Novel Magnetoresistance Oscillations in a Periodically Modulated Two-Dimensional Electron Gas

R. R. Gerhardts, D. Weiss, and K. v. Klitzing
Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-7000 Stuttgart 80, Federal Republic of Germany
(Received 12 December 1988)

A novel type of magnetoresistance oscillation is observed in high-mobility AlGaAs/GaAs heterojunctions with a holographically induced lateral periodic modulation in one direction. Theoretically, the effect is shown to result from an oscillatory dependence of the bandwidth of the modulation-broadened Landau levels of the two-dimensional electron system on the band index, which leads in high-mobility samples to a strongly anisotropic oscillating contribution to the conductivity tensor. The oscillations, periodic in B^{-1}, reflect the commensurability of the cyclotron diameter at the Fermi energy and the modulation period.

PACS numbers: 73.50.Jt, 73.20.Dx, 73.40.−c

The two-dimensional motion of electrons subjected to both a periodic potential and a perpendicular homogeneous magnetic field **B** leads to interesting commensurability problems,[1] owing to the presence of two length scales, the period a of the potential and the magnetic length $l=(c\hbar/eB)^{1/2}$. Drastic effects on measurable quantities, e.g., the magnetotransport properties of a quasi-two-dimensional electron system (2D ES), are expected if a and l are roughly of the same order of magnitude. Since transport is mainly due to electrons at the Fermi energy E_F, the Fermi wavelength $2\pi/k_F$, which is related to the mean density $N_S=k_F^2/2\pi$ of the 2D ES, becomes important as a third length scale, which complicates the situation. Even without any potential modulation the well-known Shubnikov–de Haas (SdH) oscillations, which are periodic in B^{-1} with period $\Delta_{\rm SdH}(c/B)=e/\hbar k_F^2$, result from the fact that for even-integer values of the filling factor of Landau levels, $\nu=l^2k_F^2$ (we assume the spin degeneracy is not resolved), elastic scattering leading to finite conductivity components σ_{xx},σ_{yy} is not possible.[2]

In the present Letter we consider the situation of a periodic modulation of the 2D ES in only one direction. We demonstrate both experimentally and theoretically that in high-mobility samples pronounced commensurability effects exist, which manifest themselves as a novel type of magnetic quantum oscillation, also periodic in B^{-1}, but with a period $\Delta(c/B)=ea/2\hbar k_F$, which is determined by the three lengths a, l, and k_F^{-1}, and is much larger the $\Delta_{\rm SdH}(c/B)$ for the present experiments.

The experiments were performed on conventional high-mobility AlGaAs/GaAs heterojunctions. Details of sample preparation and experimental setup will be published elsewhere.[3] Here we concentrate on the description and theoretical explanation of the basic physics. The samples were illuminated at liquid-helium temperatures with a well-defined fringe pattern of two interfering laser beams. Exploiting the persistent photoconductivity effect, a periodic modulation of the positive background charge in the doped AlGaAs layer[4] was produced with a periodic a that is well known from the laser wavelength and the geometry of the setup. The advantage of this kind of "microstructure engineering" is its simplicity and the achieved high mobility of the microstructured sample due to the absence of the defects introduced by the usual pattern transfer techniques.[5] After a short holographic illumination (30-ms duration, 1.8-mW HeNe laser with $\lambda=633$ nm) our sample had a carrier density $N_S=3.16\times10^{11}$ cm^{-2} ($N_S=2.35\times10^{11}$ cm^{-2} before illumination) and a mobility $\mu=1.3\times10^6$ cm^2/(V s) (corresponding to an elastic free path $l_e=12$ μm). For the simultaneous measurement of the resistance parallel ($\rho_\parallel$) and perpendicular ($\rho_\perp$) to the interference fringes, an L-shaped sample was used as sketched in the inset of Fig. 1.

The experimental results can be summarized as follows. In addition to the usual SdH oscillations, pronounced novel oscillations appear in $\rho_\perp$ at magnetic fields lower than 1 T and at low temperatures. No additional structure appears in the Hall resistance, and only weak oscillations with a phase shift of 180° relative to those of $\rho_\perp$ are visible in $\rho_\parallel$, as is shown in Fig. 1(a).[6] The extrema of $\rho_\perp$ occur at B values

$$\frac{c}{B_\lambda}=\frac{ea}{2\hbar k_F}(\lambda+\phi)\,,\quad \lambda=1,2\ldots\,, \tag{1}$$

with $\phi=-0.25\pm0.06$ for the minima and $\phi=+0.17\pm0.06$ for the maxima. The validity of Eq. (1) has been confirmed by performing these experiments on different samples, by changing the carrier density with an applied gate voltage, and by using another laser wavelength in order to vary the period a.[3] As the temperature is increased from 2.2 to 5 K, the SdH oscillations are strongly damped whereas the novel oscillations are apparently unaffected.

For the theoretical discussion, we describe the modulation by a harmonic potential

$$V(x)=V_0\cos(Kx)\,,\quad K=2\pi/a\,. \tag{2}$$

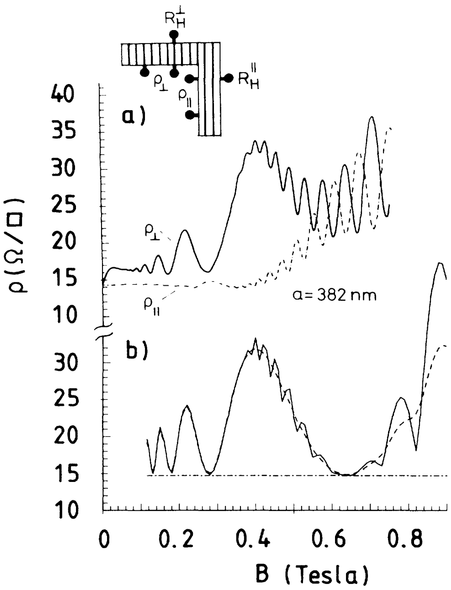

FIG. 1. Magnetoresistivities for current perpendicular ($\rho_\perp$, solid lines) and parallel ($\rho_\parallel$, dash-dotted lines) to the interference fringes, as indicated in the inset, for a sample with $N_S = 3.16\times10^{11}$ cm^{-2}, $\mu = 1.3\times10^6$ cm^2/(V s), and $a = 382$ nm: (a) measured at temperature $T = 2.2$ K, (b) calculated for $T = 2.2$ K and for 4.2 K ($\rho_\perp$, dashed line; $\rho_\parallel$ shows no temperature dependence), using $V_0 = 0.3$ meV.

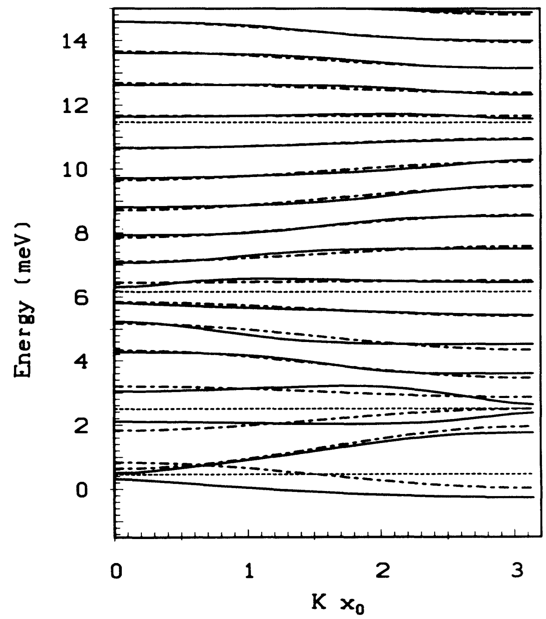

FIG. 2. Calculated energy bands $\epsilon_n(x_0)$ for $B = 0.5$ T, $V_0 = 1.5$ meV, $a = 100$ nm, and material parameters of GaAs: solid lines, numerical diagonalization; thick dash-dotted lines, first-order approximation; thin dotted lines, energies of zero bandwidth.

This is convenient, although not essential, for the analytical calculation and presumably realistic. Furthermore, we assume a weak modulation, $V_0/E_F \sim 0.1$, since in the experiments no anisotropy is observed for zero magnetic field, and a period a much larger than the Fermi wavelength, $ak_F \sim 10$, also in view of the experimental situation.

For $B = 0$, $V(x)$ introduces a Bloch-band structure in the x direction, the well known Mathieu problem.[7] The Fermi energy is in a band with large index ($n \sim 10$), where the band gaps[7] and the deviation from free-electron behavior are extremely small. Because of the free-electron spectrum in the y direction, all the occupied bands overlap at the Fermi level, and we cannot use the Peierls substitution to describe magnetic field effects.[1] The magnetic field strongly couples states of different, energetically overlapping bands, leading to strong magnetic breakdown effects, which may be the reason for the strong positive magnetoresistance observed in ρ_{xx} ($B < 0.03$ T).

We include $V(x)$ and the vector potential $\mathbf{A} = (0, xB, 0)$ in the 2D Hamiltonian, exploit the translation invariance in the y direction, and write the energy eigenfunctions in the form $\psi_{k_y n}(x,y) \propto \exp(ik_y y)\varphi_n(x;x_0)$, where $\varphi_n(x;x_0)$ is the eigenfunction of the 1D Hamiltonian

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{m}{2}\omega_c^2(x-x_0)^2 + V(x)\,, \tag{3}$$

with the eigenvalue $\epsilon_n(x_0)$, and $x_0 = -l^2 k_y$ is a center coordinate. For the homogeneous Landau system, $V(x) \equiv 0$, the eigenfunctions $\varphi_n^{(0)}(x-x_0)$ are the well known oscillator functions and the eigenvalues $\epsilon_n^{(0)} = \hbar\omega_c(n + \frac{1}{2})$ are independent of x_0. Since the matrix elements of $V(x)$ between functions $\varphi_n^{(0)}$ can be calculated analytically, the $\varphi_n^{(0)}$ form a convenient basis for the numerical diagonalization of H. A typical energy spectrum is presented by the solid lines of Fig. 2; note the symmetry $\epsilon_n(x_0 + a) = \epsilon_n(x_0) = \epsilon_n(-x_0)$. The potential $V(x)$ lifts the degeneracy of the Landau levels, and yields eigenstates $|x_0 n\rangle$ which carry current in the y direction,

$$\langle x_0 n | v_y | x_0 n\rangle = -\frac{1}{m\omega_c}\frac{d\epsilon_n}{dx_0} = \frac{1}{\hbar}\frac{d\epsilon_n}{dk_y}\,, \tag{4}$$

whereas $\langle x_0 n | v_x | x_0 n\rangle = 0$, since the $\varphi_n(x;x_0)$ are localized in the x direction. This is the origin of the anisotropic transport coefficients observed. The width of the Landau bands oscillates (cf. Fig. 2), which is most easily understood within first-order perturbation theory with respect to V_0. This yields for the energy spectrum (dashed-dotted lines of Fig. 2)

$$\epsilon_n^{(1)}(x_0) = \hbar\omega_c(n + \tfrac{1}{2}) + V_0\cos(Kx_0)e^{-X/2}L_n(X)\,, \tag{5}$$

where $X = \frac{1}{2}K^2 l^2$, and exhibits qualitatively the same features as the exact energy spectrum. For fixed magnetic field (fixed X), the Laguerre polynominal $L_n(X)$ oscillates as a function of its index n.[7] The flat-band condition, $L_n(X)=0$, is obtained from the asymptotic formula[7] for the λth zero $X_\lambda^{(n)} \approx [\frac{1}{2}\pi(\lambda-\frac{1}{4})]^2/(n+\frac{1}{2})$. In terms of the cyclotron radius $R_n = l(2n+1)^{1/2}$, this can be expressed as

$$2R_{n_\lambda} = a(\lambda - \tfrac{1}{4}), \quad \lambda = 1,2,\ldots . \tag{6}$$

The flat-band energies $\hbar\omega_{n_\lambda}$ are indicated for $\lambda=1,\ldots,4$ as dotted lines in Fig. 2. The origin of the oscillations is that the wave functions, having a spatial extent of approximately $2R_n$, sense effectively the average value of the periodic potential over an interval of length $2R_n$. This gives rise to oscillations of the bandwidth, which also lead to oscillations of the density of states.[8]

To investigate the commensurability effects on the conductivity tensor, we evaluate the Kubo-type formula[9]

$$\sigma_{\mu\nu}(\omega) = \frac{2e^2\hbar}{iL_xL_y}\sum_{\alpha,\alpha'}\frac{f(E_\alpha)-f(E_{\alpha'})}{(E_\alpha - E_{\alpha'})(E_\alpha - E_{\alpha'} + \hbar\omega + i0^+)} \times \langle \alpha | v_\mu | \alpha' \rangle \langle \alpha' | v_\nu | \alpha \rangle , \tag{7}$$

where $f(E)$ is the Fermi function, within a simple damping approximation replacing the frequency ω by i/τ, where $\tau^{-1} = \gamma/\hbar$ is a scattering rate. This yields no SdH oscillations, but demonstrates the origin of the novel oscillations. With the eigenstates $|x_0 n\rangle$ of H, the velocity matrix elements are diagonal with respect to x_0, and the off-diagonal matrix elements with respect to n yield Drude-type formulas, which for the unmodulated Landau system yield exactly the Drude results $\sigma_{xx}^D = \sigma_{yy}^D = \sigma_0/[1+(\omega_c\tau)^2]$, with $\sigma_0 = e^2N_S\tau/m$ and $\sigma_{yx}^D = \omega_c\tau\sigma_{xx}^D$. Their evaluation for the modulated system gives results similar to those for the unmodulated system. The nonvanishing matrix elements (4) lead, however, to an additional contribution to σ_{yy}, cf. Eq. (7),

$$\Delta\sigma_{yy} = -\frac{2e^2\hbar}{2\pi l^2}\int_0^a \frac{dx_0}{a}\sum_n \frac{1}{\gamma}\frac{df}{dE}(\epsilon_n(x_0)) \times |\langle x_0 n | v_y | x_0 n\rangle|^2 , \tag{8}$$

which becomes important for high-mobility systems [$\gamma \ll |\epsilon_n(x_0) - \epsilon_{n\pm1}(x_0)| \approx \hbar\omega_c$]. Then, this contribution of current-carrying states at the Fermi level, which has no counterpart in σ_{xx}, accounts for the anisotropy of the transport coefficients and leads to oscillations with minima if flat bands occur at the Fermi energy. For the small magnetic field values of interest, we may estimate the band index at the Fermi level as $n_F + \frac{1}{2} = E_F/\hbar\omega_c = \frac{1}{2}l^2k_F^2$ so that the cyclotron radius $R_{n_F} = l^2k_F$ results. Inserting this into Eq. (6), we derive for the minima of σ_{yy} exactly Eq. (1) with $\phi = -\frac{1}{4}$. Since σ_{yx} shows no noticeable oscillations and since $\sigma_{yx}^2 \gg \sigma_{xx}\sigma_{yy}$, one has $\rho_{xx} \approx \sigma_{yy}/\sigma_{yx}^2$, $\rho_{yy} \approx \sigma_{xx}/\sigma_{yx}^2$, and the minima of σ_{yy} coincide with those of ρ_{xx}. The maxima of ρ_{xx} do not occur with phases $\phi = +\frac{1}{4}$ in Eq. (1), since the exponential prefactor, cf. Eq. (5), shifts the maxima to smaller values of c/B.

Figure 1(b) shows the result of a numerical evaluation of Eq. (7). The system parameters are adapted to the experiment of Fig. 1(a), with $\gamma = 0.013$ meV, corresponding to the mobility at $B=0$. The amplitude of $V(x)$, which is not known from experiment, is chosen as $V_0 = 0.3$ meV. This is about one-quarter of the most optimistic estimate assuming that all holographically induced electrons in the 2D ES come from a totally modulated ionization pattern[10] and that for the small B values of interest screening in the 2D ES is given by the Thomas-Fermi dielectric function for $B=0$.[11,12] The result for ρ_{xx} depends sensitively on V_0, which enters $\Delta\sigma_{yy}$ quadratically. Obviously, the novel oscillations of $\rho_\perp = \rho_{xx}$ are nicely reproduced by the calculation, which for $\rho_\parallel = \rho_{yy}$ yields essentially the Drude result (independent of B). The value of the background magnetoresistance seen in Fig. 1(a) is sample dependent and probably due to an inhomogeneous carrier density. The Hall resistance ρ_{xy} was also calculated and shows the linear B dependence of the Drude result, without any noticeable oscillations, consistent with experiment.[3] The temperature dependence of the novel oscillations is much weaker than that of the SdH oscillations, since the relevant energy is the distance between flat bands, which is much larger than the mean distance between adjacent bands.

A microscopic calculation, e.g., in the self-consistent Born approximation,[2,9] will show that the scattering rate, owing to the interaction of the electrons with randomly distributed impurities, exhibits the same kind of oscillations as the density of states, and will yield the SdH oscillations and the novel commensurability oscillations on the same footing. The detailed discussion of such a theory, which requires a nontrivial generalization of the existing theory for homogeneous systems,[2,9] will be left for a future publication. Preliminary results indicate that commensurability oscillations also occur in the scattering rate, which may be the reason for the weak phase-shifted oscillations seen in the measured $\rho_\parallel$ at low fields, $B \lesssim 0.3$ T.

We are grateful to D. Heitmann for stimulating suggestions, to L. Eaves, J. Hajdu, A. MacDonald, P. Streda, and C. Zhang for useful discussions, and to K. Ploog and G. Weimann for providing us with high-quality samples. The work was supported in part by the Bundesministerium für Forschung und Technologie, West Germany (under Grant No. NT-2718-C).

[1]D. R. Hofstadter, Phys. Rev. B **14**, 2239 (1976).

[2]T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).

[3]D. Weiss, K. v. Klitzing, K. Ploog, and G. Weimann, Europhys. Lett. (to be published); in "The Application of High Magnetic Fields in Semiconductor Physics," edited by G.

Landwehr, Springer Series in Solid-State Sciences (Springer-Verlag, Berlin, to be published).

[4]K. Tsubaki, H. Sakaki, J. Yoshino, and Y. Sekiguchi, Appl. Phys. Lett. **45**, 663 (1984).

[5]For recent work in this field, see *Physics and Technology of Submicron Structures,* edited by H. Heinrich, G. Bauer, and F. Kuchar, Springer Series in Solid-State Sciences Vol. 83 (Springer-Verlag, Berlin, 1988).

[6]The SdH oscillations in $\rho_{\parallel}$ and $\rho_{\perp}$ show by chance an antiphase behavior due to a slightly different carrier density in the two legs of the L-shaped sample. The antiphase behavior of the novel oscillations ($B \approx 0.28$ T) cannot be explained in this way, since the required differences of the densities would be much too large (about 1.2×10^{11} cm^{-2}).

[7]*Handbook of Mathematical Functions,* edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1972).

[8]A level broadening should be included to round off the quasi-one-dimensional Van Hove singularities of the density of states. Suitable experiments, e.g., capacitance measurements, should reveal commensurability oscillations for small magnetic fileds and a double-peak structure of individual Landau levels for high magnetic fields.

[9]See, e.g., R. R. Gerhardts, Z. Phys. B **22**, 327 (1975), and further references therein.

[10]Temperature-dependent hopping processes during illumination seem to smear out the modulation. At lower temperature a stronger modulation was obtained.

[11]U. Wulf, V. Gunmundsson, and R. R. Gerhardts, Phys. Rev. B **38**, 4218 (1988).

[12]U. Wulf and R. R. Gerhardts, in Ref. 5, p. 162.