

One-Dimensional Graphene Superlattices



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Julia Monika Amann

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Die Arbeit wurde angeleitet von: PD Dr. Jonathan Eroms

Prüfungsausschuss:

Vorsitzender: Prof. Dr. Klaus Richter

Erstgutachter: PD Dr. Jonathan Eroms

Zweitgutachter: Prof. Dr. Sergey Ganichev

Weiterer Prüfer: Prof. Dr. Ferdinand Kuemmeth

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1 Introduction

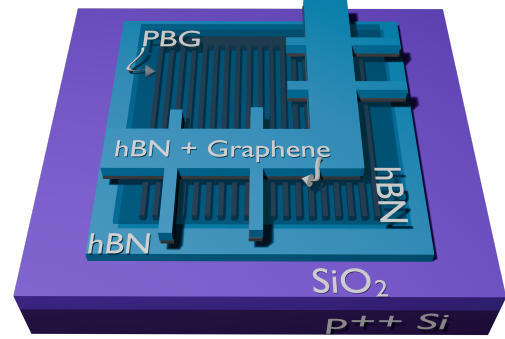
"There's Plenty of Room at the Bottom", the title of Richard Feynman's visionary 1959 talk, is widely regarded as the foundation of nanotechnology. He envisioned the manipulation of matter at the nanoscale, an idea that eventually led to the development of techniques such as electron beam lithography (EBL), which allows patterning of materials with nanometer precision [1]. Such capabilities, building on decades of progress in semiconductor technology and materials research, have enabled precise control over the electronic properties of materials at the nanoscale. In combination with the emergence of high-quality two-dimensional crystals, they allow the design of structures with tailored functionalities and open access to quantum phenomena far beyond those occurring in naturally available systems.

Within this framework, two-dimensional materials have emerged as a versatile platform for investigating quantum phenomena under well-controlled conditions. The most prominent and best-studied example of this class is graphene, a one-atom-thick crystal of carbon atoms arranged in a honeycomb lattice. Its charge carriers behave as massless Dirac fermions with a relativistic-like energy dispersion, resulting in high carrier mobility and unusual transport phenomena [2, 3].

A particularly powerful way to modify the electronic states in graphene is the application of spatially periodic potentials. For instance, by placing a series of EBL-defined, parallel gate stripes beneath the graphene layer, the electrostatic potential varies periodically along one direction, creating regions of higher and lower potential. Charge carriers moving in the graphene sheet therefore experience a modulation in a single spatial dimension, which is why such arrangements are referred to as one-dimensional (1D) superlattices. The ability to design and control such 1D superlattices opens the door to a rich playground of novel quantum phenomena. The electronic behaviour can be tuned in a highly controlled manner, enabling the exploration of regimes that were previously accessible only in theory.

Theoretical studies have predicted a wide range of striking effects: the emergence of additional (satellite) Dirac points, the formation of nearly flat bands, anisotropic transport, modified Landau quantisation, and altered quantum Hall sequences [4-6].

Fig. 1.1: Schematic of the sample design used in this thesis: L-shaped Hall bar of graphene encapsulated in hexagonal boron nitride (hBN) is positioned on a patterned bottom gate (PBG) of few-layer graphene, etched into stripes with a 50 nm period. The entire stack is placed on a SiO_2 layer that insulates it from the global silicon back gate. By applying a voltage to the PBG, a 1D superlattice modulation is induced to the graphene layer and its electronic band structure is altered.



However, these works have almost exclusively considered perfectly sinusoidal potentials with ideal symmetry. Experimental realisations, in contrast, inevitably generate more complex potential profiles. The precise form of the potential, together with its strength relative to the intrinsic energy scales of graphene, can drastically alter the electronic properties of the system. These considerations highlight the central question of this thesis, bridging the gap between theoretical predictions and experimental realisations: how do realistic, gate-defined 1D superlattice potentials affect the electronic band structure and transport properties of graphene, and how do the precise form and strength of the potential influence the experimentally observable phenomena? Using EBL-defined bottom gates, 1D superlattice potentials with a period of 50 nm are created in high-quality, hBN-encapsulated monolayer graphene devices (see Fig. 1.1). In combination with a global back gate, this design enables independent tuning of the carrier density and the superlattice modulation strength. By systematically varying these two parameters, the experiments presented in this work investigate how realistic gate-defined superlattice potentials reshape the band structure and modulate magnetotransport. The interplay with a perpendicular magnetic field reveals characteristic signatures such as the evolution of satellite Dirac points, modified Landau quantisation, and Weiss oscillations. L-shaped Hall bar geometries enable simultaneous measurements along and across the modulation axis, providing direct access to anisotropic transport.

The utilisation of bilayer graphene as opposed to monolayer graphene introduces an additional degree of tunability. While theoretical studies predict that 1D superlattice potentials can open and control band gaps, generate new Dirac points, and strongly influence magnetotransport [5, 6], experimental studies of bilayer superlattices remain scarce. Extending the experimental approach to bilayer graphene in this work enables a direct comparison with monolayer behaviour and provides experimental insight into regimes that had previously been explored only theoretically.

By systematically studying transport in these engineered superlattices, this thesis eluci-

dates how periodic potentials beyond the idealised sinusoidal case shape the electronic band structure and magnetotransport in both monolayer and bilayer graphene. In doing so, it establishes the experimental basis for understanding the role of potential strength and shape in determining fundamental quantum phenomena in graphene superlattices.

This thesis is organised into seven chapters:

Chapter **1** outlines the motivation and scientific context of this work.

Chapter **2** provides the fundamental theoretical groundwork for both monolayer and bilayer graphene, detailing their electronic band structures, transport properties, and the influence of a perpendicular magnetic field on phenomena such as the quantum Hall effect. This foundation is crucial for interpreting the data presented in later chapters.

Chapter **3** gives a detailed theoretical overview of superlattice effects in graphene. It explains how a periodic potential modifies the band structure, leading to new Dirac points and flattened bands. It also addresses how these effects manifest in transport at zero magnetic field and in magnetotransport through commensurability oscillations and altered quantum Hall plateaus, thus setting the stage for the experimental observations.

The experimental methods and fabrication processes used in this thesis are described in chapter **4**. It covers the stacking of van der Waals heterostructures, the advanced fabrication of patterned gates, and the setup of the low-temperature measurement system.

Chapter **5** then presents the core experimental results obtained from monolayer graphene superlattices. The chapter directly addresses the central question of how the exact form of the 1D potential reshapes the band structure and manifests in transport. It is organised into three distinct sections: first, a detailed analysis of the transport properties at zero magnetic field ($B = 0$), focusing on anisotropic transport and the emergence of satellite Dirac points as direct signatures of band structure engineering. Second, it explores commensurability oscillations in a perpendicular magnetic field. Finally, the chapter analyses the impact of potential asymmetry on the quantum Hall regime, showing pronounced electron–hole asymmetries in the quantum Hall plateau sequence.

Extending the investigation, chapter **6** discusses the experimental results for bilayer graphene superlattices. It highlights the high sensitivity of the bilayer band structure to superlattice modulation, particularly regarding its tunable band gap. Similar to chapter 5, this chapter includes an analysis of $B = 0$ transport properties, the characteristics of commensurability oscillations, and a thorough examination of magnetotransport phenomena in the quantum Hall regime.

Finally, Chapter **7** summarises the main findings and contributions of this thesis, consolidating the experimental and theoretical insights gained from the systematic study of 1D superlattices. It concludes with an outlook on potential future research directions.

2 Theory of Graphene

This chapter introduces the fundamental concepts of charge transport in graphene, focusing on monolayer (MLG) and bilayer (BLG) systems. It begins with the crystal structure and electronic properties of monolayer graphene, highlighting the behaviour of massless Dirac fermions, and then discusses bilayer graphene, where the band structure can be tuned by external fields. These properties make graphene an ideal system to study carrier dynamics under various conditions. The chapter further covers field-effect tuning and scattering mechanisms that influence transport, and concludes with an overview of magnetotransport phenomena, where magnetic fields modify carrier motion and give rise to characteristic quantum transport signatures. These concepts form the theoretical foundation for the later analysis of graphene superlattices, where the interplay of the intrinsic band structure with an external periodic potential and magnetic fields determines the transport behaviour studied in this thesis.

2.1 Crystallographic and Band Structure of Graphene

Crystal structure

Graphene is a two-dimensional material consisting of a single layer of carbon atoms arranged in a hexagonal lattice, which is schematically depicted in Fig. 2.1(a). Each carbon atom has six electrons, occupying the atomic orbitals $1s^2 2s^2 2p^2$ in its ground state. The $2s$, $2p_x$ and $2p_y$ orbitals combine to form three energetically favourable in-plane sp^2 hybrid orbitals, which arrange themselves at 120° angles, giving rise to the atomic honeycomb structure. The resulting strong covalent σ -bonds between neighbouring atoms have a binding energy of 4.3 eV, providing mechanical stability to the material. The remaining p_z orbitals of different atoms hybridise and form delocalised π -bonds, which are responsible for the unique electronic properties of graphene, discussed in the following sections. [7–10]

The binding energy between graphene layers is only about 70 meV, which makes it

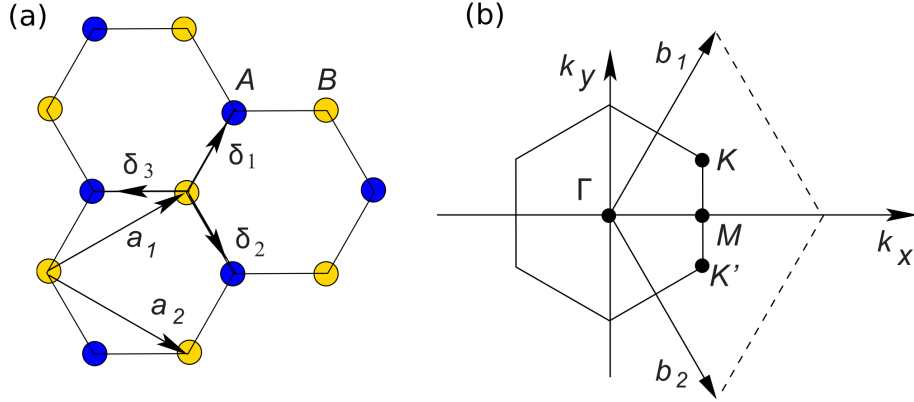


Fig. 2.1: (a) Lattice structure of graphene in real space and (b) in reciprocal space. The two sublattices A and B are marked in blue and yellow, respectively, along with the lattice vectors $\vec{a}_1$ and $\vec{a}_2$ and the σ -bonds between nearest neighbours. In (b), the corresponding reciprocal lattice vectors $\vec{b}_1$ and $\vec{b}_2$ are shown. Adapted from [9].

possible to separate the layers easily from each other. This was demonstrated in the Nobel Prize-winning experiment by Geim and Novoselov in 2004, who used Scotch tape to mechanically exfoliate few-layer and even monolayer graphene onto a substrate. [9, 11]

The honeycomb structure of graphene is not a Bravais lattice but can be described by two triangular Bravais sublattices (A and B), or equivalently, by a triangular lattice with a basis of two atoms. The unit cell is shown in Fig. 2.1 and is defined by the following lattice vectors

$$\vec{a}_1 = \frac{a}{2} (3, \sqrt{3}), \quad \vec{a}_2 = \frac{a}{2} (3, -\sqrt{3}), \quad (2.1)$$

with the corresponding unit vectors in the reciprocal lattice

$$\vec{b}_1 = \frac{2\pi}{3a} (1, \sqrt{3}), \quad \vec{b}_2 = \frac{2\pi}{3a} (1, -\sqrt{3}). \quad (2.2)$$

Here, $a = a_0/\sqrt{3} \approx 1.42 \text{ \AA}$ is the nearest-neighbour distance and a_0 is the lattice constant [9]. The two sublattices A and B are shown colour-coded in Fig. 2.1(a) in real space, and in (b) the reciprocal lattice is depicted with the inequivalent K and K' points. These points, located at the corners of the first Brillouin zone, are referred to as K and K' valleys or, more commonly, as **Dirac points**, which will be explained in the following section on the band structure of graphene. [7–9]

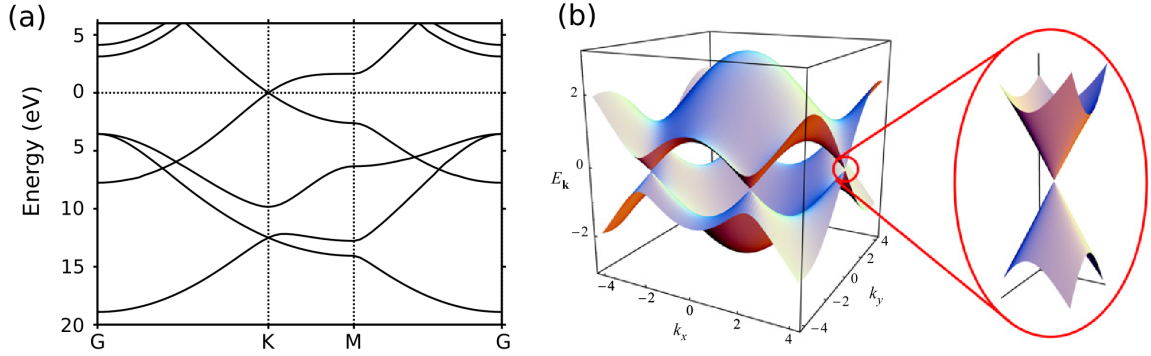


Fig. 2.2: (a) Band structure of monolayer graphene along high-symmetry directions, showing the linear dispersion near the Dirac points. (b) 3D representation of the energy dispersion in the first Brillouin zone, highlighting the six Dirac cones located at the corners of the hexagonal Brillouin zone. Adapted from [7].

Band structure

The electronic band structure can be calculated using a tight-binding approach, which was first applied by Wallace to a single layer of graphite in 1947 [12]. The unique band structure of graphene arises from its two interpenetrating sublattices, which lead to cosine-shaped energy bands that touch at the edges of the Brillouin zone. The Bloch functions φ_A and φ_B of the two sublattices A and B are constructed from the atomic p_z orbitals $X(\vec{r})$ [12]:

$$\varphi_A(\vec{k}, \vec{r}) = \frac{1}{\sqrt{N}} \sum_{\vec{R}_A} e^{i\vec{k} \cdot \vec{R}_A} X(\vec{r} - \vec{R}_A), \quad \varphi_B(\vec{k}, \vec{r}) = \frac{1}{\sqrt{N}} \sum_{\vec{R}_B} e^{i\vec{k} \cdot \vec{R}_B} X(\vec{r} - \vec{R}_B). \quad (2.3)$$

The total Bloch function $\Psi_{A,B}$ can then be expressed as a linear combination of the two contributions,

$$\Psi_{A,B}(\vec{k}, \vec{r}) = \varphi_A + \lambda \varphi_B, \quad (2.4)$$

where $\vec{k} = (k_x, k_y)$ is the wave vector, $\vec{r}$ the real space coordinate, N the number of unit cells, $\vec{R}_{A,B}$ denote the atomic positions on the respective sublattices, and $\lambda = \pm 1$ distinguishes between the bonding ($\lambda = -1$, valence band) and antibonding ($\lambda = +1$, conduction band) combinations of the sublattice Bloch functions [12]. Considering only nearest-neighbour hopping, which occurs between atoms on different sublattices, the tight-binding Hamiltonian can be written as [13]

$$H(\vec{k}) = \begin{pmatrix} 0 & tS(\vec{k}) \\ tS^*(\vec{k}) & 0 \end{pmatrix}, \quad (2.5)$$

with the nearest-neighbour hopping parameter $t \approx 2.8 \text{ eV}$ [9], which describes the energy cost for an electron to hop between adjacent atoms and

$$S(\vec{k}) = \sum_{\vec{\delta}} \exp(i\vec{k} \cdot \vec{\delta}) = 2 \exp\left(\frac{ik_x a}{2}\right) \cos\left(\frac{k_y a \sqrt{3}}{2}\right) + \exp(-ik_x a). \quad (2.6)$$

Here, $\vec{\delta}$ are the vectors to the three nearest neighbours (also shown in Fig. 2.1(a)), and $a = 1.42 \text{ \AA}$ is the nearest-neighbour distance.

By solving the corresponding Schrödinger equation $H\Psi_{A,B}(\vec{k}, \vec{r}) = E(\vec{k})\Psi_{A,B}(\vec{k}, \vec{r})$, the resulting energy spectrum is given by [9]

$$\begin{aligned} E(\vec{k}) &= \pm t |S(\vec{k})| = \pm t \sqrt{3 + f(\vec{k})} \\ &= \pm t \sqrt{3 + 2 \cos(\sqrt{3}k_y a) + 4 \cos\left(\frac{\sqrt{3}}{2}k_y a\right) \cos\left(\frac{3}{2}k_x a\right)}. \end{aligned} \quad (2.7)$$

The positive sign corresponds to the conduction (π^*) band and the negative to the valence (π) band. At the high-symmetry points $\vec{K} = (\frac{4\pi}{3a}, 0)$ and $\vec{K}' = (-\frac{4\pi}{3a}, 0)$, the function $S(\vec{k})$ vanishes, which results in the conduction and valence bands touching and is a defining feature of graphene's semimetallic character. [13]

Figure 2.2 shows the electronic band structure of monolayer graphene. In (a), the touching π bands at the K point are visible, while (b) shows the full energy spectrum. Expanding the Hamiltonian (2.5) near the Dirac points $\vec{K}$ and $\vec{K}'$ by setting $\vec{q} = \vec{k} - \vec{K}$ or $\vec{k} - \vec{K}'$, one obtains the low-energy effective Hamiltonian [13]:

$$H(\vec{q}) = \hbar v_F \begin{pmatrix} 0 & q_x \mp iq_y \\ q_x \pm iq_y & 0 \end{pmatrix}. \quad (2.8)$$

Here, $v_F = \frac{3a|t|}{2\hbar} \approx 1 \cdot 10^6 \text{ m/s}$ is the Fermi velocity at the Dirac points, and $\hbar = \frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ Js}$ is the reduced Planck constant. [13]

The corresponding linear dispersion relation near the Fermi level is

$$E(\vec{q}) = \pm \hbar v_F |\vec{q}|, \quad (2.9)$$

with $\vec{q}$ measured from the K and K' points [13]. This linear spectrum is visible in the zoom-in of Fig. 2.2(b) and defines the so-called Dirac points, also known as the charge neutrality points (CNP), located at $\vec{q} = \kappa \vec{K}$, where κ is the valley index. Because the conduction and valence bands touch at the Dirac points, graphene has no band gap, and its low-energy excitations are described by a massless Dirac Hamiltonian. This gives rise to relativistic-like charge carriers moving with a constant Fermi velocity v_F , which is why

they are commonly referred to as massless Dirac fermions. It should be noted, however, that although the band structure is gapless and the charge carriers are massless in the sense of a linear dispersion, the charge carriers in graphene possess a finite cyclotron mass that depends on the Fermi energy. In contrast, conventional two-dimensional electron gases exhibit a parabolic dispersion and massive charge carriers. [10, 14–16]

Density of states

The linear energy dispersion in graphene also has a significant impact on its density of states (DOS). Unlike conventional two-dimensional electron gases (2DEGs), which exhibit a constant DOS, graphene's DOS increases linearly with the absolute value of energy E and exhibits a square-root dependence on the carrier density n . Close to the Dirac points, the DOS per unit area can be approximated by

$$D(E) = \frac{2|E|}{\pi(\hbar v_F)^2}, \quad (2.10)$$

which includes a factor of 2 for spin degeneracy and another factor of 2 for the valley degeneracy at K and K' . [7]

Here, v_F is the Fermi velocity introduced earlier. The linear dependence of the DOS is a direct consequence of the conical band structure around the Dirac points. The resulting DOS is illustrated in Fig. 2.3, where (a) depicts the DOS per unit cell $D(E)$ as a function of energy E and (b) shows a zoom-in close to $E = 0$, illustrating the almost linear DOS near the Dirac point. [7, 17]

Effective mass and cyclotron effective mass

In conventional systems, the effective mass of charge carriers is typically defined through the curvature of the energy dispersion relation, $m^* = \hbar^2 / (\partial^2 E / \partial k^2)$. This approach is not applicable to graphene, which exhibits a linear (non-parabolic) energy dispersion near the Dirac points, so that carriers behave as massless Dirac fermions. Instead, their dynamics in magnetic fields are governed by the cyclotron mass, which plays the role of an effective mass in experiments. For graphene, the cyclotron mass can be expressed in several equivalent ways [3, 18–20]:

$$m_C^* = \frac{\hbar^2}{2\pi} \frac{\partial S(E)}{\partial E} = \frac{\hbar k_F}{v_F} = \frac{\hbar \sqrt{\pi n}}{v_F}, \quad (2.11)$$

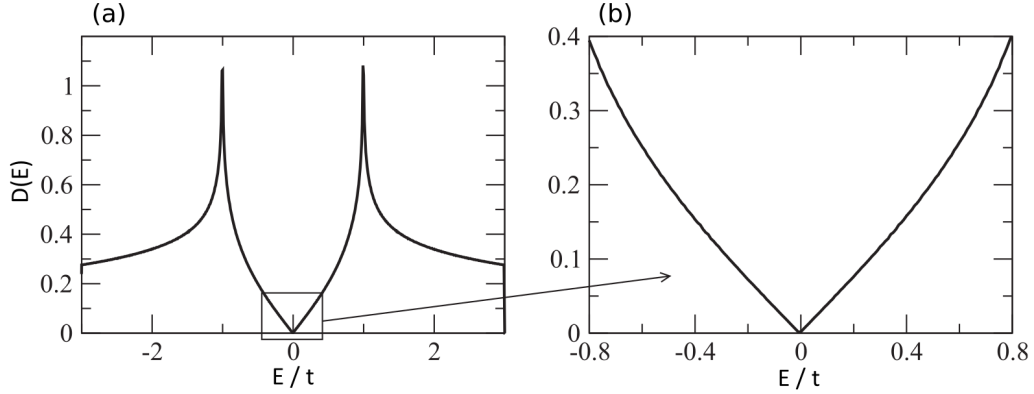


Fig. 2.3: (a) Calculated DOS per unit cell $D(E)$ as a function of energy E , shown in units of the hopping parameter t . The DOS vanishes linearly at the charge neutrality point, characteristic of the Dirac-like band structure of monolayer graphene. (b) Zoom-in of the region around $E = 0$, illustrating the linear energy dependence of the DOS near the Dirac point, where $D(E) \propto |E|$. This behaviour reflects the linear dispersion relation of massless Dirac fermions in the vicinity of the K and K' points. Adapted from [7].

where $S(E)$ is the area enclosed by the constant-energy contour in momentum space, k_F the Fermi wavevector, n the carrier density, and v_F the Fermi velocity. This equivalence arises from the linear dispersion and the circular Fermi surface near the Dirac points, and allows m_C^* to be directly extracted from magnetotransport measurements.

In the presence of sufficiently strong magnetic fields, Shubnikov–de Haas (SdH) oscillations can be observed. These quantum oscillations reveal a square-root dependence of the cyclotron mass on the carrier density, $m_C^* \propto \sqrt{n}$, confirming the linear relation $E \propto k$ and providing direct experimental evidence for the Dirac-like nature of charge carriers in graphene. The temperature dependence of the SdH oscillation amplitude allows extraction of the cyclotron mass using the Lifshitz-Kosevich formula

$$A(T) \propto \frac{T}{\sinh\left(\frac{2\pi^2 k_B T m_C^*}{\hbar e B}\right)}, \quad (2.12)$$

where $A(T)$ is the SdH amplitude, B the magnetic field, and m_C^* the cyclotron mass. [3] Experimental studies report values ranging from $\sim 0.02 m_e$ to $0.07 m_e$, with $m_e = 9.109 \times 10^{-31} \text{ kg}$ being the free electron mass. [3, 19, 21]

Figure 2.4 illustrates this process: Panel (a) presents the temperature dependence of the SdH amplitude, which can be fitted using Eq. (2.12) to extract the cyclotron mass. Panel (b) shows the extracted effective mass as a function of carrier density, following a $\propto \sqrt{n}$ dependence consistent with the linear energy dispersion near the Dirac points.

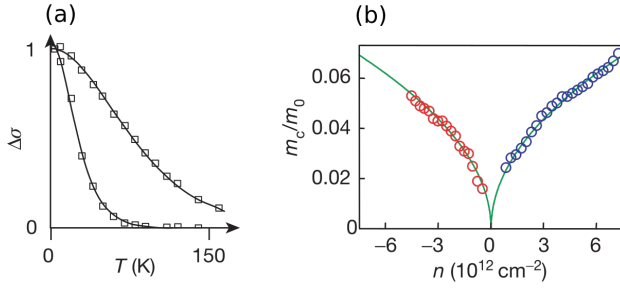


Fig. 2.4: (a) Temperature dependence of SdH oscillation amplitude $\Delta\sigma$, fitted with the Lifshitz–Kosevich formula (solid lines). (b) Extracted cyclotron mass m_C as a function of carrier density n . The data follow the relation $m_C \propto \sqrt{n}$, consistent with a linear dispersion near the Dirac point. Adapted from [3].

2.2 Crystallographic and Band Structure of Bilayer Graphene

Bilayer graphene can occur in several different crystallographic configurations, but this work focuses exclusively on Bernal (AB) bilayer graphene, which is the thermodynamically most stable form. In this arrangement, the two graphene layers are parallel, with the top layer shifted laterally relative to the bottom layer, as depicted in Fig. 2.5(a). This configuration places the atoms of the A sublattice of the top layer (A2) directly above the atoms of the B sublattice of the bottom layer (B1). In contrast, the atoms of the B sublattice of the top layer (B2) and those of the A sublattice of the bottom layer (A1) do not have a counterpart directly above or below.

This geometry facilitates strong interlayer hopping between the A2 and B1 sites, characterised by the interlayer coupling parameter $\gamma_1 = t_\perp \approx 0.4 \text{ eV}$, which is about an order of magnitude smaller than the nearest-neighbour intralayer hopping parameter $\gamma_0 = t \approx 2.7 \text{ eV}$. Hopping between other sublattice sites is negligible in the simplest model. The crystal structure, showing the sublattices A (black) and B (grey), is illustrated in Fig. 2.5(a). [22–24]

The electronic properties of AB-stacked bilayer graphene can be described using a four-band tight-binding Hamiltonian in the basis $(A1, B1, A2, B2)$ [23, 27, 28]

$$H(\vec{k}) = \begin{pmatrix} 0 & tS(\vec{k}) & 0 & 0 \\ tS^*(\vec{k}) & 0 & t_\perp & 0 \\ 0 & t_\perp & 0 & tS^*(\vec{k}) \\ 0 & 0 & tS(\vec{k}) & 0 \end{pmatrix}, \quad (2.13)$$

where $S(\vec{k})$ is defined as in Eq. (2.6). The intralayer couplings are described by the diagonal 2×2 blocks, while the interlayer coupling $t_\perp$ connects only the B1 and A2 sites.

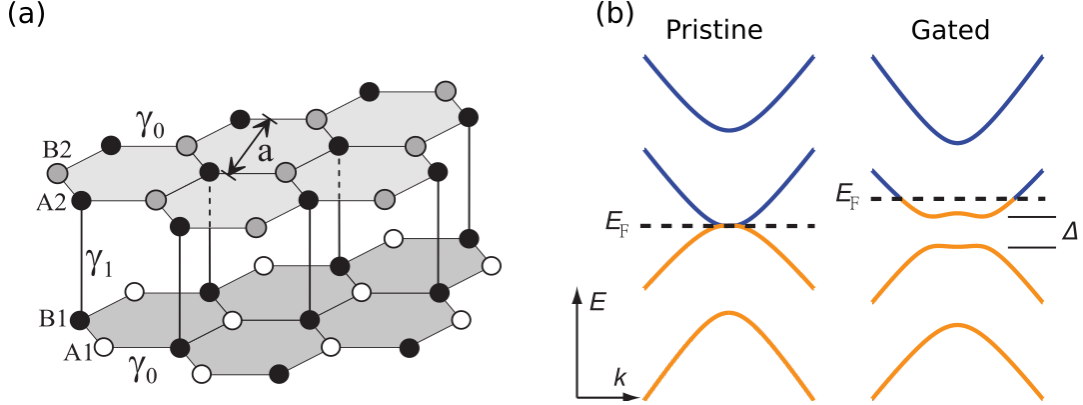


Fig. 2.5: (a) Crystal structure of BLG, where the two layers A and B are shifted relative to each other, and the A2 atom is located directly above the B1 atom. The interlayer coupling is characterised by the hopping parameter γ_1 , while the nearest-neighbour intralayer coupling is described by γ_0 . Adapted from [25] (b) Band structure of pristine BLG (left), showing a gapless parabolic dispersion near the K points. The right panel shows the case of gated BLG, where a tunable band gap opens due to the breaking of layer symmetry. Adapted from [26].

Diagonalising this Hamiltonian leads to four energy bands

$$E_i(\vec{k}) = \pm \frac{t_{\perp}}{2} \pm \sqrt{\frac{t_{\perp}^2}{4} + t^2 |S(\vec{k})|^2}, \quad (2.14)$$

where the two $\pm$ signs are independent. At low energies near the K points, only the A1 and B2 sublattices contribute significantly, leading to a simplified low-energy effective Hamiltonian [23, 27, 28]

$$H_K = -\frac{1}{2m^*} \begin{pmatrix} 0 & (\hat{p}_x - i\hat{p}_y)^2 \\ (\hat{p}_x + i\hat{p}_y)^2 & 0 \end{pmatrix}, \quad (2.15)$$

with $\hat{p}_{x,y} = -i\hbar \partial_{x,y}$. The corresponding energy spectrum is approximately parabolic

$$E_{1,2}(\vec{k}) \approx \pm \frac{t^2 |S(\vec{k})|^2}{t_{\perp}} \approx \pm \frac{\hbar^2 q^2}{2m^*}, \quad (2.16)$$

where the effective mass is given by $m^* = \frac{|t_{\perp}|}{2v_F^2} \approx 0.03 m_e$. [22]

This dispersion is shown in Fig. 2.5(b) for the pristine case (left). The spectrum features a gapless parabolic dispersion near the K points; the higher-energy bands $E_{3,4}$ are not shown.

In pristine AB-stacked bilayer graphene, inversion symmetry is present, as both layers are equivalent. However, applying an electric field perpendicular to the graphene layers

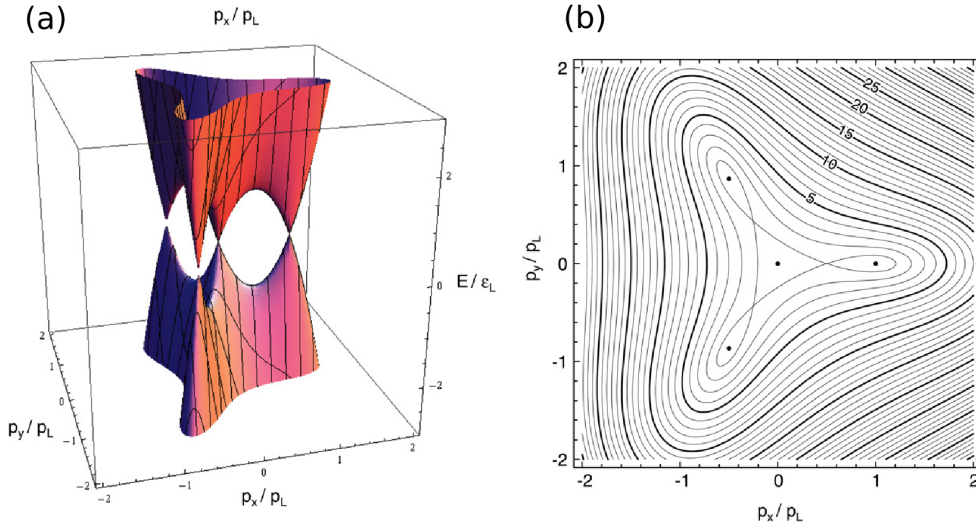


Fig. 2.6: (a) Three-dimensional representation of the low-energy band structure of bilayer graphene, illustrating the emergence of four Dirac points at low energies due to trigonal warping. (b) Top view showing iso-energy contours around the K point. The deviation from circular symmetry highlights the trigonal warping effect, which results from skew interlayer hopping. Here, p_L denotes the characteristic momentum scale and ϵ_L the corresponding energy scale associated with trigonal warping. Adapted from [31].

breaks this symmetry by introducing a potential difference V between the layers, which opens a tunable band gap, as illustrated in the right panel of Fig. 2.5(b). The modified Hamiltonian reads [22, 29]

$$H(\vec{k}) = \begin{pmatrix} \frac{V}{2} & tS(\vec{k}) & 0 & 0 \\ tS^*(\vec{k}) & \frac{V}{2} & t_{\perp} & 0 \\ 0 & t_{\perp} & -\frac{V}{2} & tS^*(\vec{k}) \\ 0 & 0 & tS(\vec{k}) & -\frac{V}{2} \end{pmatrix}, \quad (2.17)$$

which leads to eigenvalues

$$E_i^2(\vec{k}) = t^2 |S(\vec{k})|^2 + \frac{t_{\perp}^2}{2} + \frac{V^2}{4} \pm \sqrt{\frac{t_{\perp}^4}{4} + (t_{\perp}^2 + V^2)t^2 |S(\vec{k})|^2}. \quad (2.18)$$

Near the K and K' points, the dispersion forms a "Mexican hat" shape, given by [30]:

$$E(\vec{k}) \approx \pm \left(\frac{V}{2} - \frac{V\hbar^2 v_F^2}{t_{\perp}^2} k^2 + \frac{\hbar^4 v_F^4}{t_{\perp}^2 V} k^4 \right). \quad (2.19)$$

Trigonal warping

When long-range hopping processes are taken into account, the low-energy Hamiltonian of bilayer graphene is further modified. In particular, interlayer hopping between the B1 and B2 sublattices, described by the parameter $\gamma_3 \approx 0.3 \text{ eV}$, introduces additional linear terms. Although γ_3 is weaker than the dominant γ_1 coupling between A1 and B2, which primarily determines the splitting of the low-energy bands, it significantly affects the low-energy dispersion near the Dirac points. The Hamiltonian then reads as [30, 32, 33]

$$H = \begin{pmatrix} 0 & \frac{(\hat{p}_x - i\hat{p}_y)^2}{2m^*} + \frac{3\gamma_3 a}{\hbar}(\hat{p}_x + i\hat{p}_y) \\ \frac{(\hat{p}_x + i\hat{p}_y)^2}{2m^*} + \frac{3\gamma_3 a}{\hbar}(\hat{p}_x - i\hat{p}_y) & 0 \end{pmatrix}. \quad (2.20)$$

Assuming $|\gamma_3| \ll |\gamma_0|$, and expanding the dispersion near the K point, the energy spectrum becomes [30]

$$E(\vec{k}) \approx \sqrt{(3\gamma_3 a)^2 k^2 + \frac{3\gamma_3 a \hbar^2 k^3}{m^*} \cos(3\phi_{\vec{k}}) + \left(\frac{\hbar^2 k^2}{2m^*}\right)^2}, \quad (2.21)$$

where $k = |\vec{k}|$ and $\phi_{\vec{k}}$ is the polar angle of the wave vector $\vec{k}$.

The first term under the square root (proportional to k^2) describes the isotropic parabolic dispersion, while the second term (proportional to $k^3 \cos(3\phi_{\vec{k}})$) introduces an angular anisotropy known as *trigonal warping*. This effect distorts the constant-energy contours near the charge neutrality point, leading to a splitting of the central Dirac point into four separate band touching points. These appear at $\vec{k} = 0$ and at finite $|\vec{k}| \approx 6m^* \gamma_3 a / \hbar^2$ along directions where $\cos(3\phi_{\vec{k}}) = \pm 1$, i.e. along the crystallographic axes.

As illustrated in Fig. 2.6(b), the distortion of the equi-energy contours results in three satellite Dirac points surrounding the original central point. These multiple touching points merge into a single parabolic band at higher energies, where the linear terms become negligible, as shown in Fig. 2.6(a). The corresponding crossover occurs at the characteristic Lifshitz energy ϵ_L , below which the parabolic band touching is reconstructed into four Dirac cones, leading to a change in the topology of the Fermi surface (Lifshitz transition). The associated momentum scale is p_L , which defines the distance of the satellite Dirac points from the K point. [9, 30, 31, 34]

Density of states and effective mass of BLG

Bilayer graphene exhibits a fundamentally different DOS compared to monolayer graphene due to its quadratic low-energy band dispersion, which results in a constant DOS at low

energies. This difference arises from the interlayer coupling, which modifies the electronic structure and leads to massive chiral quasiparticles. Consequently, the DOS in bilayer graphene does not vanish at the Dirac point, unlike in monolayer graphene, but remains finite instead. This has significant implications for its electronic transport properties, including screening and conductivity. [31, 35]

The effective mass of charge carriers in bilayer graphene is given by

$$m^* = \frac{\gamma_1}{2v_F^2}, \quad (2.22)$$

which leads to an effective mass of approximately $m^* \approx 0.03m_e$, where m_e is the free electron mass. [21]

This expression is valid in the high-energy regime, where the dispersion can be approximated as parabolic. When a perpendicular electric field is applied, opening a band gap, the effective mass becomes asymmetric for electrons and holes. Furthermore, at low energies, trigonal warping distorts the Fermi surface, leading to a momentum-direction-dependent m^* . Zou et al. [26] experimentally confirmed this asymmetry by analyzing the temperature dependence of the SdH oscillations (see Eq. (2.12)), showing that the effective mass of electrons is 20–30 % smaller than that of holes. [26, 35]

2.3 Pseudospin and Klein Tunnelling

Each valley in monolayer graphene is associated with a distinct wave function, allowing the Dirac cones to be treated separately. The eigenfunctions are two-component spinors, where the term “spinor” refers to the sublattice degree of freedom rather than the intrinsic spin of the electron. This degree of freedom is called **pseudospin** and describes the occupation of the A and B sublattices in graphene. The pseudospin is intrinsically linked to the chirality of the carriers, defined as the projection of the propagation direction onto the pseudospin orientation, and plays a crucial role in graphene’s transport properties. [30, 37]

The momentum-space pseudospinor eigenfunctions near the K points in monolayer graphene can be obtained by solving the Dirac equation (Eq. 2.7) with the appropriate Hamiltonian (Eq. 2.5). Here, $\theta_q = \arctan(q_y/q_x)$ denotes the polar angle of the wave vector $\vec{q}$ in momentum space. The corresponding eigenfunctions are given by [9, 30]

$$\Psi_K(\vec{q}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\theta_q/2} \\ \pm e^{i\theta_q/2} \end{pmatrix}, \quad \Psi_{K'}(\vec{q}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta_q/2} \\ \pm e^{-i\theta_q/2} \end{pmatrix}, \quad (2.23)$$

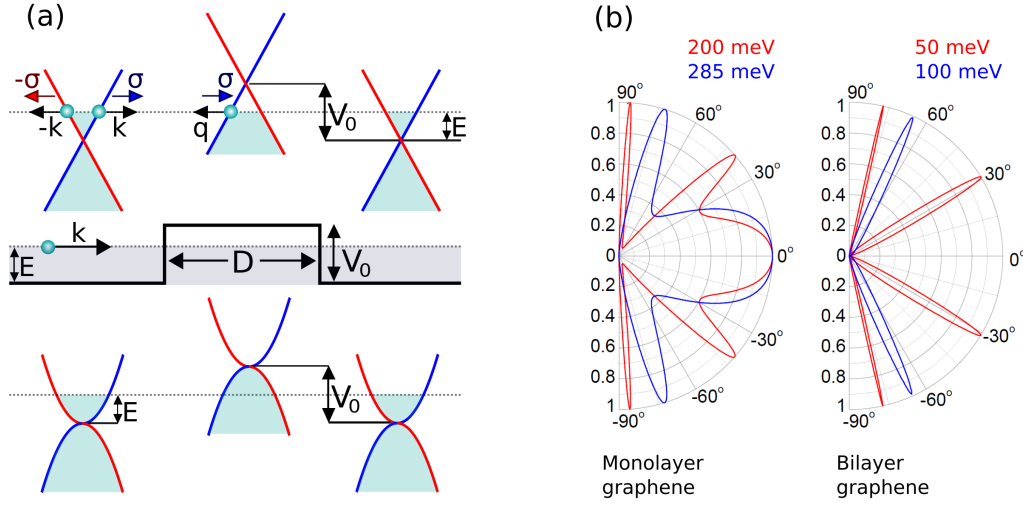


Fig. 2.7: (a) Band structure of MLG and BLG along a potential step of height V_0 and width D . Red and blue indicate the propagation direction, which is coupled to the pseudospin σ . The grey dotted line marks the Fermi level. In MLG, perfect Klein tunnelling occurs at normal incidence (0°), where pseudospin conservation forbids backscattering: an incident electron wave function transforms into a hole wave function within the barrier region without reflection. (b) Transmission probabilities for MLG and BLG across the potential step. In BLG, the pseudospin winds differently, preventing perfect alignment at normal incidence and thereby suppressing transmission. However, finite-angle transmission is possible due to partial pseudospin matching. (b) adapted from [36].

with $\pm$ corresponding to the conduction (valence) bands [14]. Rotating the wave vector by 2π changes the wave function's phase by π , which is characteristic of spinors. This phase is referred to as the **Berry phase**, and it is fundamental to graphene's unique electronic properties, influencing the behaviour of charge carriers in external fields and in topological phenomena such as the quantum Hall effect. [7, 9, 38]

Figure 2.7(a) shows the energy dispersion for monolayer (top) and bilayer (bottom) graphene and displays the coupling of the pseudospin to the propagation direction (red and blue). Further, the case of a potential step is shown, which will be described in the following.

Notably, in MLG at a potential step, there is no backscattering for an incident angle of 0° , resulting in perfect **Klein tunnelling**. This phenomenon is a relativistic effect: a sufficiently high potential step, which is above the Fermi energy, creates an energy region inside the barrier where the quasiparticles are holes rather than electrons (depicted by the grey dotted line in Fig. 2.7(a)). As a result, the conservation of pseudospin prevents backscattering and allows the electron wave function to transition into a hole wave function at a potential barrier with opposite velocity to the electron. Perfect Klein tunnelling occurs specifically for normal incidence (0°), where the pseudospin aligns with

the direction of propagation. For oblique incidence or more complex barrier shapes, the tunnelling probability depends on additional factors such as the angle of incidence and the barrier parameters, and perfect transmission may no longer occur. This is shown in the left panel of Fig. 2.7(b). [36, 37, 39]

In bilayer graphene, Klein tunnelling exhibits distinct properties due to the parabolic band structure and the associated pseudospin texture, which differs from monolayer graphene: while in MLG the pseudospin is locked to the propagation direction, in BLG the pseudospin rotates twice as fast with momentum, leading to a Berry phase of 2π and preventing perfect alignment at normal incidence. Unlike monolayer graphene, where tunnelling is governed by linear dispersion and chirality, bilayer graphene shows angle-dependent tunnelling probabilities that are significantly suppressed for normal incidence. This angle dependence is much sharper in BLG than in MLG, and the pronounced features at certain oblique angles can give rise to Fabry-Pérot-like resonances for particular barrier configurations. The difference in transmission probabilities between monolayer and bilayer graphene is shown in Fig. 2.7(b). This is because the pseudospin in bilayer graphene rotates in a way that prevents perfect alignment at normal incidence, thereby inhibiting perfect Klein tunnelling. However, transmission can be enhanced for certain oblique angles and barrier configurations, reflecting the interplay between the pseudospin orientation and the applied potential. [36, 40]

The low-energy pseudospinor eigenfunctions near the K and K' points in bilayer graphene are given by

$$\Psi_K(\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i2\theta_k} \\ \pm e^{i2\theta_k} \end{pmatrix}, \quad \Psi_{K'}(\vec{k}) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i2\theta_k} \\ \pm e^{-i2\theta_k} \end{pmatrix}, \quad (2.24)$$

where $\theta_k = \arctan(k_y/k_x)$ denotes the polar angle of the wave vector $\vec{k}$ in momentum space [22]. These eigenfunctions determine the pseudospin orientation, which in turn governs the tunnelling properties of charge carriers.

Compared to monolayer graphene, the factor of 2 in the phase indicates that the pseudospin rotates twice as fast with momentum. As a result, at normal incidence the pseudospin of incoming and transmitted states cannot perfectly align, leading to a strong suppression of Klein tunnelling. For oblique incidence or particular barrier configurations, partial alignment can occur, enhancing the transmission probability.

In bilayer graphene, the Berry phase also plays a crucial role in determining the behaviour of charge carriers. Due to the doubled winding of the pseudospin around the Dirac point, the Berry phase is 2π , in contrast to π in monolayer graphene, which significantly

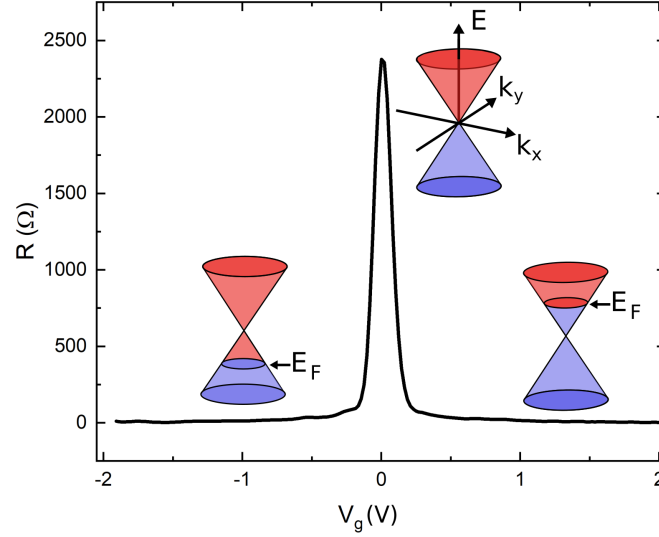


Fig. 2.8: Measured longitudinal resistance of a monolayer graphene device from this thesis as a function of applied gate voltage V_g . The pronounced resistance peak at $V_g = 0$ V indicates the Dirac point (or charge neutrality point), where the density of states vanishes and no free charge carriers are available for conduction. The insets illustrate the shift of the Fermi level within the Dirac cone as the gate voltage is varied, tuning the system from hole conduction (left) to electron conduction (right).

influences transport properties. [40, 41]

Recent experimental work by Du et al. demonstrates that this behaviour can be tuned by electrostatic gating in gapped bilayer graphene [42]. By employing Fabry-Pérot interference measurements, they show that the Berry phase can be continuously tuned from 2π to approximately 0.68π . This tuning leads to a substantial increase in the transmission probability at normal incidence, effectively enabling a transition from suppressed to near-perfect Klein tunnelling.

2.4 Ambipolar Field Effect

The application of a gate voltage enables precise control over both the type and concentration of charge carriers in graphene. This property is fundamental to the operation of field-effect transistors (FETs), which are essential components in nearly all modern electronic devices [43]. Owing to graphene's gapless electronic structure, the Fermi level can be continuously adjusted from the electron regime to the hole regime. [44]

The carrier density can be quantitatively determined using a parallel plate capacitor model, offering a direct relationship between the applied gate voltage and the resulting carrier concentration:

$$n = \frac{\epsilon_0 \epsilon_r}{ed} V = C_g |V_g - V_{CNP}| \quad (2.25)$$

Here, the graphene transport layer and the gate electrode can be considered as a plate capacitor, separated by a dielectric such as SiO₂ or hBN with a thickness of d . $\epsilon_0 = 8.85 \cdot 10^{-12} \frac{As}{Vm}$ and ϵ_r are the dielectric constants of the vacuum and of the insulating material used in the sample. Due to intrinsic doping, the absolute value of the difference between the applied voltage V_g and V_{CNP} , the voltage at the charge neutrality point, must be taken into account. The sign of this difference indicates the type of carrier involved in the transport, with hole transport for $V_g - V_{CNP} < 0$ and electron transport for $V_g - V_{CNP} > 0$. Figure 2.8 shows the measured resistance R plotted as a function of the applied gate voltage V_g . The charge neutrality point corresponds to the point of maximum resistance, where the carrier density is minimal and transport is strongly suppressed. At negative (positive) gate voltages, the charge carrier transport is governed by holes (electrons) and the Fermi level is shifted into the valence (conduction) band. [11, 45]

For heterostructures consisting of various insulating layers, the total capacitance is given by:

$$C_{layers} = \left(\sum_{i=1}^N \frac{1}{C_i} \right) \quad (2.26)$$

In the case of thin insulating layers on the single-digit nanometre scale, a quantum correction to the classical capacitor model becomes important. This correction arises because the DOS in graphene is small near the Dirac point, meaning that adding charge carriers requires an increase in the Fermi energy. The quantum capacitance C_Q acts as an additional capacitance in series with the geometric capacitance, thereby reducing the total capacitance of the system. The quantum correction depends primarily on the DOS, which varies strongly with energy in graphene [46]:

$$C_Q = e^2 D(E_F) \quad (2.27)$$

The resulting expression for the charge carrier density is

$$n = n_{class} + \text{sgn}(n_{class}) n_Q \left(1 - \sqrt{1 + 2 \frac{|n_{class}|}{n_Q}} \right) + \text{sgn}(n_0) \sqrt{2 n_Q |n_0|}, \quad (2.28)$$

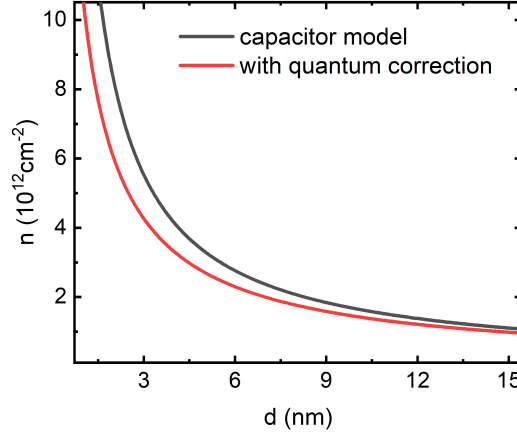


Fig. 2.9: Calculated charge carrier density as a function of dielectric thickness using the classical parallel-plate capacitor model (grey) and including quantum capacitance corrections (red), for an insulator with relative permittivity $\epsilon_r = 3$ and an applied gate voltage of $V_g = 1 \text{ V}$. At a dielectric thickness of 6 nm, the carrier density calculated with quantum capacitance is approximately 20% lower than the classical value, highlighting the relevance of quantum corrections at small thicknesses.

with n_{class} being the classical contribution calculated by the plate capacitor model, n_0 stemming from doping, and n_Q the quantum capacitance contribution [46]

$$n_Q = \frac{\pi}{2} \left(\frac{\hbar v_F C_g}{e^2} \right). \quad (2.29)$$

Figure 2.9 compares the calculation of the charge carrier density n as a function of the dielectric thickness d for a material with $\epsilon_r = 3$, such as hBN, using both the classical capacitor model and a model that incorporates quantum corrections. For dielectric thicknesses in the single-digit nanometer range, the quantum-corrected curve (red) deviates significantly, predicting a notably lower charge carrier density. For instance, at a thickness of 6 nm, as employed in this thesis, the quantum-corrected value is approximately 20% lower than the classical prediction.

Drude model

To explain the unique transport properties of graphene, it is essential to first consider the fundamental transport model for electrical conductivity in metals, as described by the Drude model. Developed in 1900 by P. Drude [47], this theory provides a basis for understanding electrical conduction and successfully accounts for the linear relationship between current and electric field $\vec{E}$, known as Ohm's law. The Drude model is based

on the assumption that the motion of charge carriers can be described using the kinetic theory of gases. In this framework, electrons are treated as free particles moving at thermal velocities and undergoing frequent scattering events with impurities or lattice vibrations, which limit their mean free path [20, 48–51]:

$$m \frac{d\vec{v}}{dt} = -e\vec{E} - \frac{m\vec{v}_d}{\tau} \quad (2.30)$$

Key parameters in this model include the drift velocity $\vec{v}_d$ and the mean momentum relaxation time (or scattering time) τ , which are essential for formulating the classical equations of motion of the charge carriers [49, 52].

For $\frac{d\vec{v}}{dt} = 0$, i.e. in the stationary case, the drift velocity becomes

$$\vec{v}_d = -\frac{e\tau}{m}\vec{E} = -\mu\vec{E}, \quad (2.31)$$

where the charge carrier mobility is given by $\mu = \frac{e\tau}{m}$, the current density $\vec{j} = -en\vec{v}_d = \frac{ne^2\tau}{m}\vec{E} = ne\mu\vec{E}$, and n is the charge carrier density. The electrical conductivity then follows as [49]:

$$\sigma = \frac{j}{E} = \frac{ne^2\tau}{m} = ne\mu. \quad (2.32)$$

The mean free path l_{mfp} is an important length scale for transport measurements. It is a measure of the average distance a charge carrier can travel before it is scattered. It can be expressed as:

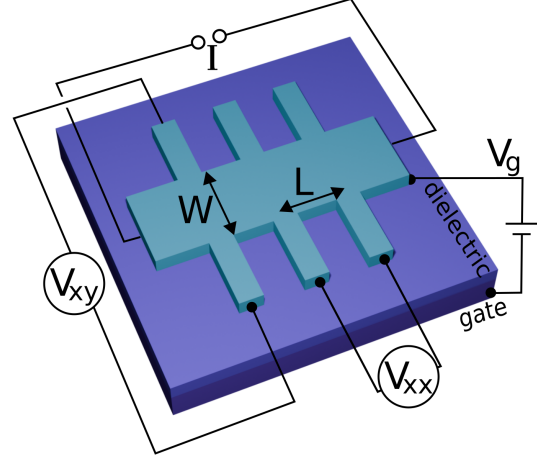
$$l_{mfp} = \frac{\hbar}{e}\mu\sqrt{\pi n} \quad (2.33)$$

Electrical transport

Figure 2.10 illustrates a typical transport measurement setup, the so-called Hall bar geometry. This configuration enables the measurement of voltage drops both along and perpendicular to the direction of current flow. Transport experiments are a versatile tool for probing fundamental material properties — in the case of graphene, for instance, the behaviour of Dirac fermions and the tunability of its intrinsic parameters.

In this geometry, a current I is applied between the outer contacts, and the longitudinal (V_{xx}) and transverse (V_{xy}) voltages are recorded. Here, W and L denote the width and length between the corresponding voltage probes, respectively. The resulting resistances

Fig. 2.10: Schematic of the transport measurement setup: A Hall bar device with a conducting transport layer on a substrate, separated from a global back gate (biased with a voltage V_g) by an insulating dielectric layer. The applied current I is indicated, along with the voltage probes for measuring the longitudinal voltage V_{xx} and the transverse (Hall) voltage V_{xy} . The geometry parameters width W and length L are marked for calculating the sheet conductivity.



are defined by Ohm's law [53]:

$$R_{xx,xy} = \frac{V_{xx,xy}}{I}. \quad (2.34)$$

For a two-dimensional system like graphene, the resistivities can be obtained by including the geometry factor W/L of the Hall bar [48, 54]:

$$\rho_{xx} = R_{xx} \frac{W}{L}, \quad \rho_{xy} = R_{xy}, \quad (2.35)$$

which represent components of the resistivity tensor:

$$\boldsymbol{\rho} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix}. \quad (2.36)$$

The conductivity tensor $\boldsymbol{\sigma}$ is the inverse of the resistivity tensor, $\boldsymbol{\sigma} = \boldsymbol{\rho}^{-1}$, with the components given by [9]:

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}, \quad \sigma_{xy} = -\frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}. \quad (2.37)$$

The current density $\vec{j}$ can then be expressed in terms of the applied electric field $\vec{E}$ as [48]:

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & -\sigma_{xy} \\ \sigma_{xy} & \sigma_{xx} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}. \quad (2.38)$$

2.5 Scattering Processes

In perfect graphene, electrons can propagate nearly undisturbed due to the absence of significant momentum-relaxing electron-electron interactions, although electron-electron scattering can be significant at elevated temperatures. Deviations from the ideal lattice, such as lattice defects or random strain variations induced from the substrate, are considered the most significant scatterers in high-mobility graphene. Sources of disorder include substrate irregularities, residues from fabrication processes (e.g. polymers or resists), intrinsic defects, or ripples in the graphene sheet [50, 55]. These scattering mechanisms can be described and combined using Matthiessen's rule [48, 54]:

$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \dots = \sum_i \frac{1}{\tau_i}. \quad (2.39)$$

At elevated temperatures, the mobility of charge carriers in high-quality, encapsulated graphene is primarily limited by scattering with acoustic phonons. In contrast, scattering at lattice defects or charged impurities typically involves short-range interactions, which can, depending on the type of defect, induce localised states near the charge neutrality point. Such scattering processes are elastic, meaning that the charge carriers' direction changes while their energy remains approximately constant. This applies both to short-range defects and to long-range scatterers such as charged impurities. [11, 13, 55]

Phonon scattering in graphene is generally inelastic, as electrons exchange energy with the lattice through the creation or annihilation of phonons. For acoustic phonons, these energy changes are typically small, whereas optical phonons involve larger transfers. A distinction is made between normal and Umklapp processes. [48, 49]

In normal processes, momentum is conserved within the Brillouin zone, so that scattering is less effective in relaxing current. In Umklapp processes, the final electron momentum lies outside the first Brillouin zone and is folded back by a reciprocal lattice vector G , transferring momentum to the lattice and thereby contributing to resistivity, especially at elevated temperatures (Fig. 2.11). [17, 56–58]

Charged impurities create a long-range Coulomb potential that can significantly reduce carrier mobility, particularly near the Dirac point due to weak screening [14, 59]. Remote phonons from substrates such as SiO_2 can also couple to the graphene sheet and affect transport at elevated temperatures [60, 61]. Finally, optical phonons become relevant at high temperatures or under high bias, providing additional inelastic scattering channels. [61]

In ultrapure graphene samples or at very short length scales (typically in the micrometre range or below), electronic transport can be **ballistic**. In this regime, electrons move

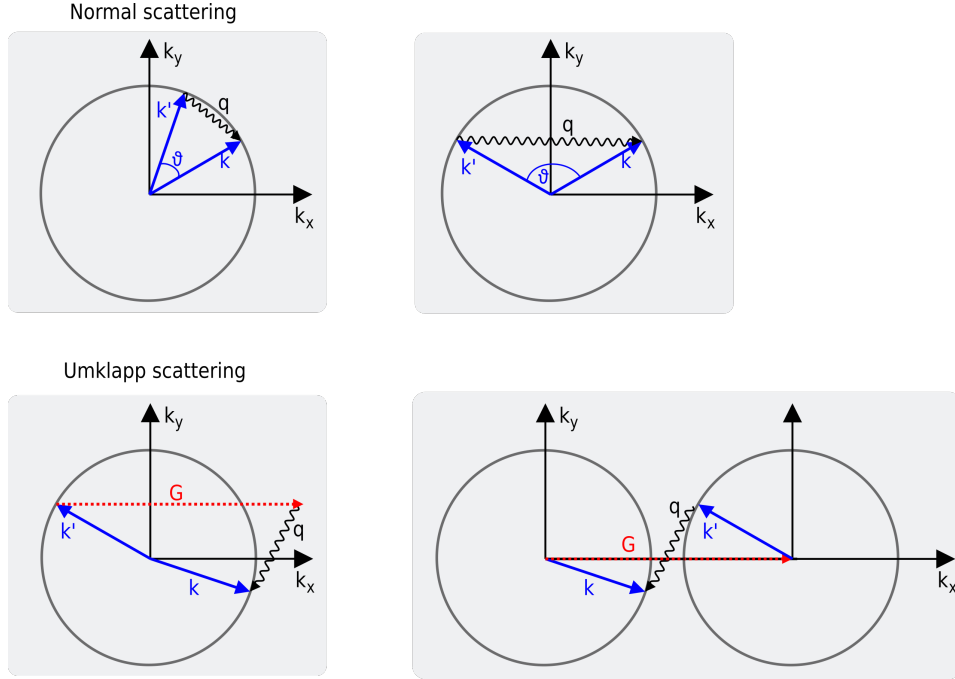


Fig. 2.11: Normal scattering: The scattering angle θ depends on temperature; at low temperatures, small-angle scattering dominates, while at elevated temperatures, larger-angle scattering becomes more probable. k and k' denote the electron wavevectors before and after the scattering event, with q representing the momentum of the emitted or absorbed phonon. **Umklapp scattering:** In this case, the final electron wavevector k' lies outside the first Brillouin zone (indicated by the circle). A reciprocal lattice vector G is added to bring the wavevector back into the first Brillouin zone, effectively transferring momentum to the lattice and enabling resistive scattering.

along the entire length of the sample without significant scattering, and the resistance is mainly determined by the contacts and the sample geometry. Ballistic transport occurs when the mean free path l_{mfp} of the charge carriers is significantly larger than the dimensions (L, W) of the sample. In encapsulated graphene, the mean free path is particularly large due to the high intrinsic mobility, which can enable ballistic transport even at room temperature. [11, 55]

For $l_{mfp} < L$, transport is in the **diffusive** regime, where charge carriers experience scattering processes along the transport channel.

In **bilayer** graphene, scattering processes are significantly influenced by the presence of the second layer. Electron-electron interactions are significant in bilayer graphene, because the interlayer coupling modifies the Coulomb interaction between charge carriers, leading to enhanced dielectric screening and altered long-range Coulomb scattering behaviour. Moreover, interlayer scattering can modify conventional scattering mechanisms

under external perturbations, such as an applied electric field, by redistributing electrons between the layers. At low energies, trigonal warping modifies the Fermi surface, leading to anisotropic scattering rates that vary with momentum direction. The presence of additional phonon modes further enriches the scattering landscape, contributing to increased electron-phonon interactions and inelastic scattering events. Additional phonon modes lead to more scattering channels, which enhance electron-phonon interactions and increase inelastic scattering. [22, 62, 63]

Defect scattering is also more complex than in monolayer graphene, as defects in one layer can influence charge carriers in the other due to interlayer coupling, leading to correlated disorder effects.

Moreover, when an external electric field induces a bandgap, the available scattering channels and carrier transport are affected. The tunability of the band structure introduces additional scattering mechanisms that depend on doping and gating conditions. Furthermore, the interplay between different scattering mechanisms, such as Coulomb and phonon scattering, can lead to nontrivial modifications of carrier relaxation times. As a result of these combined effects, charge carrier mobility in bilayer graphene is generally lower than in monolayer graphene. [22, 62, 64, 65]

2.6 Magnetotransport

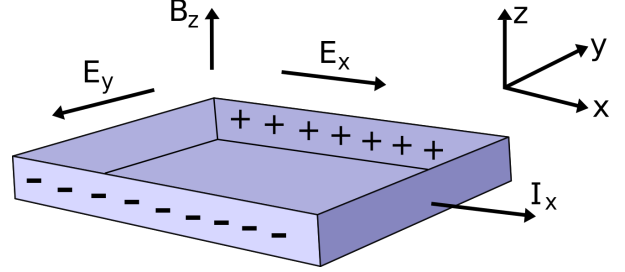
In the presence of a magnetic field, the transport properties of two-dimensional systems differ fundamentally from the zero-field case. Understanding magnetotransport in such systems is crucial for interpreting a wide range of experimental phenomena, including both classical and quantum effects. In particular, monolayer and bilayer graphene exhibit distinctive behaviours under magnetic fields, which are closely tied to their unique electronic structures. The following section provides the theoretical framework for magnetotransport in two-dimensional systems, with a focus on graphene.

2.6.1 Classical Magnetotransport

If a weak homogeneous magnetic field is applied perpendicular to a 2DEG, the charge carriers experience a force known as the Lorentz force, given by [20, 49, 66–68]

$$\vec{F}_L = q(\vec{E} + \vec{v} \times \vec{B}), \quad (2.40)$$

Fig. 2.12: Classical Hall Effect: A schematic diagram showing the mechanism of the classical Hall effect. An applied magnetic field B_z induces a Lorentz force on the charge carriers, causing them to accumulate on one side of the sample. This charge build-up generates a transverse Hall field, E_y , which in turn balances the Lorentz force. This dynamic equilibrium leads to a measurable potential difference, or Hall voltage, across the sample. The resulting Hall resistivity, ρ_{xy} , is a direct measure of this effect.



where $q = \pm e$ denotes the charge of the carriers, $\vec{E}$ is the applied electric field vector, and $\vec{B} = (0, 0, B_z)$ is the magnetic field vector perpendicular to the plane of the 2DEG. In the following, B will denote the z -component B_z for simplicity. An essential quantity in this context is the scattering time τ , which characterises the average time between two scattering events. The equation of motion for a charge carrier with an effective mass m^* then reads

$$m^* \left(\frac{d\vec{v}}{dt} + \frac{\vec{v}}{\tau} \right) = \vec{F}_L. \quad (2.41)$$

Solving this equation leads to cyclotron motion, with the cyclotron frequency and drift velocity [66]:

$$\omega_c = \frac{eB}{m^*}, \quad \vec{v}_d = \frac{\vec{E} \times \vec{B}}{B^2}. \quad (2.42)$$

The corresponding cyclotron radius is given by [66]

$$R_c = \frac{v_F}{\omega_c} = \frac{\hbar k_F}{eB}. \quad (2.43)$$

As a consequence of the Lorentz force, charge carriers accumulate on one side of the sample (depicted in Fig. 2.12). This accumulation generates an electric field in the y -direction that counteracts the Lorentz force, eventually leading to a Hall voltage V_{xy} . The corresponding resistivities are given by:

$$\rho_{xx} = \frac{1}{ne\mu}, \quad \rho_{xy} = \frac{B}{qn}. \quad (2.44)$$

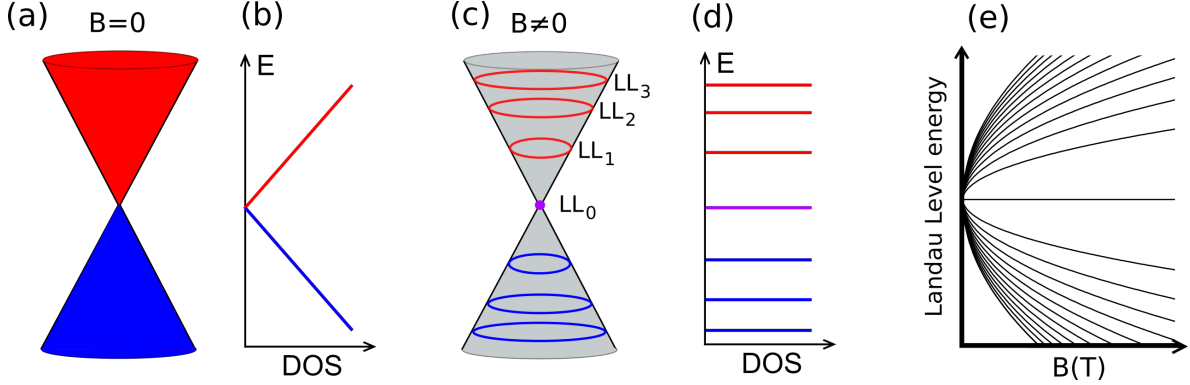


Fig. 2.13: (a) Band structure and (b) DOS at zero magnetic field ($B = 0$); red and blue denote electron and hole states, respectively. (c) In the presence of a magnetic field, the continuous energy spectrum is quantised into discrete Landau levels, illustrated schematically as quantised orbits in k -space. (d) Corresponding DOS, showing discrete peaks at the Landau level energies. (e) Landau level energies as a function of magnetic field B , exhibiting a characteristic square-root dependence $E_N \propto \sqrt{B|N|}$ that reflects the relativistic-like dispersion of charge carriers in MLG. (e) adapted from [69].

The expected linear dependence of the Hall resistivity on B allows the extraction of the charge carrier density and mobility [49, 66, 67]:

$$n = \frac{1}{e \left| \frac{d\rho_{xy}}{dB} \right|}, \quad \mu = \frac{1}{en\rho_{xx}(B=0)}. \quad (2.45)$$

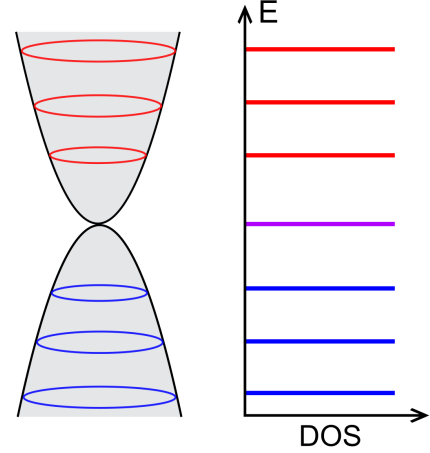
2.6.2 Landau Quantisation in Graphene

The wave character of the carriers comes into play as soon as a high magnetic field ($\omega_c\tau = \mu B \gg 1$) perpendicular to a 2DEG is applied. This leads to quantisation and entry into the Quantum Hall Effect (QHE) regime.

At sufficient magnetic fields, the electrons move in complete orbits, i.e. they interfere with themselves before participating in a scattering event. These orbits lead to discrete energy levels known as Landau levels (LL). The spectrum of these levels in graphene can be calculated by solving the Dirac equation with the additional perpendicular magnetic field B and the Peierls substitution of the momentum $\vec{p} \rightarrow \vec{p} + e\vec{A}$ [70–72]:

$$H = v_F \vec{\sigma} (i\hbar \nabla + e\vec{A}). \quad (2.46)$$

Fig. 2.14: Parabolic band structure of bilayer graphene with Landau quantisation. The discrete Landau levels arising in a perpendicular magnetic field are indicated by circles, red and blue corresponding to electrons and holes. Due to the quadratic dispersion, the level spacing is approximately linear in B and denser near the charge neutrality point compared to monolayer graphene. The $N = 0$ Landau level in bilayer graphene is eightfold degenerate due to spin, valley, and orbital degeneracy.



Here, the Landau gauge $\vec{A} = (0, Bx, 0)$ ensures $\nabla \times \vec{A} = \vec{B}$. This leads to graphene's unique Landau level spectrum [19]

$$E_N(B) = \text{sgn}(N)v_F\sqrt{2e\hbar B|N|}, \quad (2.47)$$

with the Landau level index N being an integer number. This spectrum is shown in Fig. 2.13, where the quantisation is visualised in both band structure and density of states in (c) and (d) and compared to the case at $B = 0$ in (a) and (b).

This is in contrast to normal 2DEGs with parabolic dispersion and linear dependence on B , i.e. equal spacing between Landau levels. In graphene, the square-root dependence on B leads to an increased spacing, making it possible to measure the quantum Hall effect in graphene even at room temperature. This square-root dependence is depicted in Fig. 2.13(e), where each branch shows a Landau level as a function of magnetic field B . This discrete energy level spectrum is only the ideal case, but in real samples with the influence of disorder, impurity scattering, phonon interactions and strain, these Landau levels are broadened.

Another difference from other 2DEGs is the Landau level at $N = 0$, which is equally occupied by electrons and holes. This, together with the Berry phase of π in monolayer graphene, gives rise to graphene's unique half-integer quantum Hall effect, in contrast to the conventional integer QHE in standard 2DEGs. [71, 72]

Bilayer graphene exhibits a fundamentally different quantisation due to its quadratic low-energy band dispersion. Unlike monolayer graphene, where charge carriers behave as massless Dirac fermions, bilayer graphene hosts massive chiral quasiparticles. Further, one has to take the Berry phase of 2π into account. One can calculate a low-energy Landau level spectrum by using the 2×2 Hamiltonian and $E_{2,3}$ parabolic approximation.

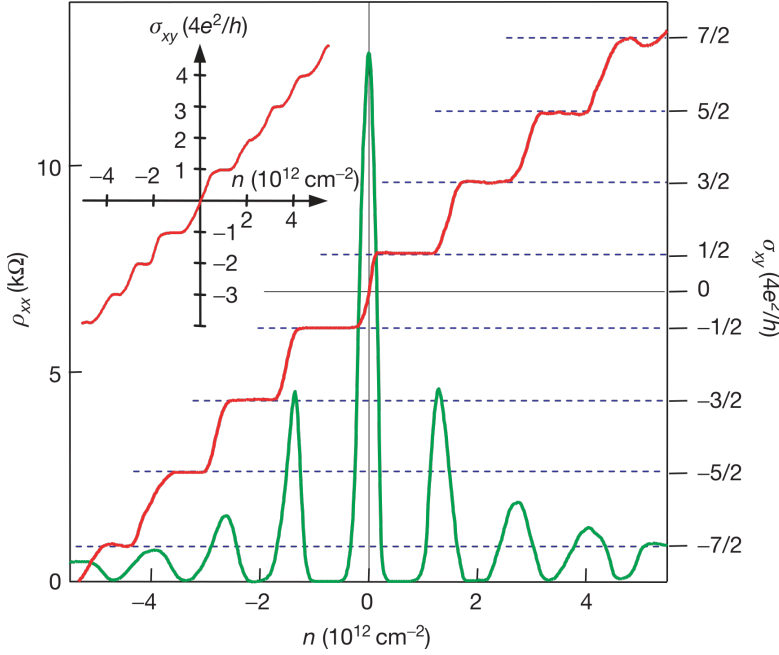


Fig. 2.15: Hall conductivity σ_{xy} and longitudinal resistivity ρ_{xx} of a MLG sample as a function of charge carrier density at $B = 14 \text{ T}$ and $T = 4 \text{ K}$. The characteristic half-integer quantum Hall effect of MLG is observed: σ_{xy} exhibits quantised plateaus at values $\sigma_{xy} = \pm(4e^2/h)(N + 1/2)$, corresponding to the filling of Landau levels. Simultaneously, ρ_{xx} shows peaks at the transitions between plateaus, corresponding to SdH oscillations. Adapted from [8].

This leads to a spectrum for the low energy Landau levels [23, 73]:

$$E_N = \text{sgn}(N)\hbar\omega_c\sqrt{N(N-1)}. \quad (2.48)$$

ω_c stands for the cyclotron frequency $\omega_c = eB/m^*$ with $m^* = \gamma_1/(2v_F^2)$ being the effective mass for bilayer graphene. The resulting Landau levels are depicted in Fig. 2.14. Each level is fourfold degenerate due to spin and valley degrees of freedom, while the $N = 0$ Landau level exhibits an eightfold degeneracy. This increased degeneracy arises from the presence of two orbital wavefunctions that contribute to the zero-energy states. In this approximation, the Landau levels depend linearly on the magnetic field B for sufficiently large N . Specifically, for $N \geq 1$ the energy spectrum asymptotically approaches the form [73]:

$$E_N \approx \text{sgn}(N)\hbar\omega_c \left(|N| + \frac{1}{2} \right). \quad (2.49)$$

At very low energies, additional effects such as trigonal warping introduce deviations from the simple parabolic approximation, leading to a modified Landau level structure near the charge neutrality point. [30, 74, 75]

2.6.3 Quantum Hall Effect in Graphene

The manifestation of Landau quantisation in transport measurements can be observed through SdH oscillations in the longitudinal resistance and the QHE in the transverse resistance. SdH oscillations appear as $1/B$ -periodic variations in the longitudinal resistivity ρ_{xx} , reflecting the density of states modulated by Landau quantisation. In contrast, the transverse conductivity σ_{xy} exhibits quantised Hall plateaus, where the SdH oscillations reach their minima. [3, 19, 30, 76]

The QHE can be understood within the edge-channel model. In the presence of disorder, Landau levels broaden due to scattering processes, leading to mobility edges that separate localised bulk states from delocalised edge states. Near the sample boundaries, Landau levels bend due to the spatial confinement potential, forming chiral edge channels. These edge states propagate without backscattering, ensuring dissipationless transport along the sample boundaries. [3, 19, 77]

When the chemical potential lies within the region of localised states, the longitudinal conductivity vanishes, i.e., $\sigma_{xx} = 0$, while the Hall conductivity remains quantised, forming a plateau in σ_{xy} . In contrast, when the Fermi level crosses a Landau level and enters a region of delocalised states, σ_{xx} becomes finite, exhibiting a maximum in SdH oscillations, while σ_{xy} transitions between quantised values.

Figure 2.15 shows this quantisation in a typical magnetotransport measurement at a constant magnetic field of $B = 14$ T and $T = 4$ K. Here, the half-integer QHE of monolayer graphene is visible in the Hall conductance, shown in red. The longitudinal resistivity, shown in green, exhibits pronounced SdH oscillations, which reach a maximum when σ_{xy} transitions to a new quantised plateau.

The number of available edge states per Landau level is determined by the degeneracy factor $g = 4$ in graphene (each Landau level contributes a single edge state per spin and valley degree of freedom). A key feature is the electron-hole symmetry, which leads to a half-filled $N = 0$ Landau level. In monolayer graphene, this results in a Hall conductivity σ_{xy} that jumps from $-2e^2/h$ to $+2e^2/h$, without forming a plateau at $\sigma_{xy} = 0$. The bulk localised states do not contribute to transport, resulting in an insulating interior and dissipationless conduction along the edges. Figure 2.16(a) shows the Landau level spectrum as a function of position for a finite sample, as in (b). Here, the bent energy levels at the sample edges due to the confinement potential can be seen. In (b) the formed edge channels are depicted in red and blue, which lead to the quantised Hall plateaus, as well as the localised states in black.

The number of edge channels is directly linked to the Hall plateaus, as the Hall conduc-

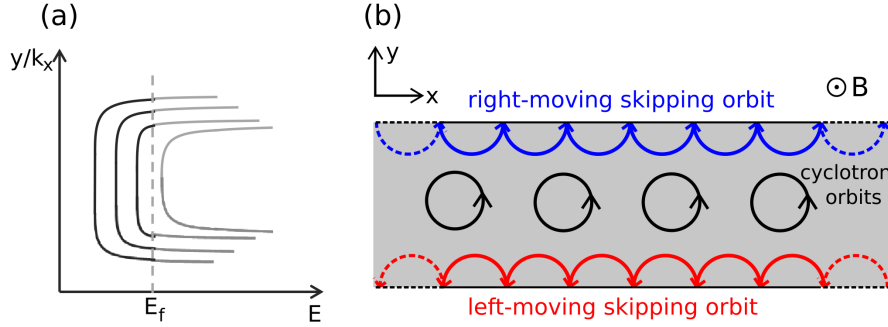


Fig. 2.16: (a) Landau level spectrum as a function of position for a sample of finite width. The Landau levels bend upward or downward near the sample edges due to the confinement potential, forming dispersive edge states. Adapted from [79]. (b) Schematic illustration of cyclotron motion (black) in the bulk and skipping orbits at the sample boundaries (red and blue). The skipping orbits at opposite edges propagate in opposite directions, forming chiral edge states that are responsible for the quantum Hall effect.

tance follows [3, 19, 30]:

$$\sigma_{xy} = \nu \frac{e^2}{h}, \quad \nu = 4\left(N + \frac{1}{2}\right), \quad (2.50)$$

where $e^2/h \approx 25813\Omega$ is the Klitzing constant. The half-integer shift originates from the Berry phase associated with Dirac fermions. In the presence of inter-channel scattering, variations in the Fermi level can induce transitions between quantum Hall plateaus, leading to an increase in σ_{xx} as σ_{xy} evolves toward the next quantised value. [78]

Quantum Hall Effect in Bilayer Graphene

Bilayer graphene exhibits a distinct quantum Hall effect due to its parabolic band structure and Berry phase of 2π . In contrast to monolayer graphene, where the Hall conductivity σ_{xy} follows a half-integer quantisation, bilayer graphene shows plateaus at integer multiples of $4e^2/h$, without the characteristic half-integer shift [23, 27, 31]:

$$\sigma_{xy} = 4N \frac{e^2}{h}. \quad (2.51)$$

The upper left inset in Fig. 2.15 shows this characteristic quantum Hall plateau sequence for bilayer graphene.

A key difference arises from the $N = 0$ Landau level, in which bilayer graphene hosts twice as many states as in monolayer graphene due to the additional orbital degree of freedom. Consequently, there is no Hall plateau at $\nu = 0$, in contrast to conventional 2DEGs. The tunability of bilayer graphene via external perpendicular electric fields

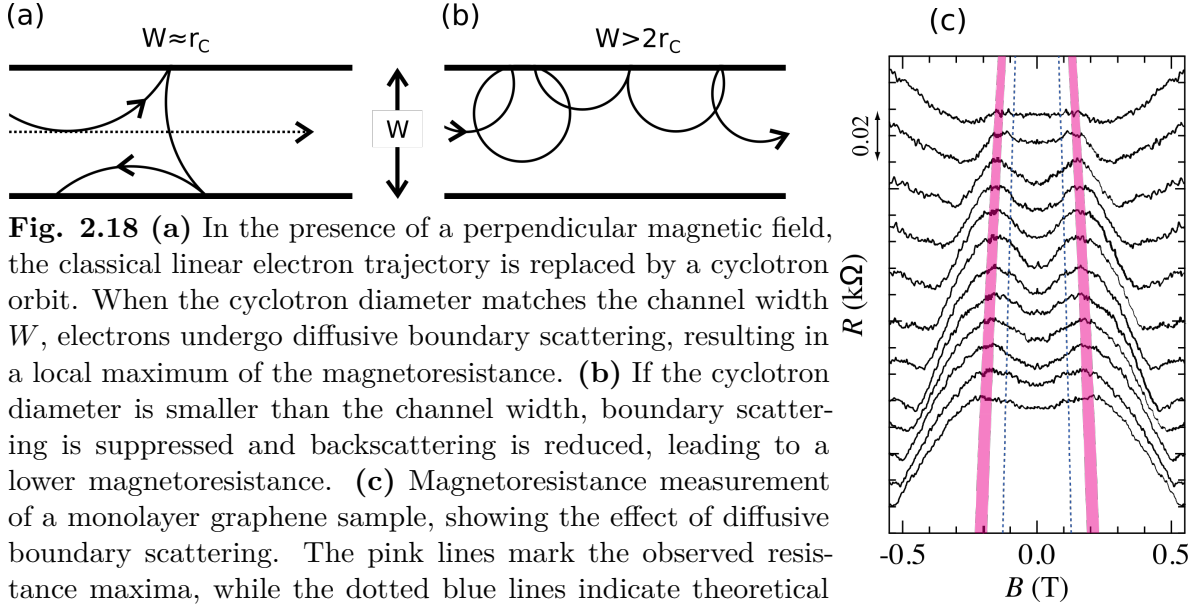


Fig. 2.18 (a) In the presence of a perpendicular magnetic field, the classical linear electron trajectory is replaced by a cyclotron orbit. When the cyclotron diameter matches the channel width W , electrons undergo diffusive boundary scattering, resulting in a local maximum of the magnetoresistance. (b) If the cyclotron diameter is smaller than the channel width, boundary scattering is suppressed and backscattering is reduced, leading to a lower magnetoresistance. (c) Magnetoresistance measurement of a monolayer graphene sample, showing the effect of diffusive boundary scattering. The pink lines mark the observed resistance maxima, while the dotted blue lines indicate theoretical values, which deviate due to the unique properties of graphene. (c) adapted from [82].

allows for the opening of a gap at charge neutrality, modifying its QHE characteristics and enabling the emergence of additional symmetry-broken states. [31, 80, 81]

2.6.4 Magneto Size Effects

In an ideal, infinite 2D system without scattering, the longitudinal resistance would be independent of the magnetic field B in the semiclassical regime. In real systems, however, scattering and imperfections lead to a finite magnetoresistance even at moderate fields. Without a magnetic field, charge carriers primarily move parallel to the sample edges. In high-quality devices, where transport is ballistic, the application of a moderate magnetic field causes charge carriers to follow cyclotron orbits due to the Lorentz force. If the cyclotron diameter r_c exceeds the channel width W , charge carriers are scattered from one edge to the opposite edge, increasing resistance, as depicted in Fig. 2.18 (a). This phenomenon is known as **diffusive boundary scattering**. As the magnetic field strength increases further, the cyclotron diameter becomes smaller than W , suppressing backscattering and causing the resistance to decrease, returning to the bulk value (see Fig. 2.18 (b)). The resulting magnetoresistance curves are shown in Fig. 2.18 (c), where the dotted blue lines indicate the condition $W/r_c \approx 0.55$, at which the resistance is expected to reach a local maximum due to diffusive boundary scattering. In contrast,

the observed maxima (pink lines) follow the relation $W/r_C \approx 0.9$, deviating from the predicted value for 2DEG systems. This discrepancy suggests that the conditions for the transition to diffusive edge scattering in graphene are more complex than initially assumed. Factors such as the quality of the graphene, the type of substrate, and the precision of the experimental methods may influence the observed behaviour. [30, 82, 83]

In small graphene devices, another effect can arise: **magnetic breakdown**. Here, electrons can tunnel between adjacent Landau levels due to perturbations in the ideal Landau-level structure. This occurs when the energy gap between Landau levels becomes comparable to the cyclotron energy, allowing electrons to overcome the gap through quantum tunnelling. Such behaviour has been observed in various two-dimensional electron systems, including graphene-based materials. For instance, studies have reported magnetic breakdown in twisted bilayer graphene, where electrons can tunnel between semiclassical k -space trajectories in different layers, leading to complex energy spectra. [31, 84]

3 Superlattice Effects

The homogeneous graphene discussed in the previous sections is now subjected to a periodic potential, forming a superlattice that enables a controlled and tunable modulation of charge carriers. This periodic potential significantly alters the electronic band structure, leading to the formation of minibands, band gaps, and tunnelling phenomena. In general, the generated superlattice can be seen as an artificial crystal, where physics are equivalent to those in a natural crystal. In the case of 1D superlattices, where the modulation occurs along a single direction, anisotropic effects arise in the band structure and transport properties, as will be discussed later. [66, 85–88]

In contrast, 2D superlattices influence the electronic states in both spatial directions, paving the way for phenomena such as Hofstadter’s butterfly. [89–93]

3.1 Periodic Potentials

We first consider the general concept of an one-dimensional superlattice, illustrated here for a system with parabolic electron dispersion such as a conventional 2DEG. A periodic modulation potential along the x -direction satisfies [66]

$$V(\vec{r}) = V(\vec{r} + \vec{R}), \quad (3.1)$$

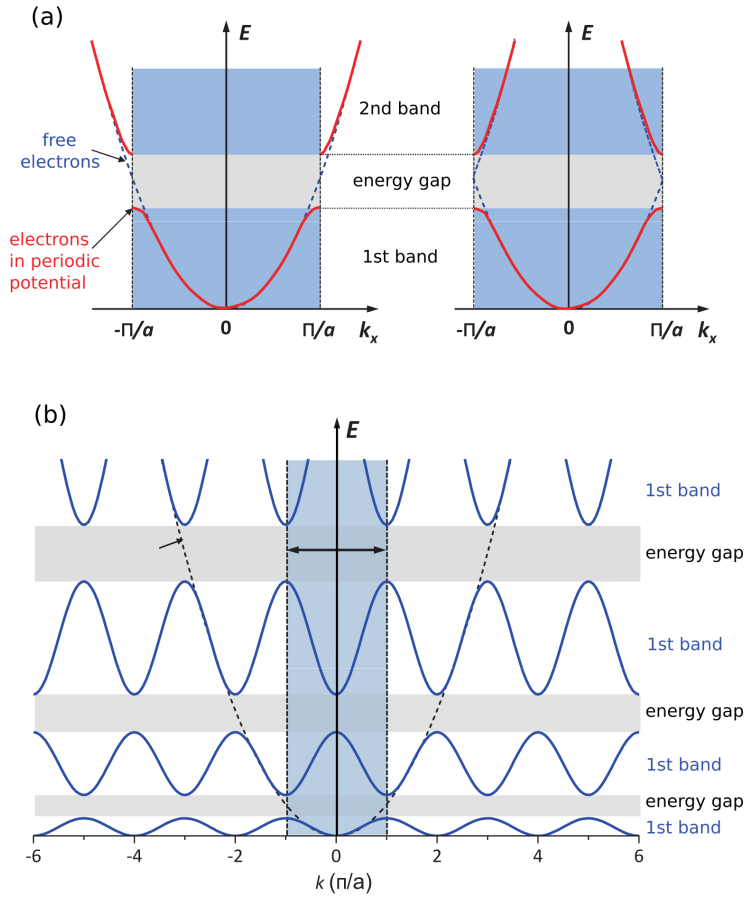
where $\vec{R}$ is a lattice vector of the imposed superlattice. The effect of this modulation on the electronic band structure can be analysed by solving the Schrödinger equation $H\Psi = E\Psi$, where $H = H_0 + V(\vec{r})$ and H_0 describes the unperturbed parabolic dispersion of the 2DEG. Solving this equation yields Bloch wave eigenfunctions [66]

$$\Psi_{n,\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{n,\vec{k}}(\vec{r}), \quad (3.2)$$

with $u_{n,\vec{k}}(\vec{r}) = u_{n,\vec{k}}(\vec{r} + \vec{R})$. Zone folding transforms the original bands into minibands $E_n(\vec{k})$ separated by energy gaps E_{gap} , forming a mini-Brillouin zone of size $2\pi/a$. At the

Fig. 3.1: (a) Extended and reduced zone scheme of a parabolic energy spectrum under the influence of a superlattice potential applied along the x -direction with lattice constant a . The folding of the original Brillouin zone leads to the formation of a mini-Brillouin zone, and energy gaps open at its boundaries.

(b) The original parabolic free-electron dispersion (black dotted line) is significantly modified by the periodic potential. In the periodic zone scheme, the resulting energy bands and band gaps become visible. Adapted from [66].



mini-Brillouin zone boundaries, Bragg reflection leads to standing waves and the opening of gaps, as schematically shown in Fig. 3.1. [66]

We now consider the specific case of graphene. As discussed in chapter 2, the low-energy Hamiltonians for monolayer (see Eq. (2.8)) and bilayer (see Eq. (2.15)) graphene are used here. The imposed superlattice potential acts equally on both sublattices and is represented as $V(\vec{r})I_2$. The full Hamiltonian is then given by $H = H_0 + V(\vec{r})I_2$ [5, 88]. The eigenfunctions of this Hamiltonian are also of Bloch form, but now as spinor Bloch waves, reflecting the sublattice pseudospin of graphene. Zone folding and the formation of minibands proceed analogously to the 2DEG case, but the underlying Dirac or bilayer Hamiltonian modifies the detailed band structure and energy gaps.

A detailed derivation of the miniband structure in graphene superlattices can be found in Refs. [5, 86, 88, 94].

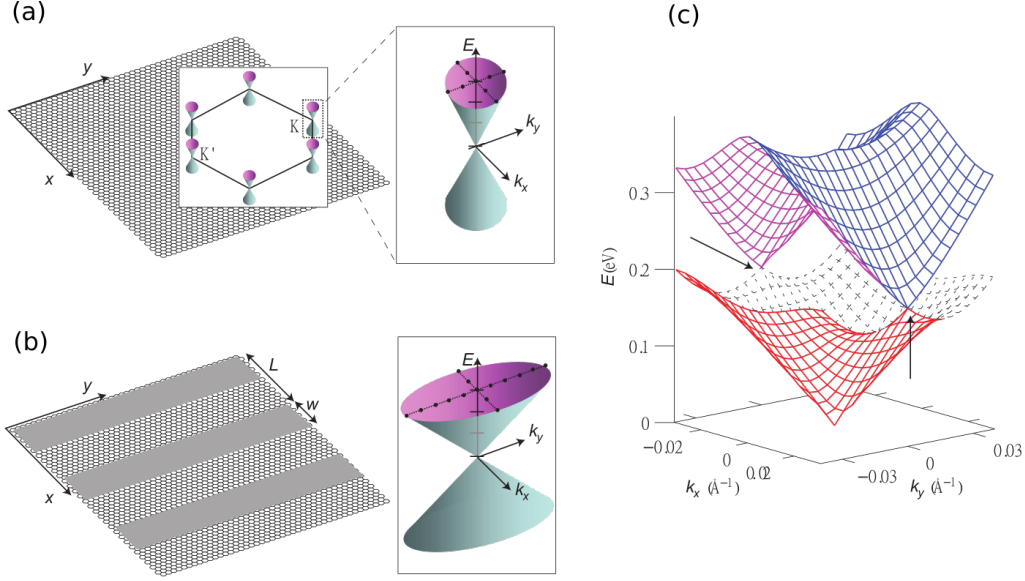


Fig. 3.2: (a) Brillouin zone and band structure of pristine graphene with the characteristic Dirac cone at the $K(K')$ points. (b) 1D graphene superlattice induced by a periodic electrostatic potential: Band structure shows an elliptically distorted Dirac cone which resembles the anisotropic renormalisation of the group velocity components v_x and v_y . (c) Band structure modified by the 1DSL, showing satellite Dirac points indicated by black arrows. Adapted from [95].

3.2 Anisotropic Transport

One interesting aspect of 1D superlattices (1DSLs) is that the modulation occurs exclusively in one direction, while the other direction remains unchanged. Fig. 3.2 shows the comparison of the unmodulated band structure (a) of monolayer graphene with a perfectly round Dirac cone. As soon as the 1D modulation with lattice period a is applied, this Dirac cone gets distorted into an elliptical shape (b) due to the modulation along the x -direction. Perpendicular to the modulation, along the y -direction, the round cone is distorted, which corresponds to a renormalisation of the Fermi velocity and therefore a reduced velocity and mobility of the charge carriers. Further, new band gaps and zero modes (indicated by the black arrows in Fig. 3.2(c)) appear. These are a result of the chiral nature of the Dirac fermions in graphene, as the emergence of these energy gaps depends on the wave vector $\vec{k}$. [95]

Moreover, the newly formed Dirac cones exhibit a modified pseudospin texture compared to the original cone. This change in pseudospin structure allows for backscattering processes within these new cones, even for slowly varying potentials, which contrasts with the suppression of backscattering in pristine graphene. [95]

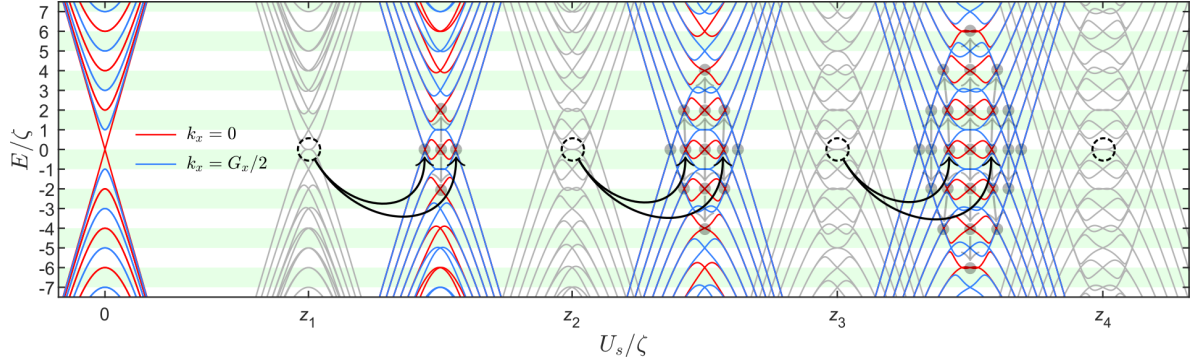


Fig. 3.3: Miniband structures of graphene under an applied 1DSL modulation. The red and blue lines represent the dispersion at fixed $k_x = 0$ and $k_x = G_x/2$, respectively. For increasing values of $z_j = U/\chi$, i.e. with increasing amplitude of the applied 1DSL potential, the Dirac point flattens (indicated by black dotted circles at each z_j), from which a new pair of zero modes emerges (black arrows). Additional zero modes also appear at higher energies, marked by grey dots. Adapted from [96].

3.2.1 Zero Modes and Flat Bands

A common realisation of 1D superlattices in graphene is based on electrostatic modulation, where a periodic variation of the capacitance along one direction induces a spatially modulated charge carrier density. This results in a periodic onsite energy profile, which can be described by e.g. a Kronig-Penney-potential or a sinusoidal potential of the form

$$V(x) = U \sin(G_x x), \quad (3.3)$$

with $G_x = \frac{2\pi}{L}$, where L is the superlattice period. Due to this periodicity, the wave vector component k_y is confined within the range $(-\frac{G_x}{2}, \frac{G_x}{2})$, whereas k_x is continuous. One of the striking effects of such a superlattice potential is the emergence of additional Dirac points in the band structure. For a sinusoidal potential, these extra Dirac points appear whenever the condition $\frac{U}{\chi} = z_j$ is fulfilled, where z_j denotes the j -th root of the zeroth Bessel function J_0 , and $\chi = \frac{\pi\hbar v_F}{L}$. The formation of these new Dirac points is accompanied by the emergence of nearly flat bands, which evolve with increasing potential strength. Furthermore, these additional Dirac points are not only created at specific energy values but also replicated at higher energies and at the superlattice Brillouin zone boundaries. [94–98]

This effect is shown in Fig. 3.3, where Kang et. al [96] calculated continuum-model-based superlattice miniband structures for the potential in Eq. (3.3). Here, the grey energy dispersions at each z_j show a flattened point inside the black dotted circle, from where an extra pair of zero modes arises with increasing 1DSL modulation. These extra Dirac

points also appear at higher energies and are shown with grey dots. The red and blue lines show the dispersion for $k_x = 0$ and $k_x = G_x/2$.

A key consequence of these flat bands is the strong anisotropy in the charge carrier dynamics. For states near these bands, the velocity component $v_y = \frac{\partial E}{\partial k_y}$ is strongly renormalised and can vanish entirely, meaning that charge carriers are effectively forced to move along the superlattice barriers with negligible dispersion in the transverse direction. This effect is a direct consequence of the chiral nature of the Dirac fermions in graphene and differs fundamentally from non-chiral systems, such as layered semiconductor superlattices. In the latter, band gaps induced by the periodic potential are nearly independent of $\vec{k}$, whereas in graphene, gap openings are highly dependent on the wave vector due to the underlying chiral symmetry. [94–97]

For the formation of these Dirac points, it is crucial that the superlattice period L is much larger than the atomic lattice spacing. This ensures that intervalley scattering between different Dirac cones is negligible, allowing the band structure to be described within a single-cone approximation. For example, in the case of a Kronig-Penney-type superlattice potential, it has been shown that the transverse group velocity v_y can be renormalised to approximately 40% of its original value. Counterintuitively, for strong modulation v_y can even approach zero, meaning that carriers exhibit highly anisotropic transport, moving preferentially along the modulation direction. This phenomenon arises due to the absence of backscattering and is a direct consequence of the chiral nature of its quasiparticles. [94–97]

3.2.2 Transport at $B = 0$

Kang et al. [96] further demonstrated the impact of drastic changes in the band structure through tight-binding-based conductance simulations for a 1D superlattice device. This device consists of a periodically etched dielectric layer that separates the global backgate from the transport graphene monolayer, while a global topgate allows investigation of transport behaviour as a function of charge carrier density and superlattice modulation strength. The employed geometry had a period of $L = 60$ nm, and the resulting charge carrier modulation is given by $n(x) = n_{tg} + \left[\frac{C_{bg}(x)}{e} \right] V_{bg}$, where the backgate capacitance $C_{bg}(x)$ depends on the local thickness of the etched dielectric. The simulations were conducted at zero temperature and without disorder.

Figure 3.4 presents the calculated conductivity maps for transport perpendicular (a) and parallel (b) to the modulation direction, with line cuts (c) and (d) taken from (b). These results are compared to the corresponding changes in band structure, depicted in (e) and (f). [96]

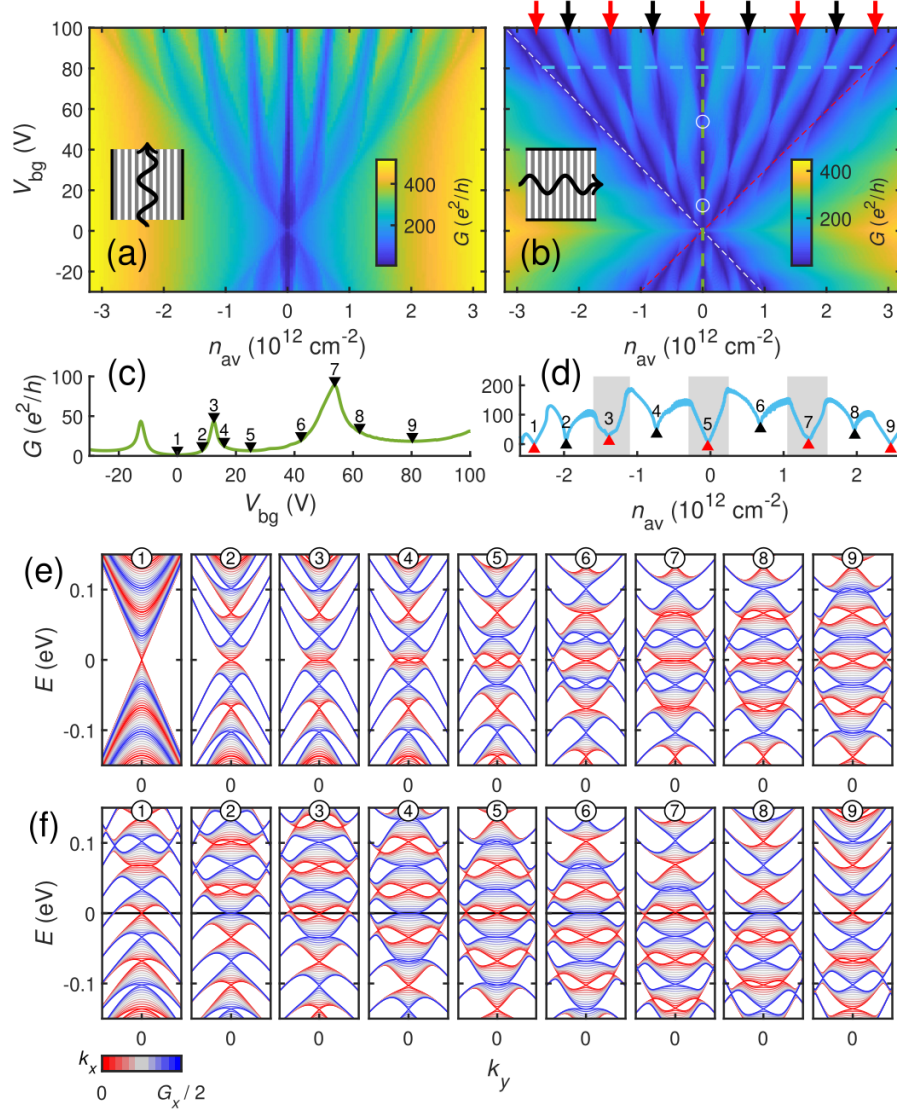


Fig. 3.4: Transport simulation showing the two-terminal conductance as a function of average charge carrier density and back gate voltage. (a) Conductance for transport parallel to the gate stripes, and (b) for transport along the modulation direction. The 1DSL period is set to 30 nm. In the modulation direction, a pattern of branching regions with increased resistivity appears as a consequence of the superlattice modulation. (c) and (d) display line cuts from (b), revealing alternating regions of high and low resistivity both along and across the branches. The origin of this pattern can be understood by referring to the band structures in (e) and (f), where the modulation strength is increased in (e) and the Fermi level is varied in (f). Adapted from [96].

In the conductance maps, the horizontal axis represents variations in the average charge carrier density, while the vertical axis corresponds to the superlattice modulation strength. This parameter space naturally divides into four quadrants: the bipolar regions (upper and lower), where the suppression of conductance and the emergence of characteristic patterns reflect significant band structure modifications, and the unipolar quadrants, where the conductance remains high. The line cuts in (c) and (d) further illustrate the rhombic pattern observed in (b).

The green line (vertical dashed line at $n = 0 \text{ cm}^{-2}$) in (b) corresponds to varying the superlattice modulation strength while keeping the Fermi level fixed. In comparison with the band structures in (e), each conductance peak coincides with a region of band flattening, where charge carriers must propagate along the modulation direction, as their group velocity in the perpendicular direction is significantly reduced. This effect, known as **supercollimation**, emerges when the miniband formation leads to strongly anisotropic transport. Notably, supercollimation can be understood as a consequence of Klein tunnelling, where normally incident carriers experience perfect transmission while oblique angles are increasingly suppressed due to the miniband structure. The conductance peaks align with minibands (3) and (7) in (e). In contrast, the blue line in (b) represents variations in the average charge carrier density at a fixed 1DSL potential strength, effectively shifting the Fermi level within a given miniband structure. Here, conductance dips occur whenever a generated pair of zero modes is crossed, with even and odd indices corresponding to $k_x = 0$ and $k_x = G_x/2$ in the energy dispersion (f). Similar conductance fringes appear in the perpendicular transport map (a), reinforcing the miniband interpretation. [96]

Li et al. [99] investigated transport in comparable samples, in which a dielectric superlattice was created by etching narrow trenches into the SiO_2 layer, resulting in a periodic modulation with lattice constants of $L = 55 \text{ nm}$ and $L = 47 \text{ nm}$. Their study combined theoretical modeling with experimental measurements and demonstrated strong agreement between the calculated and observed transport features, as shown in Fig. 3.5. The upper panels display the experimentally measured resistances along the x - and y -directions, which closely match the theoretical simulations in panels (c) and (d), respectively. Multiple satellite Dirac points were observed in the x -direction due to the 1DSL modulation, while transport along the y -direction consistently exhibited only a single Dirac point, regardless of the modulation strength.

These findings provide strong evidence for miniband formation in 1D graphene superlattices and demonstrate the ability to tailor electronic transport properties through periodic dielectric patterning.

In addition, Drienovsky et al. [100] studied transport in graphene devices with periodic potential modulations induced by patterned gates. Their experiments revealed clear

Fabry-Pérot interference patterns arising from phase-coherent transport between potential barriers, providing complementary insights into electron dynamics in periodically modulated graphene.

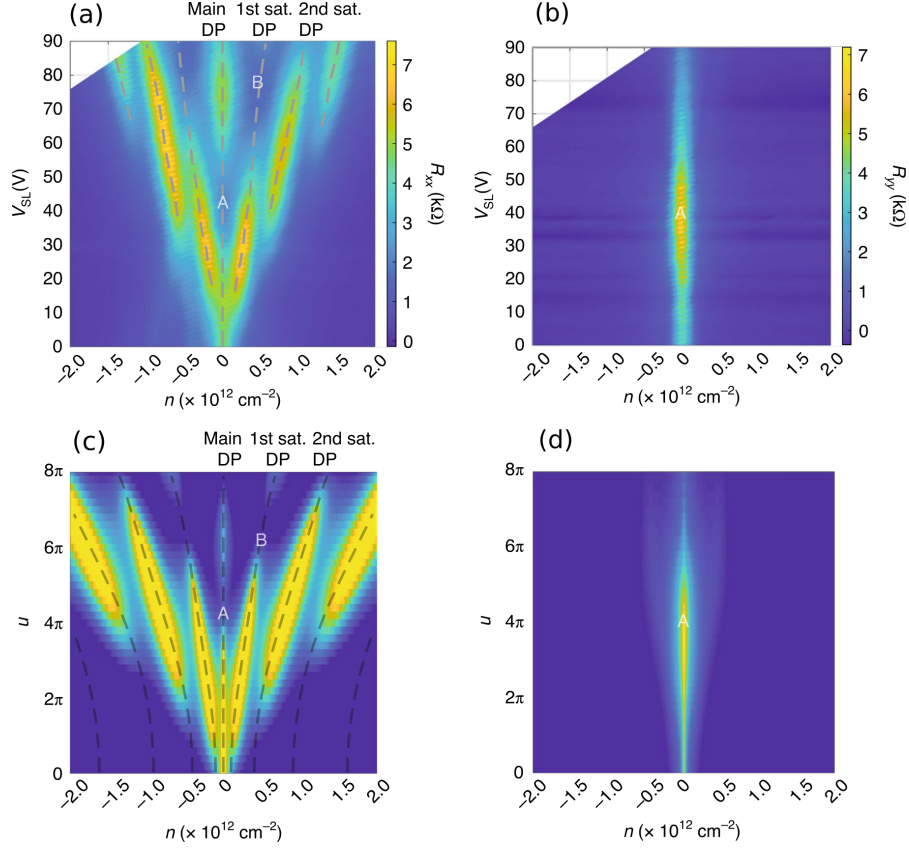


Fig. 3.5: (a) Experimental longitudinal resistance R_{xx} as a function of carrier density n and superlattice gate voltage V_{SL} for a 1DSL modulation with a period of 55 nm, by Li et al. [99]. As the modulation strength increases, satellite Dirac points emerge, corresponding to additional zero modes in the band structure. The grey dashed lines trace the features associated with each main and satellite Dirac point. (b) Resistance measured in the y -direction, where only the main Dirac point is observed due to the transport anisotropy of the 1DSL. (c) and (d) Calculated resistances as functions of the dimensionless modulation strength u and carrier density n , showing good agreement with the experimental results. Adapted from [99].

3.3 Magnetotransport in a 1D Graphene Superlattice

The introduction of a magnetic field in a 1D graphene superlattice further modifies the electronic band structure and transport properties. In pristine graphene, the Landau levels exhibit a characteristic square root dependence on the magnetic field strength.

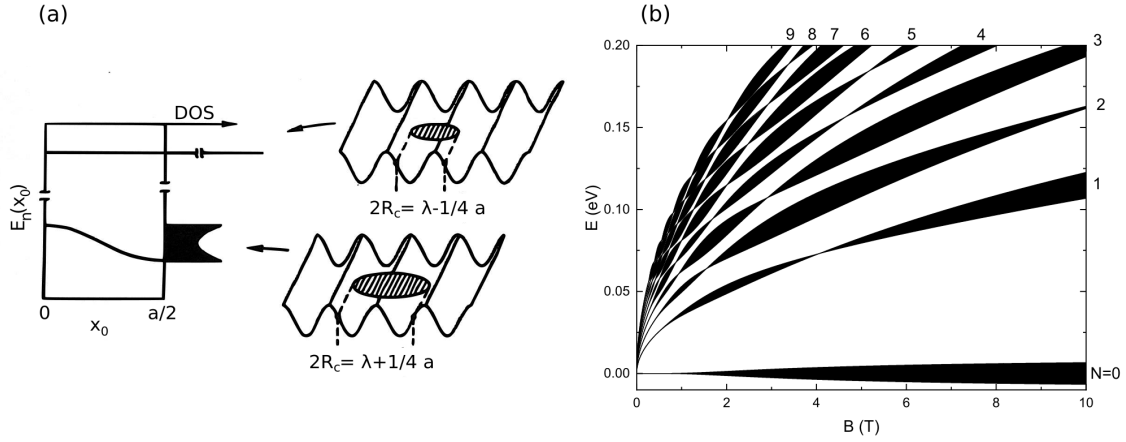


Fig. 3.6: (a) Landau level energies E_N as a function of the guiding center coordinate x_0 , along with the corresponding DOS for two representative cases shown to the right: a flat band condition at $2R_C = (\lambda - 1/4)a$, where discrete Landau levels and a maximum in the DOS occur; and the case $2R_C = (\lambda + 1/4)a$, where the cyclotron orbit overlaps with the potential extrema, resulting in maximal Landau level broadening and a minimum in the DOS. Adapted from [102]. (b) Magnetic band structure showing Landau bands up to $N = 9$, calculated for a monolayer of graphene with a one-dimensional superlattice of period 50 nm and modulation amplitude $V_0 = 10$ meV. The Landau levels are broadened due to the periodic potential, with the bandwidth depending on the magnetic field strength. At flat band conditions, the bandwidth vanishes.

However, the presence of a superlattice potential introduces an additional quasiperiodic modulation, leading to the formation of minibands and a restructuring of the Landau spectrum. These changes can have a significant impact on carrier dynamics and transport behaviour.

The interplay between the cyclotron motion and the superlattice potential allows novel control mechanisms over charge transport. Depending on the relative strength of the magnetic field and the superlattice modulation, various phenomena such as commensurability oscillations and elliptical cyclotron orbits can occur. In addition, the periodic potential increases the degeneracy of the Landau levels, leading to an altered sequence of quantum Hall plateaus. [4, 70, 101]

The following sections review theoretical models and experimental studies that investigate these effects and their implications for transport in 1D graphene superlattices.

3.3.1 Landau Bands

When a perpendicular magnetic field is applied to a 1D-modulated parabolic 2DEG, the resulting magnetic band structure exhibits Landau bands whose bandwidth varies

depending on the modulation conditions. In the weakly modulated case, i.e. $V_0/E_F \ll 1$, these bandwidths can be determined using first-order perturbation theory, applied to the unperturbed Landau energy spectrum $E_{N,0} = (N + \frac{1}{2})\hbar\omega_c$ [70]:

$$\begin{aligned} E_N(x_0) &= E_{N,0} + \langle Nx_0 | V(x) | Nx_0 \rangle \\ &= E_{N,0} + U_N \cos(Kx_0) \\ &= E_{N,0} + V_0 e^{-\frac{1}{2}X} L_N(X) \cos(Kx_0). \end{aligned} \quad (3.4)$$

Here, $V(x) = V_0 \cos(Gx)$ is the superlattice modulation potential, N is the Landau level index, and $x_0 = -l_B^2 k_y$ denotes the guiding centre of the Landau orbit, where l_B is the magnetic length. The state $|N, x_0\rangle$ represents the Landau wave function of the homogeneous 2DEG. Furthermore, $L_N(X)$ is the Laguerre polynomial of order N , and $X = \frac{1}{2}G^2 l_B^2$. The matrix element $\langle Nx_0 | V(x) | Nx_0 \rangle$ lifts the Landau level degeneracy with respect to the guiding centre coordinate, leading to the formation of Landau bands. The reason for this band formation lies in the fact that the periodic potential couples adjacent Landau orbits, breaking the continuous degeneracy present in an unmodulated system. The bandwidth U_N for a parabolic 2DEG is given by [70, 102]:

$$U_N = V_0 e^{-\frac{1}{2}X} L_N(X). \quad (3.5)$$

According to this equation, the bandwidth vanishes when the Laguerre polynomial $L_N(X)$ is zero. This means that in the N -th Landau level, there are N points where the bandwidth becomes zero. These satisfy the so-called *Landau flat band condition*, meaning that at these points, the Landau level remains discrete. The number of such points follows from the fact that the N -th order Laguerre polynomial has exactly N roots. [4, 70, 101–103]

In graphene, the Landau band width can be derived as

$$U_N = V_0 e^{-\frac{1}{2}X} \frac{1}{2} (L_N(X) + L_{N-1}(X)), \quad (3.6)$$

with $E_{N,0} = \text{sgn}(N)v_F\sqrt{2e\hbar B|N|}$. [70]

The effect of the modulation on the Landau bands in monolayer graphene is illustrated in Fig. 3.6(b), where the calculated Landau bands are shown. The black regions indicate the Landau bands and their respective bandwidths.

The zero-crossings of the Laguerre polynomials can be further approximated, leading to the following condition:

$$2l_B\sqrt{2N+1} = (\lambda - \frac{1}{4})a. \quad (3.7)$$

Here, λ is an integer parameter, and the left-hand side corresponds to twice the semiclassical cyclotron radius R_C in the N -th Landau level, given by $R_C = l_B \sqrt{2N+1}$. Thus, in a semiclassical approximation, the flat band condition can be rewritten as:

$$2R_C = \left(\lambda - \frac{1}{4}\right)a. \quad (3.8)$$

This implies that the Landau bandwidth vanishes when the cyclotron diameter $2R_C$ is commensurate with the lattice period a of the 1DSL. [4, 70, 101–103]

3.3.2 Commensurability Oscillations

The formation of Landau bands in a 1DSL leads to two distinct contributions to the conductivity. These contributions can be separated, as the longitudinal resistivity ρ_{xx} is primarily influenced by the band conductivity, while the transverse resistivity ρ_{yy} is governed by the density of states.

Band Conductivity

Band conductivity is the contribution to the electrical current that arises from the collective motion of charge carriers within the electronic bands. This motion is governed by the band structure, particularly its curvature, which determines the effective mass or, in the case of graphene, the Fermi velocity. Although scattering processes are always present and limit the mobility, band conductivity remains a fundamental contribution to the total conductivity and can dominate both in quasi-ballistic and in diffusion-dominated transport regimes.

In 1D superlattices, the periodic potential modulation results in the formation of minibands, which quantise the energy levels. This quantisation modifies the DOS, the Fermi velocity, and the dispersion relation. The band conductivity is therefore dependent on the periodicity and potential of the lattice, as well as the Landau levels in the presence of a magnetic field.

The contribution to the band conductivity for a 1D modulated 2DEG can be derived from the Kubo formalism. In the asymptotic limit of many filled Landau levels this reads as [102, 104]

$$\Delta\sigma_{yy} = \frac{1}{2} [1 - A(T/T_c)] + A(T/T_c) \cos^2 \left[2\pi \frac{R_c}{a} - \frac{\pi}{4} \right], \quad (3.9)$$

with $A(x) = \frac{x}{\sinh(x)}$ and the characteristic temperature $T_c = \frac{\hbar\omega_c k_B}{4\pi^2} a k_F$.

In this framework, the change in conductivity $\Delta\sigma_{yy}$ is added to the regular conductivity

σ_{yy} , while $\Delta\sigma_{xx}$ remains zero. This correction arises from the increased degeneracy of the Landau levels along the k_y direction. The group velocity $v_g = \langle Nx_0 | v_y | Nx_0 \rangle = \frac{1}{\hbar} \frac{dE_N}{dk_y}$ vanishes in the case of flat bands, where the dispersion is minimal and the DOS reaches a maximum. Therefore, the band conductivity correction is proportional to the group velocity, implying that broader, more dispersive bands result in higher conductivity. [105–107]

In monolayer graphene modulated by a superlattice, the band conductivity contribution is given as [70]

$$\Delta\sigma_{yy} = 8\pi^2 \frac{e^2}{h} \frac{V_0^2 \tau}{\hbar k_B T} \Phi, \quad (3.10)$$

with

$$\Phi = \frac{1}{4} X \exp(-X) \sum_{N=0}^{\infty} \frac{\exp[\beta(E - E_F)]}{(\exp[\beta(E - E_F)] + 1)^2} [L_N(X) + L_{N-1}(X)]^2. \quad (3.11)$$

The $\Delta\sigma_{yy}$ term further leads to an additional resistivity contribution $\Delta\rho_{xx}$, as the tensors are connected by $\vec{\sigma} = \vec{\rho}^{-1}$. The resulting oscillations that are visible in the longitudinal magnetoresistance are called **Weiss oscillations** or commensurability oscillations. This effect was first measured by Dieter Weiss et al. [108] in 1989 on a 1D modulated 2DEG realised with a GaAlAs/GaAs heterojunction. These oscillations arise when the cyclotron radius R_c becomes commensurate with the superlattice period a , leading to a periodic modulation of the carrier dynamics.

These oscillations are depicted in Fig. 3.7(a), where the measured magnetoresistivity (upper graph) shows SdH oscillations that are superimposed by Weiss oscillations with lower frequency in the longitudinal resistance (solid lines) and transversal (dashed lines). The lower graph shows the numerically calculated longitudinal magnetoresistivity for $T = 4.2$ K and $T = 2.2$ K, which reproduce the experimentally observed oscillations.

In graphene, the higher Fermi velocity of the Dirac electrons compared to a 2DEG with parabolic dispersion leads to more pronounced Weiss oscillations. [70, 101]

Scattering Conductivity

In contrast to band conductivity, which usually provides the dominant contribution, scattering processes can also modulate the conductivity. These include long-range, short-range, and interface scattering. In 1DSLs, the periodic potential primarily affects transport along the modulation direction, leading to anisotropic mobility: charge carriers are less mobile in modulation direction than perpendicular to it. [97]

The influence of scattering on the conductivity is closely tied to the DOS at the Fermi level. Oscillations in the bandwidth lead to oscillations in the DOS, which in turn

modulate the scattering contribution to the conductivity. These oscillations manifest in ρ_{yy} and are in phase opposition to the Weiss oscillations observed in ρ_{xx} . The oscillations in ρ_{yy} are related to the maxima in the DOS, which occur at flat-band positions in the band structure. While scattering also contributes to ρ_{xx} , this contribution is typically much weaker than the band-conductivity oscillations. [85, 104]

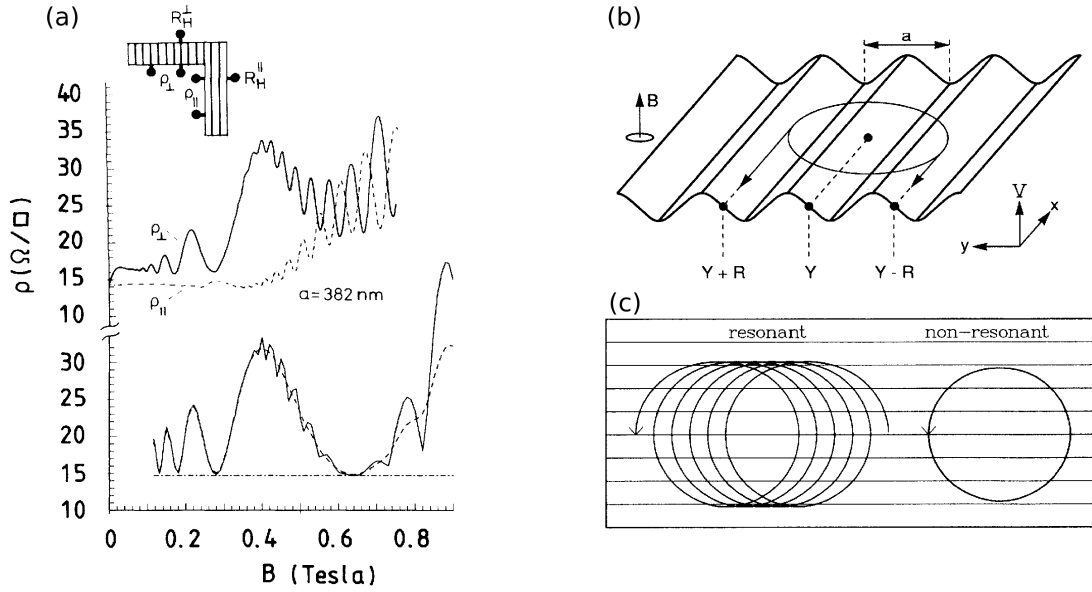


Fig. 3.7: (a) Upper panel: Measured longitudinal (solid) and transverse (dashed) magnetoresistivity of a 2DEG with a one-dimensional modulation. The SdH oscillations are superimposed by lower-frequency Weiss oscillations. The inset shows the sample geometry. Lower panel: Numerically calculated longitudinal resistivity $\rho_\perp \equiv \rho_{xx}$ at $T = 2.2\text{ K}$ and $T = 4.2\text{ K}$ (solid and dashed lines, respectively), using parameters adapted from the experiment. The calculation reproduces the Weiss oscillations. Adapted from [105]. (b) Cyclotron orbits of charge carriers in a magnetic field, modulated by a one-dimensional electrostatic potential (illustrated as a potential grating). The arrows indicate the $\vec{E} \times \vec{B}$ drift that arises when the cyclotron diameter satisfies the resonance condition. (c) Comparison of the resonant case ($2R_c = (\lambda + 1/4)a$), where the guiding center drift of the cyclotron orbit is maximal, and the non-resonant case, in which no net drift occurs. Adapted from [109].

Semi-classical picture: guiding centre drift

The previously introduced Weiss oscillations can also be explained within a semi-classical framework, as described by Beenakker [109]. This explanation is based on the motion of the guiding center of electrons moving with a cyclotron radius R_c , given by $x_0 = x - \frac{v_y}{\omega_c}$, $y_0 = y + \frac{v_x}{\omega_c}$.

Here, the cyclotron motion is superimposed by a drift motion in the y - or $-y$ -direction,

depending on the initial conditions. The combined effect of the perpendicular magnetic field B (along the z -direction) and an electric field $E(x)$ (along the x -direction) leads to an $\vec{E} \times \vec{B}$ -drift, with velocity $v_d = \frac{E(x)}{B}$.

Since $E(x)$ varies periodically with x , the drift velocity v_d is also modulated, leading to oscillatory behaviour in the longitudinal resistance. For $R_c \gg a$ (where a is the lattice period), the motion must be averaged over multiple lattice periods. This averaging leads to the same result as the quantum mechanical treatment, including the occurrence of minima in the longitudinal resistance at positions where the drift velocity is maximized. This indicates that the group velocity in the quantum mechanical description corresponds to the drift velocity in the semi-classical picture.

This picture is shown in Fig. 3.7(b) and (c), where (b) shows the cyclotron orbit compared to the potential grating of the 1DSL, that in combination with the magnetic field leads to the $\vec{E} \times \vec{B}$ drift, indicated by the arrows. In (c) the difference between the resonant case ($2R_c = \lambda + 1/4 a$), where the guiding-centre drift is maximal and the non-resonant case, without drift, is depicted. The straight lines show the equipotentials of the 1DSL; the cyclotron trajectories are numerically calculated.

However, this semi-classical explanation is only valid for Weiss oscillations in ρ_{xx} . The oscillations in ρ_{yy} require a quantum mechanical approach, as they depend on the scattering time τ , which is determined by the DOS. In this case, the oscillatory variation in DOS leads to corresponding oscillations in τ , which ultimately modulate the resistivity ρ_{yy} . [109–111]

Drienovsky et al. [106] presented the first experimental observation of commensurability oscillations in MLG samples with a one-dimensional superlattice potential. The devices featured lattice periods of 80 nm and 200 nm, as shown in Fig. 3.8. The MLG was encapsulated in hBN and subjected to a 1D periodic potential created by a patterned few-layer graphene bottom gate, in combination with a global back gate.

Clear signatures of commensurability oscillations were observed in the longitudinal resistance R_{xx} for both electrons and holes. The authors also investigated the temperature dependence of these oscillations to evaluate their robustness.

Panels (a)–(c) show R_{xx} for the 200 nm device at various carrier densities, controlled via different combinations of top and back gate voltages. In panels (a) and (c), the minima associated with commensurability oscillations are marked by blue dashed lines and correspond to the calculated flat-band conditions. In contrast, panel (b) shows data from the unmodulated region of the sample, where only conventional SdH oscillations are visible.

Panels (d)–(f) present measurements from the 80 nm device at high hole densities ($n \approx 4.5\text{--}4.8 \times 10^{16} \text{ m}^{-2}$) under various gate and temperature conditions. At $T = 40 \text{ K}$, the SdH

oscillations are significantly damped, which enhances the visibility of commensurability features, as seen in (d) and (e). Panel (f) demonstrates that commensurability oscillations persist up $\lambda = 6$, proving the high quality of the superlattice and the robustness of the effect.

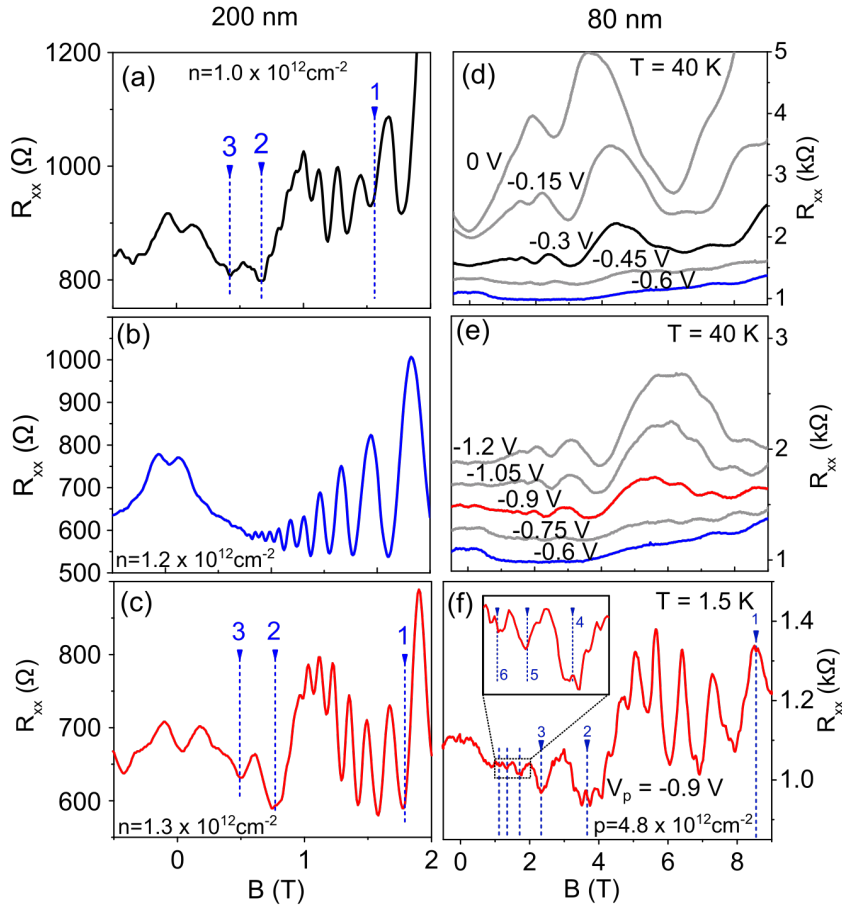
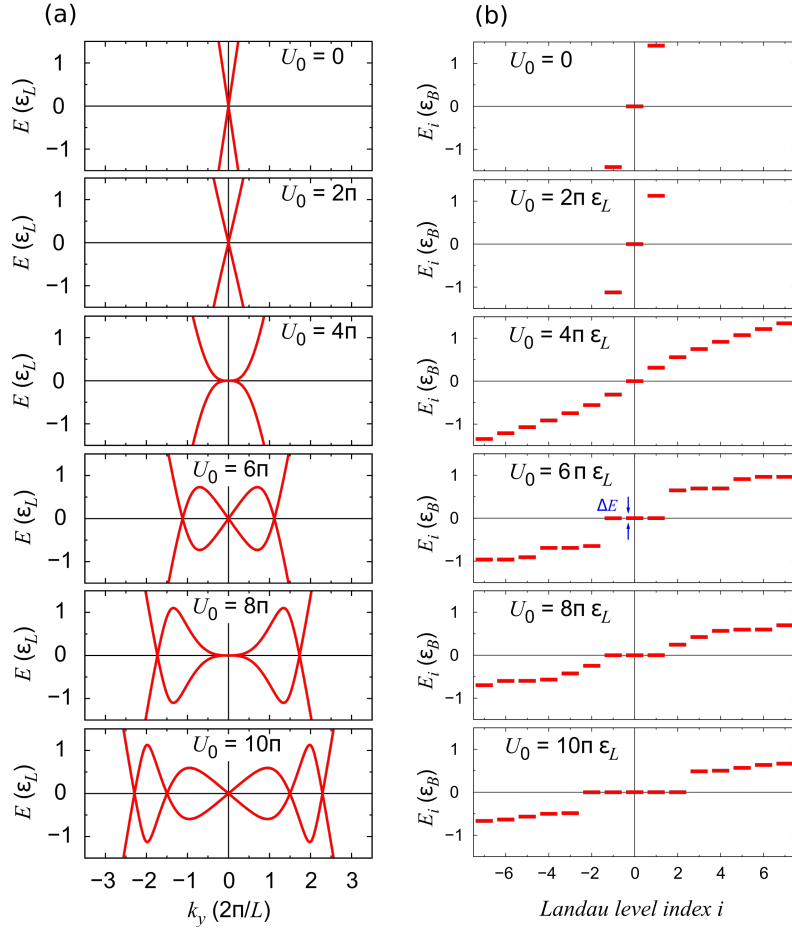


Fig. 3.8: (a)–(c): Commensurability oscillations in the longitudinal resistance R_{xx} of an MLG sample with a 1DSL period of 200 nm for different gate voltage and carrier density configurations. In (b), the sample is in the unmodulated regime, and only SdH oscillations are visible. Blue dotted lines mark the calculated flat-band conditions, which coincide with the observed minima caused by Weiss oscillations. (d)–(e): Longitudinal resistance in the 80 nm device for different gate voltage combinations. At $T = 40 \text{ K}$, the SdH oscillations are strongly damped, enhancing the visibility of Weiss oscillations. (f): Weiss oscillations observed in the hole regime, persisting up to $\lambda = 6$, as indicated by the blue dotted lines. Adapted from [106].

Fig. 3.9: (a) Energy dispersion as a function of k_y at fixed $k_x = 0$ for increasing 1DSL amplitudes U_0 . As U_0 increases, additional zero modes emerge, becoming clearly visible at $U_0 = 6\pi$, where extra Dirac points appear at the same energy as in pristine graphene. (b) Corresponding LL spectra. With increasing U_0 , the LL spacing decreases due to the renormalised (reduced) band velocity. At $U_0 = 6\pi$, the zeroth LL becomes three-fold degenerate due to the appearance of the new zero modes; at $U_0 = 10\pi$, it becomes fivefold degenerate. This modifies the quantum Hall sequence around $E = 0$. Adapted from [4].



3.3.3 Quantum Hall Effect and Landau Level Spectrum

The QHE in pristine monolayer graphene is characterised by quantised Hall conductivity plateaus at $\sigma_{xy} = 4 \left(N + \frac{1}{2}\right) \frac{e^2}{h}$, where N is an integer (compare Eq. (2.50)), and the factor of 4 arises from spin and valley degeneracy. When a 1DSL is applied to graphene, this quantisation pattern can be significantly altered due to modifications in the electronic band structure and LL spectrum. [97, 112, 113]

The introduction of a periodic superlattice potential leads to the formation of additional massless Dirac fermions. The physical potential can be written as $V(x) = V_0 \cos(Gx)$, where V_0 is the potential amplitude in energy units and $G = 2\pi/a$ is the reciprocal lattice vector. For theoretical analysis, it is convenient to introduce a dimensionless modulation amplitude U , defined as $U = 4V_0/(\hbar v_F G)$, so that the 1DSL potential entering the Hamiltonian can be expressed in the scaled form $V(x) = U \cos(Gx)$, where U is now dimensionless. These additional Dirac points appear symmetrically in the momentum space and share the same electron-hole crossing energy as the original Dirac point in

unmodulated graphene, as described by Park et al. [112]. The number of emergent Dirac cones increases with modulation amplitude U due to zone folding and coupling effects. [97, 112, 113]

This increase in Dirac points alters the LL degeneracy at charge neutrality. Specifically, the zero-energy LL becomes $4(2N + 1)$ -fold degenerate, where N is the number of Dirac cones on one side of the Brillouin zone. As a result, the quantised Hall conductivity no longer changes in steps of $4e^2/h$, but rather exhibits larger jumps of $4(2N + 1)e^2/h$, effectively skipping intermediate plateaus. These steps serve as a hallmark of the modified band topology.

At intermediate modulation strengths, the spacing between adjacent LLs decreases due to a reduction in the effective group velocity. At a certain critical value of U , the zero-energy LL becomes triply degenerate. This increased degeneracy is robust over a finite range of potential strengths, indicating that the associated quantum Hall plateaus are topologically protected and insensitive to small variations in the modulation amplitude, as calculated in detail by Park et al. [112]. This effect on the LL spectrum is illustrated in Fig. 3.9, where panel (a) shows the band structure for increasing 1DSL modulation amplitudes, and panel (b) displays the corresponding LL spectra. At a modulation strength of $U = 6\pi$, the extra pair of Dirac points generated by the 1DSL in (a) gives rise to a triply degenerate zero-energy LL, as seen in the corresponding panel of (b). For a 1DSL period $a = 50$ nm, as used in this thesis, the critical value $U = 6\pi$, which generates the additional Dirac points, corresponds to a modulation amplitude of $V_0 = U\hbar v_F G/4 \approx 368$ meV.

Field-Dependent Behaviour of the Landau Level Spectrum

The response of the Landau level spectrum to a 1DSL modulation in graphene is strongly dependent on the relative strength of the cyclotron energy $\hbar\omega_c$ and the modulation amplitude U , which denotes the physical amplitude of the superlattice potential ($V(x) = \frac{U}{2} \cos\left(\frac{2\pi x}{\lambda}\right)$). Following the classification of Wu et al. [97], three distinct magnetic field regimes can be identified: weak, intermediate, and strong field. The evolution of the Landau level spectrum in the weak and intermediate regimes for different modulation strengths is shown in Fig. 3.10.

Weak field regime ($\hbar\omega_c \ll U$): In this regime, where the cyclotron energy is much smaller than the superlattice potential. As a result, the matrix elements of the Hamiltonian are only weakly dependent on the guiding centre coordinate y_0 , and the LL spectrum becomes nearly flat in k space, reflecting negligible dispersion. For small modulation strengths (e.g. $U = 1$), the spectrum contains only a single Dirac point and the LLs closely resemble those of pristine graphene, but with an anisotropically renormalised

Fermi velocity. As the modulation increases (e.g. $U = 3$), two additional Dirac points appear, leading to a threefold degeneracy of the zero-energy LL. However, the Dirac point at K has a different average Fermi velocity compared to the newly generated ones, leading to a breaking of the degeneracy. This results in a splitting of the higher-order LL, which is most noticeable at $n = 1$ and $n = 2$. [97, 112]

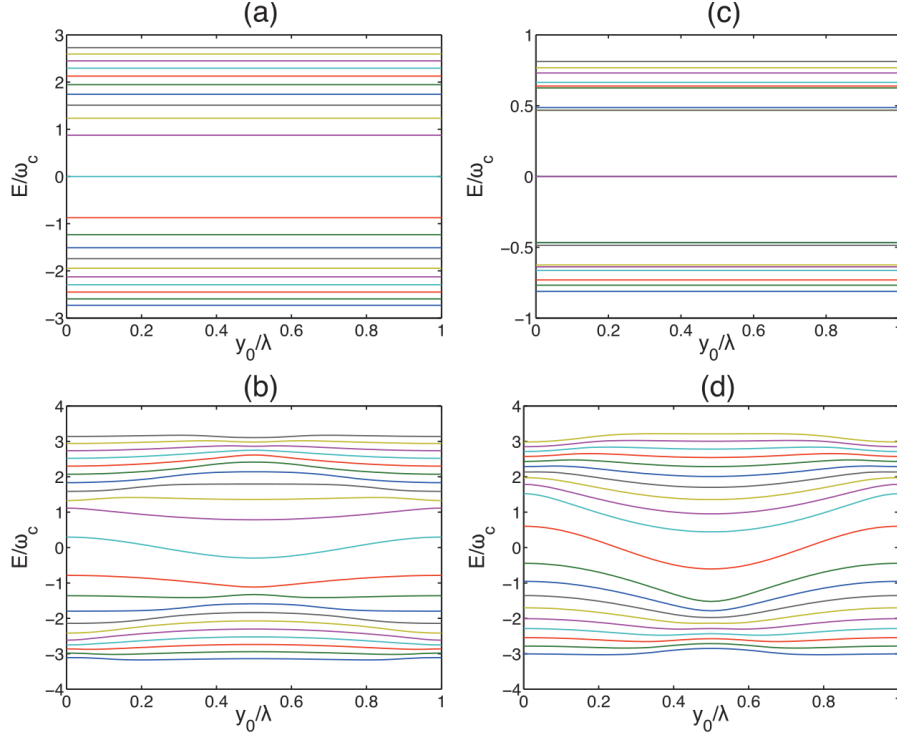


Fig. 3.10: Landau level spectrum of monolayer graphene with a 1DSL potential under varying modulation strengths U and magnetic field regimes. (a) and (b) show the spectra for $U = 1$, where the band structure still contains a single Dirac point. (c) and (d) correspond to $U = 3$, where the band structure exhibits three Dirac points at the same energy. The upper panels (a) and (c) represent the weak-field regime, where the magnetic length is $l_B = 2\lambda$. In this case, the LLs are split due to the 1DSL modulation. At $U = 3$, the zeroth LL hosts three times as many states as for $U = 1$, consistent with the number of Dirac points. (b) and (d) show the spectra in the intermediate magnetic field regime. The LLs become dispersive and modulate with the periodic potential. The modulation amplitude is three times larger for $U = 3$ than for $U = 1$. Additionally, the low-energy LLs for $U = 3$ overlap significantly, leading to a modified quantum Hall conductivity. Adapted from [97].

Intermediate field regime ($\hbar\omega_c \sim U$): In this regime, the 1DSL potential induces a strong dispersion in the LL spectrum. The energies of the low index LLs ($n = 0, 1, 2$) are modulated as a function of y_0 , and this modulation closely follows the shape of the underlying cosine potential of the 1DSL. For $U = 3$, this modulation is about three times

stronger than for $U = 1$. At these modulation strengths, the low energy LLs begin to overlap, significantly affecting the conductivity and making it complicated to observe distinct quantum Hall plateaus. In contrast, the higher LLs deviate from the simple cosine-like behaviour. This is attributed to the increased LL density at higher energies, where the LL spacing becomes comparable to the matrix elements of the 1DSL potential. In this case, first-order perturbation theory is no longer sufficient to describe the induced dispersion, and second-order effects lead to a breakdown of the cosine-like modulation. [97, 112]

Strong field regime ($\hbar\omega_c \gg U$): In the strong field limit, the LL spectrum reverts to that of unmodulated graphene. The familiar $\sqrt{nB}$ energy dependence is restored, and only one zero energy LL remains. At such high magnetic fields, the 1DSL potential acts merely as a small perturbation. Nevertheless, the LL energies still show a weak spatial dispersion that traces the shape of the superlattice potential. The ratio of modulation amplitude to LL spacing becomes extremely small, giving rise to Weiss oscillations which modulate the Shubnikov-de Haas oscillations in magnetotransport experiments, as described in section (3.3.2). [97, 112]

These findings are consistent with the earlier work of Matulis and Peeters [70], who applied perturbation theory to graphene in a weak 1DSL and demonstrated the emergence of Weiss oscillations due to the field-dependent broadening of Landau levels.

Influence on longitudinal and Hall conductivities

The impact of a 1DSL potential on the magnetotransport properties of graphene has been calculated by Wu et al. [97] using the Kubo formalism. The resulting longitudinal conductivities σ_{xx} (modulation direction) and σ_{yy} , as well as the Hall conductivity σ_{xy} , exhibit strongly field-dependent behaviour over the weak, intermediate and strong magnetic field regimes.

In the *weak field regime* the conductivities are strongly anisotropic, with $\sigma_{xx} > \sigma_{yy}$. This anisotropy arises both from the anisotropic renormalisation of the Fermi velocity in the absence of a magnetic field and from the nearly flat LL bands. The dominant contribution to the diagonal conductivities comes from off-diagonal velocity matrix elements. Since the velocity component v_x is always larger than v_y due to the anisotropic dispersion, σ_{xx} dominates over σ_{yy} in this regime. The Hall conductivity σ_{xy} shows well defined plateaus resulting from the almost flat energy bands as shown in Fig. 3.11(a). The values of the Hall conductivity around the Dirac points, taking into account spin and

valley degeneracies, are $\pm 1/2 \cdot (4e^2/h)$ for weak modulation ($U = 1$, Fig. 3.11(a)) and $\pm 3/2 \cdot (4e^2/h)$ for stronger modulation ($U = 3$, Fig. 3.11(c)), reflecting the increase in degeneracy due to the additional Dirac points. Further away from the Dirac point, conventional quantum Hall plateaus reappear as the chemical potential crosses higher LLs. [70, 97, 112]

In the *intermediate field regime*, the LLs acquire significant dispersion due to the influence of the 1DSL potential, and the conductivities remain anisotropic. In this case, σ_{yy} becomes larger than σ_{xx} because v_y acquires additional matrix elements due to the band dispersion, while v_x remains relatively unaffected.

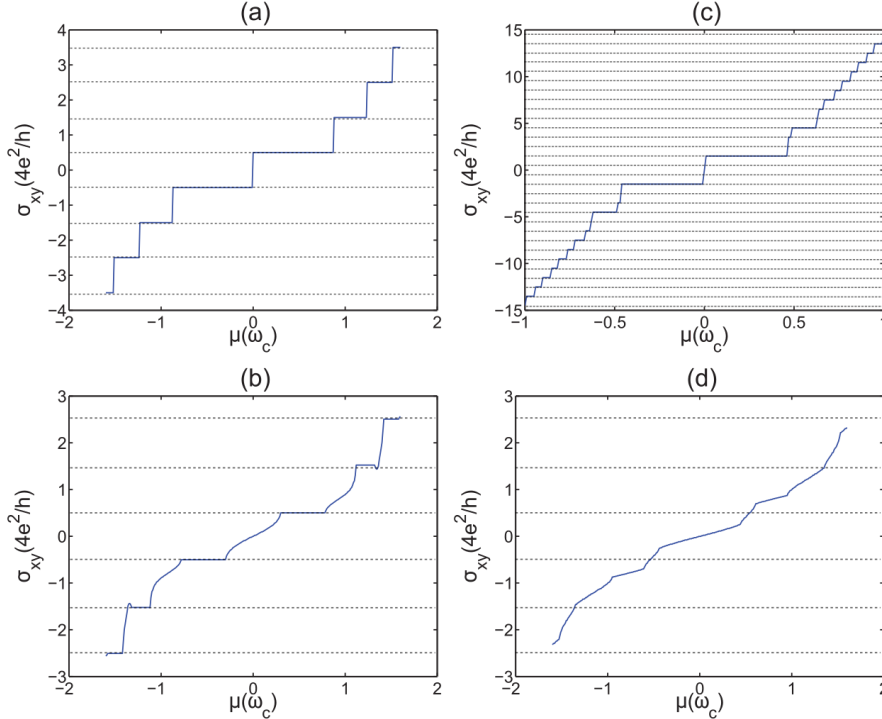


Fig. 3.11: Calculated Hall conductivity σ_{xy} for monolayer graphene with a 1DSL modulation. Grey dotted lines indicate the expected plateau values of pristine MLG at $\sigma_{xy} = (N + 1/2)4e^2/h$. (a) and (b) correspond to a modulation strength of $U = 1$, where the band structure contains a single Dirac point. (c) and (d) show the case of $U = 3$, where three Dirac points are present. In the weak-field regime ((a) and (c)), σ_{xy} exhibits well-defined QH plateaus due to nearly flat LLs. For $U = 1$, the plateaus follow the standard graphene sequence with steps of $(\pm 1/2)(4e^2/h)$ around the Dirac point. For $U = 3$, the zeroth LL is threefold degenerate, leading to plateau steps of $(\pm 3/2)(4e^2/h)$ around zero energy. In the intermediate-field regime ((b) and (d)), the LLs become dispersive and begin to overlap, which weakens the visibility of the QH plateaus. For $U = 1$, the LLs still remain sufficiently separated for small residual plateaus to form when the chemical potential lies between adjacent bands. Adapted from [97].

The position of the σ_{xx} peaks now coincides with the extrema of the energy bands where the DOS is highest. In contrast, σ_{yy} shows minima at the band edges, where the velocity matrix element $\langle v_y \rangle$ vanishes. These dips indicate suppressed intra-LL contributions at the band boundaries. For weak modulations, σ_{yy} may approach zero if there is no overlap between LLs. For strong modulation, however, LL overlap ensures that σ_{yy} remains finite and dispersive. The Hall conductivity no longer shows sharp quantised plateaus as the LLs are broadened and overlapped. For weak 1DSL modulation (Fig. 3.11(b)), small residual plateaus may still appear when the chemical potential lies between well-separated LLs. For strong modulation (Fig. 3.11(d)), however, the band dispersion prevents the formation of distinct quantised steps.[70, 97, 112]

In the *strong field regime*, the LL structure of pristine graphene is effectively restored, as the magnetic length becomes smaller than the 1DSL period and the cyclotron energy dominates over the modulation. Consequently, both σ_{xx} and σ_{yy} become isotropic, and the Hall conductivity σ_{xy} exhibits the well-known half-integer quantum Hall plateaus. The 1DSL potential acts only as a small perturbation, with minor influence on the spectrum. [70, 97, 112]

Symmetry of the 1DSL potential

The symmetry of the 1DSL potential plays a crucial role in determining the structure of the low-energy spectrum and the associated quantum Hall features. As discussed in the supplemental material of Park et al. [112], deviations from perfect symmetry in the superlattice potential can significantly alter the emergence and alignment of massless Dirac fermions.

In the case of a superlattice potential with ideal odd symmetry (see Fig. 3.12(a) to (c)), such as a Kronig-Penney potential, additional branches of massless Dirac fermions are generated, and their electron-hole crossing energies coincide exactly with the Dirac point of pristine graphene. This energetic alignment leads to an increased degeneracy of the zero-energy LL, which becomes threefold degenerate for each spin and valley degree of freedom, as discussed earlier.

However, if the odd symmetry is broken - for example, by introducing higher harmonics or asymmetry in the potential shape - additional zero modes can still be generated, but their energies no longer coincide with the original Dirac point (see Fig. 3.12(d) to (f)). As a result, the zero-energy LL is no longer triply degenerate. Instead, higher degeneracies may occur in other low index LLs, depending on how the shifted zero modes align with

the LL spectrum. This results in modified Hall conductivity steps where plateaus can still be skipped, but at different energies and with less regularity.

In contrast, breaking the even symmetry of the potential has a more drastic effect (see Fig. 3.12(g) to (i)). If the even symmetry is not preserved, the formation of new Dirac points is completely suppressed. As a result, no additional zero modes appear and the LL degeneracy remains at the standard fourfold value (from spin and valley), following the quantum Hall sequence of unmodulated graphene without any anomalous steps. [96, 112]

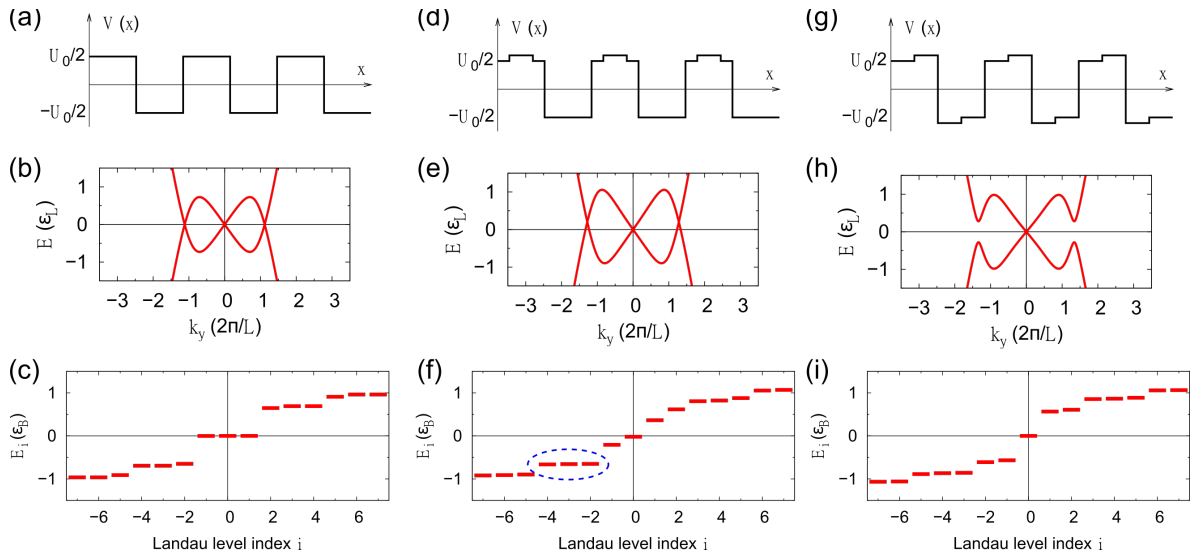


Fig. 3.12: (a)–(c): Kronig–Penney-type periodic potential with lattice period L and the corresponding electronic band structure and LL spectrum. The emergence of additional zero modes increases the degeneracy of the zeroth LL. (d)–(f): Periodic potential with broken *odd* symmetry due to a perturbation. The resulting zero modes no longer align in energy with the Dirac point of pristine graphene. Consequently, the zeroth LL does not exhibit enhanced degeneracy. However, some lower-index LLs may still show increased degeneracy, as indicated by the dashed blue circle. (g)–(i): Periodic potential with broken *even* symmetry. In this case, symmetry breaking prevents the formation of additional zero modes, and the LL degeneracy remains unchanged compared to pristine graphene. Adapted from [4].

3.4 Bilayer Graphene Superlattices

Compared to monolayer graphene, bilayer graphene superlattices exhibit fundamentally different physics due to their quadratic low-energy dispersion, increased sensitivity to external fields, and the absence of Klein tunnelling protection. In monolayer graphene, backscattering is strongly suppressed at normal incidence due to chirality and the

associated Klein tunnelling. In bilayer graphene, however, this effect is absent, and backscattering remains possible even at normal incidence, owing to the different pseudospin structure and reduced symmetry. Furthermore, the parabolic band structure of bilayer graphene—unlike the linear Dirac cones in monolayer graphene—is highly susceptible to external perturbations, as discussed in Section 2.2. An applied electrostatic 1DSL potential can affect the band structure in two main ways: (i) by creating an interlayer bias through different voltages on the two layers, a tunable band gap can be opened; and (ii) by applying the same voltage to both layers, the band structure can be rigidly shifted. In realistic bilayer 1DSL samples, these two effects are typically superimposed, and intrinsic doping or substrate-induced asymmetries may further alter the band structure even before any external field is applied.

The following section was written mainly based on the theoretical frameworks and numerical studies from [6, 113–116], complemented by experimental insights and band structure analyses from [117–119], as well as recent comprehensive reviews such as [120].

3.4.1 Band structure of a Bilayer Graphene 1DSL

To analyse miniband formation in bilayer graphene under a 1DSL modulation, we follow the approach by Killi et al. [6], who formulated the problem using the effective two-band model in a Fourier spinor basis. The low-energy Hamiltonian near the K point for unbiased BLG is given by equation (2.15). The presence of a 1DSL potential introduces additional periodic modulation, which may act identically or differently on the two graphene layers.

The superlattice potentials on the top and bottom layers are described by two periodic functions $V_1(x)$ and $V_2(x)$, which can be expanded in Fourier components:

$$V_\alpha(x) = \sum_G V_\alpha(G) e^{iGx}, \quad G = \frac{2\pi n}{\lambda}, \quad \alpha = 1, 2. \quad (3.12)$$

The eigenstates are expanded in a plane-wave spinor basis

$$\Psi_p(\mathbf{r}) = \frac{1}{\sqrt{A}} e^{i(k_y y + (k_x + G_p)x)} \chi_p, \quad (3.13)$$

where $G_p = 2\pi p/\lambda$ and χ_p are two-component spinors in the (A_1, B_2) basis. In this basis, the total Hamiltonian becomes a matrix in both momentum and pseudospin space

$$H = \sum_p \psi_p^\dagger H_0(k_x + G_p, k_y) \psi_p + \sum_{p,g} \psi_p^\dagger W_{p,g} \psi_{p+g}. \quad (3.14)$$

The coupling terms $W_{p,g}$ arise from the superlattice potential and couple states with momenta differing by reciprocal lattice vectors $G = G_g$. Following Killi et al., they take the form

$$W_{p,g} = \begin{pmatrix} V_1(G_g) + V_2(G_g) & V_1(G_g) - V_2(G_g) \\ V_1(G_g) - V_2(G_g) & V_1(G_g) + V_2(G_g) \end{pmatrix}, \quad (3.15)$$

which acts on the spinor $\psi_p = (\psi_{A_1,p}, \psi_{B_2,p})^T$. The structure of this matrix reflects the symmetry (or asymmetry) of the potential acting on the two layers.

This formalism allows one to distinguish between different types of superlattice modulations. For a *chemical potential* superlattice, the modulation is identical on both layers: $V_1 = V_2$, and $W_{p,g}$ becomes diagonal. In this case, inversion symmetry is preserved, and the periodic potential leads to the appearance of new Dirac points at finite momentum depicted in Fig. 3.13(c) and (d). These additional Dirac points emerge due to Bragg scattering at the boundaries of the mini-Brillouin zone and remain gapless due to the symmetry of the system. Their exact location and shape depend on the strength and period of the modulation.

For an *electrostatic bias* superlattice, the modulation is opposite on the two layers: $V_1 = -V_2$, and $W_{p,g}$ becomes purely off-diagonal. This breaks inversion symmetry and opens a band gap at the original Dirac point, turning the system into a semiconductor. The opening band gap with rising 1DSL potential strength is shown in Fig. 3.13(a) and (b). Physically, this corresponds to a periodic sequence of regions with alternating layer bias, often called *kinks* and *antikinks*, where the direction of the interlayer potential flips. Each kink or antikink can host edge-like states that propagate along the domain wall. In a periodic structure, these localised states couple and form minibands, whose properties are determined by the superlattice parameters.

In realistic samples, however, the potential is rarely perfectly symmetric. In devices with only one patterned gate, the modulation acts more strongly on the nearby graphene layer, leading to $V_1(G) \neq \pm V_2(G)$. The resulting band structure thus reflects a hybrid modulation, with both diagonal and off-diagonal contributions, and cannot be captured by pure chemical or electrostatic models alone.

Such asymmetries may lead to the opening of band gaps, a flattening of minibands, or more complex modifications of the dispersion, depending on the amplitude and symmetry of the potential.

In addition to the sinusoidal superlattice potential considered by Killi et al., we also take into account the work of Barbier et al. [5], who investigate the effects of a square-wave periodic potential. They use a four-band model to describe the low-energy physics of bilayer graphene, which incorporates the full set of hopping terms between the layers, unlike the two-band approximation in Killi et al. While the two-band model suffices for most low-energy analysis, the inclusion of the higher-energy bands provides additional

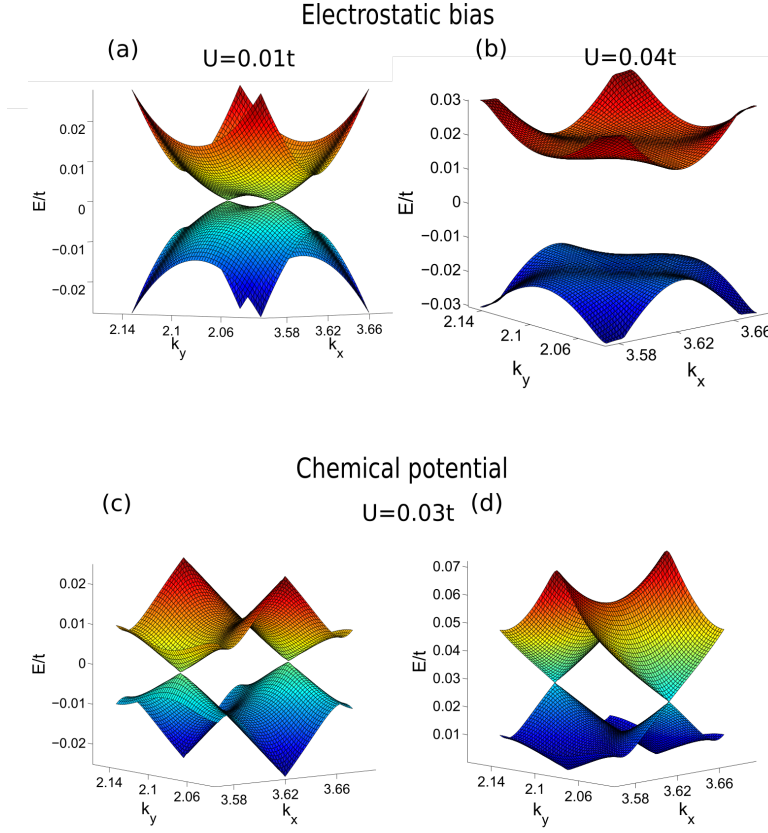


Fig. 3.13: (a) and (b): Energy spectrum of bilayer graphene under a *electrostatic bias* 1DSL modulation with modulation strengths of $U = 0.01t$ and $U = 0.04t$, respectively, where t is the nearest-neighbour hopping energy. In (a), the two Dirac points begin to split along the k_y -direction near the K point. With increasing modulation strength, a full band gap opens at the K point, as shown in (b).

(c) and (d): Energy spectrum of bilayer graphene under a symmetric *chemical potential* 1DSL with modulation strength $U = 0.03t$. In (c), a pair of Dirac points appears at zero energy, while in (d) an additional pair emerges at finite energy. Adapted from [113].

insight into the band structure, especially in the presence of strong superlattice modulations. The four-band model also accounts for more complex interlayer coupling, which can influence the miniband formation, particularly when asymmetric gating conditions are applied.

The authors further explore the effects of these modulations on transmission across the superlattice. They calculate the transmission probability for electrons through periodic barriers, showing how the periodicity and shape of the potential modulate the band structure and lead to the opening of gaps or the formation of flat bands.

3.4.2 Magnetotransport in a Bilayer Graphene 1DSL

Landau bands and commensurability oscillations in BLG 1DSLs

In contrast to monolayer graphene, where the Weiss oscillations and Landau minibands are governed by a linear dispersion and a Berry phase of π , bilayer graphene features a quadratic low-energy band structure and a Berry phase of 2π , leading to distinct modifications in the oscillation phase and Landau level broadening.

In bilayer graphene with a 1DSL, the LL structure is significantly modified, similar to monolayer graphene. The originally sharp LLs are broadened by the periodic potential and evolve into dispersive minibands. Tahir et al. [121] studied this system using a Hamiltonian of the form $H_{\text{SL}} = V_0 \cos(Kx)$, with eigenenergies $\epsilon_{n,x_0} = V_n \cos(Kx_0)$, where $x_0 = k_y l_B^2$ is the guiding center coordinate and V_n is the LL-dependent modulation amplitude.

The Landau level bandwidth is given by

$$U_N = \frac{V_0}{2} \exp\left(-\frac{1}{2}X\right) [L_N(X) + L_{N-2}(X)], \quad (3.16)$$

where $L_N(X)$ are Laguerre polynomials and $X = \frac{1}{2}(Ka)^2 l_B^2$. Compared to monolayer graphene, where the terms $L_N + L_{N-1}$ appear, the use of L_{N-2} in bilayer graphene reflects its different underlying band structure.

To evaluate the conductivity, Tahir et al. employed the Kubo formalism in the linear response regime. In the semiclassical limit ($N \gg 1$), the Weiss oscillations take the form

$$\Delta\sigma_{xx} = \frac{T}{4\pi^2 T_c} \left[1 - A\left(\frac{T}{T_c}\right) + 2A\left(\frac{T}{T_c}\right) \cos^2\left(2\pi \frac{R_c}{a} - \frac{1}{8}\right) \right], \quad (3.17)$$

with the temperature damping factor $A(x) = \frac{x}{\sinh(x)}$, and the characteristic temperature $T_c = \frac{\omega_c k_F a^2}{4\pi^2}$.

In bilayer graphene, the LL energies scale as $\sqrt{N(N-1)}$, in contrast to the $\sqrt{N}$ scaling in monolayer graphene. This difference reflects the underlying quadratic low-energy band structure and the Berry phase of 2π in BLG.

The Weiss oscillations in bilayer graphene arise from the same semiclassical commensurability condition as in monolayer graphene and therefore do not exhibit a Berry-phase-induced phase shift.

The temperature damping of Weiss oscillations in bilayer graphene lies between that of a conventional 2DEG and monolayer graphene: they are less strongly damped than in a 2DEG but more so than in MLG with 1DSL.

The resonance condition for Weiss oscillations in bilayer graphene 1DSLs is similar to monolayer graphene, where minima in the conductivity are observed when the cyclotron radius R_c matches integer multiples of the superlattice period. Thus, the position of the oscillations is determined by the condition

$$2R_c = \left(\lambda - \frac{1}{4}\right) a, \quad (3.18)$$

with $\lambda \in \mathbb{N}$, where a is the superlattice period and R_c is the cyclotron radius. In bilayer graphene, the Fermi wavevector k_F behaves similarly to monolayer graphene at

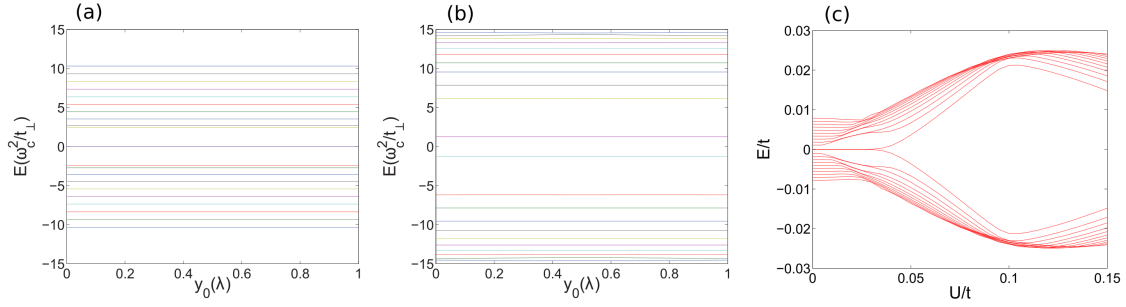


Fig. 3.14: Energy spectrum for BLG with a chemical potential 1D superlattice and a weak magnetic field. **(a)** shows the spectrum for a 1DSL potential strength of $U = 0.01t$, where degenerate pairs of Landau levels appear due to the anisotropic Dirac cones at zero and finite energy. At higher energies, the spacing between Landau levels increases. **(b)** corresponds to $U = 0.04t$. **(c)** shows the evolution of energy levels as a function of the 1DSL potential strength V . Adapted from [113].

low energies, with $k_F \propto n^{1/2}$. Therefore, the Weiss oscillations in bilayer graphene are expected to occur at similar positions in $1/B$ or n as in monolayer graphene, with the main differences being the broader Landau level structure. [121]

Landau Levels in bilayer graphene 1DSLs

Next, we examine the behaviour of Landau levels in bilayer graphene in the presence of a 1DSL potential. The influence of such modulations has been studied in detail by Killi et al. [6] and Si Wu et al. [97], who analyse both chemical potential and electrostatic superlattices in the presence of a perpendicular magnetic field.

In the absence of a superlattice, the low-energy spectrum of BLG is quadratic, and the system supports massive charge carriers with an effective mass m and cyclotron frequency $\omega_c = eB/m = 1/(ml_B^2)$. The corresponding LLs are equally spaced, and the zero energy level is fourfold degenerate due to spin and valley symmetry.

Chemical potential superlattices ($V_1 = V_2$) significantly alter this picture. At low magnetic fields and small modulation strength, the band structure develops two anisotropic Dirac cones, leading to a low-energy spectrum similar to that of monolayer graphene, but with anisotropic Fermi velocities v_x and v_y . In this case, the cyclotron frequency becomes $\omega_c = \sqrt{2v_x v_y}/l_B$.

These cones give rise to two nearly degenerate sets of Landau levels, both at zero energy and at finite energy. The levels appear in pairs due to the valley symmetry and the presence of two Dirac cones per mini-Brillouin zone. At higher energies, the LL spectrum deviates from the Dirac-like behaviour, and the LL spacing becomes non-uniform, as depicted in Fig. 3.14(a).

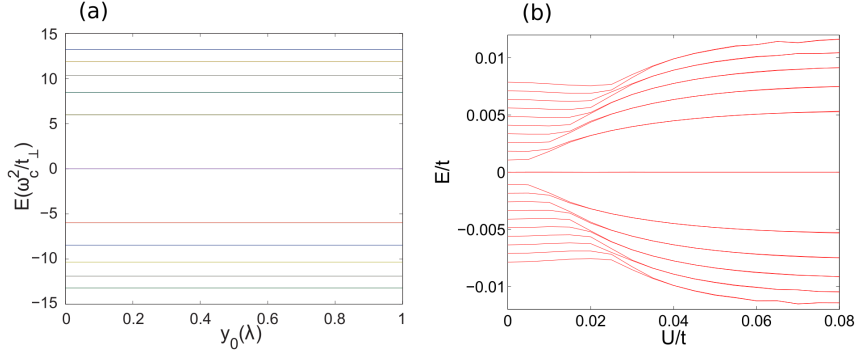


Fig. 3.15: (a) Energy spectrum for bilayer graphene with an electric field 1D superlattice and a weak magnetic field. (b) Evolution of energy levels in the electric field superlattice as a function of the potential strength U . Adapted from [113].

As the 1DSL potential increases, the system undergoes a crossover from a non-relativistic regime (with parabolic dispersion) to a relativistic regime (with linear Dirac-like dispersion). In this intermediate regime, both energy scales — the superlattice potential and the magnetic field — compete. The zero-energy LLs remain doubly degenerate, but the higher levels shift and develop structure. Eventually, for large modulation strength, the Dirac cones move away from zero energy, gapping the zero modes and pushing the spectrum away from the Dirac point, shown in Fig. 3.14(b). Fig. 3.14(c) displays the evolution of the LLs with increasing 1DSL potential.

At very large superlattice amplitudes, new Dirac cones reappear, and the spectrum becomes doubly degenerate again. However, due to the bilayer nature of the system, the resulting LLs are only weakly dispersive, unlike the highly dispersive levels in monolayer graphene superlattices.

Electrostatic bias superlattices ($V_1 = -V_2$), on the other hand, always support a pair of zero-energy Dirac points, associated with the topological kink and antikink states localised at domain walls in the interlayer bias. These localised modes hybridise in the periodic structure and give rise to a doubly degenerate zero-energy Landau level.

At small 1DSL potential, the LL spectrum resembles that of a chemical potential SL, with equally spaced levels and a crossover at intermediate modulation strength, as depicted in Fig. 3.15(a). However, a key difference is that the linear Dirac approximation remains valid over a wider energy range, and thus the relativistic behaviour of the system persists to higher LLs. Figure 3.15(b) displays the evolution of the LLs with increasing 1DSL potential.

In the strong 1DSL limit, the Dirac cones dominate the low-energy physics, and the higher Landau levels become doubly degenerate, reflecting the presence of two well-defined cones. The LL spectrum in this regime strongly deviates from the non-relativistic BLG

picture, and the structure is largely determined by the coupling of the kink and antikink modes to the magnetic field. [6, 97, 113, 122]

4 Sample Fabrication and Measurement Setup

In order to realise the discussed superlattice effects, a dual-gated sample design was used [123, 124], which has already been utilised successfully to show 1D and 2D SL effects in graphene as Weiss oscillations, Landau bands and the Hofstadter butterfly [92, 106, 125]. In this work a further improved design and fabrication for gate-tunable 1D superlattices has been developed. Superlattice periods were minimised down to 50 nm and high mobilities up to $\sim 90\,000\text{ cm}^2/\text{Vs}$ were achieved, leading to well-defined band structure modifications. A first approach towards 1D superlattices in bilayer graphene was made to study the complex band structure and band gap modification in this system. Furthermore, steps towards perfectly symmetric potentials were investigated. In two bachelor theses, various new resist systems were investigated in the aim for reaching even smaller periods in our samples [126, 127].

The fabrication techniques developed in this work were also employed for the design and fabrication of graphene-based samples used for the investigation of ratchet effects under terahertz excitation, see Refs [128–130].

The following chapter gives an overview of all of the fabrication steps used for the samples that will be discussed later. First, the fabrication of the gates is explained, followed by the Hall bar and contacting processes are discussed. A brief overview is given of further, promising fabrication steps towards symmetrical superlattices. Finally, the measurement setup is shown.

A complete and detailed process list can be found in the Appendix.

4.1 Preparation of Few-layer Graphene Stripes

The patterned bottom gate (PBG) is the basis for our 1D graphene superlattice structures and is the first step in fabrication. For this purpose, a few-layer graphene (FLG) of 2-5

layers is either directly exfoliated onto a silicon chip with an insulating layer of 285 nm SiO_2 or stacked on hBN. The exfoliation of graphene and hBN is done using Nitto tape (ELP BT - 150ECM) and the famous Scotch tape method of Geim and Novoselov [11]. The chips are then annealed in a vacuum (pressure ~ 10 mbar) at 400°C for about 1 hour. The chosen FLG flake should be large enough to define a square of $5\text{ }\mu\text{m} \times 5\text{ }\mu\text{m}$ on it, where the stripe pattern can be etched. The limit on the size of the pattern field is a result of the proximity effect during electron beam lithography (EBL), as a larger size would lead to overexposure [131]. The stripes are written as single lines and have a period down to 50 nm. A thin layer (~ 30 nm) of PMMA (polymethylmethacrylate) 950k 2% was used as the resist and a cooled ($\sim 10^\circ\text{C}$) mixture of water and isopropyl alcohol in a ratio of 3:7 was used for development for a high resolution [132]. The EBL process was performed with the highest available acceleration voltage (30 kV), a small aperture (10 μm) and a small working distance (~ 4 mm). For better results, several stripe fields were written into a FLG flake with a slight variation of the dose value, as the process is so sensitive. After a development step, a short (~ 16 s) oxygen plasma etching process at RIE (reactive ion-etching) was performed. It was found that the most favourable outcomes were generally achieved when the exposed resist stripes were underdeveloped. The residues between the stripes then serve to stabilise the pattern. The subsequent oxygen process etched all the residues and the underlying FLG. With that, the thin resist stripes and thus the FLG underneath remained after this etching step.

As mentioned above, for some samples a hBN layer was used underneath the FLG for etching into PBG stripes. This further improved the fabrication as the hBN leads to a higher dose value due to the reduced backscattering of electrons. [133]

As a result, the dose range for a suitable stripe pattern became wider and the yield increased.

For a uniform superlattice electric field, the PBG and the contact area to the graphene-hBN stack should be as clean as possible. A strong resist stripper (Remover PG from Microchem) was used at 60°C for ~ 20 h. This was followed by annealing at vacuum and 400°C for ~ 1 h and the whole cleaning process was repeated.

The PBG was then etched with oxygen plasma with RIE into a square of about $6\text{ }\mu\text{m} \times 6\text{ }\mu\text{m}$ so that the stripes were connected on both sides and the cleaning steps were applied again. The reduced field size of the PBG is needed for the subsequent stack of hBN and MLG on top: the contacts to the MLG have to be outside of the PBG to prevent any short circuits. Figure 4.1 shows an atomic force microscope (AFM) image of a finished PBG and the schematic working principle of the dual-gate design. Here, the PBG mainly controls the average carrier density in the MLG and screens the global silicon BG, which mainly induces the periodic modulation. The shape of this SL modulation depends on the width and period of the PBG stripes and the thickness of the thin hBN layer between the

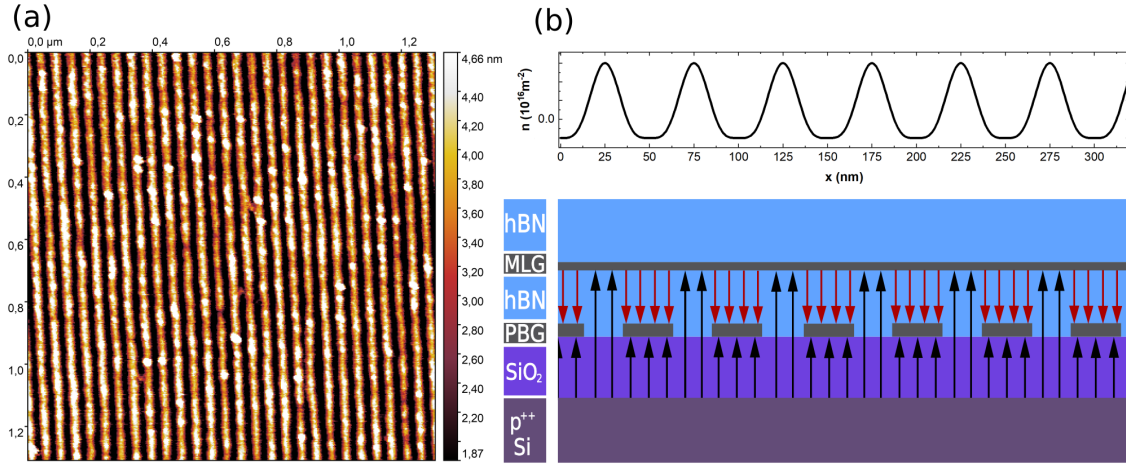


Fig. 4.1: (a) AFM image of a finished PBG stripe pattern with a period of 50 nm. (b) Schematic of carrier density modulation along the gate stripes with voltage applied to both PBG and global silicon BG. Black and red arrows indicate the electric field from both gates. Since the BG is mainly shielded by the PBG stripes, a periodic modulation is induced in the MLG. The average carrier density in the MLG is primarily controlled by the PBG. The resulting carrier modulation is asymmetric along the x -axis, which affects the band structure modulation.

PBG and MLG - and also on possible residuals from the fabrication processes. For the samples in this work, this potential was always quite asymmetric, as shown in the upper graph of Fig. 4.1(b). Here, asymmetric means that the peaks and dips of the potential shape differ, leading to an asymmetric distortion of the band structure as discussed in chapter 3.

4.2 Stacking and Etching of Graphene-hBN Heterostructures

Immediately after the last annealing step, the stack of hBN and graphene is placed onto the PBG to ensure a clean interface. For this purpose, a dry van der Waals stacking process is used [134]. This process was also used for the samples that had an additional hBN layer underneath the PGB, as mentioned above. A thin layer of PC (polycarbonate) is stretched over a drop of PDMS (polydimethylsiloxane) on a microscope slide. This stamp can be built into an optical microscope with an additional heatable stage. This allows a pick-up process of the selected flakes by heating the sample to $\sim 110^\circ\text{C}$, making

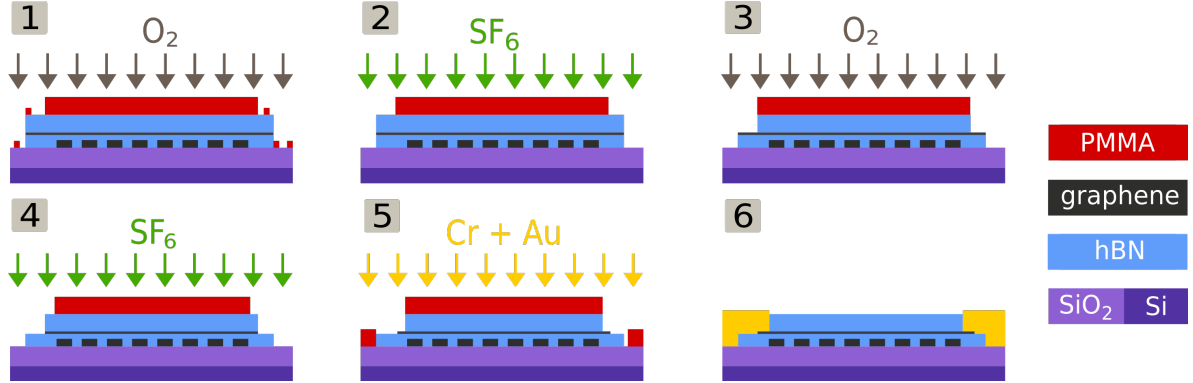


Fig. 4.2: Schematic of all process steps after placing the hBN-graphene-hBN stack on the PBG. (1) After Hall bar lithography with PMMA, a mild oxygen plasma is applied to reduce resist residues that could cause uneven etching. (2) RIE with SF_6 is used to etch the top hBN, stopping at the encapsulated graphene layer. (3) RIE with oxygen removes only the graphene layer, leaving the bottom hBN insulating the PBG. (4) After lithography with two layers of PMMA for contacts, RIE with SF_6 etches only hBN to enable area contacts. (5) Evaporation of chromium and gold is performed to form the contacts. (6) Side view of the finished sample.

them stick to the PC. The sample stage can be lifted by a screw and gently brought into contact with the PC film while monitoring the process through the microscope. When released, the flake should adhere to the PC film or to previously picked up flakes, as this step can be repeated for multiple flakes. The microscope stage allows precise placement and stacking of the flakes. By heating the sample to $\sim 180^\circ\text{C}$ the PC melts and the stack can be placed onto the desired chip or in our case the prepared PBG. The flakes used for the samples in this thesis were - from top to bottom - a thicker ($\sim 25\text{ nm}$) hBN flake, then a graphene flake, and lastly a thin ($\sim 2\text{-}6\text{ nm}$) hBN flake. It is important to ensure that the lower thin hBN between PBG and graphene covers the entire PBG square as it acts as an insulating layer. The graphene layer should also cover the PBG because the sequential etching in the next step stops at the graphene. Otherwise one would etch down to the PBG and risk a short circuit there.

The next step is to etch the stack into a Hall bar and for that, a standard EBL process with PMMA 950k 5% resist is applied with an acceleration voltage of 30 kV and a $20\text{ }\mu\text{m}$ aperture. MIBK (methyl isobutyl ketone) dissolved in isopropyl alcohol with a ratio of 1:3 is used for development. To remove any residue of the resist in the exposed areas, a short and mild oxygen plasma at RIE is applied (see panel (1) in Fig. 4.2), which is necessary for a clean and even etching result [135]. The stack is now etched sequentially layer by layer, starting with SF_6 removing the top hBN (panel (2)). This process stops at the graphene layer, which is removed by oxygen plasma in the next step (panel (3)). The RIE chamber must be cleaned and properly pumped between etching steps to ensure that no residual gases remain in the chamber, which ensures that the etching is anisotropic.

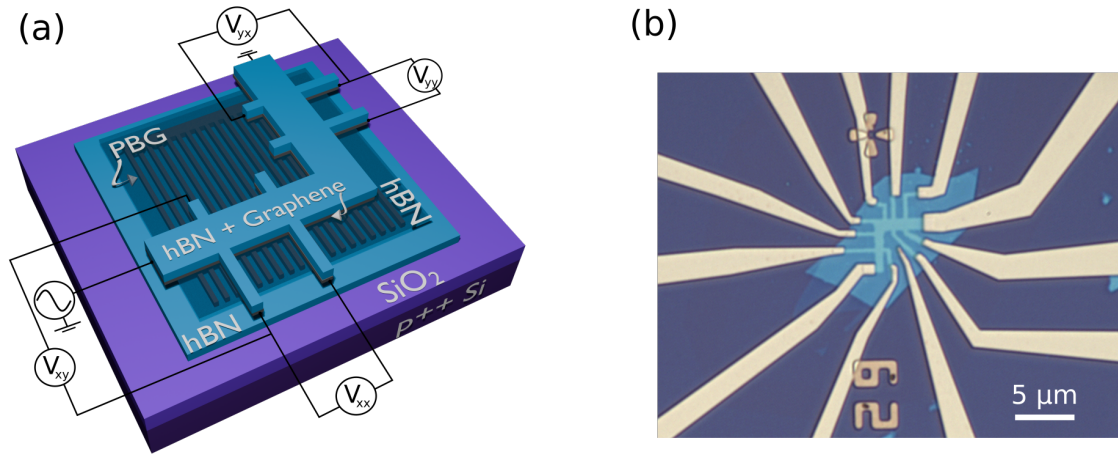


Fig. 4.3: (a) Schematic overview of the sample structure showing an L-shaped Hall bar of MLG encapsulated in hBN and insulated from the underlying PBG by a thin hBN flake. The different combinations of voltage probes along and perpendicular to the modulation direction are also indicated. (b) Optical microscope image of a finished 1DSL sample including gold contacts.

The sample is designed as a L-shaped Hall bar for simultaneous measurements in and perpendicular to the modulation direction of the 1DSL.

4.3 Contacts and final Steps

To make electrical connection to the sample, a system of two PMMA resists is used. This creates an undercut which eases the lift-off process of the evaporated materials. PMMA 200k 9% and PMMA 950k 5% are used and developed with MIBK dissolved in isopropyl alcohol. Any residual resist is then removed with a mild oxygen plasma (not shown in Fig. 4.2), followed by a SF_6 RIE process to expose the MLG at the tips of the Hall bar arms (panel (4)). Different approaches for contacting graphene samples, including edge contacts and the surface contacts shown here, have been compared in an bachelor thesis by Joachim Jurenka under the supervision of the author [136]. The results showed no measurable difference in contact resistance between these approaches, and as area contacts were easier to integrate into the fabrication process, they were used in all samples presented in this work.

The sample is loaded into the evaporation chamber immediately after the SF_6 etching process for a clean interface, where 0.5 nm of chromium and ≈ 50 nm of gold are deposited (panel (5)). This is followed by a lift-off process in acetone on a heating plate at 60°C for

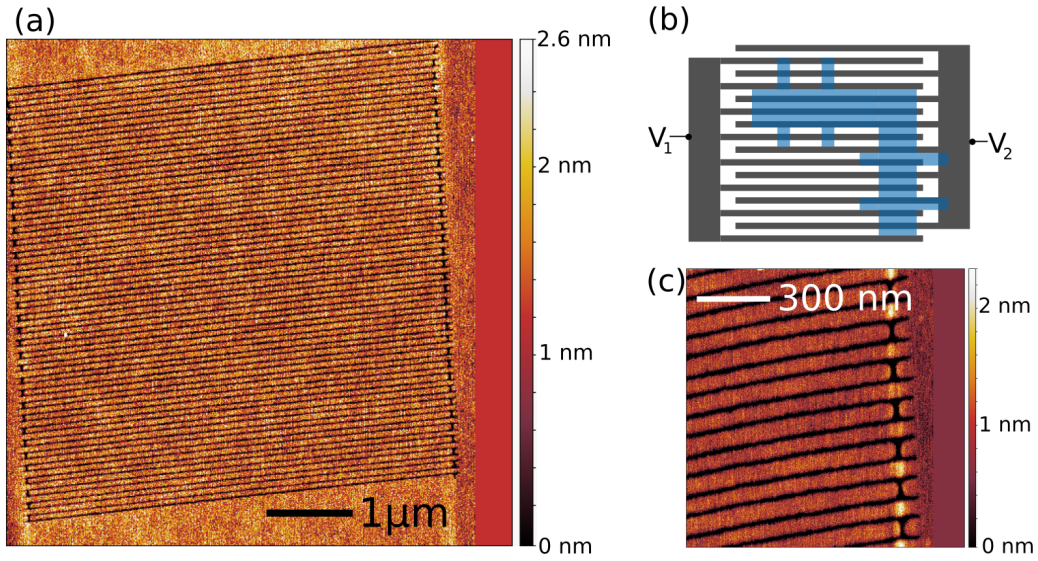


Fig. 4.4: (a) AFM image of an etched double-comb structure into FLG, with a stripe period of 75 nm. Applying alternating gate voltages between neighbouring stripes creates a superlattice with an overall period of 150 nm. Importantly, no electrical contact or short circuits between the two combs are observed. (b) Schematic sample design with interlocking combs enabling fully symmetric 1DSL potential profiles, where voltages of opposite sign are applied to the two combs ($V_1 = -V_2$). (c) Zoom-in of panel (a) showing the clear spacing between the stripes, as any short circuit between the combs would cause gate failure. The slight bending of the stripes is an artefact caused by the slow scanning speed used for high-resolution AFM imaging.

≈ 20 h. The loosened gold layer can then be removed by rinsing the sample with acetone, which has been filled into a syringe (panel (6)). After a final cleaning step in isopropyl alcohol, the sample is ready to be mounted into the chip carrier. An optical microscope image of the finished sample A, which will be presented in the next chapter, is shown in Fig. 4.3(b). Here, the L-shaped Hall bar and the evaporated gold contacts are visible. Figure 4.3(a) shows a schematic 3D model of the sample geometry and all combinations of voltage probes in and perpendicular to the modulation direction.

The final fabrication step is to adhere the sample to a chip carrier using a conductive silver epoxy glue, which electrically connects the global silicon backgate. Ultrasonic bonding using an aluminium wire is used to connect the samples to the chip carrier pads via the gold contacts.

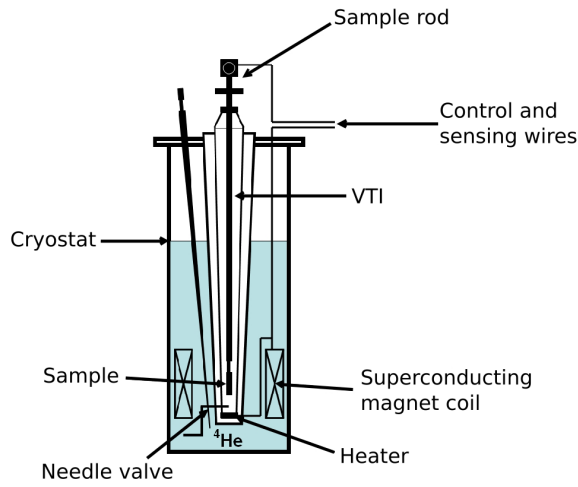


Fig. 4.5: Cross-section of the cryostat setup with the main components highlighted. The sample rod holds the sample and provides electrical access. The VTI allows controlled thermal coupling to the helium bath. The needle valve regulates the helium flow into the VTI, while the heater enables precise temperature tuning. The superconducting coil surrounding the insert generates the magnetic field for the magnetotransport measurements. Adapted from [137].

4.4 Symmetrical Double-Gates

Kang et al. proposed that a patterned gate designed as interlocking combs (see Fig. 4.4(b)) can realise a symmetric potential profile, enhancing 1DSL effects [96]. When voltages of equal magnitude but opposite sign are applied to the two combs ($V_1 = -V_2$), the potential peaks generated by one comb and the potential dips from the other comb have identical shapes and amplitudes. Since the combs interlock and the stripe widths and magnitude of the applied voltages are identical, this configuration produces a perfectly mirror-symmetric potential along the periodic direction.

For this reason, the design was implemented using the well-known PBG stripes of FLG. The stripes were arranged with a slightly increased spacing to prevent short circuits between the two combs, which would lead to a complete failure of the gate. By applying voltages of opposite sign to alternating stripes, the resulting 1DSL has a total period of 150 nm. As shown in Fig. 4.4 the fabrication of this design was realised, a full view of the structure is shown in (a), a zoom in to show the clear spacing between the stripes can be seen in (c). The gate was tested at the probe station where no leakage current was measured. Unfortunately, the delicate gate structure was probably broken during stacking, as there was an instantaneous leakage current between the two combs in the finished sample. This was tested on several samples and this design was not pursued for subsequent ones.

4.5 Measurement Setup

The measurements were conducted in a ^4He cryostat from *Oxford Instruments* with a base temperature of $T = 1.4\text{ K}$. A schematic overview of the cryostat and its operating principle is shown in Fig. 4.5. The Hall bar sample was mounted onto a sample rod and inserted into the variable temperature insert (VTI), which is connected to the helium bath via a needle valve. When the valve is opened, helium flows into the VTI, which is pumped to a low pressure (typically a few millibar), allowing for evaporative cooling down to 1.4 K . A superconducting magnet located in the helium bath generates a magnetic field perpendicular to the sample plane, enabling measurements up to $B = 10\text{ T}$. The sample rod connects the Hall bar to the measurement setup via various control and sensing lines. AC measurements were performed using a master lock-in amplifier operating at 13 Hz . A series resistor of $10\text{ M}\Omega$ was used to apply a constant current of $I_{SD} = 10\text{ nA}$ from source to drain. Additional lock-in amplifiers were used to measure the voltage drops V_{xx} , V_{xy} , V_{yy} , and V_{yx} . Voltages to the global back gate and the patterned bottom gate were applied using source-measure units (SMUs). The entire measurement process was controlled via a Perl-based script using the *Lab::Measurement* framework [138].

5 Monolayer Graphene Superlattices

This chapter presents a thorough investigation into the electronic transport properties of monolayer graphene devices when subjected to an electrostatically induced 1DSL potential. First, we conduct transport measurements at zero magnetic field to characterise the samples, revealing the significant band structure modifications imposed by the superlattice, such as the emergence of satellite Dirac points and anisotropic transport behaviour. Next, we examine magnetotransport measurements, in which the interplay between the 1DSL potential and an external magnetic field leads to a variety of quantum phenomena. These include distinct Landau fans originating from the main and satellite Dirac points, Weiss oscillations, and modified quantum Hall plateaus. The experimental findings are presented using data from two representative monolayer graphene 1DSL samples, labelled A and B, the design of which is shown in Fig. 5.1(a). Their key parameters are listed in the following table:

Sample	$d_{hBN,bottom}$	$d_{hBN,top}$	1DSL period
A	6 nm	25 nm	50 nm
B	2 nm	20 nm	50 nm

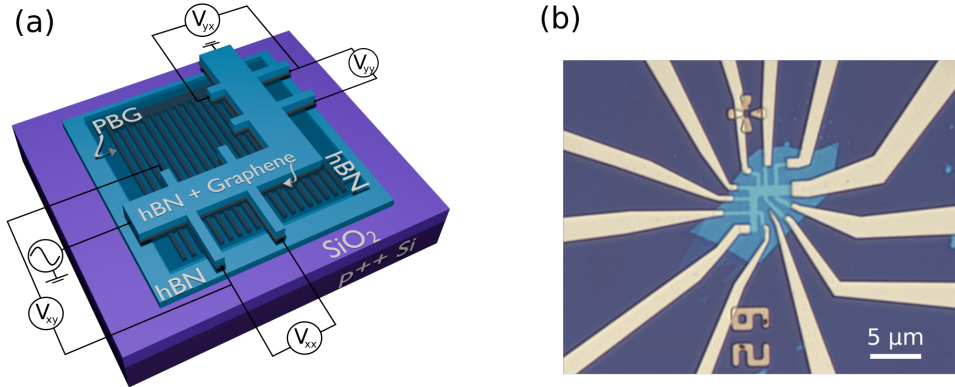


Fig. 5.1: (a) Schematic design of a 1DSL device: L-shaped Hall bar enabling simultaneous transport measurements along the x - and y -directions. A monolayer graphene flake is encapsulated in hBN and placed on top of a PBG with a period of 50 nm. (b) Optical microscope image of the fabricated sample A, showing the contacted structure with chromium/gold electrodes.

5.1 Transport Measurements at $B = 0$

Transport measurements at zero magnetic field ($B = 0$) offer direct insights into the electronic band structure and carrier density distribution in graphene, particularly when subjected to a perpendicular electric field. In monolayer graphene, the characteristic resistance peak marks the charge neutrality point, corresponding to the main Dirac point. When a 1DSL potential is introduced, the periodic modulation leads to significant modifications of this pristine band structure, including the anisotropic deformation of the main Dirac point and the generation of new satellite Dirac points. This section presents a detailed analysis of the longitudinal resistances as a function of gate voltages, showcasing these 1DSL-induced band structure alterations through gate characterisation maps. These measurements were conducted on both samples A and B, with transport properties investigated both parallel and perpendicular to the direction of the superlattice modulation.

5.1.1 Characterisation of Sample A

The following measurements characterise the 1DSL in **sample A** (optical microscope image in Fig. 5.1(b)), which has a superlattice period of 50 nm. The PBG is separated from the MLG transport layer by a 6 nm thick hBN flake. A global back gate is realised

by the silicon substrate, located beneath 285 nm of SiO_2 . The geometric aspect ratio for the modulation direction, defined here as the x -direction (perpendicular to the gate stripes), is $W/L \approx 0.5$; for the y -direction (parallel to the gate stripes), the aspect ratio is approximately 0.42. All measurements were performed at a temperature of 1.4 K and zero magnetic field, with an AC current of 10 nA at a frequency of 13 Hz.

Figure 5.2 shows the four-point resistance as a function of the PBG voltage V_{PBG} , for different values of the back gate voltage V_{BG} , comparing R_{xx} (black), measured along the modulation direction, with R_{yy} (red), measured parallel to the stripes. While V_{BG} mainly influences the 1DSL modulation strength, V_{PBG} mainly changes the average charge carrier density. Here, R_{yy} consistently shows a single Dirac peak, while the number and height of the Dirac peaks in R_{xx} vary with the applied back gate voltage V_{BG} . At $V_{\text{BG}} = -35$ V, both directions show a single dominant peak, accompanied by indications of two very small side peaks in the x -direction, suggesting that this voltage corresponds to the minimal 1DSL modulation. At $V_{\text{BG}} = 70$ V, R_{xx} exhibits five peaks, which originate from the emergence of zero modes in the band structure induced by the 1DSL modulation, as described in section 3.2. By varying V_{PBG} , we effectively change the Fermi level and can thus access all of the generated zero modes. This is a common tool to investigate and characterise superlattice structures.

The mobilities in this sample estimated using the field effect are $90\,000\text{ cm}^2/\text{Vs}$ in the x -direction (unmodulated case, $V_{\text{BG}} = -35$ V), while the y -direction shows lower mobilities around $40\,000\text{ cm}^2/\text{Vs}$ due to reduced sample quality in that region. Using equation (2.33), the high mobility in the x -direction leads to a mean free path $l_{\text{mfp}} = \frac{\hbar}{e}\mu\sqrt{\pi n} \approx 1\text{ }\mu\text{m}$ for an average charge carrier density $n = 1 \times 10^{16}\text{ m}^{-2}$, which means the charge carriers travel along approximately 20 superlattice periods, before scattering occurs.

The observed shift of the charge neutrality point to $V_{\text{BG}} = -35$ V indicates a finite intrinsic doping of the sample. In pristine, undoped graphene, the neutrality point would be expected at $V_{\text{BG}} = 0$ V. The significant displacement observed here suggests the presence of residual impurities introduced during fabrication, which act as unintentional dopants. Nevertheless, the sample still exhibits high mobilities, demonstrating that the overall crystalline quality remains good despite the doping. It should also be noted that the effective back-gate coupling is reduced by screening from the PBG structure, so that comparatively larger gate voltages are required to compensate the doping compared to a uniformly gated device.

The variation in peak positions results from the fact that both gates lie beneath the transport layer and therefore simultaneously affect the 1DSL modulation strength and the Fermi level.

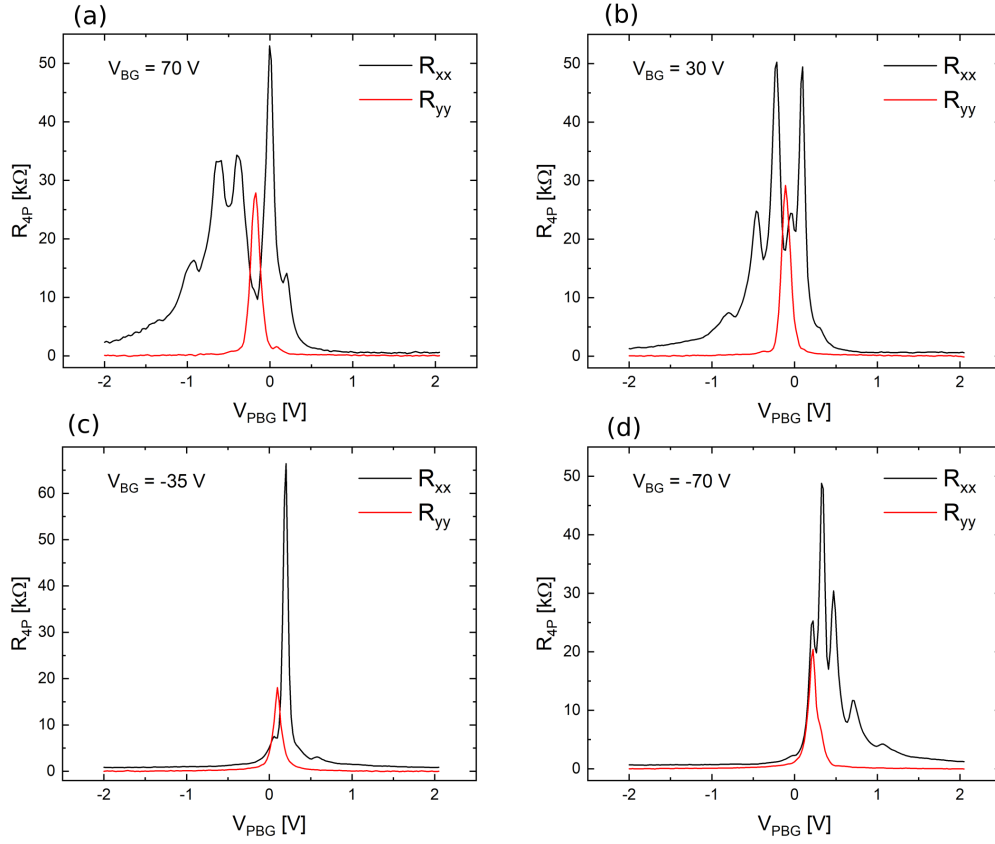


Fig. 5.2: Measured four-terminal resistance in x - and y -direction for different V_{BG} combinations: **(a)** $V_{BG} = 70$ V: the x -direction (modulation direction) exhibits five Dirac peaks, indicating the presence of the 1DSL modulation; the y -direction shows a single peak. **(b)** $V_{BG} = 30$ V: five peaks appear in x -direction with altered intensities compared to (a), while y -direction still shows a single peak. **(c)** $V_{BG} = -35$ V: both x - and y -direction show a single peak, consistent with the 1DSL modulation being almost absent. **(d)** $V_{BG} = -70$ V: four peaks are visible in x -direction, confirming reactivation of the 1DSL modulation; y -direction again shows a single peak. The emergence of multiple peaks exclusively in x -direction demonstrates that the zero modes generated by the 1DSL potential are confined to the modulation direction, resulting in anisotropic transport.

This is visualised in more detail in Fig. 5.3, where the four-point resistances in and perpendicular to the modulation direction are shown as colour plots. In (a), the emerging zero modes induced by the 1DSL potential are visible as satellite Dirac points. The white arrow indicates the estimated axis along which the 1DSL modulation is varied, as the gate response changes from a single to multiple Dirac points along this direction.

Perpendicular to the modulation, shown in (b), the main Dirac point remains unchanged. At $(V_{PBG}, V_{BG}) = (0 \text{ V}, 70 \text{ V})$, marked by the white dot, a feature consistent with *super-collimation* appears in (a), where the main Dirac point becomes flattened and charge

carriers move along the modulation direction, lowering the resistance [96, 139]. This pattern is in good agreement with the calculations by Kang et al. [96], where a similar pattern of alternating branches of Dirac points is shown (see also Fig. 3.4(b)).

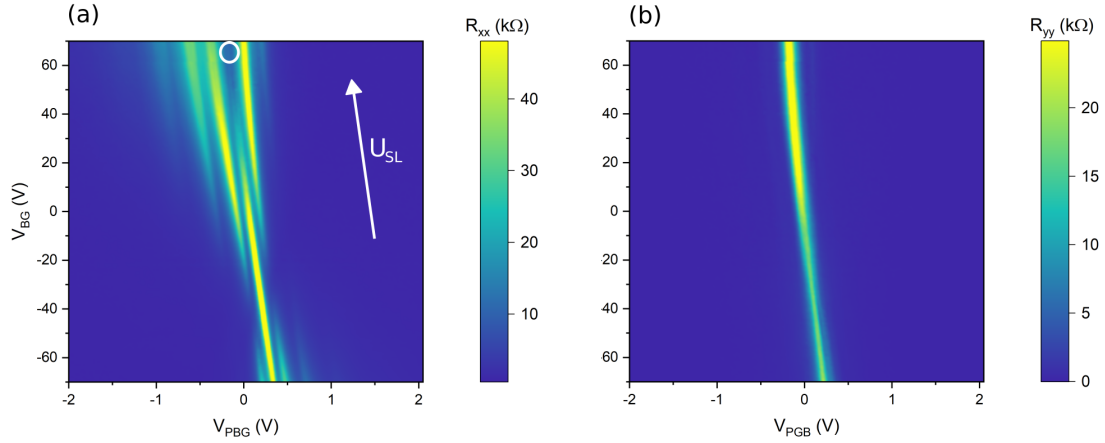


Fig. 5.3: Gatemap characterisation of the four-terminal resistance shown as colour plots: **(a)** Resistance in x -direction, where the 1DSL modulation leads to the emergence of multiple Dirac points with increasing modulation strength. The white arrow indicates the estimated axis of increasing 1DSL modulation strength, as both gates are placed underneath the MLG transport layer. The white dots shows the point of reduced resistance, where the Dirac point gets flattened and the velocity in y -direction renormalises to zero. **(b)** Resistance in y -direction, where a single peak remains unchanged, illustrating the anisotropic transport induced by the 1DSL.

Electrostatic characterisation of sample A

As mentioned before, both gates are located beneath the MLG, and since the PBG is separated from the MLG by only 6 nm of hBN, the system is highly sensitive to variations in the gate parameters. Furthermore, the insulating layer thickness can be affected by residues from the stacking or exfoliation process, and the precise dimensions of the gate stripes and gaps remain uncertain, as AFM measurements are limited by the tip diameter, which is on the same order as the stripe width.

To navigate this gate map and identify gate voltage combinations that selectively tune either the charge carrier density *or* the 1DSL modulation, we estimated the ratio of the gate capacitances to be approximately 1/90 based on the gate response map. In graphene the Fermi wave vector is given by $k_F = \sqrt{\pi n}$, leading to a Fermi energy $E_F = \hbar v_F k_F$. Using a simple parallel plate capacitor model, we calculated lines of

constant 1DSL modulation by combining the effects of both gates: $E = E_{\text{BG}} - E_{\text{PBG}} = \hbar v_F \cdot \sqrt{\pi |n_{\text{BG}}|} - \hbar v_F \cdot \sqrt{\pi |n_{\text{PBG}}|}$ with $n_{\text{BG,PBG}} = C_{\text{BG,PBG}} \cdot (V_{\text{BG,PBG}} - V_{\text{BG,PBG};0})$ [9, 11]. Here, $V_{\text{BG,PBG};0}$ denotes the voltage at the CNP in the absence of modulation and $C_{\text{BG}} \approx 6.5 \times 10^{14} \text{ m}^{-2} \text{ V}^{-1}$ is the back gate capacitance. From the slope of the main Dirac point depending on both gate voltages, a ratio of $C_{\text{BG}}/C_{\text{PBG}} \approx 1/100$ was estimated. These equipotential lines, along which the modulation is expected to remain nearly constant, are shown as an overlay in Fig. 5.4(a).

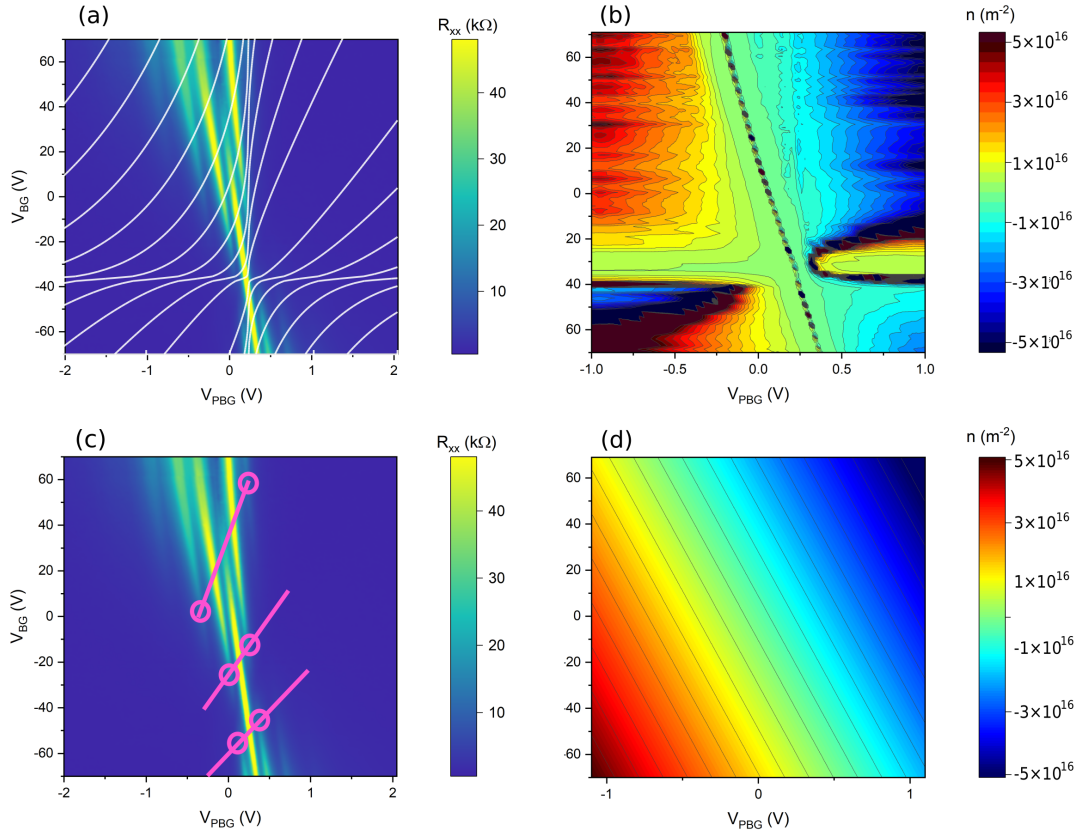


Fig. 5.4: (a) Estimated equipotential lines based on a simple parallel-plate capacitor model. These were used to define trace lines along which both gate voltages were swept simultaneously in order to maintain approximately constant 1DSL modulation during the measurements. (b) Experimentally determined charge carrier density map, calculated from $(R_{xy}(B = 1 \text{ T}) - R_{xy}(B = -1 \text{ T}))/2$ as the Hall slope. (c) Locations (pink circles) where satellite Dirac points first become visible in the x -direction gate response. The changing angle of these onset points (solid lines) indicates an asymmetry in the electrostatic potential with respect to the x -axis. (d) Simulated charge carrier density based on a classical plate capacitor model, showing notable deviations from the measured density in (b). Due to uncertainties in the electrostatics of the PBG, the experimental charge carrier density was used in all subsequent evaluations.

To extract the charge carrier density, we performed a simplified Hall measurement by recording the transverse resistance R_{xy} at magnetic fields of $+1$ T and -1 T, and calculating the antisymmetric component via $R_{xy}^{\text{asym}} = \frac{R_{xy}(+1\text{ T}) - R_{xy}(-1\text{ T})}{2}$. This value was then treated as an effective Hall slope to estimate the carrier density using $n = \frac{B}{e \cdot R_{xy}^{\text{asym}}}$ assuming a single carrier type. The resulting carrier density is shown as a colour plot in Fig. 5.4(b).

A comparison with values calculated from a simple parallel-plate capacitor model, shown in Fig. 5.4(d), reveals clear differences in both the slope of the charge neutrality line and the absolute carrier density. For the calculation, we used $n_{\text{total}} = 0.5 n_{\text{PBG}} + 0.5 n_{\text{BG}}$, where the factor of 0.5 for each gate reflects that the PBG covers roughly half of the device area and simultaneously screens the global BG beneath it. Here, $n_{\text{PBG}}, n_{\text{BG}} = C_{\text{PBG}}, C_{\text{BG}} \cdot (V_{\text{PBG}}, V_{\text{BG}} - V_{\text{PBG,CNP}}, V_{\text{BG,CNP}})$. This discrepancy highlights the strong influence of electrostatic parameters, such as gate geometry and dielectric thicknesses, that are not precisely known due to fabrication and measurement limitations, and also reflects the limitations of the simple parallel-plate capacitor model. Therefore, for all subsequent analyses, we used the charge carrier density determined from the Hall measurements.

Figure 5.4(c) highlights, by pink circles, the positions in the gate maps along the x -direction where each generation of satellite Dirac points emerges. Connecting the corresponding pairs reveals a change in slope, indicating that the electron-hole asymmetry varies with increasing 1DSL modulation strength. This observation further underlines the sensitivity of the band structure modifications induced by the 1DSL to the local electrostatic environment of the sample.

A possible origin of this asymmetry could lie in a non-symmetric shape of the modulating potential. Since the gate stripes could be wider than the gaps in between, the resulting 1DSL potential may deviate from an ideal sinusoidal or symmetric square-wave profile. Such an asymmetry in the peak and valley shapes of the potential would naturally lead to an imbalance in how electrons and holes experience the modulation, and a misalignment of the generated zero-modes, as described in section 3.3.3 [4, 96]. This interpretation is supported by electrostatic simulations of the device geometry, which will be discussed in the following section.

5.1.2 Transport Simulation of Sample A

As previously mentioned, the exact parameters of the gate geometry and insulator thickness are difficult to determine experimentally. To address this, quantum transport simulations were conducted by Ming-Hao Liu. These simulations, calibrated against the experimental gate response, provided a clearer understanding of the gate parameters and

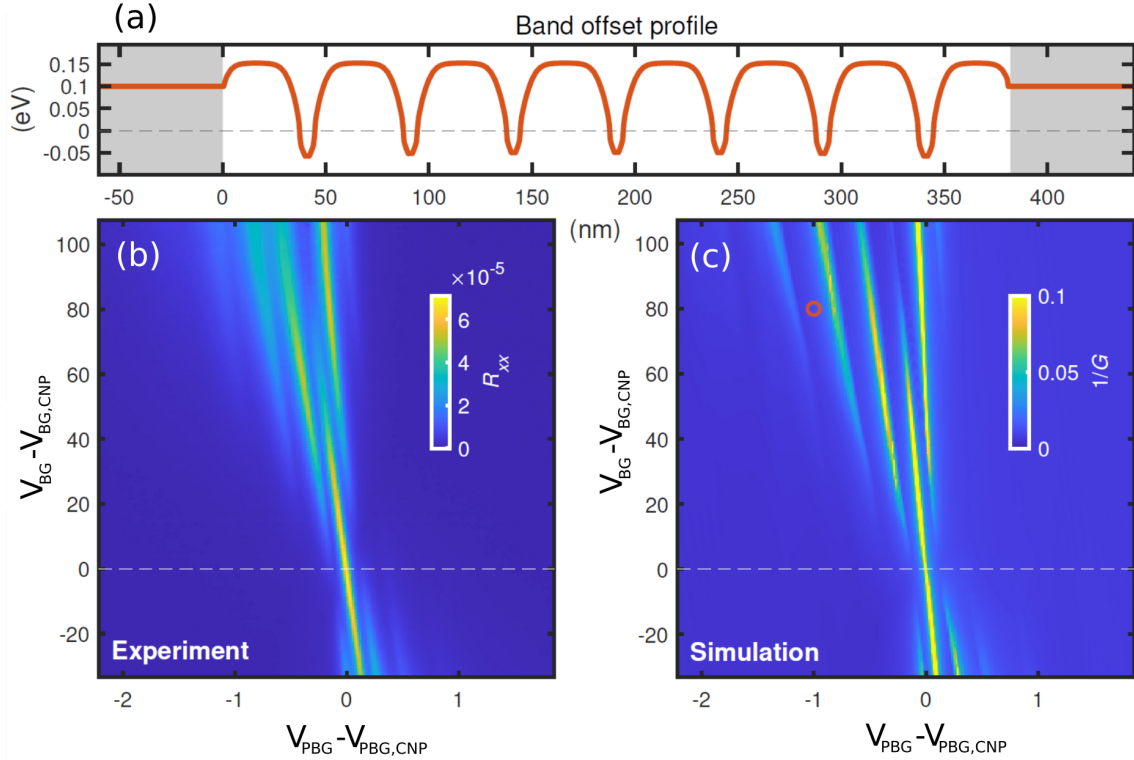


Fig. 5.5: (a) Potential profile extracted from the electrostatic simulation shown in (c), taken at the position of the red circle. The potential is asymmetric with respect to the x -axis, which leads to a misalignment of the generated zero modes in the band structure and to an electron–hole asymmetry. (b) Experimental gate response in x -direction. (c) Quantum transport simulation of the inverse conductance ($1/G$) based on the optimal device geometry, matching the measurement in (b). The best-fitting parameter set suggests a PBG stripe width of ~ 30 nm and a gap of ~ 20 nm between the stripes, resulting in the observed asymmetric potential profile. The simulations were provided by Ming-Hao Liu.

the resulting 1DSL potential profile.

By systematically varying combinations of insulator thicknesses, gate stripe widths, gap widths, and the number of barriers, the simulation results were compared to the experimental data. This optimisation process aimed to identify the parameter set that best matched the measurements.

Figure 5.5(a) shows the extracted band offset profile, which is proportional to the potential profile, based on the optimal set of geometry parameters. The resulting transport simulation in (c) is compared to the experiment in (b), revealing a good match. The appearance and precise positions of the satellite Dirac peaks were reproduced, including the previously mentioned electron-hole asymmetry. The shown potential profile corresponds to the gate combination marked by the red circle in the simulated gate map.

The simulations further revealed a possible PBG stripe width of approximately 30 nm, with gaps of around 20 nm between them. This combination leads to an asymmetric potential shape, where the stripes create broader potential peaks than the gaps do dips – and vice versa. This is crucial information, as it indicates an asymmetric change in the band structure, implying that the generated zero-modes will not align symmetrically. This resulting asymmetry is also visually evident, as the changing slope between the generated Dirac points for electrons and holes was accurately reproduced compared to Fig. 5.4(c).

Further, the calculations revealed that at least five lattice periods are required to observe a pronounced pattern of satellite Dirac points in the gate response. This implies that the charge carriers should traverse several superlattice periods without significant scattering, which is compatible with the independently estimated mean free path of $l_{\text{mfp}} \approx 1 \mu\text{m}$.

5.1.3 Characterisation of Sample B

Sample B has the same geometry as Sample A, with the only difference being that the hBN flake insulating the PBG from the MLG is only 2 nm thick in this case. The idea behind this design was to achieve a stronger 1DSL modulation by reducing the insulator thickness. The sample showed mobilities of $\mu \approx 30\,000 \text{ cm}^2/\text{Vs}$.

The gate maps of R_{xx} and R_{yy} as functions of V_{PBG} and V_{BG} are shown in Fig. 5.6(a) and (b). The overall gate response is qualitatively similar to that of Sample A, with satellite Dirac peaks appearing in R_{xx} at large positive and negative V_{BG} values, indicating the presence of 1DSL modulation. However, at $V_{\text{BG}} \approx -18 \text{ V}$, where the 1DSL modulation is expected to vanish, R_{xx} still shows a double peak. This feature is not observed in Sample A and is not expected in the absence of superlattice modulation.

One possible explanation could be local strain, which might arise from the thin hBN layer and a slight corrugation of the graphene. However, this type would not only cause a splitting around the neutrality point, but also generate additional oscillatory features in the unipolar regime. [140–144]

Since no such features are present in the measurements, strain can be ruled out as the primary mechanism. Instead, the more consistent interpretation is that the sample exhibits inhomogeneities, such as two regions with slightly different electronic properties. These may originate from variations in local doping, small differences in gate capacitance due to hBN thickness fluctuations or trapped bubbles, charge inhomogeneities at the graphene–hBN interface, or doping gradients induced by the contacts. Each of these regions would contribute its own neutrality point, and their superposition naturally explains the observed double peak. [145–148]

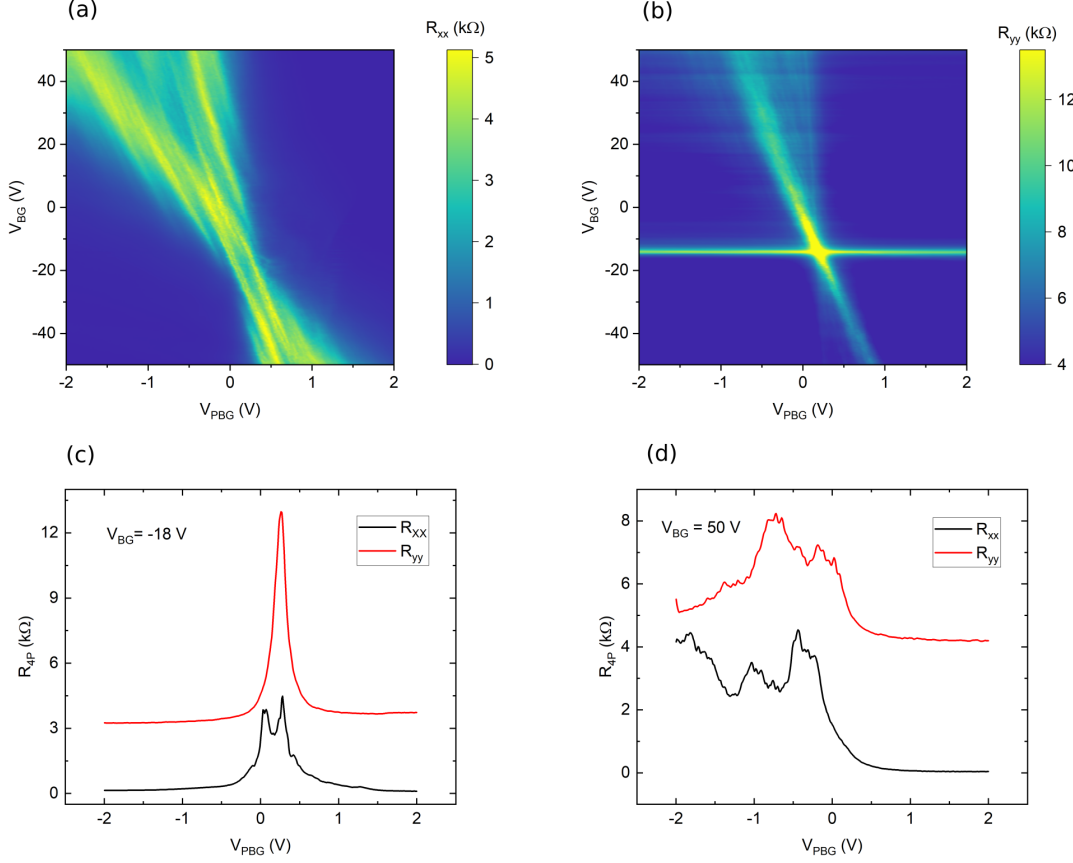


Fig. 5.6: (a) Gate map characterisation of sample B: Measured four-terminal resistance as a function of both gate voltages. The response resembles that of sample A, with the emergence of multiple satellite Dirac points as the 1DSL modulation strength increases. At $V_{\text{BG}} \approx -18$ V, where the 1DSL modulation is approximately absent, a double-peak feature appears. This may indicate inhomogeneities in the transport layer. (b) Gate map in y -direction, showing a single Dirac point that remains largely unaffected. The horizontal feature suggests a disruption of the PBG in the region of the Hall bar used for the y -direction measurement. (c) Line cuts at $V_{\text{BG}} = -18$ V from (a) and (b), showing a clear Dirac peak in y -direction and the unexpected double-peak structure in x -direction. (d) Line cuts at $V_{\text{BG}} = 50$ V, where satellite peaks appear in x -direction due to the 1DSL modulation. The y -direction also shows a distorted peak, attributed to reduced sample quality in that region.

Figure 5.6(c) and (d) show line cuts at $V_{\text{BG}} = -18 \text{ V}$ (no modulation) and $V_{\text{BG}} = 50 \text{ V}$ (strong modulation), respectively. In panel (c), the unexpected double peak in R_{xx} contrasts with the single Dirac peak in R_{yy} , suggesting that the feature originates from sample inhomogeneities in the part of the Hall bar where the transport along the x -direction is measured. In panel (d), multiple satellite peaks appear in R_{xx} due to the formation of zero modes in the modulated band structure. A small side peak is also visible in R_{yy} , which could either also result from inhomogeneities or from a mixing of the R_{xx} signal into the R_{yy} measurement.

Another artefact observed in this sample is a horizontal line of high resistance in R_{yy} , visible as a flat feature independent of V_{BG} . This suggests a region of insufficient gating. Since the PBG consists of FLG and undergoes multiple stacking steps, structural damage is possible. If the gate is partially broken, the applied V_{PBG} might not reach the entire area of the PBG, particularly in the region where R_{yy} is measured.

Electrostatic characterisation of sample B

To investigate the charge carrier density as a function of the gate voltages, we performed the same measurement as for Sample A: the Hall resistance slope was approximated by calculating $(R_{xy}(1 \text{ T}) - R_{xy}(-1 \text{ T}))/2$, and the resulting carrier density was plotted as a colour map in Fig. 5.7(a).

For comparison, we also calculated the carrier density using a parallel-plate capacitor model with weighted gate contributions. Based on the assumption that the PBG stripes are slightly thicker than the gaps in between, we used the weighting $n_{\text{total}} = 0.6 n_{\text{PBG}} + 0.4 n_{\text{BG}}$, with $n_{\text{PBG}}, n_{\text{BG}} = C_{\text{PBG}}, C_{\text{BG}} \cdot (V_{\text{PBG}}, V_{\text{BG}} - V_{\text{PBG,CNP}}, V_{\text{BG,CNP}})$. The resulting calculated carrier density is shown in Fig. 5.7(b).

The two methods show significant discrepancies not only in the slope of the charge neutrality line, but also in the absolute carrier densities, differing by nearly a factor of two. This illustrates the strong sensitivity of the electrostatics in these samples to small geometric variations and fabrication-induced uncertainties, such as gate dimensions, hBN thicknesses, or possible residues of the stacking process.

Due to these deviations, we relied on the experimentally determined carrier density for all further analyses and measurement plots.

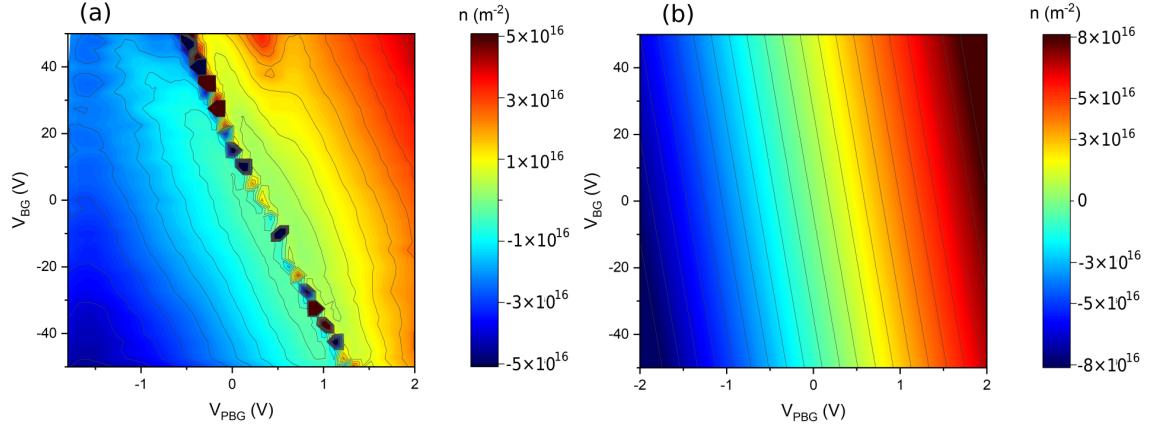


Fig. 5.7: (a) Measured charge carrier density map of sample B as a function of both gate voltages. The Hall slope was estimated using the antisymmetric part of the Hall resistance: $(R_{xy}(B = 1 \text{ T}) - R_{xy}(B = -1 \text{ T}))/2$. (b) Calculated charge carrier density map based on a simple parallel-plate capacitor model. Due to discrepancies in slope and absolute values compared to the measured map, the experimental data were used in all further analyses to ensure accurate charge carrier densities.

5.1.4 Temperature Dependence in Sample B

We performed temperature-dependent measurements on sample B with a finite 1DSL modulation applied. To keep the 1DSL modulation as constant as possible while varying only the charge carrier density, we swept the gate voltages along the white dotted line shown in the inset of Fig. 5.8(a). This line serves as a refined approximation of the equipotential lines shown in Fig. 5.4(a), which were analogously estimated for sample B. The resulting data, presented in Fig. 5.8(a), exhibit a clear and distinct temperature dependence for different Dirac features.

At the main Dirac point, we observe an increase in resistance with rising temperature. While commonly intrinsic graphene shows decreasing resistance with increasing temperature due to thermally activated carriers, in our sample, this increase likely primarily stems from enhanced phonon scattering at elevated temperatures. Stronger lattice vibrations increase scattering, and thus increase resistivity. Additionally, the diminishing satellite features at higher temperatures likely contribute to a more pronounced appearance of the main Dirac point, as their overlapping influence lessens.

Figure 5.8(b) summarises the temperature dependence by plotting the maximum resistance values of the main Dirac point and the two satellite peaks. While the resistance at

the main Dirac point rises with temperature, the satellite peak heights drop significantly. These satellite features arise from the periodic 1DSL potential, which folds the band structure and generates miniband gaps or a modulated density of states at specific carrier densities. The temperature-induced reduction of resistance peaks may be consistent with Umklapp scattering processes, as described in section 2.5. However, given the large period of our 1DSL (50 nm), the efficiency of such Umklapp processes is expected to be reduced compared to e.g. moiré superlattice systems, and their contribution remains less well documented. Therefore, other mechanisms such as loss of miniband coherence, thermally broadened occupation, or increased disorder-related scattering could also play a significant role in the observed suppression of resistance maxima at higher temperatures. Consequently, the satellite peaks diminish with increasing temperature. [149–152] Due to a breakdown of the back gate in sample A, it was not possible to measure the temperature dependence in this sample.

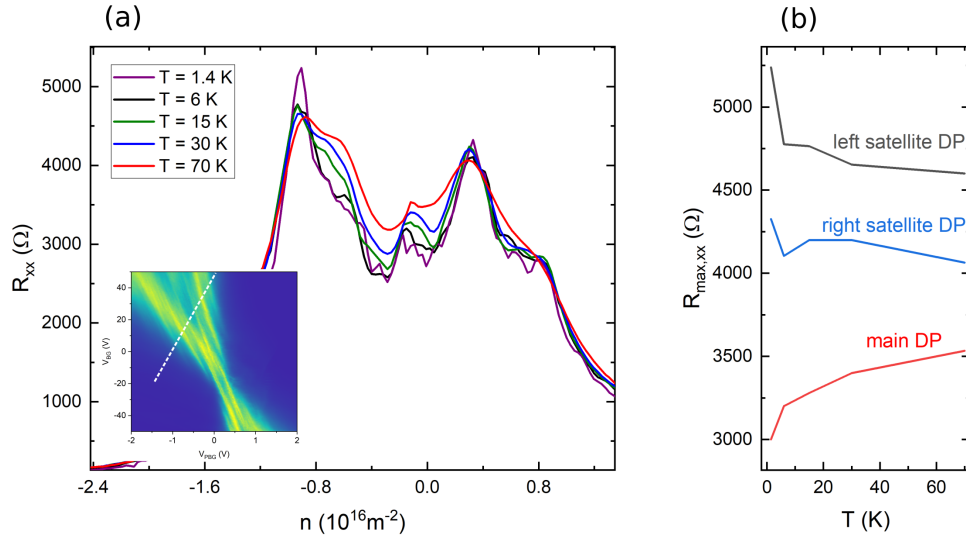


Fig. 5.8: (a) Four-terminal resistance in modulation direction for an applied 1DSL modulation. The gate voltage sweep was performed along the direction indicated by the white dashed line in the inset. With increasing temperature, the satellite Dirac peaks (left and right) decrease in height, while the main Dirac peak increases. (b) Temperature dependence of the peak heights, summarising the contrasting behaviour of main and satellite Dirac points.

5.2 Commensurability Oscillations

Magnetotransport measurements serve as a powerful tool to probe the electronic properties and band structure modifications in graphene under periodic potentials. When a perpendicular magnetic field is applied, Landau quantisation occurs, leading to the formation of discrete Landau levels (see section 2.6.2), which can be visualised in Landau fan diagrams for both modulated and unmodulated regions. In addition to these quantum oscillations, commensurability oscillations (also known as Weiss oscillations, see section 3.3.2) emerge when the cyclotron orbit diameter becomes commensurate with the period of the one-dimensional superlattice potential. These oscillations offer unique insight into the interplay between the superlattice modulation and the electronic band structure. This chapter focuses on the detailed analysis of magnetotransport data from monolayer graphene devices with electrostatically induced 1DSL modulation, highlighting the regime dominated by commensurability oscillations as well as the underlying Landau quantisation.

The measurements were performed on samples A and B previously characterised in section 5.1, using an AC current of $I = 10$ nA at a frequency of 13 Hz.

5.2.1 Sample A

Unmodulated regime in sample A

Figure 5.9 shows magnetotransport measurements performed on sample A at $T = 1.5$ K for the case of no 1DSL modulation. The longitudinal resistance was measured along the modulation direction (perpendicular to the stripes), shown in (a), and perpendicular to it, shown in (b), as a function of the average charge carrier density determined experimentally in the characterisation measurements presented in the previous section. Panel (d) shows the slope along which both gates were varied to keep the 1DSL modulation as close to zero as possible, also estimated from the characterisation.

The modulation direction in (a) shows a pronounced pattern of SdH oscillations due to Landau quantisation. These oscillations reflect the transformation of the continuous DOS into discrete Landau levels. The pattern is symmetric around $n = 0$, as Landau quantisation occurs equally for electrons and holes, and the zeroth Landau level is half-filled with electrons and holes. The SdH pattern becomes visible at around $B = 1$ T, indicating high carrier mobilities, as expected in the unmodulated regime. Below $B = 1$ T, the Landau levels broaden beyond their spacing, which limits the resolution of the SdH oscillations. Figure 5.9 (c) displays the same R_{xx} data as panel (a). The pink lines overlaid on

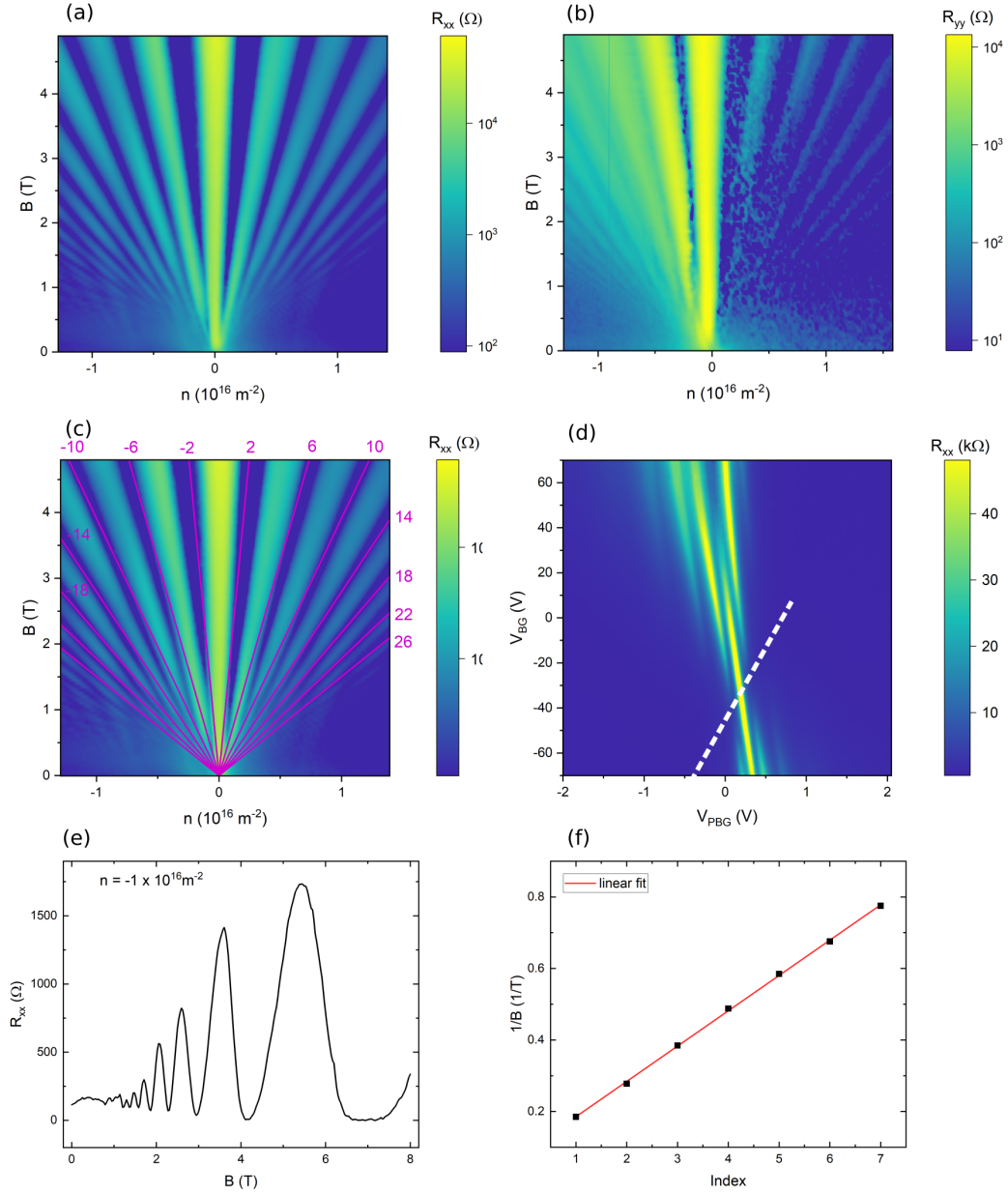


Fig. 5.9: (a) Landau fan for the unmodulated case in sample A: four-terminal resistance as a function of average charge carrier density and perpendicular magnetic field. The slope along which both gates were varied simultaneously is shown in panel (d). (b) Landau fan in the y -direction, less pronounced due to lower sample quality in this region. (c) Same plot as (a), with pink lines indicating the calculated filling factors, which agree well with the measured minima of the SdH oscillations. (e) Single sweep from (a) at a charge carrier density of $n = -1 \times 10^{16} \text{ m}^{-2}$, showing pronounced SdH oscillations. (f) SdH peak positions from (e) plotted against inverse magnetic field. The fit can be used to determine the charge carrier density.

the experimental data represent the theoretically calculated traces of the R_{xx} minima, corresponding to specific Landau level filling factors ν , given by $B = \frac{nh}{\nu e}$. The good agreement observed between the experimental data and these theoretical lines confirms two key aspects: firstly, the accuracy of our approximations for determining the charge carrier density from the measured Hall slope map in Fig. 5.4 (b), and secondly, that the observed filling factor sequence precisely matches that expected for pristine unmodulated monolayer graphene ($\nu = \pm 4(N + 1/2)$). [4, 77]

The y -direction in (b) appears asymmetric for electrons and holes, which is not expected. Furthermore, the SdH pattern is much less pronounced than along the modulation direction, indicating significantly lower sample quality in this region of the Hall bar, which is in agreement with the mobilities obtained in section 5.1.

Figure 5.9(e) shows a single line cut from the colour plot in (a) at $n = -1.0 \times 10^{16} \text{ m}^{-2}$. The SdH peaks increase in height with increasing B , and their positions scale with $1/B$, reflecting the Landau quantisation modulating the DOS. These peak positions can be used to extract the charge carrier density. In Fig. 5.9(f), the positions in $1/B$ are plotted versus their index. From its slope s of the linear fit the charge carrier density can be calculated via $n = \frac{gsqv}{s} \frac{e}{h}$ [8]. Here, a slope of $s = 0.1$ yields a charge carrier density of $n = 1.03 \times 10^{16} \text{ m}^{-2}$, which matches perfectly with the value extracted from the colour plot in Fig. 5.7.

Weakly modulated regime in sample A

Figure 5.10 (a)-(c) show the longitudinal resistance in x -direction as a function of average charge carrier density and magnetic field, with an applied 1DSL modulation. Panel (f) shows the sweep path in the gate voltage map along which the gate voltages were varied to keep the 1DSL modulation as constant as possible. This line was approximated using the equipotential lines shown in Fig. 5.4 (a). In this regime, the generated additional Dirac points at higher energies are already visible, while the main Dirac point becomes anisotropically flattened.

This band structure modification is also reflected in the magnetotransport measurements along the modulation direction, where satellite Landau fans appear symmetrically at $n = \pm 0.3 \times 10^{16} \text{ m}^{-2}$ around the main fan at $n = 0$.

Additionally, the SdH oscillations are modulated by a superimposed pattern, from which darker traces of lower resistance emerge. For better visibility, these traces are highlighted in (b) with grey lines.

This modulation of the SdH oscillations arises from commensurability oscillations, de-

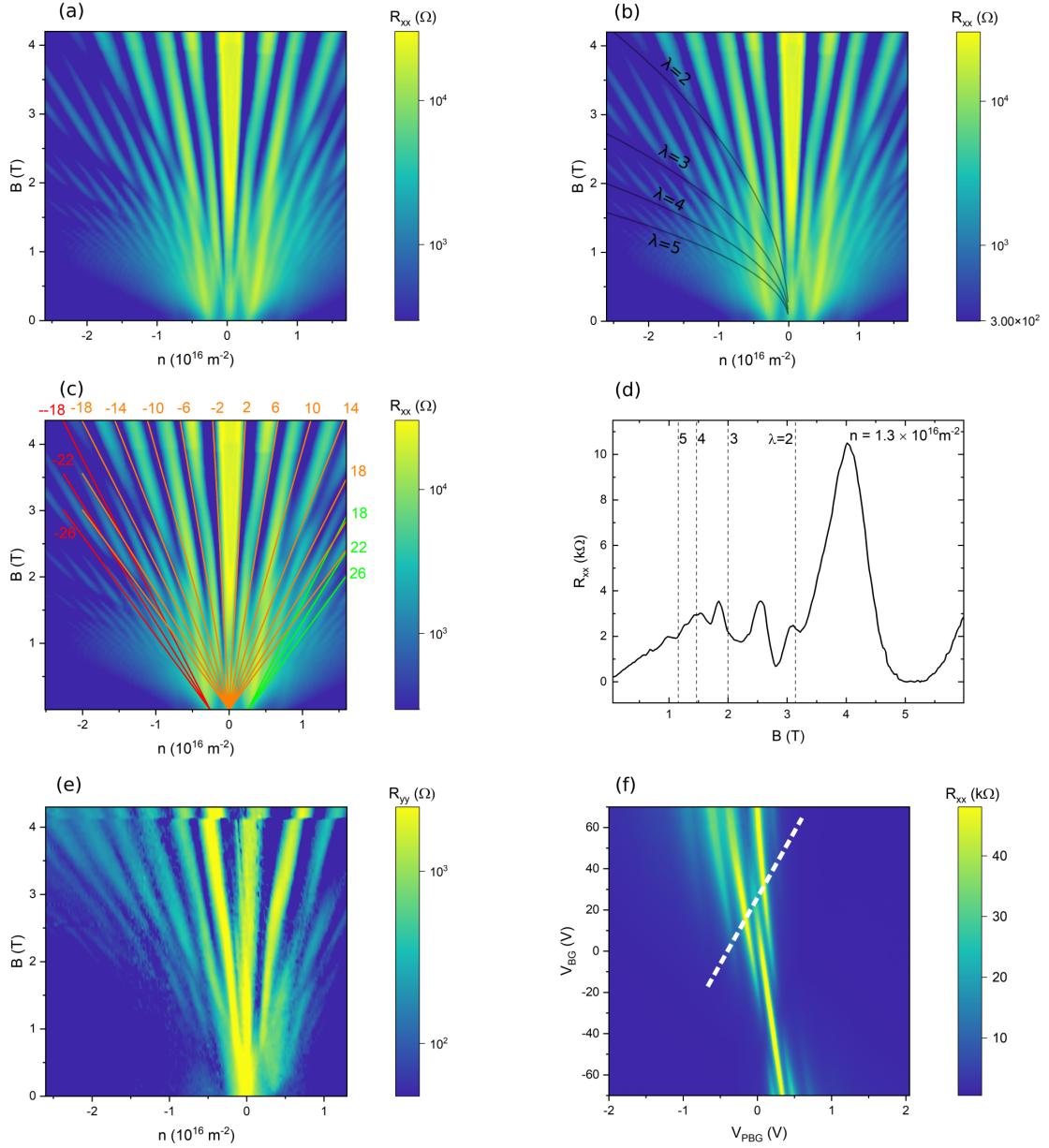


Fig. 5.10: (a) Landau fan for the low-modulation case in sample A, showing three fans originating from the main and satellite Dirac points. The SdH pattern is still visible, but superimposed by commensurability oscillations, marked by grey lines in panel (b). Whenever the cyclotron radius matches the 1DSL period ($R_C = (\lambda - 1/4)a$), the resistance is lowered. (c) Same measurement as in (a), with calculated traces for several filling factors starting at the main and satellite Dirac points. (d) Single sweep from (a) at $n = 1.3 \times 10^{16} \text{ m}^{-2}$, showing the superposition of SdH and commensurability oscillations. The grey dashed lines indicate the expected positions for different values of λ , which match well with the measurement. (e) Landau fan in y -direction, which shows lower quality and no visible superimposed oscillation pattern. (f) Gate map with white dashed line indicating the sweep trace used to maintain a nearly constant 1DSL modulation.

scribed in section 3.3.2. Whenever the cyclotron orbit is commensurate with the superlattice period a and fulfills the flat-band condition $2R_c = (\lambda - 1/4)a$, the longitudinal resistance exhibits a minimum. This can be understood either as a result of destructive interference between scattered cyclotron orbits or as a maximisation of the drift velocity due to averaging over multiple superlattice periods, both leading to enhanced conductivity. [102, 106]

The values λ for which this condition is fulfilled are marked in panel (b), and they coincide well with the experimentally observed resistance minima. The persistence of these commensurability oscillations up to $\lambda = 5$ indicates high carrier mobilities and long mean free paths. This is in good agreement with the estimated mean free path in section 5.1 of $l_{mfp} \approx 1\mu\text{m}$.

Figure 5.10 (d) shows a single sweep at a hole density of $n = 1.3 \times 10^{16}\text{m}^{-2}$ taken from the Landau fan in Fig. 5.10(a). Here, the difference to the unmodulated sweep in Fig. 5.9(c) is visible, as the steadily increasing height of the SdH peaks is altered by the superimposed commensurability oscillations. The flat-band condition is indicated by the grey dashed line, where the resistance is lowered.

The absence of such oscillations in the unmodulated regime (Fig. 5.9) confirms that these features originate solely from the electrostatically induced 1DSL modulation in the patterned gate design. This excludes significant contributions from intrinsic strain or moiré-type modulations.

Figure 5.10 (c) illustrates the Landau fan traces of the R_{xx} minima. The main Landau fan, originating at $n = 0$, is traced in orange, while additional satellite fans, stemming from the 1DSL-generated Dirac points, are visible around $n = \pm 0.3 \times 10^{16}\text{m}^{-2}$ and are highlighted in red and green. Notably, at filling factors higher than $|\nu| = 18$, several traces deviate from the expected sequence of the main Landau fan. These deviations can be unambiguously attributed to the influence of the satellite Dirac points, directly reflecting the significant band structure modification induced by the 1DSL potential. This observed behaviour confirms that these features are indeed a consequence of the superlattice and not trivial Fabry-Pérot resonances. Our findings are consistent with prior reports on MLG 1DSL samples. Drienovsky et al. [106] similarly observed commensurability oscillations in magnetotransport in 1D modulated MLG samples, which was realised with patterned stripe gates of 80 nm and 200 nm period. Their work also noted the presence of Fabry-Pérot resonances in the gate response, alongside the commensurability oscillations in magnetotransport. Furthermore, Li et al. [99] reported the clear observation of satellite Dirac points in both gate response and magnetotransport measurements, as well as commensurability oscillations, in a MLG 1DSL sample created using a patterned SiO_2 dielectric.

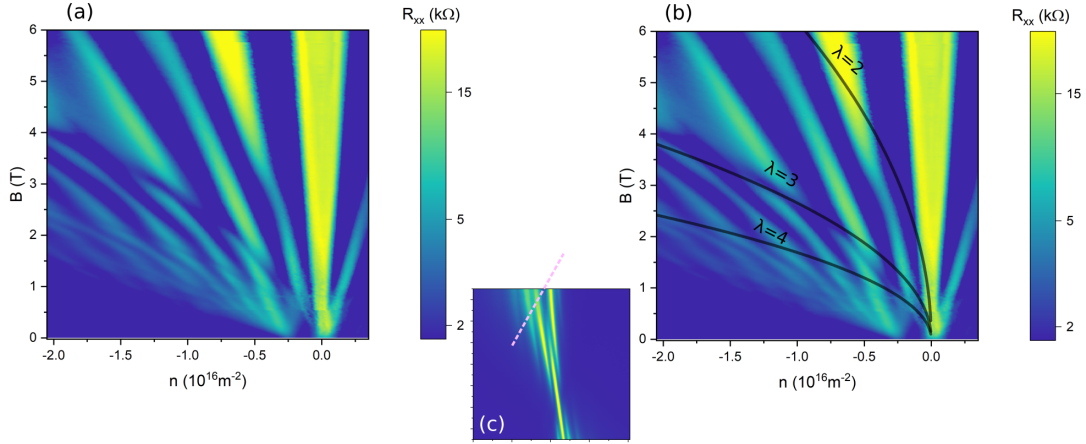


Fig. 5.11: (a) Landau fan in modulation direction for the high modulation case. The trace of the gate sweep is shown in panel (c) by the white dashed line. The SdH and commensurability oscillations are now less pronounced due to the stronger modulation potential, which distorts the Landau levels and leads to increased broadening of the energy spectrum. (b) Calculated traces for the commensurability oscillations, which match well with the measured features. (c) Gate map with the sweep direction used in (a) indicated by the white dashed line.

High modulation regime in sample A

Figure 5.11 (a) and (b) present magnetotransport measurements conducted in the regime of strong 1DSL modulation. Panel (c) of Fig. 5.11 illustrates the experimental trajectory (pink line in the gate characterisation map) along which both gate voltages were swept simultaneously. The presented data correspond to the maximum achievable modulation strength in this sample, as further increasing the gate voltages would exceed the dielectric breakdown voltage, posing a risk of irreversible sample damage. In this regime, the point of supercollimation, characterised by the anisotropic flattening of the main Dirac cone and a significant reduction in Fermi velocity, has been surpassed, as discussed in section 3.2. Theoretically, this regime is associated with the emergence of additional Dirac points at $E = 0$ and the opening of miniband gaps in the periodically modulated band structure. Since the modulation strength is only slightly beyond the supercollimation point, these new Dirac points are weakly developed. Experimentally, signatures of the main and satellite Landau fans are still visible. However, at elevated modulation strengths the discrete Landau levels broaden into Landau bands. This broadening reduces the visibility of both SdH and commensurability oscillations, while at lower modulation strengths, the minibands remain flatter and the oscillations are better resolved. [97, 106, 153–156]

These effects are distinctly evident in Fig. 5.11, where both SdH and commensurability oscillations are significantly less pronounced compared to the weakly modulated regime

presented in Fig. 5.10. Specifically, the commensurability oscillations are only discernible up to a Weiss index of $\lambda = 4$ (see panel (b)), whereas in the weak modulation case they persisted up to $\lambda = 5$. This reduction in the observable oscillation order serves as a compelling indication of a shortened mean free path, directly attributable to the increased scattering within the strongly modulated potential landscape.

5.2.2 Sample B

Sample B, which showed the double Dirac peak feature at zero 1DSL modulation due to inhomogeneities, was also investigated in magnetotransport. The aim was to analyse how SdH and commensurability oscillations manifest in such a structure. Additionally, temperature-dependent measurements were possible in this sample, which were not feasible in sample A before the back gate broke.

Unmodulated regime

The double feature visible in the gate map of sample B in Fig. 5.6(a) also presents as two Landau fans symmetrically around $n = 0$. Figure 5.12(a) displays the Landau fan at zero 1DSL modulation, which was measured by simultaneously varying both gates along the white line in the gate map shown in panel (c). This confirms that the two peaks originate from two zero modes in the band structure. Since the 1DSL modulation is absent at this point, the observed signature is attributed to inhomogeneities in the sample, such as regions with slightly different doping levels or gate capacitances. These inhomogeneities can cause the appearance of two distinct neutrality points, which manifest as two overlapping Landau fans.

This line was approximated using a simple capacitor model by calculating equipotential lines of the 1DSL potential, analogous to Fig. 5.4(a).

The Landau fans of the Dirac points overlap, making it impossible to unambiguously trace the SdH minima as in Fig. 5.9(c). The overlapping Landau fans and the lower mobility lead to a reduced visibility of the SdH oscillations compared to sample A. At higher charge carrier densities of $n \approx 3 \times 10^{16} \text{ m}^{-2}$, some darker features indicate hints of commensurability oscillations, which are not expected at zero 1DSL modulation. These features, however, cannot be unambiguously assigned either to residual sample inhomogeneity or to limitations of the equipotential line approximation at higher densities.

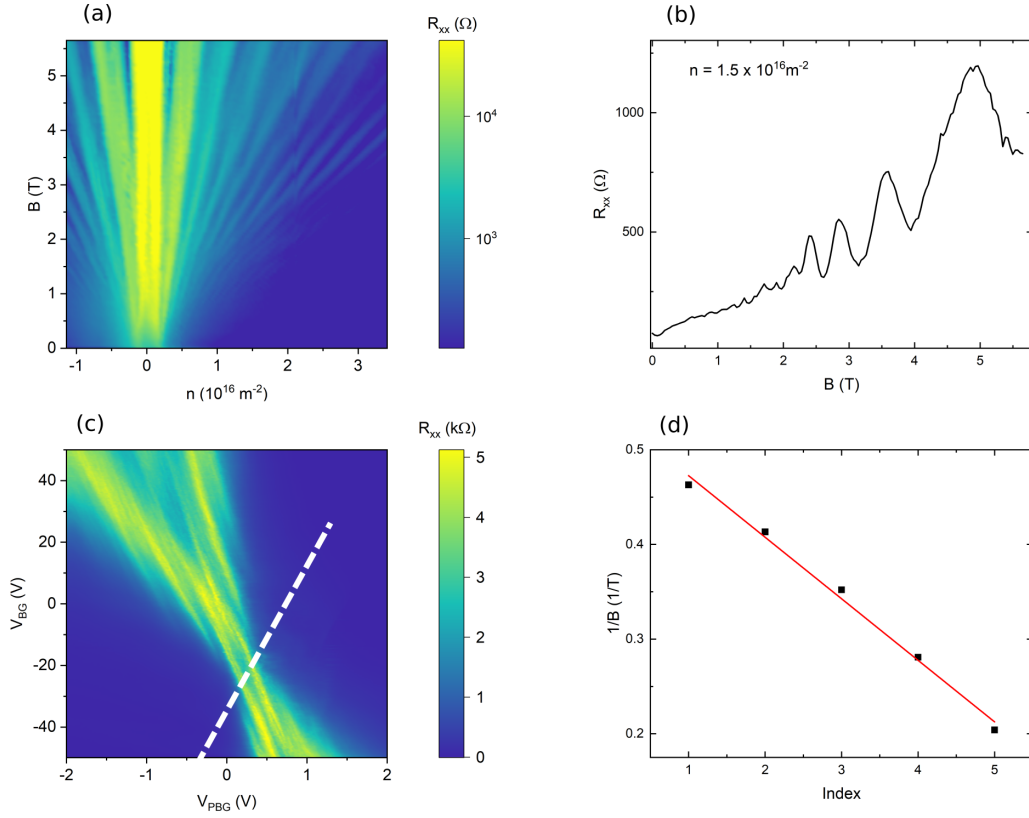


Fig. 5.12: (a) Landau fan of the unmodulated case in sample B, with two fans likely caused by inhomogeneities. The SdH pattern is visible, along with features resembling commensurability oscillations. These could originate from an inaccurate estimation of the equipotential lines, potentially leading to a residual modulation at the edges of the sweep range. The sweep slope is indicated in panel (c) by the white dashed line. (b) Single line cut from (a) at a fixed carrier density, showing clear SdH oscillations. (c) Gate map used to define the sweep trajectory shown in (a). (d) Peak positions of the SdH oscillations plotted against $1/B$, used to extract and verify the charge carrier density.

The single sweep in Fig. 5.12(b) shows SdH oscillations superimposed on a rising background, which appear less pronounced compared to sample A, likely due to differences in sample quality. From these peaks, the charge carrier density can be estimated using the fit in panel (d), providing a check of the approximations made from the measured charge carrier density map via the Hall slope in Fig. 5.7. A slope of $s = 0.065$ yields $n = 1.58 \times 10^{16} \text{ m}^{-2}$, in good agreement with the value of $n = 1.50 \times 10^{16} \text{ m}^{-2}$ obtained from the Hall slope approximation.

Modulated regime

The magnetotransport measurements in the modulated regime are shown in Fig. 5.13 for transport along the direction of the 1DSL modulation. The sweeps were performed along the white dashed line in panel (c), where the 1DSL potential is expected to be approximately constant. In this region there are two satellite Dirac points, and the main Dirac point in the band structure is already flattened, indicating the initial stage before additional Dirac points develop under stronger modulation. In this regime the condition for supercollimation is expected to be approximately fulfilled.

In addition to the periodic 1DSL potential, the response of sample B is affected by sample inhomogeneities that do not follow the ideal periodic pattern. As discussed above, the double peak in the unmodulated regime indicates the presence of regions with slightly different neutrality points, for instance due to local doping variations, fluctuations in gate capacitance caused by the thin hBN layer, or trapped charges at the interface. When the periodic 1DSL potential is applied, these inhomogeneities act in parallel with the modulation, effectively broadening and multiplying the observed features in the magnetotransport.

Figure 5.13(a) shows the resulting Landau fan pattern under finite 1DSL modulation. No clear origins of individual fans at $B = 0$ are distinguishable, nor can the number of fans be directly inferred due to their strong overlap. This complexity is characteristic of a superposition of multiple neutrality points together with the external periodic modulation. To resolve the features, panel (e) shows the second derivative of the longitudinal resistance with respect to B , enhancing the visibility of oscillatory features. Now, five distinct Landau fans become apparent, indicated by red arrows. These fans are arranged symmetrically around $n = 0$, which suggests that both electrons and holes are equally affected by the effective potential landscape.

This observation stands in contrast to the gate sweep in panel (c), where mainly two crossing Dirac peaks are visible. The higher number of Landau fans most likely originates from the interplay of the electrostatic superlattice potential with the intrinsic inhomogeneities of the sample, which effectively replicate neutrality points that cannot be resolved directly in zero-field gate maps but become evident through magnetotransport. For comparison, in sample A at a comparable modulation strength, only three Landau fans were observed. This supports the interpretation that the reduced sample quality and inhomogeneity play a dominant role in sample B.

Figure 5.13(d) shows a single sweep from panel (a) at a hole density of $n = 2.5 \times 10^{16} \text{ m}^{-2}$. In this regime, SdH oscillations are superimposed with commensurability oscillations. These occur whenever the cyclotron radius fulfills the so-called flat-band condition $2R_c = \left(\lambda - \frac{1}{4}\right)a$, discussed in section 3.3.2. At these points, destructive interference between scattered cyclotron orbits suppresses backscattering, leading to minima in the

longitudinal resistance. These oscillations confirm effective modulation and coherent transport across multiple periods. [102, 108]

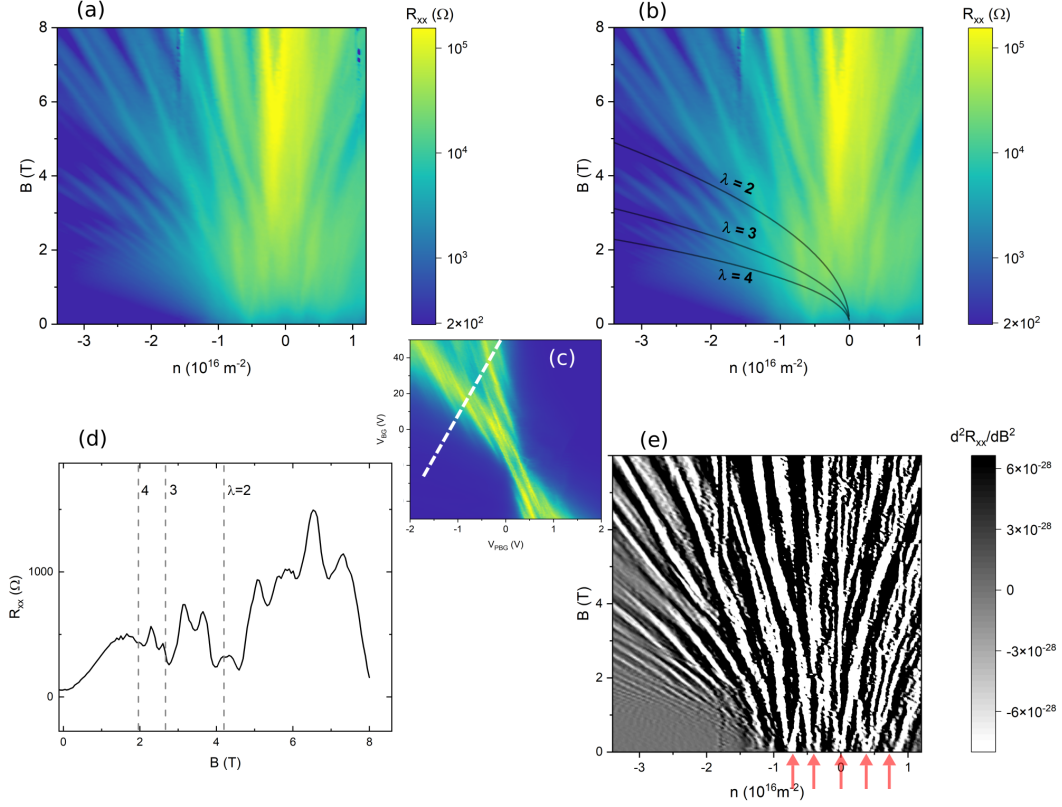
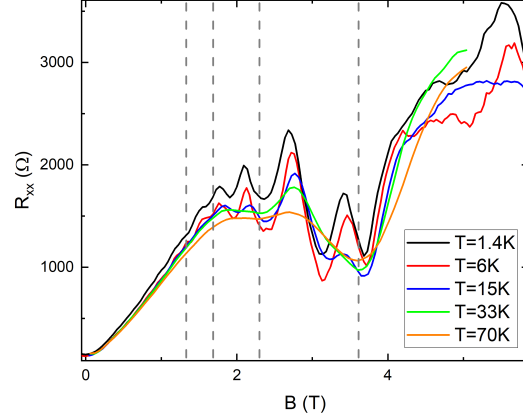


Fig. 5.13: (a) Landau fan for the low-modulation case in sample B, showing satellite Landau fans and superimposed commensurability oscillations. (b) Calculated trajectories for the commensurability oscillations, which are in good agreement with the measured features. (c) Gate sweep trajectory used in (a), chosen to keep the 1DSL modulation as constant as possible. (d) Single line cut from (a) at a charge carrier density of $n = -2.5 \times 10^{16} \text{ m}^{-2}$, where SdH oscillations are clearly modulated by commensurability oscillations. The grey dashed lines mark the calculated positions of the flat-band condition. (e) Second derivative of (a), where the number and positions of the satellite Landau fans become more distinct, as indicated by the red arrows.

5.2.3 Temperature Dependence in Sample B

To investigate the robustness of the commensurability features, magnetotransport measurements were performed at various temperatures between 1.5 K and 70 K. Figure 5.14 shows the longitudinal resistance R_{xx} as a function of magnetic field for fixed hole density

Fig. 5.14: Temperature dependence of the SdH and commensurability oscillations. The sweeps were performed at the same 1DSL modulation strength as in Fig. 5.13(a), at a fixed charge carrier density of $n = 2 \times 10^{16} \text{ m}^{-2}$. With increasing temperature, the SdH oscillations gradually decrease and vanish around $T = 33 \text{ K}$, while the commensurability oscillations remain visible at higher temperatures.



$n = 2 \times 10^{16} \text{ m}^{-2}$ and current flow along the direction of the 1D superlattice.

At low temperatures ($T \leq 15 \text{ K}$), both SdH oscillations and Weiss oscillations are clearly visible. The SdH oscillations are periodic in $1/B$, whereas the Weiss oscillations follow a different periodicity. With increasing temperature, the SdH oscillations are strongly suppressed and become practically unresolvable above $T \approx 30 \text{ K}$ due to thermal broadening of the Landau levels.

In contrast, the Weiss oscillations persist up to significantly higher temperatures. Even at $T = 70 \text{ K}$, weak commensurability features are still detectable, as shown in Fig. 5.14. This behaviour is consistent with the semiclassical origin of Weiss oscillations, which arise from the commensurability between the cyclotron radius and the periodic potential. Their temperature dependence is governed by a thermal damping factor of the form $A(T) \propto \frac{T}{\sinh(2\pi^2 k_B T / \hbar \omega_c)}$ [157]. In addition, impurity scattering leads to an additional exponential damping factor $\exp\left(-\frac{\pi}{\omega_c \tau}\right)$ [157]. However, a quantitative analysis of their temperature-dependent amplitude was not performed here due to the overlap with SdH oscillations, which complicates an unambiguous extraction of the Weiss oscillation peaks. The persistence of Weiss oscillations beyond the disappearance of SdH oscillations confirms their fundamentally different origin: while SdH oscillations rely on Landau quantisation and coherent quantum interference, Weiss oscillations are a semiclassical phenomenon based on orbit geometry. Their visibility at elevated temperatures indicates that the charge carriers maintain a mean free path spanning several modulation periods, which is sufficient for the semiclassical commensurability condition [102, 109, 158].

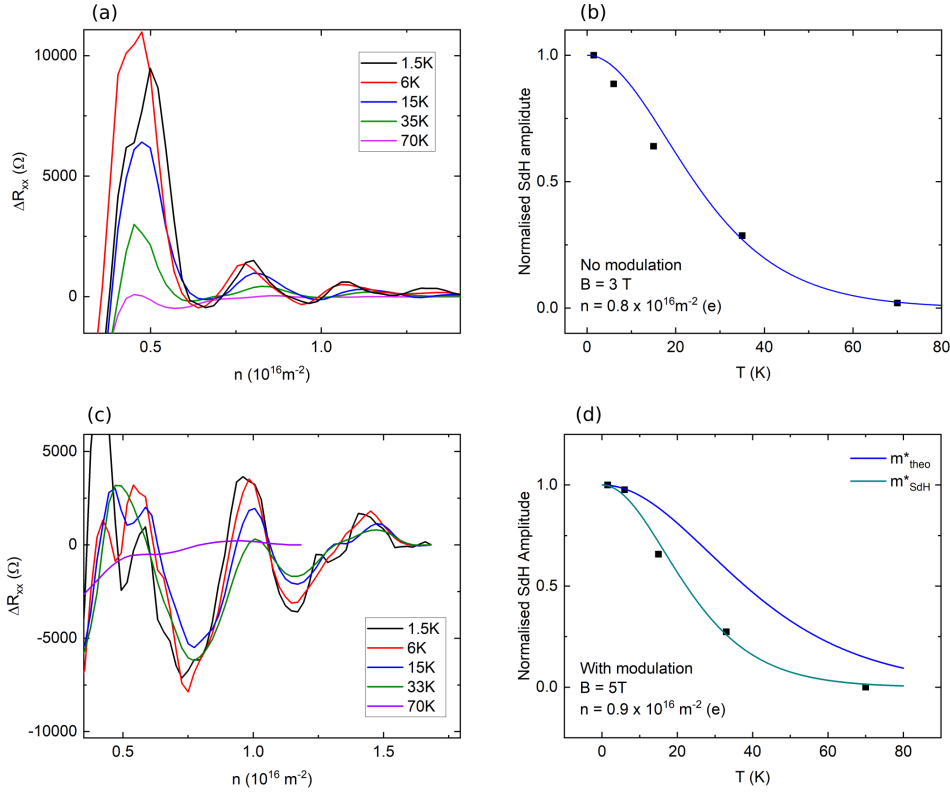


Fig. 5.15: (a) Isolated SdH oscillations for the unmodulated case in sample B at different temperatures, obtained after background subtraction. (b) Peak amplitude of the SdH oscillation at $n = 0.8 \times 10^{16} \text{ m}^{-2}$ and $B = 3 \text{ T}$ as a function of temperature. The trend is fitted using the Lifshitz-Kosevich formula to estimate the charge carrier density. The blue line shows the expected temperature dependence, which agrees well with the experimental data. (c) Isolated SdH oscillations for the modulated case at various temperatures. (d) Temperature dependence of the SdH peak from (c) at $n = 0.9 \times 10^{16} \text{ m}^{-2}$ and $B = 5 \text{ T}$, with a Lifshitz-Kosevich fit for the expected cyclotron mass m_{theo}^* (blue line) that does not describe the data well. The green line shows the fit, revealing a higher cyclotron mass m_{SdH}^* caused by the elliptical distortion of the Fermi contour.

The temperature-dependent amplitude of the SdH oscillations allows the determination of the effective cyclotron mass. In our 1DSL graphene sample, the periodic modulation induces an anisotropic deformation of the Fermi contour, resulting in an elliptical Fermi surface, as discussed in section 3.1. Consequently, the effective mass is expected to increase in the y -direction while remaining approximately unchanged in the x -direction. Notably, the SdH oscillations originate from carriers executing complete cyclotron orbits in the magnetic field, effectively averaging over momentum space directions. This implies that the measured cyclotron mass from SdH oscillations along the x -direction inherently

reflects properties of the entire anisotropic Fermi contour, not solely the x -direction alone.

We analysed the temperature dependence of the SdH oscillations measured along the modulation direction, as shown in Fig. 5.15. Panel (a) displays the longitudinal resistance R_{xx} in the unmodulated regime as a function of average charge carrier density n at a fixed magnetic field $B = 3$ T for various temperatures. After subtracting a smooth background by interpolation, the oscillatory SdH component was isolated and the amplitude extracted. Panel (b) shows the normalized SdH amplitudes versus temperature at $n = 0.8 \times 10^{16} \text{ m}^{-2}$, with the blue line representing the expected Lifshitz-Kosevich temperature dependence [3]:

$$A(T) \propto \frac{T}{\sinh\left(\frac{2\pi^2 k_B T m_c}{\hbar e B}\right)}. \quad (5.1)$$

The effective cyclotron mass m_c can be calculated as

$$m_c = \frac{\hbar k_F}{v_F} = \frac{\hbar \sqrt{\pi n}}{v_F}. \quad (5.2)$$

Fitting this expression yields consistent expected (m_{theo}^*) and extracted (m_{SdH}^*) effective masses in the unmodulated regime of $m_{\text{SdH}}^* = m_{\text{theo}}^* = 1.67 \times 10^{-32} \text{ kg} \approx 0.018 m_e$.

Figure 5.15(c) shows the measurements under 1DSL modulation. The direction of the gate sweep is indicated in Fig. 5.13(c). In this modulated regime, the Dirac cone is elliptically distorted, leading to an anisotropic increase in the effective mass, primarily along the y -direction, while the x -direction remains unchanged. The carrier density and magnetic field were adjusted compared to the unmodulated case to find suitable SdH oscillations with the best visibility.

Panel (d) displays the normalized SdH amplitudes extracted from (c) (black dots), together with the expected temperature dependence (blue line), calculated using the measurement parameters. The green line shows the fitted curve, revealing an effective mass of $m_{\text{SdH}}^* = 3.1 \times 10^{-32} \text{ kg}$, which is approximately 1.7 times the expected value for pristine monolayer graphene at the same carrier density, $m_{\text{theo}}^* = 1.86 \times 10^{-32} \text{ kg}$.

Although the fit quality is somewhat reduced compared to the unmodulated case due to the superposition of commensurability oscillations, it clearly indicates an increased effective mass. This provides strong evidence for a larger and anisotropically distorted Fermi contour, reflecting the modified band structure and significantly affecting the electron dynamics.

Assuming that the 1DSL modulation induces anisotropy only in one direction, we interpret the measured effective mass as the geometric mean of the effective masses in the directions parallel and perpendicular to the modulation $m_{\text{SdH}}^* = \sqrt{m_y^* \cdot m_x^*}$. [3, 18, 20, 66]

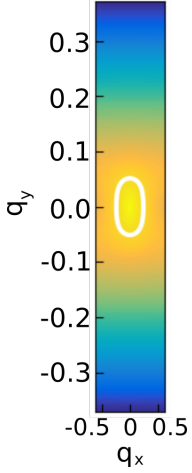


Fig. 5.16: Theoretical estimated elliptical distortion of the Fermi contour for a comparable modulation strength and density. The simulations were provided by Alina Mreńca-Kolasińska.

Using $m_x^* = m_{\text{theo}}^*$, we extract the modulated mass component as

$$m_y^* = \frac{(m_{\text{SdH}}^*)^2}{m_x^*} = 5.17 \times 10^{-32} \text{ kg}. \quad (5.3)$$

This corresponds to an anisotropy in the Fermi contour of

$$\frac{k_{F,y}}{k_{F,x}} = \sqrt{\frac{m_y^*}{m_x^*}} \approx 1.67, \quad (5.4)$$

indicating a significant elliptical distortion induced by the 1DSL modulation. For comparison, Alina Mreńca-Kolasińska calculated band structures for a similar estimated modulation strength and carrier density. Figure 5.16 shows the top view of the distorted Dirac cone in the (q_x, q_y) -plane. The two principal axes of the resulting ellipse differ by a factor of 2.5, which is even larger, but of the same order of magnitude as the value obtained from fitting the temperature dependence of the experimental SdH oscillations.

5.3 Quantum Hall Effect

In this section, we present detailed magnetotransport measurements focusing on the QHE in MLG devices with electrostatically induced 1DSL modulation. By comparing measurements performed at gate voltages corresponding to a finite 1DSL modulation with those at gate voltages effectively yielding no modulation on the same sample, we reveal how the presence of the 1DSL potential alters the formation, sequence, and robustness of quantum Hall plateaus. Particular attention is given to changes in plateau widths and the emergence of additional features related to the superlattice potential. Moreover, we discuss how the spatial profile and symmetry of the 1DSL influence the magnetic quantisation and corresponding transport signatures. These observations provide important insights into the complex interplay between periodic potential modulation and Landau quantisation in graphene.

5.3.1 Sample A

Unmodulated regime

In the unmodulated regime, we expect the QHE sequence to follow that of pristine graphene. In Fig. 5.17(a), the Hall conductivity σ_{xy} is shown as a function of charge carrier density n for a perpendicular magnetic field of $B = 3.5$ T; panel (c) depicts the slope of this measurement in the gate map along a region where the 1DSL modulation is expected to be absent. The Hall plateaus in (a) follow the $(4(N + 1/2))e^2/h$ sequence of the half-integer QHE, consistent with the chiral nature of Dirac fermions in MLG, as discussed in section 2.6.3. Panel (b) shows σ_{xy} as a function of n and B in a colour plot, where each Hall plateau is colour-coded with a small interval around the quantised value to suppress noise. This specific colour scheme was applied consistently for all QHE measurements. Only the expected QHE plateaus were assigned colours, with a small delta around each plateau, allowing a clear representation and distinction of both the plateau positions and their evolution, including higher-order plateaus. In contrast, a classical rainbow colour scale blends colours and does not assign equal width to each colour, making the identification of plateaus and their widths less clear. Continuous rainbow-like transitions also limit the number of distinguishable plateaus that can be displayed without causing visual confusion. For comparison, the QHE data are additionally presented in the Appendix using a continuous colour scale.

As visible in Fig. 5.17(b), the QHE plateaus evolve linearly and symmetrically with n and B , as expected for pristine MLG. Small deviations or gaps in the plateaus around

$n = -1.0 \times 10^{16} \text{ m}^{-2}$ may indicate a residual modulation at the edge of the nominally unmodulated region, possibly due to a misalignment in the estimated equipotential lines. Despite these minor deviations, the observation of well-developed, symmetric Hall plateaus confirms that Landau quantisation is fully preserved in the unmodulated regime of the sample. This indicates a sufficiently high sample quality and low disorder level, allowing the characteristic QHE physics of MLG to be clearly resolved.

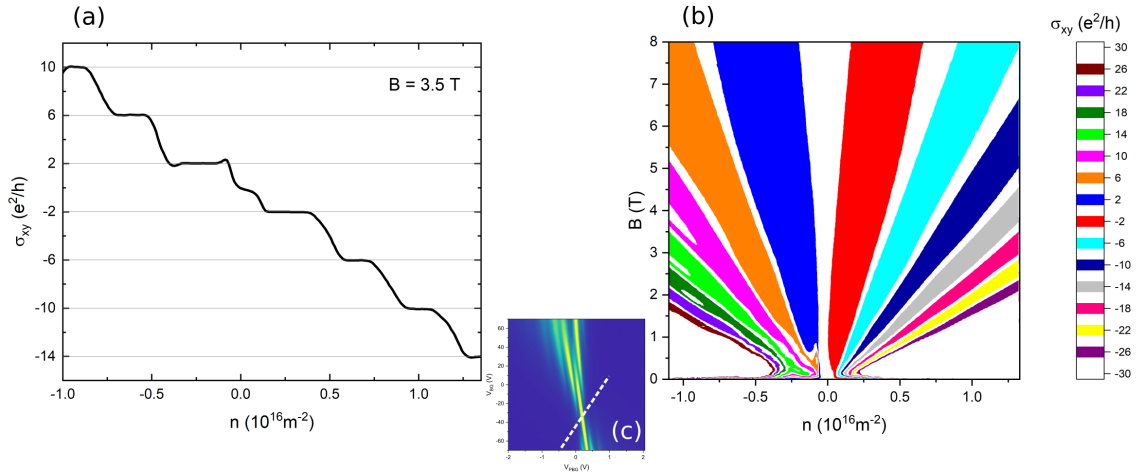


Fig. 5.17: (a) Transverse conductivity at $B = 3 \text{ T}$ and no 1DSL modulation, showing the QHE in MLG: The conductivity exhibits plateaus at integer values of the filling factor $\nu = 4(N + 1/2)$, consistent with the relativistic Landau level structure. (b) Colour plot of the transverse conductivity as a function of charge carrier density and magnetic field. Each plateau region corresponding to a specific ν is colour-coded, with a small range around each value used to suppress noise. The plateaus fan out linearly and symmetrically, as expected for unmodulated MLG. (c) Sweep trace of the gate voltages used in the measurements shown in (a) and (b).

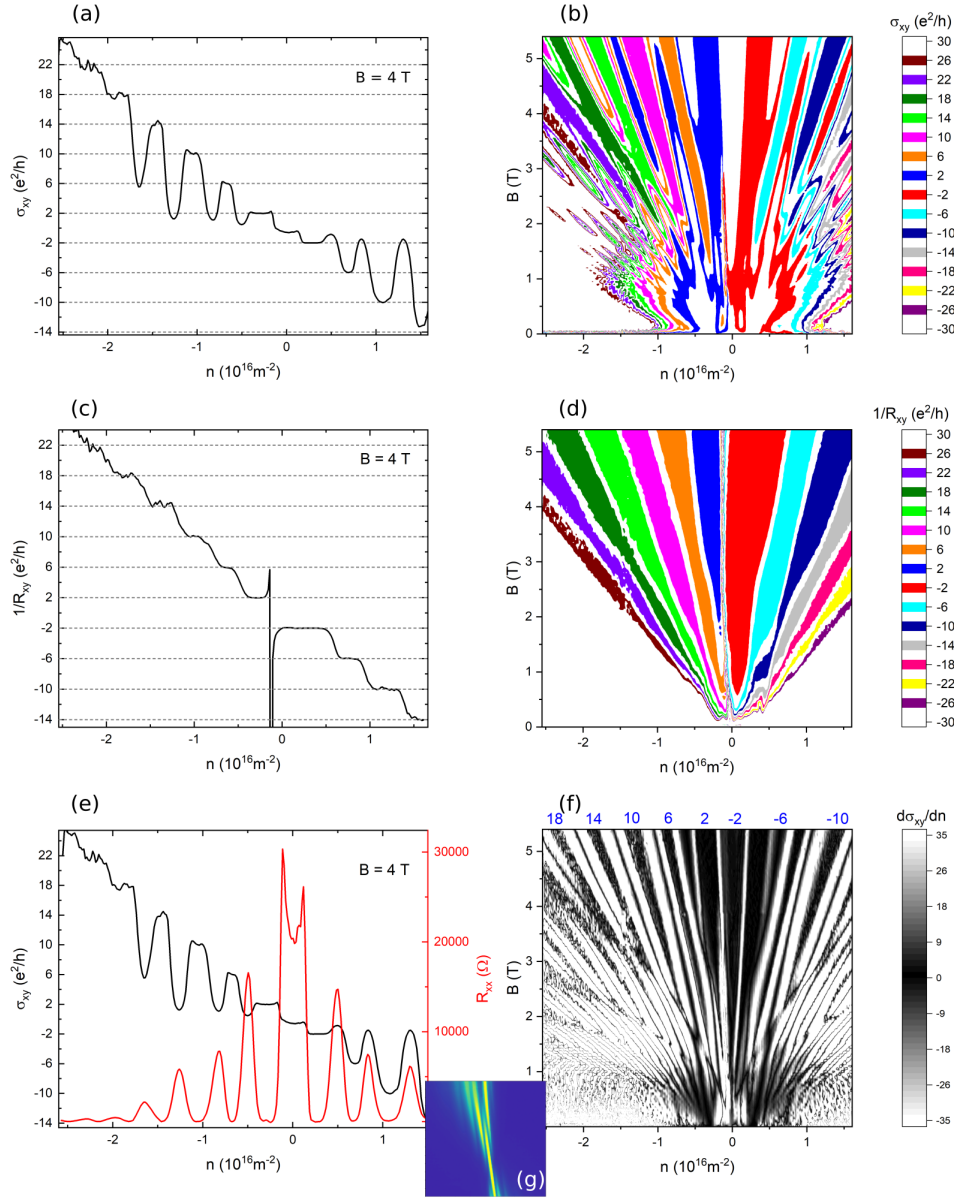


Fig. 5.18: QHE in the low-modulation regime in sample A: **(a)** Transverse conductivity as a function of carrier density at $B = 4$ T, significantly modified compared to the unmodulated case. The QHE steps exhibit pronounced dips between plateaus, indicating a dominant contribution of the longitudinal resistivity to the conductivity tensor. **(b)** Colour plot of the transverse conductivity, with each QHE plateau assigned a distinct colour, reflecting the same behaviour. **(c)** Inverse transverse resistance showing clear Hall steps at the expected values for MLG. **(d)** Colour plot of the inverse resistance with colour-coded Hall plateaus; a strong asymmetry between the $\nu = +2$ and $\nu = -2$ plateaus is visible, caused by the asymmetric change in band structure. **(e)** Transverse conductivity (black) compared to the longitudinal resistance (red), highlighting the origin of the pronounced dips between the QHE steps. **(f)** First derivative of σ_{xy} , with QHE plateau indices in blue, illustrating the strong asymmetry between the $\nu = +2$ and $\nu = -2$ plateaus and revealing traces of Weiss oscillations. **(g)** Gate voltage trace used for the QHE measurement, shown as a white dashed line in the gate map.

Low modulation regime

Figure 5.18 shows the investigation of the QHE in the low-modulation regime. The slope of the measurement is shown in panel (g): here, the two satellite Dirac points are already visible, and satellite Landau fans and commensurability oscillations were observed, as shown in the previous section.

Panel (a) depicts the Hall conductivity σ_{xy} as a function of carrier density n at a magnetic field of $B = 4$ T. The result differs drastically from the unmodulated case: the linear connections between Hall plateaus are disrupted, and pronounced dips in σ_{xy} down to nearly zero appear. This indicates a significant contribution of the longitudinal resistance R_{xx} to the Hall signal, which enters the denominator of the resistivity tensor and causes the dips. Panel (b) shows the Hall plateaus colour-coded as a function of n and B , where the plateaus no longer evolve linearly. Multicoloured regions emerge due to the strong variations in σ_{xy} between the quantised values.

To assess the role of longitudinal resistance in the Hall conductivity ($\sigma_{xy} = \rho_{xy}/(\rho_{xx}^2 + \rho_{xy}^2)$), panel (c) shows $1/R_{xy}$ from the same measurement as (a). Here, linear transitions between plateaus are still visible, although the overall plateau structure is less pronounced than in the unmodulated case. In (d), the colour plot of $1/R_{xy}$ as a function of n and B reveals a strong asymmetry between the $\nu = +2$ and $\nu = -2$ plateaus. This reflects the asymmetric potential profile of the 1DSL modulation: since the potential peaks and dips differ in shape and width, the resulting band structure is asymmetric with respect to electron and hole doping. This leads to differing QHE behaviour on either side of the charge neutrality point. Panel (e) compares σ_{xy} with the longitudinal resistance. The dip positions in σ_{xy} align with the peaks in the longitudinal signal, confirming that σ_{xx} dominates the Hall response at these points.

Panel (f) shows the first derivative $\frac{d\sigma_{xy}}{dn}$, where the influence of σ_{xx} is again evident. Traces of commensurability oscillations can be seen on the left side of the plot. Darker regions indicate a small derivative, i.e. flat σ_{xy} regions corresponding to QHE plateaus. The blue numbers above the plot mark the quantised Hall values, as the colour-coding is not used here.

This is in good agreement with the calculations in Fig. 3.11 by Wu et al. [97], where the transition from the unmodulated to the modulated regime leads to a deformation of the well-defined QHE plateau structure. This results from the broadening and eventual overlap of Landau levels under the influence of the 1DSL potential. In our case, the applied modulation corresponds roughly to a potential strength between $U = 1$ and $U = 3$ in their model, as no additional zero-energy Dirac points have formed yet and only the original Dirac cone is slightly flattened. The resulting band anisotropy and Landau level broadening increase the longitudinal conductivity σ_{xx} , as more extended and dispersive states become available for transport along the transverse direction. This leads to a

reduced visibility of the Hall plateaus despite the underlying Landau quantisation still being present.

High modulation regime

Figure 5.19 presents magnetotransport measurements investigating the QHE in the high-modulation regime. As indicated by the pink slope in panel (e), this regime is characterised by modulation strengths just above the point of supercollimation, leading to the emergence of four satellite Dirac points. It is also possible that a pair of zero-energy Dirac points is generated within the band structure, which could significantly alter graphene's electronic properties and affect the LL degeneracy, thereby modifying the QHE response.

Panel (a) shows the Hall conductivity σ_{xy} as a function of carrier density n at $B = 3.5$ T. Compared to the low-modulation regime (Fig. 5.18(a)), the trace exhibits even more pronounced dips between the plateaus. Notably, the $\nu = 14$ plateau is entirely absent, $\nu = 18$ reappears, and $\nu = 22$ is significantly weakened. These features raise the question whether the skipping of plateaus results from a modified LL degeneracy, possibly caused by the formation of new zero modes, as suggested by Park et al. [4], or whether it stems from an interplay of dominant longitudinal resistance and commensurability effects.

Panel (b) displays the Hall conductivity as a function of n and B with plateaus colour-coded. The high amount of blue areas, corresponding to $\sigma_{xy} = 2e^2/h$, reflects the deep dips between plateaus due to enhanced longitudinal resistance R_{xx} . As $\sigma_{xy} = \rho_{xy}/(\rho_{xx}^2 + \rho_{xy}^2)$, a large ρ_{xx} leads to a suppression of σ_{xy} , causing these low values. In this regime, plateaus above $\nu = 10$ are barely visible, further highlighting the strong influence of the 1DSL modulation.

To examine the influence of R_{xx} in more detail, panels (c) and (d) show the inverse Hall resistance $1/R_{xy}$. Panel (c) confirms that the $\nu = 14$ plateau is missing, while $\nu = 18$ and $\nu = 22$ are also significantly changed. Panel (d) supports this observation, as the light green region associated with $\nu = 14$ appears notably thinner. This suggests that the absence of plateaus is not solely due to enhanced R_{xx} but also to the transformation of discrete LLs into dispersive Landau bands caused by the strong 1DSL potential. This broadening reduces the LL gaps and diminishes the visibility of quantised Hall features. A prominent asymmetry is observed between the $\nu = +2$ and $\nu = -2$ plateaus: while $\nu = +2$ becomes visible around $B \approx 2$ T, the $\nu = -2$ plateau appears already at $B \approx 1$ T and is much wider. This reflects the strong asymmetry of the electrostatic modulation, which leads to an asymmetric band structure for electrons and holes. Such strong asymmetry also makes an interpretation in terms of increased degeneracy less likely, as the generated zero modes would be spatially mismatched.

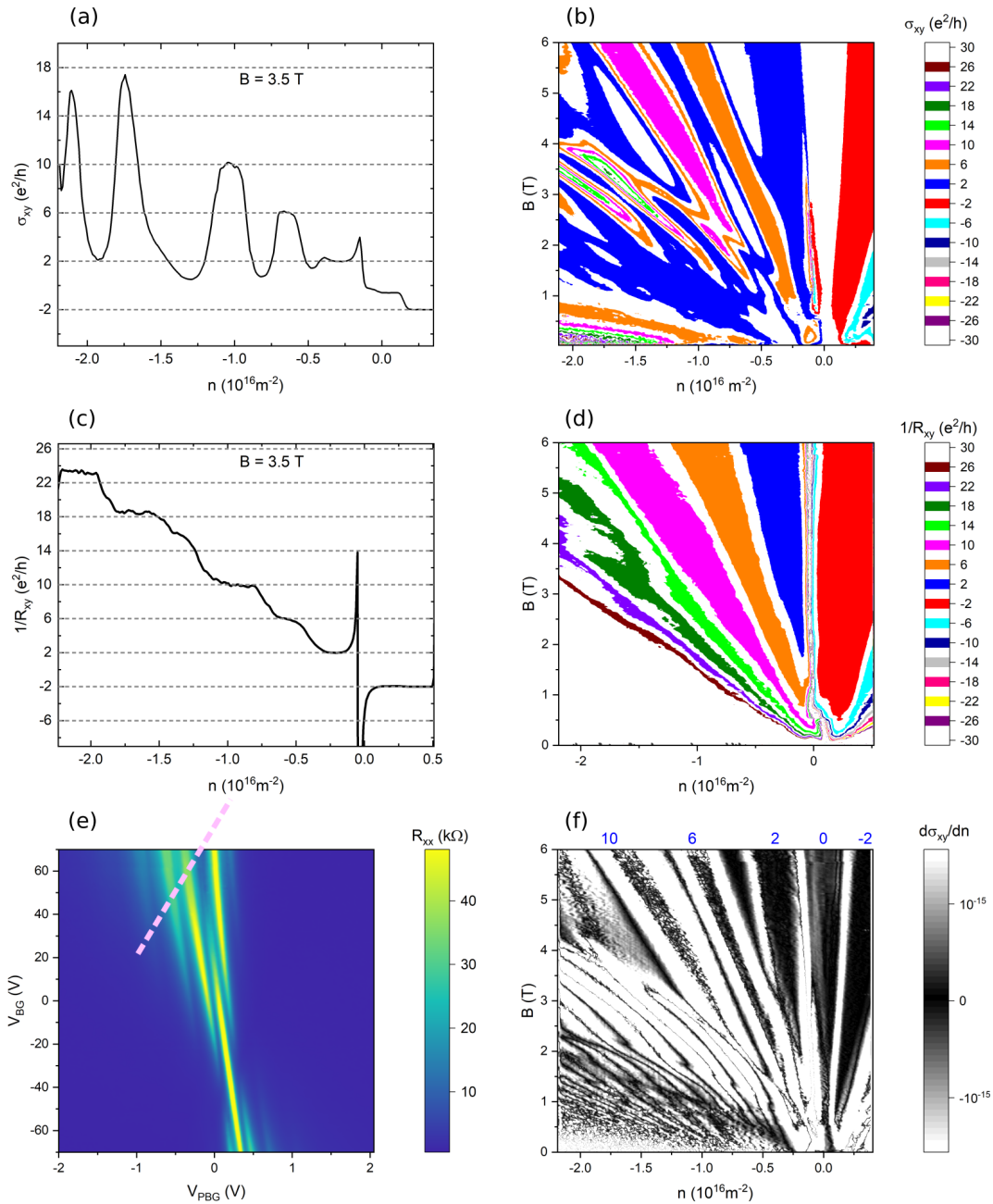


Fig. 5.19: QHE in the high-modulation regime in sample A: **(a)** Transverse conductivity σ_{xy} as a function of carrier density at $B = 3.5 \text{ T}$. The dips between the QHE plateaus are so pronounced that some plateaus appear suppressed. **(b)** Corresponding colour plot of σ_{xy} , showing the disappearance of certain plateaus and a pronounced asymmetry between electrons and holes. **(c)** Inverse Hall resistance $1/R_{xy}$, demonstrating that even without the contribution of longitudinal resistance, the step at $\nu = 14$ is missing. **(d)** Corresponding colour plot of $1/R_{xy}$, highlighting both the strong asymmetry and the absence of the step at $\nu = 14$. **(e)** Pink dashed line indicates the gate voltage trace used for the QHE measurement, as shown in the gate map. **(f)** First derivative of σ_{xy} , revealing a strong asymmetry, a significant influence of the longitudinal resistance, and traces of Weiss oscillations.

Panel (f) shows the first derivative $d\sigma_{xy}/dn$. In addition to the asymmetry, it clearly reveals the influence of σ_{xx} in the form of faint commensurability oscillations. These oscillations overlap with the missing $\nu = 14$ plateau, further supporting the idea that their superposition with broadened LLs masks the plateau. The blue numbers on top of the graph indicate the assigned quantised Hall values.

A plateau-like feature near $\nu = 0$ becomes visible in $d\sigma_{xy}/dn$, which may result from a suppression of the Hall conductivity around the charge neutrality point. This is likely caused by a combination of strong longitudinal resistance R_{xx} and a flattening of $\sigma_{xy}(n)$ near $n = 0$, rather than a genuine quantised $\nu = 0$ state. The asymmetry and strength of the modulation potential could further enhance this suppression.

Taken together, these observations confirm that the Hall quantisation is strongly altered in the high-modulation regime. The disappearance of plateaus is likely caused by a combination of broadened Landau bands, dominant σ_{xx} contributions, and commensurability effects, rather than by a change in LL degeneracy.

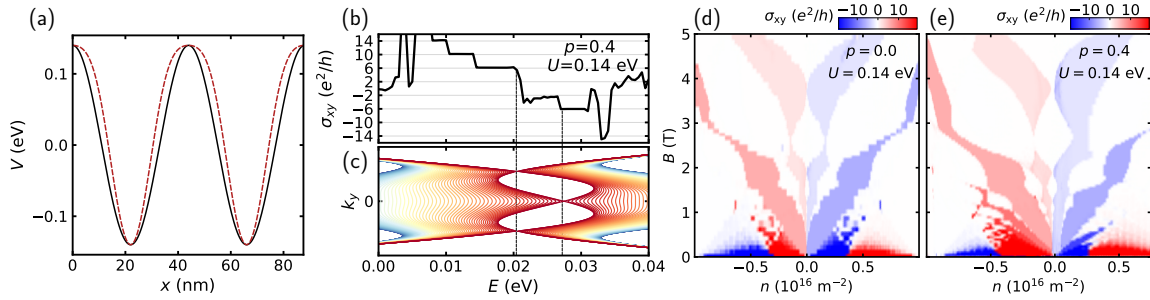


Fig. 5.20: Comparison of magnetotransport for symmetric and asymmetric modulation potentials. Simulations provided by Alina Mreńca-Kolasińska. (a) Potential profile at $U = 0.14$ for the symmetric case (black solid) and the asymmetric case with $p = 0.4$ (red dashed). (b) Hall conductance for the asymmetric potential at $B = 0.5$ T and the corresponding miniband structure in (c), illustrating the effect of misaligned Dirac points. (d) Conductance map for the symmetric case and (e) for the asymmetric case as a function of carrier density and magnetic field. The comparison highlights the asymmetry between the $+2$ and -2 plateaus caused by the shifted Dirac points.

5.3.2 Theoretical Calculations of the QHE in MLG 1DSLs

Calculations for the QHE at high modulation strength in this 1DSL sample were carried out by Alina Mreńca-Kolasińska. These simulations employed a five-terminal device model with dimensions similar to those of the real sample (width of Hall bar of 600 nm and width of leads of 350 nm), which is crucial for accurately capturing the Hall response. A

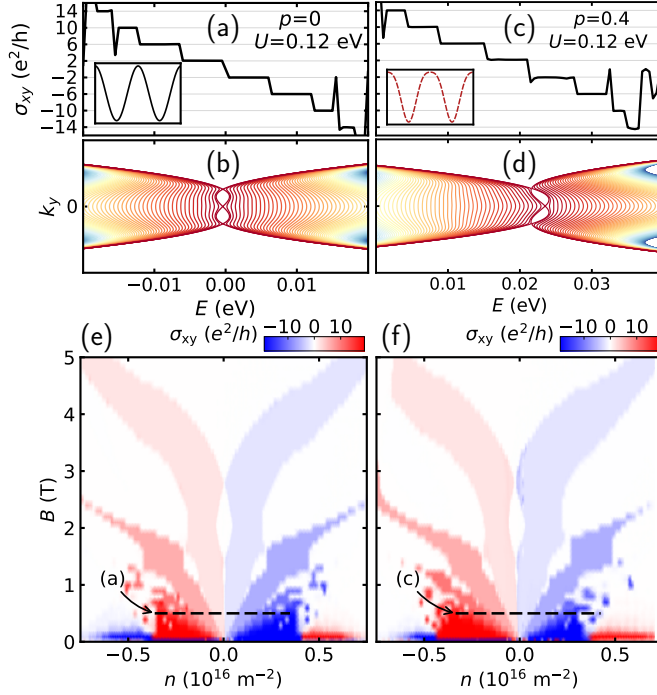


Fig. 5.21: Comparison of conductivity quantisation for the symmetric case (left column) and the asymmetric case (right column) at $U = 0.12$ eV. Simulations provided by Alina Mreńca-Kolasińska. **(a)** Hall conductivity at $B = 0.5$ T with well-quantised, symmetric plateaus, and the corresponding miniband structure in **(b)**. For the asymmetric case with $p = 0.4$, the Hall conductivity and miniband structure are shown in **(c)** and **(d)**. The conductivity maps in **(e)** and **(f)** highlight the electron-hole asymmetry induced by the asymmetric modulation potential.

scaled tight-binding model was used, directly incorporating the electrostatically simulated potential profile of the sample, previously obtained by Ming-Hao Liu and shown in Fig. 5.5. This potential defines the periodic landscape experienced by the charge carriers in the transport simulations and enables a realistic modelling of the quantum Hall regime.

To understand the origin of the observed features, particularly the asymmetry between the $\nu = +2$ and $\nu = -2$ plateaus, model potentials with different asymmetries were considered. This allowed for the systematic investigation of the influence of asymmetry and modulation strength on the band structure and Hall conductivity.

A periodic model potential of the form

$$V(x) = U \cos\left(\frac{2\pi x}{\lambda} - p^2 \cos^2\left(\frac{2\pi x}{\lambda}\right) + p\right) \quad (5.5)$$

was used in these first calculations, where U denotes the modulation amplitude, λ the superlattice period, and p an asymmetry parameter. In the symmetric case ($p = 0$), this expression reduces to a pure cosine modulation. For sufficiently large modulation strengths, it is known that such symmetric 1DSL potentials lead to the formation of multiple additional Dirac points, which in turn increase the Landau level degeneracy and result in quantised Hall steps of $4(2N + 1)e^2/h$. [94]

Figure 5.20 illustrates the two idealised potential shapes in panel (a), with the symmetric case in black and the asymmetric case in red. Panel (c) shows the calculated band structure for the asymmetric potential ($p = 0.4$), where the generated satellite Dirac

points are shifted. This misalignment directly manifests in the Hall conductivity shown in panel (b) for $U = 0.14$, leading to modified plateau sequences. Panels (d) and (e) display the Hall conductivity for the symmetric ($p = 0$) and asymmetric ($p = 0.4$) cases, respectively.

In the symmetric case, all expected plateaus appear and are symmetric in field and density. Even in this idealised potential, the observation of the enhanced degeneracy steps predicted for strong 1DSL modulation is limited to low magnetic fields (here below ≈ 0.5 T), due to the requirement that the magnetic length l_B is large compared to the superlattice period ($2\pi l_B \gg \lambda$).

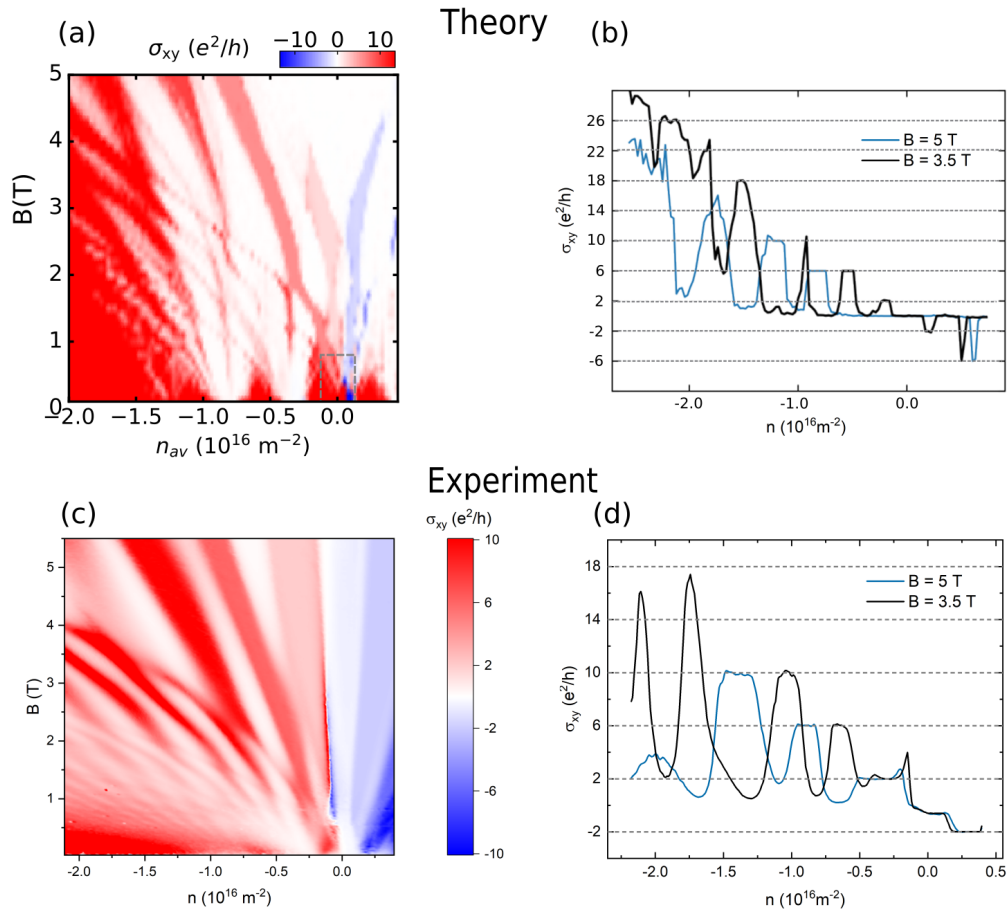


Fig. 5.22: QHE modeling with realistic onsite modulation and comparison to experiment. Simulations provided by Alina Mreńca-Kolasińska. (a) Calculated Hall conductance as a function of carrier density and magnetic field for the high-modulation regime, compared to the experimental data in (c). The model reproduces the electron–hole asymmetry, the visibility of the longitudinal resistance, and traces of Weiss oscillations. (b) and (d) show line cuts at $B = 3.5$ T and $B = 5$ T from simulation (in (b)) and experiment (in (d)), demonstrating good agreement, including the pronounced dips between plateaus.

In contrast, for $p = 0.4$, only the $\nu = +2$ plateau is strongly suppressed and even absent up to $B \approx 0.5$ T. This effect arises from a misalignment in energy between the main Dirac point at $k_y = 0$ and the satellite cones at $k_y \neq 0$, as depicted in Fig. 5.20. The resulting $\nu = -2$ plateau reflects a mixed contribution from the satellite ($-4e^2/h$) and main ($+2e^2/h$) Dirac cones, leading to a broader and asymmetric feature.

The impact of potential asymmetry is already visible at relatively low modulation strength. Figure 5.21 shows calculations for $U = 0.12$, which corresponds approximately to the maximum modulation strength achieved in sample A, close to the onset of additional Dirac point formation. The $\nu = -2$ plateau becomes broader than the $\nu = +2$ one (see panels (c) and (f)), a consequence of the misalignment of the newly generated Dirac points shown in panel (d). In the symmetric case, all Hall plateaus (panels (a) and (e)) remain symmetric and well resolved. This demonstrates that even before fully developed satellite Dirac cones appear (panel (b)), asymmetry alone can induce pronounced electron-hole symmetry breaking in the quantum Hall response.

While these model potentials highlight the qualitative role of asymmetry, a more quantitative comparison requires incorporating the realistic electrostatic profile of the device. Fig. 5.22 presents the simulation results for the electrostatically extracted potential of the sample in panel (a) and (b), with the corresponding experimental results shown in panels (c) and (d) for comparison. The experimental data are identical to those presented in Fig. 5.19. The calculated Hall conductivity reproduces several key features observed in the experiment: dips between plateaus, an asymmetry between the $\nu = \pm 2$ plateaus, visible satellite Landau fans, as well as traces of both SdH and commensurability oscillations. The lighter regions in the colour map correspond to suppressed σ_{xy} due to enhanced longitudinal resistance R_{xx} , confirming that the observed dips result from dissipation rather than broken quantisation. At higher filling factors, the disappearance of well-defined plateaus is also well reproduced, which is attributed to the increased miniband dispersion and the resulting broadening of Landau levels into overlapping Landau bands.

Overall, these simulations demonstrate how strong modulation, potential asymmetry, and realistic superlattice profiles together reshape the quantum Hall landscape.

5.3.3 Sample B

Sample B exhibited behaviour in $1/R_{xy}$ very similar to that of sample A. Due to the double feature in the gate response, the longitudinal resistance displayed numerous satellite Landau fans and a broad region of enhanced resistance (see Fig. 5.13), which complicated the analysis of σ_{xy} , particularly in the region of the Hall conductivity steps

near zero. Therefore, we focus on the differences in $1/R_{xy}$ between the unmodulated and modulated regions of sample B, as shown in Fig. 5.23.

In the unmodulated region (panel (a)), the inverse Hall resistance is plotted as a function of charge carrier density at a fixed magnetic field of 5 T. The expected sequence of Hall plateaus for MLG is clearly visible, indicating that the double feature in Fig. 5.6, likely arising from local variations in doping or potential fluctuations across the sample, does not affect the quantum Hall sequence. Panel (c) shows the inverse Hall resistance as a colour plot, where each colour represents a quantised Hall value. Here, the plateaus fan out evenly, linearly, and symmetrically with charge carrier density and magnetic field, as expected.

In contrast, the modulated region (panel (b)) exhibits a more complex behaviour. The $+2$ and -2 plateaus differ significantly in width, with the $+2$ plateau being barely visible. The -14 plateau is absent, similar to observations in sample A, while the subsequent plateaus reappear. Additionally, several steps appear between the expected quantised values. Panel (d) shows the corresponding colour plot of the inverse Hall resistance, where the pattern is significantly altered compared to the unmodulated case. The plateaus no longer spread linearly and vary noticeably in width. The asymmetry between the $+2$ and -2 plateaus is also clearly visible: the -2 plateau (red) emerges at lower magnetic fields and is significantly broader than the $+2$ plateau (blue), which only starts to form at higher fields.

These results demonstrate that even in sample B, despite its lower overall quality and the additional feature of a double Dirac point, the presence of a 1DSL potential substantially alters the MLG band structure and leads to observable deviations from conventional quantum Hall behaviour.

5.4 Conclusion

In this chapter, two samples were presented, both consisting of a gate-tunable 1DSL in MLG with a period of 50 nm and an L-shaped Hall bar for simultaneous measurements along and perpendicular to the modulation. In $B = 0$ measurements in section 5.1, the predicted anisotropic transport was observed [96]: the generation of satellite Dirac points with rising 1DSL modulation strength along the modulation direction, and a single Dirac point perpendicular to it. Furthermore, the gate maps already showed signs of asymmetry between electrons and holes, as the generated satellites did not appear perfectly symmetric. To investigate the potential profile in more detail, Ming-Hao Liu rebuilt the measured gate map in quantum transport simulations. His reconstruction

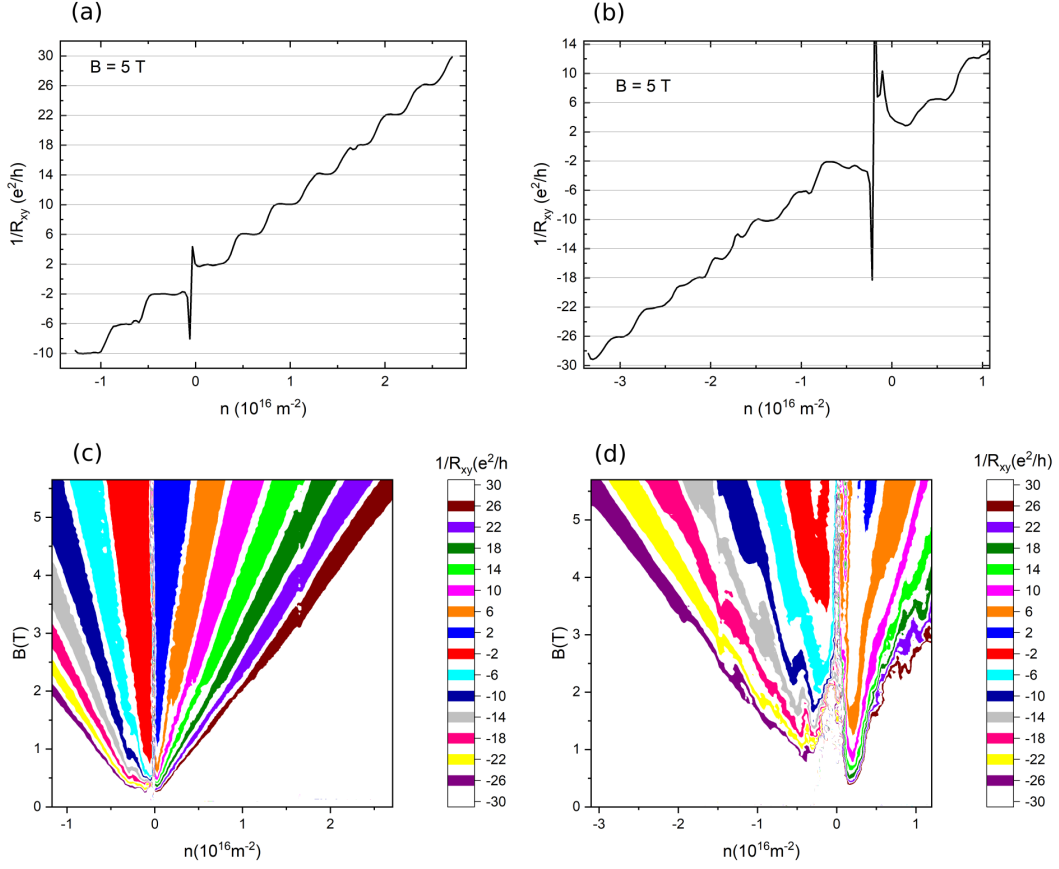


Fig. 5.23: QHE in sample B: (a) Inverse Hall resistance $1/R_{xy}$ without 1DSL modulation as a function of carrier density at $B = 5 \text{ T}$. All QHE plateaus are clearly visible and match the expected values. This is also reflected in the corresponding colour plot of $1/R_{xy}$ in (c). (b) $1/R_{xy}$ for the 1D-modulated case, where the clear plateau structure has vanished and the plateaus at $\nu = -14$ and $\nu = -18$ are missing. (d) Corresponding colour plot of $1/R_{xy}$, showing the drastic modification of the QHE sequence due to the 1DSL modulation.

revealed a highly asymmetric potential profile, in which the shape of the peaks differed from that of the dips, and an estimated gate geometry of 30 nm stripe width and 20 nm gaps was found. This could not be verified directly by AFM measurements due to the limited tip diameter. This raised the question of how such asymmetry would affect further experiments, as most theoretical and experimental studies so far have considered only sinusoidal or Kronig–Penney-type potentials.

Overall, the $B = 0$ measurements already demonstrated how sensitive changes in the band structure are to the precise potential profile, as also noted by Kang et al. [96]. In recent work, anisotropic transport in similar 1DSL samples with dielectric patterning was investigated, but the symmetry of the potential shape and its influence on the gate

maps were not discussed [99].

In section 5.2, magnetotransport measurements in the modulated regime then showed pronounced Weiss oscillations and satellite Landau fans with well-quantised SdH oscillations in both samples, confirming both the applied 1DSL potential and the high sample quality. The absence of Weiss oscillations at specific gate voltage combinations further demonstrated that the 1DSL modulation can be tuned completely on and off, providing a reliable reference. From the temperature dependence of the SdH oscillations, the effective mass and the distortion of the Fermi contour by the 1DSL potential were extracted. The analysis revealed an anisotropic distortion of the Fermi circle into an ellipse with an axis ratio of ≈ 1.7 . Band structure calculations by Alina Mreńca-Kolasińska, based on the potential profile reconstructed by Ming-Hao Liu, yielded a ratio of ≈ 2.5 , which is in good agreement.

Finally, the effects in the QHE regime were investigated in section 5.3. Park et al. [112] predicted that the first QHE plateaus would be skipped as soon as satellite Dirac points appear at $E = 0$, which would modify the degeneracy. However, in the supplemental material, they already noted that this effect may be highly sensitive to the potential shape – an aspect that has so far not been considered in detail for MLG 1DSLs. The two samples characterised in the first section showed well-quantised QHE plateaus in the unmodulated regime. As soon as even a weak modulation was applied, this picture changed completely: pronounced dips between the QHE plateaus appeared, indicating a strong contribution of the longitudinal resistance to the Hall conductivity tensor. At higher modulation strengths, some of the higher-index plateaus disappeared, whereas the $\nu = \pm 2$ plateaus remained but became asymmetric. Furthermore, Weiss oscillations appeared in the Hall conductivity and were responsible for the suppression or weakening of certain plateaus. Band structure and magnetotransport simulations by Alina Mreńca-Kolasińska, using both idealised asymmetric potentials and the exact extracted potential of sample A, reproduced the main experimental findings. These included the sensitivity of the $\nu = \pm 2$ plateaus to the potential symmetry, as their survival relies on the degeneracy of the zero modes, which becomes lifted in the case of an asymmetric potential. The resulting asymmetry of the $\nu = \pm 2$ plateaus can be attributed to the misalignment of the generated zero modes around $E = 0$. The simulations also reproduced the dips between the QHE plateaus and the Weiss oscillations observed experimentally.

Overall, these samples highlight that not only the strength of the 1DSL modulation but also its symmetry in realistic devices crucially influence the band structure, reshaping both $B = 0$ transport and the QHE regime.

6 Bilayer Graphene Superlattices

1DSLs in BLG have so far only been studied theoretically, with no corresponding experimental investigations reported to date. In contrast to monolayer graphene, the presence of two coupled layers introduces additional complexity: both a global displacement field (chemical potential) and an interlayer bias (electrostatic potential) can arise. These two mechanisms affect the band structure in different ways, and in realistic samples they typically act in combination. Because the BLG band structure is highly sensitive to external electric fields, which can open a tunable band gap, periodic potentials are expected to generate a rich variety of modifications to the electronic spectrum. In realistic devices, internal doping, gate-induced asymmetries, and the spatial profile of the applied potential all interplay in shaping the resulting miniband structure. [97, 115]

In this chapter, we investigate two BLG samples modulated by a 1DSL potential. Both samples consist of hBN-encapsulated BLG coupled to a patterned FLG gate forming a PBG with a stripe period of 50 nm.

6.1 Transport at $B = 0$

6.1.1 Characterisation of Sample C

Sample C has the same geometry as the MLG samples shown in Fig. 5.1: an L-shaped Hall bar for simultaneous measurements in and perpendicular to the modulation direction, a 50 nm FLG PBG made from approximately five layers of graphene, separated by 6 nm of hBN from the BLG. All measurements were performed at $T = 1.4$ K with an AC current of 10 nA and a frequency of 13 Hz.

Figure 6.1 shows the gate map characterisation of sample C, i.e. the longitudinal resistance in modulation direction (panel (a)) and perpendicular to it (panel (b)), as a function of the global back gate voltage V_{BG} and the PBG voltage V_{PBG} . The overall gate response qualitatively resembles that of the MLG case shown in Fig. 5.3. In the

bipolar regime, panel (a) shows the emergence of spreading satellite Dirac features with increasing modulation strength, which indicates modifications to the band structure that give rise to additional zero-energy crossings at finite carrier densities. Sample C showed mobilities of $\mu \approx 35\,000\text{ cm}^2/\text{Vs}$. For a charge carrier density of $1.0 \times 10^{16}\text{ m}^{-2}$, this leads to a mean free path $l_{mfp} \approx 0.4\text{ }\mu\text{m}$.

A key difference to MLG, however, becomes apparent around zero modulation, located approximately at $(V_{\text{PBG}}, V_{\text{BG}}) = (0.2\text{ V}, -10\text{ V})$. With increasing 1DSL modulation strength in either direction, the central Dirac point initially vanishes and later reappears. Unlike in the MLG case, this reappearance does not involve a clear splitting into satellite Dirac points. Instead, the behaviour suggests that the longitudinal resistance at charge neutrality is governed by local opening and closing of the band gap, consistent with the field-induced tunability of the BLG band structure discussed in section 3.4.1.

In BLG, a perpendicular electric field breaks the layer symmetry and opens a tunable band gap [159]. In the presence of a periodic field modulation, as imposed by the PBG, the effective interlayer bias becomes spatially modulated. This creates alternating regions with locally opened and closed gaps, which in turn cause the resistance at the central Dirac point to vary non-monotonically. This effect could either be caused by internal doping or a different stripe width, as in comparable samples, which will be presented in the next section.

Thus, we observe a hallmark of 1DSLs in BLG: the interplay of miniband formation, as in MLG, with the spatially modulated band gap that arises from the interlayer bias in BLG. As discussed before, realistic devices will generally exhibit a superposition of these effects due to screening between the layers and asymmetries introduced by the gate geometry and internal doping. [113]

The perpendicular direction in panel (b) exhibits a similar behaviour around the Dirac point at zero 1DSL modulation: in both directions of increasing modulation strength, the central Dirac point first disappears and later reappears. However, in contrast to the modulation direction, no additional satellite Dirac points emerge at higher modulation strengths. This observation is consistent with the anisotropic influence of the 1DSL on the band structure, which modifies the electronic spectrum primarily along the modulation axis and thus leads to an anisotropic transport response. The disappearance and subsequent reappearance of the central Dirac point is attributed to the modulation-induced opening and closing of a band gap, as discussed in section 3.4.1.

Panels (c) to (f) show individual gate voltage sweeps extracted from panel (a), as indicated by the white dashed lines. Panel (c) displays the resistance peak at the central Dirac point under zero modulation. Panel (d) shows the reappeared Dirac point at finite modulation strength, characterised by an enhanced resistance peak and the first indications of satellite peaks.

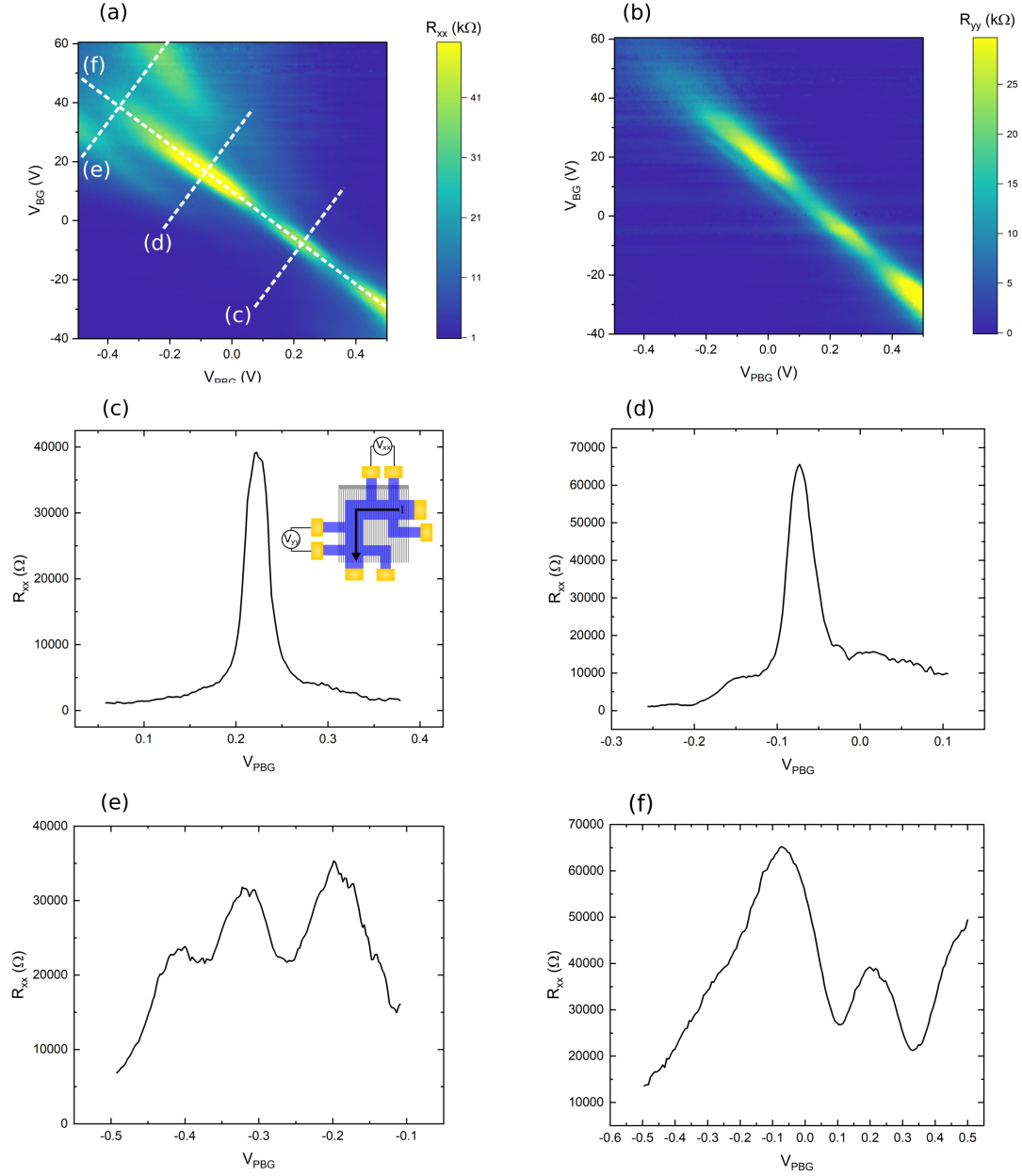


Fig. 6.1: Gate response in sample C: (a) Longitudinal resistance along the modulation direction R_{xx} as a function of both gate voltages. In the upper left, the modulation strength is sufficient to generate satellite Dirac points. The disappearance and reappearance of the central Dirac point are attributed to a modulation-induced band gap formation, enhanced by intrinsic doping. (b) Resistance perpendicular to the modulation R_{yy} as a function of both gate voltages. No satellite features are observed, consistent with the anisotropic transport expected in 1DSLs. (c)–(f) Gate voltage sweeps corresponding to the traces marked in (a). The inset in (c) shows a schematic top view of the sample.

Panel (e) corresponds to the regime of strong modulation, where two additional resistance peaks emerge due to the formation of satellite Dirac points from extra zero modes in the band structure. In total, three resistance peaks are observed.

Finally, panel (f) presents a sweep along the 1DSL axis, where the disappearance and reappearance of the central Dirac point are clearly visible, in agreement with the expected opening and closing of a modulation-induced band gap.

The observation that the main Dirac point in both transport directions initially appears at $(V_{\text{PBG}}, V_{\text{BG}}) \approx (0.2 \text{ V}, -10 \text{ V})$ instead of at zero gate voltages suggests the presence of an intrinsic interlayer asymmetry in the system. This asymmetry most likely arises from internal doping effects, such as trapped charges in the dielectric environment or residual built-in fields from fabrication, which induce a finite electric field between the two graphene layers. As a consequence, the band structure of the bilayer is already gapped in the absence of any externally applied 1DSL modulation. The disappearance of the Dirac point with increasing V_{PBG} and its reappearance at higher modulation strengths can thus be interpreted as the external modulation either enhancing or compensating the intrinsic bias field. This interplay between internal doping and externally applied potentials highlights the complexity of realistic bilayer graphene superlattices, where multiple mechanisms simultaneously contribute to band structure modification and gap formation. [5, 6]

6.1.2 Characterisation of Sample D

Figure 6.2 shows the characterisation of a second BLG 1DSL in sample D, which was fabricated in the same batch as sample C. This means that the dimensions of the Hall bar and the hBN dielectrics were chosen as similar as possible to those in sample C, and that both devices were etched and contacted in the same fabrication steps. All measurements were performed at $T = 1.4 \text{ K}$ with an AC current of 10 nA and a frequency of 13 Hz .

Sample D showed mobilities of $\mu \approx 40\,000 \text{ cm}^2/\text{Vs}$. The gate maps along and perpendicular to the 1DSL modulation (panels (a) and (b)) show a markedly different behaviour compared to sample C. Along the y -direction, i.e., perpendicular to the modulation, a single Dirac peak is observed for all back gate voltages, which increases in resistance with increasing displacement field, as expected for gapped BLG. Along the modulation direction, the main Dirac point also never vanishes, but likewise increases in resistance with stronger fields (see panels (c)–(e)). At higher modulation strengths, satellite peaks appear on both sides of the main Dirac point.

This distinctly different behaviour indicates a different internal doping level in sample D compared to sample C, implying that the initial band structure before applying any

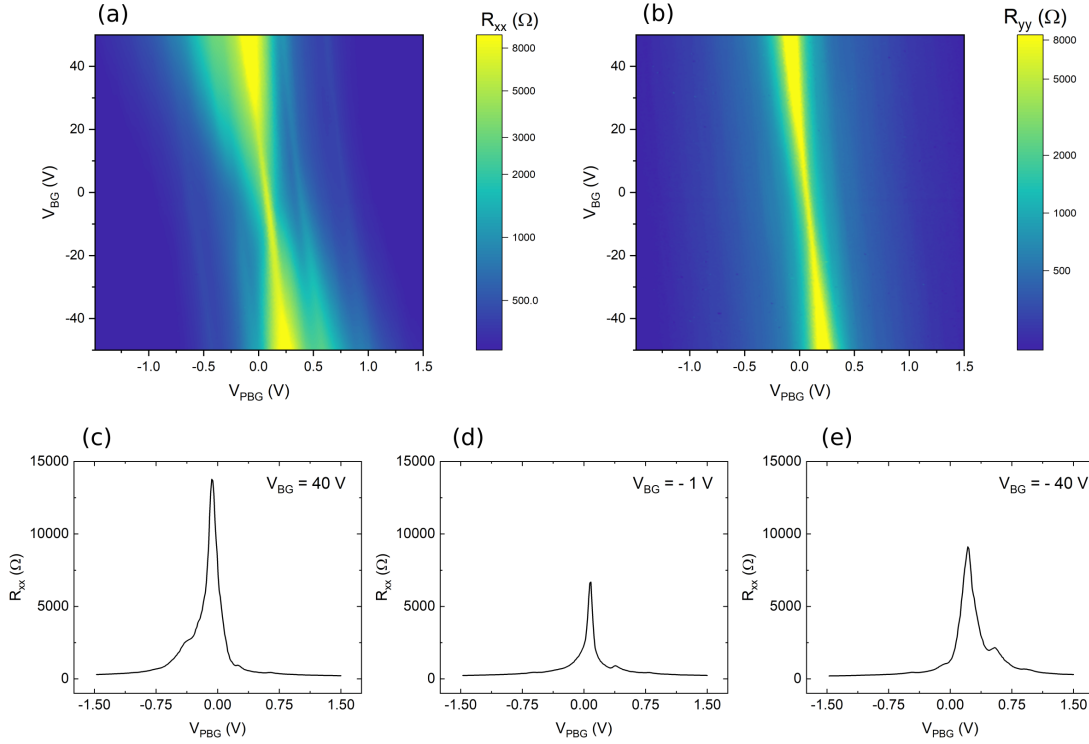


Fig. 6.2: Gate response in sample D: **(a)** Longitudinal resistance along the modulation direction R_{xx} as a function of both gate voltages. Satellite features are visible, but the overall response differs from that of sample C, likely due to different internal doping and gate geometry. **(b)** Resistance perpendicular to the modulation R_{yy} as a function of both gate voltages. No satellite features are observed, consistent with the anisotropic transport expected in 1DSLs. **(c)–(e)** PBG voltage sweeps for different back gate voltages, showing the evolution of the Dirac peak and the emergence of satellite Dirac points.

1DSL or displacement field was fundamentally different. The data suggests that the intrinsic displacement field in sample D was close to zero, resulting in a nearly gapless band structure, which was then gapped by the applied modulation field, as reflected in the rising resistance at the Dirac point. Simultaneously, the lateral modulation appears to induce additional Dirac-like features in the band structure, which manifest as satellite resistance peaks in the gate response. The region around $(V_{PBG}, V_{BG}) = (-0.2 \text{ V}, -1 \text{ V})$ shows the lowest resistance at the Dirac point and likely corresponds to the condition where both the band gap and the superlattice modulation are minimal or absent.

In BLG 1DSLs, both the initial electrostatic configuration and the gate geometry imposing the 1DSL potential can influence the band structure. Slight variations in the stripe width or spacing of the PBG, which can arise from fabrication tolerances, modify the effective potential landscape, potentially contributing to the steeper slope or distinct shape of the main Dirac point in sample D. Thus, both differences in intrinsic doping

and small variations in the 1DSL potential profile provide plausible explanations for the observed deviations from sample C.

As shown in Ref. [113], these two mechanisms (periodic modulation of the local band gap and of the chemical potential) typically appear simultaneously in experimental devices and cannot be tuned independently. A systematic investigation of this aspect could be enabled by introducing a third global gate to future devices, which—together with the other two gates—would allow independent control over the band gap, the 1DSL modulation of the band structure, and the Fermi level.

6.2 Commensurability Oscillations

6.2.1 Sample C

To investigate the influence of the 1DSL on Landau quantisation in BLG, we first examine the **unmodulated regime** in sample C, measured at the gate configuration corresponding to that used in Fig. 6.1(c). In this regime, only a single Dirac point is present, from which additional satellite Dirac points emerge symmetrically as the 1DSL modulation strength increases. The resistance along the modulation direction as a function of charge carrier density and magnetic field is depicted in Fig. 6.3. In panel (a), a single Landau

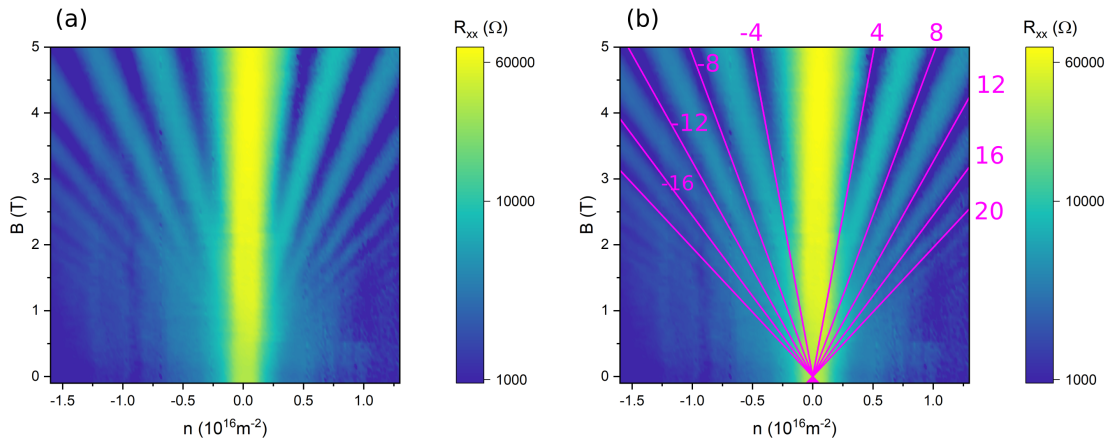


Fig. 6.3: (a) Longitudinal resistance R_{xx} as a function of carrier density and magnetic field, showing the expected Landau fan pattern with pronounced SdH oscillations. The measurement was taken along line 1 in Fig. 6.1(c), where the 1DSL modulation was approximately absent. The calculated positions of the SdH minima for different filling factors are indicated by pink lines in panel (d), which agree well with the experimental data.

fan originating from the main Dirac point is clearly visible, with well-defined Landau levels manifested as pronounced SdH oscillations. In panel (b), the SdH minima are labeled by the pink numbers, and the calculated positions of the minima for each Landau level are shown as solid traces.

In bilayer graphene, the Landau levels give rise to quantum Hall plateaus at multiples of $4e^2/h$, as discussed in section 3.4.2, which is consistent with the observed resistance oscillations. As in the MLG 1DSL sample, the charge carrier density axis was calibrated experimentally by comparing the Hall slopes, since the parallel-plate capacitor model did not yield accurate results for this configuration.

Importantly, no signatures of an additional periodic potential, such as commensurability oscillations, are observed in this regime. This confirms that the measurement is taken in the unmodulated part of the device, where only a single Dirac point contributes and no superlattice modulation is present.

The direction perpendicular to the 1DSL did not show clear Landau quantisation in this sample due to lower quality in this region of the sample and is thus not shown here.

The **modulated regime** is shown in Fig. 6.4 for two different modulation strengths. Panel (a) displays the resistance along the modulation direction as a function of charge carrier density and magnetic field, measured in the regime where a second single Dirac point emerges. The measurement was performed at the gate configuration corresponding to that used in Fig. 6.1(d).

The superimposed pattern visible around $n = -3 \times 10^{16} \text{ m}^{-2}$ consists of commensurability oscillations, which are a hallmark of the 1DSL modulation. These occur when the cyclotron diameter of the charge carriers becomes commensurate with the 1DSL period, leading to a reduction in resistance and a modulation of the SdH oscillations, as discussed in section 3.3.2. The regions around zero charge carrier density appear less well quantised, which could be a sign of emerging satellite Dirac points that were not clearly resolved in the gate map. In this regime, both the band gap and the emergence of satellite Dirac points are jointly tuned by the patterned bottom gate and the global back gate.

Panel (b) shows the resistance along the modulation direction, measured at the gate configuration used in Fig. 6.1(e). In this regime, two satellite Dirac points are clearly visible. This is also reflected in the magnetotransport measurement, where a total of three resistance peaks can be observed. However, no well-defined Landau fans are visible. This could indicate that the 1DSL modulation is strong enough to significantly broaden the Landau bands, thereby suppressing clear quantisation.

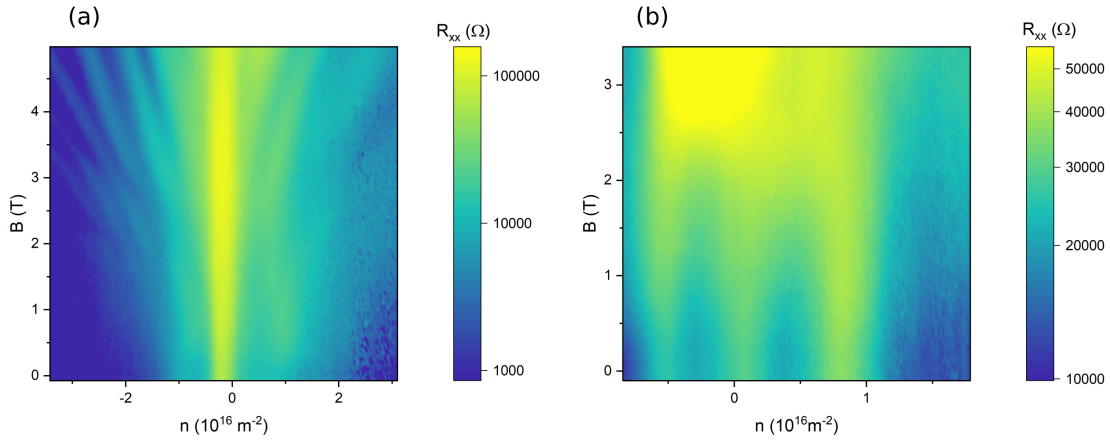


Fig. 6.4: (a) Longitudinal resistance R_{xx} as a function of carrier density and magnetic field, measured along line 2 in Fig. 6.1(c), where the 1DSL modulation is present. The regular SdH pattern fades, and traces of Weiss oscillations emerge, indicating the presence of the 1D modulation. (b) Landau fan of R_{xx} in the high-modulation regime, corresponding to line 3 in Fig. 6.1(c). Here, the SdH oscillations are fully suppressed due to Landau level broadening, while two satellite fans appear, originating from the additional Dirac points generated by the 1DSL.

6.2.2 Sample D

Magnetotransport measurements in the **unmodulated regime** of sample D are shown in Fig. 6.5. Panel (c) displays the gate voltage trajectory (white dashed line) used to minimise the influence of the 1DSL modulation.

Panel (a) shows the longitudinal resistance in x -direction as a function of charge carrier density and magnetic field, revealing a clear quantisation of Landau levels, visible as pronounced SdH oscillations. Panel (d) displays the calculated positions of SdH minima expected for BLG, which agree well with the experimental data in panel (a).

At lower magnetic fields ($B \approx 2 \text{ T}$), signatures of commensurability oscillations appear: the darker blue traces indicate regions of reduced resistance. This suggests that either the chosen gate trajectory did not completely suppress the 1DSL modulation, or that internal doping combined with the sensitivity of the BLG band structure prevented a full cancellation of the effective modulation.

In fact, all Landau fan measurements in this sample exhibited signs of commensurability oscillations, indicating that no gate voltage combination could be found where the modulation was entirely absent. Additionally, the SdH oscillations show an extra minimum between the $\nu = \pm 4$ plateaus, which may reflect the lifting of valley or spin degeneracy

in BLG rather than arising from a residual 1DSL modulation.

The corresponding resistance in y -direction is shown in panel (b). The pattern appears qualitatively similar, with well-developed SdH oscillations matching the calculated traces in panel (e). However, no clear commensurability oscillations are observed in this direction. Nevertheless, a small additional minimum between the $\nu = \pm 4$ plateaus is again visible.

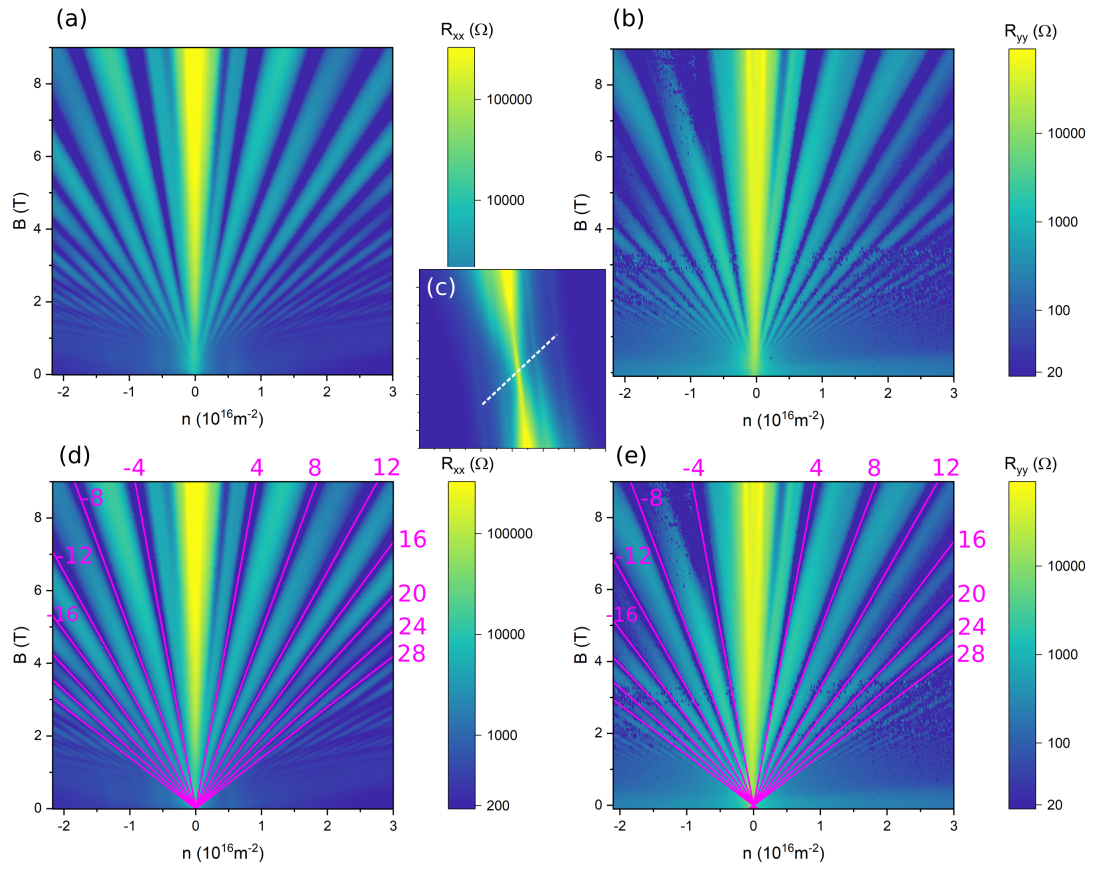


Fig. 6.5: (a) Longitudinal resistance R_{xx} as a function of carrier density and magnetic field, measured along the white dashed line in (c), where the 1DSL modulation is approximately absent. Pronounced SdH oscillations are visible, along with signs of Weiss oscillations, indicating that the modulation is not entirely absent. (b) Landau fan of R_{yy} , also showing pronounced SdH oscillations. (d) and (e) display the calculated SdH minima positions for the corresponding measurements in (a) and (b), which agree well with the experimental data in both transport directions.

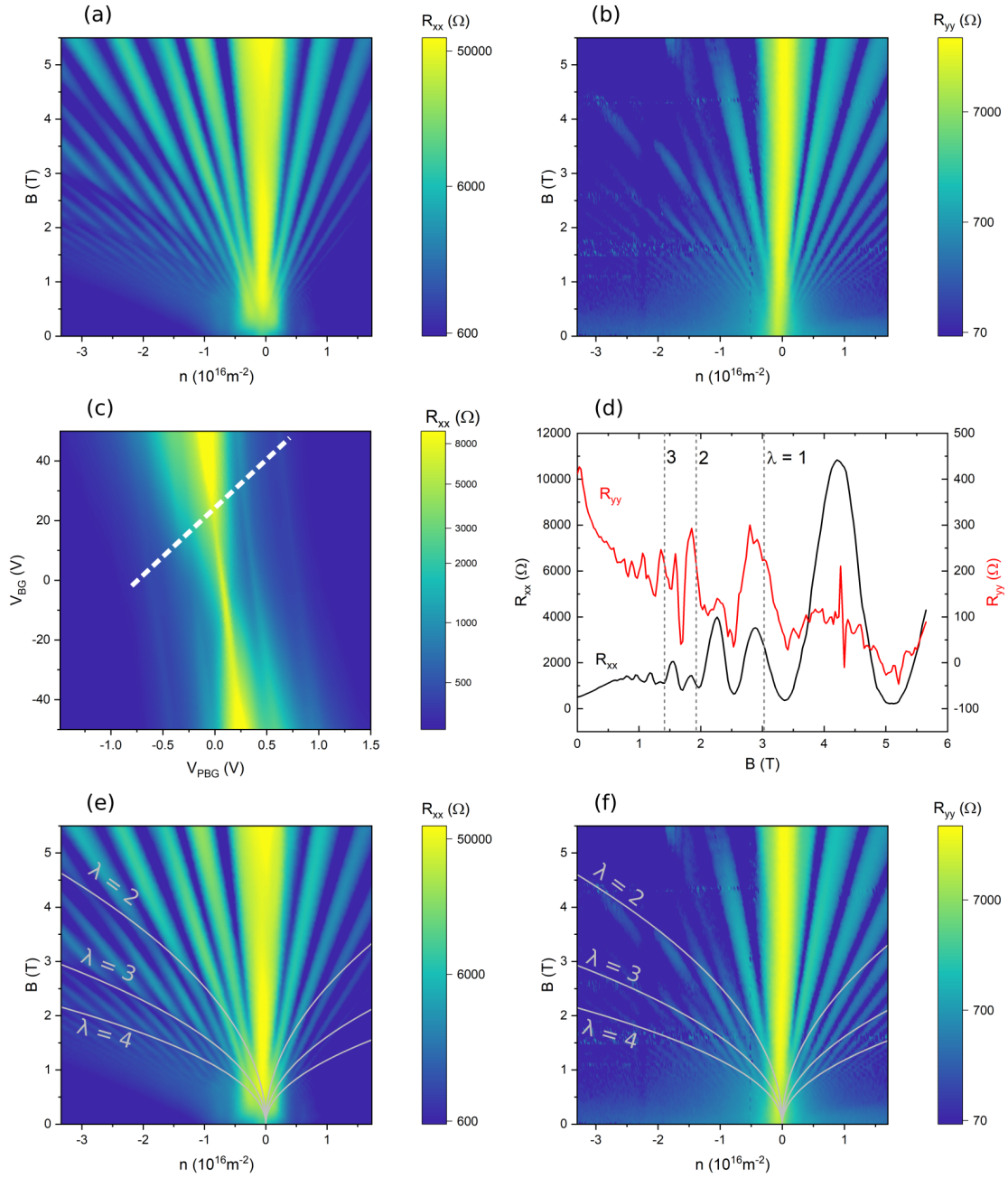


Fig. 6.6: (a) Longitudinal resistance R_{xx} as a function of carrier density and magnetic field, measured along the white dashed line in (c), where the 1DSL modulation is present. Pronounced SdH oscillations are visible, along with clear Weiss oscillations, confirming the presence of one-dimensional modulation. At low magnetic fields signs of satellite Landau fans are visible, resulting from the generated Dirac points. (b) Landau fan of R_{yy} , also showing pronounced SdH oscillations and Weiss oscillations that are in counterphase to those in (a). (d) Comparison of R_{xx} and R_{yy} at $n = -2 \times 10^{16} \text{m}^{-2}$, with the expected flat-band positions indicated by dashed lines. These match well with the experimental data and highlight the antiphase behaviour between x - and y -direction. (e) and (f) Calculated flat-band conditions for Weiss oscillations, showing resistance minima in R_{xx} and corresponding maxima in R_{yy} .

The **low modulation regime** is depicted in Fig. 6.6, where panel (c) shows the gate voltage trace used for this measurement. The resistance along the x -direction is shown in panel (a), where multiple Landau fans emerge from different charge neutrality points. The SdH oscillations are clearly superimposed by commensurability oscillations (see section 3.3.2), which are indicated by grey lines in panel (d). The red and black curves show single sweeps from (a) and (b) at an average charge carrier density of $n = 2 \times 10^{16} \text{ m}^{-2}$. Here, λ denotes the integer index of the flat band condition, which occurs when $R_C = (\lambda - \frac{1}{4})a$ is satisfied, with R_C being the cyclotron radius and a the 1DSL period. The calculated resonance conditions are in good agreement with the experimental data.

Panel (b) shows the corresponding measurement in the y -direction, where only one Landau fan is visible. While the Landau quantisation is less pronounced in the hole regime, distinct signatures of commensurability oscillations are still visible. The same flat-band resonance conditions as in panel (e) are marked in panel (f) and compared to the experimental data from (b). Notably, the COs appear in antiphase relative to the x -direction, consistent with the expectation that in the direction perpendicular to the 1DSL modulation, resistance maxima rather than minima occur at the flat band condition (see section 3.3.2). This phase shift arises from fundamentally different physical mechanisms: along the 1DSL modulation, COs result from a classical resonance between the cyclotron orbit and the periodic potential, leading to minima in resistance when the drift velocity is maximised. Perpendicular to the modulation, however, the effect is quantum mechanical in nature. The periodic potential modulates the DOS at the Fermi energy, which in turn affects the scattering rate and thus the resistivity via the scattering conductivity. This results in resistance maxima at the same resonance condition where minima occur in x -direction. [104, 105, 160]

The Landau fans in the **high-modulation regime** are shown in Fig. 6.7, where panel (c) indicates the trace of gate voltage combinations used to keep the 1DSL modulation as constant as possible. Panel (a) shows the longitudinal resistance along the modulation direction. Both SdH oscillations and commensurability oscillations are visible, although less pronounced than in the low-modulation measurement. This observation is consistent with the results from the MLG 1DSL sample in section 5.2: when the 1DSL modulation is strong enough, the Landau levels broaden and begin to overlap, leading to a suppression of SdH oscillations. This broadening arises due to the formation of magnetic minibands, as the periodic potential couples adjacent Landau orbits and thereby introduces a dispersion to the otherwise flat Landau levels. As a result, the density of states becomes smoother, and the quantisation is less pronounced in magnetotransport. Panel (b) shows the resistance in the direction perpendicular to the modulation. Here, the SdH oscillations are also less distinct, and no commensurability features are observed.

In summary, samples C and D exhibit qualitatively similar behaviour in their Landau fan measurements. In both devices, the 1DSL modulation leads to a tunable Landau level spectrum, with the emergence of satellite Landau fans and commensurability oscillations as clear signatures of miniband formation. This confirms that the periodic potential introduced by the patterned bottom gate effectively modifies the band structure of bilayer graphene.

Interestingly, while the zero-field gate responses of samples C and D differ significantly, likely due to variations in intrinsic doping and initial displacement field, the high-field magnetotransport features are remarkably consistent. This underlines the robustness of the underlying miniband formation mechanisms and the general influence of the 1DSL modulation on Landau quantisation in bilayer graphene.

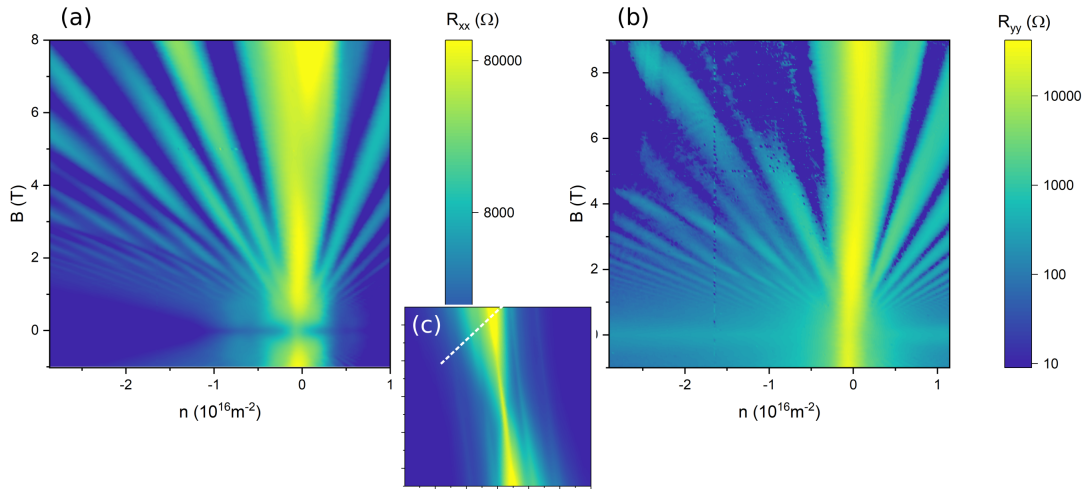


Fig. 6.7: (a) Longitudinal resistance R_{xx} as a function of carrier density and magnetic field, measured along the white dashed line in (c), where the 1DSL modulation is strong. SdH oscillations are less pronounced due to Landau level broadening caused by the modulation, while Weiss oscillations remain visible. (b) Landau fan of R_{yy} , showing similarly reduced SdH oscillations consistent with Landau level broadening from the 1DSL modulation.

6.3 Quantum Hall Effect

This section presents QHE measurements on sample D. Measurements were also performed on sample C, but the plateaus were less pronounced even in the unmodulated regime, preventing an unambiguous analysis of the 1DSL influence on the QHE in BLG.

The QHE is a further tool to investigate the influence of the 1DSL modulation on Landau quantisation in BLG. As seen in the previous measurement on sample D, the 1DSL modulation was never entirely absent, likely due to residual modulation or insufficient gate-induced screening. This is particularly evident in Fig. 6.5, where the Landau fan recorded along the approximated gate trace of a single Dirac point and nominally without 1DSL modulation already shows signs of commensurability oscillations, indicating that the modulation is still present.

Figure 6.8 displays the corresponding transverse conductivity and inverse Hall resistance along the same trace (also shown in the inset of panel (b)). Here, both in the conductivity and inverse resistance plateaus appear at the expected values of $4e^2/h$ for BLG, with small additional plateaus at $\nu = \pm 2$, indicating partial lifting of spin and valley degeneracy and reflecting the high sample quality [23, 161]. As seen in panel (a), the conductivity already exhibits pronounced dips between the QHE plateaus. This further confirms the presence of a residual 1DSL modulation, as the same measurement in MLG devices showed no such dips in the unmodulated regime (see Fig. 5.17).

The full colour plot of the transverse conductivity in panel (c), where each QHE plateau is coloured with a slight delta to reduce noise, also reflects this behaviour by showing a pattern of colour changes between plateaus, caused by the conductivity dips.

For comparison, panel (b) shows the inverse Hall resistance, which is not influenced by the longitudinal resistance ρ_{xx} in the same way as the transverse conductivity σ_{xy} , and thus reveals clean QHE steps even in the presence of residual modulation. This is also reflected in the colour plot in panel (d), where the QHE plateaus appear as expected: evenly spaced in carrier density and symmetric around charge neutrality.

For completeness, the QHE measurements, shown here using a discrete colour scheme, are also presented with a continuous colour scale in the Appendix.

The QHE in the **high modulation regime** is shown in Fig. 6.9; the inset in panel (b) indicates the trace along which both gates were varied. As visible in panel (a), the transverse conductivity is now even more strongly influenced by pronounced dips between the QHE plateaus. These arise from the contribution of the longitudinal resistivity to the conductivity tensor. The steps at $\nu = -16$ and $\nu = -24$ are not clearly visible in the experiment.

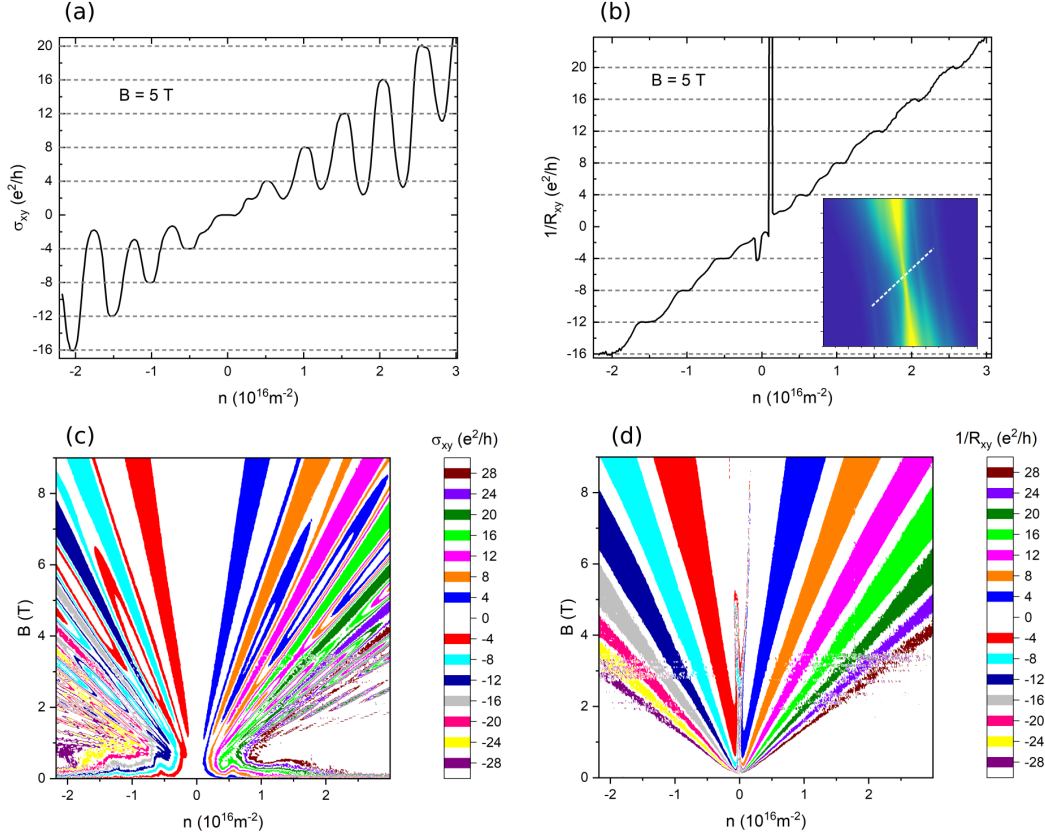


Fig. 6.8: (a) Hall conductivity for the unmodulated case at $B = 5$ T. The plateaus appear at the expected values of $4Ne^2/h$ for BLG, but pronounced dips between the plateaus are also visible. This contrasts with unmodulated MLG 1DSL measurements and suggests that the 1DSL modulation is not entirely absent in this measurement. (b) Inverse Hall resistance shows clear plateaus at the expected values and confirms that the dips originate from the influence of the longitudinal resistance. The inset shows the gate voltage trace used in this measurement. (c) Colour plot of the Hall conductivity, revealing a pattern caused by the dips between the plateaus. (d) Colour plot of the inverse Hall resistance, showing that the plateaus evolve linearly and symmetrically.

In addition, the $\nu = +4$ and $\nu = -4$ plateaus differ in width, which resembles the asymmetry observed in the MLG 1DSL case. This asymmetry is attributed to the asymmetric shape of the modulation potential, which leads to an unequal modification of the band structure for electrons and holes. Since the same gate geometry and fabrication process were used for the BLG and MLG samples, it is reasonable to assume that a similar asymmetry in the potential exists here.

This asymmetry is also visible in panel (c), which shows the Hall conductivity as a colour plot, with each plateau coloured using a small delta to suppress noise. The $\nu = -4$ plateau appears at higher magnetic field and broader than the $\nu = +4$ plateau.

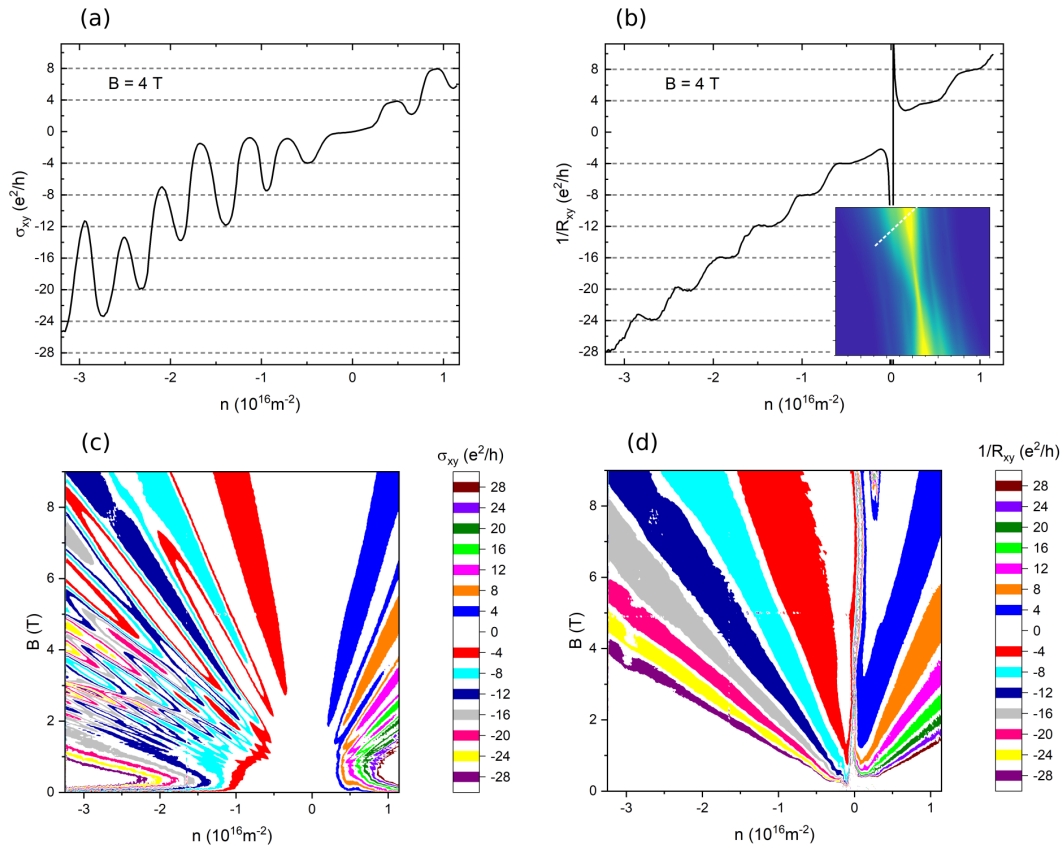


Fig. 6.9: (a) Hall conductivity for the modulated case at $B = 4 \text{ T}$. Not all plateaus appear at the expected values of $4Ne^2/h$, and pronounced dips between plateaus are visible. This indicates that the influence of the 1DSL alters the QHE sequence compared to the unmodulated case. (b) Inverse Hall resistance shows clear plateaus at the expected values and confirms that the missing QHE steps originate from the strong influence of the longitudinal resistance. The inset shows the gate voltage trace used in this measurement. (c) Colour plot of the Hall conductivity, revealing a pattern caused by the dips between the plateaus and an asymmetry between the $+4$ and -4 plateaus, attributed to the asymmetric 1DSL potential profile. (d) Colour plot of the inverse Hall resistance, also showing a modified pattern compared to the unmodulated case and an asymmetry between the $+4$ and -4 plateaus.

Panel (b) shows the inverse Hall resistance, allowing for analysis of which features in the QHE sequence are affected by the influence of the longitudinal resistivity. Here, all QHE steps are clearly visible and match the expected values for BLG. This indicates that the missing plateaus in the Hall conductivity are not due to a change in degeneracy (e.g. from additional Dirac points induced by the 1DSL), but rather a consequence of the non-negligible longitudinal resistance including Weiss oscillations.

Panel (d) presents the corresponding colour plot of the inverse resistance. All steps are visible, yet a pronounced asymmetry between the electron and hole side remains,

reflecting the asymmetrically modified band structure due to the 1DSL potential.

6.4 Conclusion

This chapter investigated transport in BLG 1DSLs, which had not been demonstrated experimentally before. Two samples were fabricated as L-shaped Hall bars for simultaneous measurements along and perpendicular to the modulation, with a 1DSL period of 50 nm. BLG 1DSLs are predicted to be highly sensitive to the applied 1DSL potential, as two mechanisms can play a role: the potentials applied to the two graphene layers can be of equal magnitude, opposite in sign, or a mixture of both in real devices (see section 3.4.1). These scenarios may combine two predicted effects: the opening of a band gap at the main Dirac point and the generation of satellite Dirac points [5, 6].

The first sample showed a sequence of disappearing and reappearing resistance peaks at the main Dirac point, followed by the emergence of satellite Dirac points at higher modulation strength. In contrast, the second sample always exhibited a resistance maximum at the main Dirac point, accompanied by satellites once a 1DSL potential was applied. Two factors may account for this difference: (i) the internal doping in the second sample was significantly lower than in the first, which may already have opened a band gap before applying a 1DSL potential, or (ii) the gate geometries were not exactly identical, leading to variations in stripe and gap width, which alter the potential profile and strongly influence the BLG band structure.

Magnetotransport revealed pronounced Weiss oscillations that were in antiphase along and perpendicular to the modulation, confirming both the tunability of the applied 1DSL and the high sample quality. Well-quantised SdH oscillations were also observed, and the overall Landau quantisation was found to be strongly tunable by the 1DSL potential.

The QHE response resembled that of the MLG case in section 5.3: pronounced dips between QHE plateaus appeared in the modulated regime, and the $\nu = \pm 4$ plateaus showed a tunable asymmetry under the applied 1DSL potential. Since the fabrication of the PBGs in these samples was identical to the MLG case, the 1DSL potential is likewise expected to be asymmetric. Consequently, electron–hole asymmetry due to misaligned satellite Dirac points was also present in the QHE regime of these samples. A complete absence of dips was never observed, highlighting the strong sensitivity of the BLG band structure to 1DSL potentials, as no parameter combination was found that completely suppressed the modulation effect.

Overall, these experiments demonstrate the rich physics of BLG 1DSLs. A promising direction for future studies would be the integration of a third global gate, enabling inde-

pendent control of the displacement field, the 1DSL modulation, and the carrier density. Compared to the MLG case, the BLG devices reveal an even higher sensitivity of the band structure to the exact potential profile and the initial electrostatic configuration.

7 Summary and Outlook

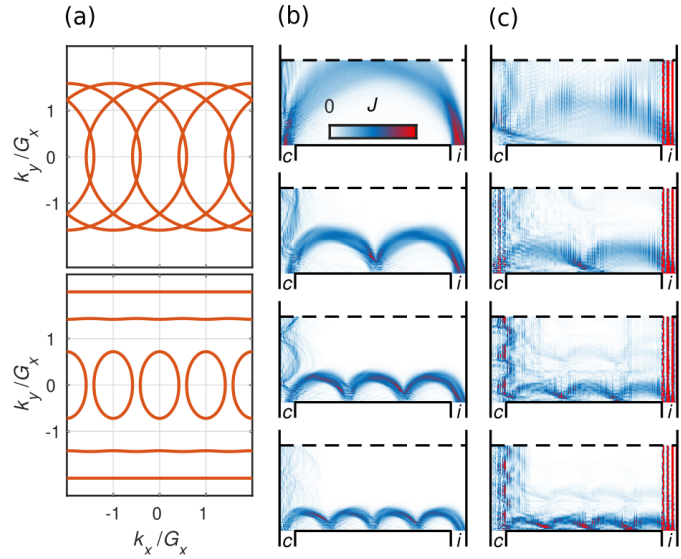
This thesis investigates a central question: how does a one-dimensional superlattice potential reshape the band structure and transport properties of graphene, and what role does the potential's exact form play? We particularly explore how such modulations manifest in anisotropic transport, the emergence of additional Dirac points, commensurability oscillations, and the quantum Hall effect. A second guiding question concerns the differences between monolayer and bilayer graphene, where the unique band structure of BLG offers additional possibilities such as tunable band gaps, but where experimental studies of 1D superlattices were largely absent.

We fabricated MLG and BLG devices with patterned bottom gates creating lattice periods of 50 nm. This gate-tunable approach enables independent control over modulation strength and Fermi level, providing a versatile platform to explore the influence of 1DSLs. We investigated two MLG samples with different hBN thicknesses insulating the PBG from the MLG. The high-quality device, with mobilities up to $90\,000\text{ cm}^2/\text{Vs}$, showed clear signatures of anisotropic transport, the emergence of satellite Dirac points, and supercollimation in zero-field measurements. These results provide direct evidence of band structure engineering. While transport along the modulation direction revealed the reshaped band structure, measurements perpendicular to the modulation consistently demonstrated anisotropy, with the central Dirac point remaining preserved.

Electrostatic simulations supported our experimental findings, providing insight into the potential profile and its effects on the band structure. The simulations also helped to determine the effective gate stripe dimensions, which are difficult to extract from AFM measurements. They further highlighted the asymmetric nature of the potential, which manifests experimentally as a pronounced electron–hole asymmetry in the gate characterisation. This asymmetry originates from the misaligned generation of zero modes in the band structure.

In magnetotransport, we observed robust Weiss oscillations in all MLG samples once a finite modulation was applied, persisting across a wide range of modulation strengths up to the regime of pronounced miniband formation. The oscillations exhibited a clear periodicity in $1/B$, with resistance minima occurring whenever the cyclotron radius

Fig. 7.1: (a) Fermi contours of a MLG 1DSL for the unmodulated case (upper panel) and the modulated case (lower panel), where the contour exhibits an elliptical shape. (b) Simulated local current density J revealing the TMF pattern for the unmodulated case, showing circular trajectories for several TMF states. (c) TMF pattern for the modulated case, illustrating the elliptical distortion caused by the 1DSL modulation.



became commensurate with the superlattice period. Their temperature dependence was consistent with a classical origin, showing the expected decrease in amplitude with increasing temperature.

The 1DSL had an even stronger impact on the quantum Hall effect. While theory had long predicted higher degeneracies and plateau skipping [112], our work demonstrates that the key reason for their absence in previous experiments lies in the asymmetry of the realistic potential shapes. The imperfect alignment of zero modes prevents the emergence of higher degeneracies. Nevertheless, our QHE measurements displayed clear electron–hole asymmetries with excellent agreement to simulations. Furthermore, the longitudinal resistance contributed strongly to the Hall conductivity tensor, leading to pronounced dips between successive QHE steps.

We also investigated BLG 1DSLs, revealing a rich and highly sensitive response of the BLG band structure to modulation. Two nominally identical devices exhibited qualitatively different behaviours, which can be explained by the combined effects of the 1DSL shape and an intrinsic band gap. The modulation locally closed and reopened this gap, thereby reshaping the accessible band structure. These results highlight the unique tunability of BLG, where 1DSLs can not only generate zero modes but also control gap opening. In magnetotransport, BLG samples showed pronounced Weiss oscillations and QHE plateaus with distinct dips, confirming the persistent influence of the 1DSL. Similar to MLG, the BLG devices exhibited an asymmetry between the +4 and −4 plateaus, underscoring the decisive role of the potential’s shape and symmetry.

Beyond these detailed results, this work provides several broader insights. First, it

demonstrates that the shape and asymmetry of the 1D potential are decisive for the observable physics, bridging the gap between theoretical predictions for ideal modulations and experimental findings. Second, it shows that bilayer graphene, with its unique ability to host tunable band gaps, offers an additional and highly sensitive degree of freedom for band structure engineering with superlattices.

The findings also open new questions. The anisotropic band structure with elliptic Fermi contours remains to be probed directly, which could be realised in transverse magnetic focusing (TMF) experiments [96]. TMF allows charge carriers to be injected and collected along trajectories controlled by the applied magnetic field, providing a direct probe of the Fermi contour. Such measurements could reveal how the 1DSL reshapes the Fermi surface. Figure 7.1 illustrates how such anisotropy in band structure (panel (a)) would appear in TMF (panels (b) and (c)), providing a promising route for future experiments. More broadly, this thesis demonstrates that 1DSL potentials not only reshape the band structure and transport properties of graphene, but that their exact form decisively determines the outcome. This central insight, together with the added tunability of bilayer graphene, highlights how superlattices provide a versatile playground for engineering electronic states in 2D materials. In line with Feynman’s vision of tailoring matter at the nanoscale, these results show how superlattices in graphene enable the engineering of band structures and transport properties far beyond those found in nature.

8 Appendix

8.1 Frequently used abbreviations

1DSL	One-dimensional superlattice
2DEG	Two-dimensional electron gas
AFM	Atomic force microscopy
BLG	Bilayer graphene
CNP	Charge neutrality point
DOS	Density of states
EBL	Electron beam lithography
FET	Field-effect transistor
FLG	Few-layer graphene
hBN	Hexagonal Boron Nitride
LL	Landau level
MIBK	Methyl isobutyl ketone
MLG	Monolayer graphene
PBG	Patterned bottomgate
PMMA	Poly(methyl methacrylate)
PC	Polycarbonate
PDMS	Polydimethylsiloxane
QHE	Quantum Hall effect
RIE	Reactive ion-etching
SdH(O)	Shubnikov-de-Haas (oscillations)
SF₆	Sulfur hexafluoride
SiO₂	Silicon dioxide
SMU	Source measure unit
TMF	Transverse magnetic focusing
VTI	Variable temperature insert
WO	Weiss oscillation

8.2 Fabrication Details and Recipes

Preparation and Exfoliation

- Dicing of p^{++} -doped Si wafer with 285 nm of SiO_2 on top into 1×1 cm chips or using prepared $4.5 \text{ mm} \times 4.5 \text{ mm}$ chips with coordinate system of evaporated 5 nm Cr and 60 nm Au
- Cleaning of chips in $2 \times$ acetone, isopropyl alcohol and oxygen plasma for 5 min
- Micromechanical exfoliation of graphite or hBN with Nitto tape ELP BT - 150 ECM
- Identification of suitable flakes at an optical microscope (2-5 layers of graphene for PBGs, monolayers of graphene as transport layer, 3-30 nm of hBN for insulation and encapsulation)
- Annealing at 400°C in vacuum ($\approx 8 \text{ mbar}$) for ≈ 1 h of FLG flakes used for PBG

Fabrication of PBG

- Spin coating selected FLG chips with PMMA 950k 2% at 3000 rpm for 5 s and 6000 rpm for 30 s
- Bakeout of resist at 150° for 5 min
- EBL with Auriga SEM at $\text{EHT} = 30 \text{ kV}$, working distance = 4 mm, aperture $10 \mu\text{m}$ and dose 400-500 pC/cm for FLG on SiO_2 and 550-650 pC/cm for FLG on hBN, line spacing 10 nm. Stripes are written as single lines in a field limited to $5 \times 5 \mu\text{m}$ with a period of 50 nm
- Developing in cooled ($\approx 10^\circ\text{C}$) isopropyl alcohol and water (7:3) for 60 s and short dip into pure isopropyl alcohol before drying with nitrogen
- RIE with O_2 at 100 sccm, 30 mTorr and 50 W for 17 s
- Cleaning in Remover PG at 60°C for ≈ 20 h, annealing at 400°C in vacuum for ≈ 1 h
- AFM image to search for suitable stripe field, if successful repeat last cleaning step
- Spincoating with PMMA 950k 5% at 3000 rpm for 5 s and 6000 rpm for 30 s, bakeout of resist at 150° for 5 min
- Removing surrounding FLG and leave only $\approx 6 \times 6 \mu\text{m}$ of stripe field. EBL with Auriga SEM at $\text{EHT} = 30 \text{ kV}$, working distance = 4 mm, aperture $20 \mu\text{m}$ and dose $240 \mu\text{C/cm}^2$, line spacing 5 nm
- O_2 RIE at 100 sccm, 30 mTorr and 50 W for 30 s
- Cleaning step with Remover PG at 60°C for ≈ 20 h, annealing at 400°C in vacuum for ≈ 1 h

Van der Waals stacking

- Prepare PDMS: (polydimethylsiloxane; Dow Corning, Sylgard 184): mixing of elastomer and curing agent (10:1), remove air bubbles by ultrasonic treatment for ~ 10 min
- Place a cleaned microscope glass slide on hot plate (150°C) and use pipette to place drops of PDMS mixture onto glass slide. Bake for 5 min. Repeat this step if desired for a higher "double-drop".
- Prepare PC (polycarbonate; Aldrich, Goodfellow, CT306310): dissolve PC in chloroform (3%)
- Use pipette to cover a cleaned glass slide with PC and gently put second glass slide on top and move in circles and remove by sweeping down quickly. A thin and even film of PC should remain on the first glass slide.
- Use punch to make a hole into a piece of Scotch Magic Tape. Pick up a piece of PC by placing the Scotch tape onto it, cutting around it with a scalpel and remove it gently.
- Place the suspended PC over the prepared PDMS droplet and make sure it is stretched and there are no creases. For a better stretch use more tape closely around PDMS droplet.
- Place the PC/PDMS "stamp" in the appropriate holder for the optical microscope, with a heatable, height-adjustable vacuum sample chuck underneath.
- Instantly after last annealing step, move sample to heated transfer chuck to avoid contamination.
- Pick up desired flakes by heating the sample chuck to $\approx 110^{\circ}\text{C}$ and place finished stack (hBN - graphene - hBN) on prepared PBG by heating the sample chuck to $\approx 185^{\circ}\text{C}$.
- Remove PC layer by chloroform bath for ~ 5 min and isopropyl alcohol.
- AFM image of finished stack for control of proper placement and isolation of PBG and graphene.

Hall bar etching

- Spin coating of resist PMMA 950k 5% at 3000 rpm for 5 s and 6000 rpm for 30 s. Bakeout at 150°C for 5 min
- EBL at Auriga: acceleration voltage 30 kV, aperture $20\mu\text{m}$, working distance 10 mm, dose $240\mu\text{C}/\text{cm}^2$, line spacing 5 nm
- Develop resist for 90 s in MIBK:IPA and 30 s isopropyl alcohol
- Selective etching of hallbar at RIE:
O₂ at 20 sccm, 30 mTorr and 20 W for 30 s

Conditioning chamber and etching hBN with same recipe: SF_6 at 3 sccm, 50 mTorr and 10 W for 60 s

Check in optical microscope if top hBN was successfully etched and remove resist with acetone and isopropyl alcohol

Conditioning chamber and etching graphene with same recipe: O_2 at 20 sccm, 30 mTorr and 20 W for 60 s

Contacts and final steps

- Spin coating of resist PMMA 200k 9% and PMMA 950k 5%, both at 3000 rpm for 5 s and 6000 rpm for 30 s. Bakeout inbetween and after at 150°C for 5 min
- EBL at Auriga: acceleration voltage 30 kV, aperture $20\mu\text{m}$ for fine wiring and $120\mu\text{m}$ for coarse wiring, working distance 10 mm, dose $440\mu\text{C}/\text{cm}^2$ and line spacing 5 nm for fine wiring and 15 nm for coarse wiring
- Develop resist in 90 s MIBK:IPA and 30 s isopropyl alcohol
- Etching of contact areas at RIE:
 O_2 at 20 sccm, 30 mTorr and 20 W for 30 s
 Conditioning chamber and etching hBN with same recipe: SF_6 at 3 sccm, 50 mTorr and 10 W for 60 s
- Quickly after etching build into Univex B chamber to guarantee a clean interface. Evaporate 0.5 nm Cr and 50 nm of Au
- Lift off for ~ 20 h in acetone at 60°C . Remove residual gold by rinsing the sample with acetone loaded into syringe
- Glue chip into chip carrier by two component silver epoxy adhesive
- Ultrasonic wire bonding with aluminium wire to connect sample with chip carrier

8.3 QHE measurements

For completeness, Figures 8.1–8.6 present the same QHE measurements discussed in the main text in sections 5.3 in MLG and 6.3 in BLG, now shown using a continuous colour scale. In the main chapters, a discrete colour scheme was used to clearly highlight individual Hall plateaus and their shape. Here, the continuous scale allows for an alternative visualisation of the same data.

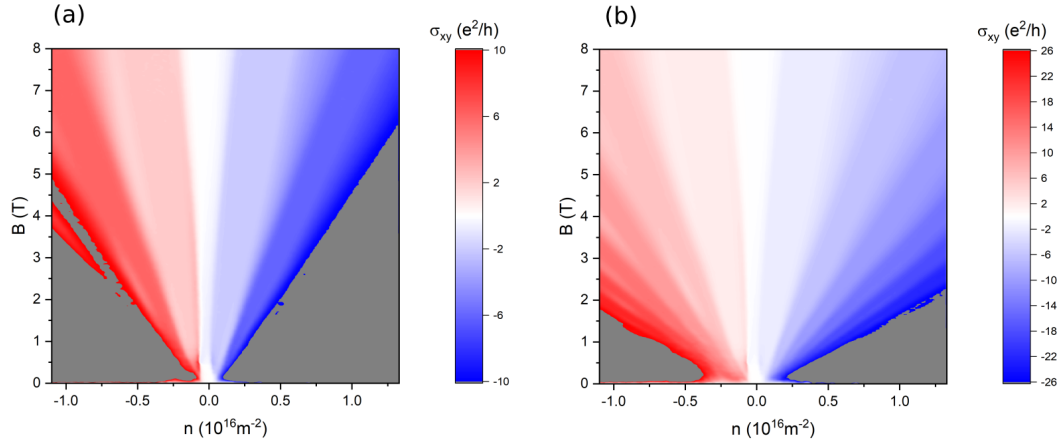


Fig. 8.1: QHE in sample A in the unmodulated case shown using a continuous colour scale: (a) Well-developed and symmetric Hall plateaus, colour scale limited to $\nu = 10$. (b) Same data with extended colour scale up to $\nu = 26$. Data identical to Fig. 5.17.

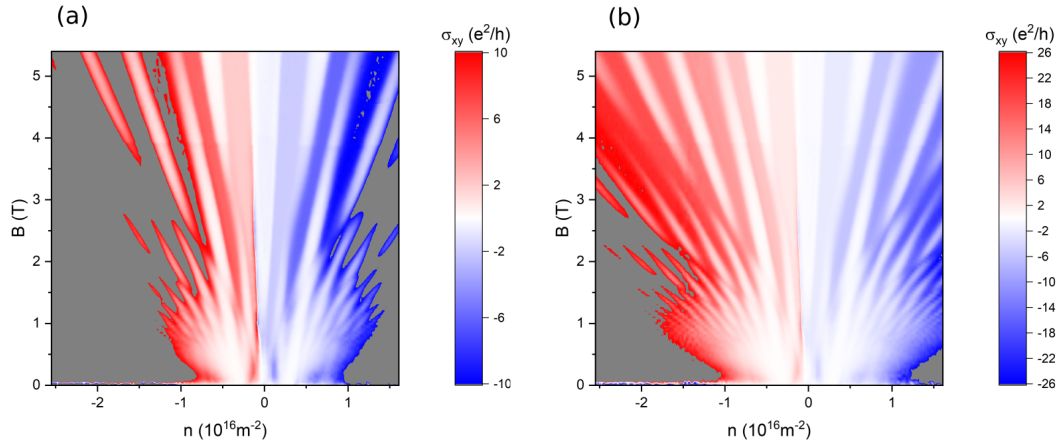


Fig. 8.2: QHE in sample A in the low-modulated case shown using a continuous colour scale: (a) Pronounced dips between Hall plateaus are visible as white lines due to enhanced longitudinal resistance. An asymmetry between the $\nu = +2$ and $\nu = -2$ plateaus is also visible, resulting from misaligned Dirac points in the band structure. Colour scale limited to $\nu = 10$. (b) Same data with extended colour scale up to $\nu = 26$. Data identical to Fig. 5.18.

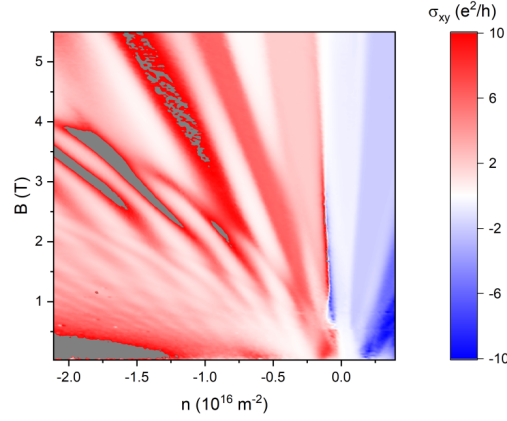


Fig. 8.3: QHE in sample A in the high-modulated case shown using a continuous colour scale: Pronounced dips between Hall plateaus are visible as white lines due to enhanced longitudinal resistance. An asymmetry between the $\nu = +2$ and $\nu = -2$ plateaus is also visible, resulting from misaligned Dirac points in the band structure. Additionally, the visibility of higher plateaus is reduced. Colour scale limited to $\nu = 10$. Data identical to Fig. 5.19.

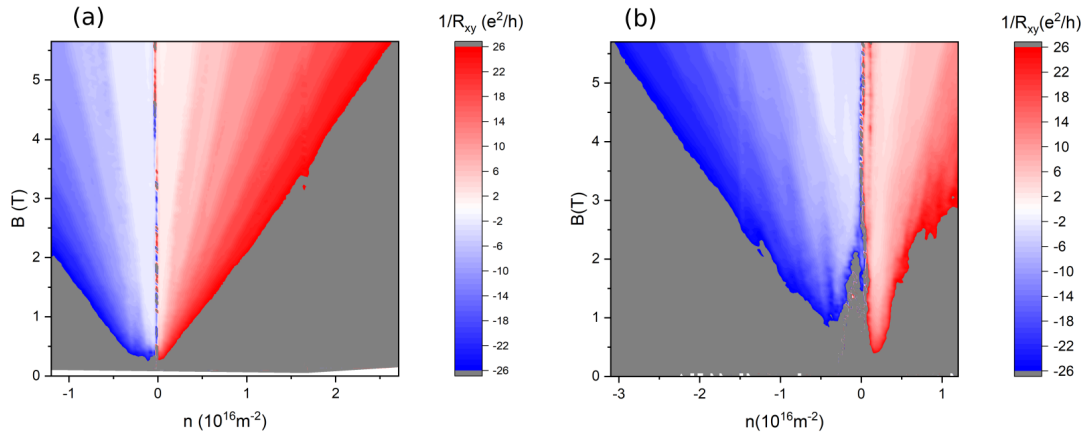


Fig. 8.4: QHE in sample B shown as inverse Hall resistance using a continuous colour scale: (a) **unmodulated case** Well-developed and symmetric Hall plateaus, colour scale limited to $\nu = 26$. (b) **modulated case** Pronounced asymmetry between $\nu = +2$ and $\nu = -2$ plateaus due to misaligned generated Dirac points. Data identical to Fig. 5.23.

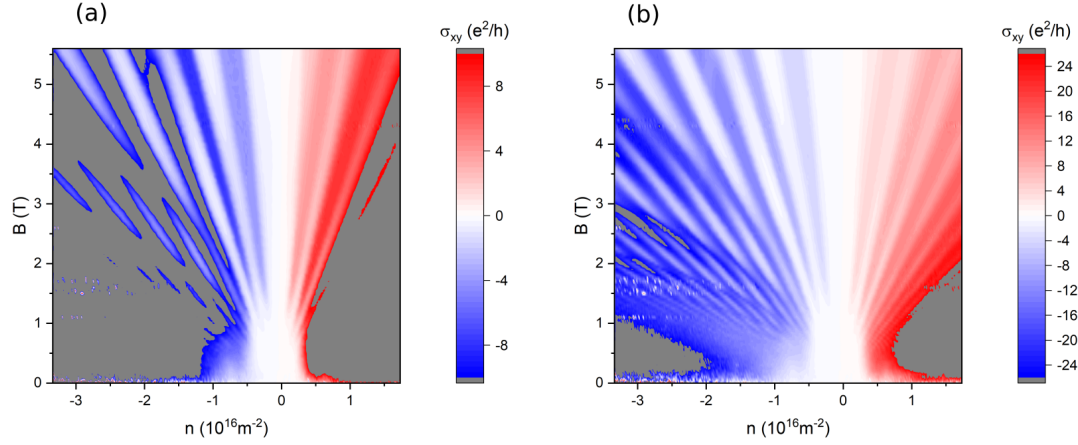


Fig. 8.5: QHE in sample D (BLG) in the unmodulated case shown using a continuous colour scale: (a) Pronounced dips between Hall plateaus appear as white lines due to enhanced longitudinal resistance, indicating that the modulation was not entirely absent. Colour scale limited to $\nu = 10$. (b) Same data with extended colour scale up to $\nu = 26$. Data identical to Fig. 6.8.

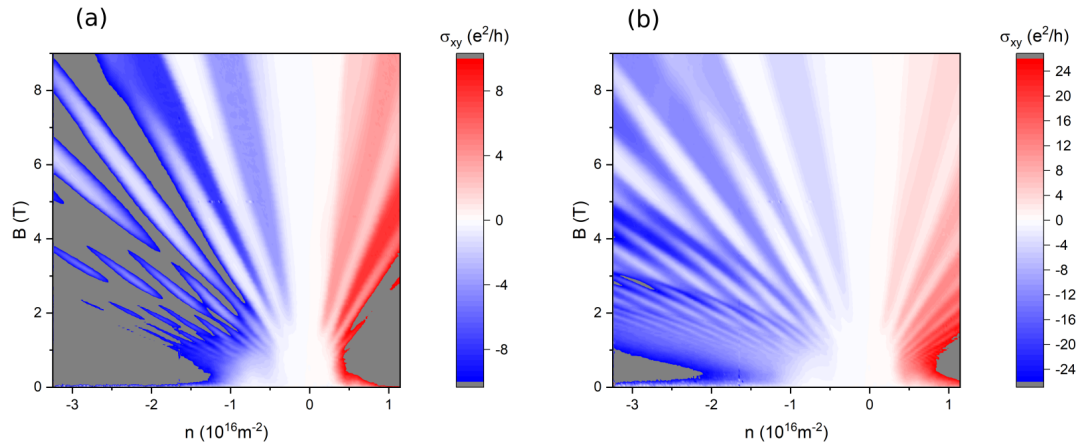


Fig. 8.6: QHE in sample D (BLG) in the modulated case shown using a continuous colour scale: (a) Pronounced dips between Hall plateaus appear as white lines due to enhanced longitudinal resistance, and a strong asymmetry between the $\nu = +2$ and $\nu = -2$ plateaus arises from misaligned generated Dirac points. Colour scale limited to $\nu = 10$. (b) Same data with extended colour scale up to $\nu = 26$. Data identical to Fig. 6.9.

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