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Density of states in a two-dimensional electron gas in the presence of a one-dimensional superlattice potential

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Magnetocapacitance measurements are performed on a holographically microstructured $Al_xGa_{1-x}As$-GaAs heterostructure, in order to obtain direct information about the density of states of a periodically modulated two-dimensional electron gas. Comparison with microscopic calculations shows that the observed pronounced modulation of the magnetocapacitance oscillations has the same origin as the novel resistance oscillations recently reported for this system.

Recently, we have reported a new type of magnetoresistance oscillations which occur when the two-dimensional electron gas (2D EG) in GaAs-$Al_xGa_{1-x}As$ heterostructures is periodically modulated in one direction.[1] The modulation was realized at liquid-helium temperatures by superimposing two interfering plane light waves on top of the sample surface (holographic illumination).[1,2] Because of the persistent photoconductivity effect in GaAs-$Al_xGa_{1-x}As$ heterojunctions, the resulting interference pattern with a period a on the order of the laser wavelength produces a modulation of the two-dimensional carrier density N_s. In addition to the usual Shubnikov–de Haas (SdH) oscillations at larger values of the magnetic field ($B \geq 0.4$ T), we found pronounced low-field oscillations ($B \leq 0.8$ T) of the resistivity ρ_{xx} relating the components of electric field and current in the direction of the modulation, whereas ρ_{yy} showed only much weaker oscillations with a phase shift of 180° with respect to those of ρ_{xx}. Similar to the SdH oscillations, which appear for 0.4 T $< B <$ 0.8 T as superimposed on the novel low-field oscillations, these new oscillations are periodic in $1/B$, but with a larger period depending on both N_s and a. This period is obtained from the minima of ρ_{xx}, which can be characterized by the commensurability condition[1]

$$2R_c=(\lambda-\tfrac{1}{4})a,\ \ \lambda=1,2,3,\ldots, \tag{1}$$

between the cyclotron diameter at the Fermi level, $2R_c=2v_F/\omega_c=2l^2k_F$, and the period a of the modulation. Here $k_F=(2\pi N_s)^{1/2}$ is the Fermi wave number, $l=(\hbar c/eB)^{1/2}$ the magnetic length, and $\omega_c=\hbar/ml^2$ the cyclotron frequency with the effective mass $m=0.067m_0$ of GaAs. Theoretically,[3] these experimental findings have been explained by a modification of the Landau energy spectrum due to a weak modulation potential, $V(x)=V_0\cos Kx$, with period $a=2\pi/K$ created by the holographic illuminations ($V_0 \ll E_F$, the Fermi energy). The modulation lifts the degeneracy of the Landau levels (LL's) and leads to modulation-broadened Landau bands with eigenstates which carry current in the y direction but not in the x direction. The width of these band is an oscillatory function of the band index n due to the fact that the eigenstates effectively average the periodic potential over an interval of the order of the cyclotron diameter $2R_c=2l(2n+1)^{1/2}$, the extent of the wave functions in the x direction.[3] With these ingredients, the anisotropy of the magnetoresistance and the strong oscillations of ρ_{xx} have been explained on the basis of Kubo's formula for the conductivity tensor.[3] Recently, similar experimental results have also been reported for conventionally microstructured samples, and are explained by essentially the same theoretical picture.[4]

The key for the explanation of the novel magnetotransport oscillations is the oscillatory linewidth of the modulation-broadened Landau bands. Since, for fixed magnetic field, the number of states per Landau band is fixed, oscillations of the bandwidth should lead to oscillations of the peak values of the density of states (DOS) with maxima for flat bands satisfying Eq. (1). In order to check this immediate consequence of the theoretical model, it is highly desirable to obtain direct experimental information on the DOS. For the unmodulated 2D EG, such information has been extracted from measurements of the magnetocapacitance.[5–7] In this Rapid Communication we present the first magnetocapacitance measurements on a holographically modulated 2D EG. We also present microscopic calculations of DOS and magnetocapacitance, based on a generalization of the well-known self-consistent Born approximation[8,9] (SCBA), which includes both the modulation broadening of the LL's due to the periodic modulation potential, and the usual collision broadening due to scattering from randomly distributed impurities. The comparison of experiment and theory strongly supports the theoretical model used to explain the novel magnetoresistance oscillations.

The modulation-doped GaAs-$Al_xGa_{1-x}As$ heterostructure studied in this Rapid Communication is grown by molecular-beam epitaxy and consists of a semi-insulating GaAs substrate, followed by a 3.5-μm-thick undoped GaAs buffer layer, an undoped $Al_xGa_{1-x}As$ spacer (33 nm), Si-doped $Al_xGa_{1-x}As$ (33 nm), and an undoped GaAs top layer (22 nm). The active region of the sample was defined by etching a Hall-bar geometry and evaporating an 8-nm-thick semitransparent NiCr film as a gate electrode. Alloyed AuGe pads contact the 2D EG. Prepared in such a way, the sample shows a low-temperature mobility of 240 000 cm^2/Vs and a carrier density of 1.5×10^{11} cm^{-2}, as has been determined by SdH oscillations and low-field Hall measurements. After holographic illumination (duration 90 ms) using a 5-mW HeNe laser, which creates a periodic modulation with a period $a=365$ nm, the carrier density was 3.2×10^{11} cm^{-2} with a mobility of 450 000 cm^2/Vs. The modulation amplitude V_0 created by holographic illumination is typically on the order of 0.5 meV.[3] The capacitance between the semitransparent gate and the 2D EG is measured with an ac technique as sketched in Fig. 1.[6,7] The oscillations of the capacitance, as a function of the magnetic field, are directly connected to the DOS at the Fermi energy.[5-7,10] In a homogeneous 2D EG it has been shown experimentally that the LL linewidth Γ has a magnetic field dependence of the form $\Gamma\propto B^{\alpha}$ with $0\leq\alpha\leq0.5$,[7,11] whereas theoretically $\Gamma\propto\sqrt{B}$ is expected for short-range scatterers and Γ independent of B for long-range scatterers.[8,9] Since the Landau degeneracy is proportional to B, the peak values of the DOS in the individual LL's and, as a consequence, the peak values of the capacitance are expected to increase monotonically with a structure less envelope with increasing magnetic field. On the other hand, the envelope of the magnetocapacitance minima decreases monotonically with B due to the increasing LL separation $\hbar\omega_c$. In Fig. 1 the magnetocapacitance data after an initial holographic illumination (90 ms long) (a) is compared with the capacitance measured after an additional

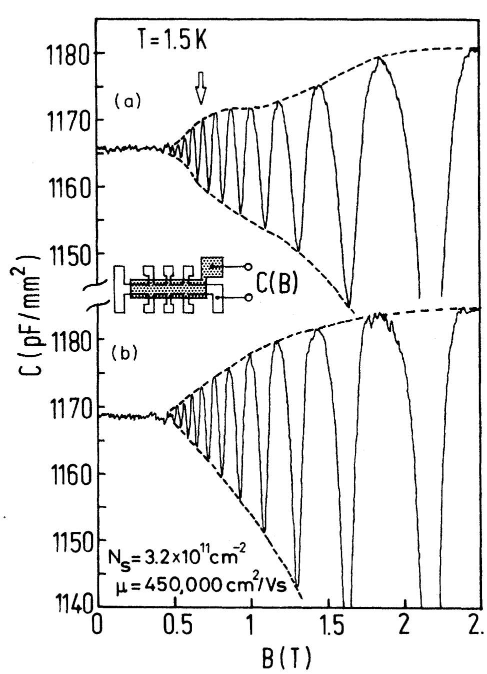

FIG. 1. Measured capacitance of a (a) modulated ($a=365$ nm) and (b) not modulated sample. The arrow corresponds to the magnetic field value satisfying Eq. (1) for $\lambda=1$.

illumination which essentially smears out the periodic modulation (b). The carrier density in (b) has been adjusted to the same value as before the additional illumination using a negative gate voltage. In contrast to Fig. 1(b), where the magnetocapacitance behaves as usually observed in a 2D EG, the capacitance oscillations in Fig. 1(a) display a pronounced modulation of both the minima and maxima, which in the following is explained as a consequence of the oscillatory bandwidth of modulation-broadened LL's. It should be mentioned that the modulation effect is observed for different angles between the one-dimensional modulation and the long axis of the Hall bar, as is expected for a thermodynamic quantity.

For the theoretical description of the system two aspects of the impurity distribution are important, which can be characterized by different length scales. On a "microscopic" scale ($\sim$10 nm) the impurities, i.e., mainly ionized donors behind the spacer layer, seem to be distributed randomly and lead to collision-broadening effects, such as imaginary parts of the self-energy. On a larger scale ($\sim$300 nm), the charged-donor density is periodic and leads, via screening effects, to a periodic effective potential seen by the electrons, and thus to a modification of the single-particle energy spectrum, i.e., a real part of the self-energy. In principle, it should be possible to treat both aspects on the same footing, taking into account a suitable correlation of the impurity distribution. In practice, however, such a theory has not yet been worked out, and we will treat both aspects differently.

We describe the electron-impurity system by the 2D Hamiltonian

$$H=\frac{1}{2m}\left[-\hbar^2\frac{d^2}{dx^2}+\left(\frac{\hbar}{i}\frac{d}{dy}+\frac{e}{c}Bx\right)^2\right]+V_0\cos(Kx)+\sum_j u(\mathbf{r}-\mathbf{R}_j)\,, \quad (2)$$

where $u(\mathbf{r}-\mathbf{R}_j)$ is the potential of an impurity at $\mathbf{R}_j$. Without impurities, the energy eigenfunctions are of the form $\psi_{kn}(x,y)\propto\exp(iky)\phi_{nx_0}(x)$, and $\phi_{nx_0}(x)$ is the eigenfunction of the one-dimensional Hamiltonian

$$H_{x_0}=-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}+\frac{1}{2}m\omega_c^2(x-x_0)^2+V_0\cos(Kx)\,, \quad (3)$$

with eigenvalue $E_n(x_0)$. In the following, we will use the result of the first-order perturbation theory with respect to V_0,[12] $E_n(x_0)=\epsilon_n+U_n\cos(Kx_0)$ with $\epsilon_n=\hbar\omega_c(n+\frac{1}{2})$, $U_n=V_0\exp(-\frac{1}{2}X)L_n(X)$, and $X=\frac{1}{2}l^2K^2$, where $L_n(X)$ is a Laguerre polynomial.[13] This is a reasonable approximation[3] provided the magnetic field is not too small. In addition to this modulation broadening, we include the collision-broadening effects in the simplest reasonable approximation. For a homogeneous 2D EG, this is the SCBA for short-range scatterers, which leads to a self-energy $\Sigma(E)$ independent of the quantum numbers.[8,9] Since we are not interested in a sophisticated quantum-number-dependent self-energy, we write for the impurity-averaged Green's function $G_{nx_0}^{-}(E)=[E-E_n(x_0)-\Sigma^{-}(E)]^{-1}$ and calculate the effective self-energy from the simple self-consistent equation

$$\Sigma^-(E) = \Gamma_0^2 \sum_n \int_0^a \frac{dx_0}{a} G^-_{nx_0}(E)$$
$$= \sum_n \frac{\Gamma_0^2}{\{[E-\epsilon_n-\Sigma^-(E)]^2 - U_n^2\}^{1/2}}, \qquad (4)$$

where a high-energy cutoff $n \le 2E_F/\hbar\omega_c$ is used, and both $G^-_{nx_0}$ and $\Sigma^-(E)$ are analytical functions with nonnegative imaginary parts in the complex half-plane $\mathrm{Im}E<0$. For the nonmodulated system, $U_n \equiv 0$, Eq. (4) reduces to the usual SCBA for δ impurities with $\Gamma_0^2 = (2/\pi)\hbar\omega_c(\hbar/\tau)$, where τ is the corresponding lifetime for zero magnetic field.[8] The solution of Eq. (4) yields the DOS (including spin degeneracy)

$$D(E) = \frac{2}{2\pi l^2}\frac{1}{\pi}\mathrm{Im}\left(\frac{\Sigma^-}{\Gamma_0^2}\right) = \frac{1}{\pi^2 l^2}\sum_n \frac{\sqrt{w_n - \eta_n}}{\sqrt{2}w_n}, \qquad (5)$$

with

$$\eta_n = [E-\epsilon_n-\Delta(E)]^2 - U_n^2 - [\tfrac{1}{2}\Gamma(E)]^2,$$
$$w_n = \{\eta_n^2 + \Gamma(E)^2[E-\epsilon_n-\Delta(E)]^2\}^{1/2},$$

and

$$\Sigma^-(E) = \Delta(E) + \frac{i}{2}\Gamma(E).$$

In the limit $\Gamma_0^2 \to 0$, Eq. (5) exhibits 1D van Hove singularities at the low- and high-energy edges of the modulation-broadened Landau levels.

Figure 2 shows the numerical results of $D(E)$ for three different values of the lifetime τ expressed in terms of the zero-field mobility $\mu_0 = e\tau/m$. The double-peak structure of the Landau levels due to the van Hove singularities is resolved if the collision broadening is sufficiently small compared to the modulation broadening. The oscillatory n dependence of the modulation-broadened LL's leads to an oscillation of the peak value of the DOS with maxima for the narrowest levels. The energy values of the maxima can be estimated from Eq. (1) with $R_c = l(2n+1)^{1/2}$. The sharp band edges obtained for nonoverlapping LL's are an artifact of the SCBA and would smear out, if coherent multicenter scattering would be taken into account.[10] For sufficiently large collision broadening (upper part of Fig. 2), the LL's overlap and the double-peak structure is not resolved, but now the minima of the density of states also oscillate, reflecting again the oscillatory width of the modulation-broadened LL's.

In order to compare with the experiment, we calculate the magnetic-field-dependent capacitance $C(B)$ according to[7,8]

$$\frac{1}{C(B)} = \frac{1}{C(0)} - \frac{1}{e^2 D_T(0)} + \frac{1}{e^2 D_T(B)}, \qquad (6)$$

with $D_T(0) = m/(\pi\hbar^2)$, with the zero-field value $C(0)$ taken from experiment, and, using Eq. (5), with

$$D_T(B) = \frac{\partial N_s}{\partial\mu} = \int dE\, D(E)\frac{df(E-\mu)}{d\mu}, \qquad (7)$$

where $f(x) = [\exp(x/kT)+1]^{-1}$ is the Fermi function and the chemical potential μ is evaluated for given electron density from $N_s = \int dE\, D(E) f(E-\mu)$. The results of Fig. 3 are obtained for a B-independent value $\Gamma_0 = 0.3$

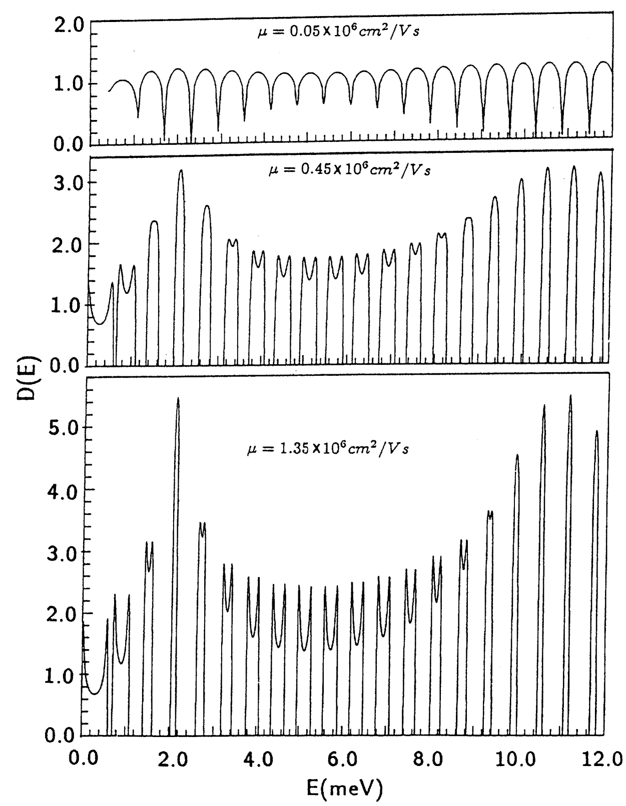

FIG. 2. Calculated DOS in units of $m/(\pi\hbar^2)$ vs energy for $B=0.35$ T and $V_0=0.35$ meV.

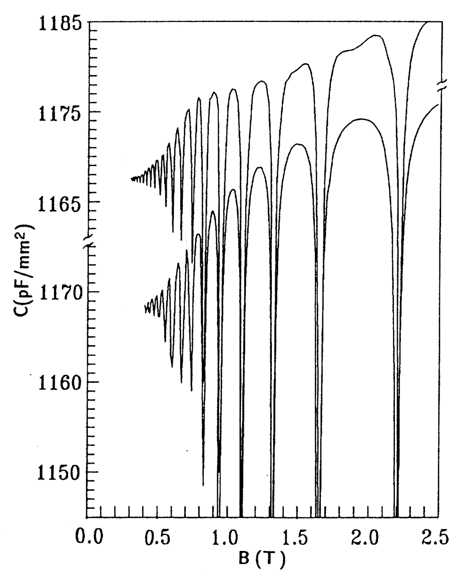

FIG. 3. Calculated magnetocapacitance vs magnetic field for $N_s = 3.2\times10^{11}$ cm^{-2}. A B-independent linewidth is chosen to be $\Gamma_0 = 0.3$ meV. The upper curve is for $V_0 = 0.7$ meV, and the lower one for the weak modulation $V_0 = 0.1$ meV.

meV, which yields a much better agreement with the experimental capacitance results than a linewidth calculated from the mobility μ_0 according to the relation $\Gamma_0 \sim (B/\mu_0)^{1/2}$ valid for short-range scatterers. This phenomenon is well known for samples of comparable mobility[8] and indicates the importance of long-range scatterers. A systematic calculation of transport coefficients and DOS for the modulated electron gas, in the presence of realistic long-range scatterers (ionized donors), requires much more effort than the approximation (4) and must be left for future work.

We think that our calculations demonstrate satisfactorily that the pronounced modulation of both maxima and minima of the measured magnetocapacitance is explained by the broadening of LL's due to a modulation potential, and is another manifestation of the same physics that leads to the novel magnetoresistance oscillations discussed recently.[1,3] Quite recently, Beenakker[14] attempted to explain the novel magnetoresistance oscillations within a semiclassical model as a resonance between the periodic cyclotron orbit motion and the oscillatory $\mathbf{E}\times\mathbf{B}$ drift of the orbit center in the periodic potential. Energy-level quantization is not taken into account and, as a consequence, the DOS effects discussed in the present work cannot be described in this semiclassical approach. Our quantum-mechanical approach, on the other hand, shows that the novel DOS and magnetoresistance oscillations have the same origin, the oscillatory width of modulation-broadened Landau levels, and is supported by the experiment.

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