODEL

Let us represent the Hamiltonian $\hat{H}$ in the σ-DVR basis obtained in the previous section in order to derive an effective tight-binding model.

Using the $\{|\zeta, m, j, \sigma\rangle_{\gamma,j}\}$ basis the Hamiltonian $\hat{H}$ can be written as

$$\hat{H} = \sum_{\substack{\zeta,m,j,\sigma \\ \zeta',m',j',\sigma'}} \, _{\gamma,j'}\langle \zeta', m', j', \sigma'|\hat{H}|\zeta, m, j, \sigma\rangle_{\gamma,j} \times \\ \times |\zeta', m', j', \sigma'\rangle_{\gamma,j'} \, _{\gamma,j}\langle \zeta, m, j, \sigma|, \tag{23}$$

with the matrix

$$_{\gamma,j'}\langle \zeta', m', j', \sigma'|\hat{H}|\zeta, m, j, \sigma\rangle_{\gamma,j} = \sum_{l, k_{\mathrm{B}}, \eta} \varepsilon_{\gamma;l,\eta}(k_{\mathrm{B}}) \times \\ \times \, _{\gamma,j'}\langle \zeta', m'|l, k_{\mathrm{B}} + \sigma' k_{\mathrm{so}}\rangle_{\gamma,j'} \times \\ \times \, _{\gamma,j}\langle l, k_{\mathrm{B}} + \sigma k_{\mathrm{so}}|\zeta, m\rangle_{\gamma,j} \, \theta_{\gamma;l,k_{\mathrm{B}},\eta}(j', \sigma') \times \\ \times \theta^*_{\gamma;l,k_{\mathrm{B}},\eta}(j, \sigma). \tag{24}$$

The tight-binding approximation of (23) is obtained if one assumes that the matrix elements (24) with $|m' - m| > 1$ are negligibly small.

We consider temperatures low enough and assume that electrons populate only the lowest Bloch sub-bands with $l = 1$ (*i.e.*, $N_B = 1$). Under this condition the periodic potential can be of arbitrary shape and the only limitation on it is the validity of the tight-binding approximation.

Below we thoroughly study the case where the four lowest Bloch sub-bands are the ones with $l = 1$, $\eta = 1, 2, 3, 4$ and the only ones which are populated with electrons. For simplicity we consider weak orbit-orbit coupling and calculate the corresponding eigen-energies $\varepsilon_{\gamma;l,\eta}(k_B)$ and eigen-spinors $\theta_{\gamma;l,k_B,\eta}(j, \sigma)$ retaining only the first two transverse modes, that is $j = 0, 1$. In this case $\hat{H}$ has the form

$$
\hat{H} = \sum_m \Bigg[\sum_{j,\sigma} \varepsilon_{\gamma;j,\sigma} |m, j, \sigma\rangle_{\gamma,j} \; _{\gamma,j}\langle m, j, \sigma| +
$$
$$
+ \sum_{j,\sigma' \neq \sigma} \Delta^{\text{intra}}_{\gamma;j,\sigma';j,\sigma}(m) |m, j, \sigma'\rangle_{\gamma,j} \; _{\gamma,j}\langle m, j, \sigma| +
$$
$$
+ \sum_{j' \neq j,\sigma',\sigma} \Delta^{\text{intra}}_{\gamma;j',\sigma';j,\sigma}(m) |m, j', \sigma'\rangle_{\gamma,j'} \; _{\gamma,j}\langle m, j, \sigma| + \quad (25)
$$
$$
+ \sum_{j',j,\sigma',\sigma} \Big(\Delta^{\text{inter,b}}_{\gamma;j',\sigma';j,\sigma}(m) |m, j', \sigma'\rangle_{\gamma,j'} \; _{\gamma,j}\langle m+1, j, \sigma| +
$$
$$
+ \Delta^{\text{inter,f}}_{\gamma;j',\sigma';j,\sigma}(m) |m+1, j', \sigma'\rangle_{\gamma,j'} \; _{\gamma,j}\langle m, j, \sigma| \Big) \Bigg],
$$

where

$$
|m, j, \sigma\rangle_{\gamma,j} \equiv |\zeta = 1, m, j, \sigma\rangle_{\gamma,j}, \quad (26)
$$

and we have defined the on-site energies $\varepsilon_{\gamma;j,\sigma}$ and hopping matrix elements $\Delta^{\text{intra}}_{\gamma;j',\sigma';j,\sigma}(m)$, $\Delta^{\text{inter,b}}_{\gamma;j',\sigma';j,\sigma}(m)$ and $\Delta^{\text{inter,f}}_{\gamma;j',\sigma';j,\sigma}(m)$ as follows

$$
\varepsilon_{\gamma;j,\sigma} \equiv \; _{\gamma,j}\langle m, j, \sigma| \hat{H} |m, j, \sigma\rangle_{\gamma,j},
$$
$$
\Delta^{\text{intra}}_{\gamma;j',\sigma';j,\sigma}(m) \underset{(j',\sigma') \neq (j,\sigma)}{\equiv} \; _{\gamma,j}\langle m, j', \sigma'| \hat{H} |m, j, \sigma\rangle_{\gamma,j},
$$
$$
\Delta^{\text{inter,b}}_{\gamma;j',\sigma';j,\sigma}(m) \equiv \; _{\gamma,j'}\langle m, j', \sigma'| \hat{H} |m+1, j, \sigma\rangle_{\gamma,j}, \quad (27)
$$
$$
\Delta^{\text{inter,f}}_{\gamma;j',\sigma';j,\sigma}(m) \equiv \; _{\gamma,j'}\langle m+1, j', \sigma'| \hat{H} |m, j, \sigma\rangle_{\gamma,j}.
$$

Note that

$$
[\Delta^{\text{intra}}_{\gamma;j',\sigma';j,\sigma}(m)]^* = \Delta^{\text{intra}}_{\gamma;j,\sigma;j',\sigma'}(m), \quad (28)
$$

$$
[\Delta^{\text{inter,b}}_{\gamma;j',\sigma';j,\sigma}(m)]^* = \Delta^{\text{inter,f}}_{\gamma;j,\sigma;j',\sigma'}(m). \quad (29)
$$

Introducing the notations

$$
\{\xi\} \equiv \{(j, \sigma)\},
$$
$$
\xi = 1 \Leftrightarrow (0, 1), \; \xi = 2 \Leftrightarrow (0, -1), \quad (30)
$$
$$
\xi = 3 \Leftrightarrow (1, 1), \; \xi = 4 \Leftrightarrow (1, -1),
$$

we finally have

$$
\hat{H} = \sum_m \Bigg[\sum_{\xi=1}^{4} \varepsilon_{\gamma;\xi} |m, \xi\rangle_{\gamma,\xi} \; _{\gamma,\xi}\langle m, \xi| +
$$
$$
+ \sum_{\xi \neq \xi'=1}^{4} \Delta^{\text{intra}}_{\gamma;\xi',\xi}(m) |m, \xi'\rangle_{\gamma,\xi'} \; _{\gamma,\xi}\langle m, \xi| + \quad (31)
$$
$$
+ \sum_{\xi,\xi'=1}^{4} \Big(\Delta^{\text{inter,b}}_{\gamma;\xi',\xi}(m) |m, \xi'\rangle_{\gamma,\xi'} \; _{\gamma,\xi}\langle m+1, \xi| +
$$
$$
+ \Delta^{\text{inter,f}}_{\gamma;\xi',\xi}(m) |m+1, \xi'\rangle_{\gamma,\xi'} \; _{\gamma,\xi}\langle m, \xi| \Big) \Bigg].
$$

Equation (31) represents a tight-binding model which can now be used to perform actual calculations of quantum transport in a dissipative system.

To conclude this section, we would like to note that because of the simultaneous presence of the harmonic confinement and RSOI the system splits into two subsystems. The first subsystem is characterized by $\xi = 1, 4$ and the second one by $\xi = 2, 3$. These subsystems are totally decoupled: there is no electron exchange between them. Such a state of affairs persists if one considers more than two transverse modes. In this work, for simplicity, we only consider one subsystem, namely the one with $\xi = 1, 4$. Such uncoupled subsystems also appear within the hard wall model of the transverse confinement[33].

V. TRANSITION RATES

The tight-binding model introduced in Section IV relies upon the fact that the hopping matrix elements (27) are small. In this case the second-order approximation for the averaged transition rates in Eq. (14) can be used giving[4,34]

$$
\bar{\Gamma}^{m',m}_{\gamma;\xi',\xi} = \frac{|\Delta^{m',m}_{\gamma;\xi',\xi}|^2}{\hbar^2} \times
$$
$$
\times \int_{-\infty}^{\infty} d\tau e^{-[(x_{\gamma;m,\xi} - x_{\gamma;m',\xi'})^2/\hbar] Q(\tau) + i[(\varepsilon_{\gamma;\xi} - \varepsilon_{\gamma;\xi'})/\hbar]\tau} \times \quad (32)
$$
$$
\times J_0 \Bigg[\frac{2F(x_{\gamma;m,\xi} - x_{\gamma;m',\xi'})}{\hbar\Omega} \sin\left(\frac{\Omega\tau}{2}\right) \Bigg],
$$

where $x_{\gamma;m,\xi} \equiv x_{\gamma;\zeta=1,m,\xi} = mL + d_{\gamma;\xi}$ with $d_{\gamma;\xi} \equiv d_{\gamma;1,j}$, $\Delta^{m',m}_{\gamma;\xi',\xi} \equiv \; _{\gamma,\xi'}\langle m', \xi'| \hat{H} |m, \xi\rangle_{\gamma,\xi}$ the hopping matrix element between the states $|m', \xi'\rangle_{\gamma,\xi'}$ and $|m, \xi\rangle_{\gamma,\xi}$, $J_0(x)$ the zero-order Bessel function and $Q(\tau)$ the twice integrated bath correlation function[32]:

$$
Q(\tau) = \frac{1}{\pi} \int_0^\infty d\omega \frac{J(\omega)}{\omega^2} \Bigg[\coth\left(\frac{\hbar\omega\beta}{2}\right) \times
$$
$$
\times [1 - \cos(\omega\tau)] + i \sin(\omega\tau) \Bigg], \quad (33)
$$

where $J(\omega)$ is given by Eq. (7) and β is the inverse temperature.

The transition rates are functions of the orbit-orbit coupling strength γ because the Bloch amplitudes as well as

the difference $\Delta d_\gamma \equiv d_{\gamma;1,0} - d_{\gamma;1,1}$ depend on γ. Within the context of the tight-binding model the eigen-values $d_{\gamma;1,0}$ and $d_{\gamma;1,1}$ tend to zero and fulfil $\Delta d_\gamma / l_r \ll 1$, where $l_r = \min[L, \sqrt{\hbar/m\omega_0}, \hbar\Omega/F, \dots]$. Consequently, the transition rates depend on γ predominantly through the Bloch amplitudes, and in this work we pay no regard to terms of order $\mathcal{O}(\Delta d_\gamma / l_r)$. This is also consistent with our model taking into account only the first two transverse modes. Keeping terms of order $\mathcal{O}(\Delta d_\gamma / l_r)$ would mean that the strength γ of the orbit-orbit coupling is large enough so that one would need to consider more than just the first two transverse modes because in this case the non-diagonal elements would be comparable with the diagonal ones.

Using the notations,

$$
\begin{aligned}
\bar{\Gamma}^{m,m}_{\gamma;\xi',\xi} &\equiv \bar{\Gamma}^{\text{intra}}_{\gamma;\xi',\xi}, \quad \xi' \neq \xi, \\
\bar{\Gamma}^{m,m+1}_{\gamma;\xi',\xi} &\equiv \bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi',\xi}, \\
\bar{\Gamma}^{m+1,m}_{\gamma;\xi',\xi} &\equiv \bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi',\xi},
\end{aligned}
\tag{34}
$$

from (32) one obtains

$$
\bar{\Gamma}^{\text{intra}}_{\gamma;\xi',\xi} = 0, \tag{35}
$$

and

$$
\begin{aligned}
\bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi',\xi} &= |\Delta^{\text{inter,b}}_{\gamma;\xi',\xi}(m)|^2 J_{\gamma;\xi',\xi}, \\
\bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi',\xi} &= |\Delta^{\text{inter,f}}_{\gamma;\xi',\xi}(m)|^2 J_{\gamma;\xi',\xi},
\end{aligned}
\tag{36}
$$

where

$$
\begin{aligned}
J_{\gamma;\xi',\xi} = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} d\tau e^{-\frac{L^2}{\hbar}Q(\tau) + \mathrm{i}[(\varepsilon_{\gamma;\xi} - \varepsilon_{\gamma;\xi'})/\hbar]\tau} \times \\
\times J_0\left[\frac{2FL}{\hbar\Omega} \sin\left(\frac{\Omega\tau}{2}\right)\right].
\end{aligned}
\tag{37}
$$

Note that $\bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi',\xi}$ and $\bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi',\xi}$ do not depend on m due to the Bloch theorem which leads to an m-dependence of

$\Delta^{\text{inter,b}}_{\gamma;\xi',\xi}(m)$ and $\Delta^{\text{inter,f}}_{\gamma;\xi',\xi}(m)$ only through a phase factor as it is shown in Appendix D. From (29) and (36) it follows that

$$
\bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi,\xi} = \bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi,\xi}, \tag{38}
$$

$$
\bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi',\xi} \bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi,\xi'} = \bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi',\xi} \bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi,\xi'}. \tag{39}
$$

To calculate the charge and spin currents we additionally need the transition rates

$$
\bar{\Gamma}_{\gamma;\xi,\xi'} \equiv \bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi,\xi'} + \bar{\Gamma}^{\text{intra}}_{\gamma;\xi,\xi'} + \bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi,\xi'}. \tag{40}
$$

As pointed out at the end of Section IV, the system is split into two subsystems isolated from each other. Since electron exchange between the subsystems is absent one can write

$$
\begin{aligned}
\bar{\Gamma}_{\gamma;1,2} = \bar{\Gamma}_{\gamma;1,3} = \bar{\Gamma}_{\gamma;2,1} = \bar{\Gamma}_{\gamma;2,4} = \\
= \bar{\Gamma}_{\gamma;3,1} = \bar{\Gamma}_{\gamma;3,4} = \bar{\Gamma}_{\gamma;4,2} = \bar{\Gamma}_{\gamma;4,3} = 0.
\end{aligned}
\tag{41}
$$

The last equalities are very useful because they allow us to significantly simplify the expressions for the charge and spin currents which are derived in the next section.

VI. CHARGE AND SPIN CURRENTS

The expressions for the stationary averaged charge and spin currents,

$$
\bar{J}^{\infty}_C \equiv \lim_{t\to\infty} \bar{J}_C(t), \quad \bar{J}^{\infty}_S \equiv \lim_{t\to\infty} \bar{J}_S(t), \tag{42}
$$

can be found from the averaged master equation (14) which we rewrite here using the σ-DVR indices and tight-binding approximation introduced in Section IV and utilizing the notations of Section V for the transition rates:

$$
\begin{aligned}
\frac{d}{dt}\bar{P}^m_{\gamma;\xi}(t) = \sum_{\substack{\xi'=1 \\ (\xi'\neq\xi)}}^4 \left[\bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi,\xi'}\bar{P}^{m-1}_{\gamma;\xi'}(t) + \bar{\Gamma}^{\text{intra}}_{\gamma;\xi,\xi'}\bar{P}^m_{\gamma;\xi'}(t) + \bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi,\xi'}\bar{P}^{m+1}_{\gamma;\xi'}(t)\right] - \sum_{\substack{\xi'=1 \\ (\xi'\neq\xi)}}^4 \left[\bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi',\xi} + \bar{\Gamma}^{\text{intra}}_{\gamma;\xi',\xi} + \bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi',\xi}\right]\bar{P}^m_{\gamma;\xi}(t) + \\
+ \left[\bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi,\xi}\bar{P}^{m-1}_{\gamma;\xi}(t) + \bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi,\xi}\bar{P}^{m+1}_{\gamma;\xi}(t)\right] - \left[\bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi,\xi} + \bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi,\xi}\right]\bar{P}^m_{\gamma;\xi}(t),
\end{aligned}
\tag{43}
$$

From (12) and (43) one finds

$$
\bar{J}^{\infty}_C = -eL \sum_{\xi,\xi'=1}^4 \left[\bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi,\xi'} - \bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi,\xi'}\right] p^{\infty}_{\gamma;\xi'}, \tag{44}
$$

$$
\begin{aligned}
\bar{J}^{\infty}_S = \sum_{\xi,\xi'=1}^4 \left[\left(d_{\gamma;\xi}\sigma_\xi - d_{\gamma;\xi'}\sigma_{\xi'}\right)\left(\bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi,\xi'} + \bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi,\xi'}\right) + \right. \\
\left. + L\sigma_\xi\left(\bar{\Gamma}^{\text{inter,f}}_{\gamma;\xi,\xi'} - \bar{\Gamma}^{\text{inter,b}}_{\gamma;\xi,\xi'}\right)\right] p^{\infty}_{\gamma;\xi'},
\end{aligned}
\tag{45}
$$

where we have used Eq. (18). To derive Eq. (45) we have additionally made use of Eq. (35). In Eq. (45) $\sigma_\xi \equiv \sigma_{\zeta=1,m,\xi}$ and $\sigma_1 = \sigma_3 = 1$, $\sigma_2 = \sigma_4 = -1$ as it follows from Eq. (30).

The quantities $p_{\gamma;\xi}^\infty$ are defined as

$$p_{\gamma;\xi}(t) \equiv \sum_m \bar{P}_{\gamma;\xi}^m(t), \quad p_{\gamma;\xi}^\infty \equiv \lim_{t\to\infty} p_{\gamma;\xi}(t), \qquad (46)$$

and they satisfy the constraint

$$p_{\gamma;1}(t) + p_{\gamma;2}(t) + p_{\gamma;3}(t) + p_{\gamma;4}(t) = 1, \quad \forall\, t. \qquad (47)$$

As already mentioned at the end of Section IV, we only consider the subsystem with $\xi = 1, 4$. The properties of the stationary averaged transport do not depend on initial conditions. We choose the following ones:

$$p_{\gamma;1}(t=0) = 1, \quad p_{\gamma;4}(t=0) = 0. \qquad (48)$$

Because of the constraint (47) $p_{\gamma;2}(t=0) = p_{\gamma;3}(t=0) = 0$ and since there is no electron exchange between the subsystems, the states of the subsystem with $\xi = 2, 3$ remain empty at any time, $p_{\gamma;2}(t) = p_{\gamma;3}(t) = 0$, $\forall t$. This leads to $p_{\gamma;2}^\infty = p_{\gamma;3}^\infty = 0$. Then from the master equation (43) with the initial conditions (48) and using (40), (41) one obtains

$$p_{\gamma;1}^\infty = \frac{\bar{\Gamma}_{\gamma;1,4}}{\bar{\Gamma}_{\gamma;1,4} + \bar{\Gamma}_{\gamma;4,1}}, \quad p_{\gamma;4}^\infty = \frac{\bar{\Gamma}_{\gamma;4,1}}{\bar{\Gamma}_{\gamma;1,4} + \bar{\Gamma}_{\gamma;4,1}}. \qquad (49)$$

Using Eqs. (35), (38)-(40) and (49) it follows from (44)

$$\bar{J}_\mathrm{C}^\infty = 0, \qquad (50)$$

that is the absence of the stationary averaged charge transport. However, using Eqs. (35), (38), (40) and (49) we get from Eq. (45)

$$\bar{J}_\mathrm{S}^\infty = \frac{2L}{\bar{\Gamma}_{\gamma;1,4} + \bar{\Gamma}_{\gamma;4,1}} \left(\bar{\Gamma}_{\gamma;1,4}^{\mathrm{inter,f}} \bar{\Gamma}_{\gamma;4,1}^{\mathrm{inter,b}} - \bar{\Gamma}_{\gamma;1,4}^{\mathrm{inter,b}} \bar{\Gamma}_{\gamma;4,1}^{\mathrm{inter,f}} \right). \qquad (51)$$

The last expression can be rewritten in terms of the hopping matrix elements $\Delta_{\gamma;\xi',\xi}^{\mathrm{inter,f}}(m)$. Making use of Eqs. (29), (35), (36) and (40) we derive the stationary averaged spin current:

$$\bar{J}_\mathrm{S}^\infty = 2L \frac{J_{\gamma;1,4} J_{\gamma;4,1}}{J_{\gamma;1,4} + J_{\gamma;4,1}} \times$$
$$\times \left(|\Delta_{\gamma;1,4}^{\mathrm{inter,f}}(m)|^2 - |\Delta_{\gamma;4,1}^{\mathrm{inter,f}}(m)|^2 \right). \qquad (52)$$

Using Eqs. (52) and (D2) the non-equilibrium stationary averaged spin current can be written as

$$\bar{J}_{\mathrm{n-e,S}}^\infty = -2 \left(\frac{J_{\gamma;1,4} J_{\gamma;4,1}}{J_{\gamma;1,4} + J_{\gamma;4,1}} - \frac{J_{\gamma;1,4}^{(0)} J_{\gamma;4,1}^{(0)}}{J_{\gamma;1,4}^{(0)} + J_{\gamma;4,1}^{(0)}} \right) \times$$
$$\times \frac{L\hbar^3 k_{\mathrm{so}}^2 \omega_0}{m} \sum_{k_\mathrm{B}, k_\mathrm{B}'} \sin[(k_\mathrm{B} - k_\mathrm{B}')L] \, \mathrm{Im}[F_{\gamma;k_\mathrm{B}, k_\mathrm{B}'}], \qquad (53)$$

where $J_{\gamma;\xi',\xi}^{(0)}$ is given by Eq. (37) with $F = 0$ and the function $F_{\gamma;k_\mathrm{B}, k_\mathrm{B}'}$ is defined by Eq. (D3). Note the structure of Eq. (53). It is the product of two factors of different physical origin. The factor in the second line describes the isolated system and the factor in the first line is purely due to the dissipative

coupling to an external environment. To get Eq. (53) we have eliminated from $\bar{J}_\mathrm{S}^\infty$ the equilibrium spin current arising due to the non-compensation[35] of the spin currents from different bands of the Rashba-Bloch spectrum of the isolated system. It turns out that this effect is strong enough to indenture in a dissipative system. Below we only consider the non-equilibrium spin current, $\bar{J}_{\mathrm{n-e,S}}^\infty$, and not the full one, $\bar{J}_\mathrm{S}^\infty$.

Let us at this point also mention the dependence of the spin current $\bar{J}_{\mathrm{n-e,S}}^\infty$ on the magnetic field H_0. Since the magnetic field is applied along the z-axis, it couples to the system through the $\hat{\sigma}_z$ operator and thus the hopping matrix elements $\Delta_{\gamma;1(4),4(1)}^{\mathrm{inter,f}}(m)$ do not depend on H_0. It then follows that the spin current depends on the magnetic field only through its dissipative prefactor. The dependence on H_0 comes into play through the on-site energies $\varepsilon_{\gamma;1(4)}$. The difference $\varepsilon_{\gamma;4} - \varepsilon_{\gamma;1}$ which enters the integrals $J_{\gamma;1(4),4(1)}$ and $J_{\gamma;1(4),4(1))}^{(0)}$ can be written as:

$$\varepsilon_{\gamma;4} - \varepsilon_{\gamma;1} = \frac{1}{N} \sum_{k_\mathrm{B}} \left[\varepsilon_{\gamma;1;1}^{(0)}(k_\mathrm{B}) - \varepsilon_{\gamma;0;1}^{(0)}(k_\mathrm{B}) \right] +$$
$$+ \hbar\omega_0 + 2g\mu_\mathrm{B} H_0, \qquad (54)$$

where N is the number of the elementary cells and $\varepsilon_{\gamma;j;l}^{(0)}(k_\mathrm{B})$ are the eigen-values of the truly 1D Hamiltonian

$$\hat{H}_{0;\gamma,j}^{\mathrm{1D}} \equiv \frac{\hbar^2 \hat{k}_x^2}{2m} + U(\hat{x}) \left[1 + \gamma \frac{\hbar}{m\omega_0 L^2} \left(j + \frac{1}{2} \right) \right]. \qquad (55)$$

Therefore, in the presence of a transverse in-plane uniform stationary magnetic field the existence of the spin current is possible under the same conditions which were discussed in Ref. 20. For completeness we list these conditions below.

From (53) one finds, as mentioned in Section II, that when the electronic states become localized, the stationary averaged spin current vanishes. Indeed, in this insulating limit the function $F_{\gamma;k_\mathrm{B}, k_\mathrm{B}'}$ does not depend on the quasi-momenta k_B and k_B' and Eq. (53) gives zero.

When the spin-orbit interaction is absent, that is $k_{\mathrm{so}} = 0$, we get from (53)

$$\bar{J}_{\mathrm{n-e,S}}^\infty \big|_{k_{\mathrm{so}}=0} = 0. \qquad (56)$$

Further, if the orbital degrees of freedom x and z are not coupled, that is $\gamma = 0$, it follows from Eqs. (53) and (D6) that

$$\bar{J}_{\mathrm{n-e,S}}^\infty \big|_{\gamma=0} = 0. \qquad (57)$$

Finally, if the periodic potential is symmetric, the Bloch amplitudes are real and we find from Eqs. (53) and (D4)

$$\bar{J}_{\mathrm{n-e,S}}^\infty = 0, \quad \textit{for symmetric periodic potentials.} \qquad (58)$$

Summarizing the results of this section we conclude that in order to generate a finite stationary averaged spin current three conditions must simultaneously be fulfilled: 1) presence of the spin-orbit interaction in the isolated system; 2) finite coupling between the orbital degrees of freedom x and z; 3) absence of the real space inversion center in the isolated system. Finally, one observes that a transverse in-plane uniform
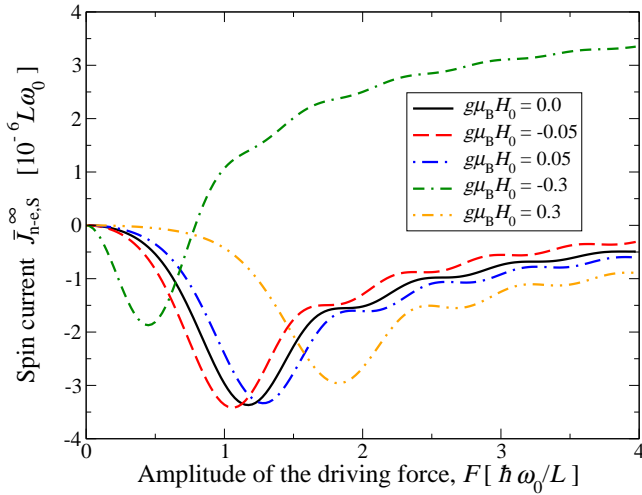
FIG. 1: (Color online) Non-equilibrium spin current, $\bar{J}_{\text{n-e,S}}^{\infty}$, as a function of the amplitude, F, of the driving force for different values of the z-projection of the magnetic field H_0. Further parameters: temperature $k_{\text{Boltz.}}T = 0.5$, spin-orbit coupling strength k_{so} with $k_{\text{so}}L = \pi/2$, orbit-orbit coupling strength $\gamma = 0.08$, driving frequency $\Omega = 0.2$, viscosity coefficient $\eta = 0.08$.

stationary magnetic field alone is not enough to produce the spin current in a driven dissipative system. The magnetic field can only affect the magnitude of the spin current when the properties of the isolated system meet the three conditions derived above.

VII. RESULTS

In this section we show some results obtained numerically for the theoretical model developed in the previous sections. As an example we consider an InGaAs/InP quantum wire structure. The values of the corresponding parameters used to get the results are similar to the ones from the work of Schäpers et al.[36] In particular, $\hbar\omega_0 = 0.225$ meV, $\alpha \equiv \hbar^2 k_{\text{so}}/m = 9.94 \times 10^{-12}$ eV·m (which gives $k_{\text{so}} = 4.82 \times 10^6$ m^{-1}), $m = 0.037 m_0$ (m_0 is the free-electron mass). The value, $g = 7.5$, of the electron spin g-factor (in our notations $g \equiv -g^*/2$, where g^* is the effective gyroscopic factor measured experimentally) is taken from Ref. 37. From these parameters and for example for the period of the super-lattice $L = 2.5\sqrt{\hbar/m\omega_0} \approx 0.24$ μm, which is easily achievable technologically at present[38], it follows that $k_{\text{so}}L \approx 0.368\pi$.

The asymmetric periodic potential is

$$U(x) = \hbar\omega_0\left\{2.6\left[1 - \cos\left(\frac{2\pi x}{L} - 1.9\right)\right] + 1.9\cos\left(\frac{4\pi x}{L}\right)\right\}. \tag{59}$$

The bath is assumed to be Ohmic with exponential cutoff:

$$J(\omega) = \eta\omega\exp\left(-\frac{\omega}{\omega_c}\right), \tag{60}$$
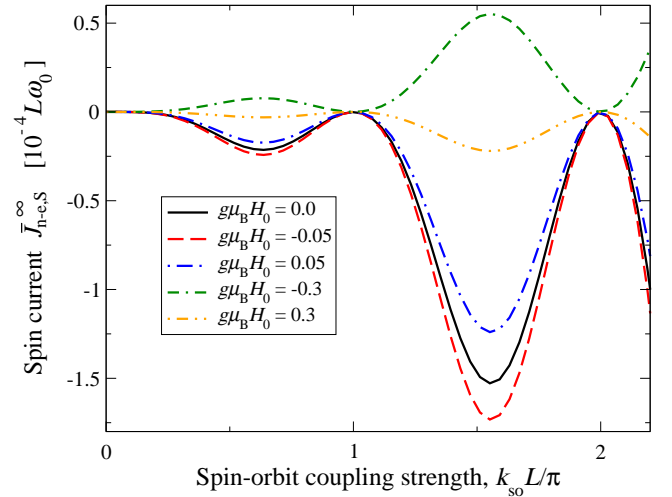

FIG. 2: (Color online) Non-equilibrium spin current, $\bar{J}_{\text{n-e,S}}^{\infty}$, as a function of the spin-orbit coupling strength, k_{so}, for different values of the z-projection of the magnetic field H_0. The driving amplitude is $F = 1.0\,\hbar\omega_0/L$. The other parameters are as in Fig. 1.

where η is the viscosity coefficient and ω_c the cutoff frequency. We use $\omega_c = 10\,\omega_0$.

To present the results we use in all the figures the units of $\hbar\omega_0$ and ω_0 for energies and frequencies, respectively. The viscosity coefficient is taken in units of $m\omega_0$.

Let us discuss possible values of the driving parameters. In a dissipationless system (or in a system with weak dissipation) of size L one should restrict possible values of the driving amplitude and frequency, $0 < FL < \hbar\omega_0$ and $0 < \Omega < \omega_0$, in order to stay within the validity of the model with the first two transverse modes opened. In a strongly dissipative system, as in our case, it is not necessary to fulfil the last inequalities because an electron loses a huge amount of its energy due to intensive dissipative processes. In general, our model of a driven strongly dissipative system taking into account the first four Bloch sub-bands remains valid if at long times the electron energy averaged over one period of the driving force, $\epsilon_{av}(F, \Omega, \eta)$ (which is a function of the driving and dissipation parameters), is smaller than $\hbar\omega_0$, $\epsilon_{av}(F, \Omega, \eta) < \hbar\omega_0$. This can take place even if $FL > \hbar\omega_0$ and $\Omega > \omega_0$ because even at such driving the strong dissipation (large values of η) will consume major amount of the electron energy.

In Fig. 1 the non-equilibrium spin current as a function of the amplitude of the external driving is shown for different values of the z-projection of the magnetic field. For small values of the driving amplitude and small magnetic fields it is seen that if the magnetic field has the same direction as the z-axis, the spin current decreases, while the opposite direction of the magnetic field amplifies the spin current. This behavior can be physically understood from Eq. (54). Positive values of H_0 can be equivalently considered as larger values of $\hbar\omega_0$, that is of the distance between the transverse modes. This in turn leads to a decrease of the transition probabilities which suppresses the spin current. On the contrary, negative values of H_0 correspond to smaller values of $\hbar\omega_0$ leading to
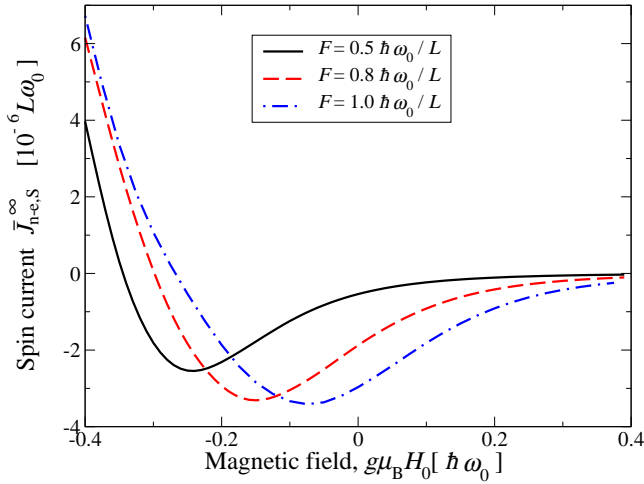
FIG. 3: (Color online) Non-equilibrium spin current, $\bar{J}_{\text{n-e,S}}^{\infty}$, as a function of the magnetic field, $g\mu_B H_0$, for different values of the amplitude of the driving force, F. The other parameters are as in Fig. 1.

FIG. 4: (Color online) Non-equilibrium spin current, $\bar{J}_{\text{n-e,S}}^{\infty}$, as a function of the magnetic field, $g\mu_B H_0$, for different values of the viscosity coefficient, η. The driving amplitude is $F = 1.0\hbar\omega_0/L$. The other parameters are as in Fig. 1.

an increase of the transition rates and thus the spin current is enhanced. Another physical explanation is that the magnetic field aligns the spins along its direction. Therefore, when H_0 is positive or negative the spins are forced to point in the direction of the z-axis or in the opposite direction, respectively. The spin current gets more polarized in the direction of the z-axis for $H_0 > 0$ or in the opposite direction for $H_0 < 0$. As a consequence its magnitude decreases for $H_0 > 0$ or increases for $H_0 < 0$ since it was polarized in the direction opposite to the one of the z-axis in the absence of the magnetic field.

The same dependence of the spin current on the magnetic field with small values of its magnitude (as well as for a small value of the driving amplitude $FL = 1.0\hbar\omega_0$) is found in Fig. 2 in view of its dependence on the spin-orbit interaction strength k_{so}. Again for $H_0 > 0$ the magnitude of the spin-current gets smaller and for $H_0 < 0$ it gets larger. Additionally, one can see that the presence of the magnetic field does not change the locations of minima and maxima of the spin current as a function of k_{so}. This has the following physical explanation. The minima and maxima in Fig. 2 are related to the periodicity of the energy spectrum in the $\mathbf{k}$-space. In terms of the band energy versus the quasi-momentum $\mathbf{k}$ dependence RSOI produces a horizontal (that is the energy of the bands does not change) split of the energy bands as well as their hybridization. Due to the periodicity this split can be minimal or maximal for some values of k_{so} which leads to the corresponding minima and maxima in Fig. 2. The role of the hybridization is that the split is never zero and thus the minima of the spin current are not exactly equal to zero. In contrast to this horizontal split the magnetic field produces a vertical (that is along the energy axis) split and it also produces hybridization. This vertical split is not correlated with the periodicity of the energy bands in the $\mathbf{k}$-space and thus the locations of minima and maxima remain untouched by the magnetic field.

However, the picture explained above is only valid for small values of the driving amplitude F and magnitude of the mag-
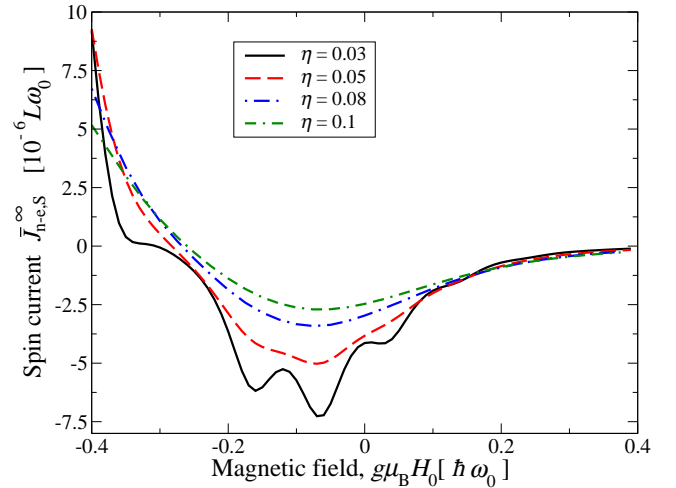
netic field $|H_0|$ where the spin current has a linear response to the magnetic field. When $|H_0|$ increases further, the spin current depends non-linearly on H_0 and a complicated interplay between the magnetic field, driving and dissipative processes develops. This dependence of the spin current on the magnetic field is depicted in Fig. 3 for different values of the amplitude of the driving force. In order to stay within the validity of our model, where only the first two transverse modes are opened, the magnitude of the magnetic field must satisfy the condition:

$$g\mu_B|H_0| \leqslant 0.5(\hbar\omega_0 + \Delta\varepsilon_{\gamma;4,1}), \qquad (61)$$

where $\Delta\varepsilon_{\gamma;4,1} \equiv \sum_{k_B}\left[\varepsilon_{\gamma,1;1}^{(0)}(k_B) - \varepsilon_{\gamma,0;1}^{(0)}(k_B)\right]/N$. For the values of the parameters used to obtain the numerical results we have $\Delta\varepsilon_{\gamma;4,1} = -0.07\hbar\omega_0$. Thus $g\mu_B|H_0| \leqslant 0.465\hbar\omega_0$. As it can be seen from Fig. 3 the magnitude of the spin current decays for large positive values of H_0. This happens because the distance between the Bloch sub-bands becomes large and thus the transition processes are less probable. For a certain negative value of H_0 the magnitude of the spin current has a maximum after which it starts to decrease and vanishes at some point $H_0^{(0)} < 0$. After this point and for $H_0 < H_0^{(0)}$ the spin current reverses its sign and its magnitude increases again. This behavior clearly demonstrates that the magnetic field can, without changing its direction, act in phase (*i.e.* destroy the spin transport) with the dissipative processes as well as out-of-phase (*i.e.* intensify the spin kinetics) with them. Mathematically it comes from the fact that in Eq. (32) for the transition rates the magnetic field H_0 and the imaginary part of the twice integrated bath correlation function $\text{Im}[Q(\tau)]$ enter the arguments of the same trigonometric functions. This is clarified by Eq. (37) appropriately rewritten below for the

case $\xi' = 1$, $\xi = 4$:

$$J_{\gamma;1,4} = \frac{2}{\hbar^2} \int_0^\infty d\tau e^{-\frac{L^2}{\hbar} Q_R(\tau)} \times$$

$$\times \cos\left[\left(\frac{\Delta\varepsilon_{\gamma;4,1}}{\hbar} + \omega_0 + \frac{2g\mu_B H_0}{\hbar}\right)\tau - \frac{L^2}{\hbar} Q_I(\tau)\right] \times \quad (62)$$

$$\times J_0\left[\frac{2FL}{\hbar\Omega} \sin\left(\frac{\Omega\tau}{2}\right)\right],$$

where $Q_R(\tau) \equiv \mathrm{Re}[Q(\tau)]$, $Q_I(\tau) \equiv \mathrm{Im}[Q(\tau)]$. The physical explanation of why in our system the magnetic field interacts only with the friction part of the dissipation and not with its noise part is rooted in the roles which the magnetic field and dissipation play for quantum coherence. On the one side quantum coherence in a dissipative system dies out due to the noise part of the Feynman-Vernon influence weight functional. On the other side, within the Feynman path integral formalism, we see that in our system a transverse in-plane uniform stationary magnetic field cannot produce the additional phase due to the integral of the vector potential along the Feynman paths (see Appendix C). Thus in our system quantum coherence is totally insensitive to the magnetic field and as a result cannot interact with the noise part of the Feynman-Vernon influence weight functional.

The mutual impact of the magnetic field and quantum dissipative processes on the spin current in the system is shown in Fig. 4 where the spin current is plotted versus the magnetic field, H_0, and the viscosity coefficient, η, plays a role of a parameter. Again for large positive values of H_0 the spin current vanishes. As expected, the spin current gets smaller if the dissipation in the system gets stronger. When the dissipation gets weaker ($\eta = 0.05$ and $\eta = 0.03$ curves) the oscillations of the spin current become observable. These oscillations are related to the interaction between the magnetic field and driving and can be described in terms of the photon emission/absorption processes[31] since changing H_0 is equivalent to changing the distance between the corresponding Bloch sub-bands.

The minima in Figs. 3 and 4 at negative values of H_0 appear as a result of a cooperative action of the orbit-orbit coupling, confinement, magnetic field, driving and dissipation. Its location changes when the strength of the driving and dissipation varies.

For completeness in Fig. 5 we also show the spin current as a contour plot using the variables H_0 and η.

VIII. CONCLUSION

In conclusion, we have studied averaged stationary quantum transport in a driven dissipative periodic quasi-one-dimensional (1D) system with Rashba spin-orbit interaction (RSOI) and placed in a transverse in-plane uniform stationary magnetic field. For the case of moderate-to-strong dissipation it has been shown that the averaged stationary charge transport is well suppressed as soon as it is restricted within the Bloch sub-bands grown out of the same Bloch band of the corresponding truly 1D problem without RSOI. However in the
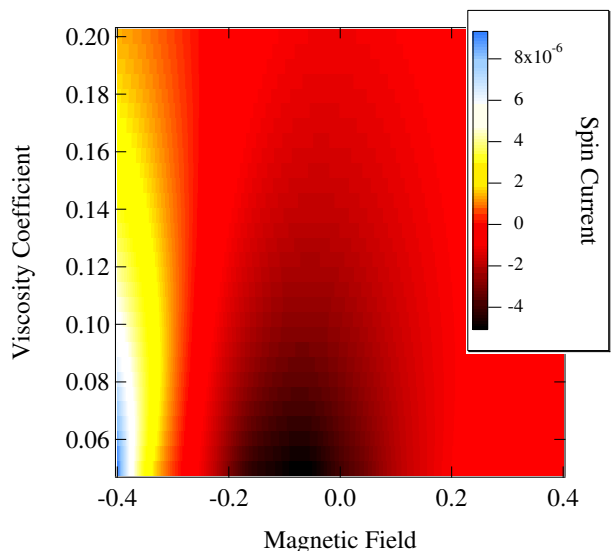

FIG. 5: (Color online) Contour plot of the non-equilibrium spin current, $\bar{J}_{n-e,S}^\infty$ [$L\omega_0$], as a function of the magnetic field, $g\mu_B H_0$ [$\hbar\omega_0$], and viscosity coefficient, η. The other parameters are as in Fig. 1.

same situation the averaged stationary spin transport is activated. The analytical expression for the spin current has been derived and its behavior as a function of the driving parameters, dissipation, spin-orbit interaction strength, orbit-orbit coupling strength and a transverse in-plane uniform stationary magnetic field has been analyzed. Our results on the spin transport in the system have been presented and thoroughly discussed. It has been found that the spin current as a function of the magnetic field shows a highly non-trivial dependence for different values of the dissipation and driving parameters. In particular, increasing the magnitude of the magnetic field does not always lead to a monotonous response in the magnitude of the spin current. The magnitude of the spin current can have maxima after which its dependence on the magnitude of the magnetic field changes to the opposite one. Moreover, the spin current as a function of the amplitude of an external longitudinal ac electric field has reversals of its direction when the system is placed in a finite transverse in-plane uniform stationary magnetic field. Also as a function of this magnetic field the spin current changes its direction at finite values of the amplitude of the ac electric field. Such behavior is undoubtedly related to a deep correlation between the dissipative processes and magnetic field effects in the system.

APPENDIX A: EIGEN-VALUE STRUCTURE OF THE COORDINATE OPERATOR IN A SUBSPACE GENERATED BY BLOCH STATES OF A FINITE NUMBER OF BANDS

In this appendix we consider a physical property with the corresponding quantum mechanical operator which when operating in the Hilbert space has a continuum spectrum and show how this continuum spectrum can turn into a discrete one under a certain restriction of the Hilbert space. To be specific we constrict the Hilbert space to a subspace using some of the Bloch states and consider how the coordinate operator changes its spectrum.

1. Introduction

In many problems of condensed matter theory one is not usually interested in the full band structure of a solid but rather in a few bands most important for the relevant physics of a system. For example in metals one or a few bands with an energy range containing the Fermi energy are most important since the main contributions to transport properties come almost only from those bands. Taking into account only a few Bloch bands leads to a restriction of the Hilbert space to a subspace which is then used to describe physical properties.

2. Truncation of the Hilbert space using Bloch states

Let $\mathcal{H}$ be the Hilbert space of all possible states and let us choose in this space the basis of Bloch's states $\{|l, k_{\mathrm{B}}\rangle\}$:

$$
\begin{aligned}
&\langle x|l, k_{\mathrm{B}}\rangle = e^{ik_{\mathrm{B}}x}u_{l,k_{\mathrm{B}}}(x), \\
&u_{l,k_{\mathrm{B}}}(x+L) = u_{l,k_{\mathrm{B}}}(x), \\
&\forall\, k_{\mathrm{B}} \in \text{B.Z.}, \, l = 1, 2, \ldots,
\end{aligned} \tag{A1}
$$

where L is the period of the Bloch amplitude $u_{l,k_{\mathrm{B}}}(x)$ and B.Z. stands for the first Brillouin zone.

Any vector $|\psi\rangle \in \mathcal{H}$ represents a linear combination

$$
|\psi\rangle = \sum_{l=1}^{\infty} \sum_{k_{\mathrm{B}} \in \text{B.Z.}} c_{l,k_{\mathrm{B}}}|l, k_{\mathrm{B}}\rangle. \tag{A2}
$$

Another basis $|\alpha\rangle$ is obtained using a transformation

$$
|\alpha\rangle = \hat{U}^{-1}|l, k_{\mathrm{B}}\rangle, \quad \forall\, k_{\mathrm{B}} \in \text{B.Z.}, \, l = 1, 2, \ldots, \tag{A3}
$$

where $\hat{U}$ is an arbitrary unitary operator.

Let us consider an operator $\hat{\mathcal{O}}$ corresponding to an observable $\mathcal{O}$. Its matrix representations in the two bases (A1) and (A3) are

$$
\begin{aligned}
&\mathcal{O}_{\mathrm{B}} = \langle l', k'_{\mathrm{B}}|\hat{\mathcal{O}}|l, k_{\mathrm{B}}\rangle, \\
&\forall\, k_{\mathrm{B}}, k'_{\mathrm{B}} \in \text{B.Z.}, \, l, l' = 1, 2, \ldots, \\
&\mathcal{O}_{\alpha} = \langle \alpha'|\hat{\mathcal{O}}|\alpha\rangle, \quad \forall\, \alpha, \alpha'.
\end{aligned} \tag{A4}
$$

The eigen-values $\{\lambda_i\}$ of the two matrices (A4) are the same and represent all possible values of the observable $\mathcal{O}$.

Now let us consider a subspace $\mathcal{S} \subset \mathcal{H}$ generated by Bloch's states corresponding to a finite number, N_{B}, of bands. A vector $|\psi^{\mathcal{S}}\rangle \in \mathcal{S}$ has the form:

$$
|\psi\rangle = \sum_{i=1}^{N_{\mathrm{B}}} \sum_{k_{\mathrm{B}} \in \text{B.Z.}} c_{l_i,k_{\mathrm{B}}}|l_i, k_{\mathrm{B}}\rangle. \tag{A5}
$$

In this subspace the operator $\hat{\mathcal{O}}$ has the matrix representation:

$$
\begin{aligned}
&\mathcal{O}_{\mathrm{B}}^{\mathcal{S}} = \langle l_{i'}, k'_{\mathrm{B}}|\hat{\mathcal{O}}|l_i, k_{\mathrm{B}}\rangle, \\
&\forall\, k_{\mathrm{B}}, k'_{\mathrm{B}} \in \text{B.Z.}, \, i, i' = 1, 2, \ldots N_{\mathrm{B}}.
\end{aligned} \tag{A6}
$$

Now the eigen-values $\{\lambda_n^{\mathcal{S}}\}$ of (A6) do not represent all possible values of the observable $\mathcal{O}$ but they only give approximate values of some of them. If the operator $\hat{\mathcal{O}}$ corresponds to a continuous observable with the spectrum from $-\infty$ to ∞, the eigen-values $\{\lambda_n^{\mathcal{S}}\}$ are some of the eigen-values $\{\lambda_i\}$, that is in this case $\{\lambda_n^{\mathcal{S}}\} \subset \{\lambda_i\}$.

A new basis $\{\alpha^{\mathcal{S}}\}$ of the subspace $\mathcal{S}$ is related to the Bloch one as:

$$
\begin{aligned}
&|\alpha^{\mathcal{S}}\rangle = \hat{U}_{\mathcal{S}}^{-1}|l_i, k_{\mathrm{B}}\rangle, \\
&\forall\, k_{\mathrm{B}} \in \text{B.Z.}, \, i = 1, 2, \ldots, N_{\mathrm{B}},
\end{aligned} \tag{A7}
$$

where now $\hat{U}_{\mathcal{S}}^{-1}$ is not an arbitrary unitary operator, but a unitary operator with the following property:

$$
\hat{U}_{\mathcal{S}}: \quad |v\rangle \in \mathcal{S} \; \Rightarrow \; \hat{U}_{\mathcal{S}}|v\rangle \in \mathcal{S}, \quad \forall\, |v\rangle \in \mathcal{S}. \tag{A8}
$$

In this case the matrix

$$
\mathcal{O}_{\alpha}^{\mathcal{S}} = \langle \alpha'^{\mathcal{S}}|\hat{\mathcal{O}}|\alpha^{\mathcal{S}}\rangle, \quad \forall\, \alpha'^{\mathcal{S}}, \alpha^{\mathcal{S}} \tag{A9}
$$

has the same set of eigen-values $\{\lambda_n^{\mathcal{S}}\}$ as the matrix $\mathcal{O}_{\mathrm{B}}^{\mathcal{S}}$ in (A6).

3. Example: coordinate

Let us specify the observable $\mathcal{O}$ from the preceding section to be particle's coordinate q with the corresponding operator denoted as $\hat{q}$. We consider the operator $\hat{q}$ in the subspace $\mathcal{S}$. Its matrix with respect to the Bloch basis is

$$
\begin{aligned}
&q_{\mathrm{B}}^{\mathcal{S}} = \langle l_{i'}, k'_{\mathrm{B}}|\hat{q}|l_i, k_{\mathrm{B}}\rangle, \\
&\forall\, k_{\mathrm{B}}, k'_{\mathrm{B}} \in \text{B.Z.}, \, i, i' = 1, 2, \ldots, N_{\mathrm{B}}.
\end{aligned} \tag{A10}
$$

Let us choose the translational operator as the unitary operator $\hat{U}_{\mathcal{S}}$ from the preceding section, that is

$$
\hat{U}_{\mathcal{S}}(a) = e^{\frac{i}{\hbar}a\hat{p}}. \tag{A11}
$$

It is obvious that for an arbitrary value of a the operator $\hat{U}_{\mathcal{S}}(a)$ does not satisfy the property (A8). However, in the case $a = L$ a Bloch state $|l, k_{\mathrm{B}}\rangle$ is translated into a Bloch state with the same l, k_{B} and thus (A8) is fulfilled. Hence, the matrix

$$
\begin{aligned}
&\tilde{q}_{\mathrm{B}}^{\mathcal{S}} = \langle l_{i'}, k'_{\mathrm{B}}|\hat{U}_{\mathcal{S}}(L)\hat{q}\hat{U}_{\mathcal{S}}^{-1}(L)|l_i, k_{\mathrm{B}}\rangle, \\
&\forall\, k_{\mathrm{B}}, k'_{\mathrm{B}} \in \text{B.Z.}, \, i, i' = 1, 2, \ldots, N_{\mathrm{B}}
\end{aligned} \tag{A12}
$$

has the same eigen-values as the matrix q_B^S in (A10). But due to the equality

$$\hat{U}_S(L)\hat{q}\hat{U}_S^{-1}(L) = \hat{q} + L \tag{A13}$$

the two matrices q_B^S and $\tilde{q}_B^S$ are related as follows:

$$\langle l_{i'}, k_B'|\hat{U}_S(L)\hat{q}\hat{U}_S^{-1}(L)|l_i, k_B\rangle =$$
$$= \langle l_{i'}, k_B'|\hat{q}|l_i, k_B\rangle + L\delta_{i',i}\delta_{k_B',k_B}, \tag{A14}$$
$$\forall\, k_B, k_B' \in B.Z.,\ i, i' = 1, 2, \ldots, N_B.$$

From (A14) it follows that the eigen-values of the matrix q_B^S are invariant under a shift equal to jL with j being an integer. That is for any $\lambda_k^S \in \{\lambda_n^S\}$ there exists $\lambda_m^S \in \{\lambda_n^S\}$ such that

$$\lambda_k^S = jL + \lambda_m^S. \tag{A15}$$

Let us denote through $\{d_k^S\}$ those eigen-values of q_B^S the distance between which is less than L,

$$|d_r^S - d_{r'}^S| < L, \quad \forall\, d_r^S, d_{r'}^S \in \{d_k^S\}, \tag{A16}$$

and which are in the zeroth elementary cell. Then each of the eigen-values $\{\lambda_n^S\}$ of the matrix q_B^S is obtained from its corresponding eigen-value $d_m \in \{d_k^S\}$ by a shift jL with a proper integer j. It means that each elementary cell contains the same number of eigen-values of the coordinate operator. Since the total number of the eigen-values $\{\lambda_n^S\}$ is equal to N_BN where N is the number of the elementary cells, we conclude that there are N_B eigen-values in each elementary cell. This gives us the final expression for the eigen-values of the matrix q_B^S (N is even to be definite):

$$\lambda_{m,j}^S = jL + d_m^S, m = 1, 2, \ldots, N_B,$$
$$j = -\frac{N}{2}+1, -\frac{N}{2}+2, \ldots, -1, 0, 1, \ldots, \frac{N}{2}-1, \frac{N}{2} \tag{A17}$$

and $N \to \infty$ afterwards.

APPENDIX B: POLARIZATION OF THE SPIN CURRENT ALONG THE x AND y DIRECTIONS

In Ref. 29 it was shown that in an isolated system (without any external magnetic field) the only non-vanishing spin polarization is along the confinement direction, that is along the z-axis. A natural question is then what is going on in an open driven system in a uniform stationary magnetic field applied along the z-axis. The external force (4) and bath Hamiltonian (6) couple to the longitudinal orbital degree of freedom, that is to the x-coordinate of our system. Because of the spin-orbit coupling the external force and bath affect the spin dynamics of electrons in the quasi-1D system. The magnetic field also influences the spin transport. Can it then happen that the longitudinal spin current acquires components polarized along the x and y axes? Below we show that the components $J_S^{x,y}(t)$ of the spin current (11) identically vanish.

1. Longitudinal spin current components $J_S^{x,y}(t)$ polarized along the x and y axes

The expressions for the spin currents

$$J_S^{x,y}(t) = \frac{d}{dt}\text{Tr}\big(\hat{\sigma}_{x,y}\hat{x}\hat{\rho}(t)\big) \tag{B1}$$

can easily be found using the σ-DVR basis $\{|\zeta, m, j, \sigma\rangle_{\gamma,j}\}$ introduced in Section III:

$$J_S^x(t) = 2\frac{d}{dt}\text{Tr}_B \sum_{\zeta,m,j} (mL + d_{\gamma;\zeta,j})\times$$
$$\times \text{Re}\big(_{\gamma,j}\langle\zeta, m, j, \sigma' = +1|\hat{W}(t)|\zeta, m, j, \sigma = -1\rangle_{\gamma,j}\big), \tag{B2}$$

$$J_S^y(t) = -2\frac{d}{dt}\text{Tr}_B \sum_{\zeta,m,j} (mL + d_{\gamma;\zeta,j})\times$$
$$\times \text{Im}\big(_{\gamma,j}\langle\zeta, m, j, \sigma' = +1|\hat{W}(t)|\zeta, m, j, \sigma = -1\rangle_{\gamma,j}\big), \tag{B3}$$

where we have explicitly written the trace over the bath degrees of freedom in order to work further with the σ-DVR matrix elements of the full statistical operator $\hat{W}(t)$.

2. Selection rules for the σ-DVR matrix elements of the full statistical operator

It turns out that the case of a harmonic confinement allows one to formulate selection rules for the σ-DVR matrix elements of the full statistical operator. These selection rules are very useful for understanding some of the properties of the spin transport.

To find the selection rules mentioned above let us decompose the Hamiltonian $\hat{H}$ in (2) into

$$\hat{H} = \hat{H}_0 + \hat{H}_{R-Z}, \tag{B4}$$

where

$$\hat{H}_0 = \frac{\hbar^2\hat{\mathbf{k}}^2}{2m} + \frac{m\omega_0^2\hat{z}^2}{2} + U(\hat{x})\left(1 + \gamma\frac{\hat{z}^2}{L^2}\right), \tag{B5}$$

$$\hat{H}_{R-Z} = -\frac{\hbar^2 k_{so}}{m}\big(\hat{\sigma}_x\hat{k}_z - \hat{\sigma}_z\hat{k}_x\big) - g\mu_B\hat{\sigma}_z H_0 =$$
$$= -\frac{\hbar^2 k_{so}}{m}\big(\hat{\sigma}_x\hat{k}_z - \hat{\sigma}_z\hat{k}_x'\big), \tag{B6}$$

and $\hat{k}_x' = \hat{k}_x - g\mu_B H_0 m/\hbar^2 k_{so}$. The full statistical operator has the form $\hat{W}(t) = \hat{U}(t, t_0)\hat{W}(t_0)\hat{U}^\dagger(t, t_0)$, where the evolution operator $\hat{U}(t, t_0)$ is given as the time-ordered exponent

$$\hat{U}(t, t_0) = \text{T}\exp\left[-\frac{i}{\hbar}\int_{t_0}^t dt'\hat{H}_{full}(t')\right] =$$
$$= \sum_{n=0}^\infty \left(-\frac{i}{\hbar}\right)^n\int_{t_0}^t dt_n\cdots\int_{t_0}^{t_2} dt_1\hat{H}_{full}(t_n)\cdots\hat{H}_{full}(t_1). \tag{B7}$$

Only the terms of $\hat{H}_{\mathrm{R-Z}}$ with odd powers contain the spin operators. These terms are linear in $\hat{\sigma}_x$ and $\hat{\sigma}_z$ or bilinear in these spin operators which is equivalent to being linear in $\hat{\sigma}_y$. Contributions to the matrix elements $_{\gamma,j}\langle\zeta,m,j,\sigma' = +1|\hat{W}(t)|\zeta,m,j,\sigma=-1\rangle_{\gamma,j}$ come from the first order terms in $\hat{\sigma}_x$. It is easy to see that these terms represent products of the factors $(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_k) + \hat{H}_{\mathrm{bath}})$ ordered chronologically (we mean the chronological ordering on the Keldysh contour[39] and thus operators from $\hat{U}^\dagger(t,t_0)$ are also included under this terminology), an odd number of factors $\hat{k}_z$ distributed in between $(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_k) + \hat{H}_{\mathrm{bath}})$ in all possible ways and a number (even or odd) of factors $\hat{k}'_x$ also distributed in between $(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_k) + \hat{H}_{\mathrm{bath}})$ in all possible ways. Such a structure is related to the fact that the Rashba-Zeeman Hamiltonian, $\hat{H}_{\mathrm{R-Z}}$, is bilinear in the operators $\hat{\sigma}_x$ and $\hat{k}_z$. To clarify our above statement we write down the third order term coming for example from $\hat{U}(t,t_0)$ (a similar result is obtained for products which are composed from different, $\hat{U}(t,t_0)$, $\hat{U}^\dagger(t,t_0)$ or $\hat{W}(t_0)$, parts of the full statistical operator):

$$
\begin{aligned}
\hat{H}_{\mathrm{full}}(t_3)\hat{H}_{\mathrm{full}}(t_2)\hat{H}_{\mathrm{full}}(t_1) &= \hat{H}_{\mathrm{R-Z}}^3 + \hat{H}_{\mathrm{R-Z}}^2\big(\hat{H}_0+ \\
&+ \hat{H}_{\mathrm{ext}}(t_1) + \hat{H}_{\mathrm{bath}}\big)+\hat{H}_{\mathrm{R-Z}}\big(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_2)+ \\
&+ \hat{H}_{\mathrm{bath}}\big)\hat{H}_{\mathrm{R-Z}}+\hat{H}_{\mathrm{R-Z}}\big(\hat{H}_0+\hat{H}_{\mathrm{ext}}(t_2) + \hat{H}_{\mathrm{bath}}\big)\times \\
&\times(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_1) + \hat{H}_{\mathrm{bath}})+(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_3)+ \\
&+ \hat{H}_{\mathrm{bath}})\hat{H}_{\mathrm{R-Z}}^2+(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_3)+\hat{H}_{\mathrm{bath}})\hat{H}_{\mathrm{R-Z}}\times \\
&\times(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_1) + \hat{H}_{\mathrm{bath}})+(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_3)+ \\
&+ \hat{H}_{\mathrm{bath}})(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_2) + \hat{H}_{\mathrm{bath}})\hat{H}_{\mathrm{R-Z}} + (\hat{H}_0+ \\
&+ \hat{H}_{\mathrm{ext}}(t_3)+\hat{H}_{\mathrm{bath}})(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_2) + \hat{H}_{\mathrm{bath}})(\hat{H}_0+ \\
&+ \hat{H}_{\mathrm{ext}}(t_1) + \hat{H}_{\mathrm{bath}}).
\end{aligned} \tag{B8}
$$

Since for a harmonic confinement all the factors $(\hat{H}_0 + \hat{H}_{\mathrm{ext}}(t_k) + \hat{H}_{\mathrm{bath}})$ and $\hat{k}'_x$ couple states with indices j and j' only of identical parity and the factors $\hat{k}_z^{2m+1}$ couple states with indices j and j' only of opposite parity, we conclude that the matrix elements $_{\gamma,j}\langle\zeta,m,j,\sigma' = +1|\hat{W}(t)|\zeta,m,j,\sigma = -1\rangle_{\gamma,j}$, being diagonal in j, are equal to zero:

$$
_{\gamma,j}\langle\zeta,m,j,\sigma' = +1|\hat{W}(t)|\zeta,m,j,\sigma = -1\rangle_{\gamma,j} = 0. \tag{B9}
$$

The selection rules (B9) represent a specific property of systems with a harmonic confinement. From (B9) one gets

$$
J_{\mathrm{S}}^{x,y}(t) = 0. \tag{B10}
$$

In spite of the fact that this result is only valid for the case of a harmonic confinement it is still general in two respects: 1) it is valid not only for the stationary state but for all times $t \geqslant t_0$; 2) the external force $F(t)$ is arbitrary.

APPENDIX C: EIGEN-ENERGIES AND EIGEN-SPINORS IN THE PRESENCE OF ORBIT-ORBIT COUPLING AND A UNIFORM STATIONARY MAGNETIC FIELD ALONG THE z-AXIS

In Ref. 29 periodic structures formed in a 2DEG with RSOI have been considered. However the influence of an external homogeneous stationary magnetic field on the energy spectrum has not been studied. Here we generalize the results of Ref. 29 to the case of a uniform stationary magnetic field applied along the z-axis. Afterwards we discuss the orbit-orbit coupling introduced in the main text in Eq. (2).

1. System's Hamiltonian and the general eigen-value equation

For an arbitrary potential $V(z)$ (not necessarily confinement) along the z-axis and a uniform stationary magnetic field applied along the z-axis (2DEG is in the $x - z$ plane) the Hamiltonian reads

$$
\hat{H} = \frac{\hbar^2\hat{\mathbf{k}}^2}{2m} + V(\hat{z}) - \frac{\hbar^2 k_{\mathrm{so}}}{m}\big(\hat{\sigma}_x\hat{k}_z - \hat{\sigma}_z\hat{k}_x\big)+ \\
+ U(\hat{x}) - g\mu_{\mathrm{B}}\hat{\sigma}_z H_0. \tag{C1}
$$

In Eq. (C1) H_0 is the z-component of the magnetic field $\mathbf{H} = (0,0,H_0)$ and the Landau gauge, $\mathbf{A} = (-H_0 y, 0, 0)$, has been chosen. Additionally we have used the fact that in a 2DEG $y = 0$. This choice effectively gives only the Zeeman term. The eigen-states of Hamiltonian (C1) are Bloch spinors with the spinorial amplitude given as (see Ref. 29)

$$
u_{l,k_{\mathrm{B}},\eta}(x;j,\sigma) = u_{l,k_{\mathrm{B}}+\sigma k_{\mathrm{so}}}(x)\theta_{l,k_{\mathrm{B}},\eta}(j,\sigma), \tag{C2}
$$

where $u_{l,k_{\mathrm{B}}}(x)$ is the Bloch amplitude of the corresponding truly 1D problem without the magnetic field and without RSOI, and $\theta_{l,k_{\mathrm{B}},\eta}(j,\sigma)$ is the eigen-spinor. This eigen-spinor is obtained from the solution of the eigen-value equation for Hamiltonian (C1):

$$
\begin{aligned}
\sum_{j',\sigma'}\Big\{&\delta_{j,j'}\delta_{\sigma,\sigma'}\Big[\varepsilon_l^{(0)}(k_{\mathrm{B}} + \sigma k_{\mathrm{so}}) - g\mu_{\mathrm{B}}\sigma H_0 + \varepsilon_j^z- \\
&- \frac{\hbar^2 k_{\mathrm{so}}^2}{2m}\Big]-\frac{\hbar^2 k_{\mathrm{so}}}{m}(1-\delta_{\sigma,\sigma'})\langle j|\hat{k}_z|j'\rangle\Big\}\theta_{l,k_{\mathrm{B}},\eta}(j',\sigma') = \\
&= \varepsilon_{l,\eta}(k_{\mathrm{B}})\theta_{l,k_{\mathrm{B}},\eta}(j,\sigma).
\end{aligned} \tag{C3}
$$

2. Harmonic confinement

The case of a harmonic confinement is characterized by the following matrix elements of the operator $\hat{k}_z$:

$$
\langle j|\hat{k}_z|j'\rangle = \pm\mathrm{i}\delta_{j,j'\pm1}\sqrt{\frac{(j + \frac{1}{2} \mp \frac{1}{2})m\omega_0}{2\hbar}}, \tag{C4}
$$

and eigen-energies ε_j^z:

$$
\varepsilon_j^z = \hbar\omega_0\Big(j + \frac{1}{2}\Big). \tag{C5}
$$

Therefore the only change in comparison with Ref. 29 is in the diagonal matrix elements of the Hamiltonian. Reproducing the same calculations as in Ref. 29, that is taking into account only the first two transverse modes ($j = 0, 1$, $\sigma = \pm 1$, $\eta = 1, 2, 3, 4$), one finds that the only change in the final results for the eigen-energies and eigen-spinors consists in replacing the function $\varepsilon_l^-(k_B)$ with

$$\varepsilon_l^-(k_B; H_0) = \frac{\varepsilon_l^{(0)}(k_B + k_{so}) - \varepsilon_l^{(0)}(k_B - k_{so})}{2} - \qquad \text{(C6)}$$
$$- g\mu_B H_0,$$

where we have explicitly shown the dependence on the z-component H_0 of the magnetic field. The expressions for the eigen-energies and eigen-spinors written through the function $\varepsilon_l^-(k_B)$ in Ref. 29 are unchanged. Also the structure (that is the zero and non-zero components) of the four dimensional eigen-spinors is the same.

The time reversal symmetry is now broken and as a result the symmetry relations between the eigen-energies and eigen-spinors hold only if one simultaneously changes the direction of the magnetic field. For the eigen-energies we have:

$$\varepsilon_{l,\eta=1}(k_B; H_0) = \varepsilon_{l,\eta=2}(-k_B; -H_0),$$
$$\varepsilon_{l,\eta=3}(k_B; H_0) = \varepsilon_{l,\eta=4}(-k_B; -H_0). \qquad \text{(C7)}$$

For the eigen-spinors the symmetry relations are written as:

$$\theta_{l,k_B,\eta=1}(j = \{0,1\}, \sigma = \{+1, -1\}; H_0) =$$
$$= \theta_{l,k_B,\eta=2}(j = \{0,1\}, \sigma = \{-1, +1\}; -H_0),$$
$$\theta_{l,k_B,\eta=3}(j = \{0,1\}, \sigma = \{-1, +1\}; H_0) = \qquad \text{(C8)}$$
$$= \theta_{l,k_B,\eta=4}(j = \{0,1\}, \sigma = \{+1, -1\}; -H_0),$$

where it is also taken into account that the z-projection of the spin operator (and as a result its eigen-values) changes its sign under the time reversal. The only non-vanishing polarization is again the one along the confinement (and also magnetic field) direction. The symmetry relations for its components are:

$$P_{l,\eta=1,4}^{(z)}(k_B; H_0) = -P_{l,\eta=2,3}^{(z)}(-k_B; -H_0). \qquad \text{(C9)}$$

Finally, we would like to note that since for the model with the first two transverse modes an operator even with respect to $\hat{z}$ is effectively diagonal, the results obtained above remain valid with the following change. The corresponding truly 1D problem without RSOI and transverse confinement has to be solved now not for the periodic potential $U(x)$ but for the periodic potential $U_{\gamma,j}(x) \equiv U(x)[1 + \gamma\hbar(j+1/2)/m\omega_0 L^2]$. Thus the solution of that truly 1D problem acquires a dependence on the transverse mode quantum number j through the periodic potential dependence on that quantum number: $\varepsilon_l^{(0)}(k_B) \to \varepsilon_{\gamma,j;l}^{(0)}(k_B)$, $|l, k_B\rangle \to |l, k_B\rangle_{\gamma,j}$. This does not change the structure (location of zero and non-zero entries) of the resulting 4×4 matrix which is thus diagonalized in the same manner as in Ref. 29. We label the eigen-energies and eigen-spinors obtained from this diagonalization as $\varepsilon_{\gamma;l,\eta}(k_B; H_0)$ and $\theta_{\gamma;l,k_B,\eta}(j, \sigma; H_0)$ to stress their dependence on the orbit-orbit coupling strength γ. The symmetry

relations (C7)-(C9) are, of course, unchanged.

APPENDIX D: BLOCH STATES IN THE DVR REPRESENTATION

The scalar products $_{\gamma,j}\langle\zeta, m|l, k_B\rangle_{\gamma,j}$ are nothing else than the Bloch states of the corresponding truly 1D problem without the magnetic field and without RSOI in the representation of the coordinate operator $\hat{x}$ operating on the subspace $\mathcal{S} \subset \mathcal{H}$, see Appendix A. Thus using the eigen-values (given in Appendix A) of this coordinate operator we have:

$$_{\gamma,j}\langle\zeta, m|l, k_B\rangle_{\gamma,j} = e^{ik_B(mL+d_{\gamma;\zeta,j})} u_{\gamma,j;l,k_B}^{DVR}(d_{\gamma;\zeta,j}), \quad \text{(D1)}$$

where we denoted the Bloch amplitude with the abbreviation DVR in order to stress that it originates from the discrete variable representation and differs from the one which originates from the continuum variable representation.

The difference of the squares of the absolute values of the hopping matrix elements, $|\Delta_{\gamma;1,4}^{\text{inter,f}}(m)|^2$ and $|\Delta_{\gamma;4,1}^{\text{inter,f}}(m)|^2$, in (52) can now be expressed in terms of the DVR Bloch amplitudes as

$$|\Delta_{\gamma;1,4}^{\text{inter,f}}(m)|^2 - |\Delta_{\gamma;4,1}^{\text{inter,f}}(m)|^2 =$$
$$= -\frac{\hbar^3 k_{so}^2 \omega_0}{m} \sum_{k_B, k_B'} \sin[(k_B - k_B')L]\text{Im}[F_{\gamma;k_B,k_B'}], \quad \text{(D2)}$$

where we have introduced a function $F_{\gamma;k_B,k_B'}$ defined as

$$F_{\gamma;k_B,k_B'} = u_{\gamma,0;1,k_B+k_{so}}^{DVR}(d_{\gamma;1,0}) u_{\gamma,1;1,k_B'-k_{so}}^{DVR}(d_{\gamma;1,1}) \times$$
$$\times \left[u_{\gamma,1;1,k_B-k_{so}}^{DVR}(d_{\gamma;1,1}) u_{\gamma,0;1,k_B'+k_{so}}^{DVR}(d_{\gamma;1,0})\right]^*. \quad \text{(D3)}$$

The function $F_{\gamma;k_B,k_B'}$ has two useful properties which directly follow from its definition (D3). The first property comes from the fact that $F_{\gamma;k_B,k_B'}$ is real if the Bloch amplitudes are real:

$$\text{Im}[u_{\gamma,j;1,k_B}^{DVR}(d_{\gamma;1,j})] = 0 \Rightarrow \text{Im}[F_{\gamma;k_B,k_B'}] = 0. \quad \text{(D4)}$$

The second property is that $F_{\gamma=0;k_B,k_B'}$ is an even function in both of its arguments. Indeed, when $\gamma = 0$, we have $u_{\gamma,j;l,k_B}(x) = u_{l,k_B}(x)$, $d_{\gamma;\zeta,j} = d_\zeta$, that is

$$F_{\gamma=0;k_B,k_B'} = u_{1,k_B+k_{so}}^{DVR}(d_1) u_{1,k_B'-k_{so}}^{DVR}(d_1) \times$$
$$\times \left[u_{1,k_B-k_{so}}^{DVR}(d_1) u_{1,k_B'+k_{so}}^{DVR}(d_1)\right]^*. \quad \text{(D5)}$$

One then finds from Eq. (D5) that $F_{\gamma=0;-k_B,k_B'} = F_{\gamma=0;k_B,k_B'}$ and $F_{\gamma=0;k_B,-k_B'} = F_{\gamma=0;k_B,k_B'}$. As a consequence, from this property one gets

$$\text{Im}[F_{\gamma=0;-k_B,k_B'}] = \text{Im}[F_{\gamma=0;k_B,k_B'}],$$
$$\text{Im}[F_{\gamma=0;k_B,-k_B'}] = \text{Im}[F_{\gamma=0;k_B,k_B'}], \quad \text{(D6)}$$

which means that $\text{Im}[F_{\gamma=0;k_B,k_B'}]$ is even in k_B and k_B'. The same is also valid for $\text{Re}[F_{\gamma=0;k_B,k_B'}]$.

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